

DUE DATE: MONDAY, OCTOBER 19, 2020 by 11:59 PM on Gradescope

1. If $S = \{0, 1, 3, 4, 6, 7, 9, 10, 11\}$ and $A = \{0, 4, 6, 10\}$, $B = \{1, 3, 7, 9, 11\}$, $C = \{4, 6, 7, 9\}$, and $D = \{0, 1, 10, 11\}$, list the elements of the sets corresponding to the following events:

(a) $A \cup C = \{0, 4, 6, 7, 9, 10\}$

(c) $(C^c \cap D) \cup B = \{0, 1, 10, 11, 3, 7, 9\}$

(b) $A \cap B = \{\phi\}$

(d) $(S \cap C)^c = \{0, 1, 3, 10, 11\}$

2. In a high school graduating class of 100 students, 47 studied mathematics, 61 studied physics, and 25 studied both mathematics and physics. If one of these students is selected at random, find the probability that

(a) the student took mathematics or physics.

$$\frac{47}{100} + \frac{61}{100} - \frac{25}{100} = 0.83$$

(b) the student did not take either of these subjects.

$$1 - 0.83 = 0.17$$

(c) the student took physics but not mathematics.

$$\frac{61}{100} - \frac{25}{100} = 0.36$$

Are studying mathematics and physics mutually exclusive events? Why or why not?

Hint: You may want to draw a Venn diagram. NO, math and physics are not mutually exclusive. Because some students are studying both.

$$P(M \cap P) \neq 0$$

3. It is common in many industrial areas to use a filling machine to fill boxes full of product. This occurs in the food industry as well as other areas in which the product is used in the home, for example, detergent. These machines are not perfect, and indeed they may A , fill to specification, B , underfill, and C , overfill. Generally the practice of underfilling is that which one hopes to avoid. Let $P(C) = 0.052$ while $P(A) = 0.940$.

(a) What is the probability that the box is underfilled, $P(B)$?

$$\begin{aligned} 0.940 + P(B) + 0.052 &= 1 \\ &= P(B) + 0.940 = 1 \\ &= P(B) = 1 - 0.940 \\ &= 0.008 \end{aligned}$$

(b) Find $P(A \cap B)$.

$$P(A \cap B) = 0$$

(c) Are A and B mutually exclusive events? Why or why not?

$$\begin{aligned} 0.940 + 0.008 &= 0 \\ &= 0.948 \end{aligned}$$

(d) Find $P(A \cup B)$.

(e) What is the probability that the machine does not overfill?

(f) What is the probability that the machine either overfills or underfills?

$$\begin{aligned} &= 1 - P(C) \\ &= 1 - 0.052 \\ &= 0.948 \end{aligned}$$

$$P(B \cap C) = 0$$

$$\begin{aligned} &= P(B) + P(C) \\ &= 0.008 + 0.052 \\ &= 0.06 \end{aligned}$$

Yes, A is the machine of filling to specification, and B is the machine of underfilling. There is no common between them



4. A random sample of 200 adults are classified below by gender and their level of education attained.

Education	Male	Female	Total
Elementary	36	47	83
Secondary	26	52	78
College	20	19	39
Total	82	118	200

If a person is picked at random from this group, find the probability that

- (a) the person has only an elementary education, given that a person is female.
 (b) the person does not have only a secondary education, given that a person is female.
 (c) the person is female, given that they do not have a only a secondary education.

$$\frac{47}{118} = 0.398$$

$$\frac{19}{118} = 0.161$$

$$\downarrow \frac{19}{39} = 0.487$$

5. Let C be the event that a computer crashed and O the event that a computer overheated. Suppose we have the following probabilities:

$$P(C) = 0.45$$

$$P(C \cap O) = 0.30$$

$$P(C^c \cap O) = 0.20$$

$$P(O) = 0.35$$

$$P(C \cap O^c) = 0.10$$

$$P(C^c \cap O^c) = 0.40$$

Answer the questions below. Round your answers below to 2 decimals.

- (a) Find the probability that the computer crashed, given that the computer overheated.
 (b) Find the probability that the computer crashed, given that the computer did not overheat.
 (c) Find the probability that the computer did not overheat, given that the computer did not crash.
 (d) Are the events of a computer crashing and a computer overheating independent events? Explain.

$$P(C|O) = \frac{P(C \cap O)}{P(O)} = \frac{0.30}{0.35} = 0.86$$

$$P(C|O^c) = \frac{P(C \cap O^c)}{P(O^c)} = \frac{0.10}{0.65} = 0.15$$

$$P(C \cap O) = 0.30$$

$$P(C) \cdot P(O) = 0.45 \cdot 0.35$$

$$= 0.1575$$

$$P(C \cap O) \neq P(C) \cdot P(O)$$

in the event of a computer crashed and the computer overheats are not independent

$$P(O^c|C^c) = \frac{P(O^c \cap C^c)}{P(C^c)}$$

$$= \frac{0.40}{1 - 0.45} = \frac{0.40}{0.55} = 0.7272$$