A structural meta-analysis of welfare reform experiments and their impacts on children

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Abstract

Using a model of maternal labor supply and investment in children, this paper synthesizes the findings from five separate welfare reform experiments across eleven sites. The proposed model relates the variation in experimental design across sites to variation in their average treatment effects (ATEs) on household income, labor supply, and the cognitive and socioemotional development of children through a low dimensional set of model parameters. These parameters define labor supply behavior, as well as the importance of time and money in the development of child skills. The statistical methods employed here amount to a *structural meta-analysis* in which the model's parameters are estimated by combining auxiliary panel data with publicly available reports on the experiments' average treatment effects. Thus, while the model can be used to jointly rationalize different experimental impacts, this set of ATEs also provides crucial information for the credible identification of the model's key causal parameters, permitting counterfactual experiments that shed light on which welfare program features prove to be most influential in shaping child outcomes.

1 Introduction

2 Model

2.1 Technology

First, let's introduce a general technology which allows the possibility that investments may not be perfectly substitutable across intra-time periods. Suppose that there are L stages within time

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period t. We define production with the following three equations:

$$\theta_{t+1} = \theta_t^{\delta_\theta} I_t^{\delta_I} \tag{2.1}$$

$$I_t = \left(\sum_L I_{lt}^{\eta} \right)^{1/\eta} \tag{2.2}$$

$$I_{lt} = z_{lt} \left(\phi x_{lt}^{\rho} + (1-\phi) \tau_{lt}^{\rho} \right)^{1/\rho} \tag{2.3}$$

where x are expenditures on investment goods and τ is time investment. My expectation is that $0 < \eta \leq 1$, but in principle this will be decided by the data. z_{lt} will vary across periods based on who is caring for the child. For example, we can assume that mothers have a specific factor productivity, z_{mt}, while the care option they use will have another z_{ct}.

For expositional purposes, let's just assume that $L = 2$ and that mothers can choose to spend 1 period working. Let Y_0 indicate income if they choose not to work, and Y_1 be their income if they do work in this period. Later, we can generalize this to part-time and full-time work, and add coefficients to reflect that these sub-periods reflect different amounts of time.

Finally, hours in any period can be converted to material resources using a linear technology at rate m.

2.2 Preferences

Periodic preferences are given by:

$$U(C, \theta) = \log(C) + \alpha_\theta \log(\theta)$$

which gives the simplifcations that we are used to getting with respect to the dynamic value of investment. Let α_V stand in for the recursive solution to this problem, given the current age of the child.

2.3 Child Care

For the period spent working, mothers can choose either informal or formal child care. If choosing informal care, this provides a pre-determined input, I_n, at no cost. If providing formal care, the mother decides how much to pay, Y_c. The childcare provider then maximizes investment subject to this budget constraint:

$$\max_{x,\tau} \left(\phi x^{\rho} + (1-\phi) \tau^{\rho} \right)^{1/\rho} \text{ s.t. } x + w_c \tau = Y_c$$

where w_c is the prevailing wage in the childcare sector. This yields the solution:

$$\frac{x}{\tau} = \left(\frac{\phi w_c}{1-\phi}\right)^{1/(\rho-1)} = \varphi_c \tag{2.4}$$

$$Y_c = p_c I_c \tag{2.5}$$

$$p_c = \left(\phi^{\varepsilon} + (1-\phi)^{\varepsilon} w_c^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{2.6}$$

$$\varepsilon = \frac{1}{1-\rho} \tag{2.7}$$

2.4 Maternal Investment

Women face a budget and a time constraint:

$$\tau_{lt} + q_{lt} = 1 - h_{lt} \tag{2.8}$$

$$c + x + p_c I_{c,t} = Y + m(q_{1t} + q_{2t}) \tag{2.9}$$

where h_{lt} is equal to 1 if they work in sub-period l.

Here we have three cases. The mother can choose not to work, she can work and use informal care, or she can work and use formal care.

2.4.1 No Work

First order conditions for investment lead to:

$$\frac{x}{\tau} = \left(\frac{\phi m}{1-\phi}\right)^{1/(\rho-1)} = \varphi_m \tag{2.10}$$

and the problem simplifies to:

$$\max_{I_m} \log(c) + \delta_I \alpha_V \log(2I_m) \text{ s.t. } c + 2p_m I_m = Y_0 + 2m$$

where

$$p_m = \left(\phi^{\varepsilon} + (1-\phi)^{\varepsilon} m^{1-\varepsilon}\right)^{1/(1-\varepsilon)}$$

Log preferences imply a solution in which a constant share of income is spent on investment:

$$2p_m I_m = \frac{\delta_I \alpha_V}{1+\delta_I \alpha_V}$$

2.4.2 Informal Care

First order conditions require the same input mixture from mothers in the first period. The problem becomes:

$$\max_{I_m} \log(c) + \delta_I \alpha_V \log(I) \text{ s.t. } c + p_m I_m = Y_1 + m$$

where p_m takes the same form as above, and:

$$I = (I_m^{\eta} + I_n^{\eta})^{1/\eta}$$

In this case, we do not get the proportional investment rule. The first order condition is:

$$\frac{p_m}{Y_1 + m - p_m I_m} = \delta_I \alpha_V \frac{I_m^{\eta-1}}{I_m^{\eta} + I_n^{\eta}}.$$

If investments are substitutes over sub-periods, then home investment will decrease in response to an increase in the quality of informal care.

2.4.3 Formal Care

The problem can be stated now as:

$$\max_{I_m, I_c} \log(c) + \delta_I \alpha_V \log(I) \text{ s.t. } c + p_m I_m + p_c I_c = Y_1 + m$$

where

$$I = (I_m^{\eta} + I_c^{\eta})^{1/\eta}.$$

In this case, first order conditions dictate:

$$\frac{I_m}{I_c} = \left(\frac{p_m}{p_c}\right)^{1/(\eta-1)}.$$

As is always the case for CES technologies, the problem can now be re-written as:

$$\max_{I} \log(c) + \delta_I \alpha_V \log(I) \text{ s.t. } c + p_I I = Y_1 + m$$

where

$$p_I = \left(p_m^{\frac{\eta}{\eta-1}} + p_c^{\frac{\eta}{\eta-1}}\right)^{\frac{\eta-1}{\eta}}$$

3 Full Model and Itemizing Parameters

Parts of the model:

- Program participation
- Choice of whether to work
- Choice of which care option to solicit.
- Investment decision (as described above).

4 Key Equations

Here are the key equations, but maybe they'll go elsewhere eventually.

$$p_I = \left((L-h)p_m^{1-\varepsilon} + h p_c^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{4.1}$$

$$p_I I_t = \varphi_a (b + (w - m)h) \tag{4.2}$$

$$b = N + mL \tag{4.3}$$

So we can write outcomes as

$$\log(\theta_{t+1}) = \delta_I \left(\log(b + (w - m)h) - \frac{1}{1-\varepsilon}\log((L-h) + h\tilde{p}_c^{1-\varepsilon}) - \frac{1}{1-\varepsilon}\log(p_m) - \log(\varphi_a) \right) + \delta_\theta \log(\theta_t)$$

where $\tilde{p_c} = p_c/p_m$ is the relative price of childcare. This helps us to see that there are three key parameters that dictate the change in skills when a mother goes to work. There is the wage, net of production at home $w - m$, there is the relative price of childcare $\tilde{p}_c$, and there is the elasticity of substitution across within-time periods, ε.

Below is the expression for total childcare expenditures:

$$p_c I_c = \frac{hp_c^{1-\varepsilon}}{(L-h)p_m^{1-\varepsilon} + hp_c^{1-\varepsilon}} X$$

where X is total expenditure. This expression allows us to derive a compensated elasticity of expenditure on childcare with respect to its price:

$$\mathcal{E}_{X_c,p_c} = (1-\varepsilon)\frac{(L-h)p_m^{1-\varepsilon}}{(L-h)p_m^{1-\varepsilon} + hp_c^{1-\varepsilon}}$$

which we can write in terms of the relative price of childcare:

$$\mathcal{E}_{X_c,p_c} = (1-\varepsilon)\frac{(L-h)}{(L-h) + h\tilde{p}_c^{1-\varepsilon}}.$$

This gives us a simple framework to think about the effect on child outcomes of:

- Subsidies to employment
- Subsidies to childcare
- Programs that jointly subsidize both employment and childcare.
- The generic effect of other programs that effect employment.

Remember that programs have the following components:

- Subsidies to employment.
- Childcare subsidies.
- Time limits.
- Work requirements.

For the latter two items, we need to think about a bigger picture (i.e. dynamic) model of labor supply. All this allows us to do other stuff (i.e. welfare calculations from welfare reform).

Let $\overline{\alpha}_{c,a} = \alpha_c + \alpha_{V,a}$. We can calculate the indirect utility of a choice h of hours, as:

$$U_h = \overline{\alpha}_{c,a}\log(b + m(L-h) + w_h h) - \alpha_{V,a}\log\left([L + h\tilde{p}_c^{1-\varepsilon}]^{1/(1-\varepsilon)}\right) - \alpha_{V,a}\log(p_m)$$

Here, we write w_h due to the non-linearity of the budget set induced by transfer programs. Allowing for h to be in a finite grid, $\mathcal{H}$, we can write the probability of employment as:

$$P[H > 0] = \frac{\sum_{h>0}\exp(U_h)}{\sum_h \exp(U_h)}$$

Now, consider the extensive marginal elasticity with respect to a proportional increase in each post-tax wage, w_h. We get:

$$\mathcal{E}_{H,w} = \overline{\alpha}_{c,a}(1 - P_H) \sum_{h>0} \frac{w_h h}{b + w_h h} P[H = h | H > 0]$$

4.1 Estimation

4.2 Identification

5 Extensions

- Extend to more than two sub-periods.
- Could we extend to a generic number of hours? i.e. 112 hours in a time period.