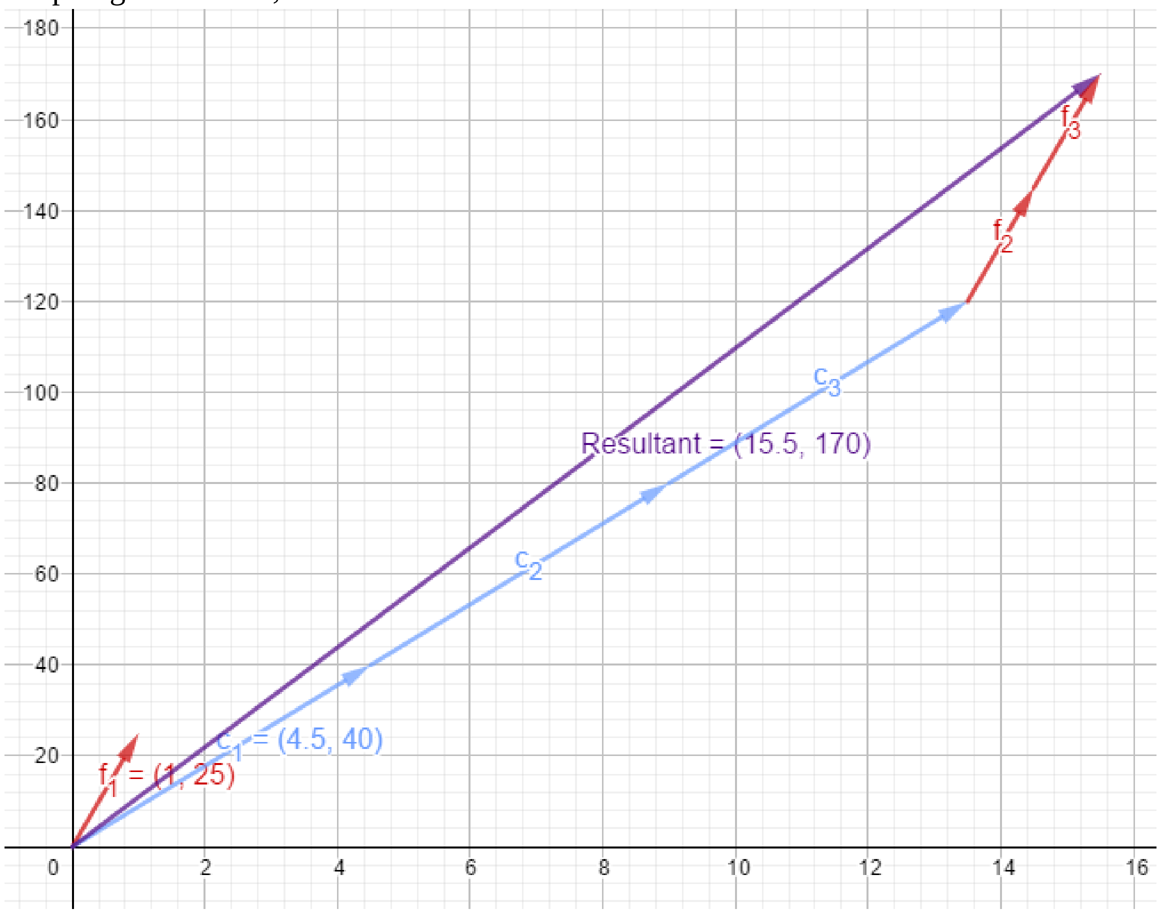
Item 1

a. The expression 2f + 3c represents the total daily requirement percentages of fat and carbohydrate taken when consuming two servings of Fiber One and three servings of Cheerios protein.

b. Graphing the vectors,



c. Solving for $2f + 3c$,

$$\begin{aligned} 2f + 3c &= 2\langle 1, 25\rangle + 3\langle 4.5, 40\rangle \\ &= \langle 2, 50\rangle + \langle 13.5, 120\rangle \\ &= \langle 15.5, 170\rangle \end{aligned}$$

d. The expression $5f - c$ represents the total daily requirement percentages of fat and carbohydrate taken equivalent to one serving of Cheerios protein less five servings of Fiber One.

Item 2

Mixing two different types of cereal to produce a set number of fat and carbohydrate values means only having positive values for the number of servings.

$$x\langle 1,25\rangle + y\langle 4.5,40\rangle = \langle 10,10\rangle$$
$$\langle x,25x\rangle + \langle 4.5y,40y\rangle = \langle 10,10\rangle$$

A system of two linear equations is formed.

$$\begin{aligned} x + 4.5y &= 10 \\ 25x + 40y &= 10 \end{aligned}$$

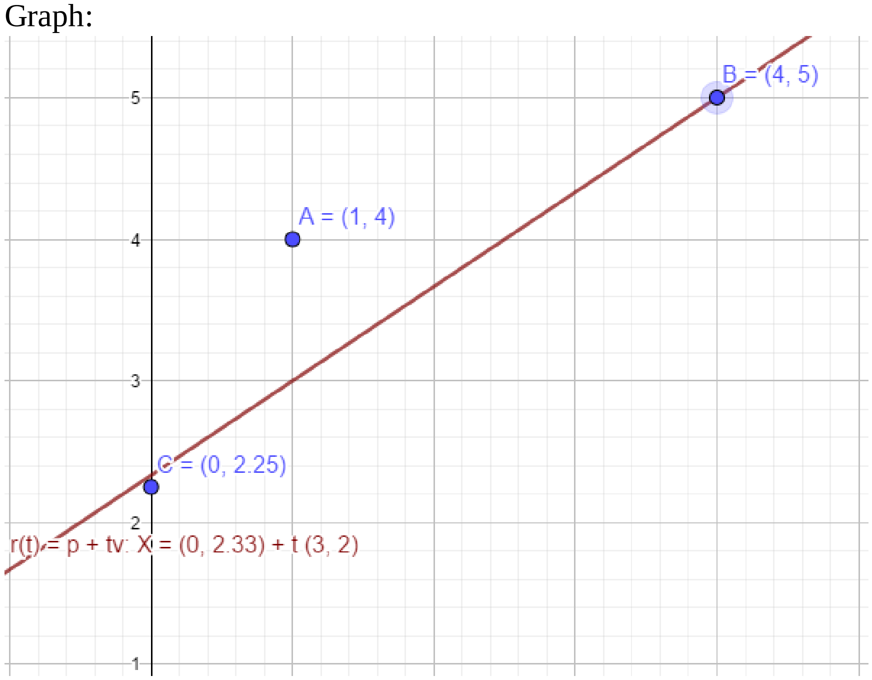
Solving for *x* and *y*,

$$\begin{aligned} x &= 10 - 4.5y \\ 25(10 - 4.5y) + 40y &= 10 \\ -72.5y &= -240 \\ y &\approx 3.3 \text{ servings} \\ x &= 10 - 4.5(3.3) \\ x &\approx -4.9 \text{ servings} \end{aligned}$$

Since *x* has a negative value, we cannot use the cereals to obtain a mixture of 10 grams of total fat and 10 grams of total carbs.

Item 3

a. Graph:



b. Based on the graph at (a), point B seems to be located on the line *r*(*t*).

Solving for the parameter *t* of point B,

$$r(t) = p + tv$$

$$r(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$4 = -2 + 3t \rightarrow \boxed{t = 2}$$

c. For point A,

$$r(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$1 = -2 + 3t \rightarrow \boxed{t = 1}$$

$$4 = 1 + 2t \rightarrow \boxed{t = \frac{3}{2}}$$

Point A does not lie on line *r*(*t*).

For point C,

$$r(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2.25 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$0 = -2 + 3t \rightarrow \boxed{t = \frac{2}{3}}$$

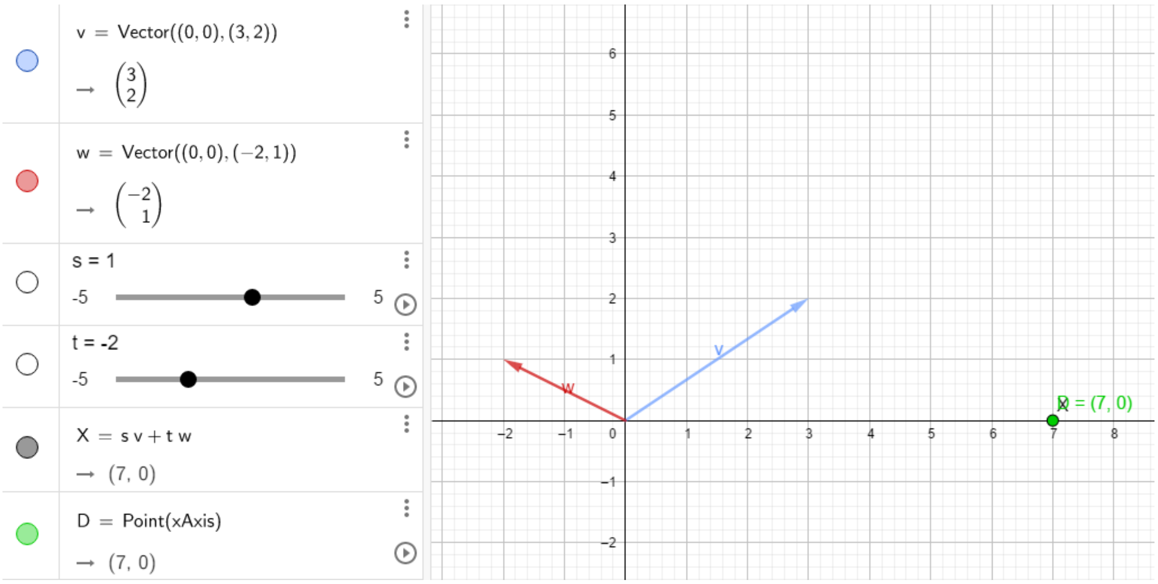
$$2.25 = 1 + 2t \rightarrow \boxed{t = \frac{5}{8}}$$

Point C does not lie on line *r*(*t*).

d. There is no scenario where I will arrive at points A and C. However, during *t* = 2, I will be located at point B(4, 5).

Item 4

a. After randomly tweaking the *s* and *t* sliders, $r(s_0, t_0) = D(7,0)$ at $(s_0, t_0) = (1, -2)$.



b. Solving for the parameters that gives the location of point D,

$$r(s,t) = sv + tw$$

$$\begin{bmatrix} s_0 \\ t_0 \end{bmatrix} = s\begin{bmatrix} 3 \\ 2 \end{bmatrix} + t\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 \\ 0 \end{bmatrix} = s\begin{bmatrix} 3 \\ 2 \end{bmatrix} + t\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

The resulting system of linear equations is:

$$7 = 3s - 2t$$

$$0 = 2s + t \rightarrow t = -2s$$

Solving for *s* and *t*,

$$7 = 3s - 2(-2s)$$

$$7 = 3s + 4s$$

$$s = 1$$

$$t = -2(1) = -2$$

c. Obtaining the quantity (7,0) can be made by combining one (1) serving of *v* and reducing it by two (2) servings of *w*.

Item 5

a. Based on the sides of the parallelogram shown in the graph, combining approximately 1.7 servings of *v* to approximately 1.3 servings of *w* will make *u*.

b. In symbolic form,

$$u \approx 1.7v + 1.3w$$

For a more accurate format,

$$u(a,b) = av + bw$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = a\begin{bmatrix} 2 \\ 1 \end{bmatrix} + b\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$2 = 2a - b$$
$$3 = a + b$$

Adding the two linear equations, the results are:

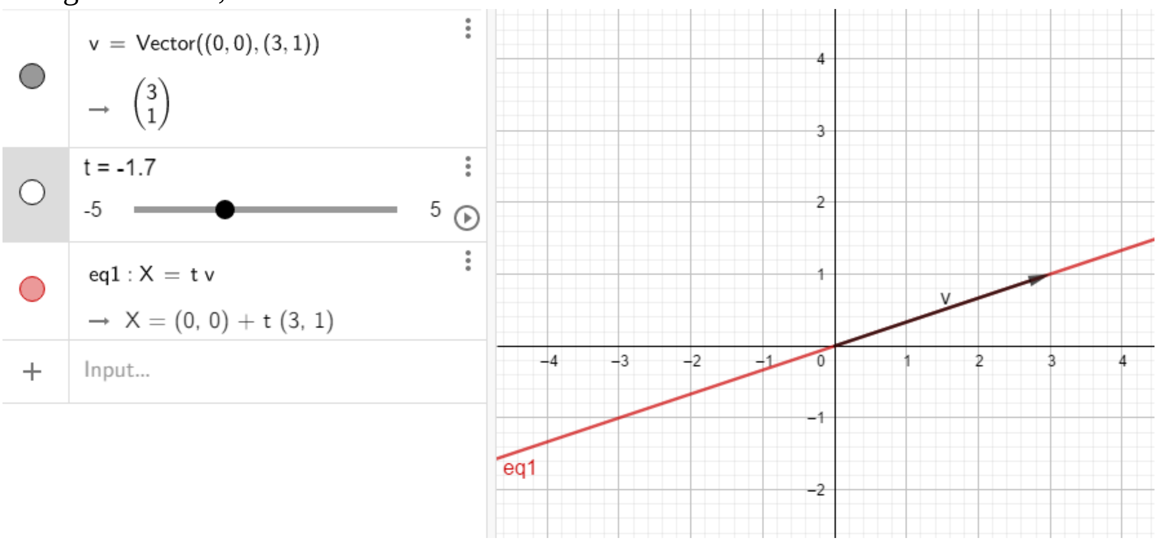
$$a = \frac{5}{3} \approx 1.67$$
$$b = \frac{4}{3} \approx 1.33$$

Hence,

$$u = \frac{5}{3}v + \frac{4}{3}w$$

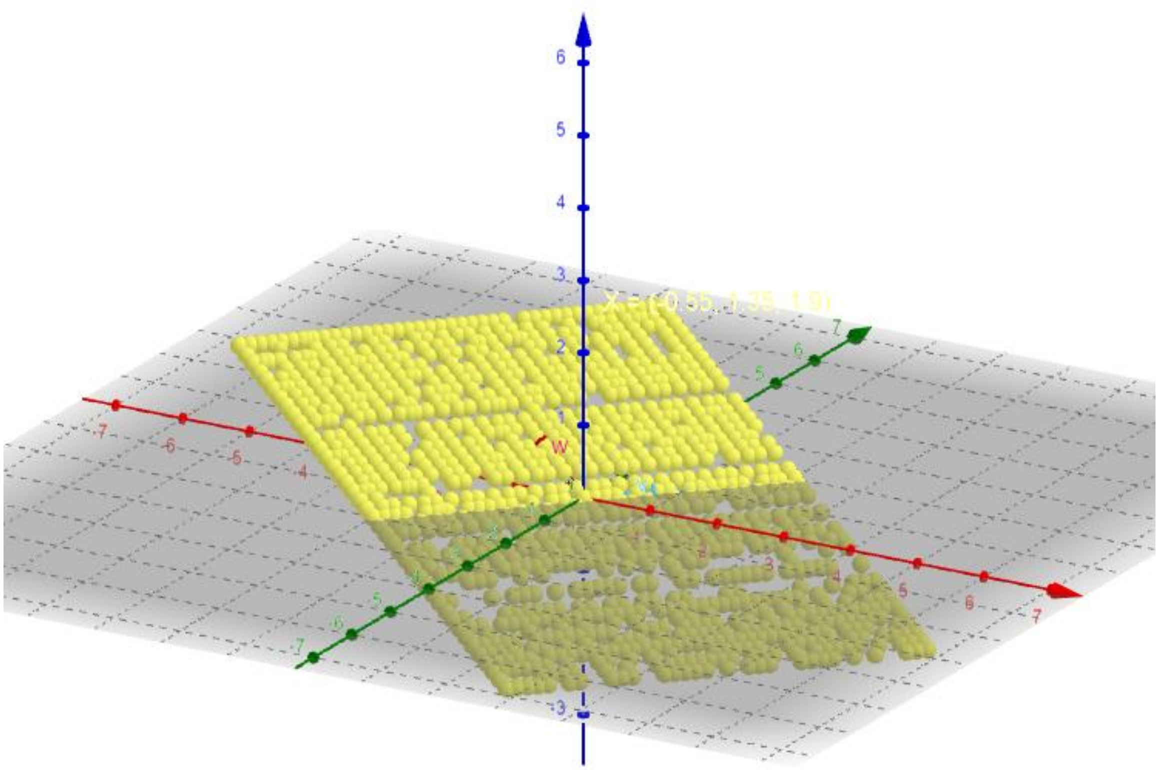
Item 6

a. Using GeoGebra,



The span of v is the line $r(t)=\begin{bmatrix}3\\1\end{bmatrix}t$ or $y=\frac{1}{3}x$ in Cartesian coordinates.

b. Using GeoGebra,



The span of *u* and *v* is the plane that contain the points (0,0,0), (1,1,0), and (-1,0,1).

Using algebraic methods,

$$\begin{aligned}\vec{n} &= \langle a,b,c\rangle \\ &= u \times w \\ &= \begin{bmatrix} i & j & k \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\ &= \langle 1,-1,1\rangle\end{aligned}$$

Hence,

$$\begin{aligned} a(x-x_0)+b(y-y_0)+c(z-z_0) &= 0 \\ 1(x-0)-1(y-0)+1(z-0) &= 0 \\ x-y+z &= 0 \end{aligned}$$

The span of *u* and *w* is the plane $x-y+z=0$