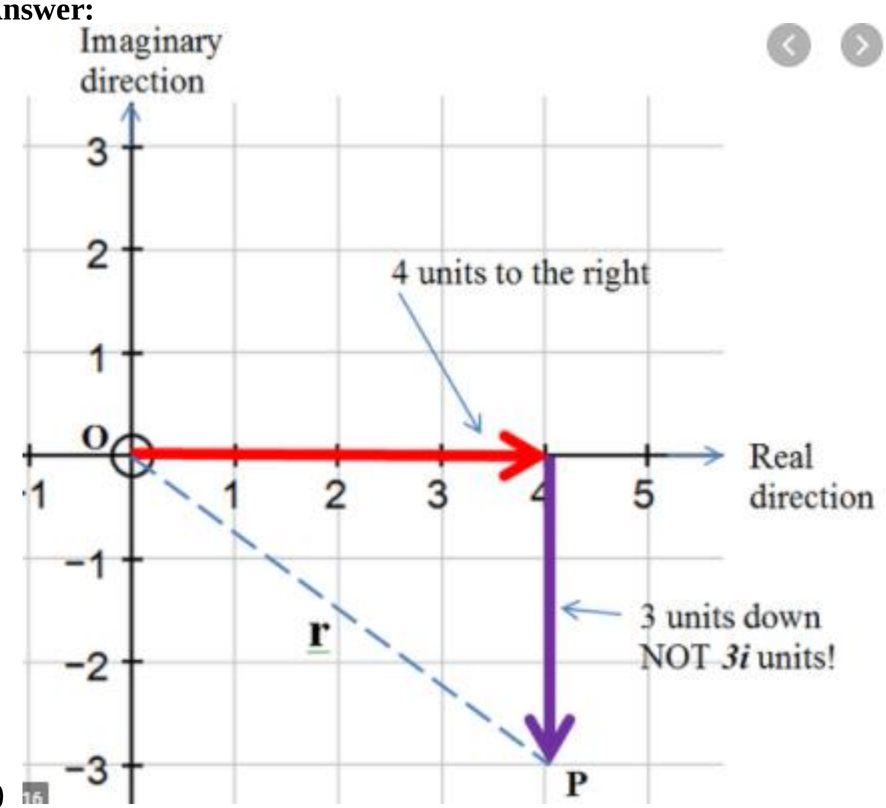
CONCEPT OF LINEAR ALGEBRA
HOMEWORK 11

Submit a single PDF to Gradescope. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. Use GeoGebra to clearly create and label all images. You are encouraged to use $\LaTeX$.

(1) Consider the complex number z = 4 + 3i.
  (a) Use GeoGebra or sketch by hand z and z. Be sure this is clearly labelled.
  (b) Find the polar representation of this complex number.
  (c) Compute z^2 using both the Cartesian and polar representations. Which one was easier?

Answer:



a)

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(1b) $z = 4 + 3i$

Find the polar representation of this complex number.

Solution

The polar form of a complex number $z = a + bi$ is $z = r(\cos\theta + i\sin\theta)$

The absolute value of r.

$$r = |z| = \sqrt{a^2 + b^2}.$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$r = 5$$

The argument θ since $a > 0$ the formula $\theta = \tan^{-1}(b/a)$

$$\theta = \tan^{-1} 3/5$$
$$\theta = 0.6.$$

The polar form of $z = 4 + 3i$ is

$$= 5(\cos(0.6) + i\sin(0.6))$$

(c) $z^2 = (4+3i)^2 = (4+3i)(4+3i) = 16 + i12 + i12 - 9$

$= 16 + 24i - 9 = 7 + 24i$ - Cartesian form

Polar form $= z = r(\cos\theta + i\sin\theta)$

$r = |z| = \sqrt{7^2 + 24^2} = \sqrt{625}$ $r = 25$

$\theta = \tan^{-1}(b/a) = \tan^{-1}(24/7) = 3.428$

Answer $= 25(\cos 3.428 + i\sin 3.428$

$= 25(\cos 3.428 + i\sin 3.428$

b)

(2) We can define a transformation of the complex plane T : C ! C where, given a complex number z, we then output iz. Recall that the complex plane C is just the plane R^2 with multiplication with the identification

z = x + iy \$ z = x y. It turns out that there exists a 2 2 real matrix A such that iz = Az.

(a) Find A.

(b) Are you surprised by this matrix? Why or why not?

(c) What if we now consider w = a + bi and a new transformation T : C ! C where, given a complex number z, we then output wz. Can you find the associated matrix A? You are encouraged to use GeoGebra in your initial exploration.

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Question 2.

Find A.

$z = x + iy$

As a 2x2 Matrix.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b. The Matrix is a 2 x 2 Matrix.

Answer

The matrix acquired is a rationalized matrix of A

As it yields

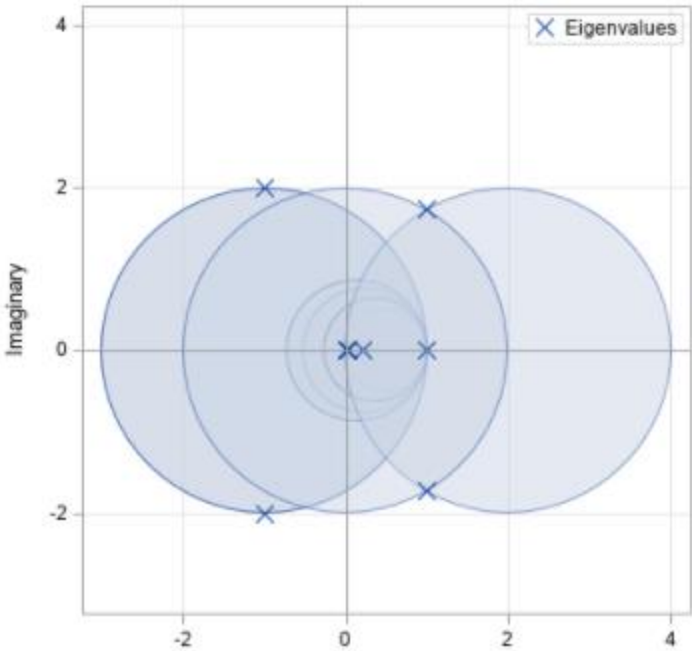
(0 1

(1 0)

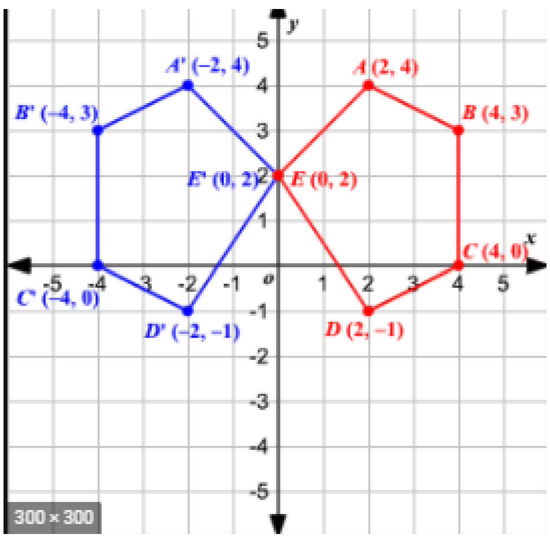
Answer A and B

(a) Compute disc(A) and then use this to determine the number of invariant lines A has in the plane.
(b) Find $p_A(l\,)$.
(c) Use the quadratic formula to find and simplify the eigenvalues l_1 and l_2.
(d) Set up, but do not evaluate, an augmented matrix whose solution sets correspond to a complex eigenspace.
(e) Find the parametric equations for the A-invariant line and pick an eigenvector v. (Note: You will have complex numbers, that is okay.)
(f) Give a complex diagonalization of A.
(g) Use v to find an ordered rotation basis B.
(h) Using complete sentences and geometric words, verbally explain how A transforms the plane.
(i) Symbolically express your rotationalization of A.
(j) Using GeoGebra, visually represent the B-coordinate grid, the B-unit square, S_B, and depict how A transforms S_B.

A


Eigenvalues
Imaginary
4
2
0
-2
-2
0
2
4

J


y
x
A' (−2, 4)
A (2, 4)
B' (−4, 3)
B (4, 3)
E' (0, 2)
E (0, 2)
C (4, 0)
C' (−4, 0)
D' (−2, −1)
D (2, −1)
o
300 × 300

3. Compute the following for the following Matrix

$$\begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix}$$

b. $P_A \lambda = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} \lambda - 2 & 4 \\ -2 & \lambda + 2 \end{bmatrix}$

$$P_A \lambda = \begin{bmatrix} \lambda - 2 & 4 \\ -2 & \lambda + 2 \end{bmatrix}$$

c. Eigen values

$$(\lambda - 2)(\lambda + 2) - (4 \times -2)$$

$$= \lambda^2 + 2\lambda - 2\lambda - 4 - -8$$

$$= \lambda^2 + 0\lambda - 4 + 12$$

$$= \lambda^2 + 8$$

using Quadratic Formula.

$$\frac{b^2 \pm \sqrt{4ac}}{2a} = \frac{0^2 \pm \sqrt{4 \times 1 \times 8}}{2}$$

$$= \frac{\pm \sqrt{32}}{2} = \frac{\sqrt{4} \times \sqrt{8}}{2}$$

$$= \frac{2}{2} + \frac{\sqrt{8}}{2} = \frac{1\sqrt{8}}{2}$$

$\lambda_1 = \frac{\cancel{-1 \pm \sqrt{}}}{2}$ $\lambda_1 = \frac{-1\sqrt{8}}{2}$ $\lambda_2 = \frac{+1\sqrt{8}}{2}$

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d. Augmented Matrix.

$$Aug_{x} = \left[\begin{array}{cc|c} 2 & -4 & x_1 \\ 2 & -2 & x_2 \end{array}\right] \text{ ---- (i)}$$

$$Aug_A = \left[\begin{array}{cc|c} 2 & -4 & y_1 \\ 2 & -2 & y_2 \end{array}\right] \text{ ---- (ii)}$$

e. $A = \begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix}$

$y = mx$

$x = t$

$y = mt$

$\begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} t \\ mt \end{bmatrix}$

$= 2t - 4mt$

$2t - 2mt$

$x = t(2-4m)$

$x = t(2-2m)$

$\frac{x}{2-4m} = t$ and $\frac{x}{2-2m} = t$

$\frac{x}{2-4m} = \frac{x}{2-2m} = \frac{2-4m}{*} = \frac{1}{2-2m}$

$= (2-4m)(2-2m) = 4 - 4m - 8m + 8m^2$

$= 4 - 12m + 8m^2$

$= 4 - 4m - 8m + 8m^2$

$= 4(1-m) - 8m(1m - 8m)$

$(4-8m)(1-m)$

$4 = 8m \quad m = 1/2$

$1 - m = \quad m = 1$

The Parametric equations are.

$m = 1/2$ and $m = 1$

F. Complex diagonalization of A.

$$= \begin{bmatrix} 2 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & -2 \end{bmatrix}$$

g. The Ordered rotation basis is.

$$\begin{bmatrix} -2 & 2 \\ 4 & 2 \end{bmatrix}$$

h. A transform the Matrix A through diagonalization.

i. Rationalization of A

$$= \begin{bmatrix} 2-\lambda & 2 \\ 4 & -2-\lambda \end{bmatrix}$$