In[1]:=
```
(* pure rotation *)

(* detector locations *)
x1 = {0, 0, 0};
x2 = {0, 0, 0};

(* unit vectors along the IFO arms *)
(* both arms in rotational plane *)
(*
u1 = {1, 0, 0};
v1 = {0,1,0};
u2={ Cos[δ],Sin[δ], 0};
v2={ -Sin[δ],Cos[δ], 0};
*)

(* one arm in rotational plane *)
u1 = {0, 1, 0};
v1 = {0, 0, 1};
u2 = { -Sin[δ], Cos[δ], 0};
v2 = { 0, 0, 1};

(* detector tensors *)
d1 = (Outer[Times, u1, u1] - Outer[Times, v1, v1]) / 2;
d2 = (Outer[Times, u2, u2] - Outer[Times, v2, v2]) / 2;

(* separation vector *)
(*s = FullSimplify[(x1-x2)/Norm[x1 - x2],{0≤δ≤2Pi}]; *)
s = {0, 0, 0};
d = Norm[x1 - x2];

(* overlap expression *)
FullSimplify[Tr[d1], {0 ≤ δ ≤ 2 Pi}]
FullSimplify[Tr[d2], {0 ≤ δ ≤ 2 Pi}]
FullSimplify[Tr[d1.d2], {0 ≤ δ ≤ 2 Pi}]
FullSimplify[s.d1.s, {0 ≤ δ ≤ 2 Pi}]
FullSimplify[s.d2.s, {0 ≤ δ ≤ 2 Pi}]
FullSimplify[s.(d1.d2).s, {0 ≤ δ ≤ 2 Pi}]
FullSimplify[(s.d1.s) (s.d2.s), {0 ≤ δ ≤ 2 Pi}]
```

$$M = \frac{1}{2\,\alpha^2}\begin{pmatrix} -5\,\alpha^2 & 10\,\alpha & 5 \\ 5\,\alpha^2 & -10\,\alpha & 5 \\ 5\,\alpha^2 & -10\,\alpha & -25 \\ -5\,\alpha^2 & 20\,\alpha & -25 \\ 5\,\alpha^2 & -50\,\alpha & 175 \end{pmatrix}.$$

```
    {SphericalBesselJ[0, α], SphericalBesselJ[1, α], SphericalBesselJ[2, α]};
γ1 = Simplify[Simplify[M[[1]] Tr[d1] Tr[d2] + 2 M[[2]] Tr[d1.d2] +
     M[[3]] (Tr[d2] s.d1.s + Tr[d1] s.d2.s) +
     4 M[[4]] s.(d1.d2).s + M[[5]] (s.d1.s) (s.d2.s)]];

(* make plot *)
γ1 = Limit[γ1, α → 0]
vars1 = {δ → ω t, ω → 2 Pi / 24};
```

```
p1 = Plot[{γ1 //. vars1}, {t, 0, 24}, PlotStyle → {Black, Thick},
  BaseStyle → {FontSize → 16}, Frame → True, FrameLabel → {"time (hrs)"},
  PlotRange → {{0, 24}, {-1, 1}}, GridLines → Automatic]
Export["pure_rotation.eps", p1];
```

Out[11]= 0

Out[12]= 0

Out[13]= $\frac{1}{8}(3+\text{Cos}[2\,\delta])$

Out[14]= 0

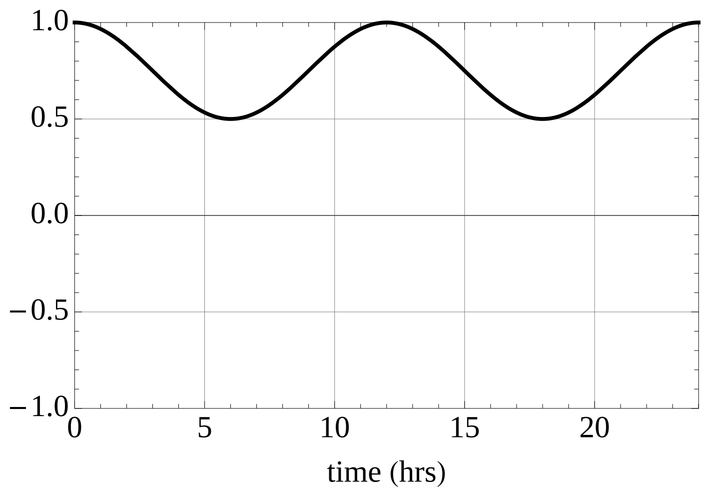
Out[15]= 0

Out[16]= 0

Out[17]= 0

Out[20]= $\frac{1}{4}(3+\text{Cos}[2\,\delta])$

Out[22]=



In[24]:=

```
(* pure translation - orbital motion *)

(* detector locations *)
x1 = {R, 0, 0};
x2 = {R Cos[δ], R Sin[δ], 0};

(* unit vectors along the IFO arms *)
u1 = {0, 1, 0};
v1 = {0, 0, 1};
u2 = { 0, 1, 0};
v2 = { 0, 0, 1};

(* detector tensors *)
d1 = (Outer[Times, u1, u1] - Outer[Times, v1, v1]) / 2;
d2 = (Outer[Times, u2, u2] - Outer[Times, v2, v2]) / 2;

(* separation vector *)
s = FullSimplify[(x1 - x2) / Norm[x1 - x2], {0 ≤ δ ≤ 2 Pi, R > 0}];
d = FullSimplify[ Norm[x1 - x2], {0 ≤ δ ≤ 2 Pi, R > 0}]

(* overlap expression *)
FullSimplify[Tr[d1], {0 ≤ δ ≤ 2 Pi, R > 0}]
FullSimplify[Tr[d2], {0 ≤ δ ≤ 2 Pi, R > 0}]
FullSimplify[Tr[d1.d2], {0 ≤ δ ≤ 2 Pi, R > 0}]
FullSimplify[s.d1.s, {0 ≤ δ ≤ 2 Pi, R > 0}]
FullSimplify[s.d2.s, {0 ≤ δ ≤ 2 Pi, R > 0}]
FullSimplify[s.(d1.d2).s, {0 ≤ δ ≤ 2 Pi, R > 0}]
FullSimplify[(s.d1.s) (s.d2.s), {0 ≤ δ ≤ 2 Pi, R > 0}]
```

$$M = \frac{1}{2\,\alpha^2}\begin{pmatrix} -5\,\alpha^2 & 10\,\alpha & 5 \\ 5\,\alpha^2 & -10\,\alpha & 5 \\ 5\,\alpha^2 & -10\,\alpha & -25 \\ -5\,\alpha^2 & 20\,\alpha & -25 \\ 5\,\alpha^2 & -50\,\alpha & 175 \end{pmatrix}.$$

```
    {SphericalBesselJ[0, α], SphericalBesselJ[1, α], SphericalBesselJ[2, α]};
γ2 = Simplify[Simplify[M[[1]] Tr[d1] Tr[d2] + 2 M[[2]] Tr[d1.d2] + M[[3]]
      (Tr[d2] s.d1.s + Tr[d1] s.d2.s) + 4 M[[4]] s.(d1.d2).s + M[[5]] (s.d1.s) (s.d2.s)]]

(* make plot *)
vars2 =
  {R → 1.5 × 10^11, c → 3 × 10^8, f → 100, α → 2 π f d / c, δ → ω t, ω → 2 Pi / (24 × 365)};
p2 = Plot[{γ2 //. vars2}, {t, 0, 1}, PlotStyle → {Black, Thick},
  BaseStyle → {FontSize → 16}, Frame → True, FrameLabel → {"time (hr)"},
  PlotRange → {{0, 1}, {-0.2, 1}}, GridLines → Automatic]
Export["pure_orbit.eps", p2];
```

Out[33]= $2\,R\,\mathrm{Sin}\left[\frac{\delta}{2}\right]$

Out[34]= 0

Out[35]= 0

Out[36]= $\frac{1}{2}$

Out[37]= $\frac{1}{4}\,(1+\text{Cos}[\delta])$

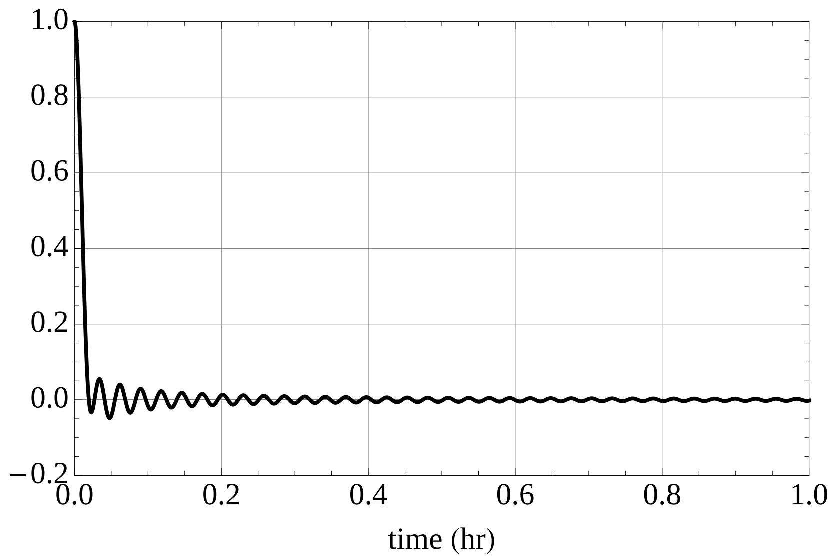
Out[38]= $\frac{1}{4}\,(1+\text{Cos}[\delta])$

Out[39]= $\frac{1}{8}\,(1+\text{Cos}[\delta])$

Out[40]= $\frac{1}{4}\,\text{Cos}\left[\frac{\delta}{2}\right]^4$

Out[42]=
$$\frac{1}{64\,\alpha^2}5\,\Big(2\,\alpha^2\,(-3+\text{Cos}[\delta])^2\,\text{SphericalBesselJ}[0,\,\alpha]-2\,\alpha\,(15-12\,\text{Cos}[\delta]+5\,\text{Cos}[2\,\delta])\,\text{SphericalBesselJ}[1,\,\alpha]+(57+60\,\text{Cos}[\delta]+35\,\text{Cos}[2\,\delta])\,\text{SphericalBesselJ}[2,\,\alpha]\Big)$$

Out[44]=



In[46]:=
```
(* rotation of IFOs on surface of earth; no orbital motion *)

(* detector locations *)
x1 = {RE, 0, 0};
x2 = {RE Cos[δ], RE Sin[δ], 0};

(* unit vectors along the IFO arms *)
u1 = {0, 1, 0};
v1 = {0, 0, 1};
u2 = { -Sin[δ], Cos[δ], 0};
v2 = { 0, 0, 1};

(* detector tensors *)
d1 = (Outer[Times, u1, u1] - Outer[Times, v1, v1]) / 2;
d2 = (Outer[Times, u2, u2] - Outer[Times, v2, v2]) / 2;

(* separation vector *)
s = FullSimplify[(x1 - x2) / Norm[x1 - x2], {0 ≤ δ ≤ 2 Pi, RE > 0}];
d = FullSimplify[ Norm[x1 - x2], {0 ≤ δ ≤ 2 Pi, RE > 0}]

(* overlap expression *)
FullSimplify[Tr[d1], {0 ≤ δ ≤ 2 Pi, RE > 0}]
FullSimplify[Tr[d2], {0 ≤ δ ≤ 2 Pi, RE > 0}]
FullSimplify[Tr[d1.d2], {0 ≤ δ ≤ 2 Pi, RE > 0}]
FullSimplify[s.d1.s, {0 ≤ δ ≤ 2 Pi, RE > 0}]
FullSimplify[s.d2.s, {0 ≤ δ ≤ 2 Pi, RE > 0}]
FullSimplify[s.(d1.d2).s, {0 ≤ δ ≤ 2 Pi, RE > 0}]
FullSimplify[(s.d1.s) (s.d2.s), {0 ≤ δ ≤ 2 Pi, RE > 0}]
```

$$M = \frac{1}{2\alpha^2}\begin{pmatrix} -5\alpha^2 & 10\alpha & 5 \\ 5\alpha^2 & -10\alpha & 5 \\ 5\alpha^2 & -10\alpha & -25 \\ -5\alpha^2 & 20\alpha & -25 \\ 5\alpha^2 & -50\alpha & 175 \end{pmatrix}.$$

```
    {SphericalBesselJ[0, α], SphericalBesselJ[1, α], SphericalBesselJ[2, α]};

γ3 = Simplify[Simplify[
   M[[1]] Tr[d1] Tr[d2] + 2 M[[2]] Tr[d1.d2] + M[[3]] (Tr[d2] s.d1.s + Tr[d1] s.d2.s) +
    4 M[[4]] s.(d1.d2).s + M[[5]] (s.d1.s) (s.d2.s)]]

(* make plot *)
vars3 = {RE → 6371 × 10^3, c → 3 × 10^8, f → 100, α → 2 π f d / c , δ → ω t, ω → 2 Pi / 24};
p3 = Plot[{γ3 //. vars3}, {t, 0, 24}, PlotStyle → {Black, Thick},
  BaseStyle → {FontSize → 16}, Frame → True, FrameLabel → {"time (hr)"},
  PlotRange → {{0, 24}, {-0.2, 1}}, GridLines → Automatic]
Export["earth_rotation.eps", p3];
```

Out[55]= $2\,\mathrm{RE}\,\mathrm{Sin}\left[\frac{\delta}{2}\right]$

Out[56]= 0

Out[57]= 0

Out[58]= $\frac{1}{8}\,(3+\mathrm{Cos}[2\,\delta])$

Out[59]= $\frac{1}{4}\,(1+\mathrm{Cos}[\delta])$

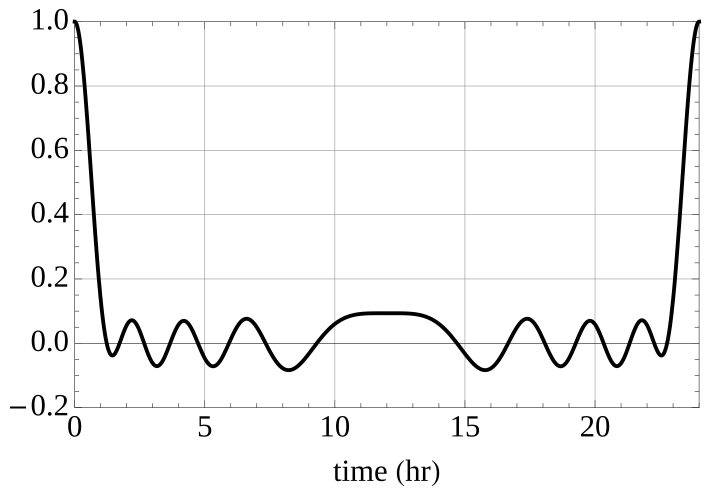
Out[60]= $\frac{1}{4}\,(1+\mathrm{Cos}[\delta])$

Out[61]= $\frac{1}{4}\,\mathrm{Cos}\left[\frac{\delta}{2}\right]^2\,\mathrm{Cos}[\delta]$

Out[62]= $\frac{1}{4}\,\mathrm{Cos}\left[\frac{\delta}{2}\right]^4$

Out[64]=
$$\frac{1}{64\,\alpha^2}5\,\Big(2\,\alpha^2\,(-3+\mathrm{Cos}[\delta])^2\,\mathrm{SphericalBesselJ}[0,\,\alpha]+2\,\alpha\,(-23+12\,\mathrm{Cos}[\delta]+3\,\mathrm{Cos}[2\,\delta])\,\mathrm{SphericalBesselJ}[1,\,\alpha]+(89+60\,\mathrm{Cos}[\delta]+3\,\mathrm{Cos}[2\,\delta])\,\mathrm{SphericalBesselJ}[2,\,\alpha]\Big)$$

Out[66]=



In[68]:=
```
(* rotation of IFOs on surface of earth plus orbital motion *)

(* position vector of center of earth *)
x01 = {R , 0, 0};
x02 = {R Cos[β], R Sin[β], 0};

(* detector locations *)
x1 = x01 + {RE, 0, 0};
x2 = x02 + {RE Cos[δ], RE Sin[δ], 0};

(* unit vectors along the IFO arms *)
u1 = {0, 1, 0};
v1 = {0, 0, 1};
u2 = { -Sin[δ], Cos[δ], 0};
v2 = { 0, 0, 1};

(* detector tensors *)
d1 = (Outer[Times, u1, u1] - Outer[Times, v1, v1]) / 2;
d2 = (Outer[Times, u2, u2] - Outer[Times, v2, v2]) / 2;

(* separation vector *)
s = FullSimplify[(x1 - x2) / Norm[x1 - x2], {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}];
d = FullSimplify[ Norm[x1 - x2], {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]

(* overlap expression *)
FullSimplify[Tr[d1], {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]
FullSimplify[Tr[d2], {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]
FullSimplify[Tr[d1.d2], {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]
FullSimplify[s.d1.s, {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]
FullSimplify[s.d2.s, {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]
FullSimplify[s.(d1.d2).s, {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]
FullSimplify[(s.d1.s) (s.d2.s), {0 ≤ β ≤ 2 Pi, 0 ≤ δ ≤ 2 Pi, R > 0, RE > 0}]


M = 1/(2 α^2) ( -5 α^2   10 α     5
                 5 α^2  -10 α     5
                 5 α^2  -10 α   -25
                -5 α^2   20 α   -25
                 5 α^2  -50 α   175 ).
   {SphericalBesselJ[0, α], SphericalBesselJ[1, α], SphericalBesselJ[2, α]};

γ4 = Simplify[Simplify[
    M[[1]] Tr[d1] Tr[d2] + 2 M[[2]] Tr[d1.d2] + M[[3]] (Tr[d2] s.d1.s + Tr[d1] s.d2.s) +
     4 M[[4]] s.(d1.d2).s + M[[5]] (s.d1.s) (s.d2.s)]];

(* make plot *)
vars4 = {R → 1.5 × 10^11, RE → 6371 × 10^3, c → 3 × 10^8, f → 100,
   α → 2 π f d / c , β → ω t, ω → 2 Pi / (24 × 365), δ → ωE t, ωE → 2 Pi / 24};
p4 = Plot[{γ4 //. vars4}, {t, 0, 1}, PlotStyle → {Black, Thick},
  BaseStyle → {FontSize → 16}, Frame → True, FrameLabel → {"time (hr)"},
  PlotRange → {{0, 1}, {-0.2, 1}}, GridLines → Automatic]
Export["rotation_and_orbit.eps", p4];
```

Out[79]= $\sqrt{(R + RE - R \,\text{Cos}[\beta] - RE \,\text{Cos}[\delta])^2 + (R \,\text{Sin}[\beta] + RE \,\text{Sin}[\delta])^2}$

Out[80]= 0

Out[81]= 0

Out[82]= $\frac{1}{8}\,(3 + \text{Cos}[2\,\delta])$

Out[83]= $\frac{(R \,\text{Sin}[\beta] + RE \,\text{Sin}[\delta])^2}{2\left((R + RE - R \,\text{Cos}[\beta] - RE \,\text{Cos}[\delta])^2 + (R \,\text{Sin}[\beta] + RE \,\text{Sin}[\delta])^2\right)}$

Out[84]= $\frac{(R \,\text{Sin}[\beta-\delta] + (R + RE)\,\text{Sin}[\delta])^2}{4\left(R^2 + R\,RE + RE^2 - R\,(R + RE)\,\text{Cos}[\beta] + R\,RE\,\text{Cos}[\beta-\delta] - RE\,(R + RE)\,\text{Cos}[\delta]\right)}$

Out[85]= $\frac{\text{Cos}[\delta]\,(R \,\text{Sin}[\beta] + RE \,\text{Sin}[\delta])\,(R \,\text{Sin}[\beta-\delta] + (R + RE)\,\text{Sin}[\delta])}{8\left(R^2 + R\,RE + RE^2 - R\,(R + RE)\,\text{Cos}[\beta] + R\,RE\,\text{Cos}[\beta-\delta] - RE\,(R + RE)\,\text{Cos}[\delta]\right)}$

Out[86]= $\frac{(R \,\text{Sin}[\beta] + RE \,\text{Sin}[\delta])^2\,(R \,\text{Sin}[\beta-\delta] + (R + RE)\,\text{Sin}[\delta])^2}{16\left(R^2 + R\,RE + RE^2 - R\,(R + RE)\,\text{Cos}[\beta] + R\,RE\,\text{Cos}[\beta-\delta] - RE\,(R + RE)\,\text{Cos}[\delta]\right)^2}$

Out[90]=