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Section A04

Executive Summary:

This project was meant as an introduction to Engineering Design and Graphics. Students were required to create a pendulum clock with an escapement wheel using lasercamms and hands-on fabrication practices.

The objective of the analysis is to determine the time taken for a full oscillation of the pendulum. This is accomplished using two types of analysis: Rigid Body and Point Mass. The main difference between these two methods is that Rigid Body considers the particles inside of the body (in our case, the pellets, pendulum, nuts, etc.) to have the same shape regardless of the stage of a movement, thus incurring a moment of inertia. A point mass on the other hand assumes there is no shape at all, and instead all the mass in concentrated at one point (usually the center of mass). Additionally, we can utilize sanity checks while making these calculations to ensure our answers make sense and aren't wildly erroneous.

In conclusion, multiple methods of analysis were used to compare timings of the pendulum to check for accuracy. In the end, point mass analysis was most similar to the actual timing with a low percent error of 4%. On the other hand, rigid body analysis had a 138% error showing an issues with it. Ultimately, there was a manufacturing error that resulted in a distorted escapement wheel affecting timings and accuracy of the actual time. We're unable to conclude whether any analysis is truly close to the actual value since the actual value is not accurate.

Theoretical Analysis: Clock Timing – Point Mass Analysis

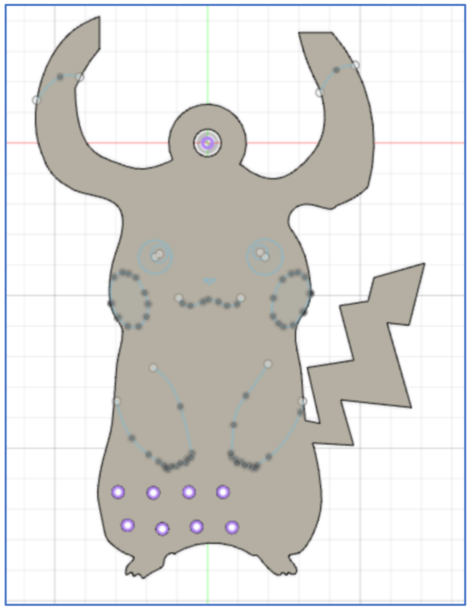

Figure 1: Pendulum Top Face View

Using Fusion 360 materials Properties, we're able to directly take the surface are of the pendulum's top face (Figure 1.) to figure out the volume of our pendulum. Using the assigned density for acrylic (under physical materials), we can calculate the mass of our pendulum. This is done using a simple equation:

Density = Mass / Volume

whereby the volume was found by multiplying the surface area by the thickness of the extrude (Figure 4). This is an accurate method since it's able to account for the holes that were placed for nuts and bolts. Therefore, we can now use the mass of the pendulum directly in our calculations.

With respect to most pendulum designs, this pendulum is significantly smaller. The calculated length to center of mass of acrylic, given by Fusion 360, is 5.795cm. This is another property provided through the materials Properties of the body. This is an opportunity to undergo an intermediate check by going on Fusion 360, creating a construction line from the pivot point (also recognized as the origin of the diagram) to an approximate 6cm towards the center of the pendulum. Doing so allows us to see quickly where the center of mass is approximately and decide whether it's indeed relatively accurate. For instance, it should be approximately along the center of the pendulum however the specific design used here might indicate the center of mass (without bolts) to be towards the left (Figure 4) since there's a "tail" section of acrylic as well.

Before going any further, it's important to define the assumptions we use in a point mass analysis. Firstly, it's assumed that the mass of the pendulum is concentrated at a single point (considered the center of mass). This is done to easily recognize the distance from the point to places of interest such as bolt holes.

$$w = \sqrt{\frac{g}{L_{com}}} \text{ (Radians/second)}$$

g = gravity

L_{com} = effective length of pendulum (distance from pivot to the center of mass of the pendulum)

Figure 6: Formulas for calculating time period of oscillation. (Source: https://canvas.ucsd.edu/courses/30128/files/folder/Lab%20Material/Week%202?preview=5131978)

This is used in the point mass analysis to find an estimated center of mass using the following formula:

Lcom_est=(((M_Calc*La)+(Mb*(SUM(C21:C28))))/Mt)/100

This finds the product of the masses with respect to its lengths, then divides by the total mass to find how far the products would be with respect to the mass of the entire pendulum. The final division by 100 is to convert from meters to centimeters. (Note: SUM(C21:C28) is taking the sum of lengths to each hole. Doing so allows us to estimate the length to the center of each hole product with the mass of two nuts and a bolt).

To calculate the actual time periods using estimated center of mass, we can use the derived formula from Figure 6. This equation gives us the angular velocity (given in radians per second) which is then converted to frequency by multiplying with the conversion factor: 1/(2*pi). To calculate the time period from frequency, we use the standard equation:

$$T = 1/f$$

which is then multiplied by the number of teeth (14) on our escapement wheel to see how long one full revolution would take (Figure 7).

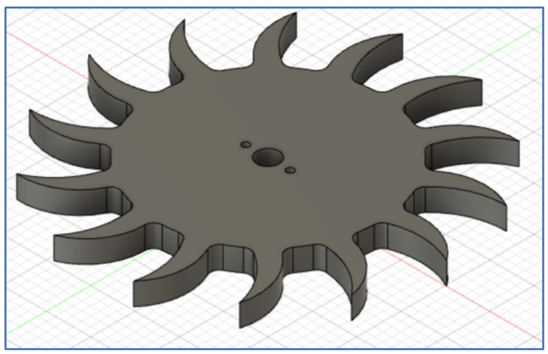

Figure 7: Escapement Wheel Isometric View

This was compared to the actual time it took for a full revolution once parts had been lasercammed and fabricated. To calculate the accuracy of our theoretical versus actual time, the difference between both times were compared with respect to actual time and converted into a percentage.

Percentage Error = ((time_meas-time_calc)/time_meas)*100

Using the values I'd calculated and measured; the Pikachu Pendulum received a percentage error of approximately 4%. Considering the assumptions, we'd made previously to create a point mass analysis, I believe this is a very good percentage of error and indicates the usefulness of point mass analysis in determining timings for similar systems.

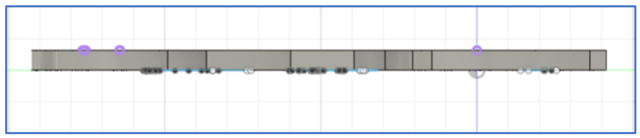

Figure 2: Pendulum Front Face View

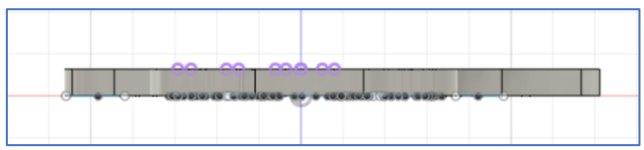

Figure 3: Pendulum Side Face View

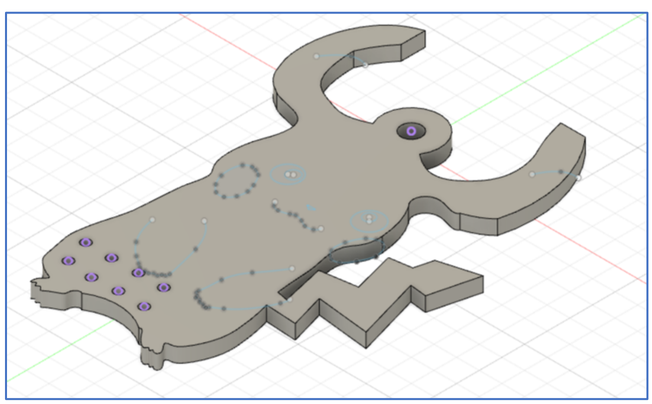

Figure 4: Pendulum Isometric View

Theoretical Analysis: Clock Timing – Rigid Body Analysis

In order to consider a rigid body analysis, we assume that there's only a small angle of oscillation. This is to consider that our pendulum oscillates back and forth. Otherwise, there'd be a consideration of circular motion as well as different moment of inertias with respect to that type of movement.

For a Rigid Body Analysis, we need to calculate the moment of inertia of the pendulum and every bolt. Fusion 360 provides the moment of inertia of the pendulum by utilizing the material Properties tab and scrolling to seek "I-zz". This gives us the moment of inertia along the z-axis. Rigid body Analysis allows us to utilize the actual distance to the bolt holes instead of vertical distance like point mass. To calculate the moment of inertia of each bolt, we utilize the following equation:

I_bolt = Mb*((L_bolt)^2)

The general formula I = M(R^2) is used as we model the pendulum as a solid cylinder with rotation around the one axis. This is the most like what we're doing with our design's even if it may not look anything like a cylinder, we're modelling it as one.

To find the sum of moments of inertia, we take the sum of each individual bolts' inertia and that of the acrylic. This is then used to find natural frequency of the pendulum. To derive this formula, we use these set of equations:

$$\text{natural frequency in radian} \quad \omega = \sqrt{\frac{\mathrm{mt} \cdot g \cdot \mathrm{Lcom}}{I}} \quad \left(\frac{\text{radians}}{\text{sec}}\right)$$

$$\text{natural frequency in Hertz} \quad f = \omega\left(\frac{\text{rad}}{\text{sec}}\right) \cdot \left(\frac{\text{cycles}}{2\pi\,\text{rad}}\right) = \frac{\omega}{2\pi}\left(\frac{\text{cycles}}{\text{sec}}\right) \quad \{\text{Hz}\}$$

$$\text{period of oscillation} \quad T = \frac{1}{f} \quad \left(\frac{\text{sec}}{\text{cycle}}\right)$$

Figure 5: Formulas for calculating time period of oscillation. (Source: https://canvas.ucsd.edu/courses/30128/files/folder/Lab%20Material/Week%202?preview=5132050)

Utilizing the same method to convert from time taken for a single oscillation to find time for one complete revolution, we can again calculate percentage error in times.

For Rigid Body Analysis, there was a significantly greater error percentage of 135%. This is over double the actual value which tells us there is a significant source of error when using rigid body analysis within this pendulum

Theoretical Analysis: Clock Timing – Comparing Methods & Impact of Assumptions

Understanding the extent of assumptions that were utilized to create a rigid body analysis is first to figuring out the discrepancy in percentage error. This is because we’ve ruled out human error by repeating multiple iterations and intermediate checks to make sure there were no outlier values. Logically understanding why the actual value was far larger than calculated time means looking into the factors that can affect timing within rigid body analysis.

1) Distance used from pivot point instead of center of mass. The distances that were used and given by the guide for calculating time period using point mass highlighted distance from pivot points. This might've led to a greater time period than what was expected to be calculated. This is a logical conclusion since both types of analysis are completely theoretical and use similar assumptions, therefore should be most similar to each other instead of the actual time. However, that is not the case and clearly shows that either rigid body or point mass assumptions were not correctly adhered to.

2) Frictional forces due to design of Pikachu Pendulum. In order to creatively design a Pikachu themed pendulum, the pellets of the pendulum were modified in the region that was recommended not to be modified. This was initially cleared through Teaching Assistants on the basis that the contact points of the pellets were to remain unchanged. However, upon prototyping this, it's clear that the extra features around the pellets did have a large impact of frictional forces towards the escapement wheel. Using the idea of gears and exact ratios, it might be possible that the gearing wasn't exact. Therefore, the pendulum with vary with time period as it went over each tooth. To test this theory, timings of each subsection within one complete revolution were taken as accurately as

possible. The results indicated that there was a delta between each section proving unevenness in the wheel movement.

3) Upon further inspection, it was noticed that the escapement wheel itself was crooked and creating a varying pattern while oscillating. After loosening and tightening each screw to realign the escapement wheel, the problem persisted. To check for manufacturing errors with the lasercamm, the wheel was placed on a flat surface and tested on random points across it's surface to check for unevenness. However, the wheel design was completely flat and highlighted the good tolerances on lasercamms.

4) The crookedness would begin as the escapement wheel was placed onto the long shaft connected to a bearing and glued-acrylic pulley. It was concluded that the uneven movement of the long shaft was caused by a uneven shaft. When looking back through the instructions, there was a requirement to remove any burrs left on the end face of the long shaft from manufacturing. For this pendulum, there was no deburring done on this shaft by its manufacturer. However, recalling the gluing process for this piece, it was known that common theft occurred between glued acrylic pulleys with shafts inserted. This seems to be the reason for incorrect movement of the escapement wheel on this clock.

Appendix: Excel Values

Pendulum Timing Analysis

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Variable Description	Variable Name	Values/Equations	Units	Comments
Acrylic Pendulum Specifications				
Area	A	111.994	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	71.11619	cm^3	
Density	p	1.188	gm/cm^3	
Calculated Mass of Acrylic	M_Calc	84.48603372	gm	
Length to Center of Mass of Acrylic	La	5.795	cm	
Calculate Total Mass of Pendulum				
Mass of One Bolt with Two Nuts	Mb	4	g	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	32	g	calculated
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	11.808	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	11.599	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	11.455	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	11.45	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	12.796	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	12.731	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	12.579	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	12.623	cm	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	0.25	**meters**	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est	0.274300177	meters	
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error	8.858972307	Percentage	
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Gravitational Constant	g	9.8	m/s^2	
Natural Frequency in radians/sec	nat_freq_rad_sec	5.977230367		
Natural Frequency in Hz	nat_freq_hz	0.951305759		
Period of Oscillation	period	1.051186741		
Number of Teeth on Escapement Wheel	nteeth	14		
Calculated Time of One Revolution of Escapement Wheel	time_calc	14.71661437		
Measured Time of One Revolution of Escapement Wheel	time_meas	15.312		
Percent Error in Clock Timing	time_error	3.888359659	Percentage	

Calculate Natural Frequency and Timing using Rigid Body Assumption				
Moment of Inertia of Pendulum	I_a	5221.304	g cm^2	
Moment of Inertia of Bolt 1	I_bolt1	557.715456		
Moment of Inertia of Bolt 2	I_bolt2	538.147204		
Moment of Inertia of Bolt 3	I_bolt3	524.8681		
Moment of Inertia of Bolt 4	I_bolt4	524.41		
Moment of Inertia of Bolt 5	I_bolt5	654.950464		
Moment of Inertia of Bolt 6	I_bolt6	648.313444		
Moment of Inertia of Bolt 7	I_bolt7	632.924964		
Moment of Inertia of Bolt 8	I_bolt8	637.360516		
Total Moment of Inertia	I_total	0.471869015	g m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	13.50175865		
Natural Frequency in Hz	rb_nat_freq_hz	2.148871629		
Period of Oscillation	rb_period	0.465360511		
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	6.515047157		
Percent Error in Clock Timing	rb_time_error	135.0251599	Percentage	

Table 1. Actual Values

Variable Description	Variable Name	Values/Equations	Units	Comments
Acrylic Pendulum Specifications				
Area	A	111.994	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	=A*t	cm^3	
Density	p	1.188	gm/cm^3	
Calculated Mass of Acrylic	M_Calc	=p*Vol	gm	
Length to Center of Mass of Acrylic	La	5.795	cm	
Calculate Total Mass of Pendulum				
Mass of One Bolt with Two Nuts	Mb	4	g	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	=Mb*Nb	g	calculated
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	11.808	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	11.599	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	11.455	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	11.45	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	12.796	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	12.731	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	12.579	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	12.623	cm	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	0.25	**meters**	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est	=(((M_Calc*La)+(Mb*(SUM(C21:C28))))/Mt)/100	meters	
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error	=(((Lcom_est-Lcom_meter)/Lcom_est)*100)	Percentage	
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Gravitational Constant	g	9.8	m/s^2	
Natural Frequency in radians/sec	nat_freq_rad_sec	=SQRT(g/Lcom_est)		
Natural Frequency in Hz	nat_freq_hz	=nat_freq_rad_sec*(1/(2*PI()))		
Period of Oscillation	period	=1/(nat_freq_hz)		
Number of Teeth on Escapement Wheel	nteeth	14		
Calculated Time of One Revolution of Escapement Wheel	time_calc	=nteeth*period		
Measured Time of One Revolution of Escapement Wheel	time_meas	15.312		
Percent Error in Clock Timing	time_error	=((time_meas-time_calc)/time_meas)*100	Percentage	
Calculate Natural Frequency and Timing using Rigid Body Assumption				
Moment of Inertia of Pendulum	I_a	5221.304	g cm^2	
Moment of Inertia of Bolt 1	I_bolt1	=Mb*((L_bolt1)^2)		
Moment of Inertia of Bolt 2	I_bolt2	=Mb*((L_bolt2)^2)		
Moment of Inertia of Bolt 3	I_bolt3	=Mb*((L_bolt3)^2)		
Moment of Inertia of Bolt 4	I_bolt4	=Mb*((L_bolt4)^2)		
Moment of Inertia of Bolt 5	I_bolt5	=Mb*((L_bolt5)^2)		
Moment of Inertia of Bolt 6	I_bolt6	=Mb*((L_bolt6)^2)		
Moment of Inertia of Bolt 7	I_bolt7	=Mb*((L_bolt7)^2)		
Moment of Inertia of Bolt 8	I_bolt8	=Mb*((L_bolt8)^2)		
Total Moment of Inertia	I_total	=SUM(C45:C52)/(10000)	g m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	=SQRT(((Mt*g*Lcom_est)/I_total))		
Natural Frequency in Hz	rb_nat_freq_hz	=rb_nat_freq_rad_sec*(1/(2*PI()))		
Period of Oscillation	rb_period	=1/rb_nat_freq_hz		
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	=nteeth*rb_period		
Percent Error in Clock Timing	rb_time_error	=ABS(((rb_time_calc-time_meas)/rb_time_calc)*100)	Percentage	

Table 2. Formulas used