Explaining known past routes and the robustness of cost functions

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"Our truth is the intersection of independent lies" – Levins (1966, p. 423)

Introduction

To explain the rationale behind past movement is often limited to explaining the material proxies of past movement. From roads to footpaths to trails, the material traces of past routes are thought to have preserved the decision-making processes of people in the past when traversing the landscape. And, without direct record, it is only through explaining these past routes that we can aim to uncover why people in the past moved where they did.

The most common approach for explaining past known routes is least-cost path (LCP) analysis. Based on the *principle of minimum effort* as proposed by Zipf (1949), it is postulated that humans will economise their behaviour when conducting a task. In terms of traversing the landscape, it is thus assumed that humans will choose a path, i.e. an LCP, that minimises the accumulated cost, with time taken or energy expended being frequently used. This cost is represented via a cost function, which is a mathematical function that expresses the cost of traversing a specific slope gradient. From here, a cost surface quantifying the difficulty of crossing between individual cells in a raster grid can be created. Finally, using the cost surface, a least-cost path can be calculated from an origin and destination point.

When aiming to explain known past routes using LCP analysis specifically, it is common for multiple least-cost paths, each derived using a different cost function, to be assessed for their ability in explaining a known route. For example, Güimil-Fariña and Parcero-Oubiña (2015) evaluated three different cost functions in their ability to explain a known Roman road in Spain, finding that the time-based Tober's Hiking Function (1993) and the energy-based cost function as proposed by Llobera and Sluckin (2007) produced adequate results. This was further corroborated by Fonte et al. (2017) when aiming to explain additional Roman roads in Spain. Similarly, Herzog (2010) compared the least-cost paths derived from six different cost functions – with three being time-based and three being energy-based – to an ancient route in Germany. The study was later expanded to include additional roads and another cost function (Herzog 2022). Lastly, Herzog (2020) aimed to identify the factors governing the construction of Roman roads in Germany using three cost functions: two time-based, one energy-based, and the other wheel-based. It is also worthwhile to note Field et al. (2022), in which multiple cost functions of the same type, that is time-based or energy-based, were used concurrently as a way to "better capture potential differences between time- and energy-based LCPs and because not all time-derived LCPs (nor all energy-derived LCPs) are anticipated to travel the same route". However, with the study focused on prediction not explanation, as evidenced by the lack of validation against a known past route, the efficacy of using multiple cost functions and explaining known past routes remains unexplored.

Whilst the comparison of multiple cost functions allows for the testing of which cost function best fits known past routes, and thus providing the means to offer potential explanations, this approach misrepresents what cost functions are, their relationship to the theories they aim to represent, and their role in explanation. The consequence of aiming to 'identify the *best* cost function' is neatly summarised by Canosa-Betés (2016, p. 419): "There have been some comparative studies of these and other cost functions […] but the archaeological community has not opted for one in particular yet". If this issue of misrepresentation remains unaddressed, particularly with regards to the choosing of a single 'best' cost function, the ability to use cost functions as support for a proposed theory when aiming to explain a past known route will continue to be weak and unjustified.

To address the issues raised, this paper aims to demonstrate that:

1. cost functions representing the same theory should not be viewed as independent and to-be tested against one another. Instead, differences in the least-cost path outputs should be attributed to the process of model idealisation. As a result, outputs derived from cost functions representing the same theory should be viewed collectively: or more simply, it is not the individual cost function that is of interest, but the theory that the multiple cost functions aim to represent
2. cost functions used to represent the theories of minimising time and energy are robust. As a result, the least-cost paths using multiple cost functions representing these two theories can be used to support the theories that the cost functions aim to represent. The mutually exclusive robustness of time- and energy cost functions is shown through the use of tactical models, that is artificial archaeological data simulated under known conditions (Lake 2014).

The Idealised Cost Function

All scientific models contain idealisations: the intentional misrepresentation of the target system. This is no less true for cost functions, with cost functions themselves idealised models representing the *cost* of traversing a certain gradient (e.g. Campbell et al. 2017, 2017, 2019; Davey et al. 1994; Herzog 2014; Irmischer and Clarke 2018; Langmuir 1984; Llobera and Sluckin 2007; Márquez-Pérez et al. 2017; Minetti et al. 2002; Naismith 1892; Rees 2004; Tobler 1993). Each cost function, regardless of the cost it aims to quantify, makes different idealising assumptions, approximations, and simplifications. These idealisations include the functional form of the cost function, such as the double exponential used by the time-based Tobler's Hiking Function (Tobler 1993) or the sixth degree polynomial used by the energy-based cost function as proposed by Herzog (2014); the set of parameters, such as the inclusion of an offset parameter to incorporate the anisotropic property of slope (e.g. Campbell et al. 2019; Tobler 1993); and the specific parameter values used within the cost function. Further to this, individual cost functions are often derived from data with their own biases: from the number of study subjects, such as the unknown number used by Tobler's Hiking Function (Tobler 1993); the 1 by Naismith (1892); or the 200 by Irmischer and Clarke (2018), to the fitness level of the participants, such as the United States Military Academy cadets used by Irmischer and Clarke (2018) or firefighters by Campbell (2017).

When discussing multiple models with different idealisations, the philosopher of science Weisberg (2007) proposed the concept of Multiple models idealisation. Following Levins (1966), multiple models idealisation posits that each model makes varying trade-offs in their accuracy, precision, generality, and simplicity of representation. As a result, no single model can simultaneously maximise *all* of these properties. To develop truer theories that can be used to explain past known routes, it is therefore necessary that a series of models – each making varying trade-offs but sharing the same theory – are used (sensu Levins 1966; Weisberg 2006).

Before moving forward, it is also necessary to reflect on the relationship between cost functions, the theories that they aim to represent, and the known past route that is being explained. When aiming to explain a known past route using LCP analysis, the cost function, itself quantifying the cost of traversing a specific slope gradient, is operationalised to represent the theory that humans minimise some cost when traversing the landscape. Thus, when the least-cost path is compared against a known past route for its ability to explain the known route, it is not the cost function used to create the least-cost path that is being tested – for which all cost functions are idealisations and therefore known to be wrong – but rather the theory that the cost function aims to represent. Through this interpretation, multiple cost functions, each different in their idealisation but representing the same theory, are viewed as one: no single cost function is more important than another for their ability to explain. It is only the theory that the cost functions aim to represent that is of direct importance.

The robustness of cost functions

When using cost functions to explain known past routes, it is necessary that the proposed modelling assumptions are robust: that the result of the models depend not on the simplifying assumptions made during the idealisation process, but on the essentials of the model (Levins 1966; Weisberg 2006). If multiple models aiming to model the same phenomenon, each similar but distinct in their assumptions, are able to generate similar results, then the specific theory that the multiple models aim to represent is called a *robust theorem.* With this, more support can be placed on the models adequately representing the theory.

As devised by Weisberg (2006), the discovery of a robust theorem using robustness analysis can be described as a three-step procedure: (1) determine whether a series of models all derive a common result, the *robust property*; (2) analyse whether the models share a *common causal structure* that generates the robust property; and (3) formulate a robust theorem that links the robust property to the common causal structure, with this more generally interpreted as mapping the mathematical structures of the models to descriptions of the empirical phenomena being investigated. More formally, Weisberg (2006, p. 738) proposes that a robust theorem can be formulated as follows:

> *"Ceteris paribus (that is 'other things being equal; other things being absent; other things being just right'* (Reiss 2016, p. 300)*), if [common causal structure] obtains, then [robust property] will obtain."*

The discovery of a robust theorem when modelling past routes can be summarised as: multiple cost functions, each representing the same theory, are tested to see whether they derive a common result. For example, multiple time-based cost functions, each an idealisation of the cost of traversing a certain gradient and representing the theory that 'humans minimise time when traversing the landscape', are used to create least-cost paths. Note that time-based cost functions are incompatible with energy-based cost functions, with the latter aiming to represent a different theory, i.e. cost in terms of energy. If the least-cost paths share a common result, then a common causal structure that produced the common result is to be identified. For example, that shallower slopes take less time to traverse than steeper slopes. From this, a robust theorem can be proposed, '*ceteris paribus, if humans minimise time when traversing the landscape, then the route will be optimised for time'*.

Tactical Modelling

Materials and Methods

To create tactical models, a simulated DEM of 1km by 1km, with a spatial resolution of 1m, was used. Generated using the spectral synthesis method (Saupe 1988) as implemented in GRASS (GRASS Development Team 2022), the simulated DEM is a fractal surface with a fractal dimension of 2.40. The fractal dimension denotes the complexity of the surface, with greater values representing more topographic variability (Tate and Wood 2001). Given that the complexity of real landscapes ranges from a fractal dimension of 2.20 to 2.60 (Hofierka et al. 2009), the value of 2.40 is deemed sufficiently representative.

The robustness of time-, energy-, and wheel-based cost functions, and the theories they aim to represent, were assessed using the following approach:

1. Two random points are selected within the extent of the simulated DEM. These represent the origin and destination used when calculating the routes
2. 324 least-cost paths using the *leastcostpath* R package (Lewis 2021) were calculated from the origin and destination using eighteen different cost functions:
    a. time-based Tobler's on-path and off-path Hiking functions (Tobler 1993), the modified Tobler's Hiking function (Márquez-Pérez et al. 2017), the Irmischer-Clarke on-path and off-path cost functions based on male military academy cadets (Irmischer and Clarke 2018), the Irmischer-Clarke on-path and off-path cost functions based on female military academy cadets (Irmischer and Clarke 2018), Rees (2004), Davey et al. (1994), Garmy et al. (2005), Kondo and Seino (2010), Naismith (1892), Campbell et al. (2017), and Campbell et al. (2019)
    b. the energy-based cost functions as proposed by Herzog (2014), Llobera and Sluckin (2007), and Minetti (2002);
    c. and the wheel-based cost function as proposed by Herzog (2013)
3. Each calculated route assigned its 'theory type', that is *time*, *energy*, or *wheel*
4. Spatial separation between the 324 least-cost path calculated using the Path Deviation Index as proposed by Jan et al. (2000)

Having calculated 324 least-cost paths and their accompanying PDI values for 1,000 simulations, the 324,000 routes were filtered to those that resulted in perfect agreement, i.e. zero spatial separation, resulting in a final number of 35,823 routes. From this, the *robust property* of the resultant routes calculated using different cost functions, and the theories that they aim to represent, was assessed. Finally, the *common causal structure* was identified, with a *robust theorem* proposed.

Results

The tactical models present very different results depending on whether the focus is on cost functions with a time, energy, or wheeled theory type. Firstly, the time-based theory type, that is the least-cost paths derived from the cost functions measuring cost in terms of time taken, shows a robust property: 92.56% of these least-cost paths, regardless of their specific model idealisation, were identical to other least-cost paths that shared the same theory type. Similarly, the energy-based theory type, that is the least-cost paths derived from cost functions measuring cost in terms of energy expended, is also robust – although less robust than the time-based theory type (Figure 2).

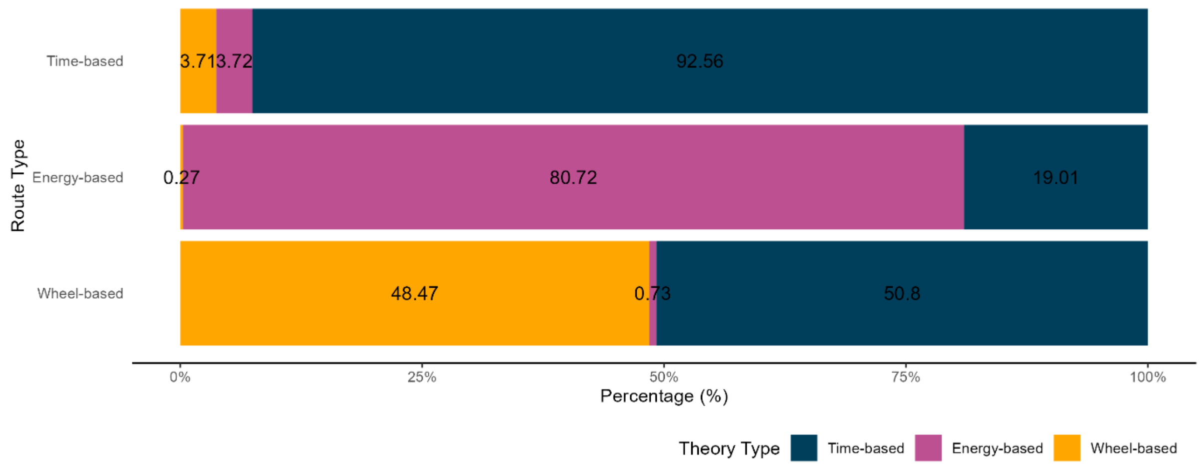

Figure 2. xxx

In contrast to the time- and energy-based theory type, the wheel-based theory type does not show a robust property. That is, the least-cost paths derived from the cost function measuring cost based on the use of wheeled vehicles can also be derived when using cost functions based on other theory types. More specifically, 50.8% of least-cost paths derived using the cost function with a wheel-based theory type can also be derived using a cost function with a time-based theory type (Figure 3).

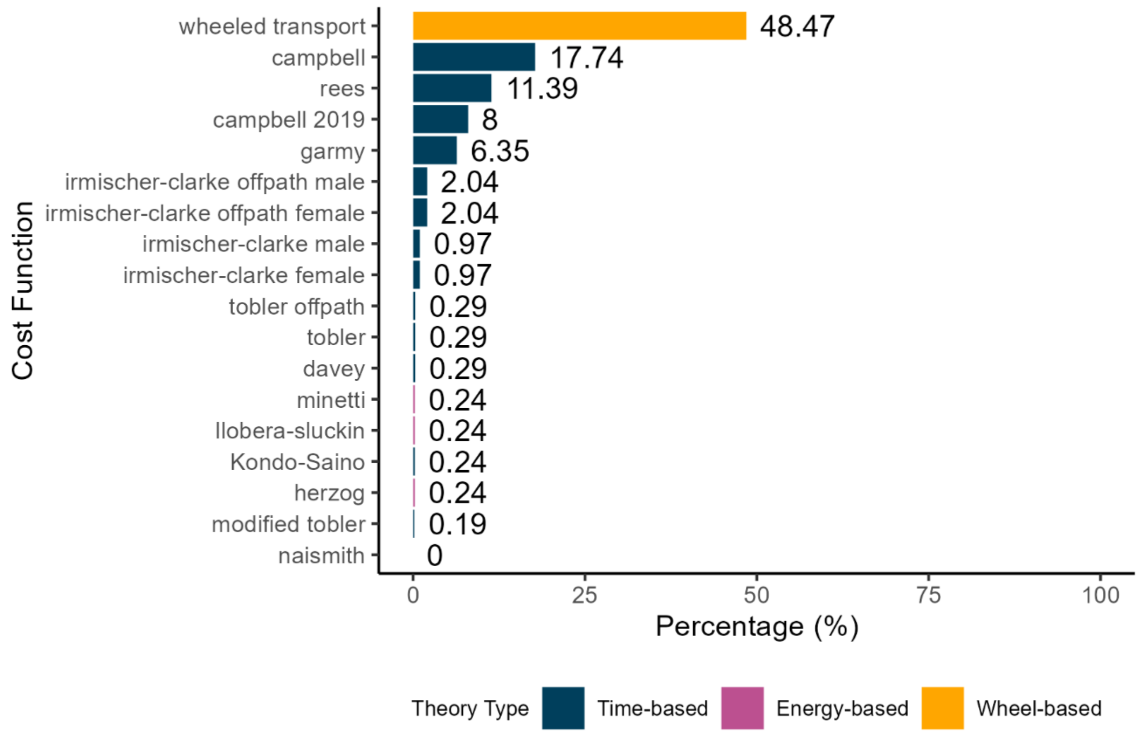

Figure 3. xxxx

With time- and energy-based cost functions shown to be robust, the following common causal structure can be identified: *shallower slopes take less time / energy to traverse than steeper slopes*. Linking this to the descriptions of the empirical phenomena being investigated, the following robust theorem can be proposed for time- and energy-based theories:

'ceteris paribus, if humans in the past minimised time taken / energy expended when traversing the landscape, then the past route will be optimised for time / energy'

Given that wheeled-based cost function has been shown to not be robust, it is not possible to identify a common causal structure. As a result, no robust theorem can be proposed. Or more simply, if humans in the past minimised the difficulty when using wheeled vehicles when traversing the landscape, it does not guarantee that the past route will be optimised for wheeled vehicles. As shown above, the same outcome can be the result of optimising for time, i.e. equifinality.

Discussion

Using the concept of multiple model idealisation, this paper has demonstrated that cost functions representing the same theory – that is, that humans minimise some cost, as operationalised through a cost function, when traversing the landscape – should be interpreted collectively, with differences in least-cost path results attributed to the process of model idealisation.

Further to this, time- and energy-based cost functions have been shown to be robust and can thus be used as support for the theory that they aim to represent. This is in contrast to the wheeled-based cost function which has been shown to not be robust. As a result, the wheel-based cost function cannot be used represent the theory that humans minimise the difficulty of using wheeled vehicles when traversing the landscape.

An element not assessed within this analysis is how landscapes with different complexity, as measured by the fractal dimension, impacts the robustness of cost functions. Whilst the value of 2.40 chosen within this study represents the complexity of real landscapes well, it has previously been shown that similarity in least-cost path results, even when using the same cost function, can be influenced by the resolution of the DEM used (e.g. Harris 2000; Herzog and Posluschny 2011). Similarly, the effect of the distance between the two points connected using least-cost path analysis on the robustness of cost functions remains to be explored.

When using least-cost path analysis for the explanation of known past routes, it is nonetheless recommended that focus shifts from individual cost functions to the hypotheses that multiple cost functions aim to represent. It is only by testing the outcomes of hypotheses against known past routes, and not individual cost functions – each unique in their idealisation – that we are able to place more support for explanations when aiming to uncover why people in the past moved where they did.

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