

Lab 11 solutions

Part I

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- 1) $C4 = 261.63 \text{ Hz}$
 $G4 = C4 * 3/2 = 392.445 \text{ Hz}$
 $F3 = C4 * 2/3 = 174.42 \text{ Hz}$
 - 2) $C8\#/D1b = (3/2)^{12} = 129.7463$
 - 3) $D8b/D1b = 2^7 = 128$
 - 4) $129.7463/128 = 1.0136$ so the difference is 1.36%

Part II

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- 1) $E4 = C4 * 5/4 = 327.0375 \text{ Hz}$
 $A3b = C4 * 4/5 = 209.304 \text{ Hz}$

Part III

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- 1) $2^{(1/12)} = 1.0595$ (semi-tone interval)

$$\begin{aligned} C4 &= 261.63 \text{ Hz} \\ C4\# (D4b) &= C4 * 2^{(1/12)} = 277.19 \text{ Hz} \\ D4 &= C4 * 2^{(2/12)} = 293.67 \text{ Hz} \\ D4\# (E4b) &= C4 * 2^{(3/12)} = 311.13 \text{ Hz} \\ E4 &= C4 * 2^{(4/12)} = 329.63 \text{ Hz} \\ F4 &= C4 * 2^{(5/12)} = 349.23 \text{ Hz} \\ F4\# (G4b) &= C4 * 2^{(6/12)} = 370.00 \text{ Hz} \\ G4 &= C4 * 2^{(7/12)} = 392.00 \text{ Hz} \\ G4\# (A4b) &= C4 * 2^{(8/12)} = 415.31 \text{ Hz} \\ A4 &= C4 * 2^{(9/12)} = 440.00 \text{ Hz} \\ A4\# (B4b) &= C4 * 2^{(10/12)} = 466.17 \text{ Hz} \\ B4 &= C4 * 2^{(11/12)} = 493.89 \text{ Hz} \end{aligned}$$

- 2) The frequencies calculated above agree (to round-off) with the frequencies in Figure 1 for the piano keyboard

- 3) Just temperament:

$$\begin{aligned} C4 &= 261.63 \text{ Hz} \\ E4 &= C4 * 5/4 = 327.04 \text{ Hz} \\ G4 &= C4 * 3/2 = 392.45 \text{ Hz} \end{aligned}$$

Equal temperament:

$$\begin{aligned} C4 &= 261.63 \text{ Hz} \\ E4 &= C4 * 2^{(4/12)} = 329.63 \text{ Hz} \\ G4 &= C4 * 2^{(7/12)} = 392.00 \text{ Hz} \end{aligned}$$

- 4) Expect the major triad in just temperament to sound better than in

equal temperament, because some of the harmonics of C4, E4, and G4 will overlap in just temperament.

Part IV

1) Start with A4 = 440 Hz and tune the other strings using fifths

Pythagorean temperament:

$$G3 = A4 * (2/3)^2 = 195.56 \text{ Hz}$$

$$D4 = A4 * 2/3 = 293.33 \text{ Hz}$$

$$A4 = 440 \text{ Hz}$$

$$E5 = A4 * 3/2 = 660 \text{ Hz}$$

Equal temperament: (from Figure 1)

$$G3 = 196.00 \text{ Hz}$$

$$D4 = 293.66 \text{ Hz}$$

$$A4 = 440 \text{ Hz}$$

$$E5 = 659.26 \text{ Hz}$$

2) Pythagorean:

$$E4 = C4 * (3/2)^4 * (1/2)^2 = 331.13 \text{ Hz}$$

Equal temperament: (from Figure 1)

$$E4 = 329.63 \text{ Hz}$$

Since these frequencies are slightly different, expect to hear beats with beat frequency $|f_2 - f_1| = 1.5 \text{ Hz}$