Interacting dark sector in the late Universe: Mapping fields and fluids, and observational signatures

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In this work, we discuss a cosmological model with dark energy – dark matter interaction. Demanding that the interaction strength Q_ν in the dark sector must have a field theory description, a unique form of interaction strength can be obtained. We show the equivalence between the fields and fluids for the $f(R,\chi)$ model where f is an arbitrary, smooth function of R and classical scalar field χ, which represents dark matter. Up to first order in perturbations, there is a one-to-one mapping between the classical field theory description and the phenomenological fluid description of interacting dark energy and dark matter, which exists only for this unique form of interaction. Different formulations of interacting dark energy models in the literature can be classified into two categories based on the field-theoretic description. Then we discuss the quantifying tools to distinguish between the interacting and non-interacting dark sector scenarios. We focus on the variation of the scalar metric perturbed quantities as a function of redshift related to structure formation, weak gravitational lensing, and the integrated Sachs-Wolfe effect and show that the difference in the evolution becomes significant for lower redshifts ($z < 20$), for all length scales.

Keywords: Dark matter - dark energy interaction, based on the work Ref. 1

1. Introduction

Dark matter dominates the galaxy mass, and dark energy forms the majority of our Universe's energy density.[2,3] However, we have little information about the properties of these two components that dominate the energy content of the Universe today.[4] The only information we have about the two components is that (i) Dark energy contributes negative pressure to the energy budget, and (ii) Dark matter has negligible, possibly zero, pressure. The above properties are based on gravitational interactions. More importantly, we do not know how they interact with each other and Baryons/Photons.

It has been shown that the dark matter-dark energy interaction can reconcile the tensions in the Hubble constant H_0.[5,6] In most interacting dark sector models, phenomenologically, the interaction is proposed between the fluid terms in the dark sector (Cf. Ref.[1]). More specifically, individually, dark matter (DM) and dark energy (DE) do not satisfy the conservation equations; however, the combined sector satisfies the energy conservation equation,[7] i. e.,

$$\nabla^\mu T_{\mu\nu}^{(\mathrm{DE,DM})} = Q_\nu^{(\mathrm{DE,DM})}, \quad Q_\nu^{(\mathrm{DE})} + Q_\nu^{(\mathrm{DM})} = 0 \tag{1}$$

where Q determines the interaction strength between dark matter and dark energy. Since the gravitational effects on dark matter and dark energy are opposite, even a small interaction can impact the cosmological evolution.[8] Since we have little information about the dark sector, in many of these models, the interaction strength Q_ν is put in by hand. However, it is unclear whether these broad classes of phenomenological models can be obtained from a field theory action.

In this work, we show that under conformal transformations, $f(R, \chi)$ is equivalent to a model with two coupled scalar fields. The dark energy - dark matter interaction, represented by the coupling between the classical scalar fields, can also be represented by the evolution equations of the dark energy (represented by a scalar field) and dark matter (represented by a fluid). We show that a one-to-one mapping exists between the field theory and fluid description for a unique interaction term.

To detect the signatures of the interacting dark sector from the observations, one needs to construct theoretical tools to distinguish interacting dark sector models from non-interacting ones. We focus on the three cosmological phenomena: (i) structure formation, (ii) weak gravitational lensing (iii) integrated Sachs Wolfe effect. We study the evolution of the relevant perturbed quantities in the redshift range $0 \leq z \leq 1500$ at different length scales. We see that there is a clear difference in the perturbed evolution in the interacting dark sector model as compared to non-interacting ones. This difference becomes significant for $z < 20$, especially at smaller length scales.

2. Dark sector interaction: Field and fluid description

Consider the following action in Jordan frame:

$$S_J = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}f(\tilde{R},\tilde{\chi}) - \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\chi}\tilde{\nabla}_\nu\tilde{\chi} - V(\tilde{\chi})\right] \tag{2}$$

where $f(\tilde{R},\tilde{\chi})$ is an arbitrary, smooth function of Ricci scalar, and scalar field $\tilde{\chi}$, and $V(\chi)$ is the self-interaction potential of the scalar field $\tilde{\chi}$. Under the conformal transformation:

$$g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}, \quad \text{where} \quad \Omega^2 = F(\tilde{R},\tilde{\chi}) \equiv \frac{\partial f(\tilde{R},\tilde{\chi})}{\partial\tilde{R}} \tag{3}$$

and a field redefinition, the action in the Einstein frame takes the following form

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - U(\phi) - \frac{1}{2}e^{2\alpha(\phi)}g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - e^{4\alpha(\phi)}V(\chi)\right). \tag{4}$$

where

$$U = \frac{F\tilde{R} - f}{2\kappa^2F^2}.$$

and $\alpha(\phi)$ denotes the interaction between dark energy and dark matter.

Defining the dark matter fluid by specifying the four velocity energy density and pressure

$$u_\mu = -\left[-g^{\alpha\beta}\nabla_\alpha\chi\nabla_\beta\chi\right]^{-\frac{1}{2}}\nabla_\mu\chi \tag{5}$$

$$p_m = -\frac{1}{2}e^{2\alpha}\left[g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi + e^{2\alpha}V(\chi)\right], \quad \rho_m = -\frac{1}{2}e^{2\alpha}\left[g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - e^{2\alpha}V(\chi)\right]. \tag{6}$$

Then the interaction function in the field theory and fluid descriptions are given by

$$Q_\nu^{(\mathrm{F})} = -e^{2\alpha(\phi)}\alpha_{,\phi}(\phi)\nabla_\nu\phi\left[\nabla^\sigma\chi\nabla_\sigma\chi + 4e^{2\alpha(\phi)}V(\chi)\right] = -\alpha_{,\phi}(\phi)\nabla_\nu\phi(\rho_m - 3p_m) \tag{7}$$

A one-to-one mapping between the field theory description and fluid description of the interacting dark sector described above exist *only* for this form of interaction function. A classification of interacting dark sector models based on the existence of this mapping is given below.[1]

Interacting DE-DM model	**DE-DM Interaction** $\nabla^\mu T_{\mu\nu}^{(\mathrm{DE,DM})} = Q_\nu^{(\mathrm{DE,DM})}$	**Is** $\boldsymbol{Q_\nu \propto Q_\nu^{(\mathrm{F})}}$?
Amendola - 1999[9]	$\dot\rho_m + 3H\rho_m = -C\rho_m\dot\phi$	Yes
Amendola - 1999[10]	$\dot\rho_m + 3H\rho_m = -C\rho_m\dot\phi$	Yes
Billyard & Coley -1999[11]	$\dot\phi(\ddot\phi + 3H\dot\phi + kV) = \frac{(4-3\gamma)}{2\sqrt{\omega+\frac{3}{2}}}\dot\phi\mu$	Yes
Olivares.etal - 2005[12]	$\frac{d\rho_c}{dt} + 3H\rho_c = 3Hc^2\left(\rho_c + \rho_x\right)$	No
Amendola.etal - 2006[13]	$\dot\rho_{DM} + 3H\rho_{DM} - \delta(a)H\rho_{DM} = 0$	No
Olivares.etal - 2007[14]	$\dot\rho_c + 3H\rho_c = 3Hc^2\left(\rho_x + \rho_c\right)$	No
Boehmer.etal - 2008[15]	$\dot\rho_c + 3H\rho_c = -\sqrt{2/3}\kappa\beta\rho_c\dot\varphi$ $\dot\rho_c + 3H\rho_c = -\alpha H\rho_c$	Yes No
Caldera-Cabral.etal - 2008[16]	$\dot\rho_c = -3H\rho_c + 3H\left(\alpha_x\rho_x + \alpha_c\rho_c\right)$ $\dot\rho_c = -3H\rho_c + 3\left(\Gamma_x\rho_x + \Gamma_c\rho_c\right)$	No No
He & Wang - 2008[17]	$\dot\rho_{DM} + 3H\rho_{DM} - \delta H\rho_{DM} = 0$ $\dot\rho_{DM} + 3H\rho_{DM} - \delta H\left(\rho_{DM} + \rho_{DE}\right) = 0$	No No
Pettorino & Baccigalupi - 2008[18]	$\phi'' + 2\mathcal{H}\phi' + a^2U_{,\phi} = a^2C_c\rho_c$	Yes
Quartin.etal - 2008[19]	$\frac{\mathrm{d}\rho_c}{\mathrm{d}N} + 3\rho_c = 3\lambda_x\rho_x + \lambda_c\rho_c$	No
Boehmer.etal - 2009[20]	$\dot\rho_c = -3H\rho_c - \frac{\alpha}{M_0}\rho_\varphi^2$ $\dot\rho_c = -3H\rho_c - \frac{\beta}{M_0}\rho_c^2$ $\dot\rho_c = -3H\rho_c - \frac{\gamma}{M_0}\rho_\varphi\rho_c$	No No No
Beyer.etal - 2010[21]	$\ddot\varphi + 3H\dot\varphi - \alpha M^3e^{-\alpha\varphi/M} = \frac{\beta}{M}\rho_\chi$	Yes
Lopez Honorez.etal - 2010[22]	$\dot\rho_{dm} + 3H\rho_{dm} = \beta(\phi)\rho_{dm}\dot\phi$	Yes
Avelino & Silva - 2012[23]	$\dot\rho_m + 3H\rho_m = \alpha Ha^\beta\rho_w$	No
Pan.etal - 2012[24]	$\dot\rho_m + 3H\rho_m = 3\lambda_mH\rho_m + 3\lambda_dH\rho_d$	No
Salvatelli.etal - 2013[25]	$\dot\rho_{dm} + 3\mathcal{H}\rho_{dm} = \xi\mathcal{H}\rho_{de}$	No
Chimento.etal - 2013[26]	$\rho_{\mathrm{m}}' + \gamma_{\mathrm{m}}\rho_{\mathrm{m}} = -\alpha\rho'\rho$	No
Amendola.etal - 2014[27]	$\dot\rho_\alpha + 3H\rho_\alpha = -\kappa\sum_i C_{i\alpha}\dot\phi_i\rho_\alpha$	Yes
Marra - 2015[28]	$\dot\rho_m + 3H\rho_m = \nu\delta_m^n\rho_m\dot\phi/M_{Pl}$	No
Bernardi & Landim - 2016[29]	$\dot\rho_m + 3H\rho_m = Q\left(\rho_\phi + \rho_m\right)\dot\phi$ $\dot\rho_m + 3H\rho_m = Q\rho_\phi\dot\phi$	No No
Pan & Sharov - 2016[30]	$\dot\rho_{dm} + 3\mathcal{H}\rho_{dm} = 3\lambda_mH\rho_{dm} + 3\lambda_dH\rho_d$	No
Bruck & Mifsud - 2017[31][a]	$\nabla^\mu T_{\mu\nu}^{DM} = Q\nabla_\nu\phi$ $Q = \frac{C_{,\phi}}{2C}T_{DM} + \frac{D_{,\phi}}{2C}T_{DM}^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ $-\nabla_\mu\left[\frac{D}{C}T_{DM}^{\mu\nu}\nabla_\nu\phi\right]$	Yes if $D = 0$
Gonzalez & Trodden - 2018[33]	$\dot\rho_\chi + 3H\rho_\chi = \alpha'\dot\phi\rho_\chi$	Yes
Barros.etal - 2018[34]	$\dot\rho_c + 3H\rho_c = -\kappa\beta\dot\phi\rho.$	Yes
Landim - 2019[35]	$\ddot\phi + 3H\dot\phi + V'(\phi) = -Q\rho_m$	Yes

[a]Violates causality condition ($D(\phi) > 0$) for the disformal transformations[32]

3. Cosmological evolution with dark energy – dark matter interaction

Consider the perturbed FRW metric in the Newtonian gauge

$$g_{00} = -\left(1 + 2\Phi\right), \quad g_{0i} = 0, \quad g_{ij} = a^2(1 - 2\Psi)\delta_{ij}\,, \tag{8}$$

For the FRW Universe, the background evolution of the scalar fields in the field theory description is given by

$$\begin{aligned}\ddot{\overline{\chi}} + 3H\dot{\overline{\chi}} + e^{2\alpha}V_{,\chi}(\overline{\chi}) + 2\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}\dot{\overline{\chi}} &= 0\\ \ddot{\overline{\phi}} + 3H\dot{\overline{\phi}} + U_{,\phi}(\overline{\phi}) + 4e^{4\alpha}\alpha_{,\phi}(\overline{\phi})V(\overline{\chi}) - e^{2\alpha}\alpha_{,\phi}(\overline{\phi})\dot{\overline{\chi}}^2 &= 0\,.\end{aligned}$$

In the fluid description, the dark energy scalar field $\bar{\phi}$ and the dark matter fluid energy density $\bar{\rho}_m$ evolve as

$$\ddot{\overline{\phi}}\dot{\overline{\phi}} + 3H\dot{\overline{\phi}}^2 + U_{,\phi}(\overline{\phi})\dot{\overline{\phi}} = \overline{Q}^{(\mathrm{F})}, \quad \dot{\overline{\rho}}_m + 3H(\overline{\rho}_m + \overline{p}_m) = -\overline{Q}^{(\mathrm{F})}$$

where the interaction term $\overline{Q}^{(\mathrm{F})}$ is given by

$$\overline{Q}^{(\mathrm{F})} = -\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}(\overline{\rho}_m - 3\overline{p}_m) = \alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}e^{2\alpha(\overline{\phi})}\left[\dot{\overline{\chi}}^2 - 4e^{2\alpha}V(\overline{\chi})\right]. \tag{9}$$

Then the dark matter fluid energy density evolves as

$$\overline{\rho}_m = \overline{\rho}_{m_0}a^{-3(1+\omega_m)}e^{[\alpha(\overline{\phi})-\alpha_0](1-3\omega_m)}, \tag{10}$$

The perturbed evolution in the Newtonian gauge is given by

$$\dot{\delta\rho_m} + 3H(\delta p_m + \delta\rho_m) + (\overline{p}_m + \overline{\rho}_m)\left[\frac{\nabla^2\delta u^s}{a^2} - 3\dot{\Psi}\right] = -\delta Q \tag{11}$$

$$\begin{aligned}\dot{\overline{\phi}}\left(\ddot{\delta\phi} - \frac{\nabla^2\delta\phi}{a^2} - 2\Phi\ddot{\overline{\phi}} + U_{,\phi\phi}(\overline{\phi})\delta\phi\right) + \dot{\delta\phi}\left(\ddot{\overline{\phi}} + 6H\dot{\overline{\phi}} + U_{,\phi}(\overline{\phi})\right)&\\ -\frac{\dot{\overline{\phi}}^2}{2}\left(3\dot{\Psi} + \dot{\Phi} + 6H\Phi\right) &= \delta Q,\end{aligned} \tag{12}$$

where

$$\delta Q^{(\mathrm{F})} = -(\delta\rho_m - 3\delta p_m)\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}} - (\overline{\rho}_m - 3\overline{p}_m)\left[\alpha_{,\phi\phi}(\overline{\phi})\dot{\overline{\phi}}\delta\phi + \alpha_{,\phi}(\overline{\phi})\dot{\delta\phi}\right] \tag{13}$$

The metric perturbations Φ and Ψ satisfy the equations

$$\Psi - \Phi = 0 \tag{14}$$

$$\dot{\Psi} + H\Phi = \frac{\kappa^2}{2}\left[\dot{\bar{\phi}}\delta\phi - (\bar{\rho}_m + \bar{p}_m)\,\delta u^s\right] \tag{15}$$

$$3H\dot{\Psi} - \frac{\nabla^2\Psi}{a^2} + 3H^2\Phi = -\frac{\kappa^2}{2}\left(\delta\rho_m + \dot{\delta\phi}\dot{\bar{\phi}} - \Phi\dot{\bar{\phi}}^2 + U_{,\phi}(\bar{\phi})\delta\phi\right) \tag{16}$$

$$3\ddot{\Psi} + \frac{\nabla^2\Phi}{a^2} + 6\Phi\left(H^2 + \dot{H}\right) + 3H\left(2\dot{\Psi} + \dot{\Phi}\right) = \frac{\kappa^2}{2}\left(\delta\rho_m + 3\delta p_m + 4\dot{\delta\phi}\dot{\bar{\phi}} - 4\Phi\dot{\bar{\phi}}^2 - 2U_{,\phi}(\bar{\phi})\delta\phi\right) \tag{17}$$

4. Observable signatures of interacting dark sector

To detect the signatures of dark energy – dark matter interaction in cosmological observation, one needs to study the evolution of perturbed quantities related to cosmological observations. For this purpose, we consider the following cosmological phenomena and the relevant perturbed quantities

(1) Structure formation: $\delta_m(t,x,y,z) \equiv \frac{\delta\rho_m(t,x,y,z)}{\overline{\rho_m}(t)}$
(2) Weak lensing : $\Phi + \Psi$
(3) Integrated Sachs-Wolfe (ISW) effect: $\Phi' + \Psi'$

where *prime* denoted derivative with respect to $N \equiv \ln a$.

To illustrate the difference between the evolution of perturbations in the interacting and non-interacting scenarios, we consider the following dark energy scalar field potential and interaction function

$$U(\phi) \sim \frac{1}{\phi}, \quad \alpha(\phi) \sim \phi \tag{18}$$

We then study the evolution of the flowing quantities in the redshift range $0 \leq z \leq 1500$ at different length scales

$$\Delta\delta_m = \delta_{m_i} - \delta_{m_{ni}}, \quad \Delta\Phi = \Phi_i - \Phi_{ni}, \quad \Delta\Phi' = \Phi_i' - \Phi_{ni}' \tag{19}$$

Evolution these quantities are given in Figs. 1 and 2.

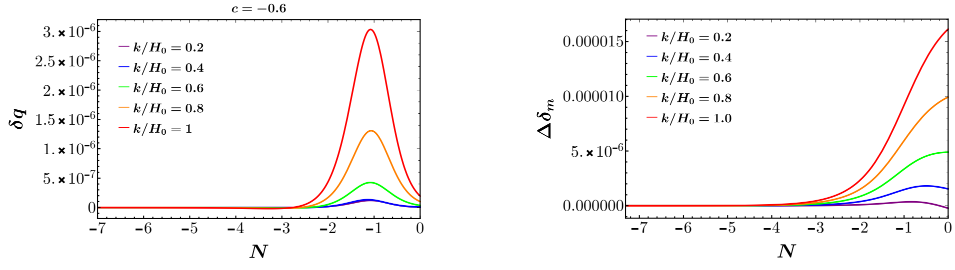

Fig. 1. Evolution of δq(left panel) and $\Delta\delta_m$(right panel) as a function of N for different values of k.

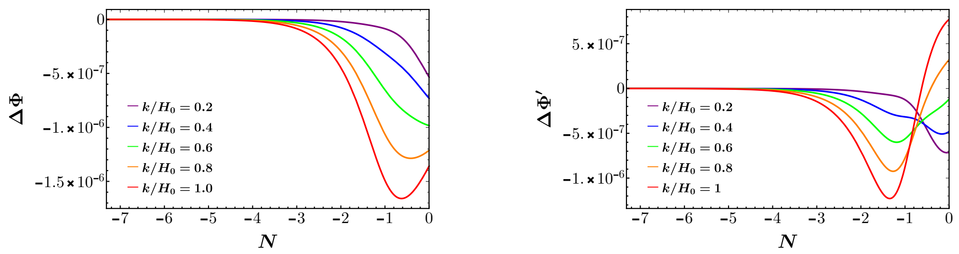

Fig. 2. Evolution of $\Delta\Phi$(left panel) and $\Delta\Phi'$(right panel) as a function of N for different values of k.

where δq is the scaled interaction function given by

$$\delta q = \frac{\delta Q}{H^3 M_{Pl}^2}. \tag{20}$$

Here we see that the evolution of perturbed quantities in the interacting dark sector models shows significant differences for $z < 20$, especially at lower length scale (larger values of k). This trend is consistent with the evolution of scaled interaction function δq.

5. Conclusion

In this work, we consider an interacting dark sector model. We demand that the energy-momentum of individual components of dark sector is conserved, but that of the individual components is not. This model is implemented through a field theory action where dark energy and dark matter are represented by classical scalar fields. This field theory description can be derived from a $f(\tilde{R}, \tilde{chi})$ by means of a conformal transformation and field redefinition. We show that there is a one-to-one mapping between the classical field theory description of the dark energy – dark matter interaction and the fluid description of the interacting dark sector. This mapping exist *only* for a unique interaction term $Q_\nu^{(\mathrm{F})}$. We then classified the popular interacting dark sector models found in literature based on whether or not this mapping is applicable to those models.

We then look at the evolution of first-order scalar perturbations in the interacting dark sector in the redshift range of $0 \leq z \leq 1500$. We consider an inverse power-law dark energy scalar field potential and a linear interaction function. To detect the potential signatures of dark energy - dark matter interaction from cosmological observations, we focus on three cosmological phenomena and related perturbed quantities: (i) structure formation (δ_m) (ii) weak gravitational lensing ($\Phi + \Psi$)(iii) integrated Sachs Wolfe effect ($\Phi' + \Psi'$). We see that the density perturbation δ_m grows at a faster rate in the interacting scenario, especially at the smaller length scales. The effect of the interaction becomes significant for redshift $z < 20$. This is consistent with the evolution of perturbed interaction function δq. We see a similar trend in the case of Φ and Φ', which indicates that the future observations of weak gravitational lensing and integrated Sachs Wolfe effect can be used to distinguish between the interacting and non-interacting models of dark sector.

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