4. Air Resonance

PURPOSE AND BACKGROUND

The concept of *resonance* in a pipe is similar to that of a string. The waves in pipes consist of compressions and rarefactions of the air, with back-and-forth motion of the air molecules in the direction of propagation or against it. The waves in air thus are *longitudinal* waves. In this laboratory we study standing waves in a pipe. They are the result of two waves traveling in opposite directions inside the pipe, with each wave being reflected at the ends of the pipe. In this way the superposition of two waves yields a standing wave, provided that in addition the *resonance conditions* are met.

For a pipe with both ends open, resonance at the *lowest* frequency (*fundamental frequency* or *first harmonic*) occurs when there are anti-nodes of the air motion at the ends – and only there, with a single velocity node at the center, see Figure 1. The motion of air molecules is highest at the anti-nodes and lowest at the nodes. For a pipe with one end closed and one end open, resonance

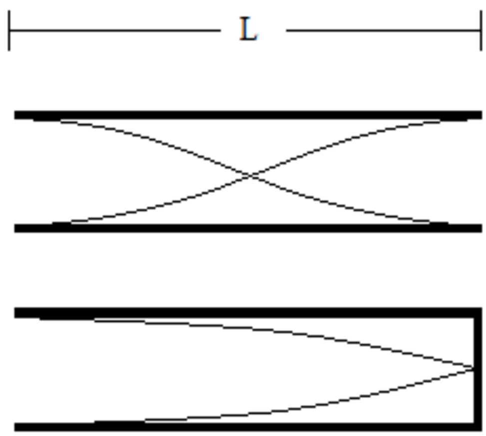

Figure 1: Air molecule displacement in an open and closed pipe.

at the lowest frequency occurs when we have a velocity node at the closed end and an anti-node at the open end. Plotted in Figure 1 is the displacement or velocity of air molecules as a function of position along the pipe. The two curves for each pipe in Figure 1 are one-half period of oscillation apart.

For the pipe with both ends open, we have $L = \lambda/2$ according to Figure 1. For the closed pipe we have $L = \lambda/4$. The fundamental frequency is given by

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} \quad \text{(both ends open)}\,, \qquad f_1 = \frac{v}{\lambda} = \frac{v}{4L} \quad \text{(one end closed)}\,, \tag{1}$$

where v is the velocity of sound.

Questions

1. For a pipe with both ends open, what are the formulas for the fundamental frequency and the frequencies f_2, f_3, f_4 of the next three overtones or harmonics? (Hint: Higher harmonics have frequencies that are integer multiples of the fundamental, and all integers are allowed for a pipe with both ends open.)

2. For a pipe with one end closed and one end open, we have $L = \lambda/4$ according to Figure 1. Write down the equations for the fundamental frequency and for the first three existing overtones. (Hint: *Only odd integers are allowed* as you can see by extending the drawings in Figure 1 to higher harmonics.)

EXPERIMENTAL PROCEDURE

In the lab, we can record the frequency spectrum of sound in a pipe using the setup shown in Figure 2. (This figure is specifically for a tube that is open at both ends.) The speaker produces *white noise* and the software Electroacoustic Toolbox can be used to record spectrum. Figure 3 shows sample recorded frequency spectra for both types of tubes.

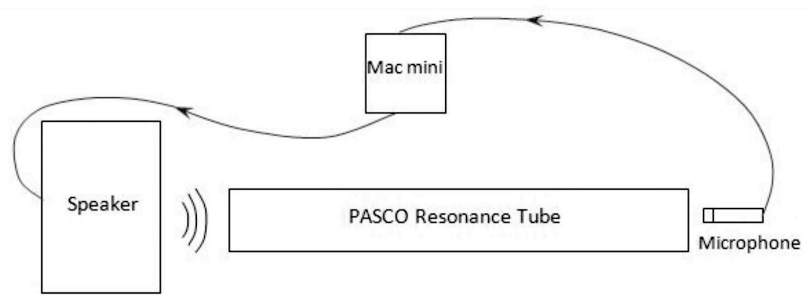

Figure 2: Set up of the resonance tube in the "open tube" configuration. White noise from the Mac mini is applied to the speaker. The sound enters the tube on the left and excites the resonances. The microphone on the right records them for display in the computer. In the "closed tube" configuration the speaker and microphone must be on the same (right) side.

I Pipe Length Correction and Determination of the Speed of Sound

If you were to compare the calculated and observed fundamental frequencies from an actual experiment, you would find that they do not agree very well. This has to do with the fact that in pipes, waves reflect from the ends of the tube by sticking out a little bit. There is an end correction that increases the wavelength. This correction is proportional to the radius of the tube. Therefore, the larger the tube radius, the more the wave will "stick out" and cause an increase in wavelength. The correction results in an extra length ΔL, given from theory by $\Delta L = 0.61r$ for each open end, where r is the radius of the pipe. Thus for a closed pipe and open pipe of length L and radius r, the effective lengths are, respectively,

$$L_{\text{eff, closed}} = L + 0.61r\,, \qquad L_{\text{eff, open}} = L + 1.22r\,. \tag{2}$$

Questions

1. Calculate the effective lengths of a closed and open tube if $L = 131$ cm and the diameter of the tube is $D = 14.4$ cm. (Recall: The diameter D equals twice the radius r.)

2. Determine the velocity of sound using the data in Figure 3 for the resonance tube open at both ends. The fundamental frequency is $f_1 = 146$ Hz and corrected pipe length is $L_{\text{eff, open}} = 1.18$ m.

3. How does your value compare with the value of 346 m/s expected for the speed of sound at at a temperature of 25 Celsius? If there is a discrepancy, what might be the reasons?

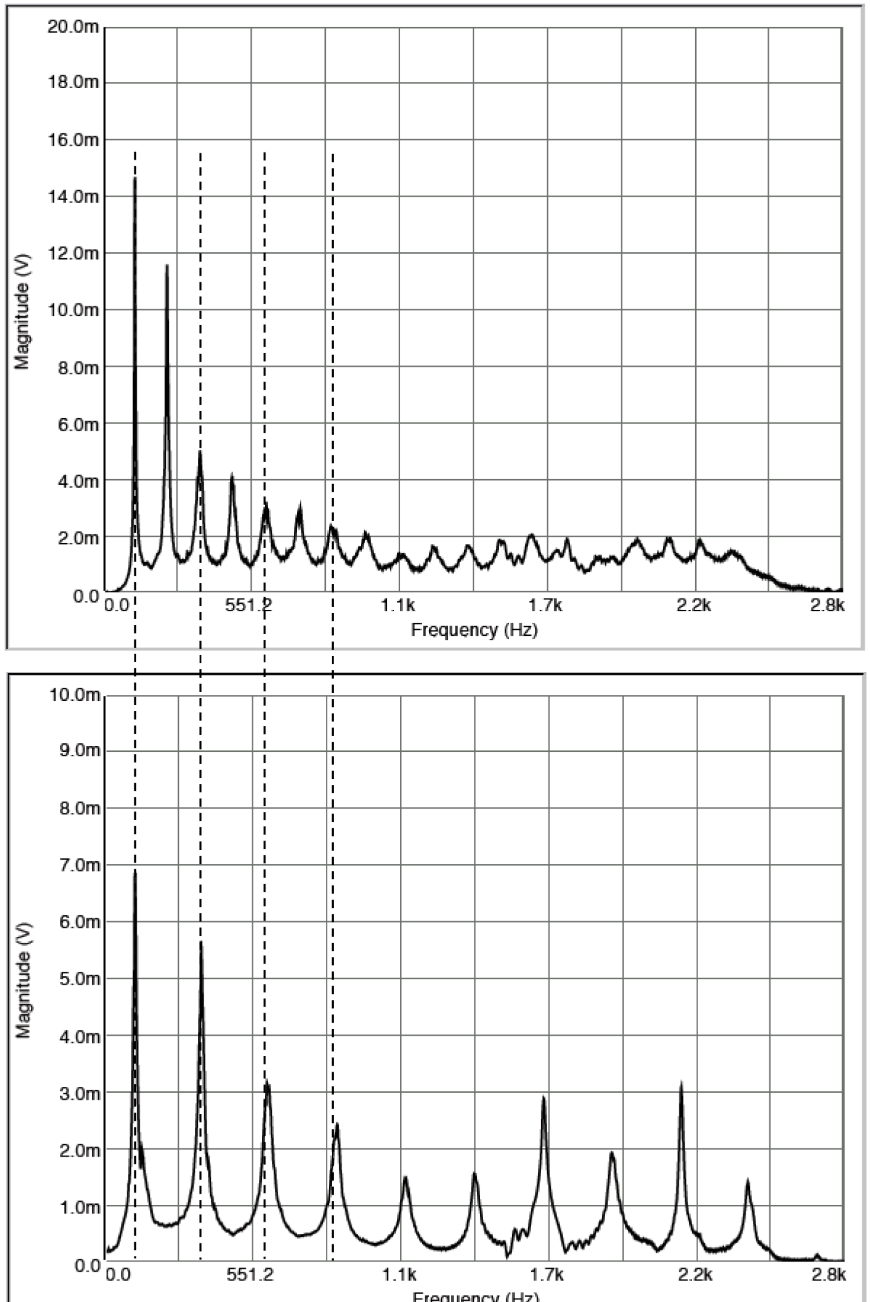

Figure 3: Resonances of a PASCO resonance tube excited with white noise. Upper figure: Tube open at both ends with an effective length $L_{\text{eff, open}} = 1.18$ m. Lower figure: Tube closed at one end with an effective length $L_{\text{eff, closed}} = L_{\text{eff, open}}/2 = 0.59$ m (with a plug in the tube to shorten its length). The fundamental frequency for both tubes is $f_1 = 146$ Hz, but only the odd harmonics are observed in the tube closed at one end.

II Experiments with the Large Cardboard PASCO Packing Tube

We can do additional experiments with the large brown cardboard packing tube from PASCO in order to study resonance and the decay of sound intensity. We close one end of the tube with a plug. Just by holding a microphone near the top of the tube, we can see a peak in the acquired FFT frequency spectrum. This peak corresponds to the resonating fundamental, which gets excited just from the broadband background noise in the room. The tube acts as a resonator that picks out its resonance frequency from the ambient noise and responds much less to the other frequencies in noise.

We can also tap the tube with its closed end on the floor to excite the tube resonances more strongly than just from the ambient noise. We can listen to the resulting resonance and record the frequency spectrum with the microphone and the FFT mode in the Electroacoustics Toolbox as usual. An example of such a resonance curve is shown in Figure 4. Note the absence of the even harmonics for a tube closed at one end.

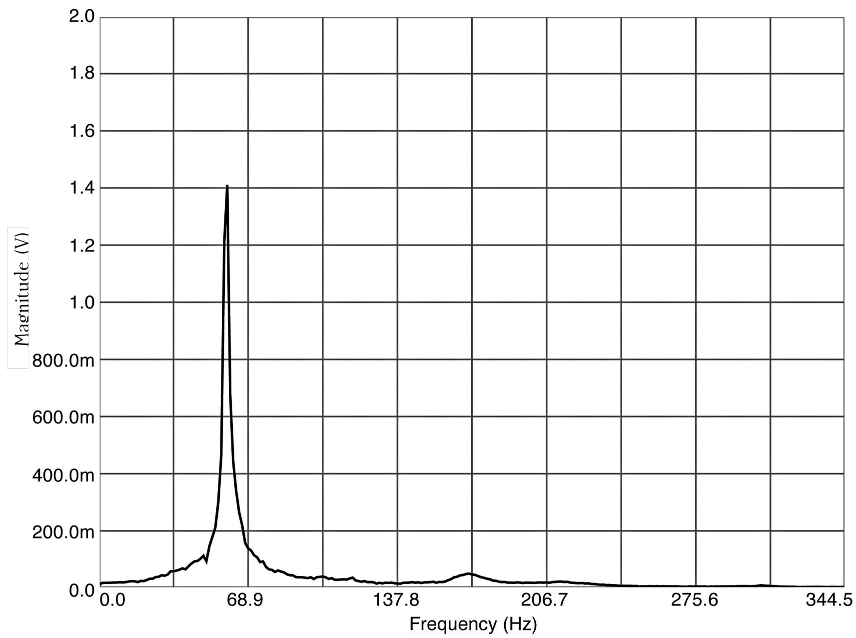

Figure 4: Frequency spectrum from a cylindrical PASCO packing tube, tapped on the floor with the closed end. The large peak is the $N = 1$ harmonic (fundamental frequency), the small peak is the $N = 3$ harmonic. The $N = 2$ harmonic is missing, as is to be expected for a tube closed at one end.

Finally, we can record the *waveform* of the damped oscillation using the Oscilloscope Tool in Electroacoustics Toolbox. Figure 5 shows the decay of the signal as a function of time after tapping the tube on the floor at time t = 400 ms. The waveform closely resembles a *damped sine wave*.

Questions

1. Look at the observed waveform in Figure 5 and estimate the so-called *exponential decay time*. This is the time it takes for the signal to decrease to 37% of its original value.

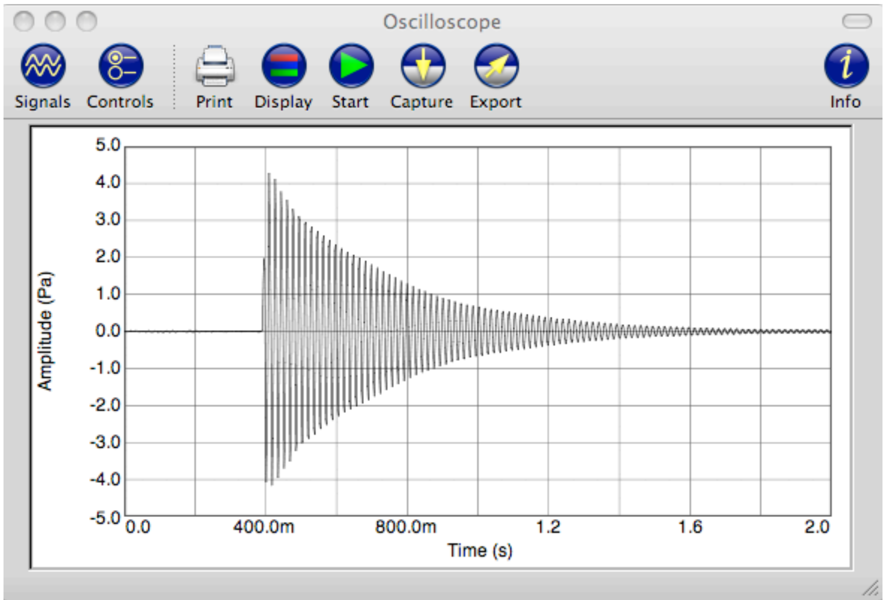

Figure 5: Decaying waveform of the packing tube.

2. An aboriginal *didgeridoo* has a length $L = 120.5$ cm and a diameter $D = 4.2$ cm. When you blow into a didgeridoo, it behaves like a tube closed at one end. Calculate the expected fundamental frequency of the didgeridoo using the appropriate corrected pipe length and $v = 346$ m/s for the speed of sound.

III Helmholtz Resonator

We can also do experiments with a simple spherical cavity called a *Helmholtz Resonator*. We have a large hollow metal sphere, with a tube protruding from one side for admitting white noise from a computer speaker. It has another smaller tube on the opposite side for listening to the resonance frequency or for recording the frequency spectrum with a shotgun microphone on a long shaft that can be inserted into this tubing.

Even without any white noise excitation, one can listen to the sound from the Helmholtz resonator when exposed to ambient room noise. It is a deep rumbling tone, corresponding to the resonant frequency of the spherical cavity. The resonance is excited from the broad noise spectrum in the room. Hermann Helmholtz (1821-1894) used a series of such "Helmholtz Resonators" of different sizes to analyze the frequency spectrum of sounds and musical instruments, all before the advent of electronic tools!

A more sophisticated setup for the Helmholtz resonator is shown in Figure 6. A speaker is connected (via a stero receiver) to the Mac mini. A resonance frequency spectrum from the Helmholtz resonator using a white noise excitation is shown in Figure 7. It has one prominent peak.

A Calculation of the Resonance Frequency of a Helmholtz Resonator

The resonance frequency of a Helmholtz resonator is given by the formula

$$f = \frac{v}{2\pi}\sqrt{\frac{A}{L_{\text{eff}} V}}\,, \tag{3}$$

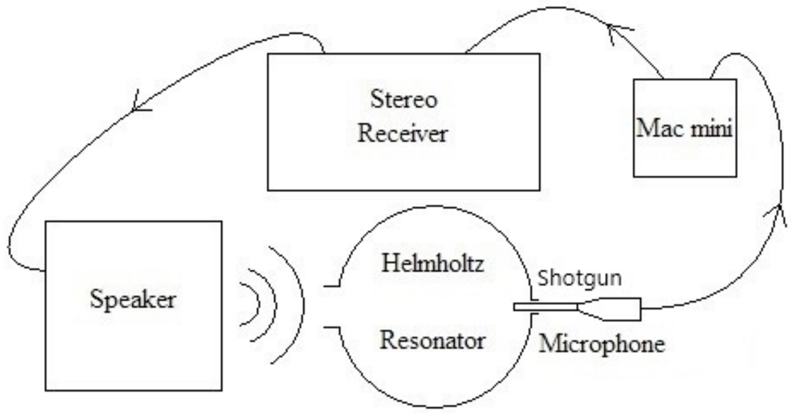

Figure 6: Experimental setup for a spherical Helmholtz resonator.

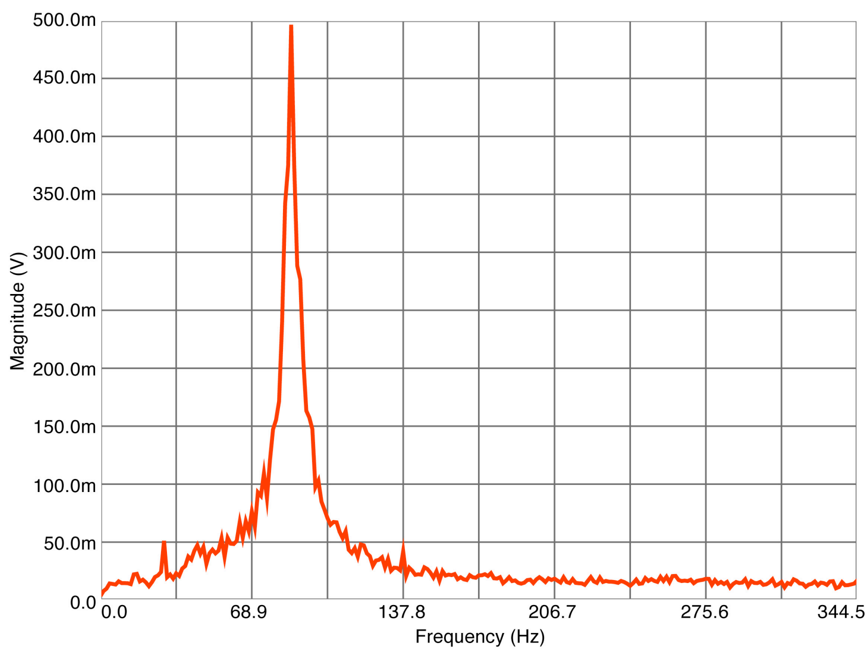

Figure 7: Helmholtz resonance curve from a large aluminum sphere. The measured and calculated values of the resonance frequency at the peak are 92 Hz and 93 Hz, respectively.

where v is the velocity of sound, A is the area of the opening of the resonator, L_{eff} is the effective length of the cylindrical neck, and V is the volume. This formula is quite general and can by used for spheres, bottles, etc.

P.S: If you have a Helmholtz resonator such as a box with just a hole in it, rather than a "bottle neck", you can still use formula (3). For the actual length we have $L = 0$. But L_{eff} is not zero. The hole is open at both ends. So use $L = 0$ in equation (2) and obtain $L_{\text{eff}} = 1.22r$ for the hole.

Questions

1. The radius of the sphere is $R = 0.150$ m, the length of the "bottle neck" is $L = 0.080$ m and its inner radius $r = 0.041$ m. Given these values calculate the values for the area A, V, and $L_{\text{effective}} = L + 1.22r$ needed in equation 3. Then calculate the resonance freqeuncy f.

B Helmholtz Resonance in Bottles

Distinct Helmholtz resonances can be obtained by blowing gently across the opening of different bottles. The resonance frequency can be measured from a frequency as in Figure 7 or Figure 8, for a 0.75-liter wine bottle. It can also be calculated from equation (3) once we determine A, L_{eff}, and V. For A we use the measured inside diameter of the bottle neck. For L_{eff} we use the measured length L of the bottle neck with a correction for the tube being open at both ends (see below). The volume V can be read off from the label on the bottle.

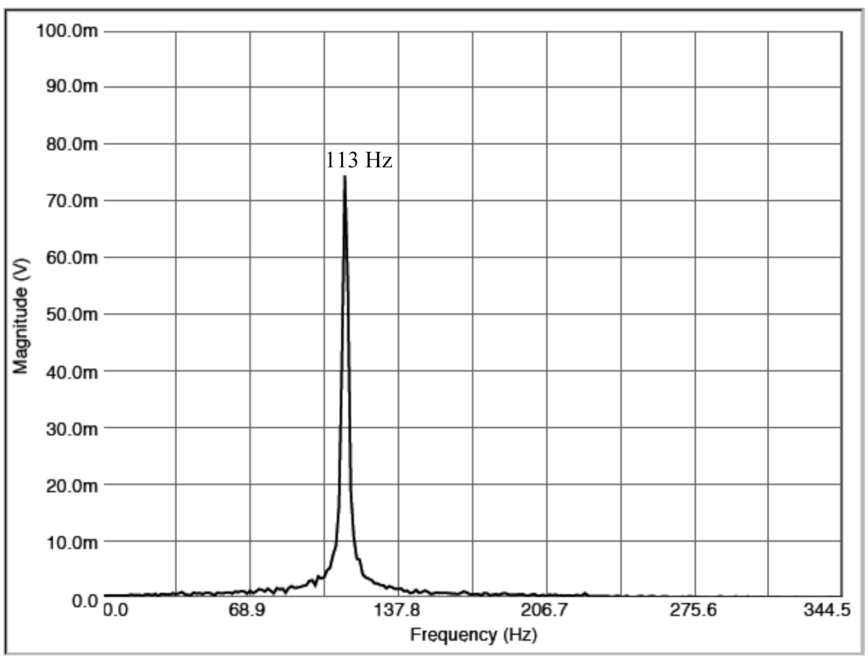

Figure 8: Helmholtz resonance from a 0.75 liter wine bottle.

Questions

1. Calculate the expected resonance frequency for a 0.75 liter wine bottle given the following data: The volume $V = 0.75$ liter $= 0.75 \times 10^{-3}$ m^3. The average radius at the middle of

the bottle neck is $r_{\text{avg}} = 10.53$ mm $= 0.01053$ m. The actual length of the bottle neck is $L = 8.5$ cm $= 0.085$ m. For the length correction at the un-baffled outside opening at top of the bottle neck take $0.61 r_{\text{avg}}$, and for the baffled opening inside the bottle take $0.85 r_{\text{avg}}$ (from physics theory). Thus the effective length of bottle neck is given by $L_{\text{eff}} = L + (0.61 + 0.85) r_{\text{avg}} = 0.085 + 1.46 \times 0.01053 = 0.1004$ m.

Given all these values calculate the resonance frequency.

2. By how much does the calculated resonance frequency differ from the measured frequency shown in Figure 8? Express the difference as a percent difference

$$\text{percent difference} = \frac{|f_{\text{calculated}} - f_{\text{measured}}|}{f_{\text{measured}}} . \tag{4}$$

3. Derive the frequency ratio $f_{1\text{ liter}}/f_{2\text{ liter}}$ for a 1-liter and 2-liter wine bottle, assuming the only thing that changes is the volume V of the bottle. This frequency interval is called a "tritone" or "devil's tone".