## Sec 6.1

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10:07 AM



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**Definition 1** (Random Variable). A random variable is a function that associates a real Use Capital letter for R.V number with each element in the sample space.

Example 1. Roll a 6-sided die. What values can it take on? Define a random variable for this scenario.

X = Roll of of the die. Small letter for the X = 1, 2, 3, 4, 5, 6

Values of the R.V.

This is an example of discrete random variable.

**Definition 2** (Discrete Random Variable). A random variable whose image is countable.

(12,1,100) 10000,..

Example 2. We wait in line to see a movie. We know that the waiting time in line can be anywhere from 0 hours to maybe 3 hours. However, we don't know how long the wait is actually going to be. Define a random variable for this scenario.

Y=waiting time

J= how long we actually wait, 0 < y < 3

This is a Continuous R.V.

**Definition 3** (Continuous Random Variable). A random variable whose image is an interval.

EX: Weight, Lime, Distance, Age, Temperature

**Definition 4** (Probability Mass Function (PMF)). A function, f(x), associated with a random variable X.

report the value that the random variable X

PMF Properties:

Can take on along with the probability of that point.

(1) P(X=2) = f(x)

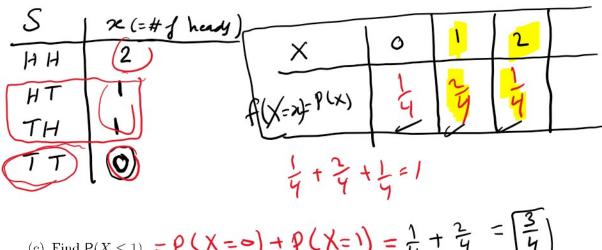
(2) & f(x)=1 # The sum of the probabilities = 1 1

By O < f(x) < n

Example 3 (Probability Calculations).

Toss a coin 2 times. Let X be a random variable representing the number of heads that occur.

- (a) Identify the sample space S.
- (b) Write the PMF for X.



(c) Find 
$$P(X \le 1)$$
. =  $P(X = 0) + P(X = 1) = \frac{1}{4} + \frac{2}{4} = \begin{bmatrix} \frac{3}{4} \\ \frac{1}{4} \end{bmatrix}$ 

(d) Find 
$$P(X < 1)$$
. =  $P(X = 0) = \frac{1}{4}$ 

(c) Find 
$$P(X \le 2)$$
. =  $P(X=0) + P(X=1) + P(X=2) =$ 

(f) Find 
$$P(X < 2)$$
. =  $P(X = 0) + P(X = 1) = 4 + 3 = 1$ 

(g) Find 
$$P(X \ge 1)$$
. =  $P(X=1) + P(X=2) = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ 

(h) Find 
$$P(X > 1)$$
.  $= P(X = Y) = \boxed{1}$ 

**Definition 5.** The mean and the standard deviation of a discrete population probability distribution are found by using these formulas:

- 1.  $\mu = \sum x P(x); \mu$  is called the expected value of x
- 2.  $\sigma = \sqrt{\Sigma(x \mu)^2 P(x)}$ ;  $\sigma$  is called the standard deviation of x

where x is the value of a random variable, P(x) is the probability of that variable, and the sum  $\Sigma$  is taken for all the values of the random variable.

**Definition 6** (Standard Deviation). The standard deviation of X is denoted by  $\sigma$  or SD(X). The standard deviation of X is the square root of the variance of X:

$$\sigma = \mathbf{SD}(X) = \sqrt{\mathbf{V}X} = \sqrt{\sigma^2}$$

Example 4 (Citrus Farmer: Expected Value / Variance / Standard Deviation). A citrus farmer observed the following distribution for X, the number of oranges per tree.

(a) What is the expected value of X?

M=EX-2xf(x)=25(0.1)+30(0.4)+35(0.3)+40(0.2) (b) What is the variance of X?

SD(x) = ((x-M).f(x)

(25-33)(6-1)+(30-33)2(0-4)+(35-33)2(6-1)+(40-33)2(0-2)

Example 5.

Let X be a random variable with probability distribution

$$\begin{array}{c|cccc} x & 0 & 1 & 3 \\ \hline f(x) & 1/3 & 1/2 & 1/6 \end{array}$$

(a) What is P(X=2)?

(b) What is 
$$P(X < 2)$$
? =  $P(X = 0) + P(X = 1) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ 

(c) What is  $\mathbf{E}(X)$ ?

(d) What is  $\mathbf{V}(X)$ ?

$$\sum (x-M)^2 f(x) = (0-1)^{\frac{3}{2}} + (1-1)^{\frac{3}{2}} + (3-1)^{\frac{3}{2}} = \boxed{1}$$

- (a) Find P(X = 2).
- (b) Find  $P(X \leq 2)$ .
- (c) Find P(X < 2).
- (d) Find P(X > 3).

(e) Find P(2 \le X \le 4). = 
$$p(X=2) + p(X=3) + p(X=4)$$
  
= 0.1+ 0.3 + 0.1 = 0.5