Bargaining, War, and Cooperation in the Long Run

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Abstract

Maintaining peace with adversaries is costly, yet conflict models typically depict cooperation as a costless byproduct of not fighting. This article provides a formal model where peace is costly and countries can diplomatically compete over the bargaining surplus. Against conventional wisdom, repeated interactions of patient countries can destabilize peace. The likelihood of war depends on fundamentals of the international order, such as the persistence of outcomes and the level of competition in diplomatic affairs. Even when cooperation is mutually preferred, wars are inevitable due to a coordination problem induced by costly peace. Competition over favorable peace settlements does not directly instigate attacks, but it reduces the gains from peace and makes inadvertent war less likely. The article offers new explanations for war, highlights the importance of institutional design in conflict resolution, and sheds light on which international orders are most likely to fare well over time.

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1 Introduction

Studies of international conflict aim to understand why countries fight wars when doing so is costly. A standard assumption in these analyses is that peace is a costless byproduct of not fighting and, hence, war is inherently inefficient.[1] This is typically reasonable; costs of peace, to the extent they exist, tend to be trivial compared to much more palpable costs of war. Nonetheless, peacetime may be costly as a direct result of the conflict situation—countries may, for example, choose to spend heavily on military investments (Powell 1993; Coe 2011). Cooperation may also be costly for reasons not endogenous to the conflict (e.g., payments to a mediating institution that are necessary to close a deal). Further, countries occasionally opt into costs of peace, especially in institutional contexts where preferential treatment is given to countries on the basis of their authority, presence, and financial contributions (Ikenberry 2001; Kuziemko and Werker 2006; Pratt 2020).

A natural question, then, is how does our understanding of war change if we account for these costs of peace directly? Given the deep relationship between costs of peace and the structure of security relations (Keohane 1984; Lake 1999), incorporating costly peace in a crisis bargaining model facilitates analysis of systemic causes of war. In particular, my analysis focuses on two features of the international system and their relationship to conflict propensity: (i) persistence of outcomes, or the time horizon for which a country expects the outcome of a crisis situation to last; and (ii) diplomatic competition, or the extent to which favorable peace settlements tomorrow can be earned by costly actions today. The former speaks to literature on systemic stability and polarity (Waltz 1979; Ikenberry, Mastanduno, and Wohlforth 2009) while the latter speaks to that of the prevailing regimes and institutional structures (Krasner 1983; Voeten 2021). These

[1] Throughout the article, I refer to peace and cooperation interchangeably, taking for granted the presence of a crisis situation.

factors will be deliberately treated as independent of country characteristics and actions, representing an external setting for the crisis taking place.

The model takes a crisis bargaining model (Fearon 1995) as a stage to a repeated game where countries can choose between a costly peace or a costlier war. Costly peace is operationalized as the necessary spending on diplomacy to reach a peace settlement, which may be arbitrarily small. When this threshold is met, cooperation becomes possible with the higher spender recognized as a proposer in ultimatum bargaining with greater probability. Ultimatum bargaining is employed because it can represent crisis situations with little loss of generality (Fey and Kenkel 2021). If the necessary amount to reach an agreement is not satisfied, on the other hand, the countries fall into war. After a settlement or war outcome is realized, the result can persist for some time until countries return to the bargaining table to repeat the game. The varying extents to which peace settlements and war outcomes persist, as well as the level of competition over favorable settlements, characterizes the international environment.

The features of the international system, in addition to country characteristics, affect a country's willingness to fight. First, even when the costs of peace are arbitrarily small, a country may prefer to attack and initiate war if peace settlements are not expected to last relative to the gains captured in war. This challenges a central result of war-as-bargaining theory: rational agents with complete and perfect information will always settle (Brito and Intriligator 1985; Fearon 1995). In this line of research, a surplus from cooperation typically exists but war can still be induced by information asymmetries (Banks 1990; Fey and Ramsay 2007, 2011; Bils and Spaniel 2017; Spaniel and Bils 2018) or commitment problems (Powell 2006; Chassang and Padró i Miquel 2009; Debs and Monteiro 2014; Chapman, McDonald, and Moser 2015; Schram 2021; Benson and Smith 2021; Carey, Bell, Ritter, and Wolford 2022). In the former, countries don't know where the bargaining range is; in the latter, countries are incapable of reaching settlements in the bargaining range—for example, if a rising power faces a liquidity problem (McDonald

2011; Krainin, Ramsay, Wang, and Ruggiero 2022) or a hegemon and rising challenger cannot commit to equitably share the gains of a growing economic pie (Monteiro and Debs 2020). In this paper, a bargaining surplus may not exist regardless of country actions, on or off the equilibrium path of play.

Second, these features affect a country's willingness to risk a war when peace is preferable to both parties. An "inadvertent" war is not the result of an attack, but of a failure to pay the necessary amount to sustain peace. Countries spend on diplomacy and, if aggregate spending is insufficient, countries cannot cooperate. Inadvertent war, therefore, is not the result of a hold-up problem, but a coordination problem: both countries would prefer to pay the costs of peace if they knew the other was not going to.[2] In equilibrium, countries either vie for proposal power by spending an amount in excess of the necessary costs of peace, creating welfare loss, or they spend nothing, risking war in hope to free-ride on the diplomatic expenses of their rival. The incentive to compete for proposal power will not directly lead countries to launch wars; however, competition decreases welfare and increases the frequency of wars. Thus, the Pareto optimal level of competition does not minimize wars.

Changes to the international order that affect how long war outcomes and settlements are expected to last, such as weakening great powers or shifts in alliances, correspond to changes in the persistence parameters of the model. These persistence parameters can be understood as a reduced form for a larger international setting to which the countries of the game are subject. For example, the persistence of peace may be informed by the willingness of major powers to involve themselves in foreign conflicts. Consider the Dayton Accords that resolved the Bosnian War in the mid 1990s: Serbia understood that defection and subsequent expropriation of territory would result in a high likelihood of conflict with the United States (US) and The North Atlantic Treaty Organization

[2]The immediacy of inadvertent war is a simplification for convenience; the substantive implications would not change if inadvertent war only happens with some probability (and costless peace otherwise) when countries fail to spend an adequate amount.

(NATO), so any agreement would be expected to last. This contrasts sharply with short-lived armistice agreements in civil conflicts, where powerful third parties are unwilling to bring about change in the event of defection.

The results may also speak to the 2022 Russian invasion of Ukraine, which I reflect on at greater length in the discussion. First, shifts in the fundamentals of the international order may instigate conflict. As the model demonstrates, increasing the persistence of war outcomes or the transience of peace agreements can induce a sufficiently patient country to fight. Then, for example, weakening global institutions and corresponding alliances (Ikenberry 2018; Mearsheimer 2019) could foment Russia's aggression. Second, war may result from a failure to coordinate a peaceful settlement. If the costs of participating in international organizations have risen, this will increase the likelihood countries forgo attempts to resolve their disputes.

Further, the model may help identify and assess the likelihood of future conflicts. Consider the possibility of war between the US and China: scholars and policy analysts continue to debate whether China's growing power poses a threat to the prevailing international order and, more specifically, whether China aspires to displace the United States as a global hegemon (Doshi 2021). A particular source of tension between the two is Taiwan's sovereignty. In 1975, during a conversation with then-Secretary of State Henry Kissinger, Mao Zedong remarked on China's hope to one day claim Taiwan: "a hundred years hence we will want it, and we are going to fight for it." Mao's statement indicates that China may be patient and that, as the model would suggest, a shift in the international order that increases the relative persistence of war outcomes may trigger a forceful attempt to annex Taiwan.

1.1 Costly Peace and Crisis Bargaining

Peace does not come free; most studies of international cooperation acknowledge that reaching peaceful arrangements is costly. A primary reason for establishing international institutions is their ability to reduce transaction costs (Keohane 1984). While it is possible or even likely that institutions—formal organizations, norms, or otherwise—can reduce these costs of peace, it is also possible that existing institutions occasionally exacerbate this problem with inefficiencies (Moravcsik 1999). Some research has argued that the nature of the costs of peace affect the way countries form alliances. Lake (1999), for example, claims that the transaction costs of international institutions determine the structure of security relations, varying from anarchic alliances to hierarchical ones (e.g., NATO and the Warsaw Pact, respectively).

Incorporating costly peace in the bargaining framework parallels with literature on international cooperation and contributes to a larger conversation on its stability. A determinant of the spending threshold, for example, may include domestic opposition to international institutions (De Vries, Hobolt, and Walter 2021) or the extent to which institutions need to change to better reflect a shifting distribution of international power (Pratt 2020; Davis and Pratt 2021). Likewise, reaching a peace settlement may come with domestic audience or reputational costs (Fearon 1994; Tomz 2007; Snyder and Borghard 2011; Crisman-Cox and Gibilisco 2018), especially when a leader's flexibility to negotiate is constrained by their willingness to use force (Kertzer and Brutger 2015). The model I present here is consistent with these ideas, reconciling them with the bargaining model of war and shedding light on underlying mechanisms.

This article is not the first to address costly peace and its implications on war's inefficiency. For example, Chiozza and Goemans (2004) point out that war need not be *ex post* inefficient from the perspective of a leader with tenure considerations. The risk of losing office is effectively a cost of peace on the decision-making leader. Coe (2011) is especially noteworthy in that it argues for costly peace as a third rational explanation for war and analyzes several cases where the costs of peace led to war.[3] Coe's work focuses on endogenous, war-related costs of peace—arming, imposition, and

[3]The cases include the Iraq War, civil conflicts in Iraq after the Gulf War, and the American Revolution.

predation—when they exceed the costs of war. Importantly, these types of costs do not improve the peacetime welfare of the country incurring the expense. This article, on the other hand, allows for costs of war that are larger than the costs of peace and studies competitive diplomatic spending in which favorable settlements are increasing in a country's expense.

Despite its differences, the model here is deeply related to many models of conflict that precede it. Powell (1993) offers a similar mechanism for direct war that relies on a specific commitment problem between the two countries: countries feel the need to arm heavily in response to each other's strength. An important difference is that, in this model, there may be no available changes to the countries' behavior that would successfully avert war—the source of a country's failure to cooperate is in part the result of structural features of the international environment that may have little to do with the two specific countries in conflict. Fearon (1998) is also relevant as it shows that a long shadow of the future can make it easier to enforce agreements but more difficult to arrive at agreements in the first place. In this paper, on the other hand, it is possible for peaceful settlements not to exist at all.

The possibility of inadvertent war also distinguishes this model from previous crisis bargaining models, including the aforementioned papers. Costly peace creates an incentive to free-ride on the necessary diplomatic expenses if the rival is spending sufficiently high amounts on diplomacy, which creates a coordination problem. Moreover, this paper studies competitive diplomacy, which is absent from previous crisis bargaining work and is consequential for both welfare and the likelihood of war.

The features of the international system (defined by the parameters of the model) determine whether countries want to fight and, when they do not, the severity of the coordination problem they face. If the environment is peace-facilitating, unfavorable shifts in these parameters can result in the outbreak of war. This is consistent with recent work that shows systemic features and instability of the international order drive territorial claims that lead to crises (Abramson and Carter 2021). The parameters that describe a fixed international order in this model can also be an abstract representation of more complicated phenomena. For example, consider the possibility of multiple, inconsistent orders that materialize simultaneously (Simmons and Goemans 2021). Then, contradictions in country relations may reduce the efficacy of peace settlements.

2 Model

Consider a game with two countries $i = 1, 2$ that have opposing preferences over the division of a unit interval. Each country's utility is continuous, increasing, and linear in their share of the pie each period. Specifically, let $u_i(x)$ denote country i's flow payoff for a share $x \in [0, 1]$ of the pie such that $u_1(x) = x$ and $u_2(x) = 1 - x$. There is a common discount factor $\delta \in (0, 1)$ over an infinite horizon, $t = 1, 2, \ldots$, where each period can be characterized by a pair of state variables $(b(t), p(t)) \in \{0, 1\} \times P$ with $P \subset [0, 1]$. The former denotes the bargaining status of the game in period t, with $b(t) = 0$ if the countries are in a settlement and $b(t) = 1$ if the countries are bargaining. The latter provides the relative strength of country 1 in period t, where P is a finite set of possibilities.[4]

A settlement is an inactive state in which each country simply consumes their share of the pie from the previous period. Bargaining states, on the other hand, permit each country a choice between cooperating with their opponent, $r_i(t) = 0$, or initiating war, $r_i(t) = 1$. If a country cooperates, they simultaneously choose an amount $s_i(t) \geq 0$ to spend in competition over proposal power. This amount can be likened to costly effort exerted in diplomatic relations, which may include embassy expenditures, ambassador salaries, etc., as well as financial contributions to an international organization that has credible domain over the conflict at hand. Exerting effort on cooperative endeavors

[4] A finite state space helps to guarantee existence. See ch. 4 of Duggan (2016) for more information.

provides countries with greater leverage in the bargaining game.

Cooperation, however, is not necessarily free. In addition to the endogenous costs of cooperation, the model allows for an exogenous, context-dependent cost that serves as a required minimum on aggregate spending for peace, denoted by $k \geq 0$. An example of such a diplomatic spending threshold is the lower bound on necessary operating costs for an institution that enforces and monitors a contract between countries.[5] Then, if $s_1(t) + s_2(t) < k$ for some period t, peace cannot be maintained and the countries fall into war. Alternatively, cooperation is possible when $s_1(t) + s_2(t) \geq k$ and, given $s_i(t) > s_{-i}(t)$, country i is recognized as proposer in period t with probability $\pi > 1/2$.[6] The proposer makes a take-it-or-leave-it offer $x(t) \in \mathbb{R}$, with acceptance leading to peaceful settlement and rejection leading to war. An accepted division results in a periodic payoff linear in their share less the costs spent on diplomacy.

The recognition probability π is deliberately independent from diplomatic spending to understand how *systemic* changes in the level of competition affect country behavior. If π were instead a function of equilibrium spending (e.g., a Tullock contest), then it would no longer represent a feature of the international system but instead a feature of the dyadic interaction. Additionally, unlike most crisis bargaining models, preventive wars due to commitment problems are ruled out by construction as offers and settlements are permitted to take any real value. This is also deliberate, as the analysis here focuses on failures to cooperate brought about by dynamic features of international interactions that do not originate with well-understood causes of war.[7]

[5]While the aggregate spending threshold k is fixed in the model, allowing it to vary over time would not change the substantive results. The conclusions reached will be compatible with the idea that investing in cooperation today may ease future cooperation, i.e., $k(t) \geq k(t+1) > 0$ for all $t \geq 1$.

[6]If $s_1(t) = s_2(t) > 0$, then proposal power is awarded to country 1 with probability 1/2. This, however, is a measure zero event in equilibrium.

[7]The possibility of preventive wars could be accommodated by restricting the domain of offers to the unit interval, in which case the only change to the solution would be an additional condition on equilibrium offers and a weakly larger set of bargaining states that lead countries to fight due to shifting power. Moreover, none of the substantive results change as a result of this feature.

War occurs in period t if and only if $b(t) = 1$ and either (i) at least one country chooses to attack, (ii) an offer is rejected, or (iii) aggregate spending is less than the exogenous spending threshold. Each country incurs a publicly known cost of war $c_i > 0$ and country 1 wins with probability given by state variable $p(t)$. The victorious country receives the entire pie for that period and subsequent periods until the game returns to bargaining. Intuitively, a country that expects to keep the gains from winning a war for longer periods of time will be more tempted to pursue those ends.

The state variables evolve as follows. First, the game begins in a bargaining state by default. Then, at the end of any period, the state variable $b(t)$ transitions to $b(t+1) = 0$ with probability $\theta \in (0, 1)$ when the current division is a war outcome and with probability $\lambda \in (0, 1)$ when the division is the result of a peaceful settlement. These represent the persistence of outcomes settled by war and those settled by diplomacy, respectively. These probabilities reflect a "conditional stability" of the two outcomes—when a persistence parameter is low, the corresponding outcome is unlikely to last for long periods of time. Further, the state variable $p(t)$ transitions according to Markov transition function $q(p(t+1)|p(t), a(t))$ where $a(t) = ((r_i(t), s_i(t), x_i(t), y_i(t)))_i$.

2.1 Timing

The timing of game is as follows.[8]

1. Nature randomly draws initial relative strength $p \in P$ and sets $b = 1$.

2. If $b = 0$, the previous period's distribution of the pie persists. If $b = 1$, each country chooses to cooperate or fight, $r_i = 0, 1$.

2.a. If both cooperate, they choose $s_i \geq 0$. If $s_1 + s_2 \geq k$, a proposer is recognized according to π, s_1, and s_2. The proposer i makes a take-it-or-leave-it offer $x_i \in \mathbb{R}$, which country $-i$ rejects or accepts $y_{-i} = 0, 1$.

[8] All notation that identifies a specific period t is suppressed when doing so does not create confusion.

2.b. War occurs if a country attacks, an offer is rejected, or $s_1+s_2<k$. Countries incur costs of war $c_i>0$ and country 1 wins with probability given by state variable p.

3. At the end of any period $t \geq 1$, the state (b,p) transitions to (b',p') according to Markov transition function $q(p'|p,a)$ as well as probabilities θ and λ.

4. Period $t+1$ proceeds from step 2 with state $(b=b', p=p')$.

3 Equilibrium

A strategy for country i is a function $\sigma_i : P \to \{0,1\}^2 \times \mathbb{R}_+ \times \mathbb{R}$, where $\sigma_i(\cdot)$ denotes behavior for country i when $b=1$ for all $p \in P$, including their war decision, diplomatic spending, bargaining offers, and bargaining responses. Then, country i's equilibrium strategy is given by quadruple

$$\sigma_i^*(p) = ((r_i^*(p), F_i^*(p), x_i^*(p), y_i^*(p)) \tag{1}$$

where, for all $p \in P$, $r_i^*(p)$ is a best response war decision, $F_i^*(p)$ is a best response distribution for diplomatic spending, $x_i^*(p)$ is a best response proposal, and $y_i^*(p)$ is a best response acceptance decision.

Note that $\sigma_i(\cdot)$ does not take b as input because, when $b=0$, the countries take no action and simply consume their share of the pie according to the previous period's division. This allows us to focus on states in which $b=1$. In slight abuse of notation, I regularly refer to state $(1,p)$ as state p when it is implied that state variable $b=1$.

I denote by $V_i(p)$ country i's *ex ante* value function, which satisfies

$$V_i(p) = r_{-i}^*(p)(W_i(p)-c_i) + (1-r_{-i}^*(p))\Big[r_i^*(p)(W_i(p)-c_i) + (1-r_i^*(p))U_i(p)\Big], \tag{2}$$

where, for all states $p \in P$, $W_i(p)$ is country i's expected value from going to war and

$U_i(p)$ is country i's expected value from cooperating.

If war occurs, each country can expect to receive the entire pie for that period with their probability of victory. In every period after war that the war settlement persists, the winning country can expect to keep receiving the entire pie while the losing country keeps receiving nothing with probability θ. The countries return to the bargaining game under a new state p' with probability $1-\theta$. Letting $p_1 := p =: 1-p_2$ without loss of generality, country i's expected value of war in state p can be expressed as $W_i(p) - c_i$, where[9]

$$W_i(p) = p_i + \frac{\delta}{1-\delta\theta}\left[\theta p_i + (1-\theta)\sum_{p'\in P} V_i(p')q(p'|p,a)\right]. \tag{3}$$

Denote by $x_m(p)$ the expected peace settlement in state p. In every period after cooperation that the peaceful settlement persists, country 1 can expect to keep receiving $x_m(p)$ and country 2 can expect to keep receiving $1-x_m(p)$ with probability λ. The countries return to the bargaining game under a new state p' with probability $1-\lambda$. Then, we can also write country i's expected value of cooperating in state p as

$$U_i(p) = u_i(x_m(p)) - \mathbb{E}[s_i^*(p)] + \frac{\delta}{1-\delta\lambda}\left[\lambda u_i(x_m(p)) + (1-\lambda)\sum_{p'\in P} V_i(p')q(p'|p,a)\right] \tag{4}$$

where $u_i(x_m(p))$ is country i's expected settlement given cooperation in p and $\mathbb{E}[s_i^*(p)]$ denotes expected diplomatic spending in state p according to i's equilibrium spending strategy.[10]

Throughout the paper, $U_i(p)$ refers to i's expected value for cooperating prior to al-

[9]Derivations are available in the appendix.

[10]The expectation $u_i(x_m(p))$ is equal to a weighted average of i's share of the pie under equilibrium offers from each country. In equilibrium with symmetric recognition, the weights are necessarily 1/2 and hence $u_i(x_m(p))$ is the value i derives from a settlement at the Nash bargaining solution. Note the subtle difference that $u_i(x_m(p))$ is the value of the settlement to either country i, which is different from $x_i(p)$, the share of the pie retained by country 1 under an offer from country i. Specifically, $u_1(x_m(p)) = 2^{-1}\sum_i x_i^*(p)$ and $u_2(x_m(p)) = 2^{-1}\sum_i (1-x_i^*(p))$.

location of proposal power. I will refer to the expected value of accepting an outstanding offer x as $U_i(x;p)$; therefore, i's expected value of cooperating given $-i$ has won proposal power can be written $U_i(x^*_{-i}(p);p)$ for equilibrium offer $x^*_{-i}(p)$.[11]

My solution concept is essentially symmetric stationary Markov perfect equilibrium (MPE).

Definition 1. *An **essentially symmetric stationary MPE** is a pair $(\sigma_1^*(\cdot), \sigma_2^*(\cdot))$ such that equations (1)-(4) hold for all $p \in P$.*

As in any stationary MPE, this solution concept satisfies the Markov property that strategies rely only on a payoff-relevant state. The term *essentially symmetric* emphasizes that countries act symmetrically in equilibrium with the exception that their behavior will necessarily rely on relative strength and potentially unequal costs of war. Countries will make different proposals and form different acceptance sets, as in standard bargaining models of war, but their equilibrium diplomatic spending (mixed) strategies will be symmetric since their incentives are identical conditional on the available bargaining surplus.

The MPE solution concept is employed instead of subgame perfect Nash equilibrium (SPE) for two reasons. First, MPE focuses attention on characteristics of the underlying institution while avoiding equilibria that are facilitated through history-dependent strategies, which are outside of this study's scope, such as strategies that rely on reputation to facilitate international cooperation. Reputation is undoubtedly an important feature of international relations—a country that fails to abide by rules and norms of other countries will likely face serious consequences either directly through war and sanctions or indirectly through the absence of mutually beneficial arrangements. Nonetheless, the aim of this paper is to understand how structural features of the international system affect the likelihood of conflict and reputational considerations in behavior manifest by

[11] Note that $U_i(x_m(p);p) = U_i(p) + \mathbb{E}[s_i^*(p)]$ because the realized s_i becomes a sunk cost at the time of the proposal. For this reason, acceptance decisions will rely on $U_i(x;p)$ instead of $U_i(p)$.

SPE, such as grim trigger and tit-for-tat strategies, are orthogonal to our parameters of interest.

Second, equilibrium refinement is necessary, as SPE in this model will allow for many unreasonable equilibria. Since both peace and war are costly and countries may prefer one or the other under different circumstances, pathological behavior can be sustained in equilibria by threatening to punish deviations from obscure sequences of actions. Consider, for example, an SPE of this game where a country initiates war in some periods and cooperates in others, with off-path behavior such that deviations to cooperate result in punishment by other countries to fight wars forever thereafter. When the set of equilibria is too large and includes possibilities that are substantively incredible, a refined solution concept is more suitable. Limiting the analysis to MPE as defined above is a natural choice and resolves these issues.

3.1 Offers and the Bargaining Range

To find equilibrium offers $x_i^*(p)$ for each country i in any state p, it is necessary to find the offer that makes each country indifferent between going to war and cooperating. By setting $U_1(\underline{x}(p);p) = W_1(p) - c_1$ and $U_2(1-\overline{x}(p);p) = W_2(p) - c_2$, we can solve for values that make countries 1 and 2 indifferent in state p, respectively. These indifference conditions yield implicit solutions for the thresholds,

$$\underline{x}(p) = (1-\delta\lambda)(W_1(p) - c_1) - \delta(1-\lambda)\sum_{p'\in P} V_1(p')q(p'|p,a) \tag{5}$$

and

$$\overline{x}(p) = 1 - (1-\delta\lambda)(W_2(p) - c_2) + \delta(1-\lambda)\sum_{p'\in P} V_2(p')q(p'|p,a). \tag{6}$$

The relationship between these indifference points and the indifference points of the standard static bargaining model of war is apparent. In particular, perfectly impatient countries are indifferent at the same settlement as countries in the standard bargaining model of war with complete and perfect information. To see this, consider when $\delta = 0$ so that equation (5) becomes $\underline{x}(p) = p - c_1$ and equation (6) becomes $\overline{x}(p) = p + c_2$. These values are equivalent to the canonical characterization of the static bargaining range as in Fearon (1995). In Fearon's static model, complete and perfect information results in country 1 offering $p + c_2$ and country 2 offering $p - c_1$, with both countries accepting offers and hence war does not occur in equilibrium. In the dynamic setting here, the bargaining surplus shrinks as δ approaches 1 and may not exist in the limit when accounting for costly peace.

The bargaining range in state p can then be defined as the set $X(p) = [\underline{x}(p), \overline{x}(p)]$ given $\overline{x}(p) \geq \underline{x}(p)$ and $X(p) = \emptyset$ otherwise. Note that $\underline{x}(p)$ and $\overline{x}(p)$ are not constrained to values in the unit interval. To avoid a particular type of conflict known as preventive war, I allow for offers to take any real value. If this were not the case, war would result from a commitment problem whenever $\overline{x}(p) < 0$ or $\underline{x}(p) > 1$ because the division that one country would have to give the other to avert war is larger than the amount they can dispense with and there is no way to credibly promise future divisions. This may happen when a country's expected relative strength in a period after peace is small relative to that after war, e.g., if $\sum p' q(p'|p, a)$ is much smaller given $r_1 = 0$ than given $r_1 = 1$. As mentioned above, while the model can accommodate these circumstances, they would only change technical details of the equilibrium characterization and have no bearing on the main results.

Unlike previous models, the bargaining range can also be empty without the typical dyadic commitment problems such that $\underline{x}(p) > \overline{x}(p)$. Then, conflict arises not because one state cannot offer an adequate division, but because they prefer not to settle given the circumstances. A trivial way this occurs in a static game is if the exogenous costs of peace are larger than the costs of war, which I assume away. This model, however, reveals that this type of problem may occur under mild conditions where the periodic costs of peace are much less than the costs of war.

Both countries prefer to fight when the bargaining range is empty. If the bargaining range is nonempty, each country will propose to allocate themselves the largest quantity that their opponent would accept to avoid war. The decision to extend and accept an offer occurs after both countries have already incurred diplomatic expenses. Then, we can define the equilibrium offers as

$$x_1^*(p) = \begin{cases} \overline{x}(p) & \text{if } \overline{x}(p) \geq \underline{x}(p) \\ x & \text{for any } x \in \mathcal{R}_2(p) \text{ if } \overline{x}(p) < \underline{x}(p) \end{cases} \tag{7}$$

and

$$x_2^*(p) = \begin{cases} \underline{x}(p) & \text{if } \overline{x}(p) \geq \underline{x}(p) \\ x & \text{for any } x \in \mathcal{R}_1(p) \text{ if } \overline{x}(p) < \underline{x}(p). \end{cases} \tag{8}$$

where $\mathcal{R}_1(p) := \{x \in \mathbb{R} : x < \underline{x}(p)\}$ and $\mathcal{R}_2(p) := \{x \in \mathbb{R} : x > \overline{x}(p)\}$ denote the rejection sets for countries 1 and 2.

3.2 Diplomatic Spending

With equilibrium offers defined, we can find equilibrium diplomatic spending. Spending is costly but provides each country with a chance to receive proposal power and hence a favorable peace settlement. Since the value of proposal power is necessarily equal to the bargaining surplus, both countries compete over what is effectively a common value good—both country 1 and 2 necessarily place an equivalent value on proposal power. In particular, the bargaining surplus is the length of the bargaining range in state p, which is $B(p) := \max\{\overline{x}(p) - \underline{x}(p), 0\}$. Given recognition probability $\pi > 1/2$, the expected gain from being the highest spender is $(2\pi - 1)B(p)$ in this period and in subsequent periods with probability λ.

There is no equilibrium in pure strategies.[12] Instead, consider equilibrium spending such that each country mixes according to a cumulative distribution function (c.d.f.) denoted by $F(\cdot)$. I look for a particular spending strategies where both countries are mixing with only nonzero amounts above k, i.e., an equilibrium spending strategy of this kind will satisfy $F^*(s) = F^*(0)$ for all $s \in [0, k)$. This is a natural place to look because when one country only spends more than k, the other country has no reason to spend nonzero amounts less than k, as there is necessarily a profitable deviation to spending zero.[13]

By spending an amount $s \geq k$, country i has expected utility

$$W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda} F^*(s;p) + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(s;p)) - s \tag{9}$$

given their opponent is spending according to c.d.f. $F^*(\cdot)$. This expression is the war payoff, which serves as a baseline value for cooperation since a country would never accept an offer less than this amount, plus their expected share of the bargaining surplus (accounting for how long they expect to hold such a share), less the costs of peace they incur.

Here, country i is the larger spender with probability $F^*(s;p)$, in which case they receive the full bargaining surplus with probability π. If they spend less, which happens with probability $1 - F^*(s;p)$, they receive the bargaining surplus with probability $1 - \pi$. The bargaining surplus is divided by $1-\delta\lambda$ to account for the expected discounted utility country i receives from the settlement as it's upheld in future periods with probability λ.

[12] This is standard for a common value all-pay contest. See the appendix for a proof of the equilibrium spending strategy.

[13] Such an equilibrium is not unique for all values of π, but does exist for all feasible values of $\pi \in (\frac{1}{2}, 1)$. When $\pi < \frac{2}{3}$, there exists a second type of equilibrium where countries occasionally spend nonzero amounts less than k. I relegate this second type of equilibrium to the appendix and show that all substantive implications continue to hold.

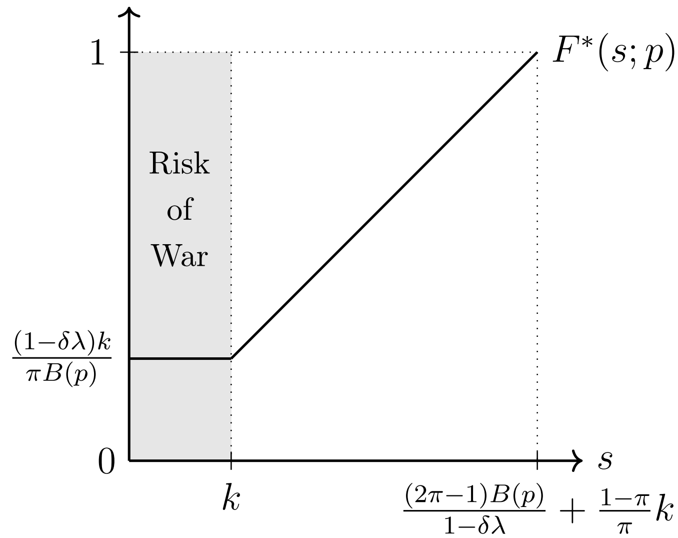

Figure 1. Equilibrium diplomatic spending according to c.d.f. $F^*(\cdot)$ with $\pi \approx 1$. Each country spends zero with probability $(1-\delta\lambda)k/(\pi B(p))$ and spends an amount greater than k with complementary probability. As $\pi \to 1/2$ from the right, countries lose the incentive to compete over the bargaining surplus and $F^*(\cdot)$ gets arbitrarily close to mixing between zero and k as in a normal coordination game. As $\pi \to 1$ from the left, countries spend more in competition over the bargaining surplus.

On the other hand, spending nothing yields an expected value of

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k;p)). \tag{10}$$

This expression is country i's war payoff plus the share of the bargaining surplus they'd expect by spending nothing. Country i knows they will not be the highest spender in cooperation if they choose $s = 0$, but they still stand a chance to be awarded the proposal power with probability $1 - \pi$.

Using the indifference condition on equations (9) and (10), each country's equilibrium spending strategy $F_i^*(s;p)$ is well-defined for all $p \in P$.[14] The result is illustrated in Figure 1 and stated formally in the following proposition.

Proposition 1 (MPE)**.** *There is an essentially symmetric stationary MPE where, for all $p \in P$, each country $i = 1, 2$ plays $\sigma_i^*(p) = (r_i^*(p), s_i^*(p), x_i^*(p), y_i^*(x;p))$ defined as*

[14] See the appendix for all proofs.

follows.

(i) Initiate war, $r_i^*(p) = 1$*, if and only if* $W_i(p) - c_i > U_i(p)$*.*

(ii) Spend $s_i^*(p)$ *according to a mixed strategy with distribution*

$$F^*(s;p) = \begin{cases} 0 & \textit{for } s < 0 \\ \frac{(1-\delta\lambda)k}{\pi B(p)} & \textit{for } s \in [0, k) \\ \frac{(1-\delta\lambda)(s - \frac{1-\pi}{\pi}k)}{(2\pi-1)B(p)} & \textit{for } s \in [k, \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k] \\ 1 & \textit{for } s > \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k \end{cases}$$

if $U_i(p) \geq W_i(p) - c_i$*, or else* $s_i^*(p) = 0$*.*

(iii) Offer $x_i^*(p)$ *as given by equations (7)-(8) when recognized as proposer*

(iv) Accept an offer x*,* $y_i^*(x;p) = 1$*, if and only if* $U_i(x;p) \geq W_i(p) - c_i$*.*

Countries will opt for cooperation if and only if the bargaining range is nonempty and whether the bargaining range is nonempty relies on the values of δ, θ, and λ.

Holding the persistence parameters θ and λ constant, increasing δ (i.e., increasing a country's patience) can reduce the gains from peace. Figure 2 illustrates this with a simple example where $\theta \approx 1$ and $\lambda \approx 0$—that is, war is an absorbing state where the victor gets to keep the pie forever and peace agreements are short-lived. The dynamic bargaining range $X(p)$ is a proper subset of the static bargaining range[15] and decreases in size as patience increases. When δ and θ are close to 1 and λ is close to 0, $X(p)$ is approximately equal to the singleton $\{p\}$. Then, any arbitrarily small cost to cooperation will instigate war, as country 1 will require settlements larger than p while country 2 will require settlements larger than $1 - p$.

In this model, countries also react to costly peace. Peace is costly in two important ways: (i) the diplomatic spending threshold creates an inefficiency of cooperation akin

[15] Refer to Fearon (1995) for the standard, static bargaining model of war.

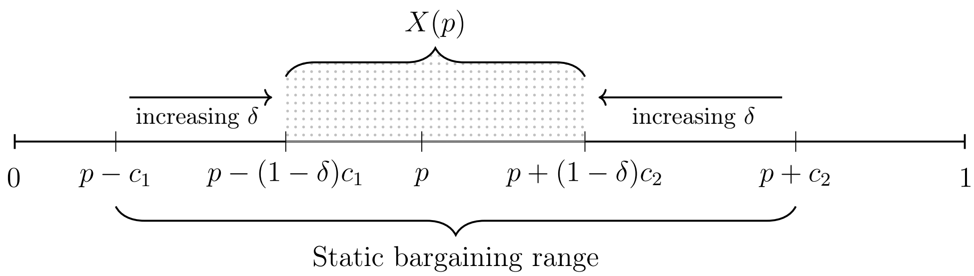

Figure 2. Patience can erode the canonical bargaining range when war outcomes are persistent and peace settlements are transient. Here is an illustrative example with $\theta \approx 1$ nand $\lambda \approx 0$. Assuming expected diplomatic spending is arbitrarily small, $X(p)$ will be arbitrarily close to $[p - (1-\delta)c_1, p + (1-\delta)c_2]$ as θ gets larger and λ gets smaller. The set of feasible settlements reduces to the singleton $\{p\}$ in the limit as $\delta \to 1$ from the left. Given an exogenous cost to cooperation $k > 0$, peaceful settlements will not exist with sufficiently high δ.

to the standard inefficiency of war and (ii) countries can now reach a more favorable settlement through diplomatic spending. Each country has incentive to continue spending in competition for proposal power and, under some conditions, they may continue spending until peace becomes almost as costly as war on average. Due to a conditional incentive to free-ride,[16] the countries will occasionally spend nothing in equilibrium even when they prefer to reach a settlement, resulting in a chance of war in any period. The outbreak of war through a coordination problem is therefore inevitable in the long run.

4 International Orders and Implications

Unlike previous bargaining models of war with complete and perfect information as well as no constraints on possible settlements, countries may prefer attacking their opponent and taking their chances in war. The logic is similar to "ripping off the band-aid"—countries would rather endure the costly pain of war today to forgo the recurring costs of peace. This occurs when countries are patient and peace settlements are not expected to last relative to war outcomes. When countries are impatient, on the other hand, they

[16]The incentive to free-ride is conditional on their expectation of being the lower spender, and hence "losing" the bargaining surplus, with some probability.

certainly prefer peace.

The bargaining surplus is endogenous to these factors. When a bargaining surplus exists, it implies that the international environment is peace-facilitating. Even in these cases, however, war may nonetheless result from a coordination problem: *ex post*, the lower spender would rather spend nothing and free-ride on the gains from cooperation if their opponent is spending at least k. Countries therefore have an incentive to spend nothing on occasion and, if both countries choose to do this simultaneously, war will result.

In addition to the persistence parameters and country patience, the extent to which countries compete over settlements affects the frequency of wars. Specifically, while changes in competitiveness do not directly instigate attacks,[17] a competitive diplomatic environment induces countries to spend more for peace since there are greater gains from being the higher spender and greater losses from being the lower spender. More spending reduces the likelihood of inadvertent wars, but it also depletes the possible gains from cooperation. In the extreme, each country's expected payoff from peace converges to that from war as π approaches 1. Therefore, the welfare implications are not straightfoward: the welfare-maximizing level of competition is not necessarily the war-minimizing one.

4.1 Patience and Outcome Persistence

Conventional wisdom in international politics tells us that peace can be achieved with repeated interactions of patient countries—reasoning that's inherited from the repeated Prisoners' Dilemma.[18] The results below suggest that the story is not necessarily that simple. While it is true that patient countries can prefer peace under some conditions,

[17]Given our assumption that $\pi B(p) > 2(1 - \delta\lambda)k$ to avoid trivial cases. Otherwise, k is too large for even the most impatient countries to consider peace.

[18]The employment of the repeated Prisoners' Dilemma in the study of international cooperation dates back at least to Axelrod (1984), Keohane (1984), and Oye (1986). See Fearon (1998) for discussion of cooperation theory in the context of the bargaining model of war.

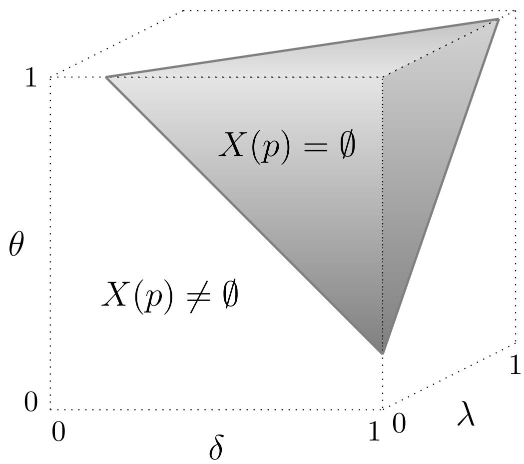

Figure 3. Parameter space $(\delta, \lambda, \theta) \in (0,1)^3$ partitioned into cases where either the bargaining range is empty, $X(p) = \emptyset$, or the bargaining range is nonempty, $X(p) \neq \emptyset$. Countries launch wars in the former case, which occurs when δ and θ are large and λ is small. Proposition 2 shows that such a partition will always divide the parameter space into two nonempty sets. This depicts a special case for illustrative purpose.

countries may also prefer war depending on the persistence of the war outcome relative to that of the peace outcome. On the other hand, impatience guarantees that countries prefer peace.

Proposition 2. *War is directly preferred by a country if war outcomes are persistent, peace settlements are transient, and countries are patient.*

Formally, there exists a $\bar{\lambda}_p$, $\bar{\theta}_p$, $\bar{\delta}_p \in (0,1)$ *for all* $p \in P$ *such that, for any* $\lambda < \bar{\lambda}_p$, $\theta > \bar{\theta}_p$, *and* $\delta > \bar{\delta}_p$, $W_i(p) - c_i > U_i(p)$ *for a country* $i = 1, 2$.

Proposition 2 shows that there is always a sufficiently high level of war outcome persistence and low level of peace outcome persistence that patient countries go to war in equilibrium.[19] Patient countries may prefer to fight wars and impatient ones will not—in contrast to the conventional wisdom from cooperation theory.[20]

This is not to say that the repeated Prisoners' Dilemma argument of cooperation the-

[19]This result is robust to cases where there are gains from cooperation (such that the size of the pie grows following agreement), as well as the possibility of permanent destruction of the pie given the destruction is not too large. See appendix for details.

[20]See the appendix for a simplified version of this model that emphasizes this point in the context of the standard bargaining model of war.

ory is wrong, per se; the logic underlying the conventional wisdom relies on reputational concerns that are ruled out by the MPE solution concept here. It is certainly possible that, given the right features of the international system, such a mechanism could be at work in international cooperation. It is true, however, that under some structural features of the international order paired with country characteristics, punishment strategies to deter war will not be possible when war is more efficient than peace. If war outcomes are persistent and peaceful settlements are transient, repeated interactions among patient countries will not necessarily facilitate peace, but could instead obstruct any hope for cooperation.

This result holds regardless of the balance of power. When allowing countries to credibly make any settlements they would like (which is done to abstract away from preventive wars), countries base their decision to attack on the persistence of outcomes, not their relative strength. An incredibly weak country may still prefer to fight a much stronger opponent if they are patient and war outcomes will likely last much longer than settlements.

The proof of Proposition 2 in the appendix demonstrates this claim. The reasoning is intuitive: it can be better to incur the larger costs of war once in hope to reap the rewards than to pay periodic diplomatic costs for settlements that don't last. Patient countries may want to "rip off the band-aid"—when war outcomes persist and settlements do not, they would rather fight at great cost today than deal with many short-lasting peace agreements regardless of their strength.

It is worth emphasizing this point with a hypothetical example: consider two countries, Weak and Strong. Weak is so much weaker than Strong that Weak expects to lose 99 out of every 100 wars between the two. Nonetheless, the international system is such that peace settlements do not last and war outcomes persist into future periods with near certainty. If Weak is patient and peace is costly, Weak may prefer to attack Strong even though losing is quite likely. This remains true even if the distribution of power is shifting in Weak's favor. Analysts and researchers tend to review wars *ex post*—after a war and subsequent Strong victory, the event would likely be chalked up to psychological or cognitive failures of Weak's leader, or some other type of miscalculation. These explanations may be plausible on a case-by-case basis, but such observed behavior is also consistent with rational choice.

4.2 Competitive Diplomacy and Inadvertent War

This model features an additional cause of war not familiar to existing crisis bargaining literature: inadvertent war as a result of an underlying coordination problem. Each country prefers to spend nothing on cooperation unless they are the higher spender, effectively creating a "conditional free-rider" problem. As a result, countries may spend zero in hope that their opponent will contribute enough to facilitate peace.

If diplomacy is not very competitive (if π is close to $1/2$), then there remains a chance the low-spending country will still receive proposal power and enjoy the bargaining surplus. Competitive environments (when π is closer to 1) make this less likely, as high-spending countries will almost surely be awarded proposal power. When a country chooses to spend nothing, they know there is a possibility their opponent spends nothing, as well, resulting in an inadvertent war. In equilibrium, each country is willing to risk war.

Since countries do not spend positive amounts less than k, the probability of inadvertent war is simply the probability that both countries spend zero.

Remark 1. *For all $p \in P$ such that a bargaining surplus exists, inadvertent war occurs with probability $(F^*(k))^2$. Equivalently, the probability of inadvertent war is*

$$\phi(p) := \left(\frac{(1-\delta\lambda)k}{\pi B(p)} \right)^2. \tag{11}$$

The probability of inadvertent war is decreasing as the international system becomes more competitive. This is intuitive: more competition mitigates the conditional free-rider effect by decreasing the amount a lower-spender would expect to gain from cooperation. As it becomes less likely that a low-spending country will be awarded proposal power and gain the bargaining surplus, countries not only want to bring about cooperation but also want to make sure they spend more than their opponent. Therefore, diplomatic competition increases the amount countries expect to spend on diplomacy and correspondingly reduces the probability of inadvertent war.

The extent to which diplomatic spending is competitive affects the probability of inadvertent war not just directly, but also through the bargaining surplus. Recall that, since $B(p)$ is decreasing in the average aggregate spending and average aggregate spending is increasing in π, $B(p)$ is decreasing in π.

Remark 1 implies that inadvertent war is inevitable in the long run, regardless of the characteristics of countries and the international system, given there is some level of diplomatic competition. Competition is key here: when there is no incentive to outspend a rival country, there also exists trivial, cooperative pure strategy equilibria where peace can be guaranteed. For example, if $\pi = 1/2$ and a bargaining surplus exists, it is clear to see that there are equilibria where one of the two countries spends k and the other spends nothing, always leading to peace. Additionally, without competitive spending, an equilibrium in which both countries spend $k/2$ can be sustained, also resulting in no inadvertent wars. These noncompetitive equilibria imply that environments free of competition can result in no inadvertent wars.[21]

The equilibrium here, on the other hand, highlights features of a competitive international order that the other situations do not. Competitive diplomatic spending does result in occasional inadvertent war, but both countries expect to strictly gain from peace when a bargaining surplus exists.

[21] Though it is worth considering whether a complete lack of diplomatic competition is plausible in international relations.

Lemma 1. *For any $\pi > 1/2$, the net gain from cooperating in state p is equal to*

$$\Psi(p) := \frac{(1-\pi)B(p)}{1-\delta\lambda} - \frac{1-\pi}{\pi}k. \tag{12}$$

Lemma 1 allows us to think about changes in country welfare as changes in their expected net gain from cooperation since, holding the structural parameters constant, changes in π do not affect each country's expected value from war. Then, a natural question is: are countries better off with a competitive international system?

Proposition 3. *Competition decreases welfare and the probability of inadvertent war. Formally, $\Psi(\cdot)$ and $\phi(\cdot)$ are decreasing in π.*

Countries fare better with a low π close to 1/2; however, low levels of competition also increase the probability of inadvertent war. Policy action is therefore not straightforward: policies aimed at reducing inadvertent wars by increasing competition would have a detrimental net effect to country welfare. Even though wars happens more often when competition is low, countries get to enjoy a larger surplus when war does not break out. This is because they do not expend as many resources on diplomacy in dispute over the bargaining surplus, but instead are more content to let their opponent be the higher spender.

5 Discussion

Since Schelling (1960) argued that conflict was essentially a bargaining situation, a long tradition of scholarship has used the bargaining framework to gain insight into the nature of international war and cooperation. The bargaining model has been extensively studied to explore mechanisms underlying problems that bring about conflict. In this paper, I develop a dynamic crisis bargaining model in which countries decide to fight or cooperate in a setting where peace is costly and diplomatic spending is rewarded with bargaining leverage.

The model establishes that patient countries will prefer to fight if war outcomes are likely to persist and peace agreements are not. Even when a bargaining surplus exists, competition over its distribution erodes the net gain from cooperation. In equilibrium, each country occasionally spends nothing on peace due to an underlying coordination problem, leading to inadvertent wars. The frequency of inadvertent war depends on the features of the international system and the shadow of the future.

War typically occurs in bargaining models due to specific features of the strategic relation at hand, most commonly information asymmetries or commitment problems between the players. The impetus for war in this paper, however, is the international system, characterized by θ, λ, and π. The interpretation of π is straightforward: larger π corresponds to greater competition over peace settlements. When π is large, it is probable that the highest spender will receive proposal power and enjoy the bargaining surplus. Hence, countries spend more than necessary on diplomacy to capture the gains from peace. When π is small, on the other hand, the lower spender is almost as likely to be recognized as their rival, making countries less inclined to spend on peace. Competition reduces the probability of war but also reduces welfare—the welfare-maximizing level of competition, therefore, is not necessarily the war-minimizing one.

Persistence parameters θ and λ, on the other hand, can be understood as a reduced form of a larger game happening on a global scale. In this sense, a small θ might reflect the expectations of a territory-seeking country when a third, unmodeled superpower has credibly stated they intend to intervene and expropriate any territory conquered in war. On the other hand, a large θ may reflect the fact that no other country is willing to intervene after territory is won in war and the international norm is to uphold the status quo. The same reasoning could be applied to λ, with small λ reflecting low enforcement capability and high λ reflecting high enforcement capability. Understanding these differences—e.g., when the United States is willing to intervene to upend an outcome that resulted from war and when they are not—can speak to which regions of the world are most prone to conflict.

The contribution of this paper is theoretical; however, the results have empirical implications that are germane to contemporary foreign affairs. In particular, the model sheds light on how changes to the international environment may give rise to conflicts we observe today. For example, the 2010s have seen the graduate weakening of Western-led international institutions—from the US withdrawal from the Paris Climate Accords in 2017 and subsequent withdrawal from the Joint Comprehensive Plan of Action (commonly known as the Iran nuclear deal) in 2018, to the UK withdrawal process from the European Union beginning in 2016. Even though the decline of these mechanisms for cooperation has not been linear (e.g., the US was re-admitted into the Paris Agreement in 2021), there remains active debate in political science over whether the end of the "liberal international order" is here or if a rebound is coming (Ikenberry 2018; Mearsheimer 2019).

In the context of this model, these changes to the international system may reasonably correspond to increases in θ and decreases in λ. While George W. Bush promoted the idea of using American influence (including troops) to promote democracy abroad,[22] both Donald Trump and Joe Biden favored the withdrawal of US troops from Afghanistan—Republican Trump initiating a withdrawal agreement in 2020 and subsequent Democrat Biden following through in 2021. A growing reluctance of leaders from both major parties in the US to engage in foreign wars corresponds to war outcomes being more persistent, as the US may be less willing to involve themselves to upend an unfavorable foreign status quo generally. Moreover, it convincingly suggests that foreign actors should not expect peace settlements to last as long as they otherwise used to, since the US and powerful allies in the West will be less willing to reinforce agreements abroad (as they did in the 1990s with the Bosnian War, for example).

[22] See George W. Bush, Inaugural Address, January 20, 2005.

Shortly after US withdrawal from Afghanistan, the capital city Kabul fell to the Taliban, a process that happened so quickly that Biden remarked that its speed "reinforced that ending US military involvement in Afghanistan now was the right decision."[23] This general trend in American foreign policy continued leading into the beginning of the Russian invasion: as Russian troops advanced on the borders of Ukraine, Biden relocated National Guard troops out of Ukraine, claiming "We have no intention of fighting Russia."[24] These actions likely sent Russia the message that the US and its Western allies are unwilling to engage in foreign conflicts with boots on the ground.[25] Given these changes to the international order, a patient[26] Russia may determine that they prefer "ripping off the band-aid" by invading Ukraine. If Russia can successfully gain control of the capital city Kyiv, they could expect to hold it for a long time (i.e., θ is high). It is the state of the international system, *not* Russia-Ukraine dyadic conditions, that lead to Russia's decision to invade.[27]

This article examines the role of the international order in conflict onset and frequency through just two of many potential channels. In particular, the model focuses on international orders that diverge with respect to their outcome persistence and the severity of their diplomatic competition. There are, on the other hand, many other components of an international order that affect war propensity. Employing formal models to better understand the consequences of changes in the international system and to form precise predictions about how those systemic changes will translate into interna-

[23]"Timeline of U.S. Withdrawal from Afghanistan" by Eugene Kiely and Robert Farley. *FactCheck.org*; August 17, 2021. Retrieved in 2022 from factcheck.org.

[24]"More U.S. troops deploying to Europe, Guard leaving Ukraine" by By Jim Garamone. *National Guard*; February 15, 2022. Retrieved in 2022 from nationalguard.mil.
"The line Biden won't cross on Ukraine" by Nahal Toosi. *Politico*; February 23, 2022. Retrieved in 2022 from politico.com.

[25]It is worth mentioning that a complete study of US willingness to engage in foreign conflicts and attempted regime changes would need to consider their willingness to engage in other tactics, such as funding foreign actors and asymmetric warfare (e.g., drone strikes).

[26]See Cohen (2018) for arguments on how Russia is a patient country.

[27]This is not to say that there are no other credible explanations for Russia's invasion, but that the mechanism in this paper is consistent with the details of this case.

tional conflict is a worthwhile enterprise for future research. The formal analysis here aims to take one step towards this better understanding.

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Online Appendix for "Bargaining, War, and Cooperation in the Long Run"

Contents

A Model Preliminaries

Payoffs

After the outbreak of war, country i wins a pie of size 1 today with probability p_i and tomorrow (discounted at rate δ) they either get, with probability θ, $Z_i^{war}(p)$, which is the *ex ante* expected future value of the war outcome persisting (i.e., prior to knowledge over whether i or i's opponent wins the war), or with complementary probability $1-\theta$ they return to bargaining under a new state p'.

Given the state $p \in P$, each country i has an expected war value of

$$W_i(p) = p_i + \delta \left[\theta Z_i^{war}(p) + (1-\theta) \sum_{p' \in P} V_i(p') q(p'|p, a) \right] \tag{A1}$$

where

$$Z_i^{war}(p) = p_i + \delta \left[\theta Z_i^{war}(p) + (1-\theta) \sum_{p' \in P} V_i(p') q(p'|p, a) \right]. \tag{A2}$$

Using equation (A2) to solve for $Z_i^{war}(p)$, equation (A1) simplifies to equation (3) in the main text.

If countries succeed in cooperating, they have an expected settlement at the Nash bargaining solution $x_m(p)$. We know this because countries win proposal power with probability $\frac{1}{2}$ in equilibrium. Then, countries receive a payoff according to the expected settlement today, less the amount they expect to spend according to their mixed diplomatic spending strategy. Tomorrow, countries receive payoffs according to the same settlement allocation with probability λ and return to the bargaining table under a new state p' with probability $1-\lambda$. Formally, we can write

$$U_i(p) = u_i(x_m(p)) - \mathbb{E}[s_i^*(p)] + \delta \left[\lambda Z_i^{peace} + (1-\lambda) \sum_{p' \in P} V_i(p') q(p'|p, a) \right] \tag{A3}$$

where

$$Z_i^{peace}(p) = u_i(x_m(p)) + \delta \left[\lambda Z_i^{peace}(p) + (1-\lambda) \sum_{p' \in P} V_i(p') q(p'|p,a) \right]. \tag{A4}$$

As before, it is necessary to account for the uncertainty over how long peace will persist with $Z_i^{peace}(p)$. Using equation (A4) to solve for $Z_i^{peace}(p)$, equation (A3) simplifies to equation (4) in the main text.

Offers

When bargaining surplus exists, a country will offer the least generous settlement that facilitates peace—that is, they will offer the point that makes their opponent indifferent. Since diplomatic expenses are sunk costs at the point at which a country makes a proposal, a proposing country $-i$ will offer an x that solves

$$W_i(p) - c_i = u_i(x) + \frac{\delta}{1-\delta} \left[\lambda u_i(x) + (1-\lambda) \sum_{p' \in P} V_i(p') q(p'|p,a) \right]. \tag{A5}$$

Solving (A5) for $u_i(x)$, we get

$$u_i(x) = (1-\delta\lambda)(W_i(p) - c_i) - \delta(1-\lambda) \sum_{p' \in P} V_i(p') q(p'|p,a),$$

which, after plugging in $u_1(x) = x$ and $u_2(x) = 1 - x$, yields equations (7) and (8) in the main text.

B Simple Example of Inefficient War

Consider the simplest bargaining model of war with complete and perfect information we can construct: country 1 wins a war with probability p and each country incurs costs of war $c_i > 0$. Without dynamics or costly peace, war never occurs in equilibrium as a proposing country 1 offers $p+c_2$, a proposing country 2 offers $p-c_1$, and both countries accept offers at least as good as these.

However, suppose we define the period in which costs of war are incurred as period 1 and include an infinite horizon over which the victor will get to enjoy the gains from war. Then, the gain from war becomes $\frac{p}{1-\delta} - c_1$ and $\frac{1-p}{1-\delta} - c_2$ for countries 1 and 2, respectively, where δ is the discount rate.

Additionally, assume there is an arbitrarily small cost of peace $k > 0$ that each country needs to incur before reaching an agreement. For simple demonstration, assume that countries need to incur these costs of peace for any period in which peace is sustained—in this case, it might be sensible to think of the costs of peace as enforcement costs. Then, for country 1 to prefer peace to war, we need a settlement $x \in \mathbb{R}$ such that

$$\frac{x-k}{1-\delta} \geq \frac{p}{1-\delta} - c_1 \tag{A6}$$

$$x - k \geq p - (1-\delta)c_1 \tag{A7}$$

and likewise, for country 2, the settlement must satisfy

$$\frac{1-x-k}{1-\delta} \geq \frac{1-p}{1-\delta} - c_2 \tag{A8}$$

$$p + (1-\delta)c_2 \geq x + k. \tag{A9}$$

From equations (A7) and (A9), we can see that as $\delta \to 1$, country 1 requires $x \geq p + k$ whereas country 2 requires $x \leq p - k$. Given $k > 0$, such an x cannot exist.

Therefore, when δ is large, even arbitrarily small costs of peace can destabilize cooperation.

C Additional MPE

It is clear to see that the equilibrium of the main text becomes the unique MPE when the cooperation game is reduced to an exogenous cost of peace. Then, countries either pay to have peace or choose to initiate conflict, but a bargaining surplus still may not exist depending on the parameters of the game. Incorporating the competitive diplomacy, however, creates multiplicity.

In particular, the equilibrium in the main text is not unique for all π. Consider the cases where π is close to 1—i.e., the institution of cooperation is responsive to donors—as opposed to that in which π is close to $1/2$—that is, the institution of cooperation is fairly egalitarian in proposer recognition. When $\pi \approx 1$, countries deplete the bargaining surplus with diplomatic spending. When $\pi \approx 1/2$, countries can recover large gains from peace even if they spend much less than their opponent. Therefore, if π is low, a country may be willing to spend lower amounts between 0 and k if their opponent does as well. This is simply due to a stronger incentive to cooperate.

For example, there is another equilibrium if $\pi < 2/3$ where countries always spend nonzero amounts, including amounts between 0 and k. This equilibrium cannot be sustained for larger π since the gain from cooperating as a lower spender is too small to justify low spending in equilibrium.

Consider, for example, π close to 1—the value of cooperation conditional on not being recognized as proposer is very close to zero. On the other hand, with π close to $1/2$, the value of cooperation conditional on not being recognized as proposer is very close to the value of cooperation conditional on being the proposer, both of which are larger than the expected war payoff. Therefore, if countries can contribute very small amounts and sustain cooperation with higher likelihood, they may prefer to even if they are very unlikely to win proposal power.

For the remainder of the appendix, I distinguish the equilibrium stated in the main text from this underspending equilibrium by referencing them as MPE I and MPE II, respectively. The following proposition provides a formal characterization for MPE II.

Proposition 4 (MPE II). *For all $\pi < 2/3$, there is an essentially symmetric stationary MPE where each country $i = 1, 2$ plays $\sigma_i^*(p) = (r_i^*(p), \tilde{s}_i^*(p), x_i^*(p), y_i^*(x; p))$ for all $p \in P$, with $r_i^*(p)$, $x_i^*(p)$, and $y_i^*(x; p)$ as defined in Proposition 1 and $\tilde{s}_i^*(p)$ as a random draw from the distribution*

$$\tilde{F}^*(s; p) = \begin{cases} 0 & \text{for } s < 0 \\ \frac{(1-\delta\lambda)(2-3\pi)s}{(1-\pi)\pi B(p)} & \text{for } s \in [0, \frac{k}{2}) \\ \frac{(1-\delta\lambda)(\pi s-(2\pi-1)k)}{(1-\pi)\pi B(p)} & \text{for } s \in [\frac{k}{2}, k) \\ \frac{(1-\delta\lambda)(s-\frac{1-\pi}{\pi}k)}{(2\pi-1)B(p)} & \text{for } s \in [k, \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k] \\ 1 & \text{for } s > \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k \end{cases}$$

if $U_i(p) > W_i(p) - c_i$, or else $\tilde{F}^*(s; p) = 1$ for all $s \geq 0$.

This result is illustrated in Figure 4. While diplomatic spending in MPE II is different than in MPE I, all other behavior (whether to launch a war, what settlement to offer when proposing, and what settlements to accept if not proposing) remains the same. Though countries are always spending a positive amount, there remains a risk of war through a coordination problem since aggregate spending can still be less than k.

Remark 2. *$\tilde{F}^*(\cdot)$ first-order stochastically dominates $F^*(\cdot)$.*

Remark 2 implies that the probability of inadvertent war, i.e., the probability that both countries spend less than k in aggregate, will necessarily be weakly larger under MPE I than under MPE II. In fact, because we know $F^*(s; p) > \tilde{F}^*(s; p)$ only for $s < k$ and $F^*(s; p) = \tilde{F}^*(s; p)$ otherwise, we know the inequality is strict. Denote $\phi(p)$ and $\tilde{\phi}(p)$ the probability of inadvertent war under MPE I and MPE II, respectively.

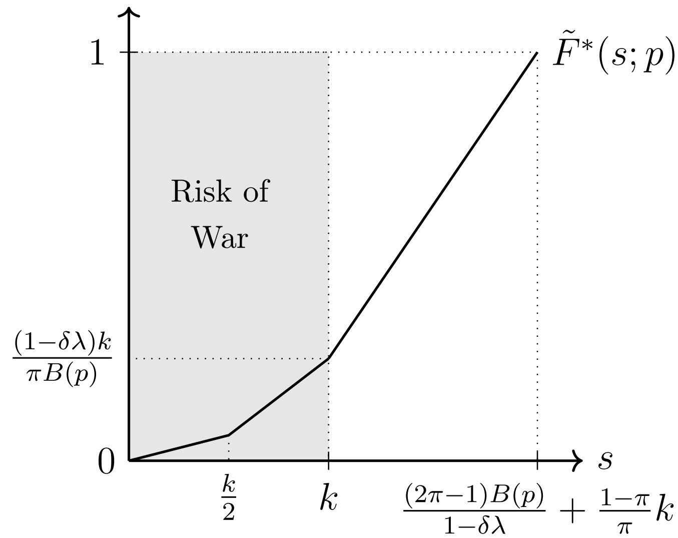

Figure 4. Equilibrium diplomatic spending according to c.d.f. $\tilde{F}^*(\cdot)$ with $\pi < \frac{2}{3}$. Now each country spends a nonzero amount with probability 1 because the gains from cooperation are large even if the country is outspent by their opponent. As $\pi \to 1/2$ from the right, each country will play a strategy arbitrarily close to mixing uniformly between 0 and k with probability $(1-\delta\lambda)2k/B(p)$ and spending k with complementary probability.

Comments on Remark 2. Recall that $\tilde{F}^*(\cdot)$ first-order stochastically dominates $F^*(\cdot)$ if and only if $F^*(s) \geq \tilde{F}^*(s)$ for all s and $F^*(s) > \tilde{F}^*(s)$ for some s. Then, note that $\tilde{F}^*(s) = F^*(s)$ for all $s \notin (0,k)$. For all $s \in [\frac{k}{2}, k)$,

$$F^*(s) = \frac{(1-\delta\lambda)k}{\pi B(p)} > \frac{(1-\delta\lambda)(\pi s - (2\pi-1)k}{(1-\pi)\pi B(p)} = \tilde{F}^*(s)$$

and for all $s \in (0, \frac{k}{2})$,

$$F^*(s) = \frac{(1-\delta\lambda)k}{\pi B(p)} > \frac{(1-\delta\lambda)(2-3\pi)s}{(1-\pi)\pi B(p)} = \tilde{F}^*(s).$$

Lemma 2. *For all $p \in P$, $\phi(p) > \tilde{\phi}(p) > 0$.*

Lemma 2 states the probability of inadvertent war under MPE I is always greater than that under MPE II, which is always greater than zero.

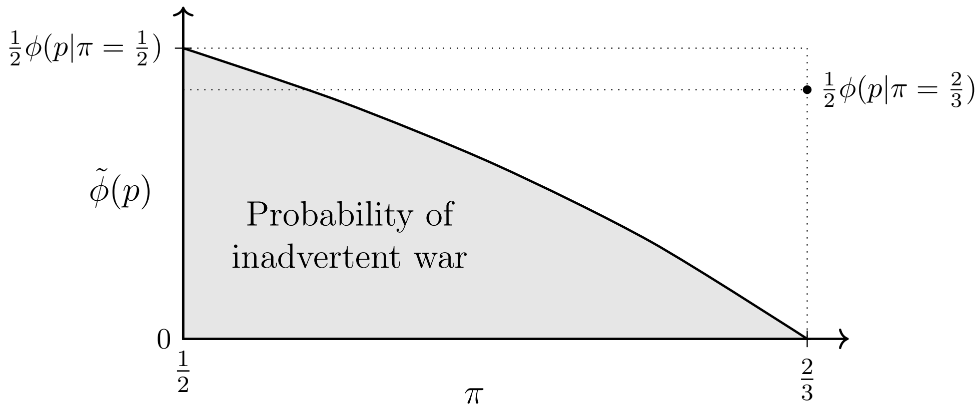

Figure 5. The probability of inadvertent war from a coordination failure under MPE II, $\tilde{\phi}(p)$, as a function of π. When π is close to $1/2$, $\tilde{\phi}(p)$ is close to $\frac{1}{2}\phi(p)$, where $\phi(p)$ is the probability of inadvertent war under MPE I. As π increases, $\tilde{\phi}(p)$ decreases, approaching zero in the limit as π approaches $2/3$ from the left. Note that $\tilde{\phi}(p)$ never reaches zero, as MPE II breaks for any $\pi \geq 2/3$. The coordinate in the upper right points out that $\phi(p)$ is also decreasing in π, though always strictly greater than $\tilde{\phi}(p)$.

Lemma 3. *A country's expected net gain from cooperating in state p equal to $\Psi(p)$ under both MPE I and MPE II.*

Lemma 3 states that the expected gain from peace in MPE I is equal to that in MPE II—neither equilibrium is preferable *ex ante* even though less inadvertent war occurs under MPE II. The reason for this is that, while inadvertent war is more likely under MPE I, the expected spending is smaller. Hence it is more likely that a nonproposing country, which will not receive the benefit of the bargaining surplus, will have spent a larger amount on diplomatic spending under MPE II.

D Proofs

Proof of Proposition 1. The war decision is straightforward. The countries seek to maximize their expected utility over the long run and if, given state $p \in P$, their war continuation value $W_i(p)$ less the costs of war c_i is larger than their continuation value from cooperating, $V_i(p)$, they will necessarily prefer to fight.

Then, if there exists a bargaining surplus, $B(p) = x_1^*(p) - x_2^*(p) > 0$, both countries will cooperate and choose an amount to spend as a function of the state p. Then, a country i that's recognized as proposer will receive the value of the bargaining surplus today and possibly in the future, with likelihood according to λ. Hence, the expected net gain of winning proposal power becomes

$$B(p) + \delta\lambda B(p) + (\delta\lambda)^2 B(p) + \ldots = \frac{B(p)}{1 - \delta\lambda} \tag{A10}$$

However, the net gain from becoming proposer is equal to the net gain from spending more on diplomacy than your opponent divided by recognition probability π. Hence, the expected net gain from spending an amount s on diplomacy is

$$\frac{\pi B(p)}{1 - \delta\lambda} \Pr(s > s_{-i}) + \frac{(1 - \pi)B(p)}{1 - \delta\lambda} \Pr(s_{-i} > s) - s. \tag{A11}$$

Clearly, the net gain is zero when $B(p) = 0$, in which case neither country will be willing to spend a positive amount in equilibrium, leading to war as a result of the unsatisfied minimum funding requirement k. Therefore, assume $B(p) > 0$.

It is straightforward that there will be no pure strategy spending in equilibrium. If a country always spends amount $s > 0$, their opponent would either deviate to an amount greater than s or zero. If the opponent deviated to zero, the country will prefer to spend less than s. If the opponent deviated to an amount greater than s, the country will prefer to move to a greater amount or deviate to zero. If both countries spend the same amount, they will either have an incentive to increase their spending a marginal amount to increase their gain by approximately double or they will prefer to deviate to zero. This is standard in common value all-pay contests.

Therefore, I proceed by looking for a mixed strategy given by cumulative distribution function (c.d.f.) $F^*(\cdot)$ that satisfies equation (A11) for both countries. Note that, because $B(p) > 0$, there is a strict gain to cooperating that is decreasing in π. In particular, the expected utility of country i spending $s = 0$ is strictly greater than their war payoff. We can write the payoff from spending zero as

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k)). \tag{A12}$$

Given a country is spending zero with positive probability, their opponent has incentive to spend at least k with positive probability, which yields an expected payoff

$$W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda}F^*(k) + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k)) - k. \tag{A13}$$

Using equations (A12) and (A13), we can solve for $F^*(k)$,

$$F^*(k) = \frac{(1-\delta\lambda)k}{\pi B(p)}. \tag{A14}$$

We know that k cannot be the top of the support because $\pi B(p) > (1-\delta\lambda)k$ by assumption and they can get a strictly higher payoff by contributing slightly more than k to get recognized as proposer. Then, a country i spending $s > k$ will receive an expected payoff

$$W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda}F^*(s) + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(s)) - s. \tag{A15}$$

Using the indifference condition for equations (A13) and (A15) and plugging in equation (A14), we find that for $s \geq k$

$$F^*(s) = \frac{(1-\delta\lambda)(s - \frac{1-\pi}{\pi}k)}{(2\pi - 1)B(p)}. \tag{A16}$$

By definition of a c.d.f., we know the largest amount a country can spend and still be indifferent is given by $\bar{s} := \inf\{s \geq 0 : F^*(\bar{s}) = 1\}$. Using equation (A16), we find that

$$\bar{s} = \frac{(2\pi - 1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k. \tag{A17}$$

These equations yield an equilibrium spending strategy presented in Proposition 1. To check for profitable deviations, consider the case where a country spends $s > \bar{s}$ with nonzero probability. By deviating, their payoff will be

$$W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda} - s < W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda} - \bar{s}, \tag{A18}$$

and therefore they do not deviate. In words, a country already wins with certainty when spending $\bar{s}$, so there is no reason to ever spend more given their opponent plays this strategy as well.

Further, consider a deviation to spending a nonzero amount less than k, $s \in (0, k)$, with some probability. By deviating, their expected payoff will be

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k-s)) - s. \tag{A19}$$

Since their opponent is playing a strategy such that $F^*(k) = F^*(k-s)$, we can plug this into equation (A19) and see that their payoff is equal to

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k)) - s \tag{A20}$$

which is strictly less than their payoff from spending zero given by equation (A12), hence this is not a profitable deviation. This is sufficient to show that c.d.f. $F^*(\cdot)$ is an equilibrium spending strategy.

Now consider a country's offer. From the main text, equations (5) and (6) yield implicit conditions for $\overline{x}(p)$ and $\underline{x}(p)$, which represent the settlements at which country 1 and 2 are left indifferent between peace and war, respectively. If this value is in the unit interval, this is the country's equilibrium offer as offering a settlement less favorable to their opponent will result in a rejection and subsequent war, whereas offering a settlement more favorable to their opponent will result in acceptance but a worse settlement for themselves. If these values are not in the unit interval, the countries offer the closest value in the unit interval by Euclidean distance. In the event this results in acceptance, this is the best the country can do and they will consume the entire pie by settlement. If this results in rejection, all other possible settlement offers would likewise result in rejection, so there is no profitable deviation.

Their decision to accept or reject and offer is also straightforward. At the point at which a proposal has been made, a proposer has already been recognized and spending has already occurred, hence diplomatic spending in that period becomes a sunk cost. Therefore, when an offer x is proposed, countries decide whether they prefer $U_i(x;p) = U_i(p) + s_i(p)$ or their war continuation value and accept or reject accordingly. □

Proof of Proposition 2. For contradiction, suppose $U_i(p) \geq W_i(p) - c_i$ for both $i = 1, 2$ for all $\theta \in (0,1)$, $\lambda \in (0,1)$, and $\delta \in (0,1)$. Let any $x \in [0,1]$ be the expected peaceful settlement in state p, under which the continuation values for peace are

$$U_1(p) = x - \int_0^\infty s dF_1^*(s;p) + \frac{\delta}{1-\delta\lambda}\left[\lambda x + (1-\lambda)\sum_{p'\in P} V_1(p')q(p'|p,a)\right]$$

and

$$U_2(p) = 1 - x - \int_0^\infty s dF_2^*(s;p) + \frac{\delta}{1-\delta\lambda}\left[\lambda(1-x) + (1-\lambda)\sum_{p'\in P} V_2(p')q(p'|p,a)\right].$$

Note that $\int_0^\infty s dF_i^*(s;p)$ relies on equilibrium behavior as a function of state p, but it is sufficient for the argument that follows to assume that $\int_0^\infty s dF_i^*(s;p) > s_i$ for some fixed and arbitrarily small $s_i > 0$. If the proof holds under such an s_i, it must also hold under true expected spending in equilibrium since $U_i(p)$ is decreasing in spending and expected spending is greater than zero to facilitate peace.

Further, we only need to look for one-shot deviations, so we can assume that $V_i(p) = U_i(p)$ for all $p \in P$ and denote $U_i(\bar{p}) := \sum_{p'\in P} U_i(p')q(p'|p,a)$. Note that if $U_1(\bar{p}) > U_1(p)$, then necessarily $U_2(\bar{p}) < U_2(p)$. Thus if we can show there exist thresholds that make $W_i(p) - c_i > U_i(p)$ for both i when $U_i(p) = U_i(\bar{p})$, it also holds for at least one country when $U_i(p) \neq U_i(\bar{p})$. Therefore, assume $U_i(p) = U_i(\bar{p})$ and look to show there exists a δ, θ, and λ such that both countries prefer war.

With this, we can rewrite continuation values as

$$U_1(p) = \frac{x - (1 - \delta\lambda)s_1}{1 - \delta}$$

and

$$U_2(p) = \frac{1 - x - (1 - \delta\lambda)s_2}{1 - \delta}.$$

We need to compare this to their war continuation values. Given our assumption that $\sum_{p' \in P} V_i(p')q(p'|p, a) = U_i(p)$, we can plug these into $W_1(p)$ and $W_2(p)$ to recover

$$W_1(p) = \frac{p}{1 - \delta\theta} + \frac{\delta(1 - \theta)}{1 - \delta\theta} \cdot \frac{x - (1 - \delta\lambda)s_1}{1 - \delta}$$

and

$$W_2(p) = \frac{1 - p}{1 - \delta\theta} + \frac{\delta(1 - \theta)}{1 - \delta\theta} \cdot \frac{1 - x - (1 - \delta\lambda)s_2}{1 - \delta}.$$

This implies that $W_1(p) - c_1 > U_1(p)$ and $W_2(p) - c_1 > U_2(p)$ if and only if

$$p - (1 - \delta\theta)c_1 + (1 - \delta\lambda)s_1 > p + (1 - \delta\theta)c_2 - (1 - \delta\lambda)s_2.$$

Analogously, there does not exist an $x \in \mathbb{R}$ that satisfies either country if

$$\theta > \bar{\theta}(\delta, \lambda) := \frac{c_1 + c_2 - (1 - \delta\lambda)(s_1 + s_2)}{\delta(c_1 + c_2)}.$$

Here $\bar{\theta}(\cdot)$ is not a function of the state p because we took an s_1 and s_2 strictly less than the expected equilibrium spending in a given state p; however, in general, $\bar{\theta}(\cdot)$ will rely on the state p since equilibrium spending will rely on the state p. It is sufficient to show that $\bar{\theta}(\delta, \lambda) < 1$ for sufficiently high δ and sufficiently low λ. Note that $\bar{\theta}(\delta, \lambda) < 1$ when $(s_1 + s_2)/(c_1 + c_2) > (1-\delta)/(1-\delta\lambda)$. The left-hand side is a positive real number that does not rely on parameters, whereas the multivariable limit of the right-hand side is

$$\lim_{(\delta,\lambda)\to(1,0)} \frac{1-\delta}{1-\delta\lambda} = 0.$$

Hence the expression is satisfied in the limit of (δ, λ), completing the proof. □

Proof of Lemma 1. This lemma follows directly from the expected payoff in peace given by equation (A12) less war payoff $W_i(p) - c_i$, which is equal to

$$\frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k)) \tag{A21}$$

Plugging in $F^*(k)$ defined by (A14) this becomes

$$\frac{(1-\pi)B(p)}{1-\delta\lambda} - \frac{1-\pi}{\pi}k. \tag{A22}$$

□

Proof of Proposition 3. It is straightforward to see welfare $\Psi(\cdot)$ is decreasing in π. In the first part of equation (12), it is clear to see that increases in π decrease $1-\pi$ directly and $B(p)$ indirectly through average aggregate spending. In the second part, increases in π increases $-k(1-\pi)/\pi$. However, since the rate of decrease in the first term is necessarily greater than the rate of increase in the second term, we know $\Psi(\cdot)$ is decreasing in π.

The probability of inadvertent war $\phi(\cdot)$ is slightly more complicated, since π directly increases the denominator but indirectly decreases it through $B(p)$. To find which force outweighs the other, first note that, when a bargaining surplus exists,

$$\sum_i \sum_{p' \in P} V_i(p') q(p'|p, a) = \frac{1 - 2(1 - \delta\lambda)\mathbb{E}[s^*(p)]}{1 - \delta} \tag{A23}$$

and therefore we can rewrite $B(p)$ in terms of average spending

$$B(p) = 1 - (1 - \delta\lambda) \sum_i (W_i(p) - c_i) + \delta(1 - \lambda)\frac{1 - 2(1 - \delta\lambda)\mathbb{E}[s^*(p)]}{1 - \delta}. \tag{A24}$$

We can plug equation (A24) into equation (11) to recover

$$\phi(p) = \left(\frac{(1 - \delta\lambda)k}{\pi \left(1 - (1 - \delta\lambda) \sum_i (W_i(p) - c_i) + \delta(1 - \lambda)\frac{1 - 2(1 - \delta\lambda)\mathbb{E}[s^*(p)]}{1 - \delta}\right)} \right)^2 \tag{A25}$$

from which we can calculate

$$\frac{\partial\phi(p)}{\partial\pi} = -\frac{2(1 - \delta\lambda)^2 k^2}{\pi^3 \left(1 - (1 - \delta\lambda) \sum_i (W_i(p) - c_i) + \frac{\delta(1-\lambda)(1-2(1-\delta\lambda)\mathbb{E}[s^*(p)])}{1-\delta}\right)^2}. \tag{A26}$$

This implies

$$\operatorname{sgn}\left(\frac{\partial\phi(p)}{\partial\pi}\right) = -\frac{\operatorname{sgn}(1 - \delta\lambda)^2 \operatorname{sgn}(k)^2}{\operatorname{sgn}(\pi)^3 \operatorname{sgn}\left(1 - (1 - \delta\lambda) \sum_i (W_i(p) - c_i) + \frac{\delta(1-\lambda)(1-2(1-\delta\lambda)\mathbb{E}[s^*(p)])}{1-\delta}\right)^2} \tag{A27}$$

and therefore $\operatorname{sgn}\left(\frac{\partial\phi(p)}{\partial\pi}\right) = -1$.

□

Proof of Proposition 4. To look for a new equilibrium diplomatic spending strategy $\tilde{F}^*(\cdot) \neq F^*(\cdot)$ such that countries occasionally spend positive amounts less than k, suppose there exists an $s \in (0, \frac{k}{2})$ such that $\tilde{F}^*(s) \neq \tilde{F}^*(s + \varepsilon)$ for any $\varepsilon \neq 0$. Then we know that

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - \tilde{F}^*(s')) - s = W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda} - \bar{\bar{s}} \tag{A28}$$

where $\bar{\bar{s}} := \inf\{s \geq 0 : F^*(s) = 1\}$ and $s' := k - s$. Solving for $\tilde{F}^*(s')$ yields

$$\tilde{F}^*(s') = \frac{(1-\delta\lambda)(\bar{\bar{s}} + s' - k)}{(1-\pi)B(p)} - \frac{2\pi - 1}{1-\pi}. \tag{A29}$$

We also know that in equilibrium

$$\begin{aligned} & W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - \tilde{F}^*(s')) - s \\ = W_i(p) - c_i + & \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - \tilde{F}^*(s')) + \frac{\pi B(p)}{1-\delta\lambda}(\tilde{F}^*(s') - \tilde{F}^*(s)) - s' \end{aligned} \tag{A30}$$

which implies

$$\tilde{F}^*(s') = \tilde{F}^*(s) + \frac{(1-\delta\lambda)(s' - s)}{\pi B(p)}. \tag{A31}$$

Using equations (A29) and (A31) we can solve for $F^*(s)$,

$$\tilde{F}^*(s) = \frac{(1-\delta\lambda)(\pi\bar{\bar{s}} + (2-3\pi)s - (1-\pi)k)}{(1-\pi)\pi B(p)} - \frac{2\pi - 1}{1-\pi}. \tag{A32}$$

To proceed we need to know the value of $\bar{\bar{s}}$. We can find it from the equilibrium condition that for any $s \geq k$, we have

$$\begin{aligned} W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - \tilde{F}^*(s)) + \frac{\pi B(p)}{1-\delta\lambda}\tilde{F}^*(s) - s \\ = W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - \tilde{F}^*(k)) \end{aligned} \tag{A33}$$

where the righthand side follows from the need to have zero in the support. If zero is not in support, there is always a profitable deviation from the infinum of the support to zero. We know from equation (A29) that $F^*(k) = (\pi B(p))^{-1}(1-\delta\lambda)k$, so we can solve the above equation to recover

$$\bar{\tilde{s}} = \frac{(2\pi - 1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k. \tag{A34}$$

Plugging this into equation (A32), we have that for any $s \in (0, \frac{k}{2})$,

$$\tilde{F}^*(s) = \frac{(1-\delta\lambda)(2-3\pi)s}{(1-\pi)\pi B(p)}. \tag{A35}$$

For this to be a valid c.d.f., we require $(2-3\pi)s > 0$, or $\pi < \frac{2}{3}$. So for sufficiently high values of π, this is not possible. Given $\pi < \frac{2}{3}$, suppose there exists an $s' \in (\frac{k}{2}, k)$ such that $\tilde{F}^*(s') \neq \tilde{F}^*(s' + \varepsilon)$ for any $\varepsilon \neq 0$. Then, letting $s = k - s'$, we have

$$\tilde{F}^*(s') = \frac{(1-\delta\lambda)(\pi s' + (1-2\pi)k)}{(1-\pi)\pi B(p)} \tag{A36}$$

For this expression to be a valid c.d.f. we require $\pi > \frac{k}{2k+s'}$, which should always hold for any $\pi \in (\frac{1}{2}, \frac{2}{3})$. By equations (A34) and (A36), we know that $\bar{\tilde{s}} = \bar{s}$ and $\tilde{F}^*(k) = F^*(k)$, which implies $\tilde{F}^*(s) = F^*(s)$ for all $s \geq k$.

There is no profitable deviation from $\tilde{F}^*(\cdot)$ for the same reasons as $F^*(\cdot)$. Countries are indifferent over spending at all amounts in the support and the country would do strictly worse by spending more than the supremum of the support. Moreover, given equilibrium spending $\tilde{s}_i(p)$ as a random draw from $\tilde{F}^*(\cdot)$, there is no profitable deviation from other actions for the same reasons as Proposition 1. □

Proof of Lemma 2. Lemma 2 follows immediately from Remark 2; however, we can calculate the quantities explicitly since we have closed-form solutions for equilibrium spending strategies. Under MPE I, no country spends a nonzero amount less than k. Thus the probability of inadvertent war is simply given by the c.d.f. evaluated at k squared, i.e., $(F^*(k;p))^2$, since this is the probability that both countries spend less than k simultaneously. Equivalently,

$$\phi(p) = \left(\frac{(1-\delta\lambda)k}{\pi B(p)}\right)^2. \tag{A37}$$

Under MPE II, on the other hand, countries now spend nonzero amounts less than k in equilibrium, implying that, conditional on a country spending an amount $s < k$, the probability of inadvertent war becomes $\tilde{F}^*(k-s;p) < \tilde{F}^*(k;p)$. We can utilize the fact that countries mix uniformly over $s \in (0, \frac{k}{2})$ and over $s \in [\frac{k}{2}, k)$ to reduce the probability of inadvertent war to the simple expression $\tilde{F}^*(k;p)\tilde{F}^*(\frac{k}{2};p)$, or equivalently

$$\tilde{\phi}(p) = \frac{(1-\delta\lambda)(2-3\pi)k}{2(1-\pi)\pi B(p)} \cdot \frac{(1-\delta\lambda)k}{\pi B(p)}. \tag{A38}$$

Together, equation (A37) and (A38) imply

$$\tilde{\phi}(p) = \frac{2-3\pi}{2-2\pi}\phi(p)$$

for all $p \in P$. Since $1 > (2-3\pi)/(2-2\pi) > 0$ and $\phi(p) > 0$ for all $\pi \in (\frac{1}{2}, 1)$, Lemma 2 always holds: there is always a nonzero probability of inadvertent war and, holding π constant, this probability is strictly smaller under MPE II. □

Proof of Lemma 3. For MPE II, the condition is the same since $\tilde{F}^*(k) = F^*(k)$. □

E Robustness

Gains from Cooperation

The results are robust to potential gains in the size of the pie from cooperation. In particular, consider a case where cooperation grows the size of the pie because, although there is an underlying conflict situation and opposing preferences over the division of resources, there may also be the existence of comparative advantages that the two countries could exploit via cooperation.

Proposition 5. *Proposition 2 continues to hold if cooperation grows the pie.*

Proof. Suppose $\Pi > 1$ is the size of the pie from cooperating, which is contrasted with the pie of 1 from war. Then, the new bargaining surplus can be expressed

$$\tilde{B}(p) = \Pi - (1-\delta\lambda)\sum_i (W_i(p) - c_i) + \delta(1-\lambda)\left(\sum_i \sum_{p' \in P} V_i(p')q(p'|p,a)\right) \tag{A39}$$

Then we know that $\tilde{B}(p) > 0$ if and only if

$$\Pi < 1 - (1-\delta\theta)(c_1 + c_2) + 2\delta(\theta - \lambda)\mathbb{E}[s^*(p)], \tag{A40}$$

as both countries will spend the same on average in equilibrium. This occurs if and only if

$$\theta > \frac{c_1 + c_2 + 2\delta\lambda\mathbb{E}[s^*(p)]}{\delta(c_1 + c_2 + 2\mathbb{E}[s^*(p)])} \tag{A41}$$

Taking the multivariable limit,

$$\lim_{(\delta,\lambda)\to(1,0)} \frac{c_1 + c_2 + \delta\lambda(s_1 + s_2)}{\delta(c_1 + c_2 + s_1 + s_2)} = \frac{c_1 + c_2}{c_1 + c_2 + s_1 + s_2} < 1 \tag{A42}$$

concluding the proof. □

Permanent Costs of War

In the baseline model, the costs of war and peace do not last forever, so countries choose to go to war or cooperate on the basis of which is more painful for them: in the extreme case, one large cost today or many small costs over a long period of time. This logic is robust to circumstances where we think damages in war are more or less destructive but not permanent. This is a natural framing, since countries do seem to recover from wars in some amount of time. Even if these recovery periods can be very long, we can always consider that amount of time (again, however long) the length of a single period in the model.

Nonetheless, we can consider the case of truly permanent costs of war and show it is robust as long as the amount of destruction is sufficiently small. Suppose, for example, that if we fight on this land, we may permanently destroy its potential for agriculture. If too large of an amount of the pie will be permanently destroyed, there may no longer exist parameters that bring about war directly. This should be obvious, since in the limit the full pie is destroyed and nothing could make you choose war, even if you were settling with virtually none of the pie each period. However, so long as not too much of the pie is getting destroyed each period, we can recreate the results of the paper.

Consider the simple case where $\lambda = 0$ and $\theta = 1$ as in the example in Appendix B. Say that $1 - \gamma$ of the pie is destroyed, leaving γ behind. Then, we know that country 1 prefers peace to war if and only if

$$\frac{x-k}{1-\delta} \geq \frac{\gamma p}{1-\delta} - c_1$$
$$x - k \geq \gamma p - (1-\delta)c_1$$

whereas country 2 prefers peace to war if and only if

$$\frac{1-x-k}{1-\delta} \geq \frac{\gamma(1-p)}{1-\delta} - c_2$$
$$\gamma p + (1-\delta)c_2 \geq x + k - (1-\gamma).$$

As $\delta \to 1$, an x such that both countries prefer peace does not exist if and only if

$$\begin{aligned} x + k - (1-\gamma) &> x - k \\ \gamma &> 1 - 2k \end{aligned}$$

In other words, holding k fixed, the result holds as long as γ is sufficiently large. If $k \geq \frac{1}{2}$, the result holds for any γ (i.e., a peaceful settlement will not exist as costs of peace are too large). On the other hand, if $k < \frac{1}{2}$, there may exist $\gamma \in (0,1)$ such that the result does not hold. As $k \to 0$, the threshold $\overline{\gamma} \to 1$, i.e. as the costs of peace become arbitrarily small, a peaceful settlement will continue to exist for even arbitrarily small destruction of the pie.