Example 1. Suppose a coin is flipped twice. The sample space is

$$S = \{HH,\ HT,\ TH,\ TT\}$$

Let

$$A = \{\text{exactly 1 head} = \{HT,\ TH\}$$

What are the chances of this event actually occurring?

The event occurs 2 out of 4 times

or $\frac{2}{4}$ or $\frac{1}{2}$

$P(A) = \frac{1}{2}$ ← The value of probability.

The probability →

Let $B = \{\text{at least 1 head} = \{HH,\ HT,\ TH\}$

$P(B) = \boxed{\frac{3}{4}}$

Q 1. What are some basic rules of probability?

1. $0 \leq \mathsf{P}(A) \leq 1$
2. $\mathsf{P}(\varnothing) = 0$; $\mathsf{P}(S) = 1$
3. If A and B are mutually exclusive events, i.e. $A \cap B = \varnothing$, then

Disjoints ↗

$$\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B)$$

In general, we calculate a probability as:

$$P(\text{event}) = \frac{\#\text{ outcomes in event}}{\text{total \# of possible outcomes}}$$

Example 2. The plastic arrow on a spinner for a child's game stops rotating to point at a color that will determine what happens next. Which of the following probability assignments are possible?

	Red	Yellow	Green	Blue	Yes/No
a)	0.25	0.25	0.25	0.25	Yes No
b)	0.10	0.20	0.30	0.40	Yes No
c)	0.20	0.30	0.40	0.50	Yes No
d)	0.00	0.00	1.00	0.00	Yes No
e)	0.10	0.20	1.20	-1.50	Yes No

$0 \le P(A) \le 1$?

When add, do we get 1?

Example 3. If a letter is chosen at random from the English alphabet, find the probability that the letter

26 Letters

(a) is a vowel exclusive of y.

vowels: a, e, i, o, u = 5

$\frac{5}{26}$

(b) is listed somewhere ahead of letter j.

Ahead of J: a, b, c, d, e, f, g, h, i, j = 9

$\frac{9}{26}$

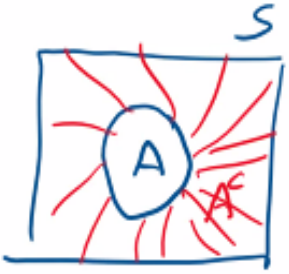
(c) is listed somewhere after g.

After G: h, i ,j, k, ..., x, y, z = 19

$\frac{19}{26}$

Theorem 1. *Complement Rule:* $P(A^c) = 1 - P(A)$

A^c or A'



Proof:

$S = A \cup A^c$

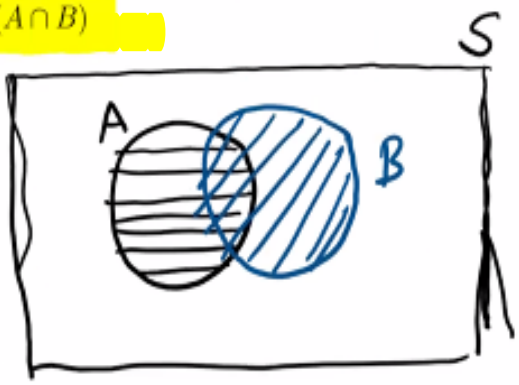
$P(S) = P(A \cup A^c)$

$1 = P(A) + P(A^c)$

$\Rightarrow \boxed{P(A^c) = 1 - P(A)}$

Theorem 2. *Additive Rule for two Events, A, B:*

$$\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B) - \mathsf{P}(A \cap B)$$



Q 2. What if events A and B did not overlap?

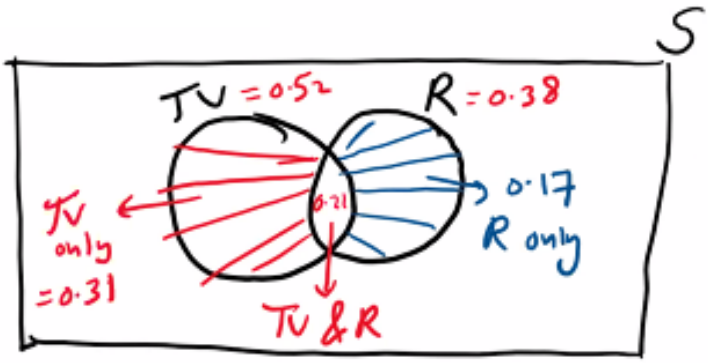
Corollary 1. *If* $A \cap B = \varnothing$, *then* $\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B)$.

$P(A \cap B) = 0$

$\Rightarrow P(A \cup B) = P(A) + P(B)$

Example 4. A check of dorm rooms on a large college campus revealed that 38% had refrigerators, 52% had TVs, and 21% had both a TV and a refrigerator.

(a) Draw a Venn Diagram for this scenario.



(b) What's the probability that a randomly selected dorm room has a TV but no refrigerator?

$0.52 - 0.21 = 0.31$

$= 31\%$

(c) What's the probability that a randomly selected dorm room has a refrigerator but no TV?

$0.38 - 0.21 = 0.17$

(d) What's the probability that a randomly selected dorm room has a TV or a refrigerator but not both?

$0.31 + 0.17 = 0.48.$

(e) What's the probability that a randomly selected dorm room has a TV or a refrigerator?

$$P(Tv \cup R) = P(Tv) + P(R) - P(Tv \cap R)$$
$$= 0.52 - 0.38 - 0.21 = \boxed{0.69}$$

(f) Are the states of having a TV and a Refrigerator disjoint events? Briefly justify your answer.

NO because $Tv \cap R \neq \emptyset$.

(g) What's the probability that a randomly selected dorm room has neither a TV nor a refrigerator?

$$P(Tv^c \cap R^c) = P(Tv \cup R)^c \quad \sim \text{DeMorgan's}$$
$$= 1 - \boxed{P(Tv \cup R)}$$
$$= 1 - 0.69 = \boxed{0.31}$$

(POSSIBLE EXAM QUESTION)

Example 5. Accidents in a factory can occur because working or conditions themselves are unsafe, or due to human error. We have the following information about accidents in a specific factory. It is broken down into the shift the worker is on, and whether the accidents were due to unsafe conditions or human error.

Shift	Unsafe Conditions	Human Error
Day	5%	32%
Evening	6%	25%
Graveyard	2%	30%

Based on the table, what is the probability that

(a) an accident occurred on the graveyard shift?

30% + 2% = 32% or 0.32

(b) an accident was due to human error?

32% + 25% + 30% = 87% or 0.87

(c) an accident was due to unsafe conditions?

5% + 6% + 2% = 13% or 0.13

(d) an accident occurred during either the evening or graveyard shifts?

6% + 25% + 2% + 30% = 63% or 0.63

Example 6 (YOU TRY). The probability that a pizza contains cheese is 0.90. The probability that a pizza contains onions is 0.08. The probability that a pizza contains cheese or onions is 0.95.

(a) What is the probability that a pizza contains cheese and onions? Find $P(C \cap O)$

$P(C) = 0.90,\ P(O) = 0.08,\ P(C \cup O) = 0.95$

$P(C \cup O) = P(C) + P(O) - P(C \cap O) \Rightarrow 0.95 = 0.90 + 0.08 - P(C \cap O)$

(b) What is the probability that a pizza contains no cheese and no onions?

$\Rightarrow P(C \cap O) = 0.03$

Example 7. The probability that a UIC undergraduate majors in Engineering is 0.15. The probability that a UIC undergraduate is a Junior is 0.20. The probability that a UIC undergraduate is a Junior and an Engineering major is 0.035.

$P(E) = 0.15$, $P(J) = 0.20$

$P(E \cap J) = 0.035$

(a) What is the probability that a UIC undergraduate is not majoring in Engineering?

$$P(E^c) = 1 - P(E) = 1 - 0.15 = \boxed{0.85}$$

(b) What is the probability that a UIC undergraduate is majoring in Engineering or is a Junior?

$$P(E \cup J) = P(E) + P(J) - P(E \cap J)$$
$$= 0.15 + 0.20 - 0.035 = 0.315$$

(c) What is the probability that a UIC undergraduate is not majoring in Engineering nor a Junior?

$$P(E^c \cap J^c) = P(E \cup J)^c$$
$$= 1 - P(E \cup J)$$
$$= 1 - 0.315 = 0.685$$