1. Introduction

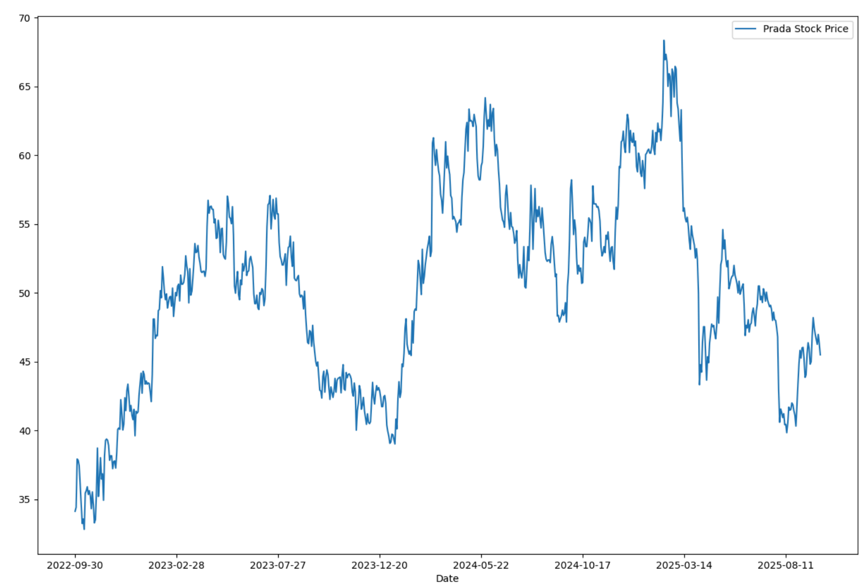
This report presents a comprehensive time-series analysis of Prada's (Ticker: 1913.HK) stock prices over the period from September 2022 to September 2025. The primary objective is to assess the effectiveness of classical econometric models—ARIMA, Holt's Linear Trend, and Exponential Smoothing (ETS)—in forecasting the stock price.

Prada's stock, representative of the luxury fashion sector, exhibits both short-term volatility and longer-term trends influenced by market sentiment, global fashion cycles, and macroeconomic conditions. Accurate forecasting of such equity prices is essential for investment decision-making, portfolio optimization, and risk management.

In addition, this report explores a Markov regime-switching model to identify distinct market regimes and assess their impact on returns and volatility.

2. Data and Preprocessing

Data were obtained from a curated global fashion and luxury equities dataset. The dataset contains daily closing prices for Prada (1913.HK) along with trading volumes.



The Prada stock price, after a powerful multi-year rally that peaked in early 2025 (reaching approximately HK$71.70), has undergone a sharp and significant correction, falling to the mid-40s by the second half of the year. This steep decline reflects the broader slowdown and anxiety in the global luxury sector, particularly in China, combined with recent analyst downgrades due to mixed brand performance (with Miu Miu still surging but the core Prada brand facing headwinds). While the company remains fundamentally strong and benefits from the long-term potential of its recent Versace acquisition, the stock is currently in a phase of consolidation and re-evaluation, trading well off its peak as the market assesses its ability to sustain growth amidst a challenging economic environment.

3. Exploratory Analysis

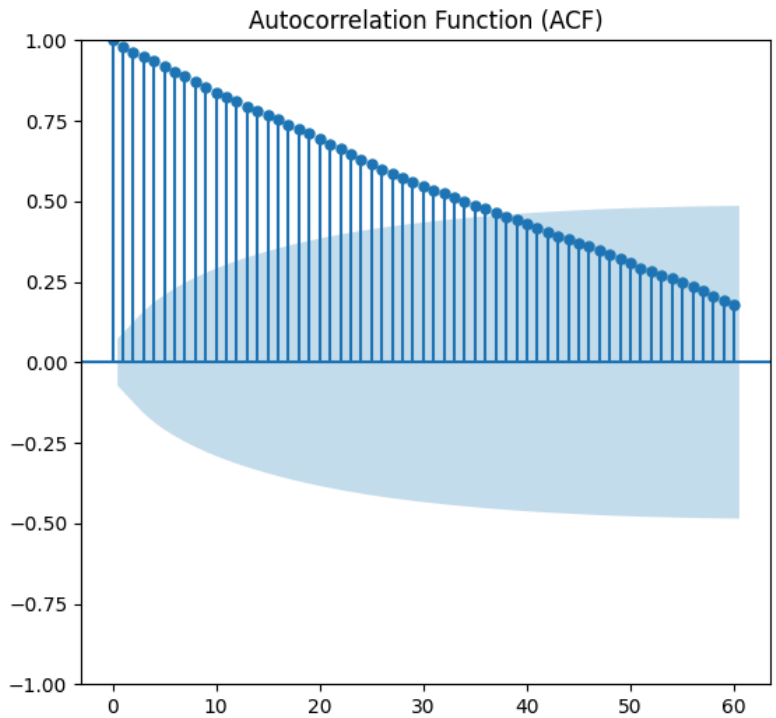
3.1 Trend and Seasonality

A STL decomposition was applied to the log-transformed prices:

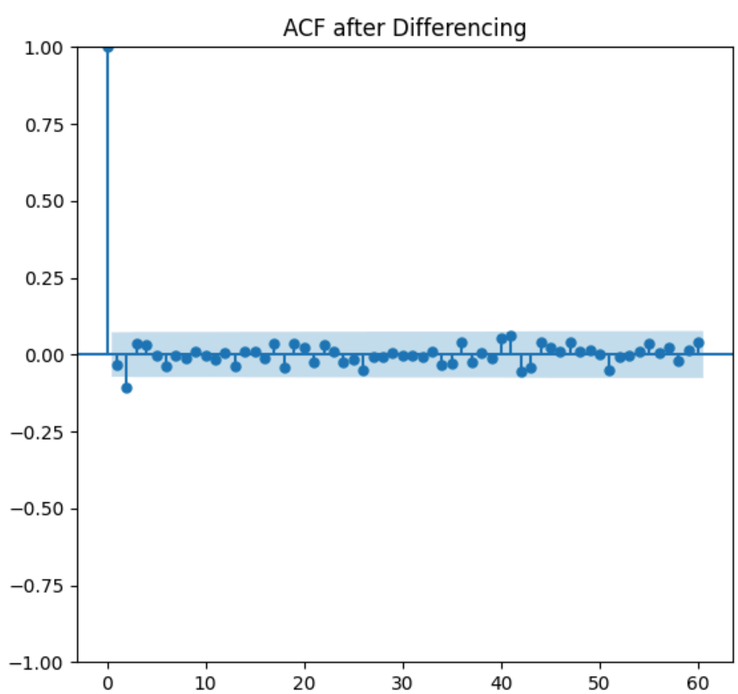
- Seasonality: No strong or consistent seasonal pattern was identified in Prada’s stock prices, consistent with the luxury retail market’s behavior.
- Residuals: Displayed occasional spikes, suggesting volatility clustering.

3.2 Autocorrelation Analysis

The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) were computed for the original and first-differenced series:

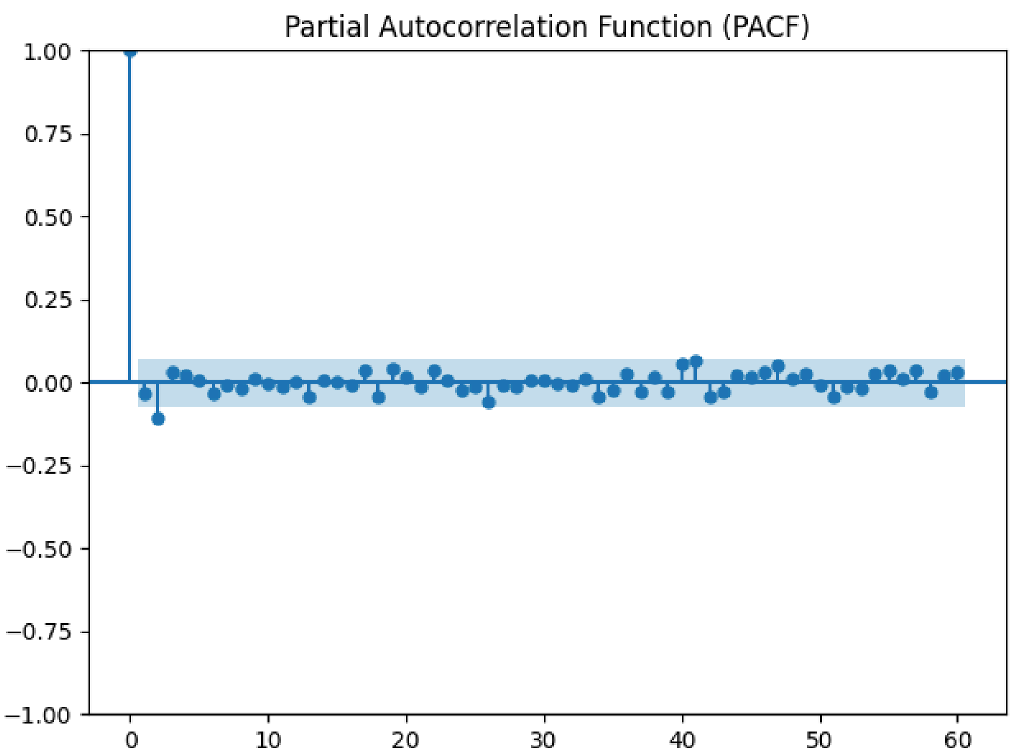


- The raw series displayed a slow decay in the ACF, confirming non-stationarity.
- After differencing, the series appeared stationary, further confirmed by the Augmented Dickey-Fuller (ADF) test:



Test Statistic	p-value
-21.73	0.0

- PACF plots suggested that there is no significant autocorrelation after lag 0, but to make sure I will optimize AIC in choosing the parameters.



4. Time-Series Modeling

4.1 ARIMA

An ARIMA model was fitted on the training set (80% of observations). After parameter grid search for (p,q) with chosen d = 1, the best model selected by AIC was ARIMA(2,1,2).

Forecast performance on the test set:

Metric	Value
MSE	229.52
MAE	14.24
MAPE (%)	30.48

Observations: ARIMA captures the general trend but tends to smooth over short-term fluctuations, which is expected given its reliance on historical linear patterns.

4.2 Holt's Linear Trend

Holt's Linear Exponential Smoothing explicitly models trends without seasonality.

- Optimal smoothing parameters were found to be alpha = 0.5 and beta = 0.2 based on MAPE minimization.

Metric	Value
MSE	37.69
MAE	4.68
MAPE (%)	9.79

Observations: Holt's Linear better captures upward momentum compared to ARIMA, producing forecasts that track the trend more closely while reacting to short-term changes.

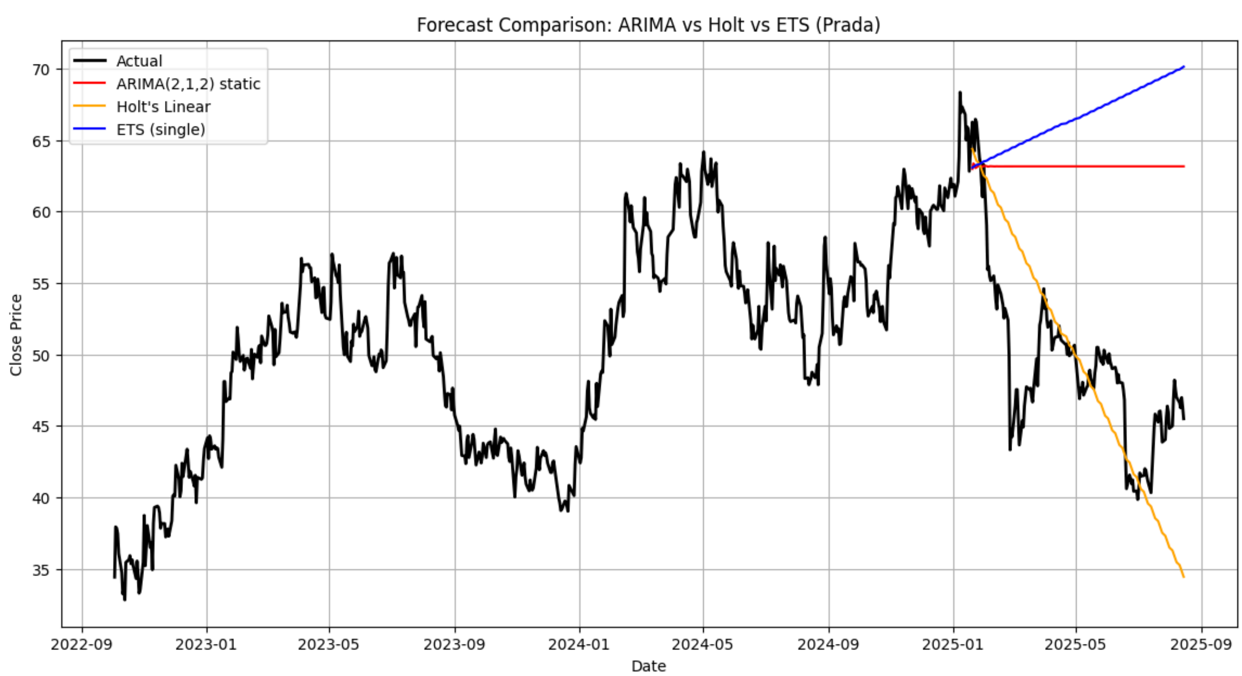
4.3 Exponential Smoothing (ETS)

A single-fit ETS model with additive trend was fitted:

Metric	Value
MSE	358.89
MAE	17.68
MAPE (%)	37.89

Observations: The ETS model underperformed relative to Holt's Linear and ARIMA, likely due to overfitting the trend and limited adaptation to recent price changes.

5. Forecast Comparison



Interpretation: Holt's Linear forecasts track the actual prices most closely. ARIMA produces slightly smoother forecasts, while ETS is overly reactive to early trends.

6. Regime-Switching Analysis

To capture market regimes, a Markov Switching Model with three regimes was fitted on the log returns. As a result, I figured out three distinct regimes as below:

- Regime 0 (Bear): Negative mean returns, moderate volatility.
- Regime 1 (Bull): Positive mean returns, low-to-moderate volatility.
- Regime 2 (Stable/Low-volatility): Returns near zero, low volatility.

Smoothed probabilities for each regime are plotted over time, and a prediction was made for the next 10 days. However, due to the Markov chain nature, long-term forecasts converge to the stationary distribution. Therefore, I think this model can be suitable to predict short-term trend (3 days ahead).

	Regime_0	Regime_1	Regime_2
2025-09-30	0.329995	0.336602	0.333404
2025-10-01	0.331994	0.334556	0.333450
2025-10-02	0.331986	0.334564	0.333450
2025-10-03	0.331986	0.334564	0.333450
2025-10-06	0.331986	0.334564	0.333450
2025-10-07	0.331986	0.334564	0.333450
2025-10-08	0.331986	0.334564	0.333450
2025-10-09	0.331986	0.334564	0.333450
2025-10-10	0.331986	0.334564	0.333450
2025-10-13	0.331986	0.334564	0.333450

Implications: ARIMA, Holt, and ETS models cannot inherently capture regime shifts. The Markov Switching Model offers a more adaptive framework, particularly valuable in volatile periods.

7. Discussion

- **ARIMA Limitations:** Smooths out volatility and is less sensitive to abrupt price movements. Suitable for linear trends but inadequate for regime changes.
- **Holt's Linear Strengths:** Incorporates trend explicitly, yielding better short-term forecasts for upward momentum.

- **ETS Weakness:** Overfitting trend and less responsive to recent changes.
- **Regime-Switching Insights:** Identifying Bear, Bull, and Stable regimes helps understand underlying market dynamics. Combining this approach with classical forecasting can enhance predictive accuracy.

Future Work:

- Explore hybrid approaches (e.g., GARCH + LSTM) to better capture volatility clustering and non-linear dynamics.
- Extend forecasting to include trading volumes or macroeconomic variables for multivariate analysis.