Explaining known past routes and the robustness of cost functions

Joseph Lewis

Department of Archaeology, University of Cambridge, Cambridge, UK

Jl2094@cam.ac.uk

0000-0002-0477-1756

Keywords: least-cost path analysis; cost functions; routes; explanation; movement; postdiction

"Our truth is the intersection of independent lies" – Levins (1966, p. 423)

Abstract

Explaining material traces of movement as proxies for past movement is fundamental for explaining the processes behind why people in the past traversed the landscape in the way that they did. For this, least-cost path analysis and the use of cost functions have become commonplace. Despite its importance, current approaches misrepresent what the models we use to measure cost are, the relationship between the models and the theories that they aim to represent, and the role that models have in explanation. As a result, conclusions drawn these models are often theoretically weak and unjustified. Here, I argue that the focus should shift from individual cost functions used when explaining known past routes to the use of a collection of cost functions all sharing the same theory. Using robustness analysis, I demonstrate that cost functions quantifying cost in terms of time and energy are robust. Thus, increased support can be placed on these cost functions – when modelled collectively with a shared theory – as connecting the theories that they aim to represent with the results obtained.

Introduction

Explaining the rationale behind past movement is often limited to explaining material traces of past movement. From roads to footpaths to trails, the material traces of past movement are thought to have preserved the decision-making processes of people when traversing the landscape in the past (Snead et al. 2009). Without direct records of these processes, it is by explaining past routes that we can aim to uncover why people in the past moved where they did. For proposed explanations to be valid, it is necessary that the approaches used are robust and sufficiently reflect the theories that they aim to represent.

A common method for explaining known past routes – termed more generally as 'postdiction' or 'retrodiction' (Arnoldus-Huyzendveld, et al. 2016) – is least-cost path (LCP) analysis (Verhagen et al. 2019). Based on the *principle of minimum effort* as proposed by Zipf (1949), it is thought that humans will economise their behaviour when conducting a task. In terms of traversing the landscape, it is therefore assumed that humans will choose a path, i.e. an LCP, that minimises an accumulated cost, with time taken or energy expended being frequently used (Herzog 2010). This cost is represented via a cost function, which is a mathematical function that expresses the cost of traversing a specific slope gradient (Herzog 2010). From here, a cost surface quantifying the ease/difficulty of crossing between individual cells in a raster grid can be created. Finally, using the cost surface, a least-cost path can be calculated from a chosen origin and destination point.

When aiming to explain known past routes using LCP analysis, it is common for multiple LCPs, each derived using a different cost function, to be assessed for their similarity to a known route. This is done on the basis that similarity between the modelled and known route can be used to infer that the proposed cost function reflects the process(es) that resulted in the known route. For example, Güimil-Fariña and Parcero-Oubiña (2015) evaluated three different cost functions in their ability to explain a known Roman road in Spain, finding that the time-based Tober's Hiking Function (1993) and the energy-based cost function as proposed by Llobera and Sluckin (2007) produced adequate results. This was further corroborated by Fonte et al. (2017) when aiming to explain additional Roman roads in Spain. Similarly, Herzog (2010) compared least-cost paths derived using six different cost functions – with three being time-based and three being energy-based – to an ancient route in Germany. The study was later expanded to include additional roads and another cost function (Herzog 2022). Lastly, Herzog (2020) aimed to identify the factors governing the construction of Roman roads in Germany using four cost functions: two time-based, one energy-based, and the other wheel-based. It is also worthwhile to note Field et al. (2022), in which multiple cost functions of the same type, that is time-based or energy-based, were used concurrently as a way to "better capture potential differences between time- and energy-based LCPs and because not all time-derived LCPs (nor all energy-derived LCPs) are anticipated to travel the same route". However, with this study focused on exploration not explanation, as evidenced by the lack of validation against a known past route, the efficacy of using multiple cost functions and explanation remains unaddressed.

Whilst the comparison of multiple cost functions as outlined above can be used to assess which cost function best fits known past routes and thus provides a potential explanation, this approach misrepresents what cost functions are, their relationship to the theories they aim to represent, and their role in explanation. The consequence of this current thinking is summarised by Canosa-Betés (2016, p. 419): "There have been some comparative studies of these and other cost functions […] but the archaeological community has not opted for one in particular yet. For this reason we decided to make the analysis with more than one function [with one being time-based and two being energy-based]".

If the issue of misrepresenting cost functions continues, particularly with regards to the seeking and choosing of a single 'best' cost function, the use of cost functions as support for a proposed theory when aiming to explain a known past route will continue to be weak.

To address the issues raised, this paper aims to demonstrate that:

1. cost functions representing the same theory should not be viewed as independent and to-be tested against one another. Instead, differences in the least-cost path outputs from cost functions

representing the same theory should be attributed to the process of model idealisation. As a result, outputs derived from cost functions representing the same theory should be viewed collectively: or more simply, it is not the individual cost function that is of interest, but the theory that the multiple cost functions aim to represent

2. cost functions used to represent the theories of minimising time and energy are robust. As a result, least-cost paths calculated using time- and energy-based cost functions can be used to support the theories that the cost functions aim to represent. The mutually exclusive robustness of time- and energy-based cost functions is shown through the use of tactical models, that is artificial archaeological data simulated under known conditions (Lake 2014).

The Idealised Cost Function

All scientific models contain idealisations: the intentional misrepresentation of the target system. This is no less true for cost functions, with cost functions themselves idealised models representing the *cost* of traversing a certain gradient (e.g. Campbell et al. 2017, 2017, 2019; Davey et al. 1994; Herzog 2014; Irmischer and Clarke 2018; Langmuir 1984; Llobera and Sluckin 2007; Márquez-Pérez et al. 2017; Minetti et al. 2002; Naismith 1892; Rees 2004; Tobler 1993). Firstly, each cost function, regardless of the cost it aims to quantify, makes different idealising assumptions, approximations, and simplifications. These idealisations include the functional form of the cost function, such as the double exponential used by the time-based Tobler's Hiking Function (Tobler 1993) or the sixth degree polynomial used by the energy-based cost function as proposed by Herzog (2014); the set of parameters, such as the inclusion of an offset parameter to incorporate the anisotropic property of slope (e.g. Campbell et al. 2019; Tobler 1993); and the specific parameter values used within the cost function. Secondly, individual cost functions are often derived from data with their own biases: from the number of study subjects, such as the unknown number used by Tobler's Hiking Function (Tobler 1993), one by Naismith (1892), or 200 by Irmischer and Clarke (2018); to the fitness level of the participants, such as the United States Military Academy cadets used by Irmischer and Clarke (2018) or firefighters by Campbell (2017). And lastly, cost functions are the product of the *MAXOUT* idealisation (Weisberg 2007), that is, the choosing of functional form, set of parameters, and parameter values that aim to best fit the data at hand. Whilst this is ideal for prediction, it does however guarantee that the cost functions will be useful for explanation.

When discussing multiple models with different idealisations, the philosopher of science Weisberg (2007) proposed the concept of Multiple models idealisation. Following Levins (1966), multiple models idealisation posits that each model makes varying trade-offs in their accuracy, precision, generality, and simplicity of representation. As a result, no single model can simultaneously maximise all properties (Matthewson and Weisberg 2009). To develop truer theories that can be used to explain known past routes, it is therefore necessary that a series of models – each making varying trade-offs but sharing the same theory – are used. In the words of Levins (1966, p. 431):

"The multiplicity of models is imposed by the contradictory demands of a complex, heterogeneous nature and a mind that can only cope with few variables at a time; by the contradictory desiderata of generality, realism,

and precision; by the need to understand and also to control; even by the opposing esthetic standards which emphasize the stark simplicity and power of a general theorem as against the richness and the diversity of living nature. These conflicts are irreconcilable. Therefore, the alternative approaches even of contending schools are part of a larger mixed strategy. But the conflict is about method, not nature, for the individual models, while they are essential for understanding reality, should not be confused with that reality itself."

Before moving forward, it is also necessary to reflect on the relationship between cost functions, the theories they aim to represent, and the known past route that is being explained. When aiming to explain a known past route using LCP analysis, the cost function, itself quantifying the cost of traversing slope gradients, is operationalised to represent the theory that humans minimise some cost when traversing the landscape. This is achieved through the creation of the cost surface and the subsequent calculation of the least-cost path. Thus, when the least-cost path is compared against a known past route for its ability to explain the known route, it is not the cost function used to create the least-cost path that is being tested – for which all cost functions are idealisations and known to be wrong – but rather the theory that the cost function aims to represent. Through this interpretation, multiple cost functions, each different in their idealisation but representing the same theory, can be viewed as one: no single cost function is more important than another. It is only the theory, represented by multiple cost functions, that is of direct importance.

The Robustness of Cost Functions

When using cost functions to explain known past routes, it is necessary that the proposed modelling assumptions are robust: that the result of the models depends not on the simplifying assumptions made during the idealisation process, but on the essentials of the model (Levins 1966; Weisberg 2006). If multiple models aiming to model the same phenomenon, each similar but distinct in their assumptions, are able to generate similar results, then the specific theory that the multiple models aim to represent can be deemed robust (Weisberg 2006). With this, increased support can be placed on the robust theorem as connecting the essentials of the models to the results obtained.

As devised by Weisberg (2006), the discovery of a robust theorem using robustness analysis can be described as a three-step procedure: (1) determine whether a series of models all derive a common result, the *robust property*; (2) analyse whether the models share a *common causal structure* that generates the robust property; and (3) formulate a robust theorem that links the robust property to the common causal structure, with this more generally interpreted as mapping the mathematical structures of the models to descriptions of the empirical phenomena being investigated. It should be noted that Weisberg (2006) includes an additional step of conducting a sensitivity analysis to determine under what conditions the connection between the common causal structure and the robust property fails. This, however, is outside the scope of this paper and will not be investigated further.

More formally, Weisberg (2006, p. 738) proposes that a robust theorem can be formulated as follows:

> *"Ceteris paribus (that is 'other things being equal; other things being absent; other things being just right'* (Reiss 2016, p. 300)*), if [common causal structure] obtains, then [robust property] will obtain."*

The discovery of a robust theorem when aiming to explain past routes can be summarised as: multiple cost functions, each representing the same theory, are tested to see whether they derive a common result. For example, multiple time-based cost functions, each an idealisation of the cost of traversing a specific gradient, are operationalised to represent the theory that 'humans minimise time when traversing the landscape' and used to calculate least-cost paths. Note that time-based cost functions are incompatible with energy-based cost functions, with the latter aiming to represent a different theory, i.e. cost in terms of energy. If the least-cost paths share a common result, then a common causal structure that produced the common result is to be identified. For example, that shallower slopes take less time to traverse than steeper slopes. From here, a robust theorem can be proposed, '*ceteris paribus, if humans minimise time when traversing the landscape, then the route will be optimised for time'*.

Tactical Modelling

Materials and Methods

To create tactical models, a simulated Digital Elevation Model (DEM) of 1km by 1km, with a spatial resolution of 1m, was used. Generated using the spectral synthesis method (Saupe 1988) as implemented in GRASS (GRASS Development Team 2022), the simulated DEM is a fractal surface with a fractal dimension of 2.40. The fractal dimension denotes the complexity of the surface, with greater values representing more topographic variability (Tate and Wood 2001). Given that the complexity of real landscapes ranges from a fractal dimension of 2.20 to 2.60 (Hofierka et al. 2009), the value of 2.40 is deemed sufficiently representative.

The robustness of time-, energy-, and wheel-based cost functions, and the theories they aim to represent, were assessed using the following approach:

1. Two random points are selected within the extent of the simulated DEM. These represent the origin and destination used when calculating the routes
2. 324 (*18x18*) least-cost paths using the *leastcostpath* R package (Lewis 2022) were calculated from the origin and destination using eighteen different cost functions (**Figure 1**):
    a. time-based Tobler's on-path and off-path Hiking functions (Tobler 1993), the modified Tobler's Hiking function (Márquez-Pérez et al. 2017), the Irmischer-Clarke male and female on-path and off-path cost functions (Irmischer and Clarke 2018), and the cost functions proposed by Rees (2004), Davey et al. (1994), Garmy et al. (2005), Kondo and Seino (2010), Naismith (1892), Campbell et al. (2017), and Campbell et al. (2019) (50th percentile).
    b. the energy-based cost functions proposed by Herzog (2014), Llobera and Sluckin (2007), and Minetti (2002);
    c. and the wheel-based cost function proposed by Herzog (2013) with a critical slope of 12%
3. Each calculated route assigned its 'theory type', that is *time*, *energy*, or *wheel*

4. Spatial separation between the 324 least-cost path calculated using the Path Deviation Index as proposed by Jan et al. (2000)

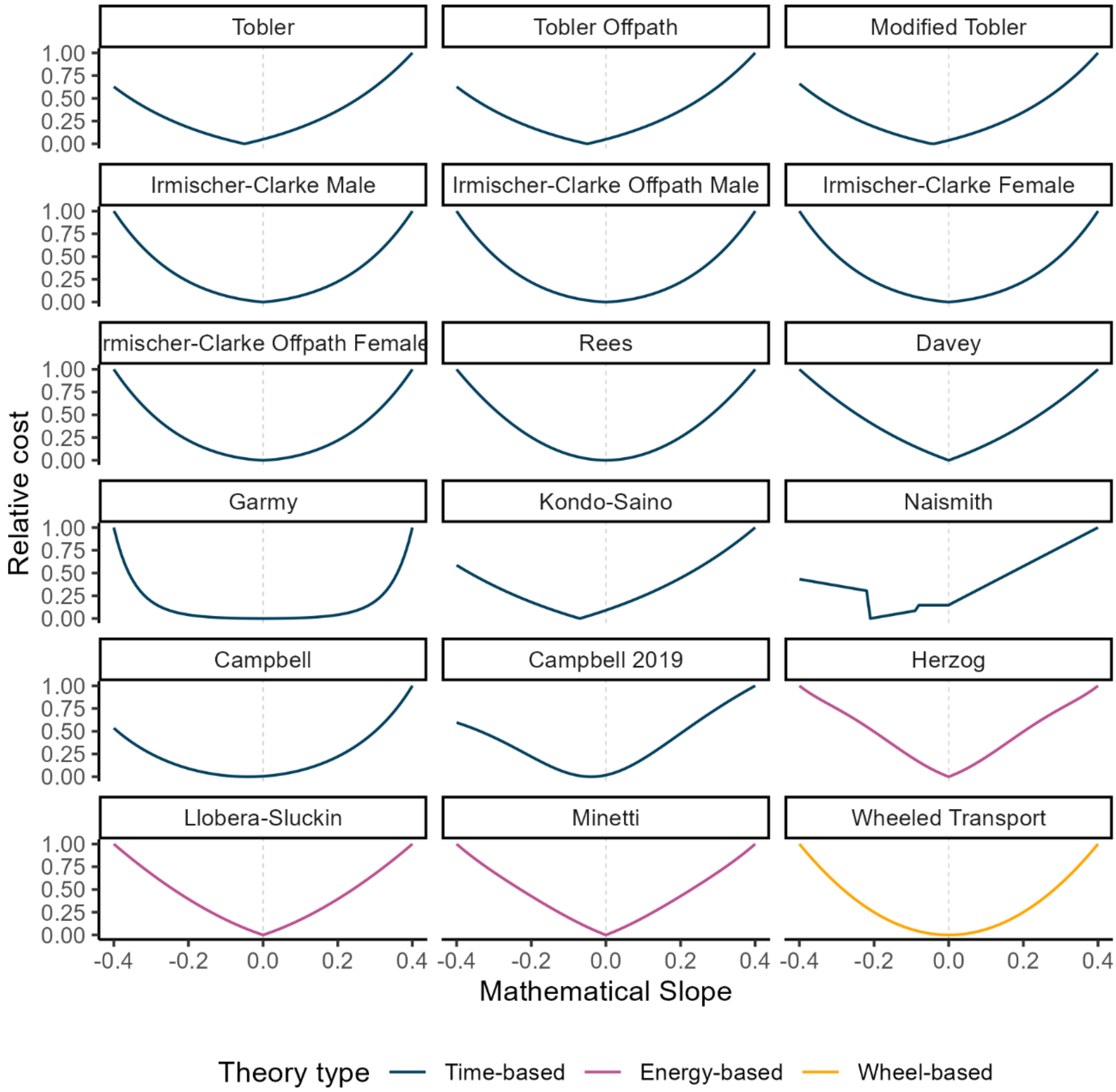

Fig 1 *Eighteen cost functions estimating time, energy expenditure, and difficulty when using wheeled vehicle as a function of mathematical slope. Uphill/downhill slopes are denoted by positive/negative slope values, respectively. The cost functions are scaled and limited to slopes of ±40% for ease of comparison. Campbell 2019 is based on the 50th percentile. Wheeled transport is calculated with a critical slope of 12%*

Having calculated 324 least-cost paths and their accompanying PDI values for 1,000 simulations, the 324,000 routes were filtered to those that resulted in perfect agreement, i.e. zero spatial separation, resulting in a final number of 35,823 routes. From this, the *robust property* of the resultant routes calculated using different cost functions, and the theories that they aim to represent, was assessed. Finally, the *common causal structures* identified, with a *robust theorem* proposed.

Results

The tactical models present very different results depending on the theory type. The time-based theory type, that is the least-cost paths derived from the cost functions measuring cost in terms of time taken, shows a robust property: 92.56% of these least-cost paths, regardless of their specific model idealisation, were identical to other least-cost paths that shared the same theory type. Similarly, the energy-based theory type, that is the least-cost paths derived from cost functions measuring cost in terms of energy expended, is also robust – although less robust than the time-based theory type (**Figure 2**).

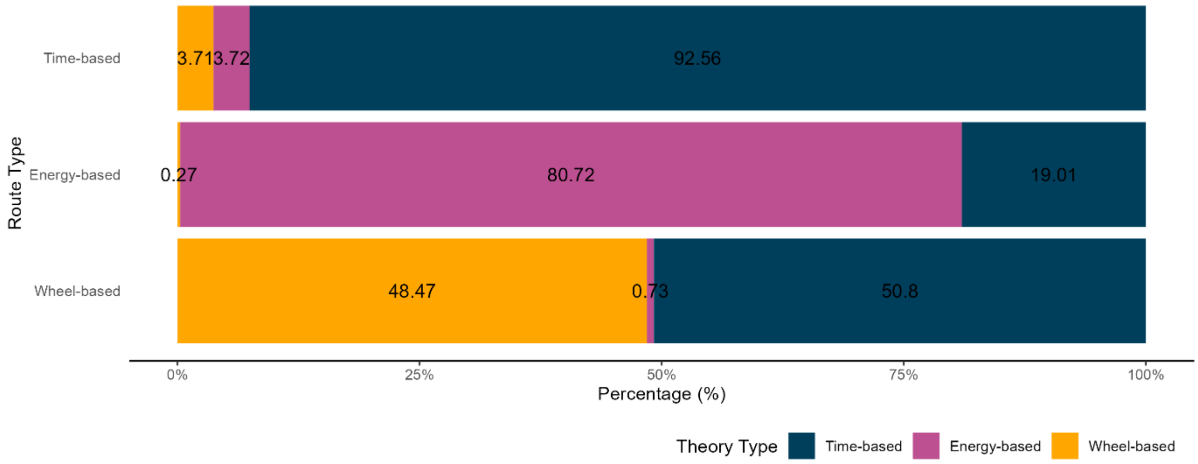

Fig.2 *Robustness of cost functions based on Route 'type' and associated Theory 'type'. Increased percentage agreement between the same Route Type and Theory Type indicates that the cost functions show a robust property regardless of their specific model idealisation. With increased agreement, more support can be placed on the robust theorem as connecting the essentials of the models to the results obtained*

In contrast to the time- and energy-based theory type, the wheel-based theory type does not show a robust property. That is, the least-cost paths derived from the cost function measuring cost based on the use of wheeled vehicles can also be derived when using cost functions based on other theory types. More specifically, 50.8% of least-cost paths derived using the cost function with a wheel-based theory type can also be derived using a cost function with a time-based theory type (**Figure 3**).

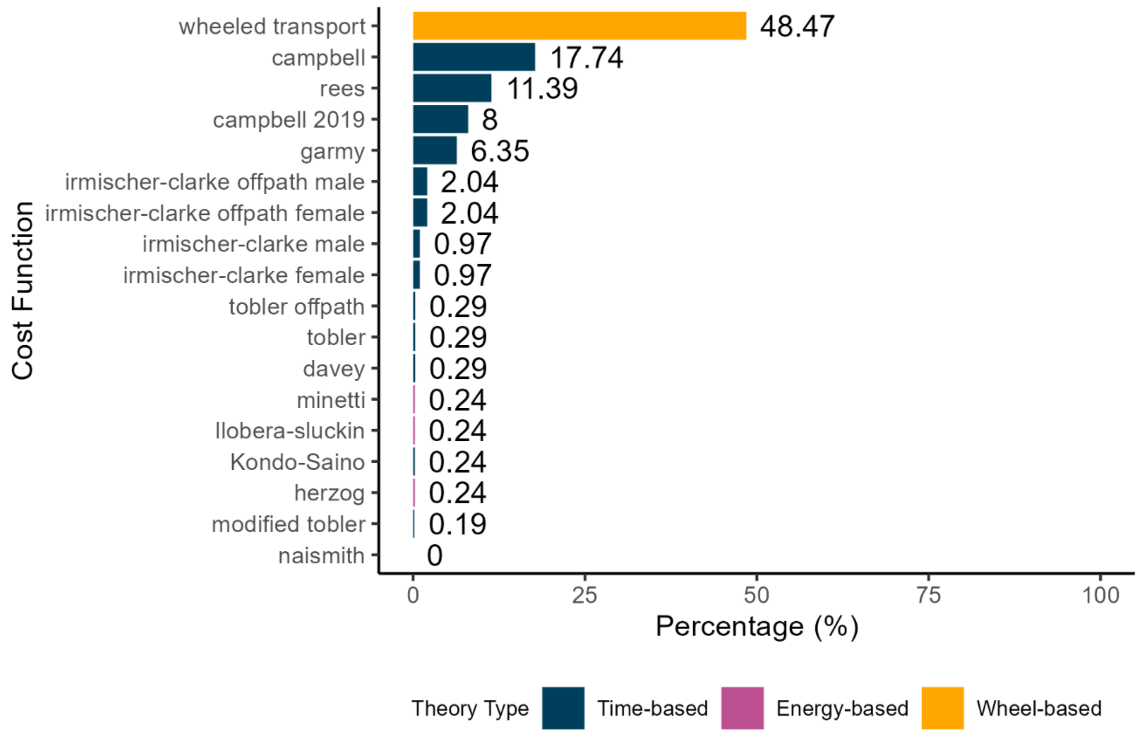

Fig.3 ***Robustness of wheel-based cost function and associated Theory 'type' of other cost functions that produce the same result (zero spatial separation)***

Discussion

Using the concept of multiple model idealisation, this paper has argued that cost functions representing the same theory – that is, that humans minimise some cost, as operationalised through a cost function, when traversing the landscape – should be interpreted collectively, with differences in the least-cost path results attributed to the process of model idealisation only.

With time- and energy-based cost functions shown to be robust, it is possible to propose the following common causal structure: *shallower slopes take less time / energy to traverse than steeper slopes*. Linking this to the descriptions of the empirical phenomena being investigated, a robust theorem can be proposed for time- and energy-based theories:

'ceteris paribus, if humans in the past minimised time taken / energy expended when traversing the landscape, then the past route will be optimised for time / energy'

Given that the wheeled-based cost function has been shown to not be robust, it is not possible to identify a common causal structure. As a result, no robust theorem can be proposed. Or more simply, if humans in the past minimised the difficulty of using wheeled vehicles when traversing the landscape, it cannot be stated credibly that the past route will be optimised for wheeled vehicles. As shown above, the same outcome can be the result of optimising for time. This can be attributed to the more general issue of equifinality whereby the same outcome can be the result of different processes. Or more simply, the archaeological record shows us what people got, not necessarily their intentions (Salmon 1989).

An element not assessed within this analysis is how landscapes with different complexity, as measured by the fractal dimension, impact the robustness of cost functions. Whilst the value of 2.40 chosen within this study aims to represent the complexity of real landscapes, it has previously been shown that similarity in least-cost path results, even when using the same cost function, can be influenced by the resolution of the DEM used (e.g. Harris 2000; Herzog and Posluschny 2011). Similarly, the effect of the distance between the two points connected using least-cost path analysis on the robustness of cost functions remains to be explored.

When using least-cost path analysis for the explanation of known past routes, this paper nonetheless concludes that focus should shift from individual cost functions to the shared theory that multiple cost functions aim to represent. It is only by assessing the outcome of theories – shown to be robust via robustness analysis – against known past routes that we can place more support on these theories as having explanatory power for uncovering why people in the past moved where they did.

Data and Materials Availability

All data and code necessary to reproduce the analyses are available on the GitHub repository https://github.com/josephlewis/Simulated_routes_and_explanation

Statements and Declarations

I have no conflicts of interest to disclose.

References

Arnoldus-Huyzendveld, A., Citter, C., & Pizziolo, G. (2016). Predictivity – Postdictivity: a Theoretical Framework. In S. Campana, R. Scopigno, G. Carpentiero, & M. Cirillo (Eds.), *Keep the Revolution Going, proceedings of 43rd Computer Applications and Quantitative Methods in Archaeology. Siena, 2015.* (pp. 593–598). Oxford: Archaeopress.

Campbell, M. J., Dennison, P. E., & Butler, B. W. (2017). A LiDAR-based analysis of the effects of slope, vegetation density, and ground surface roughness on travel rates for wildland firefighter escape route mapping. *International Journal of Wildland Fire*, *26*(10), 884. https://doi.org/10.1071/WF17031

Campbell, M. J., Dennison, P. E., Butler, B. W., & Page, W. G. (2019). Using crowdsourced fitness tracker data to model the relationship between slope and travel rates. *Applied Geography*, *106*, 93–107. https://doi.org/10.1016/j.apgeog.2019.03.008

Canosa-Betés, J. (2016). Border surveillance: Testing the territorial control of the Andalusian defense network in center-south Iberia through GIS. *Journal of Archaeological Science: Reports*, *9*, 416–426. https://doi.org/10.1016/j.jasrep.2016.08.026

Davey, R. C., Hayes, M., & Norman, J. M. (1994). Running Uphill: An Experimental Result and Its Applications. *The Journal of the Operational Research Society*, *45*(1), 25. https://doi.org/10.2307/2583947

Field, S., Glowacki, D. M., & Gettler, L. T. (2022). The Importance of Energetics in Archaeological Least Cost Analysis. *Journal of Archaeological Method and Theory*. https://doi.org/10.1007/s10816-022-09564-8

Fonte, J., Parcero-Oubiña, C., & Costa-García, J. M. (2017). A Gis-Based Analysis Of The Rationale Behind Roman Roads. The Case Of The So-Called Via Xvii (Nw Iberian Peninsula). *Mediterranean Archaeology and Archaeometry*, *17*, 163–189. https://doi.org/10.5281/zenodo.1005562

Garmy, P., Kaddouri, L., Rozenblat, C., & Schneider, L. (2005). Logiques spatiales et “systèmes de villes” en Lodévois de l’Antiquité à la période moderne. In *Temps et espaces de l’homme en société, analyses et modèles spatiaux en archéologie* (pp. 335–346). http://opac.regesta-imperii.de/id/1018915

GRASS Development Team. (2022). GRASS GIS manual: r.surf.fractal. https://grass.osgeo.org/grass78/manuals/r.surf.fractal.html. Accessed 20 January 2022

Güimil-Fariña, A., & Parcero-Oubiña, C. (2015). “Dotting the joins”: a non-reconstructive use of Least Cost Paths to approach ancient roads. The case of the Roman roads in the NW Iberian Peninsula. *Journal of Archaeological Science*, *54*, 31–44. https://doi.org/10.1016/j.jas.2014.11.030

Harris, T. (2000). Moving GIS: exploring movements within prehistoric cultural landscapes using GIS. In G. R. Lock (Ed.), *Beyond the map: archaeology and spatial technologies* (pp. 116–123). Amsterdam: IOS Press.

Herzog, I. (2010). Theory and practice of cost functions. In *Fusion of Cultures: Abstracts of the XXXVIII Conference on Computer Applications and Quantitative Methods in Archaeology* (pp. 431–434). Granada: CAA 2010.

Herzog, I. (2013). The potential and limits of Optimal Path Analysis. In A. Bevan & M. Lake (Eds.), *Computational approaches to archaeological spaces* (pp. 179–211). Walnut Creek, California: Left Coast Press.

Herzog, I. (2014). Least-cost Paths – Some Methodological Issues. *Internet Archaeology*, (36). https://doi.org/10.11141/ia.36.5

Herzog, I. (2020). Spatial Analysis Based On Cost Functions. In M. Gillings, P. Hacıgüzeller, & G. Lock (Eds.), *Archaeological Spatial Analysis* (1st ed., pp. 333–358). Routledge. https://doi.org/10.4324/9781351243858-18

Herzog, I. (2022). Issues in Replication and Stability of Least-cost Path Calculations. *Studies in Digital Heritage*, *5*(2), 131–155. https://doi.org/10.14434/sdh.v5i2.33796

Herzog, I., & Posluschny, A. (2011). Tilt – slope-dependent least cost path calculations revisited. In E. Jerem, F. Redő, & V. Szevérenyi (Eds.), *On the road to reconstructing the past: Computer Applications and Quantitative Methods in Archaeology (CAA): proceedings of the 36th International Conference, Budapest, April 2-6, 2008* (pp. 212–218). Budapest: Archaeolingua. http://proceedings.caaconference.org/paper/cd31_herzog_posluschny_caa2008/. Accessed 22 December 2019

Hofierka, J., Mitášová, H., & Neteler, M. (2009). Chapter 17 Geomorphometry in GRASS GIS. In *Developments in Soil Science* (Vol. 33, pp. 387–410). Elsevier. https://doi.org/10.1016/S0166-2481(08)00017-2

Irmischer, I. J., & Clarke, K. C. (2018). Measuring and modeling the speed of human navigation. *Cartography and Geographic Information Science*, *45*(2), 177–186. https://doi.org/10.1080/15230406.2017.1292150

Jan, O., Horowitz, A. J., & Peng, Z.-R. (2000). Using Global Positioning System Data to Understand Variations in Path Choice. *Transportation Research Record: Journal of the Transportation Research Board*, *1725*(1), 37–44. https://doi.org/10.3141/1725-06

Kondo, Y., & Seino, Y. (2010). GPS-aided walking experiments and data-driven travel cost modeling on the historical road of Nakasendō-Kisoji (Central Highland Japan). In *Making History Interactive* (pp. 158–165). Presented at the Computer Applications and Quantitative Methods in Archaeology (CAA), Proceedings of the 37th International Conference, Williamsburg, Virginia, United States of America: AR International Series, Archaeopress, Oxford.

Lake, M. W. (2014). Trends in Archaeological Simulation. *Journal of Archaeological Method and Theory*, *21*(2), 258–287. https://doi.org/10.1007/s10816-013-9188-1

Langmuir, E. (1984). *Mountaincraft and leadership: a handbook for mountaineers and hillwalking leaders in the British Isles*. Scottish Sports Council Edinburgh.

Levins, R. (1966). The Strategy of Model Building in Population Biology. *American Scientist*, *54*(4), 421–431.

Lewis, J. (2022). leastcostpath: Modelling Pathways and Movement Potential Within a Landscape (Version 2.0.0). R. https://github.com/josephlewis/leastcostpath/tree/dev

Llobera, M., & Sluckin, T. J. (2007). Zigzagging: Theoretical insights on climbing strategies. *Journal of Theoretical Biology*, *249*(2), 206–217. https://doi.org/10.1016/j.jtbi.2007.07.020

Márquez-Pérez, J., Vallejo-Villalta, I., & Álvarez-Francoso, J. I. (2017). Estimated travel time for walking trails in natural areas. *Geografisk Tidsskrift-Danish Journal of Geography*, *117*(1), 53–62. https://doi.org/10.1080/00167223.2017.1316212

Matthewson, J., & Weisberg, M. (2009). The structure of tradeoffs in model building. *Synthese*, *170*(1), 169–190. https://doi.org/10.1007/s11229-008-9366-y

Minetti, A. E., Moia, C., Roi, G. S., Susta, D., & Ferretti, G. (2002). Energy cost of walking and running at extreme uphill and downhill slopes. *Journal of Applied Physiology*, *93*(3), 1039–1046. https://doi.org/10.1152/japplphysiol.01177.2001

Naismith, W. (1892). Excursions: Cruach Ardran, Stobinian, and Ben More. *Scottish Mountaineering club journal*, *2*(3), 136.

Rees, W. G. (2004). Least-cost paths in mountainous terrain. *Computers & Geosciences*, *30*(3), 203–209. https://doi.org/10.1016/j.cageo.2003.11.001

Reiss, J. (2016). Are there social scientific laws? In *The Routledge companion to philosophy of social science* (pp. 315–329). Routledge.

Salmon, M. (1989). Efficient explanations and efficient behaviour. In V. Pinsky & A. Wylie (Eds.), *Critical traditions in contemporary archaeology: essays in the philosophy, history, and socio-politics of archaeology* (pp. 10–13). Cambridge ; New York: Cambridge University Press.

Saupe, D. (1988). Algorithms for random fractals. In H.-O. Peitgen & D. Saupe (Eds.), *The Science of Fractal Images* (pp. 71–136). New York, NY: Springer New York. https://doi.org/10.1007/978-1-4612-3784-6_2

Snead, J. E., Erickson, C. L., & Darling, J. A. (2009). 1 Making Human Space: The Archaeology of Trails, Paths, and Roads. In J. E. Snead, C. L. Erickson, & J. A. Darling (Eds.), *Landscapes of movement: trails, paths, and roads in anthropological perspective* (1st ed., pp. 1–19). Philadelphia: University of Pennsylvania Museum of Archaeology and Anthropology.

Tate, N. J., & Wood, J. (2001). Fractals and Scale Dependencies in Topography. In N. J. Tate & P. Atkinson (Eds.), *Scale in Geographical Information Systems* (pp. 35–51). New York: John Wiley & Sons.

Tobler, W. (1993). *Three Presentations on Geographical Analysis and Modeling. Technical Report 93-1*. Santa Barbara, CA: National Center for Geographic Information and Analysis. https://escholarship.org/uc/item/05r820mz. Accessed 22 December 2019

Verhagen, P., Nuninger, L., & Groenhuijzen, M. R. (2019). Modelling of Pathways and Movement Networks in Archaeology: An Overview of Current Approaches. In P. Verhagen, J. Joyce, & M. R. Groenhuijzen (Eds.), *Finding the Limits of the Limes: Modelling Demography, Economy and Transport on the Edge of the Roman Empire* (pp. 217–249). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-04576-0_11

Weisberg, M. (2006). Robustness Analysis. *Philosophy of Science*, *73*(5), 730–742. https://doi.org/10.1086/518628

Weisberg, M. (2007). Three Kinds of Idealization. *Journal of Philosophy*, *104*(12), 639–659. https://doi.org/10.5840/jphil20071041240

Zipf, G. K. (1949). *Human behavior and the principle of least effort* (pp. xi, 573). Oxford, England: Addison-Wesley Press.