Competitive Diplomacy in Bargaining and War

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Abstract

War is often viewed as a bargaining problem. However, prior to bargaining, countries can vie for leverage by expending effort on diplomacy. This article presents a dynamic model of conflict where agenda-setting power is endogenous to pre-bargaining diplomatic competition. The ability to compete for leverage generates a new channel through which the nature of potential war affects the quality of peace. First, costs of war grow the bargaining surplus, fueling the battle for leverage and reducing welfare even if war never occurs on the path of play. Second, competitive diplomacy erodes the gains from peace, making it possible that war is relatively efficient. An extension of the model finds that frictions to cooperation create a risk of costly delay but also protect against erosion of the surplus. Moreover, I find that reliable deals avert "efficient" wars but introduce a trade-off between welfare and timely settlement.

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> "Diplomacy is indispensable to identify and implement solutions to conflicts [...] short of military involvement. It helps to galvanize allies for action and marshal the collective resources of like-minded nations and organizations to address shared problems [...] We must upgrade our diplomatic capabilities to compete in the current environment and to embrace a competitive mindset."
> — US National Security Strategy (2017) on "Competitive Diplomacy"

Diplomacy can advance state interests. Secretary of State George Shultz likened diplomacy to gardening, where investments in international relations keep out the weeds to maintain fertile ground.[1] Costly sacrifices today can help secure a strong position in negotiations tomorrow. For example, under Anwar Sadat, Egypt withdrew from the Soviet-Egyptian Treaty of Friendship and Cooperation, relinquishing the associated benefits. It is difficult to reconcile how diplomatic maneuvers of this kind could *advance* a country's position with the standard model of crisis bargaining, where endogenous sources of leverage come from power and the costs of war. Clearly, Sadat believed these actions would benefit Egypt in the long run despite the negative direct effect of weakening ties with the USSR on their military strength, as well as any additional costs they would incur in a full-scale war with Israel.

I propose a theory in which countries exert diplomatic effort to gain control of the agenda in impending peace talks. If each side of a zero-sum dispute can use diplomacy to advance their bargaining position, it is necessarily at each other's expense and hence fundamentally competitive. While "competitive diplomacy" has not yet been incorporated in theories of crisis bargaining, the concept is not new to international politics—the term was used in the US National Security Strategy (2017), which called for greater investments into diplomatic capabilities to enhance the state's ability to achieve favorable outcomes through cooperation. Similarly, the US National Defense Strategy (2018) stated that "long-term strategic competition requires the seamless integration of multiple elements of national power" with "diplomacy" being the first element on the list.

Incorporating competitive diplomacy in crisis bargaining raises new theoretical questions. A

[1] https://afsa.org/tending-garden.

state's willingness to expend resources in diplomacy to improve their bargaining position will depend on their expectations about the additional value of the improved position, which in turn will depend on specifics of the conflict. Diplomacy then becomes a channel through which characteristics of a potential war can affect the quality of peace. Moreover, if diplomacy entails sacrifices during peacetime, it makes cooperation less efficient and affects a country's propensity to fight. Certain diplomatic settings may be effective at facilitating cooperation and avoiding war, but nevertheless fail to yield satisfying conditions in peace. Likewise, environments that bring good conditions in peace may be lacking in their ability to sustain cooperation in the first place. How does the incentive to compete in diplomacy affect the prospects of cooperation? Which settings fare well at averting conflict while simultaneously preserving the largest gains from peace?

To answer these questions, I develop a game-theoretic model of war that captures the core trade-off between internalizing the gains from cooperation and exerting costly effort to reach preferable peace deals. War and cooperation are modeled as two distinct technologies that can be used to divide a pie. Following previous work in crisis bargaining, war takes the form of a costly lottery over the disputed pie, while cooperation requires agreement to an offer (Morrow, 1989; Wagner, 2000; Fey and Kenkel, 2021; Qiu, 2022; Davis, 2023). Most importantly, countries can engage in diplomatic competition over agenda-setting power in the subsequent bargaining subgame.

The analysis reveals new insights. First, I find that costly wars endogenously create costly peace. Even if wars never occur on the path of play, greater costs of war imply a larger surplus from cooperation. As the surplus expands, agenda-setting power becomes more valuable, creating a stronger incentive to engage in competitive diplomacy. The extent to which costlier wars degrade the quality of peace is moderated by the decisiveness of the competition, where a competition is more decisive if the higher performer is more likely to become the agenda setter. The surplus from cooperation is least affected when the competition is less decisive, as the value of being the higher performer and hence the total incentive to compete is weaker. With more decisive competition, countries expend more effort vying for leverage and less of the gains from peace can be recovered. In extreme cases, countries may be no better off in peace than in war.

The model also identifies when war is more efficient than cooperation. After a division of the pie is reached, through war or cooperation, it persists until an opportunity to renegotiate arises. Countries are then driven to fight efficient wars by two forces: the (exogenous) differential persistence of outcomes and the (endogenous) equilibrium effort in competitive diplomacy. The logic is analogous to that of "ripping off the bandage." For example, if war outcomes persist while peace deals require constant renegotiation, states may prefer simply incurring the larger costs of war today to resolve the issue immediately. Likewise, countries will have an incentive to fight in the current period if they expect that competitive diplomacy will introduce significantly more inefficiencies after cooperation than it would after war. The precise condition that determines whether war is relatively efficient depends on how outcome persistence and competitive diplomacy interact.

There are two key ingredients of the model that establish these novel results. First, while existing models treat proposers as either predetermined or randomly recognized by exogenous probability, my approach treats agenda-setting power as endogenous to a country's performance in pre-bargaining competition. This is the core innovation of the model; before bargaining, countries choose how much effort to exert in competitive diplomacy, and greater effort is rewarded with a greater likelihood of setting the terms of a peace offering. In the case of Egypt, bearing the costs of disassociation from the USSR positioned them to recover the Sinai Peninsula by establishing good rapport with the de facto mediator, the United States.[2] Due to improving US-Egypt relations, the US pressured Israel to show restraint in the face of a surprise attack in which Egypt gained control of the Suez Canal's eastern bank.[3] Moreover, because Sadat signaled an openness to realignment, Egypt's demands took precedence over other Arab states. These actions gave Egypt the necessary bargaining leverage to demand its preferred terms of peace, which were eventually conceded in the Camp David Accords.

Note that, here, competitive diplomacy does *not* refer to diplomacy that improves a country's probability of victory in war. If the goal of diplomacy is to increase military power, it is analogous

[2] This was a key component of Sadat's diplomatic strategy from the beginning of his tenure. For example, in a 1971 meeting with US Secretary of State William Rogers, Sadat claimed that he "would like to become much closer with the West" and that "[t]here's no reason why the Arabs should be closer aligned to the Soviet Union" because "[the Egyptian] people like the West better." [https://history.state.gov/historicaldocuments/frus1969-76v23/d227].

[3] On the morning of the attack, Kissinger articulated the US position: "Israel must not attack, no matter what they think the provocation is." [https://history.state.gov/historicaldocuments/frus1969-76v25/d101, https://www.hudson.org/foreign-policy/hidden-calculation-behind-yom-kippur-war-michael-doran].

to arms building, which has been widely explored (Powell, 1993; Kydd, 2000). What has not been explored is diplomacy that positions a country to set the terms of peace. In the context of this paper, competitive diplomacy will thus satisfy three core properties: (1) it is costly, (2) it does not improve a country's war payoffs, and (3) it can improve a country's cooperation payoff. The model meets these conditions by implementing a pre-bargaining all-pay contest over a lottery for agenda-setting power. The all-pay contest captures a broad range of competitive diplomacy methods and enhances analytical clarity as it admits closed-form expressions. The lottery governs the strength of the mapping between performance in the competition and agenda-setting power, reflecting the scope of competitive diplomacy or its "decisiveness." Because agenda setters can only recover as much as their adversary is willing to concede in bargaining, diplomatic competition is notably distinct from military competition in that countries are competing over the surplus from peace, *not* the disputed pie—a fundamentally different object.

The second key ingredient is that the interaction is repeated. Importantly, this paper allows periodic outcomes to persist into the future. Crisis bargaining commonly assumes that peace deals are transient and war is game-ending (Fearon, 1995). This is usually appropriate, as these models aim to understand how an inability to commit to long-term settlements affects the incentives for conflict. In contrast, this paper adopts a more general approach to examine the incentives to compete for leverage and their implications for war onset. The model allows country strengths to evolve flexibly over time and, once an outcome ceases to endure, countries again need to resolve their dispute through cooperation or war. These dynamics are especially helpful to understand competitive diplomacy, as a country's incentive to cooperate, and hence their willingness to compete, depends not only on expectations about future power shifts but also on expectations about the relative persistence of peace deals. Naturally, the significance of winning a war is greater when war outcomes are durable than when they are transient, and the value of agenda-setting power is greater when peace deals are expected to persist than when they are not.

I then extend the model to examine how barriers to cooperation affect the incentives for diplomatic competition and conflict, bridging contractual theories of international cooperation and crisis bargaining. A core idea in international cooperation is that reaching successful deals is not always free and easy—the prevailing rationale for international institutions is their ability to improve the efficiency of the bargaining process by reducing the costs inherent in contracting, from actual expenditures and transaction costs to less tangible frictions (Keohane, 1984; Downs and Rocke, 1995; Dixit, 1998; Martin, 1999; Lake, 1999). Consequently, the extension considers small frictions that obstruct the settlement procedure if countries choose to opt out of diplomacy and, if countries fail to overcome these frictions, they experience costly delay.

With frictions, states may risk delay in attempt to free ride on their adversary's efforts in obtaining peace. The willingness to risk delay depends not only on the size of frictions but also on expectations about future shifts in power, the reliability of peace deals, war outcome persistence, and the scope of competitive diplomacy. As frictions create an incentive to free ride, they discourage diplomatic effort and simultaneously protect against the erosion of the bargaining surplus caused by competitive diplomacy. Likewise, the model suggests that the Pareto optimal level of competition counterintuitively maximizes the probability of delay when peace deals are reliable. This occurs because, while heightened competition discourages free riding and hence reduces the probability of delay, it can simultaneously erode the gains from peace. Reliable peace deals that avert efficient wars therefore create a trade-off between welfare and timely settlement.

This paper makes a number of contributions to the literature on international conflict. First, I incorporate and derive analytical implications for an important yet previously unexplored aspect of international diplomacy in crisis bargaining: pre-bargaining competition over agenda-setting power. This builds on previous work that studies pre-bargaining competition by non-diplomatic means; most notably, arming. Powell (1993) and Jackson and Morelli (2009) model the decisions to arm and initiate wars, while Fearon (2018) builds on these by incorporating a dispute in which arming also improves a country's resolution. Coe (2011) argues for costly peace due to arming, imposition, and predation as a rationalist explanation for war. Competitive diplomacy is not only conceptually different from war-related expenditures like arming, but the mechanism in this paper is technically different in a number of important ways. First, arming increases a country's strength and hence improves their expected payoff in war, while diplomatic effort never improves war payoffs. Second, war-related expenses are generally strategic complements, while diplomatic effort can be a strategic complement or substitute depending on the level of their rival's effort. Third, arming and competitive diplomacy are contests over different objects—the incentive to invest in war technology stems from the desire to recover the pie, whereas the incentive to exert diplomatic effort stems from a desire to recover the bargaining surplus.

The article also contributes to our understanding of long-run incentives for war. Work in this domain has primarily focused on commitment problems (Powell, 2006), most typically due to power shifts that create preventive motives (Kim and Morrow, 1992; Debs and Monteiro, 2014) and strategic settings that ease or exacerbate the incentive for prevention (McCormack and Pascoe, 2017; Schram, 2021; Krainin et al., 2022). This paper instead focuses squarely on the incentives to compete for agenda-setting power with diplomacy. Other related work includes that on diplomacy in crisis bargaining, though the focus on communicating private information makes it quite different from this paper in nature (Smith, 1998; Sartori, 2002; Kurizaki, 2007; Fey and Ramsay, 2010; Trager, 2010, 2011; Ramsay, 2011; Wolford, 2020; Malis and Smith, 2021).

Finally, there is a noteworthy connection between this paper and the literature on legislative bargaining with endogenous institutions (Baron and Ferejohn, 1989; Eguia and Shepsle, 2015), which has extensively studied the dynamics of proposal power. As in legislative settings, agenda-setting power has important implications on core incentives in international diplomatic settings. My approach to treat proposal power as endogenous may be useful to future work in this area.

1 Methods of Competitive Diplomacy

This paper aims to study the implications of competitive diplomacy in crisis bargaining. To isolate theoretical mechanisms, the model intentionally abstracts away from details that may otherwise be important to understand particular empirical cases (Paine and Tyson, 2020). With that said, it is helpful to briefly outline some of the ways countries can use competitive diplomacy to gain bargaining leverage in practice and identify examples that are consistent with these ideas. As already discussed in the previous example of Egypt, competitive diplomacy includes both appeals to an authority to advance one's own bargaining position and political maneuvers that undermine the position of others. There are many cases where countries engage in competitive diplomacy by cultivating a powerful mediator, such as the United States. When this occurs, the mediator essentially has the ability to improve a disputing country's agenda-setting power through its control of the conflict resolution process.

Direct lobbying is one way states can win over mediators. According to Foreign Lobby Watch, billions of US dollars have been spent on foreign lobbying over the past several years.[4] Some of the largest US lobbying firms routinely recruit former government officials and represent foreign governments, using their DC connections to benefit their clients.[5,6] For example, BGR group's clients include Azerbaijan, Bahrain, Bangladesh, India, Qatar, Uzbekistan, Saudi Arabia, Serbia, Panama, Kurdistan, Guyana, and South Korea, among others. Saudi Arabia has been especially active, spending over $142 million USD on influencing US legislators between 2016-2021 according to the Foreign Agents Registration Act (FARA) database.[7] Moreover, lobbying powerful mediators to win favorable peace outcomes extends to civil conflict—BGR also represents the Yemeni National Resistance, for instance. Huang (2016) details how rebel groups engage in competitive diplomacy by hiring law and public relations firms to lobby elected officials in the US and gain political leverage.

Competitive diplomacy can also take the form of preemptive favors. For example, Barham et al. (2023) provide evidence that states used vaccine distribution during the COVID-19 pandemic, or "vaccine diplomacy," to earn trust abroad and expand their global influence. As Russia began its invasion of Ukraine in February 2022, Argentinian President Alberto Fernández visited Moscow and proclaimed that "Argentina is indebted to Russia because it was the first country to help Argentina access vaccines, and we'll always be grateful."[8] While they initially called on Russia to

[4] https://www.opensecrets.org/fara.
[5] https://www.law.com/nationallawjournal/2019/01/08/akin-gump-adds-audit-watchdog-modesti-in-latest-dc-hire.
[6] https://quincyinst.org/research/foreign-lobbying-in-the-u-s/#h-the-facilitators.
[7] https://www.opensecrets.org/news/2022/10/saudi-arabia-ramped-up-foreign-influence-operations-in-the-us-during-bidens-presidency/.
[8] https://www.wilsoncenter.org/blog-post/latin-america-vaccine-diplomacy-recipient-now-vaccine-donor-too.

cease military operations in Ukraine, Argentina has since refused to impose sanctions on Russia, refused to send Ukraine weapons they requested, and abstained from a UN vote to formally condemn Russia's actions.[9,10,11] Earning influence over relevant third parties, such as UN member states like Argentina, can improve a country's ability to set the agenda in peace negotiations.

Finally, competitive diplomacy can be used to worsen the bargaining position of other states. Because conflict settings are zero sum, a state taking actions to weaken their rival's position effectively advances their own. An example is President Biden's visit to Saudi Arabia in July 2022 to exchange international legitimacy[12] for an opportunity to persuade Crown Prince Mohammed bin Salman to produce more oil. Although this effort failed,[13] it would have reduced European dependence on Russian energy, diminishing Russia's ability to leverage supply shocks to make credible take-it-or-leave-it demands in their conflict with Ukraine. Another example is China keeping Taiwan out of international organizations by making pledges to UN member states with voting power in the General Assembly. For example, China has given $150 million USD in aid and $500 million USD in development projects to El Salvador,[14] as well as hundreds of millions USD for infrastructure projects in Honduras and Nicaragua.[15,16] These states have cut their diplomatic relations with Taiwan and issued statements recognizing Taiwan as a Chinese territory. These efforts provide China with more control of the agenda in future cooperative agreements with Taiwan.

2 Model

This section presents the model, with additional details available in Appendix A. Two countries $i = 1, 2$ are in a dispute over the division of the unit interval, or "pie." They have opposing

[9] https://carnegieendowment.org/research/2023/11/argentina-in-the-emerging-world-order?lang=en.

[10] https://buenosairesherald.com/world/argentina-abstains-from-un-vote-condemning-russia.

[11] https://www.batimes.com.ar/news/argentina/back-and-forth-argentinas-changing-rhetoric-on-russias-war-in-ukraine.phtml.

[12] Saudi Arabia's broad diplomatic strategy has been to improve its public image through reputation laundering [e.g., https://www.theguardian.com/football/blog/2021/oct/08/saudi-takeover-of-newcastle-leaves-human-rights-to-fog-on-the-tyne]. From BBC News: "They wanted the rest of the Middle East, and the international community, to sit up and take notice of the favour they had been granted by the US president." [https://www.bbc.com/news/world-middle-east-62189543].

[13] https://apnews.com/article/russia-ukraine-2022-midterm-elections-biden-inflation-business-fa45f3023af51b1b7201f0c06d86f72d.

[14] https://www.reuters.com/article/world/china-pledges-150-million-aid-to-el-salvador-as-relationship-deepens-idUSKCN1ND0IS/.

[15] https://www.reuters.com/world/americas/honduras-china-set-275-mln-cooperation-agreement-2024-03-22/.

[16] https://thediplomat.com/2023/12/chinas-growing-strategic-position-in-nicaragua/.

preferences over an outcome $x \in \mathbb{R}$, with $u_1(x) = x$ and $u_2(x) = 1 - x$ denoting the flow payoff for country 1 and 2. Time is discrete and discounted by $\delta \in (0, 1)$ over an infinite horizon.

Each country faces a choice between cooperating or attacking, $a_{it} \in \{0, 1\}$. If either country attacks, war occurs as a costly lottery.[17] Specifically, a war in period t is won by country 1 with probability given by the state variable $s_t \in S$, where S is a finite subset of $(0, 1)$. The costs of war are allowed to vary by country and the state of the world, and are assumed to be strictly positive, $c_i(s_t) > 0$. Additionally, a country's flow payoff after war reflects the winner controlling the entire pie of value 1 and the loser having nothing.

On the other hand, if countries cooperate, they exert effort $e_{it} \geq 0$ in competitive diplomacy. The country that exerted the most effort is recognized as the agenda setter with probability $\pi \in (\frac{1}{2}, 1]$. This characterization lends a straightforward intuition: those that excel at competitive diplomacy can expect to recover π share of the bargaining surplus ex ante. Larger π reflects a competition with greater decisiveness, which translates to the contest's winner being able to expect more of the surplus for themselves. If $\pi = 1$, the all-pay contest is deterministic with the higher performer receiving agenda-setting power and capturing the entire surplus.

Upon recognition, the agenda setter i extends the peace offering of their choice, $x_{it} \in \mathcal{X}_i = \mathbb{R}$.[18] The receiving country $-i$ chooses whether to accept the offer, $y_{-it}(x_{it}) \in \{0, 1\}$, with rejection leading to war. The ultimatum bargaining protocol is employed as it is prevalent in the crisis bargaining literature and has been shown to be with little loss of generality (Fey and Kenkel, 2021). A peace deal results in a flow payoff that includes the utility over their accepted allocation as well as any costs of effort they incurred in the process of competitive diplomacy.

After an allocation is determined through either cooperation or war, the game proceeds to the next period. The outcome in period t may persist into period $t + 1$ and beyond: each period, the country controlling the pie after war retains possession with probability $\theta \in [0, 1]$, while a peace

[17] War as a costly lottery is the preferred approach for comparability with existing crisis bargaining models. The results continue to hold for war with endogenous military effort and with infinite costs of war (or analogously cooperation that grows the pie). Related propositions and proofs are available directly from the author.

[18] Note that we eschew preventive war by removing budget constraints. If states are unconstrained in their settlements, they can always offer large enough side payments to dissuade preventive war (by, for example, taking out sovereign debt to extend a cash transfer as in Krainin et al. (2022)). This choice streamlines the analysis of the main text and is not crucial for the results. All proofs in the Appendix consider the general case where countries are potentially constrained in their offers.

Table 1. Notation

Model Parameters		Actions (country i, period t)	
Discount factor	$\delta \in (0, 1)$	War decision	$a_{it} \in \{0, 1\}$
Peace deal persistence	$\lambda \in [0, 1]$	Effort in diplomacy	$e_{it} \in \mathbb{R}_{\geq 0}$
War outcome persistence	$\theta \in [0, 1]$	Peace deal offering	$x_{it} \in \mathbb{R}$
Decisiveness of competition	$\pi \in (\frac{1}{2}, 1]$	Peace deal acceptance	$y_{it}(x) \in \{0, 1\}$
Key State Variable			
Relative strength	$s_t \in S \subset (0, 1)$		

deal arrived at through cooperation remains intact with probability $\lambda \in [0, 1]$. With complementary probabilities, the countries return to crisis bargaining. As a technical necessity to accommodate these dynamics, the state of the world additionally includes whether the countries are in an active crisis bargaining stage and the status quo allocation. The persistence parameters λ and θ are probabilities that manage the transitions between active and passive states, reflecting the reliability of peace deals and the durability of war outcomes, respectively.

Power transitions may occur over time. In particular, the state variable s_t may evolve flexibly over time according to a Markov transition function $q : S \times A \to S$, where $a_t \in A \equiv \{0, 1\}^2$ are the war actions taken in period t. Throughout the article, I opt for a general exposition and do not impose additional constraints on the transition function. The virtue of this approach is to explicitly bridge the current model with previous work that explores shifting power in a variety of ways. This ensures the results are robust to a large set of transition functions that allow but do not require dependencies between actions and the evolution of power. Consequently, many of the expressions are implicitly defined, as explicit characterization in terms of parameters requires selecting a specific q, which is not necessary to recover the results.

The solution concept is Markov Perfect Equilibrium (MPE), henceforth simply equilibrium.[19] All proofs are relegated to Appendix B. In subsequent sections, I suppress all notation that identifies a specific period t when doing so does not create confusion.

[19] MPE is chosen over Subgame Perfect Equilibrium because folk-theorem results allow for unreasonable equilibria in this game due to its infinite horizon. MPE is the standard refinement in this case (Maskin and Tirole, 2001; Fudenberg and Tirole, 1991, p. 501-505). Refer to Slantchev (2002) for useful discussion of MPE in crisis bargaining.

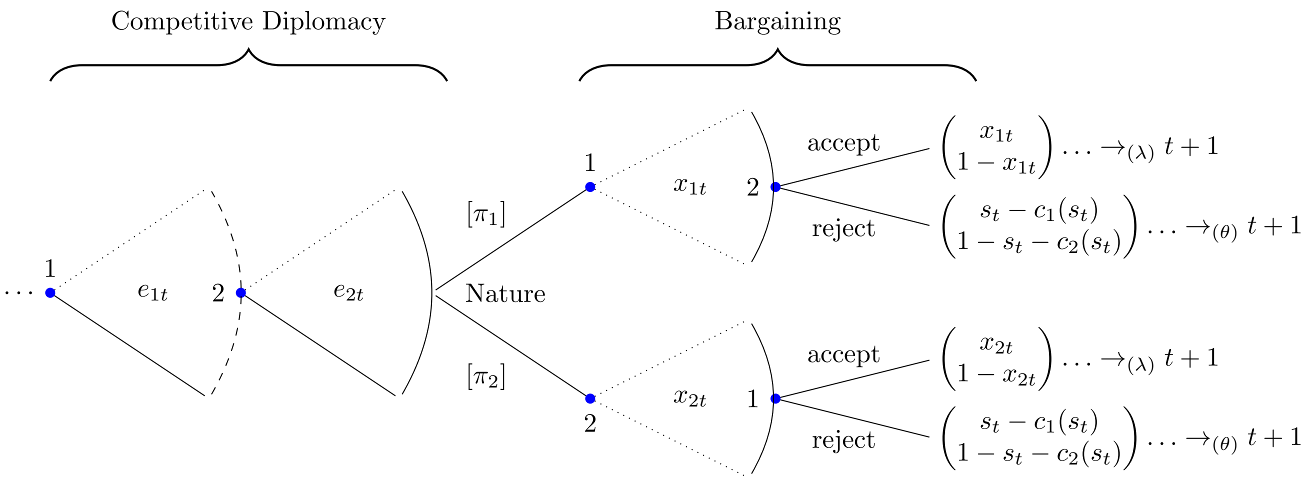

Figure 1. Cooperation in state s_t with $\pi_1 = 1 - \pi_2 \in \{\pi, 1 - \pi\}$

2.1 Continuation Values and the Bargaining Surplus

A strategy for country i is a function $\sigma_i : S \to \{0,1\}^2 \times \mathbb{R}_+ \times \mathcal{X}_i$ given by quadruple

$$\sigma_i(s) = (a_i(s), e_i(s), x_i(s), y_i(x; s)) \tag{1}$$

corresponding to i's decision to launch a strike, an amount to exert on competitive diplomacy, a bargaining offer if recognized, and acceptance decisions for all $x \in \mathbb{R}$, respectively.

If war occurs, each country can expect to receive the entire pie for that period with their probability of victory. In every period after war that the war outcome persists, the winning country can expect to hold onto the entire pie with probability θ. With probability $1 - \theta$, the countries return to the bargaining game under a new state s'. Letting $s_1 := s$ and $s_2 := 1 - s_1$ without loss of generality, country i's expected value of war in state s under strategies σ can be expressed[20]

$$W_i(s) = \underbrace{s_i}_{\substack{\text{expected war} \\ \text{outcome in } s}} - \underbrace{c_i(s)}_{\substack{\text{costs of} \\ \text{war in } s}} + \frac{\delta}{1 - \delta\theta}\left[\theta \underbrace{s_i}_{\substack{\text{war outcome} \\ \text{persists}}} + \underbrace{(1-\theta) \sum_{s' \in S} V_i(s') q(s'|s, a_W)}_{\substack{\text{expected value in return} \\ \text{to bargaining after war}}}\right]. \tag{2}$$

[20] Refer to Appendix A.3 for full derivations. Notation denoting dependence on a strategy σ is suppressed in continuation values and objects containing them where doing so does not create confusion.

where a_W denotes actions that involve a country choosing to fight a war.

On the other hand, countries may cooperate. Denote by $N(s)$ the expected peace settlement in state s under strategies σ.[21] In every period after cooperation that the peaceful settlement persists, country 1 can expect to keep receiving $N(s)$ and country 2 can expect to keep receiving $1 - N(s)$ with probability λ, or else return to the bargaining game under a new state s'. Then, country i's expected value of cooperating in state s given equilibrium strategies σ is

$$U_i(s) = \underbrace{u_i(N(s))}_{\substack{\text{expected peace} \\ \text{deal in } s}} - \underbrace{\int_0^\infty e dF_s(e)}_{\substack{\text{exp. diplomatic} \\ \text{effort in } s}} + \frac{\delta}{1-\delta\lambda}\left[\lambda \underbrace{u_i(N(s))}_{\substack{\text{peace deal} \\ \text{persists}}} + (1-\lambda) \underbrace{\sum_{s' \in S} V_i(s')q(s'|s, a_U)}_{\substack{\text{expected value in return} \\ \text{to bargaining from cooperation}}}\right] \tag{3}$$

where a_U denotes actions that involve countries choosing to cooperate and F_s is the cumulative distribution of diplomatic effort in state s given strategies σ.

Then, prior to any declarations of war or recognition of agenda-setting power, each country i's ex-ante value for being in any state s given strategies σ satisfies

$$V_i(s) = \begin{cases} U_i(s) & \text{if } \max a_i^*(s) = 0 \\ W_i(s) & \text{otherwise} \end{cases} \tag{4}$$

in equilibrium. A country prefers fighting a war when $W_i(s) > U_i(s)$, or else they attempt to cooperate. When choosing to cooperate, countries will also need to choose an amount to exert on diplomacy, which will depend on how much they expect to be able to extract upon becoming the agenda setter. Intuitively, a country will be willing to expend more effort in competition if agenda-setting power yields larger returns.

The agenda setter always extracts as much as their opponent is willing to concede, taking the entire surplus from cooperation for themselves on the path of play. The receiver's continuation value of accepting an offer x is given by $U_i(x; s)$, which differs from $U_i(s)$ in that it considers a specific proposal and does not include the expected costs of effort expended in diplomacy, as these

[21] $N(s)$ will correspond to the Nash bargaining solution due to symmetric nature of each country's competitive diplomacy in equilibrium.

become sunk at the time of the proposal. Then, each country i is indifferent between accepting and rejecting an offer $\bar{x}_i(s)$ that solves $U_i(\bar{x}_i(s); s) = W_i(s)$. Mutually satisfactory peace deals exist in equilibrium if a country prefers to settle at their opponent's "indifference deal" than their own, $\bar{x}_2(s) \geq \bar{x}_1(s)$. On the other hand, agreement is impossible if countries prefer their own indifference deal to their adversary's, $\bar{x}_1(s) > \bar{x}_2(s)$. In equilibrium, offers that get accepted are always equal to an opponent's indifference deal.

Denoting the equilibrium offer of country i in state s by $x_i^*(s)$, the expected bargaining surplus from cooperation in state s can then be denoted by $B(s) := x_1^*(s) - x_2^*(s)$, reflecting the amount an agenda setter can extract. Then, $U_i(s) \geq W_i(s)$ for country $i = 1, 2$ if and only if

$$B(s) = 1 - (1 - \delta\lambda)W(s) + \delta(1 - \lambda) \sum_{s' \in S} V(s')q(s'|s, a_U) \geq 0 \tag{5}$$

where $W(s) := W_1(s) + W_2(s)$ and $V(s') := V_1(s') + V_2(s')$ for any state s and transitions q.

2.2 Comments on the Model

Let us briefly discuss several aspects of the model. First, competitive diplomacy takes the form of an all-pay contest over a lottery for agenda-setting power. This is a natural choice for both substantive and technical reasons. First, the all-pay contest captures many of the competitive diplomacy methods of interest that were outlined above. All-pay contests are regularly used to model a large range of competitive environments, including lobbying (Baye, Kovenock, and de Vries, 1993; Fang, 2002), battles and races (Konrad and Kovenock, 2009), and market power competition (Siegel, 2009). Second, all-pay contests are unique in the fact that they yield closed-form expressions that facilitate a clean exposition and comparative statics. Moreover, the central logic derived from all-pay contests is robust to a wide array of contests. Although Tullock contests do not exhibit continuous equilibrium effort strategies and generally do not admit closed-form expressions, making it a suboptimal choice for the model here, the key implications continue to hold for decisive Tullock contests.[22] Additionally, by the Revenue Equivalence Theorem (RET), the implications also hold under all standard auction contests where the largest bid is awarded the surplus (Myerson, 1981). Because the RET also applies to auctions with private valuations, the model can be straightforwardly extended to bargaining models of war with information asymmetries (e.g., with privately known costs of war).

Second, the scope or decisiveness of the contest is given by parameter π. This provides a straightforward way of understanding variation in the ability to use competitive diplomacy to make cooperative gains. Cooperative protocols can yield any of an infinite number of mutually preferred peace deals within the bargaining range (Binmore and Dasgupta, 1987), and the lottery introduced by π allows us to explore various mappings without implementing different protocols. For example, we can compare settings where higher performance in competitive diplomacy is certain to return the entire surplus ($\pi = 1$) to those where the contest yields interior expectations ($\pi < 1$). Moreover, while π is independent of power in the model, Appendix C.2 demonstrates that the results are robust to state-dependent differential advantages, $\pi_i(s_t) > \pi_{-i}(s_t) > \frac{1}{2}$. Thus, the model is consistent with recognition probabilities that depend on the relative strength. The intuition is that country incentives are necessarily tied to the *net* gain from higher performance in competitive diplomacy, which is always the same for both players: $\pi_1(s_t) - (1 - \pi_2(s_t)) = \pi_2(s_t) - (1 - \pi_1(s_t))$. The model therefore avoids the notational burden without loss in this regard.

Third, the model introduces persistence parameters $\lambda \in [0, 1]$ and $\theta \in [0, 1]$. This generalizes the canonical setting from Fearon (1995), where peace deals are short-lived and war outcomes are permanent ($\lambda = 0$ and $\theta = 1$). As mentioned above, the classic assumptions are strong but sensible given the aim to understand problems pertaining to the inability to commit to peace deals (e.g., after a power shift). This paper, in contrast, does not focus on this type of commitment problem and therefore does not need to unnecessarily restrict itself. More importantly, these parameters are substantively interesting to the analysis as they provide insight into a country's incentive to employ competitive diplomacy. For example, the classic assumptions are too rigid to account for settings

[22] Refer to discussion around Proposition 4 and Appendix C.1 for related propositions and proofs.

where war outcomes are not permanent and peace settlements are not transient. Consider, for example, the back-and-forth claims to Alsace-Lorraine through wars between France and Germany. In the model, the possibility of this to occur affects equilibrium behavior. Thus, I relax the typical constraints on λ and θ to understand how the incentive to compete for agenda-setting power varies across a wider range of settings. The analysis treats λ and θ as exogenous primitives; however, it is plausible that country choices can affect the reliability of a peace deal. This may be a productive consideration for future work.

Finally, while the results can be established with a finite horizon, an infinite horizon is the most appropriate setting as the focus is on the incentive to compete for agenda-setting power, which should always depend on expectations about the future. If the horizon were finite, countries would no longer make future considerations at some period T, which would likewise affect all decisions in preceding periods by backward induction. In this way, behavior in all periods would be unduly influenced by expectations of the final period, undermining the intention of repeated play. Lastly, a finite horizon necessarily implies a contradiction to any interior $\lambda \in (0, 1)$ or $\theta \in (0, 1)$.

3 Benchmark: No Competitive Diplomacy

When there is a positive bargaining surplus, $B(s) > 0$, states will have incentive to compete for agenda-setting power in hope of recovering it for themselves. First, consider a benchmark without competitive diplomacy. This is analogous to the model with the condition that $\pi = \frac{1}{2}$.

A useful frame of reference will be the canonical Fearon (1995) repeated crisis bargaining game, in which country 1 is endowed with agenda-setting power (instead of competitive diplomacy), war outcomes are permanent $\theta = 1$, and peace deals do not persist $\lambda = 0$. Because (Fearon, 1995) is especially interested in an inability to commit to long-term settlements, it imposes budget constraints that prevent any side payments, $\bigcap_i \mathcal{X}_i = [0, 1]$. The main text deviates from this to avoid preventive war, though it is accommodated with additional notation in the Appendix. Also noteworthy is Powell (2006), which models deterministically alternating agenda setters over an infinite horizon.

First, this article builds on these works by allowing different outcomes to persist. Rather than war being game-ending and peace deals only lasting one period, war outcomes can now persist into future periods with probability $\theta \in [0, 1]$ and peace deals with probability $\lambda \in [0, 1]$. Since expectations about outcome persistence affect the relative value of going to war over settling things cooperatively, it also affects the value of setting the agenda.

Lemma 1. *The value of being the agenda setter is directly increased by peace deal persistence λ and directly reduced by war outcome persistence θ.*

The case in Fearon (1995) is therefore a worst-case scenario for agenda setters: when peace deals are more persistent and war outcomes less so—greater λ and smaller θ—it is likely that they can extract even greater surplus from their adversaries. This is because, as the outcome of war becomes less persistent relative to the cooperative settlement, war becomes an even less desirable outside option. Since the outside option is worsened, agenda-setters can demand greater proportions from their adversaries. Of course, whether the total effect of persistence parameters on agenda-setting power is positive depends on how those parameters affect the corresponding continuation values, which in turn depends on the transition function.

Allowing outcome persistence to vary flexibly has distributional consequences in peace, but alone does not change the qualitative nature of equilibrium behavior from earlier work.

Proposition 1. *In any equilibrium of the game without competitive diplomacy, countries will cooperate without exerting effort.*

If countries cannot compete with diplomacy to improve their bargaining position, there is no longer an incentive to exert diplomatic effort. Cooperation must be completely efficient when it occurs. Consequently, the sum of country payoffs are not affected by changing the costs of war in a state that does not result in war, as this sum is guaranteed to be the full pie of value 1.

4 Implications of Competitive Diplomacy

Next, we explore the implications of including competitive diplomacy. There are two new features of equilibrium behavior. First, countries now compete with strictly positive diplomatic effort in cooperation, and second, countries may now choose to initiate war because it is relatively efficient compared to competitive cooperation.

Definition 1. *A war in state $s \in S$ is efficient if and only if there is no mutually preferred settlement, $\bar{x}_1(s) > \bar{x}_2(s)$.*

Proposition 2 (Equilibrium). *There is a unique equilibrium with strategies σ^* in which countries either fight efficient war or cooperate by mixing diplomatic effort uniformly between zero and $\frac{(2\pi-1)B(s)}{1-\delta\lambda}$.*

Figure 2 illustrates how countries compete in state s according to a mixed strategy defined by cumulative distribution F_s^*. In terms of the cases previously discussed, we can think about a draw from this mixed strategy as the amount of money Saudi Arabia spends lobbying US legislators or the amount of pledges China makes to El Salvador, for example. Let us focus on understanding the process of cooperation before turning to efficient wars in the following section.

When choosing to cooperate, a country always extends the offer that is best for them in their adversary's acceptance set, and that offer is always accepted. Then, the difference between the two equilibrium offers defines the surplus from cooperation, which is the value of being a recognized agenda setter. Importantly, the nature of the hypothetical war affects each country's acceptance set, which in turn affects the value of being the agenda setter. Since countries choose how to engage in competitive diplomacy with the value of becoming an agenda setter in mind, the nature of a hypothetical war affects their equilibrium diplomatic strategies and therefore the quality of peace, even if war never occurs on the path of play.

Proposition 3. *Costs of war make peace costlier and reduce welfare.*

Let $C(s) := c_1(s) + c_2(s)$. Then, $\frac{\partial}{\partial C(s)} \int e dF_s^*(e) \geq 0$ and $\frac{\partial}{\partial c_i(s)} V_i(s) \leq 0$ for each country $i = 1, 2$ and for any state s and transition function q.

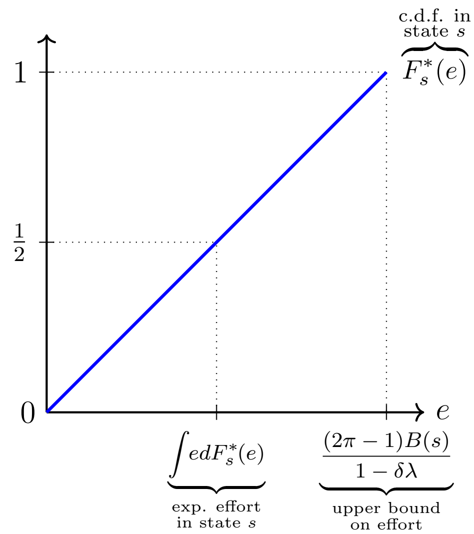

Figure 2. Cumulative distribution for diplomatic effort F_s^* with no frictions $\mu = 0$

Usually, increasing the costs of war in the bargaining model makes the peaceful settlement better for the agenda setter and worse for the receiver, with a net effect of zero on aggregate welfare. However, because agenda-setting power is now endogenous to a pre-bargaining diplomatic competition, the larger surplus that results from larger costs of war also provides countries with an incentive to exert greater effort in competitive diplomacy. While the increased surplus still serves to reallocate wealth from the receiver to the recognized proposer in zero-sum fashion, the total net effect is no longer zero, as countries now exert increased effort trying to become the agenda setter. This increased effort is an inefficient waste, guaranteeing a reduction of expected welfare in the process. Crucially, this does not depend on the outbreak of war, but simply the fact that countries will reduce their acceptance standards in response to a worse outside option.

In addition to the characteristics of war, the extent to which the competition is decisive has important implications on equilibrium behavior and consequently outcomes.

Proposition 4. *Welfare is decreasing in a competition's decisiveness. Moreover, completely decisive competition guarantees the full erosion of the gains from peace.*

Formally, for all states s and transition functions q, (i) $\frac{\partial}{\partial \pi} V_i(s) \leq 0$ and (ii) $\pi = 1$ implies $U_i(s) \leq W_i(s)$ for each country $i = 1, 2$.

This proposition reveals how detrimental competitive diplomacy can be. The competition's decisiveness heightens the incentive to compete, introducing inefficiencies to cooperation and reducing welfare. Under completely decisive contests, the desire to earn agenda-setting power to capture the benefits of peace can drive countries to exert effort until all of the surplus is destroyed. In particular, a country expects to do no better in peace than in war when $\pi = 1$. Furthermore, it remains possible that the surplus is fully eroded before the point of complete decisiveness, $\pi < 1$.

As discussed above, the all-pay contest is preferred for our main analysis; however, the result is also robust to other common contests. Most importantly, consider a Tullock contest. Let country i receive the better end of the lottery governed by π with probability $\frac{e_i^d}{e_i^d + e_{-i}^d}$ for any effort $e_1, e_2 > 0$ and some decisiveness parameter $d > 2$.[23] Appendix C.1 shows how diplomatic competition by any such Tullock contest continues to erode the full bargaining surplus as $\pi \to 1$, consistent with Proposition 4. In fact, equilibrium behavior in the all-pay contest is equivalent to that in the limiting Tullock contest as $d \to \infty$. The robustness of the result is due to the prevailing logic that if disproportionate gains in cooperation can be made by marginal increases in competitive effort, each country will exert even more effort until there is no longer an expected gain from peace.

4.1 Efficient Wars

Unlike most game-theoretic bargaining models of war, countries may prefer fighting to cooperating despite an ability to settle peacefully today. When this occurs, war is called "efficient" as it entails fewer inefficiencies on the path of play.

Figure 3 illustrates how different couples of equilibrium indifference deals (the deals at which countries are indifferent between war and peace) lead to cooperation or efficient war. If country 2 prefers settlements that are larger than those demanded by country 1 to keep them indifferent (above the dashed blue line), countries will choose to cooperate. Also visible are shaded areas where we would have countries choosing to fight preventive wars due to an inability to settle at a

[23] At slight abuse of terminology, parameter π captures the decisiveness of the lottery over agenda-setting power and d captures the decisiveness of the Tullock contest. The condition $d > 2$ guarantees non-degenerate mixed effort strategies. See Appendix C.1 for more detail, explanation, results, and proofs.

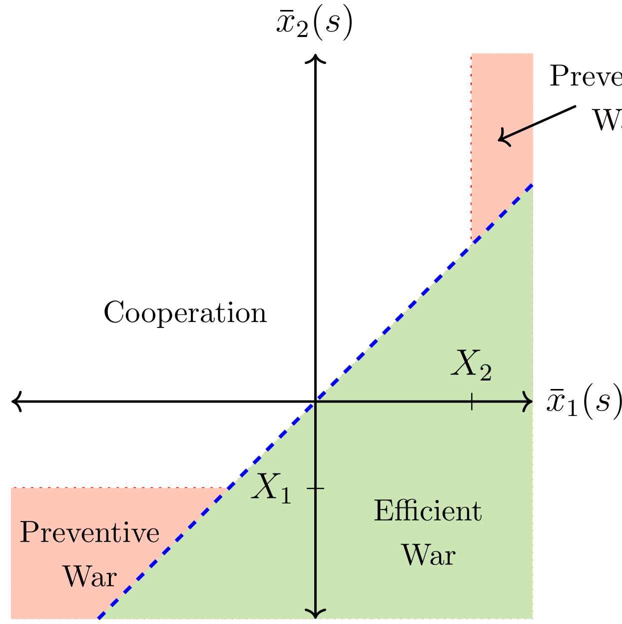

Figure 3. Efficient wars in indifference settlements

mutually preferred deal if we were to allow for standard convex budget constraints, $\mathcal{X}_1 = [X_1, \infty)$ and $\mathcal{X}_2 = (-\infty, X_2]$; most typically, $X_1 = 0$ and $X_2 = 1$.

If country 2 does *not* prefer settlements that are larger than those demanded by country 1 to keep them indifferent (below the dashed blue line), efficient war will break out. These indifference deals come out of equilibrium behavior and, while the couples that lead to efficient war are usually assumed away in standard settings, this section shows how efficient wars can arise endogenously due to competitive diplomacy. Further, the figure demonstrates how the incentives driving those wars differ from those driving preventive wars.

Before stating the general condition for efficient war, let us first define two key terms.

Definition 2. *The expected net present value of the pie after war is given by*

$$G := \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda}. \tag{6}$$

Winning a war yields the entire pie of value 1 and it is retained for each subsequent period with probability θ. The expected present value of this is reflected in the first term of equation (6). The second term in the expression reflects the aggregate expected present value of the pie prior to a return to bargaining if they cooperate instead, persisting with probability λ. The value G then reflects the aggregate value countries expect to recover from war over peace specifically with regard to the differential tendencies of their outcome persistence, leaving aside their continuation values that rely on strategies upon returning to bargaining.

On the other hand, we also need to account for the discrepancy between war and cooperation upon a return to bargaining.

Definition 3. *The total expected net present value of returning to bargaining after fighting in state s is given by*

$$\Delta V(s) := \sum_{s' \in S} (V_1(s') + V_2(s')) \left[\frac{\delta(1-\theta)}{1-\delta\theta} q(s'|s, a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda} q(s'|s, a_U) \right] \tag{7}$$

for any transition function q.

The present value of returning to the bargaining table after a war in state s is given by $\frac{\delta(1-\theta)}{1-\delta\theta} \sum_{s'} V_i(s') q(s'|s, a_W)$ for country $i = 1, 2$, whereas country i's present value of returning to bargaining after cooperating in state s is given by $\frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s'} V_i(s') q(s'|s, a_U)$. Equation (7) defines $\Delta V(s)$ as the difference between the aggregates of these two objects, reflecting the net present value of returning to bargaining after fighting. Unlike G, the term $\Delta V(s)$ reflects the total additional value countries can expect to get by returning to bargaining from war instead of from an existing peace deal, which depends not only on the model parameters but also on the current state of the world and the transition function.

Together, the expression $G + \Delta V(s)$ yields the total net present value of going to war, where G is an exogenous component that reflects to the differential persistence of outcomes and $\Delta V(s)$ is an endogenous component attributable to variation in country strategies across different states of the world (and the likelihood of their realizations) after a return to bargaining.

Equipped with this notation, we can concisely state the following proposition.

Proposition 5. *An efficient war occurs if and only if the total expected net present value of fighting is greater than the total costs of war. Formally,* $\bar{x}_1(s) > \bar{x}_2(s)$ *for any state* s *with transition function* q *if and only if* $G + \Delta V(s) > C(s)$.

Proposition 5 has an intuitive interpretation but is not possible in typical dynamic conflict models, as both G and $\Delta V(s)$ are usually assumed to be weakly negative while $C(s)$ is assumed to be strictly positive. In this article, however, the total expected net present value of fighting may be positive and possibly larger than the total costs of war. In particular, the expression demonstrates how there exist two paths to efficient war: one through the exogenous component G and another through the endogenous component $\Delta V(s)$.

First, because the model allows for war outcomes and peace deals to persist at varying rates according to θ and λ, it is possible that a much greater persistence of war outcomes relative to peace deals can drive countries to fight. The logic parallels that of "ripping off the bandage," as countries prefer taking their chances in war today if it means they can avoid having to regularly incur the smaller costs of peace for the rest of time. Given $\lim_{(\lambda,\theta)\to(0,1)} G = \frac{\delta}{1-\delta}$, it becomes clear that enough patience can result in an arbitrarily large expected net present value of the pie after fighting. However, while exogenous factors can drive efficient war, their interaction with competitive diplomacy is crucial for the result—as the benchmark above demonstrates, the absence of competitive diplomacy implies the absence of efficient wars. This is because, without competitive diplomacy, the value for cooperation is always large enough relative to the value for war that the equation can never be satisfied (i.e., $\Delta V(s) \to -\infty$ as $\delta \to 1$ at a faster rate than $G \to \infty$).

Second, variation in countries' equilibrium strategies across different states of the world can bring about efficient war if fighting today is more likely to transition into preferable states of the world tomorrow. Each possible state of the world comes with a corresponding value for each country that depends on both the primitives of the model and their behavior in equilibrium, some of which are preferable to others. If war is expected to return countries to bargaining under better conditions, they may choose to incur the greater costs of fighting today in order to improve the likelihood they transition to better states of the world tomorrow.

To see how endogenous effort to compete for agenda-setting power can create efficient wars, consider a simple example. Let there be three states of the world $S = \{s, \underline{s}, \bar{s}\}$, with s as the current state and both $\underline{s}$ and $\bar{s}$ as absorbing states where countries cooperate. If war guarantees a transition to state $\underline{s}$ while cooperation guarantees a transition to state $\bar{s}$, countries may prefer to fight an efficient war today when the expected inefficiency from competitive diplomacy in $\bar{s}$ is sufficiently larger than that in $\underline{s}$. This can occur if the surplus from cooperation in state $\bar{s}$ is much larger than that in state $\underline{s}$.

Formally, let $q(\underline{s}|s, a_W) = 1$ and $q(\bar{s}|s, a_U) = 1$, where s is the current state. Fighting ensures the state transitions to $\underline{s}$ whereas the choice to cooperate ensures the state transitions to $\bar{s}$. By Proposition 5, an efficient war results in state s under these conditions if and only if

$$G + \frac{\delta(1-\theta)}{1-\delta\theta} V(\underline{s}) - \frac{\delta(1-\lambda)}{1-\delta\lambda} V(\bar{s}) > C(s). \tag{8}$$

Because $\underline{s}$ and $\bar{s}$ are absorbing states, we can write this expression solely in terms of model primitives.[24] If we choose costs of war $C(s)$, $C(\underline{s})$, and $C(\bar{s})$ such that this condition holds given δ, λ, θ, and π, then countries will prefer to fight an efficient war as a result of expecting to exert too much effort under cooperation.

Condition (8) shows how increasing the costs of war in a cooperative state can endogenously cause countries to exert greater effort in competitive diplomacy, creating an incentive for efficient war. Intuitively, a country has the incentive to exert greater effort in state s when $C(s)$ is large in addition to being more likely to choose war state s if $C(s)$ is small. In this example, efficient wars are more likely to emerge when there are large costs of war in state $\bar{s}$ and low costs of war in states s and $\underline{s}$. To demonstrate numerically, consider the primitives $\delta = 0.9$, $\theta = \lambda = 0.7$, $\pi = 0.9$, and $C(s) = C(\underline{s}) = 0.1$. Then, an approximate condition for efficient war in state s is $C(\bar{s}) > 0.146$.

In general, the total effect of θ and λ on $\Delta V(s)$ is not straightforward as it depends on how the transition function maps different actions today into different states in the future. Naturally, greater persistence of either outcome can increase $\Delta V(s)$ through country continuation values. Persistence

[24] Refer to Appendix A.7 for supporting details. Specifically, equation (A20) can be used to plug in for $V(\underline{s})$ and $V(\bar{s})$.

avoids inefficiencies, allowing countries to do better upon a return to bargaining if outcomes are expected to last. However, persistence can also decrease $\Delta V(s)$ as the present value accounts for temporal considerations—the more an outcome persists, the longer it will take for a country to recover the gains that come with a return to bargaining, and hence the smaller its present value ceteris paribus.

Nonetheless, the model provides a path for understanding the conditional effects of persistence on countries' willingness to fight efficient wars. For example, the following corollary demonstrates that in the event of unreliable peace deals and permanent war outcomes, the effect of war outcome persistence on G overwhelms any possible countervailing effects on $\Delta V(s)$, guaranteeing an efficient war in equilibrium.

Corollary 1. *Patient countries will fight an efficient war if peace deals do not persist and war outcomes are permanent. Formally, for any state s and transitions q, $\lambda = 0$ and $\theta = 1$ implies that $\lim_{\delta \to 1^-} (\bar{x}_1(s) - \bar{x}_2(s)) > 0$ for both countries $i = 1, 2$.*

This result demonstrates a sufficient condition for countries to prefer "ripping off the bandage" and trying their luck in war. Notably, this condition is the typical assumption in dynamic crisis bargaining models, going back at least to the analysis of preventive motives in Fearon (1995) where peace deals do not persist into the subsequent periods but war is game-ending. If peace entails constant renegotiation based on changes to the underlying state of the world, whereas war outcomes settle a dispute absolutely, then countries will necessarily prefer fighting an efficient war to cooperation regardless of the state's expected evolution. Importantly, this will be true for any country irrespective of their relative strength today or their expected strength in the future.

Moreover, additional implications can be derived under mild assumptions on the transition function. The relative efficiency of war depends heavily on the expected downstream consequences of actions in the current period. If, for example, a war today would be sure to produce a detrimental state of the world tomorrow, the inefficiencies associated with cooperation would need to be vastly greater than if war today is sure to transition into a superior state of the world. For this reason, conditions on state transitions can allow us to recover more precise conditions for when efficient war will or will not arise.

For example, suppose the expected value of returning to bargaining after war is no better than that of returning to bargaining after peace. Then, a corollary follows from Proposition 5.

Corollary 2. *If the expected value of a return to bargaining after war is no better than the expected value of returning to bargaining after cooperation, efficient wars are averted by peace deals that are more persistent than war outcomes.*

Take any transition function $q : S \times A \to S$ *that satisfies* $\sum_{s'} V(s')(q(s'|s, a_U) - q(s'|s, a_W)) \geq 0$ *for all states* $s \in S$. *Then,* $\lambda > \theta$ *ensures that* $\bar{x}_2(s) > \bar{x}_1(s)$.

Countries may prefer war or cooperation for one of two reasons: either they are associated with less inefficiencies or they tend to produce superior states of the world in the future. By making the above assumption on state transitions, we rule out the possibility that countries have an inherent bias towards war specifically because they tend to transition to preferable states of the world. This requires that returns to bargaining after war cannot be more favorable in the aggregate than they would be after cooperation. As a result, the only way to favor efficient war is if there is less inefficiency associated with fighting. Since country choices of diplomatic effort are endogenous, the condition that $\lambda > \theta$ is sufficient to guarantee this does not occur.

On the other hand, $\theta > \lambda$ is necessary but not sufficient for efficient war, as a preference for war also requires sufficiently small costs of war under the current state and a sufficiently small expected value of returning to bargaining after cooperation. Like Corollary 1, this result also demonstrates the logic of "ripping off the bandage" at play.

5 Frictions and Costly Delay

Consider an extension of the model in which there are frictions, $\mu \geq 0$, that serve to potentially obstruct cooperation. Specifically, if aggregate effort exceeds these frictions, the game proceeds as before and the country that exerted the most effort will be recognized as the agenda setter with probability π. If aggregate effort does not exceed frictions, however, each country will incur a cost of delay $\kappa_i(s) > 0$ before settling at the Nash bargaining solution.[25] These frictions may be considered a lower bound on transaction costs in the sense of Keohane (1984) and related work in international cooperation theory. For simplicity, I allow each country's cost of delay to be proportional to the expected gains it stands to lose by going to war. In particular, let $\kappa_i(s) := \rho(U_i(s) - W_i(s))$ for all $s \in S$, so that $\rho \in (0, 1]$ captures the damages each country expects to incur from delay.

In equilibrium, countries take this risk into account when choosing their actions, including their effort in competitive diplomacy. Although countries could guarantee cooperation by exerting the bare minimum necessary, μ, each country occasionally attempts free riding on the efforts of the other despite creating the possibility of costly delay.

Proposition 6 (Equilibrium)**.** *Take any $\mu > 0$. Given equilibrium strategies σ^*, countries will either fight an efficient war, or else they will cooperate by exerting zero effort with probability $\frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}$ and otherwise mixing uniformly between μ and $\frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu$.*[26]

Figure 4 illustrates how countries compete in state s when $\mu > 0$ according to a mixed strategy defined by cumulative distribution F_s^*. The c.d.f. reveals how equilibrium behavior with frictions results in an occasional costly delay.

Proposition 7. *The results in Propositions 3, 4, and 5 continue to hold with frictions $\mu > 0$.*

Barriers to peace actually protect against the erosion of peace's surplus caused by competitive diplomacy. This occurs because frictions introduce a countervailing incentive to occasionally exert zero effort, effectively forfeiting the competition in lieu of internalizing the passive gains of cooperating for free. The extent to which frictions serve to insulate countries from the erosion of competition is therefore proportional to their ability to free ride on their adversary's efforts without fear of delay.

Remark 1. *Given equilibrium strategies σ^*, the probability of costly delay in state s is equal to $\omega(s) := \left[\frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}\right]^2 > 0$ for any q such that $B(s) > 0$.*

[25] This approximates any protocol in which one side does not have an ex ante advantage in recovering agenda-setting power after delay.

[26] Cooperation in state s therefore requires $\mu < \frac{(2\pi-(1-\rho))B(s)}{2(1-\delta\lambda)}$. The article focuses on strategies σ^* as it is unique for $\mu = 0$ and always valid for any configuration of parameter values when $\mu > 0$. Under special cases of parameter configurations and $\mu > 0$, strictly positive effort strategies can also be sustained, but all of the results continue to hold. Related propositions and proofs are available directly from the author.

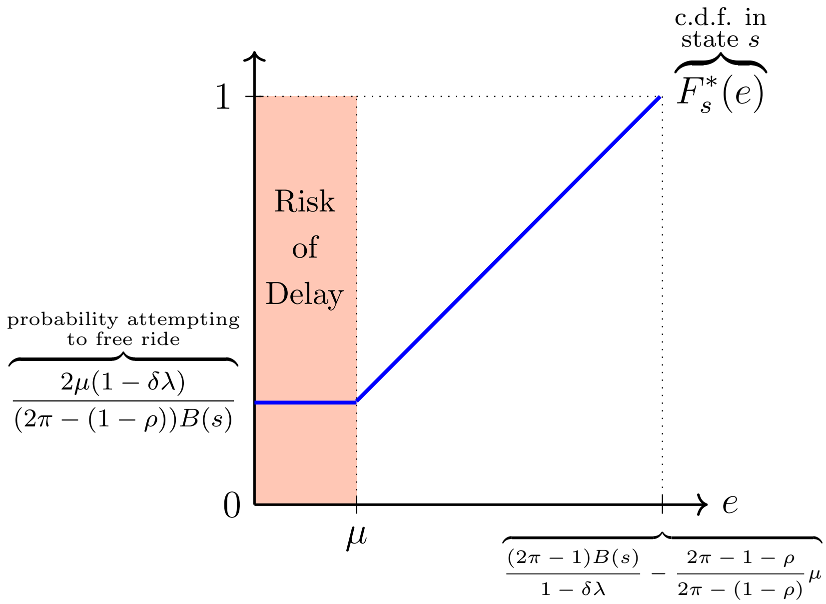

Figure 4. Cumulative distribution for diplomatic effort F_s^* with frictions $\mu > 0$

Even arbitrarily small frictions to cooperation introduce an incentive to occasionally attempt free riding on an adversary's efforts. In general, countries want to perform well in diplomacy and receive a greater chance at recovering the surplus from cooperation, given their exerted effort is not larger than the gain. However, conditional on their relative performance, a country prefers not to exert unnecessary additional effort. This incentive to reduce effort conditional on diplomatic performance can therefore create a dilemma for countries who need to cumulatively exert at least $\mu > 0$ to reach an agreement. When countries simultaneously try to free ride on each other, they delay the settlement.

With the probability of delay characterized as a function of equilibrium strategies, we can learn how it is affected by changes in the form of competitive diplomacy. In general, the effect of competition's decisiveness on the probability of delay depends on the selected transition function, as the expected gain from war relative to peace depends not only on the specific values of θ and λ, but also on how current actions correspond to expected transitions into different states tomorrow. However, for any state space and transition function we can choose, decisiveness reduces the occurrence of delay when peace deals are reliable.

Proposition 8. *The probability of delay is decreasing in the decisiveness of competitive diplomacy when peace deals are reliable. Formally,* $1 = \lambda > \theta$ *implies* $\frac{\partial}{\partial \pi}\omega(s) \leq 0$ *for all* s *and* q.

Countries are less willing to free ride under decisive competition, as doing so often results in forfeiting agenda-setting power to their adversary. Instead, countries respond to increased decisiveness by exerting greater effort in competitive diplomacy, consequently diminishing the probability of delay in equilibrium. However, this effect can be overruled when the persistence of war outcomes exceeds the reliability of peace deals, in which case increases to decisiveness may cause an erosion of the bargaining surplus that reinforces free-riding incentives.

However, note that by Proposition 4, decisiveness also reduces welfare. Thus, the Pareto optimal level of competition *maximizes* the probability of delay when peace deals are reliable. Decisive contests endogenously cause countries to exert greater effort relative to the bargaining surplus, not only in the current period but also in every future period where there is an expected net gain from cooperation. However, these same equilibrium efforts contribute to an erosion of the bargaining surplus, making peace less valuable. Because countries only free ride to the extent they can gain by doing so, their willingness to free ride is inversely proportional to the severity of competitive diplomacy. This is most clearly seen in the limit: fully decisive competition discourages attempts to free ride, but coincides with the complete dissipation of the bargaining surplus, resulting in payoffs from cooperation that are no better than payoffs from war.

Recall also that $\lambda > \theta$ is the same condition that ensures efficient wars are avoided under mild conditions in Corollary 2. This implies a dilemma: improving the reliability of peace deals can reduce the outbreak of efficient wars; however, it also ensures that there will be tension between promoting country welfare and reaching timely settlements.

How changes in other parameters, including the persistence parameters θ and λ and the harm from delay ρ, affect the probability of delay depends on how those changes affect the bargaining surplus. The intuition is as follows: the faster the surplus grows (or the slower it reduces) in response to changes in a parameter, the more it heightens the incentive to engage in competitive diplomacy and, thus, reduces the probability of delay. Refer to Appendix A.8 for a full discussion.

6 Conclusion

I propose a dynamic bargaining theory of war where countries can use diplomacy to compete over agenda-setting power. When competitive diplomacy is present, both the quality and prospects of peace are negatively affected. Increasing the costs of potential war increases the stakes of the dispute by reducing the value of a country's outside option, therefore encouraging more diplomatic competition and reducing welfare. Under these conditions, it is possible for war to be relatively efficient. Further, I extend the model to examine the role of settlement frictions, finding that they introduce a trade-off between welfare and timely settlement.

The contributions of this paper are theoretical; however, the analysis has empirical implications. For example, the model predicts that there will not be an association between competitive diplomatic efforts and cooperative gains in equilibrium. This is borne out in Blair, Marty, and Roessler (2022), which finds that Chinese aid to Africa does not increase their support among beneficiaries, while the US does enjoy greater support after aid provisions. Despite this, China considers foreign aid essential to the development of their ability to achieve favorable outcomes through cooperation (Yoshihara and Holmes, 2008). This is consistent with my analysis: by Propositions 2 and 6, the observed effort in competitive diplomacy correlates with the stakes of the dispute and the scope of the competition, but does not govern the equilibrium success in recovering bargaining leverage.

The analysis also highlights that changes to the international environment, as opposed to specific characteristics of a bilateral relationship, can give rise to war. Recent work has re-emphasized the importance of the systemic conditions (Wolford, 2020; Abramson and Carter, 2021). By Proposition 5, a country's propensity to fight depends on the scope of competitive diplomacy to improve bargaining positions π, as well as how it interacts with the reliability of peace deals λ and the durability of war outcomes θ. External factors often play a role in shaping these determinants of country incentives. For example, the decisiveness of competition may reflect the rules, norms, and procedures of prevailing institutions, and the persistence of outcomes may depend on the nature and interests of third-parties, such as a great power's willingness to intervene to uphold a previous agreement in the event of defection. Consider, for example, how war outcomes in disputes not involving the United States may be more (or less) durable as a result of their willingness (or reluctance) to intervene and upend unfavorable outcomes. Studying the interaction of bilateral factors and systemic-level changes is productive path for future work on international cooperation.

Finally, the theory suggests three points of guidance for the effective design of international institutions. First, it highlights the need for institutions to reduce the scope of competitive diplomacy. If states can improve their bargaining position through diplomatic effort, not only can this cause significant reductions in aggregate welfare, but it can also cause war to be relatively efficient from the perspective of the disputing states. Second, while barriers to cooperation introduce the possibility of costly delay, the analysis demonstrates that frictions protect against an erosion of the gains from peace caused by competitive diplomacy. As a result, cost-reducing institutions may do more harm than good: it can be welfare-enhancing to intentionally impose frictions when high levels of competition are unavoidable. Third, the paper identifies a dilemma: reliable peace deals avert efficient wars, but they can simultaneously introduce an unavoidable tension between safeguarding the gains from peace and preventing costly delay. Institutions that aim to maximize welfare while simultaneously reaching immediate peace may prefer to construct peace deals that are not intended to be permanent, especially if the barriers are large.

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Online Appendix for "Competitive Diplomacy in Bargaining and War"

Contents

A Additional Model Details and Discussion

A.1 State Variables and Transitions

There are three state variables. The first simply traces the previous period's distribution of the pie, $z_t = o_{t-1} \in \mathbb{R}$ for each period $t \geq 2$ where o_t gives the outcome in period t and without loss of generality $z_1 = 0$. The outcome of a period corresponds to the interaction's resulting division of the pie. In particular, period t's outcome is given by $o_t = 1$ if country 1 wins a war, $o_t = 0$ if country 2 wins a war, and $o_t = x$ for a peace deal reached at $x \in \mathbb{R}$.

Second, the state variable $s_t \in S$ provides the measure of country 1's relative strength, where S is a finite subset of $(0, 1)$. Strengths may evolve flexibly over time according to a Markov transition function $q : S \times A \to S$, where $a_t \in A \equiv \{0, 1\}^2$ are the war actions taken in period t.

Finally, the state variable b_t denotes the crisis bargaining status of the game in period t. In particular, $b_t = 0$ is an inactive state where an outcome persists and each country consumes their status quo share of the pie, z_t. On the other hand, $b_t = 1$ implies countries have an opportunity to engage in diplomatic competition and crisis bargaining.

A key feature of the model is that these outcomes can persist into future periods. Specifically, the game begins in bargaining $b_t = 1$ and, at the end of every period, the state variable b_t transitions to $b_{t+1} = 0$ with probability $\theta \in [0, 1]$ when the current division is a war outcome and with probability $\lambda \in [0, 1]$ when the division is the result of a peace deal.

Note that a strategy σ_i does not take b_t or z_t as input because (1) when $b_t = 0$, countries simply consume their share of the pie according to the previous period's division, which is equal to the current state variable z_t, and (2) given $b_t = 1$, state variable z_t is no longer payoff relevant. Thus, I refer to state $(1, z_t, s_t)$ as state s_t when it is implied that $b_t = 1$.

A.2 Budget Constraints

Budget constraints are given by $\mathcal{X}_1 := \{x \in \mathbb{R} : x \geq X_1\}$ and $\mathcal{X}_2 := \{x \in \mathbb{R} : x \leq X_2\}$ where X_1 and X_2 are the greatest credible agreements for country 1 and 2, respectively. This is consistent with the standard approach in dynamic crisis bargaining, where a lack of liquidity prevents side payments that can placate an opponent. Most typically, $X_1 = 0$ and $X_2 = 1$. The main text focuses on $X_1 = -\infty$ and $X_2 = \infty$ to avoid preventive wars and isolate the mechanisms of interest.

A.3 Payoffs

After the outbreak of war, country i wins a pie of size 1 today with probability s_i and tomorrow (discounted at rate δ) they either get, with probability θ, $Z_i^{war}(s)$, which is the ex-ante expected future value of the war outcome persisting (i.e., prior to knowledge over whether i or i's opponent wins the war), or with complementary probability $1-\theta$ they return to bargaining under a new state s'. Given the state $s \in S$, each country i has an expected war value of

$$W_i(s) = s_i - c_i(s) + \delta \left[\theta Z_i^{war}(s) + (1-\theta) \sum_{s' \in S} V_i(s') q(s'|s, a_W) \right] \tag{A1}$$

where

$$Z_i^{war}(s) = s_i + \delta \left[\theta Z_i^{war}(s) + (1-\theta) \sum_{s' \in S} V_i(s') q(s'|s, a_W) \right]. \tag{A2}$$

and a_W denotes actions that involve countries choosing to fight a war. Using equation (A2) to solve for $Z_i^{war}(s)$, equation (A1) simplifies to equation (2) in the main text.

If countries succeed in cooperating, let the expected settlement be given by $N(s)$. In equilibrium, this expectation will correspond to the Nash bargaining solution due to symmetric effort strategies. Then, countries receive a payoff according to the expected settlement today, less the amount they expect to exert according to their mixed diplomatic effort strategy. Tomorrow, countries receive payoffs according to the same settlement allocation with probability λ and return to the bargaining table under a new state s' with probability $1-\lambda$. Formally, we can write

$$U_i(s) = u_i(N(s)) - \int e dF_s^*(e) + \delta \left[\lambda Z_i^{peace}(s) + (1-\lambda) \sum_{s' \in S} V_i(s') q(s'|s, a_U) \right] \tag{A3}$$

where

$$Z_i^{peace}(s) = u_i(N(s)) + \delta \left[\lambda Z_i^{peace}(s) + (1-\lambda) \sum_{s' \in S} V_i(s') q(s'|s, a_U) \right]. \tag{A4}$$

and a_U denotes actions that involve countries choosing to cooperate. As before, it is necessary to account for the uncertainty over how long peace will persist with $Z_i^{peace}(s)$. Using equation (A4) to solve for $Z_i^{peace}(s)$, equation (A3) simplifies to equation (3) in the main text.

A.4 Equilibrium Offers

When bargaining surplus exists, a country will offer the least generous settlement that facilitates peace—that is, they will offer the point that makes their opponent indifferent or their opponent's greatest credible settlement. First, consider the case without budget constraints. Since diplomatic effort is a sunk cost at the point at which a country makes a proposal, an agenda setter $-i$ will offer an $\bar{x}_i(s)$ that solves $U_i(\bar{x}_i(s); s) = W_i(s)$. In particular, we can solve for values that make countries 1 and 2 indifferent in state s,

$$W_i(s) = u_i(x) + \frac{\delta}{1-\delta\lambda}\left[\lambda u_i(x) + (1-\lambda)\sum_{s'\in S} V_i(s')q(s'|s, a_U)\right]. \tag{A5}$$

Solving (A5) for $u_i(x)$, we get

$$u_i(x) = (1-\delta\lambda)W_i(s) - \delta(1-\lambda)\sum_{s'\in S} V_i(s')q(s'|s, a_U),$$

which, after plugging in $u_1(x) = x$ and $u_2(x) = 1 - x$, yields implicit solutions,

$$\bar{x}_1(s) = (1-\delta\lambda)W_1(s) - \delta(1-\lambda)\sum_{s'\in S} V_1(s')q(s'|s, a_U) \tag{A6}$$

$$\bar{x}_2(s) = 1 - (1-\delta\lambda)W_2(s) + \delta(1-\lambda)\sum_{s'\in S} V_2(s')q(s'|s, a_U). \tag{A7}$$

The relationship between these indifference points and the indifference points of the standard static bargaining model of war is apparent. In particular, perfectly impatient countries are indifferent at the same settlement as countries in the standard bargaining model of war with complete and perfect information. To see this, consider when $\delta = 0$ so that equation (A6) becomes $\bar{x}_1(s) = s - c_1$ and equation (A7) becomes $\bar{x}_2(s) = s + c_2$. These are equivalent to the standard static bargaining range from the literature.

Upon making a proposal, countries will choose an offer that allocates themselves the largest quantity that their opponent would accept to avoid war. The decision to extend and accept an offer occurs after both countries have already incurred diplomatic effort. This is going to be equal to the adversary's indifference deal unless budget constraints get in the way, in which case an agenda setter will extract the greatest credible settlement. If the greatest credible settlement is worse than the agenda setter's indifference deal, however, they will prefer war to cooperation. Then, we can formally define the equilibrium offers as

$$x_1^*(s) = \begin{cases} \bar{x}_2(s) & \text{if } X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s) \\ X_2 & \text{if } \bar{x}_2(s) > X_2 \geq \bar{x}_1(s) \\ x & \text{for any } x \in \mathcal{R}_2(s) \text{ otherwise} \end{cases} \tag{A8}$$

$$x_2^*(s) = \begin{cases} \bar{x}_1(s) & \text{if } \bar{x}_2(s) \geq \bar{x}_1(s) \geq X_1 \\ X_1 & \text{if } \bar{x}_2(s) \geq X_1 > \bar{x}_1(s) \\ x & \text{for any } x \in \mathcal{R}_1(s) \text{ otherwise.} \end{cases} \tag{A9}$$

where $\mathcal{R}_1(s) := \{x \in \mathbb{R} : x < \bar{x}_1(s)\}$ and $\mathcal{R}_2(s) := \{x \in \mathbb{R} : x > \bar{x}_2(s)\}$ denote the rejection sets for countries 1 and 2 and where X_i denotes the most generous credible settlement for a country i. The receiving country then accepts offers if and only if they are at least as good as their outside option. In particular, $y_i(x; s) = \mathbb{1}\{U_i(x; s) \geq W_i(s)\}$.

Note that the bargaining surplus is given by

$$B(s) = \begin{cases} \bar{x}_2(s) - \bar{x}_1(s) & \text{if } X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s) \geq X_1 \\ \bar{x}_2(s) - X_1 & \text{if } X_2 \geq \bar{x}_2(s) \geq X_1 > \bar{x}_1(s) \\ X_2 - \bar{x}_1(s) & \text{if } \bar{x}_2(s) > X_2 \geq \bar{x}_1(s) \geq X_1 \\ X_2 - X_1 & \text{if } \bar{x}_2(s) > X_2 \geq X_1 > \bar{x}_1(s) \\ 0 & \text{if otherwise.} \end{cases}$$

Therefore, note that equation (5) in the main text specifically refers to the primary case of interest where budget constraints do not interfere in any way. Since this is a necessary condition and budget constraints are fixed, I often refer to this object as the bargaining surplus more generally, and consider violations of the budget constraint ex post.

A.5 Competing for Bargaining Surplus

Competing is costly but improves the odds of recovering the bargaining surplus, which must be of common value to both countries. If there exists a bargaining surplus, $B(s) > 0$, both countries will cooperate and choose an amount of effort as a function of the state s. Then, a country i that's recognized as proposer will receive the value of the bargaining surplus today and possibly in the future, with likelihood according to λ. Hence, the expected net gain of recovering agenda-setting power becomes

$$B(s) + \delta\lambda B(s) + (\delta\lambda)^2 B(s) + \ldots = \frac{B(s)}{1-\delta\lambda}.$$

There is no equilibrium in pure effort strategies, as (1) countries have an incentive to increase their effort insofar as they are losing the competition and there exists profitable deviations above their opponent's level, (2) countries have incentive to decrease their effort insofar as they are losing and there is no profitable deviation above their opponent's level, and (3) countries that are winning have incentive to marginally decrease their effort.

Instead, consider equilibrium effort such that each country mixes according to a cumulative distribution function (c.d.f.) denoted by F_s^* in state s. Moreover, because the baseline model is a special case of the extended model with frictions in which $\mu = 0$, I focus on the general case here. In particular, I begin by looking for equilibrium diplomatic strategies where countries only exert nonzero efforts above any frictions that exist $\mu \geq 0$. By exerting effort $e \geq \mu$, country i then has an expected utility

$$\underbrace{W_i(s)}_{\substack{\text{expected} \\ \text{value of war}}} + \frac{B(s)}{1-\delta\lambda}\Big[\underbrace{F_s^*(e)\pi}_{\substack{\text{opponent} \\ \text{exerts less}}} + \underbrace{(1-F_s^*(e))(1-\pi)}_{\text{opponent exerts more}}\Big] - e. \tag{A10}$$

Each country's war payoff serves as their baseline value for cooperation, reflecting their outside option. By choosing to exert e on competition, a country can expect a share of the bargaining surplus according to the bracketed component, and have total value for it according to their discount on time δ and how long they expect the settlement to persist λ.

On the other hand, exerting zero effort yields an expected value

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\Big[\underbrace{F_s^*(\mu)\frac{1}{2}(1-\rho)}_{\text{mutual shirking}} + \underbrace{(1-F_s^*(\mu))(1-\pi)}_{\text{opponent exerts effort}}\Big] \tag{A11}$$

recalling from the main text that delay costs each country ρ of the gains from peace. The gain from peace to a country that exerts nothing reflects the odds they are still awarded the agenda-setting power in the event their opponent exerts effort. Mixing strategies require indifference between equations (A10) and (A11), allowing us to solve for the distribution F_s^* (refer to the proof of Proposition A1).

A.6 Equilibrium Characterization

This section formally states results that support and extend the analysis of the main text.

The below proposition provides a full characterization of equilibrium with strategies σ^* under primary focus in the main text.

Proposition A1 (Equilibrium). *An equilibrium exists where, for all $s \in S$ and $q : S \times A \to S$, each country $i = 1, 2$ plays $\sigma_i^*(s) = (a_i^*(s), e_i^*(s), x_i^*(s), y_i^*(x; s))$ defined as follows.*

(i) Initiate war $a_i^*(s) = 1$ if and only if $W_i(s) > U_i(s)$.

(ii) If $U_i(s) \geq W_i(s)$, choose effort $e_i^*(s)$ according to a mixed strategy with distribution

$$F_s^*(e) = \begin{cases} \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)} & \text{for } e \in [0, \mu) \\ \frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right) & \text{for } e \in \left[\mu, \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right] \end{cases}$$

with $F_s^*(e) = 0$ for $e < 0$ and $F_s^*(e) = 1$ for $e > \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu$. Otherwise exert zero effort $e_i^*(s) = 0$.

(iii) Offer $x_i^*(s)$ as given by equations (A8)-(A9) when recognized as agenda setter.

(iv) Accept an offer x, $y_i^*(x; s) = 1$, if and only if $U_i(x; s) \geq W_i(s)$.

Proof of Proposition A1. First, consider the war decision. The countries maximize their expected utility and if, given state s, their war continuation value $W_i(s)$ is larger than their continuation value from cooperating $U_i(s)$, they will necessarily prefer to fight. However, if there is positive bargaining surplus $B(s) \geq 0$, the expected net gain from exerting effort $e \geq 0$ on diplomacy is

$$\begin{aligned} \frac{B(s)}{1-\delta\lambda}\Big[&\pi \Pr(e > \max\{e_{-i}, \mu - e_{-i}\}) + (1-\pi) \\ &\times \Pr(e_{-i} > \max\{e, \mu - e\}) + \frac{1}{2}(1-\rho)\Pr(\mu > e + e_{-i})\Big] - e. \end{aligned} \tag{A12}$$

The net gain is zero when $B(s) = 0$, in which case neither country will be willing to exert a positive amount of effort in equilibrium. Therefore, to understand strategies with nonzero amounts of effort, assume $B(s) > 0$.

To see why there are no pure strategies in equilibrium, consider the following. If a country always exerts amount $e^* > 0$, their opponent would either deviate to an amount greater than e^* or zero. If the opponent deviated to zero, the country will prefer to exert less than e^*. If the opponent deviated to an amount greater than e^*, the country will prefer to move to a greater amount or deviate to zero. If both countries exert the same amount, they will either have an incentive to increase their effort a marginal amount to increase their gain by approximately double or they will prefer to deviate to zero.

I now look for a mixed strategy corresponding to state s given by c.d.f. F_s^* that satisfies equation (A12) for both countries. We can write the payoff from zero effort as

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1-F_s^*(\mu))(1-\pi)\right]. \tag{A13}$$

On the other hand, exerting effort of μ yields an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\pi + (1-F_s^*(\mu))(1-\pi)\right] - \mu. \tag{A14}$$

Using equations (A13) and (A14), we can solve for

$$F_s^*(\mu) = \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}. \tag{A15}$$

We know that μ cannot be the top of the support by assumption that frictions are not so high as to discourage cooperation altogether. Then, a country i exerting effort $e > \mu$ will receive an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(e)\pi + (1-F_s^*(e))(1-\pi)\right] - e. \tag{A16}$$

Using the indifference condition for equations (A14) and (A16) and plugging in equation (A15), we find that for $e \geq \mu$,

$$F_s^*(e) = \frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right). \tag{A17}$$

By definition of a c.d.f., we know the largest amount a country can exert and still be indifferent is given by $\bar{e}_s := \inf\{e \geq 0 : F^*(\bar{e}_s) = 1\}$. Using equation (A17), we find that

$$\bar{e}_s = \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu.$$

Together, these equations yield an equilibrium effort strategy presented in Proposition A1. To check for profitable deviations, consider the case where a country exerts effort $e > \bar{e}_s$ with nonzero probability. By deviating, their payoff will be

$$W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - e < W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - \bar{e}_s,$$

and therefore they do not deviate. In words, a country already wins with certainty when expending $\bar{e}_s$, so there is no reason to ever exert more given their opponent plays this strategy as well.

Further, consider a deviation to expending a nonzero amount less than μ, $e \in (0, \mu)$, with some probability. By deviating, their expected payoff will be

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F^*(\mu - e)\frac{1}{2}(1-\rho) + (1 - F^*(\mu - e))(1-\pi)\right] - e. \tag{A18}$$

Since their opponent is playing a strategy such that $F^*_s(\mu) = F^*(\mu - e)$ for any $e \in (0, \mu)$, we can plug this into equation (A18) and see that their payoff is equal to

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F^*_s(\mu)\frac{1}{2}(1-\rho) + (1 - F^*_s(\mu))(1-\pi)\right] - e.$$

which is strictly less than their payoff from exerting zero given by equation (A13), hence this is not a profitable deviation. This is sufficient to show that F^*_s is an equilibrium effort strategy.

Equations (A6) and (A7) that govern equilibrium offers yield implicit conditions for $\bar{x}_2(s)$ and $\bar{x}_1(s)$, which represent the settlements at which country 1 and 2 are left indifferent between peace and war, respectively. Each country i will extend an offer equal to country $-i$'s indifference point or the greatest possible offer $-i$ can accept if and only if it is preferable to their own indifference point $\bar{x}_i(s)$ and the offer falls within their budgets $\mathcal{X}_i$. Otherwise, each country i will extend an offer that they know will get rejected, $x \in \mathcal{R}_{-i}(s)$ where $\mathcal{R}_i(s) := \{x : U_i(x; p) < W_i(s)\}$ denotes country i's rejection set in state s. A country i receiving an offer rejects offers in their rejection set and accepts all others. $\square$

Additionally, this is the unique equilibrium when there are no frictions.

Proposition A2. *If $\mu = 0$, Proposition A1 characterizes the unique equilibrium.*

Proof of Proposition A2. Since the war, offer, and accept decisions are straightforward, we only need to show that there is no equilibrium under an alternative effort strategy $F' \neq F^*$.

First note that any effort strategy must be continuous. Suppose for example there is an F' with finite probability $\xi > 0$ of exerting effort e' in the distribution. Then, there exists a profitable deviation F'' in which a country could improve their expected share of the bargaining surplus by allocating greater probability to an arbitrarily small increase above e'. Thus, we know all effort strategies must be given by continuous distributions.

Second, zero must be in the support of the distribution. Suppose otherwise, so that the lower bound of the distribution is some $\underline{e} > 0$. Then, following this distribution would imply that countries exert strictly positive amounts despite an expectation that they will have the weakest performance. As a result, there would be a profitable deviation to effort less than $\underline{e}$ that save on the expense of competition without resulting in lower expected shares of the bargaining surplus.

Moreover, the top of the support must be equal to $\bar{e}_s$ as defined above. Suppose there is an alternate distribution F' such that $F'(\bar{e}_s) < 1$, i.e., countries exert greater than $\bar{e}_s$ with positive probability in equilibrium. Because we have just shown that zero must be in the support of F', expending amounts consistent with the new upper bound $\bar{\bar{e}} > \bar{e}_s = \frac{(2\pi-1)B(s)}{1-\delta\lambda}$ needs to be consistent with the lower bound of zero. Let $\bar{\bar{e}} = \bar{e}_s + \varepsilon$ for small $\varepsilon > 0$. This implies

$$W_i(s) + \frac{(1-\pi)B(s)}{1-\delta\lambda} = W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \varepsilon$$

which implies $\varepsilon = 0$, a contradiction. On the other hand, if there exists an $\bar{\bar{e}} < \bar{e}_s$ such that $F'(\bar{\bar{e}}) = 1$, there is a profitable deviation to exerting $\bar{\bar{e}} + \varepsilon$ for an arbitrarily small $\varepsilon > 0$.

Lastly, to show that the distribution is necessarily uniform, suppose for contradiction there is an alternate distribution F' where, for some $\hat{e} \in [0, \bar{e}_s]$, $F'(\hat{e}) > F^*(\hat{e}) = \frac{(1-\delta\lambda)\hat{e}}{(2\pi-1)B(s)}$. Then, the payoff from exerting $\hat{e}$ must be

$$\frac{B(s)}{1-\delta\lambda}\bigg[F'(\hat{e})\pi + (1-F'(\hat{e}))(1-\pi)\bigg] - \hat{e} = \frac{B(s)}{1-\delta\lambda}\bigg[F'(\hat{e})(2\pi-1) + (1-\pi)\bigg] - \hat{e} > \frac{(1-\pi)B(s)}{1-\delta\lambda}$$

which is the payoff from zero effort. But we know this must be in the support, a contradiction. The analogous argument can be made for F' such that $F'(\hat{e}) < F^*(\hat{e})$ for some $\hat{e} \in [0, \bar{e}_s]$. Hence, $\mu = 0$ implies that the equilibrium detailed in Proposition A1 is unique. □

A.7 Ex-Ante Continuation Value in an Absorbing State

This section shows that the ex-ante continuation value for an absorbing state can be expressed solely in terms of model primitives. This derivation is for purposes with $\mu = 0$, but a similar procedure can be followed to derive an analogous expression for when $\mu > 0$.

Take any state $s' \in S$ to be absorbing, i.e., $q(s'|s', a) = 1$ for any $a \in A$. Then, the aggregate present value associated with arriving in state s' is given by

$$V(s') = \frac{1}{1-\delta\lambda} - 2\int e dF^*_{s'}(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\left(\frac{1}{1-\delta\lambda} - 2\int e dF^*_{s'}(e) + \dots\right).$$

Recognizing the geometric series, we can rewrite the expression as

$$V(s') = \frac{1 - 2(1-\delta\lambda)\int e dF^*_{s'}(e)}{1-\delta}. \tag{A19}$$

Because s' is absorbing, we have a closed-form expression for its bargaining surplus. In particular, the bargaining surplus in an absorbing state is equal to

$$B(s') = 1 - (1-\delta\lambda)\left[\frac{1}{1-\delta\lambda} - C(s') + \frac{\delta(1-\lambda)}{1-\delta\lambda}V(s')\right] + \delta(1-\lambda)V(s') = (1-\delta\lambda)C(s')$$

where recall that $C(s) := c_1(s) + c_2(s)$. Consequently, we can now express a country's expected effort in an absorbing state s' in terms of the total costs of war in that state and model parameters,

$$\int e dF^*_{s'}(e) = \frac{1}{2}(2\pi - 1)C(s').$$

Plugging this into equation (A19), we have

$$V(s') = \frac{1 - (1-\delta\lambda)(2\pi - 1)C(s')}{1-\delta} \tag{A20}$$

yielding the continuation value for being in s' solely in terms of model primitives.

A.8 Costly Delay and Bargaining Surplus

There is a strong connection between the probability of costly delay and the surplus from cooperation: to understand the effect of a parameter on the former often requires an understanding of its effect on the latter. For example, the cost of delay in the event of coordination failure has a direct exogenous effect on the probability of costly delay, but also an indirect endogenous effect through the bargaining surplus and corresponding equilibrium behavior. The same is true for the persistence parameters. As a result, it is not straightforward to make claims about how changes to these parameters affect the probability of costly delay.

Whether one force or the other will prevail inevitably depends on the expected downstream consequences. For example, the total probability of costly delay is decreasing in the cost of delay if the changes to equilibrium behavior result in a large enough increase to the cooperative surplus.

Moreover, we can also see this inverse relationship borne out in the effect of peace deal reliability and war outcome durability.

Proposition A3. *Let $\lambda = \alpha\theta$. Then, (i) $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\theta}B(s) \geq 0$,*

(ii) $\frac{\partial}{\partial\alpha}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\alpha}B(s) \geq \frac{-\delta\theta B(s)}{1-\delta\lambda}$, and

(iii) $\frac{\partial}{\partial\rho}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\rho}B(s) \geq \frac{B(s)}{2\pi-(1-\rho)}$.

Proof of Proposition A3. **(i) $\boldsymbol{\theta}$:** Taking the partial derivative of $\omega(s)$ with respect to θ,

$$\frac{\partial}{\partial\theta}\omega(s) = \frac{-8\mu^2(1-\delta\lambda)^2}{(2\pi-(1-\rho))^2 B(s)^3} \cdot \frac{\partial}{\partial\theta}B(s)$$

and because the first term must be negative, we can conclude that $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\theta}B(s) \geq 0$. From the definition of $B(s)$, we can conclude $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if

$$\frac{\partial}{\partial\theta}B(s) = \delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) - (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s) \geq 0$$

or equivalently $\delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) > (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s)$.

(ii) $\boldsymbol{\alpha}$: Plugging in $\alpha\theta$ for λ and taking the partial derivative of $\omega(s)$,

$$\frac{\partial}{\partial\alpha}\omega(s) = \frac{-8\mu^2(1-\delta\alpha\theta)(\delta\theta B(s) + (1-\delta\alpha\theta)\frac{\partial}{\partial\alpha}B(s))}{(2\pi-(1-\rho))^2 B(s)^2}.$$

This expression is negative if and only if $\frac{\partial}{\partial\alpha}B(s) \geq \frac{-\delta\theta B(s)}{1-\delta\theta}$ for any valid parameter values.

(iii) $\boldsymbol{\rho}$: Taking the partial derivative of $\omega(s)$ with respect to ρ,

$$\frac{\partial}{\partial\rho}\omega(s) = \frac{-8\mu^2(1-\delta\lambda)^2(B(s) + (2\pi-(1-\rho))\frac{\partial}{\partial\rho}B(s))}{(2\pi-(1-\rho))^3 B(s)^3)}.$$

This expression is negative if and only if $\frac{\partial}{\partial\rho}B(s) \geq \frac{B(s)}{2\pi-(1-\rho)}$. Refer to the supplementary files for verification of these derivatives and inequalities. □

To see how these results depend on downstream expectations, it is useful to observe that the effect of a parameter on the bargaining surplus is inextricably tied to its differential effect on a country's incentive to fight. In the case of θ, we can see that $\frac{\partial}{\partial\theta}B(s) \geq 0$ if and only if $\delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) \geq (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s)$ for any state $s \in S$, by definition. On the left-hand side of this equation reflects the effect of war outcome durability on the expected continuation values upon a return to bargaining after cooperation in state s, whereas the right-hand side reflects its effect on the continuation values for war in state s. The probability of costly delay is inversely related to the surplus, and the surplus is increasing if future values after cooperating are expanding at a fast enough rate relative to that of the current war value. Clearly, the way in which actions in the current state of the world map onto probability distributions over future states of the world is paramount in determining whether the inequality will hold.

B Proofs

Proof of Lemma 1. Without loss of generality, suppose country 1 is the agenda setter. Then, recall from Appendix A.4 that the equilibrium offer is equal to

$$x_1^*(s) = 1 - (1-\delta\lambda)W_2(s) + \delta(1-\lambda)\sum_{s' \in S} V_2(s')q(s'|s, a_U)$$

if $X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s)$, or else $x_1^*(s) = X_2$ if $\bar{x}_2(s) > X_2 \geq \bar{x}_1(s)$ or $x_1^*(s) = x$ for any $x \in \mathcal{R}_2(s)$ otherwise. In the latter two cases, there is no direct effect of θ or λ. Therefore, suppose $X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s)$, which reflects the primary setting of interest: on-path equilibrium behavior that is unaffected by budget constraints. Unpacking the continuation values, we can see that

$$x_1^*(s) = \frac{\delta(\lambda-\theta)}{1-\delta\theta} - (1-\delta\lambda)c_2(s) + \sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s, a_W) + \delta(1-\lambda)q(s'|s, a_U)\right]$$

Taking the partial derivative with respect to θ:

$$\frac{\partial}{\partial\theta}x_1^*(s) = \underbrace{\frac{-\delta(1-\delta\lambda)}{(1-\delta\theta)^2}}_{\text{direct}} + \underbrace{\frac{\partial}{\partial\theta}\left(\sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s, a_W) + \delta(1-\lambda)q(s'|s, a_U)\right]\right)}_{\text{indirect}}$$

where we can see the direct effect must be negative under any configuration of parameters.

Next, taking the partial derivative with respect to λ:

$$\frac{\partial}{\partial\lambda}x_1^*(s) = \underbrace{\frac{1}{1-\delta\theta} + \delta c_2(s)}_{\text{direct}} + \underbrace{\frac{\partial}{\partial\lambda}\left(\sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s, a_W) + \delta(1-\lambda)q(s'|s, a_U)\right]\right)}_{\text{indirect}}$$

from which the direct effect must be positive. Assuming the direct effects prevail ensures that country 1 can extract greater concessions from country 2 in equilibrium when θ is lower and λ is higher. By symmetry country 2 can extract greater concessions under these same circumstances, from which we can conclude agenda-setting power is more valuable. $\square$

Proof of Proposition 1. In the absence of competitive diplomacy, we can suppose, for example, that $\pi = \frac{1}{2}$ and therefore the agenda setter is randomly recognized with probability $\frac{1}{2}$ regardless of their diplomatic effort. Any other rule that does not reward competition, such as deterministic agenda setter endowments, are also acceptable. From there, it follows that countries will not exert costly effort for no gain, and hence $e_1(s) = e_2(s) = 0$ for all states s.

Since cooperation then does not entail inefficiencies, it must be that for any period s, the sum of continuation values under an always cooperate strategy σ_U is $V^{\sigma_U}(s) = V_1^{\sigma_U}(s) + V_2^{\sigma_U}(s) = \frac{1}{1-\delta}$ for any state $s \in S$. Further, since war is inherently costly, the sum of the continuation values in under any strategy σ_W that involves some war must be strictly less than this amount, $V^{\sigma_W}(s) < V^{\sigma_U}(s)$ for any state s. Let $\hat{s}$ be a state where at least one country prefers to fight a war in equilibrium strategy σ^*. Then, we know $V^{\sigma^*}(\hat{s}) < V^{\sigma_U}(\hat{s})$ which in turn implies $V_i^{\sigma^*}(\hat{s}) < V_i^{\sigma_U}(\hat{s})$ for at least one country i. Since war occurs in state $\hat{s}$, we know that for this country $W_i^{\sigma^*}(\hat{s}) > U_i^{\sigma^*}(\hat{s})$.

Now suppose country $-i$ is recognized as agenda setter in state $\hat{s}$. By extending equilibrium offer according to σ^*, it will be rejected and war will ensue, leaving them with a payoff $W_{-i}^{\sigma^*}(\hat{s})$. However, we know that, due to the inefficiency of war, countries must lose out on a total surplus of $\frac{1}{1-\delta} - V(\hat{s}) > 0$. Therefore, there is a strictly profitable deviation to strategy σ^{**} with offer $x^{**} \in \mathbb{R}$ such that $U_i(x^{**}; \hat{s}) = W_i(\hat{s})$, allowing $-i$ to consume the additional surplus $U_{-i}^{\sigma^{**}}(\hat{s}) = W_{-i}^{\sigma^*}(\hat{s}) + \frac{1}{1-\delta} - V^{\sigma^*}(\hat{s})$. Country $-i$ would be strictly better off by playing this strategy and country i would accept this offer as it satisfies their indifference condition. Further, since country $-i$ is willing to extend this offer, country i will be able to recover a settlement at least as good as x^{**} in the event they are recognized as agenda setter. $\square$

Proof of Proposition 2. By Proposition A1, $\mu = 0$ implies

$$\frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right) = \frac{(1-\delta\lambda)e}{(2\pi-1)B(s)}$$

for all $e \in [0, \bar{e}_s^{\mu=0}]$ and some $\bar{e}_s^{\mu=0} \geq 0$. The upper bound can be solved by observing that $F_s^*(\bar{e}_s^{\mu=0}) = 1$, which yields

$$\bar{e}_s^{\mu=0} = \frac{(2\pi-1)B(s)}{1-\delta\lambda}.$$

By Proposition A2, this equilibrium is unique. $\square$

Proof of Proposition 3. First, define the following set of terms for notational convenience. Let $C(s) := c_1(s) + c_2(s)$ reflect the total costs of war in state s, $W(s) = \sum_i W_i(s)$ the sum of the war continuation values in state s, $\hat{W}(s) := W(s) + C(s)$ the sum of the war continuation values less the total costs of war in state s, $U(s) := \sum_i U_i(s)$ the sum of the cooperation continuation values in state s, and $V(s) := \sum_i V_i(s)$ the sum of the ex-ante continuation values in state s.

Part 1. Costs of war: Recall that total expected effort $\int edF_s^*(e) = \frac{(2\pi-1)B(s)}{2(1-\delta\lambda)}$, where

$$B(s) = 1 - (1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s' \in S} V(s')q(s'|s, a_U)$$

defines the bargaining surplus for all $s, s' \in S$, for both countries $i = 1, 2$. Then, plugging in for $B(s)$, total effort allocated to competitive diplomacy in state s is

$$E(s) := 2\int edF_s^*(e) = \frac{2\pi - 1}{1-\delta\lambda}\left[1 - (1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s' \in S} V(s')q(s'|s, a_U)\right].$$

Note that $\frac{\partial}{\partial C(s)}E(s) = 0$ for any $s \in S$ such that $V(s) = W(s)$ and hence $E(s) = 0$. Letting $s_{min} := \arg\min_s\{\frac{\partial}{\partial C(s)}E(s) : V(s) = U(s)\}$ and $C(s_{min}) := C$, we can take the partial derivative,

$$\begin{aligned}
\frac{\partial}{\partial C}E(s_{min}) &= \frac{2\pi-1}{1-\delta\lambda}\left[(1-\delta\lambda)\left(1 - \frac{\partial}{\partial C}\hat{W}(s_{min})\right) + \delta(1-\lambda)\sum_{s' \in S}\frac{\partial}{\partial C}U(s')q(s'|s_{min}, a_U)\right] \\
&= \frac{2\pi-1}{1-\delta\lambda}\left[(1-\delta\lambda)\left(1 - \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial C}U(s_{min})\right) + \delta(1-\lambda)\frac{\partial}{\partial C}U(s_{min})\right] \\
&= 2\pi - 1 + \left[\frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\right]\frac{\partial}{\partial C}U(s_{min})
\end{aligned}$$

where we assume that $q(s_{min}|p, a) = 1$ for all $s \in S$ to impose the worst case scenario if it is possible for $\frac{\partial}{\partial C(s)}E(s) < 0$. If the partial derivative of $E(s_{min})$ with respect to C is positive under these conditions, then it must be positive in all cases.

Plugging in for $\frac{\partial}{\partial C}U(s_{min})$, we have

$$\begin{aligned}\frac{\partial}{\partial C}E(s_{min}) &= 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\frac{\partial}{\partial C}U(s_{min})\right) \\ &= 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda} \\ &\qquad \times \left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\left(\dots\right)\right)\right).\end{aligned}$$

Noting the geometric series, we can express the above as

$$\frac{\partial}{\partial C}E(s_{min}) = 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\left(\frac{-(1-\delta\lambda)\frac{\partial}{\partial C}E(s_{min})}{1-\delta}\right) \tag{A21}$$

Solving for $\frac{\partial}{\partial C}E(s_{min})$,

$$\frac{\partial}{\partial C}E(s_{min}) = \left[1 - \frac{\delta(1-\delta)^2(2\pi-1)(\lambda-\theta)}{(1-\delta\lambda)^2(1-\delta\theta)}\right]^{-1}(2\pi-1) > 0$$

which is strictly positive for all valid parameter values.

Part 2. Welfare: Define $s_{i,max} := \arg\max_s \frac{\partial}{\partial c_i(s)}V_i(s)$. If $V_i(s_{i,max}) = W_i(s_{i,max})$, it is straightforward to see that

$$\begin{aligned}\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max}) &= -1 + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'\in S}\frac{\partial}{\partial c_i(s_{i,max})}V(s')q(s'|s_{i,max}, a_W) \\ &\le -1 + \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max})\end{aligned}$$

implying

$$\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max}) \le -\frac{1-\delta\theta}{1-\delta} < 0.$$

On the other hand, if $V_i(s_{i,max}) = U_i(s_{i,max})$, we have

$$\begin{aligned}
\frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) &= -\frac{\partial}{\partial c_i(s)} \int e dF_s^*(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s' \in S} \frac{\partial}{\partial c_i(s_{i,max})} V(s') q(s' | s_{i,max}, a_W) \\
&\leq -\frac{\partial}{\partial c_i(s)} \int e dF_s^*(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) \\
&\leq \frac{\delta(1-\lambda)}{1-\delta\lambda} \frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}),
\end{aligned}$$

where the final inequalty follows from the first part of this proof. This condition requires

$$\frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) = 0.$$

□

Proof of Proposition 4.

Part 1. Partial derivative: *Step 1. Show that $\frac{\partial}{\partial \pi} B(s) \leq 0$ and $\frac{\partial}{\partial \pi} \int e dF_s^*(e) \geq 0$ for all $s \in S$.* Defining $E(s) := 2 \int e dF_s^*(e)$, we can expand

$$E(s) = \left(\frac{(2\pi - 1)B(s)}{1-\delta\lambda} - \frac{2\pi - 1 - \rho}{2\pi - (1-\rho)} \mu + \mu \right) \times \left(1 - \frac{2\mu(1-\delta\lambda)}{(2\pi - (1-\rho))B(s)} \right).$$

This implies the partial derivative with respect π is

$$\begin{aligned}
\frac{\partial}{\partial \pi} E(s) = &\left(1 - \frac{2\mu(1-\delta\lambda)}{(2\pi - (1-\rho))B(s)} \right) \left(\frac{2B(s)}{1-\delta\lambda} + \frac{(2\pi-1)\frac{\partial}{\partial \pi} B(s)}{1-\delta\lambda} - \frac{4\mu\rho}{2\pi - (1-\rho))^2} \right) \\
&- \left(2\mu(2\mu\rho(\delta\lambda - 1) - (2\pi - 1)(2\pi - (1-\rho))B(s))(2B(s) + (2\pi - (1-\rho))B(s)) \right) \\
&\Big/ \left((2\pi - (1-\rho))^3 B(s)^2 \right) \qquad \text{(A22)}
\end{aligned}$$

Equation (A22) implies that $\frac{\partial}{\partial \pi} B(s) \geq 0$ is a sufficient condition[27] to show that $\frac{\partial}{\partial \pi} E(s) \geq 0$ for all $s \in S$ and for any transition function $q : S \times A \to S$.

Suppose there exists s and q such that $\frac{\partial}{\partial \pi} E(s) < 0$. By definition of $B(s)$, we can write

$$\frac{\partial}{\partial \pi} B(s) = \delta(1-\lambda) \sum_{s'} \frac{\partial}{\partial \pi} V(s') q(s' | s, a_U) - (1-\delta\lambda) \frac{\partial}{\partial \pi} W(s).$$

[27] See the supplementary files for the precise threshold that gives a necessary and sufficient condition in terms of parameters, which must be strictly negative.

The expression is decreasing in the partial derivative of $W(s)$ and increasing in the partial derivative of the expected ex-ante continuation value tomorrow from state s. Note that

$$\frac{\partial}{\partial \pi} W(s) = \frac{\delta(1-\theta)}{1-\delta\theta} \sum_{s'} \frac{\partial}{\partial \pi} V(s') q(s'|s, a_W)$$
$$\frac{\partial}{\partial \pi} U(s) = -\frac{\partial}{\partial \pi} E(s) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s'} \frac{\partial}{\partial \pi} V(s') q(s'|s, a_U).$$

Let $s_{min} := \arg\min_s \frac{\partial}{\partial \pi} B(s)$. Hence, because $-\frac{\partial}{\partial \pi} E(s) > 0$, the minimum value of $\frac{\partial}{\partial \pi} B(s_{min})$ cannot exceed the case where $V(s_{min}) = W(s_{min})$ and $q(s_{min}|s_{min}, a_W) = 1$, or equivalently $\frac{\partial}{\partial \pi} B(s_{min}) = 0$.

Therefore, since $\frac{\partial}{\partial \pi} E(s) < 0$ implies $\frac{\partial}{\partial \pi} B(s) \geq 0$, which in turn implies $\frac{\partial}{\partial \pi} E(s) \geq 0$, we can conclude $\frac{\partial}{\partial \pi} E(s) \geq 0$. By the same logic as above, replacing s_{min} for $s_{max} := \arg\max_s \frac{\partial}{\partial \pi} B(s)$ and solving, we have $\frac{\partial}{\partial \pi} B(s) \leq 0$ for all $s \in S$.

Step 2. Show that $\frac{\partial}{\partial \pi} V_i(s) \leq 0$ for all $s \in S$. Let $s_{i,max} = \arg\max_s \frac{\partial}{\partial \pi} V_i(s)$. If $V_i(s_{i,max}) = W_i(s_{i,max})$ then

$$\begin{aligned}\frac{\partial}{\partial \pi} W_i(s_{i,max}) &= \frac{\delta(1-\theta)}{1-\delta\theta} \sum_{s' \in S} \frac{\partial}{\partial \pi} V_i(s') q(s'|s_{i,max}, a_W) \\ &\leq \frac{\delta(1-\theta)}{1-\delta\theta} \frac{\partial}{\partial \pi} W_i(s_{i,max})\end{aligned}$$

which is only satisfied by $\frac{\partial}{\partial \pi} W_i(s_{i,max}) = 0$.

On the other hand, $V_i(s_{i,max}) = U_i(s_{i,max})$ implies

$$\begin{aligned}\frac{\partial}{\partial \pi} U_i(s_{i,max}) &= -\frac{\partial}{\partial \pi} E(s_{i,max}) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s' \in S} \frac{\partial}{\partial \pi} V_i(s') q(s'|s_{i,max}, a_U) \\ &\leq -\frac{\partial}{\partial \pi} E(s_{i,max}) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \frac{\partial}{\partial \pi} U_i(s_{i,max})\end{aligned}$$

which implies

$$\frac{\partial}{\partial \pi} U_i(s_{i,max}) \leq \frac{-(1-\delta\lambda)\frac{\partial}{\partial \pi} E(s_{i,max})}{1-\delta} \leq 0.$$

Part 2. Completely decisive contests: First note that if $U_i(s) < W_i(s)$ then the result is trivially satisfied. Suppose then that $U_i(s) \geq W_i(s)$.

Recall $U_i(s)$ can be expressed by the expected value of exerting zero effort

$$U_i(s) = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1-F_s^*(\mu))(1-\pi)\right]. \tag{A23}$$

Additionally, recall from Proposition A1 the definition of $F_s^*(\mu)$. Plugging into (A23) yields

$$U_i(s) = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[1-\pi + \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}\left(\frac{1}{2}(1-\rho)-(1-\pi)\right)\right]$$

and setting $\pi = 1$ yields $U_i(s) = W_i(s) + \frac{1-\rho}{1+\rho}\mu$, concluding the proof. □

Proof of Proposition 5. By Definition 1, $\bar{x}_1(s) > \bar{x}_2(s)$ implies that war is efficient. By equations (A6) and (A7), this is equivalent to

$$\frac{1}{1-\delta\lambda} - \frac{1}{1-\delta\theta} + \sum_{s'\in S} V(s')\left[\frac{\delta(1-\lambda)}{1-\delta\lambda}q(s'|s,a_U) - \frac{\delta(1-\theta)}{1-\delta\theta}q(s'|s,a_W)\right] + C(s) < 0,$$

yielding the desired equation by definition of G and $\Delta V(s)$. □

Proof of Corollary 1. Recall that efficient wars break out if and only if $\bar{x}_1(s) > \bar{x}_2(s)$, or equivalently

$$1-(1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s'} V(s')q(s'|s,a_U) < 0.$$

Plugging in for $W(s)$, this yields the following condition in terms of the total costs of war,

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'} V(s')q(s'|s,a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'} V(s')q(s'|s,a_U).$$

Letting $\lambda = 0$ and $\theta = 1$, the condition becomes

$$C(s) < \frac{\delta}{1-\delta} - \sum_{s'} V(s')q(s'|s,a_U).$$

Then, we can expand the expression to

$$C(s) < \frac{\delta}{1-\delta} - \sum_{s'}\left[1-\tau(s') + \delta\sum_{s''} V(s'')q(s''|p',a_U)\right]q(s'|s,a_U)$$

where $\tau(s) := 2\int edF_s^*(e)$ for all s such that $V(s) = U(s)$ and $\tau(s) := C(s)$ for all s such that $V(s) = W(s)$. Letting $\hat{\tau}_s := \sum_{s'} \tau(s')q(s'|s, a_U)$ denote the expected inefficiency upon returning to bargaining after cooperation in state s (across both diplomatic effort and costs of war depending on which arises), we can rewrite the condition as

$$C(s) < \frac{\delta}{1-\delta} + \hat{\tau}_s - 1 - \delta \sum_{s'} \sum_{s''} V(s'')q(s''|s', a_U)q(s'|s, a_U)$$

Choose any $\tau \in (0, \hat{\tau}_s)$. Then, we have a sufficient condition

$$\begin{aligned} C(s) &< \frac{\delta}{1-\delta} + \tau - 1 - \delta \sum_{s'} \sum_{s''} V(s'')q(s''|p', a_U)q(s'|s, a_U) \\ &< \frac{\delta}{1-\delta} + \tau - 1 - \delta \sum_{s'} \sum_{s''} \left[1 - \hat{\tau}_{s''} + \delta \sum_{s'''} V(s''')q(s'''|p', a_U) \right] q(s''|p', a_U)q(s'|s, a_U). \end{aligned}$$

Again choosing a $\tau \in (0, \hat{\tau}_{s''})$, we can write a sufficient condition as

$$C(s) < \frac{\delta}{1-\delta} + (\tau - 1)(1+\delta) - \delta^2 \sum_{s'} \sum_{s''} \sum_{s'''} V(s''')q(s'''|p', a_U)q(s''|p', a_U)q(s'|s, a_U).$$

Following this geometric series, war is directly preferred to cooperation if

$$C(s) < \frac{\delta + \tau - 1}{1-\delta} \tag{A24}$$

Because $\lim_{\delta \to 1^-} \frac{\delta+\tau-1}{1-\delta} = \tau\infty$ and $\tau > 0$, inequality (A24) holds for sufficiently large δ. □

Proof of Corollary 2. Recall from the proof of Corollary 1 that a general condition for efficient war is

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{\delta(1-\theta)}{1-\delta\theta} \sum_{s'} V(s')q(s'|s, a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s'} V(s')q(s'|s, a_U)$$

for any $s \in S$ and $q : S \times A \to S$.

If we know that q satisfies $\sum_{s'} V(s')q(s'|s, a_U) \geq \sum_{s'} V(s')q(s'|s, a_W)$ for all s, then we can deduce the following sufficient condition,

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \left[\frac{\delta(1-\theta)}{1-\delta\theta} - \frac{\delta(1-\lambda)}{1-\delta\lambda} \right] \sum_{s'} V(s')q(s'|s, a_W).$$

We know that $C(s) > 0$ and $\sum_{s'} V(s')q(s'|s, a_W) \leq \frac{1}{1-\delta}$ for all $s \in S$. When $\lambda > \theta$, the first component (G) is strictly negative and the second bracketed component is strictly positive. Then, a necessary condition for efficient war becomes

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{1}{1-\delta}\left[\frac{\delta(1-\theta)}{1-\delta\theta} - \frac{\delta(1-\lambda)}{1-\delta\lambda}\right] = 0,$$

which cannot hold. □

Proof of Proposition 6. Refer to the proof of Proposition A1. □

Proof of Proposition 7. Refer to the proofs of Propositions 3, 4, and 5, which consider $\mu > 0$. □

Proof of Proposition 8. By the definition of $\omega(s)$, we have

$$\frac{\partial}{\partial\pi}\omega(s) = \frac{-8\mu^2(1-\delta\lambda)^2}{(2\pi-(1-\rho))^2 B(s)^2}\left[\frac{2}{2\pi-(1-\rho)} + \frac{\frac{\partial}{\partial\pi}B(s)}{B(s)}\right]$$

For this condition to be negative, we require

$$\frac{\partial}{\partial\pi}B(s) \geq \frac{-2B(s)}{2\pi-(1-\rho)} := D_*$$

where D_* denotes the floor of $\frac{\partial}{\partial\pi}B(s)$ if the probability of war is to decrease in competition. Note that, for given any valid set of parameters $(\delta, \lambda, \theta, \rho, \mu)$, any transition function q, and any consequent bargaining surplus $B(s)$, it must be that $D_* < 0$.

Next, recall that by the definition of $B(s)$, we have

$$\begin{aligned}
\frac{\partial}{\partial\pi}B(s) &= \delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\pi}V(s')q(s'|s,a_U) - (1-\delta\lambda)\frac{\partial}{\partial\pi}W(s) \\
&= \sum_{s'}\frac{\partial}{\partial\pi}V(s')\left[\delta(1-\lambda)q(s'|s,a_U) - \frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s,a_W)\right] \\
&\geq \frac{\partial}{\partial\pi}V(s_{min})\delta(1-\lambda) - \frac{\partial}{\partial\pi}V(s_{max})\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}
\end{aligned}$$

where $s_{min} := \arg\min_s \frac{\partial}{\partial\pi}B(s)$ and $s_{max} := \arg\max_s \frac{\partial}{\partial\pi}B(s)$. Then, with $\lambda = 1$ this becomes a lower bound of $-\frac{\partial}{\partial\pi}V(s_{max})\frac{\delta(1-\theta)(1-\delta)}{1-\delta\theta}$. Recall by Proposition 4 that $\frac{\partial}{\partial\pi}V(s) \leq 0$ for all s and q and hence the lower bound on $\frac{\partial}{\partial\pi}B(s)$ is weakly positive when $\lambda = 1$. As a result, we know that $1 = \lambda \geq \theta$ implies $\frac{\partial}{\partial\pi}B(s) \geq 0$, which additionally implies that $\frac{\partial}{\partial\pi}B(s) \geq D_*$. □

C Robustness

C.1 Competitive Diplomacy as a Tullock Contest

In the main text, a country is recognized as proposer with probability $\pi > \frac{1}{2}$ when they exert greater effort in diplomacy. Work in economic theory has demonstrated that many contests are strategically equivalent (see Chowdhury and Sheremeta (2015), for example). In this section, I show that the core incentives that drive strategic behavior in the all-pay contest of the main text carries into any decisive Tullock contest.

In particular, consider instead that endogenous country efforts govern the likelihood of having the upper-hand in the bargaining game, so that the probability that country i recovers proposal power is π times i's share of total diplomatic efforts. Then, country i's payoff to exerting effort $e_i \geq 0$ when their adversary exerts $e^*_{-i} \geq 0$ becomes

$$W_i(s) + \xi_i(e_i, e^*_{-i}; d)\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} + \frac{(1 - \pi)B(s)}{1 - \delta\lambda} - e_i$$

where

$$\xi_i(e_i, e_{-i}; d) = \begin{cases} \frac{e_i^d}{e_i^d + e_{-i}^d} & \text{if } e_i + e_{-i} > 0 \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

for some decisiveness parameter $d \geq 0$. Importantly, note that the decisiveness parameter d refers to the decisiveness of the Tullock contest, not to be confused with the decisiveness parameter π refers to the decisiveness of performance on recognition of agenda setting power. Tullock contests are defined by d, but our parameter of interest in the main text is π.

The next proposition immediately follows Theorem 4.1 of Ewerhart (2015), with the only change being that competition is now over the long-run expected value of the bargaining surplus instead of a unit. The proof is rederived and stated for completeness.

Proposition A4. *In any equilibrium with $2 < d < \infty$, the support of the distribution of effort has zero as an accumulation point. The equilibrium is characterized by a sequence $e_1 > e_2 > \ldots > 0$*

with $\lim_{k\to\infty} e_k = 0$ *chosen with respective probabilities* f_1*,* f_2*,* $\ldots$ *such that* $\sum_k f_k = 1$*. Moreover,*

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_k \frac{f_k e_j^d}{e_j^d+e_k^d} - e_j = 0 \tag{A25}$$

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_k \frac{f_k d e_j^{d-1} e_k^d}{(e_j^d+e_k^d)^2} - 1 = 0 \tag{A26}$$

for any integer $j \geq 1$*,* $s \in S$*.*

Proof of Proposition A4. This proof is derivative of Ewerhart (2015) with the only change being that competition is over the long-run expected value of the bargaining surplus instead of a unit, and is restated here only for the sake of completeness.

Let $e_1 > e_2 > \ldots > e_L$ be the mass points of the equilibrium effort distribution, used with respective probabilities $f_1, \ldots, f_L$, $\sum_{k=1}^L f_k = 1$. Suppose for contradiction that zero is not an accumulation point of the support. From the first-order condition at e_L,

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^L \frac{f_k d e_L^{d-1} e_k^d}{(e_L^d+e_k^d)^2} = 1,$$

we recover

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^L \frac{2 f_k e_k^d e_L^d}{(e_L^d+e_k^d)^2} - e_L = \frac{(2-d)e_L}{d}.$$

But since $e_k \geq e_L$ for all $k = 1, \ldots, L$, it follows that there is an upper bound on the expected payoff of exerting effort e_L,

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^L \frac{f_k e_L^d}{e_L^d+e_k^d} - e_L \leq \frac{(2-d)e_L}{d}.$$

Therefore, $d > 2$ implies a negative expected payoff for any equilibrium level of effort $e_L > 0$, a contradiction.

To prove equation (A25), taking any index $j \geq 1$, letting $j \to \infty$, and subsequently exchanging the sum and the limit via Lebesgue's Dominated Convergence Theorem, we recover that the expected equilibrium payoff is zero,

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^\infty \frac{f_k e_j^d}{e_j^d+e_k^d} - e_j = 0,$$

completing the proof. □

While equation (A26) defines the first-order conditions, equation (A25) demonstrates that all decisive ($d > 2$) Tullock contests erode the net gain from being recognized as the agenda setter, which is increasing in π and results in complete depletion of the entire bargaining surplus at $\pi = 1$, consistent with behavior in the main text. Straightforwardly we can see that the results from the paper continue to apply, as costlier wars (which increase the bargaining surplus) continue to create costlier peace by establishing greater incentive to exert diplomatic effort, and the expected aggregate payoff in peace continues to be eroded by steeper competition such that in the limit it is equal to the expected aggregate payoff in war.

Additionally, because zero is necessarily an accumulation point, it must be the case that there continues to be a nonzero risk of delay in the equilibrium of any game with settlement frictions $\mu > 0$. This, however, relies on non-degenerate mixed strategies, and hence $d > 2$.

C.2 Competitive Diplomacy with Flexible Recognition Probabilities

In this section, I allow the recognition probabilities to vary by country and state. A country i exerting greater effort than their opponent, $e_i > e_{-i}$, in state $s \in S$ results in country i being recognized as the agenda setter with probability $\pi_i(s)$. Specifically, these can vary across countries such that $\pi_i(s) \neq \pi_{-i}(s)$ as well as across states such that $\pi_i(s) \neq \pi_{-i}(s')$ for $s \neq s'$. One reasonable setting would be that $\pi_i(s)$ is increasing in a country i's relative strength, although I impose no restrictions of this kind. To demonstrate the core similarities and differences, I focus on the case without frictions $\mu = 0$.

A natural expectation would be that a country's incentive to exert effort in competitive diplomacy depends on the advantage they have in the competition. On the contrary, I find that there is a unique equilibrium in which countries play the identical strategies. This is because country incentives to compete are determined by their net gain from being the greatest performer, which is proportional to $\pi_i(s) - (1 - \pi_{-i}(s)) = \pi_{-i}(s) - (1 - \pi_i(s))$ for both countries given any couple of $(\pi_i(s), \pi_{-i}(s))$. The following proposition characterizes the equilibrium.

Proposition A5 (Equilibrium with Flexible Recognition)**.** *There is a unique equilibrium to a flexible recognition game without frictions where, for all states s and transitions q, each country $i = 1, 2$ plays $\sigma_i^{rec}(s) = (a_i^*(s), e_i^{rec}(s), x_i^*(s), y_i^*(x; s))$ defined as follows.*

(i) Decisions $a_i^*(s)$, $x_i^*(s)$, and $y_i^*(x; s)$ are defined in Proposition A1.

(ii) Effort $e_i^{rec}(s)$ is drawn from the cumulative distribution

$$F_s^{rec}(e) = \frac{(1-\delta\lambda)e}{(\pi_1(s)+\pi_2(s)-1)B(s)}$$

with $F_s^{rec}(e) = 0$ for $e < 0$ and $F_s^{rec}(e) = 1$ for $e > \frac{(\pi_1(s)+\pi_2(s)-1)B(s)}{1-\delta\lambda}$ if $U_i(s) \geq W_i(s)$. Otherwise exert zero effort $e_i^{rec}(s) = 0$.

Proof of Proposition A5. The proof follows the structure of the proof of Proposition A1 with the imposition that $\mu = 0$. The derivation of $a^*(s)$, $x^*(s)$, and $y^*(x; s)$ are identical to the derivation from Proposition A1. Now, the expected net gain to country i from exerting effort $e \geq 0$ on diplomacy is

$$\frac{B(s)}{1-\delta\lambda}\Big[\pi_i(s)\Pr(e > e_{-i}) + (1-\pi_{-i}(s))\Pr(e_{-i} > e)\Big] - e. \tag{A27}$$

The net gain is zero when $B(s) = 0$, in which case neither country will be willing to spend a positive amount in equilibrium. Therefore, to understand strategies with nonzero amounts of effort, assume $B(s) > 0$.

There are still no pure strategies under the identical logic as before. I now look for a mixed strategy corresponding to state s given by c.d.f. F_s^{rec} that satisfies equation (A27) for both countries.

We can write the payoff from zero effort as

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}(1-\pi_{-i}(s)). \tag{A28}$$

On the other hand, exerting effort of $e > 0$ yields an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\Big[F_s^{rec}(e)\pi_i(s) + (1-F_s^{rec}(e))(1-\pi_{-i}(s))\Big] - e. \tag{A29}$$

Using equations (A28) and (A29), we can solve for

$$F_s^{rec}(e) = \frac{(1-\delta\lambda)e}{(\pi_1(s)+\pi_2(s)-1)B(s)}. \tag{A30}$$

which implies an upper bound on equilibrium effort of $\bar{e}_s = \frac{(\pi_1(s)+\pi_2(s)-1)B(s)}{1-\delta\lambda}$.

Uniqueness follows from the same logic as Proposition A2. Consider the case where a country exerts effort $e > \bar{e}_s$ with nonzero probability. If both countries are, there is a profitable deviation to zero effort. If only one country is, by deviating back to $\bar{e}_s$, their payoff will be strictly more as they continue to gain only $W_i(s) + \frac{\pi_i(s)B(s)}{1-\delta\lambda}$ but expend strictly less effort. Further, consider a strategy where zero effort is expended with nonzero probability. Then, the country must be indifferent between expending zero and expending nonzero amounts of effort. However, this would imply the country is always the worst performer in equilibrium, in which case there is a strictly profitable deviation to never expending effort, which cannot be the case in equilibrium.

Next, consider a strategy where there is an atom on a nonzero amount of effort. Then, there is a strictly positive deviation to marginal levels of effort that disproportionately increase their expected win probability. Lastly, to show that zero must be the lower bound of the support, suppose otherwise that one country i always exerts at least effort $e' > 0$. Then, their opponent $-i$ will never exert amounts between zero and e' as exerting zero is strictly better. However, this implies that country i has a strictly profitable deviation to reducing effort levels below e' without altering the probability of being the stronger performer.

As before, equilibrium offers are given by equations (A8) and (A9), and a country receiving an offer rejects offers in their rejection set and accepts all others. □

The result is especially interesting because both countries play the same mixed strategy in equilibrium, which is counterintuitive given they have different absolute values for being the greatest performer. This is in contrast to other contests with different valuations (e.g., Hillman and Riley (1989)), where countries with a lower valuation tend to exert less effort on average. The reason for the difference is that, while their absolute values for being the greatest performer is different, these are offset by their differing absolute values for being the weakest performer. Consequently, regardless of how much larger one country's advantage is over the other, their expected net gain from becoming the agenda setter is equal to $\pi_1(s) + \pi_2(s) - 1$ times the expected value of recovering the bargaining surplus in the current period.

Appendix References

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