

Summary of GMM Estimates

your old pal Jo

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Preliminaries

- Let there be N children indexed by n . Let $g(M_n, Z_n)$ indicate the moment conditions with marital status $M_n = 1$ if married.
- Data used here is the same as the data from the first draft, not yet using the updated dataset.
- Let the vector of production parameters to be estimated be $(\rho, \gamma, \phi, \phi_\theta, \delta)$ where (ρ, γ) are the elasticity parameters, ϕ represents the coefficients on observables that determine the factor shares, ϕ_θ is the vector of coefficients that determine θ (TFP), and δ is the vector of cobb-douglas factor shares in the outer aggregator.
- We use $\tilde{\cdot}$ to indicate the *perceived* value of a production parameter as it can be inferred from the demand system.
- As in the original paper, we exploit that:

$$\mathcal{J}_{n,t} = \Phi_{\mathcal{J},m}(\Pi_{n,t}, Z_{n,t})\tau_{m,n,t}, \quad \mathcal{J} \in \{Y_C, g, \tau_f\}$$

where $\Pi_{n,t}$ is the vector of prices for child n at time t and $Z_{n,t}$ is the vector of observables that can plausibly shift factor shares. We suppress dependence of Φ on the parameters $(\tilde{\rho}, \tilde{\gamma}, \tilde{\phi})$.

Relative Demand: Quick Summary of Specifications

As before, relative demand is summarized by the parameters $(\tilde{\rho}, \tilde{\gamma}, \tilde{\phi})$ which can be estimated by forming moment conditions on the difference between observed input ratios and those predicted by prices (with deviations explained only by measurement error in inputs).

We try a few specifications, as described below.

Specification (1) replicates the approach from the first draft of the paper, where moments for single and married individuals are stacked on top of each other:

$$g(Z_n) = \begin{bmatrix} (1 - M_n)g(0, Z_n) \\ M_n g(1, Z_n) \end{bmatrix}.$$

Results are slightly different to the draft (found some small errors there).

Specifications (2)-(6) arranges the moment function g differently, without distinguishing by marital status (which is added to the set of instruments), and instead assigns values of zero to relative demand residuals that do not apply (those involving father's time) and sets values of instruments that do not apply (those involving father's education) to zero also. Remaining differences between specifications (2)-(6) pertain to observables that affect factor shares, which can be read from the table. In this version moments involving relative demand for childcare on mother's time, for example, will be found in the same position within g regardless of marital status.

Specifications (3)-(6) use grouped fixed effect estimates from the mother's wage equation, assuming the model:

$$\log(W_{mt}) = \mu_{k(m)} + X_{mt}\beta + \epsilon_{mt}$$

where X_{mt} includes education dummies, age and age squared, and $k(m) \in \{1, 2, \dots, K\}$ is mother m 's type. It is estimated via an iterative clustering algorithm.

Specifications (3) and (4) uses dummies for Types (2) and (3) from a clustering routine (using the above approach) with $K = 3$, where types are sorted in ascending order by their estimated value μ_k .

Specifications (5) and (6) instead uses the estimated value of $\mu_{k(m)}$ for each m , with $K = 10$.

Estimation Results - No Skill Outcomes

Adding Skill Outcomes

Next, the intertemporal moment conditions are expanded to include intertemporal moment conditions on skill outcomes. Differently from the original approach, the overall input $X_{n,t}$ must be imputed using separate relative demand parameters. For the case of single mothers (for example) this is:

$$X_{n,t}(\Pi_{n,t}) = \left((a_m(\phi_m, Z_{n,t}) + a_{g,t}(\phi_g, Z_{n,t})\Phi_{g,m}(\Pi_{n,t}, Z_{n,t})^\rho)^{\gamma/\rho} + \Phi_{Y,m}(\Pi_{n,t}, Z_{n,t})^\gamma \right)^{1/\gamma} \tau_{m,n,t}$$

where the Φ functions depend on the perceived production parameters $(\tilde{\rho}, \tilde{\gamma}, \tilde{\phi})$ and total investment additional depends on the true values of these parameters. For simplicity, let $\kappa = (\phi_m, \phi_f, \phi_g, \rho, \gamma)$, so we can write:

$$X_{n,t} = \mathcal{X}(\Pi_{n,t}, Z_{n,t}, \tilde{\kappa})\tau_{m,n,t}.$$

As in the previous draft, we must iterate the outcome equation forward and using a modeling assumption to impute investment in missing periods. Let $\bar{p}(\Pi_{n,t}, Z_{n,t}, \tilde{\kappa})$ indicate the *perceived* composite price of X . In the case of no binding borrowing constraints (for example), will result in a final estimating equation:

$$\lambda_S^{-1} S_{n,5} = Z_{n,0} \tilde{\phi}_\theta + \sum_{t=0}^4 \delta_2^{4-t} \delta_1 \ln \left(\frac{\bar{p}(\Pi_{n,0}, Z_{n,0}, \tilde{\kappa}) \mathcal{X}(\Pi_{n,t}, Z_{n,t}, \tilde{\kappa}) \tau_{m,n,0}^0}{\bar{p}(\Pi_{n,t}, Z_{n,t}, \tilde{\kappa})} \right) + \delta_2^5 \lambda_S^{-1} S_{n,0} + \xi_{S,n}$$

where $\xi_{S,n}$ collects measurement error in $\tau_{m,n,0}$ as well as measurement error in $S_{n,5}$ and $S_{n,0}$. To account for measurement error in both time investment and skills, future observations of time investment ($\tau_{m,n,5}$) are used to instrument for $\tau_{m,n,0}$, and the second measure of skills (applied problems if letter-word is used as the outcome, and vice versa) is used to instrument for the outcome score.

Note on testing To test the cross-equation restrictions that $\kappa = \tilde{\kappa}$, we can estimate the full GMM problems in the restricted case as well as the

Table 1: GMM Estimation of Relative Demand System

	(1)	(2)	(3)	(4)	(5)	(6)
ρ	-1.67 (0.50)	-1.56 (0.47)	-1.90 (0.62)	-2.44 (0.90)	-2.66 (1.15)	-2.56 (0.99)
γ	-1.07 (0.39)	-0.94 (0.37)	-1.09 (0.38)	-1.03 (0.40)	-1.17 (0.43)	-1.09 (0.44)
ϕ_m : Mother's Time						
Married	3.05 (1.77)	3.43 (2.01)	5.97 (2.73)	8.15 (4.19)	4.33 (3.11)	4.64 (3.14)
Single	2.60 (1.27)	2.79 (1.40)	5.10 (2.07)	6.86 (3.19)	3.28 (2.15)	3.47 (2.14)
Mother: Some College	-0.62 (0.39)	-0.62 (0.39)	-	-0.90 (0.57)	-	-0.98 (0.62)
Mother: College+	0.16 (0.35)	-0.02 (0.35)	-	-0.02 (0.50)	-	-0.10 (0.52)
Child Age	0.15 (0.09)	0.15 (0.10)	0.18 (0.12)	0.22 (0.15)	0.21 (0.16)	0.23 (0.16)
Num. Children 0-5	-0.40 (0.30)	-0.40 (0.30)	-0.41 (0.34)	-0.50 (0.46)	-0.54 (0.48)	-0.47 (0.46)
Type 2	-	-	-2.58 (1.09)	-2.93 (1.41)	-	-
Type 3	-	-	-2.82 (1.12)	-3.30 (1.48)	-	-
μ_k	-	-	-	-	-1.00 (0.70)	-1.17 (0.71)
ϕ_f : Father's Time						
Const.	0.70 (1.71)	0.96 (1.93)	0.43 (1.81)	1.68 (2.81)	1.27 (2.67)	1.41 (2.82)
Father: College+	-0.72 (0.51)	-0.77 (0.52)	-0.87 (0.59)	-1.41 (0.79)	-1.07 (0.78)	-1.24 (0.80)
Father: Some College	0.13 (0.41)	0.02 (0.42)	0.11 (0.49)	-0.22 (0.57)	-0.02 (0.61)	-0.27 (0.60)
Child Age	0.33 (0.13)	0.33 (0.13)	0.45 (0.18)	0.52 (0.23)	0.49 (0.24)	0.53 (0.24)
Num. Children 0-5	-0.23 (0.42)	0.10 (0.43)	0.01 (0.49)	-0.02 (0.63)	0.11 (0.63)	0.14 (0.63)
ϕ_g : Goods						
Married	-3.42 (1.39)	-2.51 (1.65)	-1.61 (2.02)	-0.80 (2.93)	-2.67 (2.20)	-2.42 (2.39)
Single	-4.31 (1.07)	-3.66 (1.16)	-3.01 (1.57)	-2.85 (2.12)	-4.54 (1.64)	-4.45 (1.66)
Mother: Some College	-0.11 (0.39)	-0.22 (0.40)	-	-0.36 (0.56)	0.06 (0.64)	-0.33 (0.58)
Mother: College+	0.90 (0.55)	0.64 (0.52)	-	0.99 (0.75)	1.31 (0.93)	1.11 (0.83)
Father: College+	-0.36 (0.45)	-0.33 (0.46)	-0.19 (0.51)	-0.53 (0.63)	-0.66 (0.69)	-0.55 (0.66)
Father: Some College	0.25 (0.46)	0.21 (0.46)	0.72 (0.57)	0.25 (0.61)	0.23 (0.66)	0.10 (0.62)
Child Age	0.50 (0.15)	0.44 (0.15)	0.54 (0.19)	0.66 (0.26)	0.67 (0.30)	0.68 (0.28)
Num. Children 0-5	-0.39 (0.37)	-0.47 (0.38)	-0.59 (0.46)	-0.71 (0.58)	-0.87 (0.65)	-0.73 (0.60)
Type 2	-	-	-1.56 (1.02)	-1.70 (1.29)	-	-
Type 3	-	-	-1.31 (0.96)	-1.36 (1.20)	-	-
μ_k	-	-	-	-	0.47 (0.69)	0.21 (0.65)
Residual Correlation Test						
p-value	0.34	0.36	0.30	0.41	0.31	0.39