Part 5. Hearing and Auditory Effects

Auditory System Parts

Outer Ear
The *pinna* collects and concentrates the sound from the outside into the *auditory canal*.

Eardrum
The eardrum separates the outer from the middle ear, so that pressure fluctuations in the sound will not equalize rapidly. Instead, the eardrum vibrates with these fluctuations. (N.B.: Slow pressure changes such as those occurring in an elevator are equalized by the Eustachian tube.)

Middle Ear
The middle contains the three ossicles called *hammer (malleus), anvil (incus), and stirrup (stapes)*. These three tiny bones amplify the vibrations of the eardrum and apply them to the *oval window* between middle and inner ear.

Inner Ear
The principal hearing organ of the inner ear is the coil-like *cochlea.* Vibrations from the stirrup of the middle ear are transmitted at the *oval window* to the fluid in the cochlea.
Traveling waves move through this fluid in the *scala vestibuli.* The cochlea has an opening at the end of the scala vestibuli, called the *helicotrema.* The waves travel through there and on through the *scala timpani* to the *round window*. This is a flexible membrane vibrating with opposite phase to the oval window. The elasticity of the round window allows the fluid in the cochlea to move. This in turn lets the "*hair cells"*, actually nerve cells, of the *basilar membrane* move and be stimulated. The round window also dampens and absorbs the oscillations and minimizes reflections.

Question
Why does the membrane of the round window have to be flexible?

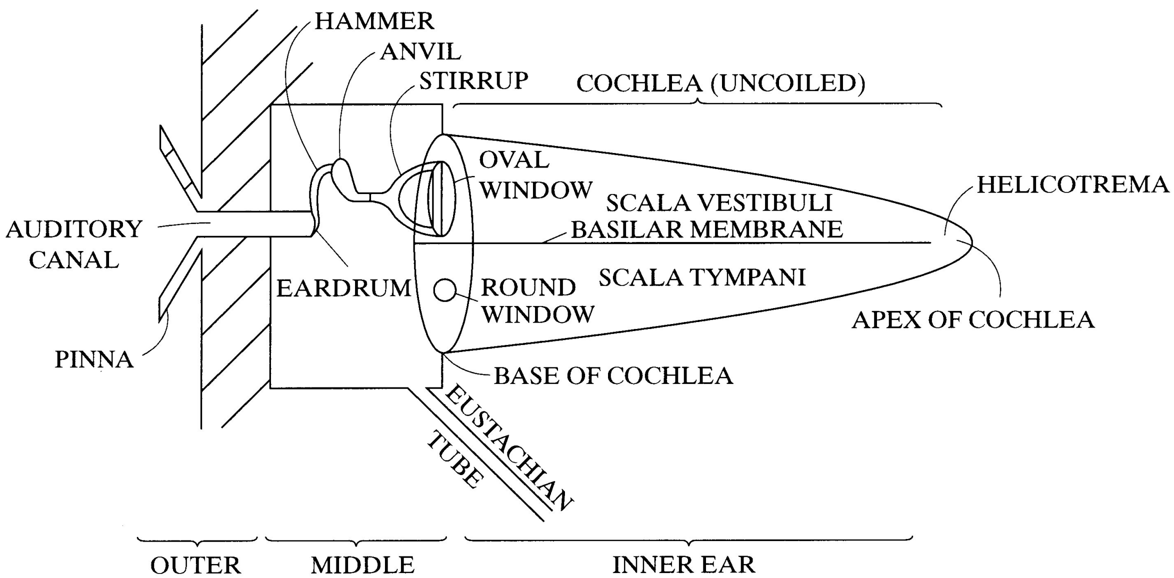
Answer

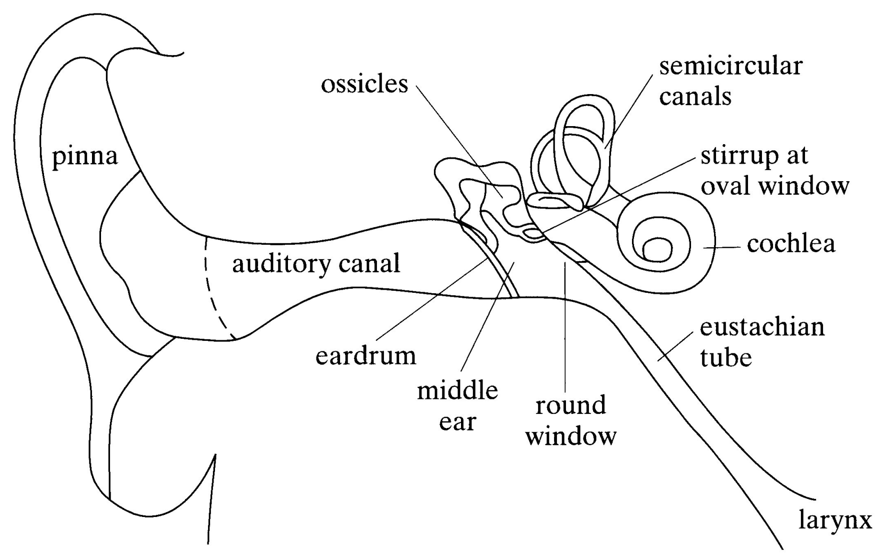
If the round window were rigid, the stapes would push against an incompressible fluid, which then would not move, and therefore hearing of sound would be impossible. (A case of hearing loss of this kind actually exists.)

Basilar Membrane
The basilar membrane separates the scala vestibuli and scala tympani. The *organ of Corti* is the region along the basilar membrane that contains the nerve endings or "hair cells". These generate electrical signals from the traveling waves. The signals are collected by the *auditory nerve* and conducted to the brain.

The organ of Corti is about 35 mm long and contains 30,000 "hair cells".
The brain compares the composite electrical signal from the auditory nerve to sound patterns previously stored. Speech, music, and other sounds are then recognized in this way.

Cross Section - Auditory System





Lower Figure: Ear and peripheral auditory system.
Upper Figure: Cross section and schematic of the peripheral auditory system.
(From Berg & Stork, 3rd edition, Fig. 6-1, p.146.)

Exercises

1. In the lower figure, clearly mark in color the main three parts of the peripheral auditory system.
2. Add to the lower figure the basilar membrane, even though it is hidden.

Schematic of Ear

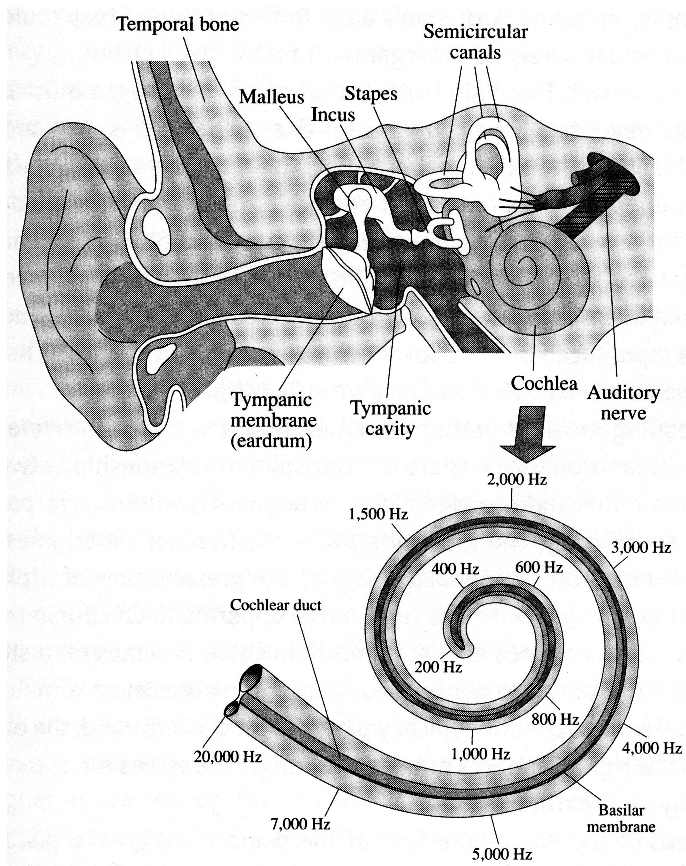


Figure. Schematic diagram of the ear and the distribution of frequencies along the basilar membrane. “Hair cells” (not shown) protrude above the surface of the membrane. Vibrations in the fluid of the cochlea make the hairs move. This opens small pores in the cell walls into which charged ions from the fluid can enter. In this way the charge state of the cells is altered and generates signals in the nerves that lead to the brain.
(From Philip Ball, The Music Instinct, Fig. 3.2, p. 37, Oxford University Press, 2010.)

Engineering Schematic of the Ear. Frequency Response

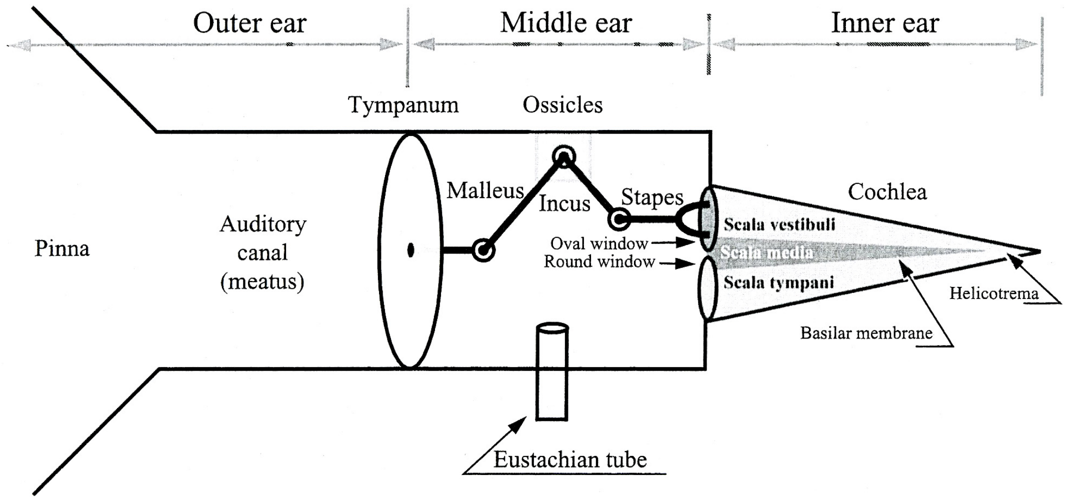


Figure. "Mechanics" of the human ear. Note the levers (ossicles) of the middle ear and the cochlea with the basilar membrane of the inner ear.
(From: Gareth Loy, Musimathics, volume 1, Fig. 6.1, p. 151, The MIT Press, 2006.)

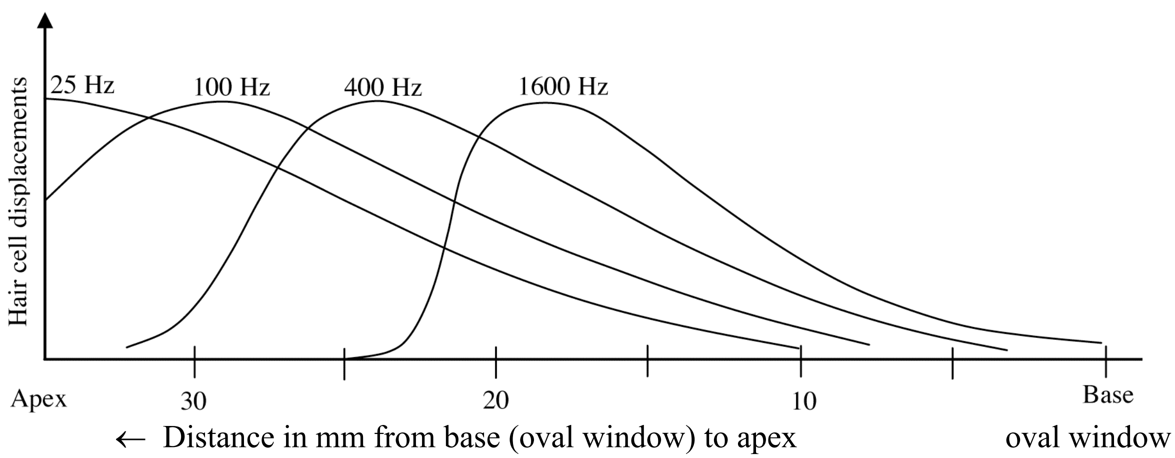


Figure. Response of the basilar membrane for various frequencies. Note the peak response for excitation frequencies of 25, 100, 400, and 1600 Hz. The base is at the oval window where the high frequencies are detected. The apex is farthest away from it and responds to the lowest frequencies.

Basilar Membrane and Place Theory of Hearing

A direct correlation exists between the frequency of a pure tone and the location of the maximum nerve cell response on the basilar membrane.

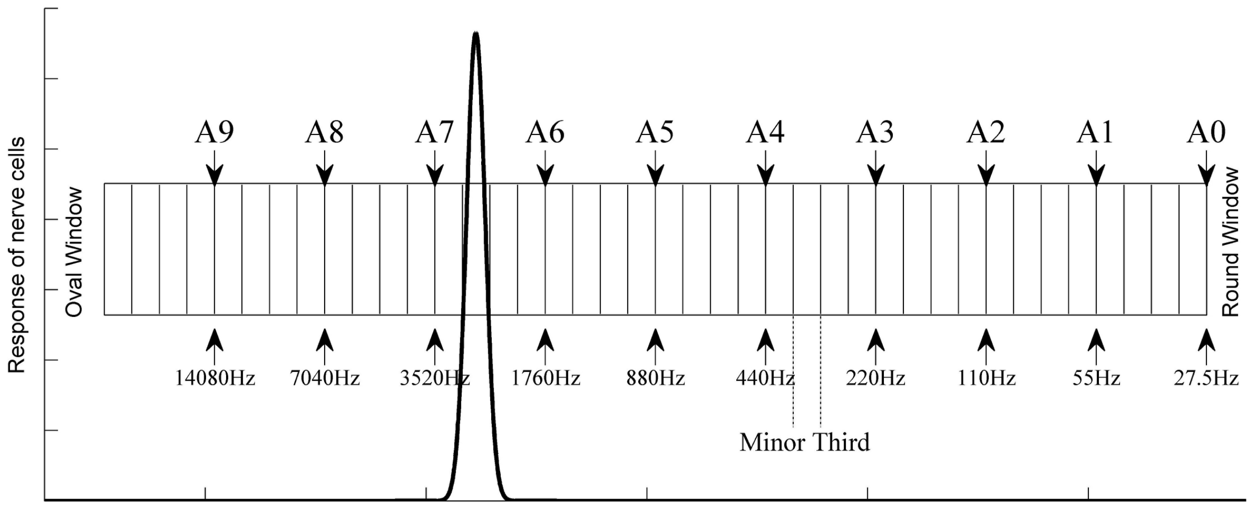


Figure. Schematic diagram of the uncoiled basilar membrane of the cochlea in the inner ear. The bell shaped curve is a simplified schematic of the *"critical band"* and shows the region of nerve cells that respond to a pure sinusoidal sound. The width of the critical band at half the height of the maximum is about a minor third over much of the audible range in the middle.
Note the logarithmic frequency scale in the figure, which corresponds to the arrangements of frequencies on the basilar membrane.
(Adapted from Ian Johnston, Measured Tones, 3rd edition, p. 238-9, CRC Press.)

Acoustic vibrations from the stirrup of the middle ear are transferred to the oval window on the left in the above figure and travel along the basilar membrane. The audible range extends logarithmically over the 35 mm long basilar membrane and contains about 30000 hair cells ("resonators") over the audible range of 10 octaves. The region over which the nerve endings show a large response to a pure tone (sine wave) is the *critical band*. The notes A1 to A9 are spaced octaves apart.

According to the *place theory of hearing,* the maximum response for a given note occurs at a specific location on the basilar membrane. This is shown in the figure by arrows for the maxima of the notes "A", where A4 = 440 Hz is the "concert A". The range of hearing spans almost 10 octaves. The highest frequencies are sensed near the oval window, the lowest frequencies farthest away from it near the end of the basilar membrane (apex of the cochlea). Given the nearly logarithmic frequency response of the cochlea, the spacing between adjacent octaves (or any corresponding musical intervals) is the same, although the frequency increases exponentially.

Critical Band, Auditory Sharpening, Musical Intervals

Critical Band and Place Theory of Hearing

According to the *place theory of hearing*, each frequency is sensed at a certain place on the basilar membrane. This is the place where a wave of given frequency “breaks” or “crashes” in the cochlear fluid, similar to ocean waves nearing a beach. Octaves occupy equal lengths along the basilar membrane. The spacing of octaves and other frequency intervals is nearly *logarithmic*. Each octave covers a length of 3 to 4 mm or about 3000 hair cells.

With 12 semitones to an octave, there are about 250 cells to a semitone. There are 3 semitones to a minor third and hence 750 hair cells. One octave covers about 3.5 mm of the basilar membrane. Therefore, one semitone spacing is about 1/12 of this or about 0.3 mm = 300 μm.

The *critical band* on the basilar membrane is defined as the number of hair cells that respond to a *pure sinusoidal tone*. The critical band is about a *musical minor third* (frequency interval 6/5), or 3 musical half steps over most of the audible range. This corresponds to about 1 mm on the basilar membrane or close to 750 hair cells. On the other hand, at frequencies below 200 Hz the critical band is almost an octave wide or about 12 semitones. For low notes such as G2 on the piano, minor thirds therefore sound very rough, and composers rarely use small musical intervals at very low frequencies.

Demonstrations

1. Select the “sine tone” mode on the keyboard. Play minor thirds in the middle of the keyboard to demonstrate the width of the critical band.
2.Use sine waves and play the minor thirds G4-B^{b}4 and G2-B^{b}2. Note that the critical band at low frequencies is wider. Where does the minor third sound less coarse and more pleasing?
3. Select “grand piano” or “square wave” mode on the keyboard. Repeat the above. There is less coarseness in the sound now for the minor thirds. This is due to the *sharpening effect.* Sharpening is more acute for complex waves because of the additional frequency information from the harmonics. Tones less than one semitone apart may thus be resolved.
4. Play musical fifths, fourths, and thirds at various locations on the keyboard and listen whether they “sound similar”.

Auditory Sharpening

How can we hear a *pure tone* or sine wave with one single sharp frequency if the critical band is a minor third wide? This is a major puzzle. Apparently our neural system and brain narrow down a large range of frequencies to a tone that is perceived with single sharp pitch! This process is called *auditory sharpening*.

Research Paper

Write a research paper on the mechanism of auditory sharpening.

Hearing on a Log-Frequency Scale, Musical Intervals

Equal musical intervals such as octaves, minor thirds etc., occupy the same length on the basilar membrane. For a musical third this distance is about 1 mm. This shows that our "hearing" takes place on a compressed, logarithmic frequency scale on the basilar membrane. Musical intervals sound the same over much of the basilar membrane, independent of pitch.

Piano Keyboard Analogy

The arrangement of hair cells on the basilar membrane is analogous to the arrangements of keys on the piano. An octave occupies 3.5 mm on the basilar membrane and 160 mm on the keyboard.

Demonstration of Musical Intervals on a Log-Frequency Scale

1. Show on the keyboard that a musical interval "sounds the same" irrespective of where the interval is played. Listen to octaves, fifths, fourths, thirds at different locations on the keyboard.
2. Select a musical interval such as a major third E4-C4 with two frequency generators. Select *"log-scale"* on the frequency axis of the computer display. Note the spacing E4-C4 in the spectrum. Next select the major third E5-C5. Again note the spacing on the logarithmic frequency scale. They are the same! While the frequency differences E4-C4 and E5-C5 are different on a linear scale, the ratios E4/C4 and E5/C5 are the same and have the same separation between them on the logarithmic scale. The log-scale for frequency in this demonstration simulates how we hear musical intervals. Mathematically speaking we have:
$f_{E4}/f_{C4} = f_{E5}/f_{C5} \rightarrow \log(f_{E4}/f_{C4}) = \log(f_{E5}/f_{C5})$ and, therefore, on the log-scale
$\log f_{E4} - \log f_{C4} = \log f_{E5} - \log f_{C5}$, while on a linear frequency scale we have $f_{E4} - f_{C4} \neq f_{E5} - f_{C5}$.

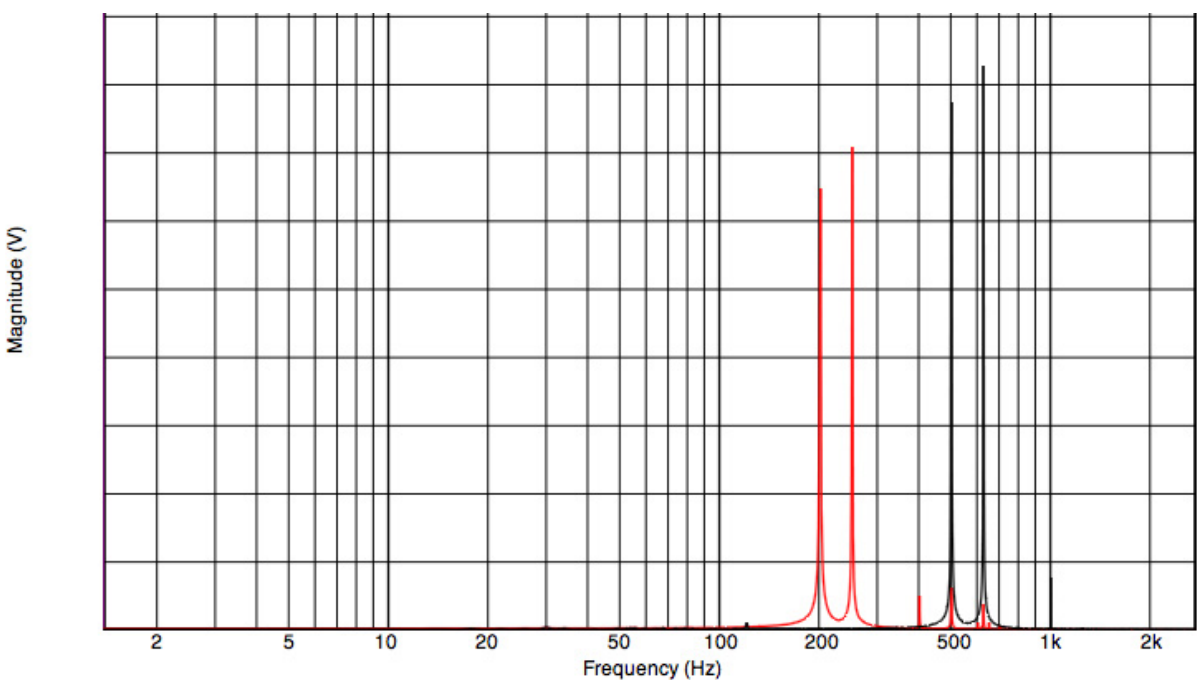


Figure. Two musical major thirds on a logarithmic frequency scale, namely 250Hz/200Hz = 5/4 and 625Hz/500Hz = 5/4. The width of the intervals on the logarithmic scale is the same, and this is the way we hear them (in contrast to a linear frequency scale).

Frequency Discriminations by the Ear (Pure Tones)

Critical Bandwidth (CB)	Limit of Frequency Discrimination (LFD)	Just Noticeable Difference (Frequency JND)
Single pure tone	Two simultaneous pure tones	Two sequential pure tones
15% at high f to 100% at low f	approximately 10%	0.5% at high f to 3% at low f
(minor third to octave)	(Semitone to full tone)	(0.1 to 0.5 semitone)

Example: **f = 2000 Hz**

Critical bandwidth	Limit of Frequency Discrimination	Just noticeable difference
300 Hz	200 Hz	10 Hz

Example: **f = 200 Hz**

Critical bandwidth	Limit of Frequency Discrimination	Just noticeable difference
100 Hz	25 Hz	3 Hz

Demonstrations

1. **Critical bandwidth (CB)**

Play simultaneously two sine waves of the same frequency and amplitude. Gradually increase one frequency. We first hear the average frequency as the pitch as well as beats with the difference frequency. When we increase one of the frequencies further, the beats become more rapid and both frequencies become distinguishable. A low frequency rumble may also be heard. When this disappears with an increasing frequency difference, we have reached the *critical band*.

Critical bandwidth measured: _______ Hz, or _____%, at f = ________ Hz

2. **Limit of frequency discrimination (LFD)**

Related to the above, play *simultaneously* two pure tones of different frequencies. Can you hear two tones? Bring one of the frequencies closer to the other until beating starts. The difference between the two frequencies where this occurs is the *limit of frequency discrimination.*

Limit of frequency discrimination: ______Hz, or _____ %, at f = ________ Hz

3. **Just noticeable difference in frequency (frequency JND)**

Play two pure tones *sequentially*. Start with the same frequency. Increase one frequency slightly and keep alternating between the two frequencies. Note when you hear the *just noticeable difference* in frequency.

Measured just noticeable difference: _______ Hz, or _____%, at f = ________ Hz

4. **Touch analogy for the JND and LFD**

Touch the forearm with two pencil tips as close together as possible. Keep moving away with one tip while raising and lowering it. Note when you can feel both pencil tips separately. You have reached the *"just noticeable difference"* (JND) for touch! Now touch the arm with the two pencil tips simultaneously far enough apart so that you can feel them individually. Move closer with one tip towards the other while still touching the skin. Stop when you no longer can feel the tips individually. The distance between them could be called the *"limit of touch discrimination",* corresponding to the LFD above. Note that JND < LFD, as for hearing!

Logarithms, Exponents, Bel and Decibel (dB), Hearing on a Log-Intensity Scale

We have seen that the ear has a logarithmic response to frequency. It also has a logarithmic response to sound intensity. It is time for some practice with logarithms.

Logarithms to the base of 10
Let $y = 10^x \rightarrow \log y = \log 10^x = x \log 10 = x \cdot 1 \rightarrow x = \log y$

Examples
1. Let $y = 1 = 10^0$ $\rightarrow$ $x = 0$, $\log y = \log 1 = 0$
2. Let $y = 100 = 10^2$ $\rightarrow$ $x = 2$, $\log y = 2$
3. Let $y = 1{,}000{,}000 = 10^6$ $\rightarrow$ $x = 6$, $\log y = 6$
4. Let $y = 0.001 = 10^{-3}$ $\rightarrow$ $x = -3$, $\log y = -3$

The advantage of logarithms is that a large range of numbers is compressed into a small range of logarithms. This is important for understanding the response of our auditory system.

Example
Consider the range of numbers from 1 to 1,000,000.
The logarithms of this range are 0 to 6.
Obviously the number 6 is more easily written than 1,000,000.

The Bel and Decibel
Consider an increase in the intensity of a sound from 1 to 10, i.e. a 10-fold increase.
Take the log: $\log(10/1) = \log 10 - \log 1 = 1 - 0 = 1$
This increase is called **1 Bel** in honor of Alexander Graham Bell.
A smaller unit, used more often, is the **decibel (dB)**, with **1 bel = 10 dB** or **1 dB = 0.1 Bel**
A 10-fold increase in the value of a quantity corresponds to an increase of 10 dB.

Examples
1. A quantity increases 100-fold.
The increase is $\log(100/1) = \log 10^2 = 2 \cdot \log 10 = 2$ Bel $= 20$ dB
2. A quantity increases 2-fold.
The increase is $\log(2/1) = \log 2 = 0.301$ Bel $= 3.01$ dB ≈ 3 dB increase.
3. A certain quantity increases by 5 dB.
In other words, the quantity has increased by a factor of $10^{5/10} = 3.19$ times larger.
4. A quantity does not change, i.e. 0 dB.
This means $10^{0/10} = 1$, or in other words, no change.

Table of Some Logarithms (remember approximate values!)

log 1	log 2	log 3	log 4	log 5	log 6	log 7	log 8	log 9	log 10
0.000	0.301	0.477	0.602	0.699	0.778	0.845	0.903	0.954	1.000 Bel
0.00	3.01	4.77	6.02	6.99	7.78	8.45	9.03	9.54	10.00 dB

Equal Loudness Curves (Fletcher-Munson Curves)

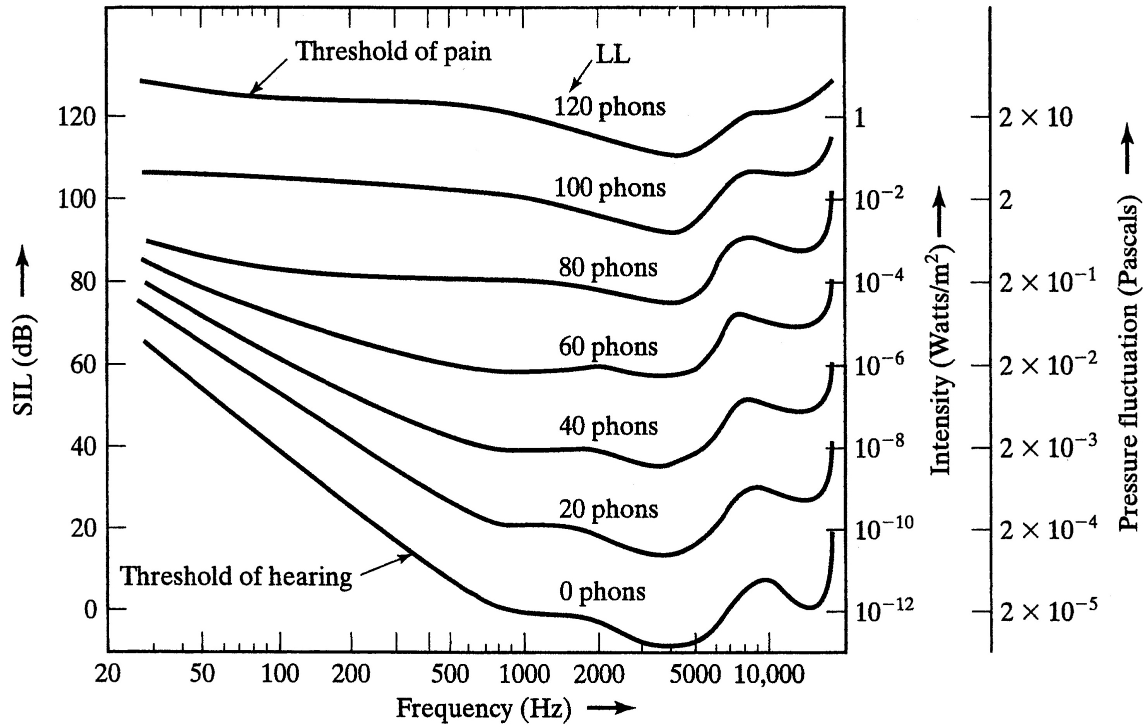


Figure. Fletcher-Munson curves, or Equal Loudness Curves, showing the response of the human ear to sound intensity as a function of frequency. For each given curve the ear perceives the same loudness over the entire frequency range. Note the logarithmic scales for the response of the human ear to frequency and intensity. (From Berg & Stork, Fig. 6-4, p. 152.)

The psychoacoustic *unit of loudness* is the *phon*. The physical *unit of sound intensity* is power in Watt per square meter or *W/m²*. The threshold of hearing is 10^{-12} W/m^2 at 1000 Hz, while threshold of pain is 1 W/m^2 at 1000 Hz for the "average person". The *sound intensity level SIL* is a logarithmic scale for sound intensity and is expressed in *decibel (dB)*. The sound intensity varies over 12 orders of magnitude from 10^{-12} W/m^2 to 1 W/m^2. Accordingly, the SIL varies from 0 to 120 dB between the threshold of hearing and the threshold of pain.

The loudness level in *phon* is chosen to have the same value as the sound intensity level SIL in dB at f = 1000 Hz. Note that for a given equal-loudness curve, for instance 60 phon, the SIL in dB varies greatly, namely from 85 dB at 30 Hz, to a minimum of 60 dB at 4000 Hz, and to 80 dB at 20000 Hz. This means that the ear is most sensitive around 4000 Hz, and much less sensitive at the lowest and highest frequencies. At all the different SIL values for the 60-phon curve, the ear perceives the same loudness, namely 60 phon. The difference in SIL of about 85 dB – 60 dB = 25 dB means that the sound intensity has to be about 30 times higher at 30 Hz than at 4000 Hz in order to be heard with equal loudness.
The intensity range of 120 dB corresponds to an amplitude range of 6 orders of magnitude in the sound pressure waves. The highest amplitude of 20 Pa at the threshold of pain still is 5000-times smaller than the static atmospheric pressure of 100,000 Pa. The ear is a sensitive organ.

Hearing best at the Minima in the Equal Loudness Curves

Two local minima can be seen in each of the Fletcher-Munson equal loudness curves. The ear is more sensitive and hears best around these minima. The more important minimum for all curves occurs around 4000 Hz and the other near 13,000 Hz. These frequencies correspond to the first two existing resonances in the auditory canal.

Exercise
Estimate the resonance frequencies for the two minima in the equal loudness curves. Assume an effective length L = 2 cm for the ear canal.
Answer
Consider the ear canal as a cylindrical tube, closed at the ear drum.
We know for the fundamental resonance of such a tube that $L = \lambda/4$ and $f_1 = v/\lambda$.
Hence $\lambda = 4L = 8\text{cm} = 0.08$ m, and $f_1 = 346/0.08 = 4330$ Hz.
This is in good agreement with the observed first minimum in the equal loudness curves.
The first resonance N = 1 in the ear canal makes our hearing most acute around 4000 Hz.
The next existing resonance is N = 3 with $f_3 = 3f_1 = 3 \cdot 4330$ Hz = 12990 Hz ≈ 13000 Hz.
This is in good agreement with the second minimum in the equal loudness curves. Sound near this frequency will also be heard loudly (by those who can still hear such high frequencies).

Exercise
From the Fletcher-Munson curves determine the sound intensity level (SIL) for a loudness of 50 phon at f = 500 Hz. Interpolate between the 40 phon and 60 phon curves. **Answer:** _______dB

Vocal Tract Analogy
The resonance frequencies of the vocal tract (see also later) can be calculated in a similar way as for the ear canal. Very roughly, the overall length of the vocal tract can be considered a closed tube of effective length L = 17.3 cm.
Exercise
Verify that this length yields the resonances at $f_1 = 500$, $f_3 = 1500$, $f_5 = 2500$, $f_7 = 3500$ Hz etc. These are the center frequencies of the so-called *formant regions* of the human voice. The resonances actually are not sharp, but are broadened because of the soft tissues of the oral tract.

Demonstrations
1. Set two signal generators to 100 Hz and 4000 Hz sine waves. With a sound level meter, adjust the volume of the two tones to the same sound intensity level, e.g. SIL = 75 dB. Switch between the two tones. The 4000 Hz tone should sound louder according to the Fletcher-Munson curves.
2. Conversely, adjust the two signal generators to the same perceived loudness.
Read the SIL values for the two tones. The 4000 Hz tone should show a lower SIL reading according to the Fletcher-Munson curves.
3. Strike a brass rod longitudinally producing a 3000 to 4000 Hz sound. Note the SIL, or more accurately the increase in the SIL from the background. Produce a 70- Hz tone with a loudspeaker set at the same SIL. Compare your perceived loudness in both cases. The brass rod should sound much louder than the speaker.
4. Blow into an emergency whistle at about 3000 Hz. Note how loud and shrill it sounds.

Amplitude-Frequency Spectra in Linear-Linear and Log-Log Displays

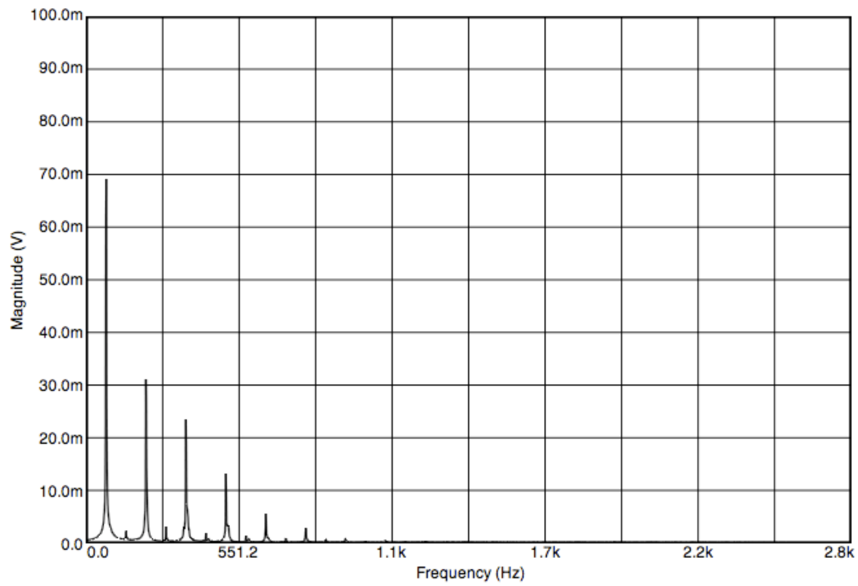


Figure. Amplitude-frequency spectrum from a didgeridoo in a double-linear display. The note played is D2. (The odd harmonics dominate for the tube "closed" by the lips at one end.) This type of display is tidy. However, it does not take into account the human ear response.

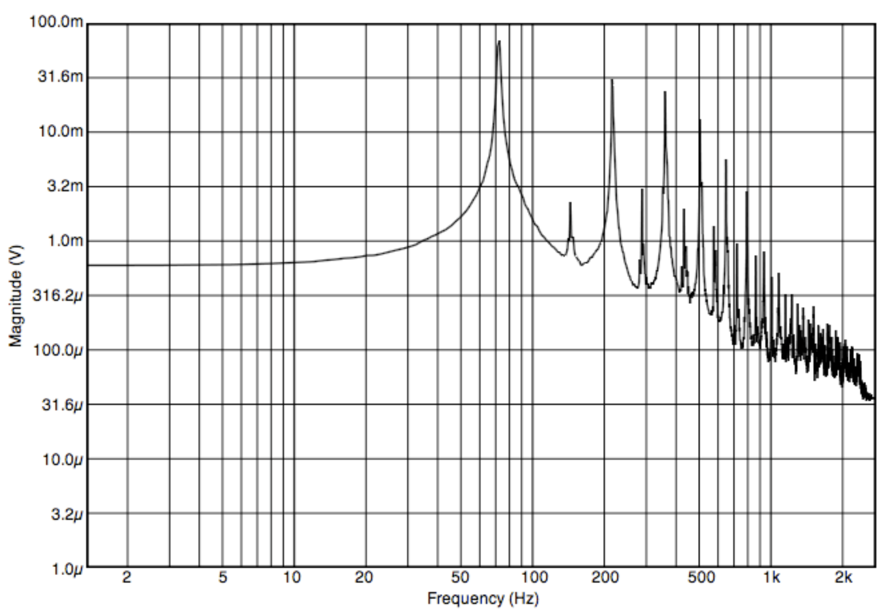


Figure. The amplitude–frequency spectrum from above, now shown in a double-logarithmic display. This representation more closely approximates the human ear response. For instance, the log-amplitudes (perceived loudness) of the first 4 harmonics are more similar in height than on the linear display above. The log-spectrum also is richer in the number of noticeable harmonics. Finally, we hear better at higher frequencies. We therefore perceive the higher harmonics even louder than shown on the log-scale in the figure.

Sound Intensity Level (SIL) and Sound Intensity (I)
The *sound intensity I* is defined as the acoustic power passing through an area of one square meter. It is measured in units of Watt per square meter (W/m^2). The *sound intensity level SIL* is the logarithm (in dB) of the *sound intensity I*. The relationship between *SIL and I* is

$$\mathbf{SIL = SIL_0 + 10\log(I/I_0)},$$

where SIL_0 is the sound intensity level of a reference intensity I_0.

Examples
1. At the threshold of hearing $I_0 = 1\text{x}10^{-12}$ W/m^2 (see Fletcher-Munson curves)
2. At the threshold of pain: $I = 1$ W/m^2 (see Fletcher-Munson curves)
3. For the threshold of hearing, we have $I = I_0$ and $SIL = SIL_0 + 10\log(I_0/I_0) = SIL_0 + 0$.
One can choose $SIL_0 = 0$ dB at the threshold of hearing at a frequency f = 1000 Hz.
4. For the threshold of pain, we have $I = 1$ W/m^2.
Then $SIL = 0 + 10\log(1/10^{-12}) = 10\log 10^{12} = 10\cdot 12 = 120$ dB, as we already know.

Important Note
The above equation for the sound intensity level SIL holds for any levels I_0 and SIL_0, not just the threshold of hearing. It holds for instance when a noise increases from an initial reference level SIL_0 =50 dB to higher sound intensity level SIL = 90 dB.

Demonstration
Three students in class applaud the instructor. We measure $SIL_0 = 60$ dB.
Now 30 students applaud, each equally loud as the three students before.
Question: What is the SIL, and what is the increase in SIL?
Answer:
$SIL = SIL_0 + 10\log(I/I_0) = 60 + 10\log(30/3) = 60 + 10\log 10 = 60 + 10 = 70$ dB
The increase in SIL is 10 dB, from 60 dB to 70 dB.

Exercise
If a sound source such as a fan produces an $SIL_0 = 50$ dB, how many identical fans will produce an SIL = 70 dB?
Answer: Use $SIL = SIL_0 + 10\log(I/I_0)$. Then $70 = 50 + 10\log(I/I_0)$, or $\log(I/I_0) = 20$, or $I/I_0 = 100$. This means 100 fans.

Demonstration - Logarithmic Response of the Ear to Intensity Changes
Increase the SIL of a sound in equal steps, e.g. 60→70→80→90 dB. For each step, the perceived increase in loudness sounds similar. For such a 10 dB increase, many listeners sense this as "twice as loud". (P.S.: Others may already sense a 6 dB increase as "twice as loud".)

Fechner's Law
An empirical observation says that our senses respond logarithmically to the intensity of an external stimulus. This is ***Fechner's law***.

Exercise: Give some examples where our other senses respond logarithmically.

Change in Sound Intensity Level

We saw earlier that the sound intensity level (SIL) for a sound *intensity I* is defined as

$$\mathbf{SIL = SIL_0 + 10\ log(I/I_0)}$$

SIL is the sound intensity level in dB and *I* the corresponding sound intensity *I* in W/m^2.
SIL_0 is the sound intensity level of a reference intensity I_0.

Change in Sound Intensity Level
Let the sound intensity change from I_0 to I.
Question: What is the change ΔSIL in the sound intensity level?
Answer: The change is

$$\mathbf{\Delta SIL\ = SIL - SIL_0 = 10\ log(I/I_0)}$$

Example
You increase the sound intensity from a loudspeaker from I_0 to an intensity 5-times greater, i.e. $I = 5I_0$.
Question: What is the corresponding change in the sound level intensity (in dB)?
Answer: $\Delta SIL = 10\log(I/I_0) = 10\log(5I_0/I_0) = 10\log 5 = 10 \cdot 0.699 \approx 7$ dB.

On the other hand, if we are given the change ΔSIL, we invert this equation and obtain for the intensity ratio

$$\mathbf{I/I_0 = 10^{\Delta SIL/10}}$$

Example
When 3 students applaud the instructor in class, we measure $SIL_0 = 85$ dB.
When 24 students applaud, we measure SIL = 94 dB.
Question: How many times has the sound intensity changed?
Answer: ΔSIL = 94 dB – 85 dB = 9 dB. Thus $I/I_0 = 10^{9/10} = 10^{0.9} = 7.9$
The intensity has increased approximately 8-fold.
Many people perceive such an increase as about "twice as loud".

Challenge Question
Why did the sound intensity not quite increase 10-fold, as was to be expected, when 30 instead of 3 students where applauding?
Answer: ____

A Challenging Example: Absolute Sound Intensity I
Question: What is the *absolute* sound intensity (in W/m^2) when 3 students applaud with a sound intensity level SIL = 85 dB?
Answer: We must use a reference level such as the threshold of hearing, i.e. use $SIL_0 = 0$ dB.
Then $SIL - SIL_0 = SIL - 0 = 10\ \log(I/I_0) = 85$ dB or $I/I_0 = 8.5$ Bel $= 10^{8.5} = 3.16 \cdot 10^8$
At the threshold of hearing $I_0 = 10^{-12}$ W/m^2.
Therefore $I = 10^{-12}$ W/m$^2 \cdot 3.16 \cdot 10^8 = 3.1 \cdot 10^{-4}$ W/m^2 = 0.000316 W/m^2 = 0.316 mW/m^2.

Sound Intensity *I*, Sound Intensity Level *SIL*, Fletcher-Munson Curves

Questions
1. What is the combined SIL of two tones, each having an SIL of 50 dB?
Answer:
The intensity of both tones played together is twice the intensity of each: $I = 2I_0$
Hence $SIL = SIL_0 + 10\log(2I_0/I_0) = 50 + 10\log2 = 50 + 3 = 53$ dB.

2. Similar to above, but take three tones with 50 dB each.
Answer: $SIL = 50 + \log(3I_0/I_0) = 50 + 10\log3 = 50 + 5 = 55$ dB.

3. **Challenge Question:**
Find the combined SIL for two tones, one with $SIL_1 = 50$ db, the other with $SIL_2 = 60$ db.
Answer: $(I_1 + I_2)/I_1 = (10^5 + 10^6)/10^5 = 11$.
Then $SIL = 50 + 10\log11 = 50 + 10.4 = 60.4$ dB.

Demonstrations
1. Set two sine generators to 500 Hz and the same SIL for each. (This means equal loudness as the frequency is the same.) First play one tone, then both tones together. The measured increase in the SIL should be 3 dB.
Question
How much louder subjectively do the two tones sound together?
They will not sound "twice as loud". This is consistent with the logarithmic response of the ear to sound intensity.

2. Play some sound tracks from the CD "Auditory Demonstrations" by the Acoustical Society of America, for instance the track "Decibel Scale and Intensity".

Fletcher-Munson Curves (Equal Loudness Curves) Revisited
Understand the Fletcher-Munson curves and answer the corresponding questions in the homework!

Example:
From the equal loudness curve of 40 phon, what is the SIL at f = 100 Hz?
Answer: SIL = 60 dB.

Environmental Sound Intensity Levels

Table. Approximate sound intensity levels encountered in various environments, corresponding sound intensities, and human reaction. These are typical values; individual cases might give readings 10 dB higher or lower.

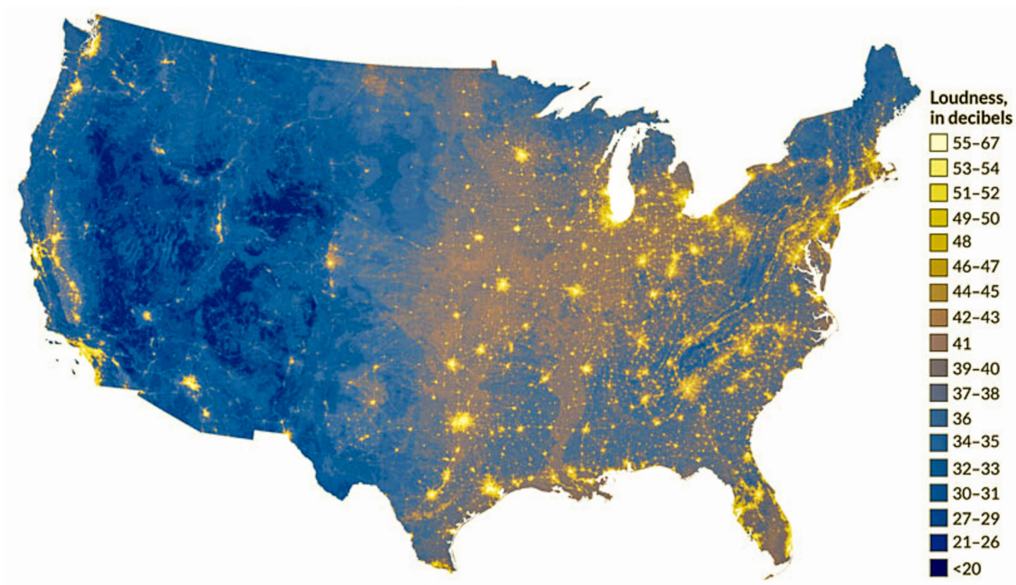
Sound Source	Sound Level (dB)	Intensity (W/m^2)	Human Reaction
Jet Engine at 10 m	150	10^3	Serious damage
	140		
	130		
SST takeoff at 500 m	120	1	Very painful
Amplified Rock Music	110		
Machine Shop	100		
Subway Train	90	10^{-3}	
Factory	80		
City Traffic	70		
Quiet Conversation	60	10^{-6}	Musically Useful
Quiet Auto Interior	50		
Library	40		
Empty Auditorium	30	10^{-9}	
Whisper at 1 m	20		
Falling pin	10		
	0	10^{-12}	Inaudible

Table. United States limitations on permissible daily occupational noise exposure and the more conservative limits suggested by OSHA for avoidable, non-occupational exposure. Some countries allow only a 3 dB level increase for each halving of the exposure time.

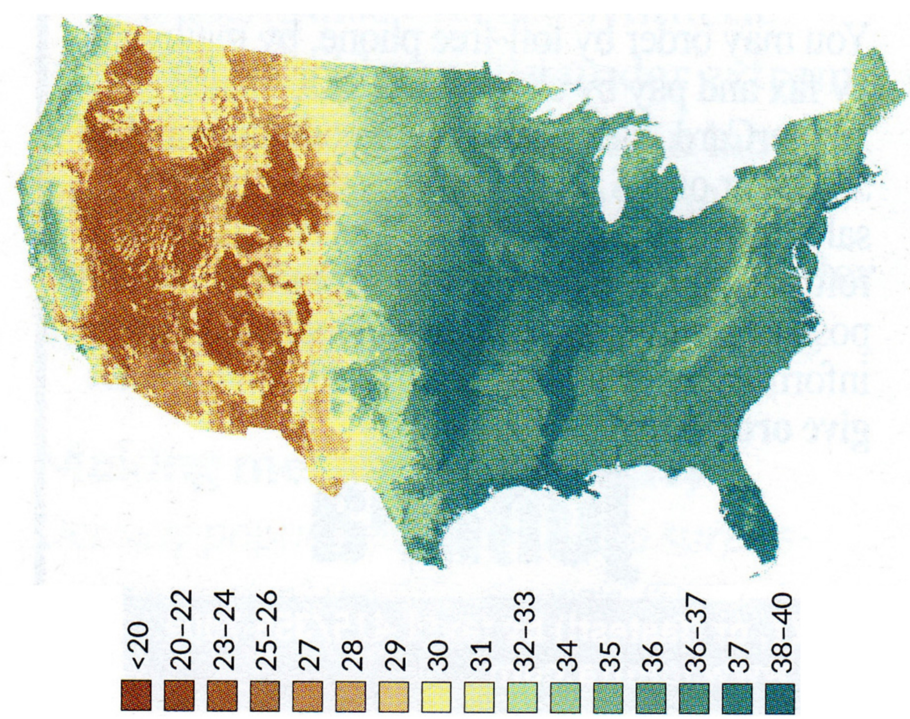
	Maximum 24 Hour Exposure	
Sound Level (dBA)	Occupational	Non-occupational
80		4 hr
85		2 hr
90	8 hr	1 hr
95	4 hr	30 min
100	2 hr	15 min
105	1 hr	8 min
110	30 min	4 min
115	15 min	2 min
120	0 min	0 min

P.S.: Collapse of Atmospheric Pressure
Suppose the entire atmospheric pressure of 100,000 Pa were to collapse suddenly to a vacuum. What would the dB-level of the resulting pressure wave be? As 120 dB corresponds to a pressure amplitude of 20 Pa, an amplitude of 100,000 Pa would result in an SIL = 194 dB (Exercise)!

Sound Map USA



Loudness levels of a typical summer day, including people and machinery.
The decibel levels apply to loudness that is exceeded half the time at given spots.



Loudness levels of the natural environment on a summer day, excluding people.
(Reference: Science News, February 21, 2015, p. 32.)
Machinery and other man-made noises are also exluded from the lower map.

Sound Intensity Level and Human Annoyance Curve

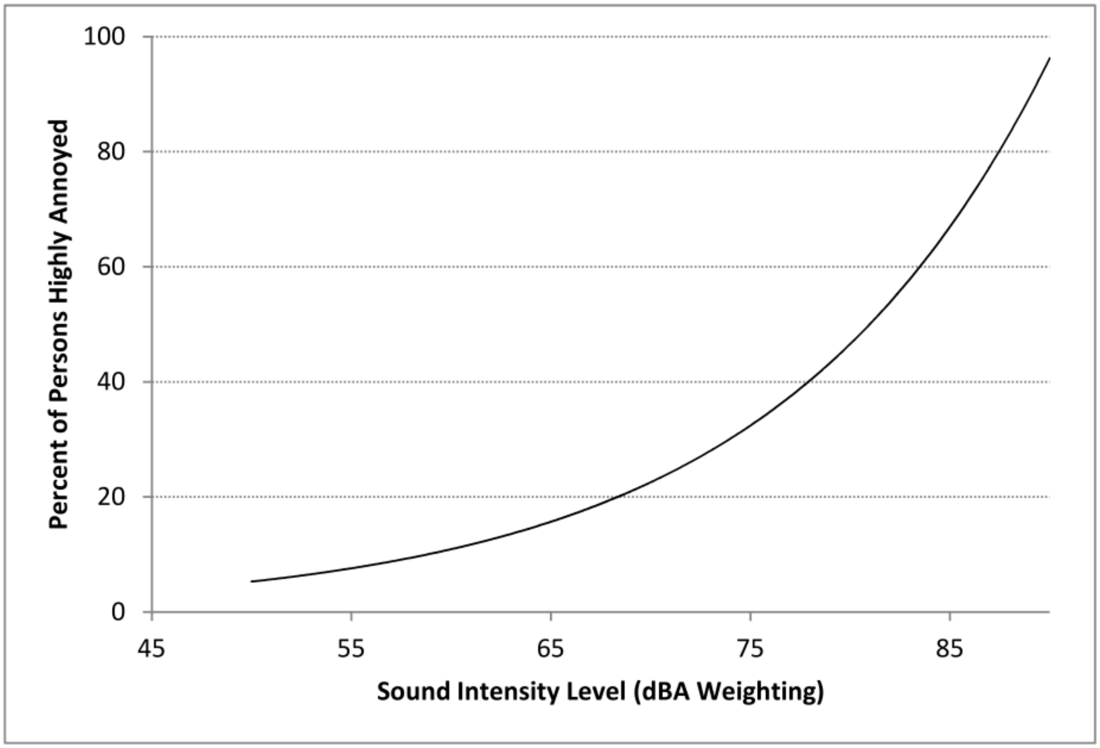


Figure: The degree of human annoyance as a function of average environmental noise intensity level. The graph is an average of the results of numerous surveys, as reported by T.J. Schultz, JASA, 64, 377 (1978). Nighttime levels as low as 55 dB can cause serious disturbance of sleep.

Just Noticeable Difference (JND) in Sound Intensity Level

Let us consider a change in sound intensity of a sine tone and a more complex tone.
The question is:
How much does the intensity have to change to become "just noticeable" to us?
This difference is called the just noticeable intensity difference or **intensity JND**.
Tests show that the difference lies in the range $\Delta SIL = 0.5$ to 1.5 dB.

Demonstrations
1. Play a 500 Hz sine tone through a loudspeaker. Measure the sound intensity level SIL with a sound level meter. Slowly increase the intensity and stop when you hear a change. (The same can be done with a keyboard set to the "sine wave" mode.)
Note the just noticeable increase ΔSIL:
Answer: ΔSIL = _______dB

2. Set the piano keyboard to "sine wave". Keep tapping a key near 500 Hz while increasing the SIL. Note the just noticeable difference ΔSIL:
Answer: ΔSIL = _______dB

Question
From what you have found from the two demonstrations above, is the ear more sensitive to abrupt or gradual changes in intensity?
Answer: ____

3. Set the piano keyboard to "grand piano". Then tap again as in the preceding demonstration. What is the just noticeable difference this time?
Answer: ΔSIL = _______dB

Question
Is the ear more sensitive to intensity changes in pure tones (sine waves) or complex tones containing several overtones?
Answer: ____

Question
Assume that the just noticeable difference in sound intensity level is 1 dB.
Find the percentage change in the sound intensity *I* (not in sound intensity level *SIL*) that corresponds to this $\Delta SIL = 1$ dB.
Answer: $\Delta SIL = 10\log(I/I_0) = 1$ dB $= 0.1$ Bel, or $I/I_0 = 10^{0.1} = 1.26$
The sound intensity in this case has to change by the rather large amount of 26% to become "just noticeable".

Demonstration
Use a sound level meter and record the SIL of several students applauding. How many more students have to applaud for you to hear a noticeable change in intensity?
Data and Answer

Initial number of students applauding	____________
Number of students applauding for a "just noticeable difference"	____________
Percentage change in the number of students applauding	____________%
Decibel change in the relative number of students	____________dB
Measured change in the SIL	____________dB
Compare the two dB-values. Are they similar, as they should be?	____________

Comparison between Intensity JND and Frequency JND

Intensity JND:	0.5dB – 1.5 dB or 12% - 41% change	(Exercise)
Frequency JND:	0.5% – 3% or change of 0.022 dB – 0.13 dB	(Exercise)

Exercise
Verify the above percentage ranges in the intensity JND from the dB-ranges given.
Exercise
Verify the above dB-ranges in the frequency JND ranges given.

Comment on the Sensitivity of the Ear to Frequency and Intensity Changes
From the numbers obtained we see that the ear is about 10-times more sensitive to frequency changes than to intensity changes. The ear can determine much more acutely whether two tones are out of tune compared to when they differ in intensity.

Dissonance and Consonance

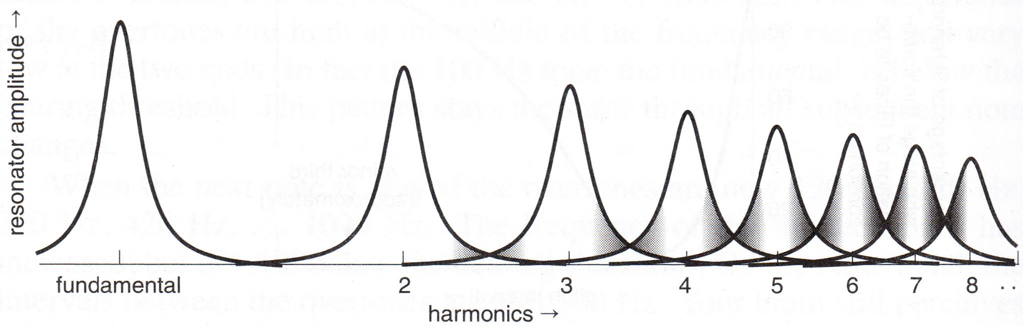


Figure. The frequency spectrum of a single complex tone with harmonics displayed along the basilar membrane on a logarithmic scale. Increasing overlap of the critical bands occurs for the higher harmonics, but the sound is still consonant.

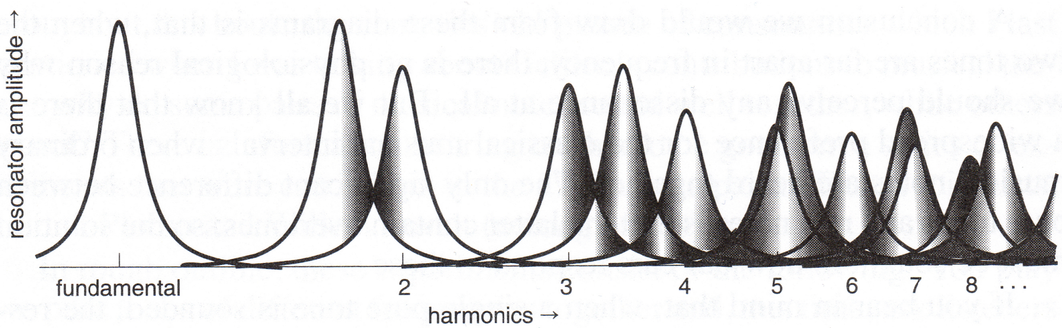


Figure. Two randomly chosen tones separated by more than a minor third and played together. Significant overlap occurs for the higher harmonics. The sound is dissonant and rough.

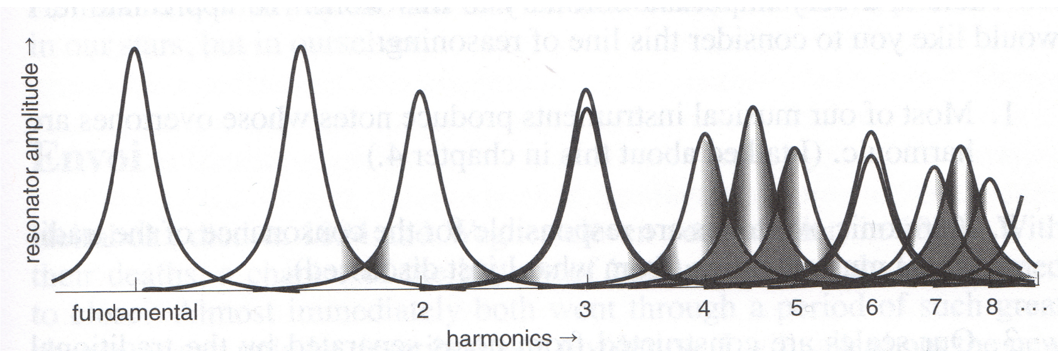


Figure. Two complex tones a perfect fifth (ratio 3/2) apart. Every 2^{nd} peak of the higher tone coincides with every 3^{rd} peak of the lower tone. The total overlap is reduced and the sound is rather consonant, but less consonant than for the harmonics of the single complex tone above. (From: "Measured Tones" by Ian Johnston, 3^{rd} edition, CRC Press, 2009, pp. 248, 249.)

Challenge question: For which musical interval (i.e. pair of notes) do you expect to hear the least roughness in the sound?

Answer: ____

A Dissonance Curve

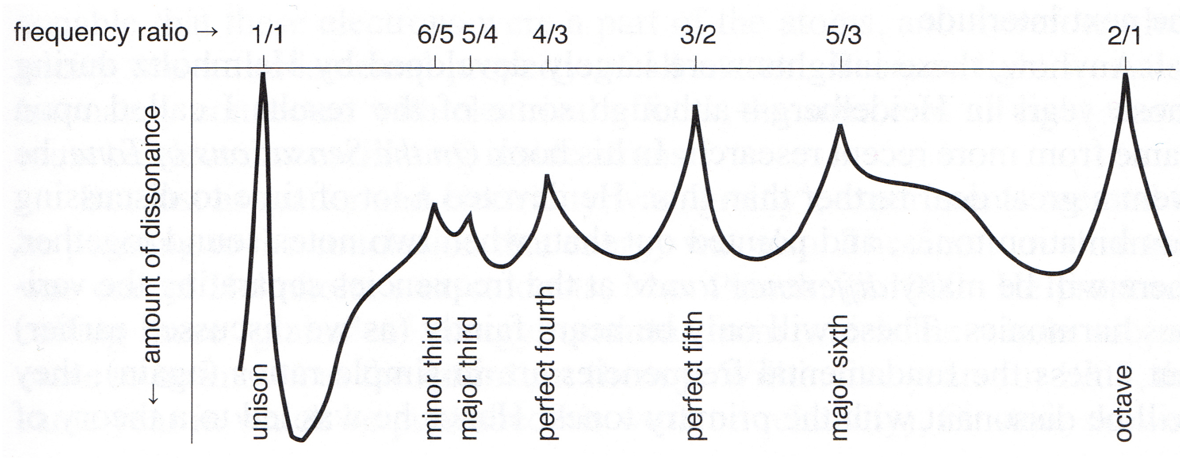


Figure. A dissonance curve calculated by Plomp and Levelt from the overlap of the critical bands of pairs of notes. The historic musical intervals are relatively free of dissonance, especially fourth, fifth, major sixth, and octave. A half step and full step immediately to the right of the unison show the largest dip and thus the greatest dissonance.
(From: Ian Johnston, "Measured Tones", 3rd edition, DRC Press, 2009, p. 249.)

The overlap of critical bands on the basilar membrane of the ear thus may explain the perception of dissonance. Conversely, we may say that

Consonance = Absence of dissonance

Some Related Comments
Consonance is highest near the just musical intervals.
Musical scales are constructed from traditional intervals.
Harmonic overtones are responsible for the consonance of traditional musical intervals.
Many musical instruments produce notes whose overtones are harmonic.

Our scales are what they are because of the kind of instruments we play.
Is this a musical anthropic principle?

Compare with **Cosmological Anthropic Principle** (Carter):
The Universe and the fundamental parameters on which it depends must be such as to admit the creation of observers…. (Descartes: *Cogito ergo mundus talis est*.)

Listening Interludes - When is Sound "Musical"?

For a sound to be perceived as "musical", several conditions must be met:
a) The sound must be of sufficient duration, e.g. several periods long (~10T).
b) The spectrum must show distinct frequency spikes.
c) The spikes have narrow widths, i.e. they are "sharp" and not resembling "pink noise".
d) Play a minor third such as G2 – B2b with sine waves and with piano sound. The interval sounds more musical with piano sound, because of narrower critical bands of the overtones and more sensitivity of the ear to higher frequencies than to the low-frequency fundamental.

Demonstrations
1. Play a percussion instrument, for instance a djembe (drum) or cymbal. Observe the Fourier spectrum. Play the instrument to sound "musical". Then play it to sound more "percussive". Compare the frequency spectra.
Question: What are the common and different features in the two spectra?
Answer: ____

2. Play a wind instrument, for instance a recorder or whistle.
Compare the frequency spectrum with the percussion instrument. Describe the pronounced differences in the spectra.
Answer: ____

3. Make some vocal sounds and observe the frequency spectra.
a) Vowel sounds
b) Buzzing sounds
c) Sibilant or hissing sounds
What changes do you see in the spectra when changing from "musical" to "non-musical" sound?
Answer: ____

4. Play Sabian and Wuhan cymbals.
Display the rich Fourier spectra. Many individual densely packed spikes can be seen over almost the entire audible frequency range.
Question: Do cymbals sound musical? Which of the two cymbals sounds more "musical". Why?
Answer: ____

5. Play a snare drum.
Does it sound "noisier" than the Sabian Cymbals? Explain by inspecting the frequency spectra.

Sound Tracks of Auditory Effects
Listen to some CD tracks from "Auditory Demonstrations" by the American Acoustical Society:
a) Decibel scale and intensity
b) Cancelled harmonics
c) Filtered noise
d) Logarithmic and linear frequency scales
e) Effect of spectrum on timbre