

Part 1**Introduction to PHYSICS OF SOUND AND MUSIC**

Please always bring this Course Guide with you to class.

Add your own notes and mark in the Guide what we have discussed in class.

Survey of Course Topics

Basics of acoustics, sound, and music

Waves and harmonics

Harmonic analysis and frequencies in sound

Voice and hearing

Environmental sound and noise

Architectural acoustics

Some elementary music theory, intervals, scales, temperament

Microphones and loudspeakers

CDs and DVDs

Lecture demonstrations for sound, acoustics, and musical instruments.

Music performances by faculty from the TTU School of Music and by students in class.

Oldest Known Musical Instrument.

A stone age flute was found in 2008 at Hohle Fels cave in Germany that was made from a griffon vulture's thin radial bone 35,000-40,000 years ago.



The flute dates to a time when modern humans first migrated from Africa to Europe. The flute here is held by the researcher Nicholas J. Conrad of the University of Tübingen. Beautiful songs and melodies can be played on a reproduction of this flute by Friedrich Seeberger. (See CD “Klangwelten der Altsteinzeit, for instance the song “Spiel mir das Lied vom Knochen” or “Play for me the Song of the Bone”.) (For a related article about the discovery of the flutes, see Science News, page 13, July 18, 2009.)

What is Sound? What is Music?

Sound consists of pressure waves in air, more specifically, sound can be understood as organized vibrations of air molecules.

We can distinguish between general sound, noise, and musical sound.

“Without music, life would be a mistake” (Friedrich Nietzsche, 1844 – 1900).

Demonstrations

1. Produce sound with some musical instruments.
2. Listen to voice.
3. Produce your own sound.
4. Produce some noise.
5. Distinguish between periodic general sound and periodic musical sound.
6. When does a sound represent a musical sound?
7. Produce sound with a percussion instrument. Does it sound musical?

Questions

1. Describe the “quality” of sound from speech, musical sound, and noise.
2. Can you describe in physical terms the difference between sound from singing a tone and from a lawn mower, both being periodic and having the same pitch?

“Consonance” and “Dissonance” in Sensory Perceptions

Hearing: Harmony, melody, rhythm – noise.

Sight: Pleasing colors – color mismatch.

Taste: “Consonant” – “dissonant” food combinations.

Touch: Gentle touch – scratching.

Smell: Perfume, i.e. “consonance”, and “dissonance”: _____

Perception of External Reality

See examples above for humans.

The acute sense of smell of dogs corresponds to our sense of hearing in terms of sensitivity. Some smell combinations can be “harmonious” (without a “melody” or “rhythm” of course).

Bats perceive external reality from echoes of textured objects (objects at night, insects).

Sense of Hearing versus Seeing

The human range of hearing extends over 10 octaves in frequency and 12 powers of ten in intensity. In contrast, our faculty of sight covers only about 1 octave in frequency (i.e. colors) and 5 powers of ten in intensity.

Pleasant Sound

The ancient Greek philosopher Pythagoras (570-495 B.C.) noticed pleasing sound when tones were played together whose pitches stood in simple ratios. He used a single string (“monochord”) with a movable bridge. Pleasant musical intervals were heard when the two lengths of the divided string were in the ratios of 2:1 octave, 3:2 musical fifth, 4:3 musical fourth, 5:4 musical major third, etc. Such ratios were used to build an 8-note musical scale closely related to the diatonic scale in Western music today.

Demonstrations

1. “Play” a monochord and keyboard to demonstrate Pythagorean musical intervals.
2. Divide the string of a monochord into two sections with a wedge. Pluck the two sections together and listen to the prevalent dissonant and rare consonant musical intervals while you move the wedge under the string. Note the octave, fifth, fourth, third corresponding to ratios of the two string sections of 2:1, 3:2, 4:3, 5:4, respectively.
3. Listen to the consonance in these intervals, and dissonance in others where the lengths of the two string sections are not in such simple ratios.
4. Show Galilean pendulums and the corresponding simple musical intervals.
5. Play some notes and musical intervals on a Native American flute.
6. Show the vibrational modes on a vibrating vertical circular wire ring.

What is Music? Some Possible Answers

Music is organized sound.

Music is a sequence of notes that are varied to sound pleasant.

Music is speech without words, a universal form of communication.

“Music is above all wisdom” (Ludwig van Beethoven).

Music amplifies human emotions and may be the highest art form.

Elements of Music

Pitch is the fundamental frequency of a periodic sound. The sound repeats itself after a period T . The pitch is $f = 1/T$. For instance, “concert A4” has a pitch of 440 Hz.

Rhythm results when notes are played with different durations (see also body rhythms).

Melody results from a combination of notes with different pitches and rhythms.

Harmony results when different notes are played simultaneously to create chords.

Dynamics relate to the loudness of notes and how it changes over several notes.

Timbre is tonal quality and makes a piano sound different from a violin, etc.

Tempo is the speed of a musical piece, often similar to heart rate or breathing.

Meter is a pattern in which rhythmic pulses are organized, e.g. 3/4, 4/4, 6/8, etc.

(For more details, see: “Birth of the Beat”, Science News, August 14, 2010, pp. 19.)

Range of Hearing

Demonstration on Range of Hearing

Use a frequency generator and loudspeaker. Start with a very low frequency and increase it. Ask students at which high frequency they no longer can hear the sound. This varies from student to student and depends on age and other factors.

Write down the range of hearing from class: _____ Hz to _____ kHz.

For a young person the range of hearing is about **20 to 20,000 Hz** or **20 Hz to 20 kHz**.

Question

How did this particular range of hearing evolve in humans?

A Possible Answer

Our hearing has evolved so that we are able to hear the frequencies of running and falling water and thus be able to find this commodity essential for life.

(See also Trevor Cox, “The Sound Book”, p. 187).

Concept of frequency: f = number of cycles per second

The physical unit of frequency is Hertz (abbreviated Hz).

Example: A heart beats 60 times per minute or once per second.

Therefore the frequency is $f = 1$ Hz.

Example: The frequency of the household voltage in the USA is $f = 60$ Hz.

Example: The piano has one of the widest frequency ranges of all musical instruments.

The frequency range of the 88 keys extends from key A0 = 27 Hz to key C8 = 4186 Hz.

(See also the figure of piano keys, their names, and frequencies.)

The relation between frequency f and period of oscillation T is **$f = 1/T$** .

Question

Given a frequency of oscillation $f = 50$ Hz for a deep bass tone, what is the period of oscillation T ?

Answer: $T = 1/f = 1/50 \text{ Hz} = 0.02 \text{ s} = 20 \text{ millisecond} = 20 \text{ ms}$

Note that we have used the notation $1 \text{ ms} = 0.001 \text{ s} = 10^{-3} \text{ s}$.

Demonstrations

1. When does a periodic sound start to sound “musical”?

Drive a mechanical vibrator with a frequency generator. Increase the frequency from 10 Hz to 100 Hz. When does it sound “musical”?

Answer: The approximate frequency is $f =$ _____ Hz.

2. A rattle: Why does it not sound “musical” despite the fact that the sound has a pitch?

3. Maracas for rhythmic accompaniment produce “periodic” noise.

4. Play a percussion triangle: Does it sound “musical”?

Simple Harmonic Motion (SHM) and Applications

Simple Harmonic Motion versus General (Non-Simple) Periodic Motion

General Periodic Motion: The motion repeats itself after a period T .

Examples

Heartbeat T = about 1 second

Sounding the horn of an automobile

Special Case: Simple Harmonic Motion (SHM)

The motion follows a simple *sine- or cosine-curve* with period T .

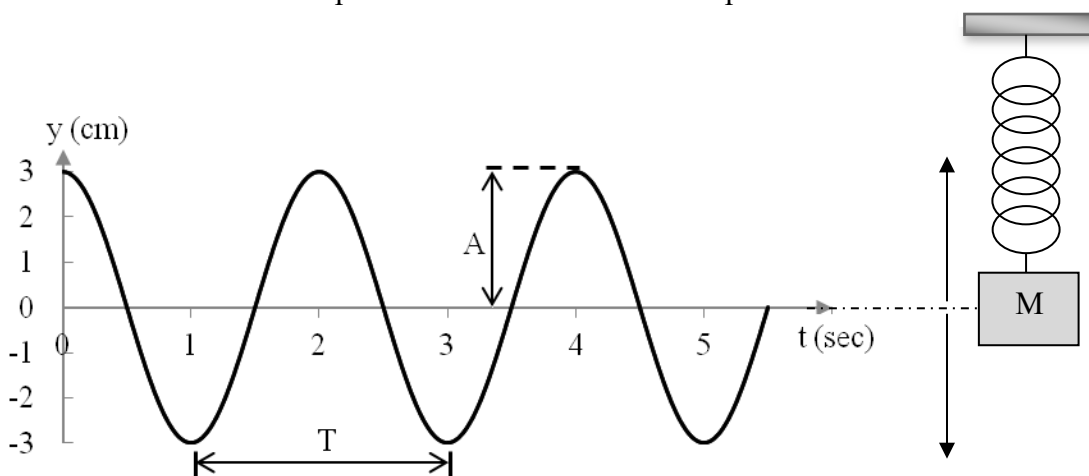


Figure. Simple harmonic motion (SHM) showing the displacement y (in cm) from equilibrium as a function of time t . The oscillating spring on the right would show such a displacement as a function of time.

Exercise: Read the amplitude $A =$ _____ cm and period $T =$ _____ s

Conditions for Simple Harmonic Motion

An equilibrium position must exist about which the medium oscillates.

A restoring force F exists that is proportional to the displacement y from equilibrium, i.e.

$F \propto y$ is expressed by **Hooke's Law** $F = -ky$.

(The minus sign indicates that F is in the opposite direction to y ; k is the spring constant.)

Demonstrations

1. Hooke's law: Weights on a scale or postal scale. Read the force in Newton (N).
2. SHM: Spring with a weight suspended and oscillating up and down.
3. A simple pendulum swinging back and forth.
4. "Walking" a spring pendulum: Follow the sine wave traced by the oscillating weight.
5. A pure sine-tone is played on a keyboard synthesizer: Hear a sine wave.
6. A flute and recorder approximately produce sine tones, with air molecules moving back and forth about an equilibrium position while the wave travels.
7. Perpetual motion pendulum driven by a photovoltaic cell.
8. Chaotic double pendulum: Show SHM, non-SHM, and chaotic motion ("noise").

Comparison of Simple Harmonic Motion with General Periodic Motion

Simple Harmonic Motion

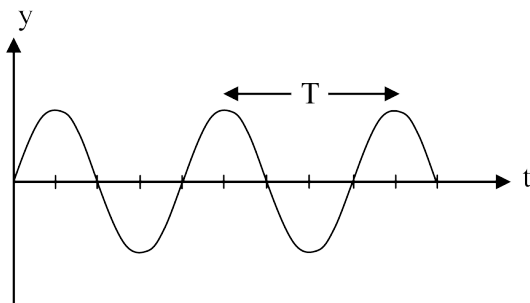
Periodic with period T

Single frequency (fundamental)

Pure sine wave

Angel singing

A sine wave show of period T :



Lecture Demonstration

Listen to a sine wave
(pure tone)

Mechanical Example:

Simple pendulum
Spring pendulum

General Periodic Motion

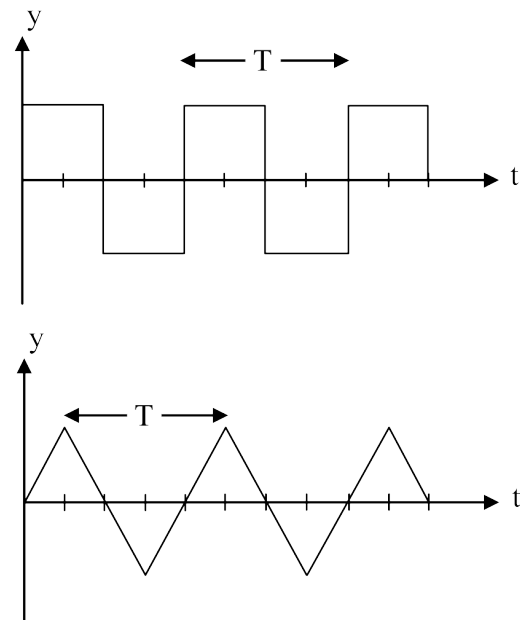
Periodic with period T

Fundamental plus overtones

Complex wave

Tone from a bassoon, violin

A square and triangular wave of period T :



Lecture Demonstration

Listen to a square wave and
triangular wave (composite tone
with overtones)

Mechanical Example:

Windshield wiper
Heartbeat
Skateboarding a "half-pipe"

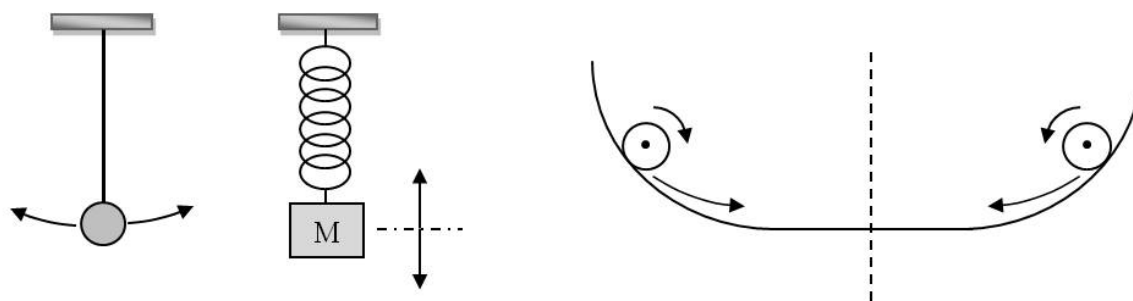


Figure. Left: Simple harmonic motion of a ballistic pendulum and a spring pendulum.
Right: Periodic (not simple) harmonic motion of a skateboarder in a “half pipe”.

Challenge question

a) What shape would the lowest part of a half-pipe have to be so that a skater moves with simple harmonic motion (SHM)? (Hint: Consider the motion of a simple pendulum where a ball is attached to a string.)

Answer: _____

b) For the skate boarder in the half pipe, is the motion simple harmonic? Is it periodic?

Answer: _____

Demonstration

1. Show a cylinder or a ball bearing rolling up and down a “half pipe” of semi-circular shape.
2. Show a cylinder or ball bearing rolling up and down a “half pipe” having circular arcs on the sides and a flat bottom.
3. Let the cutout part from the semi-circular half pipe rock back and forth and observe the motion.
4. Show a double pendulum and its harmonic and inharmonic motions.
5. Show a physical pendulum driven by a solar cell and magnet.
6. Discuss in all the above cases whether the motion is harmonic, periodic, or inharmonic.

Various Wave Forms

A sine wave corresponds to simple harmonic motion (SHM). It is the most elementary wave as it contains only a single frequency, the so-called *fundamental frequency*. Many other waveforms exist. Some of these are shown here:

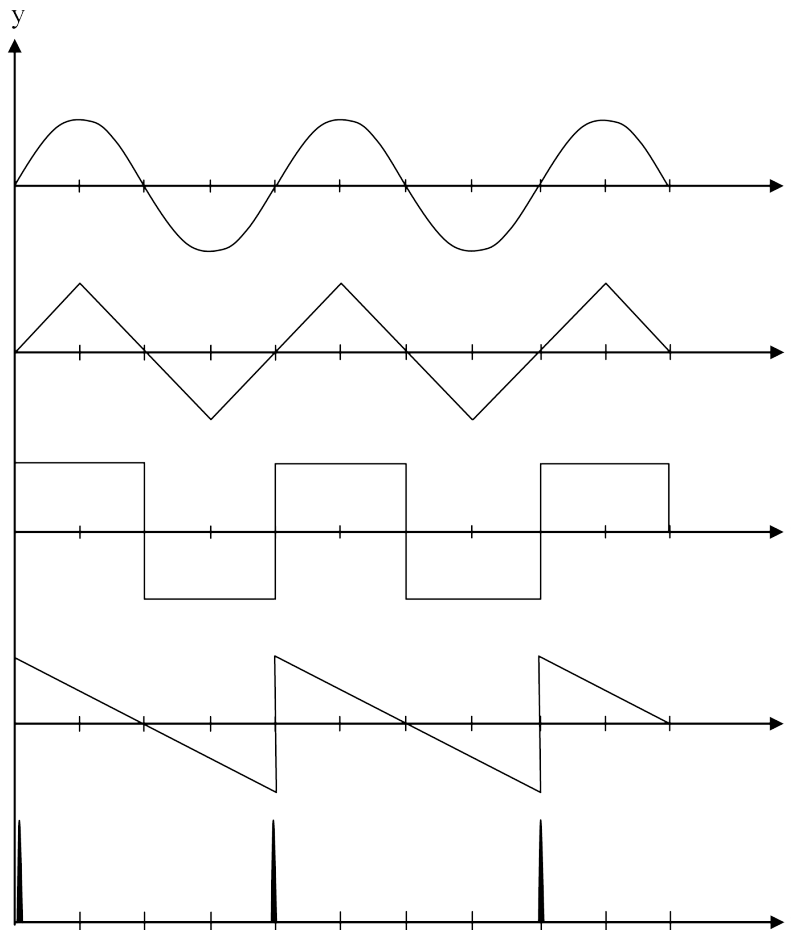


Figure. Sine wave (SHM), triangular wave, square wave, saw tooth wave or ramp, and pulse train. All waves have the same period, fundamental frequency, and amplitude. The waveforms shown are the most basic ones. They are for instance used for testing audio equipment.

Question Concerning Harmonics and Overtones

Guess what causes the differences in the shape of these waveforms, even though they all have the same fundamental frequency.

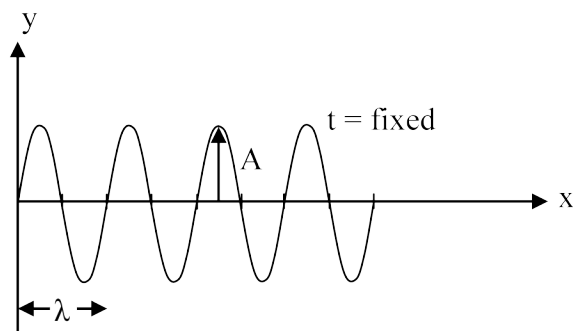
Answer: _____

Demonstration

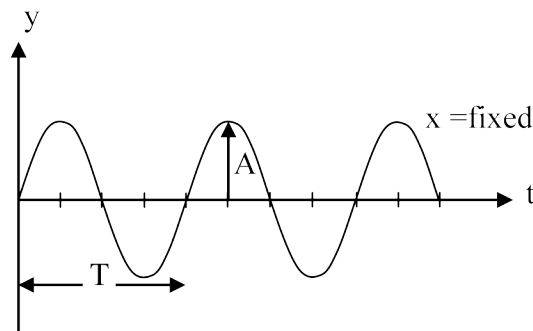
1. Listen to a sine wave and observe it on a monitor.
2. Listen to the different *quality or timbre* of the sound of various waveforms from a signal generator. Alternatively, listen to some synthesized waveforms from a keyboard.

Representation of a Sine Wave as a Function of Frequency and Distance

Draw a sine wave in the *spatial domain*.
Show the wavelength λ :



Draw a sine wave in the *time domain*.
Show the period T :



Speed of Sound

Speed or velocity is given by distance x traveled, divided by time t , or $v = x/t$.

Rewrite this as $v = x/t = \lambda/T$, where λ is the wavelength, and T the period of oscillation.
We know that “period = inverse of frequency”, $T = 1/f$, and so we have the result

$$v = \lambda f \quad (\text{Speed} = \text{wavelength times frequency})$$

We shall use this formula on the following pages.
The speed of sound depends on the temperature.

Example: At typical class room temperatures of 25°C (77°F) we have $v = 346.1 \text{ m/s}$

Example: At the freezing point 0°C (32°F) of water we have $= 325.2 \text{ m/s}$

We shall take the value of $v = 346 \text{ m/s}$ at 25°C (77°F) if not otherwise specified.

Determination of the Speed of Sound with a Plosive Aerophone (“Slap Tube”)

Use a common 1-inch plastic pipe, slap it with the open hand or a rubber glove. Listen to the pitch of the emitted tone. Obtain the pitch (frequency) from the key on a keyboard closest in frequency to the sound from the tube. The effective length L_{eff} of the pipe gives us the wavelength $\lambda = 4 \cdot L_{\text{eff}}$ of the sound. (See Figure below.) The speed of sound then follows from $v = \lambda f$. This is a very simple and fun experiment!

Consider a plastic pipe (length $L = 38.5$ cm, radius $R = 1.25$ cm). After the hand slaps the tube, the latter is closed, as see on the left of the Figure below. For the fundamental air resonance in the tube, we see that $L = \lambda/4$. Also shown is a wave representing the air movement or displacement in the tube. The air moves most vigorously at the “antinode” at the open end and least at the “node” at the closed end of the tube.

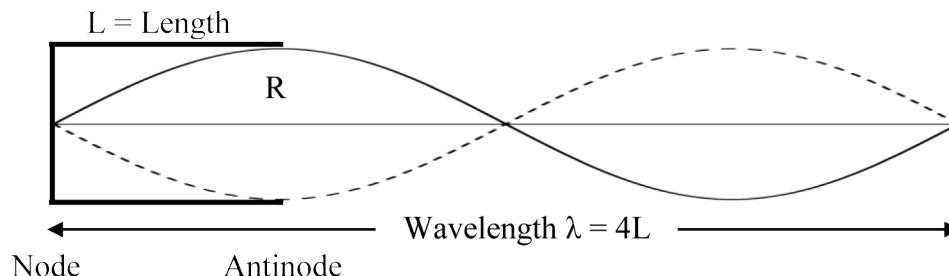


Figure. A tube closed at one end (node) and open at the other end (antinode).

The pitch of the sound from the pipe is close to the key A3 on the piano. Therefore the frequency is $f = 220$ Hz.

Now substitute the numbers: $L = \lambda/4 \rightarrow \lambda = 4 \times 0.385 \text{ m} = 1.54 \text{ m}$.

Then $v = \lambda f = 1.54 \text{ m} \times 220 \text{ Hz} = 339 \text{ m/s}$.

This is a good result.

But we can do even better by using the **effective length** L_{eff} of the tube, which includes a correction for the open end. The formula for this is:

$$L_{\text{eff}} = L + 0.6 \cdot R, \text{ where } L \text{ is the actual length and } R \text{ the radius of the tube.}$$

Use the values for L and R and obtain: $L_{\text{eff}} = 0.385 \text{ m} + 0.6 \times 0.0125 \text{ m} = 0.3925 \text{ m}$

The wavelength with the end correction then is $\lambda = 4 L_{\text{eff}} = 1.57 \text{ m}$.

From this follows the speed of sound as $v = \lambda f = 1.57 \text{ m} \times 220 \text{ Hz} = \mathbf{345.4 \text{ m/s}}$.

This is very close to the speed of sound $v = 346 \text{ m/s}$ in air at 25°C .

Demonstration

1. Students play a familiar tune on a set of “slap tubes”.
2. When the hand is released quickly, the emitted sound is an octave higher. Hold a $2 \times 220 = 440$ Hz tuning fork at the open end and listen to the resonance.

Determination of the Speed of Sound with a Didgeridoo

Use a plastic model of an aboriginal Australian didgeridoo and play it.

The fundamental frequency of our didgeridoo is close to the note D2 ($f = 73.4$ Hz). A more exact determination for our tube yields $f = 71.5$ Hz.

The length of the didgeridoo is $L = 1.208$ m.

Finding the speed of sound in this case is very similar to the “slap tube”. We again are dealing with a tube, closed at one end by the mouth (“closed” by the buzzing lips).

Here we use the actual length L for the effective length L_{eff} and ignore the end correction from the diameter of the tube, i.e. we ignore the extra length $\Delta L = 0.6 \cdot R$. We can justify this because the tube radius is very small compared to the tube length. We also ignore the flared shape of the open end of the didgeridoo as well as the slightly undulating shape over its entire length.

We then have $L = \lambda/4$ and $\lambda = 4L = 4 \cdot 1.208 \text{ m} = 4.832 \text{ m}$.

For the fundamental frequency (pitch) of the didgeridoo we take $f = 71.5$ Hz.

Then the speed of sound is $v = \lambda f = 4.832 \text{ m} \cdot 71.5 \text{ Hz} = 345.5 \text{ m/s}$.

This is a very good value for the speed of sound, considering the simplicity of the experiment. Unless otherwise specified, we shall always use $v = \mathbf{346 \text{ m/s}}$.

Demonstrations

1. Play a second didgeridoo of the *same length* and listen to the pitch. Compare the pitch with the first didgeridoo. It should be very similar.
2. Use a straight plastic pipe of similar length and diameter and “play” it. Is the pitch similar, as it should be?

Question

The didgeridoos in the above demonstrations produce a very similar pitch. But do they sound the same? In other words, is the *quality of the sound or timbre* the same?

If not, what could cause the difference in the timbre?

Answers: _____

Demonstration - Determination of the Speed of Sound with a Cardboard Tube

We use a cardboard tube and excite standing waves of the air in it. Place a loudspeaker near one opening of the tube and tune the frequency to the lowest resonance. Write down the frequency for which the sound from the tube is loudest.

For the tube with *two* open ends, the wavelength for the lowest or fundamental vibrational mode is given by $\lambda = 2L$, where L is the effective length of the pipe.

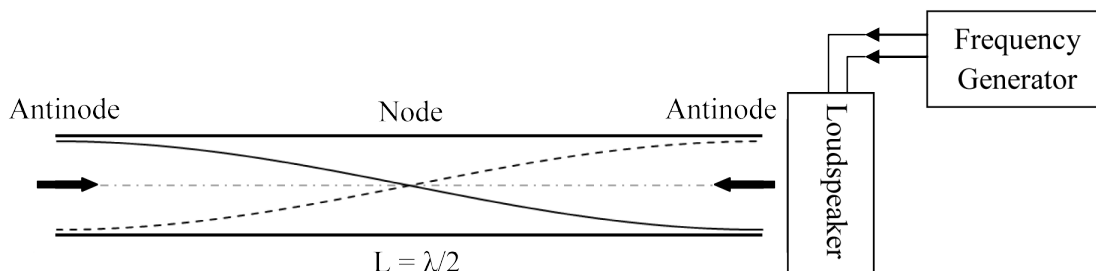


Figure. Sketch of the apparatus, including cylindrical open tube, loudspeaker, and frequency generator. At the ends of the tube we have maximum air movement (antinodes).

The longitudinal air displacement in the tube is shown in the vertical direction for clarity.

Data:

Measured length $L = 1.32$ m (see later for using the *effective length*!)

Wavelength from $\lambda = 2L$

$$\lambda = 2.64 \text{ m}$$

Measured resonance frequency (fundamental mode):

$$f = 123 \text{ Hz}$$

Calculate the speed from $v = \lambda f$ and verify

$$v = 325 \text{ m/s}$$

Question

Compare the value for the speed from this experiment with the value of $v = 346$ m/s.

Discuss possible causes in the discrepancies between the two values (e. g. temperature effects, diameter of tube, i.e. end correction).

Including the End Correction of the Tube

The diameter of the tube contributes a correction to the length of the tube at its open end (but not at its closed end):

$$\Delta L = 0.6 \cdot R, \text{ where } R = \text{inside radius of tube}$$

For our tube with two open ends we have the effective length

$$L_{\text{eff}} = L + 2 \cdot \Delta L = L + 1.2 \cdot R$$

The radius of the tube is $R = 6.9 \text{ cm} = 0.069 \text{ m}$.

This yields $L_{\text{eff}} = L + 1.2 \cdot R = 1.32 + 2 \cdot 0.6 \cdot 0.069 = 1.403 \text{ m}$.

Hence $\lambda = 2L_{\text{eff}} = 2.806 \text{ m}$.

The speed of sound then is $v = \lambda f = 2.806 \cdot 123 = 345 \text{ m/s}$.

This is in excellent agreement with the value $v = 346 \text{ m/s}$ at a temperature of

Table. Values for the Speed of Sound

Speed of Sound in Air		Hydrogen	Helium	
T(°C)	v _{vair} (m/s)	v (m/s)	v (m/s)	
-10	325.2			
-5	328.3			
0	331.3	1286	972	
5	334.3			
10	337.3			
15	340.3			
20	343.3			
25	346.1			
30	349.0			
35	351.9			
40	354.7			
Liquids, 25°C	v (m/s)	Solids	v _{vair} (m/s)	
Glycerol	1904	Diamond	12000	
Seawater	1533	Pyrex glass	5640	
Water	1493	Iron, steel	5000-6000	
Mercury	1450	Steel rod	5000	
Kerosene	1324	Aluminum	5100	
Methyl-		Brass	4700	
alcohol	1143	Gold	3240	
Carbon-		Lucite	2680	
tetrachloride	926	Lead	1960	
		Rubber	1600	
The values given are for longitudinal waves in bulk materials.				
Speeds for longitudinal waves in thin rods are smaller.				

Temperature dependence:
$$v_{\text{air}} = 331.3 \sqrt{1 + \frac{T}{273.15}} \approx 331.3 + 0.606 \cdot T$$

where v_{air} is in meter/second (m/s) and the temperature T in degree Celsius (°C).

Example: At $T = 25^\circ\text{C}$ (classroom temperature): $v_{\text{air}} = 331.3 + 0.606 \cdot 25 = 346.1 \text{ m/s}$

Example: Where does the change of the speed of sound with temperature play a role?

Answer: In the pitch change of wind instruments with change in ambient temperature:

Play a note A4 on a wind instrument such as a flute or bassoon with $f = 440 \text{ Hz}$ at 20°C .

Play it later at 30°C . The relative frequency change is $f_{30}/f_{20} = v_{30}/v_{20} = 349.0/343.3 = 1.0166$. The frequency has risen from $f_{20} = 440 \text{ Hz}$ to $f_{30} = 1.0166 \times 440 \text{ Hz} = 447 \text{ Hz}$, a noticeable change in pitch. You have to retune the instruments!

Basic Physics for Acoustics

Physical Quantities, Symbols, and Units

Quantity	Symbol	Unit	
Distance, length	x, L	meter	m
Time	t	second	s
Velocity, speed	v	meter/second	m/s
Acceleration	a	change of v/time	m/s ²
Mass	m	kilogram	kg
Force	F	Newton	N
Pressure	p	force/area	N/m ² , Pascal Pa

Distance, Length

1 meter = 1 m = 100 cm = 1000 mm
 1 cm = 0.01 m = 10^{-2} m
 1 mm = 0.001 m = 10^{-3} m
 1 micrometer = 1 μ m = 10^{-6} m
 1 kilometer = 1 km = 1000 m = 10^3 m

Conversions:

1 inch = 2.54 cm
 1 foot = 12 inches = 30.5 cm
 1 mile = 1.609 km = 1609 m

Note:

The physical quantities “distance”, “velocity”, “force”, and “acceleration” are vectors that include a value, physical unit, and direction, for instance 30 m northwest. The quantity “length” only includes a value and unit, irrespective of the direction, and similarly for “speed”. Nonetheless, we shall use distance and length, and also velocity and speed, interchangeably in this course.

Exercises

1. The height of a person is 6 ft. 2 in. Convert this to meters and give the answer with three (3) significant figures (for instance 1.73 m).

Answer: Height = _____ m

2. The distance between your apartment to our lecture hall at TTU is 1.7 miles. Express this in kilometers.

Answer: x = _____ km

3. The length of a cello string is 71 cm. Convert this to millimeters.

Answer: L = _____ mm

Speed and Velocity

Speed = distance/time: $v = x/t$

Alternatively: $x = v \cdot t$ and $t = x/v$

Examples

1. The speed of sound at 25°C is $v = 346$ m/s.
2. The speed of light in vacuum or air is $v = 300,000$ km/s.

Exercises

1. A car travels a distance of 390 miles from Lubbock to Austin in 7 hours. What is the average speed in m/s?

Answer: $v = 24.9$ m/s

2. What distance does sound travel in one minute? Use $v = 346$ m/s.

Answer: $x = 20.8$ km

3. A car travels at a speed of 26.8 m/s. Express this in kilometers and miles per hour.

Answer: $v = 96.5$ km/hr $v = 60.0$ mi/hr

4. The Earth-Moon distance is about 384,000 km. How long does it take a laser pulse or radio signal to travel to the moon and back? (This is the minimum time lag for communication with astronauts on the moon.) Use 300,000 km/s for the speed of light.

Answer: $t = 2.56$ s

Remark

Speed has a number and a unit, without indicating the direction, for instance 100 km/hr.

Velocity is a vector whose magnitude is the speed, but it also includes the direction of travel, for instance 100 km/hr *north*, e.g. going from Lubbock to Amarillo.

We usually only need *speed*, but may use the concepts of *speed* and *velocity* interchangeably.

Acceleration

Acceleration = change of speed/time or $a = \Delta v / \Delta t$

Example

The acceleration of a freely falling body in Earth's gravitational field is $a = g = 9.8 \text{ m/s}^2$, neglecting air resistance. This means that the falling body experiences an increase in speed of 9.8 m/s per second. This results in the unit of m/s^2 for acceleration.

Example

A car accelerates from 0 to 60 mi/hr in 8 seconds. Express this result in units of m/s^2 and compare with the value of g .

Answer: Convert the increase in speed to m/s and show that

$$\Delta v = 60 \text{ mi/hr} = 26.8 \text{ m/s}$$

Then use $\Delta t = 8 \text{ s}$ and obtain the acceleration

$$a = \Delta v / \Delta t = 3.35 \text{ m/s}^2$$

Compare this with the acceleration $g = 9.8 \text{ m/s}^2$ in Earth's gravitational field and obtain the result as a fraction of g and percentage:

$$a/g = 0.34 = 34\%$$

Where do we need acceleration in this course?

Acceleration is of interest when we consider vibrating strings in string instruments or vibrating air columns in wind instruments. Vibrations with accompanying accelerations also occur in percussion instruments.

Comment about Sound Generation, Sound Radiation, and Detection

For sound to be radiated from a source (voice, musical instrument, loudspeaker) or be detected by a receiver (ear, microphone), it is necessary that the air molecules accelerate and not just move with constant speed. Any vibration with its back-and-forth motion is associated with an acceleration.

Mass and Mass Density

The mass m of an object is a measure of its amount of matter and the number of atoms.

Unit of mass: 1 kilogram = 1 kg

1 kg = 1000 gram = 1000 g

Conversion factors

1 kg = is 2.2046 pound mass

1 pound mass = 0.45359 kg

1 ounce = 28.35 gram = 0.002835 kg

Example

1 liter of water at 4⁰ C has a mass of 1 kg.

Mass Density

Density of a substance = mass/volume. The symbol for it is the Greek letter ρ “rho”.

Hence we have the formulas

$$\rho = m/V, \text{ hence also } m = \rho V, \quad V = m/\rho$$

The unit of density ρ is kg/m³. The densities of some substances are:

Steel, Iron	$\rho = 7900 \text{ kg/m}^3$
Aluminum	2700
Copper	8930
Gold	19300
Earth average	5520
Concrete, brick, glass	~2000
Water	1000
Air at 1 atm, 0 ⁰ C	1.293

Exercises

1. A steel string has a length of 1 m and a diameter of 1 mm. What is its mass in kg and gram?

Answer: Mass $m = \rho \pi r^2 L = 0.00620 \text{ kg} = 6.20 \text{ g}$

2. Find the volume in cubic centimeters of 1 kg of gold. (Note that $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$.)

Answer: Volume $V = 51.8 \text{ cm}^3$

Demonstration

Show 2 ounces ($\approx 60 \text{ cm}^3$) of water in a beaker. What would this be worth in gold?

Answer: About \$60,000. (Verify this!)

Note that we will need the concepts of *mass* and *density* when discussing the frequencies of vibrating strings and waves.

Force

Force is “push” or “pull”.

Unit of force = 1 Newton = **1 N**

Conversion factors: 1 N = 0.2250 pound force or 1 pound force = 4.445 N

For accelerating bodies we have Newton’s 2nd law: **$F = ma$**

Exercises

1. A car accelerates with an acceleration $a = 2 \text{ m/s}^2$. The mass of the car is 1500 kg. What is the magnitude of the force that accelerates the car, friction neglected?

Answer: Force $F = 3000 \text{ N}$

2. A person has a mass of 72.6 kg. What is his weight in Newton? (Use $g = 9.8 \text{ m/s}^2$.)

Answer: $W = mg = 711 \text{ N}$

3. What is the person’s weight in pounds?

Answer: $W = 160 \text{ lb force}$

Weight of a Body

You are standing on a scale. What is your weight? The short answer is: $W = mg$.

But there are some subtle details in case you are interested:

Gravity pulls you down. The reaction force from a scale, the so-called “normal force”, pushes you up. The two forces are in equilibrium as long as you don’t accelerate. In this case the scale reads the reaction force with a compressed spring or its equivalent, and this is your weight! The absolute values of the gravitational force and the normal force are the same, but their directions are opposite.

Gravity pulls down with a force $F = mg$. The reaction force from the scale when standing still is $(-mg)$ up. We call this the “weight”. Its magnitude is $W = mg$, but we cavalierly omitted the minus sign. Did you ever think of your weight in this way?

Note that a person standing on a scale and falling freely is *weightless*. What happened?

There is no reaction force up from the scale, and so the scale reads nothing. You are “weightless”. The only force acting on the object is the gravitational force down - *not* the weight.

Demonstrations

1. Suspend a mass on a spring. See how gravity pulls the mass down and the reaction force from the spring pulls it up.

2. Use a spring scale and suspend a mass from it. Read the weight directly in Newton from the dial on the scale.

Pressure

Pressure = Force/Area: $p = F/A$

Unit of pressure = $1 \text{ N/m}^2 = 1 \text{ Pascal} = 1 \text{ Pa}$

Examples

1. Average atmospheric pressure at sea level:

1 atmosphere = 1 atm = 101,325 Pa = 1013.25 millibar (mb)

1 atm = 14.70 lb/in² = 14.7 PSI

2. The pressure in the tires of a car is $p = 32 \text{ PSI} = 32/14.70 = 2.18 \text{ atm}$.

Pressure Fluctuations in Sound Waves

Sound waves in air cause small pressure fluctuations about the average atmospheric pressure. We hear these fluctuations as sound.

Typical amplitudes of pressure fluctuations in sound waves in air are:

Smallest audible sound $\Delta p \approx 0.00002 \text{ Pa}$

Soft music $\Delta p \approx 0.002 \text{ Pa}$

Moderately loud music $\Delta p \approx 0.02 \text{ Pa}$

Threshold of pain $\Delta p \approx 20 \text{ Pa}$

Obviously the pressure fluctuations in sound waves are very small compared to the static atmospheric pressure of about 100,000 Pa.

Exercises

1. Calculate the ratio $\Delta p/p_{\text{atm}}$ for moderately loud music and interpret your result.

Answer: $\Delta p/p_{\text{atm}} = 2.0 \cdot 10^{-7}$

2. An elephant and a woman with high-heeled shoes are standing on a beach. Who exerts a higher pressure on the ground? Estimate the pressure in both cases.

Hint: Calculate the weight from $F = mg$ and use $p = F/A$. Estimate the mass m of the elephant and of the woman, and the total area A in contact with the ground. Don't forget that the elephant has four feet and the woman only two. You should find that the pressure on the ground from each of the woman's heels is on the order of magnitude $p \sim 50 \text{ atm}$!

Answer (do it yourself)

Mass of the elephant $m = \underline{\hspace{2cm}} \text{ kg}$,

Weight of elephant $W = mg = \underline{\hspace{2cm}} \text{ N}$

Total area A : Elephant $A = \underline{\hspace{2cm}} \text{ m}^2$

Pressure: Elephant $p = \underline{\hspace{2cm}} \text{ N/m}^2$

mass of the woman $m = \underline{\hspace{2cm}} \text{ kg}$

Weight of woman $W = \underline{\hspace{2cm}} \text{ N}$

Woman $A = \underline{\hspace{2cm}} \text{ m}^2$

Woman $p = \underline{\hspace{2cm}} \text{ N/m}^2$

Vibrating Strings - Some Basics

“There is geometry in the humming of the strings. There is music in the spacing of the spheres.” (Pythagoras, 570–495 B.C.)

A string under tension accelerates and radiates sound when bowed or plucked. The pitch (fundamental frequency) of the sound depends on the tension T in the string, its length L , and linear mass density μ (mass/length, $\mu = m/L$ - see later).

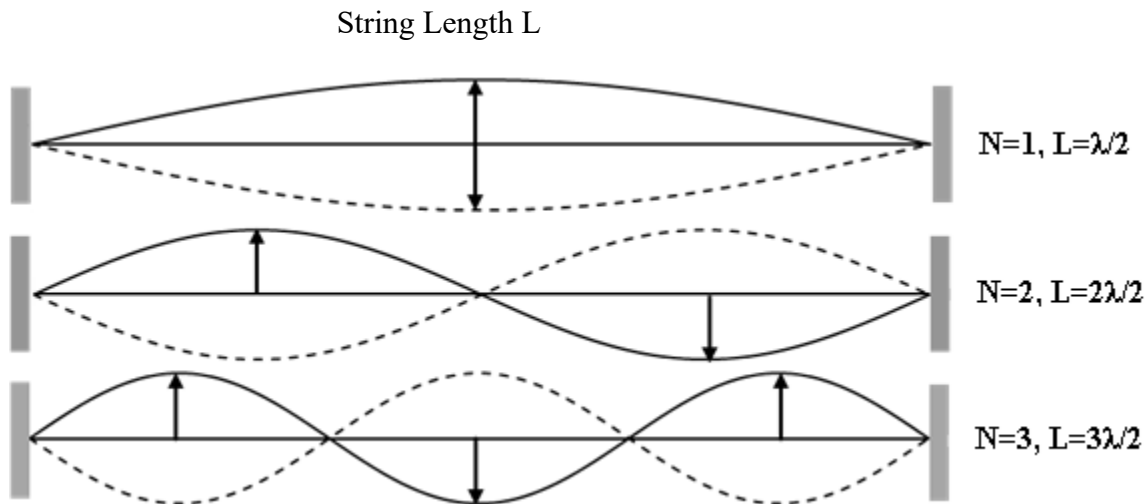


Figure. The lowest three vibrational modes of a stretched string fastened at its ends and the relationship between string length L and wavelength λ .

N = 1 first harmonic or fundamental mode, $L = (1/2) \cdot \lambda_1$ or $\lambda_1 = (2/1) \cdot L$
 N = 2 second harmonic or first overtone, $L = (2/2) \cdot \lambda_2$ or $\lambda_2 = (2/2) \cdot L$
 N = 3 third harmonic or second overtone, $L = (3/2) \cdot \lambda_3$ or $\lambda_3 = (2/3) \cdot L$
 etc.... Generally we have:

$\lambda_N = 2L/N$, where $N = 1, 2, 3, 4, \dots$ is the so-called **harmonics number**.

For the harmonic frequencies of the vibrating string, we then have $f_N = v/\lambda_N = N(v/2L)$, or $f_N = Nf_1$, with $f_N = f_1, 2f_1, 3f_1, 4f_1, 5f_1, \dots$, where $f_1 = v/\lambda_N = (v/2L)$ is the fundamental frequency (or “pitch”).

Demonstrations

1. Drive a stretched cord with a vibrator and show the different vibrational modes.
2. Use a sonometer or monochord with a stretched string fastened at its two ends. Vary the length or tension of the string and listen to the change in frequency. Divide the string into two equal parts with a wedge and listen to the resulting *octave*. Divide the string into two appropriate parts and listen to the musical intervals of a *fifth*, *fourth*, and *third*.
3. Show the strings on a violin. Bow and pluck them.
4. Show a “slinky” and simulate the transverse vibrations of a string.
5. With the slinky, also simulate the longitudinal vibrations in air.

Resonance

Resonance generally occurs when a driving force of frequency f excites vibrations in an object whose own natural frequency of vibration is the same as that of the driver, or

$$f_{\text{driver}} = f_{\text{vibrator}} = f_{\text{resonator}}$$

The distinction between vibrator and resonator may not always be obvious.

Examples

1. A child is on a swing: His parent (driver) pushes on the swing with the same frequency as the natural frequency of the swing (resonator). There is no obvious vibrator.
2. You hit a metal bar with a mallet (driver). The bar is the vibrator and resonator.
3. You strike a marimba bar (vibrator) with a mallet (driver). The tube (resonator) below amplifies the sound from the vibrating bar
3. You slap a plastic tube with your hand (driver) on one of its open ends. The impulse from your hand contains a range of frequencies. The tube is the resonator and briefly resonates with its own natural frequency. The resonance frequency of the fundamental vibrational mode then is $f = v/\lambda = v/4L$.). There is no obvious vibrator (maybe the air).
4. The bow (driver) of a violin excites a string (vibrator). The vibrations are transferred to the body (resonator) of the violin. The body radiates the sound. Little sound comes from the vibrating strings themselves.

Demonstrations

1. Observe the different resonance frequencies of 5 washers suspended from a rod.
2. Move a spring pendulum up-and-down. Note the resonance when $f_{\text{driver}} = f_{\text{resonator}}$.
3. Place a vibrating A4 tuning fork in front of a 38.5 cm long plastic pipe (resonator) tuned to 440 Hz and note the increase in loudness caused by the resonance.
4. Excite the vibrations in a large cardboard packing tube by tapping it on the ground.
5. Place wooden bars or tile strips on top of a tuned closed tube and note the resonance.

Helmholtz Resonators (Hermann von Helmholtz, 1821–1894)

Enclosed cavities (resonators) such as boxes, cylinders, and bottles with small openings have characteristic resonances when excited by tapping, blowing, or striking with sticks or mallets (drivers).

Demonstrations

1. Blow across the opening of a bottle and listen to the resonance resembling a deep and “throaty” tone. Blowing produces a broad frequency spectrum. The air in the bottle responds by vibrating selectively at the frequency of the Helmholtz resonance.
2. A loudspeaker enclosure is a Helmholtz resonator that enhances the low frequencies.
3. Pluck or bow the strings of a sonometer and violin. Listen to the “amplified” sound from the openings in the wooden Helmholtz resonator.
4. A tuning fork (C4) on a wooden box with one end open sounds louder than without the box. Other tuning forks than C4 do not meet the resonance condition and sound less loud.
5. Helmholtz resonator boxes with vibrating bars on top (440 Hz, 441 Hz). Cover the hole of one box and note the diminished sound intensity. Remove the cover again and hear the sound come back louder again.

Summary of Wave Parameters

A sine wave is characterized by:

Frequency f

Amplitude A

Phase ϕ

Wavelength λ

Wave speed $v = \lambda f$

Mathematical Expression for a Traveling Sine Wave

For a simple sine wave, the displacement of the vibrating medium at a location x and at time t is given by

$$y(x,t) = A \cdot \sin[2\pi(x/\lambda - f \cdot t) - \phi],$$

where $y(x,t)$ is the displacement of the vibrating medium from equilibrium at position x and time t , λ the wavelength, f the frequency, and ϕ the phase difference when y is not the maximum displacement (amplitude A) at position $x = 0$ and time $t = 0$.

The above expression is the mathematical representation of a sine wave traveling in the x -direction.

Transverse Wave

The medium vibrates perpendicular to the direction of wave propagation.

Example: A string under tension.

Longitudinal Wave (Pressure Wave)

Medium vibrates in the direction of motion. Example: sound waves in air.

Demonstrations

1. Transverse wave traveling along a rope or long spring.
2. Longitudinal waves on a slinky.
3. Longitudinal waves on a vertically oriented spring driven by a vibrator.

Remember the Formulas?

Wave speed $v = x/t = \text{wavelength}/\text{period} = \lambda/T \rightarrow v = \lambda/T$

Frequency $f = \text{inverse period} = 1/T \rightarrow v = \lambda f$

Example

A wave travels on a stretched string. The period of oscillation is $T = 5 \text{ ms} = 0.005 \text{ s}$ and the wavelength is $\lambda = 1.2 \text{ m}$. What is the speed of the wave on the string?

Answer: Method 1: $v = \lambda/T = 1.2 \text{ m}/0.005 \text{ s} = 240 \text{ m/s}$

Answer: Method 2: Convert to $f = 1/T = 1/0.005 \text{ s} = 200 \text{ Hz}$

$v = \lambda f = 1.2 \text{ m} \times 200 \text{ Hz} = 240 \text{ m/s}$, same as it must be.