QUESTIONS

Question. Every Riemannian isometry is a distance isometry. Is every distance isometry also a Riemannian isometry?

Question. The differential of a function is a covector field. Is every covector field the differential of some function?

Answer. No. Consider $\omega = xdx + xdy$.