Why should I learn this stuff?

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General remarks: Since very few of you will go on to careers that will require you to use classical mechanics as part of your job, one is justified in asking why you should spend a whole semester learning graduate-level classical mechanics, when you could be learning other "more practical" things. One answer to this question (which applies as well to other core courses in physics) is that the problem-solving skills and mathematical techniques that you will learn in this course are applicable to *other* areas of physics and science. Having extra "tools in your toolbox" is often very helpful when it comes to tackling unsolved research problems. One never knows when a certain technique or approach will lead to a solution.

Another answer is that, although the number may be small, some of you *will* actually end up working in fields where certain aspects of classical mechanics will be required or at least be very helpful. For example, a theoretical physicist studying studying classical or quantum field theory (particle physics, relativity, etc.) will often write down a Lagrangian as the first step in defining or solving a problem. Knowing the associated classical Hamiltonian and Poisson bracket structure then allows one to transition to quantum theory with its Hamiltonian operator and commutators.

Many mechanical, civil, and aerospace engineers will also use classical mechanics in their work, as an analysis of the forces and torques acting on a system determines whether or not a bridge will collapse or a rocket will leave the launch pad, etc. Astrophysicists make use of concepts from central force motion when studying the gravitational interaction between binary star systems, galaxy rotation curves, etc.

And, finally, don't forget the numerous applications of classical mechanics in your everyday lifes. An apple falling from a tree, the orbit of the Moon, a spinning ice skater, the trajectory of football pass, the impact of a tennis racket with a tennis ball, . . ., are all described by the laws of classical mechanics. Having a solid understanding of classical mechanics will therefore give you a better understanding and appreciation of much of the world around you.

1 Lagrangian mechanics (§1-5)

Lagrangian mechanics is an alternative formulation of Newton's laws applied to a system of interacting particles. It is formulated in terms of scalar quantities (kinetic and potential energies) as opposed to vectors (accelerations, forces, torques), and as such is mathematically easier to work with. Constraint (reaction) forces can be ignored in the Lagrangian formalism by working with generalized coordinates that specify the true degrees of freedom of the system. If desired, constraint forces can be included in the Lagrangian formalism by using the method of Lagrange multipliers.

Conservation of energy, momentum, and angular momentum are simply described in the Lagrangian formalism, being related to symmetries of the Lagrangian with respect to time translations, space translations, and rotations, respectively.

2 Conservation laws (§6-10)

Conserved quantities are useful since they don't change during the motion of a system of particles. They reduce the number of equations of motion that we need to integrate. For example, using conservation of energy and angular momentum for central force problems allows to us to solve *first-order* differential equations for r and ϕ, instead of more complicated second-order equations.

The motion of the center of mass (COM) of a system of particles can be thought of as the motion of the system *as as whole*. For a closed system, the COM moves with constant velocity, which can be set to zero by working in the COM frame. This simplifies the description of the motion of the individual particles.

3 Hamiltonian mechanics (§40)

Hamiltonian mechanics is a reformulation of Lagrangian mechanics which takes the generalized coordinates q_i and generalized momenta $p_i \equiv \partial L/\partial q_i$ as the fundamental variables. Hamilton's equations are first-order differential equations for both the p's and q's, which are often simpler to solve than the corresponding second-order Lagrange's equations for the q's.

The true value of the Hamiltonian formulation arises when one considers transformations that mixes the q's and the p's. Such transformations, which preserve Hamilton's equations, are called *canonical transformations*, and by judicious choice of these transformations, the system of equations is easier to solve. In the Hamiltonian formulation, conserved quantities turn out to be generators of symmetry transformations of the Hamiltonian. Unfortunately, we will not have time this semester to cover these latter items. They are discussed in Chapter VII, §42, 45, 47.

4 Central force motion (§11, 13-15)

Central force motion arises whenever you have two bodies that interact via a potential that depends only on the distance between the two bodies. It describes, for example, the motion of the planets around the Sun, the orbits of communication satellites around the Earth, etc. Kepler's laws of planetary motion are all derivable using the formalism of central force motion for the potential $U(r) = -\alpha/r$, with $\alpha > 0$.

5 Collisions and scattering (§16-20)

Using only conservation of momentum and energy for elastic collisions or disintegrations of particles, we can determine a lot about the end state of a collision without knowing the explicit form of the interaction that causes it. Additionally knowing the scattering potential allows us to fully determine the scattering angles and differential cross section for the incident and target particles, as a function of the incident energy and impact parameter of the incoming particle.

Much of experimental high-energy physics involves the scattering of high-energy subatomic particles (e.g., think of the Large Hadron Collider). Gravitational "sling-shot" trajectories, which are used to get satellites to distant planets without the use of additional fuel, are a combination of inverse-square-law-force bound orbits and scattering of one mass off of another.

6 Small oscillations (§21-23)

Simple harmonic motion (like that for a mass attached to a spring, $F = -kx$) occurs whenever one has small oscillations about a position of stable equilibrium. This occurs in many different circumstances, as it only requires that the system of interacting particles be slightly perturbed away from a stable equilibrium configuration. Mathematically, such oscillations occur because the potential is well-approximated by a quadratic function in the neighborhood of a position of stable equilibrium.

7 Rigid body motion (§31-36, 38)

To describe the complicated translational and rotational motion of a football (or any extended object) as it moves through space, we need to go beyond the particle approximation, which was used in the previous sections. The angular velocity, angular momentum, and any external torques that may act on the body are the key quantities that enter such an analysis. Spinning tops, gyroscopes, and the precession of the Earth's axis are examples that fall within the realm of rigid-body motion.

8 Non-inertial reference frames (§39)

Newton's law $\mathbf{F} = m\mathbf{a}$ and the Lagrangian $L = T - U$ are valid only in an inertial frame of reference. But most reference frames, like one attached to the surface of the Earth, are non-inertial (due to translational acceleration or rotational motion). In a non-inertial frame, Newton's law acquires additional *fictitious* force terms associated with the acceleration of the reference frame. We must include these fictitious force terms (e.g., the Coriolis force) to properly describe projectile motion on the surface of the Earth. Firing artillery shells on a battle field without taking into account the deflection caused by the Coriolis force would lead you to miss the target. In addition, the precession of the plane of oscillation of a pendulum on the surface of the Earth is a direct demonstration of Earth's rotational motion (Foucault's pendulum).