

Foundations of Robotics- Final Project

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Instructions :

Just press run. The code should load.

1. Defining the Scope

Basic Requirements :

- Perform Forward Kinematics
- Draw the robot
- Perform Inverse Kinematics

2. Forward Kinematics

The forward kinematics function calculates the position and orientation of the robot's end-effector given the joint angles. This is achieved using the Denavit-Hartenberg (DH) convention and homogeneous transformation matrices.

Build the DH Table

Inputs:

Theta - rotation (in radians)

d - translation along the z-axis

a- translation along the x-axis

alpha - rotation along the x-axis (in radians)

Output:

T - 4x4 transformation matrix

Define the parameters

```
%scaled down by 10
l1 = 26.7;
l2 = 28.45;
l3 = 5.35;
l4 = 34.25;
l5 = 7.6;
l6 = 9.7;

theta = [pi/2, pi, 0, pi, -pi, 0];
d = [l1, 0, 0, l4, 0, l6];
```

```
a = [0, 28.9489, 7.75, 0, 15, 0];
alpha = [-pi/2, 0, -pi/2, pi/2, -pi/2, 0];
```

Building Transformation Matrix

```
T = eye(4);
T_i = cell(1,6);

for i = 1:6
    T_i{i} = [cos(theta(i)) -sin(theta(i))*cos(alpha(i))
              sin(theta(i))*sin(alpha(i)) a(i)*cos(theta(i));
              sin(theta(i)) cos(theta(i))*cos(alpha(i))
              -cos(theta(i))*sin(alpha(i)) a(i)*sin(theta(i));
              0 sin(alpha(i)) cos(alpha(i)) d(i);
              0 0 0 1];
    T = T * T_i{i};
end

T0_6 = T;
disp('The DH transformation matrix is:');
```

The DH transformation matrix is:

```
disp(T0_6)
```

```
-0.0000 -1.0000 0.0000 -0.0000
-1.0000 0.0000 0.0000 -44.2989
-0.0000 -0.0000 -1.0000 51.2500
0 0 0 1.0000
```

Compute the links

The final transformation matrix T_{0_6} is computed by multiplying these individual matrices:

$$T_{0_6} = T_{0_1} \cdot T_{1_2} \cdot T_{2_3} \cdot T_{3_4} \cdot T_{4_5} \cdot T_{5_6} \quad T_{0_6} = T_{0_1} \cdot T_{1_2} \cdot T_{2_3} \cdot T_{3_4} \cdot T_{4_5} \cdot T_{5_6}$$

This matrix represents the position and orientation of the end-effector relative to the base frame.

```
T0_1 = T_i{1}
```

```
T0_1 = 4x4
0.0000 -0.0000 -1.0000 0
1.0000 0.0000 0.0000 0
0 -1.0000 0.0000 26.7000
0 0 0 1.0000
```

```
T1_2 = T_i{1} * T_i{2}
```

```
T1_2 = 4x4
-0.0000 0.0000 -1.0000 -0.0000
-1.0000 -0.0000 0.0000 -28.9489
-0.0000 1.0000 0.0000 26.7000
```

0	0	0	1.0000
---	---	---	--------

$T2_3 = T1_2 * T_i\{3\}$

$T2_3 = 4 \times 4$

-0.0000	1.0000	-0.0000	-0.0000
-1.0000	-0.0000	-0.0000	-36.6989
-0.0000	0	1.0000	26.7000
0	0	0	1.0000

$T3_4 = T2_3 * T_i\{4\}$

$T3_4 = 4 \times 4$

0.0000	-0.0000	1.0000	-0.0000
1.0000	-0.0000	-0.0000	-36.6989
0.0000	1.0000	0.0000	60.9500
0	0	0	1.0000

$T4_5 = T3_4 * T_i\{5\}$

$T4_5 = 4 \times 4$

-0.0000	-1.0000	0.0000	-0.0000
-1.0000	0.0000	0.0000	-44.2989
-0.0000	-0.0000	-1.0000	60.9500
0	0	0	1.0000

$T5_6 = T4_5 * T_i\{6\}$

$T5_6 = 4 \times 4$

-0.0000	-1.0000	0.0000	-0.0000
-1.0000	0.0000	0.0000	-44.2989
-0.0000	-0.0000	-1.0000	51.2500
0	0	0	1.0000

$T0_6 = T0_1 * T1_2 * T2_3 * T3_4 * T4_5 * T5_6$

$T0_6 = 4 \times 4$

0.0000	0.0000	1.0000	19.6989
-0.0000	-1.0000	0.0000	-43.3511
1.0000	-0.0000	-0.0000	63.2489
0	0	0	1.0000

3. Visualization: Plotting the points

```
figure;
hold on;
grid on;
title('3D Visualization')
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');

% Define colors for links and axes
link_colors = {'k', 'y', 'b', 'g', 'r', 'b'};
axis_colors = {'r', 'g', 'b'}; % Red for X, Green for Y, Blue for Z
axis_length = 5;
```

```

% Plot links
plot3([0, T0_1(1,4)], [0, T0_1(2,4)], [0, T0_1(3,4)], [link_colors{1} '-'],
'LineWidth', 3)
plot3([T0_1(1,4), T1_2(1,4)], [T0_1(2,4), T1_2(2,4)], [T0_1(3,4), T1_2(3,4)],
[link_colors{2} '-o'], 'LineWidth', 3)
plot3([T1_2(1,4), T2_3(1,4)], [T1_2(2,4), T2_3(2,4)], [T1_2(3,4), T2_3(3,4)],
[link_colors{3} '-o'], 'LineWidth', 3)
plot3([T2_3(1,4), T3_4(1,4)], [T2_3(2,4), T3_4(2,4)], [T2_3(3,4), T3_4(3,4)],
[link_colors{4} '-o'], 'LineWidth', 3)
plot3([T3_4(1,4), T4_5(1,4)], [T3_4(2,4), T4_5(2,4)], [T3_4(3,4), T4_5(3,4)],
[link_colors{5} '-o'], 'LineWidth', 3)
plot3([T4_5(1,4), T5_6(1,4)], [T4_5(2,4), T5_6(2,4)], [T4_5(3,4), T5_6(3,4)],
[link_colors{6} '-o'], 'LineWidth', 3)

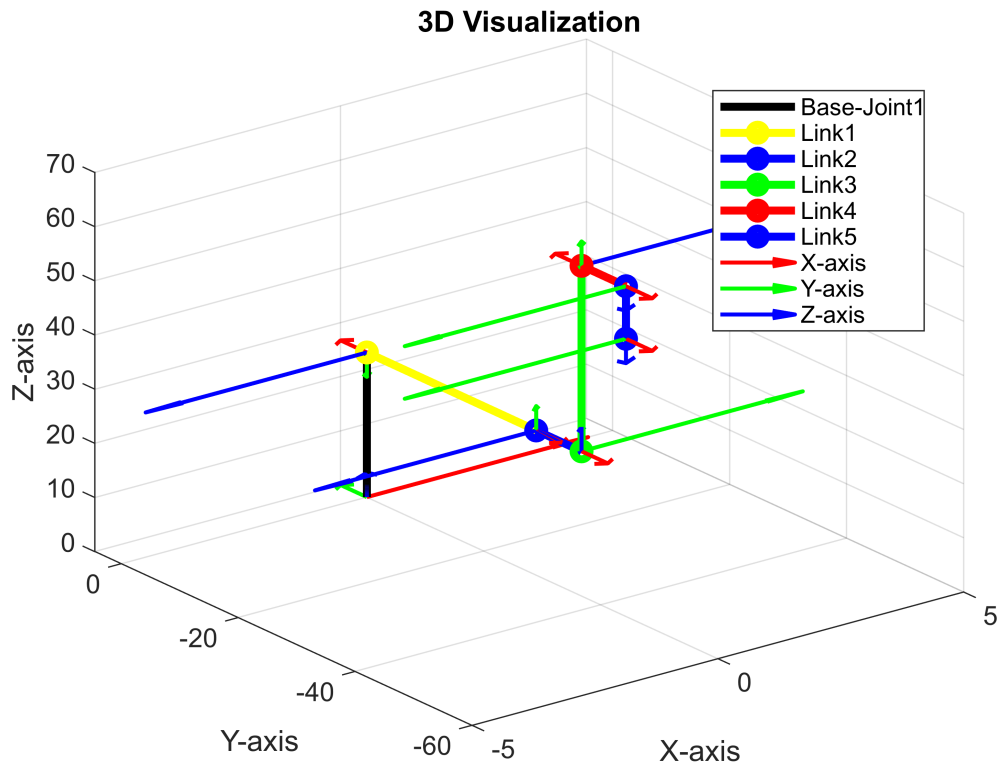
% Plot reference frames
joint_positions = {[0;0;0], T0_1(1:3,4), T1_2(1:3,4), T2_3(1:3,4), T3_4(1:3,4),
T4_5(1:3,4), T5_6(1:3,4)};
transformations = {eye(4), T0_1, T1_2, T2_3, T3_4, T4_5, T5_6};

for i = 1:7
    current_pos = joint_positions{i};
    current_T = transformations{i};
    R = current_T(1:3, 1:3);

    for j = 1:3
        quiver3(current_pos(1), current_pos(2), current_pos(3), ...
            R(1,j)*axis_length, R(2,j)*axis_length, R(3,j)*axis_length, ...
            axis_colors{j}, 'LineWidth', 1.5, 'MaxHeadSize', 0.5);
    end
end

legend('Base-Joint1', 'Link1', 'Link2', 'Link3', 'Link4', 'Link5', 'X-axis', 'Y-
axis', 'Z-axis', 'Location', 'best');
view(3);
hold off;

```



Inverse Kinematics

The inverse kinematics function calculates the joint angles required to achieve a desired end-effector position and orientation. This project uses a geometric approach combined with trigonometry.

These methods allow for the accurate computation of both forward and inverse kinematics, enabling the robot to move to desired positions and orientations in 3D space.

```
% Define offsets
offset_T2 = -1.384;
offset_T3 = -1.384;

% Inverse Kinematics function
function q = inverseKinematics(T0_6, offset_T2, offset_T3)
    % Extract position and orientation
    px = T0_6(1,4); py = T0_6(2,4); pz = T0_6(3,4);
    R = T0_6(1:3,1:3);

    % Calculate joint angles (with offsets)
    q1 = atan2(py, px)
    r = sqrt(px^2 + py^2);
    q2 = atan2(pz - 26.7, r - 28.9489) + offset_T2
    q3 = atan2(60.95 - pz, 44.2989 - r) + offset_T3
```

```

% Wrist angles
q4 = atan2(R(2,1), R(1,1))
q5 = atan2(sqrt(R(3,1)^2 + R(3,2)^2), R(3,3))
q6 = atan2(R(3,2), -R(3,1))

q = [q1; q2; q3; q4; q5; q6];
end

% Calculate inverse kinematics with offsets
joint_angles = inverseKinematics(T0_6, offset_T2, offset_T3);

```

```

q1 =
-1.1443
q2 =
-0.2854
q3 =
-3.9197
q4 =
-0.2268
q5 =
1.5708
q6 =
-3.1416

```

References

1. UFACTORY xArm 6 DoF Robotic Arm (6 DoF) - https://www.robotshop.com/products/xarm-6-dof-robotic-arm?srsId=AfmBOopPyiBaEuVI-cDxjGoZkpzS28sKnXH5GVZxOjf3XAPvWnr_pYeP
2. Manipulator Differential Kinematics, J.Haviland, P.Corke - https://magazines.ieee.org/ra/library/page/december_2024/155/?mkt_tok=NzU2LUdQSC04OTkAAAGXfbzun0i2_TeMzo3ZqHD1XUddjCQor4rOSn68r-Mm9pq_QVdyYu8ft8GrA08DINL2EuLDJOPWGHbJFwgaTNmHtE6xwFkAZYAWkLOB-G5w8r2BejOb_q6tdB
3. Inverse Kinematics of Robots | Robotics 101 - <https://www.youtube.com/watch?v=1-FJhmey7vk>