Symmetry breaking in competing single-well linear-nonlinear potentials

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Bannoe, Magnitogorsk

Setup

One-dimentional Gross-Pitaevskii equation:

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - P(x)|\Psi|^2\Psi, \quad V(x), P(x) \in \mathbb{R}.$$

Optics: implanting nonlinearity-inducing dopants[1].
BEC: locally applied Feshbach resonance[2].

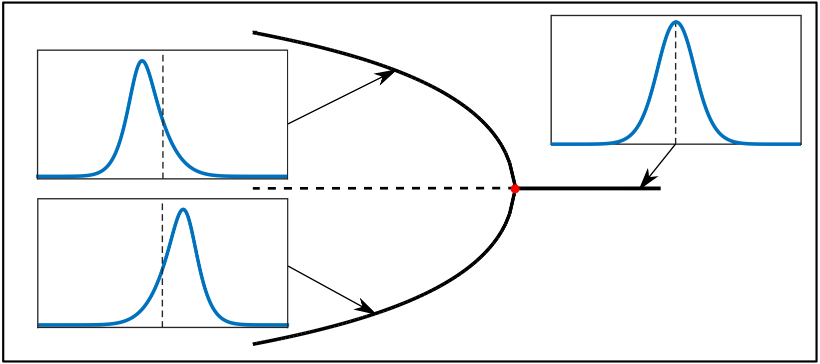
- $V(x)$ – linear potential;
- $P(x)$ – nonlinear (pseudo) potential.

Stationary localized nonlinear modes: $\left\| \begin{array}{l} \Psi(t,x) = e^{-i\mu x} u(x) \\ \lim\limits_{x\to\infty} u(x) = 0 \end{array} \right.$

[1] J. Hukriede, D. Runde, and D. Kip, Phys. D **36**, R1-R16 (2003)
[2] D. M. Bauer, M. Lettner, C. Vo, G. Rempe and S. Dürr, Nature Phys. 5, 339-342 (2009)

SSB

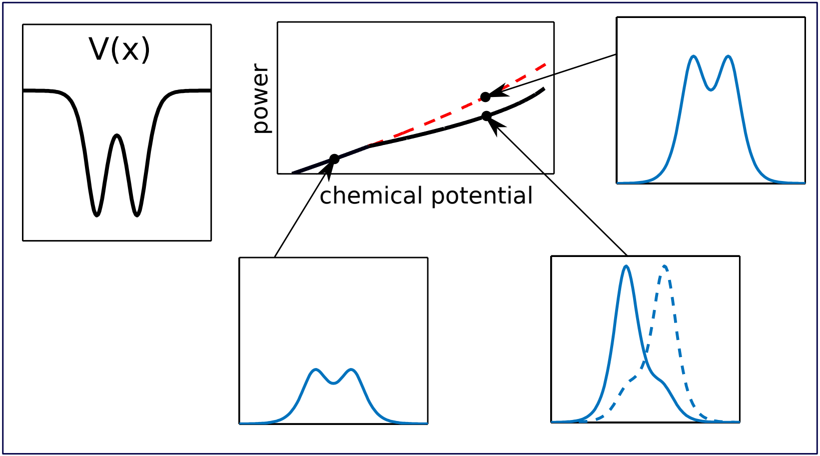
Spontaneous symmetry breaking (SSB) — type of bifurcation when one symmetric solution loses its stability and two stable asymmetric solutions arise.



SSB: examples (1)

GNLS (Generalized NLS), $V(x)$ – double-well[3]:

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - |\Psi|^2\Psi + 0.25|\Psi|^4\Psi.$$

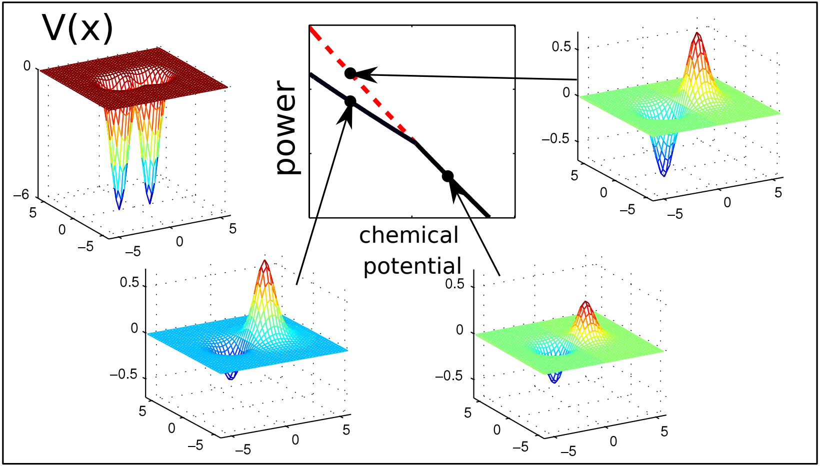


[3] Jianke Yang, Phys. D **244**, 50-67 (2013)

SSB: examples (2)

2D GNLS, $V(x, y)$ – double-well[3]:

$$i\Psi_t = -\Psi_{xx} - \Psi_{yy} + V(x, y)\Psi + |\Psi|^2\Psi.$$

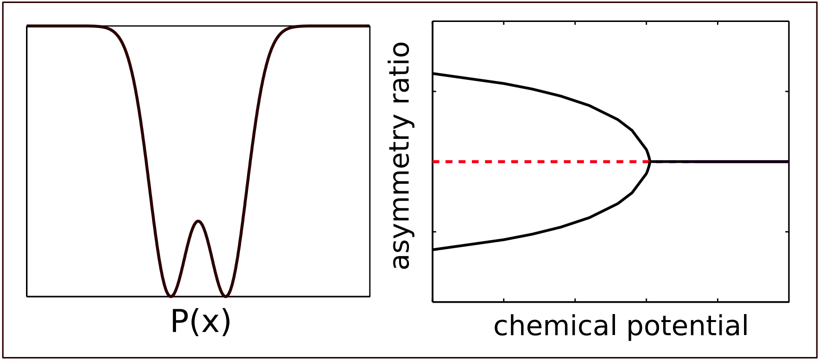


[3] Jianke Yang, Phys. D **244**, 50-67 (2013)

SSB: examples (3)

NLS, $V(x) = 0$, $P(x)$ – double-well[4]:

$$i\Psi_t = \Psi_{xx} + P(x)|\Psi|^2\Psi.$$

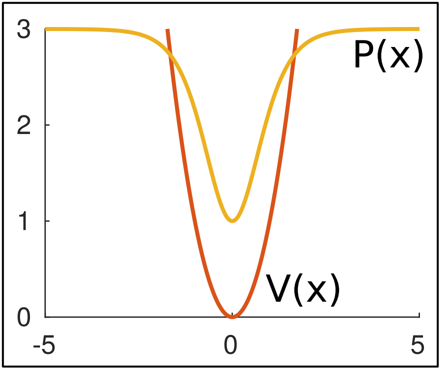


[4] T. Mayteevarunyoo, B. A. Malomed, and G. Dong, Phys. Rev. A **78**, 053601 (2008)

SSB without double-well structure?

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - P(x)|\Psi|^2\Psi;$$

$$V(x) = \frac{1}{2}\omega^2 x^2, \qquad P(x) = 1 + A\tanh^2 x.$$

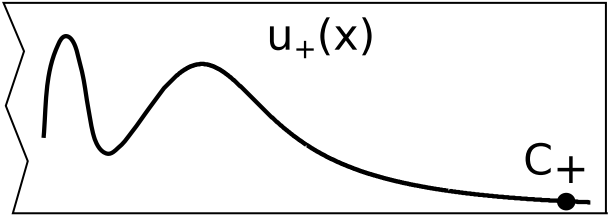


Interplay between linear and nonlinear potentials?

(γ_+, γ_-) diagrams[5]

$$u_{xx} + (\mu - V(x))u + P(x)u^3 = 0; \qquad (1)$$

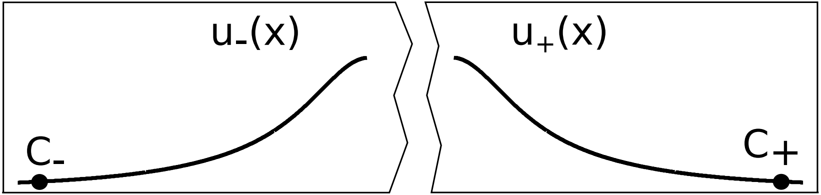
$$S_+ = \{u(x) | \lim_{x \to +\infty} u(x) = 0\}; \quad u(x) \sim C_+ x^{\frac{1}{2}(\mu-1)} e^{-\frac{\omega^2 x^2}{4}}.$$



[5] G. L. Alfimov and D. A. Zezyulin, Nonlinearity **20**, 2075–2092 (2007)

(γ_+, γ_-) diagrams

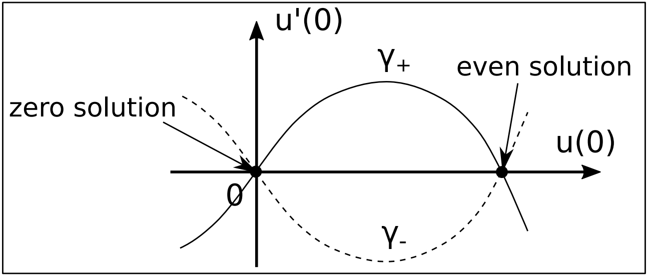
$$S_- = \{u(x)| \lim_{x \to -\infty} u(x) = 0\}; \quad u(x) \sim C_-(-x)^{\frac{1}{2}(\mu-1)} e^{-\frac{\omega^2 x^2}{4}}$$



$$u(x) - \text{localized} \quad \leftrightarrow \quad u(x) \in S_+ \cap S_-;$$
$$u(0) = u_+(0; C_+) = u_-(0; C_-), \quad u'(0) = u'_+(0; C_+) = u'_-(0; C_-).$$

(γ_+, γ_-) diagrams

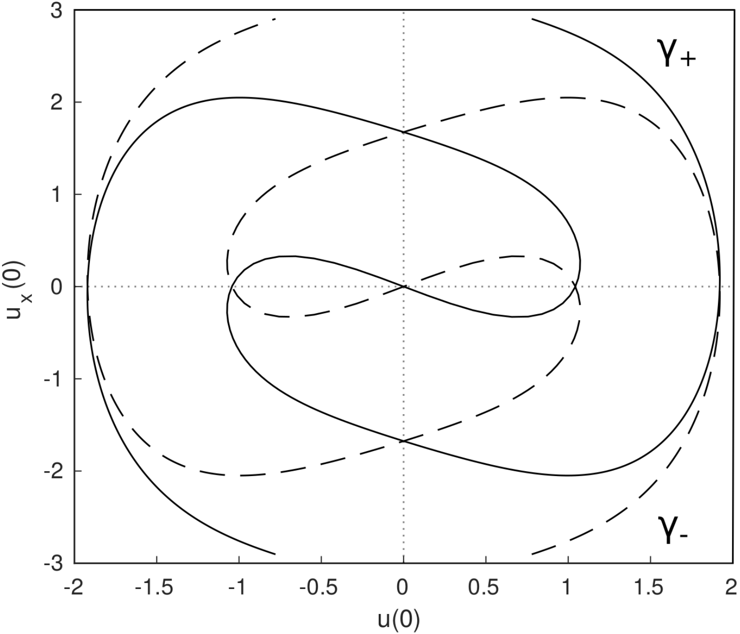
$$\gamma_+ = \{(u_+(0; C_+); u'_+(0; C_+)), \quad C_+ \in [-\tilde{C}_+, \tilde{C}_+]\};$$
$$\gamma_- = \{(u_-(0; C_-); u'_-(0; C_-)), \quad C_- \in [-\tilde{C}_-, \tilde{C}_-]\}$$



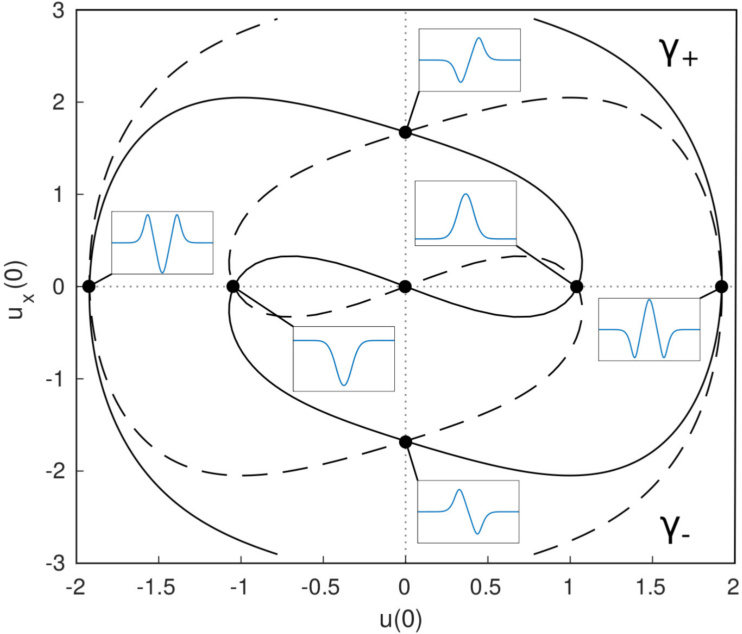
Properties:

- each point of intersection $\gamma_+ \cap \gamma_-$ corresponds to a solution;
- symmetry of equation $x \to -x$ leads to a symmetry of $\gamma_\pm$ curves about u' axis;
- intersections of $\gamma_\pm$ with u, u' axes correspond to even and add localized modes.

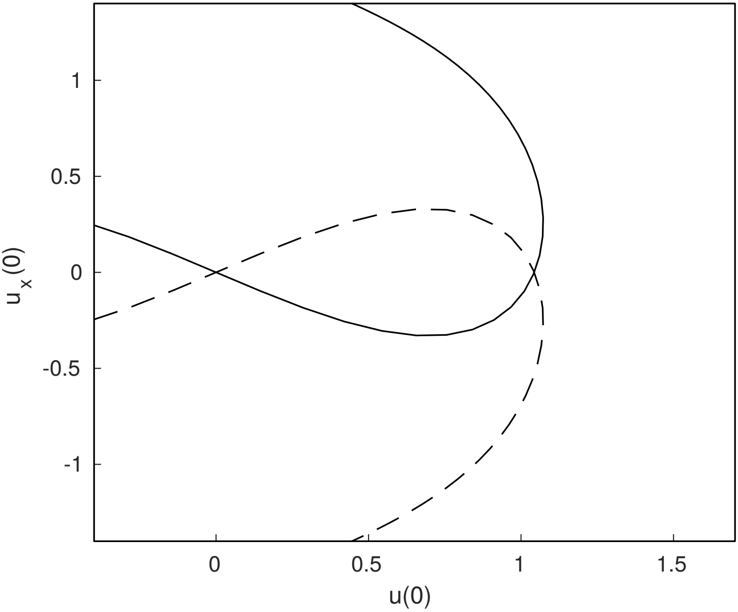
Application: $\mu = 0$, $A = 2$



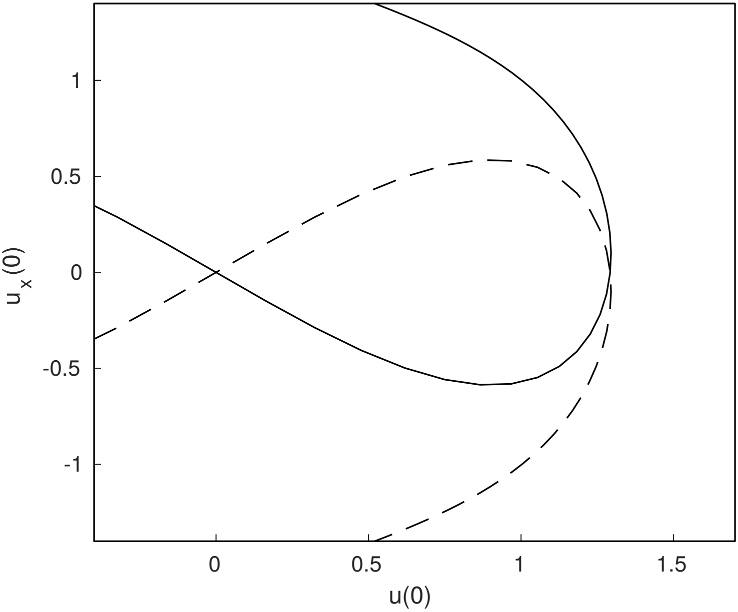
Application: $\mu = 0$, $A = 2$



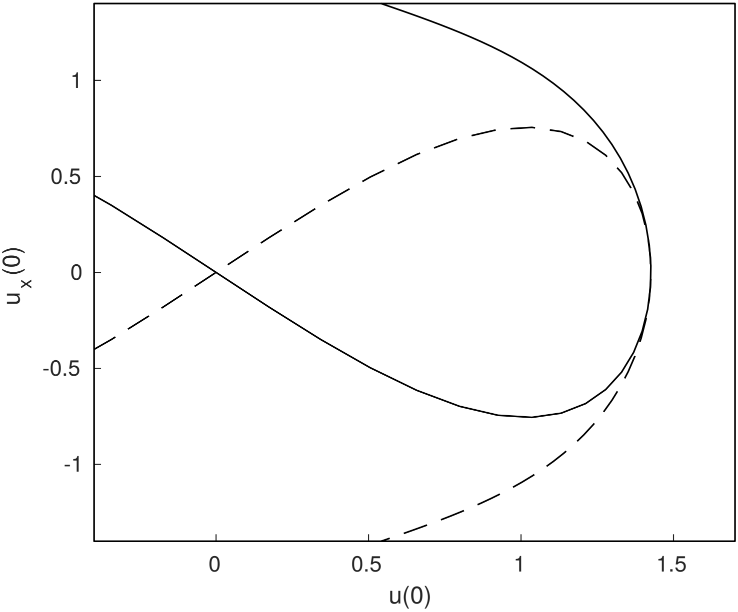
Application: $\mu = 0$, $A = 2$



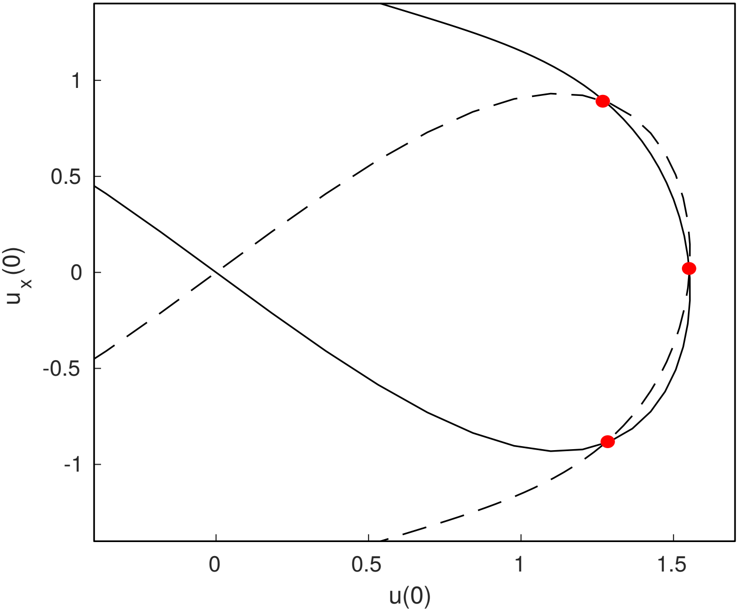
Application: $\mu = -0.5$, $A = 2$



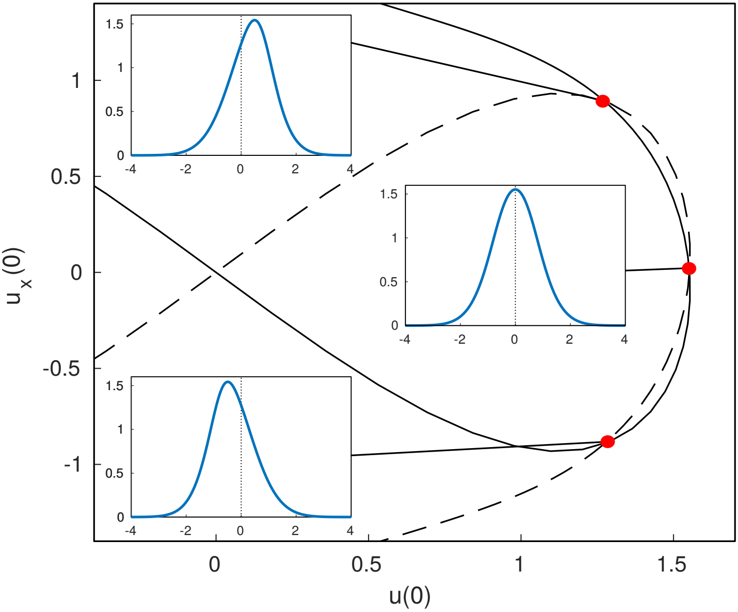
Application: $\mu = -0.8$, $A = 2$



Application: $\mu = -1.1$, $A = 2$



SSB bifurcation?

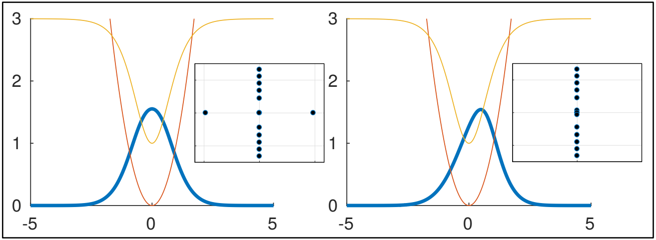


Stability

Considering eqigenvalue problem for operator[5]:

$$\mathcal{L} = i \begin{pmatrix} 0 & \mathcal{L}_- \\ \mathcal{L}_+ & 0 \end{pmatrix} \tag{2}$$

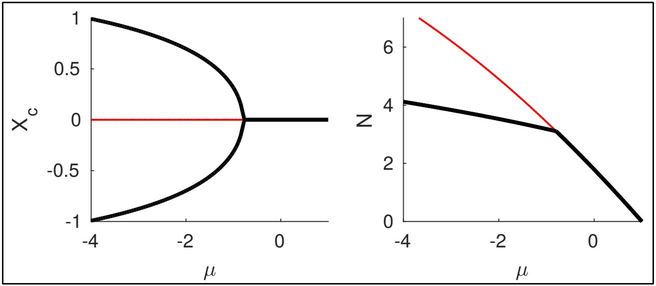
where $\mathcal{L}_\pm = \dfrac{d^2}{dx^2} + \mu - \dfrac{1}{2}\omega^2 x^2 + +(2 \pm 1)P(x)u^2$.



[6] Jianke Yang, Nonlinear Waves in Integrable and Nonintegrable Systems, Society of Industrial and Applied Mathematics (2010)

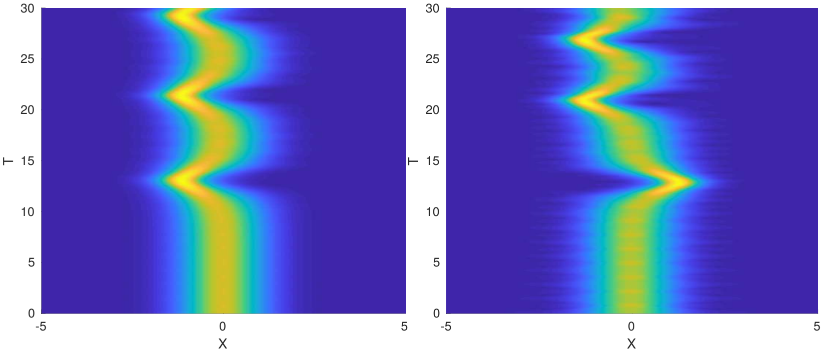
SSB bifurcation

$$N = \int_{-\infty}^{+\infty} u^2(x)dx, \quad X_c = N^{-1} \int_{\infty}^{+\infty} xu^2(x)fx$$



Evolution of unstable symmetric mode

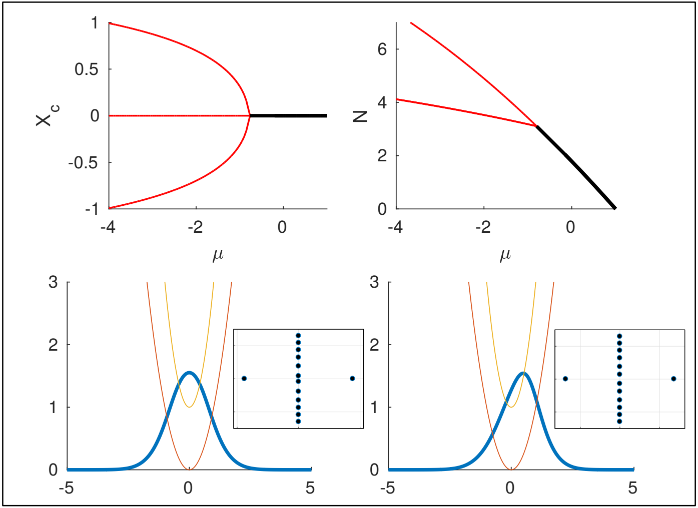
We use conservative Trofimov-Peskov finite-difference scheme[7] to calculate time evolution of the nonlinear modes.



[7] V. A. Trofimov, N. V. Peskov, Mathematical Modelling and Analysis, Volume 14 Number 1, 2009, pages 109-126

Unbounded nonlinear potential

$$V(x) = \frac{1}{2}\omega^2 x^2; \quad P(x) = 1 + Ax^2$$



Conclusion

- GPE equation with signle-well linear and nonlinear potentials was considered:

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - P(x)|\Psi|^2\Psi, \quad V(x), P(x) \in \mathbb{R}.$$

- Method of (γ_+, γ_-) diagrams helps us to understand the structure of nonlinear modes for this case.
- SSB bifurcation was found.

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The combination of linear and nonlinear potentials, both shaped as a single well, enables competition between the confinement and expulsion induced by the former and latter potentials, respectively. We demonstrate that this setting leads to spontaneous symmetry breaking (SSB) of the ground state in the respective generalized nonlinear Schrödinger (Gross–Pitaevskii) equation, through a spontaneous off-center shift of the trapped mode. Two different SSB bifurcation scenarios are possible, depending on the shape of the nonlinearity-modulation profile, which determines the nonlinear potential. If the profile is bounded (remaining finite at $|x| \to \infty$), at a critical value of the integral norm the spatially symmetric state loses its stability, giving rise to a pair of mutually symmetric stable asymmetric ones via a direct pitchfork bifurcation. On the other hand, if the nonlinear potential is unbounded, two unstable asymmetric modes merge into the symmetric metastable mode and destabilize it via an inverted pitchfork bifurcation. Parallel to a systematic numerical investigation, basic results are obtained in an analytical form. The settings can be realized in Bose–Einstein condensates and nonlinear optical waveguides.

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[8]D. A. Zezyulin, M. E. Lebedev, G. L. Alfimov, and Boris A. Malomed, Phys. Rev. E **98**, 042209 (2018)

Thanks for your attention!