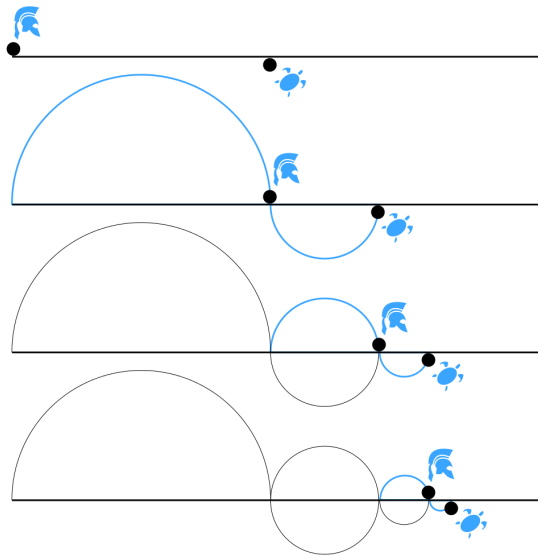# Zeno's Paradox

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With the Olympics around the corner, competition is again coming to the world's stage. Many popular events are races, whether on land or in the water. Recently, I was gifted the book *The Puzzle Universe*, by Ivan Moscovich[1], which includes a fun paradox involving a race. Zeno's paradox of Achilles and the Tortoise is over 2000 years old and goes roughly as follows:

Achilles (in Position 0) sees a tortoise in the distance (in Position 1) and decides to chase after it. The tortoise then flees and the race is on. Zeno argues that by the time Achilles reaches Position 1, the tortoise will have moved on further away to Position 2. Then by the time Achilles reaches position 2, the tortoise will have moved to position 3, and so on and so on to infinity. Zeno reasons that Achilles will therefore never be able to catch the tortoise.



Figure 1: Illustration of Achilles' Race.[2]

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[1]https://www.amazon.com/Puzzle-Universe-History-Mathematics-Puzzles/dp/1770854754

[2]By Martin Grandjean - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=39999636

There are several ways to disprove this paradox, but the most mathematically interesting, and the most direct response to Zeno involves infinite series. Assume for example that Achilles starts 1 meter behind the tortoise, and is twice as fast. Then, the distance between Position k and Position k+1 is $(1/2)^k$ for k from 0 to infinity. Then, going to infinity, we can use infinite series to show that the total distance Achilles would have to run to catch to the tortoise is $\sum_0^\infty (1/2)^k = 2$. Obviously, Achilles, being a legendary athlete, can keep up his pace for two meters, and catch the tortoise. As long as Achilles is faster than the tortoise, and the tortoise has a finite lead, we know from geometric series that Achilles will catch the tortoise in a finite distance.

This paradox is a neat way to introduce infinite series and convergence, but was also thought provoking for me in a non-mathematical sense. In my doctoral studies, I sometimes feel like I am chasing down the field of statistics, my tortoise. Every time I learn something new and make progress, I am left with more questions than before. With so many brilliant people in the field, I know that statistics will progress faster than I, or any one person, can keep up. However, in thinking about this relationship, I realized that catching the tortoise is not the goal of my studies. The true goal is simply running the race and seeing how far I can go. I need to keep in mind where I started, so that when I stop running at the end of the day/year/career, even if the tortoise has left me in the dust, I can look back and take pride in my progress. If the race was any different, it wouldn't be fun.