Lecture #14: Thu 10/7

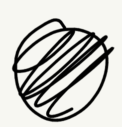
Next two lecture — collisions
Following 3 lectures — scattering

1.) Spontaneous disintegration of a single mass $M = m_1 + m_2$ into ~~t~~two particles m_1 and m_2

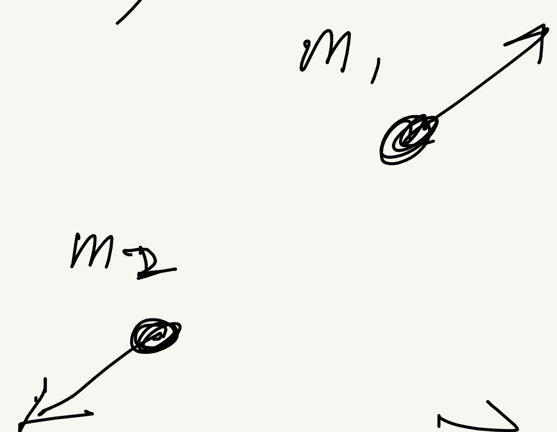
closed system $\longrightarrow$ COM Frame (conservation of total linear momentum)

(after)

M



m_1 $\vec{p}_1 \equiv \vec{p}_0$



m_2

(before)

$\vec{p} = 0$

$\vec{p}_2 = -\vec{p}_0$

$\vec{p}_1 + \vec{p}_2 = 0 \rightarrow \vec{p}_2 = -\vec{p}_1$

Cons. of energy:

$$\rightarrow \quad \underset{\uparrow}{E_i} = E_{1i} + \underset{\text{COM}}{T_{10}} + E_{2i} + T_{20}$$

internal energy of M

$$= E_{1i} + E_{2i} + \frac{p_0^2}{2m_1} + \frac{p_0^2}{2m_2}$$

$$= E_{1i} + E_{2i} + \frac{p_0^2}{2}\underbrace{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}_{\frac{m_1+m_2}{m_1 m_2} = \frac{1}{m}}$$

$$\underbrace{E_i - E_{1i} - E_{2i}}_{\mathcal{E}\,:\ \text{disintegration energy}} = \frac{p_0^2}{2m}$$

$$\rightarrow \quad p_0 = \sqrt{2m\mathcal{E}} \quad \rightarrow \quad \boxed{v_{10} = \frac{p_0}{m_1}, \quad v_{20} = \frac{p_0}{m_2}}$$

Spontaneous disintegration into N masses $m_1, m_2, \ldots, m_N$

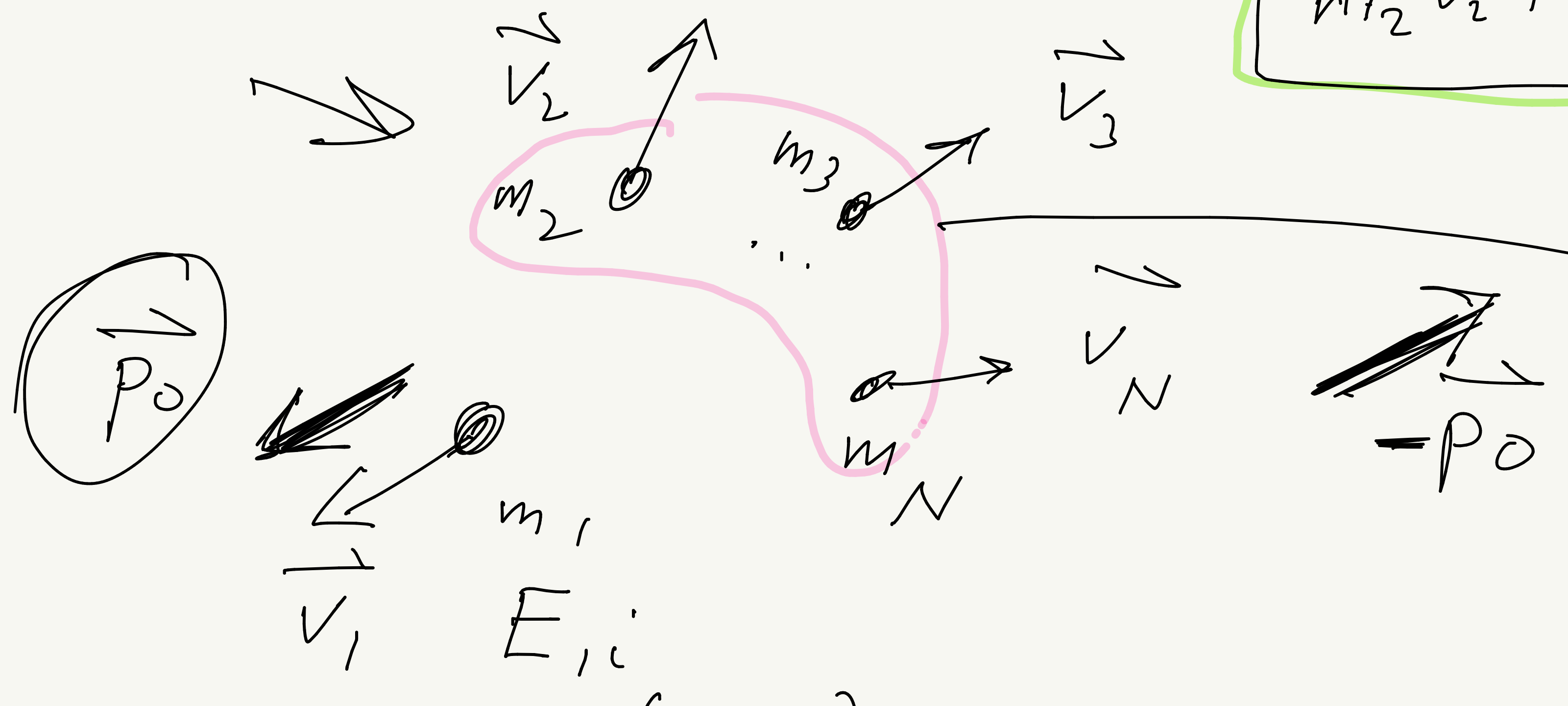
$$\boxed{m_2 \vec{v}_2 + m_3 \vec{v}_3 + \cdots = -\vec{p}_0}$$

COM frame:

E_i M

(before)

$\vec{p} = 0$



represent by their COM

$M - m_1$
$= m_2 + m_3 + \ldots m_N$

E_i'

Upper limit on potential of a single particle m_1

cons. of energy

$$\Rightarrow E_i = \frac{p_0^2}{2m_1} + E_{1i} + \frac{p_0^2}{2(M-m_1)} + E_i'$$

$$= \underline{E_{1i} + E_i'} + \frac{p_0^2}{2}\left(\frac{1}{m_1} + \frac{1}{M-m_1}\right)$$

$$\frac{M}{m_1(M-m_1)}$$

$$E_i - E_{1i} - E_i' = \frac{p_0^2}{2m_1}\left(\frac{M}{M-m_1}\right)$$

$$E_{2i}' + E_{3i}' + \cdots$$

for maximum KE for m_1

$$\rightarrow \underbrace{\frac{p_0^2}{2m_1}}_{T_{01}} = (E_i - E_{1i} - E_i')\left(\frac{M-m_1}{M}\right)$$

just a number

T_{01} maximum when E_i' is minimum

$$E_i' + \frac{p_0^2}{2(M-m_1)} = E_{2i} + \frac{1}{2}m_2|\vec{v}_2|^2 + E_{3i} + \frac{1}{2}m_3|\vec{v}_3|^2 + \cdots$$

$$= \underbrace{E_{2i} + E_{3i} + \cdots}\ + \underbrace{\frac{1}{2}m_2(\vec{v}_2)^2 + \frac{1}{2}m_3(\vec{v}_3)^2 + \cdots}$$

$$-\vec{p}_0 = m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots \rightarrow p_0^2 = |m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots|^2$$

$$E_i' = \underbrace{E_{2i} + E_{3i} + \cdots}_{\text{possible}} + \frac{1}{2} m_2 |\vec{v}_2|^2 + \frac{1}{2} m_3 |\vec{v}_3|^2 + \cdots$$

$$- \frac{1}{2(m_2 + m_3 + \cdots)} \| m_2 \vec{v}_2 + m_3 \vec{v}_3 + \cdots \|^2$$

$$| m_2 \vec{v}_2 + m_3 \vec{v}_3 + \cdots |^2 = m_2^2 |\vec{v}_2|^2 + m_3^2 |\vec{v}_3|^2 + \cdots$$

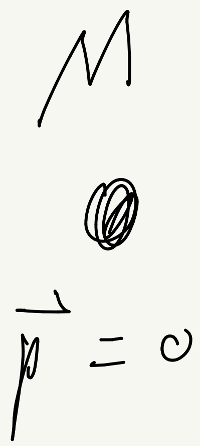
$$+ 2 m_2 m_3 \vec{v}_2 \cdot \vec{v}_3 + \cdots$$

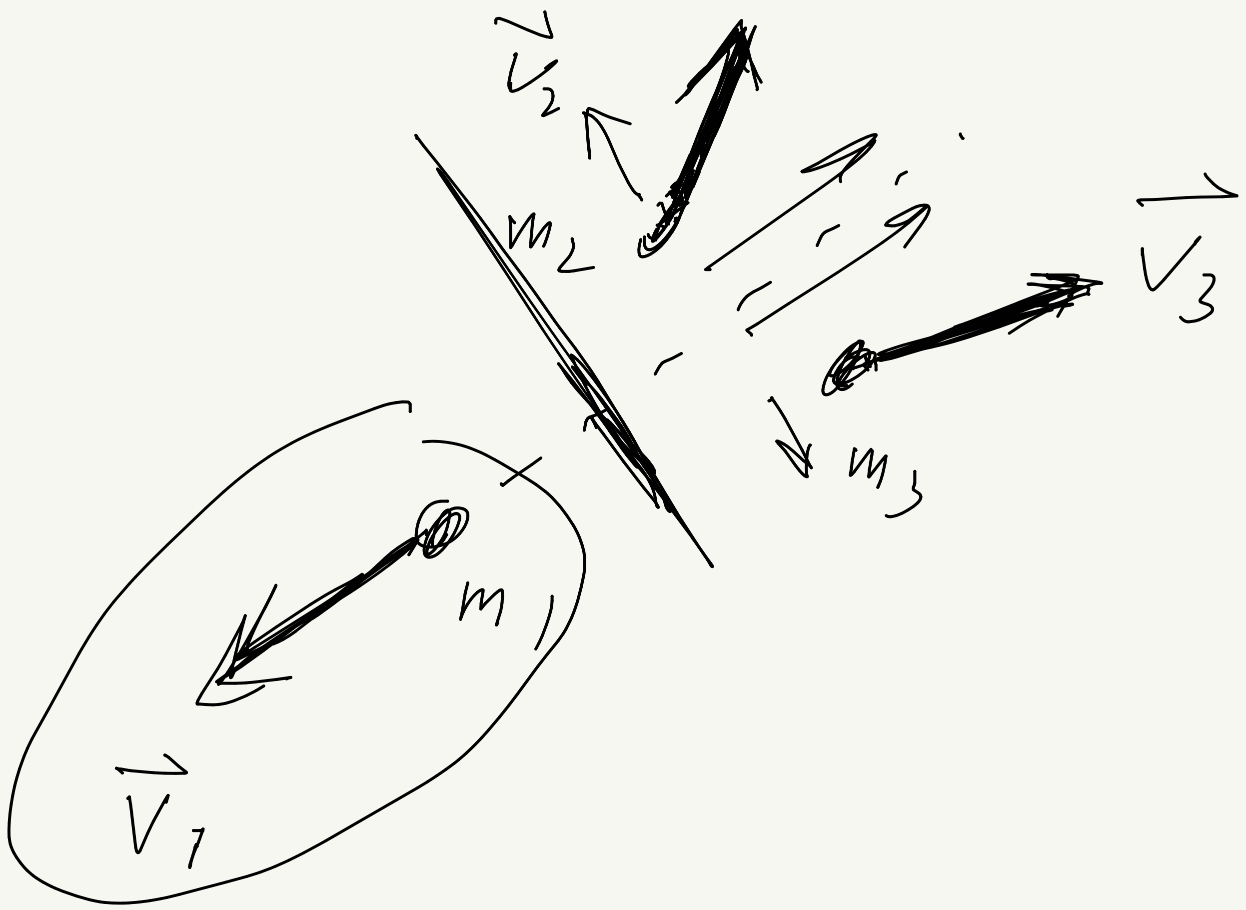
$$\boxed{\text{Suppose: } \quad \vec{v}_2 = \vec{v}_3 = \cdots \equiv \vec{v}}$$

$$\frac{1}{2} m_2 |\vec{v}_2|^2 + \frac{1}{2} m_3 |\vec{v}_3|^2 + \cdots = \frac{1}{2}(m_2 + m_3 + \cdots) |\vec{v}|^2$$

$$- \frac{1}{2(m_2 + m_3 + \cdots)} (m_2 + m_3 + \cdots)^2 |\vec{v}|^2 = -\frac{1}{2}(m_2 + m_3 + \cdots) |\vec{v}|^2$$

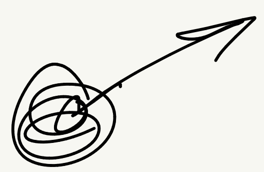
$$\rightarrow \quad E_i' = E_{2i} + E_{3i} + \cdots$$





$$\frac{p_0^2}{2m_1} = T_{1o}$$

$$\underbrace{m_1\vec{V}_1}_{\vec{p}_0} + \underbrace{m_2\vec{V}_2 + m_3\vec{V}_2}_{-\vec{p}_0} = 0$$



$$E_1' = E_{2,i} + E_{3,i} + \dots$$

Added discussion: (meaning of E_i')

Conservation of total energy:

$$E_i = \underbrace{E_{1i} + \tfrac{1}{2} m_1 |\vec{v}_1|^2}_{\text{internal + KE of mass } m_1} + \underbrace{E_{2i} + \tfrac{1}{2} m_2 |\vec{v}_2|^2}_{\text{same for } m_2} + \underbrace{E_{3i} + \tfrac{1}{2} m_3 |\vec{v}_3|^2}_{\text{same for } m_3} + \cdots$$

Let: $T' \equiv \frac{1}{2} m_2 |\vec{v}_2|^2 + \frac{1}{2} m_3 |\vec{v}_3|^2 + \cdots$ (= KE of $m_2, m_3, \ldots$)

$$= \frac{1}{2} m_2 |\vec{V}' + \vec{v}_2{}'|^2 + \frac{1}{2} m_3 |\vec{V}' + \vec{v}_3{}'|^2 + \cdots$$

where $\vec{V}'$ = velocity of COM of $m_2, m_3, \ldots$

$\vec{v}_2{}'$ = velocity of m_2 with respect to COM of $m_2, m_3, \ldots$

$\vec{v}_3{}'$ = " m_3 "

...

$$\rightarrow T' = \frac{1}{2} m_2 \left(|\vec{V}'|^2 + |\vec{v}_2{}'|^2 + 2 \vec{V}' \cdot \vec{v}_2{}' \right) + \cdots$$

$$= \frac{1}{2} (m_2 + m_3 + \cdots) |\vec{V}'|^2 + \frac{1}{2} m_2 |\vec{v}_2{}'|^2 + \frac{1}{2} m_3 |\vec{v}_3{}'|^2 + \cdots$$

$$+ \underbrace{\left(m_2 \vec{v}_2{}' + m_3 \vec{v}_3{}' + \cdots \right)}_{= 0 \text{ (by definition of COM for } m_2, m_3, \ldots)} \cdot \vec{V}'$$

T'_{COM} = KE of COM of $m_2, m_3, \ldots$

T'_0 = KE of $m_2, m_3, \ldots$ w.r.t. COM of $m_2, m_3, \ldots$

so $T' = T'_{com} + T_0'$

$\rightarrow$ $E_i = E_{1i} + \frac{1}{2} m_1 |\vec{v}_1|^2 + (E_{2i} + E_{3i} + \cdots) + T'_{com} + T_0'$

$= E_{1i} + \frac{1}{2} m_1 |\vec{v}_1|^2 + (E_{2i} + E_{3i} + \cdots) + \frac{p_0^2}{2(m_2+m_3+\cdots)} + T_0'$

compare to

$$E_i = \left(E_{1i} + \frac{1}{2} m_1 |\vec{v}_1|^2\right) + \left(E_i' + \frac{p_0^2}{2(\underbrace{M-m_1}_{m_2+m_3+\cdots})}\right)$$

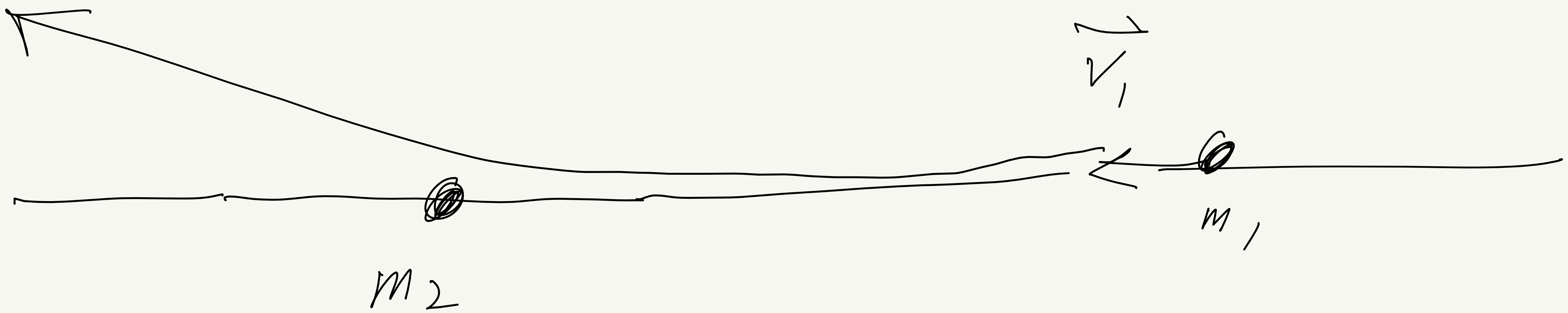
$\rightarrow$ $\boxed{E_i' = E_{2i} + E_{3i} + \cdots + T_0'}$

$= (\text{internal energy of } m_2, m_3, \cdots) + \left(\text{KE of } m_2, m_3, \cdots \text{ w.r.t. com of } m_2, m_3, \cdots\right)$

NOTE: when $m_2, m_3, \cdots$ all move with the same velocity, they are moving together with the com of $m_2, m_3, \cdots$. Then $T_0' = 0$ and E_i' = internal energy of $m_2, m_3, \cdots$

COM frame vs. lab frame

Example:

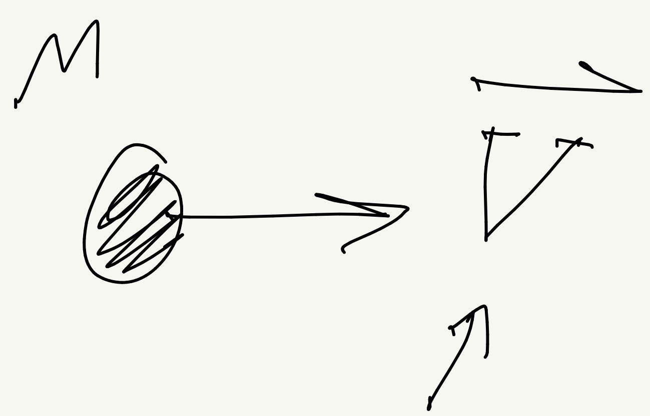


(at rest)

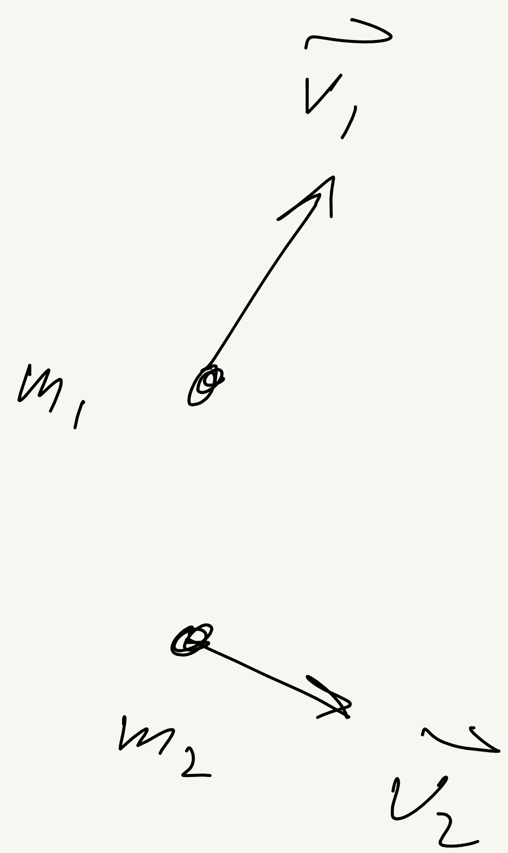
lab frame

$\vec{p}_{tot} = m_1 \vec{v}_1 + \cancel{m_2 \vec{v}_2}^{\,0} = m_1 \vec{v}_1 \neq 0$

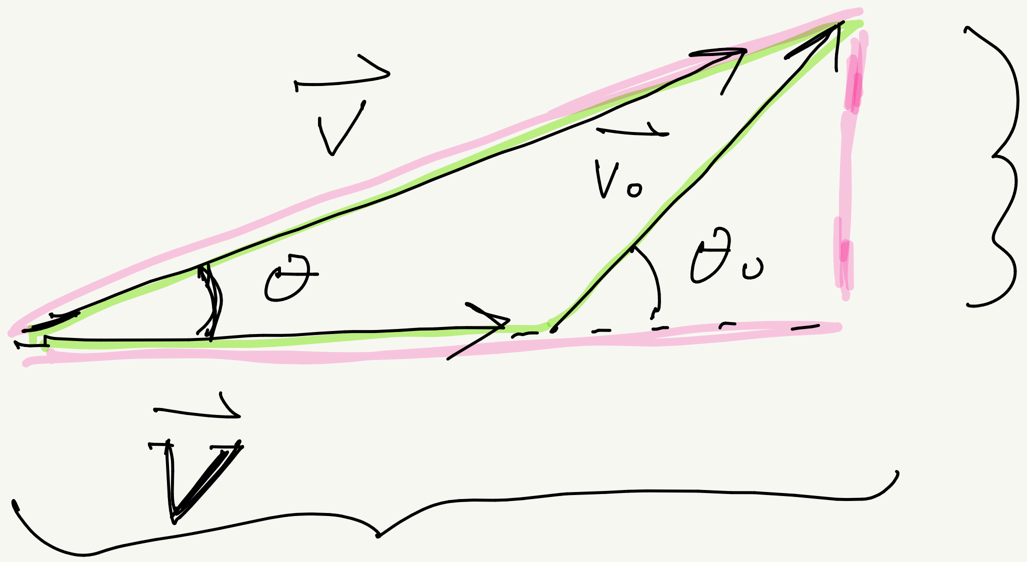
lab frame:



velocity of M w.r.t lab frame



$M\vec{V} = m_1 \vec{v}_1 + m_2 \vec{v}_2$



θ : w.r.t lab frame

θ_0 : w.r.t COM frame

$\vec{v}_0$: velocity of m_1 (or m_2) w.r.t COM frame

$\vec{V}$: velocity of M w.r.t lab frame

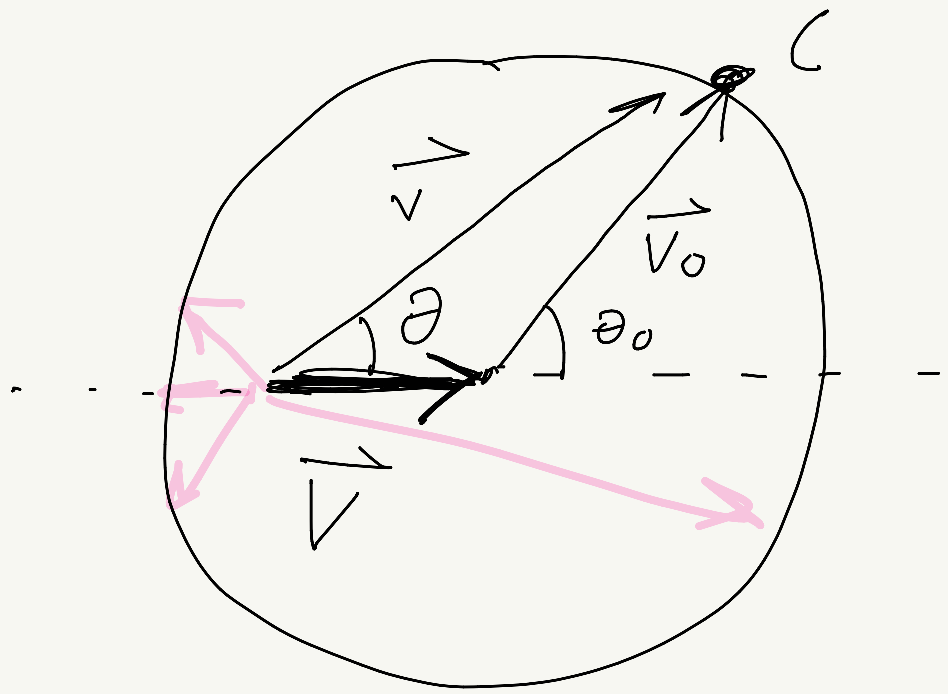
$\vec{v}$: velocity of m_1 (or m_2) wrt lab frame

$$\boxed{v_0^2 = v^2 + V^2 - 2vV\cos\theta}$$

$$v \sin\theta = v_0 \sin\theta_0$$

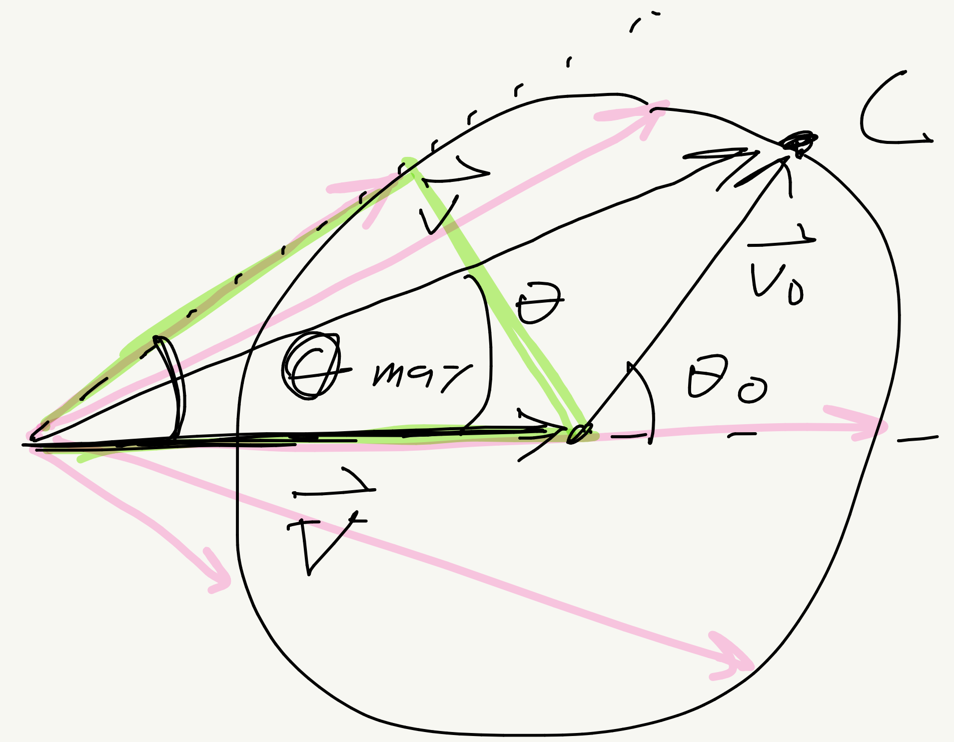
$$v\cos\theta = V + v_0\cos\theta_0$$

$\div \rightarrow$ $$\boxed{\tan\theta = \frac{v_0 \sin\theta_0}{V + v_0\cos\theta_0}}$$



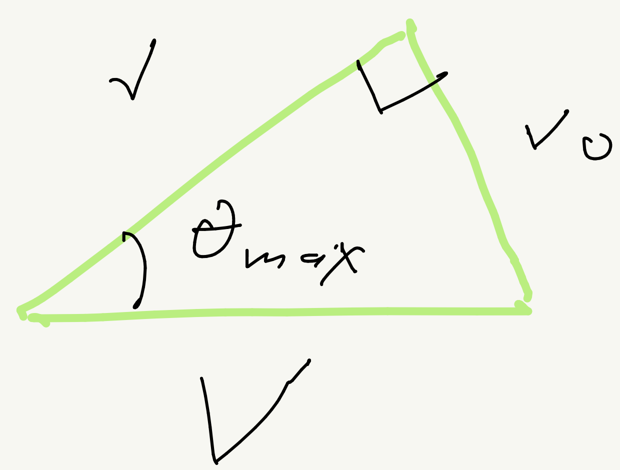
$V < v_0$

$\vec{v}$ Can point in any direction



$V > v_0$

$\vec{v}$ can only point in the forward directio



$$\sin\theta_{max} = \frac{v_0}{V}$$