

Homework 1

Chapter 2 (2.8, 2.11 a,d , 2.26, 2.33 a, f, 2.39 a,b)

2.8)

a) What is the largest positive number one can represent in an 8-bit 2's complement code? Write your result in binary and decimal.

Binary: $(01111111)_2$

$$\begin{aligned}\text{Decimal: } (01111111)_2 &= 2^6 \cdot 1 + 2^5 \cdot 1 + 2^4 \cdot 1 + 2^3 \cdot 1 + 2^2 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1 \\ &= 64 + 32 + 16 + 8 + 4 + 2 + 1 \\ &= 127\end{aligned}$$

b) What is the greatest magnitude negative number one can represent in an 8-bit 2's complement code? Write your result in binary and Decimal.

Binary: $(10000000)_2$

$$\begin{aligned}\text{Decimal: } (10000000)_2 &= 2^7 \cdot 1 + 2^6 \cdot 0 + 2^5 \cdot 0 + 2^4 \cdot 0 + 2^3 \cdot 0 + 2^2 \cdot 0 + 2^1 \cdot 0 + 2^0 \cdot 0 \\ &= 128 \\ &= -128 \text{ because the sign bit is negative so the decimal value is negative.}\end{aligned}$$

c) What is the largest positive number one can represent in n-bit 2's complement code?

The largest positive number that can be represented as 1's complement code using n bits is $2^{n-1} - 1$

d) What is the greatest magnitude negative number one can represent in n-bit 2's complement code?

The greatest magnitude negative number represented as 2's complement code using n bits is -2^{n-1}

2.11) Convert these decimal numbers to 8-bit 2's complement binary numbers.

a) 102

$$102 = a_6 \cdot 2^6 + a_5 \cdot 2^5 + a_4 \cdot 2^4 + a_3 \cdot 2^3 + a_2 \cdot 2^2 + a_1 \cdot 2^1 + a_0 \cdot 2^0$$

$$102 = a_6 \cdot 2^6 + a_5 \cdot 2^5 + a_4 \cdot 2^4 + a_3 \cdot 2^3 + a_2 \cdot 2^2 + a_1 \cdot 2^1$$

$$51 = a_6 \cdot 2^5 + a_5 \cdot 2^4 + a_4 \cdot 2^3 + a_3 \cdot 2^2 + a_2 \cdot 2^1 + a_1 \cdot 2^0$$

$$50 = a_6 \cdot 2^5 + a_5 \cdot 2^4 + a_4 \cdot 2^3 + a_3 \cdot 2^2 + a_2 \cdot 2^1$$

$$25 = a_6 \cdot 2^4 + a_5 \cdot 2^3 + a_4 \cdot 2^2 + a_3 \cdot 2^1 + a_2 \cdot 2^0$$

$$24 = a_6 \cdot 2^4 + a_5 \cdot 2^3 + a_4 \cdot 2^2 + a_3 \cdot 2^1$$

$$12 = a_6 \cdot 2^3 + a_5 \cdot 2^2 + a_4 \cdot 2^1 + a_3 \cdot 2^0$$

$$12 = a_6 \cdot 2^3 + a_5 \cdot 2^2 + a_4 \cdot 2^1$$

$$6 = a_6 \cdot 2^2 + a_5 \cdot 2^1 + a_4 \cdot 2^0$$

$$6 = a_6 \cdot 2^2 + a_5 \cdot 2^1$$

$$3 = a_6 \cdot 2^1 + a_5 \cdot 2^0$$

$$3 = a_6 \cdot 2^1$$

$$1 = a_6 \cdot 2^0$$

$$a_6 = 1$$

8 bit 2's complimentary number for 102 is 01100110

d) - 128

$$128 = a_6 \cdot 2^6 + a_5 \cdot 2^5 + a_4 \cdot 2^4 + a_3 \cdot 2^3 + a_2 \cdot 2^2 + a_1 \cdot 2^1 + a_0 \cdot 2^0$$

$$128 = a_6 \cdot 2^6 + a_5 \cdot 2^5 + a_4 \cdot 2^4 + a_3 \cdot 2^3 + a_2 \cdot 2^2 + a_1 \cdot 2^1$$

$$64 = a_6 \cdot 2^5 + a_5 \cdot 2^4 + a_4 \cdot 2^3 + a_3 \cdot 2^2 + a_2 \cdot 2^1 + a_1 \cdot 2^0$$

$$64 = a_6 \cdot 2^5 + a_5 \cdot 2^4 + a_4 \cdot 2^3 + a_3 \cdot 2^2 + a_2 \cdot 2^1$$

$$32 = a_6 \cdot 2^4 + a_5 \cdot 2^3 + a_4 \cdot 2^2 + a_3 \cdot 2^1 + a_2 \cdot 2^0$$

$$32 = a_6 \cdot 2^4 + a_5 \cdot 2^3 + a_4 \cdot 2^2 + a_3 \cdot 2^1$$

$$16 = a_6 \cdot 2^3 + a_5 \cdot 2^2 + a_4 \cdot 2^1 + a_3 \cdot 2^0$$

$$16 = a_6 \cdot 2^3 + a_5 \cdot 2^2 + a_4 \cdot 2^1$$

$$8 = a_6 \cdot 2^2 + a_5 \cdot 2^1 + a_4 \cdot 2^0$$

$$8 = a_6 \cdot 2^2 + a_5 \cdot 2^1$$

$$4 = a_6 \cdot 2^1 + a_5 \cdot 2^0$$

$$4 = a_6 \cdot 2^1$$

$$2 = a_6 \cdot 2^0$$

$$a_6 = 0$$

8 bit 2's complimentary number for -128 is 10000000

2.26) You wish to express - 64 as a 2's complement number.

a) How many bits do you need (the minimum number)?

2's complement of $(-64)_{10}$ is also $(10000000)_2$.

The minimum number of bits to express $(-64)_{10}$ as 2's complement is 7 bits.

b) With this number of bits, what is the largest positive number you can represent? (Please give answer in both decimal and binary).

Binary: 0111111

$$\begin{aligned} \text{Decimal: } (0111111)_2 &= 2^6 \cdot 0 + 2^5 \cdot 1 + 2^4 \cdot 1 + 2^3 \cdot 1 + 2^2 \cdot 1 + 2^0 \cdot 1 \\ &= 32 + 16 + 8 + 4 + 2 + 1 \\ &= 63 \end{aligned}$$

The largest positive 7-bit binary number is $(0111111)_2$ and decimal number is $(63)_{10}$

c) With this number of bits, what is the largest unsigned number you can represent? (Please give answer in both decimal and binary).

Binary: 1111111

$$\begin{aligned} \text{Decimal: } (1111111)_2 &= 2^6 \cdot 1 + 2^5 \cdot 1 + 2^4 \cdot 1 + 2^3 \cdot 1 + 2^2 \cdot 1 + 2^0 \cdot 1 \\ &= 64 + 32 + 16 + 8 + 4 + 2 + 1 \\ &= 127 \end{aligned}$$

The largest unsigned 7-bit binary number is $(1111111)_2$ and decimal number is $(127)_{10}$

2.33) Compute the following:

a. 01010111 OR 11010111

f) 0101 OR (1100 OR 1101)

1100 \longrightarrow (1)

1101 \longrightarrow (2)

— — — — —

1101

Applying the OR function to inputs (1) and (2), the output will be 1101

0101 OR 1101

0101 \rightarrow (3)

1101 \rightarrow (4)

— — — — —

1101

Applying the OR function to the inputs (3) and (4), the output will be 1101

2.39) Write IEEE floating point representation of the following decimal numbers.

a) 3.75

The binary value for 3 is 011

The binary value for decimal 0.75: $0.75 \cdot 2 = 1.50$

$0.50 \cdot 2 = 1.00$

The binary value is: $(011.11)_2$

The sign bit 0 reflects 3.75 is positive.

Normalizing the binary value: $+1.111 \cdot 2^1$

The exponent is unsigned number 128, and is represented as 10000000 in binary. The real exponent is $+1 (128 - 127 = +1)$

<i>Sign</i>	<i>Exponent</i>	<i>Fraction</i>
0	10000000	1110000000000000000000

The representation is 0 1000000 1110000000000000000000

b) $-55\frac{23}{64}$

The fraction can be written as -55.359

The binary value for 55 is 110111

The binary value for decimal 0.349: $0.359 \cdot 2 = 0.718$

$$0.718 \cdot 2 = 1.436$$

$$0.436 \cdot 2 = 0.872$$

$$0.872 \cdot 2 = 1.744$$

The binary value is $(110111.0101)_2$

The sign bit 1 reflects the fact that $-55\frac{23}{64}$ is negative.

Normalizing the binary value is $-1.101110101 \cdot 2^5$

The exponent is unsigned number 132, represented as 10000100 in binary, This reflects the real exponent is $+5 (132 - 127 = +5)$

The fraction is 23 bits of precision, removing the leading 1. The fraction is 10111010100000000000000.

<i>Sign</i>	<i>Exponent</i>	<i>Fraction</i>
1	10000100	10111010100000000000000

The representation is 1 10000100 10111010100000000000000

Chapter 3 (3.9, 3.10, 3.23)

3.9) Fill in the truth table for the logical expression NOT(NOT(A) OR NOT(B)). What single logic gate has the same truth table?

A	B	NOT(NOT(A) OR NOT(B))
0	0	NOT(NOT(0) OR NOT (0)) = NOT (1 OR 1) = NOT 1 = 0
0	1	NOT(NOT(0) OR NOT (1)) = NOT (1 OR 0) = NOT 1 = 0
1	0	NOT(NOT(1) OR NOT (0)) = NOT (0 OR 1) = NOT 1 = 0
1	1	NOT(NOT(1) OR NOT (1)) = NOT (0 OR 0) = NOT 0 = 1

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

3.10) Fill in the truth table for a two-input NOR gate.

A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

3.23) Given the logic circuit in Figure 3.38, fill in the truth table for the output value Z.

A	B	C	Z
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

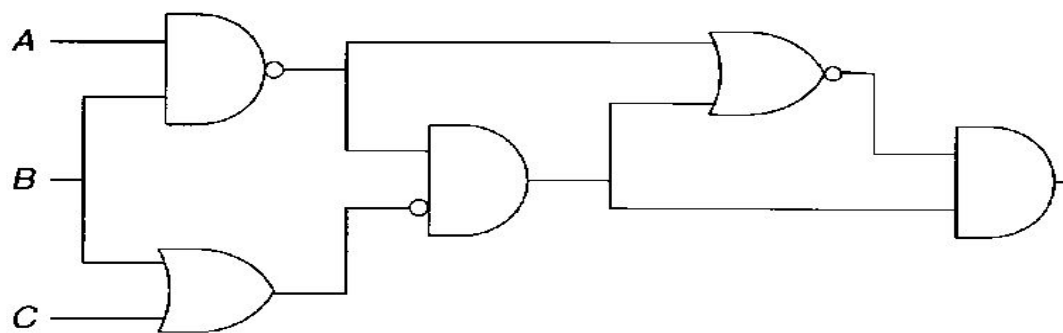


Figure 3.38 Diagram for Exercise 3.23

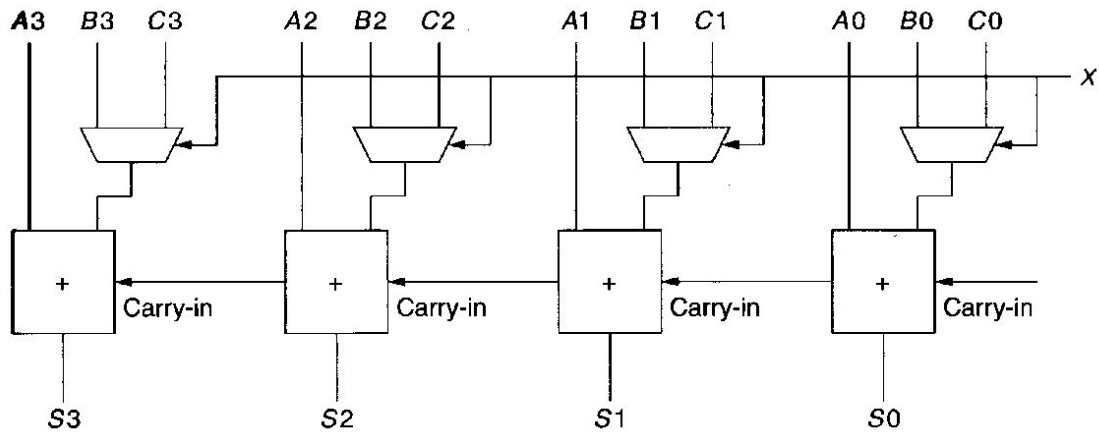


Figure 3.39 Diagram for Exercise 3.24

The value of Z is $\overline{((\overline{A.B}) + ((\overline{A.B})(B + C)))} \cdot ((\overline{A.B})(B + C))$

A	B	C	A.B	$\overline{A.B}$	B + C	$\overline{B + C}$	$(\overline{A.B}) \cdot (\overline{B + C})$	$F = \overline{A.B} + (\overline{A.B}) \cdot (\overline{B + C})$	$\overline{F} = \overline{AB + (\overline{A.B}) \cdot (\overline{B + C})}$	$Z = \overline{F} \cdot (\overline{A.B}) \cdot (\overline{B + C})$
0	0	0	0	1	0	1	1	1	0	0
0	0	1	0	1	1	0	0	1	0	0
0	1	0	0	1	1	0	0	1	0	0
0	1	1	0	1	1	0	0	1	0	0
1	0	0	0	1	0	1	1	1	0	0
1	0	1	0	1	1	0	0	1	0	0
1	1	0	1	0	1	0	0	0	1	0
1	1	1	1	0	1	0	0	0	1	0