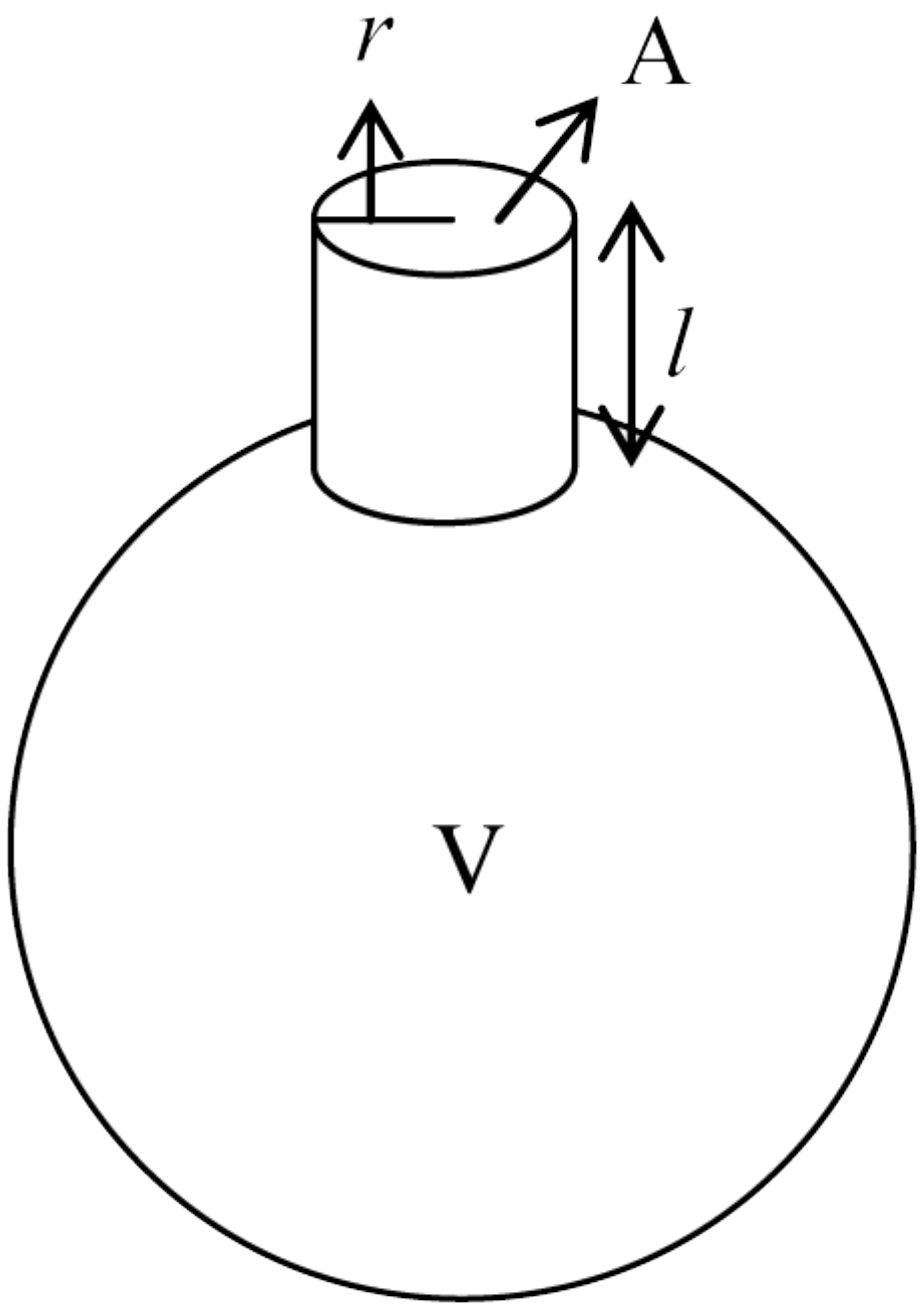
Helmholtz resonator



$$f = \frac{v}{2\pi}\sqrt{\frac{A}{l_{\text{eff}}V}}$$

- Example:

$r = 1\ \text{cm}\,,\ l = 2.7\ \text{cm}\,, V = 425\ \text{mL}\,, v = 346\ \text{m/s}$

$A = \pi r^2,\ 1\ \text{mL} = 10^{-6}\ \text{m}^3 \quad \Rightarrow \quad f = 239\ \text{Hz}$

4. Fourier analysis & synthesis

Fourier's theorem

- **standing wave vibrations** are the **"building blocks"** for any complex vibration
- any complex periodic wave can be written as a **sum of harmonics**:

$$y(t) = A_1 \sin(2\pi f_1 t + \phi_1) + A_2 \sin(2\pi f_2 t + \phi_2) + \cdots$$

$$f_N = N f_1, \qquad N = 1, 2, \cdots$$

- **Ohm's law of hearing**: Phases have little effect on the timbre of the sound
- **Fourier analysis**: decomposing a complex periodic wave into its contributing harmonics
- **Fourier synthesis**: constructing a complex periodic wave by combining harmonics

5. String instruments

Plucked versus bowed strings

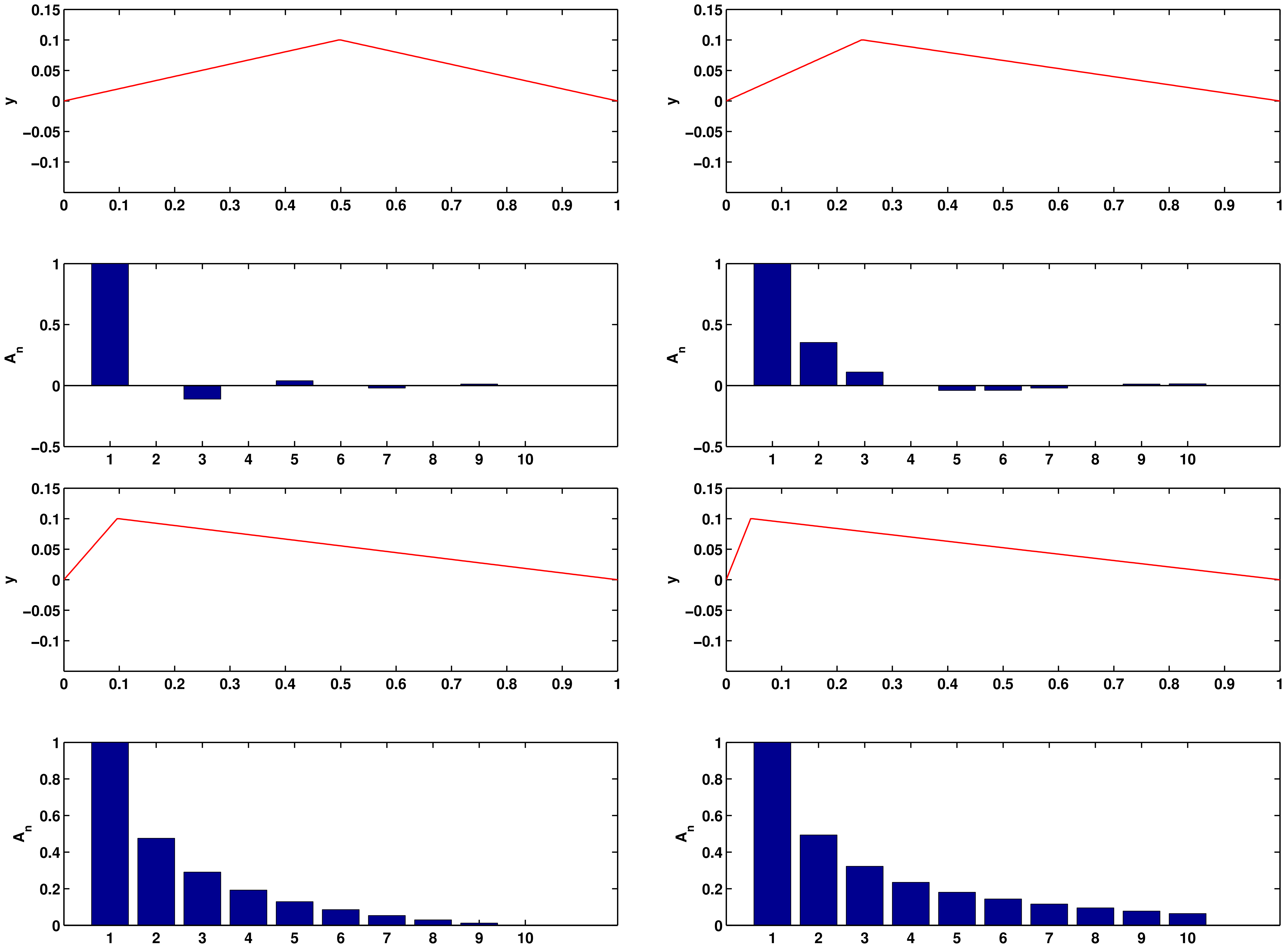
- Plucked string: https://www.youtube.com/watch?v=_X72on6CSL0
- Bowed string: https://www.youtube.com/watch?v=6JeyiM0YNo4
- iPhone guitar video: https://www.youtube.com/watch?v=TKF6nFzpHBU

- NOTE: the iPhone guitar video does not show the wave pulses on the strings as they really are. Rather one sees multiple images of the same pulse shape on the string due to the "rolling shutter" effect of the iPhone camera. The actual pulses on a guitar string behave as shown in the first video.

Fourier coefficients of a plucked string

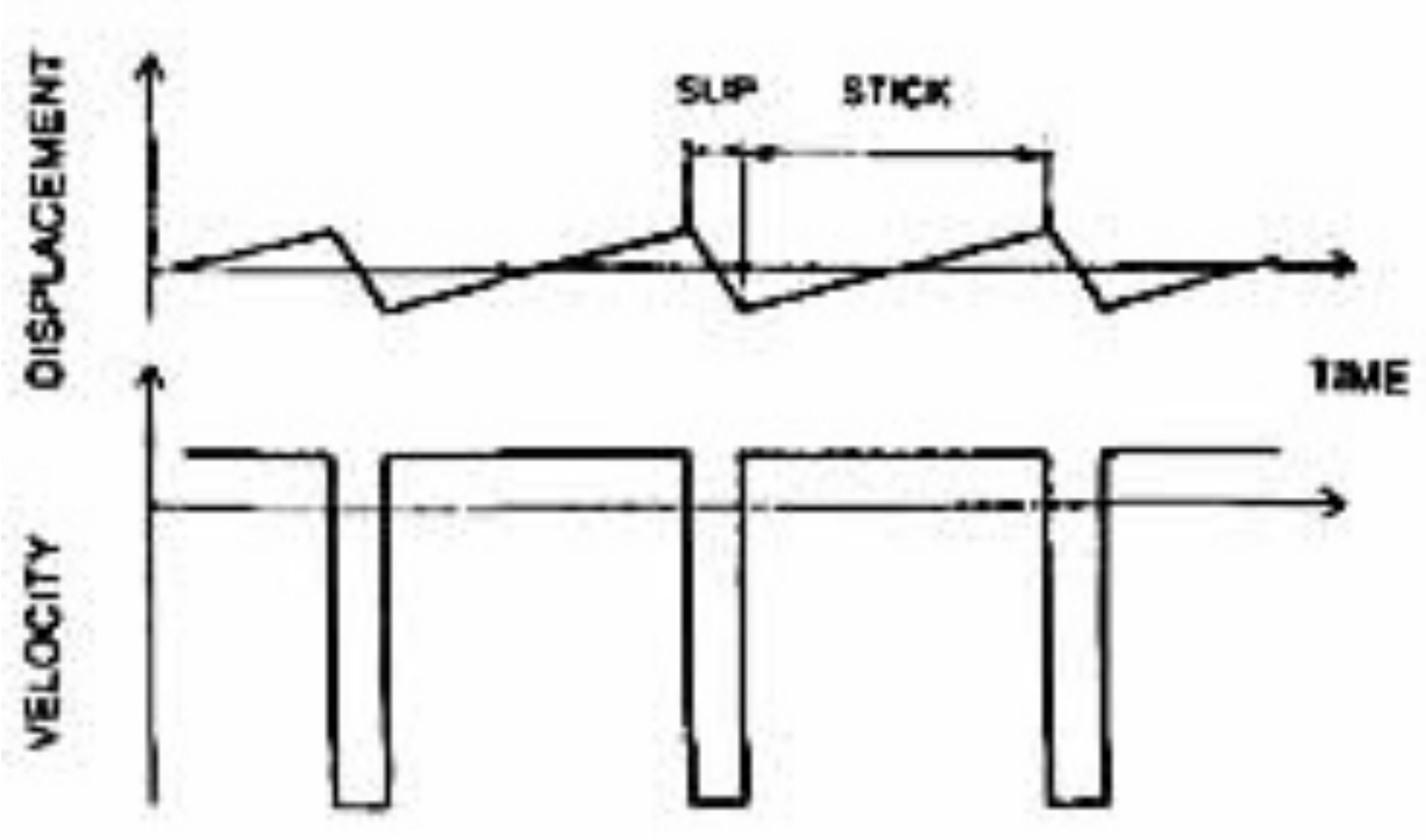
- Sounds are **richer** when the string is plucked **closer to the bridge**

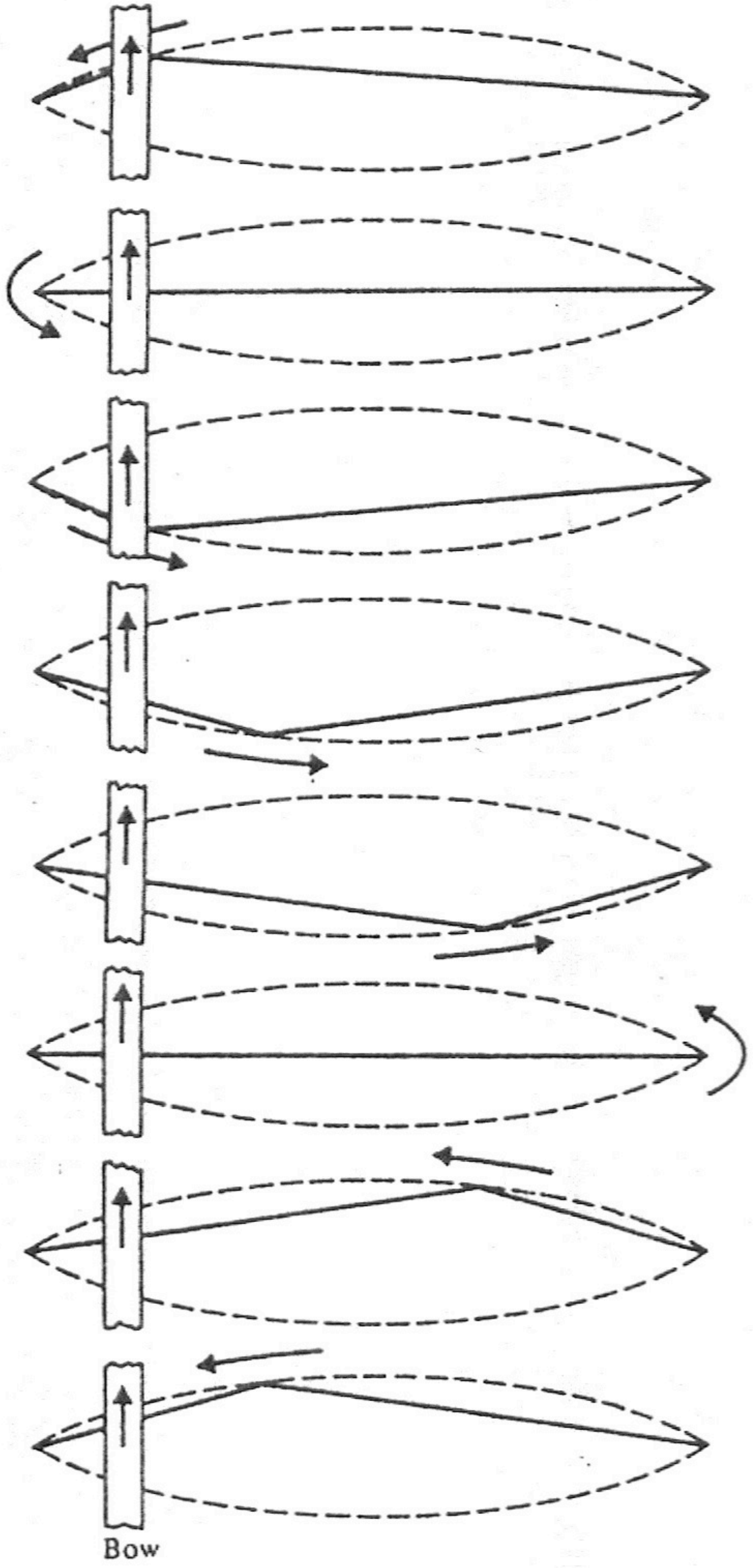
- If the string is plucked in the **middle**, there at **no even harmonics**



Stick-slip motion of a bowed string

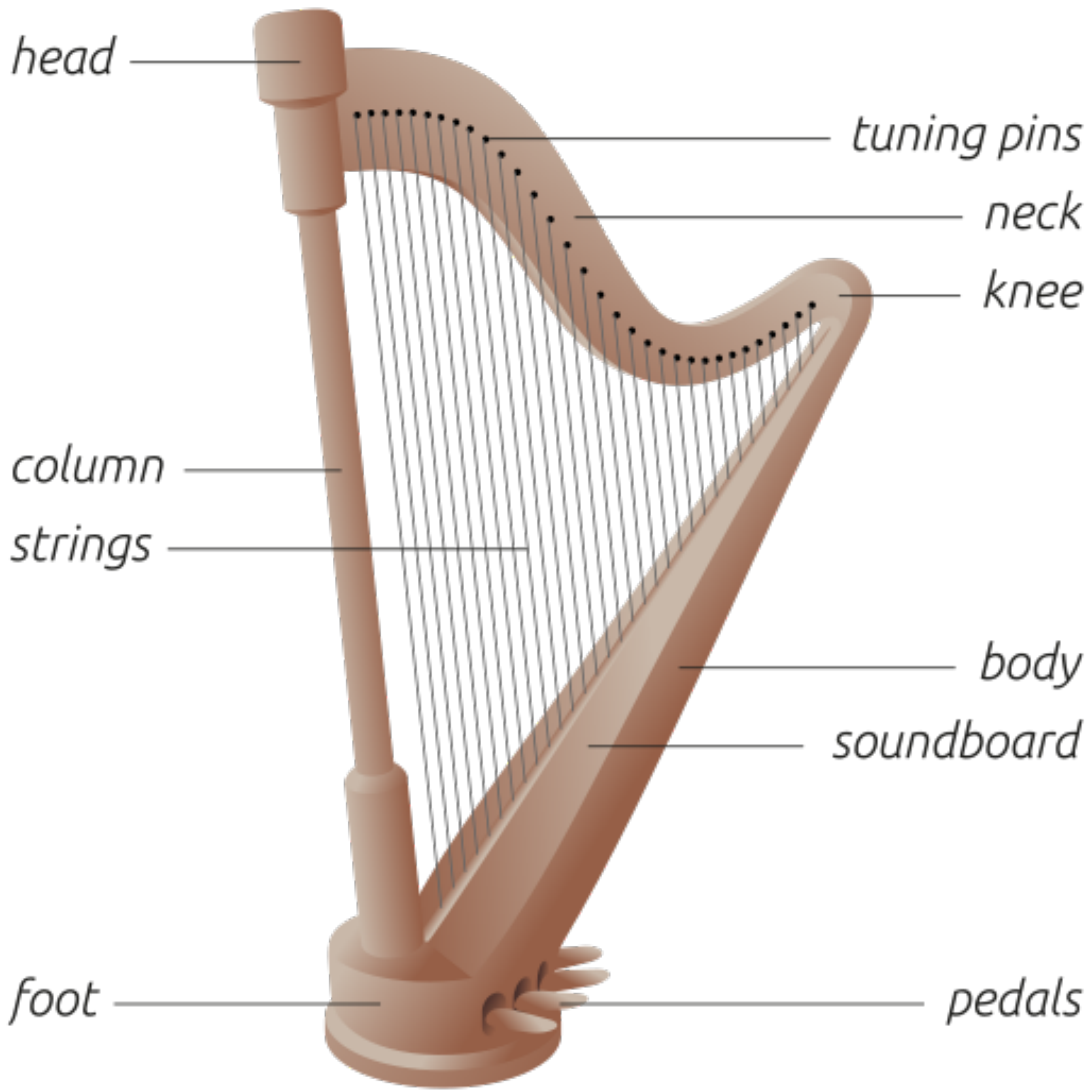
- The violin string alternately “sticks” and then “slips” against the bow hundreds of times per second





Harp

- the strings have **different fixed lengths** and are plucked
- only **one note** per string -> need lots of strings
- foot pedal can change the note, but only by **only a semitone**

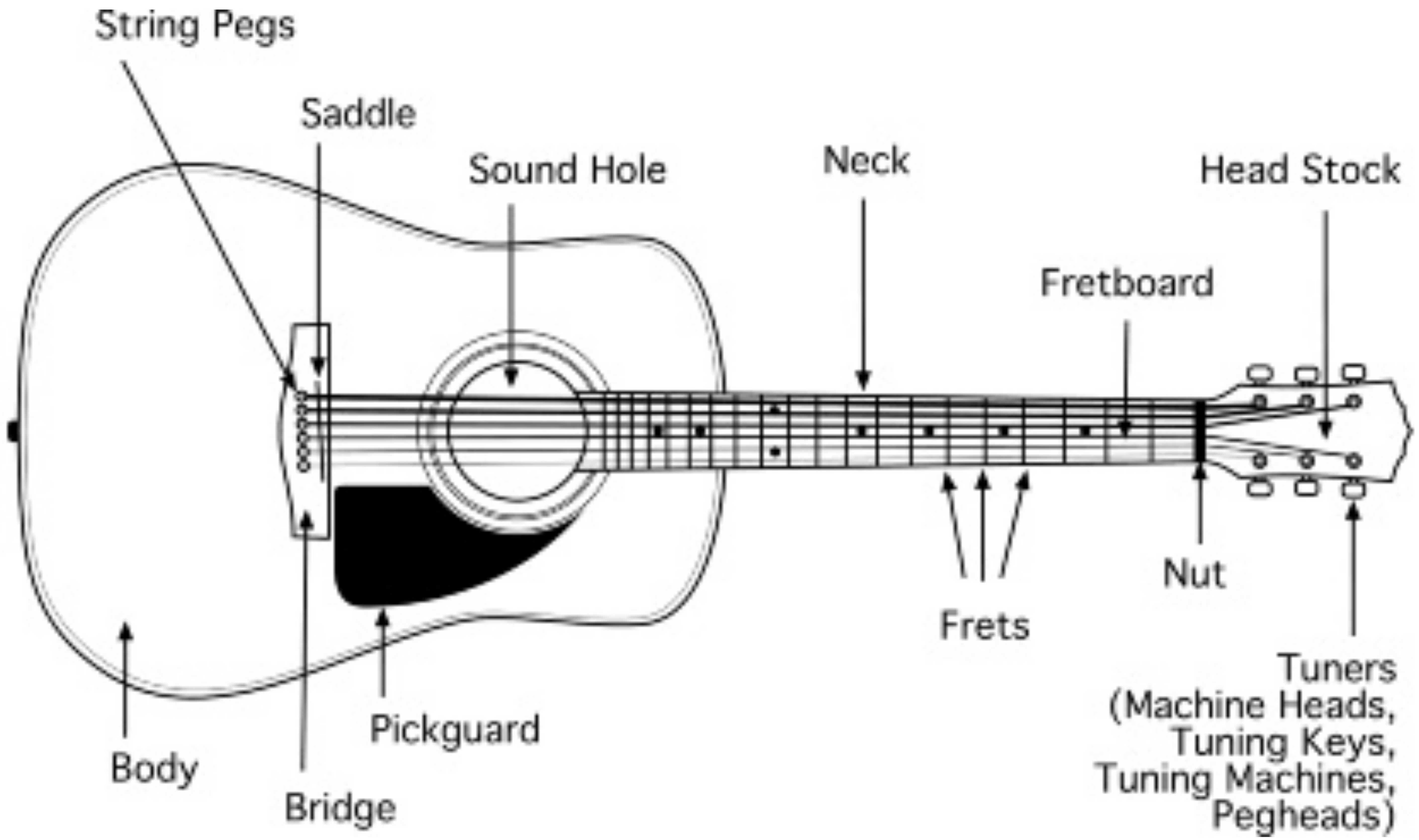


Guitar

- strings are all the **same length**, but are made of different materials and are under different tensions

- get **multiple notes** per string by pressing against a fret

- frets -> **fixed notes** (like a piano keyboard)



Violin

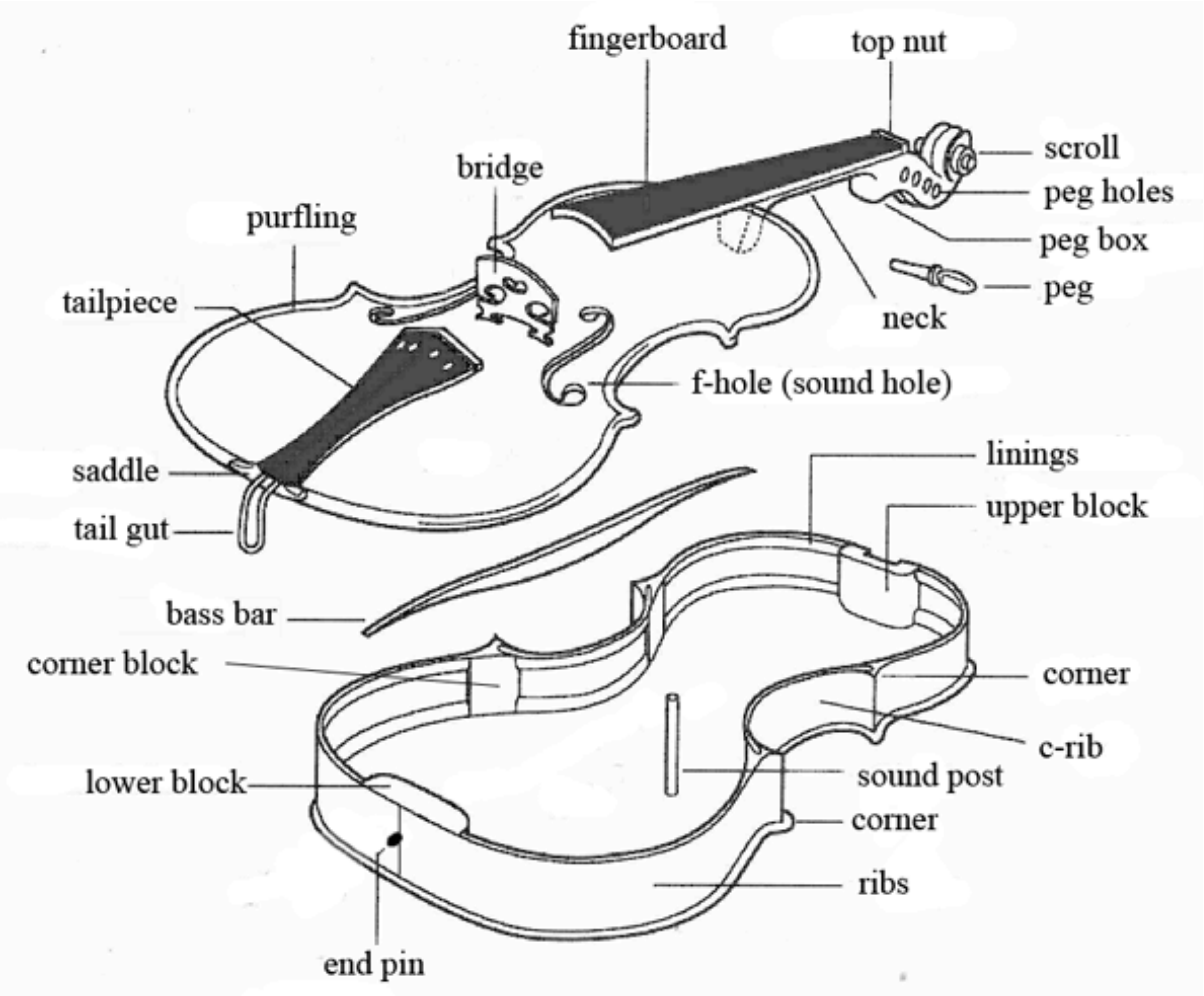
- strings are all the **same length**, but are made of different materials and are under different tensions

- get **multiple notes** per string by pressing against the neck

- no frets -> **no fixed notes**

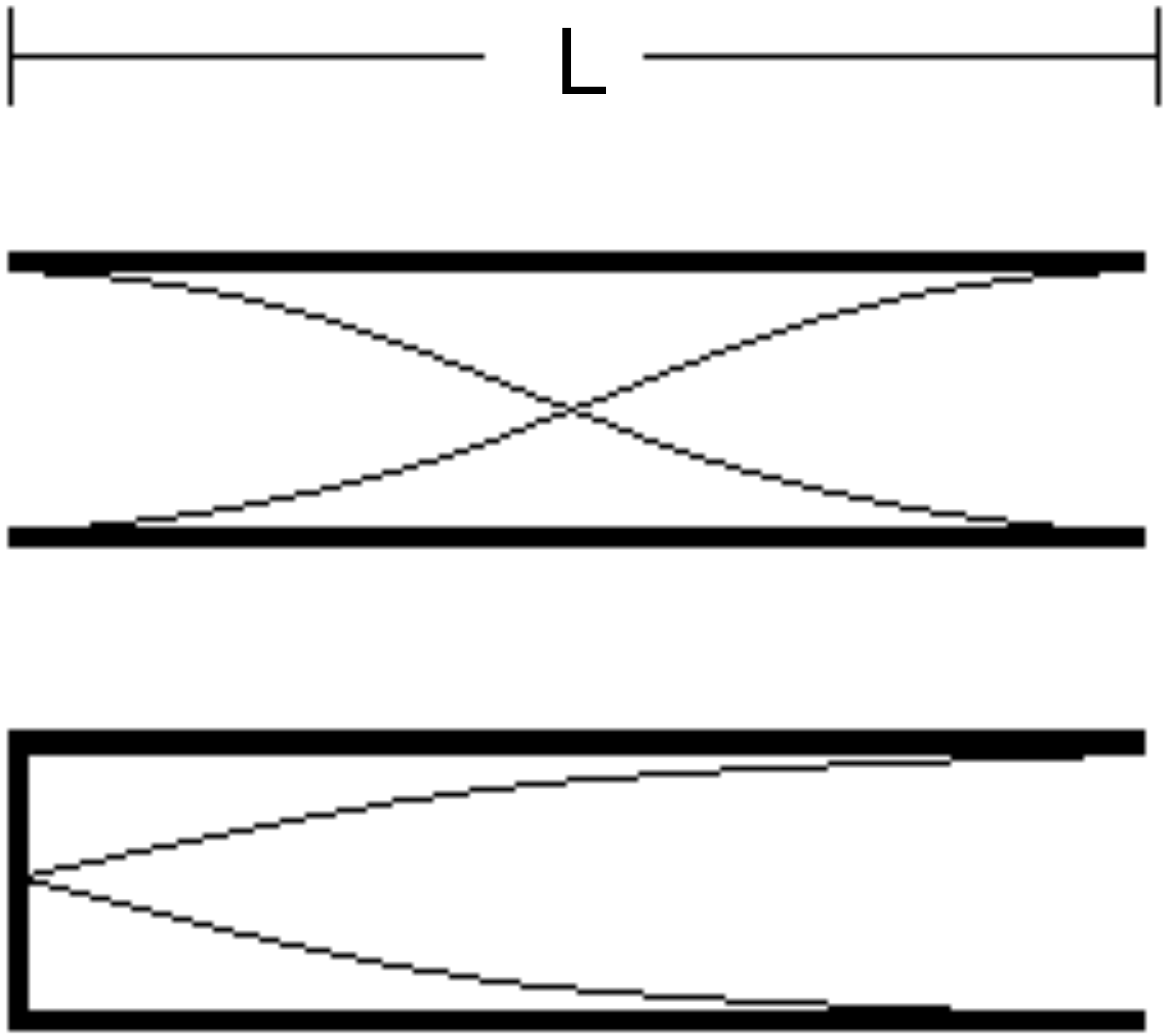
- string vibrations are **quickly damped** if strings are plucked -> bowed instead

- can **vary tone quality** by adjusting the intensity of bowing



6. Wind instruments

Open and closed tubes (recall previous discussion)



$$\lambda_N = \frac{2L}{N} \qquad f_N = N f_1 \qquad f_1 = \frac{v}{2L} \qquad N = 1,2,\cdots$$

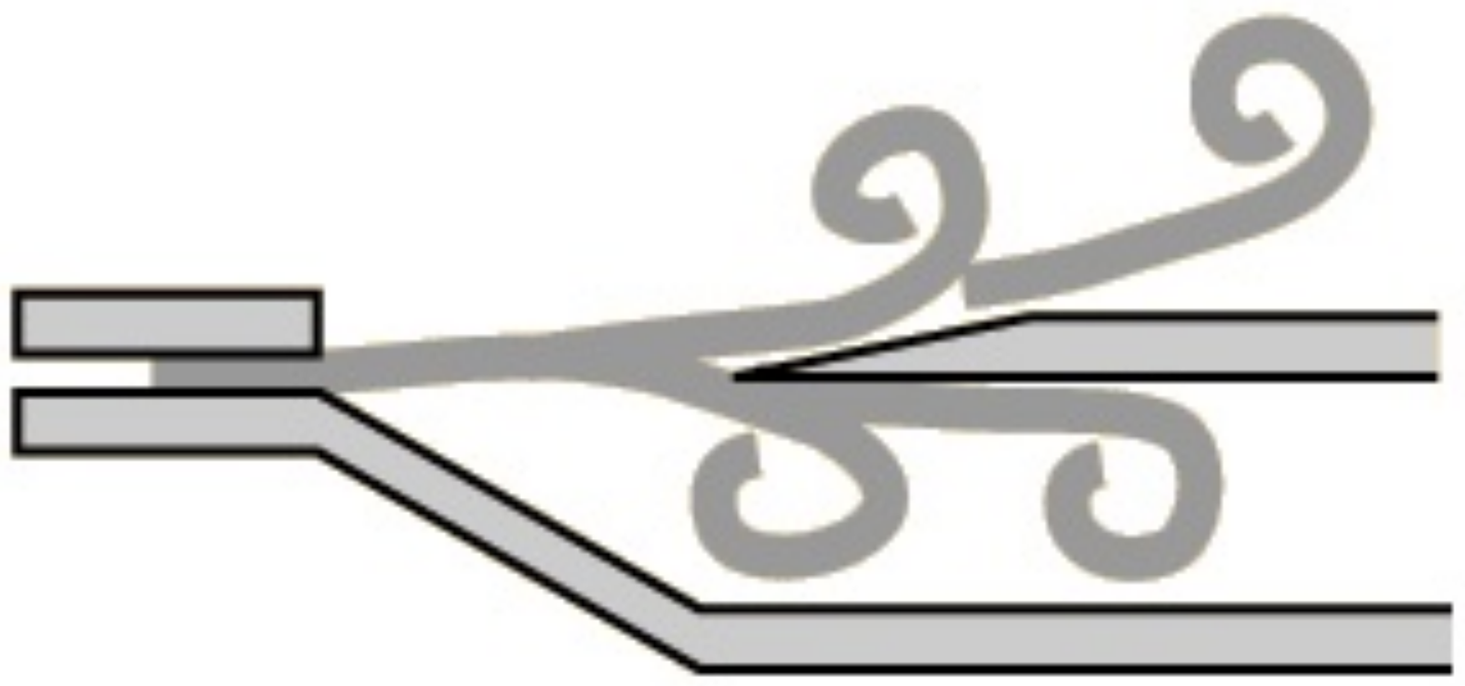
(both even and odd harmonics)

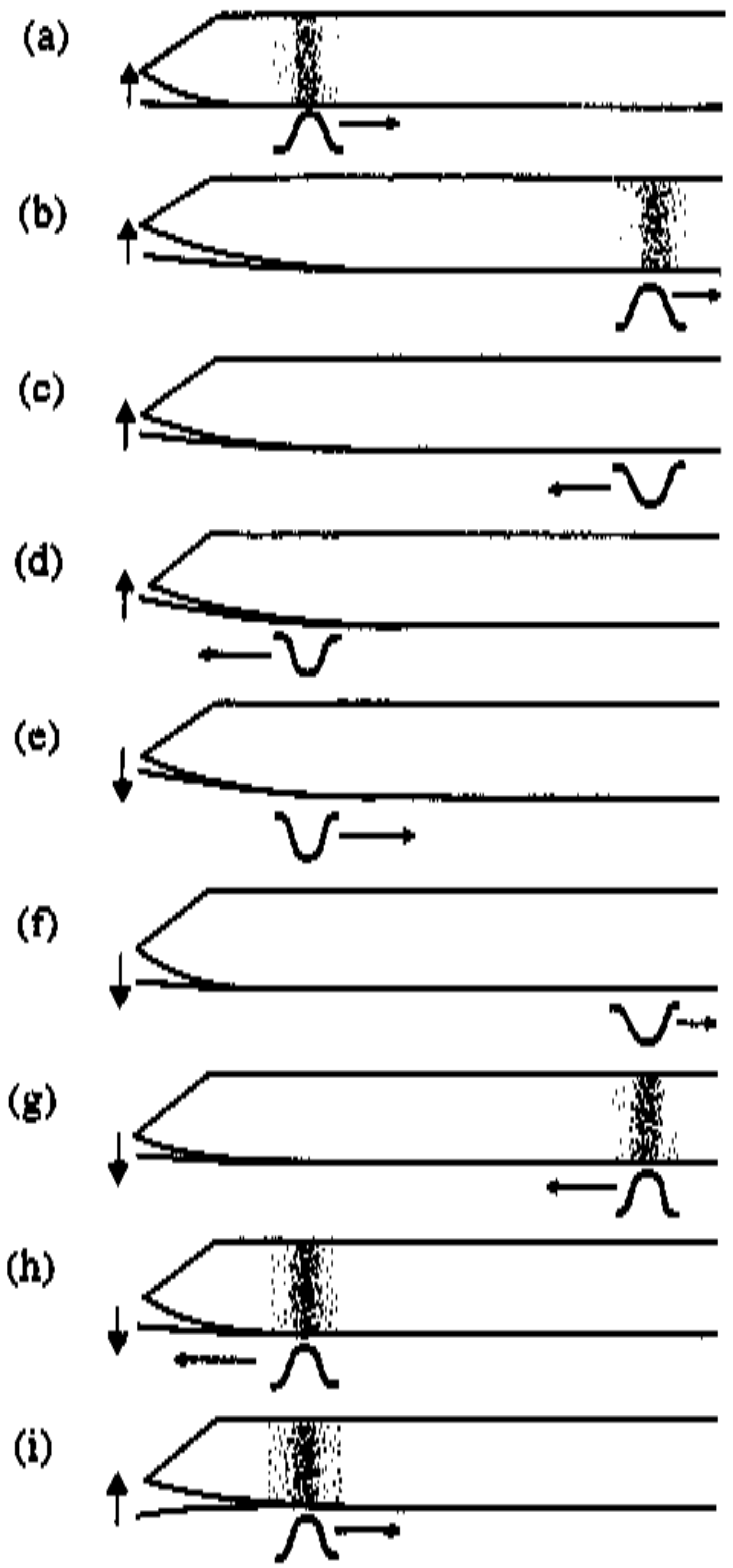
$$\lambda_N = \frac{4L}{N} \qquad f_N = N f_1 \qquad f_1 = \frac{v}{4L} \qquad N = 1,3,5,\cdots$$

(only odd harmonics)

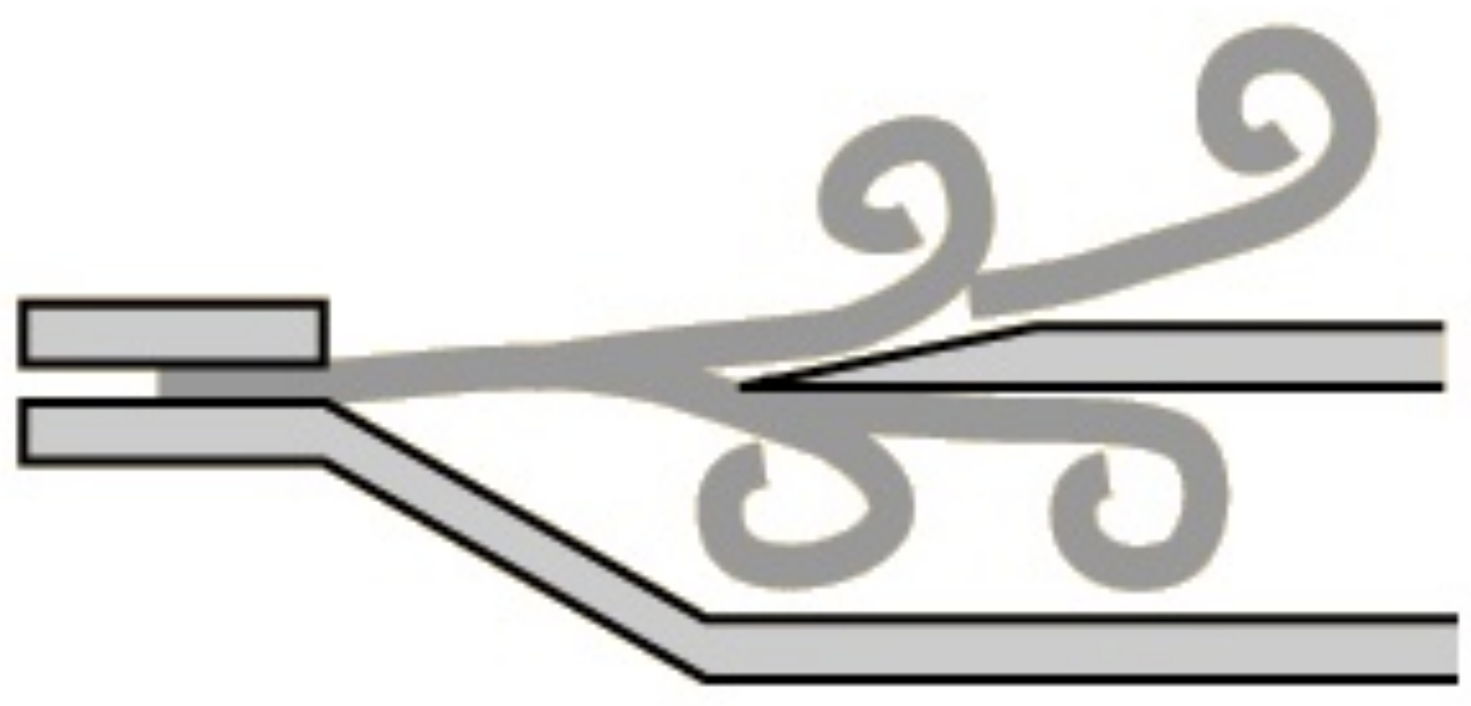
(air molecule displacements)

Excitations produced by an oscillating air stream or a vibrating reed





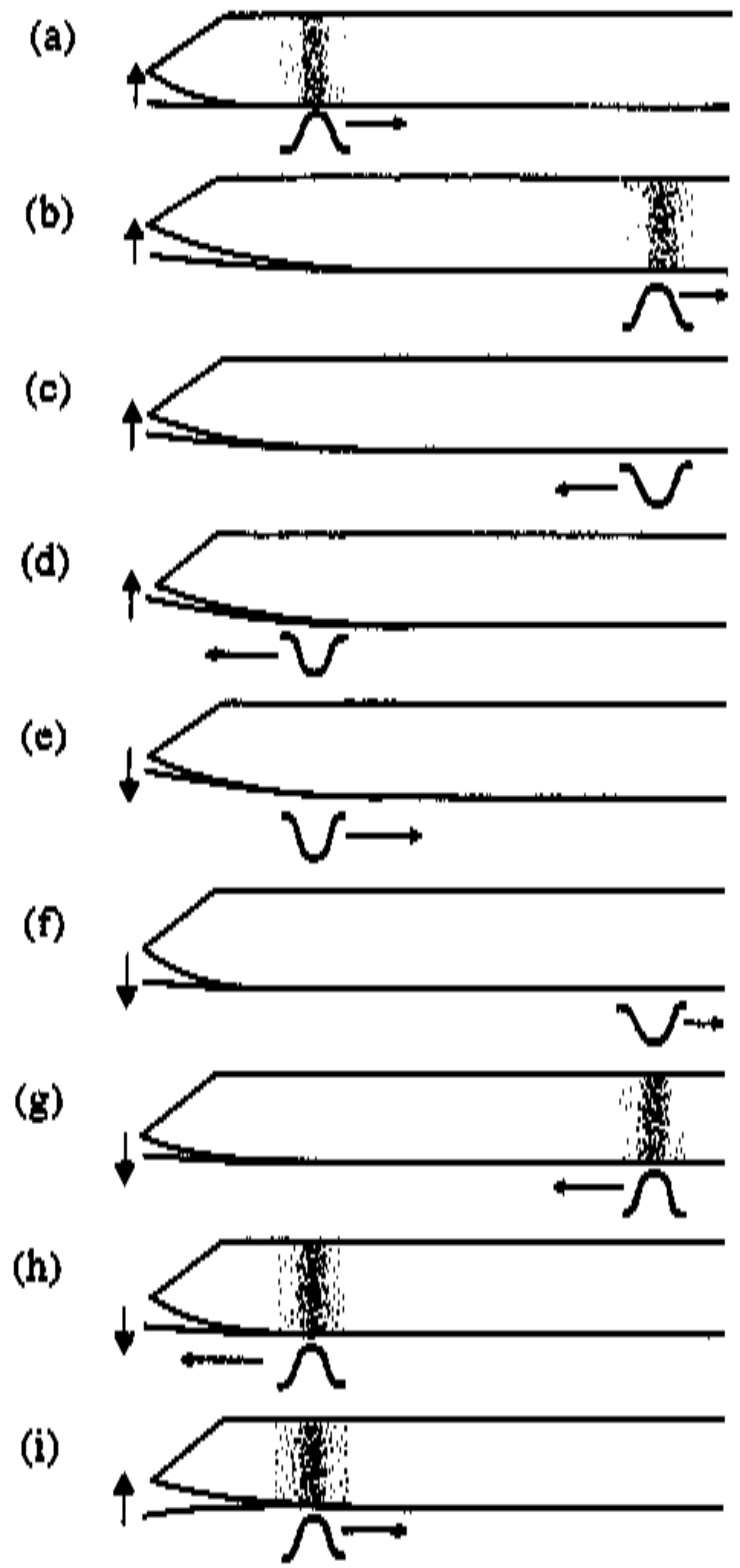
Oscillating air stream



- “**Flow-controlled**” excitation
- Used in recorders, flutes, penny whistles, etc.
- When the air stream hits the edge, it creates tiny **whirpools** (or “vortices”) of air, which **alternate** going either into or out of the tube
- The frequency of the alternating whirlpools is determined by the **natural frequencies** of the remainder of the tube (resonance phenomena)

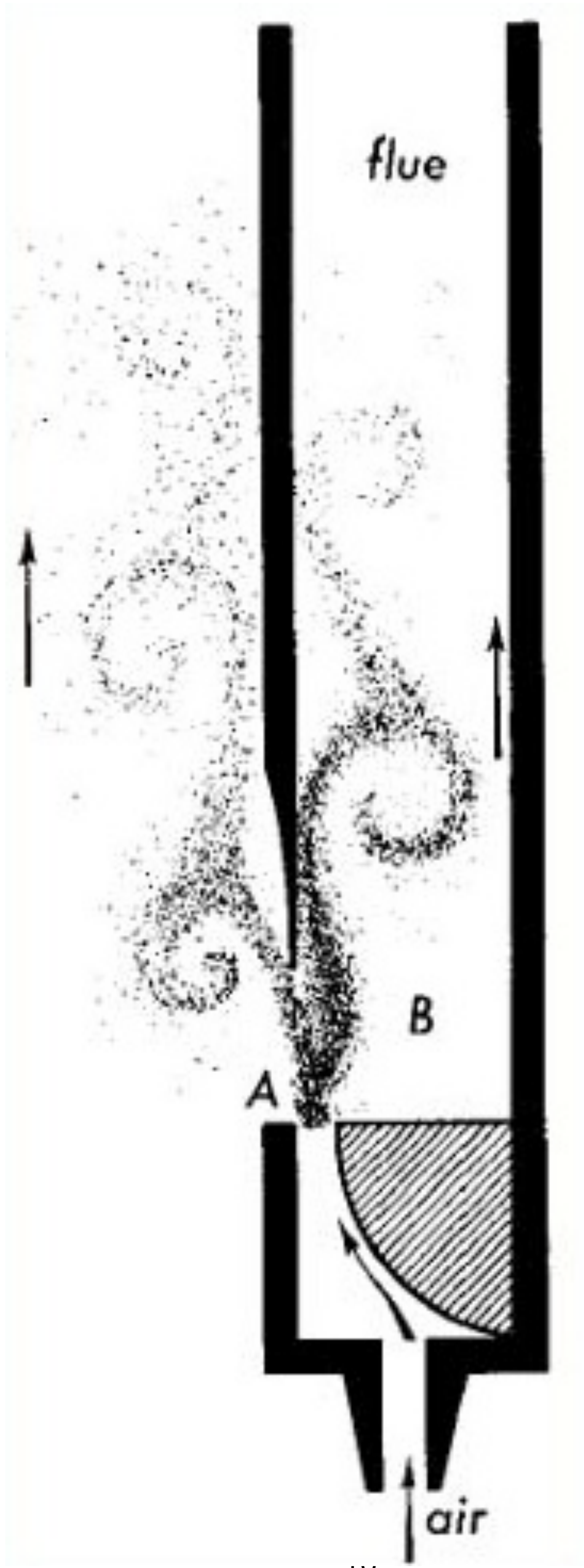
Vibrating reed

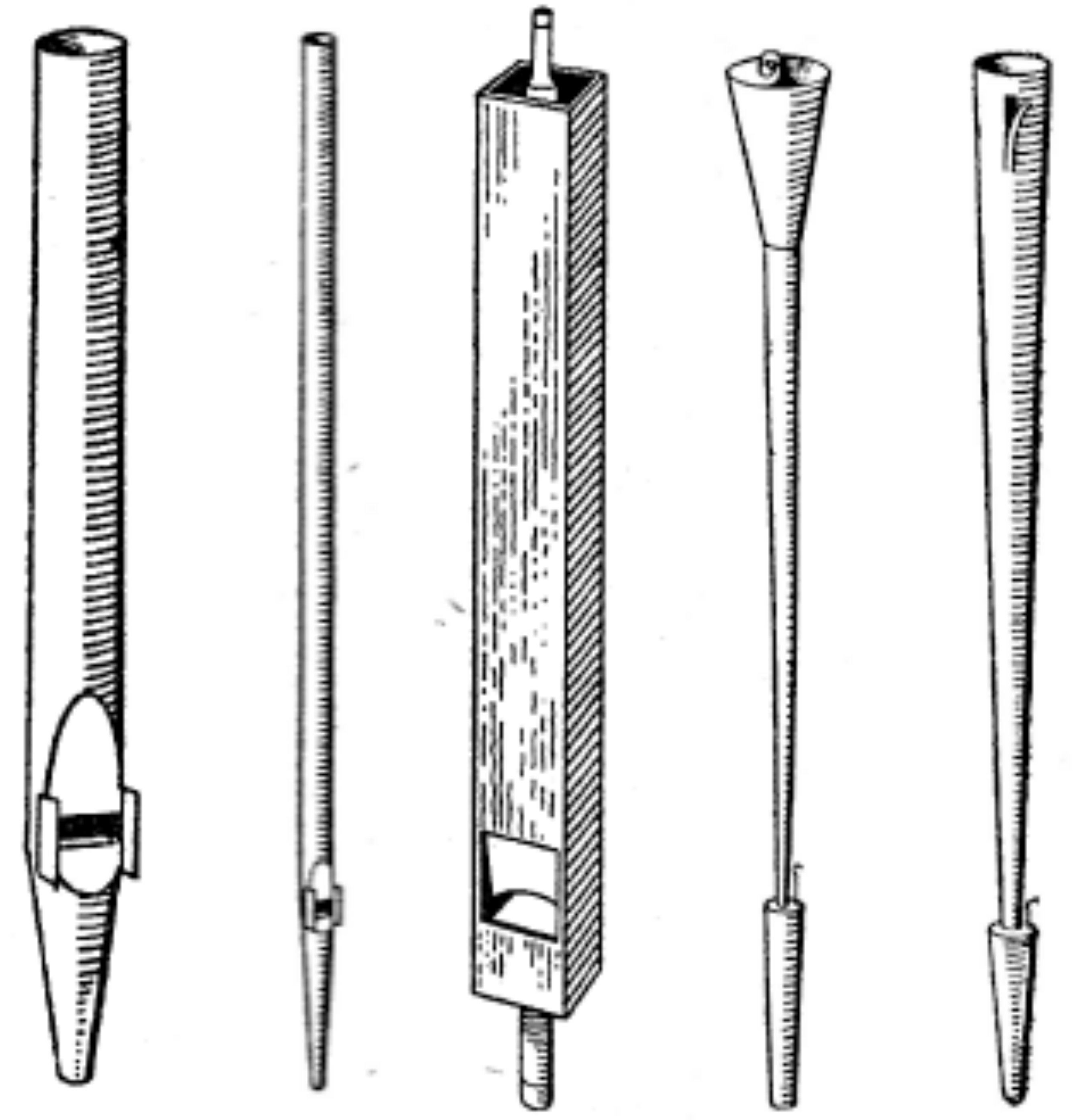
- "**Pressure-controlled**" excitation
- Used in clarinets, saxophones, bassoons, etc. (i.e., any instrument with a reed)
- Frequency of the opening and closing of the reed is determined by the **natural frequencies** of the remainder of the tube (resonance phenomenon)
- Also applies to the didgeridoo and brass instruments where the musicians **lips** play the role of a reed



Flue-organ pipe

- Excitation can be either an oscillating air stream or a vibrating reed
- Pipe has **fixed length**
- **One note** per pipe -> need many pipes (e.g., church organ)

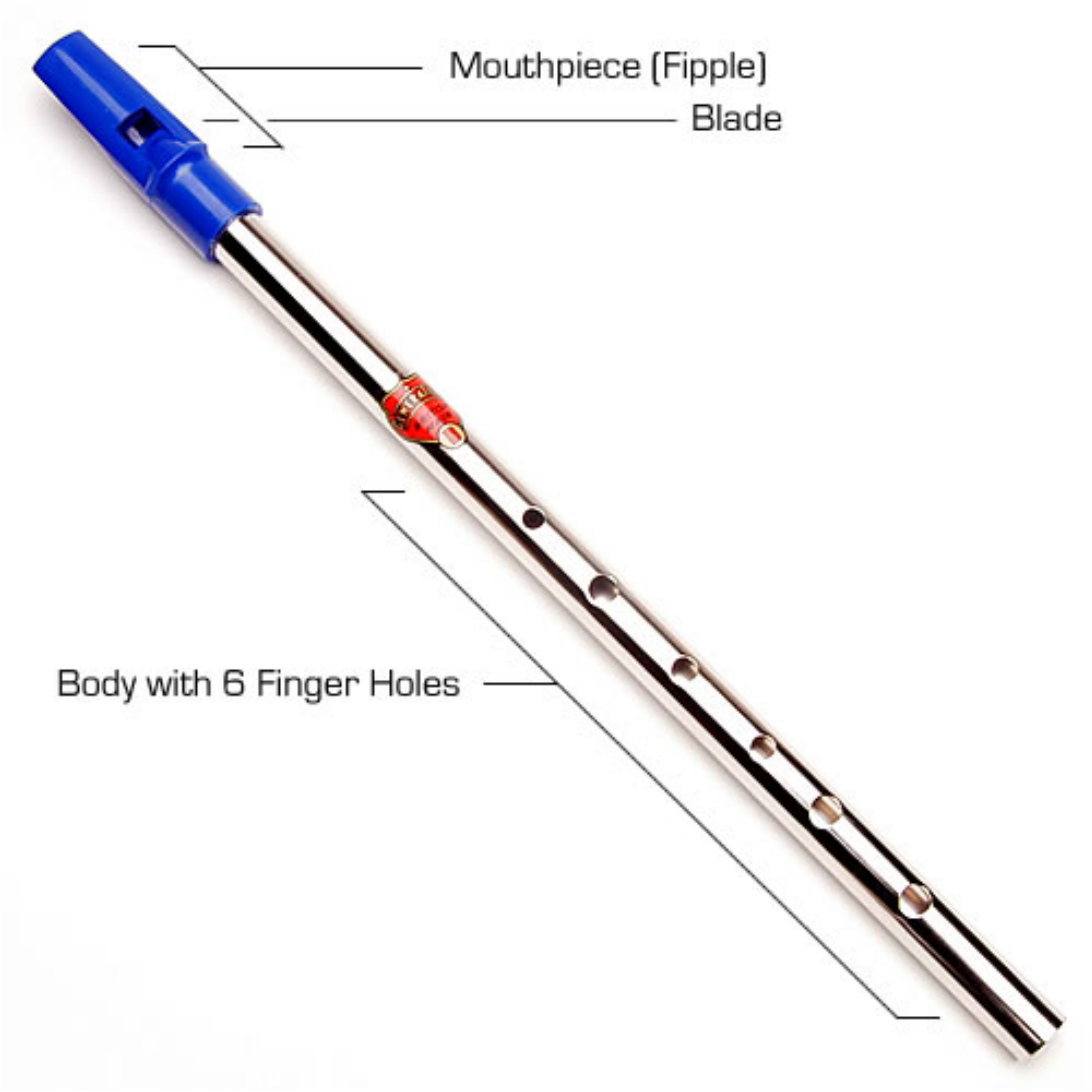




Different types of pipe
1. Open Diapason
2. Dulciana
3. Stopped Diapason
4. Oboe (reed pipe)
5. Trumpet (reed pipe)

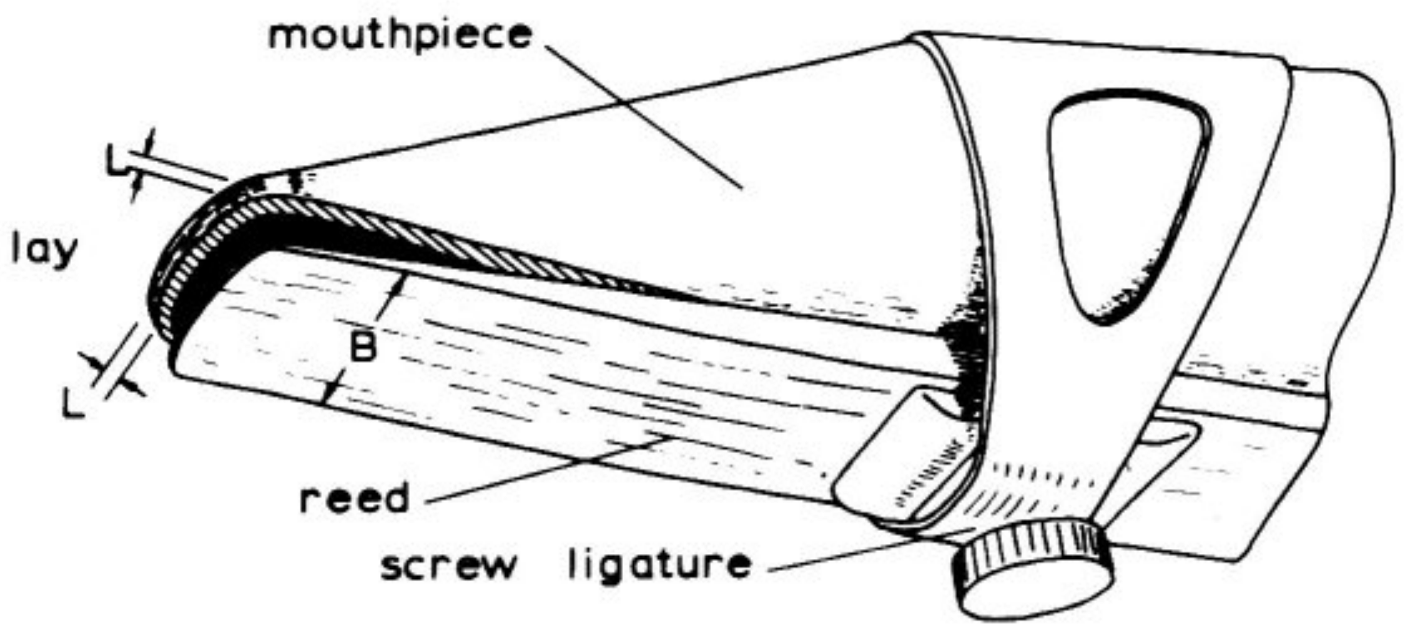
Penny whistle

- Oscillating air stream -> open at both ends
- **Tone holes** allow playing **multiple notes**



Clarinet

- Vibrating **reed** -> **closed** at one end
- Tone holes and register keys -> **multiple notes**
- Bell at end creates contribution from even harmonics



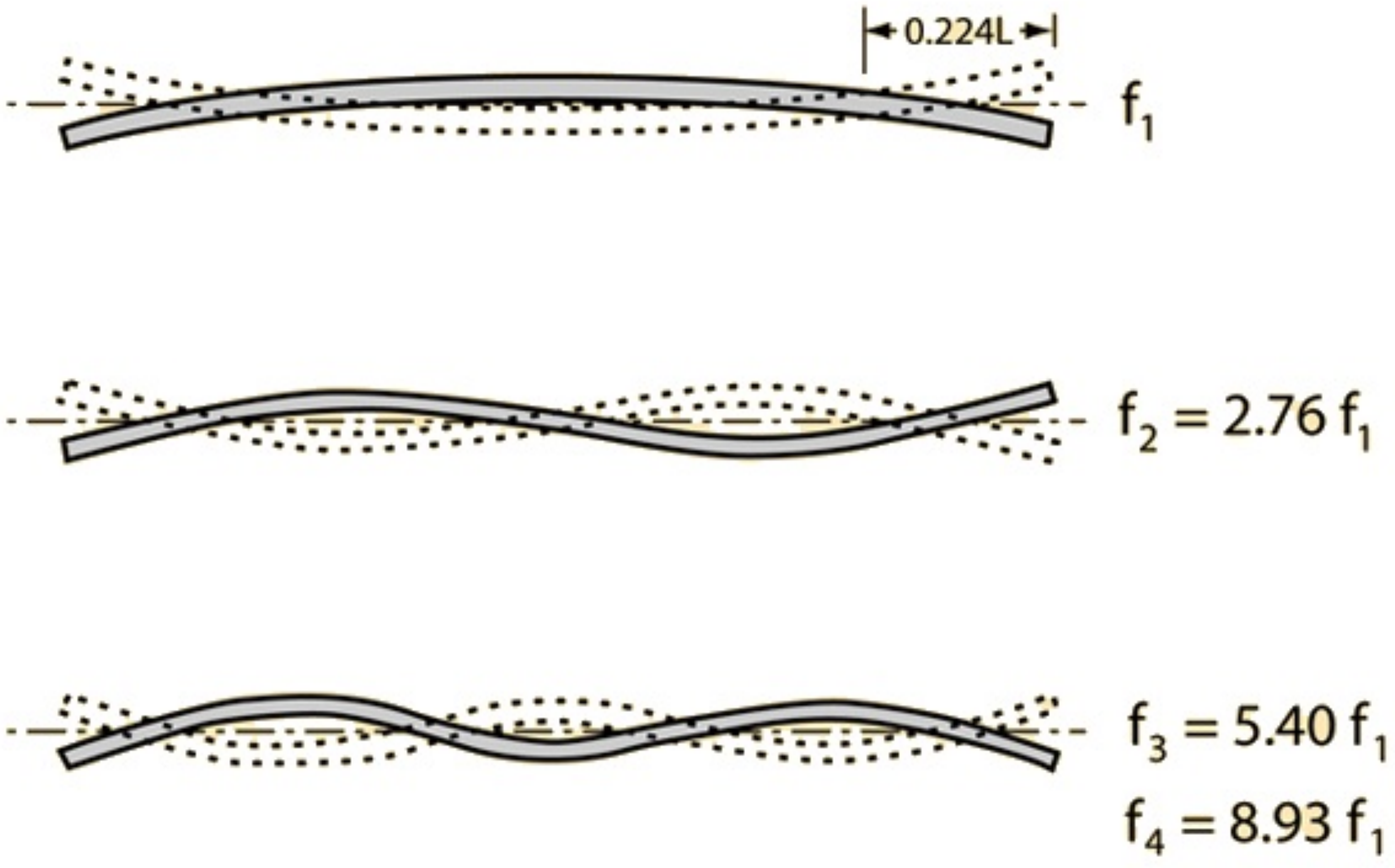
Brass instruments

https://www.youtube.com/watch?v=Bo7VRSQLpfY

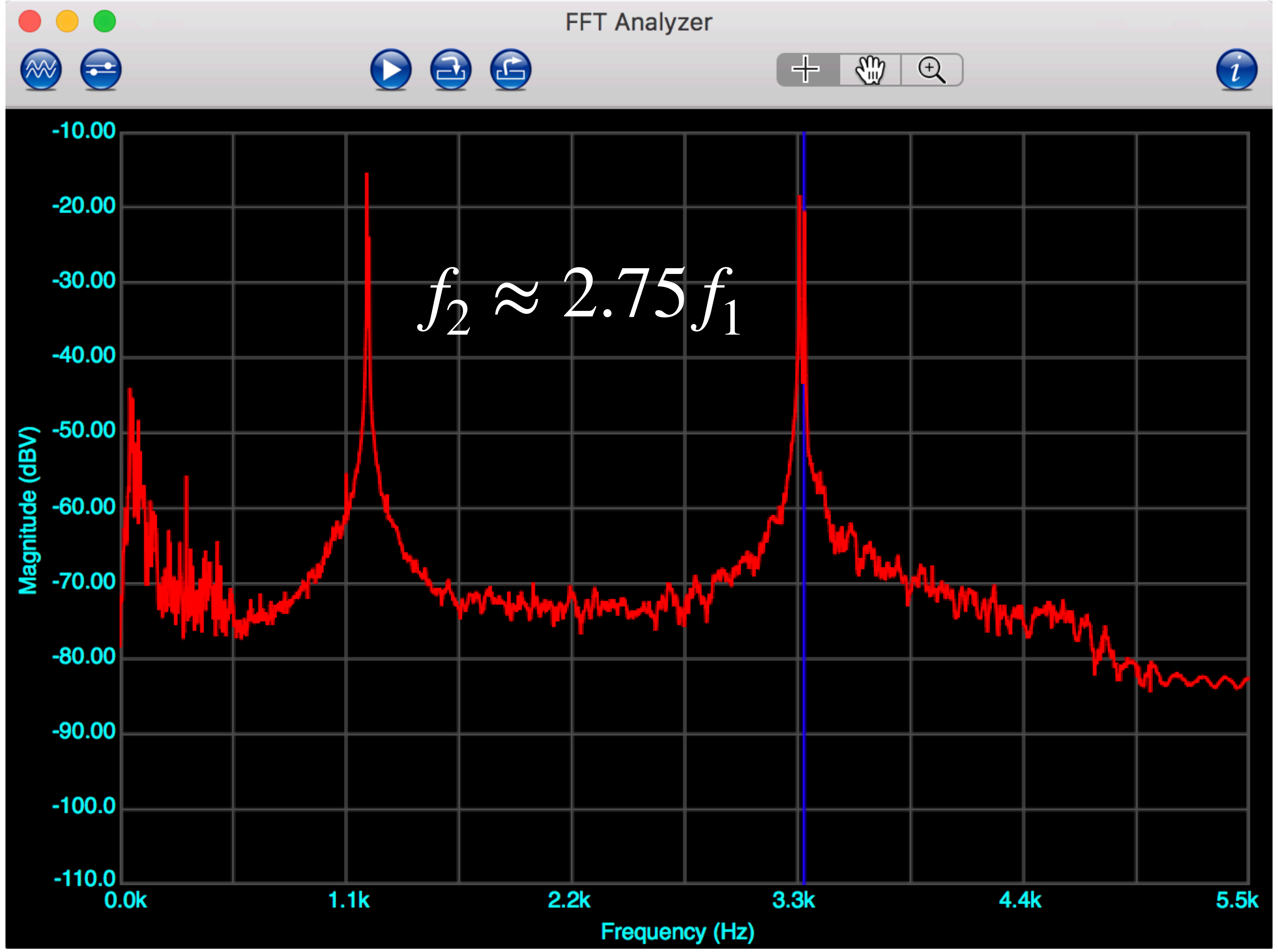
7. Percussion instruments (briefly)

Inharmonicity

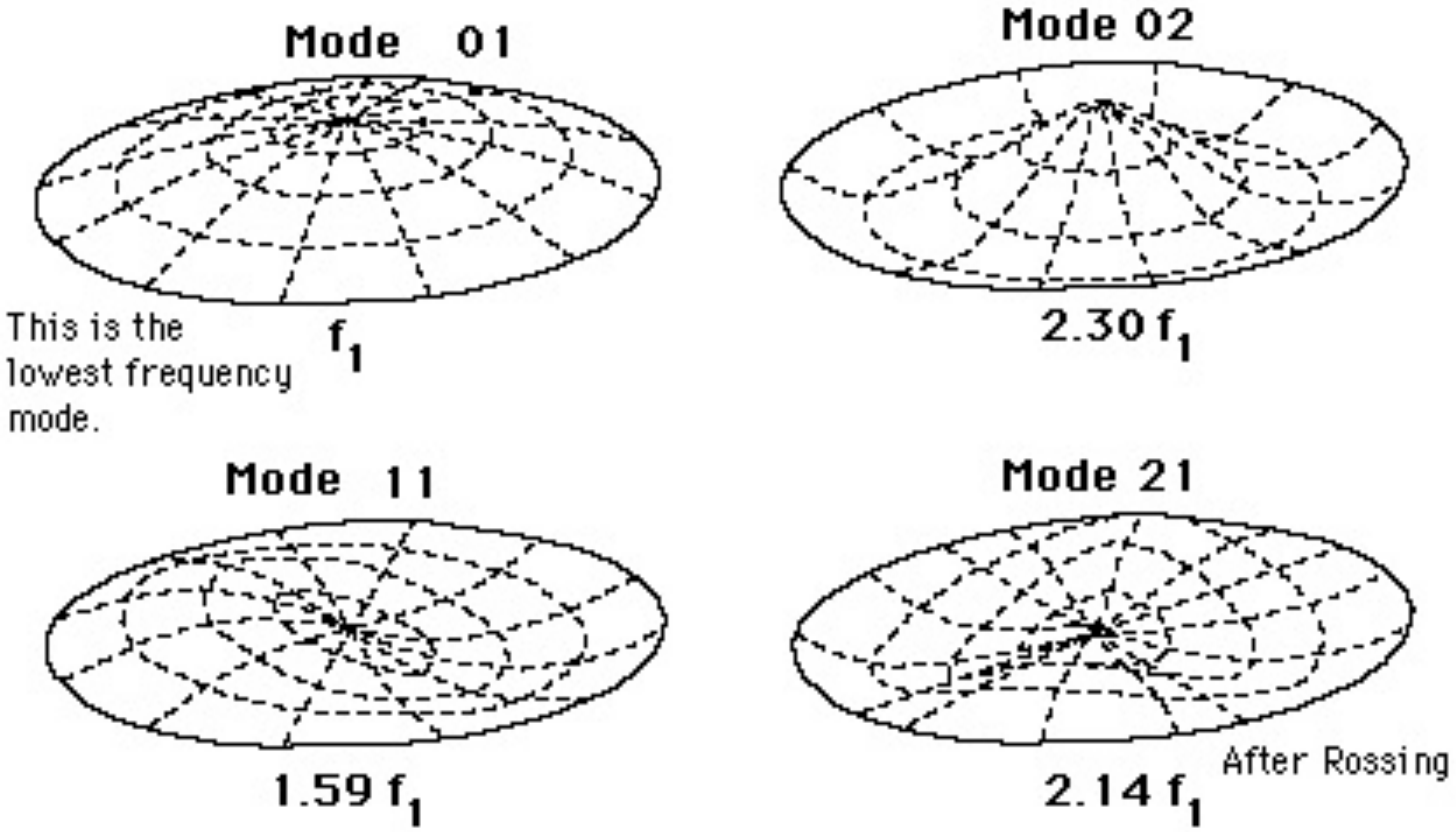
- Presence of **overtones** that are **not harmonically** related to the fundamental frequency



Example: Bell



Vibrating drum head (2-d standing waves)



mode = (# nodal diameters, # nodal circles)

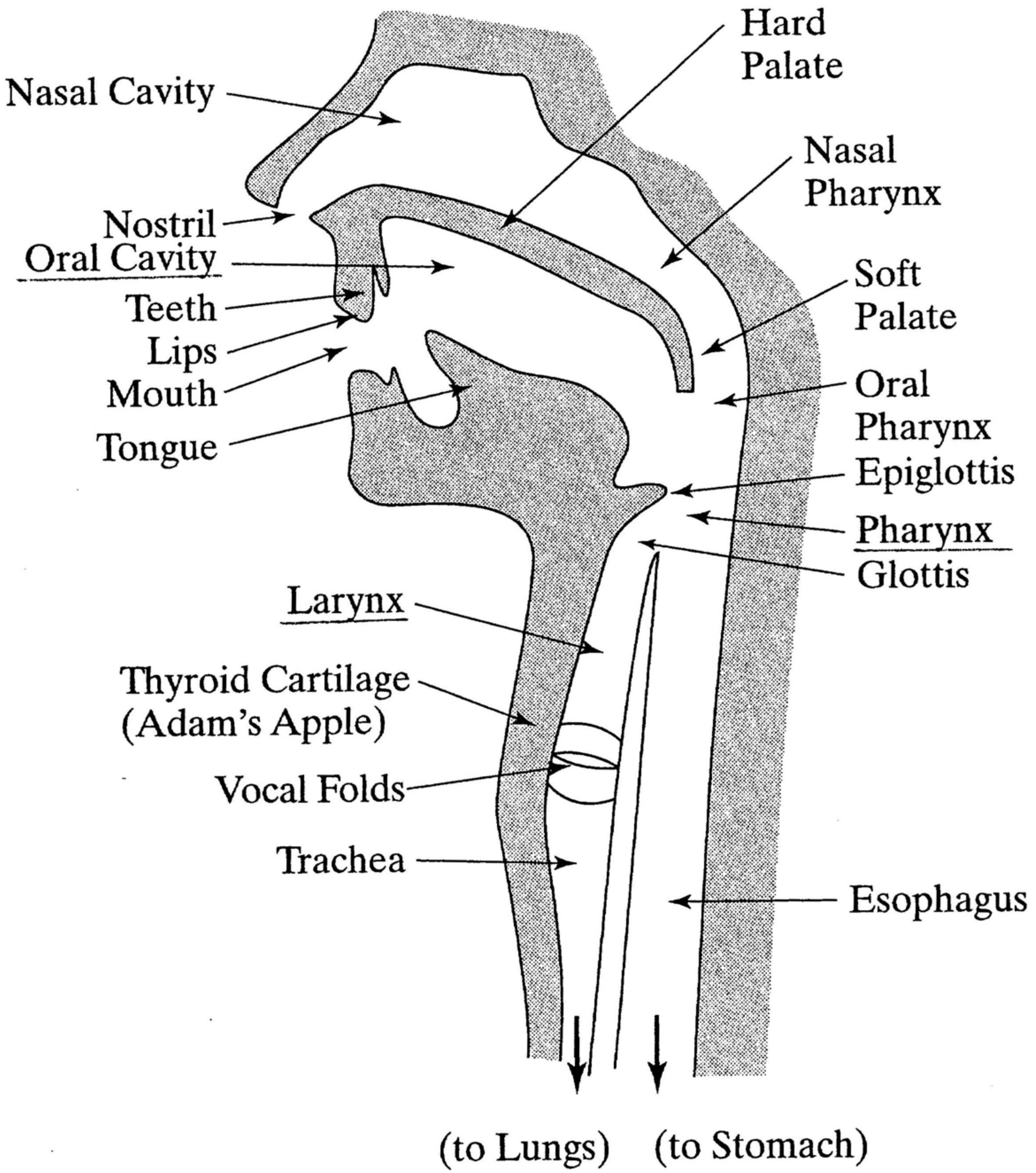
https://commons.wikimedia.org/wiki/Category:Drum_vibration_animations

https://josephromano.github.io/PHYS1406/labs/S2021/modified/Chladni_patterns.mov

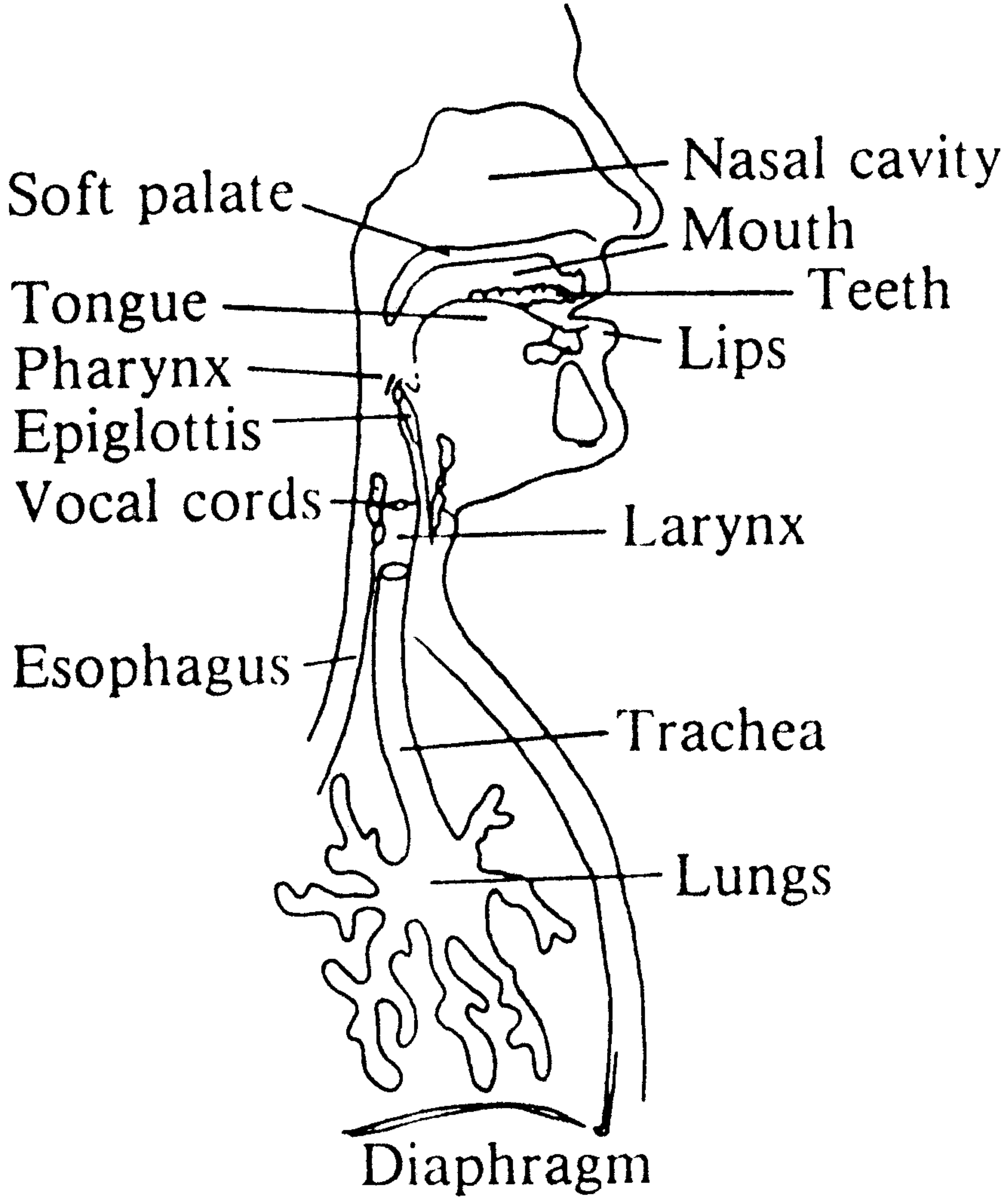
8. Voice

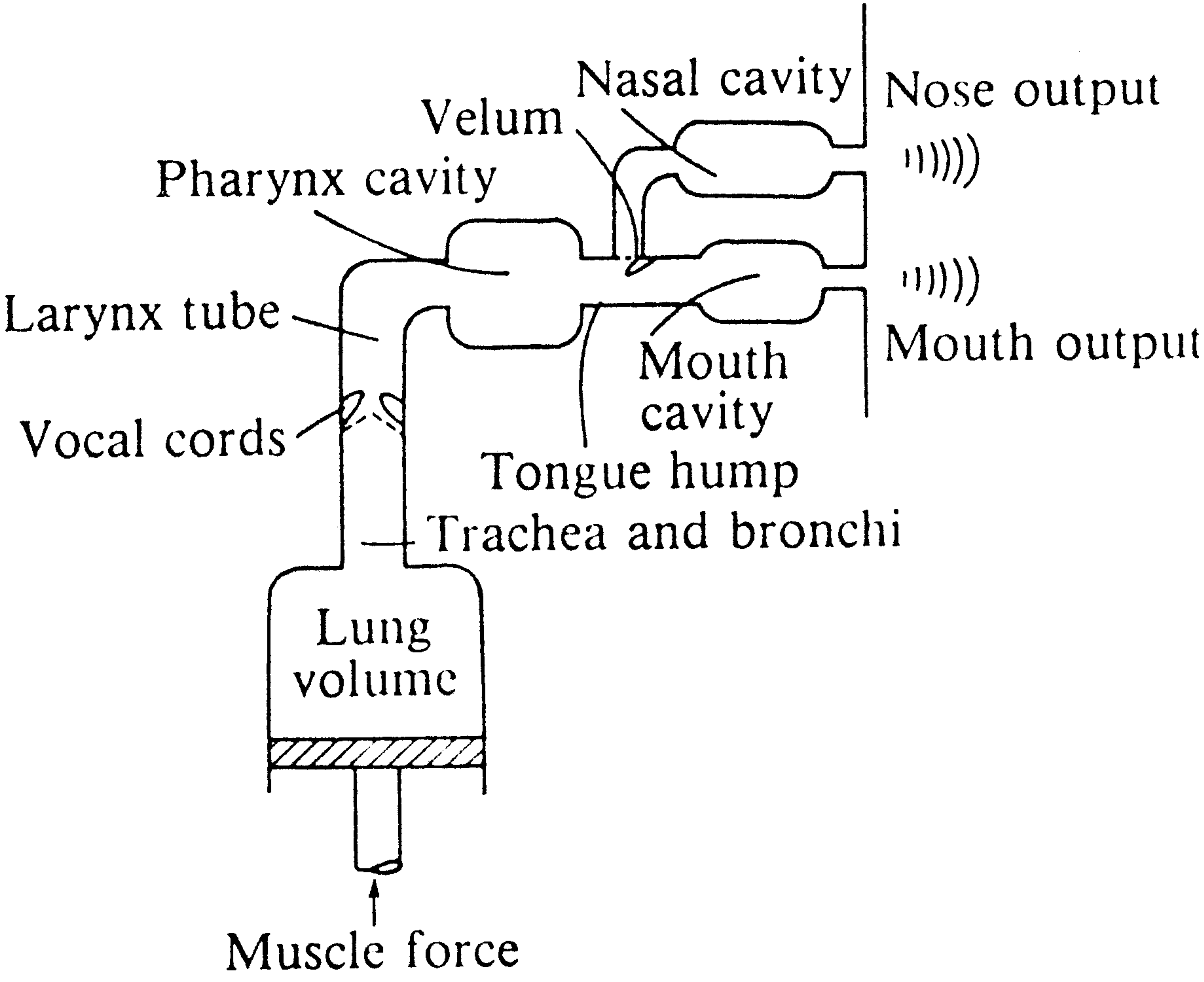
Vocal organs

- power supply: lungs
- generator/vibrator: vocal folds
- resonator: vocal tract (larynx, pharynx, oral and nasal cavities)
- radiator: mouth/lips and nostrils
- vocal folds:
    - women: ~10 mm, ~220 Hz
    - men: ~15-20 mm, ~110 Hz
    - forced open by air pressure, come together due to **Bernoulli effect** (demo)



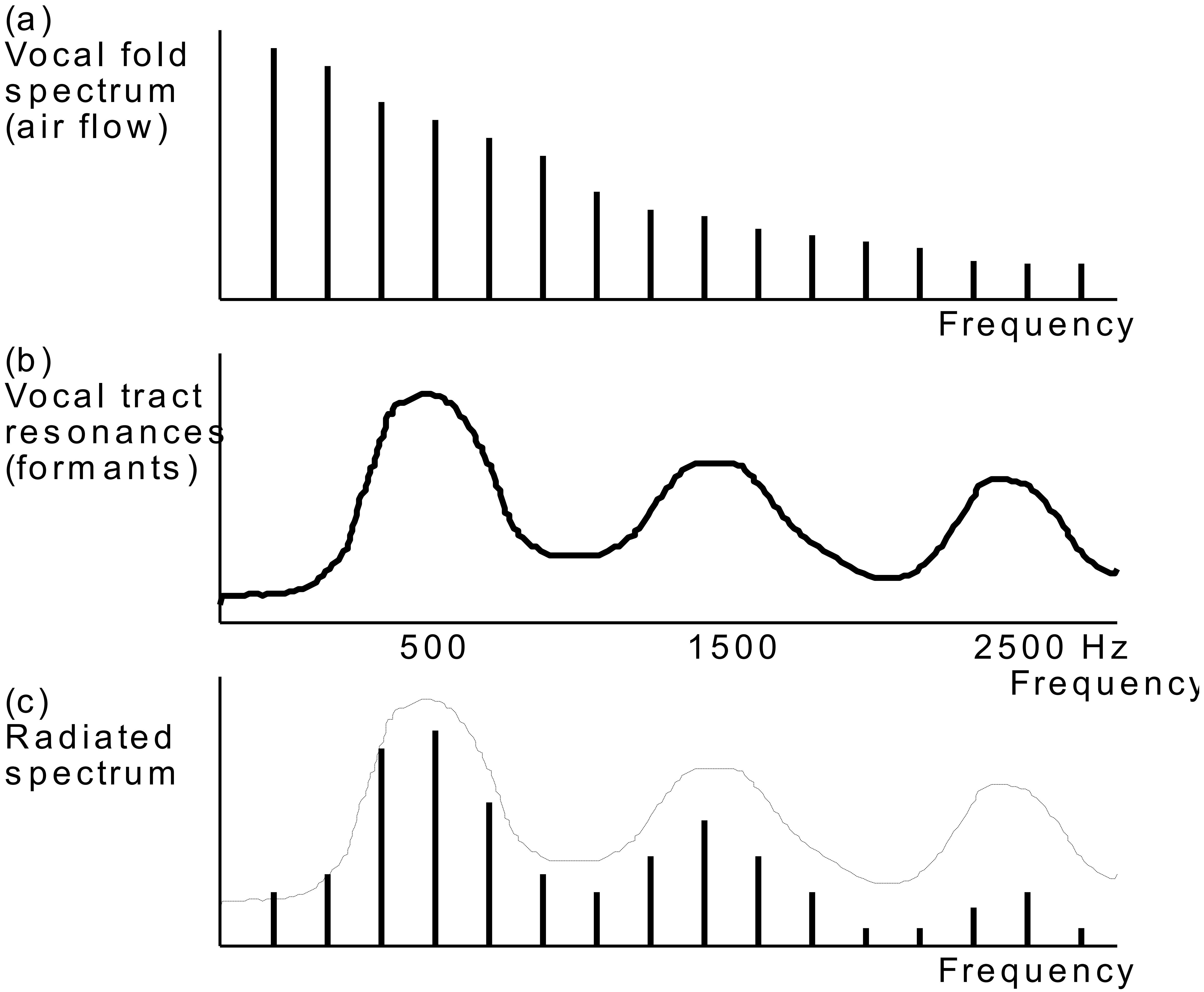
Physics / engineering model



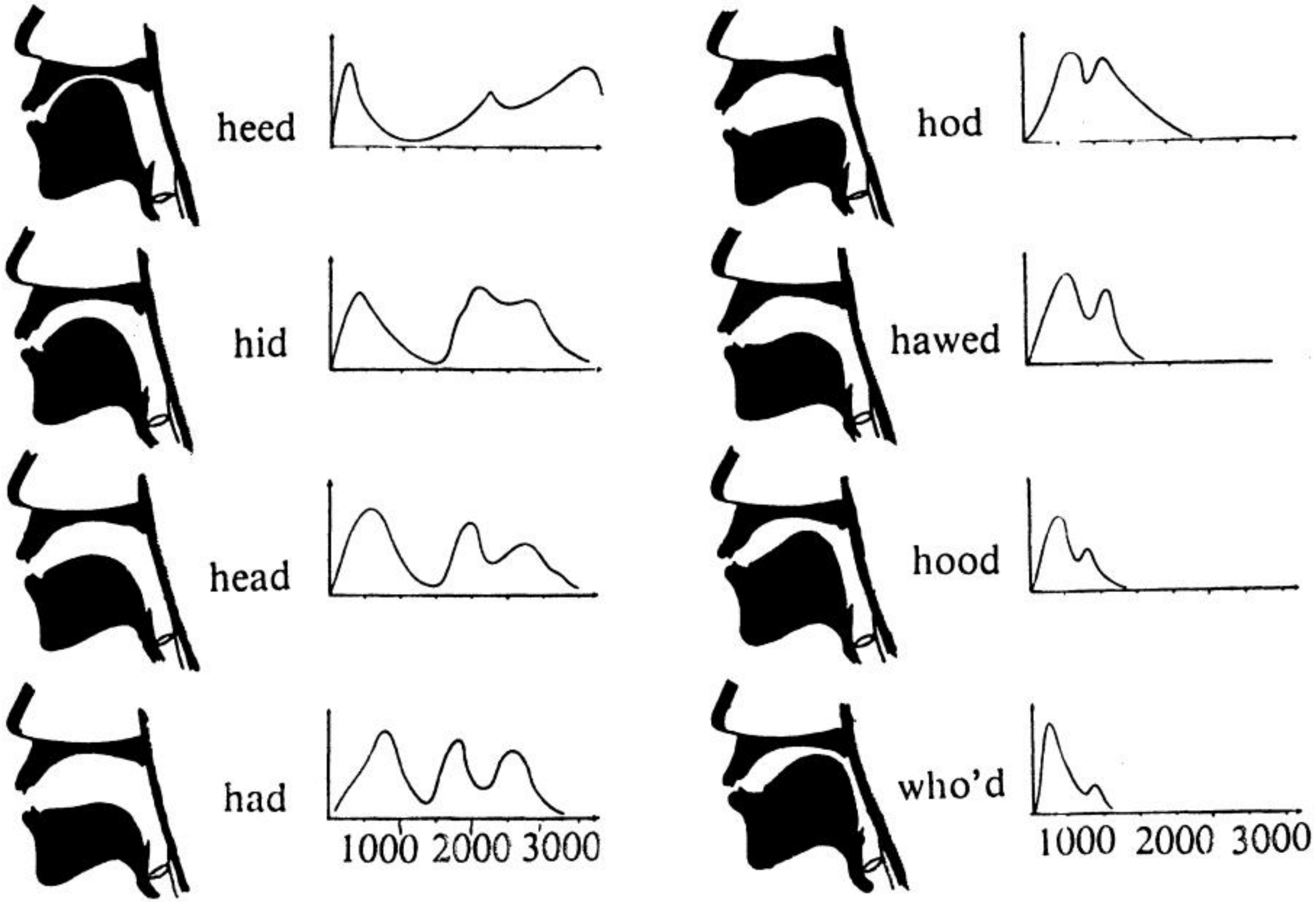


Formants

- Spectrum of vocal folds ~$1/N^2$
- Vocal tract acts as a **filter**
    - ~17cm cylindrical tube
    - $f_n = Nv/4L\,, \quad N = 1,3,\cdots$
    - ~500 Hz, 1500 Hz, 2500 Hz, …
    - "Donald duck" effect if one inhales helium

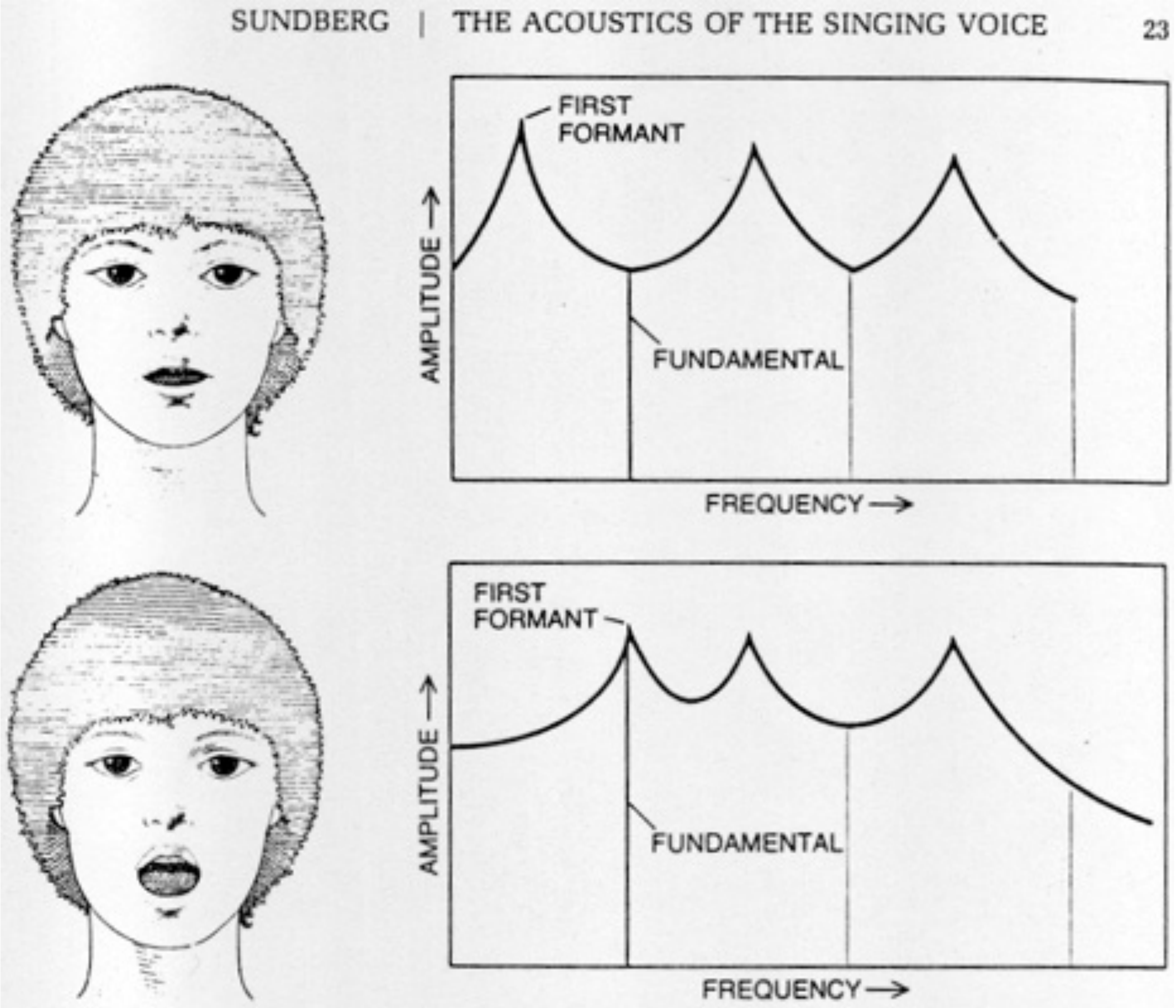


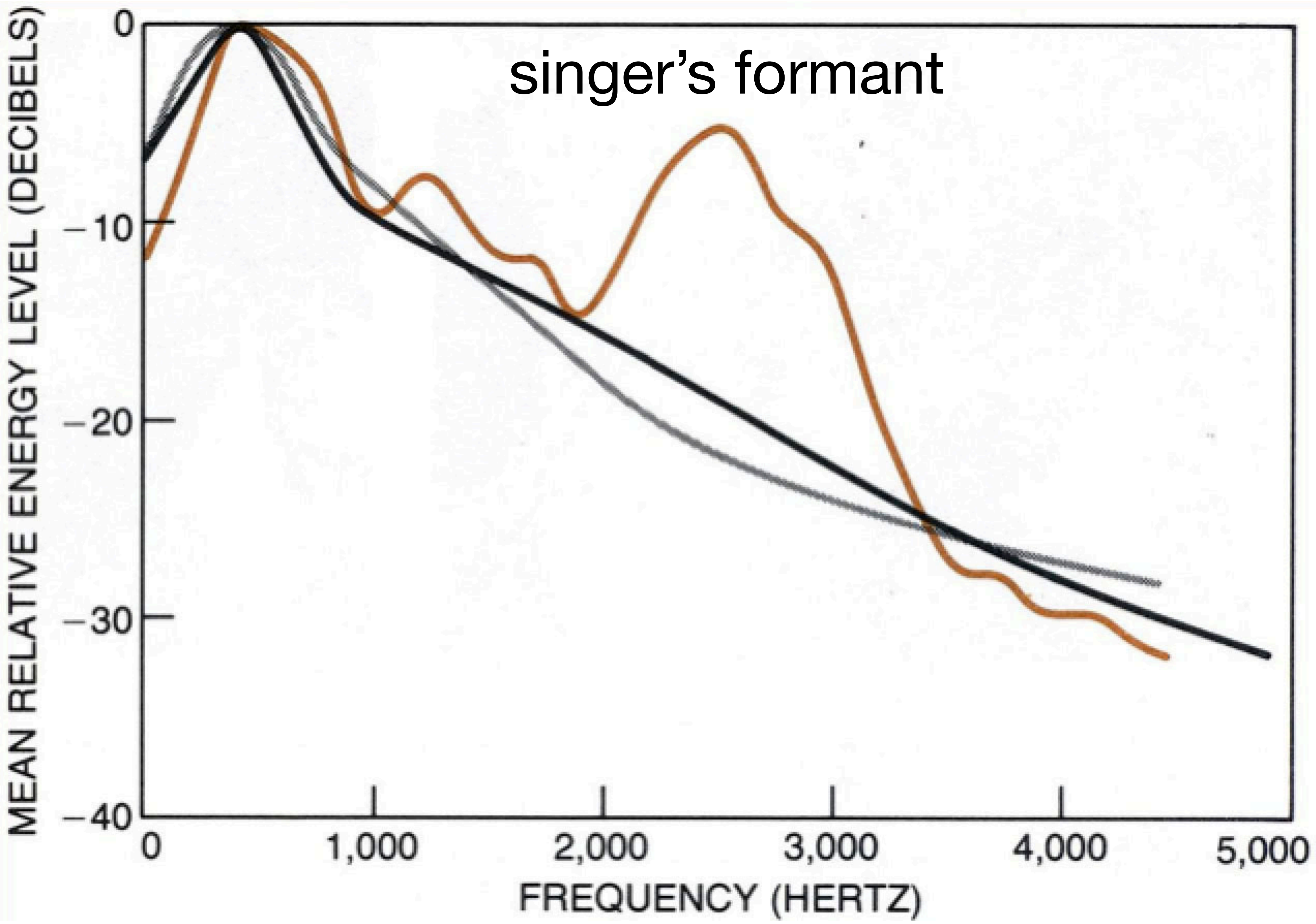
Formant regions for different vowel sounds



Demonstration: compare "who'd", "hod", and "heed" using spectrogram

Singing

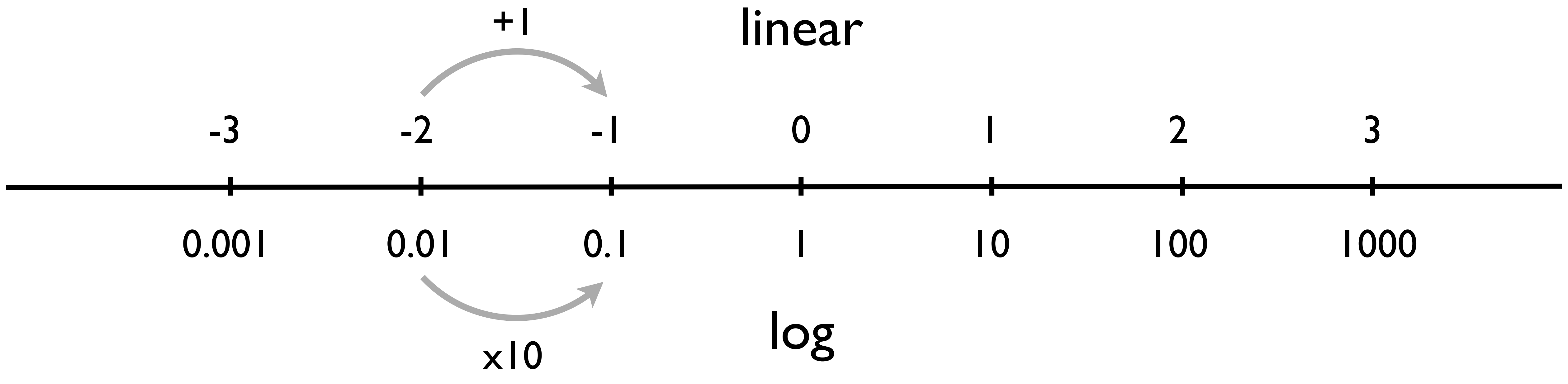

SUNDBERG | THE ACOUSTICS OF THE SINGING VOICE
23
FIRST
FORMANT
AMPLITUDE →
FUNDAMENTAL
FREQUENCY →
FIRST
FORMANT
AMPLITUDE →
FUNDAMENTAL
FREQUENCY →


singer's formant
MEAN RELATIVE ENERGY LEVEL (DECIBELS)
0
−10
−20
−30
−40
0
1,000
2,000
3,000
4,000
5,000
FREQUENCY (HERTZ)

9. Hearing

Fechner’s law and range of human hearing

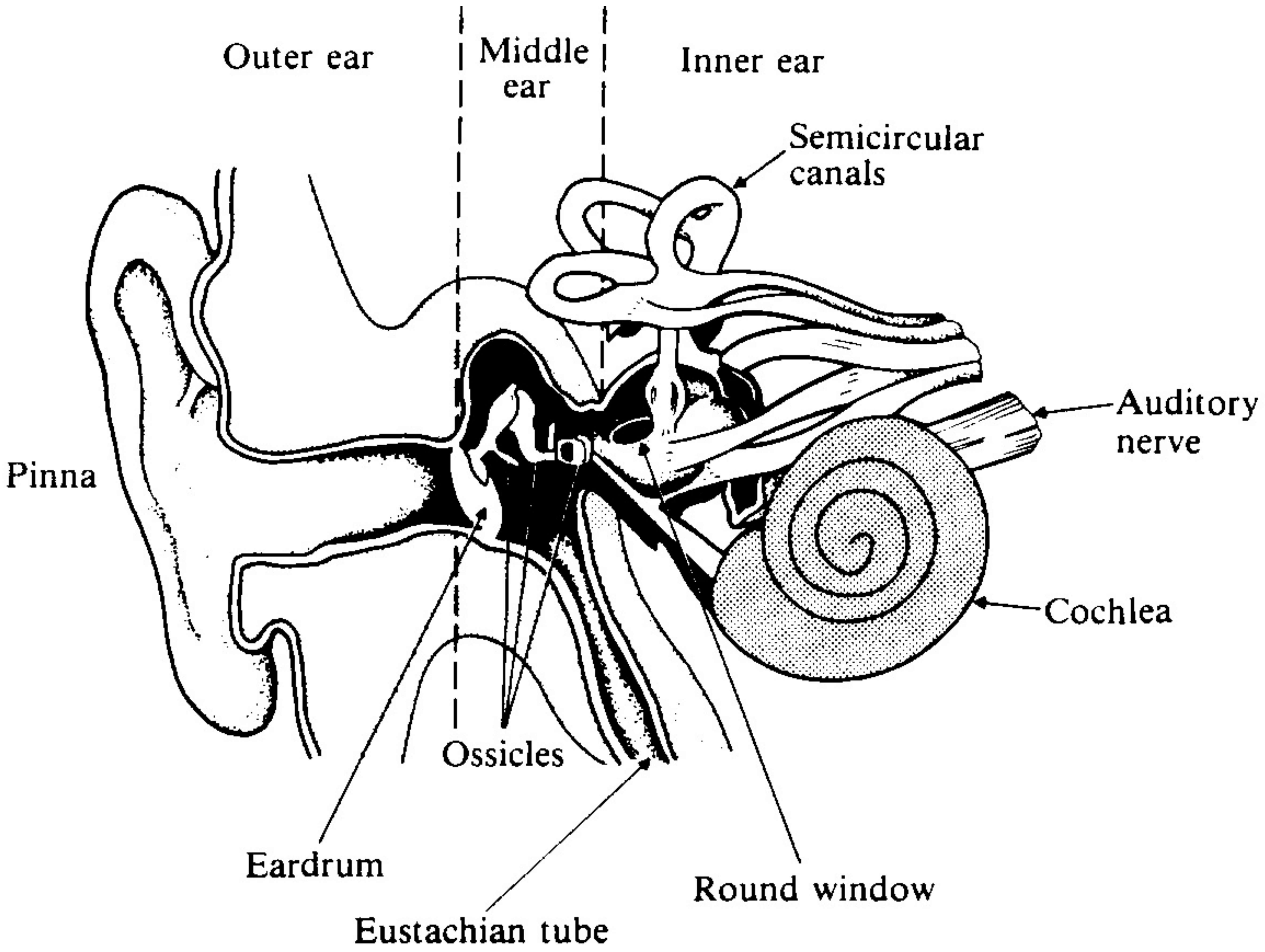
- Fechner’s law: “As **stimuli** are increased by **multiplication**, **sensation** increases by **addition**”



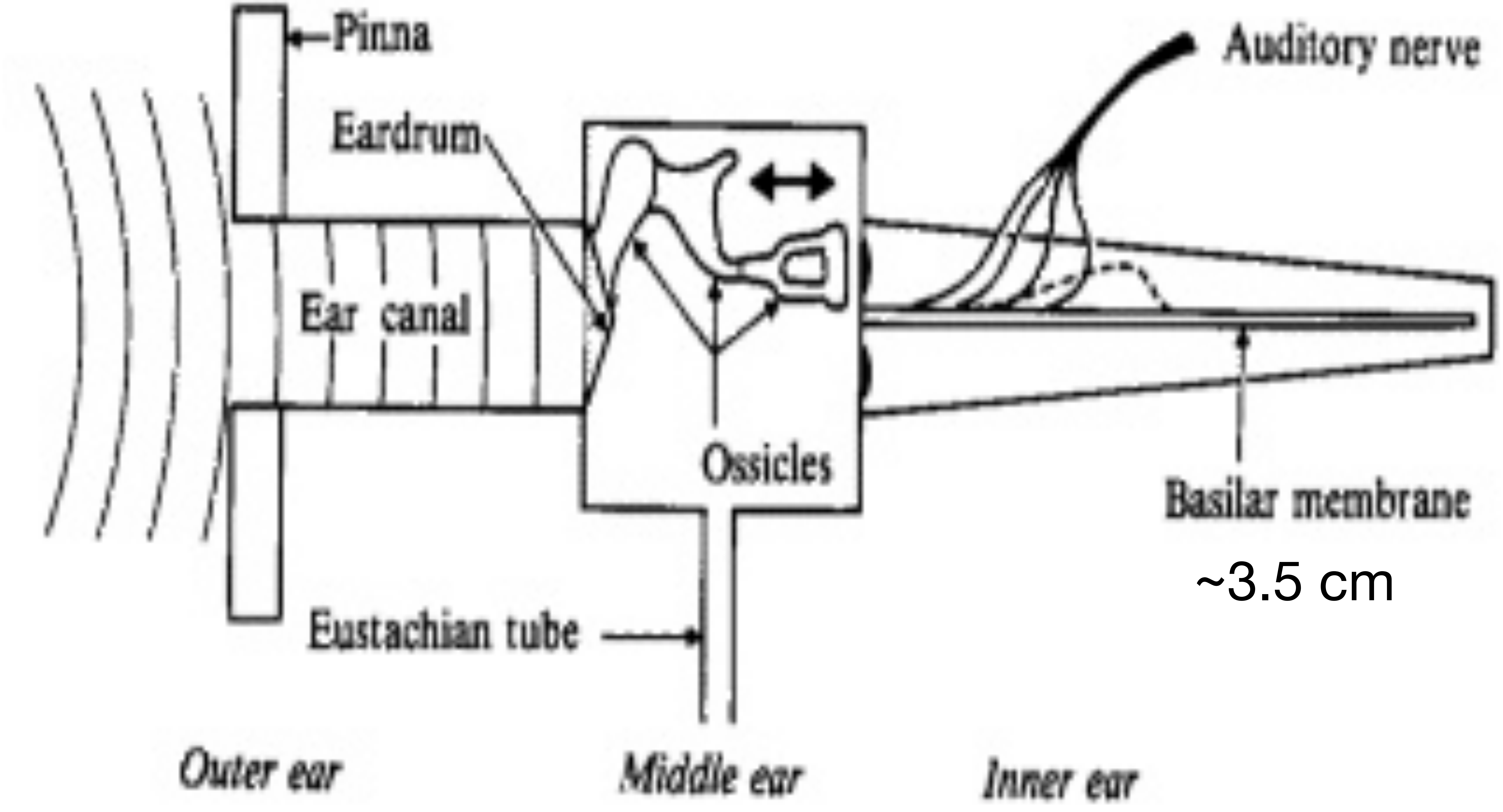
$$y = \log x \iff 10^y = x$$

- Range of hearing
  - **Pitch** (frequency): 20 – 20,000 Hz (~10 octaves)
  - **Loudness** (intensity): $10^{-12} - 1\ \mathrm{W/m^2}$ (12 orders of magnitude)
- Eye: sensitive to ~1 octave in color (frequency) and 5 orders of magnitude in brightness (intensity)

Anatomy of human ear



Anatomy of human ear - continued



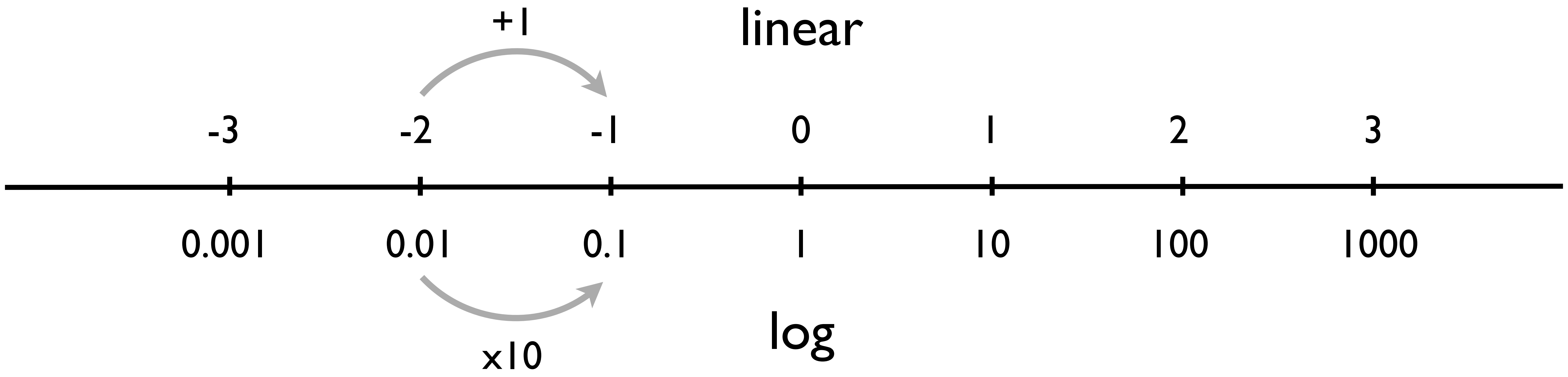
Place theory of pitch

- A pure tone excites a ~1.3 mm region of the basilar membrane (**critical band**)
- There are ~**24 critical bands** on the basilar membrane spanning 20-20,000 Hz
- Center frequencies of critical bands are **spaced logarithmically** on the basilar membrane like a piano keyboard (called **"place theory of pitch"**)
- Critical bands:
  - ~100 Hz for center frequencies below 500 Hz
  - ~3 semitones (1/4 octave) above 500 Hz

Hearing via air vs bone conduction and sound localization

- Air vs bone conduction:
    - Q: Why do you sound differently when you listen to a recording of your voice?
- Sound localization (binaural hearing)
    - **High frequency** sounds (> 4000 Hz): **intensity difference**
    - **Low frequency** sounds (< 1000 Hz): **time of arrival**

Logarithms



$$y = \log x \;\;\Leftrightarrow\;\; 10^y = x$$

$$\log(ab) = \log a + \log b$$

$$\log 2 \approx 0.3\,, \quad \log 3 \approx 0.5\,, \quad \log 4 \approx 0.6\,, \quad \log 5 \approx 0.7\,, \quad \log 10 = 1$$

10. Loudness

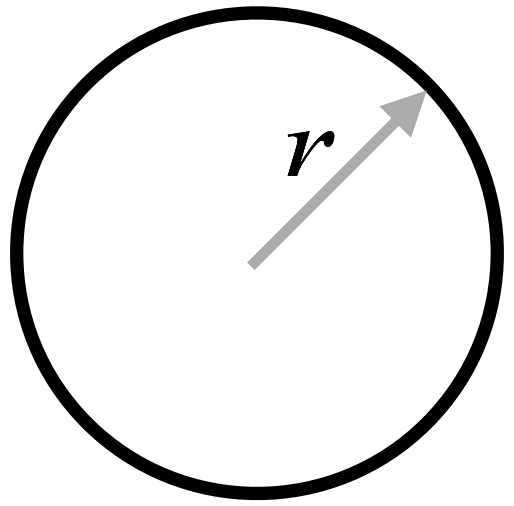
Loudness — overview

(compare two sounds, or compare one sound to the threshold of hearing)

- our perception of the **relative** strength of a sound
- depends on the **intensity** and **frequency** of the sound
- **logarithmic response** to intensity (consistent with Fechner's law)
- several different ways of quantifying loudness:
    - intensity
    - sound intensity level
    - sound loudness level
    - subjective loudness

Intensity

- Intensity is the power in a sound wave divided by the area it passes through (Watts/m^2)



$$I = \frac{P}{4\pi r^2}$$

(sound is less intense the farther you are away from the source of the sound)

- Range of intensities:
    - $I_0 = 10^{-12}$ W/m^2 (**threshold of hearing** at $f = 1000$ Hz)
    - $I = 1$ W/m^2 (**threshold of pain** at $f = 1000$ Hz)
- Intensity is proportional to the **square of the amplitude** of the sound wave: $I \propto (\Delta p)^2$
- Intensities add: (intensity of 2 violins = twice the intensity of 1 violin)

Sound Intensity Level (SIL, dB)

- Sound intensity level is the logarithm of the intensity compared to the threshold of hearing:

$$\boxed{\text{SIL} = 10\ \log(I/I_0)\ \text{dB}}$$

- Threshold of hearing: $I = I_0 = 10^{-12}\ \text{W/m}^2 \ \Rightarrow\ \text{SIL} = 0\ \text{dB}$

- Threshold of pain: $I = 1\ \text{W/m}^2 \ \Rightarrow\ \text{SIL} = 120\ \text{dB}$

$$\boxed{\Delta\text{SIL} = 10\ \log(I_2/I_1)\ \text{dB}}$$ (comparing two intensities)

- 2x intensity: $\Delta\text{SIL} = 10\ \log(2)\ \text{dB} = 3\ \text{dB}$

- 10x intensity: $\Delta\text{SIL} = 10\ \log(10)\ \text{dB} = 10\ \text{dB}$ (perceived as “twice as loud”)

- Just noticeable difference (JND): $\Delta\text{SIL} = 1\ \text{dB} \ \Leftrightarrow\ I_2 = 1.26\, I_1$

Sound Loudness Level (L_L, phon)

- Human ear **responds differently to different frequencies** (Fletcher-Munson curves)
- Most sensitive to frequencies **~4000 Hz**
- Equal loudness curves labeled by **phon** (not the same as dB)

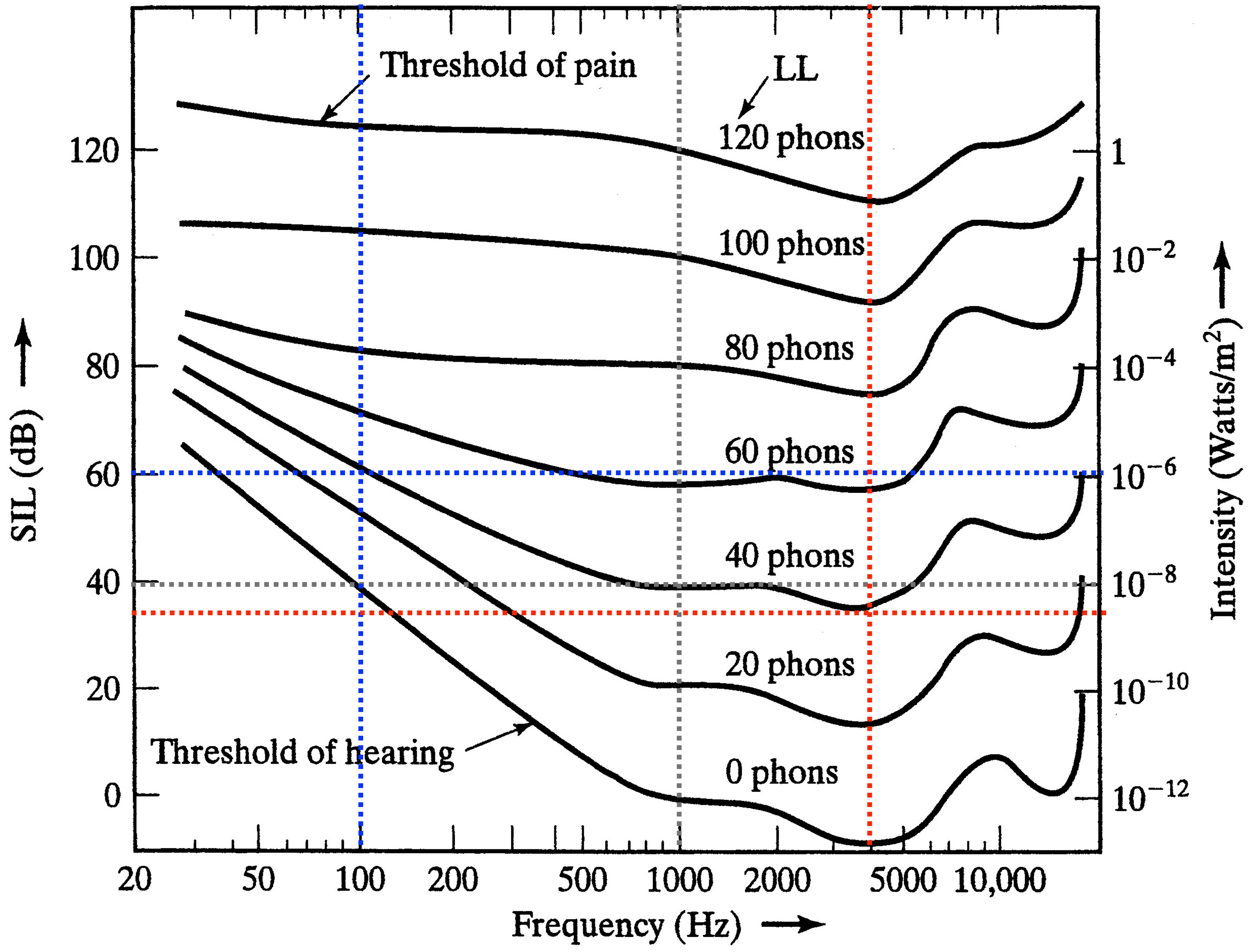
$$L_L \text{ (phon)} = \text{SIL (dB)} \quad \text{at} \quad f = 1000 \text{ Hz}$$

- Examples:

$\text{SIL}(40 \text{ phon}, 1000 \text{ Hz}) = 40 \text{ dB}$ (grey)

$\text{SIL}(40 \text{ phon}, 100 \text{ Hz}) = 60 \text{ dB}$ (blue)

$\text{SIL}(40 \text{ phon}, 4000 \text{ Hz}) = 35 \text{ dB}$ (red)



Subjective Loudness (S, sone)

- Measure of loudness where "**twice as loud**" corresponds to "**multiply by 2**"
- Most people perceive **10x increase in intensity** as "**twice as loud**"
  - Recall: 10x intensity –> $\Delta\mathrm{SIL} = 10\ \mathrm{dB}$, $\Delta L_L = 10\ \mathrm{phon}$
  - Some values:

$$L_L = 40\ \mathrm{phon} \quad \Leftrightarrow \quad S = 1\ \mathrm{sone}$$
$$L_L = 50\ \mathrm{phon} \quad \Leftrightarrow \quad S = 2\ \mathrm{sone}$$
$$L_L = 60\ \mathrm{phon} \quad \Leftrightarrow \quad S = 4\ \mathrm{sone}$$
$$L_L = 30\ \mathrm{phon} \quad \Leftrightarrow \quad S = 1/2\ \mathrm{sone}$$

General formula:

$$S = 2^{(L_L - 40)/10}\ \mathrm{sone}$$

  - Questions:

$$L_L = 70\ \mathrm{phon} \quad \Leftrightarrow \quad S = ??\ \mathrm{sone}$$
$$L_L = 20\ \mathrm{phon} \quad \Leftrightarrow \quad S = ??\ \mathrm{sone}$$

Different measures of loudness and safe noise levels

	SIL (dB) at 1000 Hz	L_L (phon)	S (sone)
Threshold of hearing	0	0	1/16
Recording studio	20	20	1/4
Quiet office	40	40	1
Ordinary conversation	60	60	4
Normal piano practice	80	80	16
Piano fortissimo	100	100	64
Threshold of pain	120	120	256

Duration per day, hours	Sound level dBA slow response
8	90
6	92
4	95
3	97
2	100
1 1/2	102
1	105
1/2	110
1/4 or less	115