Problem 1

Part 1:

1.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.4 + 0.5 - 0 \\ &= 0.9 \end{aligned}$$

2.

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ &= (0.4)(0.5) \\ &= 0.2 \end{aligned}$$

3.

$$\begin{aligned} P\left(\frac{A}{B}\right) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{0.2}{0.5} \\ &= 0.4 \end{aligned}$$

Part 2:

Let A = majors in Nursing and B = is a junior

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.15 + 0.20 - 0.085 \\ &= 0.265 \end{aligned}$$

Problem 2

1.

$$\begin{aligned} \text{98\% confidence interval} &= \bar{x} \pm E \\ &= \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \\ &= 2.2 \pm 2.326\left(\frac{0.4}{\sqrt{49}}\right) \\ &= 2.2 \pm 0.133 \\ &= [2.067,\ 2.333] \end{aligned}$$

The 98% confidence interval is between 2.067 g/ml and 2.333 g/ml.

2.

The 3% of 2.2 g/ml is 0.066 g/ml (margin of error)

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$n = \left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2$$

$$= \left[\frac{(2.326)(0.4)}{0.066}\right]^2$$

$$\approx 199 \rightarrow \text{rounded up}$$

Problem 3

Part 1:

1.

$$\mu_{\bar{x}} = \mu$$

$$= 40$$

2.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{10}{\sqrt{49}}$$

$$= 1.429$$

3.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$$= \frac{38-40}{1.429}$$

$$= -1.4$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$$= \frac{44-40}{1.429}$$

$$= 2.8$$

Hence,

$$P\left(38 \le \bar{x} \le 44\right) = P\left(-1.4 \le z \le 2.8\right)$$
$$= P\left(z \le 2.8\right) - P\left(z \le -1.4\right)$$
$$= 0.9974 - 0.0808$$
$$= 0.9167$$

Part 2:

1.

$$P\left(X = 4\right) = C_4^8 \left(0.7\right)^4 \left(0.3\right)^4$$
$$= 0.13614$$

2.

$$P\left(\text{not less than } 2\right) = P\left(X \ge 2\right)$$
$$= 1 - P\left(X = 0\right) - P\left(X = 1\right)$$
$$= 1 - C_0^8 \left(0.7\right)^0 \left(0.3\right)^8 - C_1^8 \left(0.7\right)^1 \left(0.3\right)^7$$
$$= 0.99871$$

Problem 4

Part 1:

1.

$$P\left(1 \le X < 3\right) = P\left(X = 1\right) + P\left(X = 2\right)$$
$$= 0.4 + 0.1$$
$$= 0.5$$

2.

$$P\left(X > 0\right) = P\left(X = 1\right) + P\left(X = 2\right) + P\left(X = 3\right)$$
$$= 0.4 + 0.1 + 0.3$$
$$= 0.8$$

3.

$$E\left(X\right) = 0\left(0.2\right) + 1\left(0.4\right) + 2\left(0.1\right) + 3\left(0.3\right)$$
$$= 1.5$$

4.

$$V\left(X\right) = \sum X^2 P\left(X\right) - \left[E\left(X\right)\right]^2$$
$$= \left[0\left(0.2\right) + 1\left(0.4\right) + 4\left(0.1\right) + 9\left(0.3\right)\right] - 2.25$$
$$= 1.25$$