Chpt 24

(1.) $\oint_S \vec{E} \cdot \hat{n}\, dA$ = electric flux thru surface

$= \dfrac{q_{enc}}{\epsilon_0}$ (by Gauss's law)

S_1 : $q_{enc} = -2Q + Q = -Q$

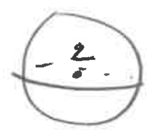
$\rightarrow \Phi_1 = -\dfrac{Q}{\epsilon_0}$

S_2 : $q_{enc} = Q - Q = 0 \rightarrow \Phi_2 = 0$

S_3 : $q_{enc} = -2Q + Q - Q = -2Q \rightarrow \Phi_3 = \dfrac{-2Q}{\epsilon_0}$

S_4 : $q_{enc} = 0 \rightarrow \Phi_4 = 0$

(2.) $q = 16.2\ \mu C$, $R = 25.5\,cm$



a) $\Phi = \dfrac{q_{enc}}{\epsilon_0} = \dfrac{16.2 \times 10^{6}\ C}{8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}}$

$k_e = \dfrac{N \cdot m^2}{C^2} = \dfrac{1}{4\pi\epsilon_0}$

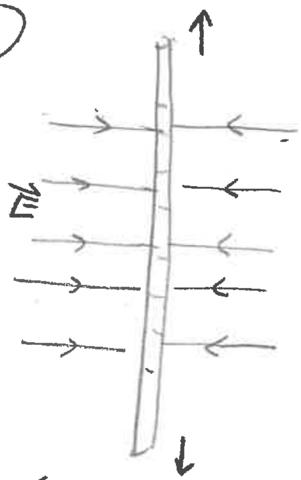
$= 1.83 \times 10^{6}\ \dfrac{N\,m^2}{C}$

b) Total electric flux thru hemisphere $= \frac{1}{2}$ above answer

$= 9.15 \times 10^{5}\ \dfrac{N \cdot m^2}{C}$

c) Result is independent of radius

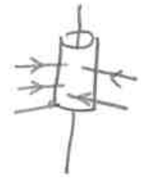
(3.)



($\vec{E}$ radially inward indep. of position along ~~filament~~ filament)

$\lambda = -90.2 \frac{\mu C}{m}$

Take cylindrical surface with filament being axis of cylinder



radius r
height h

$$\oint_S \vec{E} \cdot \hat{n}\, dA = \frac{q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi r \cdot \cancel{h} = \frac{|\lambda| \cancel{h}}{\epsilon_0}$$

(take $|\lambda|$ since $\vec{E}$ points toward filament)

$$E(r) = \frac{|\lambda|}{2\pi\epsilon_0 r}$$

a) For $r = 10.0$ cm, $\lambda = -90.2 \mu C$

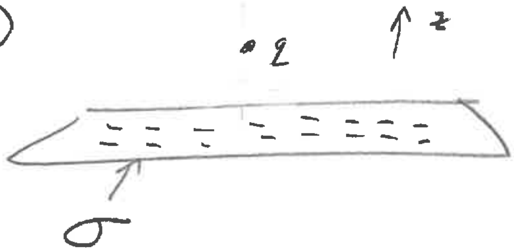
$$E(r) = \frac{90.2 \times 10^{-6} \frac{C}{m}}{2\pi \cdot 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \cdot 0.1 m}$$

$$= \boxed{1.62 \times 10^7 \frac{N}{C}} \quad \left(= 16.2 \frac{MN}{C}\right)$$

b) For $r = 23.5$ cm, $\boxed{E(r) = 6.90 \times 10^6 \frac{N}{C}}$ $\left(= 6.9 \frac{MN}{C}\right)$

c) For $r = 150$ cm, $\boxed{E(r) = 1.08 \times 10^6 \frac{N}{C}}$ $\left(= 1.08 \frac{MN}{C}\right)$

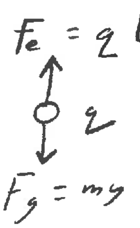
(4.)



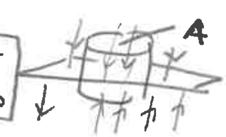
$q = -0.654 \mu C$
$m = 18.6$ gm

$\vec{F}_{tot} = 0$ means that $\vec{F}_e$ should point vertically upward, so $\sigma < 0$.

Force diagram



$$E \cdot 2A = \frac{\sigma A}{\epsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

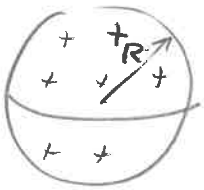


Thus, $mg = qE = q\frac{\sigma}{2\epsilon_0}$

$$\rightarrow \sigma = \frac{2\epsilon_0 mg}{q} = \frac{2 \times 8.85 \times 10^{-12} (18.6 \times 10^{-3} kg)(9.8 \frac{m}{s^2})}{-0.654 \times 10^{-6} C}$$

$$= \boxed{4.96 \mu C/m^2}$$

5.



— spherical shell

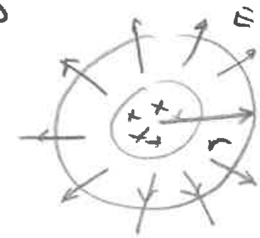
$R = 13.0$ cm

$Q = 34.0\ \mu C$ (uniformly distributed)

a) For $r = 10.0$ cm from center, $r < R$

$\rightarrow \boxed{\vec{E} = 0}$ (because no charges enclosed)

b) For $r = 25.0$ cm from center, $r > R$

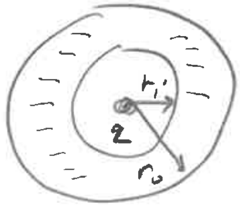


$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$= 4.89 \times 10^6\ N/C$$

$$= \boxed{4.89\ MN/C}$$

6.



$r_i = 20.0$ cm

$r_o = 32.0$ cm

$q = -60.0$ nC
$= -60 \times 10^{-9}$ C

$$\rho = \frac{Q}{\text{Volume}} = -2.92\ \frac{\mu C}{m^3}$$

just outside of shell ($r = r_o$)

$$E = \frac{1}{4\pi\epsilon_0}\frac{|Q+q|}{r_o^2}$$

$$Q = \rho \cdot \text{volume} = \rho \cdot \frac{4}{3}\pi(r_o^3 - r_i^3)$$

$\rightarrow \boxed{Q = -3.03 \times 10^{-7}\ C}$

proton ($m_p = 1.67 \times 10^{-27}$ kg, $q_p = e = 1.602 \times 10^{-19}$ C) moves in circular orbit with ~~unif~~ constant speed $v = \omega r_o$ ($a_c = \frac{v^2}{r_o}$)

Centripetal Force: $F_{cent} = \frac{m_p v^2}{r_o} = q_p E$

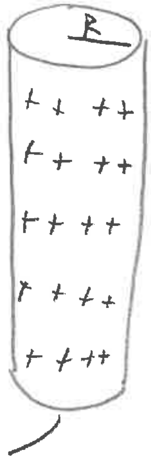
$$\rightarrow v = \sqrt{\frac{q_p E r_o}{m_p}}$$ where $E = \frac{1}{4\pi\epsilon_0}\frac{|Q+q|}{r_o^2} = \boxed{3.19 \times 10^4\ \frac{N}{C}}$

Thus, $\boxed{v = 9.90 \times 10^5\ \frac{m}{s}}$

(7.)

$R = 4.90$ cm

$\lambda = 31.2 \; \frac{nC}{m}$



metal

— charge on outside of metal rod
$(\vec{E} = 0$ inside rod$)$

Recall
$E \cdot 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\epsilon_0}$
$E = \frac{\lambda}{2\pi\epsilon_0 r}$

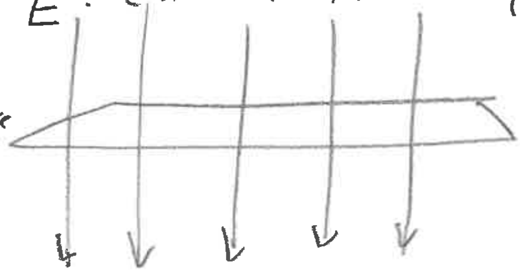
a) $r = 3.50$ cm $< R \rightarrow \boxed{\vec{E} = 0}$

b) $r = 20.0$ cm $> R \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2.81 \times 10^3 \frac{N}{C}}$
(radially outward)

c) $r = 200$ cm $> R \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{281 \; N/C}$
(radially outward)

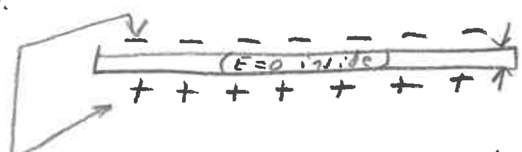
(8.) $\vec{E}$: external field $(E = 82 \; kN/C)$

perspective view



square metal plate
$s = 55$ cm (side length)

side view:



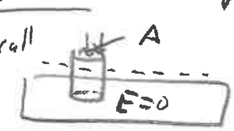
thickness of plate

induced charges on outside of plate are such as to cancel the electric field inside metal plate

a) top of metal plate $-E = \frac{\sigma_-}{\epsilon_0}$ ← negative

$\rightarrow \sigma_- = -E\epsilon_0$
$= \boxed{-283 \; nC/m^2}$

Recall



$EA = \frac{\sigma A}{\epsilon_0}$
$E = \sigma/\epsilon_0$

bottom of metal plate $\rightarrow \sigma_+ = E\epsilon_0 = \boxed{+283 \; nC/m^2}$

b) Total charge on top of plate: $Q_- = \sigma_- A$, $A = s^2$

$= \sigma_- s^2$

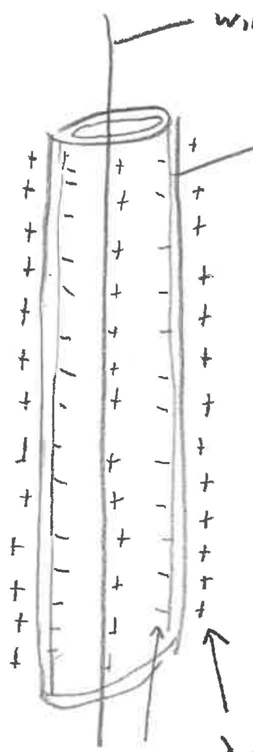
$\boxed{= -85\,nC}$

Total charge on bottom of plate: $Q_+ = \sigma_+ A$

$= \boxed{+85\,nC}$

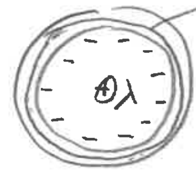
9.

wire : λ = charge/length



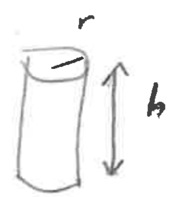
: 2λ = net charge/length

a)



$$0 = \oint_S \vec{E} \cdot \hat{n}\, dA = \frac{q_{enc}}{\epsilon_0}$$

(since $\vec{E} = 0$ inside cylinder)



$\Rightarrow q_{enc} = 0$

$\lambda h + \lambda_i h = 0$

$\rightarrow \boxed{\lambda_i = -\lambda}$ (negative)

b) since total charge/length $= 2\lambda$

$2\lambda = \lambda_0 + \lambda_i$

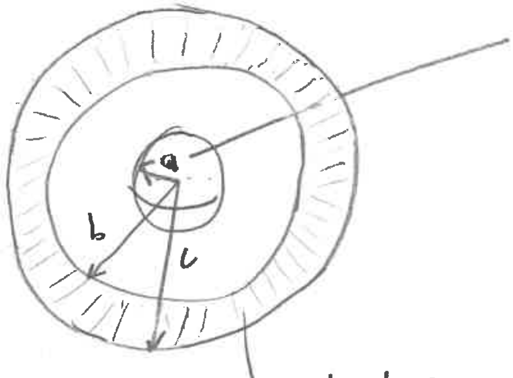
$2\lambda = \lambda_0 - \lambda$

$\rightarrow \boxed{\lambda_0 = 3\lambda}$ (positive)

c) Electric field outside cylinder a distance r from axis

$$E \cdot 2\pi r \cancel{h} = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda_{enc} \cancel{h}}{\epsilon_0} \rightarrow E = \frac{\lambda_{enc}}{2\pi\epsilon_0} = \boxed{\frac{3\lambda}{2\pi\epsilon_0}}$$

(10.) (c)



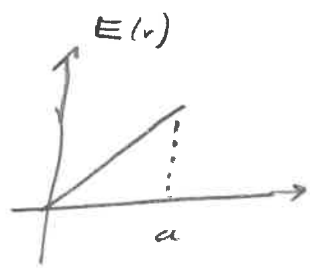
insulator : ρ = unif. charge density $= \dfrac{Q}{\frac{4}{3}\pi a^3}$

conductor : uncharged, hollow sphere
inner / outer radii : b, c

a) charge contained with sphere of radius $r<a$

$$q_{enc} = \rho \, \frac{4}{3}\pi r^3 = \frac{Q}{\cancel{\frac{4}{3}}\cancel{\pi} a^3} \cdot \cancel{\tfrac{4}{3}}\cancel{\pi} r^3 = \boxed{Q\left(\frac{r}{a}\right)^3}$$

b) $$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{r<a}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\left(\frac{r}{a}\right)^3 = \boxed{\frac{1}{4\pi\epsilon_0}\,Q\,\frac{r}{a^3}}$$



c) For $a<r<b$, $q_{enc} = \boxed{Q}$

d) For $a<r<b$, $E = \boxed{\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}}$

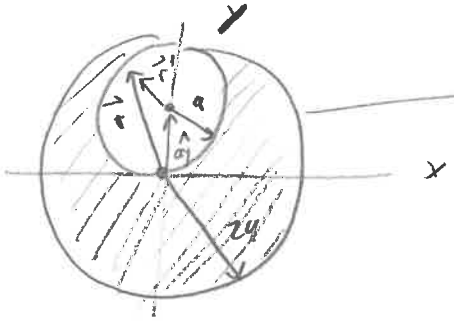
e) For $b<r<c$, $E = \boxed{0}$ (since inside conductor)

f) Induced charge on inner surface must be $\boxed{-Q}$ in order that $q_{enc}=0$ for $b<r<c$

g) Charge on outer surface must be $\boxed{+Q}$ since conducting sphere is not charged

h) $\boxed{\text{Surface } b}$ has largest magnitude of surface charge density since $|\sigma_b| = \dfrac{|Q|}{4\pi b^2} > |\sigma_c| = \dfrac{|Q|}{4\pi c^2}$ (because $c>b$). Also, ρ is a volume charge density.

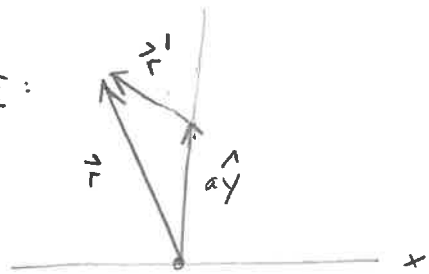
11.



sphere of non-conducting material.
Unif charge density

$$\rho = \frac{Q_{tot}}{\frac{4}{3}\pi(2a)^3}$$

NOTE:



$$\vec{r} = a\hat{y} + \vec{r}'$$

$$\vec{E}(\vec{r}) = \vec{E}_+(\vec{r}) + \vec{E}_-(\vec{r})$$

= $\vec{E}$ field for full sphere with Unif. charge density ρ

+ $\vec{E}$-field for smaller sphere with unif. charge density $-\rho$

$$\vec{E}_+(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q_{inside}}{r^2}\hat{r} \quad \left(Q_{inside} = \rho\frac{4}{3}\pi r^3\right)$$

$$= \frac{1}{4\pi}\frac{\rho\frac{4\pi}{3}r^3}{r^2}\hat{r}$$

$$= \frac{\rho}{3}r\hat{r}$$

$$= \frac{\rho}{3}\vec{r}$$

$$\vec{E}_-(\vec{r}) = \vec{E}_-(\vec{r}')$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q'_{inside}}{r'^2}\hat{r}' \quad \left(Q'_{inside} = -\rho\frac{4}{3}\pi r'^3\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{(-\rho)\frac{4\pi}{3}r'^3}{r'^2}\hat{r}'$$

$$= -\frac{\rho}{3}r'\hat{r}'$$

$$= -\frac{\rho}{3}\vec{r}'$$

Thus, $\vec{E}(\vec{r}) = \frac{\rho}{3}\vec{r} - \frac{\rho}{3}\vec{r}' = \frac{\rho}{3}(\vec{r}-\vec{r}') = \boxed{\frac{\rho a}{3}\hat{y}}$

(12.)

$\rho = \dfrac{a}{r}$, $a = \text{const}$

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q_{enc}}{r^2}\,\hat{r}$$

$$Q_{enc} = \int_0^r dV\,\rho \quad , \quad dV = d\bar{r}\;\bar{r}\,d\theta\;\bar{r}\sin\theta\,d\phi = \bar{r}^2 d\bar{r}\,\sin\theta\,d\theta\,d\phi$$

$$= \int_0^r \bar{r}^2 d\bar{r}\;\underbrace{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\sin\theta\,d\theta\,d\phi}_{4\pi}\;\frac{a}{\bar{r}}$$

$$= 4\pi a\int_0^r \frac{\bar{r}^2}{\bar{r}}\,d\bar{r}$$

$$= 4\pi a\left(\frac{1}{2}r^2\right)$$

$$= 2\pi a r^2$$

$$\vec{E}(r) = \frac{1}{4\cancel{\pi}\epsilon_0}\frac{2\cancel{\pi} a \cancel{r^2}}{\cancel{r^2}}\,\hat{r}$$

$$= \boxed{\frac{a}{2\epsilon_0}\,\hat{r}}$$