Chapter 121
Cosmological Perturbations in the Interacting Dark Sector: Mapping Fields and Fluids

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Abstract A classical field theory description (which can be obtained from an $f(R, \chi)$) of dark energy–dark matter interaction is considered. Demanding that the interaction strength Q_ν in the dark sector must have a field theory description and a fluid description, a unique form of interaction strength is obtained. It is shown that the one-to-one mapping between the *classical* field theory description and the phenomenological fluid description of dark energy–dark matter interaction exists *only* for this unique form of interaction. Then a novel autonomous system and its stability analysis for the general interacting dark sector is introduced, followed by the background analysis for a specific potential and interaction function.

121.1 Introduction

Dark matter dominates the galaxy mass, and dark energy forms the majority of our Universe's energy density [2, 3]. However, we have little information about the properties of these two components that dominate the energy content of the Universe today [4]. The only information we have about the two components is that (i) Dark energy contributes negative pressure to the energy budget, and (ii) Dark matter has negligible, possibly zero, pressure. The above properties are based on gravitational interactions. More importantly, we do not know how they interact with each other and Baryons/Photons.

It has been shown that the dark matter–dark energy interaction can reconcile the tensions in the Hubble constant H_0. In most interacting dark sector models,

Interacting dark sector, based on the work [1]

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phenomenologically, the interaction is proposed between the fluid terms in the dark sector (Cf. Ref. [1]). More specifically, individually, dark matter (DM) and dark energy (DE) do not satisfy the conservation equations; however, the combined sector satisfies the energy conservation equation [5], i.e.,

$$\nabla^{\mu} T_{\mu\nu}^{(\mathrm{DE,DM})} = Q_{\nu}^{(\mathrm{DE,DM})}, \quad Q_{\nu}^{(\mathrm{DE})} + Q_{\nu}^{(\mathrm{DM})} = 0 \tag{121.1}$$

where Q determines the interaction strength between dark matter and dark energy. Since the gravitational effects on dark matter and dark energy are opposite, even a small interaction can impact the cosmological evolution [6]. Since we have little information about the dark sector, in many of these models, the interaction strength Q_{ν} is put in by hand. However, it is unclear whether these broad classes of phenomenological models can be obtained from a field theory action.

In this talk, we show that under conformal transformations, $f(R, \chi)$ is equivalent to a model with two coupled scalar fields. The dark energy–dark matter interaction, represented by the coupling between the classical scalar fields, can also be represented by the evolution equations of the dark energy (represented by a scalar field) and dark matter (represented by a fluid). We show that a one-to-one mapping exists between the field theory and fluid description for a unique interaction term.

We define a set of dimensionless variables and construct an autonomous system that completely describes the dark energy–dark matter interaction and background evolution. We analyze the fixed points of the system and show that the system has an accelerated attractor solution. We consider a specific dark energy-dark matter interaction model and study the background evolution. We show that for a range of (both positive and negative) coupling strengths, the dark-energy dominated epoch occurs earlier with an interacting dark sector than in the non-interacting dark sector.

121.2 Dark Sector Interaction: Field and Fluid Description

Consider the following action in Jordan frame:

$$S_J = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2} f(\tilde{R}, \tilde{\chi}) - \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\nabla}_{\mu} \tilde{\chi} \tilde{\nabla}_{\nu} \tilde{\chi} - V(\tilde{\chi}) \right] \tag{121.2}$$

where $f(\tilde{R}, \tilde{\chi})$ is an arbitrary, smooth function of Ricci scalar, and scalar field $\tilde{\chi}$, and $V(\chi)$ is the self-interaction potential of the scalar field $\tilde{\chi}$. Under the conformal transformation:

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}, \quad \text{where} \quad \Omega^2 = F(\tilde{R}, \tilde{\chi}) \equiv \frac{\partial f(\tilde{R}, \tilde{\chi})}{\partial \tilde{R}} \tag{121.3}$$

and a field redefinition, the action in the Einstein frame takes the following form:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - U(\phi) - \frac{1}{2}e^{2\alpha(\phi)}g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - e^{4\alpha(\phi)}V(\chi)\right). \tag{121.4}$$

where

$$U = \frac{F\tilde{R} - f}{2\kappa^2 F^2}.$$

and $\alpha(\phi)$ denotes the interaction between dark energy and dark matter.

Defining the dark matter fluid by specifying the four velocity energy density and pressure

$$u_\mu = -\left[-g^{\alpha\beta}\nabla_\alpha\chi\nabla_\beta\chi\right]^{-\frac{1}{2}}\nabla_\mu\chi \tag{121.5}$$

$$p_m = -\frac{1}{2}e^{2\alpha}\left[g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi + e^{2\alpha}V(\chi)\right], \quad \rho_m = -\frac{1}{2}e^{2\alpha}\left[g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - e^{2\alpha}V(\chi)\right]. \tag{121.6}$$

Then the interaction function in the field theory and fluid descriptions are given by

$$Q_\nu^{(\mathrm{F})} = -e^{2\alpha(\phi)}\alpha_{,\phi}(\phi)\nabla_\nu\phi\left[\nabla^\sigma\chi\nabla_\sigma\chi + 4e^{2\alpha(\phi)}V(\chi)\right] = -\alpha_{,\phi}(\phi)\nabla_\nu\phi(\rho_m - 3p_m) \tag{121.7}$$

A one-to-one mapping between the field theory description and fluid description of the interacting dark sector described above exist *only* for this form of interaction function. A classification of interacting dark sector models based on the existence of this mapping can be found in Ref. [1].

121.3 Background Evolution with Dark Energy–Dark Matter Interaction

For the FRW background Universe, the evolution of the dark matter fluid energy density is given by

$$\overline{\rho}_m = \overline{\rho}_{m_0}a^{-3(1+\omega_m)}e^{[\alpha(\overline{\phi})-\alpha_0](1-3\omega_m)}, \tag{121.8}$$

To describe and analyze the complete background evolution, define the following dimensionless variables.

$$x = \sqrt{\frac{C_1}{6}}\frac{\dot{\phi}}{HM_{Pl}}, \quad y = \sqrt{\frac{C_1}{3}}\frac{\sqrt{U}}{HM_{Pl}}, \quad \lambda = -\frac{M_{Pl}}{\sqrt{C_1}}\frac{U_{,\phi}}{U}$$

$$\Gamma = \frac{UU_{,\phi\phi}}{U_{,\phi}^2}, \quad \alpha = \alpha(\phi), \quad \beta = -\frac{M_{Pl}}{\sqrt{C_1}}\frac{\alpha_{,\phi}}{\alpha}, \quad \gamma = \frac{\alpha\alpha_{,\phi\phi}}{\alpha_{,\phi}^2} \tag{121.9}$$

where H is the Hubble parameter and $M_{Pl} = 1/\sqrt{8\pi G}$

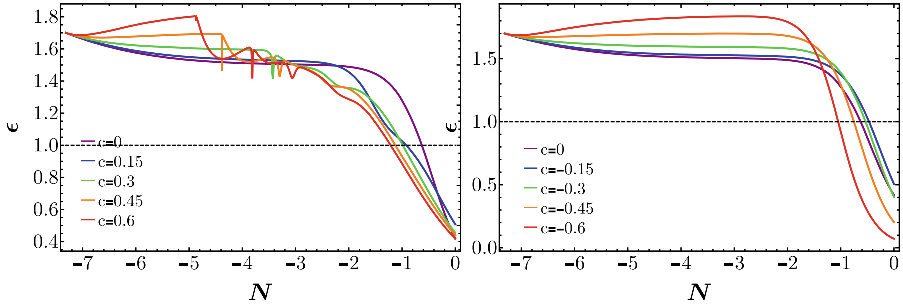

Fig. 121.1 Evolution of slow-roll parameter ϵ as a function of N for different values of interaction strength C; Left panel: $C \geq 0$, Right panel: $C \leq 0$

Stability analysis of the autonomous system describing the interacting dark sector using these variables shows that the system has a radiation-dominated saddle point, matter-dominated saddle point, and an attractor with the accelerated expansion of the Universe. Hence the model is consistent with the cosmological observations of the background evolution of the Universe.

Next we look at the background evolution for the scalar field potential $U(\phi) \sim 1/\phi$ [7] and a linear interaction function $\alpha(\phi) \sim C\phi$.

As we see in Fig. 121.1, looking at the evolution of the slow-roll parameter $\epsilon \equiv -\dot{H}/H^2$, a range of values of C (positive and negative) lead to the accelerated expansion of the Universe indicated by $\epsilon < 1$. The Universe enters the accelerated phase sooner for larger magnitude interaction strength.

121.4 Conclusion

We have introduced a classical field theory description of the dark energy–dark matter interaction with a one-to-one mapping with the fluid description of the interacting dark sector. This mapping exist *only* for a unique interaction term $Q_\nu^{(F)}$. We have then defined a set of dimensionless variables to construct an autonomous system that completely describes the background evolution of the Universe with the interacting dark sector. Stability analysis of the system shows that it has a stable attractor solution that describes the accelerated expansion of the Universe. Then for an inverse scalar field potential and a linear interaction function, we show that the model leads to the accelerated expansion of the Universe for a range of values interaction strength. Larger values of interaction strength lead to the earlier onset of the phase of accelerated expansion.

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