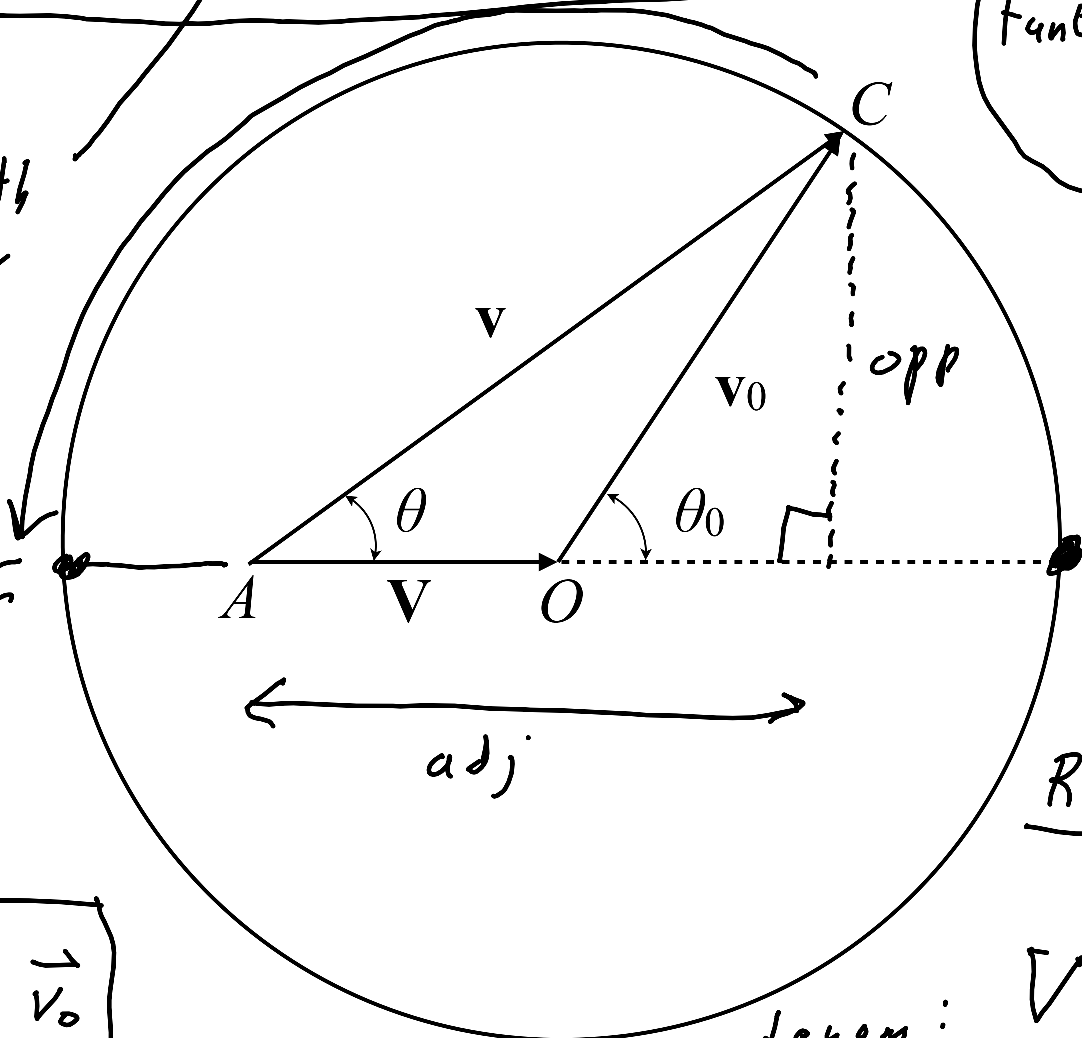
$$\boxed{\cos\theta_0 = -\frac{V}{v_0}\sin^2\theta \pm \cos\theta\sqrt{1-\left(\frac{V}{v_0}\right)^2\sin^2\theta}}$$

$$\tan\theta = \frac{opp}{adj}$$

need both +/- for $V > v_0$

For $V < v_0$, only need +



$V < v_0$

$$\tan\theta = \frac{v_0 \sin\theta_0}{V + v_0\cos\theta_0}$$

Invert

square

LHS $= \tan^2\theta$

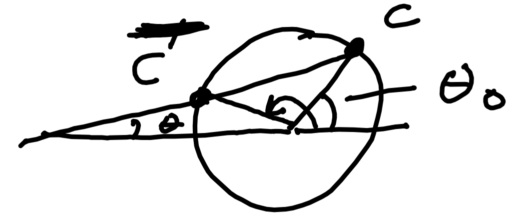
RHS: $\sin^2\theta_0 = 1-\cos^2\theta_0$

denom: $V^2 + 2Vv_0\cos\theta_0 + v_0^2\cos^2\theta_0$

$$\boxed{\vec{v} = \vec{V} + \vec{v_0}}$$

$\Longrightarrow$ Quadratic equation

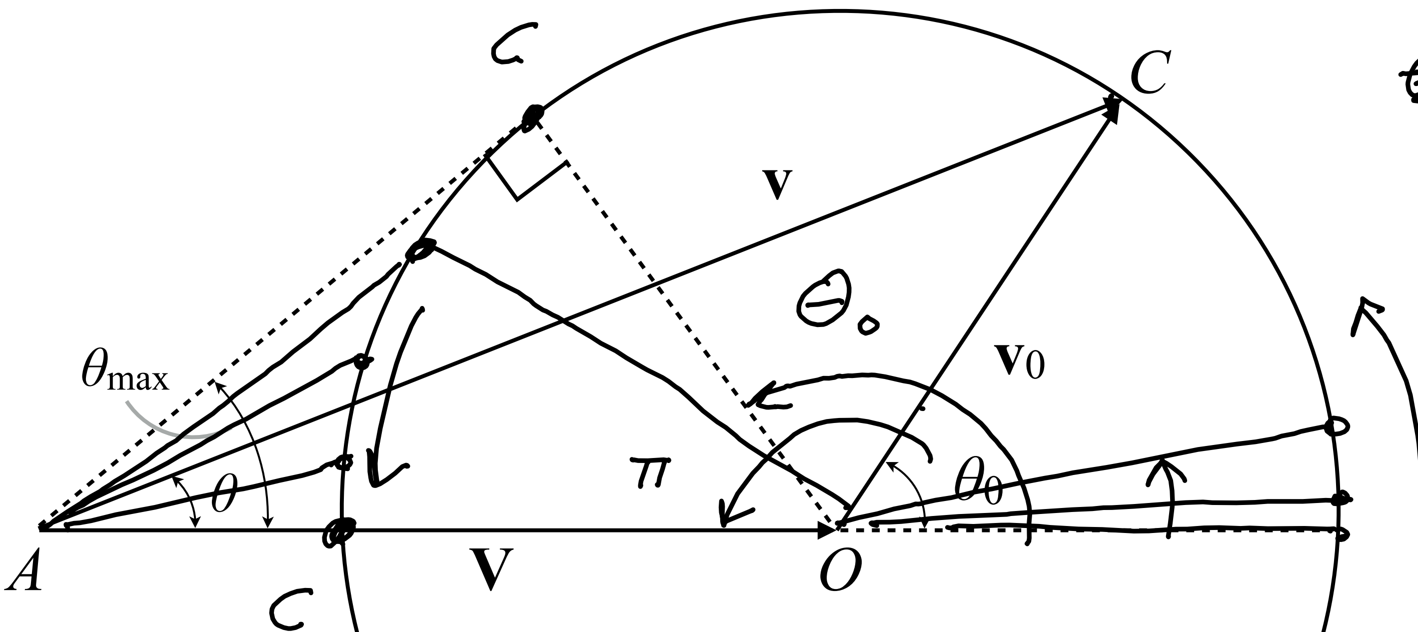
Fig 14 a,b



$\bar{\theta}_0 : \bar{C}$

$\theta_0 : C$

Θ



Example

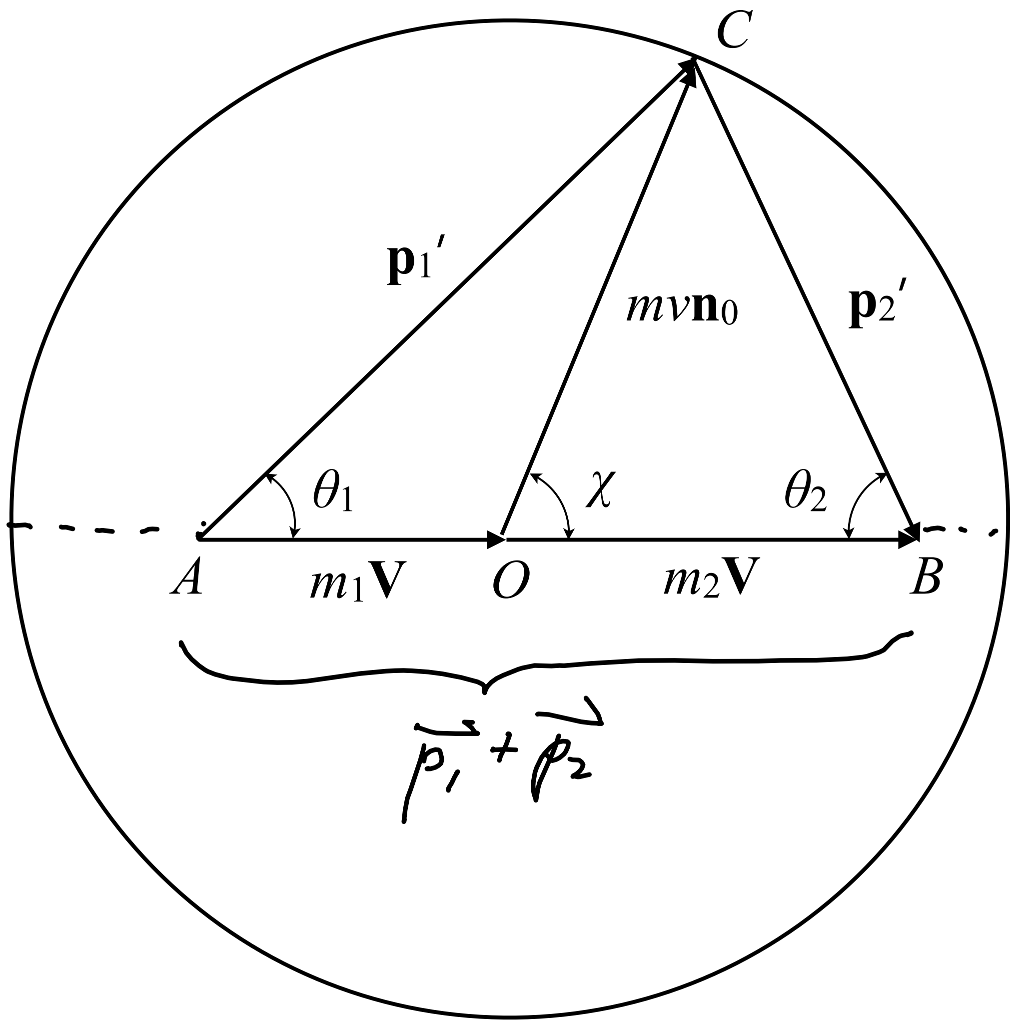
$\theta_{max} = 45°$

$\theta = 30$

$\theta_0 : 0 \to \pi$

$\theta : 0 \to \theta_{max} \to 0$ $\quad V > v_0$

Fig 15

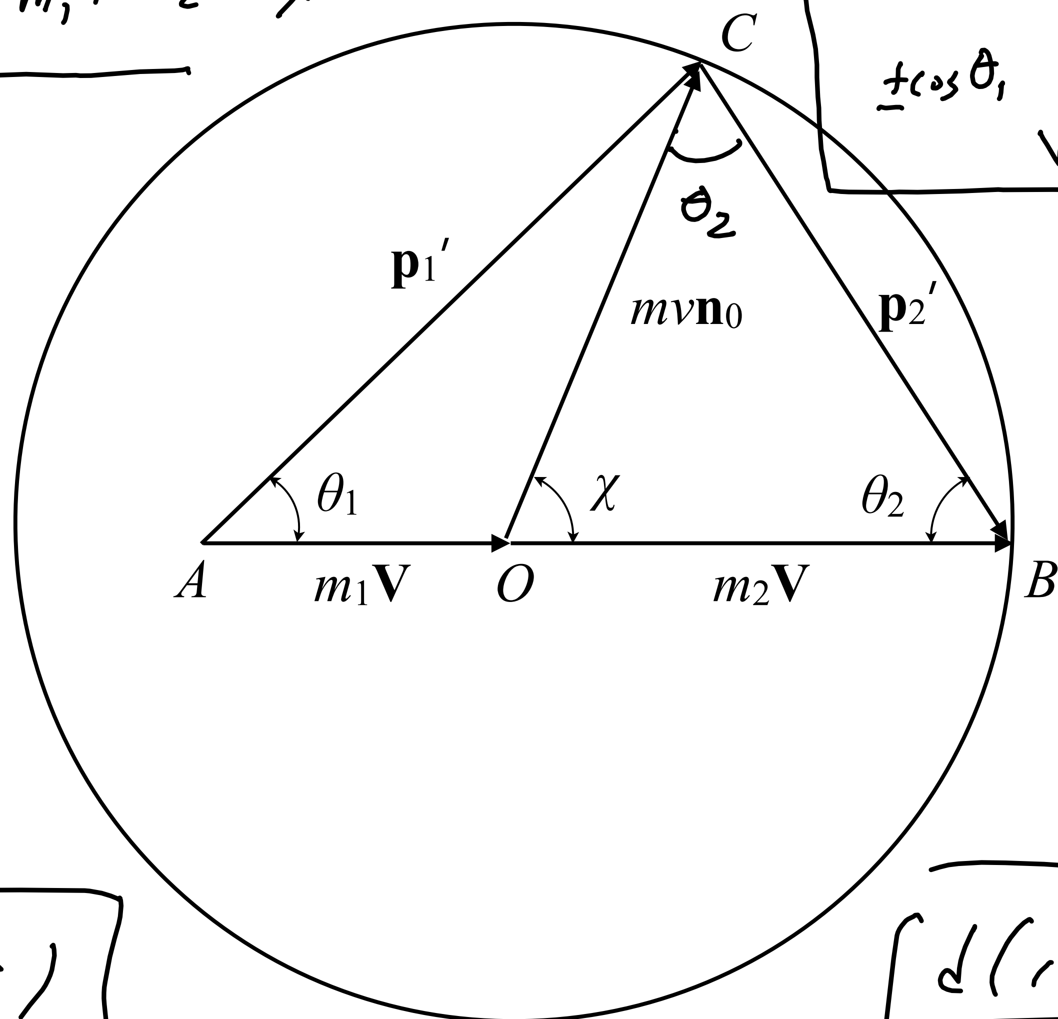

C
$\mathbf{p}_1'$
$mv\mathbf{n}_0$
$\mathbf{p}_2'$
θ_1
χ
θ_2
A
$m_1\mathbf{V}$
O
$m_2\mathbf{V}$
B
$\vec{p}_1 + \vec{p}_2$

$$\chi + 2\theta_2 = \pi \longrightarrow \boxed{\chi = \pi - 2\theta_2}$$

$$\underbrace{\tan\theta_1 = \frac{m_2 \sin\chi}{m_1 + m_2 \cos\chi}} \longrightarrow \boxed{\cos\chi = -\frac{m_1}{m_2}\sin^2\theta_1 \pm \cos\theta_1 \sqrt{1 - \left(\frac{m_1}{m_2}\right)^2 \sin^2\theta_1}}$$

quadratic

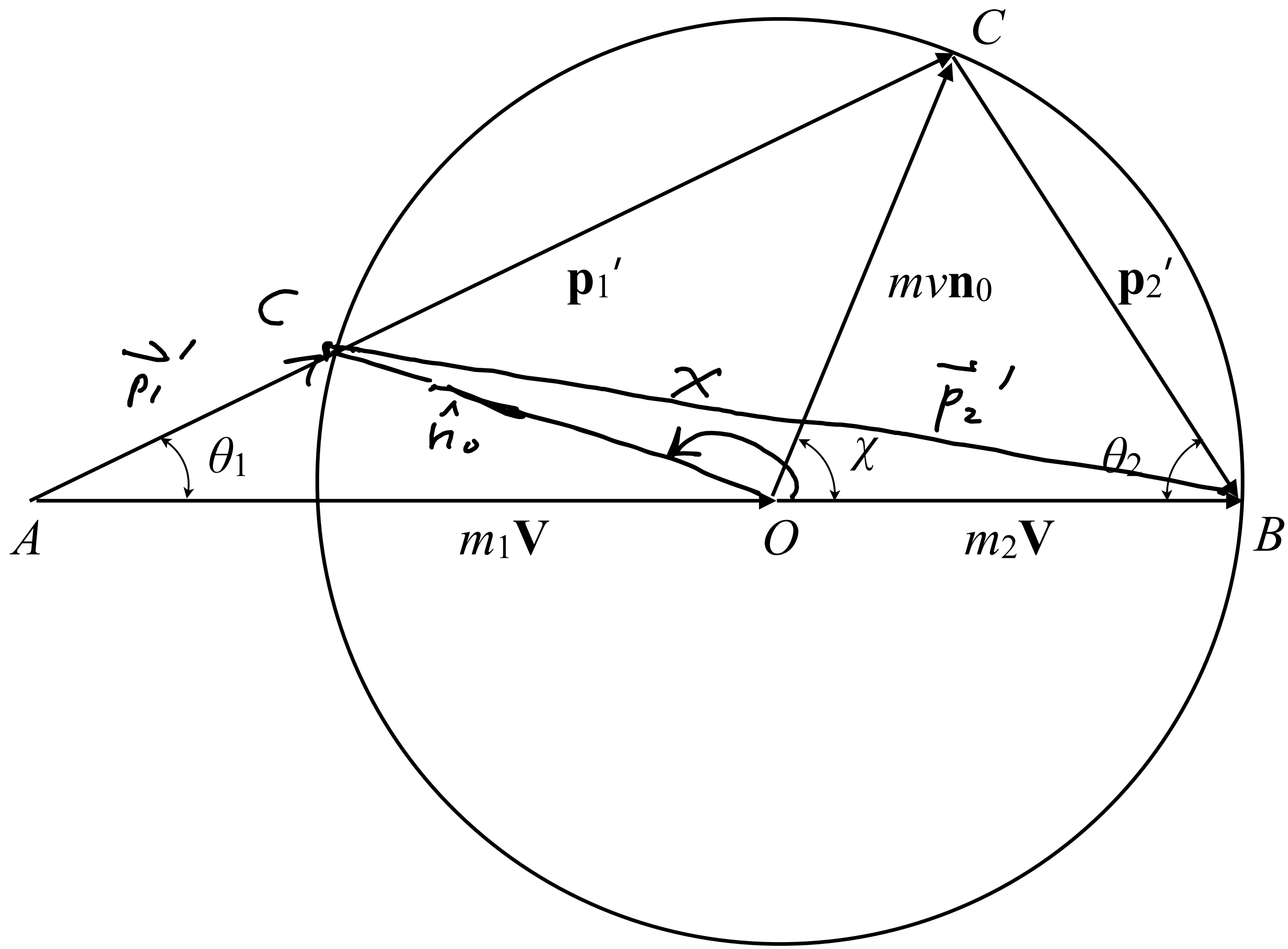
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$\boxed{\frac{d(\cos\chi)}{d(\cos\theta_1)}}$$

$m_1 < m_2$, $\underline{v_2 = 0}$

$$\boxed{\frac{d(\cos\chi)}{d(\cos\theta_2)}}$$

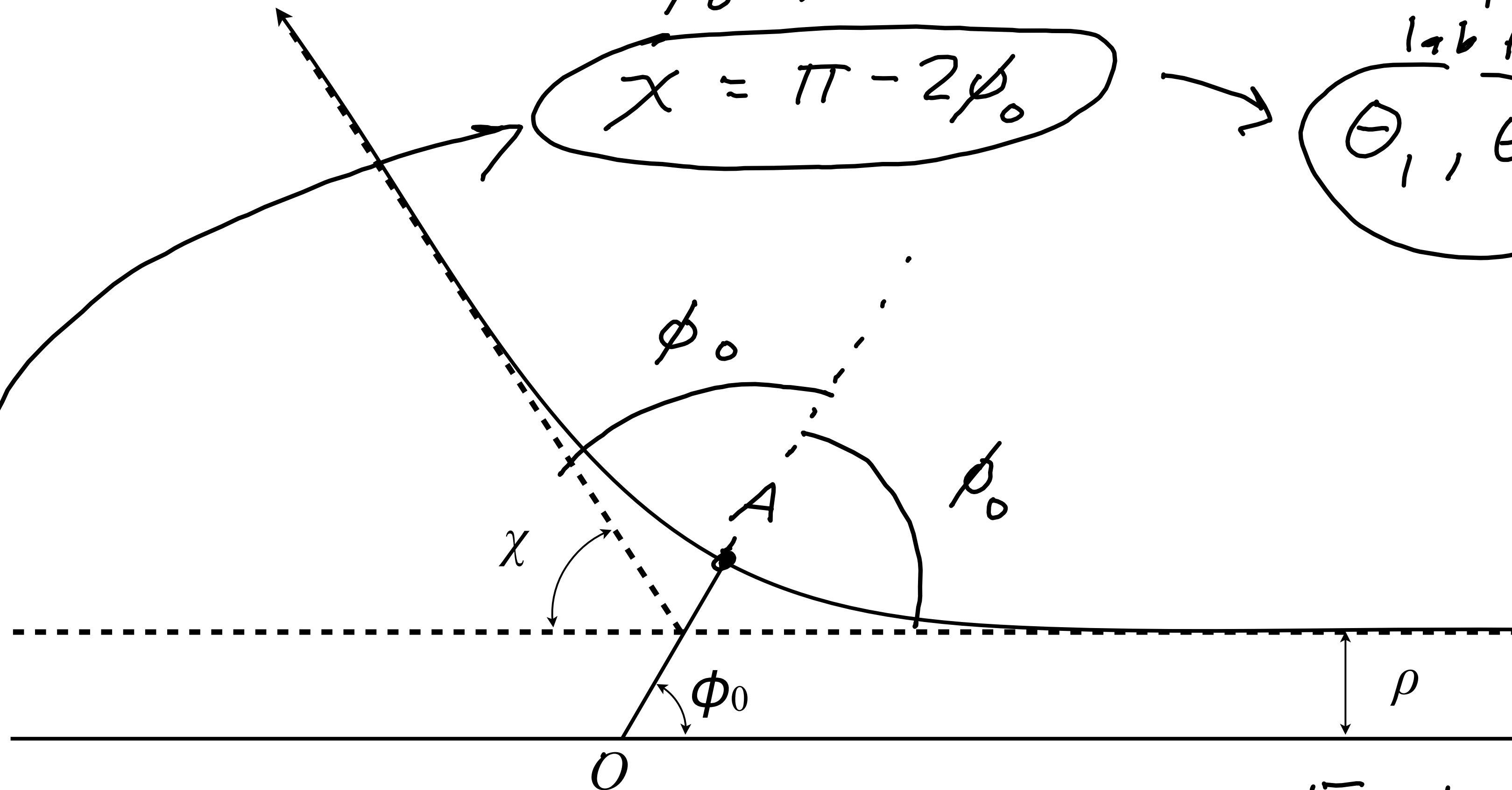

C
$\mathbf{p}_1'$
$mv\mathbf{n}_0$
$\mathbf{p}_2'$
C
$\vec{p}_1{}'$
χ
$\hat{n}_0$
$\vec{p}_2{}'$
θ_1
χ
θ_2
A
$m_1\mathbf{V}$
O
$m_2\mathbf{V}$
B

$m_1 > m_2$, $v_2 = 0$

$$2\phi_0 + \chi = \pi$$

$$\chi = \pi - 2\phi_0 \longrightarrow \theta_1, \theta_2$$

COM to lab fram



$$\boxed{\phi_0} = \int_{r_{min}}^{\infty} \frac{M\,dr/r^2}{\sqrt{\frac{2}{m}(E-U(r)) - M^2/2mr^2}}$$

$$= \int_{r_{min}}^{\infty} \frac{\rho\,dr/r^2}{\sqrt{1 - (\rho/r)^2 - 2U/mv_\infty^2}}$$

$$E = \frac{1}{2} m v_\infty^2$$

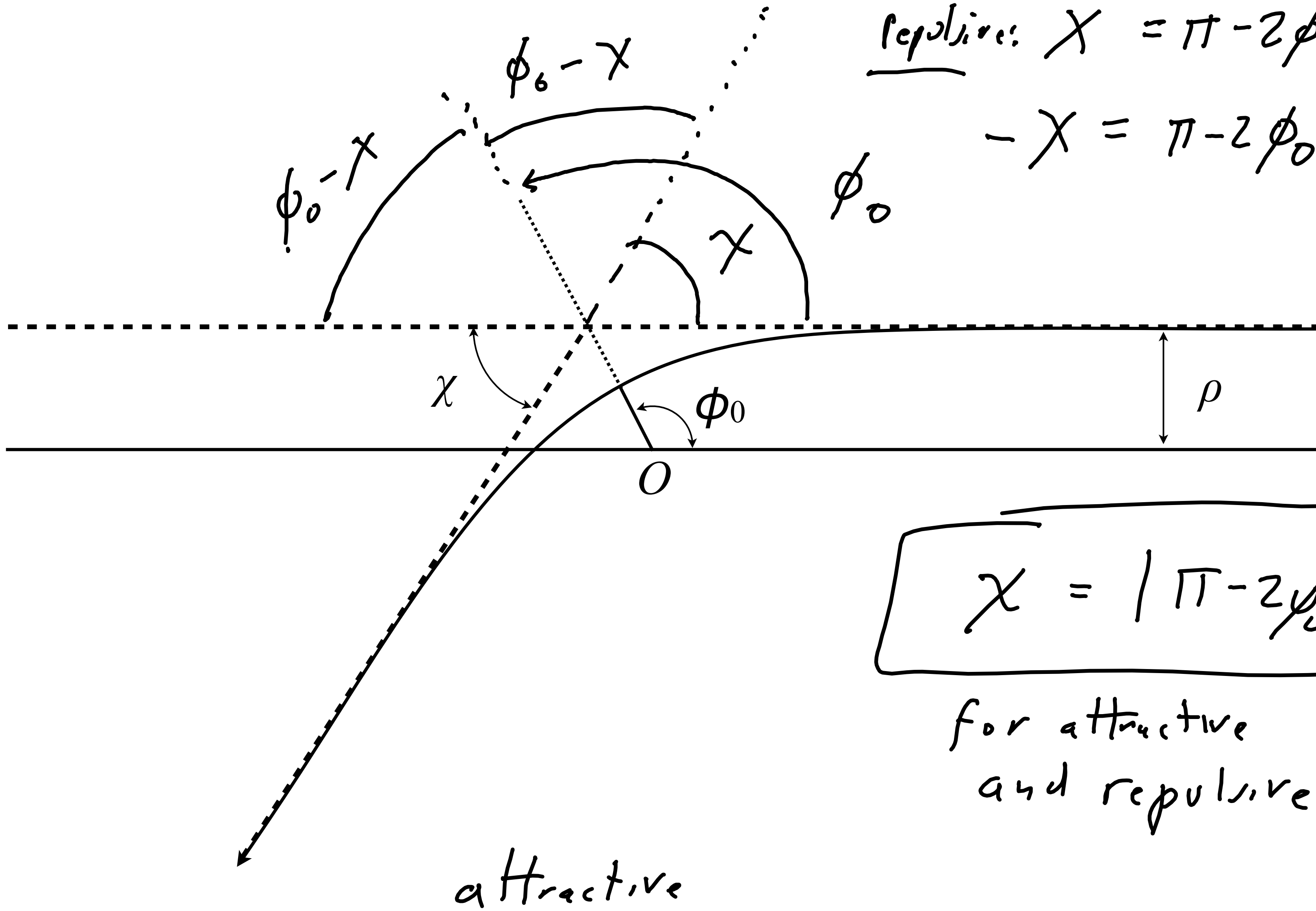
$$M = \rho m v_\infty$$

Given:
v_∞, ρ
$U(r)$

$$\pi = 2(\phi_0 - \chi) + \chi = \boxed{2\phi_0 - \chi = \pi}$$

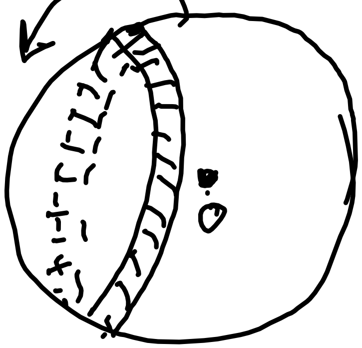
Repulsive: $\chi = \pi - 2\phi_0$

$$-\chi = \pi - 2\phi_0$$



$$\boxed{\chi = |\pi - 2\phi_0|}$$

for attractive and repulsive



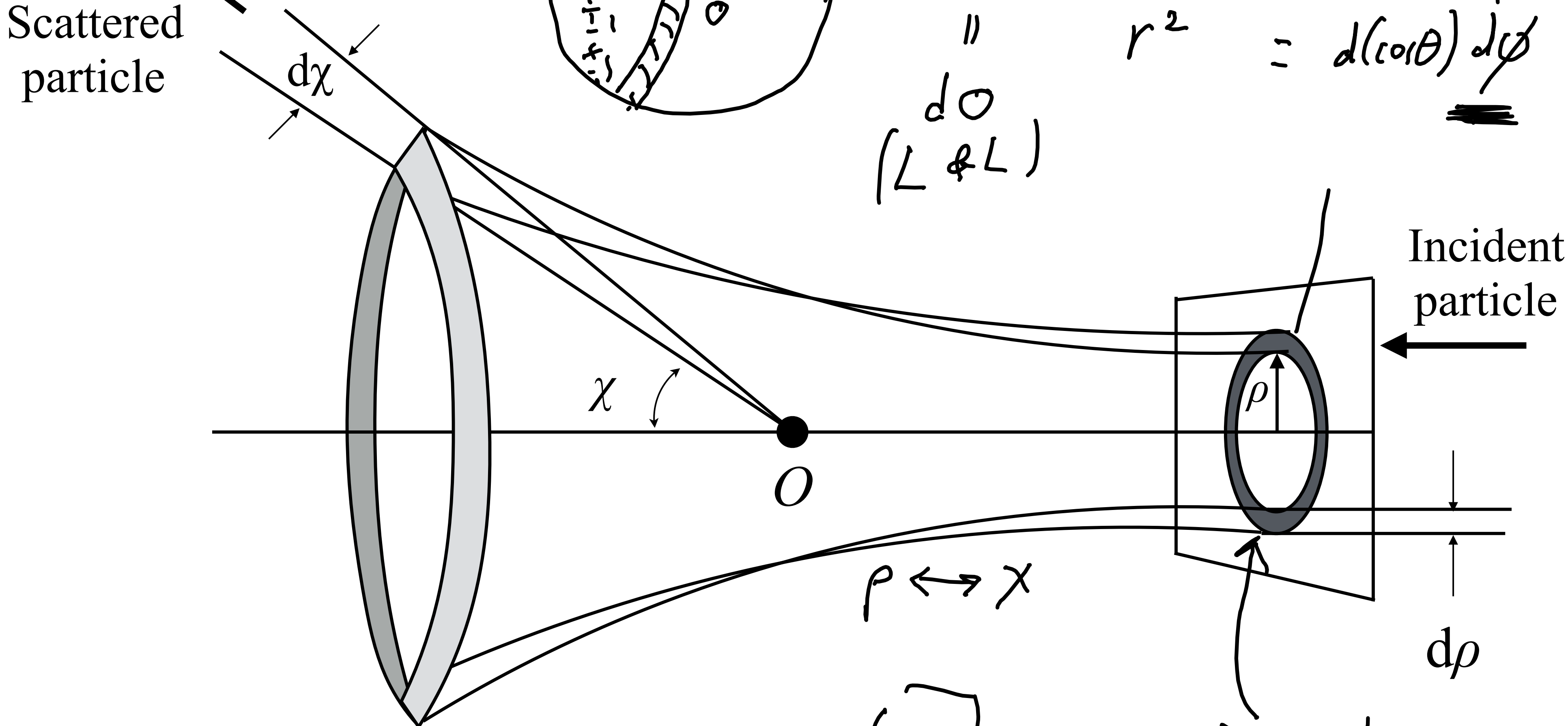
$$dA = r^2 \sin d\theta \, d\phi$$

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi = d(\cos\theta) \, d\phi$$

$\parallel$

dO

(L&L)



Differential cross section $=$ $d\sigma$ $=$ area of annulus $= 2\pi\rho \, d\rho$ $\Leftarrow$ area

$$\boxed{\begin{aligned} d\sigma &= 2\pi\rho(\chi) \left|\frac{d\rho}{d\chi}\right| d\chi \\ &= \frac{\rho(x)}{\sin\chi} \left|\frac{d\rho}{d\chi}\right| d\Omega \end{aligned}}$$

$$d\Omega = \sin\chi \, d\chi \cdot 2\pi$$

$$d\chi \cdot 2\pi = d\Omega / \sin\chi$$