The Search Model

Lecture Notes

Identification of the Model without Heterogeneity

- The McCall Search Model is an excellent introduction to identification of structural models
- Classic treatment: Flinn and Heckman (1982)

Data

- Identification arguments *always* begin with an assumption on available data
- Even though the model is dynamic, we only need a single cross-section:

$$(E_n, t_{U,n}, W_n)_{n=0}^{N}$$

Variables

- $E_n \in \{0, 1\}$: employment status of individual n
- If employed ($E_n = 1$): observe wage W_n (otherwise missing)
- If unemployed ($E_n = 0$): observe unemployment duration $t_{U,n}$ (otherwise missing)

CPS Data Example (Key Results)

- Panel dimension: more than half of individuals appear in multiple months
- Transition rates can be measured from panel:
  - Low separation rate (EU)
  - Relatively high hazard rate out of unemployment (UE)
- Observable heterogeneity matters: transition rates differ by education, race, sex

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Steady State Assumption

- Key assumption for identification: economy is in **steady state**
- Let U_t = fraction unemployed at time t
- Let $p_{\tau,t}$ = fraction with unemployment duration τ at time t

Laws of Motion

$$U_{t+1} = (1-h)U_t + \delta(1-U_{t+1}) \tag{1}$$
$$p_{\tau+1,t+1} = (1-h)p_{\tau,t} \tag{2}$$
$$p_{0,t+1} = \delta(1-U_t) \tag{3}$$

- h = hazard rate of exiting unemployment = $\lambda(1-F_W(w^*))$
- δ = separation rate

Steady State Distribution

- Enforcing constant values between t and $t+1$:

$$U_t = \frac{\delta}{\delta+h} \tag{4}$$
$$p_\tau = \frac{\delta h}{\delta+h}(1-h)^\tau \tag{5}$$

Writing Joint Probabilities

- Sampling distribution in terms of equilibrium objects:

$$\mathbb{P}(E, t_U, W) = \left(\frac{h}{h+\delta}F_W(W|W>w^*)\right)^E \left(\frac{\delta h}{\delta+h}(1-h)^{t_U}\right)^{1-E}$$

Thinking Through Identification

Step 1: Identify h from duration distribution

- Conditional distribution of unemployment durations:

$$\mathbb{P}(t_U|E=0) = h \times (1-h)^{t_U}$$

- The hazard rate h is directly identified from the distribution of durations

Step 2: Identify δ from unemployment rate

- Probability of being unemployed:

$$\mathbb{P}(E=0) = \frac{\delta}{\delta+h}$$

- Since h is now known, δ is identified from the unemployment rate

Step 3: Wage distribution and reservation wage

- Observed wages equal the offer distribution *conditional on acceptance*:

$$\mathbb{P}(W|E=1) = F_W(W|W>w^*)$$

- The conditional distribution is identified, but not offers below w^*
- Let $\underline{w}$ = lower bound on observed wage distribution
- The reservation wage is identified: $w^* = \underline{w}$

Step 4: Deeper structural parameters

- Rewrite the reservation wage equation:

$$w^* = b + \beta h \int_{w^*} \frac{1 - F(W|W > w^*)}{1 - \beta(1-\delta)} dw$$

- Infinitely many combinations of b and β rationalize the same w^*
- Common approach: **assume a plausible value for** β, then b is identified

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Credibility and a Policy Counterfactual

The Policy

- Consider a tax credit τ proportional to earnings
- Under the counterfactual, value function becomes:

$$V(w) = (1+\tau)w + \delta U + (1-\delta)V(w)$$

New Reservation Wage Equation

$$w^* = \frac{b}{1+\tau} + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*} [1 - F_W(w)] dw$$

- Effect of subsidy: equivalent to reducing flow utility of unemployment

Credibility Concerns

- Forecasting the counterfactual: re-calculate reservation wage with estimates
- **Problem:** Forecast depends on arbitrary parametric assumptions about wage distribution below w^*
- For a tax ($\tau < 0$): slightly better since it uses observable wage distribution
- But underlying assumption is stark: policy effect inferred from cross-sectional wage distribution without any policy variation

Key Observation

- Initial identification seems reasonable
- But credibility must be viewed through the lens of the **specific research question**
- Evaluate identification strengths/weaknesses relative to desired counterfactuals

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Identification with an Exclusion Restriction

Setup

- Variable Z enters model only through flow utility of unemployment

- Reservation wage equation becomes:

$$w^*(z) = b(z) + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*(z)} [1 - F_W(w)]dw$$

- Z is an **excluded variable**: affects selection into/out of jobs without moving wage offer distribution

Key Definitions

- $\underline{w}(z)$ = lower bound on conditional sampling distribution $\mathbb{P}(W|E=1,Z)$
- $\underline{w}_F$ = lower bound on support of offer distribution F_W
- As before: $w^*(z) = \underline{w}(z)$

Identification Strategy

- Suppose Z has sufficient variation such that:

$$\underline{\mathcal{Z}} = \{z : w^*(z) \leq \underline{w}_F\}$$

  has positive measure

- Then:
  - $\underline{w}_F = \inf_z \underline{w}(z) = \underline{w}$
  - The set is identified: $\underline{\mathcal{Z}} = \{z : \underline{w}(z) = \underline{w}\}$
  - F_W is nonparametrically identified:

$$F_W(w) = \mathbb{P}(w|E=1,z), \quad z \in \underline{\mathcal{Z}}$$

Key Insight

- Policy forecast now uses **articulated variation**
- Model treats wage scaling as equivalent to variation in b
- Can recreate experiment by interpolating existing variation in b

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Further Identification by Functional Form

- Suppose $b(z)$ is linear: $b(z) = b_0 + b_1 z$
- For three values (z, z', z''):

$$w^*(z') - w^*(z) = b_1(z' - z) + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*(z)}^{w^*(z')} [1 - F_W(w)]dw$$

$$w^*(z'') - w^*(z) = b_1(z'' - z) + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*(z)}^{w^*(z'')} [1 - F_W(w)]dw$$

- Two equations, two unknowns (β and b_1)
- Thus β is identified by functional form restrictions

Discussion Points

- Policy counterfactual now determined by parameters matching response to existing variation
- How does this compare to the case without variation?
- Key model implication: variation in b equivalent to variation in τ
- What would be preferable sources of variation?

Identification with Known Scale Restrictions

- If Z is a policy variable (e.g., welfare payment generosity):

$$b(z) = b_0 + Z$$

- Here β is identified directly

Importance of β

- Key parameter for matching effect of Z
- Determines response of reservation wages and hazard rates to policy

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The Whether vs How of Identification

Setup with Two Instruments

- (Z_1, Z_2): Z_1 shifts b, Z_2 shifts wages
- Z_2: labor demand shifter or marginal tax rate policy
- Assume:

$$\log(W) = \log(\omega) + \log(\mu(Z_2)), \quad Z_2 \perp \omega$$

- Reservation wage in terms of ω^*:

$$\omega^*(z_1, z_2) = \frac{b(z_1)}{\mu(z_2)} + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{\omega^*(z_1,z_2)} (1 - F_\omega(\omega))d\omega$$

Over-Identification

- Model is **over-identified**: only need subset of data
- Hazard rates:

$$h(z_1, z_2) = \lambda(1 - F_\omega(\omega^*(z_1, z_2)))$$

- Hazard rate identified from duration distribution, conditional on (z_1, z_2)
- But ratio $h(z_1', z_2')/h(z_1, z_2)$ also determined by wage data alone

Two Approaches to Over-Identification

1. **Enrich the model** using "spare" features of data
2. **Choose estimation approach** to make answer most credible (Marschak's maxim)
   - Model is misspecified
   - Cannot match every feature simultaneously
   - **The "how" of identification:** which features will you use?

Key Tensions

1. **Shape of hazard function** identified from wage data
   - Hazard rates determine unemployment rates
   - Responsiveness to Z_1, Z_2 determines how unemployment shifts
2. **Model restriction:** Z_1 and Z_2 work through same mechanism
   - Identical effect on reservation wages
   - Strong restriction unlikely to hold in extended models
   - Do we want identification to rely on this if we don't have to?

Returning to the Question of Interest

- Want to evaluate policy varying marginal tax rates
- Most convincing: show parameters replicate effects of pre-existing, functionally identical variation

Options

1. **Minimum Distance:** Match moments from joint distribution of durations and z_2
2. **Indirect Inference:** Match quasi-experimental estimates of Z_2 effects on wages/hazard rates
3. **Maximum Likelihood:** Maximize likelihood, validate ex-post that model fits observed changes in hazard rate w.r.t. Z_2

Key Point

- Even if over-identified, use variation most appropriate for the question
- Validation shows model uses conceptually appropriate sources of variation

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Identification with Unobserved Heterogeneity

Setup

- K unobserved types with population proportions $\pi = \{\pi_k\}_{k=1}^{K}$
- Types enter through value of unemployment
- Instrument Z shifts flow value in known way:

$$b_k(Z) = b_k + Z$$

Latent Reservation Wages

$$w_k^*(z) = b_k + Z + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w_k^*(z)} [1 - F_W(w)] dw$$

- Gives vector of latent hazard rates $h_k(z)$

Duration Distribution as Mixture

$$\mathbb{P}(t_U = t | E = 0, Z) = \sum_k \tilde{\pi}_k h_k(z)(1 - h_k(z))^t$$

- $\tilde{\pi}_k$ = representation of type k among unemployed:

$$\tilde{\pi}_k = \frac{\pi_k \frac{\delta}{\delta + h_k(z)}}{U(z)}$$

Identification Result (Heckman and Singer, 1984)

- For fixed K: parameters $(\tilde{\pi}_k, h_k(z))$ are identified from duration distribution

Completing the Argument (Exercise)

1. Invert π and δ from $\tilde{\pi}$ and unemployment rate $U(z)$
2. State support assumption on Z such that F_W is identified
3. Argue λ is identified in this region
4. Argue each $w_k^*(z)$ is identified from $h_k(z)$ and F_W
5. Use reservation wage equation to identify each b_k and β

Summary

Key Takeaways

- **Data Requirements:** Cross-sectional data on employment status, wages, and unemployment durations
- **Steady State:** Essential assumption for identification; gives closed-form expressions for distributions
- **Basic Identification:**
  - Hazard rate h from duration distribution
  - Separation rate δ from unemployment rate
  - Reservation wage w^* from lower bound of observed wages
  - Deep parameters (b, β) require normalization
- **Policy Counterfactuals:** Credibility depends on whether identification relies on:
  - Arbitrary parametric assumptions (less credible)
  - Articulated variation from excluded instruments (more credible)
- **Exclusion Restrictions:**

  - Variable affecting only b enables nonparametric identification of F_W
  - Known scale restrictions can identify β

- **The "How" of Identification:**
  - Over-identified models require choosing which moments to match
  - Choose estimation approach to make answer to research question most credible
  - Marschak's maxim: sufficiency for the question of interest

- **Unobserved Heterogeneity:**
  - Mixture models with latent types
  - Identified under support conditions on instruments