Sec6.2_Notes

Friday, September 25, 2020 2:35 PM

Sec6.2_No...

Example of discrete random variable.

Definition 1 (Binomial Experiment).

1. There is a fixed number of trials. We denote this number by the letter **n.**
2. The n trials are **independent** and repeated under identical conditions.
3. Each trial has only two outcomes: success, denoted by **S**, and failure, denoted by **F**.
4. For each individual trial, the probability of success is the same. We denote the probability of success by **p** and that of failure by **q**. Because each trial results in either success or failure, p+q=1 and q=1-p.

   p= Probability of success / q = Probability of failure.
5. **The central problem of a binomial experiment is to find the probability of r successes out of n trials.**

   $r = 0, 1, 2, \ldots, n$

Example 1. Analyze the following binomial experiment to determine p, q, n, and r.

Suppose 10% of the population has blood type B. Suppose we choose 15 people at random from the population and test the blood type of each. What is the probability that three of these people have blood type B?

In this experiment, we are observing whether or not a person has type B blood.

$n = 15$, $p = P(\text{success}) = P(\text{a person have type B blood}) = 0.10$

$q = P(\text{failure}) = 1 - p = 0.90$

We wish to calculate the probability of 3 successes out of 15 trials $\rightarrow$ $\boxed{r = 3}$

We have to calculate $P(r=3)$

Q 1. What are some application areas?

- Proportion of defectives in industrial processes
- Medical applications (cure or not cured)
- Military applications (hit or miss)
- ~~DATA~~ Exam question

Definition 2 (General formula for binomial probability distribution).

$$P(r) = \boxed{\frac{n!}{r!(n-r)!}} p^r q^{n-r} = C_{n,r} p^r q^{n-r}$$

$C_{n,r}$ Binomial cofficiant.

In calculater: $C_{n,r}$ is ncr function

where

1. n = number of binomial trials
2. p = probability of success on each trial
3. $q = 1 - p$ = probability of failure on each trial
4. r = random variable representing the number of successes out of n trials $(0 \le r \le n)$
5. ! = factorial notation. The factorial symbol $n!$ designates the product of all the integers between 1 and n. For instance, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Special cases are $1! = 1$ and $0! = 1$

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 =$ ___

Q 2. How do we calculate $P(r)$?

- Using the Binomial Distribution formula.
- By Calculator (*binompdf* function)
- Using a Binomial Distribution table.

Example: For example 1: $n = 15$, $p = 0.10$, $q = 0.90$, $r = 3$.

$$P(r=3) = C_{15,3} (0.10)^3 (0.90)^{15-3} = 0.129.$$

15 ncr 3

[15ncr3 * (0.10)^3 * (0.90)^12 = 0.129]

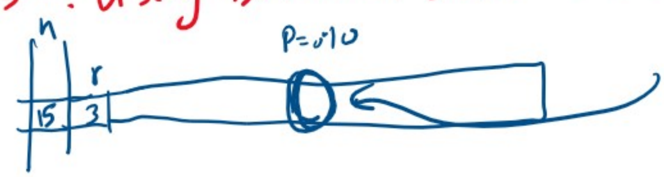
1st method.

2nd: Using binompdf(n, p, r). [or binomial pdf]

$\Rightarrow P(r=3) = \text{binompdf}(15, 0.10, 3) = \boxed{0.129}$

method is used in my calculations

3rd: Using Binomial dist table (Appendix)



0.129

Example 2. Alex uses a copy machine to make 225 copies of the exam. Suppose that for each copy of the exam the stapler independently malfunctions with probability 0.02.

(a) What is n? What is p? What is r?

$n = 225$, $P = 0.02$, $r =$ we don't know yet

it could be $0, 1, 2, 3, \ldots, 225$

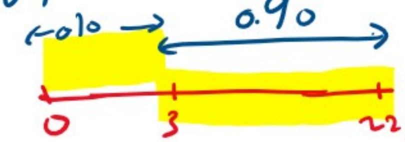
(b) Find the probability that the stapler would malfunction exactly 3 times.

$r = 3$

$$P(r=3) = \text{binompdf}(\overset{n}{225}, \overset{p}{0.02}, \overset{r}{3}) = \boxed{0.16899}$$

or

$$C_{225,3}\,(0.02)^3(0.98)^{225-3} = 0.16899 \simeq 0.169$$



(c) Find the probability that the stapler would malfunction at least 3 times.

$$P(r \geq 3) = \underbrace{P(r=3) + P(r=4) + P(r=5) + \cdots + P(r=225)}_{\text{Hard to calculate.}}$$

Complement rule

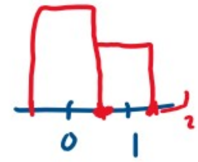
$$= 1 - P(r<3) = 1 - [P(r=0) + P(r=1) + P(r=2)]$$

$$= 1 - [\text{binompdf}(225, 0.02, 0) + \text{binompdf}(225, 0.02, 1) + \text{binompdf}(225, 0.02, 2)]$$

$$= 1 - (0.01061 - 0.04874 - 0.11140) = \boxed{0.8293}$$

How To Graph a Binomial Distribution.

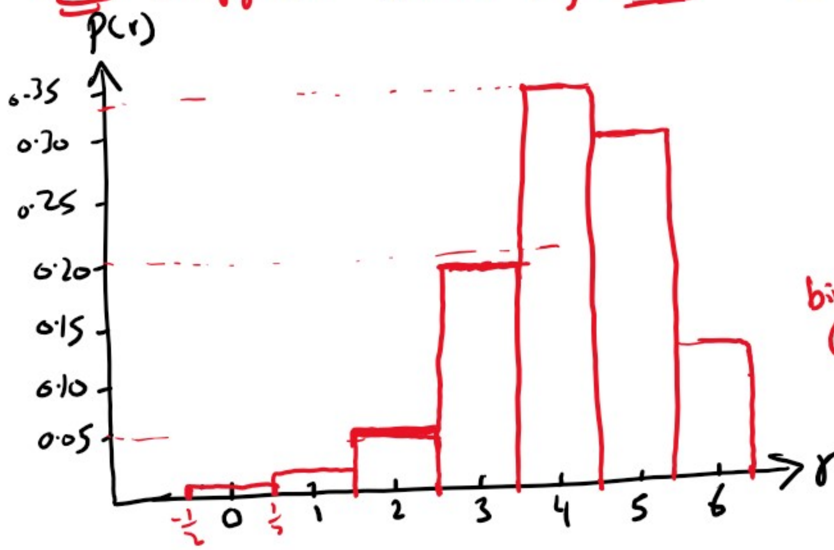
(1) replace r values on the horizontal axis.



(2) replace $p(r)$ values on the Vertical axis.

(3) construct a bar over each r value extending from $r-0.5$ to $r+0.50$. The height of the corresponding bar is $p(r)$.

Ex: Suppose $n=6$, $p=0.70$ $\Rightarrow r=0,1,2,\dots,6$.



binompdf(6, 0.70, 3)

binompdf(6, 0.70, r)

r	$p(r)$
0	0.001
1	0.010
2	0.060
3	0.185
4	0.324
5	0.303
6	0.118
Add	1

Compute the expectation (μ) and standard deviation (σ) for a Binomial Distribution.

Expectation $= \mu = np$

Standard deviation $\sigma = \sqrt{npq}$

$q = p(\text{failure})$

$q = 1-p$

Variance $= \sigma^2 = npq$

Example 3. Suppose the probability that a patient recovers from a blood disease is 0.4. 15 people are sick. What is the probability that

(a) exactly 5 survive?

(b) 2 or less survive?

(c) less than 2 survive?

(d) at least 10 survive?

(e) from 3 to 8 survive?

(f) on average, how many survive?

g) What is the variance of survive?

$p = 0.40$, $n = 15$

ⓐ $P(r=5) = \text{binompdf}(\overset{n}{15}, \overset{p}{0.40}, \overset{r}{5}) = 0.1859.$

ⓑ $P(r \le 2) = P(r=0) + P(r=1) + P(r=2)$

$= \text{binompdf}(15, 0.40, 0) + \text{binompdf}(15, 0.40, 1)$

$+ \text{binompdf}(15, 0.40, 2) = \boxed{0.0271}$ (Check!)

ⓒ $P(r<2) = P(r=0) + P(r=1) +$

$= \text{binompdf}(15, 0.40, 0) + \text{binompdf}(15, 0.40, 1)$

$= 0.0052$

ⓕ How many survive = Expectation

$= n \cdot p = 15(0.40) = \boxed{6}$

g) variance $= \sigma^2 = npq = 15(0.40)(0.6)$

$= \boxed{3.6}$

Example 4 (YOU TRY). A machine contains 10 processors that operate independently. The probability that a processor fails is 0.08.

(a) What is the probability that exactly 4 of the processors fail?

(b) What is the probability that at most 1 of the processors fails?

(c) What is the probability that at least 1 of the processors fails?

$n=10$, $p=0.08$

a) $P(r=4) = \text{binompdf}(10, 0.08, 4) = \text{----}$

b) $P(r\leq 1) = P(r=0) + P(r=1)$

c) $P(r\geq 1) = \underbrace{P(r=1) + P(r=2) + \dots + P(r=10)}_{\text{~~cdf~~ Hard}}$

$= 1 - P(r<1)$

$= 1 - P(r=0)$