HW 3 Josephine Baggen

<u>Density Parameters</u>

$$\frac{H^2}{H_0^2} = \Omega_{\gamma,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda$$

At $z = 0.5$ ignore radiation term.
Ignore curvature term $\Omega_{k,0} = 0$

$$H^2 = H_0^2\left[\Omega_{m,0}(1+z)^3 + \Omega_\Lambda\right]$$

$$H = \frac{\dot{a}}{a}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\frac{\Omega_{m,0}}{a^3} + \Omega_\Lambda\right]$$

$$\cancel{\Omega = \frac{\rho}{\rho_{crit}} = \frac{3H^2}{8\pi G}\frac{8\pi G}{3H_0^2} = \frac{H^2}{H_0^2}}$$

$$\Omega_{m,z} = \frac{\rho_{m,z}}{\rho_{crit,z}} = \frac{\rho_{m,0}(1+z)^3}{\rho_{crit,0}}\frac{\rho_{crit,0}}{\rho_{crit,z}}$$

$$= \Omega_{m,0}(1+z)^3\frac{\rho_{crit,0}}{\rho_{crit,z}}$$

$$= \Omega_{m,0}(1+z)^3\left(\frac{H_0}{H}\right)^2$$

$$\left(\frac{H}{H_0}\right)^2_{z=0.5} = 0.3(1+0.5)^3 + 0.7 = 1.7125$$

$$\Omega_{m_{z=0.5}} = 0.3(1+0.5)^3\frac{1}{1.7125} = 0.6$$

$$\Omega_{\Lambda, z=0.5} = \Omega_{\Lambda,0}\left(\frac{H_0}{H}\right)^2 = 0.7 \frac{1}{1.7125} = 0.4$$

- Luminosity Distance & Angular Diameter Distance

Flat universe $k=0$ S_k = comoving distance $= r$

$$d_L = r\cdot(1+z)$$

$$d_A = \frac{r}{1+z}$$

$$r = \frac{c}{H_0}\int_0^z \frac{dz}{E(z)}$$

$$E(z) = \sqrt{\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}}$$

Here only $\Omega_{m,0}$, $\Omega_{\Lambda,0}$

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}}$$

I write a code for a cosmology
that I specify ($\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$)
and ($\Omega_{m,0} = 1$)

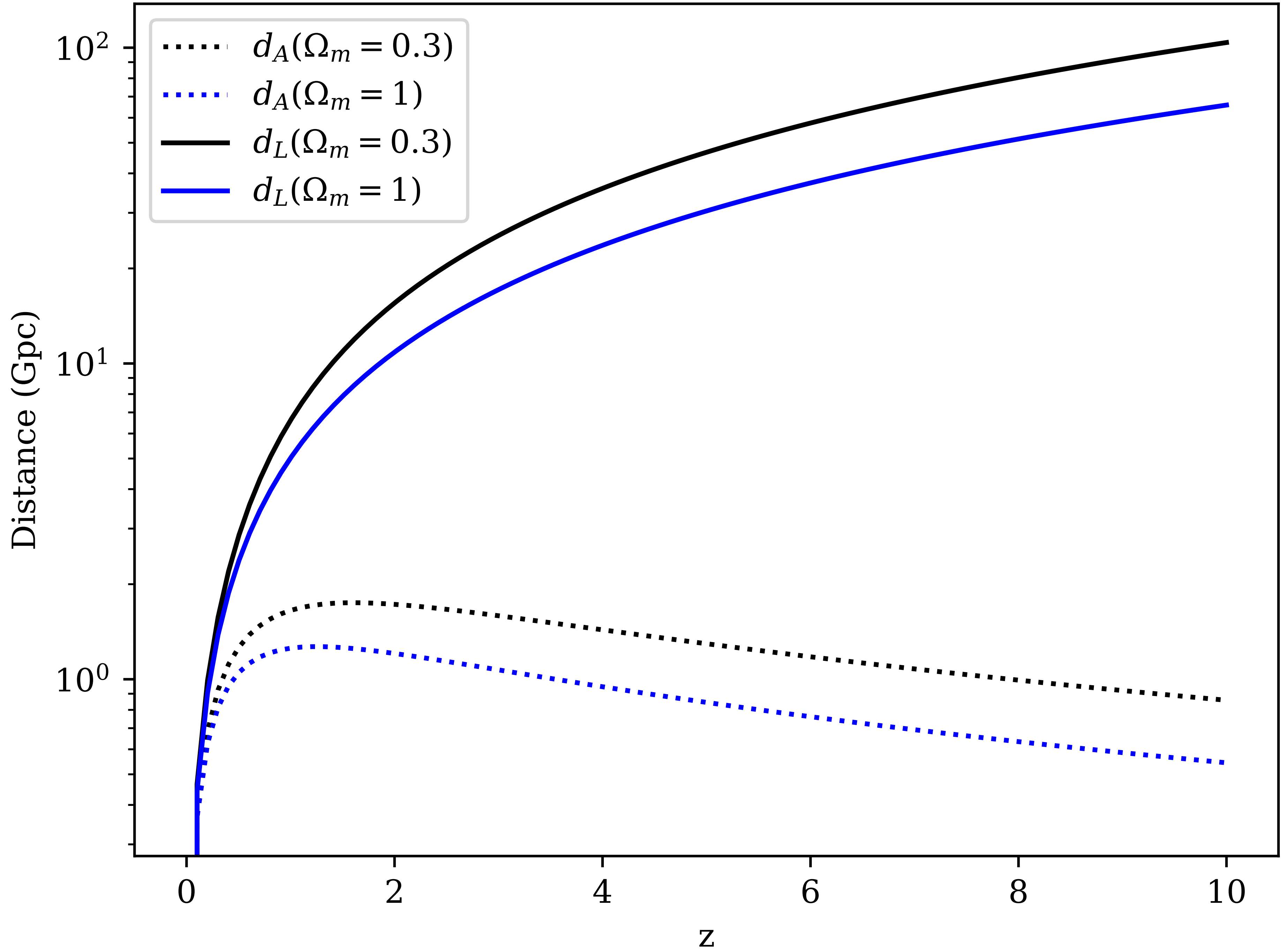
and a function that calculates the integral

$$\int_0^z \frac{1}{E(z)}\,dz$$

We see that the derivative of angular diameter distance wrt z changes signs at roughly z=1, it is first increasing and then decreasing with z > a non-monotic function. A monotonic function is either entirely non-increasing or non-decreasing.

```
import numpy as np
import matplotlib.pyplot as plt
#from ccdproc import wcs_project
import matplotlib
matplotlib.use('TKAgg')
import sys
import os
import io
import math
import scipy
import astropy.io.fits as fits
import astropy.units as u
import astropy.coordinates as coord
import pandas as pd
import matplotlib.lines as mlines
from astropy.cosmology import Planck13, z_at_value
from astropy.coordinates import SkyCoord
from astropy.coordinates import SkyCoord
from astropy.cosmology import FlatLambdaCDM
from astropy import constants as const
from scipy.integrate import quad, dblquad


plt.rcParams["font.family"] = "serif"
plt.rcParams["mathtext.fontset"] = "dejavuserif"
path ='/Users/josephine/Desktop/Yale_2/COSM/'
'''
Make a plot of the luminosity and angular diameter distances as a
function of redshift to z = 10.
Compare these (on the same plot) to the luminosity/angular diameter
distance for an Ωm,0 = 1 universe.
Make sure to label your axes. Comment on the non-monotonicity of the
angular diameter distance.
'''

'''
flat universe, so S_k = comoving distance = r
then d_L = r(1+z), d_A = r/(1+z)
'''


def E_z(x, cosm):
        Omega_r = cosm.Ogamma0
        Omega_m = cosm.Om0
        Omega_L = cosm.Ode0
        Omega_k = 1- Omega_r - Omega_m - Omega_L
        return 1/(np.sqrt(Omega_r*(1+x)**4 +Omega_m*(1+x)**3+Omega_k
*(1+x)**2 +Omega_L))

def calc_ang_diam_dist(z, cosm):
        lower_limit = 0
        upper_limit = z
        result = np.zeros(len(z))
        for i in range(len(z)):
```

```
                term1, _ = quad(E_z,
                        lower_limit, upper_limit[i], args=cosm)
                result[i] = term1
        da = const.c/cosm.H0  * result/(1+z)
        return da

def calc_lum_dist(z, cosm):
        lower_limit = 0
        upper_limit = z
        result = np.zeros(len(z))
        for i in range(len(z)):
                term1, _ = quad(E_z,
                        lower_limit, upper_limit[i], args=cosm)
                result[i] = term1
        da = const.c/cosm.H0  * result *(1+z)
        return da

z = np.linspace(0,10,100)

cosm1 = FlatLambdaCDM(H0=70, Om0=0.3)
cosm2 = FlatLambdaCDM(H0=70, Om0=1)

d_L1 = calc_lum_dist(z, cosm1).to(u.Gpc)
d_A1 = calc_ang_diam_dist(z, cosm1).to(u.Gpc)
d_L2 = calc_lum_dist(z, cosm2).to(u.Gpc)
d_A2 = calc_ang_diam_dist(z, cosm2).to(u.Gpc)

plt.plot(z, d_A1, label=r'$d_A (\Omega_m = 0.3)$', ls='dotted',
color='black')
plt.plot(z, d_A2, label=r'$d_A (\Omega_m = 1)$',   ls='dotted',
color='blue')
plt.plot(z, d_L1, label=r'$d_L (\Omega_m = 0.3)$',  color='black')
plt.plot(z, d_L2, label=r'$d_L (\Omega_m = 1)$',    color='blue')
plt.xlabel('z')
plt.ylabel('Distance (Gpc)')
plt.yscale('log')
plt.legend()
plt.savefig(path+'Distances.pdf')
plt.show()
```


$d_A(\Omega_m = 0.3)$
$d_A(\Omega_m = 1)$
$d_L(\Omega_m = 0.3)$
$d_L(\Omega_m = 1)$
10^2
10^1
10^0
Distance (Gpc)
0
2
4
6
8
10
z

- Redshift 10 billion years ago

$$t = \frac{1}{H_0} \int_0^z \frac{dz}{E(z)} \frac{1}{(1+z)}$$

$$= \frac{1}{H_0} \int_0^z \frac{dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}}\,(1+z)}$$

$$= \frac{1}{H_0} \int_0^z \frac{dz}{\sqrt{0.3(1+z)^3 + \cancel{\Omega_{\Lambda,0}}\ 0.7}\,(1+z)} = 10\ Gyr$$

$H_0 = 70\ km/s/Mpc = 2.269 \cdot 10^{-10}\ s^{-1}$

$\frac{1}{H_0} = 4.4 \cdot 10^{17}\ s = 13.98\ Gyr$

So solve for integral. z where

$$\int_0^z \frac{dz}{\sqrt{0.3(1+z)^3 + \cancel{\Omega}\ 0.7}\,(1+z)} = \frac{10}{13.98} = 0.7153$$

Gives $z = 1.85$

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PURPOSE

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NOTES

A Λ-dominated Universe

Only Λ $(\Omega_m, \Omega_r, \Omega_k = 0)$

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda$$

$$H^2 = H_0^2 \Omega_\Lambda$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_\Lambda$$

$$\frac{\dot{a}}{a} = H_0 \sqrt{\Omega_\Lambda}$$

$$\frac{1}{a} da = H_0 \sqrt{\Omega_\Lambda}\, dt$$

$$\ln(a) = H_0 \sqrt{\Omega_\Lambda}\, t + C$$

$$a(t) = e^{H_0 \sqrt{\Omega_\Lambda}\, t + C}$$

At $a(t_0)$ we want $a = 1$

$$a(t_0) = e^{H_0 \sqrt{\Omega_\Lambda}\, t_0 + C} = 1$$

$$H_0 \sqrt{\Omega_\Lambda}\, t_0 + C = 0$$

$$C = -H_0 \sqrt{\Omega_\Lambda}\, t_0$$

$$a(t) = e^{H_0 \sqrt{\Omega_\Lambda}(t - t_0)}$$

Age Universe $a \to 0$, $t \to 0$ $t_0 = ?$

SUMMARY

$$t - t_0 = \frac{\ln(a)}{\sqrt{\Omega_\Lambda} H_0} \qquad t_0 = -\frac{\ln(a)}{\sqrt{\Omega_\Lambda} H_0} + t$$

$\ln(a)$ goes to $-\infty$ for $a \to 0$ at $t \to 0$

So $\lim_{\substack{a \to 0 \\ t \to 0}} t_0 = \infty$

The age of the universe is infinite

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NOTES

Massive neutrinos*

- Neutrinos relativistic at decoupling

number density relativistic particles (fermions)

$$\rho = \frac{g}{(2\pi)^3} \int d\vec{p}\, f(p,t)\, E(p)$$

$(c = 1, \hbar = 1$, $k_B = 1)$

$$E(p) = \sqrt{p^2 + m^2}$$

$$= \frac{g}{(2\pi)^3} \cdot 4\pi \int_0^\infty dp\, p^2 \sqrt{p^2+m^2}\, f(p,t)$$

$$f(p,t) = \frac{1}{\exp\left(\frac{E(p)-\mu}{T}\right) \pm 1}$$

+ for ferm

− for bos

neglect μ and fermions

$$f(p,t) = \frac{1}{\exp(E(p)/T) + 1}$$

$$\rho = \frac{g}{2\pi^2} \int_0^\infty dp\, p^2 \sqrt{p^2+m^2}\, \frac{1}{e^{E/T}+1}$$

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Define $\xi = p/T$ $\qquad E = \sqrt{m^2+p^2}$

$p = \xi T$

$dp = T\,d\xi$

$$\rho = \frac{g}{2\pi^2}\int_0^\infty T(\xi T)^2 \sqrt{\xi^2 T^2 + m^2}\,\frac{1}{e^{\sqrt{\xi^2T^2+m^2}/T}+1}\,d\xi$$

$$= \frac{g}{2\pi^2}\int_0^\infty \frac{\xi^2 T^4 \sqrt{\xi^2 + m^2/T^2}}{e^{\sqrt{\xi^2 + m_\nu^2/T_\nu^2}}+1}\,d\xi$$

$g=2$

$$= \frac{T_\nu^4}{\pi^2}\int_0^\infty \frac{\xi^2\sqrt{\xi^2+(m_\nu/T_\nu)^2}}{e^{\xi}+1}\,d\xi$$

here I neglected $(m_\nu/T_\nu)^2$ wrt ξ^2 because the momentum at decoupling (relativistic) will be much larger than m_ν

$(m_\nu \approx 0)$

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- Small m_ν/T_ν
  so Taylor expand for $\sqrt{\xi^2+(m_\nu/T_\nu)^2}$

call this x

$$f(x)=\sqrt{\xi^2+x}$$

$$f(x)\approx f(a)+\frac{f'(a)(x-a)}{1!}+\frac{f''(a)(x-a)^2}{2!}+\ldots$$

around $a\approx 0$

$$f(x)=f(0)+f'(0)x+\frac{f''(0)x^2}{2}+\ldots$$

$$f'(x)=\frac{1}{2\sqrt{\xi^2+x}}$$

$$f(x)\approx\sqrt{\xi^2}+\frac{1}{2\sqrt{\xi^2}}x+\mathcal{O}(x^2)$$

So up to 1st order

$$\sqrt{\xi^2+(m_\nu/T_\nu)^2}=\sqrt{\xi^2}+\frac{1}{2\sqrt{\xi^2}}\left(\frac{m_\nu}{T_\nu}\right)^2$$

$$\rho_{\nu,0}=\frac{T_\nu^4}{\pi^2}\int_0^\infty d\xi\,\frac{\xi^2\sqrt{\xi^2}}{e^\xi+1}=\frac{T_\nu^4}{\pi^2}\int_0^\infty\frac{\xi^3}{e^\xi+1}$$

$$=\frac{T_\nu^4}{\pi^2}\frac{7\pi^4}{120}=\frac{7\pi^2T_\nu^4}{120}$$

evaluate this integral with Wolfram Alpha

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With Taylor expanded expression:

$$\rho_\nu = \frac{T_\nu^4}{\pi^2}\int_0^\infty \frac{\xi^2\left[\sqrt{\xi^2} + \frac{1}{2\sqrt{\xi^2}}\left(\frac{m_\nu}{T_\nu}\right)^2\right]}{e^\xi + 1}\, d\xi$$

$$= \underbrace{\frac{T_\nu^4}{\pi^2}\int_0^\infty \frac{\xi^3}{e^\xi+1}\, d\xi}_{=\rho_{\nu,0}} + \frac{T_\nu^4}{\pi^2}\int_0^\infty \frac{\frac{1}{2}\xi\left(\frac{m_\nu}{T_\nu}\right)^2}{e^\xi+1}\, d\xi$$

$$= \rho_{\nu,0} + \frac{T_\nu^4}{\pi^2}\left(\frac{m_\nu}{T_\nu}\right)^2 \frac{1}{2}\underbrace{\int_0^\infty \frac{\xi}{e^\xi+1}\, d\xi}_{\text{evaluate this again with Wolfram Alpha}}$$

$$\rho_{\nu,0} = \frac{7\pi^2}{120} T_\nu^4$$

SO $T_\nu^4 = \dfrac{120\,\rho_{\nu,0}}{7\pi^2}$

$$= \rho_{\nu,0} + \frac{120\,\rho_{\nu,0}}{7\pi^4}\left(\frac{m_\nu}{T_\nu}\right)^2 \frac{1}{2}\,\frac{\pi^2}{12}$$

$$= \rho_{\nu,0}\left[1 + \frac{5}{7\pi^2}\left(\frac{m_\nu}{T_\nu}\right)^2\right]$$

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- ρ_ν significantly larger than $\rho_{\nu,0}$

$$\rho_\nu = \rho_{\nu,0}\left(1 + \frac{5}{7\pi^2}\frac{m_\nu^2}{T_\nu^2}\right)$$

I assume say $\rho_\nu = 100\,\rho_{\nu,0}$, so that ρ_ν is significantly larger than $\rho_{\nu,0}$

$$100 = 1 + \frac{5}{7\pi^2}\frac{m_\nu^2}{T_\nu^2}$$

T_ν at ~~decoupling~~ ~~CMB~~ CMB

$$m_\nu^2 = 99\,\frac{7\pi^2}{5}\,T_\nu^2$$

$$T \propto \frac{1}{a} \propto (1+z)$$

$$\frac{T_{CMB,z=0}}{T_{CMB,z=1100}} = \left(\frac{1+0}{1+1100}\right) =$$

$$T_{CMB,z=1100} = 1101\,T_{CMB,z=0} = 1101 \cdot 0.235\text{ meV} = 258.7\text{ meV} \quad (\sim 3000\text{ K})$$

$$T_\nu \text{ at } z = 1100 = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

SUMMARY

$$= 0.160\text{ meV} \qquad \sim 2000\text{ K}$$

This gives $m_\nu^2 \simeq 39\,(\text{meV})^2$

$$m_\nu \simeq 6.2\text{ meV}$$

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$T_{\gamma,0} = 0.235$ (2.7 K)

$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0}$ (from class)

$= 0.168\ \text{meV}$ (1.9 K)

$$\frac{T_{\nu,\,z=1000}}{T_{\nu,\,z=0}} = \frac{1+1000}{1}$$

$T_{\nu,\,z=1100} = 0.168\ \text{meV} \cdot 1001$
$= 168.2\ \text{meV}$ (2100 K)

$$m_\nu^2 = 99\ \frac{7\pi^2}{5}\ T_\nu^2 = 99 \cdot \frac{7\pi^2}{5} (168.2\ \text{meV})^2$$

$m_\nu = 6.2\ \text{eV}$

(choosing other definitions of $\frac{\rho_\nu}{\rho_{\nu,0}}$ gives also orders of eV)

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NOTES

- Redshift neutrinos non relativistic
~~when they decouple~~

$T < m_\nu$

$$T \propto \frac{1}{a} \propto 1+z$$

$$\frac{T_{\nu}, z=0}{m_\nu} = \frac{1}{1+z_{NR}}$$

$$1+z_{NR} = \frac{6.2\,eV}{0.168\,meV}$$

$$z_{NR} = 3.7 \cdot 10^4$$

- From Baumann book (before decoupling in thermal eq with thermal bath)

$$n_\nu = \frac{3\zeta(3)}{4\pi^2} g_\nu T_{dec}^3$$

n_ν decays with $\frac{1}{a^3}$

T also decays as $1/a$

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NOTES So the present neutrino density

$$n_{\nu,0} = \frac{3\zeta(3)}{4\pi^2} g_\nu T_{\nu,0}^3 \qquad (g_\nu = 2)$$

1 neutrino with 2 states

$$= 8.66\cdot 10^{-9}\ \text{meV}^3$$

$T_{\nu,0} = 0.168$ meV

Full expression with $\hbar, c, k_B$:

$\cancel{n_\nu = \zeta(3)}$

$\cancel{\rho_\nu = m_\nu n_\nu}$

$\cancel{\Omega_\nu = \frac{\rho_{\nu,0}}{\rho_{crit,0}} = \frac{m_\nu n_\nu}{\rho_{crit,0}}}$

$$n_{\nu,0} = \frac{\zeta(3)}{\pi^2(\hbar c)^3}\,\frac{3}{2}\,k_B\,\frac{4}{11}\left(k_B T_{\gamma,0}\right)^3$$

$$= \cancel{\ldots}\ 1.09\cdot 10^{8}\ \cancel{kg}\ m^{-3}$$

- $\rho_\nu = m_\nu n_\nu$

$$\Omega_{\nu,0} = \frac{\rho_{\nu,0}}{\rho_{crit,0}} = \frac{\rho_{\nu,0}\,8\pi G}{3H_0^2}$$

$$= \frac{m_\nu n_{\nu,0}\,8\pi G}{3h^2(100\ km/s/Mpc)^2}$$

$$\cancel{\Omega_{\nu,0}h^2 = m_\nu \frac{1.09\cdot 10^{8}\,m^{-3}\cdot 8\pi\cdot 6.67\cdot 10^{-11}\,m^3\,kg^{-1}\,s^{-2}}{3\left(100\cdot 10^{3}\,m\,s^{-1}\,(3.086\cdot 10^{22}\,m)^{-1}\right)^2}}$$

$$\cancel{= 1.93\cdot 10^{33}\,\frac{m_\nu}{kg}}$$

$\cancel{E = mc^2}$

$\cancel{kg = \frac{eV}{c^2} = \frac{1.602\cdot 10^{-19}\,eV}{c^2}}$

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~~In planck units~~

~~$G = 6.7087 \cdot 10^{-39}$~~

$$\Omega_{\nu,0} h^2 = \frac{\zeta(3)}{\pi^2 (\hbar c)^3} \frac{3}{2} \left(\frac{4}{11}\right) (k_B T_{\gamma,0})^3 \frac{8\pi G \, m_\nu}{3 (100 \, km s^{-1} Mpc^{-1})^2}$$

$$= \frac{16 \zeta(3)}{11 \pi} \frac{G}{(\hbar c)^3} \frac{(k_B T_{\gamma,0})^3}{c^2 (100 \frac{km/s}{Mpc})^2} m_\nu \cdot c^2$$

$$G = 6.7087 \cdot 10^{-39} \hbar c \left(\frac{GeV}{c^2}\right)^{-2}$$

$$k_B T_{\gamma,0} = 0.235 \text{ meV} \qquad \hbar = 6.582 \cdot 10^{-22} \text{ MeV s}$$

$$= \frac{16 \zeta(3)}{11 \pi} \frac{6.7087 \cdot 10^{-39}}{(\hbar c)^2 \, GeV^2} \frac{(0.235 \text{ meV})^3 c^2 \, Mpc^2 \, m_\nu c^2}{(100 \, km/s)^2}$$

$$= \frac{16 \zeta(3)}{11 \pi} \frac{6.7087 \cdot 10^{-39}}{(6.582 \cdot 10^{-16} \, eV s)^2} \frac{(0.235 \cdot 10^{-3})^3 eV^3}{(10^9)^2 eV^2} \frac{(10^6 \cdot 3.086 \cdot 10^{16})^2 m^2 s^2}{(100 \cdot 10^3)^2 m^2} \cdot m_\nu c^2$$

Everything cancels, left with units $\frac{m_\nu c^2}{eV}$

$$= 0.0106514 \frac{m_\nu c^2}{eV} = \frac{m_\nu c^2}{94 \text{ eV}}$$

So indeed $\Omega_{\nu,0} h^2 = \frac{m_\nu}{94 \text{ eV}}$ ← c^2 neglect again if mass in eV

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Expansion History with Standard Candles

$$\chi = \frac{c}{H_0}\int dz \frac{1}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + \frac{(1-\Omega_{\Lambda,0}-\Omega_{m,0})}{(1+z)^2}}}$$

Taylor series for $z=0$

of $f(z) = \frac{1}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + (1-\Omega_{m,0}-\Omega_{\Lambda,0})(1+z)}}$

I use wolfram alpha
series expansion ~~of~~ gives

$$f(z) = 1 + z\left(-\frac{\Omega_{m,0}}{2} + \Omega_{\Lambda,0} - 1\right)$$

$$+ \frac{1}{8}z^2\left(3\Omega_{m,0}^2 - 12\Omega_{m,0}\Omega_{\Lambda,0} + 4\Omega_{m,0} + 12\Omega_{\Lambda,0}^2 - 20\Omega_{\Lambda,0} + 8\right)$$

$$+ \frac{1}{16}z^3\left(-5\Omega_{m,0}^3 + 6\Omega_{m,0}^2(5\Omega_{\Lambda,0}-1) + \Omega_{m,0}(-60\Omega_{\Lambda,0}^2 + 60\Omega_{m,0} - 8) + 8(\Omega_{\Lambda,0}-1)^2(5\Omega_{\Lambda,0}-2)\right)$$

$$+ O(z^4)$$

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Filling in $\Omega_{m,0} = 0.3$ $\Omega_{\Lambda,0} = 0.7$

up to z^3 order

$$\text{So } \chi \approx \frac{c}{H_0}\int dz \left[1 \cancel{+ z} - 0.45\, z - 0.14625\, z^2 - 0.1753125\, z^3\right]$$

$$= \frac{c}{H_0}\left[z - \frac{0.45}{2} z^2 - \frac{0.14625}{3} z^3\right]$$

For what z is

$$\frac{\Delta\chi}{\chi} = 0.1 \qquad \frac{\chi_{real} - \chi_{approx}}{\chi_{real}} = 0.1$$

$$\chi_{accurate} = \frac{c}{H_0}\int_0^z dz \frac{1}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + (1 - \Omega_{m,0} - \Omega_{\Lambda,0})(1+z)^2}}$$

Solving this with Wolfram Alpha by trying values for z gives $z \simeq 1.2$

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- For very low z, say $z \simeq 0$

$$\chi \simeq \int dz \frac{c}{H_0 \sqrt{\Omega_{m,0} + \Omega_{r,0} + (1-\Omega_{r,0} - \Omega_{m,0})}}$$

$$= \frac{c}{H_0}$$

The dependence of $\Omega_{m,0}$, $\Omega_{r,0}$ cancels because $z \simeq 0$

So you measure H_0 with the distance.

- Once z increases the $(1-\Omega_{r,0} - \Omega_{m,0})(1+z)^2$ doesn't cancel with $\left[\frac{\Omega_{m,0} + \Omega_{r,0}}{(1+z)^3}\right]$ anymore

so then it becomes sensitive to the parameters $\Omega_{m,0}$ and $\Omega_{r,0}$.

~~Small z, but not like $z=0$, the $(1+z)^2$ dominates.~~

~~It depends on $1-\Omega_{r,0}-\Omega_{m,0}$~~

~~$= 1-(\Omega_{r,0}+\Omega_{m,0})$~~

~~So it is a probe for the sum $\Omega_{r,0}+\Omega_{m,0}$ (or rather how their sum deviates from 1)~~

You can also see this in the Taylor expansion, the first term is 1 so in first order you probe $\frac{c}{H_0}$. Then the 2nd term $z\left(-\frac{\Omega_{m,0}}{2} + \Omega_{r,0} - 1\right)$ dominates for ~~small~~ larger z (but not too large) so we probe $\left(-\frac{\Omega_{m,0}}{2} + \Omega_{r,0} - 1\right)$ *

SUMMARY

* So we are sensitive to $-\frac{\Omega_{m,0}}{2} + \Omega_{r,0}$

$$F_{ij} = \sum_b \frac{1}{\sigma_b^2}\frac{\partial f_b}{\partial p_i}\frac{\partial f_b}{\partial p_j}$$

$p_1, p_2, \dots p_N$ N model paramerers

we have $\Omega_{m,0}, \Omega_{n,0}, H_0$

1 ~~survey~~ observable x so no sum $f = x$

$$F_{ij} = \frac{1}{\sigma^2}\frac{\partial f}{\partial p_i}\frac{\partial f}{\partial p_j}$$

$$F = \frac{1}{\sigma_x^2}\begin{pmatrix} \left(\frac{\partial x}{\partial \Omega_{m,0}}\right)^2 & \frac{\partial x}{\partial \Omega_{m,0}}\frac{\partial x}{\partial \Omega_{n,0}} & \frac{\partial x}{\partial \Omega_{m,0}}\frac{\partial x}{\partial H_0} \\ \frac{\partial x}{\partial \Omega_{n,0}}\frac{\partial x}{\partial \Omega_{m,0}} & \left(\frac{\partial x}{\partial \Omega_{n,0}}\right)^2 & \frac{\partial x}{\partial \Omega_{n,0}}\frac{\partial x}{\partial H_0} \\ \frac{\partial x}{\partial H_0}\frac{\partial x}{\partial \Omega_{m,0}} & \frac{\partial x}{\partial H_0}\frac{\partial x}{\partial \Omega_{n,0}} & \left(\frac{\partial x}{\partial H_0}\right)^2 \end{pmatrix}$$

$$\sigma_x^2 = 0.01^2 x^2$$

$$F = N_i^T C N_i$$

$$x = \frac{c}{H_0}\int_0^z dz\, f(z)$$

with $f(z)$ from before

I call this factor I

$$f(z) = 1 + z\left(-\frac{\Omega_{m,0}}{2} + \Omega_{n,0} - 1\right)$$

factor II

$$+\frac{1}{8}z^2\left(3\Omega_{m,0}^2 - 12\,\Omega_{m,0}\Omega_{n,0} + 4\,\Omega_{m,0} + 12\,\Omega_{n,0}^2 - 20\,\Omega_{n,0} + 8\right)$$

factor III

$$+\frac{1}{16}z^3\Big(-5\Omega_{m,0}^3 + 6\,\Omega_{m,0}^2(5\Omega_{n,0} - 1) + \Omega_{m,0}(-60\,\Omega_{n,0}^2 + 60\,\Omega_{m,0} - 8) + 8(\Omega_{n,0}-1)^2(5\Omega_{n,0} - 1)\Big)$$

$$\chi = \frac{c}{H_0}\left[z + \frac{1}{2}z^2\cdot I + \frac{1}{24}z^3\cdot II + \frac{1}{64}z^4\cdot III\right]$$

$$\frac{\partial\chi}{\partial\Omega_{m,0}} = \overset{\frac{c}{H_0}\cdot}{\frac{\partial\chi}{\partial I}}\frac{\partial I}{\partial\Omega_{m,0}} + \frac{\partial\chi}{\partial II}\frac{\partial II}{\partial\Omega_{m,0}} + \frac{\partial\chi}{\partial III}\frac{\partial III}{\partial\Omega_{m,0}}$$

$$= \frac{1}{\cancel{2}}z^2\cdot -\frac{1}{2} + \frac{1}{8}z^2(6\Omega_{m,0} - 12\Omega_{\Lambda,0} + 4)$$
$$+ \frac{1}{16}z^3\cdot\big(-15\Omega_{m,0}^2 + 12\Omega_{m,0}(5\Omega_{\Lambda,0} - 1)$$
$$- 60\Omega_{\Lambda,0}^2 + 120\Omega_{m,0} - 8\big)$$

$$\frac{\partial\chi}{\partial H_0} = -c\cdot\left[z + \frac{1}{2}z^2\cdot I + \frac{1}{24}z^3\, II + \frac{1}{64}z^4\cdot III\right]H_0^{-2}$$

$$\frac{\partial\chi}{\partial\Omega_{\Lambda 0}} = \frac{1}{2}z^2\cdot 1 + \frac{1}{8}z^2(-12\Omega_{m,0} + 24\Omega_{\Lambda,0} - 20)$$
$$+ \frac{1}{16}z^3\big(30\Omega_{m,0}^2 - 120\Omega_{m,0}\Omega_{\Lambda,0}$$
$$+ 16(\Omega_{\Lambda,0} - 1)(5\Omega_{\Lambda,0} - 1)$$
$$+ 40(\Omega_{\Lambda,0} - 1)^2\big)$$

At $z = 0.01$ and $\Omega_{m,0} = 0.3$ $\Omega_{\Lambda,0} = 0.7$ ~~[illegible]~~

~~[illegible]~~ $H_0 = 70$ km/s/Mpc

~~$\partial\chi/\partial\Omega_{m,0}|_{z=0.01} = -0.01003$ $\frac{\partial\chi}{\partial\Omega_{\Lambda 0}} = 0.0199$~~ $\frac{\partial\chi}{\partial H_0} = c\cdot$

~~$T = \frac{1}{0.01^2}$~~

I'm going to make my life easier and only do the I term

$$\boxed{\frac{\partial\chi}{\partial\Omega_{m,0}} = -\frac{1}{4}z^2\frac{c}{H_0} \quad \frac{\partial\chi}{\partial H_0} = \frac{-c\left(z + \frac{1}{2}z^2 I\right)}{H_0^2} \quad \frac{\partial\chi}{\partial\Omega_{\Lambda,0}} = \frac{c\, z^2}{H_0\, 2}}$$

$$F = \frac{1}{\sigma_x^2}\begin{pmatrix} \overset{0,0}{\left(\frac{\partial x}{\partial \Omega_{m,0}}\right)^2} & \overset{0,1}{\frac{\partial x}{\partial \Omega_{m,0}}\frac{\partial x}{\partial \Omega_{\Lambda,0}}} & \overset{0,2}{\frac{\partial x}{\partial \Omega_{m,0}}\frac{\partial x}{\partial H_0}} \\ \overset{1,0}{\frac{\partial x}{\partial \Omega_{\Lambda,0}}\frac{\partial x}{\partial \Omega_{m,0}}} & \overset{1,1}{\left(\frac{\partial x}{\partial \Omega_{\Lambda,0}}\right)^2} & \overset{1,2}{\frac{\partial x}{\partial \Omega_{\Lambda,0}}\frac{\partial x}{\partial H_0}} \\ \overset{2,0}{\frac{\partial x}{\partial H_0}\frac{\partial x}{\partial \Omega_{m,0}}} & \overset{2,1}{\frac{\partial x}{\partial H_0}\frac{\partial x}{\partial \Omega_{\Lambda,0}}} & \overset{2,2}{\left(\frac{\partial x}{\partial H_0}\right)^2} \end{pmatrix}$$

$$\frac{\partial x}{\partial \Omega_{m,0}} = \frac{-1}{4} z^2 \frac{c}{H_0} \qquad \frac{\partial x}{\partial \Omega_{\Lambda,0}} = \frac{1}{2} z^2 \frac{c}{H_0}$$

$$\frac{\partial x}{\partial H_0} = \frac{-c}{H_0^2}\left(z + \frac{1}{2} z^2\left(-\frac{\Omega_{m,0}}{2} + \Omega_{\Lambda,0} - 1\right)\right)$$

$$\sigma_x^2 = 0.01^2 x^2 = 0.01^2 \left(\frac{c}{H_0}\right)^2 \left(z + \frac{1}{2} z^2\left(-\frac{\Omega_{m,0}}{2} + \Omega_{\Lambda,0} - 1\right)\right)^2$$

```
##
c = 2.998e8 *u.m /u.s
H0 = (70 * u.km/(u.s*u.Mpc)).to(1/u.s)

def d_xi_domegam(z):
        return (-1/4 *z**2 *c/H0)

def d_xi_domegaL(z):
        return (0.5*z**2 * c/H0)

def d_xi_dHo(z):
        return (-c/H0**2 * (z+0.5*z**2*(-0.3/2 + 0.7-1)))

def sigma_x2(z):
        return (0.01**2 * (c/H0)**2 *(z+0.5*z**2*(-0.3/2
+0.7-1))**2)

#F = np.zeros((3,3))


zs = np.array([0.01, 0.1, 0.2, 0.3])
Ftot = np.zeros((3,3))
for i in range(len(zs)):
        F = np.zeros((3,3))
        #print(d_xi_domegam(z))
        #print(d_xi_domegaL(z))
        #print(d_xi_dHo(z))
        #print(sigma_x2(z))
        #print(d_xi_dHo(z)*d_xi_dHo(z)/sigma_x2(z))
        z = zs[i]
        F[0,0] = d_xi_domegam(z)*d_xi_domegam(z)/sigma_x2(z)  #all
units cancel
        F[0,1] = d_xi_domegam(z)*d_xi_domegaL(z)/sigma_x2(z)  #all
units cancel
        F[0,2] = d_xi_domegam(z)*d_xi_dHo(z)/(sigma_x2(z)*u.s)
#seconds left
        F[1,0] = F[0,1] #v
        F[2,0] = F[0,2] #v
        F[1,1] = d_xi_domegaL(z)*d_xi_domegaL(z)/sigma_x2(z) #all
units cancel
        F[2,1] = d_xi_domegaL(z)*d_xi_dHo(z)/(sigma_x2(z)*u.s)  #
        F[1,2] = F[2,1]
        F[2,2] = d_xi_dHo(z)*d_xi_dHo(z)/(sigma_x2(z)*(u.s)**2)


        F = np.array(F)
        #print(errors)
        Ftot += F

print(Ftot)
errors = np.array(np.sqrt(np.diagonal(1/Ftot)))
#errors[2] = (np.array(errors[2])*1/u.s).to(u.km/(u.s*u.Mpc))
print(errors)
print((1.13427275e-20/u.s).to(u.km/(u.s*u.Mpc)))
```

```
#print((2.2685455e-20/u.s).to(u.km/(u.s*u.Mpc)))

sys.exit()
```

```
[[ 9.87035138e+01 -1.97407028e+02  7.09115424e+20]
 [-1.97407028e+02  3.94814055e+02 -1.41823085e+21]
 [ 7.09115424e+20 -1.41823085e+21  7.77257644e+39]]
[1.00654615e-01 5.03273076e-02 1.13427275e-20]
0.3499999995971563 km / (Mpc s)
(base) josephine@JosephinesMBP76 COSM %
```

So the error on Omega_m = 0.1, on Omega_L = 0.05 and on H0 is 0.35