Symmetry breaking in competing single-well linear-nonlinear potentials

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Setup

One-dimentional Gross-Pitaevskii equation:

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - P(x)|\Psi|^2\Psi, \quad V(x), P(x) \in \mathbb{R}.$$

Optics: implanting nonlinearity-inducing dopants[1].
BEC: locally applied Feshbach resonance[2].

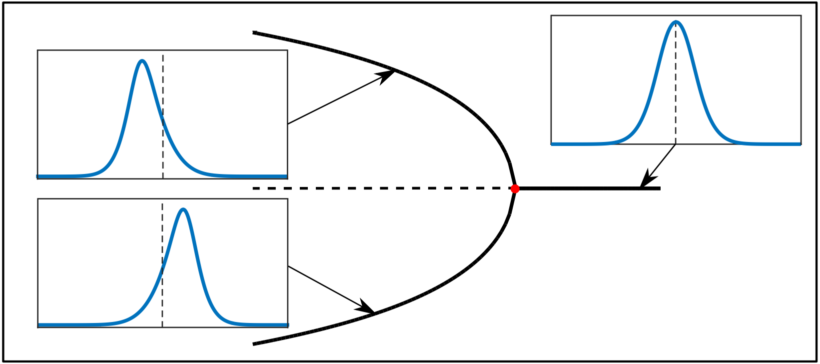
- $V(x)$ – linear potential;
- $P(x)$ – nonlinear (pseudo) potential.

Stationary localized nonlinear modes: $\left\| \begin{array}{l} \Psi(t,x) = e^{-i\mu x} u(x) \\ \lim\limits_{x\to\infty} u(x) = 0 \end{array} \right.$

[1] J. Hukriede, D. Runde, and D. Kip, Phys. D **36**, R1-R16 (2003)
[2] D. M. Bauer, M. Lettner, C. Vo, G. Rempe and S. Dürr, Nature Phys. 5, 339-342 (2009)

SSB

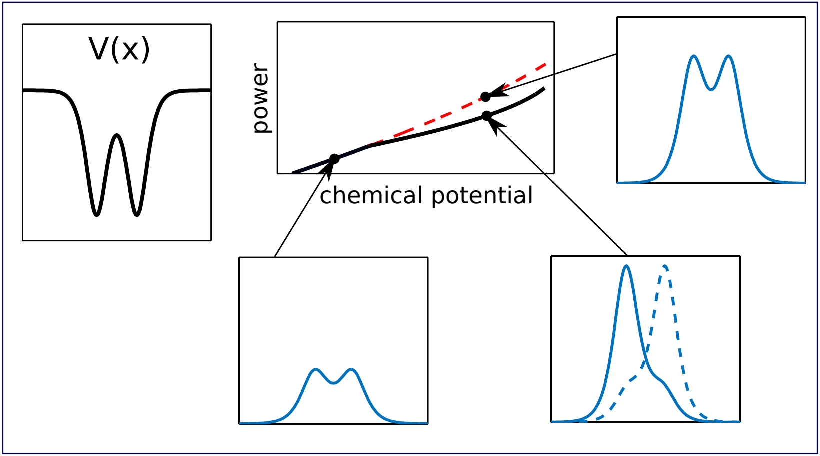
Spontaneous symmetry breaking (SSB) — type of bifurcation when one symmetric solution loses its stability and two stable asymmetric solutions arise.



SSB: examples (1)

GNLS (Generalized NLS), $V(x)$ – double-well[3]:

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - |\Psi|^2\Psi + 0.25|\Psi|^4\Psi.$$

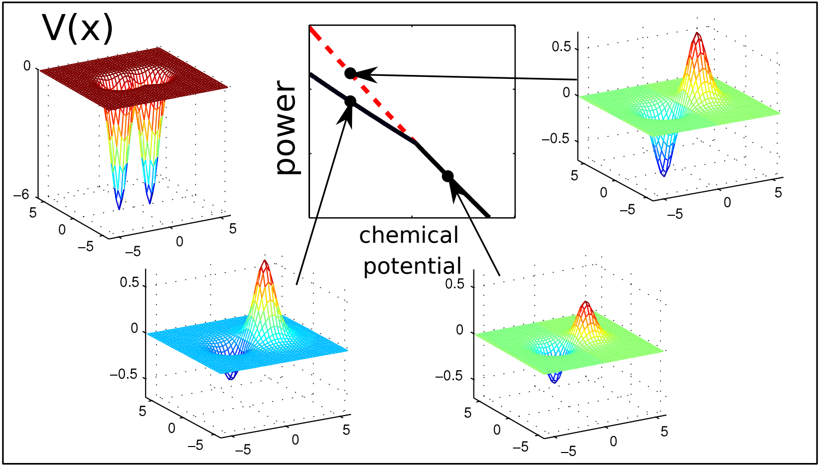


[3] Jianke Yang, Phys. D **244**, 50-67 (2013)

SSB: examples (2)

2D GNLS, $V(x,y)$ – double-well[3]:

$$i\Psi_t = -\Psi_{xx} - \Psi_{yy} + V(x,y)\Psi + |\Psi|^2\Psi.$$

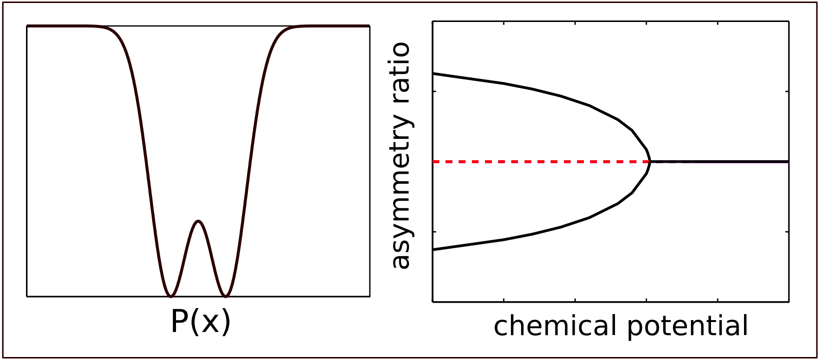


[3] Jianke Yang, Phys. D **244**, 50-67 (2013)

SSB: examples (3)

NLS, $V(x) = 0$, $P(x)$ – double-well[4]:

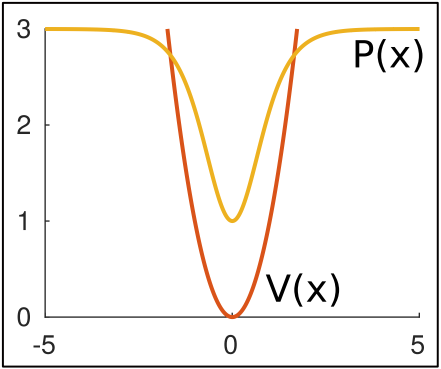
$$i\Psi_t = \Psi_{xx} + P(x)|\Psi|^2\Psi.$$



[4] T. Mayteevarunyoo, B. A. Malomed, and G. Dong, Phys. Rev. A **78**, 053601 (2008)

SSB without double-well structure?

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - P(x)|\Psi|^2\Psi;$$
$$V(x) = \frac{1}{2}\omega^2 x^2, \qquad P(x) = 1 + A\tanh^2 x.$$

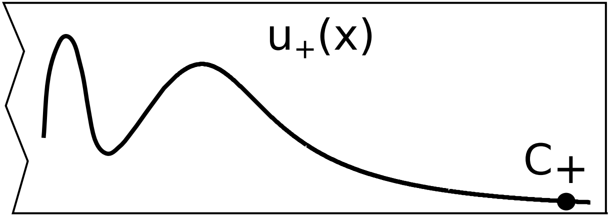


Interplay between linear and nonlinear potentials?

(γ_+, γ_-) diagrams[5]

$$u_{xx} + (\mu - V(x))u + P(x)u^3 = 0; \tag{1}$$

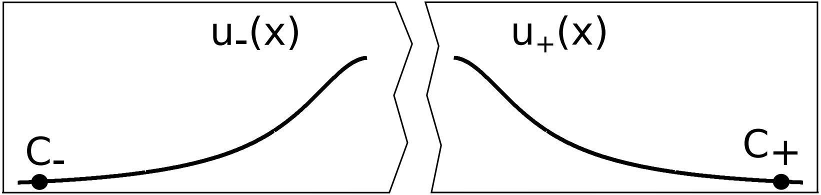
$$S_+ = \{u(x) | \lim_{x\to+\infty} u(x) = 0\}; \quad u(x) \sim C_+ x^{\frac{1}{2}(\mu-1)} e^{-\frac{\omega^2 x^2}{4}}.$$



[5] G. L. Alfimov and D. A. Zezyulin, Nonlinearity **20**, 2075–2092 (2007)

(γ_+, γ_-) diagrams

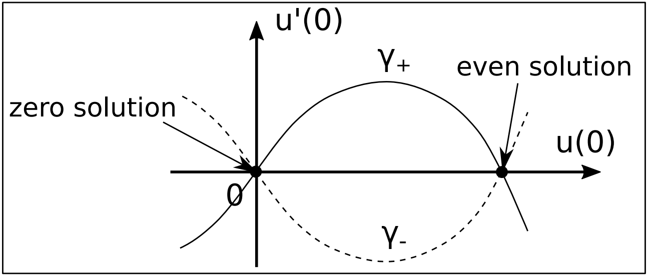
$$S_- = \{u(x)| \lim_{x\to-\infty} u(x) = 0\}; \quad u(x) \sim C_-(-x)^{\frac{1}{2}(\mu-1)} e^{-\frac{\omega^2 x^2}{4}}$$



$$u(x) \text{ -- localized} \quad \leftrightarrow \quad u(x) \in S_+ \cap S_-;$$
$$u(0) = u_+(0; C_+) = u_-(0; C_-), \quad u'(0) = u'_+(0; C_+) = u'_-(0; C_-).$$

(γ_+, γ_-) diagrams

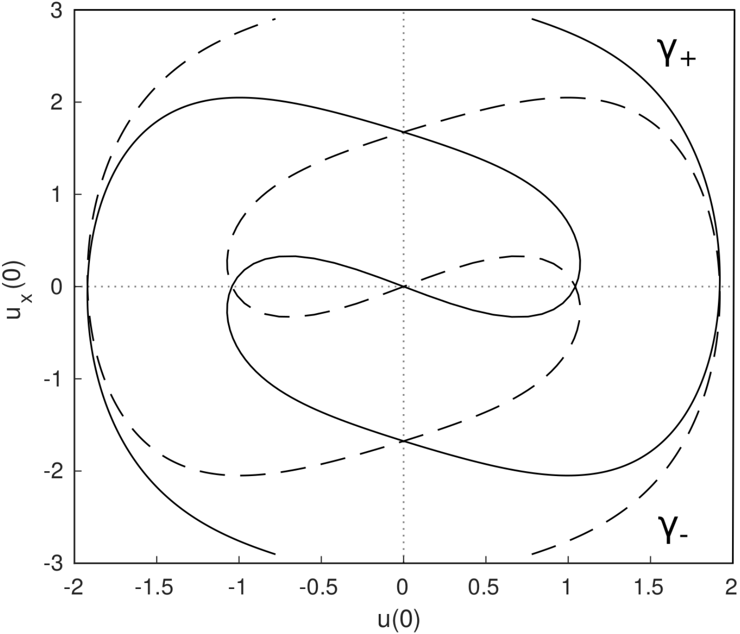
$$\gamma_+ = \{(u_+(0; C_+); u'_+(0; C_+)), \quad C_+ \in [-\tilde{C}_+, \tilde{C}_+]\};$$
$$\gamma_- = \{(u_-(0; C_-); u'_-(0; C_-)), \quad C_- \in [-\tilde{C}_-, \tilde{C}_-]\}$$



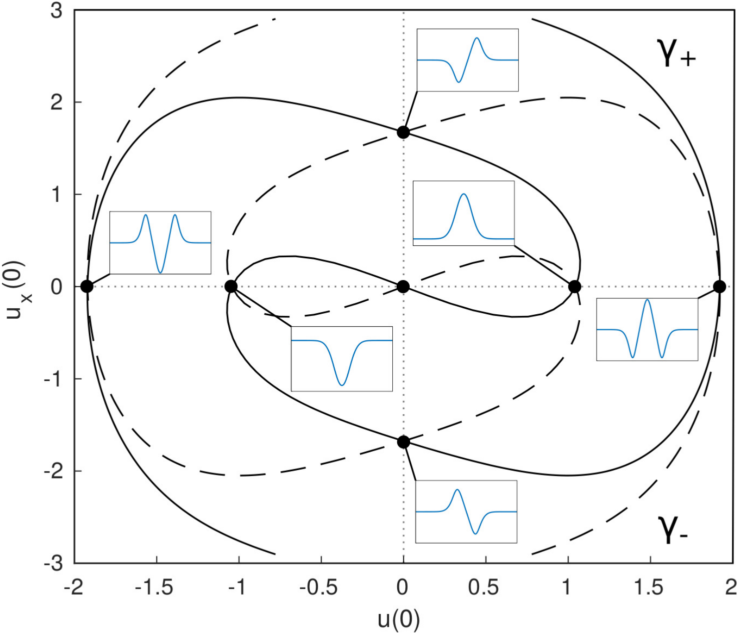
Properties:

- each point of intersection $\gamma_+ \cap \gamma_-$ corresponds to a solution;
- symmetry of equation $x \to -x$ leads to a symmetry of $\gamma_\pm$ curves about u' axis;
- intersections of $\gamma_\pm$ with u, u' axes correspond to even and add localized modes.

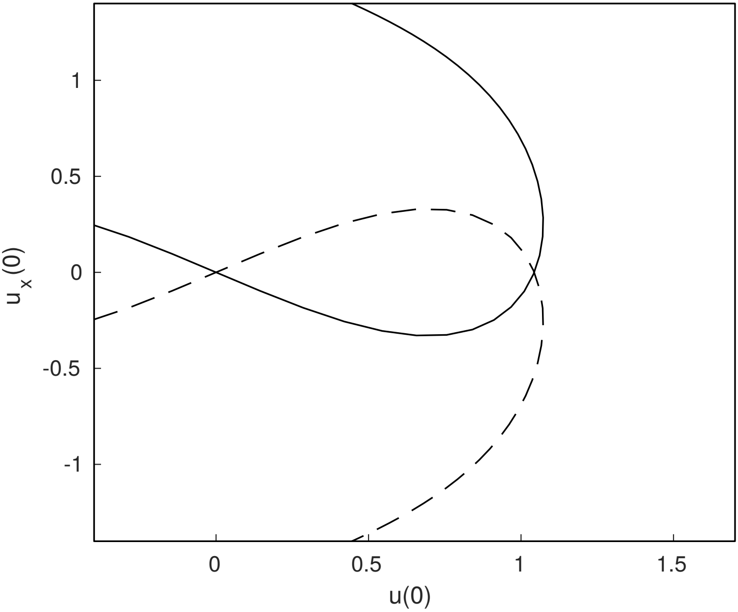
Application: $\mu = 0$, $A = 2$



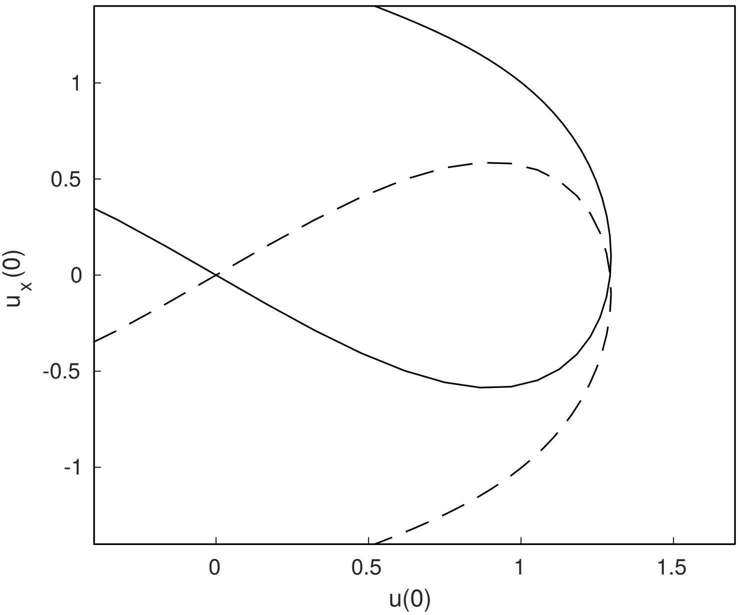
Application: $\mu = 0$, $A = 2$



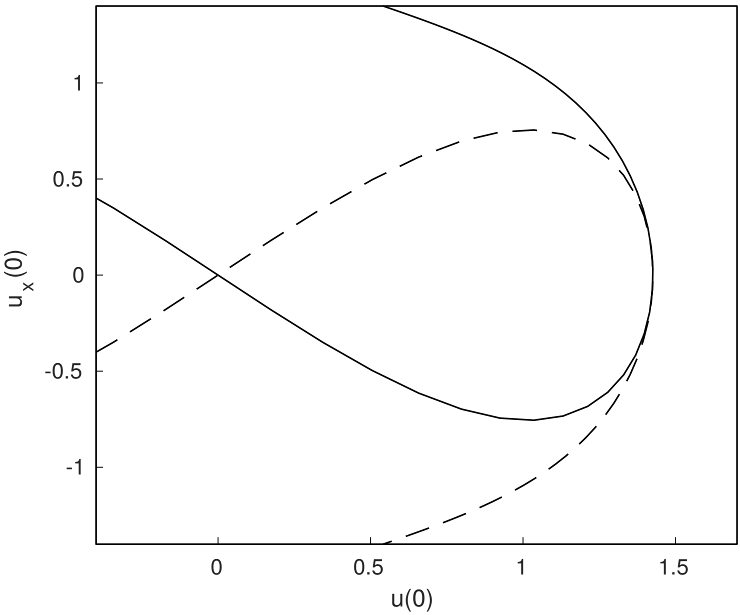
Application: $\mu = 0$, $A = 2$



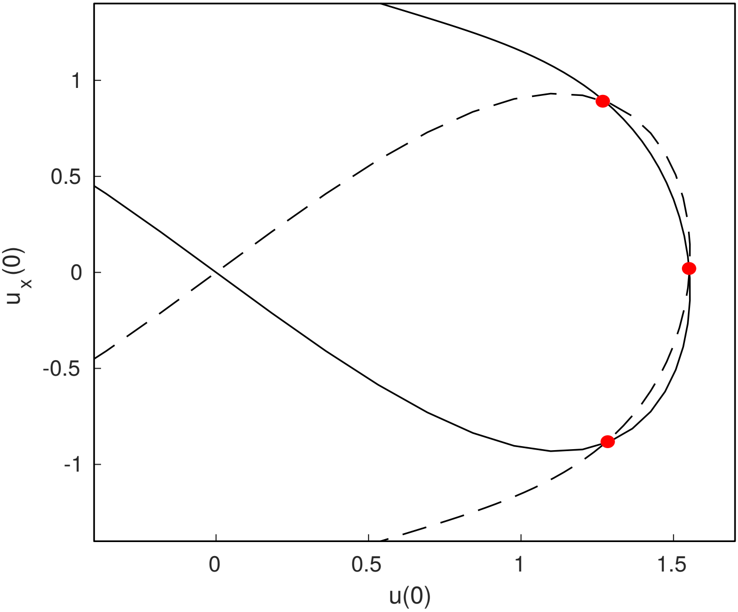
Application: $\mu = -0.5$, $A = 2$



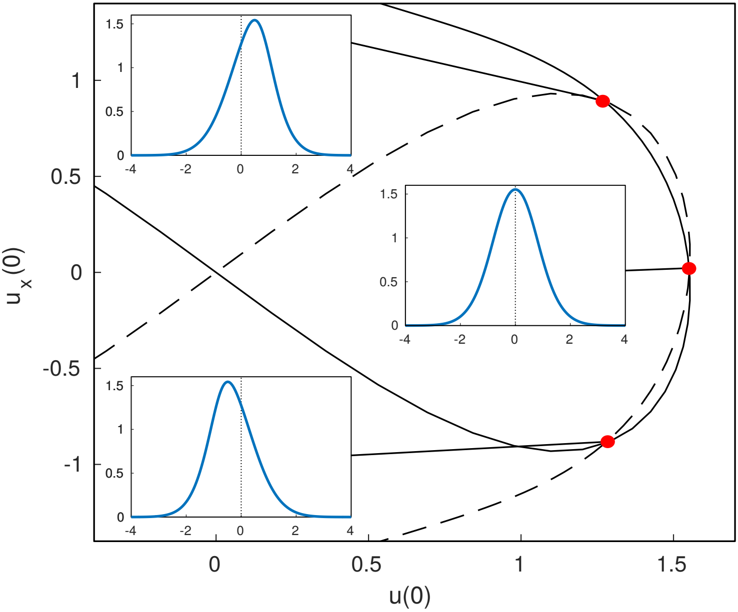
Application: $\mu = -0.8$, $A = 2$



Application: $\mu = -1.1$, $A = 2$



SSB bifurcation?

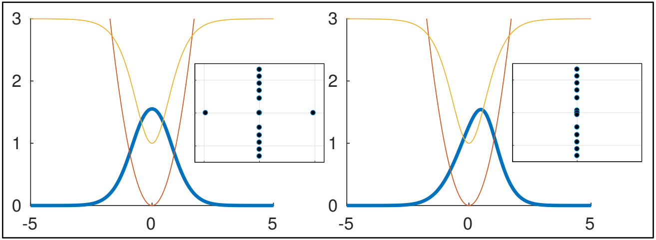


Stability

Considering eqigenvalue problem for operator[5]:

$$\mathcal{L} = i \begin{pmatrix} 0 & \mathcal{L}_- \\ \mathcal{L}_+ & 0 \end{pmatrix} \tag{2}$$

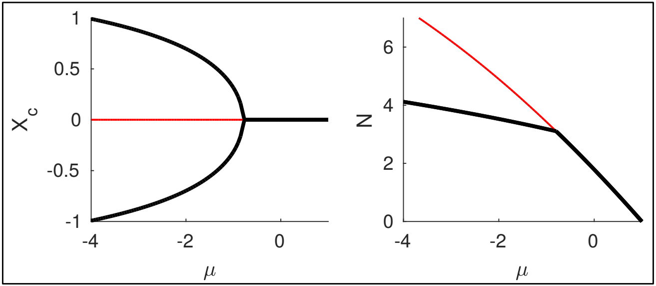
where $\mathcal{L}_\pm = \dfrac{d^2}{dx^2} + \mu - \dfrac{1}{2}\omega^2 x^2 + +(2 \pm 1)P(x)u^2$.



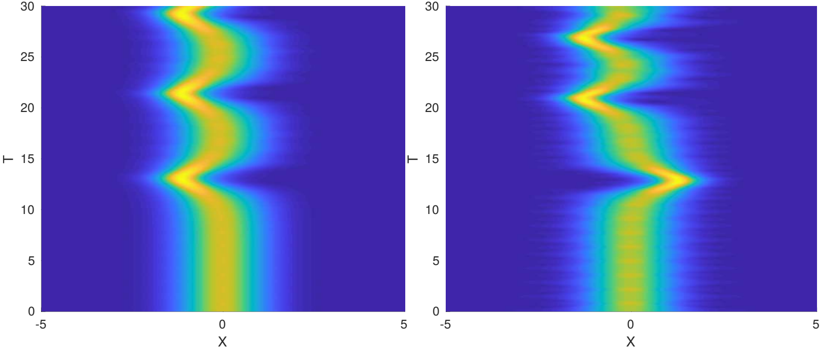
[6] Jianke Yang, Nonlinear Waves in Integrable and Nonintegrable Systems, Society of Industrial and Applied Mathematics (2010)

SSB bifurcation

$$N = \int_{-\infty}^{+\infty} u^2(x)dx, \quad X_c = N^{-1} \int_{\infty}^{+\infty} xu^2(x)fx$$

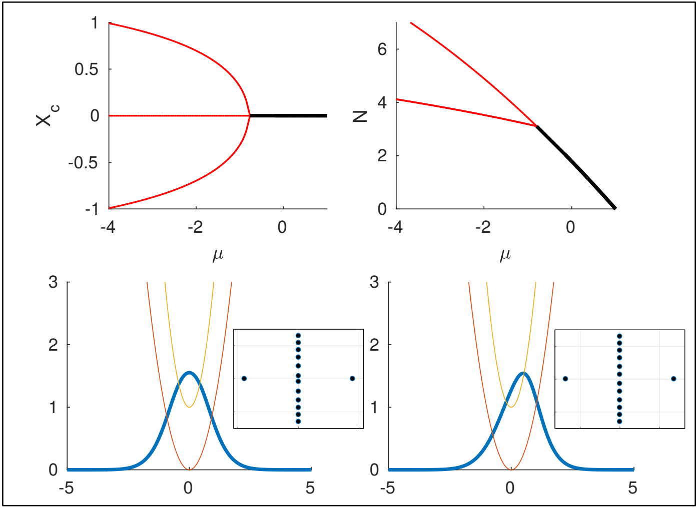


Evolution of unstable symmetric mode



Unbounded nonlinear potential

$$V(x) = \frac{1}{2}\omega^2 x^2; \quad P(x) = 1 + Ax^2$$



Conclusion

- GPE equation with signle-well linear and nonlinear potentials was considered:

$$i\Psi_t = -\Psi_{xx} + V(x)\Psi - P(x)|\Psi|^2\Psi, \quad V(x), P(x) \in \mathbb{R}.$$

- Method of (γ_+, γ_-) diagrams helps us to understand the structure of nonlinear modes for this case.
- SSB bifurcation was found.

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[7] D. A. Zezyulin, M. E. Lebedev, G. L. Alfimov, and Boris A. Malomed, Phys. Rev. E **98**, 042209 (2018)

Thanks for your attention!