

Definition 1. The **range** is the difference between the largest and smallest values of a data distribution.

Example: 13, 15, 12, 20, 9. (Find the range)

Step 1: Find max - min (order the data): Range = Max - Min = 20 - 9 = 11

Definition 2. The **sample standard deviation** is based on the difference between each data value and the mean of the data set.

Remark: The standard deviation gives an average of data spread around the mean. The larger the standard deviation, the more spread out the data are around the mean.

Q 1. How can we find the sample standard deviation?

(1st Formula) **Definition Formula**

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2 / n}{n - 1}}$$

Computation Formula

(2nd Formula)

Definition 3. The **variance** tells us the square of standard deviation.

$$S^2 = \frac{\sum (x - \bar{x})^2}{n - 1} = \frac{\sum x^2 - (\sum x)^2 / n}{n - 1}$$

Remarks:

$$1. \sum (x - \bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$$

2. For technical reason, we divide the sum by (n - 1) [not by n]

3. **Population vs Statistic:**

Statistic	Parameters
① Sample Variance = $S^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$ is statistic	① Population Variance = $\sigma^2 = \frac{\sum (x - \mu)^2}{N}$ Parameter.
② $\bar{x} = \frac{\sum x}{n}$	② $\mu = \frac{\sum x}{N}$
③ $S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$	③ $\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$

Example 1. The numbers of incorrect answers on a true-false competency test for a random sample of 9 students were recorded as follows

$$X = 1, 1, 2, 2, 2, 2, 2, 3, 3$$

Find

- a. Mean b. range c. variance d. standard deviation

$$a) \text{Mean} = \frac{\sum x}{n} = \frac{1+1+2+2+2+2+2+3+3}{9} = \frac{18}{9} = \underline{2} = \bar{x}$$

$$b) \text{Range} = \text{max} - \text{min} = 3 - 1 = \underline{2}$$

c) 1st Formula:
$$S^2 = \frac{\sum (X - \bar{X})^2}{n-1} = \frac{4}{9-1} = \frac{4}{8} = \underline{\frac{1}{2}}$$

X	$X - \bar{X}$	$(X - \bar{X})^2$
1	$1 - 2 = -1$	$(-1)^2 = 1$
1	$1 - 2 = -1$	$(-1)^2 = 1$
2	$2 - 2 = 0$	$0^2 = 0$
2	$2 - 2 = 0$	0
2	$2 - 2 = 0$	0
2	0	0
2	0	0
3	$3 - 2 = 1$	$1^2 = 1$
3	$3 - 2 = 1$	$1^2 = 1$
		$\sum (X - \bar{X})^2 = 4$

D) Standard Deviation

$$S = \sqrt{S^2} = \sqrt{\frac{1}{2}} = \underline{\frac{1}{\sqrt{2}}}$$

$$\text{Var} = 9 \Rightarrow \text{sd} = \sqrt{9} = \underline{3}$$

2nd Formula: $s^2 = \frac{\sum x^2 - (\sum x)^2/n}{n-1}$

x	x^2
1	$1^2=1$
1	$1^2=1$
2	$2^2=4$
2	$2^2=4$
2	4
2	4
2	4
3	$3^2=9$
3	9
$\sum x = 18$	$\sum x^2 = 40$

Example 2. (Calculator)

$$= \frac{40 - \frac{(18)^2}{9}}{9-1} = \frac{40 - 36}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

Example 2. (Calculator)

- Press the [STAT] key and select 1:Edit
- Enter in the data one at a time into L1
- Press the [STAT] key, scroll to highlight CALC
- Select 1: 1-Var Stats and press [ENTER]. The display will read: 1-Var Stats
- Press [2nd] followed by the [1] key to recall list 1. The display will read: 1-Var Stats L1
- Press [ENTER] to display the calculated data
- Scroll down to see all values

```

1-Var Stats
Σx=63
Σx²=504
Σx³=31876
Sx=4.208834246
σx=3.937003937
↓
n=8

```

Example 3. Consider the data set 15, 5, 17, 5, 17.

1. Find the standard deviation; S.

Solution:

$$S=6.26 \text{ (Do it!)}$$

2. Add 3 to each data value. Compute S. (POSSIBLE EXAM QUESTION)

New Data: 18, 8, 20, 8, 20

Standard Deviation = 6.26

Conclusion: Adding the same constant C to each data value results in the Standard Deviation remaining the same.

Handwritten calculations for finding the standard deviation of the data set 15, 5, 17, 5, 17.

15, 5, 17, 5, 17

$$\bar{x} = \text{mean} = \frac{\sum x}{n} = \frac{15+5+17+5+17}{5} = \frac{59}{5} = 11.8$$

$$s^2 = \frac{\sum x^2 - (\sum x)^2/n}{n-1}$$

x	x ²
15	225
5	25
17	289
5	25
17	289

$\sum x = 59$ $\sum x^2 = 853$

$$s^2 = \frac{853 - \frac{(59)^2}{5}}{5-1} = \frac{853 - \frac{3481}{5}}{4}$$

variance: $s^2 = 39.2$

$$S = \sqrt{39.2} = 6.26$$

3. Multiply each data value by 3. Compute S.

$$\text{Standard Deviation} = 3 * 6.26 = 18.78$$