Sec 6.1

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R.V.

Definition 1 (Random Variable)**.** A random variable is a function that associates a real number with each element in the sample space.

Use Capital letters for R.V such as: $X, Y, Z, \dots$

Example 1. Roll a 6-sided die. What values can it take on? Define a random variable for this scenario.

X = Roll of ~~d~~ the die.

small letters for the values of the R.V. Such as x, y, z

$x = 1, 2, 3, 4, 5, 6$

This is an example of discrete random variable.

Definition 2 (Discrete Random Variable)**.** A random variable whose image is countable.

Ex: Number of books at your home. $\{1, 2, 3, 100, 10000, \dots\}$

Example 2. We wait in line to see a movie. We know that the waiting time in line can be anywhere from 0 hours to maybe 3 hours. However, we don't know how long the wait is actually going to be. Define a random variable for this scenario.

Y = waiting time

y = how long we actually wait, $0 \le y \le 3$

This is a Continuous R.V.

Definition 3 (Continuous Random Variable)**.** A random variable whose image is an interval.

Ex: Weight, time, Distance, Age, Temperature

Step

Definition 4 (Probability Mass Function (PMF))**.** A function, $f(x)$, associated with a random variable X.

report the value that the random variable X can take on along with the probability of that point.

PMF Properties:

(1) $P(X = x) = f(x)$

(2) $\sum f(x) = 1$ * The sum of the probabilities = 1 *

(3) $0 \le f(x) \le 1$

Example 3 (Probability Calculations).
Toss a coin 2 times. Let X be a random variable representing the number of heads that occur.

(a) Identify the sample space S.

S	x (= # of heads)
HH	2
HT	1
TH	1
TT	0

(b) Write the PMF for X.

X	0	1	2
$f(X=x)=P(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$\frac{1}{4}+\frac{2}{4}+\frac{1}{4}=1$

(c) Find $P(X \leq 1)$. $= P(X=0)+P(X=1) = \frac{1}{4}+\frac{2}{4} = \boxed{\frac{3}{4}}$

(d) Find $P(X < 1)$. $= P(X=0) = \frac{1}{4}$

(e) Find $P(X \leq 2)$. $= P(X=0)+P(X=1)+P(X=2) = \boxed{1}$

(f) Find $P(X < 2)$. $= P(X=0)+P(X=1) = \frac{1}{4}+\frac{2}{4} = \boxed{\frac{3}{4}}$

(g) Find $P(X \geq 1)$. $= P(X=1)+P(X=2) = \frac{2}{4}+\frac{1}{4} = \frac{3}{4}$

(h) Find $P(X > 1)$. $= P(X=2) = \boxed{\frac{1}{4}}$

Definition 5. The mean and the standard deviation of a discrete population probability distribution are found by using these formulas:

1. $\mu = \sum xP(x)$; μ is called the expected value of x
2. $\sigma = \sqrt{\Sigma(x-\mu)^2P(x)}$; σ is called the standard deviation of x

where x is the value of a random variable, $P(x)$ is the probability of that variable, and the sum Σ is taken for all the values of the random variable.

Definition 6 (Standard Deviation)**.** The standard deviation of X is denoted by σ or $\mathbf{SD}(X)$. The standard deviation of X is the square root of the variance of X:

$$\sigma = \mathbf{SD}(X) = \sqrt{\mathbf{V}\,X} = \sqrt{\sigma^2}$$

Example 4 (Citrus Farmer: Expected Value / Variance / Standard Deviation).
A citrus farmer observed the following distribution for X, the number of oranges per tree.

x	25	30	35	40
$f(x)$	0.1	0.4	0.3	0.2

(a) What is the expected value of X?

$$\mu = EX = \sum x f(x) = 25(0.1) + 30(0.4) + 35(0.3) + 40(0.2)$$

$$= \boxed{33}$$

(b) What is the ~~variance~~ Standard deviation of X?

$$SD(X) = \sqrt{\sum (x-\mu)^2 \cdot f(x)}$$

$$= \sqrt{(25-33)^2(0.1) + (30-33)^2(0.4) + (35-33)^2(0.3) + (40-33)^2(0.2)}$$

$$= \boxed{21}$$

Check!

Example 5.
Let X be a random variable with probability distribution

x	0	1	3
$f(x)$	1/3	1/2	1/6

(a) What is $\mathsf{P}(X = 2)$? 0

(b) What is $\mathsf{P}(X < 2)$? $= P(X=0) + P(X=1) = \frac{1}{3} + \frac{1}{2} = \boxed{\frac{5}{6}}$

(c) What is $\mathbf{E}(X)$?

$\sum x f(x) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{2} + 3 \cdot \frac{1}{6} = \boxed{1} = \mu$

(d) What is $\mathbf{V}(X)$?

$\sum (x-\mu)^2 f(x) = (0-1)^2 \cdot \frac{1}{3} + (1-1)^2 \frac{1}{2} + (3-1)^2 \frac{1}{6} = \boxed{1}$

Example 6 (YOU TRY). Suppose we have the following probability distribution for X:

x	0	1	2	3	4	6
$f(x)$	0.1	0.2	0.1	0.3	0.1	0.2

(a) Find $\mathsf{P}(X = 2)$.

(b) Find $\mathsf{P}(X \le 2)$.

(c) Find $\mathsf{P}(X < 2)$.

(d) Find $\mathsf{P}(X > 3)$.

(e) Find $\mathsf{P}(2 \le X \le 4)$. $= P(X=2) + P(X=3) + P(X=4)$

$= 0.1 + 0.3 + 0.1 = \boxed{0.5}$