Competitive Diplomacy in Bargaining and War

Joseph J. Ruggiero

May 29, 2024

Abstract

War is often viewed as a bargaining problem. However, prior to bargaining, countries can vie for leverage by expending effort on diplomacy. This article presents a dynamic model of conflict where agenda-setting power is endogenous to pre-bargaining diplomatic competition. The ability to compete for leverage generates a new channel through which the nature of potential war affects the quality of peace. First, costs of war grow the bargaining surplus, fueling the battle for leverage and reducing welfare even if war never occurs on the path of play. Second, competitive diplomacy erodes the gains from peace, making it possible that war is relatively efficient. Transaction costs are a double-edged sword: they surprisingly protect against this erosion but also create a risk of inadvertent war. Moreover, I find that reliable peace deals avert "efficient" wars but introduce a trade-off between welfare and peace.

Word Count: 9,871

Ruggiero: Princeton University, Department of Politics, 300 Fisher Hall, Princeton, New Jersey 08544 (email: jruggiero@princeton.edu).

> *"Diplomacy is indispensable to identify and implement solutions to conflicts in unstable regions of the world short of military involvement. It helps to galvanize allies for action and marshal the collective resources of like-minded nations and organizations to address shared problems* [...] *We must upgrade our diplomatic capabilities to compete in the current environment and to embrace a competitive mindset."*
>
> — US National Security Strategy (2017) on "Competitive Diplomacy"

States often engage in diplomacy to advance their national interests. Secretary of State George Schultz likened diplomacy to gardening, where investments in international relationships keep the weeds out to maintain fertile ground.[1] To secure a strong position in negotiations, states must put in diplomatic work before crises occur. Considering the extraordinary amount of time and resources devoted to diplomacy (Malis and Smith, 2021), states perceive these attempts to be worthwhile. Then, this article begins with an observation: if diametrically opposed sides of a dispute can use diplomacy to promote their interests, inevitably at each other's expense, it follows that diplomacy becomes fundamentally competitive in crisis bargaining.

A state's willingness to expend resources in diplomacy to improve their bargaining position will depend on their expectations about the additional value of the improved position, which in turn will depend on specifics of the conflict. In this way, competitive diplomacy operates as an important channel through which characteristics of a potential war can affect the quality of peace. Moreover, if competition creates inefficiencies that negatively affect peacetime conditions, it may also alter a state's propensity to fight. Certain diplomatic settings may be effective at facilitating cooperation and avoiding war, but nevertheless fail to yield satisfying conditions in peace. Likewise, environments that bring good conditions in peace may be lacking in their ability to sustain cooperation in the first place. How does the incentive to compete in diplomacy affect the prospects of cooperation? Which settings fare well at averting conflict while simultaneously preserving the largest gains from peace?

To answer these questions, I develop a game-theoretic model of war that captures the core trade-off between internalizing the gains from cooperation in bargaining and exerting costly effort to improve bargaining position. In the model, war and cooperation are two distinct technologies

[1] "Tending the Garden" by Shawn Dorman. *The Foreign Service Journal.* November, 2020. Retrieved in 2024 from https://afsa.org/tending-garden.

countries can use to divide a pie. Building on previous work in crisis bargaining, war is modeled as a lottery over the pie with costs of war that reflect pure deadweight loss, while cooperation requires mutual agreement to a peace deal (Morrow, 1989; Fey, Meirowitz, and Ramsay, 2013; Wolford, 2014; Qiu, 2022; Davis, 2023; Kenkel and Schram, 2023). Successful peace offerings need to be extended by an agenda setter and accepted by a recipient in bargaining. I adopt the standard ultimatum protocol where acceptance leads to peace and rejection leads to war, as this setting represents crisis situations with little loss of generality (Fey and Kenkel, 2021).

Whereas existing models treat proposers as either predetermined or randomly recognized by exogenous probability, my approach treats agenda-setting power as endogenous to a country's performance in pre-bargaining competition. Specifically, countries choose an amount of effort to exert in competitive diplomacy, and greater effort is rewarded with a greater likelihood of setting the terms of a peace offering. The strength of the mapping between performance in the competition and the likelihood of recovering agenda-setting power is allowed to vary, reflecting the scope of competitive diplomacy or analogously its "decisiveness" in determining the agenda setter. Because agenda setters can only recover as much as their adversary is willing to concede, diplomatic competition is distinct from military competition in that it is not over the entire pie, but specifically over the surplus from peace.

A key ingredient of the model is that periodic outcomes may persist into the future. In contrast to the common approach where war is a game-ending event in which the winner receives the disputed pie for the rest of time, this model features peace deals and war outcomes that may but do not need to last forever. As time proceeds, the relative strength of countries can flexibly evolve and, once an outcome ceases to endure, countries again need to resolve their dispute through either cooperation or war. These dynamics help understand the role of competitive diplomacy as a country's incentive to cooperate, and hence their willingness to compete, depends not only on their expectations about future power shifts but also on expectations about the relative persistence of peace deals. Naturally, the significance of winning a war is greater when war outcomes are durable than when they are transient, and the value of agenda-setting power is greater when peace deals are reliable than when they are not.

The analysis reveals that costly wars endogenously create costly peace. Even if wars never occur on the path of play, greater costs of war imply a larger surplus from cooperation. As the surplus expands, agenda-setting power becomes more valuable, resulting in a stronger incentive to engage in competitive diplomacy before bargaining. The extent to which costlier wars degrade the quality of peace is moderated by the decisiveness of the competition. The gains of cooperation are more robust when the competition is less decisive (i.e., when it is less certain the winner of the competition will recover agenda-setting power), as the total incentive to compete is weaker and less responsive to changes in the surplus. With more decisive competition, countries expend more effort vying for leverage and the gains of peace can be completely eroded, leaving countries no better off in peace than they would be if they had fought a war.

Moreover, the model suggests two new explanations for war. First, the model identifies conditions for which war is relatively efficient compared to cooperation. Countries are driven to fight efficient wars by two forces: the (exogenous) differential persistence of outcomes and the (endogenous) equilibrium effort of countries in competitive diplomacy. The logic that is analogous to that of "ripping off a bandage." For example, if war outcomes persist while peace deals require constant renegotiation, states may prefer incurring the larger costs of war today to avoid the frequent need of having to deal with their adversary throughout the future. Likewise, countries will have an incentive to fight in the current period if they expect that competitive diplomacy will introduce significantly more inefficiencies after cooperation than it would after war. The precise condition that determines whether war is relatively efficient depends on how outcome persistence and competitive diplomacy interact.

The model also accounts for a core idea in international cooperation that reaching successful deals is not always free and easy. The leading rationale for building and preserving international institutions is their ability to improve the efficiency of the bargaining process by reducing the costs inherent in contracting (from literal expenditures to less tangible frictions), often referred to as the "transaction costs" of cooperation (Keohane, 1984; Downs and Rocke, 1995; Dixit, 1998; Martin, 1999; Lake, 1999). Consequently, the model allows for the possibility of small frictions that obstruct the settlement procedure if countries choose to opt out of diplomacy. If countries fail to overcome these frictions, they risk creating a momentum that drags them into war.

This leads to the second explanation: war as a result of miscoordination. This mechanism exists as long as a crisis remaining unresolved induces a nonzero probability of escalation to war, which is ensured by definition of transaction costs that are not trivially ignorable (otherwise there is a loophole where we can freely cooperate forever by never cooperating). Incorporating the possibility of inadvertent war builds on the notion that unresolved crises can spiral out of control—per Legro (1994), "states may not seek a spiral of hostility but still can stumble into escalation." A prominent example is Powell (2015), which features a defender that chooses a risk level for which a crisis escalates to war if neither state backs down. In the same vein, the risk parameter captures unmodeled dynamics of crises spiraling out of control, allowing us to focus on mechanisms of interest (Paine and Tyson, 2020). Specifically, this allows us to explore the effect of barriers to cooperation on the incentives for competition and conflict, bridging contractual theories of international cooperation and bargaining theories of war.

With transaction costs, states occasionally risk war in attempt to free ride on the efforts of their adversary in obtaining peace. The model characterizes the willingness to risk war, which depends not only on the size of transaction costs but also on expectations about future shifts in power, the reliability of peace deals, war outcome durability, and the scope of competitive diplomacy. Surprisingly, as these frictions create an incentive to free ride, they discourage diplomatic effort and simultaneously protect against the erosion of the bargaining surplus caused by competition. Likewise, the model suggests that the Pareto optimal level of competition counterintuitively maximizes the probability of war when peace deals are reliable. This occurs because, while heightened competition discourages free riding and hence reduces the probability of war, it can simultaneously erode the gains from peace. As a result, reliable peace deals that avert efficient wars create a trade-off between welfare and peace.

1 Contributions to the Literature

This article contributes to the literature on international conflict by incorporating competitive diplomacy in the context of crisis bargaining and deriving analytical results. Existing models of war that include "diplomatic" actions typically reduce them to communicating private information via cheap talk or costly signals. For example, Smith (1998) shows that communicating intentions can be effective in the presence of audience costs. Sartori (2002) provides one of the first conflict models that feature diplomacy as cheap talk, arguing that "diplomacy is the epitome of cheap talk." Related work has built on this foundation: Ramsay (2011) shows how cheap talk diplomacy can change the ex-ante probability of war, Kurizaki (2007) finds that private threats can be as credible as public threats, and Trager (2010, 2011) studies how cheap talk diplomacy can signal a state's resolve, Fey and Ramsay (2010) demonstrate that diplomatic communication can achieve any outcome that can be achieved by third-party mediation, and Wolford (2020) explores a signaling game of diplomatic support.

Diplomacy often entails information transmission as a procedural matter, but it does not follow that information transmission fully captures diplomatic practices. Previous work offers valuable insight into the role of diplomatic communication in crises; however, limiting the study of diplomacy to communication ignores the role of diplomacy by other means and may also neglect aspects of diplomatic communication in its wider context. The US National Security Strategy (2017) emphasizes that "competitive diplomacy" can "galvanize allies" and "marshal the collective resources of like-minded nations" to "identify and implement solutions [...] short of military involvement." These actions are not necessarily about the transmission of private information, but reflect any means to improve bargaining position and attain preferable outcomes through cooperation. Exploring competitive diplomacy of this kind in the crisis bargaining framework supplements recent work that studies how bargaining power and the corresponding distributional consequences affect incentives around war: Davis (2023) provides a game that incorporates a second level of domestic bargaining, Kennard, Krainin, and Ramsay (2018) explore how side payments can introduce inefficiencies and derail the bargaining process, and Leventoğlu (2023) models crisis bargaining as a principal-agent game where one country sets the rules.

Additionally, the article contributes to work studying the long-run dynamics of war onset and cooperation. Fearon (1998) is noteworthy in that it also bridges war-as-bargaining with the dynamics of cooperation theory, focusing on the enforcement of contracts rather than pre-bargaining competition over leverage. Other work on dynamic bargaining and war has centered on power shifts and the resulting commitment problems that create preventive motives (Kim and Morrow, 1992; Fearon, 1995; Powell, 1996; Debs and Monteiro, 2014; Schram, 2021). The model accommodates these mechanisms while also incorporating a novel feature that allows different outcomes in the stage game to persist into the future at varying rates. A country's incentive to fight then reflects the expected stability along different paths as well as their current interests.

In addition to preventive wars that are common to dynamic models of conflict, the model features two other types of wars. The first—"efficient" wars—are related to the finding that high costs of arming during peacetime can make war more appealing for patient states. In this vein, Powell (1993) and Jackson and Morelli (2009) model the decisions to arm and initiate wars, while Fearon (2018) builds on these by incorporating a dispute in which arming also improves a country's resolution. In addition to competitive diplomacy being conceptually different from war-related expenditures like arming, the mechanism in this paper is technically different in a number of ways. First, arming increases a country's strength and hence improves their expected payoff in both war and peace. Diplomatic effort, however, never improves a country's war payoff and can but does not necessarily increase their expected peace payoffs. Second, war-related expenses are generally strategic complements—the more a country arms, the more their opponent wants to. On the other hand, diplomatic effort can be a strategic complement or substitute depending on the level of their adversary's effort. Third, the incentive to invest in war technology stems from the desire to recover the pie in dispute, whereas the incentive to exert diplomatic effort is the result of a desire to recover the bargaining surplus, a fundamentally different object. Thus, the strategic considerations in this model are quite different from those in previous work.

The second type—inadvertent war—is less common in crisis bargaining, but is typical in the literature on nuclear deterrence and consistent with many historical explanations for conventional war. Actions short of declaring war can nevertheless create conditions that, despite a mutual desire for peace, drag countries into battle. The July Crisis is a fitting example. In the aftermath of Franz Ferdinand's assassination, failures of diplomacy led to military preparations for war that "once started, [could] not be stopped" (Schelling, 1966, p. 222). German Chancellor Bethmann Hollweg described it as a "stone [that] had started rolling" (Albertini, 1952), as "the pull of military schedules dragged them forward" (Tuchman, 1994, p. 72) until Schlieffen's "dead hand automatically pulled the trigger" (Taylor, 1964, p. 15, in reference to the Schlieffen Plan). This is not to say states had no option: Trachtenberg (1990) emphasizes that this was brought on by deliberate actions by states to take risks.

This mechanism is also related to coercive diplomacy and deterrence. While those studies focus on threats that avert war, this article studies competition that instigates them. The idea of unwanted escalation that inadvertently starts a full-scale war is familiar. Schelling (1960, 1966) coined the phrase "threats that leave something to chance," which is commonly applied to the work in nuclear politics (Nalebuff, 1986; Powell, 1987; Posen, 2014). Moreover, audience costs allow states to effectively commit to fighting by creating circumstances that are costly to back down from (Smith, 1998; Slantchev, 2006; Tarar and Leventoğlu, 2009). A classic example is Fearon (1994), where refusal to concede is conceptualized as costly escalation by, for example, preparing troops. While this work tends to focus on signaling resolve, it features the same idea that unresolved crises can cause states to become "locked" into fighting.

2 Model

This section presents the model, with additional details available in Appendix A. Two countries $i = 1, 2$ are in a dispute over the division of the unit interval, or "pie." They have opposing preferences over an outcome $x \in \mathbb{R}$, with $u_1(x) = x$ and $u_2(x) = 1 - x$ denoting the flow payoff for country 1 and 2. Time is discrete and discounted by $\delta \in (0, 1)$ over an infinite horizon.

The game proceeds as follows. Each country faces a choice between cooperating with their opponent or launching an attack, $a_{it} \in \{0, 1\}$. If either country attacks, war occurs as a costly lottery.[2] Specifically, a war in period t is won by country 1 with probability given by the state variable $s_t \in S$, where S is a finite subset of $(0, 1)$.[3] The costs of war are allowed to vary by country and the state of the world, and are assumed to be strictly positive, $c_i(s_t) > 0$. In addition to these costs of war, a country's flow payoff after war reflects the winner controlling the entire pie of value 1 and the loser having nothing.

On the other hand, if countries cooperate, they exert effort $e_{it} \geq 0$ in competitive diplomacy. There may or may not be frictions or "transaction costs," $\mu \geq 0$, which serve to potentially obstruct cooperation. If aggregate effort exceeds frictions, the country that exerted the most effort is recognized as the agenda setter with probability $\pi > \frac{1}{2}$.[4] In this way, larger π reflects a competition with greater decisiveness. If aggregate effort does not exceed frictions, the momentum

[2] War as a costly lottery is the preferred approach for comparability with existing crisis bargaining models. Refer to Appendix C.1 for a demonstration that the results are robust to war with endogenous military effort. Also, refer to Appendix C.4 for a discussion of infinite costs of war.

[3] A finite domain is assumed to avoid potential issues with existence. Refer to Duggan (2016, ch. 4).

[4] An all-pay contest is used in the main text for analytical clarity as it admits closed-form expressions, but the results are robust to reformulating competitive diplomacy as any Tullock contest or standard auction (see Appendix C.2 and discussion around Proposition 4). Also, Appendix C.3 discusses a more flexible setting where recognition probability can vary by country and the state of the world to allow for differential advantages.

Table 1. Notation

Model Parameters		Actions	
Discount factor	$\delta \in (0,1)$	War decision	$a_{it} \in \{0,1\}$
Peace deal persistence	$\lambda \in [0,1]$	Effort in diplomacy	$e_{it} \in \mathbb{R}_{\geq 0}$
War outcome persistence	$\theta \in [0,1]$	Peace deal offering	$x_{it} \in \mathcal{X}_i \subseteq \mathbb{R}$
Decisiveness of competition	$\pi \in (\frac{1}{2},1]$	Peace deal acceptance	$y_{it}(x) \in \{0,1\}$
Transaction costs	$\mu \in \mathbb{R}_{\geq 0}$		
Risk of inadvertent war	$\rho \in (0,1]$	(country i, period t)	
Key State Variable			
Relative strength	$s_t \in S \subset (0,1)$		

of the crisis drags countries into war with probability $\rho \in (0,1]$, or else countries cooperate by freely splitting the pie according to Nash bargaining with probability $1-\rho$.

Upon recognition, the agenda setter i extends the peace offering of their choice, $x_{it} \in \mathcal{X}_i \subseteq \mathbb{R}$, where $\mathcal{X}_i$ reflects any budget constraints country i may have. None of the results depend on budget constraints, however, they are included for the sake of generality and to establish a direct connection to preventive war in typical dynamic models of conflict. The receiving country $-i$ chooses whether to accept the offer, $y_{-it}(x_{it}) \in \{0,1\}$, with rejection leading to war. A peace deal results in a flow payoff that includes the utility over their accepted allocation as well as any costs of effort they incurred in the process of competitive diplomacy.

After an allocation is determined through either cooperation or war, the game proceeds to the next period. The outcome in period t may persist into period $t+1$ and beyond: each period, the country controlling the pie after war retains possession with probability $\theta \in [0,1]$, while a peace deal arrived at through cooperation remains intact with probability $\lambda \in [0,1]$. With complementary probabilities, the countries return to crisis bargaining. As a technical necessity to accommodate these dynamics, the state of the world additionally includes whether the countries are in an active crisis bargaining stage and the status quo allocation. The persistence parameters λ and θ are probabilities that manage the transitions between active and passive states, reflecting the reliability of peace deals and the durability of war outcomes, respectively.

Power transitions may also occur over time. In particular, the state variable s_t may evolve flexibly over time according to a Markov transition function $q : S \times A \to S$, where $a_t \in A \equiv \{0,1\}^2$

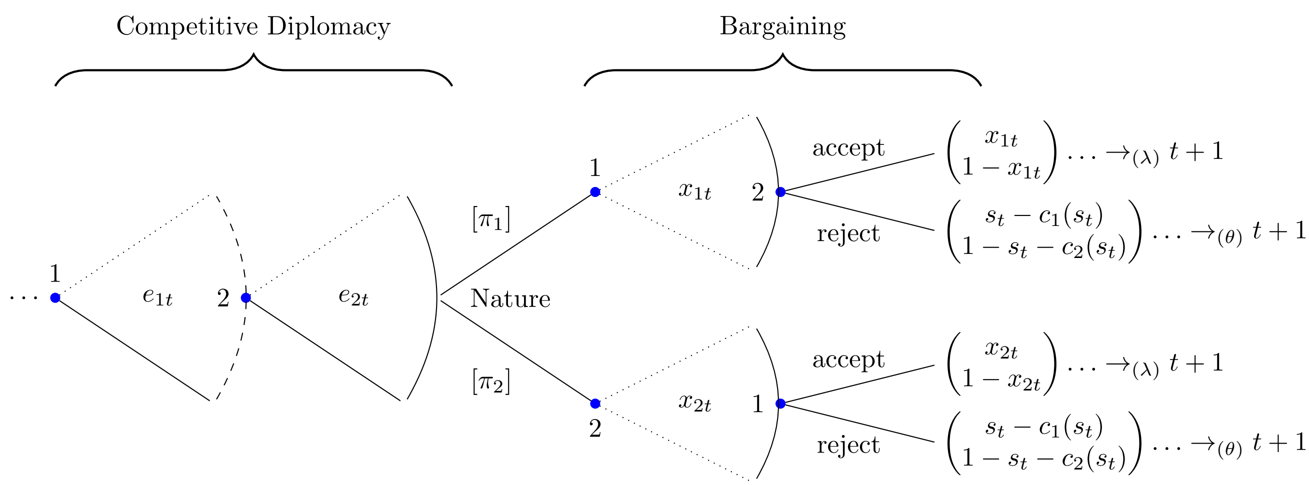

Figure 1. Cooperation in state s_t ($\mu = 0$, letting $\pi_1 = 1 - \pi_2 \in \{\pi, 1-\pi\}$)

are the war actions taken in period t. Throughout the article, I opt for a general exposition and do not impose additional constraints on the transition function. The virtue of this approach is to explicitly bridge the current model with previous work that explores shifting power in a variety of ways. This ensures the results are robust to a large set of transition functions that allow but do not require dependencies between actions and the evolution of power. Consequently, many of the expressions are implicitly defined, as explicit characterization in terms of parameters requires selecting a specific q and is not necessary to recover the results.

The solution concept is Markov Perfect Equilibrium (MPE), henceforth simply equilibrium. Appendix A.7 provides background and further discussion of the solution concept, and all proofs are relegated to Appendix B. In subsequent sections, I suppress all notation that identifies a specific period t when doing so does not create confusion.

2.1 Continuation Values and the Bargaining Surplus

A strategy for country i is a function $\sigma_i : S \rightarrow \{0,1\}^2 \times \mathbb{R}_+ \times \mathcal{X}_i$ given by quadruple

$$\sigma_i(s) = (a_i(s), e_i(s), x_i(s), y_i(x; s)) \tag{1}$$

corresponding to i's decision to launch a strike, an amount to exert on competitive diplomacy, a bargaining offer if recognized, and acceptance decisions for all $x \in \mathbb{R}$, respectively.

If war occurs, each country can expect to receive the entire pie for that period with their probability of victory. In every period after war that the war outcome persists, the winning country can expect to keep receiving the entire pie while the losing country keeps receiving nothing with probability θ. The countries return to the bargaining game under a new state s' with probability $1-\theta$. Letting $s_1 := s$ and $s_2 := 1 - s_1$ without loss of generality, country i's expected value of war in state s under strategies σ can be expressed as where[5]

$$W_i(s) = \underbrace{s_i}_{\substack{\text{expected war} \\ \text{outcome in } s}} - \underbrace{c_i(s)}_{\substack{\text{costs of} \\ \text{war in } s}} + \frac{\delta}{1-\delta\theta}\Bigg[\theta \underbrace{s_i}_{\substack{\text{war outcome} \\ \text{persists}}} + \underbrace{(1-\theta)\sum_{s' \in S} V_i(s')q(s'|s, a_W)}_{\substack{\text{expected value in return} \\ \text{to bargaining after war}}}\Bigg]. \tag{2}$$

and a_W denotes actions that involve a country choosing to fight a war.

On the other hand, countries may cooperate. Denote by $N(s)$ the expected peace settlement in state s under strategies σ, which will correspond to the Nash bargaining solution in equilibrium. In every period after cooperation that the peaceful settlement persists, country 1 can expect to keep receiving $N(s)$ and country 2 can expect to keep receiving $1 - N(s)$ with probability λ. The countries return to the bargaining game under a new state s' with probability $1-\lambda$. Then, country i's expected value of cooperating in state s given equilibrium strategies σ are

$$U_i(s) = \underbrace{u_i(N(s))}_{\substack{\text{expected peace} \\ \text{deal in } s}} - \underbrace{\int_0^\infty e dF_s(e)}_{\substack{\text{exp. diplomatic} \\ \text{effort in } s}} + \frac{\delta}{1-\delta\lambda}\Bigg[\lambda \underbrace{u_i(N(s))}_{\substack{\text{peace deal} \\ \text{persists}}} + (1-\lambda) \underbrace{\sum_{s' \in S} V_i(s')q(s'|s, a_U)}_{\substack{\text{expected value in return} \\ \text{to bargaining from cooperation}}}\Bigg] \tag{3}$$

where a_U denotes actions that involve countries choosing to cooperate and F_s yields the cumulative distribution of diplomatic effort in state s given strategies σ.

Then, prior to any declarations of war or recognition of agenda-setting power, each country i's ex-ante value for being in any state s given strategies σ satisfies

$$V_i(s) = \begin{cases} U_i(s) & \text{if } \max a_i^*(s) = 0 \\ W_i(s) & \text{otherwise} \end{cases} \tag{4}$$

in equilibrium. A country prefers fighting a war when $W_i(s) > U_i(s)$, or else they attempt to cooperate. When choosing to cooperate, countries will also need to choose an amount to exert

[5] Refer to Appendix A.3 for full derivations. Notation denoting dependence on a strategy σ is suppressed in continuation values and objects containing them where doing so does not create confusion.

on diplomacy, which will depend on how much they expect to be able to extract upon becoming the agenda setter. Intuitively, a country will be willing to expend more effort in competition if agenda-setting power yields larger returns.

Specifically, the agenda setter always extracts as much as their opponent is willing to tolerate, taking the entire expected surplus from cooperation for themselves if they can (on the path of play). The receiver's continuation value of accepting an offer $x \in \mathbb{R}$ is given by $U_i(x; s)$, which differs from $U_i(s)$ in that it considers a specific proposal and does not include the costs of effort expended in diplomacy as these become sunk at the time of the proposal. Then, each country i is indifferent between accepting and rejecting an offer $\bar{x}_i(s)$ that solves $U_i(\bar{x}_i(s); s) = W_i(s)$. Mutually satisfactory peace deals exist in equilibrium if a country prefers to settle at their opponent's indifference deal than their own, $\bar{x}_2(s) \geq \bar{x}_1(s)$, and budget constraints allow for a settlement between these two indifference deals. On the other hand, if countries prefer their own indifference deal to their adversary's, $\bar{x}_1(s) > \bar{x}_2(s)$, agreement is impossible. In equilibrium, offers that get accepted are always equal to an opponent's indifference deal or the greatest credible deal within an opponent's budget constraint.

Denoting the equilibrium offer of country i in state s by $x_i^*(s)$, the expected surplus from cooperation in state s can then be denoted by $B(s) := x_1^*(s) - x_2^*(s)$, reflecting the amount that can be extracted by an agenda setter. Using the expressions for equilibrium offers, $U_i(s) \geq W_i(s)$ for country $i = 1, 2$ if and only if

$$B(s) = 1 - (1 - \delta\lambda)W(s) + \delta(1 - \lambda) \sum_{s' \in S} V(s')q(s'|s, a_U) \geq 0$$

and budget constraints are not violated, where $W(s) := W_1(s) + W_2(s)$ and $V(s') := V_1(s') + V_2(s')$, for any state s and transitions q.

3 Benchmark: No Competitive Diplomacy

When there is a positive bargaining surplus, $B(s) \geq 0$, states will have incentive to compete for agenda-setting power in hope of recovering it for themselves. Before proceeding to the implications of competitive diplomacy, it is worthwhile to explore a benchmark where it is absent. This is analogous to the model with the condition that $\pi = \frac{1}{2}$.

Definition 1. *A war in state $s \in S$ is preventive if and only if budget constraints prevent a mutually preferred settlement, $\bar{x}_2(s) \geq \bar{x}_1(s)$ and $\bar{x}_i(s) \notin \mathcal{X}_{-i}$ for a country $i = 1, 2$.*

A preventive war is one that occurs due to a commitment problem. This is often conceptualized as a scenario where a declining state requires a large bonus for cooperating with a rising state, but the rising state does not have the liquidity to extend a satisfactory side payment. Fearon (1995) provides a canonical model of preventive war where country 1 is endowed with agenda-setting power (instead of competitive diplomacy), war outcomes are permanent $\theta = 1$, peace deals do not persist $\lambda = 0$, and budget constraints prevent side payments, $\cap_i \mathcal{X}_i = [0, 1]$. Also noteworthy is Powell (2006), which models deterministically alternating agenda setters over an infinite horizon.

First, this article builds on this by allowing different outcomes to flexibly persist. Rather than war being game-ending and peace deals only lasting one period, war outcomes can now persist into future periods with probability $\theta \in [0, 1]$ and peace deals with probability $\lambda \in [0, 1]$. Since expectations about outcome persistence affect the relative value of going to war over settling things cooperatively, it also affects the value of setting the agenda.

Lemma 1. *The value of being the agenda setter is directly increased by peace deal persistence λ and directly reduced by war outcome persistence θ.*

The case in Fearon (1995) is therefore a worst-case scenario for agenda setters: when peace deals are more persistent and war outcomes less so—greater λ and smaller θ—it is likely that they can extract even greater surplus from their adversaries. This is because, as the outcome of war becomes less persistent relative to the cooperative settlement, war becomes an even less desirable outside option. Since the outside option is worsened, agenda-setters can demand greater proportions from their adversaries. Of course, whether the total effect of persistence parameters on agenda-setting power is positive depends on how those parameters affect the corresponding continuation values, which in turn depends on the transition function.

Allowing outcome persistence to vary flexibly has distributional consequences in peace, but alone does not change the qualitative nature of equilibrium behavior from earlier work.

Proposition 1. *In any equilibrium without competitive diplomacy, countries will either (i) fight a preventive war or (ii) cooperate without effort.*

If countries cannot compete with diplomacy to improve their bargaining position, there is no longer an incentive to exert diplomatic effort. Cooperation must be completely efficient when it occurs. Consequently, the sum of country payoffs are not affected by changing the costs of war in a state that does not result in war, as this sum is guaranteed to be the full pie of value 1.

Importantly, war only occurs if it is preventive: a budget constraint must bind, $\bar{x}_i(s) \notin \mathcal{X}_{-i}$ for a country $-i$. This is typical dynamic settings that allow for budget constraints. Holding these constraints constant, changing the relative persistence of outcomes can change when preventive war would or would not occur compared to previous models where $\theta = 1$ and $\lambda = 0$. By Lemma 1 and Proposition 1, the direct effects of reducing θ and increasing λ will tend to avert preventive wars, as a worsened outside war option implies a greater willingness of the declining power to settle for less demanding side payments from the rising power.

4 Implications of Competitive Diplomacy

Next, we explore the implications of including competitive diplomacy. The first set of results hold with or without settlement frictions. Because equilibrium behavior is simpler without frictions, this section focuses mostly on behavior with $\mu = 0$ to clarify the exposition; however, the results additionally hold for $\mu > 0$ unless specifically stated.

There are two new features of equilibrium behavior. First, countries now compete with strictly positive diplomatic effort in cooperation, and second, countries may now choose to initiate war because it is relatively efficient compared to competitive cooperation.

Definition 2. *A war in state $s \in S$ is efficient if and only if there is no mutually preferred settlement, $\bar{x}_1(s) > \bar{x}_2(s)$.*

Proposition 2 (Equilibrium)**.** *When $\mu = 0$, there is a unique equilibrium with strategies σ^* in which countries can fight efficient wars in addition to preventive wars, or they cooperate by mixing diplomatic effort uniformly between zero and $\frac{(2\pi-1)B(s)}{1-\delta\lambda}$.*

Figure 2 illustrates how countries compete in state s according to a mixed strategy defined by cumulative distribution F_s^*. Let us focus on understanding the process of cooperation before returning to efficient wars in the following section.

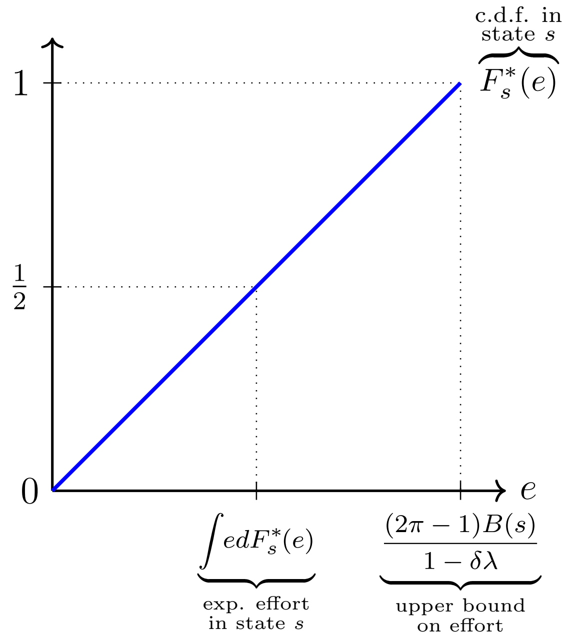

Figure 2. Cumulative distribution for diplomatic effort F_s^* with no frictions $\mu = 0$

When choosing to cooperate, a country always extends the offer that is best for them in their adversary's acceptance set, and that offer is always accepted. Then, the difference between the two equilibrium offers defines the surplus from cooperation, which is the value of being a recognized agenda setter. Importantly, the nature of the hypothetical war affects each country's acceptance set, which in turn affects the value of being the agenda setter. Since countries choose how to engage in competitive diplomacy with the value of becoming an agenda setter in mind, the nature of a hypothetical war affects their equilibrium diplomatic strategies and therefore the quality of peace, even if war never occurs on the path of play.

Proposition 3. *Costs of war make peace costlier and reduce welfare.*

Let $C(s) := c_1(s) + c_2(s)$. *Then,* $\frac{\partial}{\partial C(s)} \int edF_s^*(e) \geq 0$ *and* $\frac{\partial}{\partial c_i(s)} V_i(s) \leq 0$ *for each country* $i = 1, 2$ *and for any state* s *and transition function* q.

Usually, increasing the costs of war in the bargaining model makes the peaceful settlement better for the agenda setter and worse for the receiver, with a net effect of zero on aggregate welfare. However, because agenda-setting power is now endogenous to a pre-bargaining diplomatic competition, the larger surplus that results from larger costs of war also provides countries with an incentive to exert greater effort in competitive diplomacy. While the increased surplus still serves to reallocate wealth from the receiver to the recognized proposer in zero-sum fashion, the total net effect is no longer zero, as countries now exert increased effort trying to become the agenda setter. This increased effort is an inefficient waste, guaranteeing a reduction of expected welfare in the process. Importantly, this does not rely on the outbreak of war, but simply the fact that countries will reduce their acceptance standards in response to a worse outside option.

In addition to the characteristics of war, the extent to which the competition is decisive has important implications on equilibrium behavior and consequently outcomes.

Proposition 4. *Welfare is decreasing in a competition's decisiveness. Moreover, completely decisive competition guarantees the full erosion of the gains from peace absent transaction costs.*

Formally, for all states s and transition functions q, (i) $\frac{\partial}{\partial \pi} V_i(s) \leq 0$ and (ii) $\pi = 1$ implies $U_i(s) \leq W_i(s) + \frac{1-\rho}{1+\rho}\mu$ for each country $i = 1, 2$.

This proposition reveals how detrimental competitive diplomacy can be. The competition's decisiveness heightens the incentive to compete, introducing inefficiencies to cooperation and reducing welfare. Under completely decisive contests, the desire to earn agenda-setting power to capture the benefits of peace can drive countries to exert effort until all of the surplus is destroyed. In particular, a country is ensured to do no better in peace than in war if there are no transaction costs or if there is certain war in the event of miscoordination. Note that, while $\pi = 1$ guarantees complete erosion in the case without frictions, it remains possible that the surplus is fully eroded before the point of complete decisiveness, $\pi < 1$.

Equilibrium behavior with transaction costs will be discussed in the section on inadvertent war, but it is worth touching on the unexpected effect of frictions here. Surprisingly, transaction costs actually protect against the erosion caused by competitive diplomacy. This occurs because transaction costs introduce a countervailing incentive to occasionally exert zero effort, effectively forfeiting the competition in lieu of internalizing the passive gains of cooperating for free. The extent to which transaction costs serve to insulate countries from the erosion of competition is therefore proportional to their ability to free ride on their adversary's efforts without fear of inadvertent war. In the limit where inadvertent war is certain following coordination failure ($\rho = 1$), transaction costs are no longer effective at securing the gains of cooperation, leaving countries indifferent between war and peace.

These results are derived with competition as an all-pay contest, as this is a natural and convenient representation for the purpose of this paper. In addition to yielding closed-form expressions that facilitate comparative statics, all-pay contests have been used to reflect a large number of competitive environments, such as lobbying (Baye, Kovenock, and De Vries, 1993), battles and races (Konrad and Kovenock, 2009), and market power competition (Siegel, 2009). However, the central logic is robust to a wide array of contests. Although Tullock contests do not exhibit continuous equilibrium diplomatic strategies and generally do not admit closed-form expressions, making it a suboptimal baseline for the model here, Appendix C.2 demonstrates that the derived intuition is robust to any decisive Tullock contest. Additionally, by the Revenue Equivalence Theorem (RET), these features also hold under all standard auction contests where the largest bid is awarded the surplus (Myerson, 1981). Because the RET also applies to auctions with private valuations, the model can be straightforwardly extended to bargaining models of war with information asymmetries (e.g., with privately known costs of war).

4.1 Efficient Wars

Unlike most game-theoretic bargaining models of war, countries may prefer fighting to cooperating despite an ability to settle peacefully today. When this occurs, war is called "efficient" as it entails fewer aggregate inefficiencies on the path of play.

Figure 3 illustrates how different couples of equilibrium indifference deals (the deals at which countries are indifferent between war and peace) lead to cooperation, preventive war, or efficient war. If country 2 prefers settlements that are larger than those demanded by country 1 to keep them indifferent (above the dashed blue line), countries will either cooperate or they will have preventive war in the traditional sense due to an inability to settle at a mutually preferred deal. If not, efficient war will break out. These indifference deals come out of equilibrium behavior and, while the couples that lead to efficient war (below the dashed blue line) are usually assumed away in standard settings, this section shows how efficient wars can arise endogenously due to competitive diplomacy.

Before stating the general condition for efficient war, let us first define two key terms.

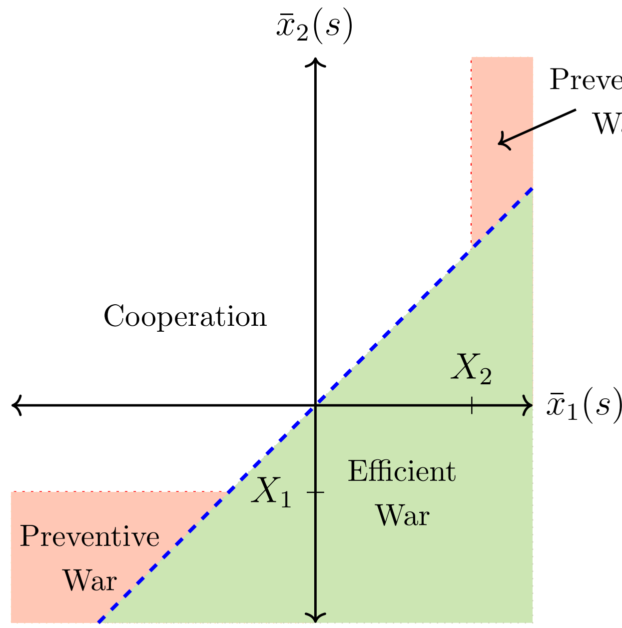

Figure 3. Efficient wars, preventive wars, and cooperation in indifference settlements

Definition 3. *The expected net present value of the pie after war is given by*

$$G := \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda}. \tag{5}$$

Winning a war yields the entire pie of value 1 and, with probability θ, it is retained for subsequent periods. The expected present value of this is reflected in the first term of equation (5). The second term in the expression reflects the aggregate expected present value of the pie prior to a return to bargaining if they cooperate instead, persisting with probability λ. The value G then reflects the aggregate value countries expect to recover from war over peace specifically with regard to the differential tendencies of their outcome persistence, leaving aside their continuation values that rely on strategies upon returning to bargaining.

On the other hand, we also need to account for the discrepancy between war and cooperation upon a return to bargaining.

Definition 4. *The total expected net present value of returning to bargaining after fighting in state s is given by*

$$\Delta V(s) := \sum_{s' \in S} (V_1(s') + V_2(s')) \left[\frac{\delta(1-\theta)}{1-\delta\theta} q(s'|s, a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda} q(s'|s, a_U) \right] \tag{6}$$

for any transition function q.

The present value of returning to the bargaining table after a war in state s is given by $\frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'} V_i(s')q(s'|s,a_W)$ for country $i = 1,2$, whereas country i's present value of returning to bargaining after cooperating in state s is given by $\frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'} V_i(s')q(s'|s,a_U)$. Equation (6) defines $\Delta V(s)$ as the difference between the aggregates of these two objects, reflecting the net present value of returning to bargaining after fighting. Unlike G, the term $\Delta V(s)$ reflects the total additional value countries can expect to get by returning to bargaining from war instead of from an existing peace deal, which depends not only on the model parameters but also on the current state of the world and the transition function.

Together, the expression $G + \Delta V(s)$ yields the total net present value of going to war, where G is an exogenous component that reflects to the differential persistence of outcomes and $\Delta V(s)$ is an endogenous component attributable to variation in country strategies across different states of the world (and the likelihood of their realizations) after a return to bargaining.

Equipped with this notation, we can concisely state the following proposition.

Proposition 5. *An efficient war occurs if and only if the total expected net present value of fighting is greater than the total costs of war. Formally, $\bar{x}_1(s) > \bar{x}_2(s)$ for any state s with transition function q if and only if $G + \Delta V(s) > C(s)$.*

Proposition 5 has an intuitive interpretation but is not possible in typical dynamic conflict models, as both G and $\Delta V(s)$ are usually assumed to be weakly negative while $C(s)$ is assumed to be strictly positive. In this article, however, the total expected net present value of fighting may be positive and possibly larger than the total costs of war. In particular, the expression demonstrates how there exist two paths to efficient war: one through the exogenous component G and another through the endogenous component $\Delta V(s)$.

First, because the model allows for war outcomes and peace deals to persist at varying rates according to θ and λ, it is possible that a much greater persistence of war outcomes relative to peace deals can drive countries to fight. The logic parallels that of "ripping off the bandage," as countries prefer taking their chances in war today if it means they can avoid having to regularly incur the smaller costs of peace for the rest of time. To see this, note that $\lim_{(\lambda,\theta)\to(0,1)} G = \frac{\delta}{1-\delta}$, from which it becomes clear that enough patience can result in an arbitrarily large expected net present value of the pie after fighting. However, while exogenous factors can drive efficient war, their interaction with competitive diplomacy is crucial for the result—as the benchmark above demonstrates, the absence of competitive diplomacy implies the absence of efficient wars. This is because, without competitive diplomacy, the value for cooperation is always sufficiently large enough relative to the value for war that the equation can never be satisfied (i.e., $\Delta V(s) \to -\infty$ as $\delta \to 1$ at a faster rate than $G \to \infty$).

Second, variation in countries' equilibrium strategies across different states of the world can bring about efficient war if fighting today is more likely to transition into preferable states of the world tomorrow. Each possible state of the world comes with a corresponding value for each country that depends on both the primitives of the model and their behavior in equilibrium, some of which are preferable to others. If war is expected to return countries to bargaining under better conditions, they may choose to incur the greater costs of fighting today in order to improve the likelihood they transition to better states of the world tomorrow.

To see how endogenous effort to compete for agenda-setting power can create efficient wars, consider a simple example. Let there be three states of the world $S = \{s, \underline{s}, \bar{s}\}$, with s as the current state and both $\underline{s}$ and $\bar{s}$ as absorbing states where countries cooperate. If war guarantees a transition to state $\underline{s}$ while cooperation guarantees a transition to state $\bar{s}$, countries may prefer to fight an efficient war today even if free cooperation is possible when the expected inefficiency from competitive diplomacy in $\bar{s}$ is sufficiently larger than that in $\underline{s}$. This can occur if the surplus from cooperation in state $\bar{s}$ is much larger than that in state $\underline{s}$. Appendix A.8 derives an analytical condition for efficient war in this case and offers a numerical example.

In general, the total effect of θ and λ on $\Delta V(s)$ is not straightforward as it depends on how the transition function maps different actions today into different states in the future. Naturally, greater persistence of either outcome can increase $\Delta V(s)$ through country continuation values. Persistence avoids inefficiencies, allowing countries to do better upon a return to bargaining if outcomes are expected to last. However, persistence can also decrease $\Delta V(s)$ as the present value accounts for temporal considerations—the more an outcome persists, the longer it will take for a country to recover the gains that come with a return to bargaining, and hence the smaller its present value ceteris paribus.

Nonetheless, the model provides a path for understanding the conditional effects of persistence on countries' willingness to fight efficient wars. For example, the following corollary demonstrates that in the event of unreliable peace deals and permanent war outcomes, the effect of war outcome persistence on G overwhelms any possible countervailing effects on $\Delta V(s)$, guaranteeing an efficient war in equilibrium.

Corollary 1. *Patient countries will fight an efficient war if peace deals do not persist and war outcomes are permanent. Formally, for any state s and transitions q, $\lambda = 0$ and $\theta = 1$ implies that $\lim_{\delta \to 1^-}(\bar{x}_1(s) - \bar{x}_2(s)) > 0$ for both countries $i = 1, 2$.*

This result demonstrates a sufficient condition for countries to prefer "ripping off the bandage" and trying their luck in war. Notably, this condition is the typical assumption in dynamic crisis bargaining models, going back at least to the analysis of preventive motives in Fearon (1995) where peace deals do not persist into the subsequent periods but war is game-ending. If peace entails constant renegotiation based on changes to the underlying state of the world, whereas war outcomes settle a dispute absolutely, then countries will necessarily prefer fighting an efficient war to cooperation regardless of the state's expected evolution. Importantly, this will be true for any country irrespective of their relative strength today or their expected strength in the future.

Moreover, additional implications can be derived under mild assumptions on the transition function. The relative efficiency of war depends heavily on the expected downstream consequences of actions in the current period. If, for example, a war today would be sure to produce a detrimental state of the world tomorrow, the inefficiencies associated with cooperation would need to be vastly greater than if war today is sure to transition into a superior state of the world. For this reason, conditions on state transitions can allow us to recover more precise conditions for when efficient war will or will not arise.

For example, suppose the expected value of returning to bargaining after war is no better than that of returning to bargaining after peace. Then, a corollary follows from Proposition 5.

Corollary 2. *If the expected value of a return to bargaining after war is no better than the expected value of returning to bargaining after cooperation, efficient wars are averted by peace deals that are more persistent than war outcomes.*

Take any transition function $q : S \times A \to S$ that satisfies $\sum_{s'} V(s')(q(s'|s, a_U) - q(s'|s, a_W)) \geq 0$ for all states $s \in S$. Then, $\lambda > \theta$ ensures that $\bar{x}_2(s) > \bar{x}_1(s)$.

Countries may prefer war or cooperation for one of two reasons: either they are associated with less inefficiencies or they tend to produce superior states of the world in the future. By making the above assumption on state transitions, we rule out the possibility that countries have an inherent bias towards war specifically because they tend to transition to preferable states of the world. This requires that returns to bargaining after war cannot be more favorable in the aggregate than they would be after cooperation. As a result, the only way to favor efficient war is if there is less inefficiency associated with fighting. Since country choices of diplomatic effort are endogenous, the condition that $\lambda > \theta$ is sufficient to guarantee this does not occur.

On the other hand, $\theta > \lambda$ is necessary but not sufficient for efficient war, as a preference for war also requires sufficiently small costs of war under the current state and a sufficiently small expected value of returning to bargaining after cooperation. Like Corollary 1, this result also demonstrates the logic of "ripping off the bandage" at play.

5 Transaction Costs and Inadvertent War

While the above results held for any amount of settlement frictions $\mu \geq 0$, the final set of results depend on nonzero settlement frictions $\mu > 0$, which may be arbitrarily small. These may be considered transaction costs in the sense of Keohane (1984) and related work.

Most importantly, transaction costs create an obstacle to cooperation, introducing an incentive to conditionally free ride on an adversary's efforts. A failure to overcome this friction in any period creates a risk that war occurs with probability $\rho \in (0, 1]$, despite countries choosing not to intentionally initiate one. This reflects the likelihood that an unresolved crisis spirals out of control. By definition of frictions that are not trivially ignorable, such a risk must be possible or else we can achieve free cooperation forever by never cooperating.

In equilibrium, countries take this risk into account when choosing their actions, including their effort in competitive diplomacy. Even though countries could guarantee cooperation by exerting a bare minimum of (potentially arbitrarily small) μ, each country occasionally attempts free riding on the efforts of the other despite creating the possibility of inadvertent war.

Proposition 6 (Equilibrium)**.** *Take any $\mu > 0$. Given equilibrium strategies σ^*, countries may choose to fight either efficient or preventive wars, or else cooperation entails exerting zero effort* *with probability* $\frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}$ *and otherwise mixing uniformly between* μ *and* $\frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu$.[6]

Figure 4 illustrates how countries compete in state s when $\mu > 0$ according to a mixed strategy defined by cumulative distribution F_s^*. The c.d.f. reveals how equilibrium behavior with frictions results in an occasional inadvertent war.

Definition 5. *A war in state* $s \in S$ *is inadvertent if and only if it occurs despite countries choosing to cooperate,* $a_1(s) = a_2(s) = 0$.

Remark 1. *Given equilibrium strategies* σ^**, the total probability of inadvertent war is equal to* $\omega(s) := \rho \left[\frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}\right]^2 > 0$ *for any state* s *and transition function* q *such that there is surplus from cooperation,* $B(s) > 0$.

Even arbitrarily small frictions to cooperation introduce an incentive to occasionally attempt free riding on an adversary's efforts. In general, countries want to perform well in diplomacy and receive a greater chance at recovering the surplus from cooperation, given their exerted effort is not larger than the gain. However, conditional on their relative performance, a country prefers not to exert unnecessary additional effort. This incentive to reduce effort conditional on diplomatic performance can therefore create a dilemma for countries who need to cumulatively exert at least $\mu > 0$ to reach an agreement. When countries simultaneously trying to free ride on each other, they fail to cooperate and risk allowing the crisis to spiral out of control.

With the total probability of inadvertent wars characterized as a function of equilibrium strategies, we can learn how changes in the form of competitive diplomacy affect the likelihood of war. In general, the effect of competition's decisiveness on the likelihood of war depends on the selected transition function, as the expected gain from war relative to peace depends not only on the specific values of θ and λ, but also on how current actions correspond to expected transitions into different states tomorrow. However, for any state space and transition function we can choose, decisiveness reduces the occurrence of inadvertent wars when peace deals are reliable.

Proposition 7. *The total probability of inadvertent war is decreasing in the decisiveness of competitive diplomacy when peace deals are reliable. Formally,* $1 = \lambda > \theta$ *implies* $\frac{\partial}{\partial \pi}\omega(s) \leq 0$ *for all states* s *and transition functions* q.

[6] Cooperation in state s therefore requires $\mu < \frac{(2\pi-(1-\rho))B(s)}{2(1-\delta\lambda)}$.

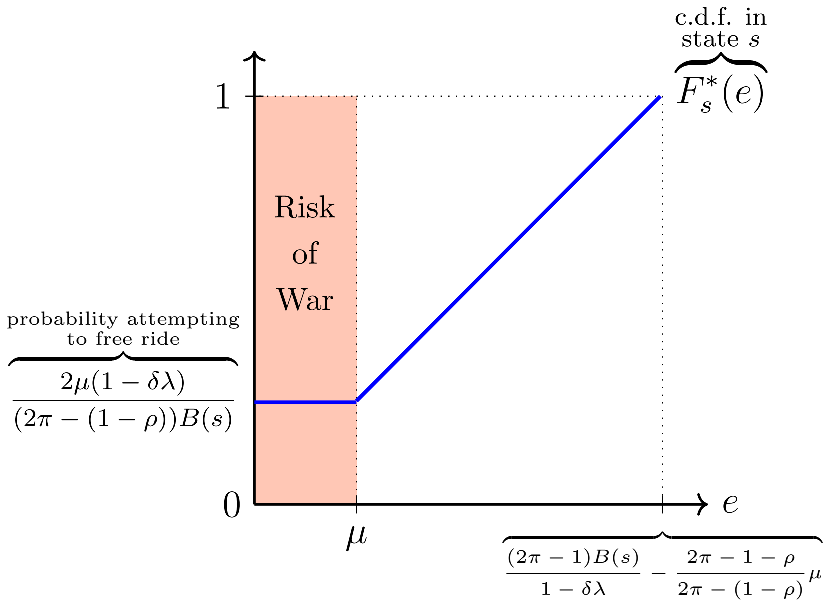

Figure 4. Cumulative distribution for diplomatic effort F_s^* with frictions $\mu > 0$

Countries are less willing to free ride under decisive competition, as doing so often results in forfeiting agenda-setting power to their adversary. Instead, countries respond to increased decisiveness by exerting greater effort in competitive diplomacy, consequently diminishing the total probability of inadvertent war in equilibrium. However, this effect can be overruled when the persistence of war outcomes exceeds the reliability of peace deals, in which case increases to decisiveness may cause an erosion of the bargaining surplus that reinforces free-riding incentives.

This result is especially surprising given that, by Proposition 4, decisiveness also reduces welfare, implying that the Pareto optimal competition maximizes the probability of war when peace deals are reliable. Decisive contests endogenously cause countries to exert greater effort relative to the bargaining surplus, not only in the current period but also in every future period where there is an expected net gain from cooperation. However, these same equilibrium efforts contribute to an erosion of the bargaining surplus, making peace less valuable. Because countries only free ride to the extent they can gain by doing so, their willingness to free ride is inversely proportional to the severity of competitive diplomacy. This is most clearly seen in the limit: fully decisive competition with certain war upon miscoordination discourages attempts to free ride, but coincides with the complete dissipation of the bargaining surplus, resulting in payoffs from cooperation that are no better than payoffs from war.

Recall also that $\lambda > \theta$ is the same condition that ensures efficient wars are avoided under mild conditions in Corollary 2. This implies a dilemma: improving the reliability of peace deals can reduce the outbreak of efficient wars; however, it also ensures that there will be tension between promoting country welfare and reducing the outbreak of inadvertent wars.

Appendix A.9 discusses how the total probability of inadvertent war interacts with other parameters of the model, including the persistence parameters θ and λ, as well as the risk of inadvertent war ρ. How changes in these parameters affect the total probability of inadvertent war depends on how those changes affect the bargaining surplus. The intuition is as follows: the faster the bargaining surplus grows (or the slower it reduces) in response to changes in these parameters, the more it heightens the incentive to engage in competitive diplomacy and, thus, reduces the total probability of inadvertent war.

6 Discussion

Competitive diplomacy is an important aspect of foreign affairs. The term "competitive diplomacy" was used in the US National Security Strategy (2017), which called for greater investments into diplomatic capabilities that can improve the state's ability to achieve favorable outcomes through cooperation. Similarly, the US National Defense Strategy (2018) stated that "long-term strategic competition requires the seamless integration of multiple elements of national power" with "diplomacy" being the first element on the list. This paper aims to understand the sources and implications of the strategic incentive to compete through diplomacy within the prevalent crisis bargaining framework.

Specifically, I propose a dynamic bargaining model of war where countries have the ability to improve bargaining leverage through diplomatic effort. Countries divide a pie by either fighting a war or reaching a peaceful agreement. War is a costly lottery over the disputed good, while cooperation entails an agenda setter extending an acceptable peace deal to their opponent in bargaining. Diplomacy is a means to compete over the right to set the terms of a peace deal, with a country's willingness depending on their expected value for said right, which in turn depends on characteristics of the potential war.

This paper primarily contributes to theoretical work in crisis bargaining, but the results also have broad implications for international relations. Most importantly, it highlights the value of accounting for costly diplomatic actions that have the potential to improve a country's payoff in cooperation without improving their expected performance in war. When states have an ability to engage in competitive diplomacy, both the quality and prospects of peace are negatively affected. Moreover, increasing the costs of potential war increases the stakes of the dispute by reducing the value of a country's outside option, therefore encouraging more competition and reducing welfare. Under these conditions, it is possible for war to be relatively efficient.

Although competitive diplomacy has not previously been modeled in crisis bargaining, work in international relations has explored the use of diplomacy to compete and influence political outcomes. For example, in studying how rebels use diplomacy to earn political leverage, Huang (2016) details how these groups regularly hire law and public relations firms to lobby elected officials in the United States. According to the Foreign Lobby Watch at OpenSecrets, billions of US dollars have been spent on foreign lobbying over the past several years.[7] In another recent example, Barham et al. (2023) provide evidence that states used vaccine distribution during the COVID-19 pandemic—in what is often called "vaccine diplomacy"—to effectively earn trust abroad and expand their global influence. Such efforts are increasingly relevant in international politics, and incorporating them in a model of crisis bargaining has major implications on our understanding of country incentives and outcomes.

The model also sheds additional light on empirical results. A relevant recent example is Blair, Marty, and Roessler (2022), who find that Chinese aid to Africa does not increase their support among beneficiaries, while the US does enjoy heightened support after aid provisions. Nonetheless, China considers foreign aid essential to the development of their ability to achieve favorable outcomes through cooperation, or "soft power" (Yoshihara and Holmes, 2008). This is consistent with expectations from the model: a country's observed effort in competitive diplomacy correlates with the stakes of the dispute and the scope of the competition, but does not govern a country's equilibrium success in recovering bargaining leverage. Moreover, Pratt (2020) provides evidence that international institutions are replaced by new ones when they fail to reflect the distribution of power. By linking competitive diplomacy to crisis bargaining, the model provides microfoundation for how this incentive arises, as competition over the bargaining surplus through

[7] Refer to https://www.opensecrets.org/fara. Accessed May 2024.

diplomacy is categorically distinct from competition over the pie by, for example, arming.

Furthermore, the analysis highlights that changes to the international environment, as opposed to specific features of a bilateral relationship, can give rise to conflicts. Recent work has begun re-emphasizing the importance of the systemic conditions. For example, Wolford (2020) highlights how system-level reputational concerns can create an incentive for restraint that alters the risk of war, and Abramson and Carter (2021) demonstrate that systemic instability can induce revisionist states to make territorial claims. In this model, a country's propensity to fight depends on the scope of competitive diplomacy to improve bargaining positions π, as well as how it interacts with the reliability of peace deals λ and the durability of war outcomes θ. External factors often play a role in shaping these determinants of country incentives. The decisiveness of competition may reflect the rules, norms, and procedures of prevailing international institutions. Alternatively, the reliability of peace deals and the durability of war outcomes may depend on the nature and interests of third-parties, such as a great power's willingness to intervene to uphold a previous agreement in the event of defection, or to prevent the re-examination of settlements that were arrived at during a past war.

Consider, for example, how war outcomes in disputes not involving the United States may be more durable as a result of their increasing reluctance to intervene and upend unfavorable outcomes.[8] Indicative of this, the US expressed that they had no intention of fighting Russia as Russian troops advanced on the borders of Ukraine in 2022.[9] Although this would not affect the propensity of war in previous crisis bargaining models, as illustrated by the benchmark results, interacting these dynamic considerations with competitive diplomacy opens up new pathways to war. In the case of Russia-Ukraine, changes to the international system could induce a patient[10] Russia to prefer "ripping off the bandage" by invading Ukraine. Therefore, by providing a theoretical basis for the role of competitive diplomacy in a crisis bargaining framework, the model motivates future empirical research on conflict onset to study the interaction of bilateral considerations and systemic-level changes to cooperative procedures.

[8] While George W. Bush promoted the use of troops to promote democracy abroad (see G.W. Bush, Inaugural Address, January 20, 2005), Donald Trump and Joe Biden favored the withdrawal of troops from Afghanistan.

[9] (1) "More U.S. troops deploying to Europe, Guard leaving Ukraine" by By Jim Garamone. *National Guard*; February 15, 2022. Retrieved in 2024 from nationalguard.mil. (2) "The line Biden won't cross on Ukraine" by Nahal Toosi. *Politico*; February 23, 2022. Retrieved in 2024 from politico.com.

[10] See Cohen (2018) for arguments on Russia's patience.

Finally, the theory suggests three points of guidance for the effective design of international institutions. First, it highlights the need for institutions to reduce the scope of competitive diplomacy. If states can improve their bargaining position through diplomatic effort, not only can this cause significant reductions in aggregate welfare, but it can also cause war to be relatively efficient from the perspective of the disputing states. Second, while transaction costs may obstruct the settlement procedure and introduce the possibility that the dispute spirals into war, the analysis demonstrates that these kinds of frictions surprisingly protect against an erosion of the gains from peace caused by competitive diplomacy. As a result, cost-reducing institutions may do more harm than good: it can be welfare-enhancing for institutions to intentionally impose transaction costs despite being otherwise unnecessary when high levels of competition are unavoidable and the risk of the dispute spiraling out of control is low. Third, the paper identifies a dilemma: reliable peace deals avert efficient wars, but they can simultaneously introduce an unavoidable tension between safeguarding the gains from peace and preventing the outbreak of wars. Institutions that aim to maximize welfare while simultaneously avoiding war may prefer to construct peace deals that are not intended to be permanent, especially if there are sizable transaction costs and risk of the dispute spiraling out of control is high.

References

Abramson, Scott F., and David B. Carter. 2021. "Systemic Instability and the Emergence of Border Disputes." *International Organization* 75(1):103–146.

Albertini, Luigi. 1952. *The Origins of the War of 1914. 3 Vols.* London: Oxford University Press.

Barham, Elena, Sarah Zukerman Daly, Julian E Gerez, John Marshall, and Oscar Pocasangre. 2023. "Vaccine Diplomacy: How COVID-19 Vaccine Distribution in Latin America Increases Trust in Foreign Governments." *World Politics* 75(4):826–875.

Baye, Michael R, Dan Kovenock, and Casper G De Vries. 1993. "Rigging the Lobbying Process: An Application of the All-Pay Auction." *The American Economic Review* 83(1):289–294.

Blair, Robert A, Robert Marty, and Philip Roessler. 2022. "Foreign aid and soft power: Great power competition in Africa in the early twenty-first century." *British Journal of Political Science* 52(3):1355–1376.

Cohen, Stephen F. 2018. *War with Russia?: From Putin & Ukraine to Trump & Russiagate.* Simon and Schuster.

Davis, Jason Sanwalka. 2023. "War as a Redistributive Problem." *American Journal of Political Science* 67(1):170–184.

Debs, Alexandre, and Nuno P. Monteiro. 2014. "Known unknowns: Power shifts, uncertainty, and war." *International Organization* 68(1):1–31.

Dixit, Avinash K. 1998. *The Making of Economic Policy: A Transaction-Cost Politics Perspective.* MIT Press.

Downs, George W, and David M Rocke. 1995. *Optimal Imperfection? Domestic Uncertainty and Institutions in International Relations.* Princeton University Press.

Duggan, John. 2016. Notes on Spatial Bargaining and Stochastic Games. Technical report.

Fearon, James D. 1994. "Domestic Political Audiences and the Escalation of International Disputes." *American Political Science Review* 88(3):577–592.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Fearon, James D. 1998. "Bargaining, Enforcement, and International Cooperation." *International Organization* 52(2):269–305.

Fearon, James D. 2018. "Cooperation, Conflict, and the Costs of Anarchy." *International Organization* 72(3):523–559.

Fey, Mark, Adam Meirowitz, and Kristopher W Ramsay. 2013. "Credibility and commitment in crisis bargaining." *Political Science Research and Methods* 1(1):27–52.

Fey, Mark, and Brenton Kenkel. 2021. "Is an ultimatum the last word on crisis bargaining?" *Journal of Politics* 83(1):87–102.

Fey, Mark, and Kristopher W Ramsay. 2010. "When is shuttle diplomacy worth the commute? Information through mediation." *World Politics* 62(4):529–560.

Huang, Reyko. 2016. "Rebel diplomacy in civil war." *International Security* 40(4):89–126.

Jackson, Matthew O, and Massimo Morelli. 2009. "Strategic Militarization, Deterrence, and Wars." *Quarterly Journal of Political Science* 4:279–313.

Kenkel, Brenton, and Peter Schram. 2023. "Uncertainty in Crisis Bargaining with Multiple Policy Options." *American Journal of Political Science* (Forthcoming).

Kennard, Amanda, Colin Krainin, and Kristopher W. Ramsay. 2018. "Coasian Bargaining in World Politics: Re-Examining the Demand for International Regimes.".

Keohane, Robert O. 1984. *After Hegemony: Cooperation and Discord in the World Political Economy.* Princeton University Press.

Kim, Woosang, and James D Morrow. 1992. "When do power shifts lead to war?" *American Journal of Political Science* pp. 896–922.

Konrad, Kai A, and Dan Kovenock. 2009. "Multi-battle contests." *Games and Economic Behavior* 66(1):256–274.

Kurizaki, Shuhei. 2007. "Efficient secrecy: Public versus private threats in crisis diplomacy." *American Political Science Review* 101(3):543–558.

Lake, David A. 1999. *Entangling Relations: American Foreign Policy in Its Century.* Princeton, NJ: Princeton University Press.

Legro, Jeffrey W. 1994. "Military Culture and Inadvertent Escalation in World War II." *International Security* 18(4):108–142.

Leventoğlu, Bahar. 2023. "Bargaining power in crisis bargaining." *Review of Economic Design* pp. 1–23.

Malis, Matt, and Alastair Smith. 2021. "State Visits and Leader Survival." *American Journal of Political Science* 65(1):241–256.

Martin, Lisa L. 1999. The political economy of international cooperation. In *Global public goods: International cooperation in the 21st century*, ed. Inge Kaul, Isabelle Grunberg, and Marc Stern. pp. 51–64.

Mattis, Jim. 2018. "Summary of the 2018 National Defense Strategy of the United States of America: Sharpening the American Military's Competitive Edge.".

Morrow, James D. 1989. "Capabilities, Uncertainty, and Resolve: A Limited Information Model of Crisis Bargaining." *American Journal of Political Science* 33(4):941–972.

Myerson, Roger B. 1981. "Optimal auction design." *Mathematics of Operations Research* 6(1):58–73.

Nalebuff, Barry. 1986. "Brinkmanship and Nuclear Deterrence: The Neutrality of Escalation." *Conflict Management and Peace Science* 9(2):19–30.

Paine, Jack, and Scott A Tyson. 2020. Uses and abuses of formal models in political science. In *The SAGE Handbook of Political Science: A Global Perspective*, ed. Dirk Berg-Schlosser, Bertrand Badie, and Leonardo Morlino. chapter 11, pp. 188–202.

Posen, Barry R. 2014. *Inadvertent Escalation: Conventional War and Nuclear Risks.* Cornell University Press.

Powell, Robert. 1987. "Crisis Bargaining, Escalation, and MAD." *American Political Science Review* 81(3):717–736.

Powell, Robert. 1993. "Guns, Butter, and Anarchy." *American Political Science Review* 87(1):115–132.

Powell, Robert. 1996. "Stability and the Distribution of Power." *World Politics* 48(2):239–267.

Powell, Robert. 2006. "War as a commitment problem." *International Organization* 60(1):169–203.

Powell, Robert. 2015. "Nuclear Brinkmanship, Limited War, and Military Power." *International Organization* 69(3):589–626.

Pratt, Tyler. 2020. "Angling for Influence: Institutional Proliferation in Development Banking." *International Studies Quarterly* .

Qiu, Xiaoyan. 2022. "State support for rebels and interstate bargaining." *American Journal of Political Science* 66(4):993–1007.

Ramsay, Kristopher W. 2011. "Cheap talk diplomacy, voluntary negotiations, and variable bargaining power." *International Studies Quarterly* 55(4):1003–1023.

Sartori, Anne E. 2002. "The Might of the Pen: A Reputational Theory of Communication in International Disputes." *International Organization* 56(1):121–149.

Schelling, Thomas C. 1960. *The Strategy of Conflict.* Harvard University Press.

Schelling, Thomas C. 1966. *Arms and Influence.* New Haven: Yale University Press.

Schram, Peter. 2021. "Hassling: How States Prevent a Preventive War." *American Journal of*

Political Science 65(2):294–308.

Siegel, Ron. 2009. "All-pay contests." *Econometrica* 77(1):71–92.

Slantchev, Branislav L. 2006. "Politicians, the Media, and Domestic Audience Costs." *International Studies Quarterly* 50(2):445–477.

Smith, Alastair. 1998. "International Crises and Domestic Politics." *American Political Science Review* 92(3):623–638.

Tarar, Ahmer, and Bahar Leventoğlu. 2009. "Public commitment in crisis bargaining." *International Studies Quarterly* 53(3):817–839.

Taylor, A.J.P. 1964. *Illustrated History of the First World War.* New York: Putnam.

Trachtenberg, Marc. 1990. "The Meaning of Mobilization in 1914." *International Security* 15(3):120–150.

Trager, Robert F. 2010. "Diplomatic Calculus in Anarchy: How Communication Matters." *American Political Science Review* 104(2):347–368.

Trager, Robert F. 2011. "Multidimensional Diplomacy." *International Organization* 65(3):469–506.

Trump, Donald J. 2017. "National Security Strategy of the United States of America.".

Tuchman, Barbara W. 1994. *The Guns of August: The Outbreak of World War I.* Random House Trade Paperbacks.

Wolford, Scott. 2014. "Power, Preferences, and Balancing: The Durability of Coalitions and the Expansion of Conflict." *International Studies Quarterly* 58(1):146–157.

Wolford, Scott. 2020. "War and diplomacy on the world stage: Crisis bargaining before third parties." *Journal of Theoretical Politics* 32(2):235–261.

Yoshihara, Toshi, and James R Holmes. 2008. "China's energy-driven 'soft power'." *Orbis* 52(1):123–137.

Online Appendix for "Competitive Diplomacy in Bargaining and War"

Contents

A Additional Model Details and Discussion

A.1 State Variables and Transitions

There are three state variables. The first simply traces the previous period's distribution of the pie, $z_t = o_{t-1} \in \mathbb{R}$ for each period $t \geq 2$ where o_t gives the outcome in period t and without loss of generality $z_1 = 0$. The outcome of a period corresponds to the interaction's resulting division of the pie. In particular, period t's outcome is given by $o_t = 1$ if country 1 wins a war, $o_t = 0$ if country 2 wins a war, and $o_t = x$ for a peace deal reached at $x \in \mathbb{R}$.

Second, the state variable $s_t \in S$ provides the measure of country 1's relative strength, where S is a finite subset of $(0, 1)$. Strengths may evolve flexibly over time according to a Markov transition function $q : S \times A \to S$, where $a_t \in A \equiv \{0, 1\}^2$ are the war actions taken in period t.

Finally, the state variable b_t denotes the crisis bargaining status of the game in period t. In particular, $b_t = 0$ is an inactive state where an outcome persists and each country consumes their status quo share of the pie, z_t. On the other hand, $b_t = 1$ implies countries have an opportunity to engage in diplomatic competition and crisis bargaining.

A key feature of the model is that these outcomes can persist into future periods. Specifically, the game begins in bargaining $b_t = 1$ and, at the end of every period, the state variable b_t transitions to $b_{t+1} = 0$ with probability $\theta \in [0, 1]$ when the current division is a war outcome and with probability $\lambda \in [0, 1]$ when the division is the result of a peace deal.

Recall from the main text that a strategy σ_i does not take b_t or z_t as input because (1) when $b_t = 0$, countries simply consume their share of the pie according to the previous period's division, which is equal to the current state variable z_t, and (2) given $b_t = 1$, state variable z_t is no longer payoff relevant. Thus, I slightly abuse notation in referring to state $(1, z_t, s_t)$ as state s_t when it is implied that $b_t = 1$ and countries are taking actions.

A.2 Budget Constraints

Budget constraints $\mathcal{X}_i$ are closed subsets of the reals that include ∞ for country 1 and $-\infty$ for country 2. Specifically, $\mathcal{X}_1 := \{x \in \mathbb{R} : x \geq X_1\}$ and $\mathcal{X}_2 := \{x \in \mathbb{R} : x \leq X_2\}$ where X_1 and X_2 are the greatest credible agreements for country 1 and 2, respectively. This is consistent with budget constraints in most dynamic crisis bargaining models, where countries do not have the liquidity to offer a large enough side payment to placate their opponent (Krainin et al., 2022). In part due to the fact that this article does not focus on preventive wars, budget constraints could be modeled more generally and, while it would complicate the notation, it would not change the results.

A.3 Payoffs

After the outbreak of war, country i wins a pie of size 1 today with probability s_i and tomorrow (discounted at rate δ) they either get, with probability θ, $Z_i^{war}(s)$, which is the ex-ante expected future value of the war outcome persisting (i.e., prior to knowledge over whether i or i's opponent wins the war), or with complementary probability $1-\theta$ they return to bargaining under a new state s'. Given the state $s \in S$, each country i has an expected war value of

$$W_i(s) = s_i + \delta \left[\theta Z_i^{war}(s) + (1-\theta) \sum_{s' \in S} V_i(s') q(s'|s, a_W) \right] \tag{A1}$$

where

$$Z_i^{war}(s) = s_i + \delta \left[\theta Z_i^{war}(s) + (1-\theta) \sum_{s' \in S} V_i(s') q(s'|s, a_W) \right]. \tag{A2}$$

and a_W denotes actions that involve countries choosing to fight a war. Using equation (A2) to solve for $Z_i^{war}(s)$, equation (A1) simplifies to equation (2) in the main text.

If countries succeed in cooperating, let the expected settlement be given by $N(s)$. In equilibrium, this expectation corresponds to the Nash bargaining solution. Then, countries receive a payoff according to the expected settlement today, less the amount they expect to exert according to their mixed diplomatic effort strategy. Tomorrow, countries receive payoffs according to the same settlement allocation with probability λ and return to the bargaining table under a new state s' with probability $1-\lambda$. Formally, we can write

$$U_i(s) = u_i(N(s)) - \int e dF_s^*(e) + \delta \left[\lambda Z_i^{peace}(s) + (1-\lambda) \sum_{s' \in S} V_i(s') q(s'|s, a_U) \right] \tag{A3}$$

where

$$Z_i^{peace}(s) = u_i(N(s)) + \delta \left[\lambda Z_i^{peace}(s) + (1-\lambda) \sum_{s' \in S} V_i(s') q(s'|s, a_U) \right]. \tag{A4}$$

and a_U denotes actions that involve countries choosing to cooperate. As before, it is necessary to account for the uncertainty over how long peace will persist with $Z_i^{peace}(s)$. Using equation (A4) to solve for $Z_i^{peace}(s)$, equation (A3) simplifies to equation (3) in the main text.

A.4 Equilibrium Offers

When bargaining surplus exists, a country will offer the least generous settlement that facilitates peace—that is, they will offer the point that makes their opponent indifferent or their opponent's greatest credible settlement. First, consider the case without budget constraints. Since diplomatic effort is a sunk cost at the point at which a country makes a proposal, an agenda setter $-i$ will offer an $\bar{x}_i(s)$ that solves $U_i(\bar{x}_i(s); s) = W_i(s)$. In particular, we can solve for values that make countries 1 and 2 indifferent in state s,

$$W_i(s) = u_i(x) + \frac{\delta}{1-\delta\lambda}\left[\lambda u_i(x) + (1-\lambda)\sum_{s' \in S} V_i(s')q(s'|s, a_U)\right]. \tag{A5}$$

Solving (A5) for $u_i(x)$, we get

$$u_i(x) = (1-\delta\lambda)W_i(s) - \delta(1-\lambda)\sum_{s' \in S} V_i(s')q(s'|s, a_U),$$

which, after plugging in $u_1(x) = x$ and $u_2(x) = 1 - x$, yields implicit solutions for the indifference conditions,

$$\bar{x}_1(s) = (1-\delta\lambda)W_1(s) - \delta(1-\lambda)\sum_{s' \in S} V_1(s')q(s'|s, a_U) \tag{A6}$$

and

$$\bar{x}_2(s) = 1 - (1-\delta\lambda)W_2(s) + \delta(1-\lambda)\sum_{s' \in S} V_2(s')q(s'|s, a_U). \tag{A7}$$

The relationship between these indifference points and the indifference points of the standard static bargaining model of war is apparent. In particular, perfectly impatient countries are indifferent at the same settlement as countries in the standard bargaining model of war with complete and perfect information. To see this, consider when $\delta = 0$ so that equation (A6) becomes $\bar{x}_1(s) = s - c_1$ and equation (A7) becomes $\bar{x}_2(s) = s + c_2$. These are equivalent to the standard static bargaining range from the literature.

Upon making a proposal, countries will choose an offer that allocates themselves the largest quantity that their opponent would accept to avoid war. The decision to extend and accept an offer occurs after both countries have already incurred diplomatic effort. This is going to be equal to the adversary's indifference deal unless budget constraints get in the way, in which case an agenda setter will extract the greatest credible settlement. If the greatest credible settlement is worse than the agenda setter's indifference deal, however, they will prefer war to cooperation. Then, we can formally define the equilibrium offers as

$$x_1^*(s) = \begin{cases} \bar{x}_2(s) & \text{if } X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s) \\ X_2 & \text{if } \bar{x}_2(s) > X_2 \geq \bar{x}_1(s) \\ x & \text{for any } x \in \mathcal{R}_2(s) \text{ otherwise} \end{cases} \tag{A8}$$

$$x_2^*(s) = \begin{cases} \bar{x}_1(s) & \text{if } \bar{x}_2(s) \geq \bar{x}_1(s) > X_1 \\ X_1 & \text{if } \bar{x}_2(s) \geq X_1 > \bar{x}_1(s) \\ x & \text{for any } x \in \mathcal{R}_1(s) \text{ otherwise.} \end{cases} \tag{A9}$$

where $\mathcal{R}_1(s) := \{x \in \mathbb{R} : x < \bar{x}_1(s)\}$ and $\mathcal{R}_2(s) := \{x \in \mathbb{R} : x > \bar{x}_2(s)\}$ denote the rejection sets for countries 1 and 2 and where X_i denotes the most generous credible settlement for a country i. The receiving country then accepts offers if and only if they are at least as good as their outside option. In particular, $y_i(x; s) = \mathbb{1}\{U_i(x; s) \geq W_i(s)\}$.

A.5 Competing for Bargaining Surplus

Competing is costly but improves the odds of recovering the bargaining surplus, which must be of common value to both countries. If there exists a bargaining surplus, $B(s) := x_1^*(p) - x_2^*(p) > 0$, both countries will cooperate and choose an amount of effort as a function of the state s. Then, a country i that's recognized as proposer will receive the value of the bargaining surplus today and possibly in the future, with likelihood according to λ. Hence, the expected net gain of recovering agenda-setting power becomes

$$B(s) + \delta\lambda B(s) + (\delta\lambda)^2 B(s) + \ldots = \frac{B(s)}{1 - \delta\lambda}.$$

There is no equilibrium in pure effort strategies, as (1) countries have an incentive to increase their effort insofar as they are losing the competition and there exists profitable deviations above their opponent's level, (2) countries have incentive to decrease their effort insofar as they are losing and there is no profitable deviation above their opponent's level, and (3) countries that are winning have incentive to marginally decrease their effort.

Instead, consider equilibrium effort such that each country mixes according to a cumulative distribution function (c.d.f.) denoted by F_s^* in state s. In particular, I begin by looking for equilibrium diplomatic strategies where countries only exert nonzero efforts above any frictions that exist $\mu \geq 0$. By exerting effort $e \geq \mu$, country i then has an expected utility

$$\underbrace{W_i(s)}_{\substack{\text{expected}\\ \text{value of war}}} + \frac{B(s)}{1-\delta\lambda}\Big[\underbrace{F_s^*(e)\pi}_{\substack{\text{opponent}\\ \text{exerts less}}} + \underbrace{(1-F_s^*(e))(1-\pi)}_{\text{opponent exerts more}}\Big] - e. \tag{A10}$$

Here, each country's war payoff serves as their baseline value for cooperation, reflecting their outside option. By choosing to exert e on competition, a country can expect a share of the bargaining surplus according to the bracketed component, and have total value for it according to their discount on time δ and how long they expect the settlement to persist λ.

On the other hand, exerting zero effort yields an expected value

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\Big[\underbrace{F_s^*(\mu)\frac{1}{2}(1-\rho)}_{\text{mutual shirking}} + \underbrace{(1-F_s^*(\mu))(1-\pi)}_{\text{opponent exerts effort}}\Big]. \tag{A11}$$

The gain from peace to a country that exerts nothing reflects both (i) the odds of mutual shirking where countries risk inadvertent war ρ, as well as (ii) the odds they are still awarded the agenda-setting power in the event their opponent exerts effort. Mixing strategies require indifference between equations (A10) and (A11), allowing us to solve for the distribution F_s^* (refer to the following section for characterization and Appendix B for full derivation).

A.6 Equilibrium Characterization

This section formally states results that support and extend the analysis of the main text. All proofs are available in Appendix B.

The below proposition provides a full characterization of equilibrium with strategies σ^* under primary focus in the main text.

Proposition A1 (Equilibrium)**.** *An equilibrium exists where, for all $s \in S$ and $q : S \times A \to S$, each country $i = 1, 2$ plays $\sigma_i^*(s) = (a_i^*(s), e_i^*(s), x_i^*(s), y_i^*(x; s))$ defined as follows.*

(i) Initiate war $a_i^*(s) = 1$ if and only if $W_i(s) > U_i(s)$.

(ii) If $U_i(s) \geq W_i(s)$, choose effort $e_i^*(s)$ according to a mixed strategy with distribution

$$F_s^*(e) = \begin{cases} \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)} & \text{for } e \in [0, \mu) \\ \frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right) & \text{for } e \in \left[\mu, \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right] \end{cases}$$

with $F_s^*(e) = 0$ for $e < 0$ and $F_s^*(e) = 1$ for $e > \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu$. Otherwise exert zero effort $e_i^*(s) = 0$.

(iii) Offer $x_i^*(s)$ as given by equations (A8)-(A9) when recognized as agenda setter.

(iv) Accept an offer x, $y_i^*(x; s) = 1$, if and only if $U_i(x; s) \geq W_i(s)$.

Proof of Proposition A1. First, consider the war decision. The countries seek to maximize their expected utility over the long run and if, given state $s \in S$, their war continuation value $W_i(s)$ is larger than their continuation value from cooperating, $U_i(s)$, they will necessarily prefer to fight. However, if there is positive bargaining surplus $B(s) \geq 0$, the expected net gain from exerting effort $e \geq 0$ on diplomacy is

$$\frac{B(s)}{1-\delta\lambda}\Bigg[\pi \Pr(e > \max\{e_{-i}, \mu - e_{-i}\}) + (1-\pi) \times \Pr(e_{-i} > \max\{e, \mu - e\}) + \frac{1}{2}(1-\rho)\Pr(\mu > e + e_{-i})\Bigg] - e. \tag{A12}$$

The net gain is zero when $B(s) = 0$, in which case neither country will be willing to exert a positive amount of effort in equilibrium. Therefore, to understand strategies with nonzero amounts of effort, assume $B(s) > 0$.

To see why there are no pure strategies in equilibrium, consider the following. If a country always exerts amount $e^* > 0$, their opponent would either deviate to an amount greater than e^* or zero. If the opponent deviated to zero, the country will prefer to exert less than e^*. If the opponent deviated to an amount greater than e^*, the country will prefer to move to a greater amount or deviate to zero. If both countries exert the same amount, they will either have an incentive to increase their effort a marginal amount to increase their gain by approximately double or they will prefer to deviate to zero.

I now look for a mixed strategy corresponding to state s given by c.d.f. F_s^* that satisfies equation (A12) for both countries. We can write the payoff from zero effort as

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1-F_s^*(\mu))(1-\pi)\right]. \tag{A13}$$

On the other hand, exerting effort of μ yields an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\pi + (1-F_s^*(\mu))(1-\pi)\right] - \mu. \tag{A14}$$

Using equations (A13) and (A14), we can solve for

$$F_s^*(\mu) = \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}. \tag{A15}$$

We know that μ cannot be the top of the support by assumption that transaction costs are not so high as to discourage cooperation altogether. Then, a country i exerting effort $e > \mu$ will receive an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(e)\pi + (1-F_s^*(e))(1-\pi)\right] - e. \tag{A16}$$

Using the indifference condition for equations (A14) and (A16) and plugging in equation (A15), we find that for $e \geq \mu$,

$$F_s^*(e) = \frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right). \tag{A17}$$

By definition of a c.d.f., we know the largest amount a country can exert and still be indifferent is given by $\bar{e}_s := \inf\{e \geq 0 : F^*(\bar{e}_s) = 1\}$. Using equation (A17), we find that

$$\bar{e}_s = \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu.$$

Together, these equations yield an equilibrium effort strategy presented in Proposition A1. To check for profitable deviations, consider the case where a country exerts effort $e > \bar{e}_s$ with nonzero probability. By deviating, their payoff will be

$$W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - e < W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - \bar{e}_s,$$

and therefore they do not deviate. In words, a country already wins with certainty when expending $\bar{e}_s$, so there is no reason to ever exert more given their opponent plays this strategy as well.

Further, consider a deviation to expending a nonzero amount less than μ, $e \in (0, \mu)$, with some probability. By deviating, their expected payoff will be

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F^*(\mu - e)\frac{1}{2}(1-\rho) + (1 - F^*(\mu - e))(1-\pi)\right] - e. \tag{A18}$$

Since their opponent is playing a strategy such that $F_s^*(\mu) = F^*(\mu - s)$, we can plug this into equation (A18) and see that their payoff is equal to

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1 - F_s^*(\mu))(1-\pi)\right] - e.$$

which is strictly less than their payoff from exerting zero given by equation (A13), hence this is not a profitable deviation. This is sufficient to show that F_s^* is an equilibrium effort strategy.

Equations (A6) and (A7) that govern equilibrium offers yield implicit conditions for $\bar{x}_2(s)$ and $\bar{x}_1(s)$, which represent the settlements at which country 1 and 2 are left indifferent between peace and war, respectively. Each country i will extend an offer equal to country $-i$'s indifference point or the greatest possible offer $-i$ can accept if and only if it is preferable to their own indifference point $\bar{x}_i(s)$ and the offer falls within their budgets $\mathcal{X}_i$. Otherwise, each country i will extend an offer that they know will get rejected, $x \in \mathcal{R}_{-i}(s)$ where $\mathcal{R}_i(s) := \{x : U_i(x; p) < W_i(s)\}$ denotes country i's rejection set in state s. A country i receiving an offer rejects offers in their rejection set and accepts all others. $\square$

Additionally, this is the unique equilibrium when there are no transaction costs.

Proposition A2. *If $\mu = 0$, Proposition A1 characterizes the unique equilibrium.*

Proof of Proposition A2. Since the war, offer, and accept decisions are straightforward, we only need to show that there is no equilibrium under an alternative effort strategy $F' \neq F^*$.

First note that any effort strategy must be continuous. Suppose for example there is an F' with finite probability $\xi > 0$ of exerting effort e' in the distribution. Then, there exists a profitable deviation F'' in which a country could improve their expected share of the bargaining surplus by allocating greater probability to an arbitrarily small increase above e'. Thus, we know all effort strategies must be given by continuous distributions.

Second, zero must be in the support of the distribution. Suppose otherwise, so that the lower bound of the distribution is some $\underline{e} > 0$. Then, following this distribution would imply that countries exert strictly positive amounts despite an expectation that they will have the weakest performance. As a result, there would be a profitable deviation to effort less than $\underline{e}$ that save on the expense of competition without resulting in lower expected shares of the bargaining surplus.

Moreover, the top of the support must be equal to $\bar{e}_s$ as defined above. Suppose there is an alternate distribution F' such that $F'(\bar{e}_s) < 1$, i.e., countries exert greater than $\bar{e}_s$ with positive probability in equilibrium. Because we have just shown that zero must be in the support of F', expending amounts consistent with the new upper bound $\bar{\bar{e}} > \bar{e}_s = \frac{(2\pi-1)B(s)}{1-\delta\lambda}$ needs to be consistent with the lower bound of zero. Let $\bar{\bar{e}} = \bar{e}_s + \varepsilon$ for small $\varepsilon > 0$. This implies

$$W_i(s) + \frac{(1-\pi)B(s)}{1-\delta\lambda} = W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \varepsilon$$

which implies $\varepsilon = 0$, a contradiction. On the other hand, if there exists an $\bar{\bar{e}} < \bar{e}_s$ such that $F'(\bar{\bar{e}}) = 1$, there is a profitable deviation to exerting $\bar{\bar{e}} + \varepsilon$ for an arbitrarily small $\varepsilon > 0$.

Lastly, to show that the distribution is necessarily uniform, suppose for contradiction there is an alternate distribution F' where, for some $\hat{e} \in [0, \bar{e}_s]$, $F'(\hat{e}) > F^*(\hat{e}) = \frac{(1-\delta\lambda)\hat{e}}{(2\pi-1)B(s)}$. Then, the payoff from exerting $\hat{e}$ must be

$$\frac{B(s)}{1-\delta\lambda}\Big[F'(\hat{e})\pi + (1-F'(\hat{e}))(1-\pi)\Big] - \hat{e} = \frac{B(s)}{1-\delta\lambda}\Big[F'(\hat{e})(2\pi-1) + (1-\pi)\Big] - \hat{e} > \frac{(1-\pi)B(s)}{1-\delta\lambda}$$

which is the payoff from zero effort. But we know this must be in the support, a contradiction. The analogous argument can be made for F' such that $F'(\hat{e}) < F^*(\hat{e})$ for some $\hat{e} \in [0, \bar{e}_s]$. Hence, $\mu = 0$ implies that the equilibrium detailed in Proposition A1 is unique. $\square$

A.7 Discussion of Solution Concept

The MPE solution concept is standard for dynamic games where decision-makers interact repeatedly over time and the focus is not on history-dependent mechanisms, especially in infinitely repeated games where folk-theorem results apply.

Most importantly, the Subgame Perfect Equilibrium (SPE) solution concept will allow for many unreasonable equilibria due to the applicability of folk-theorem results. Since both peace and war are costly and countries may prefer one or the other under different circumstances, pathological behavior can be sustained in equilibrium by threatening to punish deviations from obscure sequences of actions. Consider, for example, an SPE of this game where a country initiates war in some periods and cooperates in others, with off-path behavior such that deviations to cooperate result in punishment by other countries to fight wars forever thereafter. When the set of equilibria is too large and includes possibilities that are substantively incredible, a refined solution concept is more suitable.

MPE is therefore the natural choice as it resolves these issues and is the usual choice for games of this nature (Maskin and Tirole, 2001; Fudenberg and Tirole, 1991, p. 501-505). Slantchev (2002) offers a useful discussion of MPE in bargaining models and influential work in international relations that employ this solution concept can be found in, for example, Powell (1993, 2019), Slantchev (2003), Bueno de Mesquita (2005), Carter (2015), Paine (2016), and Leventoğlu and Metternich (2018).

A.8 Diplomacy-Induced Efficient War (Example)

This section presents a simple example to demonstrate the possibility of efficient war due to endogenous effort and provide intuition.

Take any state $s' \in S$ to be absorbing, i.e., $q(s'|s', a) = 1$ for any $a \in A$. Then, the aggregate present value associated with arriving in state s' is given by

$$V(s') = \frac{1}{1-\delta\lambda} - 2\int e dF^*_{s'}(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\left(\frac{1}{1-\delta\lambda} - 2\int e dF^*_{s'}(e) + \dots\right).$$

Recognizing the geometric series, we can rewrite the expression as

$$V(s') = \frac{1 - 2(1-\delta\lambda)\int edF^*_{s'}(e)}{1-\delta}. \tag{A19}$$

Additionally, because s' is absorbing, we have a closed-form expression for its bargaining surplus. In particular, the bargaining surplus in an absorbing state is equal to

$$B(s') = 1 - (1-\delta\lambda)\left[\frac{1}{1-\delta\lambda} - C(s') + \frac{\delta(1-\lambda)}{1-\delta\lambda}V(s')\right] + \delta(1-\lambda)V(s') = (1-\delta\lambda)C(s')$$

where recall that $C(s) := c_1(s) + c_2(s)$. Consequently, we can now express a country's expected effort in an absorbing state s' in terms of the total costs of war in that state and model parameters,

$$\int edF^*_{s'}(e) = \frac{1}{2}\left((2\pi-1)C(s') + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu + \mu\right)\left(1 - \frac{2\mu}{(2\pi-(1-\rho))C(s')}\right).$$

Plugging this into equation (A19), we have

$$V(s') = \frac{1}{1-\delta}\left(1 + \frac{(1-\delta\lambda)(2\pi-1)(2\mu - C(s')(2\pi-(1-\rho)))(2\mu + C(s')(2\pi-(1-\rho)))}{C(s')(2\pi-(1-\rho))^2}\right) \tag{A20}$$

yielding the continuation value for being in s' solely in terms of model primitives.

Now suppose $\underline{s}$ and $\bar{s}$ are two absorbing states. Moreover, let s be the current state such that $q(\underline{s}|s, a_W) = 1$ and $q(\bar{s}|s, a_U) = 1$. In words, fighting ensures the state transitions to $\underline{s}$ whereas the choice to cooperate ensures the state transitions to $\bar{s}$. By Proposition 5, an efficient war results in state s under these conditions if and only if

$$G + \frac{\delta(1-\theta)}{1-\delta\theta}V(\underline{s}) - \frac{\delta(1-\lambda)}{1-\delta\lambda}V(\bar{s}) > C(s). \tag{A21}$$

Because $\underline{s}$ and $\bar{s}$ are absorbing states, we can write this expression solely in terms of model primitives by using equation (A20) to plug in for $V(\underline{s})$ and $V(\bar{s})$. If we choose costs of war $C(s)$, $C(\underline{s})$, and $C(\bar{s})$ such that this condition holds given δ, λ, θ, π, and ρ, then countries will prefer to fight an efficient war as a result of expecting to exert too much effort under cooperation.

Condition (A21) shows how increasing the costs of war in a cooperative state can endogenously cause countries to exert greater effort in competitive diplomacy, creating an incentive for efficient war. Intuitively, a country has the incentive to exert greater effort in state s when $C(s)$ is large in addition to being more likely to choose war state s if $C(s)$ is small. In this simple example, this means that efficient wars are more likely to emerge when there are large costs of war in state $\bar{s}$ and low costs of war in states s and $\underline{s}$. Consider the numerical example where $\delta = 0.9$, $\theta = \lambda = 0.75$, $\pi = 0.9$, $\rho = 0.05$, $C(s) = C(\underline{s}) = 0.1$, and $\mu = 0.001$. Then, an approximate condition for efficient war in state s is $C(\bar{s}) > 0.139978$. Refer to supplementary materials for code.

A.9 Inadvertent Wars and Bargaining Surplus

There is a strong connection between the total probability of inadvertent war and the surplus from cooperation: to understand the effect of a parameter on the former often requires an understanding of its effect on the latter. For example, the risk parameter that governs the conditional probability of war in the event of coordination failure has a direct exogenous effect on the total probability of war, but also an indirect endogenous effect through equilibrium behavior. The same is true for the persistence parameters. As a result, it is not straightforward to make claims about how changes to these parameters affect the total probability of inadvertent war.

Increases to risk will naturally have a positive direct effect; however, it may also have a negative indirect effect if countries respond to the increase in risk by working harder to overcome the barriers to cooperation. Whether one force or the other will prevail inevitably depends on the expected downstream consequences. Specifically, the total probability of inadvertent war is decreasing in the risk of inadvertent war if the changes to equilibrium behavior result in a large enough increase to the cooperative surplus. Moreover, we can also see this inverse relationship borne out in the effect of peace deal reliability and war outcome durability.

Proposition A3. *Let $\lambda = \alpha\theta$. Then,*

(i) $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ *if and only if* $\frac{\partial}{\partial\theta}B(s) \geq 0$,

(ii) $\frac{\partial}{\partial\alpha}\omega(s) \leq 0$ *if and only if* $\frac{\partial}{\partial\alpha}B(s) \geq \frac{-\delta\theta B(s)}{1-\delta\lambda}$, *and*

(iii) $\frac{\partial}{\partial\rho}\omega(s) \leq 0$ *if and only if* $\frac{\partial}{\partial\rho}B(s) \geq \frac{(2\pi-1-\rho)B(s)}{2\rho(2\pi-(1-\rho))}$.

Proof of Proposition A3. **(i) $\boldsymbol{\theta}$:** Taking the partial derivative of $\omega(s)$ with respect to θ,

$$\frac{\partial}{\partial\theta}\omega(s) = \frac{-8\rho\mu^2(1-\delta\lambda)^2}{(2\pi-(1-\rho))^2 B(s)^3} \cdot \frac{\partial}{\partial\theta}B(s)$$

and because the first term must be negative, we can conclude that $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\theta}B(s) \geq 0$. From the definition of $B(s)$, we can conclude $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if

$$\frac{\partial}{\partial\theta}B(s) = \delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) - (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s) \geq 0$$

or equivalently $\delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) > (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s)$.

(ii) $\boldsymbol{\alpha}$: Plugging in $\alpha\theta$ for λ and taking the partial derivative of $\omega(s)$,

$$\frac{\partial}{\partial\alpha}\omega(s) = \frac{-8\rho\mu^2(1-\delta\alpha\theta)(\delta\theta B(s) + (1-\delta\alpha\theta)\frac{\partial}{\partial\alpha}B(s))}{(2\pi-(1-\rho))^2 B(s)^2}.$$

This expression is negative if and only if

$$\frac{\partial}{\partial\alpha}B(s) \geq \frac{-\delta\theta B(s)}{1-\delta\theta}$$

for any valid parameter values.

(iii) $\boldsymbol{\rho}$: Taking the partial derivative of $\omega(s)$ with respect to ρ,

$$\frac{\partial}{\partial\theta}\omega(s) = \frac{4(1-\delta\lambda)^2\mu^2((2\pi-1-\rho)B(s) - 2\rho(2\pi-(1-\rho))\frac{\partial}{\partial\rho}B(s))}{(2\pi-(1-\rho))^3 B(s)^3}.$$

This expression is negative if and only if

$$\frac{\partial}{\partial\rho}B(s) \geq \frac{(2\pi-1-\rho)B(s)}{2\rho(2\pi-(1-\rho))}.$$

Refer to the supplementary files for verification of these derivatives and inequalities. □

To see how these results depend on downstream expectations, it is useful to observe that the effect of a parameter on the bargaining surplus is inextricably tied to its differential effect on a country's incentive to fight. In the case of θ, we can see that $\frac{\partial}{\partial\theta}B(s) \geq 0$ if and only if $\delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) \geq (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s)$ for any state $s \in S$, by definition. On the left-hand side of this equation reflects the effect of war outcome durability on the expected continuation values upon a return to bargaining after cooperation in state s, whereas the right-hand side reflects its effect on the continuation values for war in state s. Inadvertent wars are inversely related to the surplus, and the surplus is increasing if future values after cooperating are expanding at a fast enough rate relative to that of the current war value. Clearly, the way in which actions in the current state of the world map onto probability distributions over future states of the world is paramount in determining whether the inequality will hold.

The above results demonstrate that it is exceedingly difficult to make strong claims about the effect of parameters on the total probability of inadvertent war without conditioning it on how the parameter affects the surplus, which in turn depends on expectations about downstream events. Future analyses that wish to study particular circumstances, such as a particular type of power shift arising from cooperation, can explore special cases of the model with additional assumptions and use the above findings to make concrete claims about their interaction of interest.

A.10 Additional Equilibrium in Special Case with Frictions

The main text focuses on equilibrium strategies σ^* as characterized by Proposition A1 because this is an equilibrium for all possible parameter values and has been shown to be unique in the absence of transaction costs. However, for nonzero transaction costs $\mu > 0$ and competition characterized by $\pi \in (\frac{1}{6}(3+\rho), \frac{1}{2}(1+\rho))$, it is possible for countries to exert nonzero effort less than μ. In particular, there exists an equilibrium with strategies $\tilde{\sigma}$ where countries always expend nonzero effort in competitive diplomacy. This section shows that the results on inadvertent war continue to hold given countries play these strategies instead.

Proposition A4 (Equilibrium)**.** *If $\mu > 0$ and $\pi \in (\frac{1}{6}(3+\rho), \frac{1}{2}(1+\rho))$, there is an equilibrium where each country $i = 1, 2$ plays $\tilde{\sigma}_i(s) = (a_i^*(s), \tilde{e}_i(s), x_i^*(s), y_i^*(x; s))$ for all $s \in S$, with $a_i^*(s)$, $x_i^*(s)$, and $y_i^*(x; s)$ as defined in Proposition A1 and $\tilde{e}_i(s)$ as a random draw from the distribution*

$$\tilde{F}_s(e) = \begin{cases} \frac{2(1-\delta\lambda)(3(2\pi-1)-\rho)e}{(2\pi-1-\rho)(2\pi-(1-\rho))B(s)} & \text{for } e \in [0, \frac{\mu}{2}) \\ \frac{2(1-\delta\lambda)(2\mu(2\pi-1)-(2\pi-(1-\rho))e)}{(2\pi-1-\rho)(2\pi-(1-\rho))B(s)} & \text{for } e \in [\frac{\mu}{2}, \mu) \\ \frac{(1-\delta\lambda)((2\pi-1-\rho)\mu+(2\pi-(1-\rho))e)}{(2\pi-1)(2\pi-(1-\rho))B(s)} & \text{for } e \in [\mu, \frac{(2\pi-1)B(s)}{1-\delta\lambda} + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu] \end{cases}$$

with $\tilde{F}_s(e) = 0$ below and 1 above if $U_i(s) \geq W_i(s)$, or else $\tilde{F}_s(e) = 1$ for all $e \geq 0$.

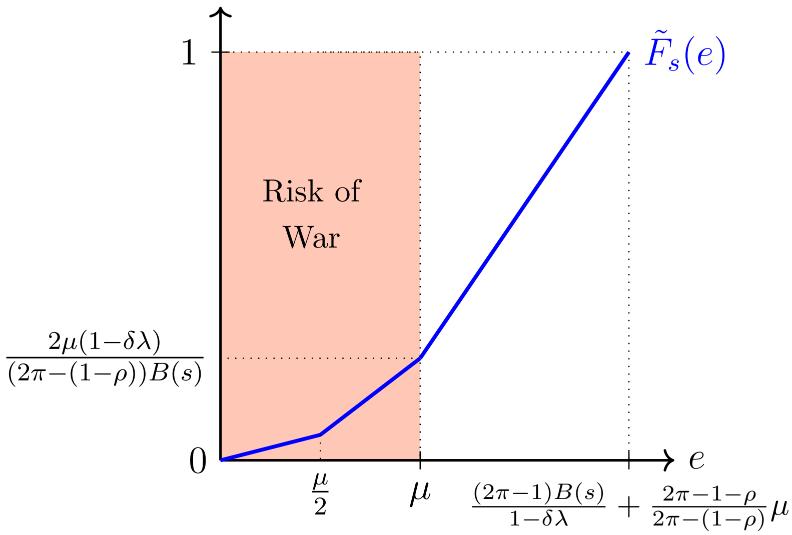

Figure A5. Cumulative distribution for diplomatic effort $\tilde{F}_s$

Proof of Proposition A4. The derivation of $a^*(s)$, $x^*(s)$, and $y^*(x;s)$ are identical to the derivation from Proposition A1. Then, to look for a new equilibrium diplomatic effort strategy $\tilde{F}_s \neq F_s^*$ such that countries occasionally exert positive amounts less than μ, suppose there exists an $e \in (0, \frac{\mu}{2})$ such that $\tilde{F}_s(e) \neq \tilde{F}_s(e+\varepsilon)$ for any $\varepsilon \neq 0$. Then,

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[\tilde{F}_s(e')\frac{1}{2}(1-\rho) + (1-\tilde{F}_s(e'))(1-\pi)\right] - e = W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - \bar{\tilde{e}}_s$$

where $\bar{\tilde{e}}_s \equiv \inf\{e \geq 0 : \tilde{F}_s(e) = 1\}$ and $e' \equiv \mu - e$. Solving for $\tilde{F}_s(e')$ yields

$$\tilde{F}_s(e') = \frac{2((2\pi-1)B(s) + (1-\delta\lambda)(\bar{\tilde{e}}_s + e' - \mu))}{(2\pi-(1-\rho))B(s)}. \tag{A22}$$

We also know that in equilibrium we need to be indifferent between efforts e and e',

$$\begin{aligned} &W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[\tilde{F}_s(e')\frac{1}{2}(1-\rho) + (1-\tilde{F}_s(e'))(1-\pi)\right] - e \\ &\quad = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[\tilde{F}_s(e)\frac{1}{2}(1-\rho) + (\tilde{F}_s(e') - \tilde{F}_s(e))\pi + (1-\tilde{F}_s(e'))(1-\pi)\right] - e' \end{aligned}$$

which implies

$$\tilde{F}_s(e) = \tilde{F}_s(e') - \frac{2(e'-e)(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)} \tag{A23}$$

Using equations (A22) and (A23) to plug in for $\tilde{F}_s(e)$, we have

$$\tilde{F}_s(e) = \frac{2((2\pi - 1)B(s) + (1-\delta\lambda)(\bar{\tilde{e}}_s - e))}{(2\pi - (1-\rho))B(s)} - \frac{2(\mu - 2e)(1-\delta\lambda)}{(2\pi - (1-\rho))B(s)} \tag{A24}$$

To proceed, we need to know the value of $\bar{\tilde{e}}_s$. We can find it from the equilibrium condition that for any $e = \mu$, we have

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[\tilde{F}_s(\mu)\frac{1}{2}(1-\rho) + (1-\tilde{F}_s(\mu))(1-\pi)\right]$$
$$= W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[\tilde{F}_s(\mu)\pi + (1-\tilde{F}_s(\mu))(1-\pi)\right] - \mu \tag{A25}$$

where the left-hand side follows from the need to have zero in the support. If zero is not in support, there is always a profitable deviation from the infinum of the support to zero. We know from equation (A23) that

$$\tilde{F}_s(\mu) = \frac{2\mu(1-\delta\lambda)}{(2\pi - (1-\rho))B(s)}.$$

Likewise, for all $e'' \geq \mu$, we must have

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[\tilde{F}_s(\mu)\frac{1}{2}(1-\rho) + (1-\tilde{F}_s(\mu))(1-\pi)\right.$$
$$\left. = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[\tilde{F}_s(e'')\pi + (1-\tilde{F}_s(e''))(1-\pi)\right] - e''\right.$$

from which we recover

$$\tilde{F}_s(e'') = \frac{(1-\delta\lambda)((2\pi - 1 - \rho)\mu + (2\pi - (1-\rho))e'')}{(2\pi - 1)(2\pi - (1-\rho))B(s)}.$$

Then, we can solve equation to recover

$$\bar{\tilde{e}}_s = \frac{(2\pi - 1)B(s)}{1-\delta\lambda} + \frac{2\pi - 1 - \rho}{2\pi - (1-\rho)}\mu. \tag{A26}$$

Plugging this into equation (A24), we have that for any $e \in (0, \frac{\mu}{2})$,

$$\tilde{F}_s(e) = \frac{2(1-\delta\lambda)(3(2\pi - 1) - \rho)e}{(2\pi - 1 - \rho)(2\pi - (1-\rho))B(s)}. \tag{A27}$$

Then, suppose there exists an $e' \in (\frac{\mu}{2}, \mu)$ such that $\tilde{F}_s(e') \neq \tilde{F}(e' + \varepsilon)$ for any $\varepsilon \neq 0$. Then, letting $e = \mu - e'$, we have

$$\tilde{F}_s(e') = \frac{2(1-\delta\lambda)(2\mu(2\pi-1) - (2\pi-(1-\rho))e')}{(2\pi-1-\rho)(2\pi-(1-\rho))B(s)}. \tag{A28}$$

For both equations (A27) and (A28) to be positive and increasing in effort, the condition $\pi \in (\frac{1}{6}(3+\rho), \frac{1}{2}(1+\rho))$ needs to be satisfied. Otherwise, this does not reflect a valid c.d.f. Additionally, by equations (A26) and (A28), we know that $\bar{\tilde{e}}_s = \bar{e}_s$ and $\tilde{F}_s(e) = F_s^*(e)$ for all $e \geq \mu$.

There is no profitable deviation from $\tilde{F}_s$ for the same reasons as F_s^*. Countries are indifferent over effort at all amounts in the support and the country would do strictly worse by exerting more than the supremum of the distribution. Moreover, given equilibrium effort $\tilde{e}_i(s)$ as a random draw from $\tilde{F}_s$, there is no profitable deviation from other actions for the same reasons as F_s^* from the proof of Proposition A1. □

Unlike the equilibrium of the main text which holds for all parameters, this equilibrium depends on nonzero transaction costs and a decisiveness of competition that is bounded by the risk of inadvertent war. In particular, when the risk of inadvertent war is close to zero, this equilibrium becomes impossible unless the competition is extremely indecisive, with π approximately $\frac{1}{2}$. As the risk of inadvertent war increases, there is an intermediate range of tolerable π that could support this equilibrium. As the risk of inadvertent war increases to 1, the competition demanded to support the equilibrium is increasingly decisive. This is sensible because, holding ρ at a constant intermediate value, we can see that very indecisive contests will make it unstable to support willingly exerting amounts between 0 and $\frac{\mu}{2}$ instead of profitably deviating to zero, whereas very decisive contests will make it unstable to support efforts between $\frac{\mu}{2}$ and μ instead of profitably deviating to amounts above μ.

This result is illustrated in Figure A5. Although diplomatic effort given equilibrium strategies $\tilde{\sigma}$ is different than those given σ^*, all other behavior (such as whether to launch a war, what settlement to offer when proposing, and what settlements to accept if not proposing) remains the same in terms of the bargaining surplus. It is important to note, however, that this does not imply behavior is identical, as the bargaining surplus is a function of equilibrium strategies and will vary across these two cases. Nonetheless, countries always exert a positive amount of effort in competitive diplomacy, there remains a risk of inadvertent war since aggregate effort can still be less than μ.

Remark A1. *Given equilibrium strategies $\tilde{\sigma}$, we can exploit linearity in the c.d.f. to express the total probability of inadvertent war by $\tilde{\omega}(s) := \rho\tilde{F}(\mu)\tilde{F}(\frac{\mu}{2}) = \rho\left(\frac{2\mu^2(1-\delta\lambda)^2(3(2\pi-1)-\rho)}{(2\pi-1-\rho)(2\pi-(1-\rho))^2(B(s))^2}\right) > 0$ for any state s and transition function q such that there is surplus from cooperation, $B(s) > 0$.*

Remark A2. *$\tilde{F}_s(x) \leq F^*_s(x)$ for all $x \leq \mu$. Because inadvertent wars require both countries to exert sufficiently low amounts, this implies $\omega(s) > \tilde{\omega}(s)$ for all states s and transitions q.*

Now, we can show that the result in Proposition 7 continues to apply when countries play this alternate equilibrium strategy profile. As the below proposition shows, all conditions in which an increase to π results in a decrease the total probability of inadvertent war given equilibrium strategies σ^* must also decrease the total probability of inadvertent war given strategies $\tilde{\sigma}$. In fact, the result is even more stark in this case, as the decisiveness of competition is guaranteed to reduce the total probability of inadvertent war under all configurations of parameters.

Proposition A5. *$\frac{\partial}{\partial\pi}\tilde{\omega}(s) < 0$.*

Proof of Proposition A5. Taking the partial derivative, we have

$$\begin{aligned}\frac{\partial}{\partial\pi}\tilde{\omega}(s) = -\Bigg(&(4(1-\delta\lambda)^2\mu^2\rho(2(3+12\pi^2+3\rho+2\rho^2-6\pi(2+\rho))B(s)\\ &+(24\pi^3-3-\rho+3\rho^2+\rho^3-4\pi^2(9+\rho)+\pi(18+4\rho-6\rho^2))\frac{\partial}{\partial\pi}B(s)\Bigg)\\ &\Bigg/\Bigg((2\pi-1+\rho)^2(2\pi-1+\rho)^3(B(s))^3\Bigg)\end{aligned}$$

which then implies $\frac{\partial}{\partial\pi}\tilde{\omega}(s) \geq 0$ if and only if

$$\frac{\partial}{\partial\pi}B(s) \geq \frac{-(24\pi-24\pi^2-6+6(2\pi-1)\rho-4\rho^2)B(s)}{18\pi-36\pi^2+24\pi^3-3+(4\pi-4\pi^2-1)\rho+3(2\pi-1)\rho^2+\rho^3} > 0.$$

However, by the proof of Proposition 4, we know that $\frac{\partial}{\partial\pi}B(s) \leq 0$, implying that $\frac{\partial}{\partial\pi}\tilde{\omega}(s) < 0$ for all possible parameters that support equilibrium strategies $\tilde{\sigma}$. □

B Proofs

Supplementary files are available for verification of algebraic steps.

Proof of Lemma 1. Without loss of generality, suppose country 1 is the agenda setter. Then, recall from Appendix A.4 that the equilibrium offer is equal to

$$x_1^*(s) = 1 - (1 - \delta\lambda)W_2(s) + \delta(1 - \lambda)\sum_{s' \in S} V_2(s')q(s'|s, a_U)$$

if $X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s)$, or else $x_1^*(s) = X_2$ if $\bar{x}_2(s) > X_2 \geq \bar{x}_1(s)$ or $x_1^*(s) = x$ for any $x \in \mathcal{R}_2(s)$ otherwise. In the latter two cases, there is no direct effect of θ or λ. Therefore, suppose $X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s)$, which reflects the primary setting of interest: on-path equilibrium behavior that is unaffected by budget constraints. Unpacking the continuation values, we can see that

$$x_1^*(s) = \frac{\delta(\lambda - \theta)}{1 - \delta\theta} - (1 - \delta\lambda)c_2(s) + \sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s, a_W) + \delta(1-\lambda)q(s'|s, a_U)\right]$$

Taking the partial derivative with respect to θ:

$$\frac{\partial}{\partial\theta}x_1^*(s) = \underbrace{\frac{-\delta(1-\delta\lambda)}{(1-\delta\theta)^2}}_{\text{direct}} + \underbrace{\frac{\partial}{\partial\theta}\left(\sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s, a_W) + \delta(1-\lambda)q(s'|s, a_U)\right]\right)}_{\text{indirect}}$$

where we can see the direct effect must be negative under any configuration of parameters.

Next, taking the partial derivative with respect to λ:

$$\frac{\partial}{\partial\lambda}x_1^*(s) = \underbrace{\frac{1}{1-\delta\theta} + \delta c_2(s)}_{\text{direct}} + \underbrace{\frac{\partial}{\partial\lambda}\left(\sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s, a_W) + \delta(1-\lambda)q(s'|s, a_U)\right]\right)}_{\text{indirect}}$$

from which the direct effect must be positive. Assuming the direct effects prevail ensures that country 1 can extract greater concessions from country 2 in equilibrium when θ is lower and λ is higher. By symmetry country 2 can extract greater concessions under these same circumstances, from which we can conclude agenda-setting power is more valuable. $\square$

Proof of Proposition 1. In the absence of competitive diplomacy, we can suppose, for example, that $\pi = \frac{1}{2}$ and therefore the agenda setter is randomly recognized with probability $\frac{1}{2}$ regardless of their diplomatic effort. Any other rule that does not reward competition, such as deterministic agenda setter endowments, are also acceptable. From there, it follows that countries will not exert costly effort for no gain, and hence $e_1(s) = e_2(s) = 0$ for all states s.

Since cooperation then does not entail inefficiencies, it must be that for any period s, the sum of continuation values under an always cooperate strategy σ_U is $V^{\sigma_U}(s) = V_1^{\sigma_U}(s) + V_2^{\sigma_U}(s) = \frac{1}{1-\delta}$ for any state $s \in S$. Further, since war is inherently costly, the sum of the continuation values in under any strategy σ_W that involves some war must be strictly less than this amount, $V^{\sigma_W}(s) < V^{\sigma_U}(s)$ for any state s. Let $\hat{s}$ be a state where at least one country prefers to fight a war in equilibrium strategy σ^*. Then, we know $V^{\sigma^*}(\hat{s}) < V^{\sigma_U}(\hat{s})$ which in turn implies $V_i^{\sigma^*}(\hat{s}) < V_i^{\sigma_U}(\hat{s})$ for at least one country i. Since war occurs in state $\hat{s}$, we know that for this country $W_i^{\sigma^*}(\hat{s}) > U_i^{\sigma^*}(\hat{s})$.

Now suppose country $-i$ is recognized as agenda setter in state $\hat{s}$. By extending equilibrium offer according to σ^*, it will be rejected and war will ensue, leaving them with a payoff $W_{-i}^{\sigma^*}(\hat{s})$. However, we know that, due to the inefficiency of war, countries must lose out on a total surplus of $\frac{1}{1-\delta} - V(\hat{s}) > 0$. Therefore, there is a strictly profitable deviation to strategy σ^{**} offer an $x^{**} \in \mathbb{R}$ such that $U_i(x^{**}; \hat{s}) = W_i(\hat{s})$, allowing $-i$ to consume the additional surplus $U_{-i}^{\sigma^{**}}(\hat{s}) = W_{-i}^{\sigma^*}(\hat{s}) + \frac{1}{1-\delta} - V^{\sigma^*}(\hat{s})$. Country $-i$ would be strictly better off by playing this strategy and country i would accept this offer as it satisfies their indifference condition. Further, since country $-i$ is willing to extend this offer, country i will be able to recover a settlement at least as good as x^{**} in the event they are recognized as agenda setter. Therefore, the only way countries do not reach peace is if this settlement is not feasible for one of the countries, $x^{**} \notin \cap_i \mathcal{X}_i$. □

Proof of Proposition 2. By Proposition A1, $\mu = 0$ implies

$$\frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right) = \frac{(1-\delta\lambda)e}{(2\pi-1)B(s)}$$

for all $e \in [0, \bar{e}_s^{\mu=0}]$ and some $\bar{e}_s^{\mu=0} \geq 0$. The upper bound can be solved by observing that $F_s^*(\bar{e}_s^{\mu=0}) = 1$, which yields

$$\bar{e}_s^{\mu=0} = \frac{(2\pi-1)B(s)}{1-\delta\lambda}.$$

By Proposition A2, this equilibrium is unique. □

Proof of Proposition 3. First, define the following set of terms for notational convenience. Let $C(s) := c_1(s) + c_2(s)$ reflect the total costs of war in state s, $W(s) = \sum_i W_i(s)$ the sum of the war continuation values in state s, $\hat{W}(s) := W(s) + C(s)$ the sum of the war continuation values less the total costs of war in state s, $U(s) := \sum_i U_i(s)$ the sum of the cooperation continuation values in state s, and $V(s) := \sum_i V_i(s)$ the sum of the ex-ante continuation values in state s.

Part 1. Costs of war: Recall that total expected effort $\int edF_s^*(e) = \frac{(2\pi-1)B(s)}{2(1-\delta\lambda)}$, where

$$B(s) = 1 - (1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s' \in S} V(s')q(s'|s, a_U)$$

defines the bargaining surplus for all $s, s' \in S$, for both countries $i = 1, 2$. Then, plugging in for $B(s)$, total effort allocated to competitive diplomacy in state s is

$$E(s) := 2\int edF_s^*(e) = \frac{2\pi - 1}{1-\delta\lambda}\left[1 - (1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s' \in S} V(s')q(s'|s, a_U)\right].$$

Note that $\frac{\partial}{\partial C(s)}E(s) = 0$ for any $s \in S$ such that $V(s) = W(s)$ and hence $E(s) = 0$. Letting $s_{min} := \arg\min_s\{\frac{\partial}{\partial C(s)}E(s) : V(s) = U(s)\}$ and $C(s_{min}) := C$, we can take the partial derivative,

$$\begin{aligned}
\frac{\partial}{\partial C}E(s_{min}) &= \frac{2\pi - 1}{1-\delta\lambda}\left[(1-\delta\lambda)\left(1 - \frac{\partial}{\partial C}\hat{W}(s_{min})\right) + \delta(1-\lambda)\sum_{s' \in S}\frac{\partial}{\partial C}U(s')q(s'|s_{min}, a_U)\right] \\
&= \frac{2\pi - 1}{1-\delta\lambda}\left[(1-\delta\lambda)\left(1 - \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial C}U(s_{min})\right) + \delta(1-\lambda)\frac{\partial}{\partial C}U(s_{min})\right] \\
&= 2\pi - 1 + \left[\frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\right]\frac{\partial}{\partial C}U(s_{min})
\end{aligned}$$

where we assume that $q(s_{min}|p, a) = 1$ for all $s \in S$ to impose the worst case scenario if it is possible for $\frac{\partial}{\partial C(s)}E(s) < 0$. If the partial derivative of $E(s_{min})$ with respect to C is positive under these conditions, then it must be positive in all cases.

Plugging in for $\frac{\partial}{\partial C}U(s_{min})$, we have

$$\begin{aligned}\frac{\partial}{\partial C}E(s_{min}) &= 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\frac{\partial}{\partial C}U(s_{min})\right)\\ &= 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\\ &\quad \times\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\Big(\dots\Big)\right)\right).\end{aligned}$$

Noting the geometric series, we can express the above as

$$\frac{\partial}{\partial C}E(s_{min}) = 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\left(\frac{-(1-\delta\lambda)\frac{\partial}{\partial C}E(s_{min})}{1-\delta}\right) \tag{A29}$$

Solving for $\frac{\partial}{\partial C}E(s_{min})$,

$$\frac{\partial}{\partial C}E(s_{min}) = \left[1 - \frac{\delta(1-\delta)^2(2\pi-1)(\lambda-\theta)}{(1-\delta\lambda)^2(1-\delta\theta)}\right]^{-1}(2\pi-1) > 0$$

which is strictly positive for all valid parameter values.

Part 2. Welfare: Define $s_{i,max} := \arg\max_s \frac{\partial}{\partial c_i(s)}V_i(s)$. If $V_i(s_{i,max}) = W_i(s_{i,max})$, it is straightforward to see that

$$\begin{aligned}\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max}) &= -1 + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'\in S}\frac{\partial}{\partial c_i(s_{i,max})}V(s')q(s'|s_{i,max}, a_W)\\ &\leq -1 + \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max})\end{aligned}$$

implying

$$\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max}) \leq -\frac{1-\delta\theta}{1-\delta} < 0.$$

On the other hand, if $V_i(s_{i,max}) = U_i(s_{i,max})$, we have

$$\begin{aligned}
\frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) &= -\frac{\partial}{\partial c_i(s)} \int e dF_s^*(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s' \in S} \frac{\partial}{\partial c_i(s_{i,max})} V(s') q(s'|s_{i,max}, a_W) \\
&\leq -\frac{\partial}{\partial c_i(s)} \int e dF_s^*(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) \\
&\leq \frac{\delta(1-\lambda)}{1-\delta\lambda} \frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}),
\end{aligned}$$

where the final inequalty follows from the first part of this proof. This condition requires

$$\frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) = 0,$$

concluding the proof. $\square$

Proof of Proposition 4.

Part 1. Partial derivative: *Step 1. Show that* $\frac{\partial}{\partial \pi} B(s) \leq 0$ *and* $\frac{\partial}{\partial \pi} \int e dF_s^*(e) \geq 0$ *for all* $s \in S$. Defining $E(s) := 2\int e dF_s^*(e)$, we can expand

$$E(s) = \left(\frac{(2\pi - 1)B(s)}{1-\delta\lambda} - \frac{2\pi - 1 - \rho}{2\pi - (1-\rho)} \mu + \mu \right) \times \left(1 - \frac{2\mu(1-\delta\lambda)}{(2\pi - (1-\rho))B(s)} \right).$$

This implies the partial derivative with respect π is

$$\begin{aligned}
\frac{\partial}{\partial \pi} E(s) = &\left(1 - \frac{2\mu(1-\delta\lambda)}{(2\pi - (1-\rho))B(s)} \right) \left(\frac{2B(s)}{1-\delta\lambda} + \frac{(2\pi-1)\frac{\partial}{\partial \pi} B(s)}{1-\delta\lambda} - \frac{4\mu\rho}{2\pi - (1-\rho))^2} \right) \\
&- \Big(2\mu(2\mu\rho(\delta\lambda - 1) - (2\pi - 1)(2\pi - (1-\rho))B(s))(2B(s) + (2\pi - (1-\rho))B(s)) \Big) \\
&\Big/ \Big((2\pi - (1-\rho))^3 B(s)^2 \Big)
\end{aligned} \tag{A30}$$

Equation (A30) implies that $\frac{\partial}{\partial \pi} B(s) \geq 0$ is a sufficient condition[11] to show that $\frac{\partial}{\partial \pi} E(s) \geq 0$ for all $s \in S$ and for any transition function $q : S \times A \to S$.

[11] See the supplementary files for the precise threshold that gives a necessary and sufficient condition in terms of parameters, which must be strictly negative.

Suppose there exists s and q such that $\frac{\partial}{\partial \pi}E(s) < 0$. By definition of $B(s)$, we can write

$$\frac{\partial}{\partial \pi}B(s) = \delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial \pi}V(s')q(s'|s,a_U) - (1-\delta\lambda)\frac{\partial}{\partial \pi}W(s).$$

The expression is decreasing in the partial derivative of $W(s)$ and increasing in the partial derivative of the expected ex-ante continuation value tomorrow from state s. Note that

$$\begin{aligned}\frac{\partial}{\partial \pi}W(s) &= \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'}\frac{\partial}{\partial \pi}V(s')q(s'|s,a_W)\\ \frac{\partial}{\partial \pi}U(s) &= -\frac{\partial}{\partial \pi}E(s) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'}\frac{\partial}{\partial \pi}V(s')q(s'|s,a_U).\end{aligned}$$

Let $s_{min} := \arg\min_s \frac{\partial}{\partial \pi}B(s)$. Hence, because $-\frac{\partial}{\partial \pi}E(s) > 0$, the minimum value of $\frac{\partial}{\partial \pi}B(s_{min})$ cannot exceed the case where $V(s_{min}) = W(s_{min})$ and $q(s_{min}|s_{min},a_W) = 1$, or equivalently $\frac{\partial}{\partial \pi}B(s_{min}) = 0$.

Therefore, since $\frac{\partial}{\partial \pi}E(s) < 0$ implies $\frac{\partial}{\partial \pi}B(s) \geq 0$, which in turn implies $\frac{\partial}{\partial \pi}E(s) \geq 0$, we can conclude $\frac{\partial}{\partial \pi}E(s) \geq 0$. By the same logic as above, replacing s_{min} for $s_{max} := \arg\max_s \frac{\partial}{\partial \pi}B(s)$ and solving, we have $\frac{\partial}{\partial \pi}B(s) \leq 0$ for all $s \in S$.

Step 2. Show that $\frac{\partial}{\partial \pi}V_i(s) \leq 0$ for all $s \in S$. Let $s_{i,max} = \arg\max_s \frac{\partial}{\partial \pi}V_i(s)$. If $V_i(s_{i,max}) = W_i(s_{i,max})$ then

$$\begin{aligned}\frac{\partial}{\partial \pi}W_i(s_{i,max}) &= \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'\in S}\frac{\partial}{\partial \pi}V_i(s')q(s'|s_{i,max},a_W)\\ &\leq \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial \pi}W_i(s_{i,max})\end{aligned}$$

which is only satisfied by $\frac{\partial}{\partial \pi}W_i(s_{i,max}) = 0$.

On the other hand, $V_i(s_{i,max}) = U_i(s_{i,max})$ implies

$$\begin{aligned}\frac{\partial}{\partial \pi}U_i(s_{i,max}) &= -\frac{\partial}{\partial \pi}E(s_{i,max}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'\in S}\frac{\partial}{\partial \pi}V_i(s')q(s'|s_{i,max},a_U)\\ &\leq -\frac{\partial}{\partial \pi}E(s_{i,max}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\frac{\partial}{\partial \pi}U_i(s_{i,max})\end{aligned}$$

which implies

$$\frac{\partial}{\partial \pi}U_i(s_{i,max}) \leq \frac{-(1-\delta\lambda)\frac{\partial}{\partial\pi}E(s_{i,max})}{1-\delta} \leq 0.$$

Part 2. Completely decisive contests: First note that if $U_i(s) < W_i(s)$ then the result is trivially satisfied. Suppose then that $U_i(s) \geq W_i(s)$.

Recall $U_i(s)$ can be expressed by the expected value of exerting zero effort

$$U_i(s) = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1-F_s^*(\mu))(1-\pi)\right]. \tag{A31}$$

Additionally, recall from Proposition A1 the definition of $F_s^*(\mu)$. Plugging into (A31) yields

$$U_i(s) = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[1-\pi+\frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}\left(\frac{1}{2}(1-\rho)-(1-\pi)\right)\right]$$

and setting $\pi = 1$ yields $U_i(s) = W_i(s) + \frac{1-\rho}{1+\rho}\mu$, concluding the proof. □

Proof of Proposition 5. By Definition 2, $\bar{x}_1(s) > \bar{x}_2(s)$ implies that war is efficient. By equations (A6) and (A7), this is equivalent to

$$1-(1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s'\in S}V(s')q(s'|s,a_U) < 0$$

$$1-(1-\delta\lambda)\left[\frac{1}{1-\delta\theta 0} - C(s) + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'\in S}V(s')q(s'|s,a_W)\right] + \delta(1-\lambda)\sum_{s'\in S}V(s')q(s'|s,a_U) < 0$$

$$\frac{1}{1-\delta\lambda} - \frac{1}{1-\delta\theta} + \sum_{s'\in S}V(s')\left[\frac{\delta(1-\lambda)}{1-\delta\lambda}q(s'|s,a_U) - \frac{\delta(1-\theta)}{1-\delta\theta}q(s'|s,a_W)\right] + C(s) < 0,$$

yielding the desired equation by definition of G and $\Delta V(s)$. □

Proof of Corollary 1. Recall that efficient wars break out if and only if $\bar{x}_1(s) > \bar{x}_2(s)$, or equivalently

$$1-(1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s'}V(s')q(s'|s,a_U) < 0.$$

Plugging in for $W(s)$, this yields the following condition in terms of the total costs of war,

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'} V(s')q(s'|s, a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'} V(s')q(s'|s, a_U).$$

Letting $\lambda = 0$ and $\theta = 1$, the condition becomes

$$C(s) < \frac{\delta}{1-\delta} - \sum_{s'} V(s')q(s'|s, a_U).$$

Then, we can expand the expression to

$$C(s) < \frac{\delta}{1-\delta} - \sum_{s'}\left[1 - \tau(s') + \delta\sum_{s''} V(s'')q(s''|p', a_U)\right] q(s'|s, a_U)$$

where $\tau(s) := 2\int e dF_s^*(e)$ for all s such that $V(s) = U(s)$ and $\tau(s) := C(s)$ for all s such that $V(s) = W(s)$. Letting $\hat{\tau}_s := \sum_{s'} \tau(s')q(s'|s, a_U)$ denote the expected inefficiency upon returning to bargaining after cooperation in state s (across both diplomatic effort and costs of war depending on which arises), we can rewrite the condition as

$$C(s) < \frac{\delta}{1-\delta} + \hat{\tau}_s - 1 - \delta\sum_{s'}\sum_{s''} V(s'')q(s''|s', a_U)q(s'|s, a_U)$$

Choose any $\tau \in (0, \hat{\tau}_s)$. Then, we have a sufficient condition

$$\begin{aligned} C(s) &< \frac{\delta}{1-\delta} + \tau - 1 - \delta\sum_{s'}\sum_{s''} V(s'')q(s''|p', a_U)q(s'|s, a_U) \\ &< \frac{\delta}{1-\delta} + \tau - 1 - \delta\sum_{s'}\sum_{s''}\left[1 - \hat{\tau}_{s''} + \delta\sum_{s'''} V(s''')q(s'''|p', a_U)\right] q(s''|p', a_U)q(s'|s, a_U). \end{aligned}$$

Again choosing a $\tau \in (0, \hat{\tau}_{s''})$, we can write a sufficient condition as

$$C(s) < \frac{\delta}{1-\delta} + (\tau - 1)(1 + \delta) - \delta^2\sum_{s'}\sum_{s''}\sum_{s'''} V(s''')q(s'''|p', a_U)q(s''|p', a_U)q(s'|s, a_U).$$

Following this geometric series, war is directly preferred to cooperation if

$$C(s) < \frac{\delta + \tau - 1}{1-\delta} \tag{A32}$$

Because $\lim_{\delta\to 1^-} \frac{\delta+\tau-1}{1-\delta} = \tau\infty$ and $\tau > 0$, inequality (A32) holds for sufficiently large δ. $\square$

Proof of Corollary 2. Recall from the proof of Corollary 1 that a general condition for efficient war is

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'} V(s')q(s'|s,a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'} V(s')q(s'|s,a_U)$$

for any $s \in S$ and $q : S \times A \to S$.

If we know that q satisfies $\sum_{s'} V(s')q(s'|s,a_U) \geq \sum_{s'} V(s')q(s'|s,a_W)$ for all s, then we can deduce the following sufficient condition,

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \left[\frac{\delta(1-\theta)}{1-\delta\theta} - \frac{\delta(1-\lambda)}{1-\delta\lambda}\right]\sum_{s'} V(s')q(s'|s,a_W).$$

We know that $C(s) > 0$ and $\sum_{s'} V(s')q(s'|s,a_W) \leq \frac{1}{1-\delta}$ for all $s \in S$. When $\lambda > \theta$, the first component (G) is strictly negative and the second bracketed component is strictly positive. Then, a necessary condition for efficient war becomes

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{1}{1-\delta}\left[\frac{\delta(1-\theta)}{1-\delta\theta} - \frac{\delta(1-\lambda)}{1-\delta\lambda}\right] = 0,$$

which cannot hold. $\square$

Proof of Proposition 6. Refer to the proof of Proposition A1. $\square$

Proof of Proposition 7. By the definition of $\omega(s)$, we have

$$\frac{\partial}{\partial\pi}\omega(s) = \frac{-8\mu^2(1-\delta\lambda)^2\rho}{(2\pi-(1-\rho))^2B(s)^2}\left[\frac{2}{2\pi-(1-\rho)} + \frac{\frac{\partial}{\partial\pi}B(s)}{B(s)}\right]$$

For this condition to be negative, we require

$$\frac{\partial}{\partial\pi}B(s) \geq \frac{-2B(s)}{2\pi-(1-\rho)} := D_*$$

where D_* denotes the floor of $\frac{\partial}{\partial\pi}B(s)$ if the probability of war is to decrease in competition. Note that, for given any valid set of parameters $(\delta,\lambda,\theta,\rho,\mu)$, any transition function q, and any consequent bargaining surplus $B(s)$, it must be that $D_* < 0$.

Next, recall that by the definition of $B(s)$, we have

$$
\begin{aligned}
\frac{\partial}{\partial \pi} B(s) &= \delta(1-\lambda) \sum_{s'} \frac{\partial}{\partial \pi} V(s') q(s'|s, a_U) - (1-\delta\lambda) \frac{\partial}{\partial \pi} W(s) \\
&= \sum_{s'} \frac{\partial}{\partial \pi} V(s') \left[\delta(1-\lambda) q(s'|s, a_U) - \frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta} q(s'|s, a_W) \right] \\
&\geq \frac{\partial}{\partial \pi} V(s_{min}) \delta(1-\lambda) - \frac{\partial}{\partial \pi} V(s_{max}) \frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}
\end{aligned}
$$

where $s_{min} := \arg\min_s \frac{\partial}{\partial \pi} B(s)$ and $s_{max} := \arg\max_s \frac{\partial}{\partial \pi} B(s)$. Then, with $\lambda = 1$ this becomes a lower bound of $-\frac{\partial}{\partial \pi} V(s_{max}) \frac{\delta(1-\theta)(1-\delta)}{1-\delta\theta}$. Recall by Proposition 4 that $\frac{\partial}{\partial \pi} V(s) \leq 0$ for all s and q and hence the lower bound on $\frac{\partial}{\partial \pi} B(s)$ is weakly positive when $\lambda = 1$. As a result, we know that $1 = \lambda \geq \theta$ implies $\frac{\partial}{\partial \pi} B(s) \geq 0$, which additionally implies that $\frac{\partial}{\partial \pi} B(s) \geq D_*$. □

C Robustness

C.1 War with Endogenous Military Effort

In the main text, war is modeled as a costly lottery while competitive diplomacy is a contest with endogenous effort choices. This approach facilitates a direct connection between the model in this paper and the most relevant previous work in dynamic crisis bargaining, which primarily uses costly lotteries as the technology of war. This section demonstrates how the results are robust to other war technologies that allow for endogenous war effort.

Suppose that war is also a contest with endogenous military effort choices. First, let there be a new state of the world υ from the finite domain Υ, separate from the state s, that fully specifies the environment in which the countries play this contest. In particular, upon the outbreak of war, both countries choose a level of military effort $m_i \in \mathcal{M}_i \subseteq \mathbb{R}_{\geq 0}$ where $\mathcal{M}_i$ gives their budget constraint, and a function $\psi : \mathcal{M}_1 \times \mathcal{M}_2 \times \Upsilon \to [0, 1]$ yields the probability country 1 wins given these military efforts and the state of the world.

Then, let $m_1^*(\upsilon)$ and $m_2^*(\upsilon)$ be the (possibly expected) equilibrium military efforts for both countries that result in an equilibrium probability of victory for country 1 of $\psi(m_1^*(\upsilon), m_2^*(\upsilon); \upsilon)$ in state υ. Because the model of the main text is general in its state space and the costs of war, it is without loss of generality to relabel the equilibrium probability of victory for country 1 as $s := \psi(m_1^*(\upsilon), m_2^*(\upsilon); \upsilon)$ and impose that the costs of war are the output of a general function that takes the corresponding equilibrium objects as input, $(s, m_1^*(\upsilon), m_2^*(\upsilon)) \mapsto (c_1(s), c_2(s))$ for all states s and υ. The only assumption this imposes on the contest is that countries expect to incur costs from it in equilibrium, to be consistent with the assumption that $c_i(s) > 0$. Note that this function does not need to be injective, as it is permissible that $c_i(s) = c_i(s')$ for $s \neq s'$. Thus, the model of the main text can capture any such contest with endogenous military effort.

C.2 Competitive Diplomacy as a Tullock Contest

In the main text, a country is recognized as proposer with probability $\pi > \frac{1}{2}$ when they exert greater effort in diplomacy. This is effectively a probabilistic all-pay contest, with $\pi = 1$ being a deterministic all-pay contest. Work in economic theory has demonstrated that many contests are strategically equivalent (see Chowdhury and Sheremeta (2015), for example). In this section, I show that the core incentives that drive strategic behavior in the all-pay contest of the main text carries into any decisive Tullock contest.

In particular, consider instead that endogenous country efforts govern the likelihood of having the upper-hand in the bargaining game, so that the probability that country i recovers proposal power is π times i's share of total diplomatic efforts. Recall that each country receives their war payoff $W_i(s)$ as a baseline. Then, country i's payoff to exerting effort $e_i \geq 0$ when their adversary exerts $e_{-i}^* \geq 0$ becomes

$$W_i(s) + \xi_i(e_i, e_{-i}^*; d)\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} + \frac{(1 - \pi)B(s)}{1 - \delta\lambda} - e_i$$

where

$$\xi_i(e_i, e_{-i}; d) = \begin{cases} \frac{e_i^d}{e_i^d + e_{-i}^d} & \text{if } e_i + e_{-i} > 0 \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

for some decisiveness parameter $d \geq 0$. Importantly, note that the decisiveness parameter d refers to the decisiveness of the Tullock contest, not to be confused with the decisiveness parameter π refers to the decisiveness of performance on recognition of agenda setting power. Tullock contests are defined by d, but our parameter of interest in the main text is π. We can think of competitive diplomacy in the main text as the limiting case in which $d \to \infty$.

The next proposition immediately follows Theorem 4.1 of Ewerhart (2015), with the only change being that competition is now over the long-run expected value of the bargaining surplus instead of a unit. The proof is rederived and stated for completeness.

Proposition A6. *In any equilibrium with $2 < d < \infty$, the support of the distribution of effort has zero as an accumulation point. The equilibrium is characterized by a sequence $e_1 > e_2 > \ldots > 0$ with $\lim_{k\to\infty} e_k = 0$ chosen with respective probabilities $f_1, f_2, \ldots$ such that $\sum_k f_k = 1$. Moreover,*

$$\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} \sum_k \frac{f_k e_j^d}{e_j^d + e_k^d} - e_j = 0 \tag{A33}$$

$$\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} \sum_k \frac{f_k d e_j^{d-1} e_k^d}{(e_j^d + e_k^d)^2} - 1 = 0 \tag{A34}$$

for any integer $j \geq 1$, $s \in S$.

Proof of Proposition A6. This proof is derivative of Ewerhart (2015) with the only change being that competition is over the long-run expected value of the bargaining surplus instead of a unit, and is restated here only for the sake of completeness.

Let $e_1 > e_2 > \ldots > e_L$ be the mass points of the equilibrium effort distribution, used with respective probabilities $f_1, \ldots, f_L$, $\sum_{k=1}^{L} f_k = 1$. Suppose for contradiction that zero is not an accumulation point of the support. From the first-order condition at e_L,

$$\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} \sum_{k=1}^{L} \frac{f_k d e_L^{d-1} e_k^d}{(e_L^d + e_k^d)^2} = 1,$$

we recover

$$\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} \sum_{k=1}^{L} \frac{2 f_k e_k^d e_L^d}{(e_L^d + e_k^d)^2} - e_L = \frac{(2 - d)e_L}{d}.$$

But since $e_k \geq e_L$ for all $k = 1, \ldots, L$, it follows that there is an upper bound on the expected payoff of exerting effort e_L,

$$\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} \sum_{k=1}^{L} \frac{f_k e_L^d}{e_L^d + e_k^d} - e_L \leq \frac{(2 - d)e_L}{d}.$$

Therefore, $d > 2$ implies a negative expected payoff for any equilibrium level of effort $e_L > 0$, a contradiction.

To prove equation (A33), taking any index $j \geq 1$, letting $j \to \infty$, and subsequently exchanging the sum and the limit via Lebesgue's Dominated Convergence Theorem, we recover that the expected equilibrium payoff is zero,

$$\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} \sum_{k=1}^{\infty} \frac{f_k e_j^d}{e_j^d + e_k^d} - e_j = 0,$$

completing the proof. $\square$

While equation (A34) defines the first-order conditions, equation (A33) demonstrates that all decisive ($d > 2$) Tullock contests erode the net gain from being recognized as the agenda setter, which is increasing in π and results in complete depletion of the entire bargaining surplus at $\pi = 1$, consistent with behavior in the main text. Straightforwardly we can see that the results from the paper continue to apply, as costlier wars (which increase the bargaining surplus) continue to create costlier peace by establishing greater incentive to exert diplomatic effort, and the expected aggregate payoff in peace continues to be eroded by steeper competition such that in the limit it is equal to the expected aggregate payoff in war.

Additionally, because zero is necessarily an accumulation point, it must be the case that there continues to be a nonzero risk of war in the equilibrium of any game with settlement frictions $\mu > 0$. This, however, relies on non-degenerate mixed strategies. In indecisive contests ($d \leq 2$), countries each play a pure effort strategy and therefore never risk war unless the settlement frictions are sufficiently large to discourage diplomacy altogether. Therefore, in the case of an indecisive contest, behavior is effectively the same as assuming $\mu = 0$ with the exception that realized effort is always equal to its expectation. The results on efficient war, therefore, continue to apply to indecisive Tullock contests, as well.

C.3 Competitive Diplomacy with Flexible Recognition Probabilities

In this section, I allow the recognition probabilities to vary by country and state. A country i exerting greater effort than their opponent, $e_i > e_{-i}$, in state $s \in S$ results in country i being recognized as the agenda setter with probability $\pi_i(s)$. Specifically, these can vary across countries such that $\pi_i(s) \neq \pi_{-i}(s)$ as well as across states such that $\pi_i(s) \neq \pi_{-i}(s')$ for $s \neq s'$. One reasonable setting would be that $\pi_i(s)$ is increasing in a country i's relative strength, although I impose no restrictions of this kind. To demonstrate the core similarities and differences, I focus on the case without transaction costs $\mu = 0$ as it generates less complex behavior in equilibrium.

A natural expectation would be that a country's incentive to exert effort in competitive diplomacy depends on the advantage they have in the competition. On the contrary, I find that there is a unique equilibrium in which countries play the identical strategies. This is because country incentives to compete are determined by their net gain from being the greatest performer, which is proportional to $\pi_i(s) - (1 - \pi_{-i}(s)) = \pi_{-i}(s) - (1 - \pi_i(s))$ for both countries given any couple of $(\pi_i(s), \pi_{-i}(s))$. The following proposition characterizes the equilibrium.

Proposition A7 (Equilibrium with Flexible Recognition)**.** *There is a unique equilibrium to a flexible recognition game without transaction costs where, for all states s and transitions q, each country $i = 1, 2$ plays $\sigma_i^{rec}(s) = (a_i^*(s), e_i^{rec}(s), x_i^*(s), y_i^*(x; s))$ defined as follows.*

(i) Decisions $a_i^*(s)$, $x_i^*(s)$, and $y_i^*(x; s)$ are defined in Proposition A1.

(ii) Effort $e_i^{rec}(s)$ is drawn from the cumulative distribution

$$F_s^{rec}(e) = \frac{(1 - \delta\lambda)e}{(\pi_1(s) + \pi_2(s) - 1)B(s)}$$

with $F_s^{rec}(e) = 0$ for $e < 0$ and $F_s^{rec}(e) = 1$ for $e > \frac{(\pi_1(s)+\pi_2(s)-1)B(s)}{1-\delta\lambda}$ if $U_i(s) \geq W_i(s)$. Otherwise exert zero effort $e_i^{rec}(s) = 0$.

Proof of Proposition A7. The proof follows the structure of the proof of Proposition A1 with the imposition that $\mu = 0$. The derivation of $a^*(s)$, $x^*(s)$, and $y^*(x; s)$ are identical to the derivation from Proposition A1. Now, the expected net gain to country i from exerting effort $e \geq 0$ on diplomacy is

$$\frac{B(s)}{1 - \delta\lambda}\left[\pi_i(s)\Pr(e > e_{-i}) + (1 - \pi_{-i}(s))\Pr(e_{-i} > e)\right] - e. \tag{A35}$$

The net gain is zero when $B(s) = 0$, in which case neither country will be willing to spend a positive amount in equilibrium. Therefore, to understand strategies with nonzero amounts of effort, assume $B(s) > 0$.

There are still no pure strategies under the identical logic as before. I now look for a mixed strategy corresponding to state s given by c.d.f. F_s^{rec} that satisfies equation (A35) for both countries. We can write the payoff from zero effort as

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}(1-\pi_{-i}(s)). \tag{A36}$$

On the other hand, exerting effort of $e > 0$ yields an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\Big[F_s^{rec}(e)\pi_i(s) + (1-F_s^{rec}(\mu))(1-\pi_{-i}(s))\Big] - \mu. \tag{A37}$$

Using equations (A36) and (A37), we can solve for

$$F_s^{rec}(e) = \frac{(1-\delta\lambda)e}{(\pi_1(s)+\pi_2(s)-(1-\rho))B(s)}. \tag{A38}$$

which implies an upper bound on equilibrium effort of $\bar{e}_s = \frac{(\pi_1(s)+\pi_2(s)-(1-\rho))B(s)}{1-\delta\lambda}$.

Uniqueness follows from the same logic as Proposition A2. Consider the case where a country exerts effort $e > \bar{e}_s$ with nonzero probability. If both countries are, there is a profitable deviation to zero effort. If only one country is, by deviating back to $\bar{e}_s$, their payoff will be strictly more as they continue to gain only $W_i(s) + \frac{\pi_i(s)B(s)}{1-\delta\lambda}$ but expend strictly less effort. Further, consider a strategy where zero effort is expended with nonzero probability. Then, the country must be indifferent between expending zero and expending nonzero amounts of effort. However, this would imply the country is always the worst performer in equilibrium, in which case there is a strictly profitable deviation to never expending effort, which cannot be the case in equilibrium.

Next, consider a strategy where there is an atom on a nonzero amount of effort. Then, there is a strictly positive deviation to marginal levels of effort that disproportionately increase their expected win probability. Lastly, to show that zero must be the lower bound of the support, suppose otherwise that one country i always exerts at least effort $e' > 0$. Then, their opponent $-i$ will never exert amounts between zero and e' as exerting zero is strictly better. However, this implies that country i has a strictly profitable deviation to reducing effort levels below e' without altering the probability of being the stronger performer.

As before, equilibrium offers are given by equations (A6) and (A7), and a country receiving an offer rejects offers in their rejection set and accepts all others. $\square$

The result is especially interesting because both countries play the same mixed strategy in equilibrium, which is surprising given they have different absolute values for being the greatest performer. This is in contrast to other contests with different valuations (e.g., Hillman and Riley (1989)), where countries with a lower valuation tend to exert less effort on average. The reason for the difference is that, while their absolute values for being the greatest performer is different, these are offset by their differing absolute values for being the weakest performer. Consequently, regardless of how much larger one country's advantage is over the other, their expected net gain from becoming the agenda setter is equal to $\pi_1(s)+\pi_2(s)-1$ times the expected value of recovering the bargaining surplus in the current period.

C.4 Infinite Costs of War

In the baseline model, the costs of war are finite. As a result, countries choose to go to war or cooperate on the basis of which expected sequence of inefficiencies are more painful for them in the long run. The model considers circumstances where damages in war are destructive but not infinite. We can always consider that the amount of time it takes to recover (however long) is the basis of normalization for the length of a single period in the model.

We can also, however, consider the case of infinite costs of war via permanent destruction of part of the pie. This section shows that efficient war can continue to exist with infinite costs of war as long as the permanent damage to the pie is not too severe. This is technically the same as the case where the act of cooperation grows the pie, in which efficient war remains possible as long as cooperation does not grow the pie too much. As they are equivalent, I proceed with the former interpretation without loss of generality.

Suppose for example that, in addition to incurring finite costs of war $c_i(s) > 0$, a proportion of the pie $1-\gamma \in (0,1)$ is destroyed by war in state s. A country i's continuation value for war in state s is then given by

$$W_i^\gamma = s_i\gamma - c_i(s) + \frac{\delta}{1-\delta\theta}\left[\theta s_i\gamma + (1-\theta)\sum_{s'} V_i^\gamma(s')q(s'|s,a_W)\right]$$

where now $V_i^\gamma \in \{U_i(s), W_i^\gamma\}$ for all states s and $U_i(s)$ is still given by equation (3) in the main text. We can see that this new continuation value implies that the new expected net present value of the pie after war (recall Definition 3) is given by

$$G_\gamma(s) := \frac{\gamma}{1-\delta\theta} - \frac{1}{1-\delta\lambda}. \tag{A39}$$

Likewise, the new expected net present value of returning to bargaining after fighting in state s (recall Definition 4) is now given by

$$\Delta V_\gamma(s) := \sum_{s' \in S} (V_1^\gamma(s') + V_2^\gamma(s')) \left[\frac{\delta(1-\theta)}{1-\delta\theta} q(s'|s, a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda} q(s'|s, a_U) \right] \tag{A40}$$

for any transition function q.

Proposition A8. *Let $1-\gamma \in (0,1)$ of the pie be permanently destroyed by war in state s. Then, there is efficient war in state s, $\bar{x}_1(s) > \bar{x}_2(s)$, if and only if*

$$\gamma > \frac{(1-\delta\theta)(1-(1-\delta\lambda)(\Delta V_\gamma(s) - C(s))}{1-\delta\lambda} \tag{A41}$$

where $\Delta V_\gamma(s) := \sum_{s' \in S} (V_1^\gamma(s') + V_2^\gamma(s')) \left[\frac{\delta(1-\theta)}{1-\delta\theta} q(s'|s, a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda} q(s'|s, a_U) \right]$.

Proof of Proposition A8. Recall that by Definition 2 that wars are relatively efficient if and only if $\bar{x}_1(s) > \bar{x}_2(s)$, which under the new continuation value implies

$$1 - (1-\delta\lambda)(W_1^\gamma(s) + W_2^\gamma(s)) + \delta(1-\lambda) \sum_{s' \in S} (V_1^\gamma(s') + V_2^\gamma(s')) q(s'|s, a_U) < 0.$$

By the definitions in equations (A39) and (A40), we can rearrange to recover the expression $G_\gamma(s) + \Delta V_\gamma(s) > C(s)$. Rearranging to isolate γ on the left-hand side yields inequality (A41). $\square$

To see this equation may be satisfied, let ζ be defined implicitly by

$$\Delta V_\gamma(s) = \frac{1-\zeta}{1-\delta} \left[\frac{\delta(1-\theta)}{1-\delta\theta} - \frac{\delta(1-\lambda)}{1-\delta\lambda} \right]$$

to represent the average inefficiency over the infinite stream of expected flow payoffs. Equivalently, you could suppose state s is certain to transition to an absorbing state where cooperation occurs with an aggregate expected diplomatic effort of ζ in each period.

Then, inequality (A41) can be rewritten as

$$\gamma > \frac{1 - \delta(\zeta\theta + (1-\zeta)\lambda) + (1-\delta\lambda)(1-\delta\theta)C(s)}{1-\delta\lambda}.$$

Taking the multivariate limit of the right-hand side,

$$\lim_{(\delta,\lambda,\theta)\to(1,0,1)} \frac{1 - \delta(\zeta\theta + (1-\zeta)\lambda) + (1-\delta\lambda)(1-\delta\theta)C(s)}{1-\delta\lambda} = 1-\zeta < 1.$$

This demonstrates how the intuition from the main text carries through in the case of infinite costs of war. As long as the total aggregate loss (evaluated at present value) from going to war is sufficiently small relative to the total aggregate loss (again, evaluated at present value) from cooperation, countries can prefer to fight an efficient war.

Appendix References

Bueno de Mesquita, Ethan. 2005. “Conciliation, Counterterrorism, and Patterns of Terrorist Violence.” *International Organization* 59(1):145–176.

Carter, David B. 2015. “The compellence dilemma: International disputes with violent groups.” *International Studies Quarterly* 59(3):461–476.

Chowdhury, Subhasish M, and Roman M Sheremeta. 2015. “Strategically equivalent contests.” *Theory and Decision* 78:587–601.

Ewerhart, Christian. 2015. “Mixed equilibria in Tullock contests.” *Economic Theory* 60:59–71.

Fudenberg, Drew, and Jean Tirole. 1991. *Game Theory.* MIT Press.

Hillman, Arye L, and John G Riley. 1989. “Politically contestable rents and transfers.” *Economics & Politics* 1(1):17–39.

Krainin, Colin, Kristopher W Ramsay, Bella Wang, and Joseph J Ruggiero. 2022. “Preventive war and sovereign debt.” *Conflict Management and Peace Science* 39(5):487–519.

Leventoğlu, Bahar, and Nils W Metternich. 2018. “Born Weak, Growing Strong: Anti-Government Protests as a Signal of Rebel Strength in the Context of Civil Wars.” *American Journal of Political Science* 62(3):581–596.

Maskin, Eric, and Jean Tirole. 2001. “Markov Perfect Equilibrium I: Observable Actions.” *Journal of Economic Theory* 100(2):191–219.

Paine, Jack. 2016. “Rethinking the Conflict “Resource Curse”: How Oil Wealth Prevents Center-Seeking Civil Wars.” *International Organization* 70(4):727–761.

Powell, Robert. 1993. “Guns, Butter, and Anarchy.” *American Political Science Review* 87(1):115–132.

Powell, Robert. 2019. “Why Some Persistent Problems Persist.” *American Political Science Review* 113(4):980–996.

Slantchev, Branislav L. 2002. Markov Perfect Equilibrium in a Stochastic Bargaining Model. Technical report.

Slantchev, Branislav L. 2003. “The Principle of Convergence in Wartime Negotiations.” *American Political Science Review* 97(4):621–632.