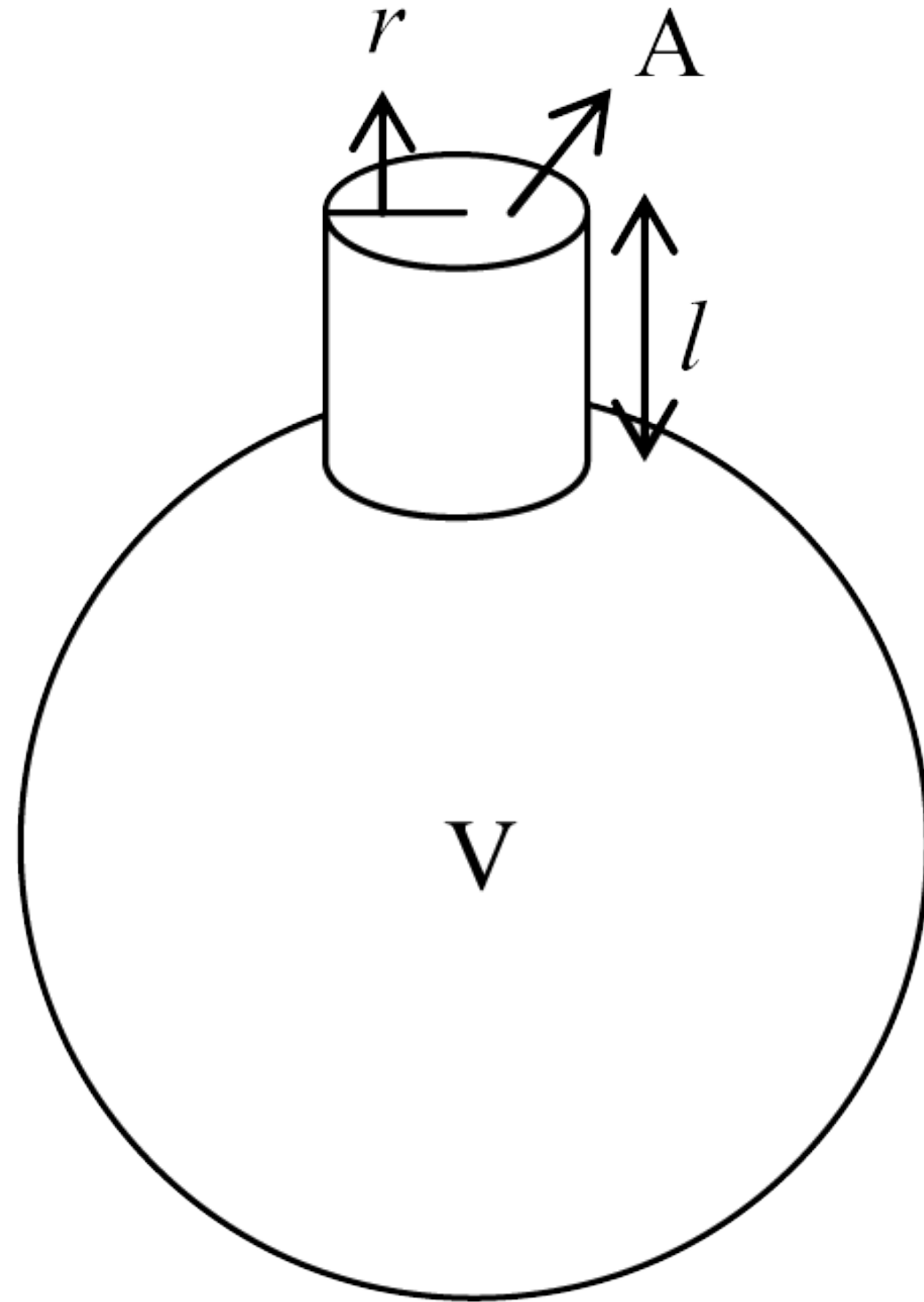


# Helmholtz resonator



$$f = \frac{v}{2\pi} \sqrt{\frac{A}{l_{\text{eff}} V}}$$

- Example:

$$r = 1 \text{ cm}, l = 2.7 \text{ cm}, V = 425 \text{ mL}, v = 346 \text{ m/s}$$

$$A = \pi r^2, 1 \text{ mL} = 10^{-6} \text{ m}^3 \quad \Rightarrow \quad f = 239 \text{ Hz}$$

# 4. Fourier analysis & synthesis

# Fourier's theorem

- **standing wave vibrations** are the “**building blocks**” for any complex vibration
- any complex periodic wave can be written as a **sum of harmonics**:

$$y(t) = A_1 \sin(2\pi f_1 t + \phi_1) + A_2 \sin(2\pi f_2 t + \phi_2) + \dots$$

$$f_N = Nf_1, \quad N = 1, 2, \dots$$

- **Ohm's law of hearing**: Phases have little effect on the timbre of the sound
- **Fourier analysis**: decomposing a complex periodic wave into its contributing harmonics
- **Fourier synthesis**: constructing a complex periodic wave by combining harmonics

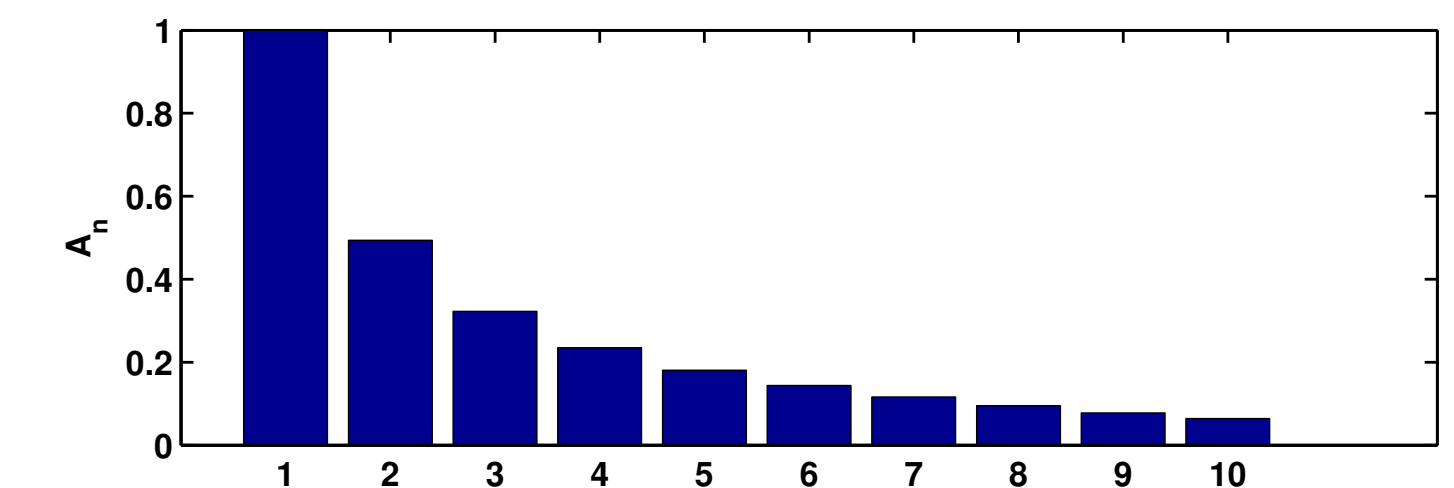
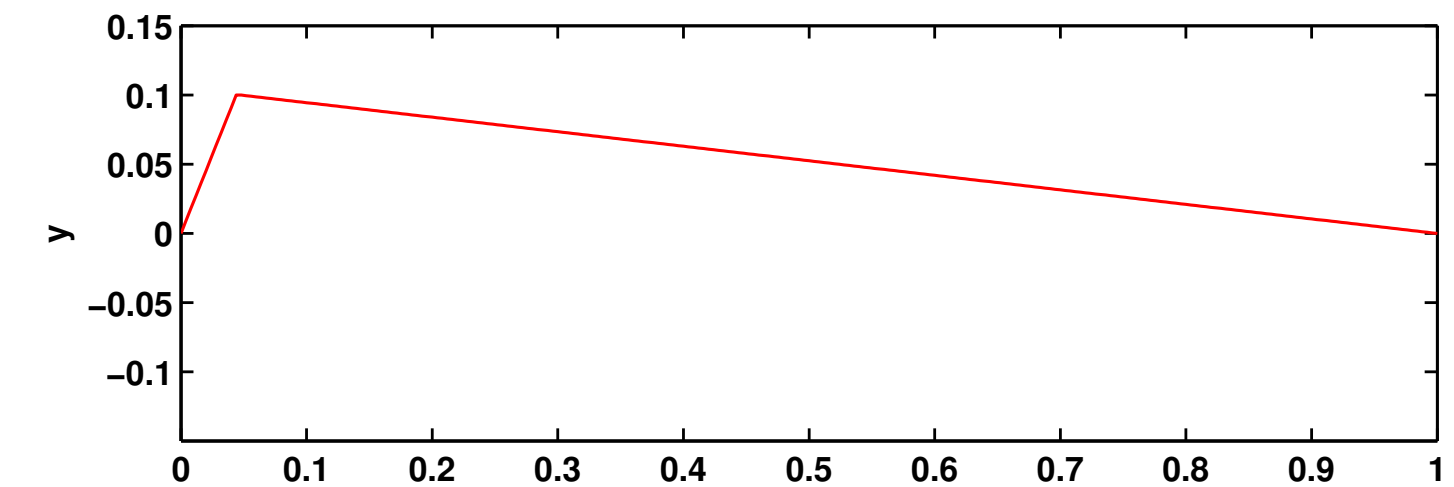
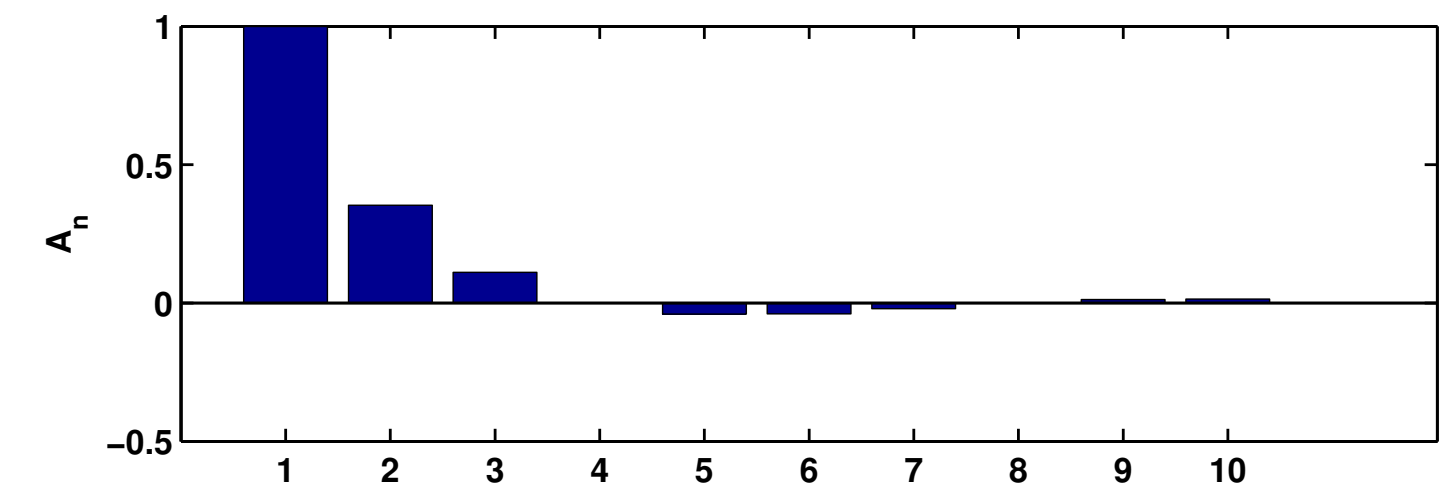
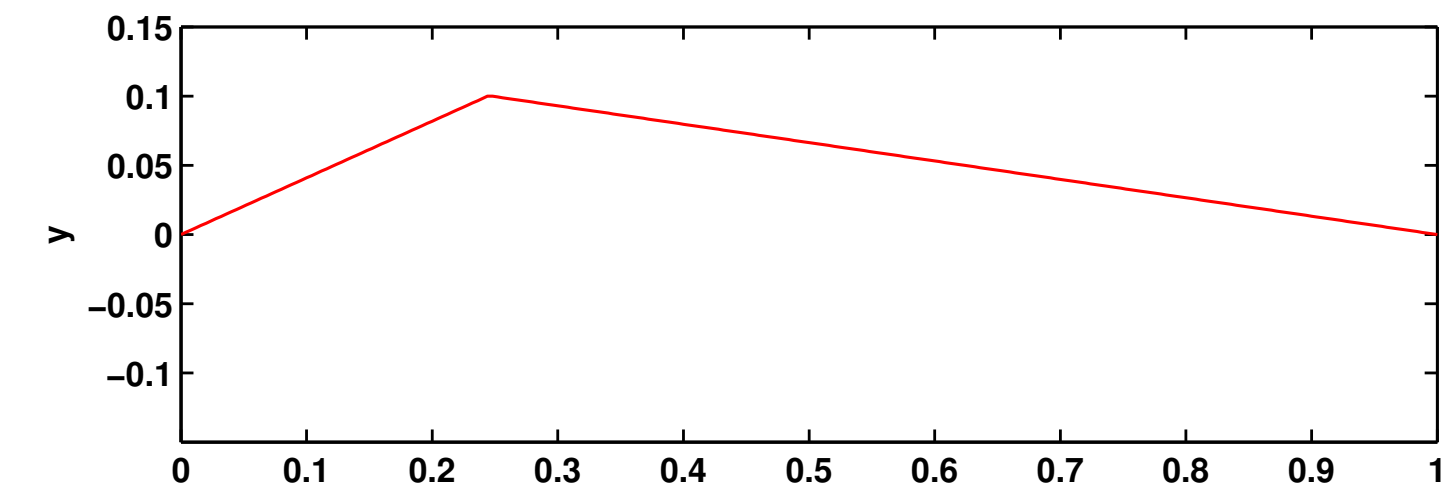
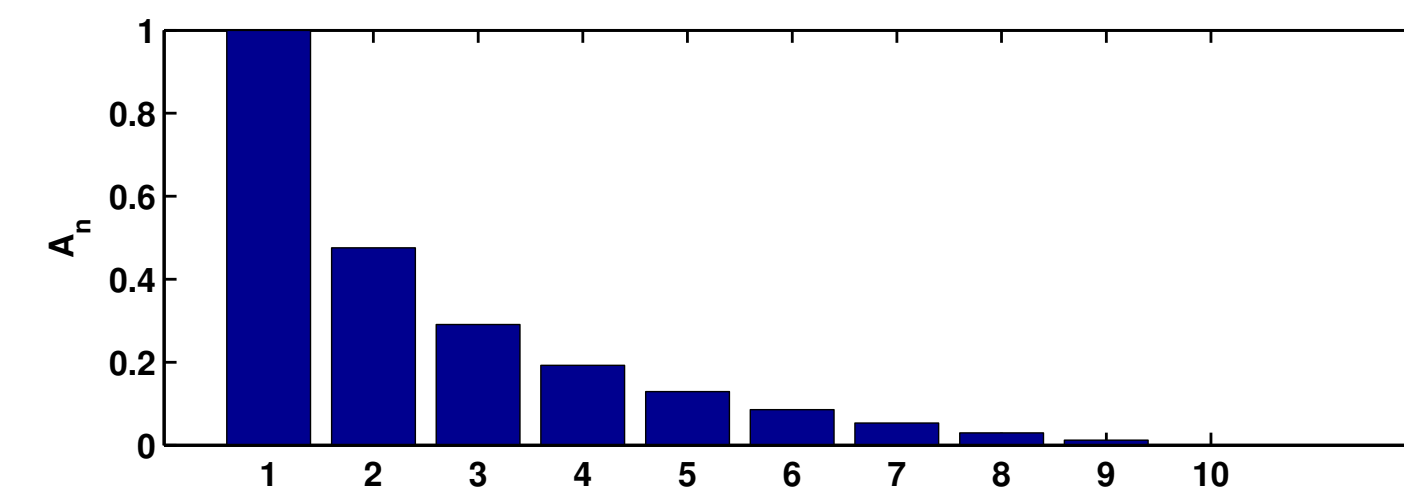
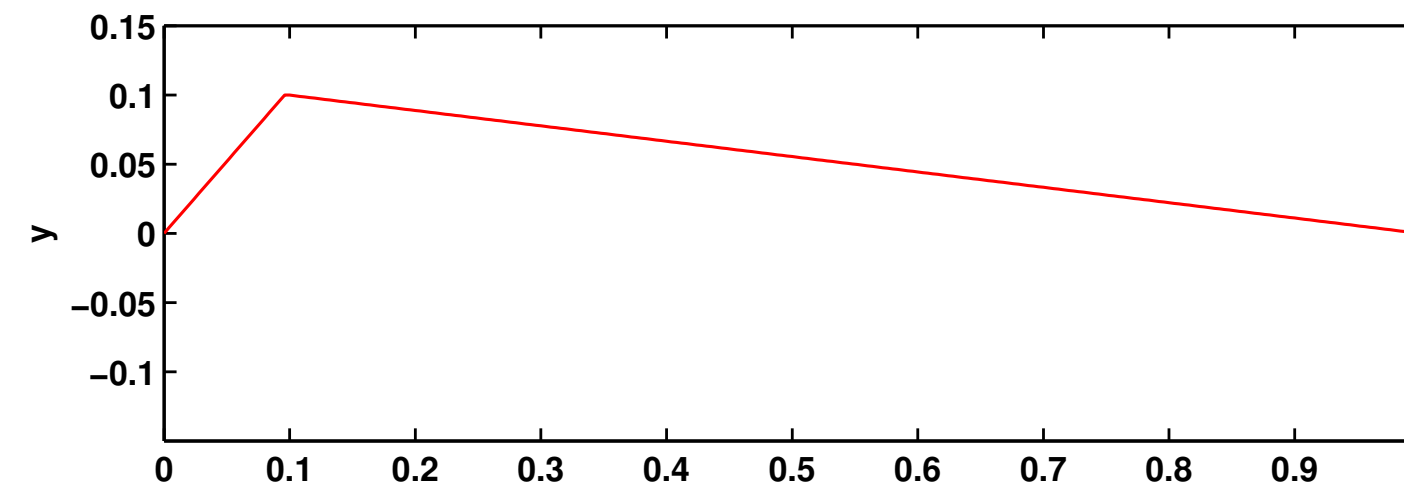
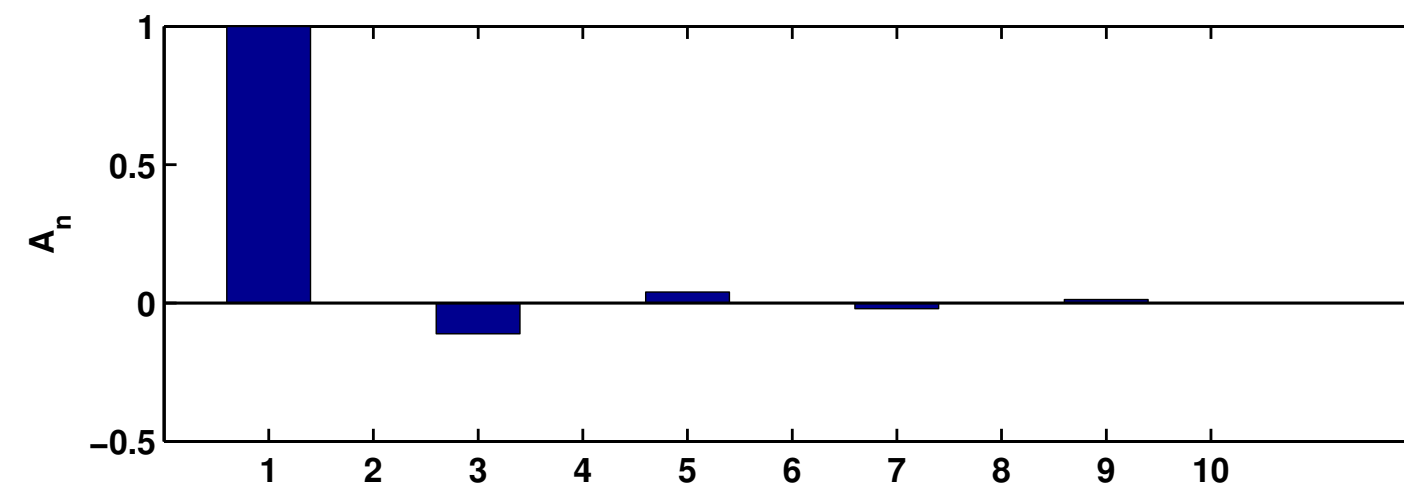
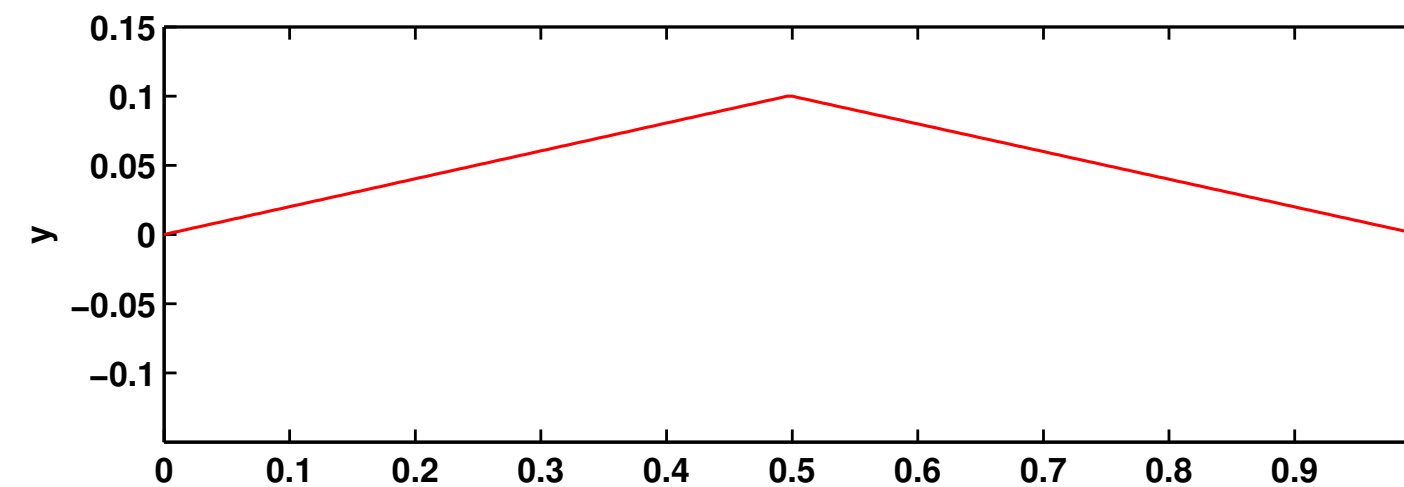
# 5. String instruments

# Plucked versus bowed strings

- Plucked string: [https://www.youtube.com/watch?v=\\_X72on6CSL0](https://www.youtube.com/watch?v=_X72on6CSL0)
- Bowed string: <https://www.youtube.com/watch?v=6JeyiM0YNo4>
- iPhone guitar video: <https://www.youtube.com/watch?v=TKF6nFzpHBU>
- NOTE: the iPhone guitar video does not show the wave pulses on the strings as they really are. Rather one sees multiple images of the same pulse shape on the string due to the “rolling shutter” effect of the iPhone camera. The actual pulses on a guitar string behave as shown in the first video.

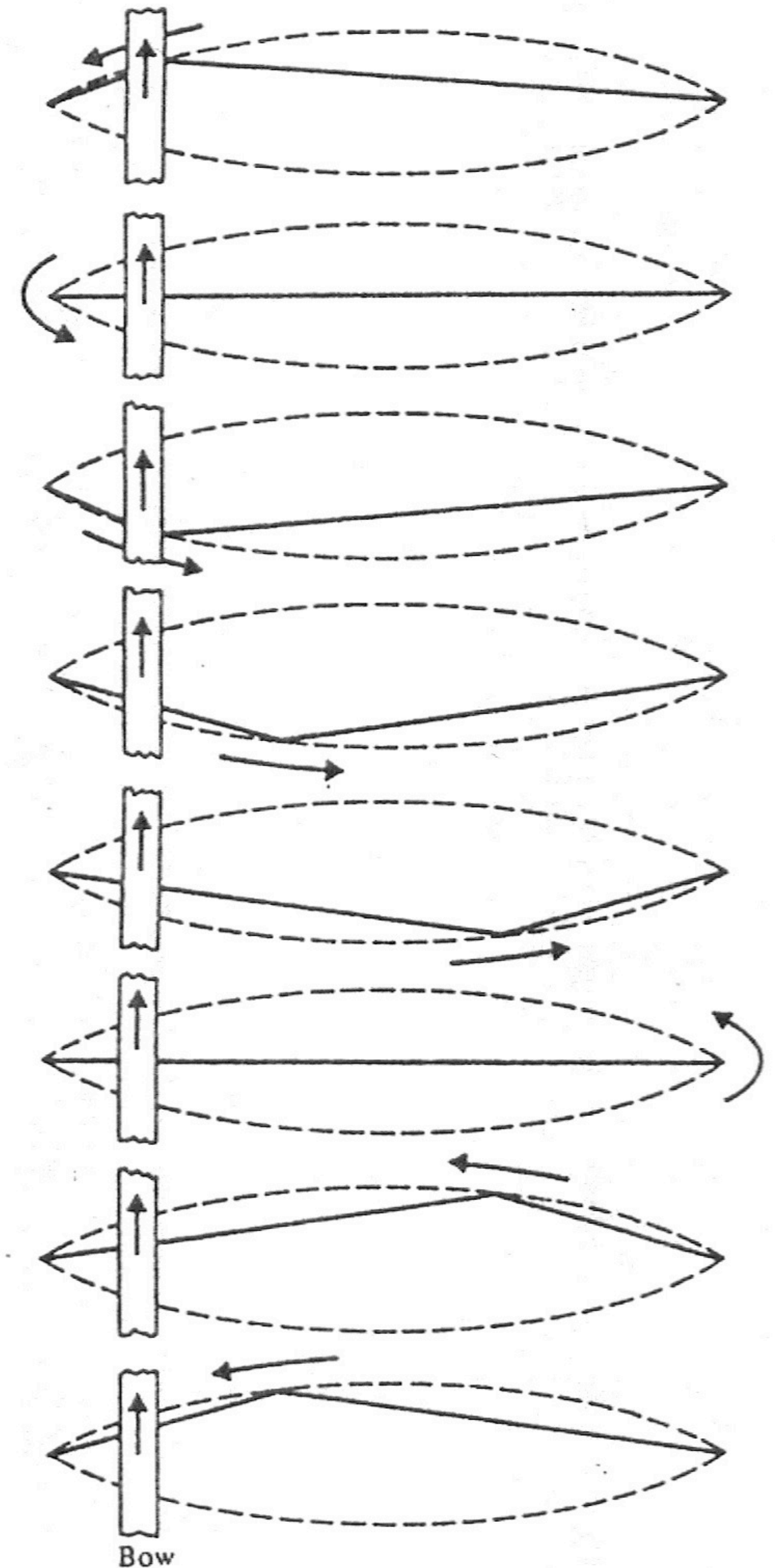
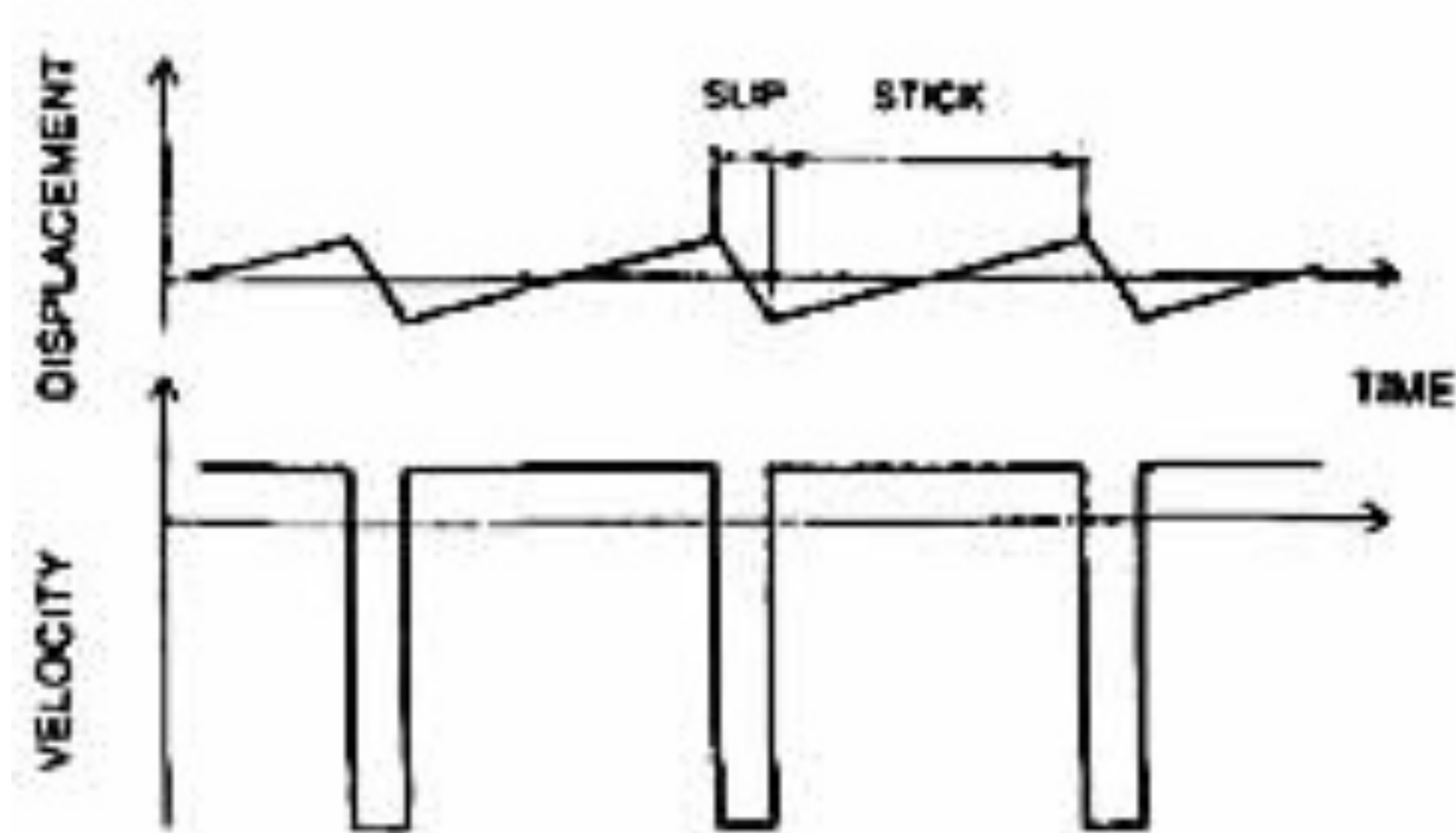
# Fourier coefficients of a plucked string

- Sounds are **richer** when the string is plucked **closer to the bridge**
- If the string is plucked in the **middle**, there are **no even harmonics**

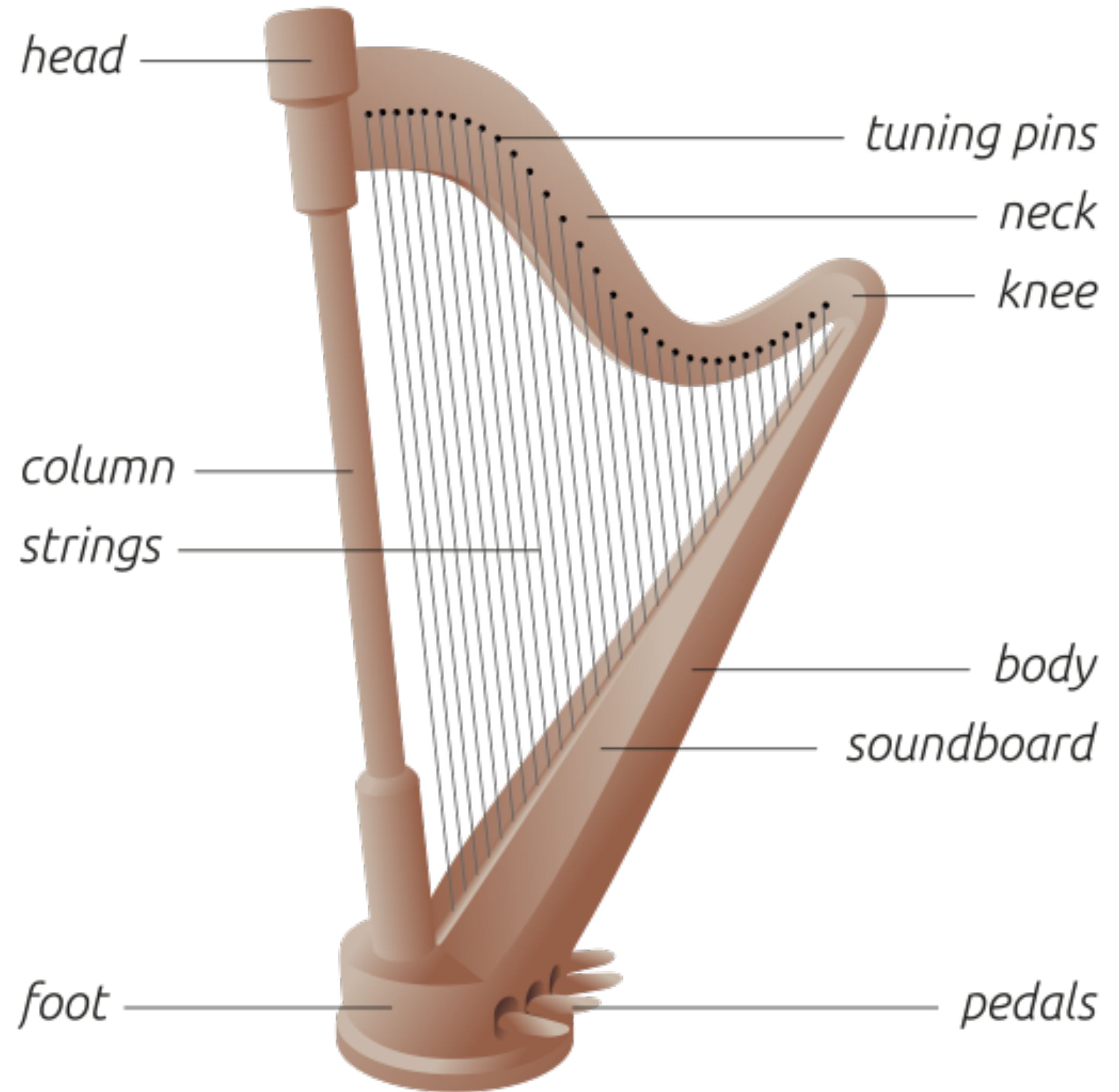


# Stick-slip motion of a bowed string

- The violin string alternately "sticks" and then "slips" against the bow hundreds of times per second

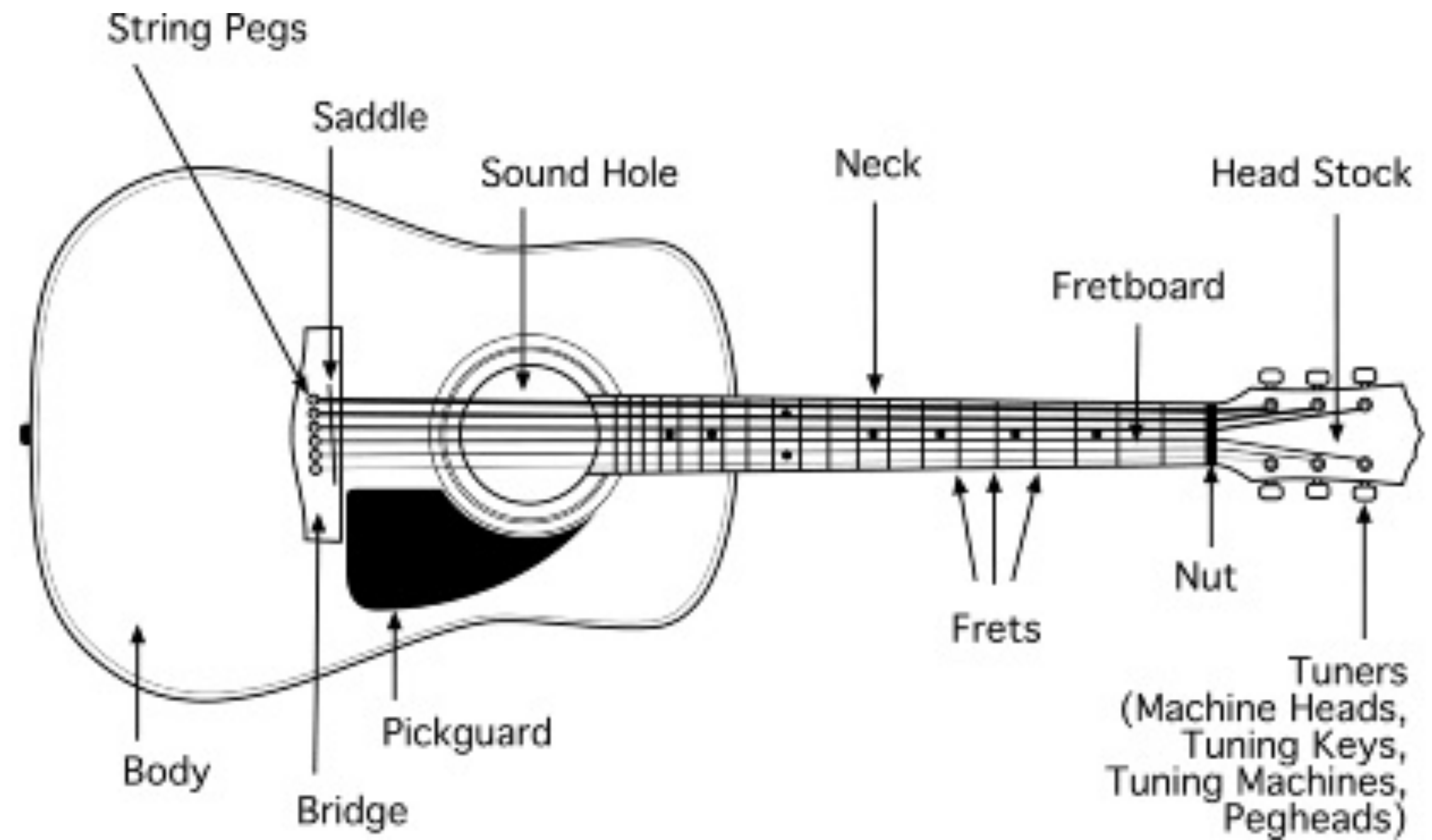


# Harp

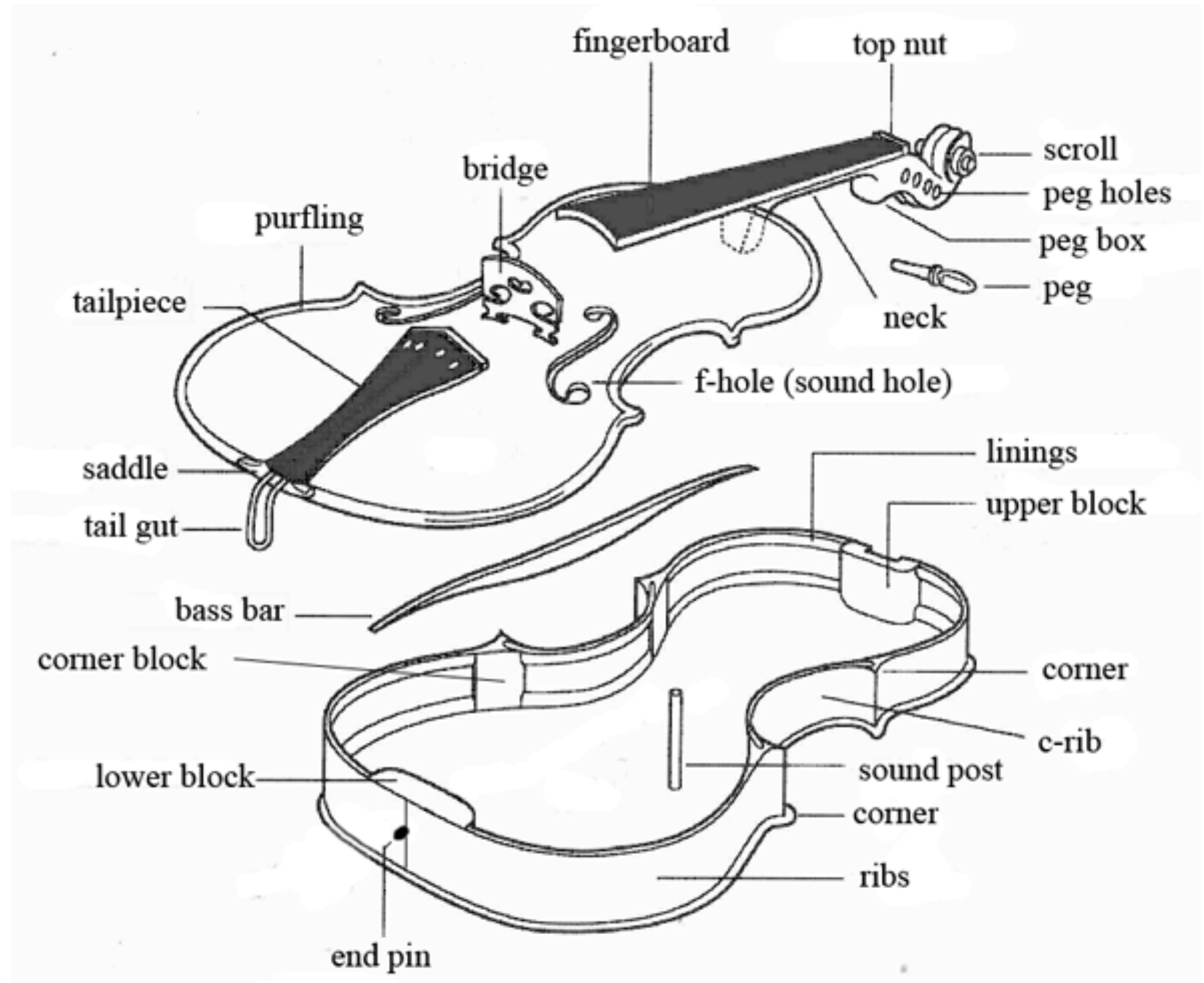




# Guitar

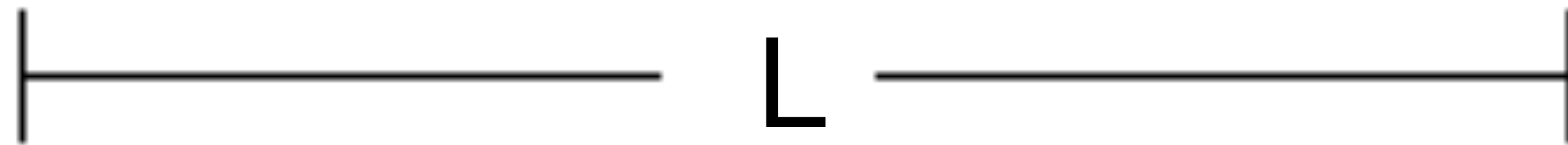


# Violin



# **6. Wind instruments**

# Open and closed tubes (recall previous discussion)



$$\lambda_N = \frac{2L}{N} \quad f_N = Nf_1 \quad f_1 = \frac{v}{2L} \quad N = 1, 2, \dots$$

(both even and odd harmonics)

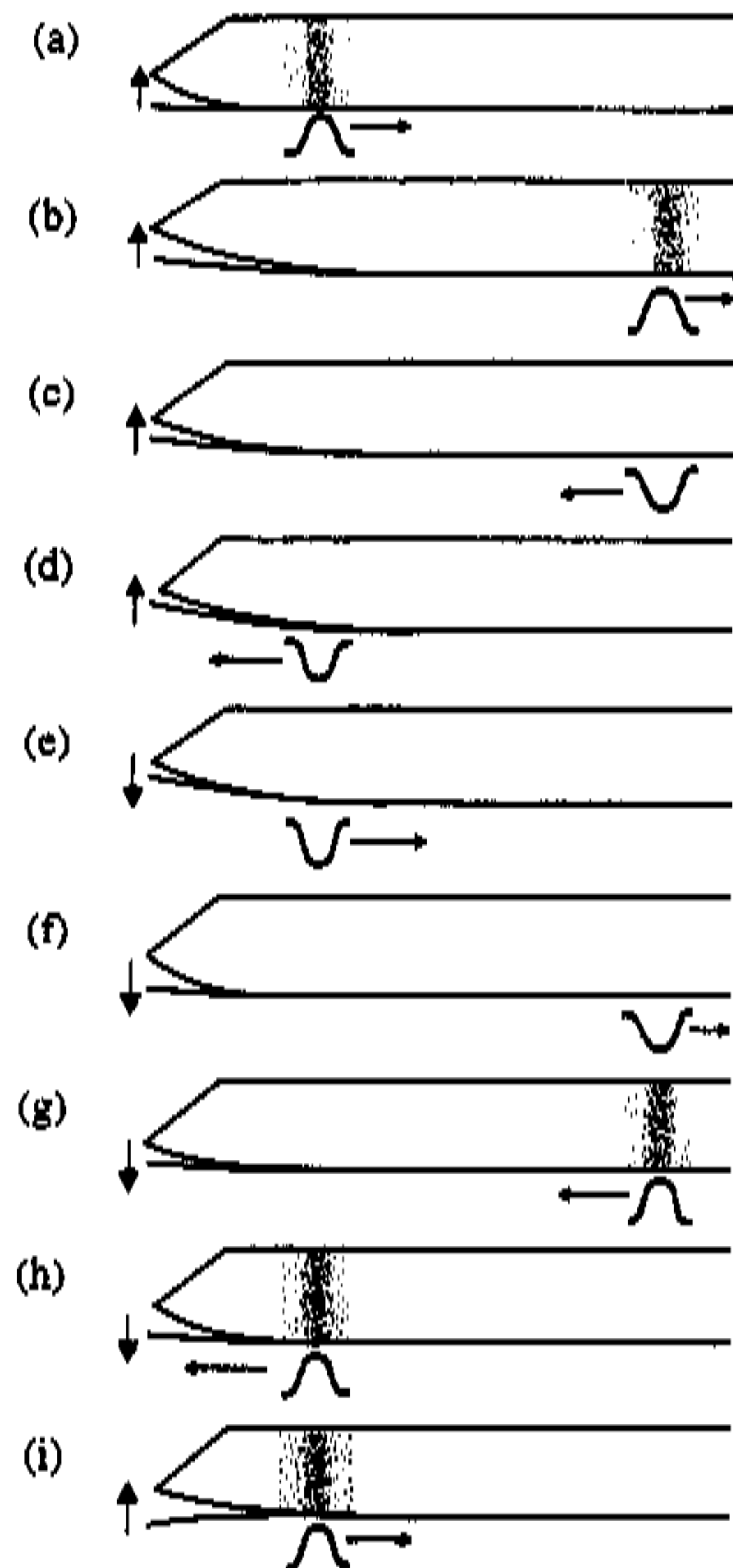
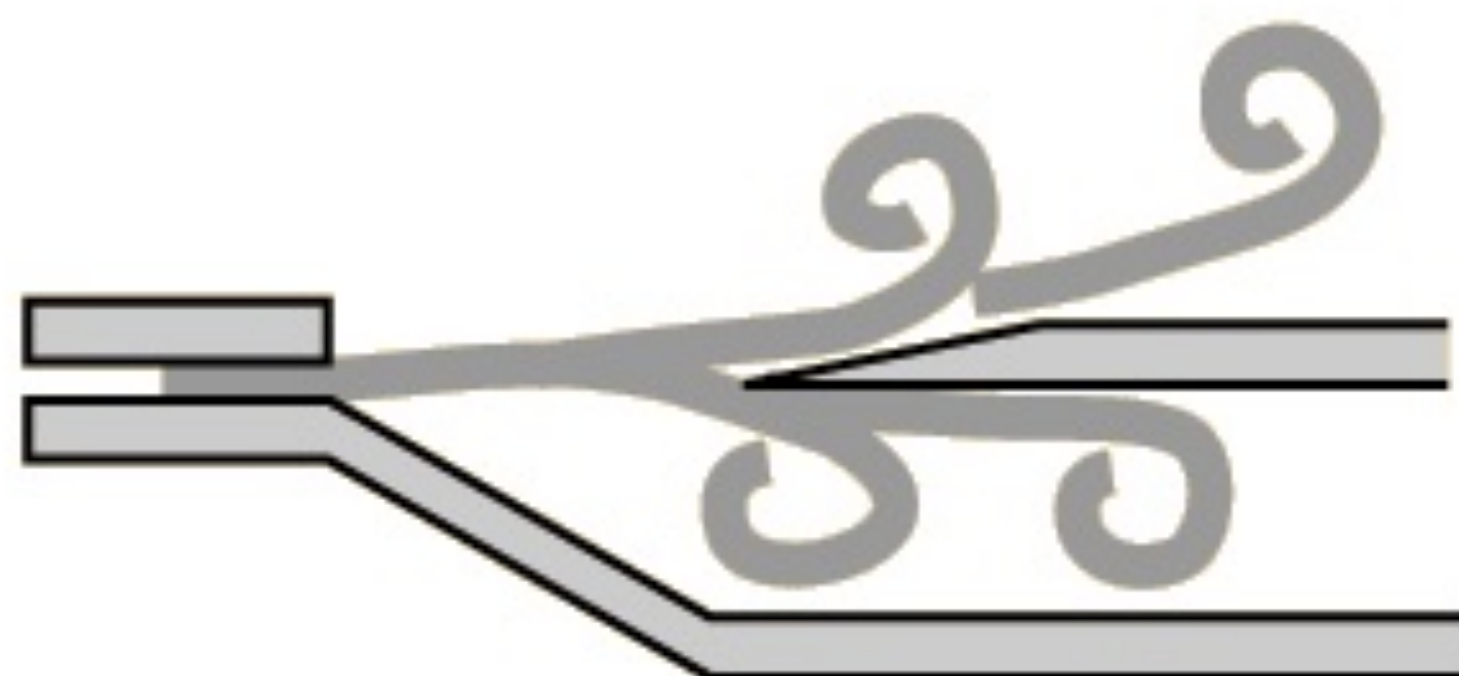


$$\lambda_N = \frac{4L}{N} \quad f_N = Nf_1 \quad f_1 = \frac{v}{4L} \quad N = 1, 3, 5, \dots$$

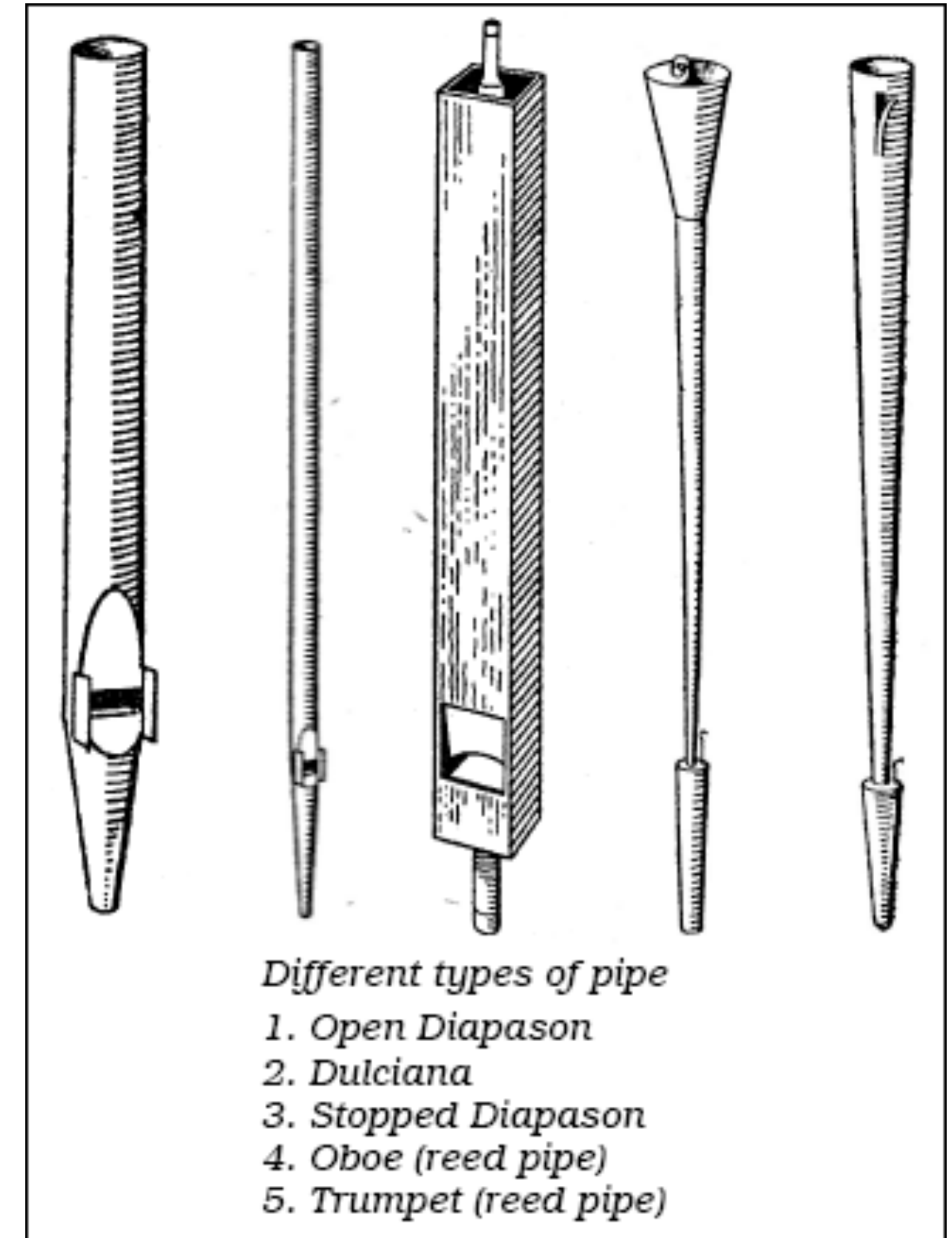
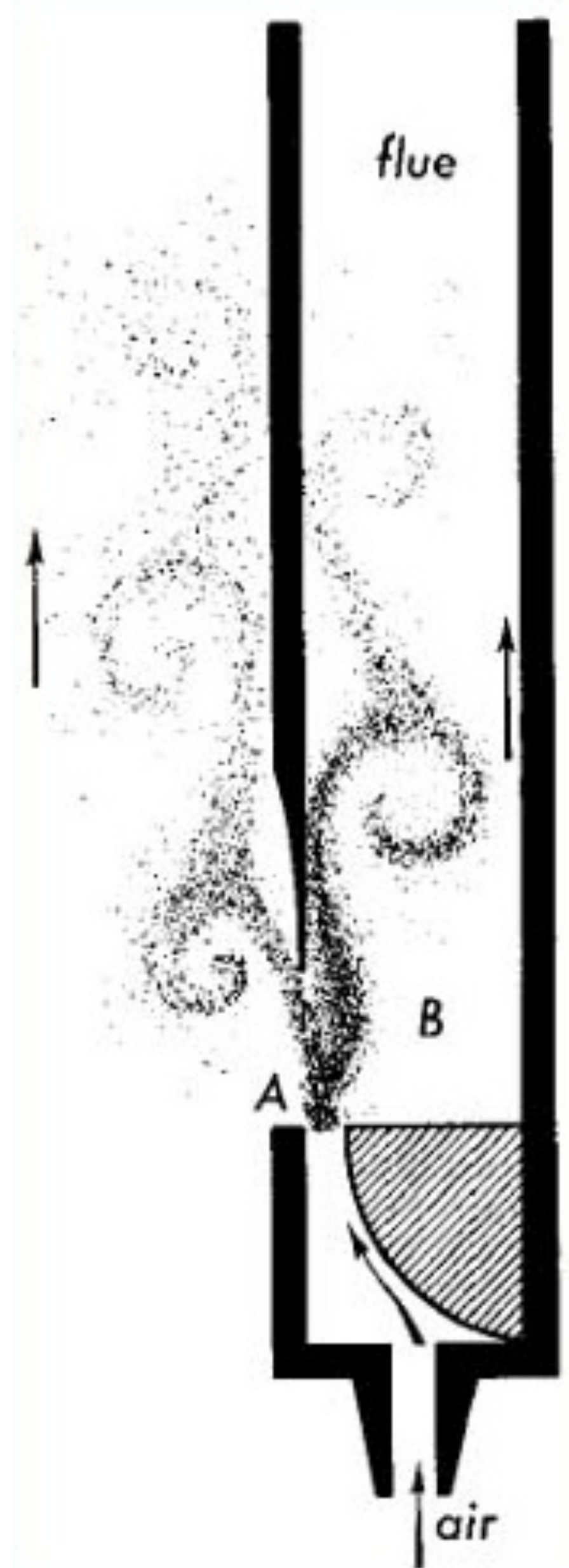
(only odd harmonics)

(air molecule displacements)

# Excitations produced by an oscillating air stream or a vibrating reed

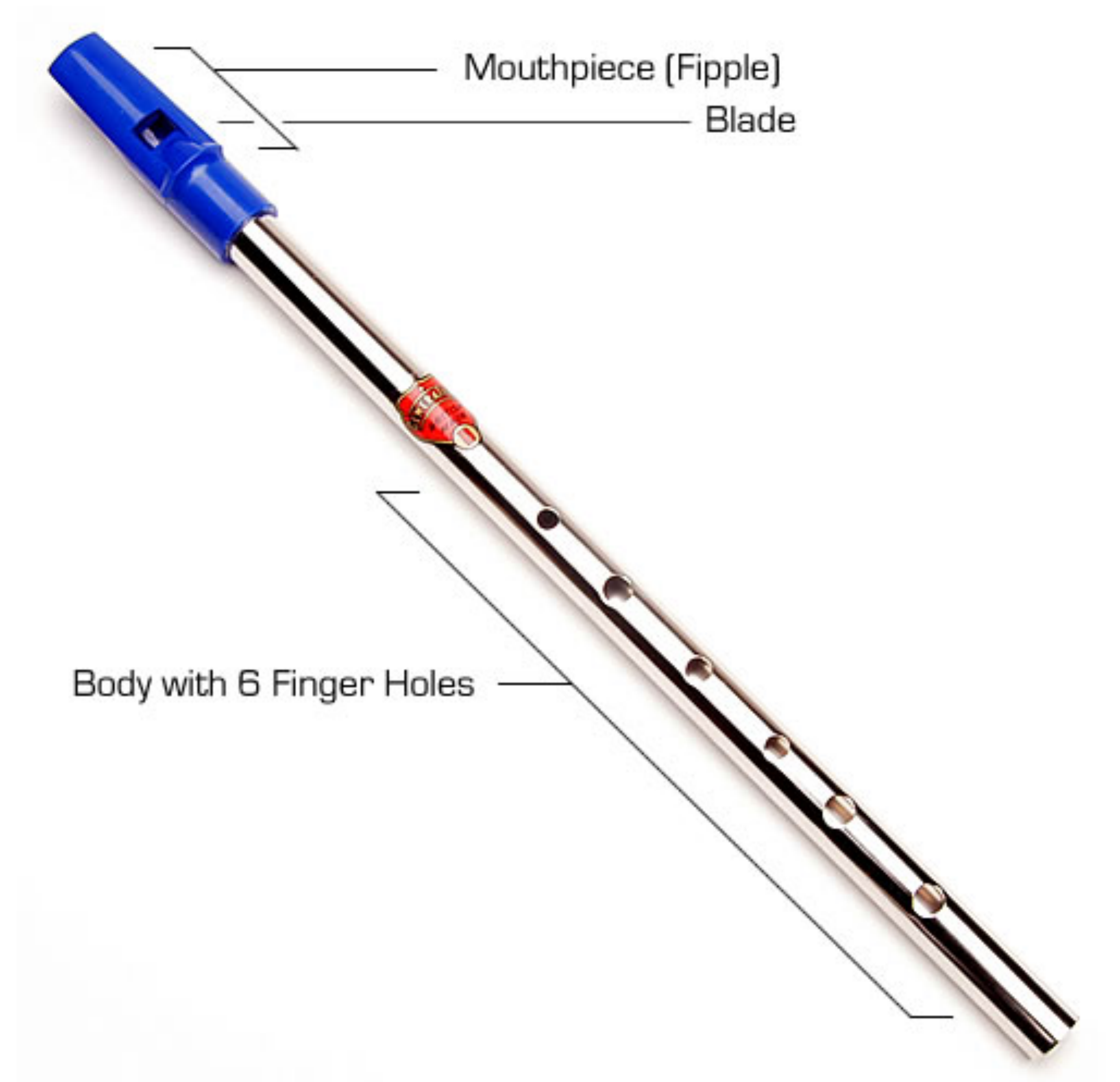


# Flue-organ pipe





# Penny whistle



# Clarinet

