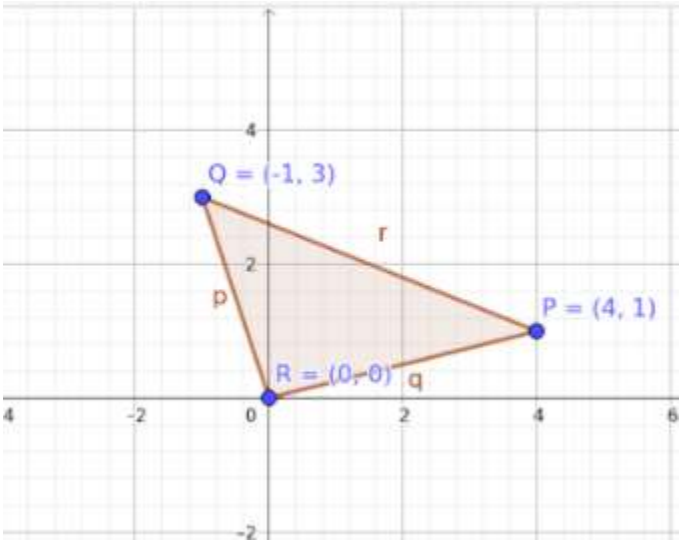
Homework 12

Linear algebra

2. a



b.

c. **solution**

$$A = \begin{pmatrix} 4 & -1 \\ 1 & 3 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 4 & 1 \\ -1 & 3 \end{pmatrix}$$

Inner product matrix = AA^T

$$= \begin{pmatrix} 4 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 16+1 & 4-3 \\ 4-3 & 1+9 \end{pmatrix}$$

$$= \begin{pmatrix} 17 & 1 \\ 1 & 10 \end{pmatrix}$$

d.

$\|V\| = \sqrt{17}$

=4.12 units

$\|W\| = \sqrt{13}$

=3.61 units

$\|Z\| = \sqrt{29}$

= 5.39 units

e. 4.12 + 3.61 = 7.73>5.39

4.12+5.39 = 11.51>3.61

3.61+5.39 = 9.0 > 4.12

Therefore, the values satisfy the inequality rule of triangles

f

$\angle$PRQ =$\cos^{-1}\frac{1}{3.61\times5.39}$

$\approx 98^0$

$\angle$PQR =$\cos^{-1}\frac{1}{3.61\times4.12}$

$\approx 50^0$

g. because vertices p is (0, 0)

sum of all angles in a triangle sum to 180^0

thus ∠RPQ = 180 – (50+98)

$\approx 32^0$

3. a

A is isometry if its transformation of a plane gives the values of the image equal to the values of the object. A is not isometry if the transformation gives a value of the image different to the object.

b.

i. $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

$Ax = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$= \begin{pmatrix} 2x + 0 \\ 0 + 3y \end{pmatrix}$

= (2x, 3y)

A transform (x, y) to (2x, 3y) thus it is not isometry as Ax≠x

ii.

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$Ax = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$= \begin{pmatrix} 0 - y \\ x + 0 \end{pmatrix}$

= (-y, x)

A transform (x, y) to (-y, x) thus it is not isometry as Ax≠x

iii.

$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$Ax = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$= \begin{pmatrix} x + 2y \\ 0 + y \end{pmatrix}$

= (2x, 3y)

A transform (x, y) to (x+2y, y) thus it is not isometry as Ax≠x

iv.

$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$Ax = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$= \begin{pmatrix} x + 0 \\ 0 - y \end{pmatrix}$

= (x, -y)

A transform (x, y) to (x, -y) thus it is not isometry as Ax≠x

v.

$A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$Ax = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$=\begin{pmatrix} 0+0 \\ 0+y \end{pmatrix}$

= (0, y)

A transform (x, y) to (0, y) thus it is not isometry as Ax≠x

c. No. This did not match my expectations because non was isometry as I thought.