An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility

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The theory of inequality and intergenerational mobility presented in this essay assumes that each family maximizes a utility function spanning several generations. Utility depends on the consumption of parents and on the quantity and quality of their children. The income of children is raised when they receive more human and nonhuman capital from their parents. Their income is also raised by their "endowment" of genetically determined race, ability, and other characteristics, family reputation and "connections," and knowledge, skills, and goals provided by their family environment. The fortunes of children are linked to their parents not only through investments but also through these endowments acquired from parents (and other family members). The equilibrium income of children is determined by their market and endowed luck, the own income and endowment of parents, and the two parameters, the degree of inheritability and the propensity to invest in children. If these parameters are both less than unity, the distribution of income between families approaches a stationary distribution. The stationary coefficient of variation is greater, the larger the degree of in-

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heritability and the smaller the propensity to invest in children. Intergenerational mobility measures the effect of a family on the well-being of its children. We show that the family is more important when the degree of inheritability and the propensity to invest are larger. If both these parameters are less than unity, an increase in family income in one generation has negligible effects on the incomes of much later descendants. However, the incomes of children, grandchildren, and other early descendants could significantly increase; indeed, if the sum of these parameters exceeds unity, the changes in income rise for several generations before falling, and the maximum increase in income could exceed the initial increase.

I. Introduction

More than a decade ago, one of us wrote: "How does one explain then that in spite of the rapid accumulation of empirical information and the persisting and even increasing interest in [the distribution of income], . . . economists have somewhat neglected the study of personal income distribution during the past generation? In my judgment the fundamental reason is the absence, despite ingenious and valiant efforts, of a theory that both articulates with general economic theory and is useful in explaining actual differences among regions, countries, and time periods" (Becker 1967, p. 1). Although the "just" distribution of income has since received an enormous amount of attention (see, e.g., Rawls [1971] and Okun [1975]), a satisfactory theory of the actual distribution has not been developed.

A full analysis of the distribution of income should include both the inequality in income between different generations of the same family—what is usually called intergenerational "social" mobility—and the inequality in income between different families in the same generation. Sociologists have been mainly concerned with intergenerational mobility and economists with inequality within a generation because they have held divergent views about the forces generating inequality. Sociologists have emphasized the role of an individual's forebears in the determination of his "socioeconomic" status through their influence on his background, class, or caste (see Blau and Duncan [1967] or Boudon [1974]). On the other hand, most models of inequality by economists have neglected the transmission of inequality through the family because they have assumed that stochastic processes largely determine inequality through distributions of luck and abilities (see Roy [1950] or Champernowne [1953]).

Two recent analytical developments suggest that a unified approach to intergenerational mobility and inequality is possible. The human-capital model shows that inequality can result from maximizing behavior without major reliance on luck and other stochastic forces.[1] The economic approach to social interactions (Becker 1974; Becker and Tomes 1976; Tomes 1978) views an individual not in isolation but as part of a family whose members span several generations. Members contribute to the production of family income and to the care of children who continue the family into the future.

The central decision makers in this essay are infinitely long-lived families with mortal members in each generation. The current generation can increase its consumption at the expense of future generations but is discouraged from doing so by its concern for the interests of its children and perhaps of other future family members. This link between generations of the same family is buttressed by family "endowments" transferred from parents to children, including a family's caste, religion, race, "culture," genes, and reputation for honesty and reliability.

Our theory incorporates the human-capital approach to inequality because parents maximize their utility by choosing optimal investments in the human and nonhuman capital of children and other members. Moreover, the theory recognizes that endowments and market rewards depend on luck, so that incomes are partly determined by the interaction between luck and maximizing behavior.

We show that the inequality in family income and intergenerational mobility over time approach equilibrium levels that depend on luck and various family parameters, especially the "inheritability" of endowments and the propensity to invest in children. They also depend, sometimes in surprising ways, on the rate of economic growth, taxes and subsidies, foresight about the incidence of "disturbances," discrimination against minorities, and family reputations. For example, even a progressive tax-subsidy system might raise the inequality in disposable incomes, and discrimination against blacks has raised their intergenerational mobility and reduced their incomes.

The theory developed in this essay is an outgrowth of our work on interactions and investments within a family (see Becker 1967, 1974; Becker and Tomes 1976; Tomes 1978). It is related to some other recent approaches to inequality that also focus on the family (esp. Loury [in press], but also see Blinder [1973, 1976], Conlisk [1974], and Parsons [1977]). However, many results appear to be new because our treatment of endowments and inheritability, the propensity to invest, family income, and some other aspects of the family is new.

II. A Single Family

The utility function of a parent is assumed to depend not only on his own consumption, but also on the number of his children and various

[1] See Mincer (1958) and Becker (1967); also see the "abilities" models of Roy (1950), Mandelbrot (1962), Houthakker (1975), and Rosen (1978).

characteristics of each child. If all his children are identical, this utility function can be written as

$$U_t = U_t(Z_t, \psi_{t+1}, n), \tag{1}$$

where Z_t is the parent's consumption, n the number of children, ψ_{t+1} the relevant characteristics of each child, and t refers to the tth generation. These children are born in and accumulate human and nonhuman capital in the tth generation and work, consume, and produce their own children in the $t+1$ generation.

To simplify the presentation, we assume that a parent only cares about his own consumption and the total characteristics of all his children, $n\psi$, and that $\psi_{t+1} = I'_{t+1}$, where I'_{t+1} equals the adult wealth of each child. This last assumption would be appropriate if parents only cared about the "quality" or economic success of their children (see the formulation in Becker and Tomes [1976]). The utility function in equation (1) can then be written as

$$U_t = U_t(Z_t, I_{t+1}), \tag{2}$$

where $I_{t+1} = nI'_{t+1}$ is the aggregate wealth of children. Some readers may prefer the assumption introduced in Appendix A that parents are altruists who care about the welfare or utility of their children. Although caring about quality is by no means the same as caring about welfare, they turn out to have quite similar implications for the distribution of income. Therefore, since the analytic development is much easier for quality, we feel justified in assuming during most of this essay that parents care about quality, as measured by the wealth of children.

A parent can change the wealth of his children by investing in their human and nonhuman capital. Initially, all capital is assumed to be homogeneous, and y_t is the total amount invested in children, measured in physical units. If Π_t is the cost in forgone consumption of each unit of y_t, the budget equation of parents can be written as

$$Z_t + \Pi_t y_t = I_t, \tag{3}$$

where I_t is their wealth. If the value to children of each unit of capital equals w_{t+1}, the rate of return on these investments is defined by the equation

$$\Pi_t y_t = \frac{w_{t+1} y_t}{1+r_t}, \tag{4}$$

where r_t is the rate of return per generation, which may encompass 20 years.

The total wealth of children equals the sum of their wealth from the capital invested in them; from their endowed capital, e_{t+1}; and from the "capital gain" due to luck in the market for income, u_{t+1}:

$$I_{t+1} = w_{t+1} y_t + w_{t+1} e_{t+1} + w_{t+1} u_{t+1}. \quad (5)$$

The government sector is ignored until Section VII, so no distinction need be made yet between before- and after-tax wealth. Since wealth can be converted into "permanent" income streams, we will treat Z_t and I_t as referring to flows of consumption and income within a generation,[2] although the basic analysis applies more directly to wealth and present values of flows.

If equations (4) and (5) are substituted into (3), the parent's budget constraint can be written in terms of the variables that enter his utility function:

$$Z_t + \frac{I_{t+1}}{1+r_t} = I_t + \frac{w_{t+1} e_{t+1}}{1+r_t} + \frac{w_{t+1} u_{t+1}}{1+r_t} = S_t. \quad (6)$$

Own consumption and the income of children are determined not by own income alone, but also by the value of the endowment and luck of children, discounted to the parent's generation. The sum of these values, denoted by S_t, will be called "family income."[3]

Parents maximize their utility with respect to Z_t and I_{t+1} subject to their expectations about family income. If they correctly anticipate both the endowment and market luck of their children, the equilibrium conditions are given by equation (6) and

$$\frac{\partial U}{\partial Z_t} \Big/ \frac{\partial U}{\partial I_{t+1}} = 1 + r_t. \quad (7)$$

If the utility function is assumed to be homothetic, so that Z_t and I_{t+1} both have unitary family income elasticities, these equilibrium conditions determine linear demand functions for Z_t, I_{t+1}, and y_t that can be written as

$$\begin{aligned} \frac{I_{t+1}}{1+r_t} &= \alpha(\gamma,\ 1+r)\, S_t, \\ Z_t &= (1-\alpha)\, S_t, \\ \frac{1}{1+r_t} w_{t+1} y_t &= \alpha S_t - \frac{1}{1+r_t} w_{t+1} e_{t+1} - \frac{1}{1+r_t} w_{t+1} u_{t+1}. \end{aligned} \quad (8)$$

The parameter γ measures the preference for the income of children relative to own consumption, where $\partial\alpha/\partial\gamma > 0$ and $\partial\alpha/\partial(1+r) \gtreqless 0$ as the elasticity of substitution between Z_t and I_{t+1} in the utility function exceeds, equals, or falls short of unity.

[2] We ignore life-cycle considerations in this essay, even though many discussions of inequality really refer only to different stages of the life-cycle variation in income; the allocation of resources over the life cycle has been rather fully analyzed elsewhere (Ghez and Becker 1975; Blinder and Weiss 1976; Heckman 1976).

[3] Family income is a special case of "social income" introduced in Becker (1974).

The equilibrium condition given by equation (7) assumes that the rate of return is independent of the amount invested in children and that parents can consume more than their own income by creating a debt to be repaid by their children. Both assumptions are maintained in the formal development of the analysis but are modified later on, especially when the distinction between human and nonhuman capital is explicitly considered (Section VIII*B*).

By substituting the definition of family income into (8), the demand function for the income of children can be written as

$$\begin{aligned} I_{t+1} &= \alpha(1 + r_t) I_t + \alpha w_{t+1} e_{t+1} + \alpha w_{t+1} u_{t+1} \\ &= \beta_t I_t + \alpha w_{t+1} e_{t+1} + \alpha w_{t+1} u_{t+1}, \end{aligned} \tag{9}$$

where $\beta_t = \alpha(1 + r_t)$. Also,

$$w_{t+1} y_t = \beta_t I_t - (1 - \alpha) w_{t+1} e_{t+1} - (1 - \alpha) w_{t+1} u_{t+1}.$$

If parents correctly anticipate their children's luck and endowment, an increase in either would not add an equal amount to the income of children because part of the increase would be spent on the parents' own consumption through reduced investment in their children; this can be seen from the negative relation between y_t and e_{t+1} (or u_{t+1}). Equation (9) shows that the equilibrium relation between I_{t+1} and e_{t+1} (and u_{t+1}) depends on α, the fraction of S_t that is spent on children. This equation also shows that I_{t+1} is related to I_t through β_t, which can be called the propensity to invest in children. This propensity links the incomes of parents and children and is one of the important building blocks in our analysis of inequality and intergenerational mobility.

The concept of the endowment is also a fundamental part of our analysis. Children are assumed to receive endowments of capital that are determined by the reputation and "connections" of their families, the contribution to the ability, race, and other characteristics of children from the genetic constitutions of their families, and the learning, skills, goals, and other "family commodities" acquired through belonging to a particular family culture. Obviously, endowments depend on many characteristics of parents, grandparents, and other family members and may also be culturally influenced by other families.

To simplify the analysis, the expected endowment is assumed to depend only on the endowments of parents and the average endowment in society. No significant generality for present purposes is lost by neglecting nonendowed incomes or the endowments of grandparents and more distant relatives (a more general formulation can be found in Becker [in press]). A simple linear endowment-generating equation can be written as

$$e_{t+1} = (1 - h + f)\bar{e}_t + he_t + v_{t+1}, \tag{10}$$

where e_t is the endowment of parents, h is a constant that measures the fraction of e_t transmitted to ("inherited by") children, $\bar{e}_t$ is the average endowment in generation t, f is the rate of growth of $\bar{e}_t$, and the term $(1 - h + f)\bar{e}_t$ is a simple way of incorporating the influence of the "culture" or "social capital" of other families (for a formulation of cultural transmission along these lines, see Cavalli-Sforza and Feldman [1973]). The difference between actual and expected endowment, v_{t+1}, measures the exogenous component in the endowment of children.

If equation (10) is substituted for e_{t+1} in the demand functions in equation (9), the income of and investments in children would depend on parents' income and endowment, children's market and endowed "luck," and the average endowment in the parents' generation:

$$\begin{aligned} I_{t+1} &= \alpha w_{t+1}(1 - h + f)\,\bar{e}_t + \beta_t I_t + \alpha h w_{t+1} e_t \\ &\quad + \alpha w_{t+1}\, v_{t+1} + \alpha w_{t+1}\, u_{t+1}, \\ w_{t+1}\, y_t &= \beta_t I_t - (1 - \alpha)\, w_{t+1}(1 - h + f)\,\bar{e}_t - (1 - \alpha) h w_{t+1} e_t \\ &\quad - (1 - \alpha)\, w_{t+1}\, v_{t+1} - (1 - \alpha)\, w_{t+1}\, u_{t+1}. \end{aligned} \tag{11}$$

Each dollar of endowed luck also raises the equilibrium income of children only by α dollars because the rest is spent on parents' consumption through their reduced investments in children.

Equation (11) shows that each dollar's worth of parent endowment directly raises the income of children by less than h dollars, the amount "inherited" by children, because some of the latter's endowment is "spent" on parents' consumption through reduced investment. However, an increase in the parent endowment also directly raises their own income, which increases their investment in children. The total effect of a change in parent endowment on the income of children is the sum of these effects:

$$\begin{aligned} \frac{dI_{t+1}}{d\, w_{t+1}\, e_t} &= \alpha h + \beta_t \frac{dI_t}{d\, w_{t+1}\, e_t} = \alpha h + \beta_t \alpha \frac{w_t}{w_{t+1}} \\ &= \alpha\left(h + \beta_t \frac{w_t}{w_{t+1}}\right). \end{aligned} \tag{12}$$

The total effect would exceed the degree of inheritability if the fraction of family income spent on children and the propensity to invest were large relative to the degree of inheritability. Indeed, if they were sufficiently large, the total effect would exceed the change in the parent endowment.

Clearly, the income of children is affected differently by the endowment and other income of parents because their endowment not only raises their income but also directly raises the income of children through the transmission of endowments. Consequently, the income of children depends on the division of their parents' income between endowment and other sources.

Parents may be able to anticipate fully the endowment luck of their children because unusual ability, motivation, or handicaps are often revealed prior to the time when most investments in children are committed. The market luck of children, however, is determined by fluctuations in production possibilities and the prices of goods and factors of production that are often revealed only after children have received their education and much of their other training and entered the labor force (although usually prior to their "inheritance" of some nonhuman capital). Therefore, parents may have to commit most of their investments before they know a great deal about their children's market luck.

If parents can fully anticipate the endowed luck but cannot anticipate the market luck of children, and if parents are risk neutral[4] and maximize utility subject to expected family income, the demand function for the expected income of children would be

$$E_t I_{t+1} = \beta_t E_t S_t, \tag{13}$$

where E_t represents expectations taken on the basis of information available at time t. If the incidence of u_{t+1} cannot be anticipated, $E_t S_t = I_t + [(w_{t+1}\, e_{t+1})/(1 + r_t)]$; hence

$$\begin{aligned} I_{t+1} &= E_t I_{t+1} + w_{t+1}\, u_{t+1} = \alpha w_{t+1}(1 - h + f)\,\overline{e}_t \\ &\quad + \beta_t I_t + \alpha h w_{t+1}\, e_t + \alpha w_{t+1}\, v_{t+1} + w_{t+1}\, u_{t+1}, \\ w_{t+1}\, y_t &= \beta_t I_t - (1 - \alpha)\, w_{t+1}(1 - h + f)\,\overline{e}_t \\ &\quad - (1 - \alpha)\, h w_{t+1}\, e_t - (1 - \alpha)\, w_{t+1}\, v_{t+1}. \end{aligned} \tag{11'}$$

The only difference between equations (11) and (11′) is in the coefficient of market luck. Increased investment cannot partially offset bad luck and reduced investment cannot partially nullify good luck if luck cannot be anticipated. Hence the coefficient of market luck is raised from α in the equation for I_{t+1} in (11) to unity in (11′) and from $-(1 - \alpha)$ in the equation for y_t in (11) to zero in (11′).

III. The Equilibrium Inequality in Income

Even if all families were basically identical, incomes would be unequally distributed because of the unequal incidence of endowment

[4] The effect of risk aversion on the amount invested in children is ambiguous in the sense that the effect depends on the third derivative of the utility function (see Loury, in press).

and market luck. The income inequality in any generation depends, of course, on the inequality of luck in that generation, but also, in a decisive way, on the luck in previous generations. Since lucky parents invest more in their children, the increase in the children's incomes would, in turn, induce them to invest more in their own children in the succeeding generation, and so on until all descendants benefit from the initial luck. Since investments depend on the parameters β and h that measure the propensity to invest in children and the degree of inheritability of endowments, the effect of luck in previous generations on the income inequality in a given generation must also depend on these parameters.

The exact relation between the income inequality in any generation, the incidence of luck in that and in previous generations, and β and h can be derived from equation (11) or equation (11′). To separate the effect of differences among families from the incidence of luck, we assume until Section VI that all families have the same utility function, degree of inheritability, and rate of return. We also avoid any discussion of factor market equilibrium by assuming that r_t and w_t are independent of the aggregate accumulation of capital and are given to the community as well as to each family. Until Section VIII*A* these parameters and the average endowment are assumed to be stationary over time, so that $r_t = r$, $w_t = w = 1$ by the choice of units, and $f = 0$. The income of children of the ith family in the $t + 1$ generation can then be expressed as

$$I^i_{t+1} = \alpha a + \beta I^i_t + \alpha h e^i_t + \alpha v^i_{t+1} + \alpha u^i_{t+1}, \quad (14)$$

[5]

where $a = \bar{e}(1 - h)$.

We assume that children have the same utility function as their parents and are produced without mating, or asexually. A given family then maintains its identity indefinitely, and its fortunes can be followed over as many generations as desired. Asexual reproduction could be replaced without any effect on the analysis by perfect assortative mating: each person, in effect, then mates with his own image.

Since all families are assumed to be identical, they would have the same income in any generation if they have had the same luck in that and in all previous generations. Therefore, the income inequality in any generation would depend on the distribution of luck in all previ-

[5] In an interesting article on social mobility, Conlisk (1974) assumes an equation structure with a reduced form similar to eq. (14) (see his eq. [16], p. 84). However, his structure is not derived from utility-maximizing behavior and does not incorporate the relations between the coefficients of I^i_t, e^i_t, v^i_{t+1}, and u^i_{t+1} implied by maximizing behavior and found in eq. (14), such as the effect of a change in r on β and α. Moreover, the coefficients in his equations are not related to market or household characteristics, such as rates of return on investments or the importance of children in parental preferences.

ous generations. This can be shown explicitly by repeatedly substituting equations (10) and (14) into (14) to relate the income of the ith family in the $t + 1$ generation to its income and endowment in the $m + 1$ prior generation and to its luck in all intervening generations:

$$\begin{aligned} I^i_{t+1} = {} & \alpha a \sum_{j=0}^{m} \beta^j \sum_{k=0}^{m-j} h^k + \beta^{m+1} I^i_{t-m} \\ & + \alpha h \left(\sum_{j=0}^{m} \beta^{m-j} h^j \right) e^i_{t-m} + \alpha \sum_{j=0}^{m} \beta^j u^i_{t+1-j} \\ & + \alpha \sum_{k=0}^{m} \sum_{j=0}^{k} \beta^j h^{k-j} v^i_{t+1-k}. \end{aligned} \tag{15}$$

Presumably, $0 < h < 1$; some, but only part, of the parent's endowment passes to his children. The rate of return, r, has the units of percent per generation, and even a modest percent per year implies a sizable percent per generation because human generations are separated by 20 or perhaps more years. So r would exceed 0.5 and might well be above unity. Therefore, $\beta = \alpha(1 + r)$ also might exceed unity because α, the fraction of family income spent on children, is far from negligible.

If, however, we assume that β, as well as h, is less than unity ($\beta > 1$ is considered in Section VIII*A*), the coefficients of both I^i_{t-m} and e^i_{t-m} approach zero as m becomes larger and larger, and the coefficient of αa approaches a constant. Since

$$\sum_{j=0}^{k} \beta^j h^{k-j} = \begin{cases} \dfrac{\beta^{k+1} - h^{k+1}}{\beta - h}, & \text{for } \beta \neq h; \\ \beta^k (k + 1), & \text{for } \beta = h, \end{cases}$$

equation (15) could be extended back through infinitely many generations and written as (for $\beta \neq h$)

$$I^i_{t+1} = \frac{\alpha a}{(1 - \beta)(1 - h)} + \alpha \sum_{k=0}^{\infty} \beta^k u^i_{t+1-k} + \alpha \sum_{k=0}^{\infty} \left(\frac{\beta^{k+1} - h^{k+1}}{\beta - h} \right) v^i_{t+1-k}. \tag{16}$$

The income of the ith family in any generation is expressed solely in terms of its luck in this and all previous generations, the family parameters α, β, and h, and the social parameter a. Starting from any initial distribution of income and endowment, the distribution of income would change over time and eventually approach the right-hand side of equation (16).

If the u_t and the v_t were identically distributed random variables with finite variances, the variance of income must approach a stationary level without any additional restrictions on the properties of u_t

and v_t or on the utility function. If u_t and v_t were also independently distributed, the stationary variance can be simply written as

$$\sigma_I^2 = \frac{\alpha^2}{1-\beta^2}\sigma_u^2 + \frac{\alpha^2(1+h\beta)\sigma_v^2}{(1-h^2)(1-\beta^2)(1-h\beta)}, \tag{17}$$

where σ_I^2, σ_u^2, and σ_v^2 are the variances of I, u, and v, respectively (see Appendix B).

Since the expected value of both endowed and market luck equals zero, equation (16) shows that expected or average income in any generation must approach the stationary level

$$\bar{I} = \frac{\alpha a}{(1-\beta)(1-h)} = \frac{\alpha \bar{e}}{1-\beta}, \tag{18}$$

since $a = \bar{e}(1-h)$ (steady-state growth is discussed in Section VIII*A*). Average income is a simple function of the family parameters α and β and the social parameter $\bar{e}$ and is independent of the inheritability of endowments. The fraction of $\bar{I}$ contributed by investments is given by

$$d = 1 - \frac{\bar{e}}{\bar{I}} = 1 - \frac{(1-\beta)}{\alpha} = 2 + r - \frac{1}{\alpha}; \tag{19}$$

not surprisingly, this fraction is positively related to the rate of return on investments and to the fraction of family income invested in children.

Writers on social "justice" and on the political process have usually been interested in relative measures of inequality, such as the Gini coefficient or the coefficient of variation. If the expression in equation (17) is divided by the square of the expression in (18), the square of the equilibrium coefficient of variation in income can be written as

$$\begin{aligned} CV_I^2 &= \frac{1-\beta}{1+\beta} CV_u^2 + \frac{(1+h\beta)(1-\beta)}{(1-h^2)(1-h\beta)(1+\beta)} CV_v^2 \\ &= \frac{1-\beta}{1+\beta} CV_u^2 + \frac{(1+h\beta)(1-\beta)}{(1-h\beta)(1+\beta)} CV_e^2, \end{aligned} \tag{20}$$

since $\sigma_v^2 = (1-h^2)\,\sigma_e^2$ (see Appendix B), where the inequality in both market and endowment luck has been measured relative to the average endowment since both u and v are assumed to have zero means: $CV_u = (\sigma_u/\bar{e})$ and $CV_v = (\sigma_v/\bar{e})$.

Of course, the equilibrium inequality in income depends on, and indeed is proportional to, the inequality in market and endowed luck. The factors of proportionality, however, are determined by families through the inheritability of endowments and the propensity to invest in children. Since $\beta < 1$, the coefficient of market luck must be less than unity, probably less than one-third because β almost certainly exceeds one-half. Therefore, the effect of market luck on inequality is greatly attenuated by the reactions of parents to its anticipated incidence.

The coefficient of endowed luck exceeds that of market luck, and the difference is large when both h and β are large; for example, the coefficient of CV_v^2 would be about 2.5 times and the coefficient of CV_e^2 would be about twice that of CV_u^2 when $\beta = .6$ and $h = .5$. Endowed luck has a much greater effect on income inequality because it is automatically inherited by children. This also explains why endowed luck has a larger effect on income inequality when h is greater.

Not only does the coefficient of endowed luck exceed the coefficient of market luck, but also the inequality in endowed luck probably exceeds the inequality in market luck. Since variations within a life cycle are ignored, the income concept is close to permanent income (see the discussion after eq. [5]). Endowed luck is determined by genetic inheritance and childhood experiences and tends to last throughout a lifetime, whereas market luck is more transitory and fluctuates from year to year. Therefore, the inequality in lifetime endowed luck would be considerably greater than the inequality in lifetime market luck if the annual inequality in market and endowed luck were about the same. Section V shows how the inequality in both endowed and market luck could be estimated from information on the permanent incomes of different generations.

An increase in the rate of return raises the propensity to invest $\beta = \alpha(1 + r)$, which, according to equation (17), would raise the equilibrium variance of income. An increase in β also raises the equilibrium level of average income (see eq. [18]); indeed, the percentage increase in the latter exceeds the percentage increase in the standard deviation, so that an increase in the rate of return and the propensity to invest lowers the coefficient of variation in income in equation (20). A well-known result from human-capital theory states the contrary, that an increase in the rate of return on human capital raises inequality, but this result only considers the *impact* of a change in the rate of return on income inequality and neglects longer-run effects on the level of income.[6] The negative relation between income inequality and the rate of return implied by equation (20) is used in later sections to determine the effect of racial discrimination, taxation, and economic growth on inequality (see Sections VI, VII, and VIII).

[6] I.e., the result neglects the effect of a change in rates of return on the average level and distribution of investments in human capital (see Becker 1964 or Chiswick 1974). One analysis that does consider the effect on the equilibrium distribution of investments does not find a clear-cut relation between income inequality and rates of return (see Becker 1967).

An increase in h raises income inequality by raising the coefficient of endowments. Perhaps the most interesting property of equation (20) is that h and β do not enter additively but multiplicatively: the effect of an increase in h on income inequality is greater when β is greater. This interaction reflects the interaction in the model between inheritability and investment in children. The parameters β and h interact through the covariance between income and endowment in any generation (see Appendix B).

The effect of utility maximization on this interaction between inheritability and investment, as well as on other properties of the equation generating income inequality (eq. [20]), can be determined from a comparison with the equation generating inequality when families do not maximize. If the amount invested in children were independent of rates of return, family income, endowments, and luck, the interaction between inheritability and investment would be eliminated, and the contribution of endowment inequality to income inequality would be greatly reduced.[7] For example, if $h = .5$ and $\beta = .6$, the coefficient of endowment inequality would be twice that of market luck with utility maximization and only the same as that of market luck without maximization. Therefore, "mechanical" models of the intergenerational transmission of inequality that do not incorporate optimizing responses of parents to their own or to their children's circumstances greatly understate the contribution of endowments and thereby understate the influence of family background on inequality.

If parents could not anticipate their children's market luck but were risk neutral and had unbiased expectations, the coefficient of CV_u^2 in equation (20) would simply be multiplied by $1/\alpha^2$. Since α is below unity, imperfectly anticipated disturbances increase the variability in individual incomes, as well as the cyclical variability in aggregate incomes (on the latter, see Sargent and Wallace [1975]). Moreover, the coefficient of market luck might then exceed the coefficient of endowed luck because parents could not offset the bad or good market luck of their children with larger or smaller investments.

[7] The equilibrium variance of income would equal

$$\sigma_I^2 = \sigma_y^2 + \sigma_e^2 + \sigma_u^2 = \sigma_y^2 + \frac{\sigma_v^2}{1-h^2} + \sigma_u^2,$$

and average income would be

$$\bar{I} = \bar{y} + \bar{e} = \frac{\bar{e}}{1-d},$$

where d is defined in eq. (19). Then

$$CV_I^2 = (1-d)^2 CV_u^2 + (1-d)^2 CV_e^2 + d^2 CV_y^2 = (1-d)^2 CV_u^2 + \frac{(1-d)^2}{1-h^2} CV_v^2 + d^2 CV_y^2,$$

where $CV_y = \sigma_u / \bar{y}$.

IV. Intergenerational Mobility

Variation in the income and status of a given family in different generations has usually been discussed under the heading of intergenerational mobility, the "circulation of elites" (Pareto 1935), or equality of opportunity. Considerable inequality among different families in the same generation is consistent with a highly stable ranking of a given family in different generations, or an unstable ranking is consistent with only moderate inequality in the same generation. An enormous literature discusses each type of inequality, yet they have rarely been brought together through a common analytic framework. This section analyzes intergenerational mobility with the framework used in the previous section for intragenerational inequality and shows that the propensity to invest in children and the degree of inheritability also are important determinants of intergenerational mobility.

The influence of the family on the income of children can be measured by the correlation between their incomes and those of their parents or grandparents. If the degree of inheritability, h, were negligible, the equilibrium correlation coefficient between the incomes of children and parents would equal β, the propensity to invest in children, regardless of the inequality in market and endowed luck.[8] If h were not negligible, the equilibrium multiple correlation coefficient between the income of children and the income *and* endowment of parents would exceed β by an amount that depends only on β and h if the inequality in market luck were small relative to the inequality in endowments (see Appendix C for a proof; also see the discussion of luck in the previous section).

We will spend the rest of this section on a different and in some ways more revealing measure of intergenerational mobility: the sequence of changes in the incomes of parents, children, grandchildren, and later descendants. If the degree of inheritability were negligible, an increase in the income of parents by δI_t because of market or endowed luck would increase the income of children by $\beta\delta I_t$, the income of grandchildren by $\beta^2\delta I_t$, and the income of the mth generation of descendants by

$$\delta I_{t+m} = \beta^m \delta I_t,\ m = 1, 2, \ldots, \tag{21}$$

when $h = 0$ (see eq. [15]). These increases decline monotonically as long as $\beta < 1$ and are close to zero after a few generations if $\beta < .8$:

[8] Since $I^i_{t+1} = \beta I^i_t + \alpha e^i_{t+1} + \alpha u^i_{t+1}$,

$$R(I_{t+1}, I_t) = \frac{\beta\sigma_{I_t}}{\sigma_{I_{t+1}}} = \beta$$

because e_{t+1} is independent of I_t if $h = 0$ and, in equilibrium, $\sigma_{I_t} = \sigma_{I_{t+1}}$.

"from shirtsleeves to shirtsleeves in four generations." Consequently, utility maximization without inheritability of endowments implies considerable intergenerational mobility unless β is close to unity: that is, unless an increase in the income of parents increases their investment in children by almost an equal amount.

If investments did not depend on income or other variables and were simply given to each family, an increase in the endowment of parents by δv_t would increase the income of their children by $h\delta v_t$, the income of grandchildren by $h^2\delta v_t$, and the income of the mth generation of descendants by

$$\delta I_{t+m} = h^m \delta v_t,\ m = 1, 2, \ldots, \tag{22}$$

when y_t is exogenous. These increases also decline monotonically if $h < 1$ and would be close to zero after a few generations because usually $h < .75$. Consequently, cultural and biological inheritance with exogenous parental investments implies considerable intergenerational mobility except where the degree of inheritability is close to unity.

If investments in children depended on family circumstances, and if the degree of inheritability were not negligible, an increase in the income of parents would not simply raise the incomes of their descendants by the sum of the increases given in equations (21) and (22) because inheritances and investments interact. In particular, the incomes of descendants could continue to rise for several generations even though h and β were both less than unity, and many generations might elapse before the increases were below 25 percent of the initial increase. Consequently, the interaction between investments and inheritances can sharply reduce the degree of intergenerational mobility, so that incomes in any generation become more dependent on the incomes and endowments of ancestors.

Consider, for example, an increase in the endowment luck of the ith family in the tth generation that is compensated by a decline in market luck so that own income, I_t^i, remains the same. Since family income, S_t^i, increases because the endowment of children increases by $h\delta v_t^i$, parents want to increase their own consumption and reduce their investment in children. Consequently, the own income of children, I_{t+1}^i, would increase only by the fraction α of their increased endowment, because the rest is spent by parents on their own consumption. The own income of grandchildren, I_{t+2}^i, would also increase, partly because the own income of their parents increased and partly because they inherit some of the increased endowment of their parents. The total increase in the income of grandchildren would be

$$\begin{aligned} \delta I_{t+2}^i = \beta \delta I_{t+1}^i + \alpha \delta e_{t+2}^i &= \alpha h \beta v_t^i + \alpha h^2 \delta v_t^i \\ &= \alpha h(\beta + h)\delta v_t^i = (\beta + h)\delta I_{t+1}^i. \end{aligned} \tag{23}$$

Therefore, if $\beta + h > 1$, if the sum of the degree of inheritability and the propensity to invest in children exceeded unity, a compensated increase in the endowment of parents would increase the income of grandchildren by more than the income of children.

The effects on the incomes of great-grandchildren, great-great-grandchildren, and still more distant descendants can be derived in the same way. The increase in the income of, say, great-great-grandchildren would also exceed that of children if $\beta + h$ were sufficiently greater than unity. A general formula relating the change in the income of the mth generation of descendants to a compensated change in the endowment of parents is given by the coefficient of e^i_{t-m} in equation (15). This coefficient can be measured relative to the equilibrium level of average income and written as

$$\frac{\delta I^i_{t+m}}{\bar{I}} = h(1 - \beta) \sum_{j=0}^{m-1} \beta^{m-1-j} h^j \frac{\delta e^i_t}{\bar{e}}$$

$$= \begin{cases} h(1 - \beta) \dfrac{\beta^m - h^m}{\beta - h} \dfrac{\delta e^i_t}{\bar{e}} = h(1 - \beta)\, g_m \dfrac{\delta e^i_t}{\bar{e}}, & \text{for } \beta \neq h; \\ h(1 - \beta)\, m\beta^{m-1} \dfrac{\delta e^i_t}{\bar{e}}, & \text{for } \beta = h. \end{cases} \quad (24)$$

The term g_m is a symmetric polynomial in β and h that has a maximum at the initial generation when $\beta + h < 1$ and rises to a peak and then declines monotonically when $\beta + h > 1$, where the peak is later the larger $\beta + h$ is (see Appendix D). Figure 1 plots the path of g_m for three sets of values β and h. In curve A both are "low," $h = .2$ and $\beta = .45$, and by the fourth generation g_m is only 16 percent of its initial value; in curve B, $h = .3$ and $\beta = .8$, and g_m rises for one generation and then declines to less than 25 percent of the initial value by the tenth generation; in curve C, $h = .7$ and $\beta = .9$, and g_m rises for five generations, then declines slowly and does not reach its initial value until the fifteenth generation and is less than 25 percent of the initial value only after the twenty-ninth generation.

The income of a given family can be well above or below average for several consecutive generations because of a run of very good or bad luck; that is, because the u^i and v^i in equation (15) have the same sign and are not negligible for several consecutive generations. Since these random variables are assumed to be independently distributed, the probability is low that more than two consecutive generations have unusually good or bad luck. However, the income of a family with unusual luck in only one generation and average luck in all subsequent generations would also be well above or below average for

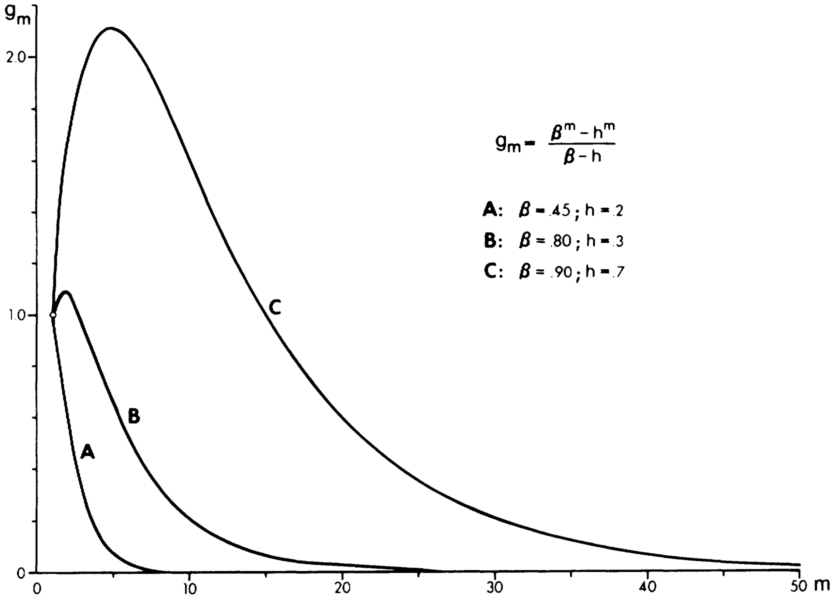

FIG. 1.—Time path of income with different values for the propensity to invest and the degree of inheritability.

several consecutive generations if the degree of inheritability and the propensity to invest in children were substantial.

Consequently, the welfare of several consecutive generations of the same family would be closely linked whenever inheritability and investments were substantial. The degree of inheritability is not rigidly determined by the biology of human inheritance but is greatly influenced by social organization. Some societies, which can be called premodern or "closed," rely heavily on family reputation in assessing various characteristics of individuals because such societies do not have accurate methods of assessing these characteristics directly. Families then have an incentive to maintain and enhance their reputations by controlling and guiding the characteristics of their members (see the fuller discussion in Becker [in press]). As a result of these efforts, members of the same family become more similar in closed than in modern "open" societies, where the behavior and development of children are less controlled by their families.

In terms of the concepts used in this essay, the endowments of children would be more similar to those of parents and other family members in closed than in open societies—that is, the inheritability of endowments would be greater in closed societies. We have shown that intergenerational mobility is smaller when endowments are more inheritable and that the degree of inheritability interacts with the propensity to invest in children. Therefore, if the propensity to invest were the same, the income of children would be more similar to the incomes of their parents, grandparents, and other relatives in closed than in open societies—which is a different way to state that mobility would be smaller—because the interaction among cousins, uncles, nephews, grandparents, grandchildren, and other family members is greater in closed societies.

Since a run of successes or failures in the same family is more likely in closed than in open societies because inheritability is greater, perhaps the Adams family in the United States has received so much attention precisely because accomplishment over many generations is unusual in this open society. Successful families are presumably more common and less worthy of attention in closed societies like traditional India or China.

V. Estimation of Family and Market Parameters

The concept of the endowment may not seem to be useful because endowments cannot readily be measured and quantified. Fortunately, equations (9) and (14) can be combined to eliminate endowments and produce a second-order stochastic difference equation that depends only on the income of the same family in three consecutive generations and its market and endowed luck.[9] The effect of the incomes of parents and grandparents on the income of children is determined only by the propensity to invest and the degree of inheritability:

$$\begin{aligned} I_{t+1}^i &= \alpha\bar{e}(1-h) + (\beta+h)I_t^i - \beta h I_{t-1}^i + \alpha\overset{*}{u}_{t+1}^i \\ &= \alpha\bar{e}(1-h) + (\beta+h-\beta h)I_t^i + \beta h(I_t^i - I_{t-1}^i) + \alpha\overset{*}{u}_{t+1}^i, \text{ where} \quad (25) \\ \overset{*}{u}_{t+1}^i &= u_{t+1}^i - hu_t^i + v_{t+1}^i, \text{ and } \beta + h - \beta h < 1 \text{ if } \beta, h < 1. \end{aligned}$$

The two roots of this equation, β and h, must be less than unity if income is to converge to the stationary level $(\alpha\bar{e})/(1-\beta)$. The positive coefficient of the difference in income between parents and grandparents measures the "momentum" that carries the growth in income between these generations into the children's generation.

The residual, $\overset{*}{u}$, is negatively correlated over time because u_t affects $\overset{*}{u}_t$ positively and $\overset{*}{u}_{t+1}$ negatively. Since the absolute value of the covariance between $\overset{*}{u}_{t+1}$ and $\overset{*}{u}_t$ is larger when h is larger, and since $h < 1$ and $E\overset{*}{u}_{t+1} = 0$, the residuals from equation (25) would have damped oscillations around the origin that would be more pronounced in closed societies because h is larger there.

[9] We are indebted to Sherwin Rosen for suggesting this formulation.

An increase in the income of grandparents (I_{t-1}) would lower the income of grandchildren (I_{t+1}) if the income of parents (I_t), the market luck of parents and grandchildren (u_t and u_{t+1}), and the endowed luck of grandchildren (v_{t+1}) were held constant. This negative relation between the incomes of grandparents and grandchildren may seem surprising because we expect an increase in the income of grandparents to raise the income of parents, which in turn would raise the income of grandchildren. The negative relation in equation (25), however, assumes that the income of parents and the stochastic terms u_{t+1}, u_t, and v_{t+1} are held constant. Since the income of grandparents can increase without changing these variables only if the endowed luck of parents, v_t, were to decrease (see eq. [14]), and since a decrease in v_t would decrease I_{t+1} because of the inheritability of endowments (see eq. [23]), such a "compensated" increase in I_{t-1} must decrease I_{t+1}.

Since both β and h could be determined if the two income coefficients in equation (25) were known,[10] the inheritability of endowments could be determined without knowing anything about endowments! Moreover, once β was determined, information on rates of return and the relation $\alpha = \beta/(1 + r)$ could be used to estimate the fraction of family income spent on children, α. The variance in endowment luck relative to the variance in market luck could also be determined without information on endowments if the variance and covariance of the residual in equation (25) were known.[11] Finally, information on β, h, α, and σ_v^2/σ_u^2 could be inserted into equation (17) to determine the variance in endowment luck (and in market luck) from information on the variance in incomes. Consequently, if the parameters in (25) could be estimated, the propensity to invest, the fraction of income spent on children, the degree of inheritability, and the inequality in market and endowed luck could all be estimated without information on endowments. That is, if (25) could be accurately estimated, all the information required to understand the de-

[10] If $\beta + h = a_1$, and $\beta h = a_2$, then $(\beta - h)^2 = a_1^2 - 4a_2$. Since, presumably, $\beta > h$,

$$\beta - h = +\sqrt{a_1^2 - 4a_2} > 0.$$

Therefore,

$$\beta = \frac{a_1 + \sqrt{a_1^2 - 4a_2}}{2}$$

and

$$h = \frac{a_1 - \sqrt{a_1^2 - 4a_2}}{2}.$$

[11] Since $\sigma_{u^*}^2 = \alpha^2[\sigma_u^2(1 + h^2) + \sigma_v^2]$, and $\mathrm{Cov}_{u_t^* u_{t+1}^*} = \alpha^2(-h\sigma_u^2)$, then

$$\frac{\sigma_v^2}{\sigma_u^2} = \frac{-h\sigma_{u^*}^2}{\mathrm{Cov}_{u_t^* u_{t+1}^*}} - (1 + h^2).$$

Consequently, σ_v^2/σ_u^2 could be determined from h, $\sigma_{u^*}^2$, and $\mathrm{Cov}_{u_t^* u_{t+1}^*}$.

terminants of inequality and intergenerational mobility would be readily attainable.

Information on the incomes of three consecutive generations in a population of homogeneous families with stable parameters and faced with a stable environment could be used to estimate equation (25) and, hence, β, h, and α. Although estimates obtained by least squares would be biased downward because the residual $\overset{*}{u}_{t+1}$ is negatively correlated with the predetermined variable I_t, the bias would be small.[12] Information on a fourth consecutive generation could be used to estimate the covariance between $\overset{*}{u}_{t+1}$ and $\overset{*}{u}_t$ and, hence, the ratio of σ_v^2 to $\sigma_{\overset{*}{u}}^2$.

Unfortunately, few data sets contain good information on the incomes of parents and children, let alone on grandparents also. If the income of children were related only to the income of their parents, the expected coefficient would be less than $\beta + h$ because the income of grandparents is omitted, and greater than β because the endowment of parents is omitted (see Appendix E). We believe that regressions relating the income of children only to the income of their parents have often given unreasonably low coefficients (see, e.g., Soltow 1965), mainly because of these and other statistical biases.

VI. Heterogeneous Families

Since income in any generation is a weighted sum of independently distributed random variables (see eq. [16]), it would be more symmetrically distributed than market and endowed luck. This conclusion is disturbing because actual income distributions are invariably quite skewed to the right. Moreover, all families would have the same long-run income, given by equation (18), although blacks in the United States and poorer families everywhere tend to have lower long-run incomes than other families in the same society.

Both difficulties can be simply overcome without changing the basic approach and the linearity of the model by dropping the assumption that all families in a given society are identical. They might have different utility functions, rates of return, average endowments, and degrees of inheritability because of market discrimination and favoritism, or differences between families in talents, abilities, and opportunities. If r, h, $\bar{e}$, and α differed among families but were the

[12] The bias in the estimate of $a_1 = \beta + h$ is

$$\beta + h - \hat{a}_1 = \alpha h b_{u_t I_t},$$

where $b_{u_t I_t}$ is the regression coefficient of market luck in t, u_t, on income in t, I_t. Since this coefficient, α, and h are all considerably less than unity, the relative bias would be quite small if β were not much smaller than h, a very reasonable assumption.

same for all generations of a given family—if parameter values were fully inherited by children—equation (16) would be modified only by introducing a superscript to indicate the ith family's parameters.

If β and h were less than unity for all families, equation (18) implies that the long-run stationary income of the ith family would be

$$\bar{I}^i = \frac{\alpha^i \bar{e}^i}{1 - \beta^i}. \tag{26}$$

Stationary income is independent of the degree of inheritability[13] but is positively related to the average endowment, fraction spent on children, and propensity to invest in children of the ith family. For example, black families in the United States have had "permanently" lower incomes than white families at least partly because they have had lower rates of return on investments in human capital (see Becker 1964).

Since the income of each family fluctuates around its own stationary level, different generations of the same family would be consistently below the average income in society (equal to $[1/n]\Sigma_{i=1}^{n} \bar{I}^i$) if its stationary income were much below average. Similarly, a family would be consistently above the average if its stationary income were much above average. Consequently, the distribution of stationary incomes would affect the degree of intergenerational mobility as measured by the correlation between the income of children and the incomes of parents, grandparents, and other ancestors (see the beginning of Section IV). This correlation is positively related to the average propensity of different families to invest and to differences in stationary incomes due to differences in endowments, the generosity toward children, and especially propensities to invest.

[13] A more reasonable assumption than full inheritability is that the fraction spent on children and the propensity to invest are only partially inherited by children. These parameters would then have generating equations similar to the endowment-generating equation in (10):

$$\alpha_{t+1}^i = (1 - b^i)\,\bar{\alpha}^i + b^i\alpha_t^i + q_{t+1}^i, \qquad 0 \leq b^i \leq 1;$$

$$\beta_{t+1}^i = (1 - c^i)\,\bar{\beta}^i + c^i\beta_t^i + m_{t+1}^i, \qquad 0 \leq c^i \leq 1,$$

where q^i and m^i are disturbances with zero means. The income-generating eq. (9) would be modified to

$$\begin{aligned} I_{t+1}^i &= \beta_t^i I_t^i + \alpha_t^i(e_{t+1}^i + u_{t+1}^i) \\ &= (1 - c^i)\bar{\beta}^i I_t^i + (1 - b^i)\bar{\alpha}^i(e_{t+1}^i + u_{t+1}^i) + (c^i\beta_{t-1}^i + m_t^i)I_t^i \\ &\quad + (b^i\alpha_{t-1}^i + q_t^i)(e_{t+1}^i + u_{t+1}^i). \end{aligned}$$

A stationary level of income would exist if $\bar{\beta}^i < 1$ and would satisfy the equation $\bar{I}^i = \bar{\beta}^i\bar{I}^i + \bar{\alpha}^i\bar{e}^i$; hence $\bar{I}^i = \bar{\alpha}^i\bar{e}^i/(1 - \bar{\beta}^i)$. Stationary income is independent of the coefficients that measure the effect of parents on the propensity to invest (c^i), the fraction spent on children (b^i), and the inheritability of endowments (h^i), but does depend on the permanent effect of the family on these parameters ($\bar{e}^i$, $\bar{\beta}^i$, and $\bar{\alpha}^i$).

If the average propensity to invest of one group of families exceeded that of a second group, and if the distribution of all the parameters were the same in both groups, intergenerational mobility would be greater within the second group. For example, since rates of return on human capital have been lower for blacks than for whites in the United States (although the differences apparently have narrowed significantly in recent years; see Freeman [1978]), the average propensity to invest would be lower for blacks than for whites if preferences are the same in black and in white families.[14] Hence, the correlation between the incomes of parents and children would also be lower for blacks if the distributions of various parameters are the same for blacks and whites. Some regressions by Richard Freeman[15] do show a lower correlation for blacks, especially for older blacks. To take another example, since families with lower rates of return on their investments in human capital tend to receive less education (see Becker 1967), intergenerational mobility should be greater in less educated families. The correlation coefficient between the incomes of parents and children does appear to be lower for less educated families in the United States.

The equilibrium distribution of income also depends on the distribution of propensities to invest, generosity toward children, and inheritability of endowments as well as the average level of these parameters and the distributions of market and endowed luck. Indeed, differences in these parameters can result in considerable inequality and skewness in the distribution of income even when market and endowed luck are negligible. For example, if the expected endowment and the fraction spent on children were the same in all families, and if endowments were not inherited ($h = 0$), the income-generating equation would be

$$I_{t+1}^i = \alpha\bar{e} + \beta^i I_t^i + \alpha(u_{t+1}^i + v_{t+1}^i). \tag{27}$$

If β^i, u_t^i, and v_t^i were normally distributed, and if $I_0^i = I_0$ for all i, I_1^i would be normally distributed with a variance equal to a weighted sum of the variances in β^i, u^i, and v^i. The variance in I_2^i, however, would exceed that in I_1^i, and the distribution of I_2^i could be quite skewed to the right because β^i and I_1^i are positively correlated. The skewness and inequality in I_3^i would exceed that in I_2^i, and both skewness and inequality would continue to increase until the equilib-

[14] The propensity to invest in children, $\beta = \alpha(1 + r)$, is both directly and indirectly affected by the rate of return, r, because an increase in r lowers or raises α as the elasticity of substitution in the parental utility function between their own consumption and the income of children is less than or greater than unity. However, β must change in the same direction as r if the income of children is not an inferior commodity.

[15] Freeman (1978) constructs crude measures of the income of parents from information on their occupations.

rium distribution of income were reached.[16] The main cause of skewness in the distribution of income is the positive correlation between parental income and parents' propensity to invest in children. A family with a relatively high propensity to invest in children also tends to have higher incomes because each generation would spend a relatively small fraction of its income on their own consumption and a relatively large fraction on the capital of subsequent generations.

VII. Government Redistribution of Income

The substantial growth of the government sector in all Western countries during this century is said to reflect an increasing concern about inequality. Fortunately, therefore, taxation, subsidies, and public expenditures can readily be incorporated into the analysis of inequality developed in previous sections. One of the surprising implications is that even a progressive tax and public expenditure system may widen the inequality in disposable income.

Assume that the government does not invest or otherwise engage in "productive activities"[17] but redistributes income by taxation, transfer payments, and other expenditures. The difference between the taxes paid and subsidies received by the ith family in the tth generation is approximated by the linear relation

$$T_t = b + sI_t^g + \Omega_t, \tag{28}$$

where I^g is "taxable" income, b and s are constants, and Ω_t is assumed to be independent of market and endowed luck[18] and hence is called the "unsystematic" component. If $b < 0$ and $s > 0$, the systematic component would be progressive in the sense that the difference between systematic taxes and subsidies would be a larger fraction of income at higher income levels. The marginal tax rate is constant, however, so that the tax system is proportional at the margin. The variable Ω partly measures the difficulty in defining income for tax purposes—for example, leisure is excluded—but probably mainly

[16] The equilibrium distribution is given by eq. (16), with $h = 0$ and $\beta = \beta^i$:

$$I_{t+1}^i = \bar{I}^i + \alpha \sum_{k=0}^{\infty} (\beta^i)^k (u_{t+1-k}^i + v_{t+1-k}^i).$$

Note that the distribution of stationary incomes, $\bar{I}^i = \alpha\bar{e}/(1 - \beta^i)$, is skewed to the right even when β^i is normally distributed because the inverse of a normally distributed variable is skewed to the right.

[17] The analysis of the family developed in this essay is also highly relevant, however, in assessing the effects of "productive" government activities (see Becker and Tomes 1976 and Becker, in press).

[18] Note that, if Ω_t were anticipated by parents, it would not be independent of I_t^g because parental investments would increase when Ω_t increased. Therefore, we assume that Ω_t is independent of u_t and v_t and possibly positively correlated with I_t^g.

measures differences in the political power of persons with similar incomes. Farmers, teachers, and truck drivers, for example, have received greater subsidies than delicatessen owners, auto mechanics, and laborers because they have had greater political power.

If all taxes and subsidies to parents and children were anticipated by parents, the income of parents net of their taxes and subsidies—their disposable income—plus the value to parents of the disposable endowments and market luck of their children would equal disposable family income:

$$S_t^d = I_t^d + \frac{(1-s)(e_{t+1}+u_{t+1})}{1+r_a} - \frac{b}{1+r_a} - \frac{\Omega_{t+1}}{1+r_a}, \quad (29)$$

where r_a is the after-tax rate of return on investments. If parents maximize their utility subject to disposable family income, the income- and investment-generating equations would be

$$\begin{aligned} I_{t+1}^d &= \beta_a I_t^d + \alpha(1-s)(e_{t+1}+u_{t+1}) - \alpha b - \alpha\Omega_{t+1}, \\ y_t &= \beta_a I_t^d - (1-\alpha)(1-s)(e_{t+1}+u_{t+1}) + (1-\alpha)b + (1-\alpha)\Omega_{t+1}, \end{aligned} \quad (30)$$

where $\beta_a = \alpha(1 + r_a)$ is the after-tax propensity to invest. Each dollar of taxes levied against children reduces their disposable income only by α dollars because their parents would increase their investments by $(1 - \alpha)$ dollars.

The government budget is assumed to be balanced in each generation to exclude government expenditures that are financed with debt repaid by subsequent generations. Then $b = -s\bar{I}^g$, and the stationary level of average disposable income (equal to average before-tax income) can be immediately derived from equation (30):

$$\bar{I}^d = \bar{I} = \frac{\alpha(1-s)\bar{e}}{1-\beta_a-\alpha sR},$$

where (31)

$$R = \frac{\bar{I}^g}{\bar{I}}.$$

An increase in government redistribution, an increase in s, would lower the numerator of equation (31), at least if the fraction of income spent on children were not significantly raised. Although the denominator might also be lowered because the increase in αsR could exceed the decrease in β_a,[19] the numerator would be more affected

[19] An increase in s reduces the after-tax rate of return, which changes the fraction of income spent on children if the elasticity of substitution between parental consumption and the disposable income of children differs from unity. The after-tax propensity to invest, $\beta_a = \alpha(1 + r_a)$, would be reduced, however, because any rise in α could not completely offset the fall in r_a.

than the denominator. An increase in redistribution lowers stationary average income because investments in children are discouraged by the reduction in after-tax rates of return.

The stationary standard deviation of disposable income is also readily derived from equation (30) (see the derivations in Section III). The stationary coefficient of variation is obtained by dividing the stationary standard deviation by stationary income:

$$\mathrm{CV}^2_{I^d} = \frac{(1-\beta_a-\alpha sR)^2}{1-\beta_a^2}\left[\mathrm{CV}_u^2 + \frac{(1+h\beta_a)}{(1-h\beta_a)}\mathrm{CV}_e^2 + \frac{\mathrm{CV}_\Omega^2}{(1-s)^2}\right], \quad (32)$$

where

$$\mathrm{CV}_\Omega = \frac{\sigma_\Omega}{\bar{e}}.$$

Although an increase in redistribution reduces the stationary standard deviation (if σ_Ω and α were not significantly raised), the coefficient of variation would be increased if average income were reduced by a larger percentage than the standard deviation.

The effect on the coefficient of variation can be shown explicitly by considering explicit definitions of taxable income. The coverage of taxable income is determined by whether investments in children can be "written off," depreciation can be deducted from taxable income, interest is taxed as it accrues, and by similar issues. Consider two definitions of taxable income:

$$\begin{aligned} I_t^{g_1} &= y_{t-1} + e_t + u_t, \\ I_t^{g_2} &= I_t^{g_1} - \frac{y_{t-1}}{1+r}. \end{aligned} \quad (33)$$

The first is the before-tax income used in previous sections, while the second essentially permits the amount invested to be "depreciated." If we maintain the assumption of previous sections that the before-tax rate of return, r, is unaffected by the accumulation of capital, the after-tax propensities to invest are

$$\begin{aligned} \alpha(1+r_{a_1}) &= \beta_{a_1} = (1-s)\alpha(1+r), \\ \alpha(1+r_{a_2}) &= \beta_{a_2} = \alpha[1+(1-s)r]. \end{aligned} \quad (34)$$

The term outside the brackets in equation (32) can be written as

$$\begin{aligned} f_1 &= \frac{(1-\beta_{a_1}-\alpha sR_1)^2}{1-\beta_{a_1}^2} = \frac{(1-\beta_{a_2})^2}{1-\beta_{a_1}^2}, \text{ since } R_1 = 1; \\ f_2 &= \frac{(1-\beta_{a_2}-\alpha sR_2)^2}{1-\beta_{a_2}^2}. \end{aligned} \quad (35)$$

An increase in s necessarily increases f_1 if α is unaffected and if r exceeds 0.52 and s exceeds +.1.[20] An increase in s also tends to increase f_2, especially if r is larger than R_2.[21]

An increase in s lowers the coefficient of CV_e^2 and raises the coefficient of CV_Ω^2 in the bracketed term in equation (32) and probably also raises the variability in Ω itself. Consequently, our analysis offers no comfort to the prevailing view that redistribution within a progressive tax-subsidy system reduces (relative) inequality in disposable income. Indeed, two explicit definitions of taxable income suggest the opposite conclusion, namely, that the inequality in disposable income is widened.

Traditional discussions of the relation between redistribution and inequality ignore "unsystematic" taxes and subsidies and usually consider only the initial impact of redistribution, whereas we have analyzed how it affects the equilibrium amount of inequality. Although increased redistribution within a progressive tax-subsidy system initially narrows inequality, the new long-run equilibrium position may well have greater inequality because parents reduce their investments in children. Perhaps this conflict between initial and long-run effects helps explain why the large growth in redistribution during the last 50 years has had very modest effects on inequality.

VIII. Some Extensions

A. Economic Growth

Average income approaches a stationary level because we have assumed that the propensity to invest in children and the degree of inheritability are less than unity, and income per unit of capital, rates of return on investments, and average endowments are stationary over time. If real income per unit of capital, w, grew over time because of, say, autonomous technological progress, equation (9) would imply that

$$\begin{aligned} I_{t+1} &= \frac{\beta}{1+g} I_t^* + \alpha w_{t+1}\, e_{t+1} + \alpha w_{t+1}\, u_{t+1} \\ &= \beta' I_t^* + \alpha w_{t+1}\, e_{t+1} + \alpha w_{t+1}\, u_{t+1}, \end{aligned} \tag{36}$$

where $I_t^* = [(w_{t+1})/(w_t)]I_t$ is the value of the capital in generation t in units of w_{t+1}, and g is the rate of growth per generation in w. The

[20] By differentiation, $\partial f_1/\partial s > 0$ if $r > (1+r)^2(1-s)\alpha(1-\alpha)$. The product $\alpha(1-\alpha)$ is maximized when $\alpha = .5$. If $s = .1$ and $\alpha = .5$, this condition requires $r > 0.225(1+r)^2$ or $r > 0.52$.

[21] By differentiation, $\partial f_2/\partial s > 0$ if $r - R_2 > r\beta_{a_2}(1 - \alpha s R_2) - R_2\beta_{a_2}^2$. This inequality necessarily holds if $r > R_2$ and $1 - \alpha s R_2 \leq \beta_{a_2} < 1$. It can also hold, however, when $1 - \alpha s R_2 > \beta_{a_2}$ or $r < R_2$.

propensity to invest in children is reduced from β to β' because the income prospects of children are raised relative to those of their parents.

If β' and h were less than unity, average income would no longer approach a stationary level but a rate of growth equal to g per generation.[22] For example, the equilibrium path of average income is readily computed from equation (36) to be

$$\bar{I}_t = \frac{\alpha w_t \bar{e}}{1 - \beta'} = \frac{\alpha w_t \bar{e}}{1 - [\beta/(1 + g)]}. \tag{37}$$

Since w_t grows at the rate of g per generation, $\bar{I}_t$ would grow at the same rate. Although an increase in g raises the rate of growth in average income, it lowers the equilibrium level of income for any given w_t because the propensity to invest in children is reduced.

The equilibrium coefficient of variation in income would still be stationary because both the standard deviation of income and average income grow at the rate of g per generation; the only change from equation (20) is that β is replaced by β'. Similarly, the relative degree of intergenerational mobility is also unchanged from equation (24), except that again β is replaced by β'.

Since an increase in g reduces the propensity to invest in children, it raises inequality within and lowers inequality between generations, for we showed in Sections III and IV that the coefficient of variation and the relative degree of intergenerational mobility are negatively related to the propensity to invest. Since an increase in g also raises the rate of growth in average income, more rapid growth would be associated with more equal opportunity between generations and less equal outcomes within a generation. The relation between growth and inequality would be more complicated if growth were associated with higher rates of return on investments or lower degrees of inheritability. Consequently, it is not surprising that the association between economic growth and inequality has not been clear cut (see Paukert 1973, esp. diagram 1).

Convergence to a stationary (relative) income distribution does not require that β is less than unity, but only that β' is less than unity. If the rate of growth in income were sizable, β' would be less than unity even when β significantly exceeded unity. Therefore, the assumption made throughout this essay that $\beta < 1$ can be replaced by the weaker assumption that

$$\alpha(1 + r) = \beta < 1 + g. \tag{38}$$

Rates of return per generation could significantly exceed unity and

[22] The proofs in Section III fully apply when w_t grows over time if β is replaced by β'.

more than half of family income could be spent on children, and yet the analysis in this essay would be fully applicable as long as the rate of growth in income was sufficiently large.

B. *Human and Nonhuman Capital*

The assumption that parents receive a constant rate of return on all additions to or subtractions from a homogeneous capital of their children is unrealistic because human and nonhuman capital are very different. Investments in human capital cannot readily be financed by borrowing and usually are self-financed by parents; rates of return on human capital are not constant but tend to decline as more is invested in any child; and the rate on a small investment in human capital typically exceeds the approximately constant rate on investments in nonhuman capital (see the discussion in Becker [1967]). Therefore, parents investing little in their children tend to invest entirely in human capital because initially it is more "profitable" than nonhuman capital. As investments in children increase, the marginal rate of return on human capital falls and eventually equals the given rate on nonhuman capital. Additional investments in children would then be entirely in nonhuman capital because the marginal rate on human capital would decline further.[23]

Even if different families had the same opportunities and preferences, richer ones would invest more in their children.[24] Hence, these families are more likely to invest in both the human and nonhuman capital of their children and to receive equal marginal rates of return on both. Poorer families, on the other hand, tend to invest only in human capital, because that yields a higher rate of return than the rate available on nonhuman capital. Therefore, richer families spend similar amounts on human capital and quite different amounts on nonhuman capital, whereas poorer families spend essentially nothing on nonhuman capital and differ mainly in human capital. Consequently, the inequality in the property income of children from rich

[23] For a fuller analysis, see Becker and Tomes (1976) and Tomes (1978). If the rate of return on any initial investment in human capital were sufficiently high, even poor parents with weak preferences for children would invest something in their children. Then the optimal investment in children would almost always be positive, and "corner" solutions or the negative investments permitted in earlier sections when $\alpha < [1/(2 + r)]$ (see eq. [19]) would not be common.

[24] From eq. (11),

$$\left.\frac{dw_{t+1}y_t}{dI_t}\right|_{e_t \text{ held constant}} = \beta > 0,$$

and

$$\left.\frac{dw_{t+1}y_t}{dI_t}\right|_{y_{t-1},\, u_t \text{ held constant}} = \beta - (1 - \alpha)h > 0 \text{ if } \beta > (1 - \alpha)h.$$

families would exceed the inequality in their earnings, whereas the inequality in the earnings of children from poor families would exceed the inequality in their property income.

The term "inheritance" is commonly restricted to nonhuman capital, although a more appropriate concept would include investments in the human capital of children. The analysis just given implies that the total inheritance so defined is more equally distributed between families than is the inheritance of nonhuman capital alone and that the total inheritance of children from poorer families may be quite unequally distributed, even though they do not inherit much nonhuman capital. Moreover, persons inheriting human capital would have greater intergenerational mobility than would those inheriting nonhuman capital.[25]

IX. Summary and Conclusions

The crucial assumption in the theory of inequality and intergenerational mobility presented in this essay is that each family maximizes a utility function spanning several generations. Utility depends on the consumption of parents and on the quantity and quality of their children. The quality of children is measured by their income when they are adults, although Appendix A shows that the implications are similar when quality is measured by their utility when they are adults.

The income of children is raised when they receive more human and nonhuman capital from their parents. Their income is also raised by their endowment of genetically determined race, ability, and other characteristics, family reputation and connections, and knowledge, skills, and goals provided by their family environment. The fortunes of children are linked to their parents not only through investments but also through these endowments acquired from parents (and other family members).

In addition, the income of children depends on stochastic terms measuring their luck in the endowment "lottery" and in the market for income. The distribution of luck is the foundation of many models of the distribution of income that ignore utility maximization. Both luck and utility maximization are important in our analysis; indeed, they interact because the optimal investment in children depends on their market and endowed luck.

Parents maximize their utility subject to their own income, the inherited endowments of children, and any anticipated endowed and market luck of children. The optimal investment in children depends

[25] These and other implications relating inheritances to intergenerational mobility are derived in Tomes (1977).

on the propensity to invest in children, an important parameter in our analysis. This propensity is positively related to the fraction of family income spent on children and to rates of return on investments in children and is negatively related to the rate of growth in income.

The equilibrium income of children is determined by their market and endowed luck, the own income and endowment of parents, and the two parameters, the degree of inheritability and the propensity to invest in children. If these parameters were both less than unity, the distribution of income between families would approach a stationary distribution. The stationary coefficient of variation would be greater, the larger the inequality in the distribution of market and endowed luck, the larger the degree of inheritability, and the smaller the propensity to invest in children. In particular, income inequality would increase if the rate of growth in average income increased or if rates of return on investments decreased.

Differences between families in rates of return, average endowments, or other parameters raise the inequality in income and stretch out its distribution because these differences interact with income and luck. For example, families with higher propensities to invest would have higher incomes, an interaction that raises inequality and skews the distribution of income to the right even if luck and all parameters were symmetrically distributed.

A progressive system of government redistribution is usually said to narrow the inequality in disposable income. One of the more surprising implications of our analysis is that progressive taxes and subsidies may well widen the inequality in the long-run equilibrium distribution of income essentially because parents are discouraged from investing in their children by the reduction in after-tax rates of return.

Intergenerational mobility measures the effect of a family on the well-being of its children. We have shown that the family is more important when the degree of inheritability and the propensity to invest are larger. If both these parameters are less than unity, an increase in family income in one generation has negligible effects on the incomes of much later descendants. However, the incomes of children, grandchildren, and other early descendants could be significantly increased; indeed, if the sum of these parameters exceeded unity, the changes in income would rise for several generations before falling, and the maximum increase in income could exceed the initial increase. Moreover, these parameters do not have independent effects: an increase, say, in the degree of inheritability raises the effect on the incomes of descendants of a change in the propensity to invest.

A larger fraction of the variance in the incomes of children from different families would be explained by the incomes and endowments of their parents when propensities to invest and other parameters differed more between families. For example, the incomes of all generations of a family with a much lower than average propensity to invest would tend to be below the average income in the same generation because this family would invest less capital in each generation.

Several of our simplifying assumptions should be modified in subsequent developments of the model. The number of children should be distinguished from the quality of children and entered as separate variables in the utility functions of parents (this is done in Becker [in press]). The number of children is determined by income, endowments, rates of return on investments in children, birth control techniques, and reproductive capacity. Some families are unable to continue their line into the future because of sterility, while others have more children than they would have had with costless and fully effective contraceptives.

Children in the same family also differ because of differences in endowment and market luck, or because parents invest more in first-born, male, abler, or handicapped children. The resulting inequality within a family contributes, along with the inequality between families, to the overall inequality in income between individuals. Elsewhere we have begun an analysis of this neglected subject of inequality within families (see Becker and Tomes 1976; Tomes 1978; Becker, in press).

The fortunes of a given family could be readily followed over generations because we assume that each adult produces children with someone having the same endowment, market luck, and family background as his own. Although the IQ, education, race, religion, family background, and many other characteristics of mates tend to be rather similar (Vandenberg 1972), this assumption of perfect assortative mating is too strong. The effect of different degrees of assortative mating on inequality is analyzed elsewhere (Becker, in press).[26] Inequality within a generation increases and intergenerational mobility decreases as the degree of assortative mating increases.

We have assumed that rates of return are independent of endowments, although rates on human capital tend to be positively related to endowments. Since a positive relation would increase the amount invested in children with larger endowments, these children would have higher incomes because they have both larger endowments and larger investments. This reinforcement of larger endowments with

[26] Also see Blinder (1973, 1976).

larger investments would increase the inequality and skewness in the distribution of income and would decrease the intergenerational mobility.

Despite these and other qualifications and extensions, we believe that the analysis in this essay firmly demonstrates that a theory of the distribution of income need not be a mixture of Pareto distributions, ad hoc probability mechanisms, and arbitrary assumptions about inheritance but can be based on the principles of maximizing behavior and equilibrium that form the core of microeconomics. At the same time, however, our theory does incorporate the effects of luck, family background, and cultural, biological, and financial inheritance on the distribution of income. We also demonstrate that inequality within a generation and inequality across generations (intergenerational mobility) do not require separate "economic" and "sociological" approaches, for both can be analyzed with a unified theory of the determination of the incomes of different families in different generations.

Appendix A

Altruism between Generations[27]

This Appendix replaces the assumption that the utility function of parents depends on the income of children with what may be a more reasonable assumption, that it depends on the utility or welfare of children. Fortunately, the implications with respect to inequality and intergenerational mobility are similar.

If the utility function of parents depends on their own consumption and the welfare of their children, as measured by a monotonic transformation of the children's utility function, then

$$U_t = V[Z_t, \psi(U_{t+1})], \tag{A1}$$

where $d\psi/dU_{t+1} > 0$. Since the children's utility function, in turn, depends on their own consumption and a transformation of the utility function of their children—the grandchildren of the tth generation—and since the utility functions of different generations are assumed to be the same, Equation (A1) can be written as

$$U_t = V(Z_t, \psi\{V[Z_{t+1}, \psi(U_{t+2})]\}) = V'[Z_t, Z_{t+1}, \phi(U_{t+2})]. \tag{A2}$$

The utility function of grandchildren also depends on their own consumption and the utility function of their children, and so on for all generations. By substituting each of these successively later utility functions into the utility function of the tth generation, the latter can be written as a function only of own consumption and that of all descendants:

$$U_t = U(Z_t, Z_{t+1}, Z_{t+2}, \dots). \tag{A3}$$

Each generation either consumes commodities or invests in children (investments in later generations are unnecessary and are not considered). If the

[27] A more complete discussion is available from the authors.

investment by the tth generation, y_t, is replaced using the budget equation of the $t + 1$ generation, the budget equation of the tth generation would become

$$Z_t + \frac{1}{1+r}Z_{t+1} + \frac{1}{(1+r)^2}y_{t+1} = I_t + \frac{1}{1+r}e_{t+1} + \frac{1}{1+r}u_{t+1}, \qquad \text{(A4)}$$

where y_{t+1} is the investment by the $t + 1$ generation in the $t + 2$ generation. By replacing y_{t+1} using the budget equation of the $t + 2$ generation, and so on for all the subsequent y_{t+i}, the fundamental budget equation of the tth generation can be written as

$$\begin{aligned} Z_t + \frac{1}{1+r}Z_{t+1} + \frac{1}{(1+r)^2}Z_{t+2} + \ldots = I_t &+ \frac{1}{(1+r)}(e_{t+1} + u_{t+1}) \\ &+ \frac{1}{(1+r)^2}(e_{t+2} + u_{t+2}) \qquad \text{(A5)} \\ &+ \ldots = W_t. \end{aligned}$$

The right-hand side gives "family wealth" at generation t, or the sum of own income at t and the present value of all subsequent endowments and market luck. The left-hand side shows that family wealth is spent on present consumption and the consumption of all descendants.

Family wealth is known only when there is perfect foresight of the market and endowment luck of all descendants into the indefinite future, a task that exceeds the capacities of the most prescient. A more reasonable approach is to go to the opposite extreme and assume that the luck of descendants cannot be anticipated at all. If they are risk neutral, each family maximizes utility subject to its expected family wealth. The equilibrium conditions imply a relation between the expected family wealths of adjacent generations, the propensity to invest, and realized market and endowed luck.

More important for present purposes is the equilibrium relation between the incomes in three consecutive generations:

$$\begin{aligned} I_t = k + (\beta + h)I_{t-1} - \beta h I_{t-2} &+ (u_t - hu_{t-1}) \\ &+ (v_t - hv_{t-1}) + \frac{h(\beta - h)}{1 + r - h}v_{t-1}. \end{aligned} \qquad \text{(A6)}$$

The coefficient of I_{t-1} is $\beta + h$, and that of I_{t-2} is $-\beta h$, exactly the same as in equation (25) in Section V. The coefficients of current and lagged market luck and current endowed luck are quite similar to those in (25), while lagged endowed luck has a negative coefficient in (A6) and does not enter (25). Since the coefficients of lagged income in (A6) and (25) are identical, utility functions that depend on the welfare of children imply exactly the same intergenerational mobility as do functions that depend on the income of children and have similar implications for the distribution of income within a generation.

Appendix B

From equation (16),

$$\begin{aligned} \sigma_I^2 &= \alpha^2\sigma_u^2\sum_{k=0}^{\infty}\beta^{2k} + \alpha^2\sigma_v^2\sum_{k=0}^{\infty}\left(\frac{\beta^{k+1} - h^{k+1}}{\beta - h}\right)^2 \\ &= \frac{\alpha^2\sigma_u^2}{1-\beta^2} + \alpha^2\sigma_v^2\sum_{k=0}^{\infty}\frac{\beta^{2(k+1)} + h^{2(k+1)} - 2h^{k+1}\beta^{k+1}}{(\beta - h)^2}, \text{ if } \beta, h < 1. \end{aligned}$$

The summation in the second term can be written as

$$\left(\frac{\beta^2}{1-\beta^2}+\frac{h^2}{1-h^2}-\frac{2h\beta}{1-h\beta}\right)\frac{1}{(\beta-h)^2}$$

or as

$$\frac{\beta^2(1-h^2)(1-h\beta)+h^2(1-\beta^2)(1-h\beta)-2h\beta(1-h^2)(1-\beta^2)}{(\beta-h)^2(1-h^2)(1-\beta^2)(1-h\beta)},$$

which equals

$$\frac{(\beta-h)^2(1+h\beta)}{(\beta-h)^2(1-h\beta)(1-h^2)(1-\beta^2)}.$$

A simpler and more transparent derivation of the equilibrium variance follows from taking the variance of both sides of equation (14):

$$\sigma^2_{I_{t+1}}=\beta^2\sigma^2_{I_t}+\alpha^2h^2\sigma^2_{e_t}+2\alpha\beta h\ \mathrm{Cov}_{I_te_t}+\alpha^2\sigma^2_v+\alpha^2\sigma^2_u. \tag{B1}$$

Moreover, since $e^i_t = a + he^i_{t-1} + v_t$,

$$\mathrm{Cov}_{I_te_t}=\beta h\ \mathrm{Cov}_{I_{t-1}e_{t-1}}+\alpha\sigma^2_e.$$

Since, in equilibrium with stationary variances and covariances,

$$\mathrm{Cov}_{I_te_t}=\mathrm{Cov}_{I_{t-1}e_{t-1}},\ \sigma^2_{I_{t+1}}=\sigma^2_{I_t}=\sigma^2_I \text{ and } \sigma^2_{e_{t+1}}=\sigma^2_{e_t}=\frac{\sigma^2_v}{1-h^2},$$

equation (B1) can be written as

$$(1-\beta^2)\sigma^2_I=\frac{\alpha^2\sigma^2_v}{1-h^2}+\frac{2\alpha^2\beta h\sigma^2_v}{(1-\beta h)(1-h^2)}+\alpha^2\sigma^2_u.$$

Hence

$$\sigma^2_I=\frac{\alpha^2}{1-\beta^2}\sigma^2_u+\frac{\alpha^2(1+\beta h)}{(1-h^2)(1-\beta h)(1-\beta^2)}\sigma^2_v.$$

Appendix C

Since $I^i_{t+1}=\beta I^i_t+\alpha he^i_t+\alpha u^i_{t+1}+\alpha v^i_{t+1}$ + a constant, then, by definition of the multiple correlation coefficient,

$$R^2(I_{t+1};I_t,e_t)=\frac{\beta^2\sigma^2_{I_t}+\alpha^2h^2\sigma^2_{e_t}+2\alpha h\beta\ \mathrm{Cov}_{I_te_t}}{\sigma^2_{I_{t+1}}}$$

$$=\beta^2+\frac{\alpha^2\sigma^2_{e_t}}{\sigma^2_I}\left(h^2+\frac{2h\beta}{1-h\beta}\right)>\beta^2,$$

since, in equilibrium,

$$\mathrm{Cov}_{I_te_t}=\frac{\alpha\sigma^2_e}{1-h\beta}$$

(see Appendix B). If $\sigma^2_u/\sigma^2_e \equiv 0$,

$$\sigma^2_I\equiv\frac{(1+h\beta)\alpha^2}{(1-h\beta)(1-\beta^2)}\sigma^2_e$$

(see eq. [17]), and then

$$R^2 \equiv \beta^2 + \frac{(1-\beta^2)h(2\beta + h - \beta h^2)}{1 + h\beta},$$

where $\partial R^2/\partial h > 0$.

Appendix D

$$\dot{g}_m = \frac{\partial g_m}{\partial m} = \frac{\beta^m \log \beta - h^m \log h}{\beta - h}.$$

If $\beta > h, \dot{g}_m \lesseqgtr 0$ as $(\beta/h)^m \gtreqless \log h/\log \beta$ since $\beta < 1$. Since the right-hand side is a constant and the left-hand side increases indefinitely as m increases, g_m must reach a single peak at a finite m and then decline monotonically. Therefore, since $g_1 = 1$ and $g_2 = \beta + h$, g_m falls for all m when $\beta + h < 1$ and reaches a peak at $m > 1$ when $\beta + h > 1$. The maximizing value of m is found from $\dot{g}_m = 0 = \beta^m \log \beta - h^m \log h$, or

$$\hat{m} = \frac{\log (\log h/\log \beta)}{\log \beta - \log h}.$$

If $\beta = kh$, $1 < k < 1/h$, then

$$\frac{\partial \hat{m}}{\partial h} = \frac{1}{h} \frac{1}{\log h \log kh} > 0,$$

or increases in β and h that keep their ratio constant would increase $\hat{m}$.

Appendix E

A regression of I_{t+1} on I_t could be said to omit I_{t-1} and u_t from equation (25). The least-squares estimate of the regression coefficient would be

$$b_{I_{t+1}I_t} = \beta + h - \beta h b_{I_{t-1}I_t} - \alpha h b_{u_tI_t},$$

where b_{yx} is the coefficient in a regression of y on x. Since $b_{I_{t-1}I_t}$ and $b_{u_tI_t}$ are both positive, and since $b_{I_{t+1}I_t} = b_{I_{t-1}I_t}$ with a stationary distribution of income,

$$b_{I_{t+1}I_t} = \frac{\beta + h - \alpha h b_{u_tI_t}}{1 + \beta h} < \beta + h,$$

and the difference might not be negligible.

A regression of I_{t+1} on I_t can also be said to omit the endowment of parents, e_t, from equation (14). Then

$$b_{I_{t+1}I_t} = \beta + \alpha h b_{e_tI_t} > \beta.$$

The correlation coefficient between I_{t+1} and I_t with a stationary distribution of income would be

$$R(I_{t+1}, I_t) = b_{I_{t+1}I_t} \frac{\sigma_{I_t}}{\sigma_{I_{t+1}}} = b_{I_{t+1}I_t},$$

because $\sigma_{I_t} = \sigma_{I_{t+1}}$. Since $0 \leq R(I_{t+1}, I_t) \leq 1$, then

$$\beta < b_{I_{t+1}I_t} < \min(\beta + h, 1).$$

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