

DUE DATE: MONDAY, OCTOBER 26, 2020 by 11:59 PM on Gradescope

1. (Binomial Distribution) The probability that a patient recovers from a delicate heart operation is 0.9. 7 patients have this operation. Of these 7, $n=7$, $p=0.9$, $q=1-p=0.1$

(a) What is the probability that exactly 5 recover?

$$P(X=5) = \binom{7}{5} (0.9)^5 (0.1)^2 = 0.124$$

(b) What is the probability that anywhere from 4 to 6 recover?

$$P(4 \leq X \leq 6) = P(X=4) + P(X=5) + P(X=6)$$

(c) what is the probability that not less than 2 recover?

$$1 = \sum_{r=4}^6 P(X=r) = \sum_{r=4}^6 \binom{7}{r} (0.9)^r (0.1)^{7-r}$$

$$P(X \geq 5) = P(X=5) + P(X=6) + P(X=7)$$

$$= \sum_{r=5}^7 \binom{7}{r} (0.9)^r (0.1)^{7-r} = 0.9743$$

2. Suppose we have the following probability distribution for X:

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= 0.1 + 0.2 + 0.1 = 0.4$$

x	0	1	2	3	4
f(x)	0.1	0.2	0.1	0.3	0.3

(a) Find $P(X \leq 2)$.

$$P(X > 3) = P(X=4) = 0.3$$

(b) Find $P(X > 3)$.

$$P(2 \leq X \leq 4) = P(X=2) + P(X=3) + P(X=4) = 0.1 + 0.3 + 0.3 = 0.7$$

(c) Find $P(2 \leq X \leq 4)$.

(d) What is $E(X)$?

$$E(X) = 0.2 + 0.2 + 0.9 + 1.2 = 2.5$$

(e) What is $V(X)$?

$$E(X^2) = 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.1 + 3^2 \times 0.3 + 4^2 \times 0.3 = 8.1 \Rightarrow V(X) = 8.1 - (2.5)^2 = 1.85$$

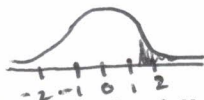
3. Given a standard normal distribution, find the area between $z = -1.84$ and $z = 0.68$.

$$Z = -1.84 \text{ and } Z = 0.68$$

$$= P(-1.84 < Z < 0.68)$$

$$= P(Z < 0.68) - P(Z < -1.84) = 0.7517 - 0.0329 = 0.7188$$

4. Find the area to the right of $z = 1.53$.



$$1 - 0.9370 = 0.0630$$

5. Convert the following Z scores to X values:

$$\bullet z = -0.23, \mu = 10, \sigma^2 = 9$$

$$\bullet z = 1.13, \mu = 2, \sigma = 2$$

$$\text{Mean} = \mu = 10$$

$$\text{SD} = \sigma^2 = 9$$

$$X = z \times \sigma + \mu$$

$$X = -0.23 \times 3 + 10$$

$$X = 9.31$$

$$\text{Mean} = \mu = 2$$

$$\text{SD} = \sigma = 2$$

$$X = z \times \sigma + \mu$$

$$X = 1.13 \times 2 + 2$$

$$X = 4.26$$