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HW 1 — Image Filtering

1 FIR Low Pass Filter

Calculate an analytical expression for $H(e^{ju}, e^{jv})$

$$H(e^{ju}, e^{jv}) = \Sigma_{m=-\infty}^{\infty}\Sigma_{n=-\infty}^{\infty}h(m,n)e^{-jum}e^{-jvm} = \frac{1}{81}\Sigma_{m=-4}^{4}\Sigma_{n=-4}^{4}e^{-jum}e^{-jvn} \tag{1-1}$$

Use Python's matplotlib to plot the magnitude of the frequency response. $\|H(e^{ju}, e^{jv})\|$

```python
import matplotlib.pyplot as plt
import numpy as np

@np.vectorize
def z(u, v):
    res = 0
    for m in range(-4, 5):
        for n in range(-4, 5):
            res +=  (1/81) * (np.e**(-1j * (m * u + n * v)))
    return abs(res)

x = np.linspace(-np.pi, np.pi, 200)
y = np.linspace(-np.pi, np.pi, 200)
X, Y = np.meshgrid(x, y)
Z = z(X, Y)

fig = plt.figure()
ax = fig.gca(projection='3d')
surf = ax.plot_surface(Y, X, Z)
ax.set_xlabel('u')
ax.set_ylabel('v')
ax.set_zlabel('z')
plt.show()
```

Modify the program Example so that it filters the red, green and blue components of img03.tif with the filter h(m, n) and generates a full color output image.

Output filtered images can be seen in the main repository. Filtered color image shown below.

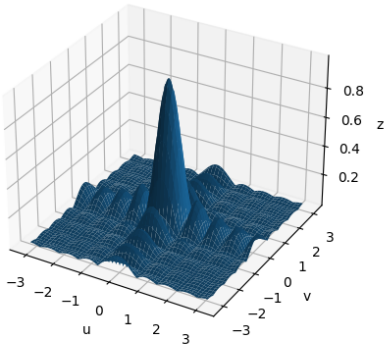

Figure 1: Magnitude of the frequency response from DSFT h(m, n).

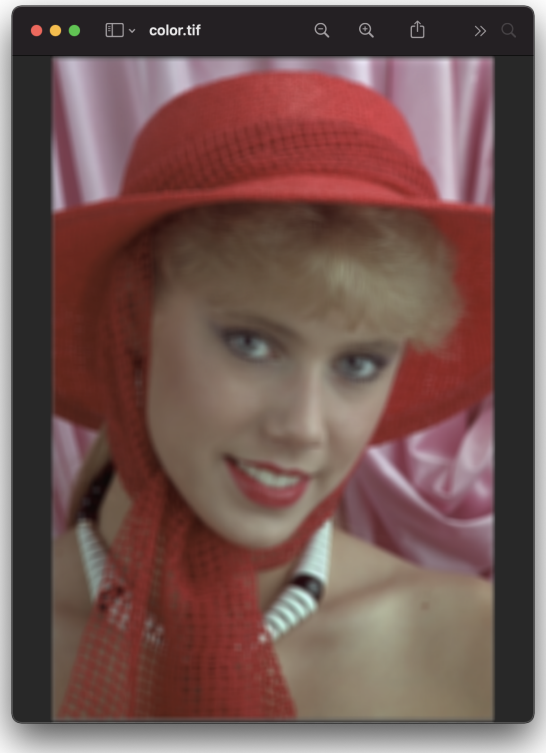

Figure 2: Filtered color image

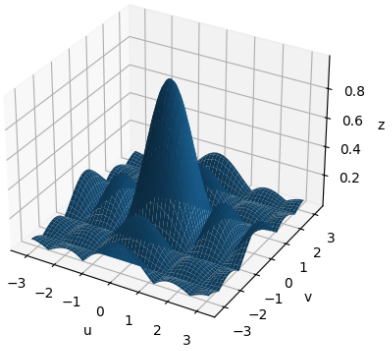

Figure 3: DSFT for sharpening filter.

$$\delta(n) \overset{DTFT}{\Leftrightarrow} 1$$

Figure 4: Useful property of DSFT from notes.

2 FIR Sharpening Filter

Calculate an analytical expression for $H(e^{ju}, e^{jv})$

$$H(e^{ju}, e^{jv}) = \Sigma_{m=-\infty}^{\infty}\Sigma_{n=-\infty}^{\infty} h(m,n)e^{-jum}e^{-jvm} = \frac{1}{25}\Sigma_{m=-2}^{2}\Sigma_{n=-2}^{2} e^{-jum}e^{-jvn} \tag{1-2}$$

Calculate an analytical expression for $G(e^{ju}, e^{jv})$

$$G(e^{ju}, e^{jv}) = \delta(m,n) + \lambda(\delta(m,n) - h(m,n)) \tag{1-3}$$

Using the property from the notes in figure 4:

$$DSFT\{\delta(m,n)+\lambda(\delta(m,n)-h(m,n))\} = 1+\lambda(1-h(m,n)) = (1+\lambda)-\lambda(\frac{1}{25}\Sigma_{m=-2}^{2}\Sigma_{n=-2}^{2} e^{-jum}e^{-jvn}) \tag{1-4}$$

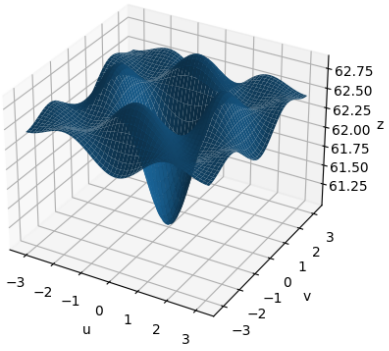

Figure 5: Graph of G with lambda=1.5

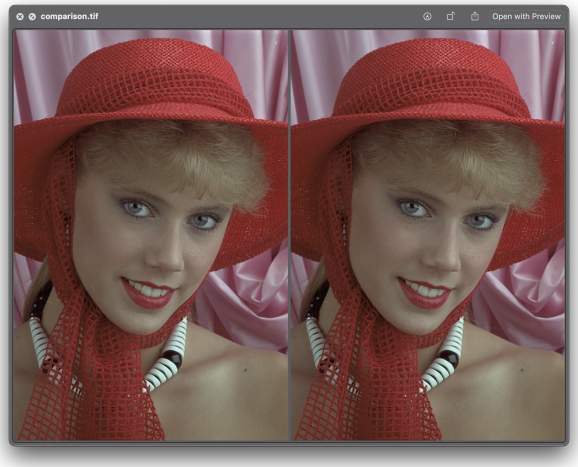

Figure 6: Lambda 0.2 (left) and lambda 1.5(right)

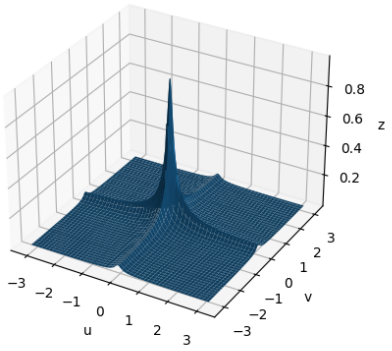

Figure 7: Magnitude of frequency response for IIR filter.

3 IIR Filter

Calculate an analytical expression for $H(e^{ju}, e^{jv})$
We calculate the transfer function:
$H(z) = \frac{Y(z)}{X(z)} = \frac{0.01}{1-0.9e^{-ju}-0.9e^{-jv}+0.81e^{-ju-jv}}$
Image of the color image after IIR filtering.

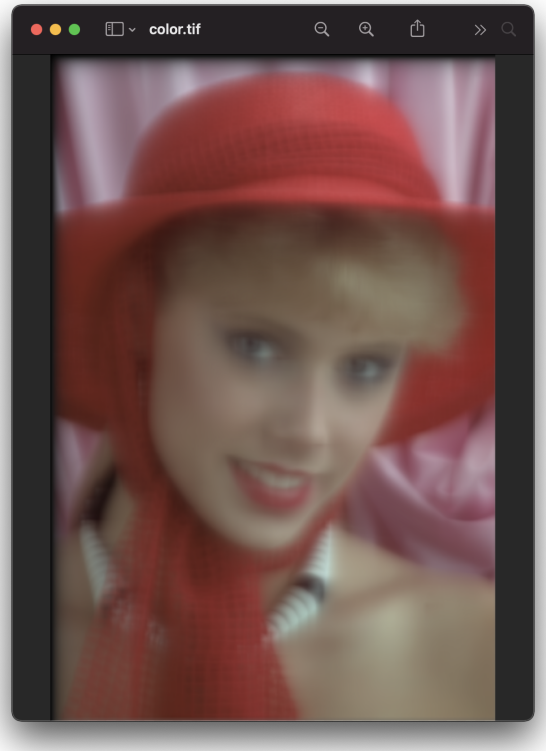

Figure 8: IIR filtered color image.

Submitted by Joseph Maa on January 20, 2024.