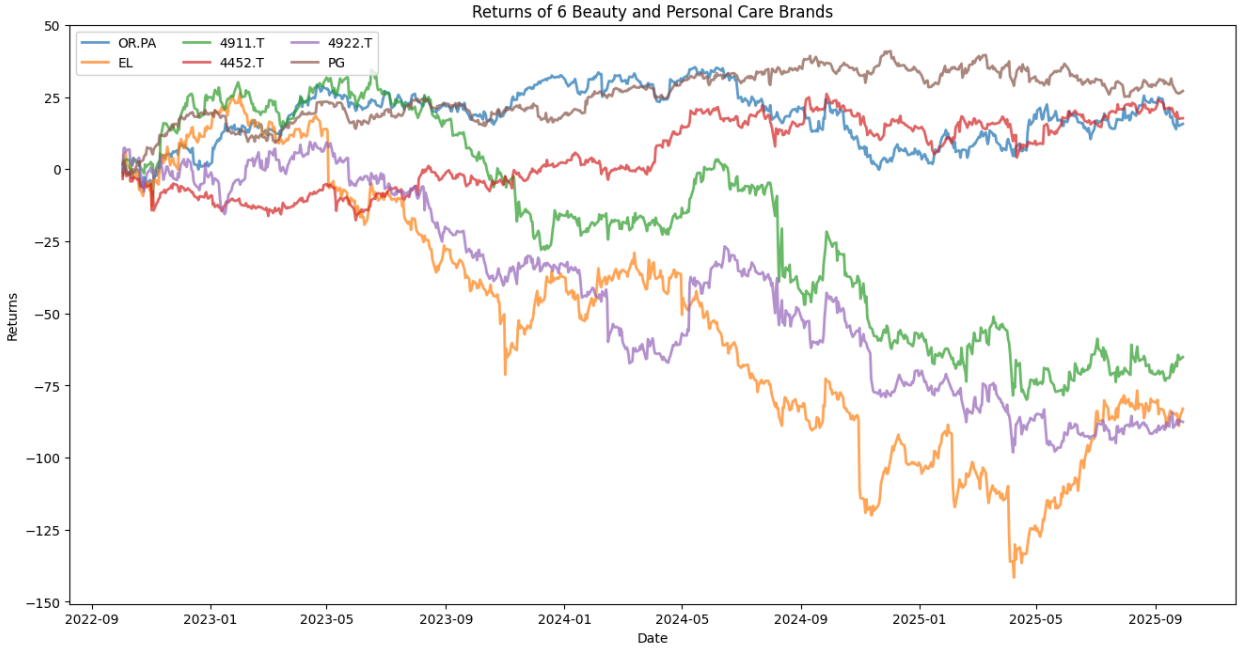
1. Personal Dataset

- I created a dataset of 100 assets in the Fashion, Beauty, and Personal Care industry all around the world. Although most of them might be some less volatile stocks (FCMG), I also included some high-growth e-commerce tickers to balance my portfolio.
- Data points: Closing price and trading volume in the past 3 years (765 trading days) so that I can do time-series analysis in the future.

2. Chapter 2: Descriptive Statistics

I chose 6 brands: Shiseido, P&G, L'Oreal Paris, Kao, KOSE, Estee Lauder as the main focus for the first week. For the missing values, I filled it with the rolling mean of the most recent 5 days.

a. Performance Overview



Visual inspection of cumulative returns shows **stable and positive returns for P&G, L'Oreal, and Kao Corporation**, while **Estee Lauder, KOSE, and Shiseido** exhibited **negative returns** over the same period. This indicates that some companies in the sample were more resilient to market fluctuations, while others experienced higher variability.

b. Descriptive Statistics

Ticker	count	mean	std	min	25%	50%	75%	max	kurtosis	skewness
EL	779.0	-0.106604	2.898912	-23.439415	-1.419619	-0.123478	1.258619	11.498241	14.739617	-1.764338
4911.T	779.0	-0.083593	2.564756	-16.866792	-1.176737	-0.077068	0.972969	18.455788	12.226447	-0.512162
4922.T	779.0	-0.112446	2.355365	-14.574546	-1.188523	-0.047074	1.088275	16.488225	8.071256	-0.302819
4452.T	779.0	0.022746	1.372461	-9.114203	-0.638806	0.014987	0.750605	6.619935	7.518617	-0.421942
OR.PA	779.0	0.020196	1.441661	-7.882967	-0.829877	0.000000	0.868311	6.730760	2.938136	0.057926
PG	779.0	0.034849	1.036256	-5.141845	-0.534347	0.065032	0.641921	4.055678	2.806064	-0.366333

- High kurtosis (fat tails / extreme events): EL, 4911.T (Kao), 4922.T (Kose), 4452.T (Shiseido) These stocks are more prone to rare, extreme returns, meaning while most days may show moderate fluctuations, there's higher likelihood of sudden large gains or losses. Investors with tail-risk sensitivity should be cautious.
- Low kurtosis (thin tails / more stable): OR.PA (L'Oreal), PG Returns are more consistent with fewer extreme outliers. This makes them relatively safer for conservative or loss-averse investors.
- Most stocks show negative skew, especially EL (-1.76), indicating occasional large losses, even if average returns are small. OR.PA (L'Oreal) has a slightly positive skew, suggesting a small chance of larger-than-expected gains.

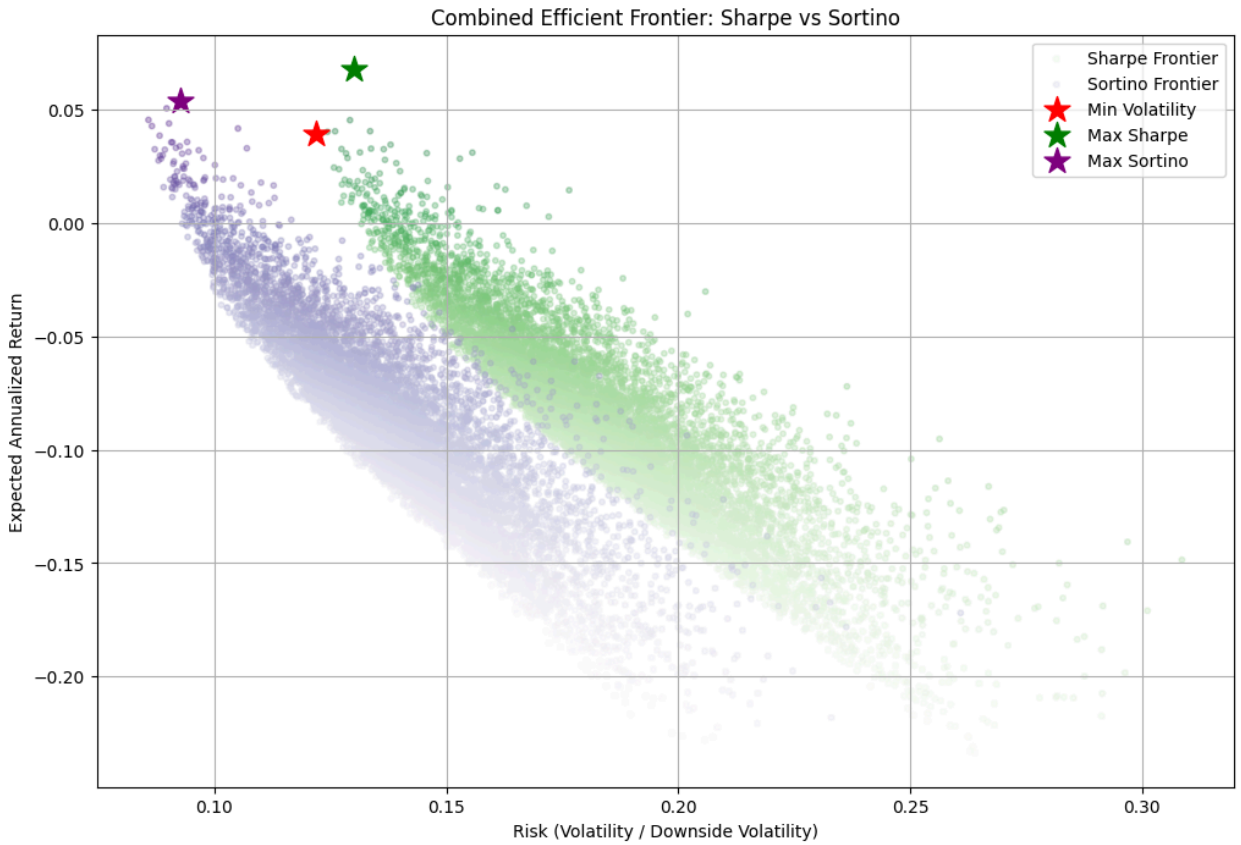
c. Risk-Adjusted Performance

Using Sharpe and Sortino ratios, I evaluated risk-adjusted returns. Annualized returns were scaled for the trading period, while volatility was considered both generally (Sharpe) and downside-only (Sortino).

- Sharpe Ratio: P&G exhibit the highest ratios, indicating strong return per unit of total risk.
- Sortino Ratio: Again, P&G leads, reflecting resilience against downside risk specifically.

d. Portfolio Selection

I made use of efficient frontier theory to find the optimal portfolio weight based on 3 different criteria:



- **Minimum Volatility Portfolio:**
  + Expected Annualized Return: 3.93%
  + This portfolio heavily favors P&G and L'Oreal, reflecting their lower volatility and more stable returns. Stocks with higher tail risk, such as EL and Shiseido, are given minimal weight. This portfolio is suitable for conservative, risk-averse investors seeking stability.
- **Maximum Sharpe Ratio Portfolio:**
  + Expected Annualized Return: 6.78%
  + Insight: This portfolio emphasizes P&G (72%) to maximize risk-adjusted returns, while underweighting high-kurtosis stocks. It represents a balanced but performance-oriented strategy, capturing upside potential while controlling overall volatility.
- **Maximum Sortino Ratio Portfolio:**
  + Expected Annualized Return: 5.37%

+ Insight: Focused on minimizing downside risk, this portfolio gives high weight to P&G and L'Oreal while avoiding stocks prone to extreme negative returns. It is ideal for investors concerned about losses more than total volatility, aligning with Post-Modern Portfolio Theory (PMPT) principles.

3. Chapter 3+ 4: Classical Linear Regression Analysis

a. Simple OLS Regression: P&G vs L'Oreal

I first estimated a simple linear regression of P&G returns on L'Oreal returns.

- R-squared = 0.031: Only 3.1% of P&G's return variation is explained by L'Oreal.
- Coefficient = 0.1256 ($p < 0.001$): P&G's returns move positively with L'Oreal, but sensitivity is low.

b. Multiple Regression: P&G vs Other Brands

- R-squared = 0.079: Explained variance increased modestly.
- Significant predictors: EL (β=0.0801) and OR.PA (β=0.0783) positively contribute to P&G returns.
- VIF Analysis: No multicollinearity issues observed (all VIF < 2).

The F-test confirms the overall significance of the regression model ($F = 13.31$, $p < 0.001$).

c. CAPM Analysis

Applying the CAPM framework, I regressed excess P&G returns on market excess returns (S&P 500):

- Beta = 0.2572 ($p \approx 0$): P&G is significantly less volatile than the market, indicating defensive characteristics.
- R-squared = 0.064: Only 6.4% of P&G's excess returns are explained by market movements, implying other idiosyncratic factors.

This aligns with descriptive insights where P&G displayed relatively stable cumulative returns.

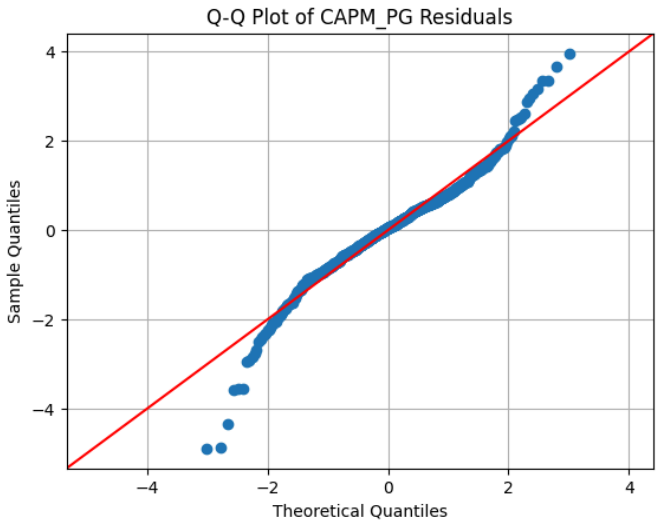
4. Chapter 5: Statistical Robustness Check

To assess the statistical robustness of CAPM, I conducted these following tests.

a. Heteroscedasticity
   - Breusch-Pagan Test: The Lagrange multiplier statistic was 4.238 with a p-value = 0.0395, indicating rejection of the null hypothesis of homoscedasticity at the 5% significance level. This suggests mild heteroscedasticity in P&G's CAPM residuals.
   - White Test: The test statistic was 19.79 with a p-value < 0.001, confirming significant heteroscedasticity. This test is robust to functional form and detects more complex variance patterns.

- Interpretation: Some variance heterogeneity exists, likely reflecting periods of market stress or large stock-specific movements. While OLS estimates remain unbiased, standard errors may be inefficient, so robust standard errors could improve inference.

b. Normality of Residuals
   - Jarque-Bera Test: The statistic strongly rejects normality ($p \approx 1.24 \times 10^{-64}$), consistent with the presence of fat tails in P&G returns.



    - Q-Q Plot Analysis: Visual inspection confirms slight deviations from a straight 45° line in both tails, indicating the residual distribution exhibits skewness and kurtosis beyond the normal distribution.

- **Interpretation:** Non-normal residuals are common in financial return data due to occasional large gains or losses. This violates classical assumptions but does not bias OLS estimates; however, it may affect the accuracy of confidence intervals and hypothesis tests.

c. Influential Observations

    - Cook's Distance: A stem plot revealed a number of 42 influential points exceeding the threshold (4/n).

- Implication: These points have high leverage or large residuals, which can disproportionately affect regression coefficients. Careful monitoring or robust regression methods may be considered to reduce their impact.

d. Autocorrelation

- Durbin-Watson Test: Statistic ≈ 1.966, close to 2, suggesting no significant autocorrelation at lag 1.
- Breusch-Godfrey Test: LM statistic = 2.466, p-value = 0.782 for up to 5 lags, confirming no significant autocorrelation over a one-week horizon.

- **Interpretation:** Residuals appear independent over time, indicating that CAPM captures most systematic dependencies, and that serial correlation does not bias standard errors significantly.

e. Structural Breaks

Financial markets are prone to regime changes, so I applied multiple structural break tests:

- CUSUM Test: No significant break detected (p = 0.41).
- Chow Test at midpoint: Evidence of a structural break (F = 9.70 > F_crit = 3.01), suggesting a shift in P&G's risk-return relationship mid-period.

  While the Chow Test offers a straightforward method for detecting breaks at known points in time, the challenge lies in the uncertainty around precise break

dates— I am not sure about the start date of the COVID-19 impact—and the likelihood of multiple influential events affecting P&G's stock returns. Consequently, relying solely on the Chow Test would necessitate conducting multiple tests at various candidate breakpoints, which may complicate the analysis and increase the risk of multiple testing issues. Similarly, the CUSUM and CUSUMSQ tests do not identify the exact timing or number of breaks, so I decided to continue with the Bai-Perron test.

- Bai-Perron Test: Identified five breakpoints, notably August 19, 2025, coinciding with P&G's 25% price increase in US products to offset tariff costs. This highlights the impact of firm-specific events on CAPM stability.