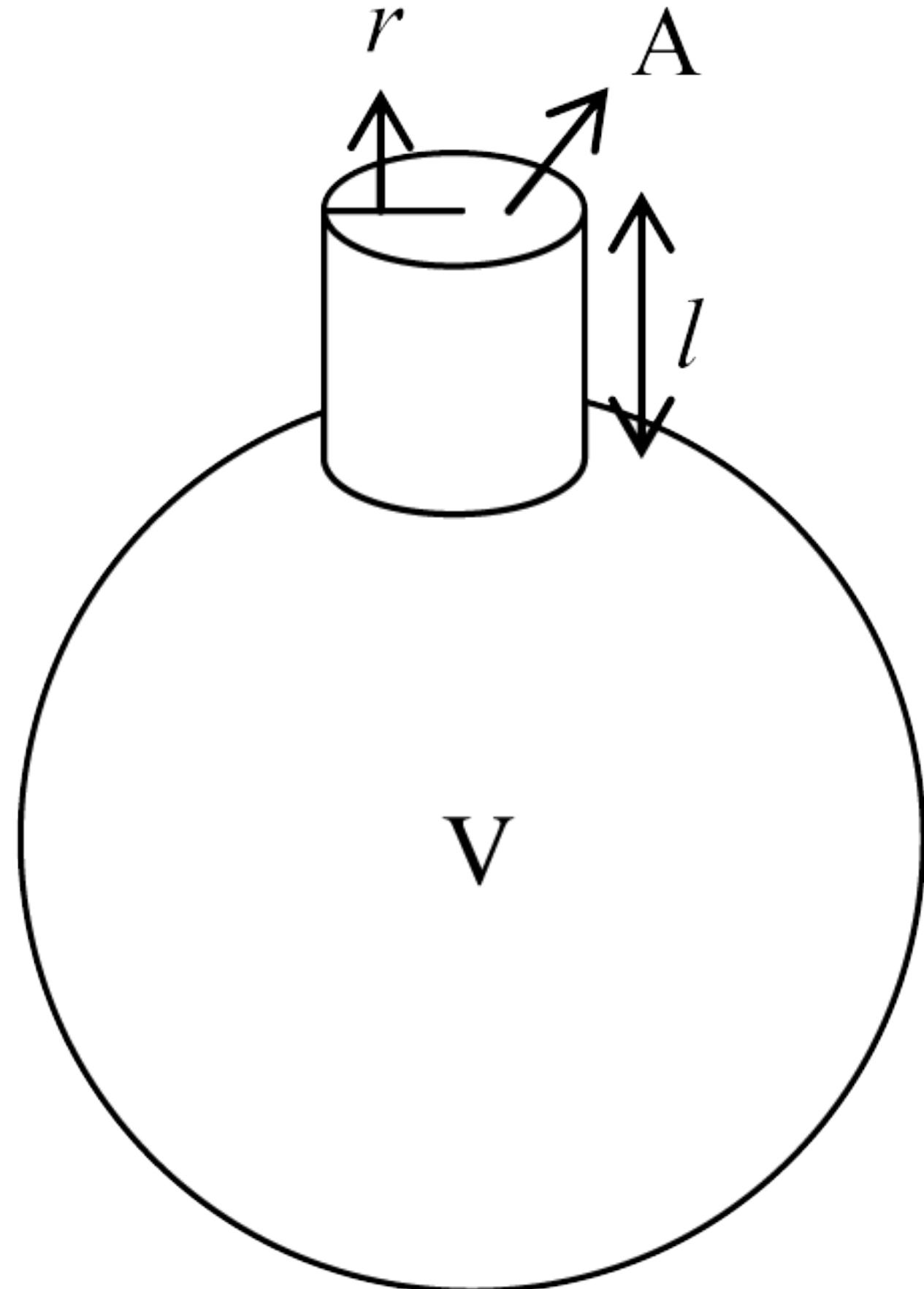


# Helmholtz resonator



$$f = \frac{v}{2\pi} \sqrt{\frac{A}{l_{\text{eff}} V}}$$

- Example:

$$r = 1 \text{ cm}, l = 2.7 \text{ cm}, V = 425 \text{ mL}, v = 346 \text{ m/s}$$

$$A = \pi r^2, 1 \text{ mL} = 10^{-6} \text{ m}^3$$

# **4. Fourier analysis & synthesis**

# Fourier's theorem

- **standing waves** are the “**building blocks**” for any complex vibration
- any complex periodic wave can be written as a **sum of harmonics**

$$y(t) = A_1 \sin(2\pi f_1 t + \phi_1) + A_2 \sin(2\pi f_2 t + \phi_2) + \dots$$

$$f_N = Nf_1, \quad N = 1, 2, \dots$$

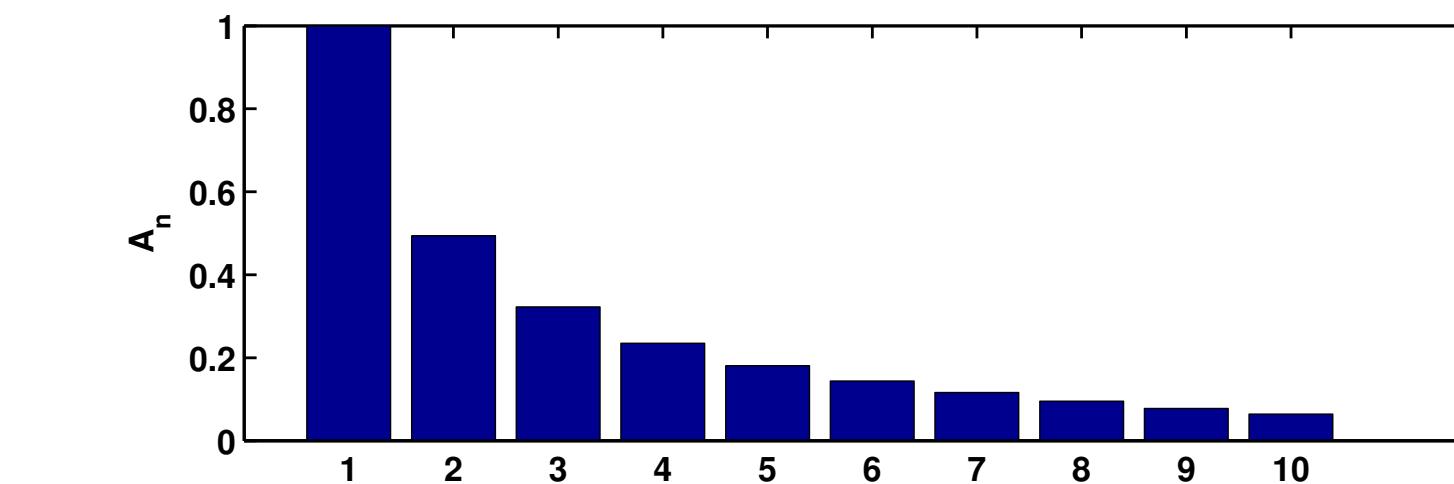
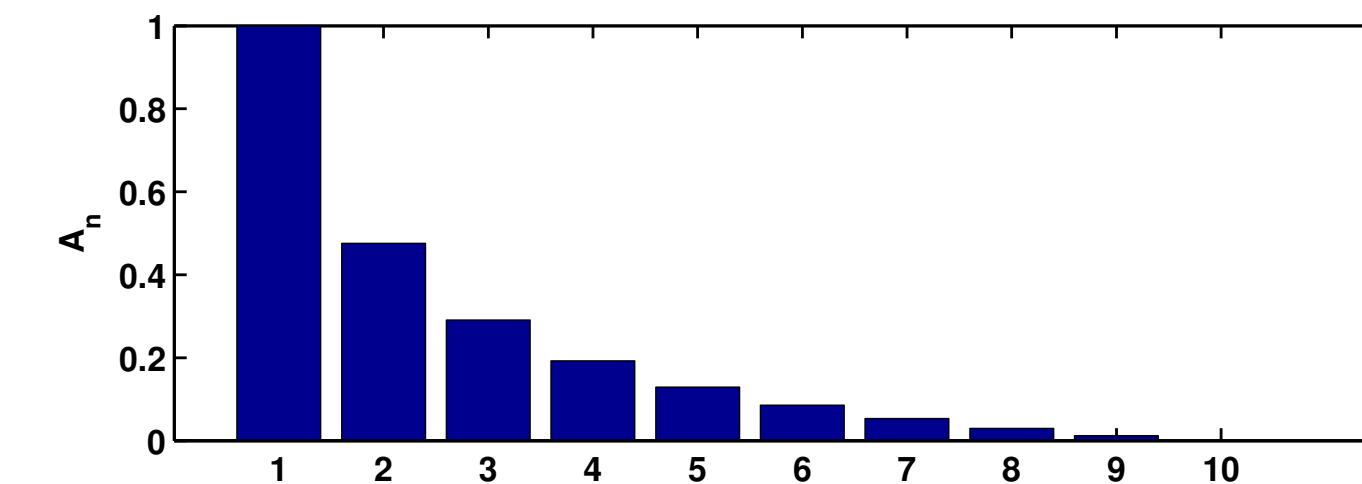
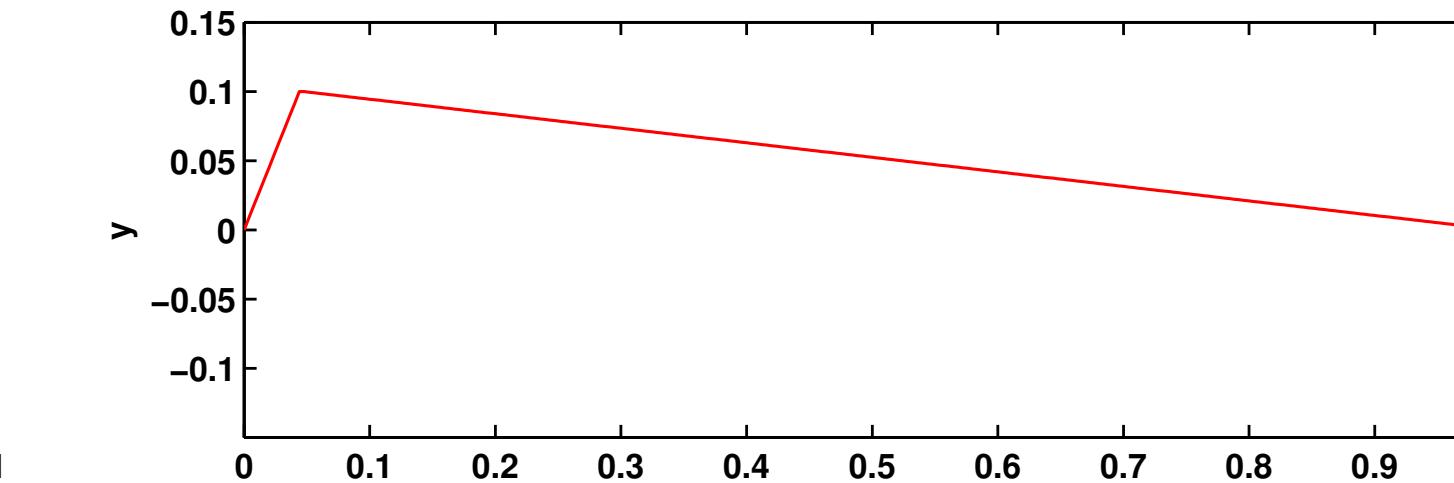
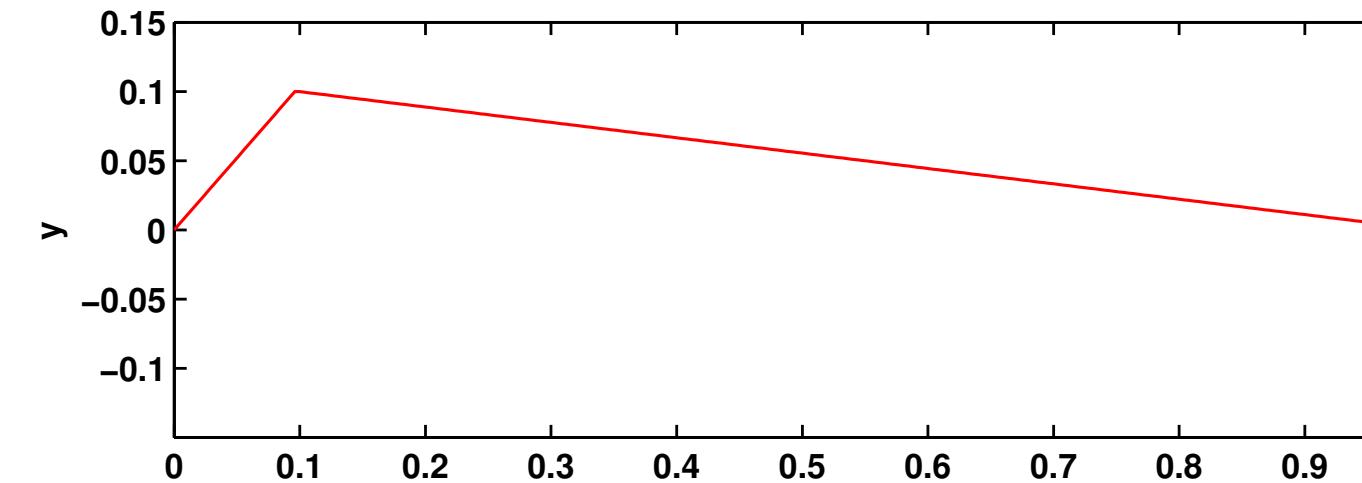
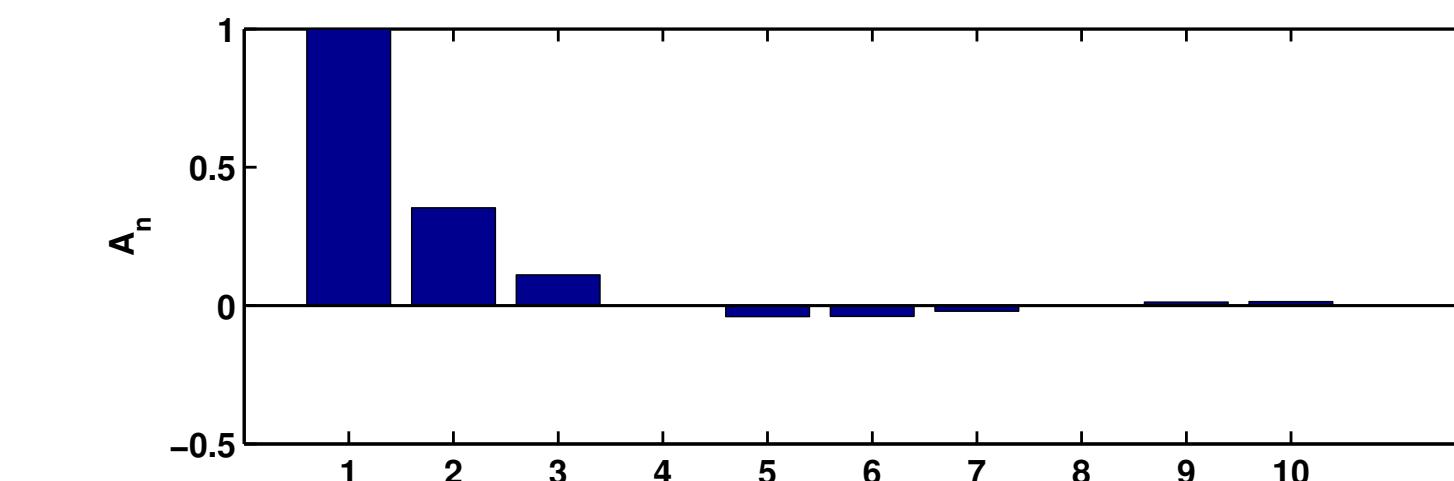
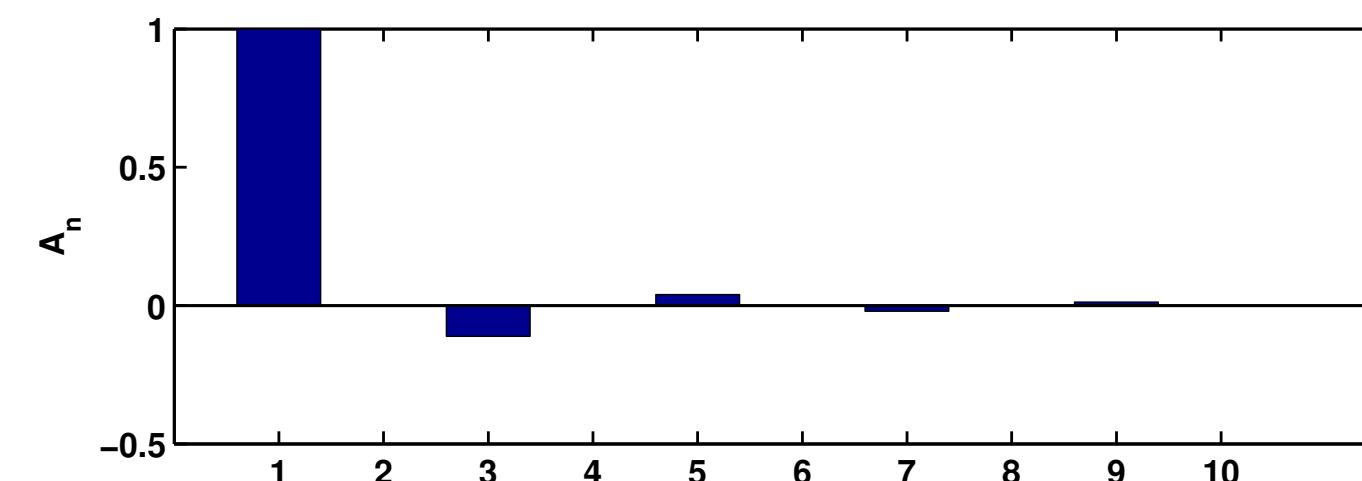
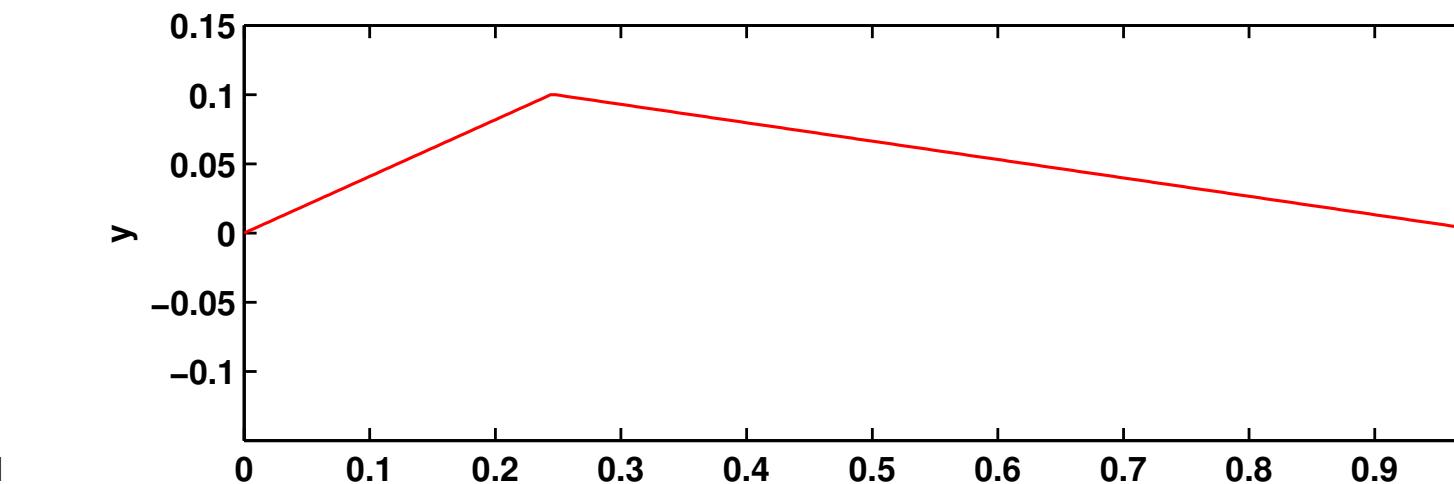
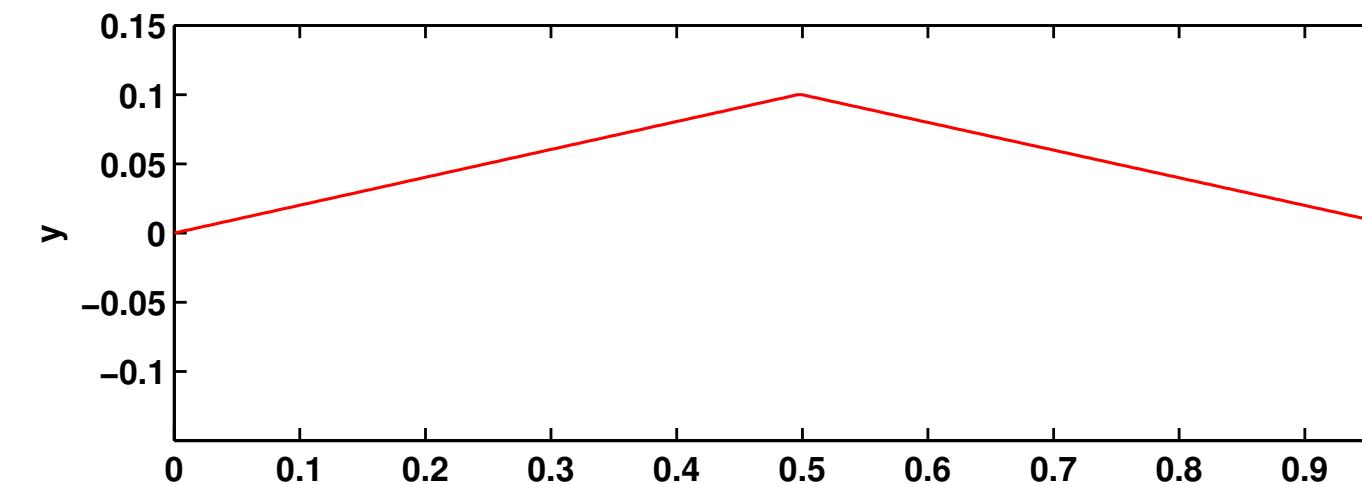
- **Ohm's law of hearing:** Phases have little effect on the timbre of the sound
- **Fourier analysis:** decomposing a complex periodic wave into its contributing harmonics
- **Fourier synthesis:** constructing a complex periodic wave by combining harmonics

# **5. String instruments**

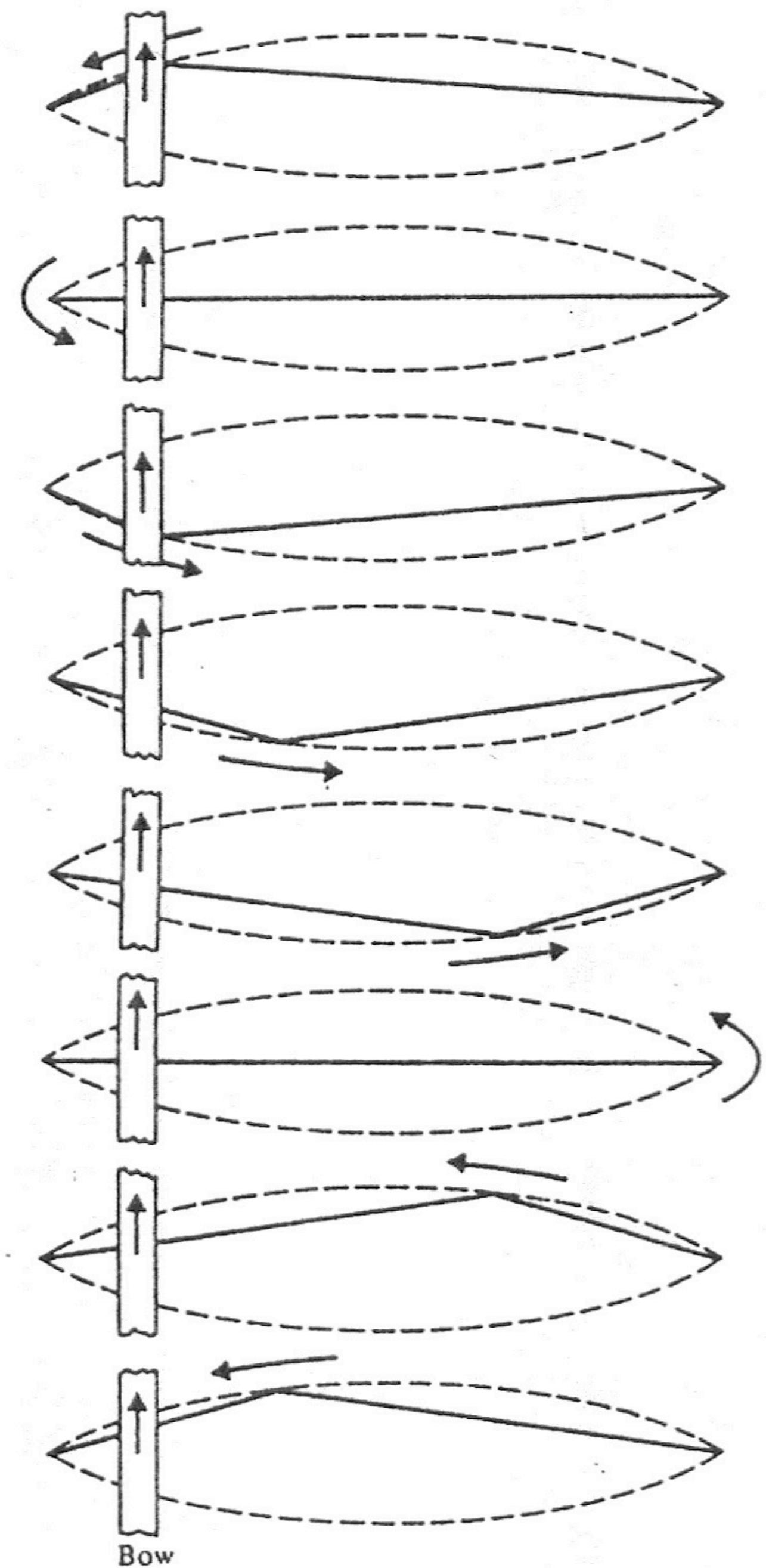
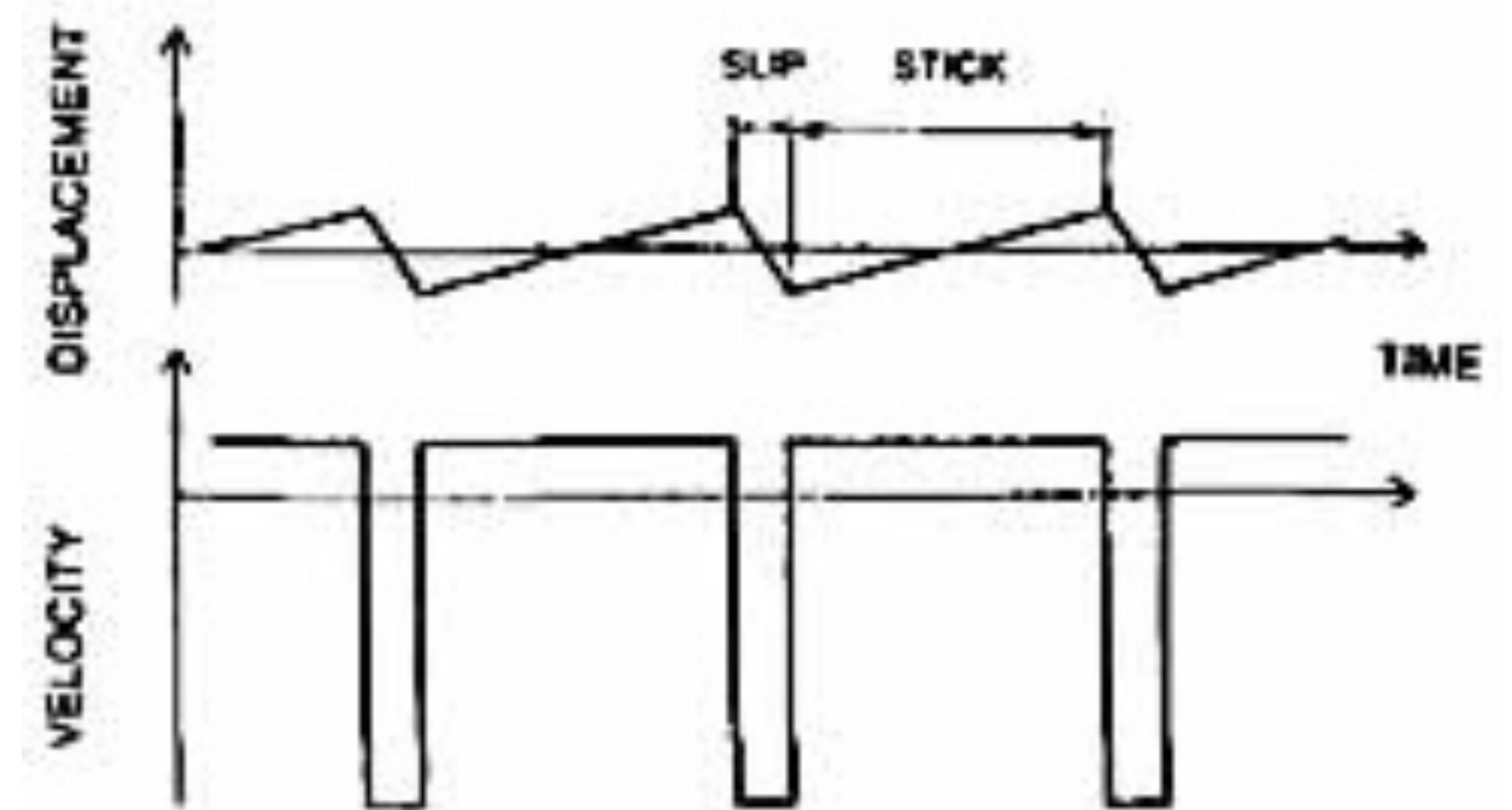
# Plucked versus bowed strings

- Plucked string: [https://www.youtube.com/watch?v=\\_X72on6CSL0](https://www.youtube.com/watch?v=_X72on6CSL0)
- Bowed string: <https://www.youtube.com/watch?v=6JeyiM0YNo4>
- iPhone guitar video: <https://www.youtube.com/watch?v=TKF6nFzpHBU>

# Fourier coefficients of a plucked string

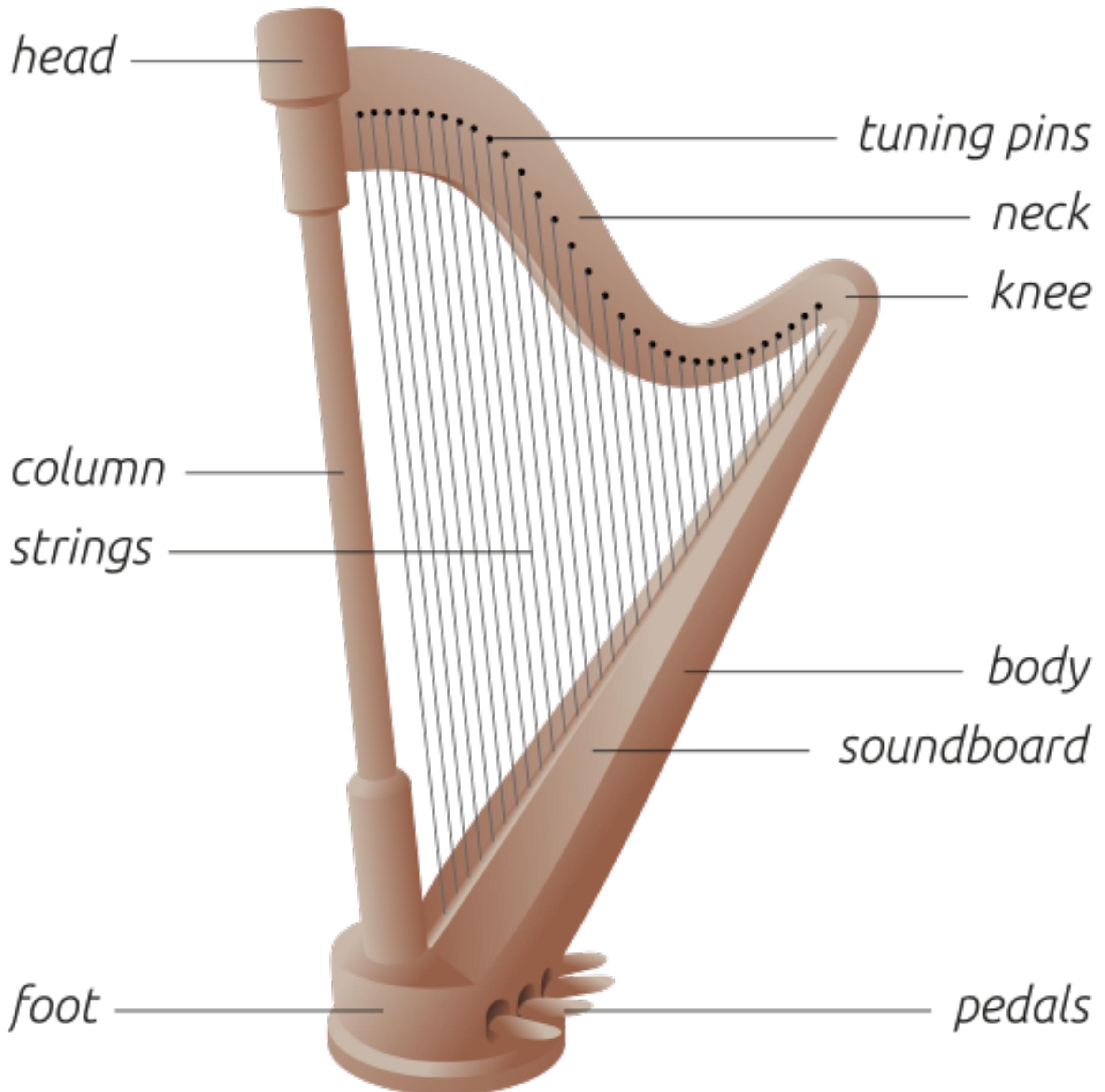


# Stick-slip motion of a bowed string



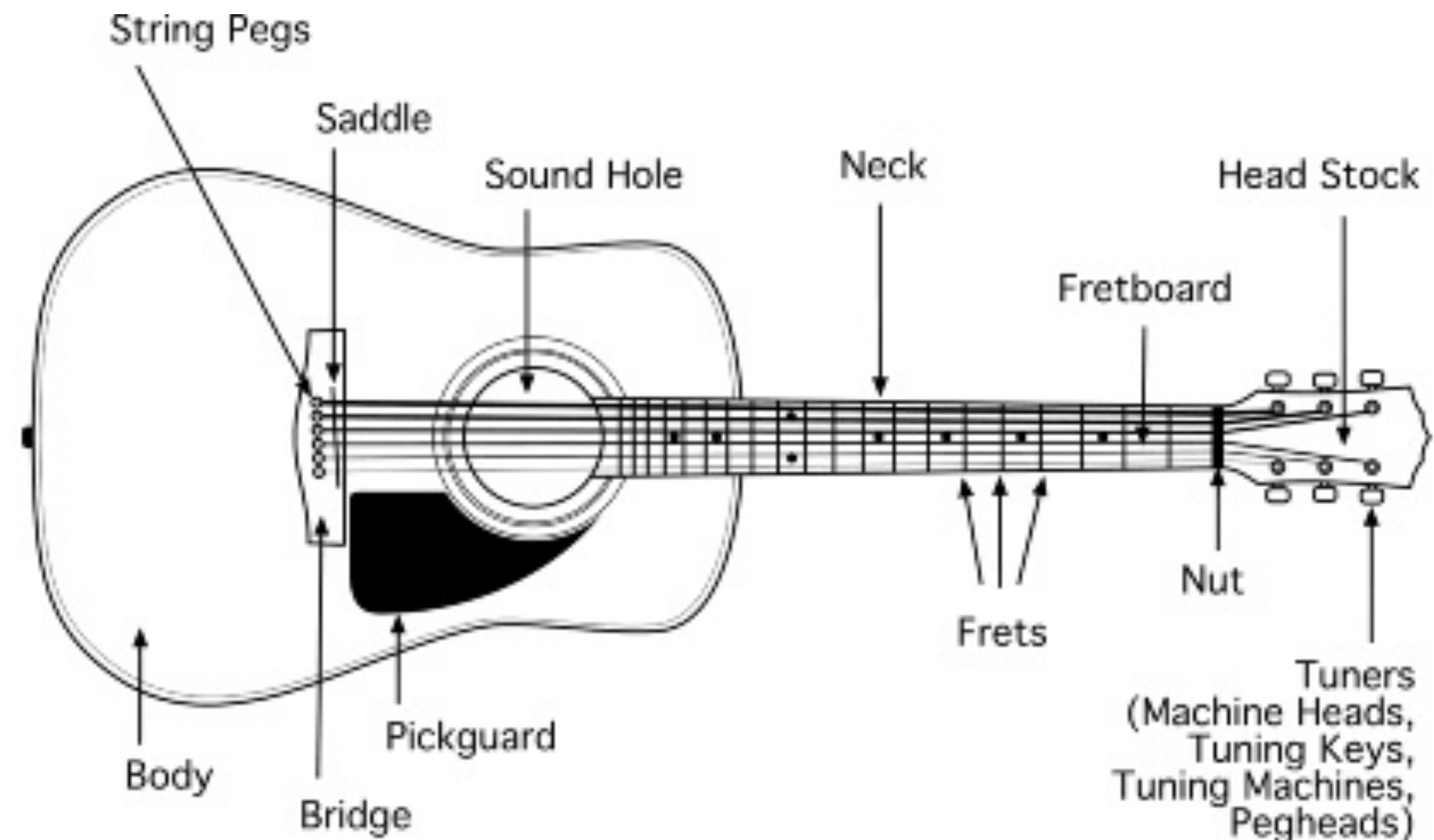
# Harp

- the strings have different fixed lengths and are plucked
- each string produces just one note -> need lots of strings
- foot pedal can change the note but only by only a semitone



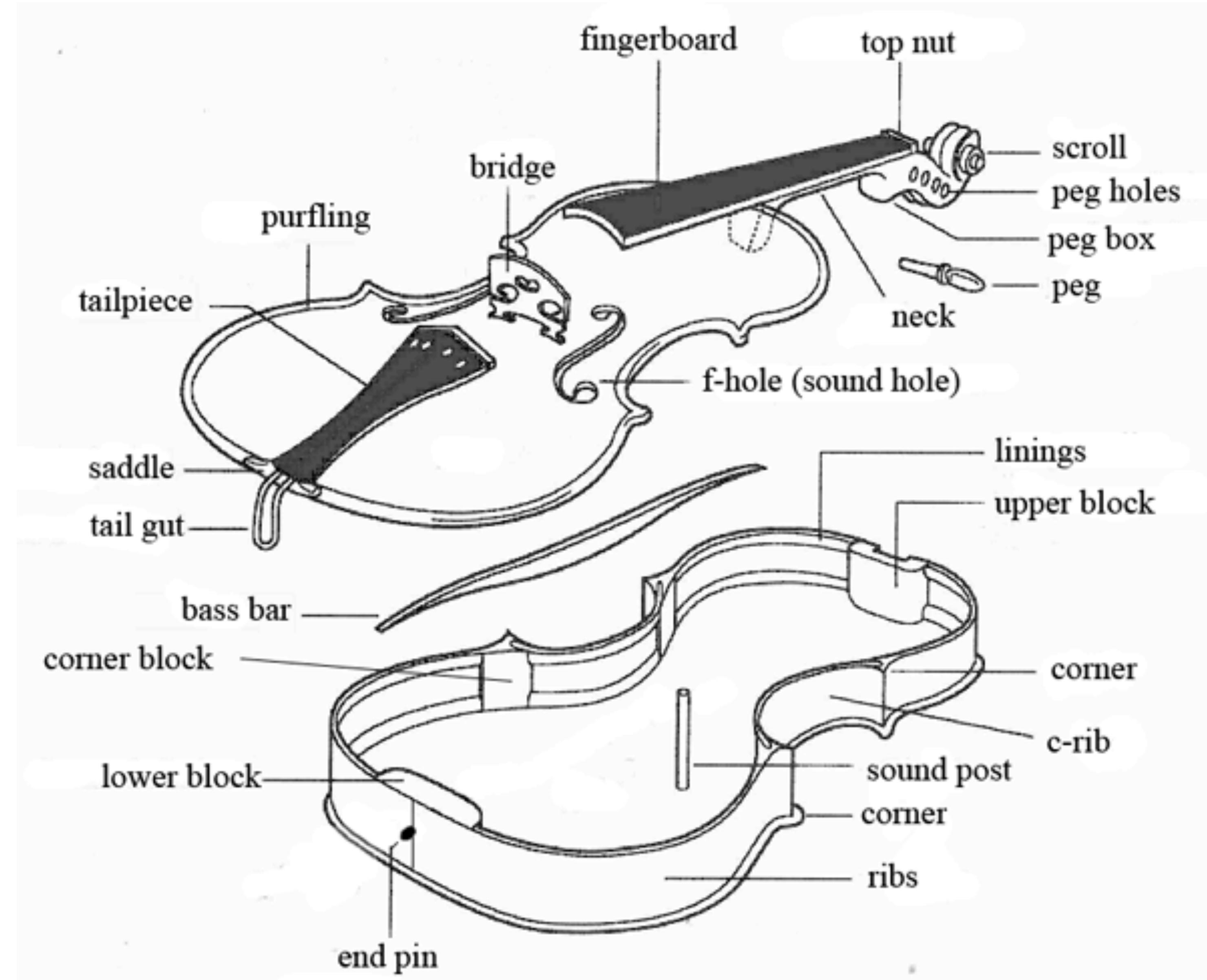
# Guitar

- strings are all the same length, but are made of different materials and are under different tensions
- get multiple notes per string by pressing against a fret
- frets -> fixed notes (like a piano keyboard)



# Violin

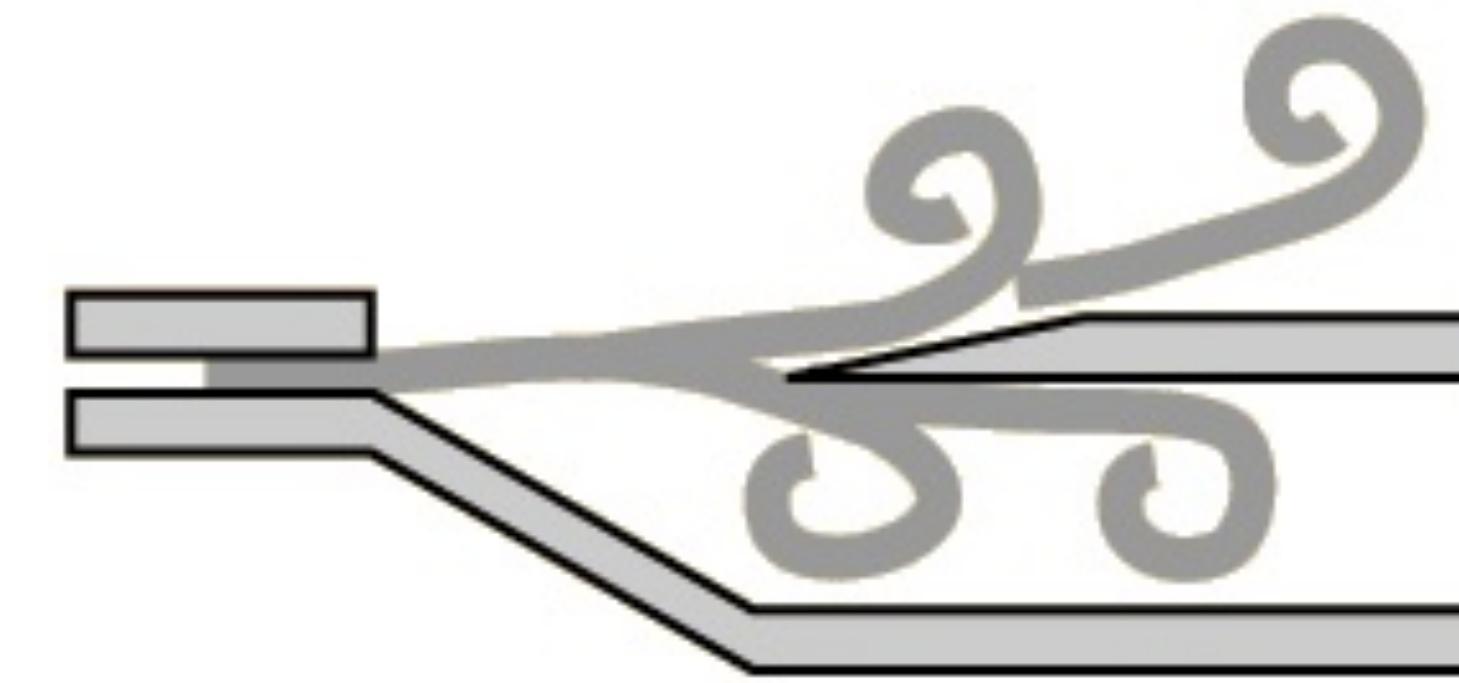
- strings are all the same length, but are made of different materials and are under different tensions
- get multiple notes per string by pressing against the neck
- no frets -> no fixed notes
- string vibrations are quickly damped if strings are plucked -> bowed instead
- can vary tone quality by adjusting the intensity of bowing

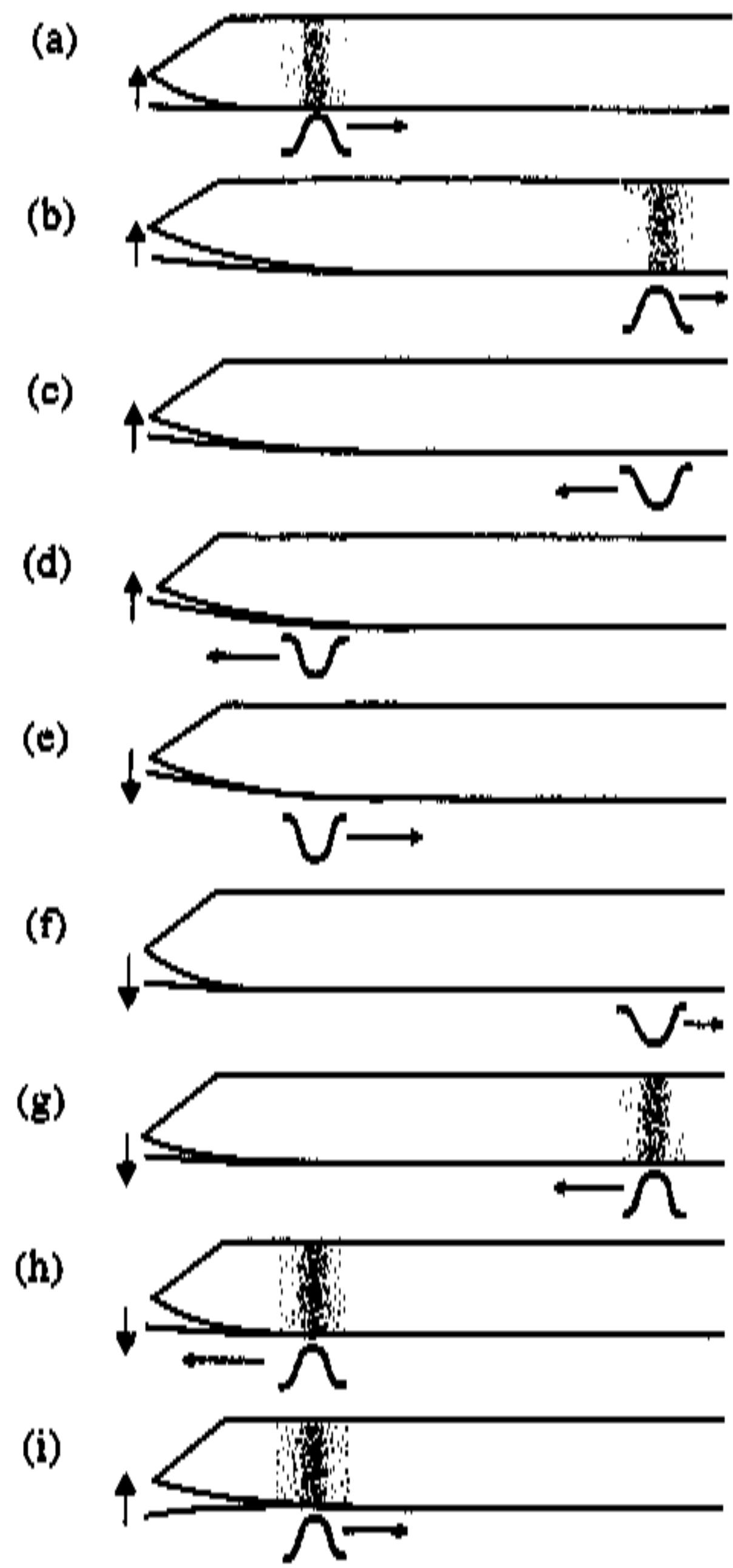


# **6. Wind instruments**

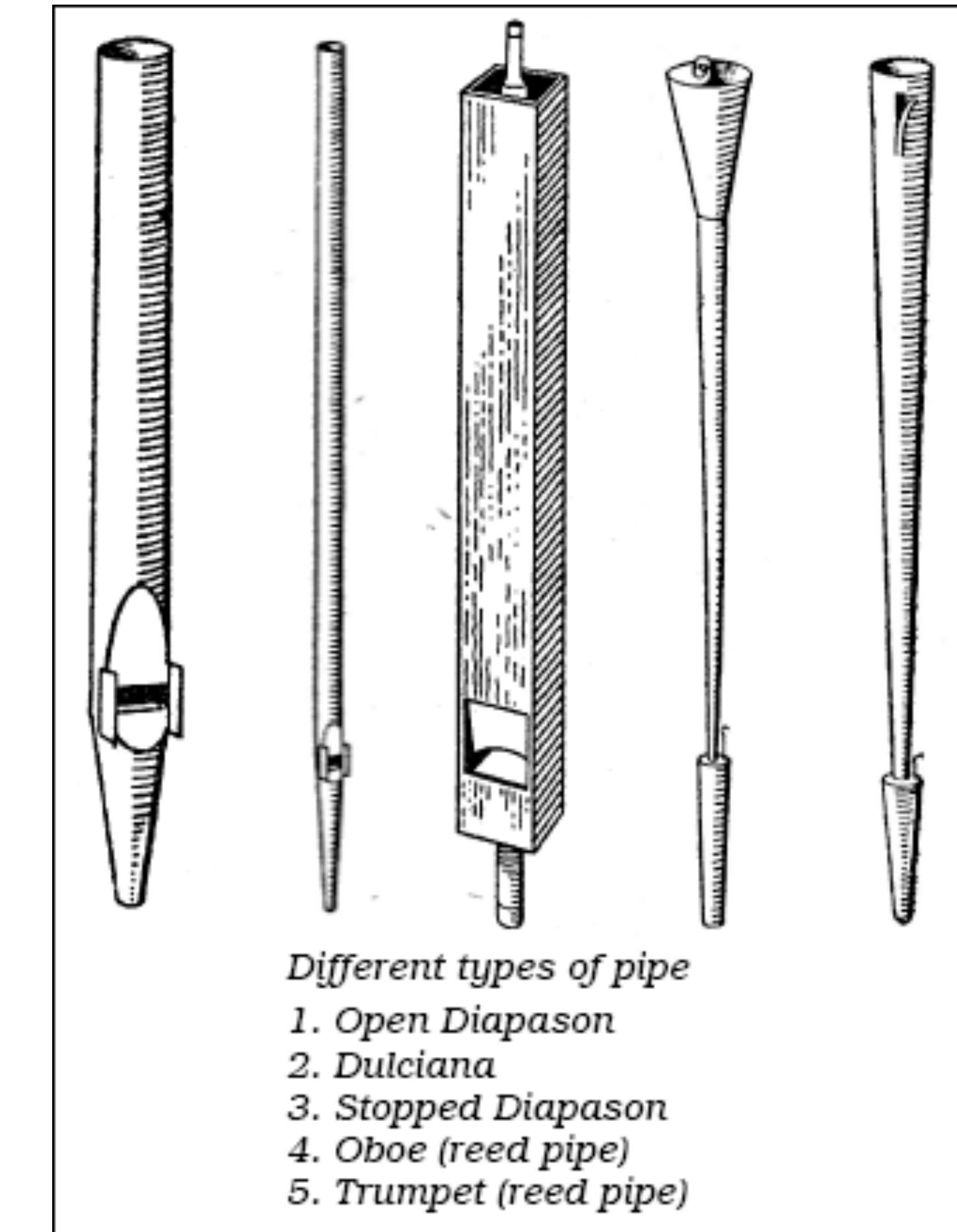
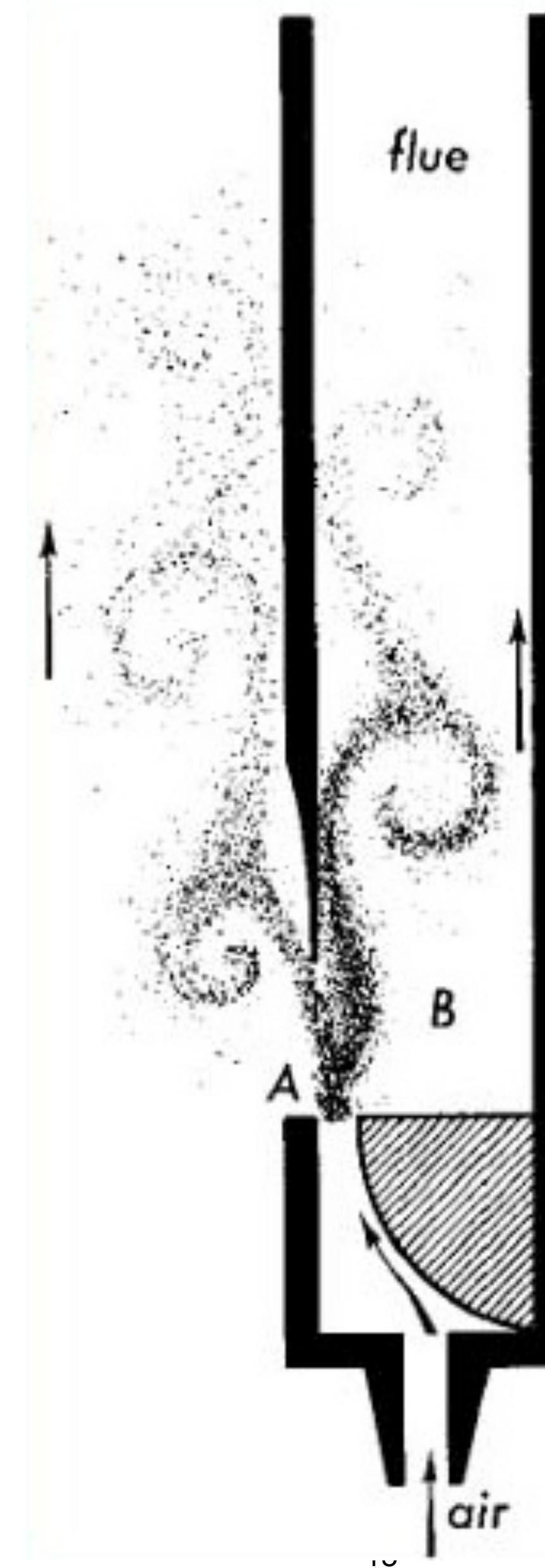
# Open and closed tubes

# Oscillating air streams vs vibrating reeds

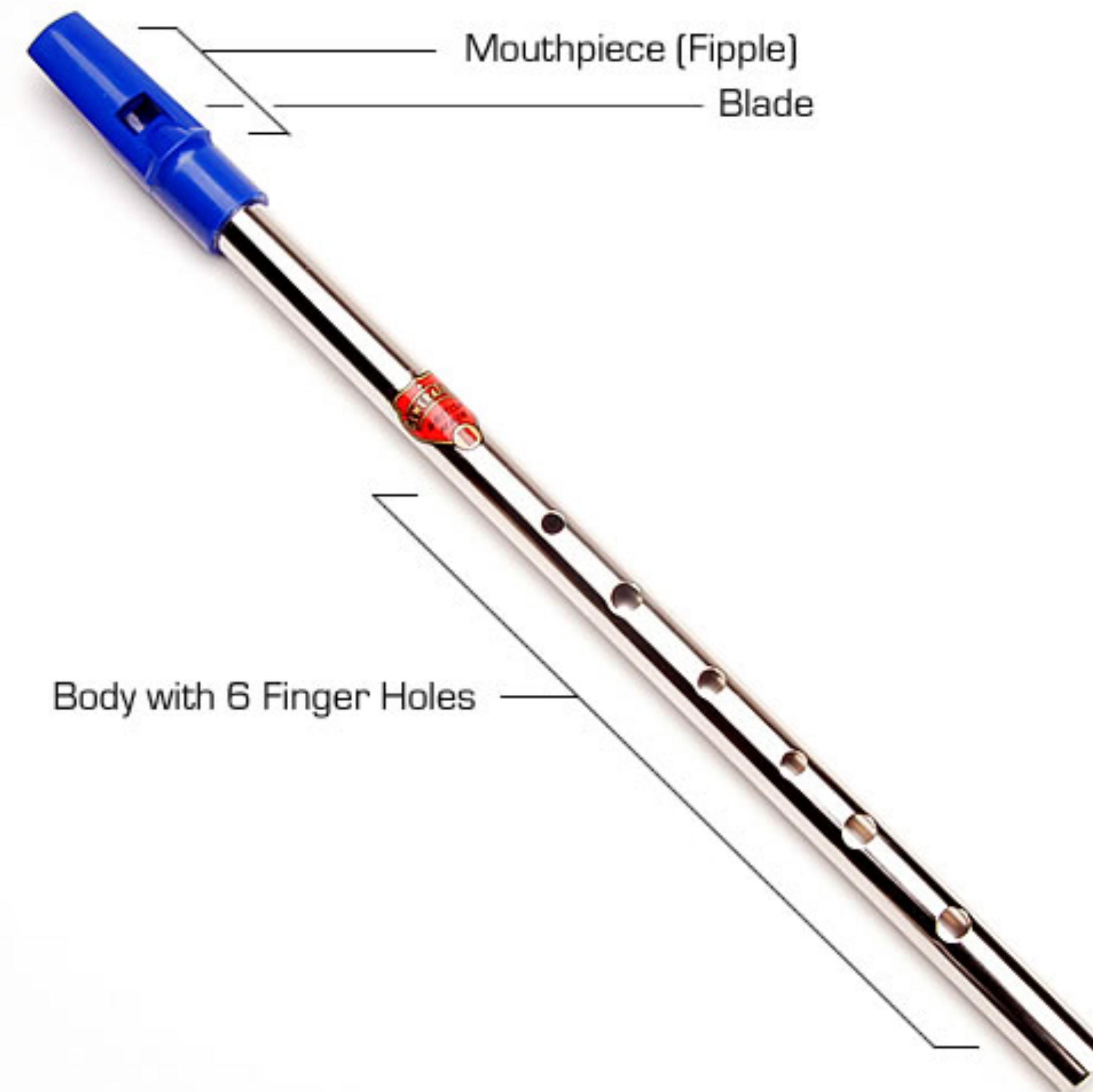




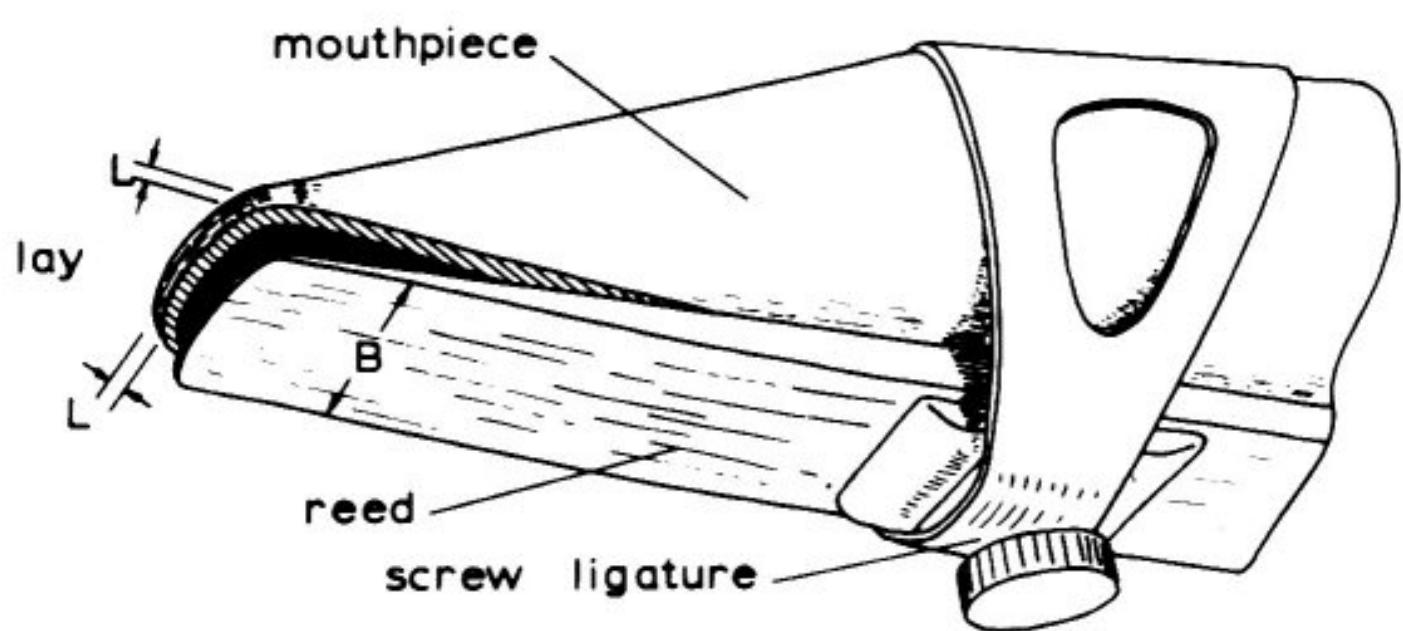
# Flue-organ pipe



# Penny whistle

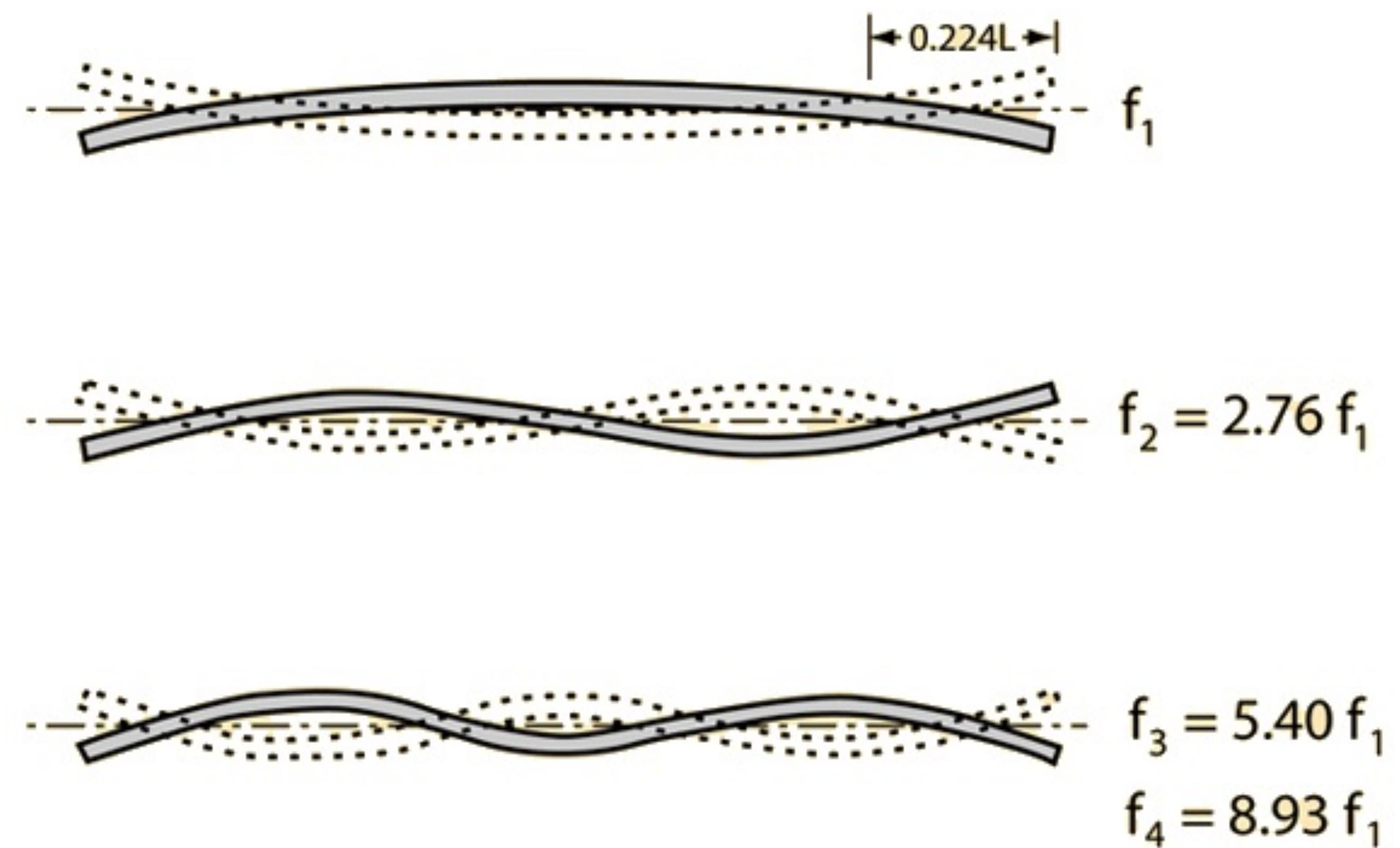


# Clarinet



# **7. Percussion instruments**

# Inharmonicity



# Vibrating drum head (2-d standing waves)

Mode 01



This is the  
lowest frequency  
mode.  
 $f_1$

Mode 02



$2.30 f_1$

Mode 11



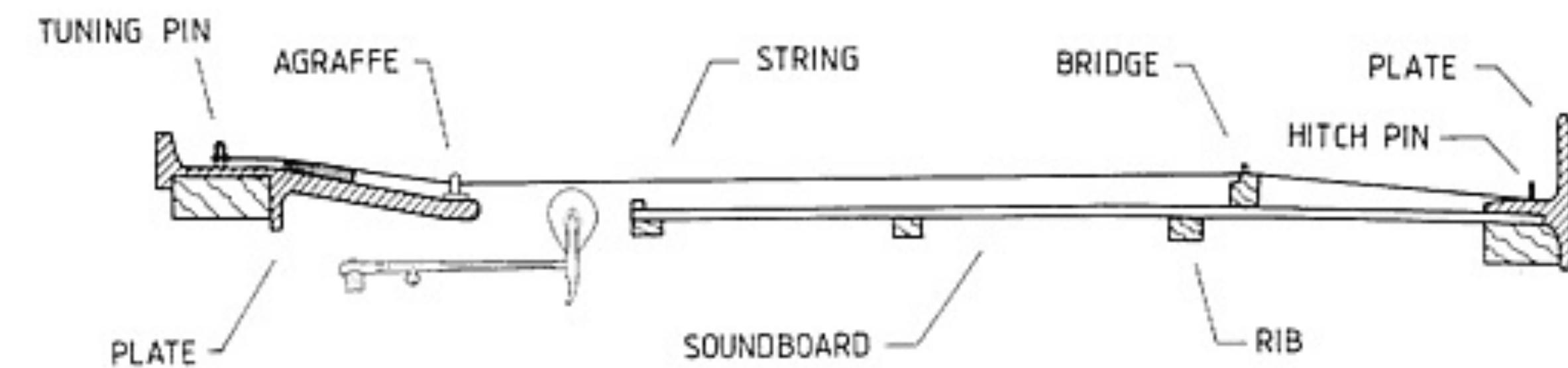
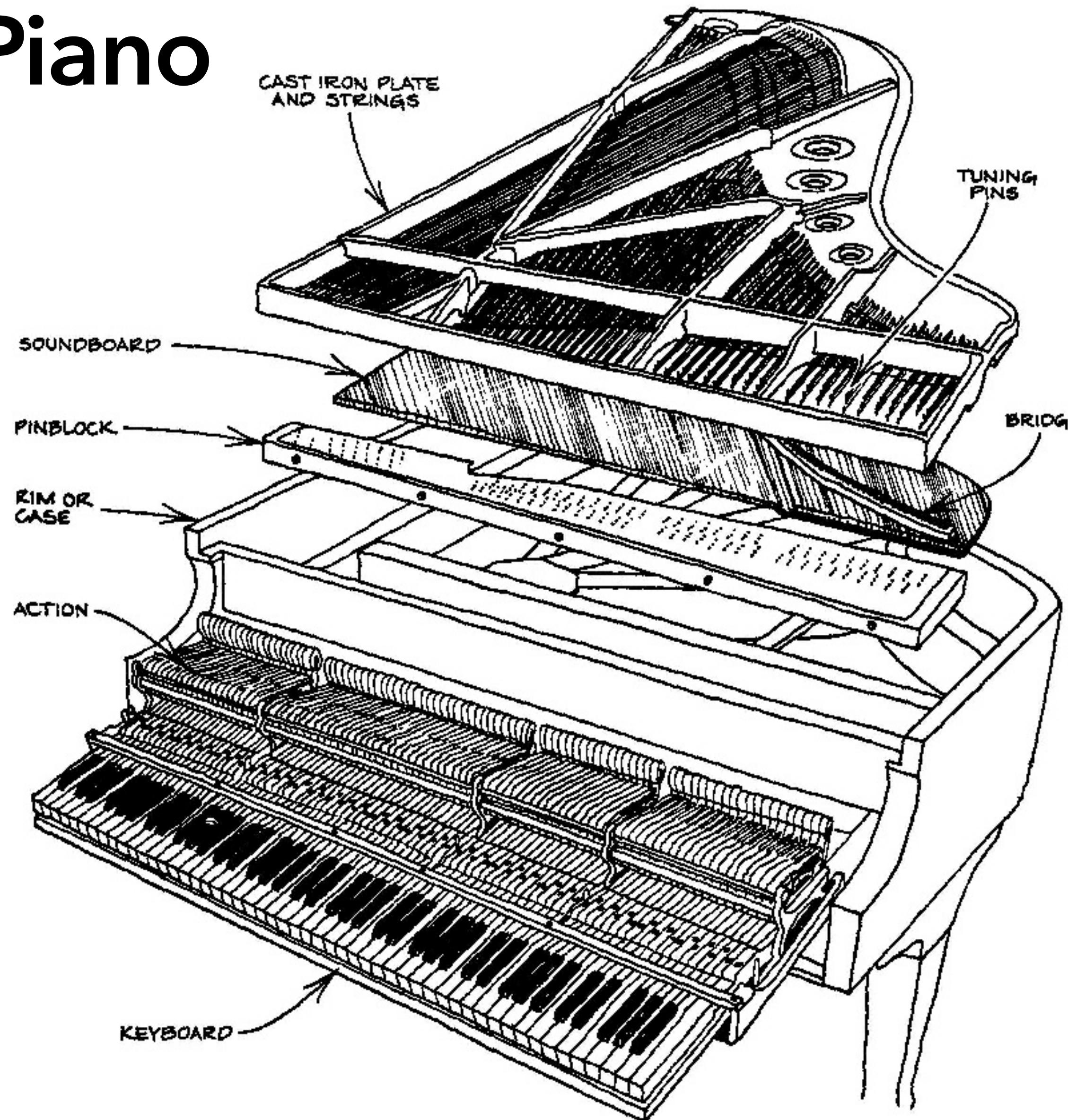
$1.59 f_1$

Mode 21



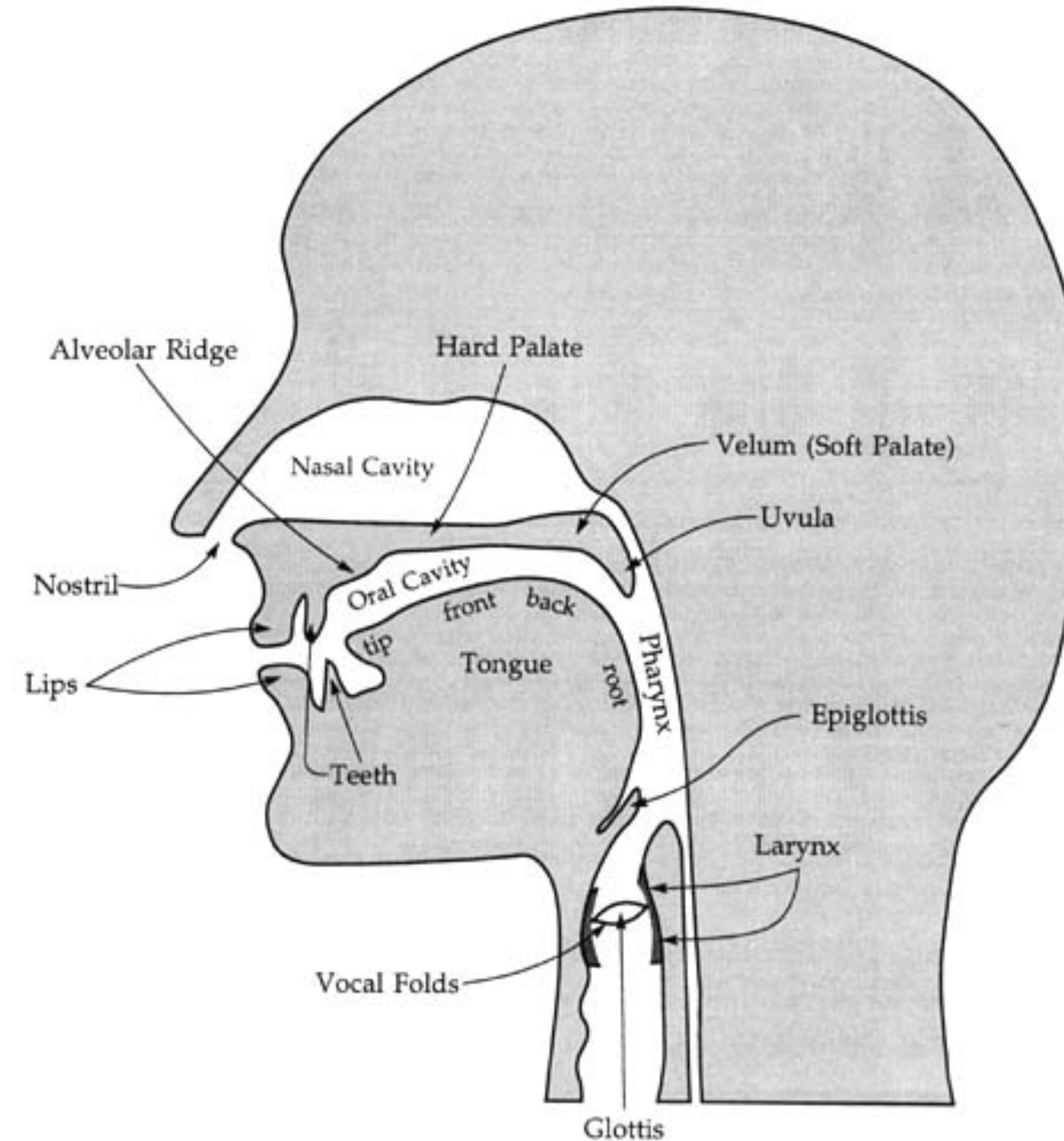
$2.14 f_1$  After Rossing

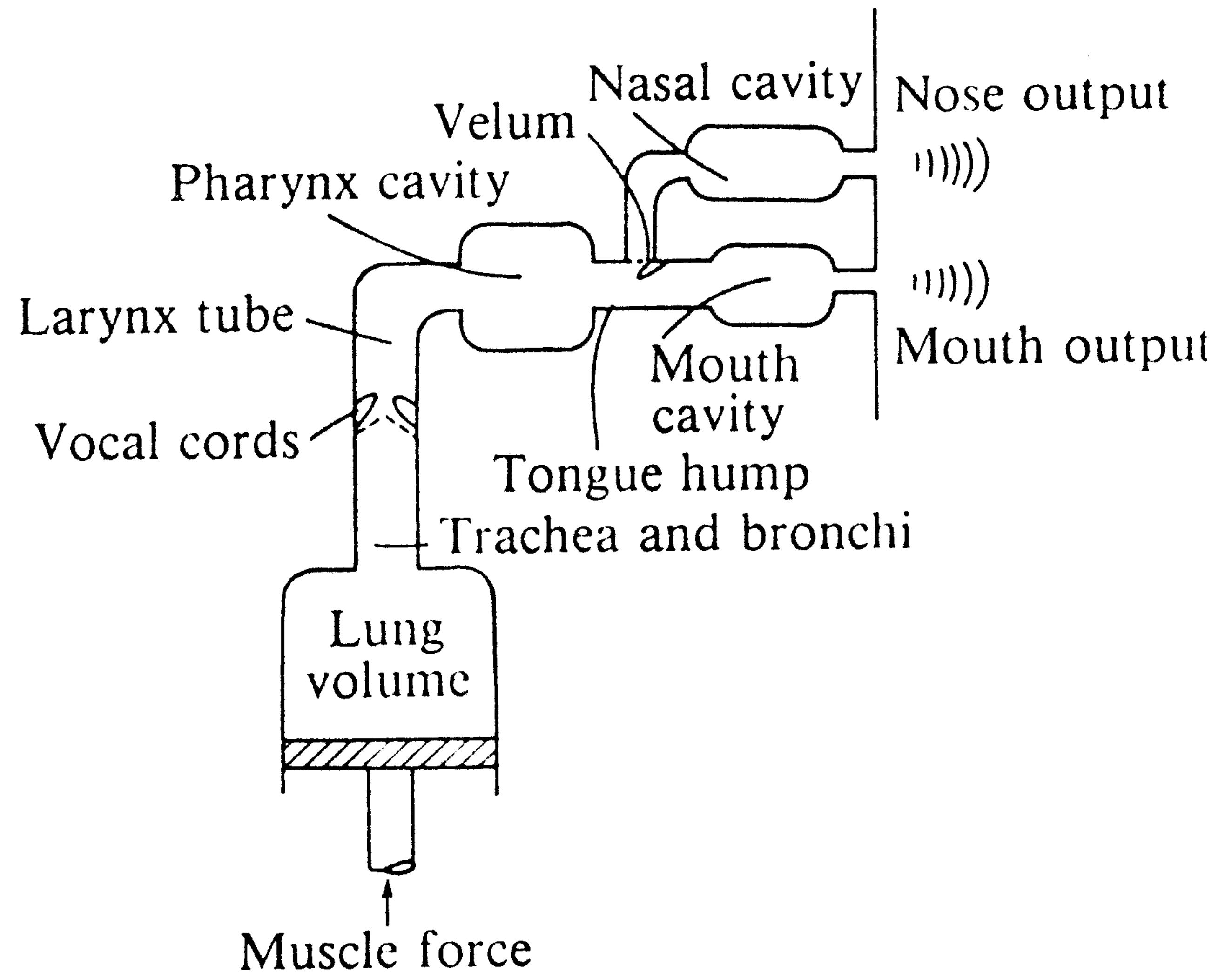
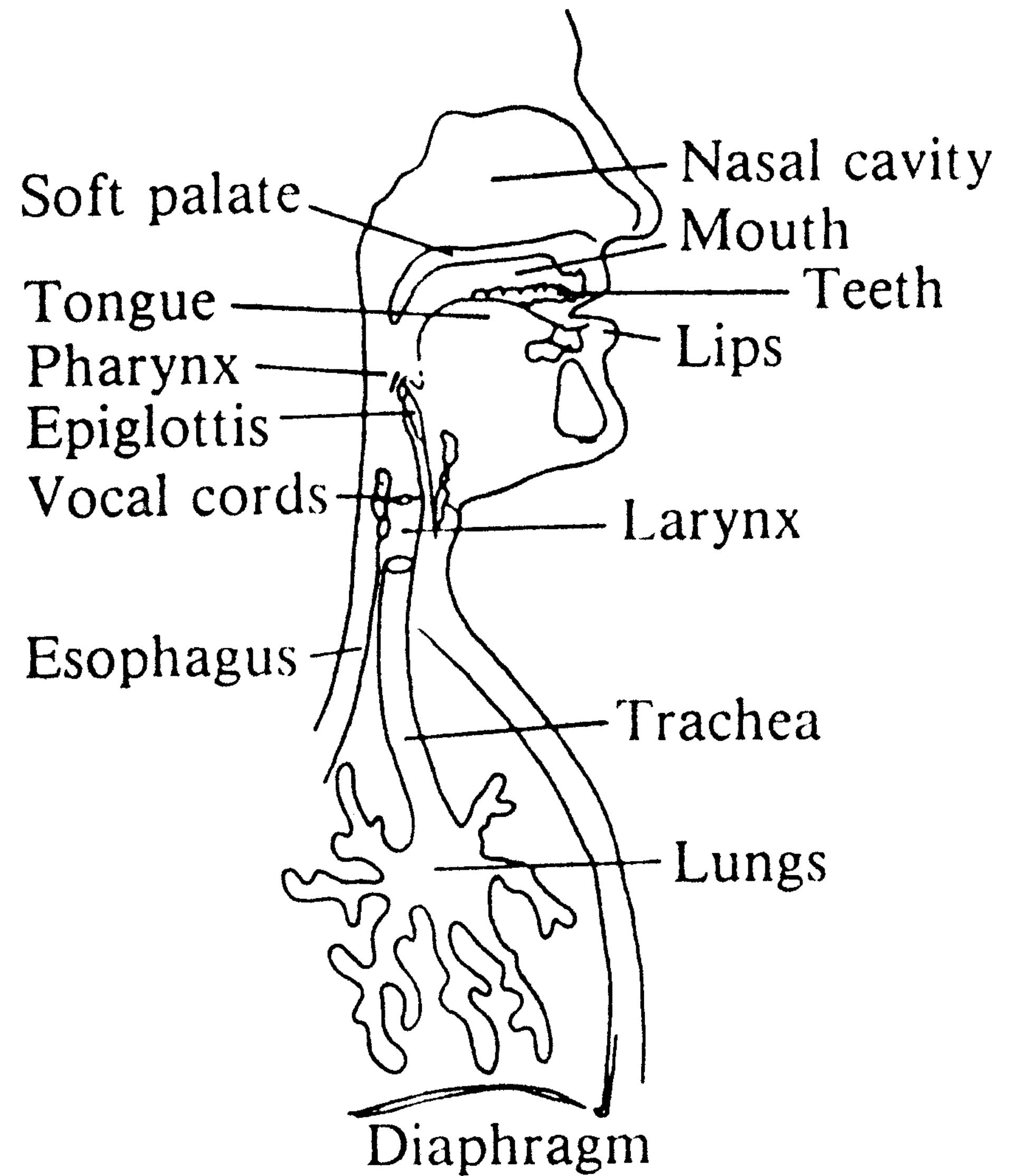
# Piano



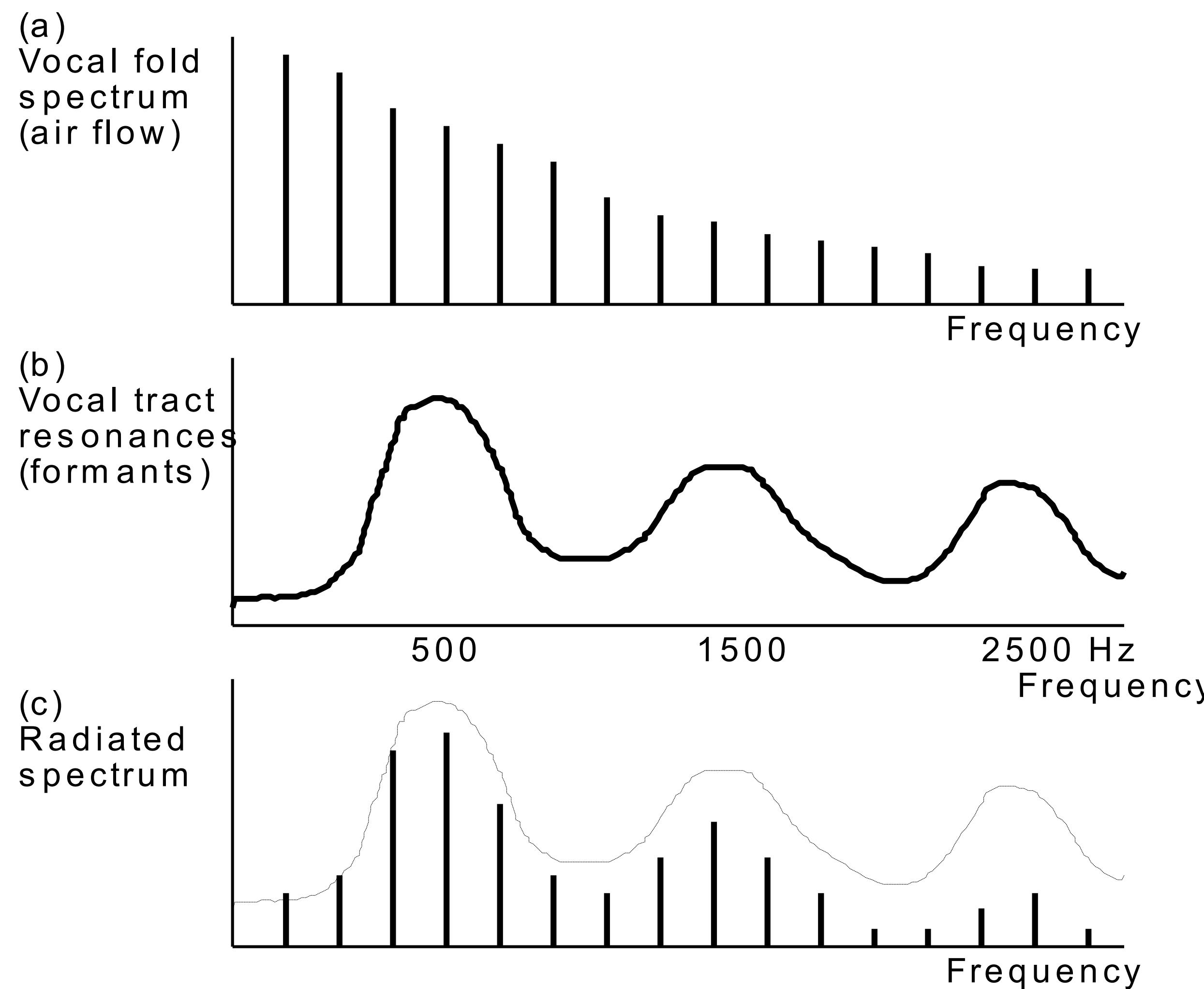
# 8. Voice

# Vocal organs





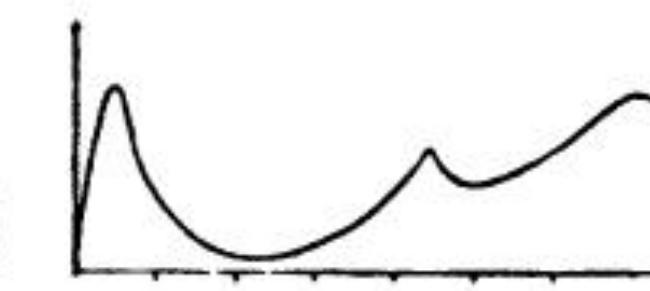
# Formants



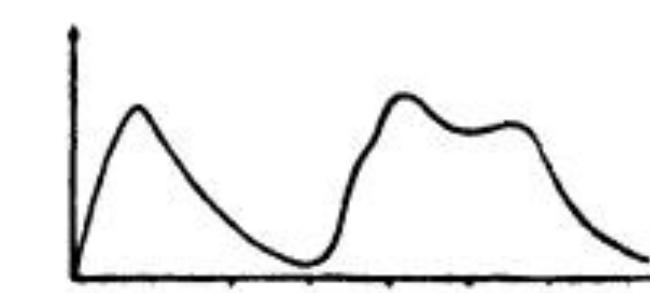
# Formant regions for different vowel sounds



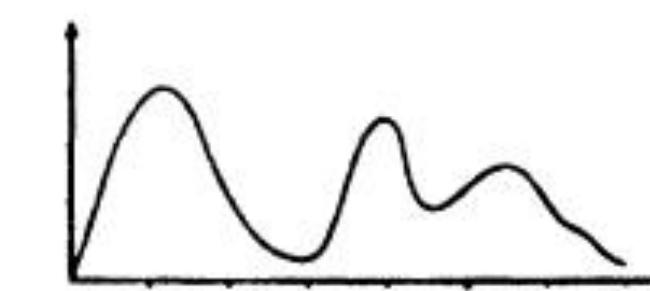
heed



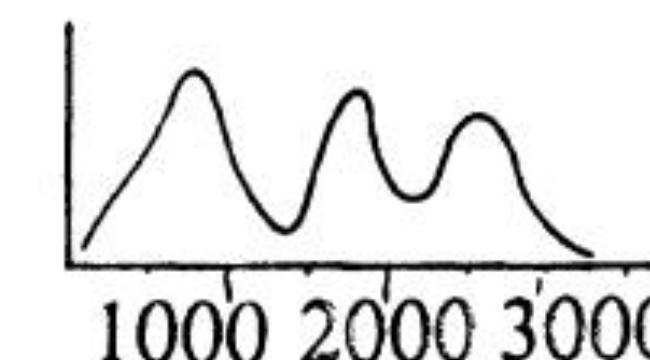
hid



head



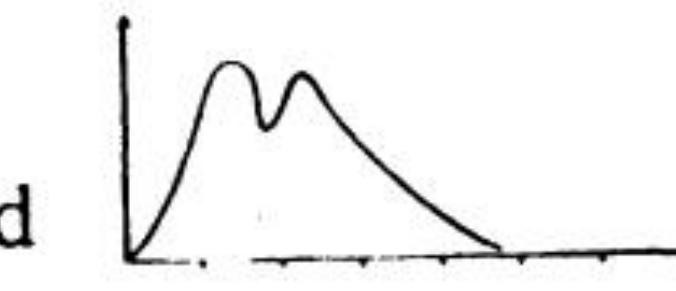
had



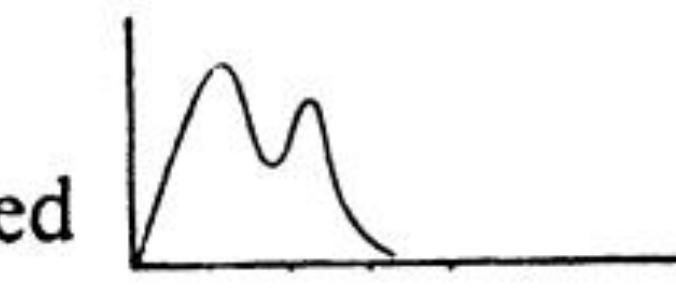
1000 2000 3000



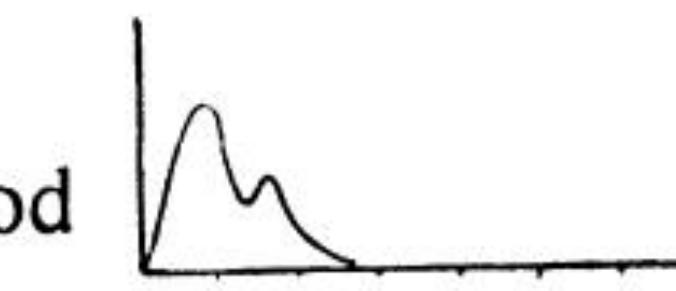
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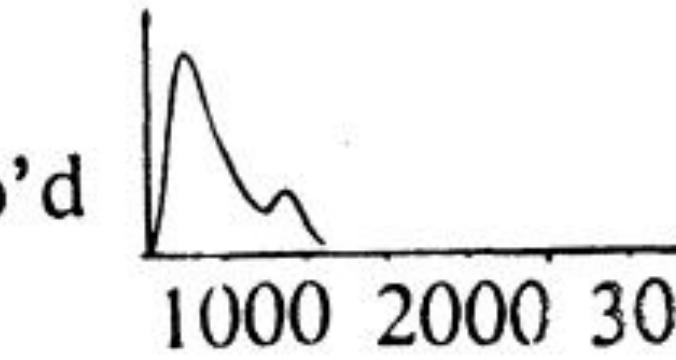
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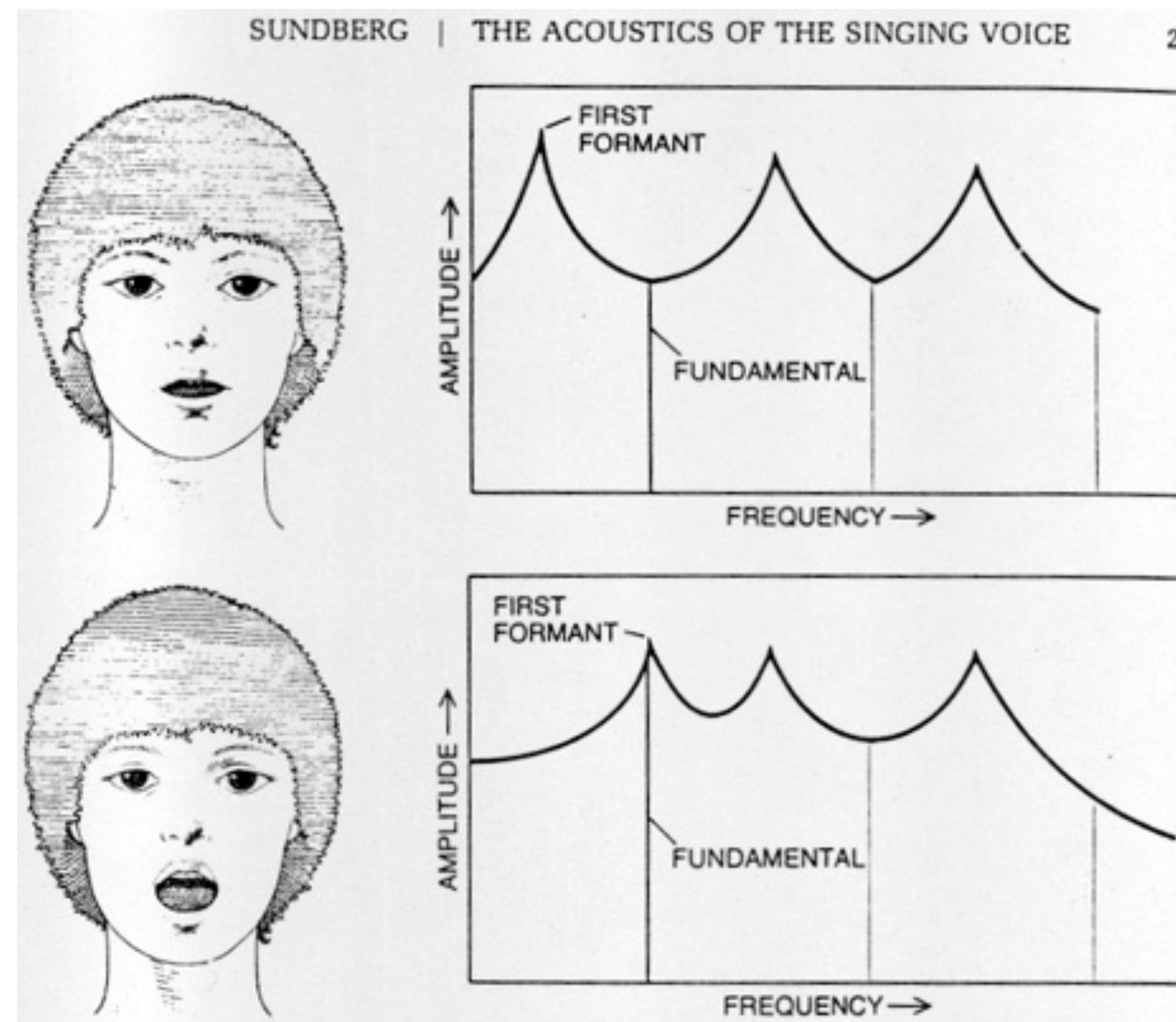
hood



who'd



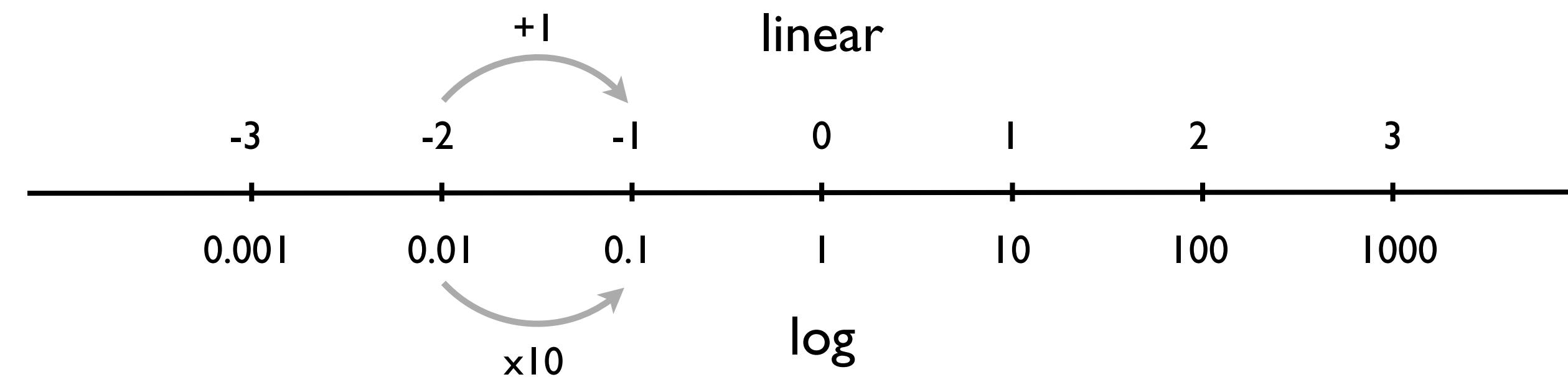
# Singing



# 9. Hearing

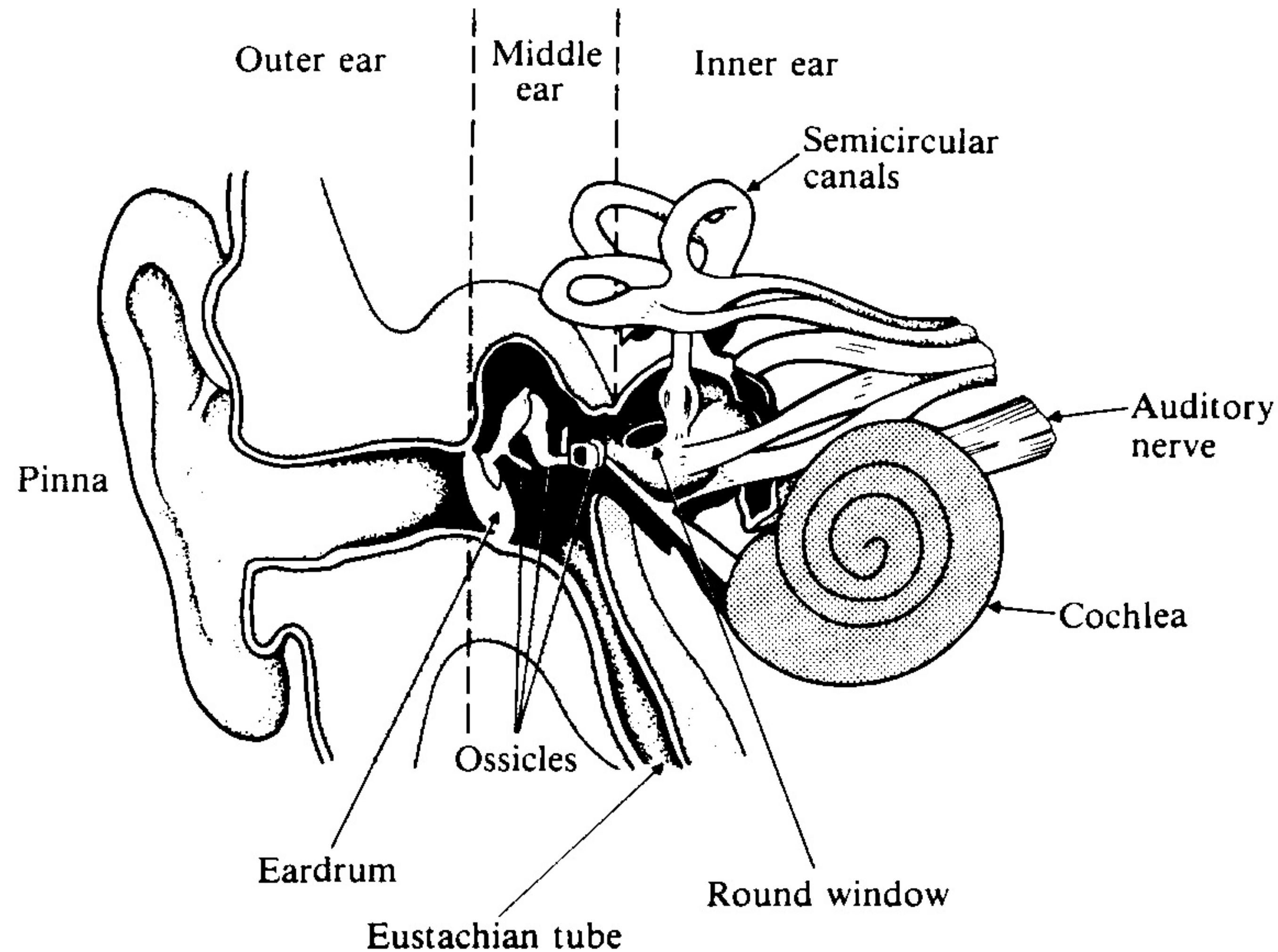
# Fechner's law and range of human hearing

- Fechner's law: Our senses respond logarithmically to external stimuli.

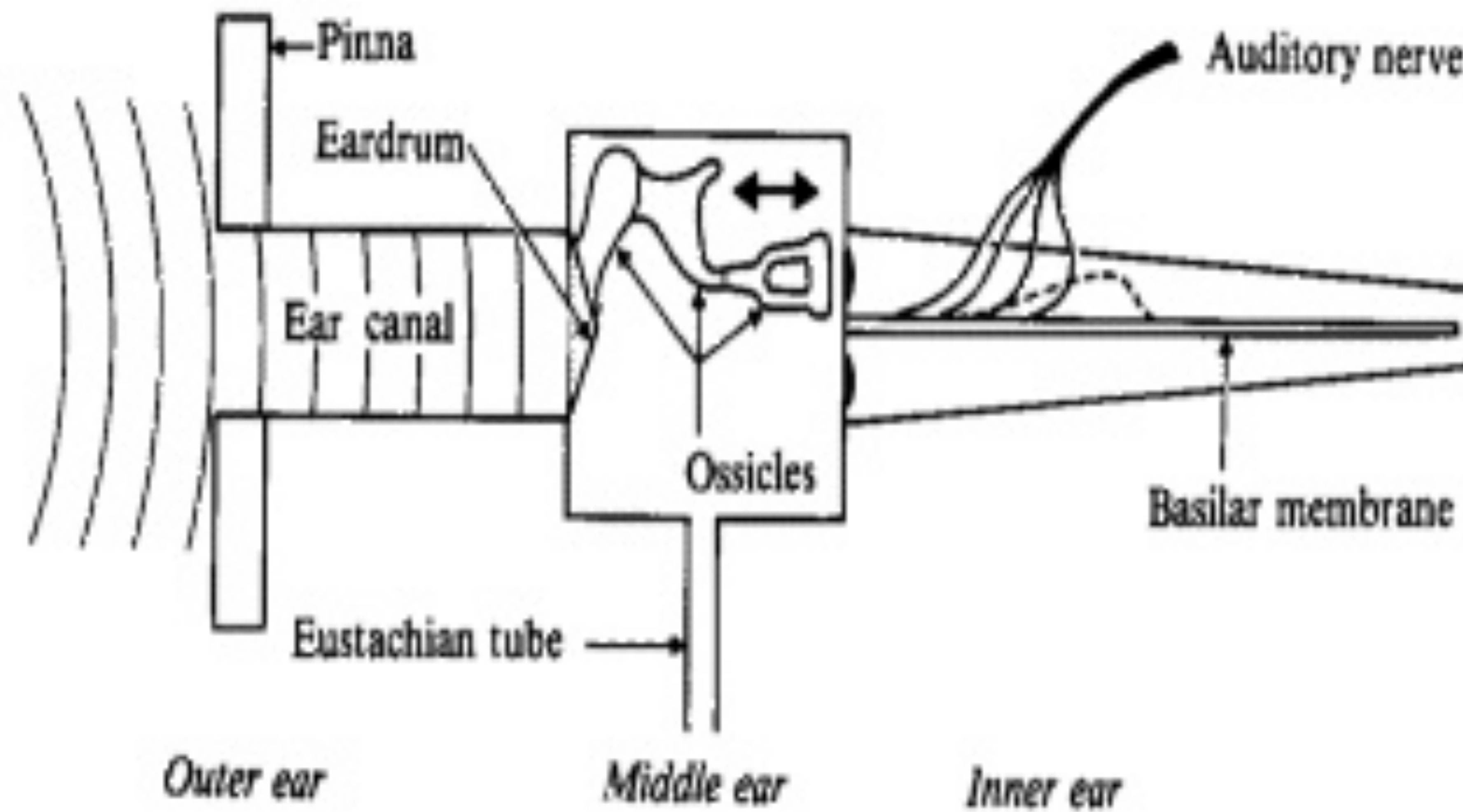


- Range of hearing
  - Pitch (frequency): 20 – 20,000 Hz (~10 octaves)
  - Loudness (intensity):  $10^{-12}$  –  $1 \text{ W/m}^2$  (12 orders of magnitude)
- Eye is sensitive to ~1 octave in colors (frequency) and 5 orders of magnitude in brightness (intensity)
- Q: So which do you think is more sensitive—our sense of sight or our sense of hearing?

# Anatomy of human ear



# Anatomy of human ear - continued



# Sound localization – binaural hearing

- High frequency ( $> 4000$  Hz) sounds – intensity difference
- Low frequency ( $< 4000$  Hz) sounds – time of arrival

# **10. Loudness**

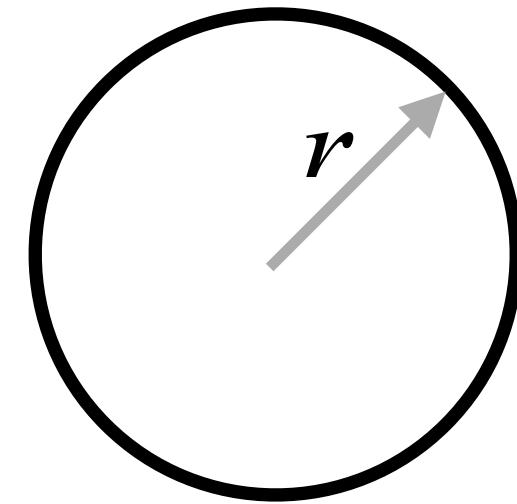
# Loudness – overview

(compare two sounds, or compare one sound to the threshold of hearing)

- our perception of the relative strength of a sound
- depends on the intensity and frequency of the sound
- logarithmic response to intensity (consistent with Fechner's law)
- several different ways of quantifying loudness:
  - intensity
  - sound intensity level
  - sound loudness level
  - subjective loudness

# Intensity

- Intensity is the power in a sound wave divided by the area it passes through (Watts/m<sup>2</sup>)



$$I = \frac{P}{4\pi r^2}$$

(sound is less intense the farther you are away from the source of the sound)

- Range of intensities:
  - $I_0 = 10^{-12} \text{ W/m}^2$  (**threshold of hearing** at  $f = 1000 \text{ Hz}$ )
  - $I = 1 \text{ W/m}^2$  (**threshold of pain** at  $f = 1000 \text{ Hz}$ )
- Intensity is proportional to the square of the amplitude of the sound wave:  $I \propto (\Delta p)^2$
- Intensities add: (intensity of 2 violins = twice the intensity of 1 violin)

# Sound Intensity Level (SIL, dB)

- Sound intensity level is the logarithm of the intensity compared to the threshold of hearing:

$$\text{SIL} = 10 \log(I/I_0) \text{ dB}$$

- Threshold of hearing:  $I = I_0 = 10^{-12} \text{ W/m}^2 \Rightarrow \text{SIL} = 0 \text{ dB}$

- Threshold of pain:  $I = 1 \text{ W/m}^2 \Rightarrow \text{SIL} = 120 \text{ dB}$

$$\Delta\text{SIL} = 10 \log(I_2/I_1) \text{ dB}$$

(comparing two intensities)

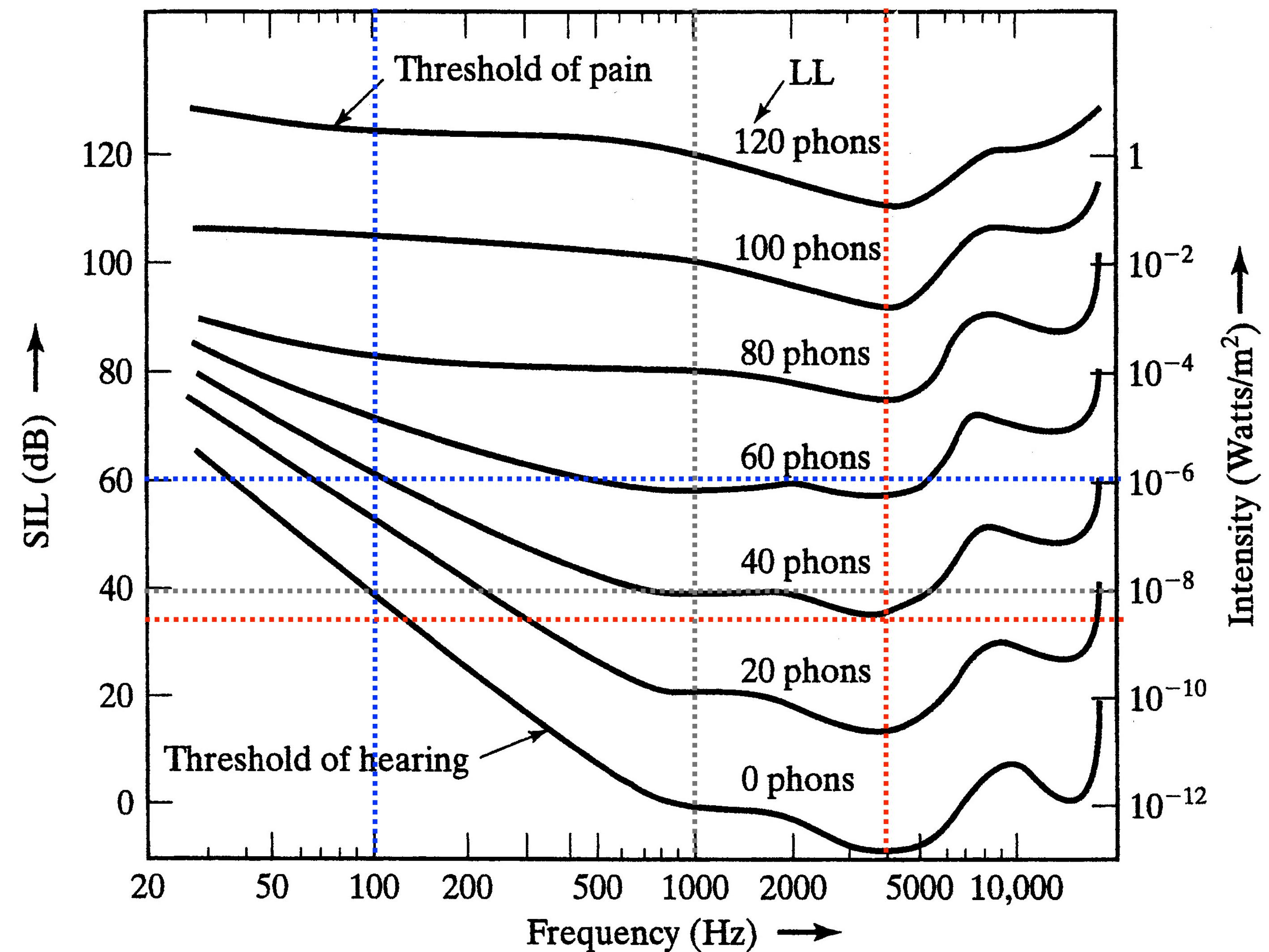
- 2x intensity:  $\Delta\text{SIL} = 10 \log(2) \text{ dB} = 3 \text{ dB}$

- 10x intensity:  $\Delta\text{SIL} = 10 \log(10) \text{ dB} = 10 \text{ dB}$  (perceived as "twice as loud")

- Just noticeable difference (JND):  $\Delta\text{SIL} = 1 \text{ dB} \Leftrightarrow I_2 = 1.26 I_1$

# Sound Loudness Level ( $L_L$ , phon)

- Human ear responds differently to different frequencies (Fletcher-Munson curves)
- Most sensitive to frequencies  $\sim 4000$  Hz
- Equal loudness curves labeled by phon (not the same as dB)  
 $L_L$  (phon) = SIL (dB) at  $f = 1000$  Hz
- Example:  
 $SIL(40 \text{ phon}, 1000 \text{ Hz}) = 40 \text{ dB}$  (grey)  
 $SIL(40 \text{ phon}, 100 \text{ Hz}) = 60 \text{ dB}$  (blue)  
 $SIL(40 \text{ phon}, 4000 \text{ Hz}) = 35 \text{ dB}$  (red)



# Subjective Loudness ( $S$ , sone)

- Measure of loudness where “twice as loud” corresponds to “multiply by 2”
- Most people perceive 10x increase in intensity as “twice as loud”
  - Recall: 10x intensity  $\rightarrow \Delta \text{SIL} = 10 \text{ dB}$ ,  $\Delta L_L = 10 \text{ phon}$
  - Some values:

$L_L = 40 \text{ phon}$	$\Leftrightarrow$	$S = 1 \text{ sone}$
$L_L = 50 \text{ phon}$	$\Leftrightarrow$	$S = 2 \text{ sone}$
$L_L = 60 \text{ phon}$	$\Leftrightarrow$	$S = 4 \text{ sone}$
$L_L = 30 \text{ phon}$	$\Leftrightarrow$	$S = 1/2 \text{ sone}$

General formula:
$$S = 2^{(L_L - 40)/10} \text{ sone}$$
  - Questions:

$L_L = 70 \text{ phon}$	$\Leftrightarrow$	$S = ?? \text{ sone}$
$L_L = 20 \text{ phon}$	$\Leftrightarrow$	$S = ?? \text{ sone}$

# Different measures of loudness and safe noise levels

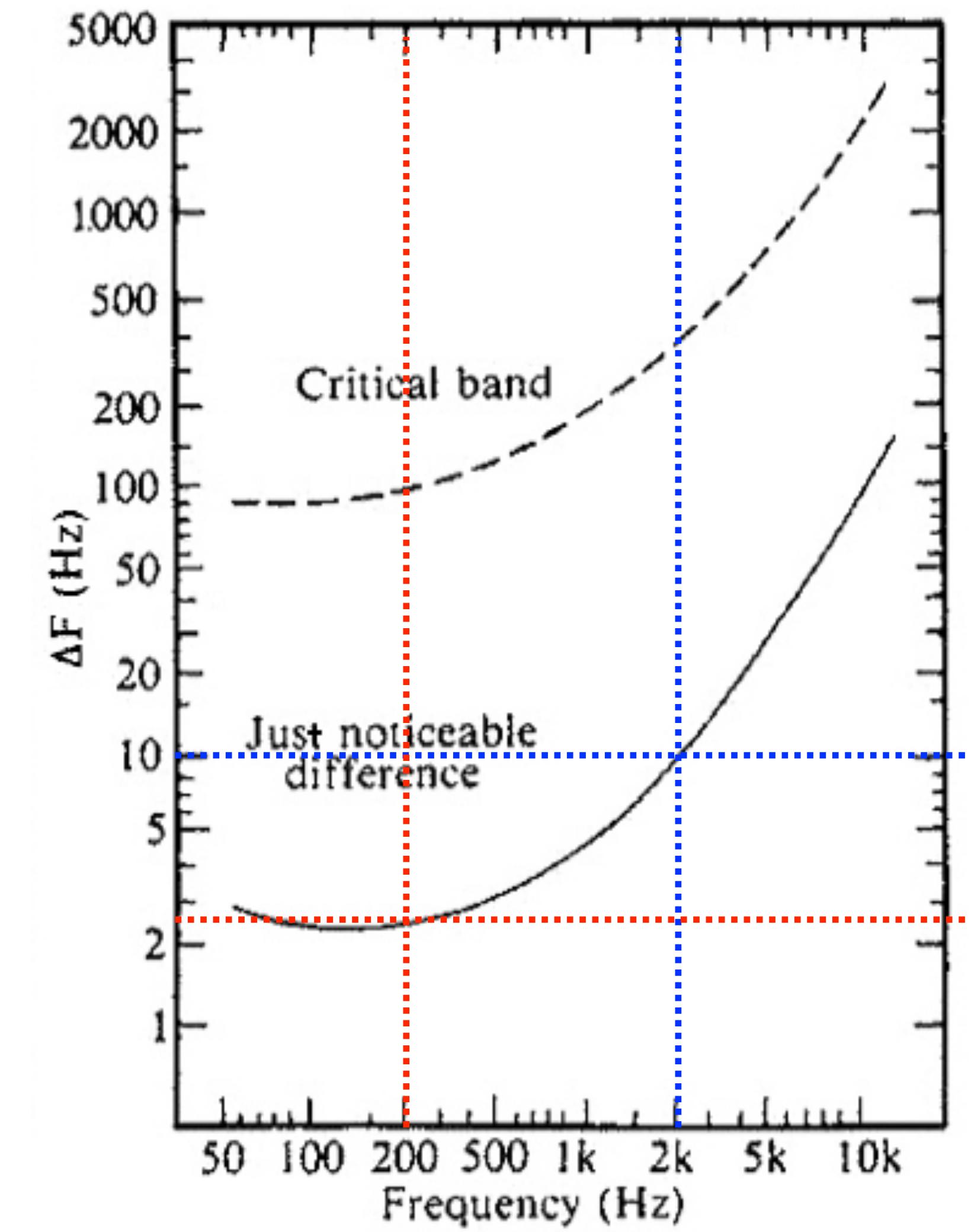
	SIL (dB) at 1000 Hz	$L_L$ (phon)	$S$ (sone)
Threshold of hearing	0	0	1/16
Recording studio	20	20	1/4
Quiet office	40	40	1
Ordinary conversation	60	60	4
Normal piano practice	80	80	16
Piano fortissimo	100	100	64
Threshold of pain	120	120	256

Duration per day, hours	Sound level dBA slow response
8.....	90
6.....	92
4.....	95
3.....	97
2.....	100
1 1/2 .....	102
1.....	105
1/2 .....	110
1/4 or less.....	115

# 11. Pitch & timbre

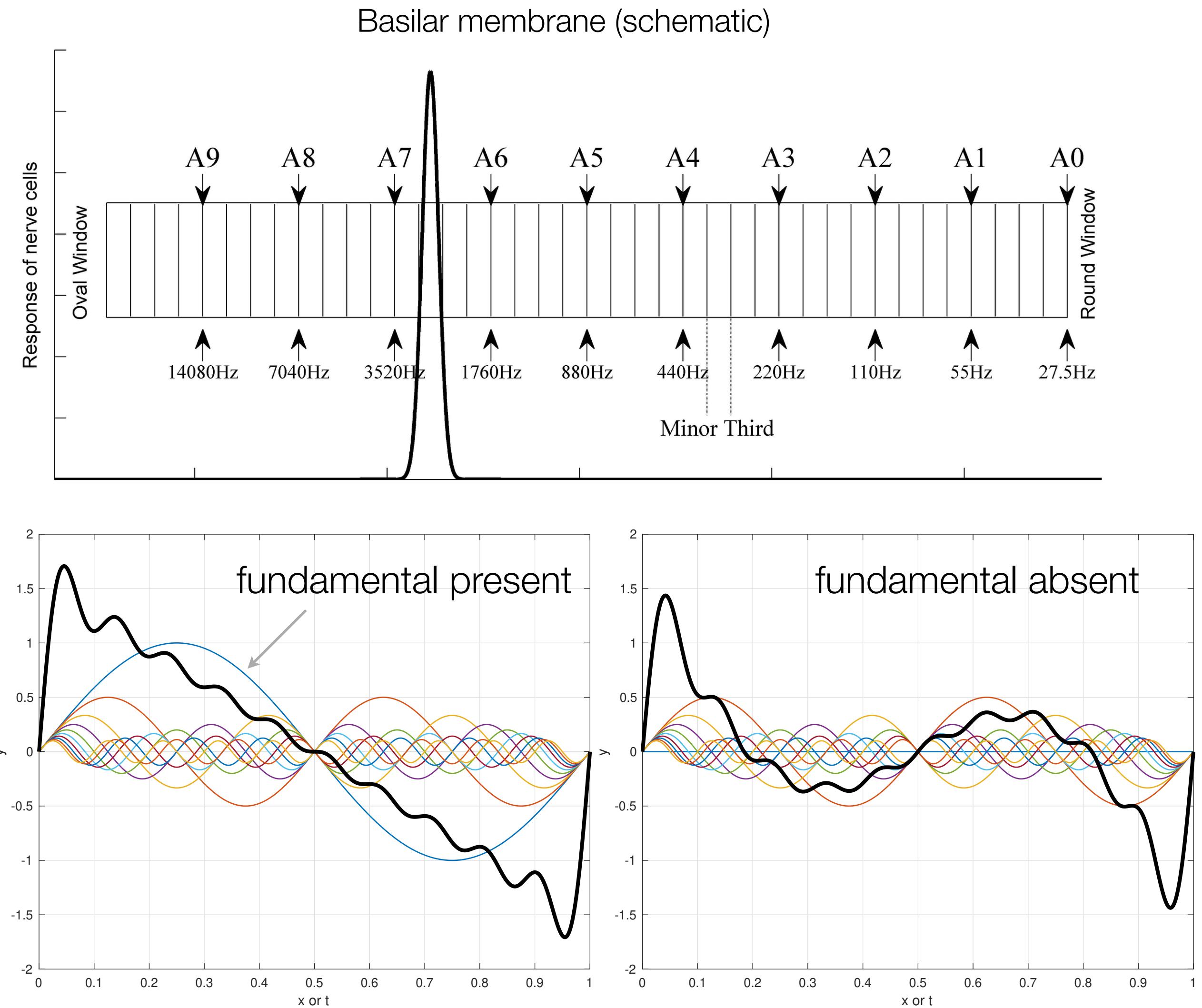
# What distinguishes one musical note from another?

- Pitch, timbre, duration, loudness (intensity), attack & decay transients
- Perfect pitch: ability to determine absolute pitch without regard to a reference (only 1 out of ~10,000 people have it)
- Pitch discrimination: ability to distinguish two different pitches
  - depends on whether you play the two notes sequentially or simultaneously
  - JND: just noticeable difference (sequential; 0.5% of center frequency; 1/10th of a semitone)
  - LFD: limit of frequency discrimination (simultaneous; 10% of center frequency; 2 semitones)
  - Analogy with sense of touch: placing two pencil points on your arm



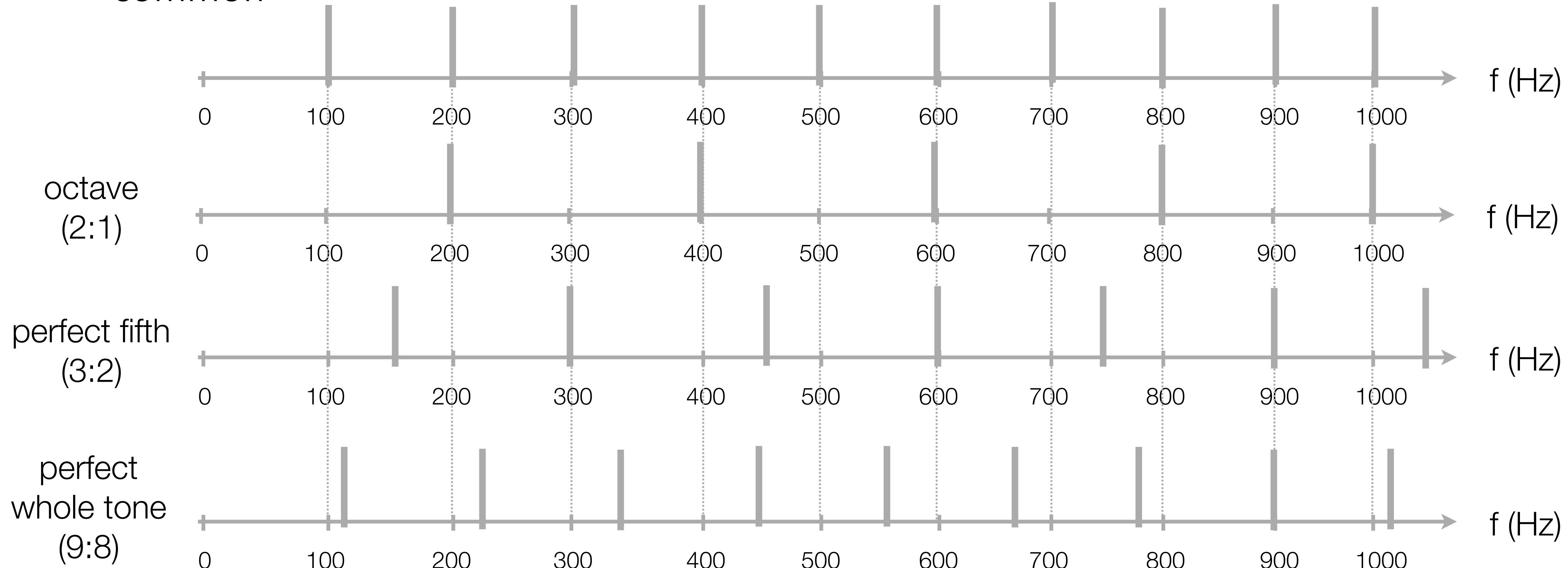
# Where does pitch determination occur, in the ear or the brain?

- Place theory: pitch determined by the location on the basilar membrane excited by the sound wave
- Periodicity theory: pitch inferred by the brain from the timing of electrical impulses triggered by the period of the sound wave
- Missing fundamental in support of periodicity theory:
  - 200 Hz, 300 Hz, 400 Hz, .... → hear 100 Hz
  - 300 Hz, 500 Hz, 700 Hz, .... → hear ??? Hz
  - <http://www.personal.psu.edu/meb26/INART50/psychoacoustics.html>



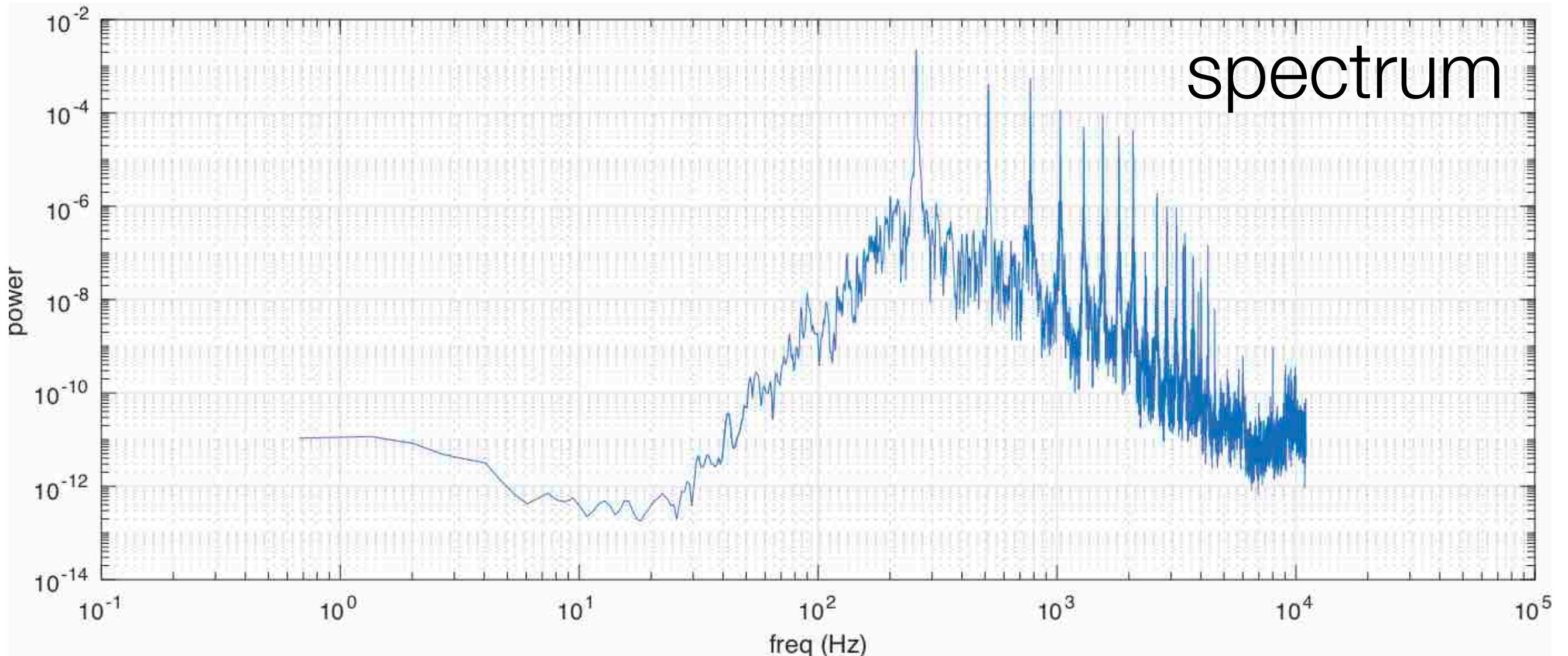
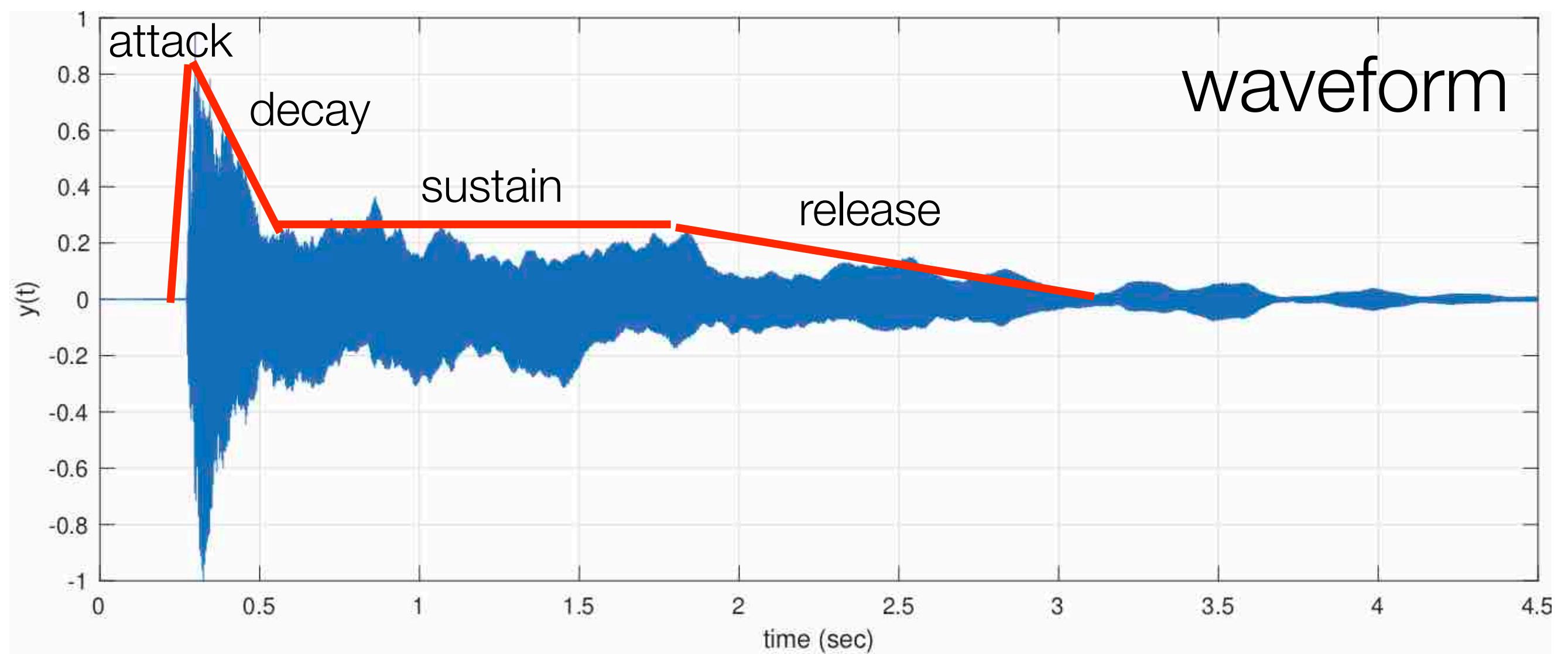
# What makes two notes pleasing when they are played together?

- Two notes are pleasing (consonant) when they have many harmonics in common
- Two notes clash with one another (dissonant) when they have very few harmonics in common



# Attack and decay transients

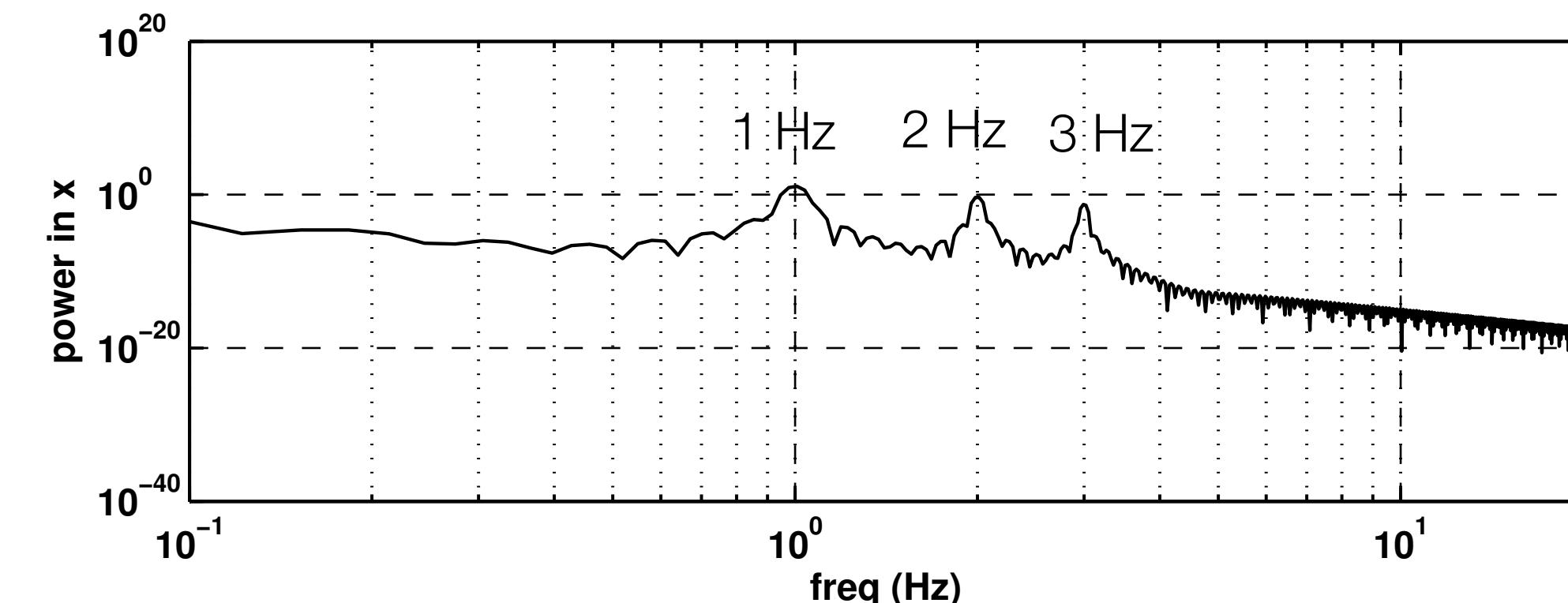
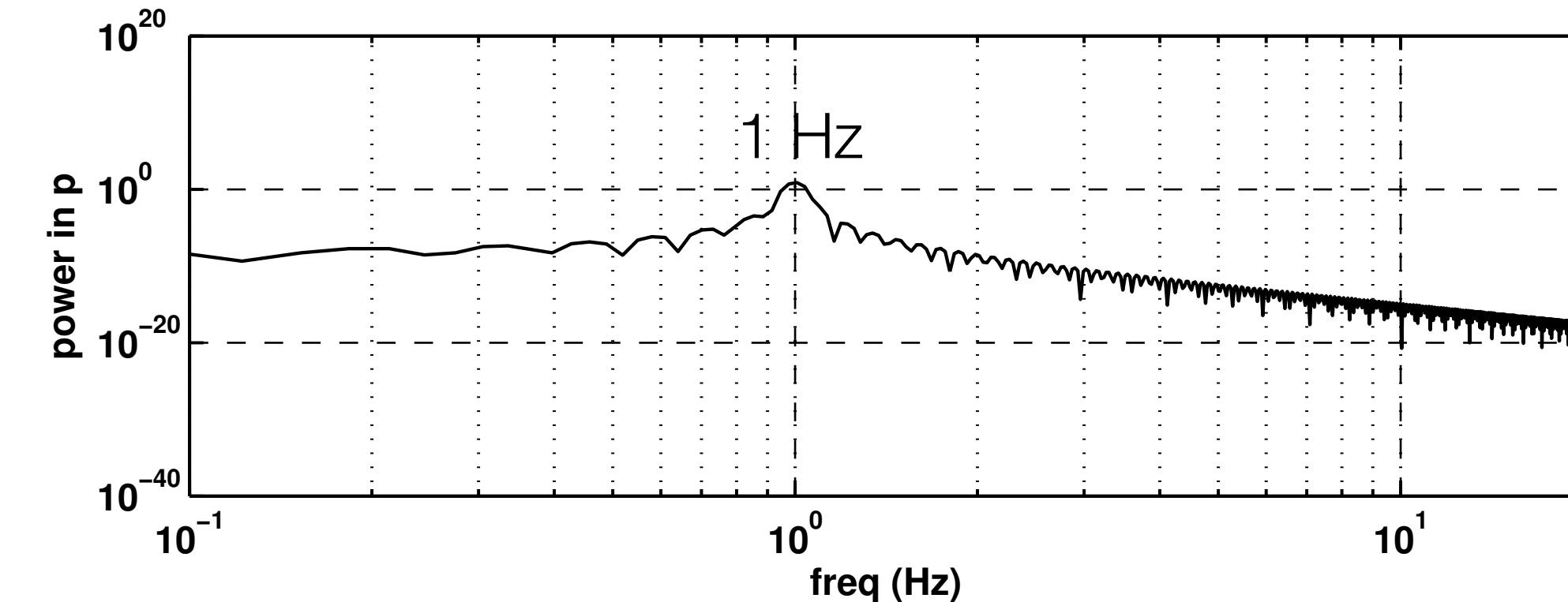
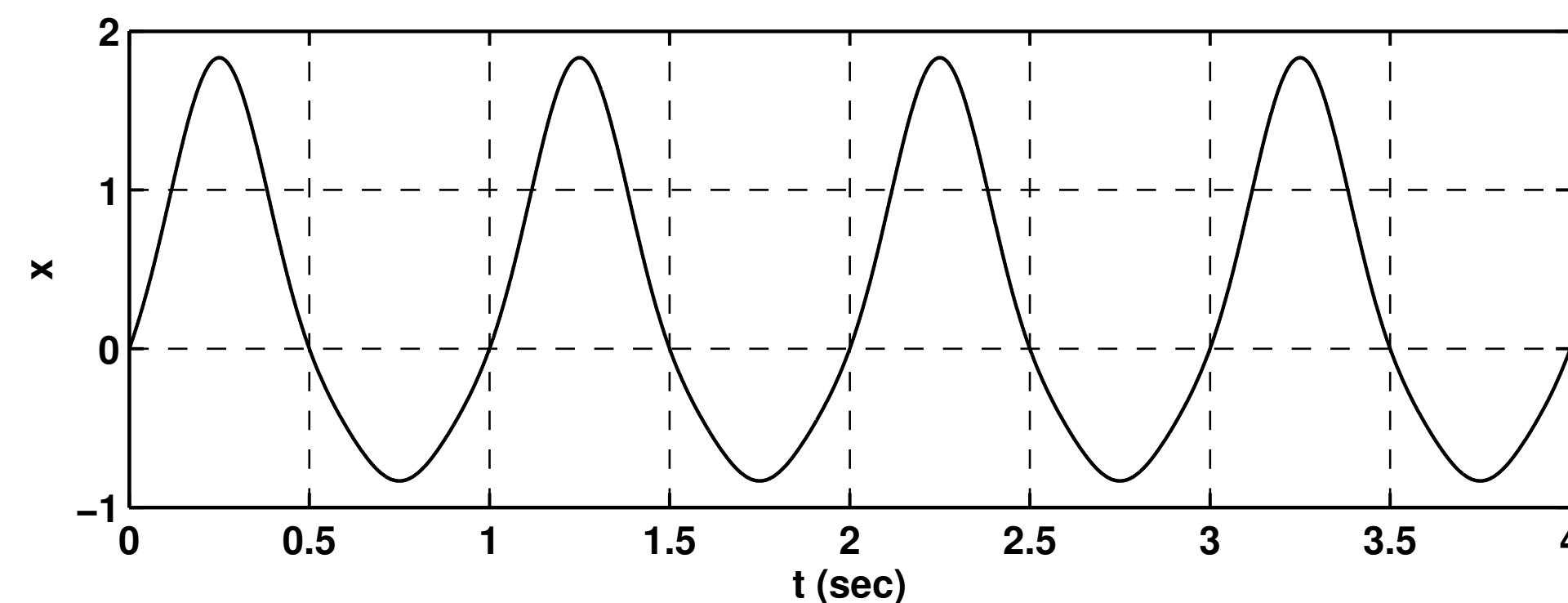
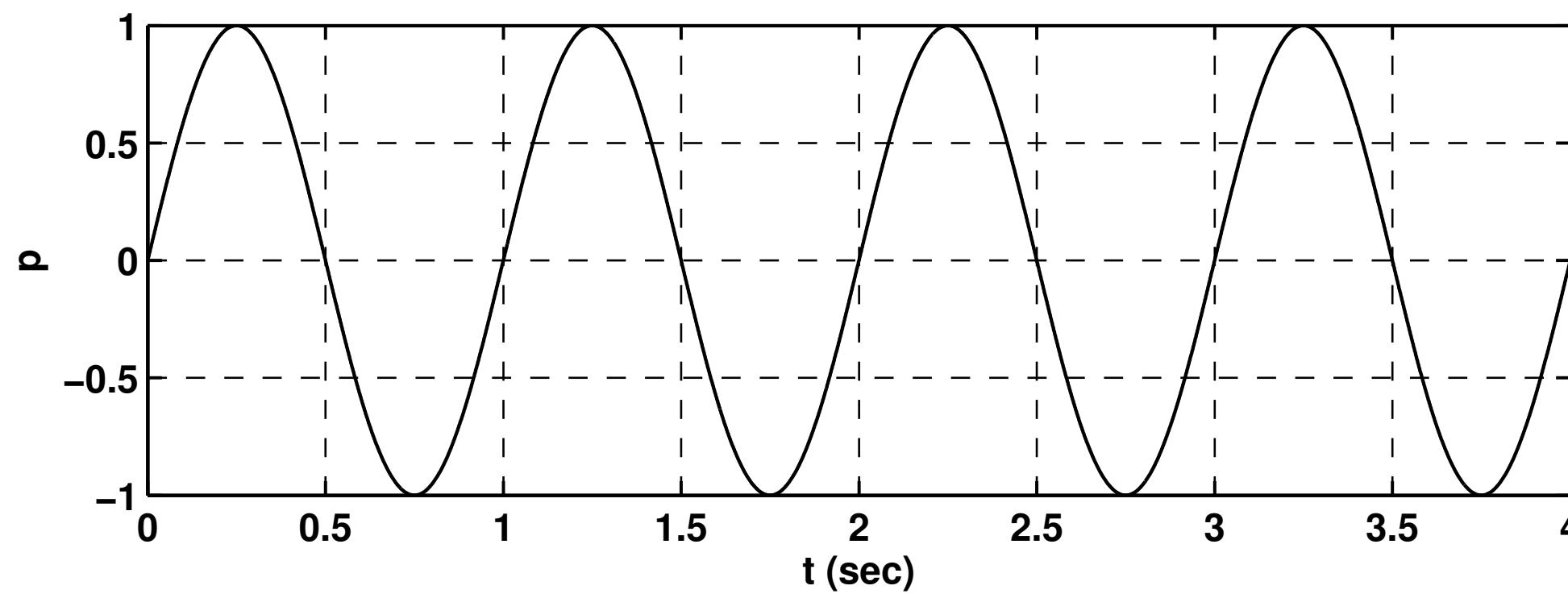
- How a note starts and ends affects how it sounds
- Piano C4
- Piano C4 (reversed)
- Happy birthday
- Happy birthday backwards
- Happy birthday backwards (reversed)



# Aural harmonics – harmonics produced by the ear

- Ear introduces distortions which converts a pure tone to one having multiple harmonics

$$x(t) = a_0 + a_1 p(t) + a_2 p^2(t) + a_3 p^3(t) + \dots$$

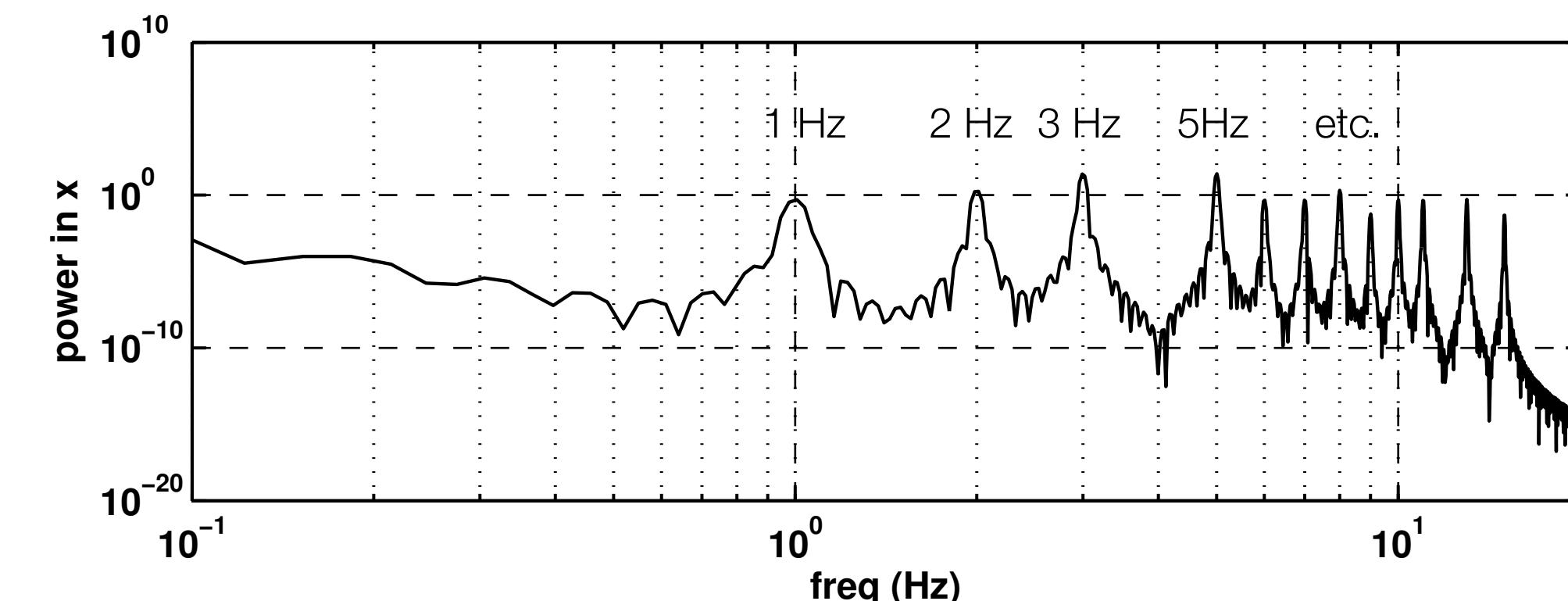
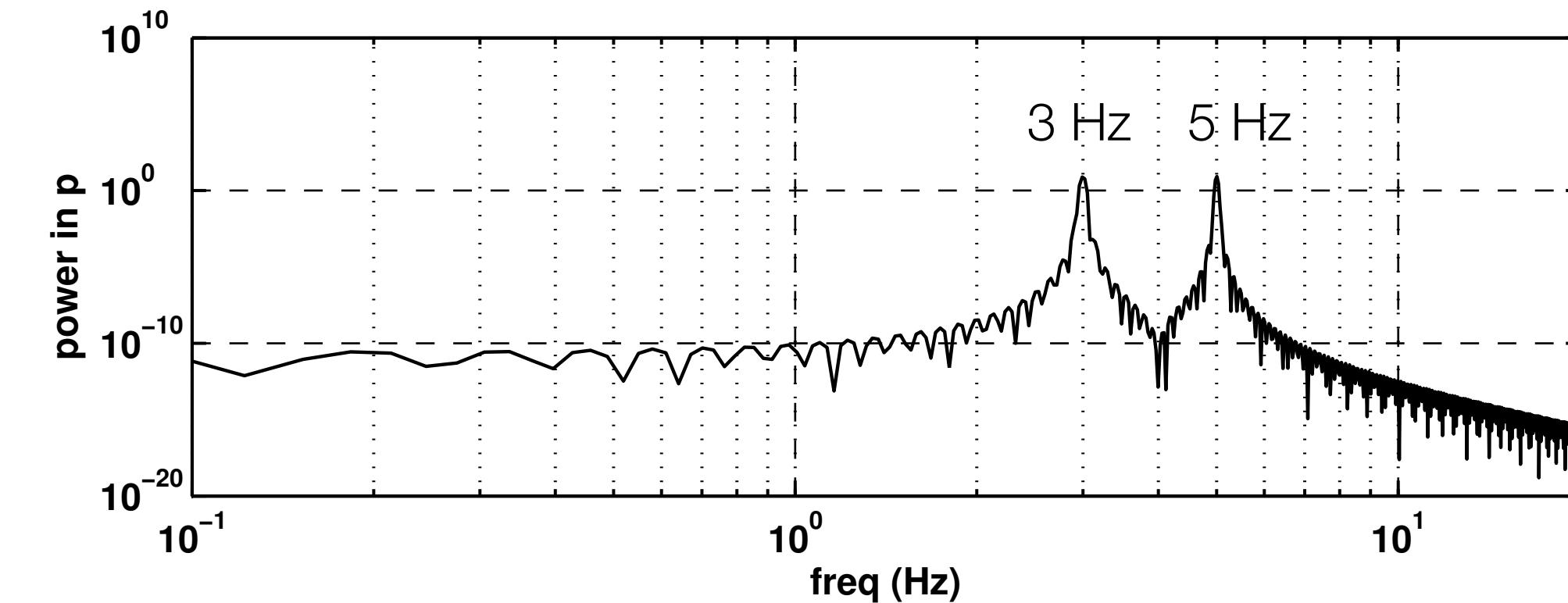
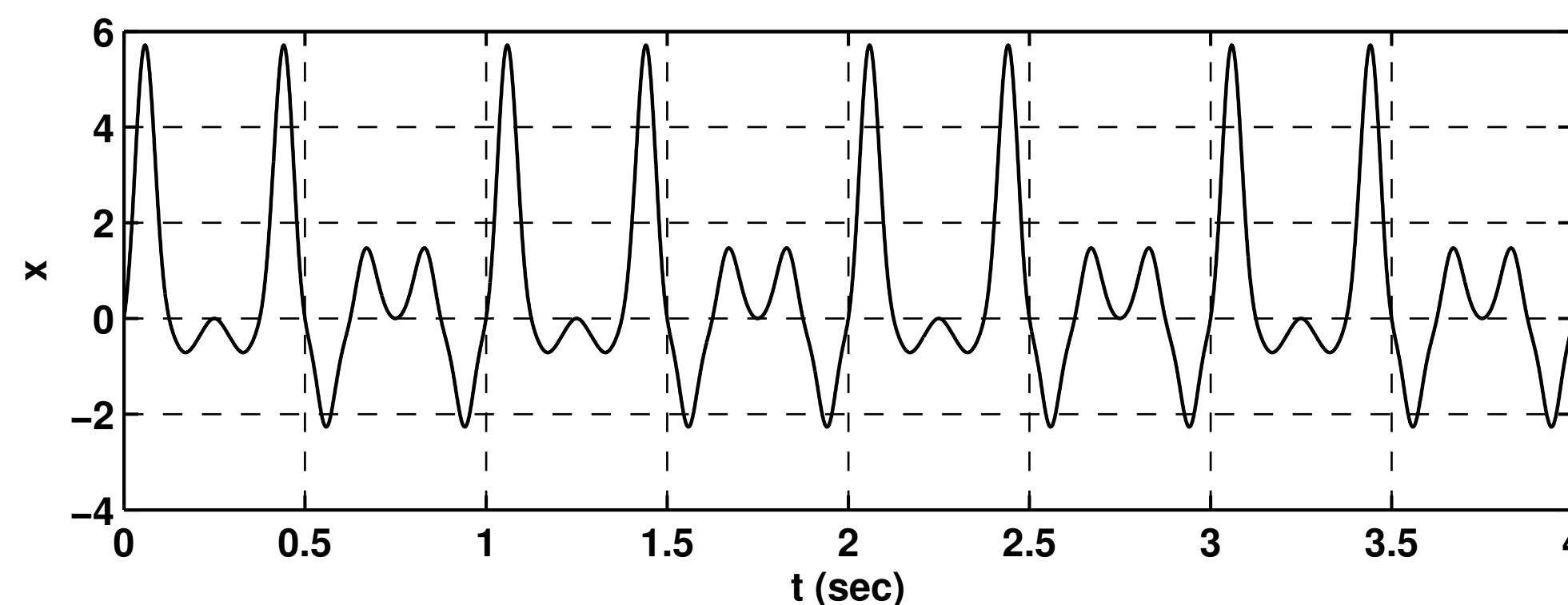
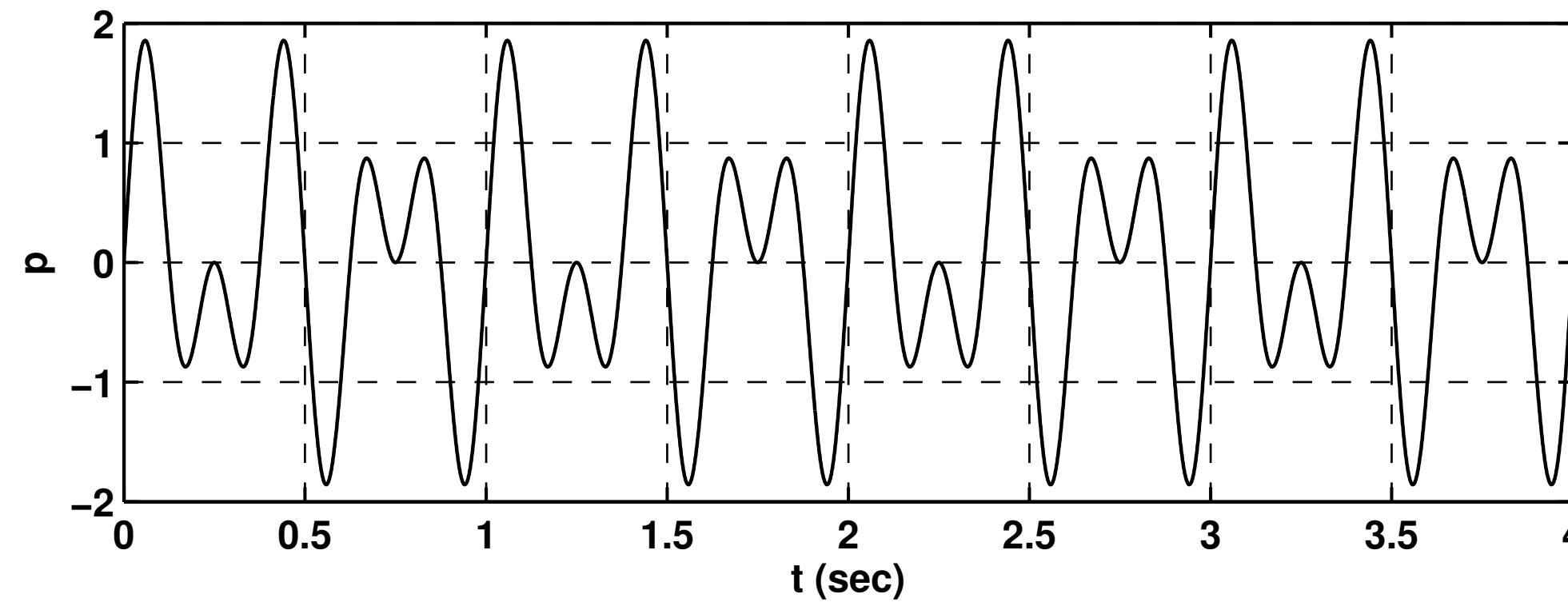


$$a_0 = 0, a_1 = 1, a_2 = 1/2, a_3 = 1/3$$

# Aural combination tones – aural harmonics for complex tones

- If two pure tones  $f_1$  and  $f_2$  are played simultaneously and sufficiently loudly, one hears sum and difference tones

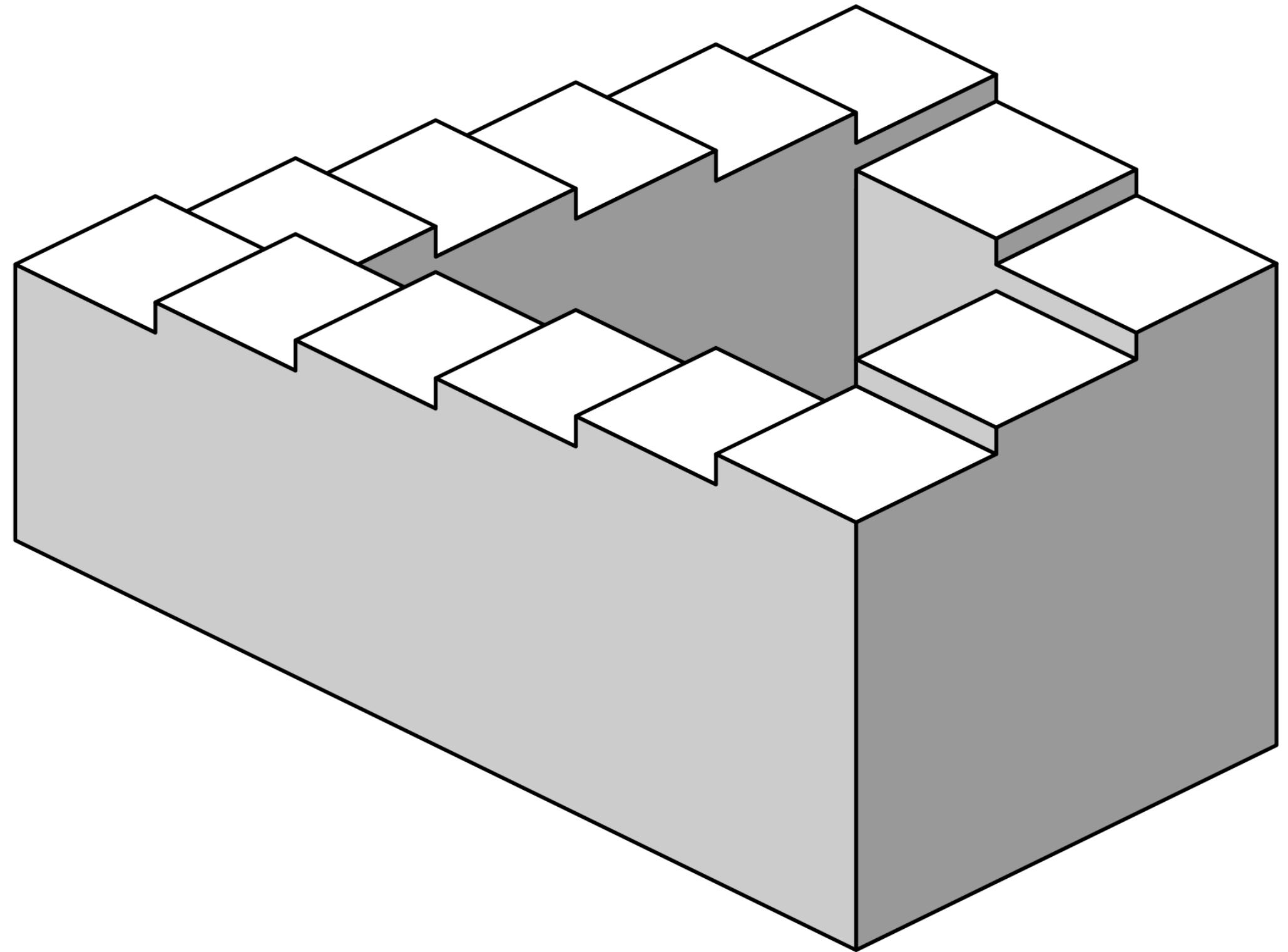
$$f_c = |mf_1 \pm nf_2| \Rightarrow |f_1 - f_2|, |2f_1 - f_2|, |3f_1 - f_2|, \dots$$



$$a_0 = 0, a_1 = 1, a_2 = 1/2, a_3 = 1/3$$

# Pitch paradox – the audio equivalent of an optical illusion

- Shepard scale: never-ending scale (pitch seems to increase indefinitely)
- YouTube videos:
  - <http://www.youtube.com/watch?v=PCs1lckF5vl>
  - <http://vimeo.com/34749558>



Never-ending staircase  
(L. Penrose; M.C. Escher)

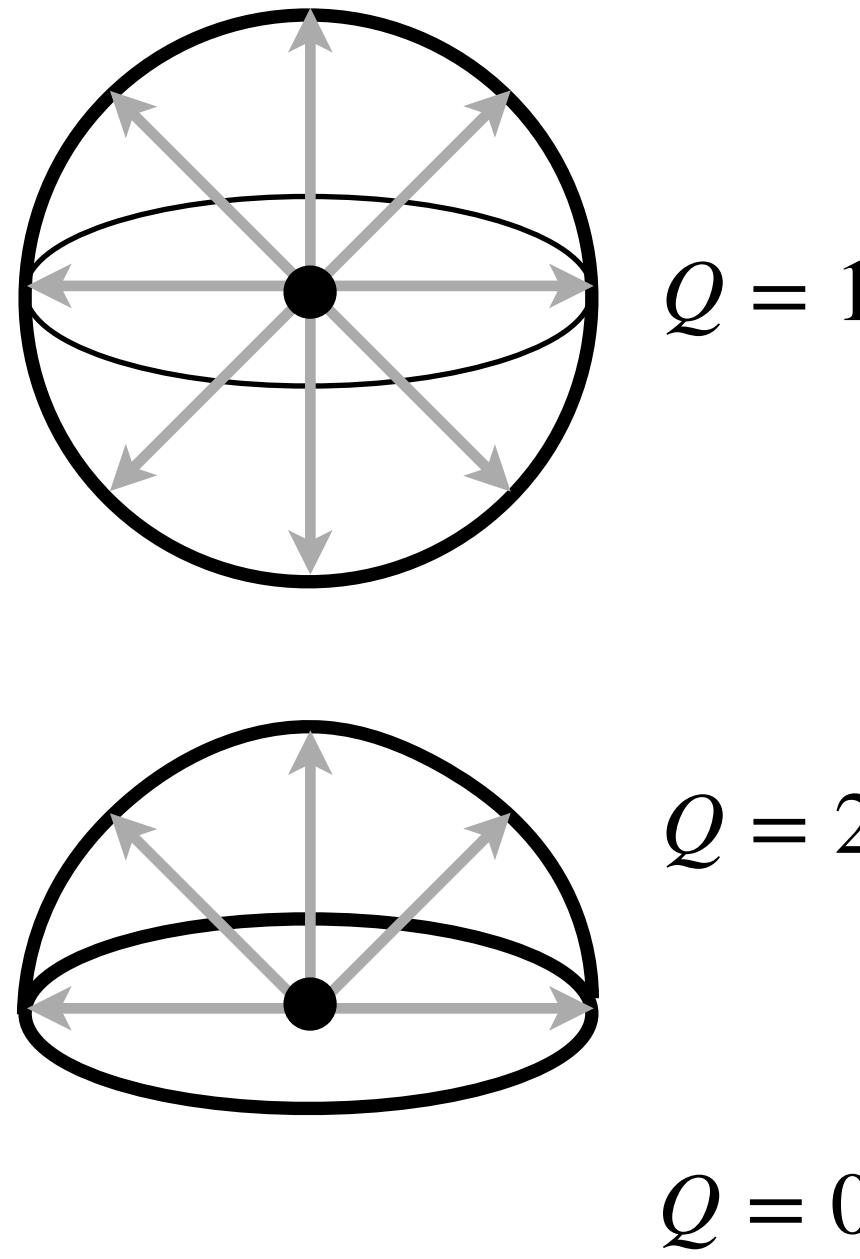
# **12. Auditorium & room acoustics**

# Auditorium and room acoustics – overview

- What makes for a good concert hall?
- Why do you sound good when you sing in a shower?
- Difference between “direct”, “reflected”, and “reverberant” sound
- Reverberation time is the most important characteristic of a room
- YouTube video / soundfile:
  - Anechoic chamber (<https://www.youtube.com/watch?v=BYBSA9v8IRE>)
  - “Sonic wonders” sound file (listen to -32:40)

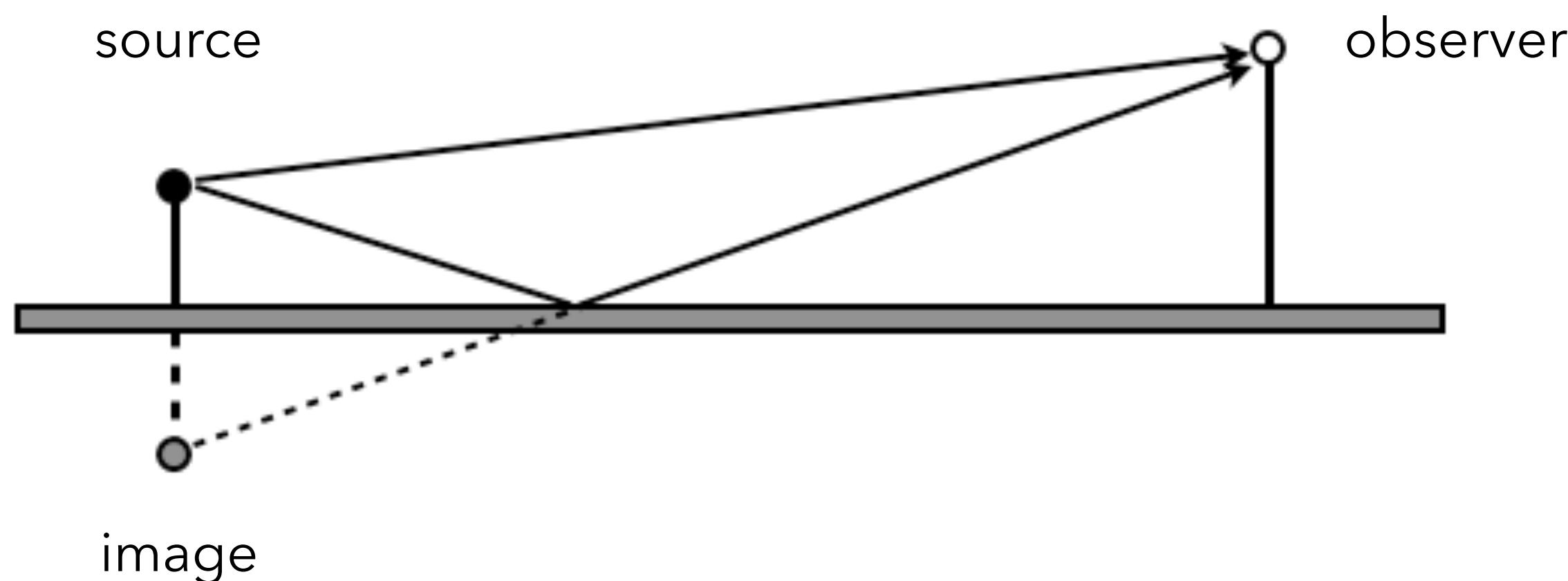
# Direct sound

- Sound received from a source in the absence of any reflections (e.g., anechoic chamber)
- Intensity:  $I = \frac{P}{4\pi r^2}$  (omni-directional);  $I = \frac{QP}{4\pi r^2}$  (directional source;  $Q$  is the directivity factor)



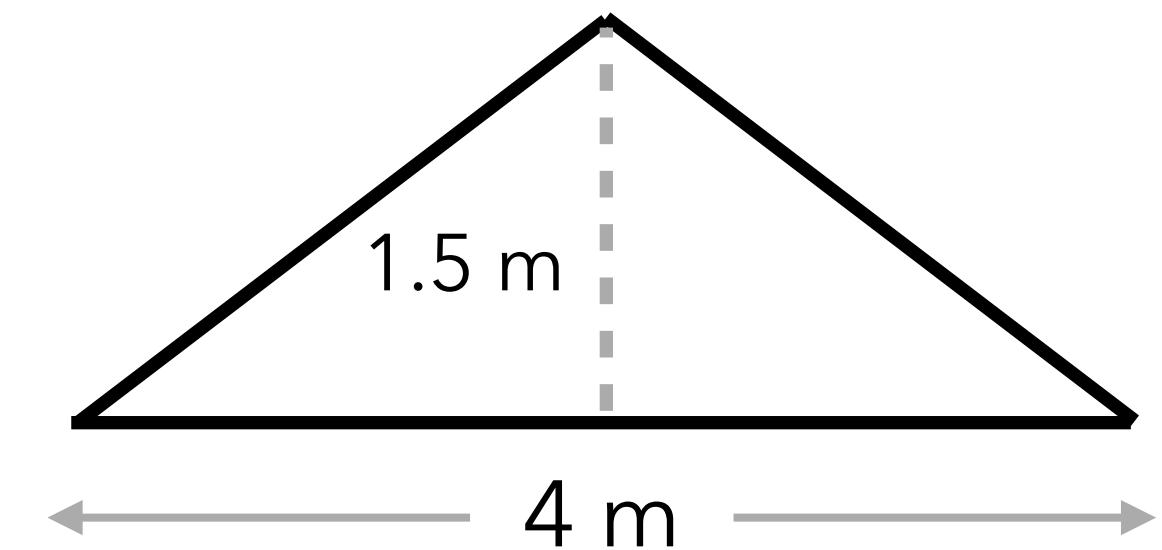
# Reflected sound

- Hear an echo if the reflected sound is heard greater than 35 msec after the direct sound
- Recall:  $v = 346 \text{ m/s} \approx 1000 \text{ ft/s} = 1 \text{ ft/msec}$
- $\text{SIL}_{\text{reflected}} < \text{SIL}_{\text{direct}}$  (reflected sound travels farther and can be partially absorbed)

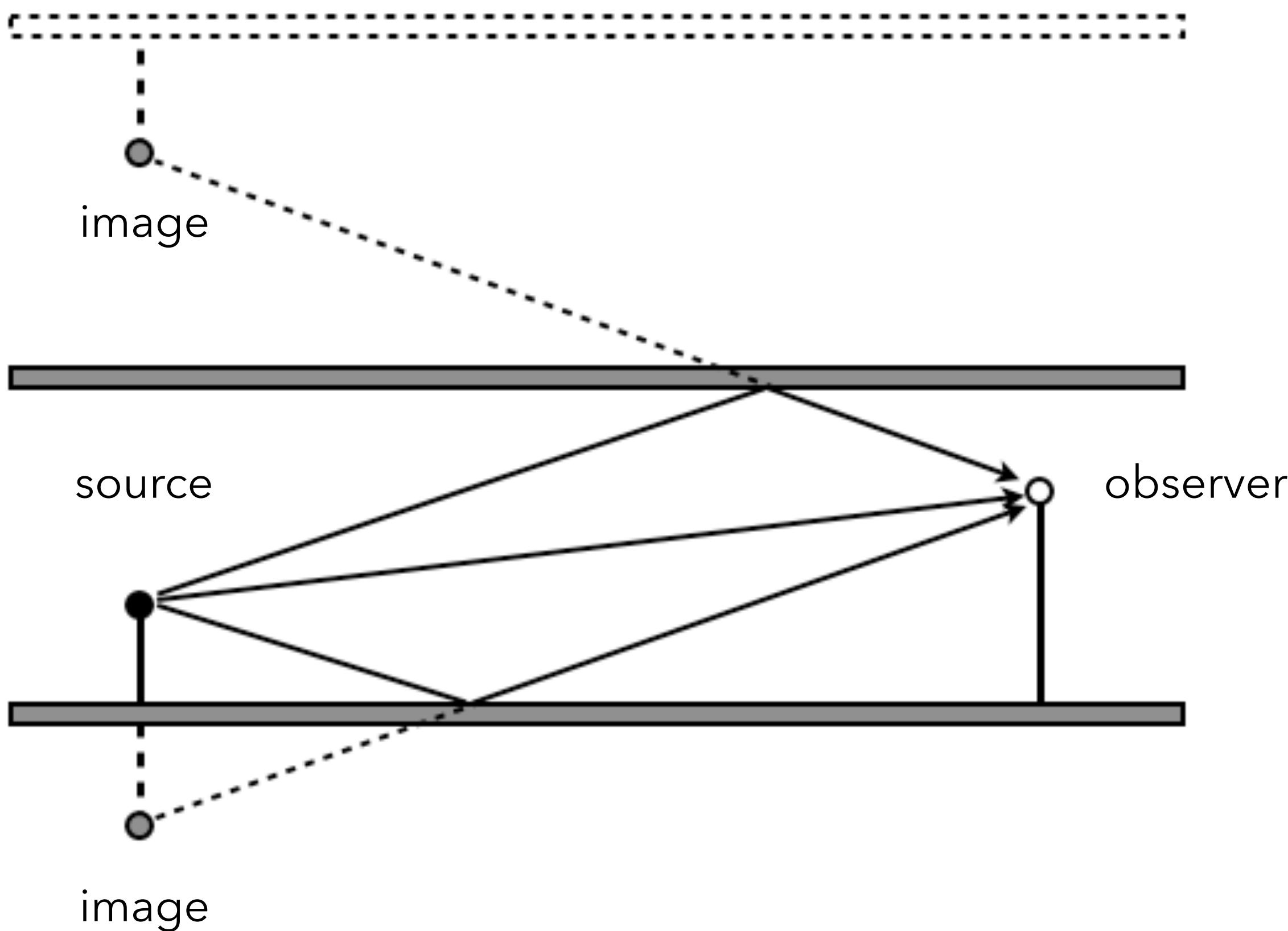


# Reflected sound - example

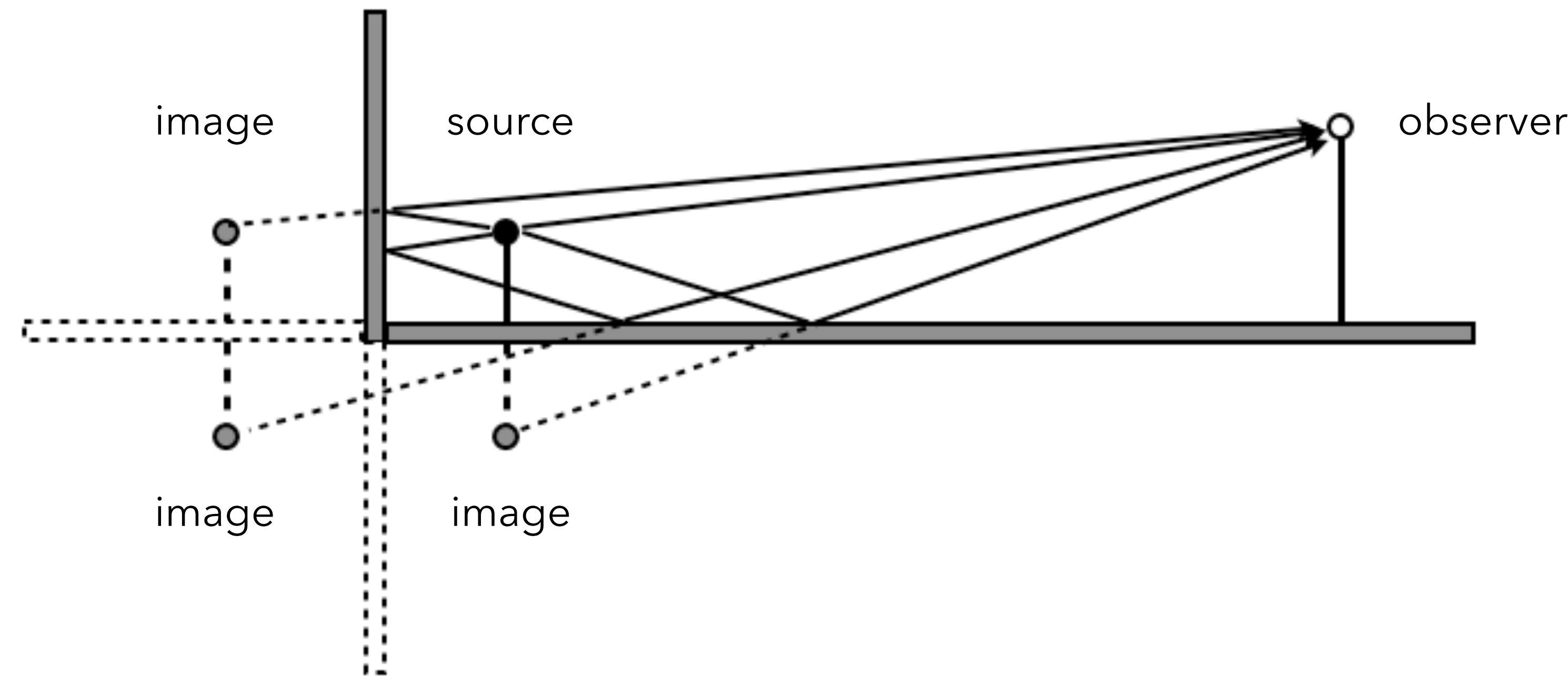
- A listener stands 4 m in front of an omni-directional loudspeaker that is 1.5 m from a reflecting wall.
- Calculate:
  - the time of arrival for both the direct and reflected sound
  - the decrease in SIL for the reflected sound due to the larger distance traveled
  - the decrease in SIL for the reflected sound assuming an absorption coefficient  $a = 0.2$  for the wall
- Answer:
  - Reflected sound travels 5 meter:  $t_{\text{direct}} = \frac{4 \text{ m}}{346 \text{ m/s}} = 12 \text{ msec}$ ,  $t_{\text{reflected}} = \frac{5 \text{ m}}{346 \text{ m/s}} = 15 \text{ msec}$
  - $\Delta \text{SIL} = 10 \log \left[ 1/(r_{\text{reflected}}/r_{\text{direct}})^2 \right] \text{ dB} = 10 \log \left[ (4/5)^2 \right] \text{ dB} = -2 \text{ dB}$
  - $\Delta \text{SIL} = 10 \log(1 - a) \text{ dB} = 10 \log(1 - 0.2) \text{ dB} = -1 \text{ dB}$



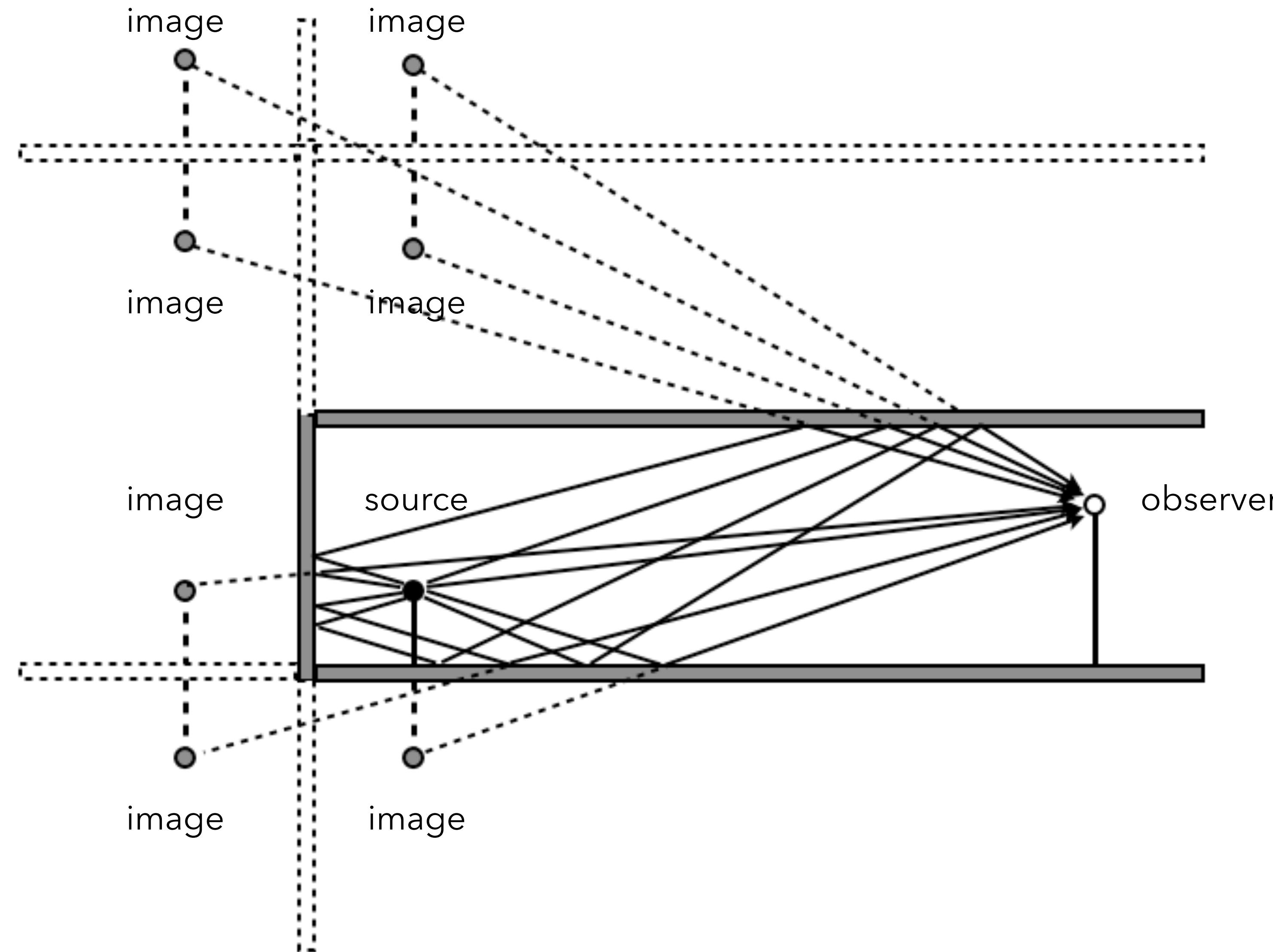
# Multiple reflections – floor and ceiling



# Multiple reflections – floor and back wall

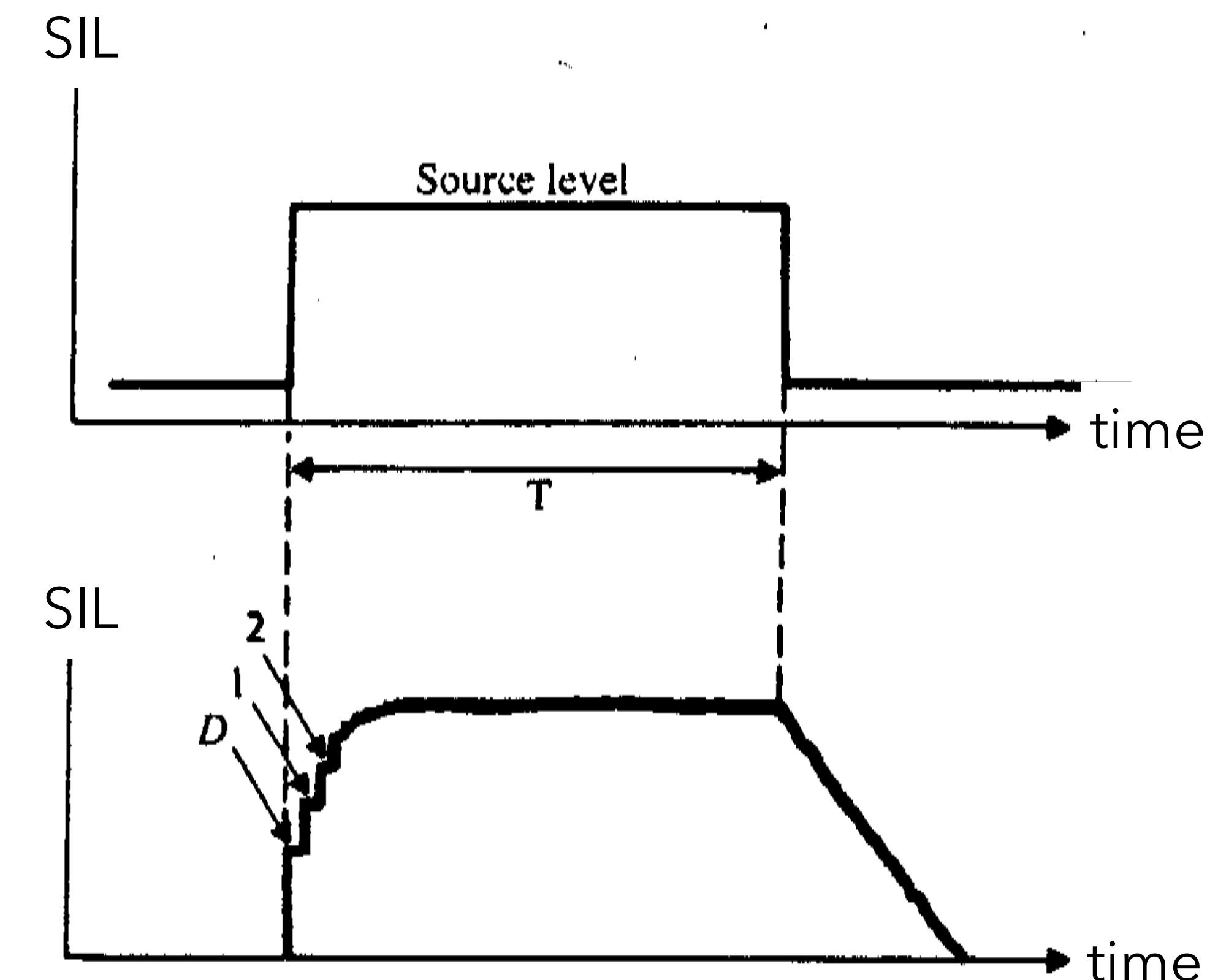
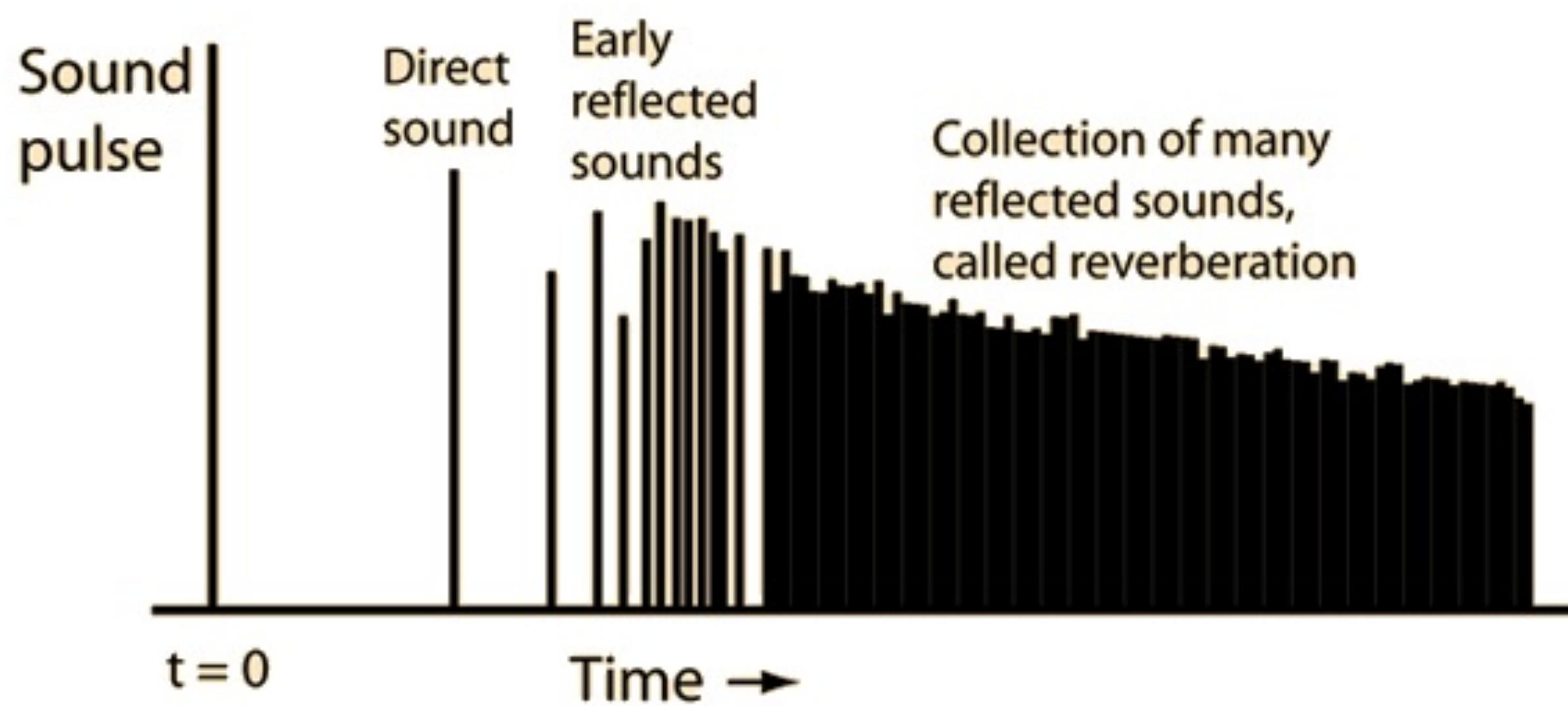


# Multiple reflections – floor, back wall, and ceiling



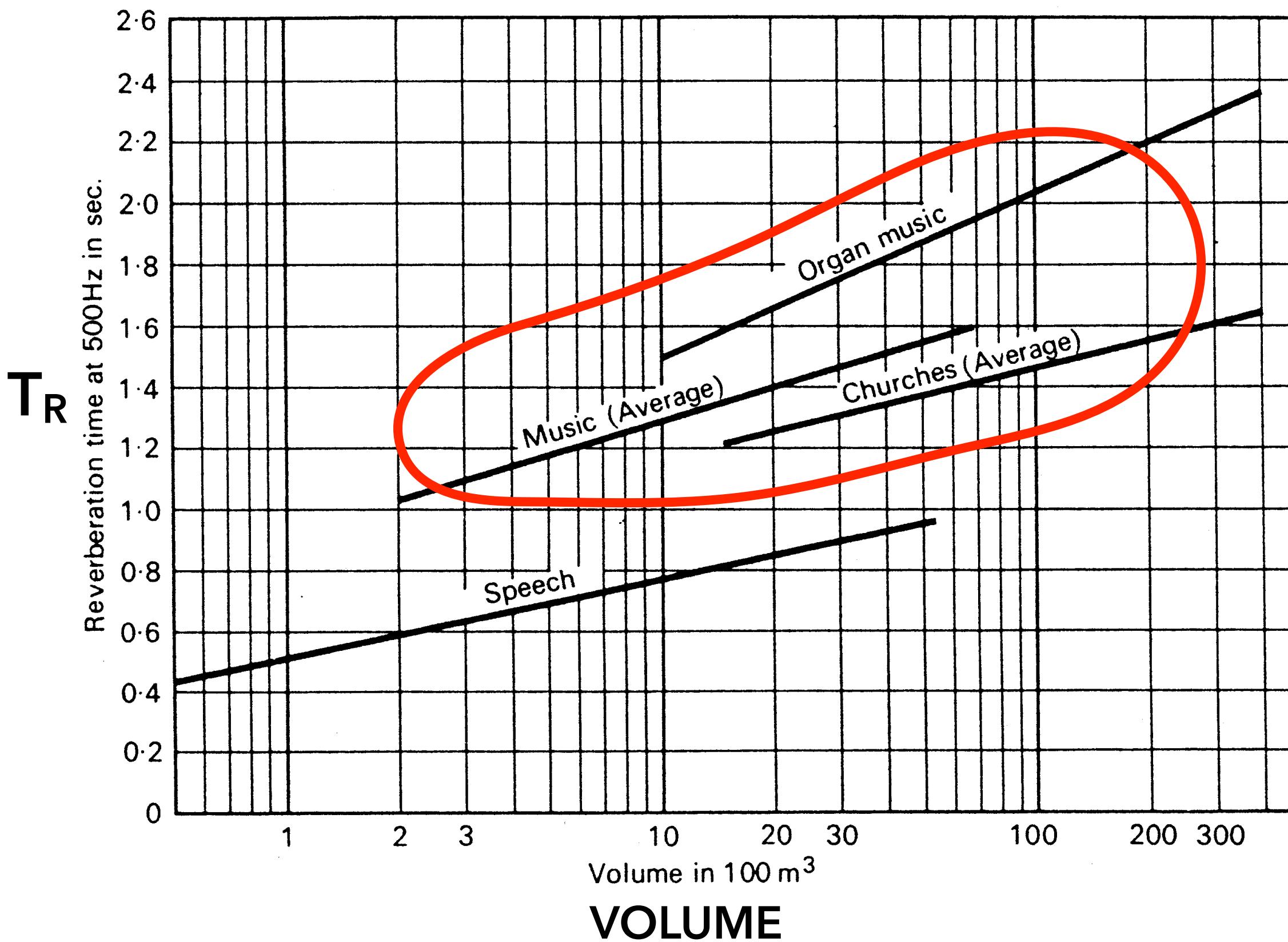
# Reverberant sound

- sound formed from multiple reflections, coming from many different directions, and overlapping in time



# Reverberation time

- time required for the reverberant SIL to decrease by 60 dB (1/106 in intensity)
- frequency dependent (low-frequency sounds typically have larger reverberation times)



Acoustical characteristics of various concert halls

	Year built	Volume ( $\text{m}^3$ )	Number of seats	Reverberation time (sec)		
				125 Hz	500 Hz	2000 Hz
Teatro alla Scala, Milan	1778	11,245	2289		1.2	
Royal Opera House	1858	12,240	2180		1.1	
Royal Albert Hall	1871	86,600	6080	3.4	2.6	2.2
Carnegie Hall, New York	1891	24,250	2760	1.8	1.8	1.6
Symphony Hall, Boston	1900	18,740	2630	2.2	1.8	1.7
Royal Festival Hall	1951	22,000	3000	1.4	1.5	1.4
Philharmonic Hall, Berlin	1963	36,030	2200		2.0	
St. David's Hall, Cardiff	1983	22,000	2200	1.8	1.9	1.8

# Calculating reverberation time

$$T_R = 0.05 \frac{V}{A_{\text{eff}}} \text{ s}$$

$V$ : volume in (ft<sup>3</sup>)

$$A_{\text{eff}} = A_1 a_1 + A_2 a_2 + \dots + B_1 + B_2 + \dots$$

- $A_{\text{eff}}$ : total absorption in sabin (1 ft<sup>2</sup> of perfectly absorbing surface)

- $A_1, A_2, \dots$  : surface area of walls, etc. (in ft<sup>2</sup>)

- $a_1, a_2, \dots$  : absorption coeffs (dimensionless, freq-dependent)

- $B_1, B_2, \dots$  : absorption for seats, people, etc. (in sabin)

**absorption coefficients (dimensionless)**

Material	Frequency (Hz)					
	125	250	500	1000	2000	4000
Concrete (painted)	0.10	0.05	0.06	0.07	0.09	0.08
Plywood panel	0.28	0.22	0.17	0.09	0.10	0.11
Plaster on lath	0.14	0.10	0.06	0.05	0.04	0.03
Gypsum board, 1/2 in.	0.29	0.10	0.05	0.04	0.07	0.09
Glass window	0.35	0.25	0.18	0.12	0.07	0.04
Curtains	0.14	0.35	0.55	0.72	0.70	0.65
Carpet (on concrete)	0.02	0.06	0.14	0.37	0.60	0.65
Carpet (on pad)	0.08	0.24	0.57	0.69	0.71	0.73
Acoustical tile, suspended	0.76	0.93	0.83	0.99	0.99	0.94

**absorption (in m<sup>2</sup>) [multiply by 10.8 to convert to sabin]**

Material	Frequency (Hz)					
	125	250	500	1000	2000	4000
Wood or metal seat, unoccupied	0.014	0.018	0.020	0.036	0.035	0.028
Upholstered seat, unoccupied	0.13	0.26	0.39	0.46	0.43	0.41
Adult	0.23	0.32	0.39	0.43	0.46	—
Adult in an upholstered seat	0.27	0.40	0.56	0.65	0.64	0.56

# Example

Exercise: Calculate the reverberation time at 500 Hz for a room with dimensions  $20 \text{ m} \times 15 \text{ m} \times 8\text{m}$  (high). The walls are painted concrete, the ceiling is plaster, and the floor is carpet on pad. Also, assume that there are 200 upholstered seats, and that they are half-filled with people.

$$L = 20 \text{ m} \times 3.28 \text{ ft/m} = 65.6 \text{ ft}$$

$$W = 15 \text{ m} \times 3.28 \text{ ft/m} = 49.2 \text{ ft}$$

$$H = 8 \text{ m} \times 3.28 \text{ ft/m} = 26.24 \text{ ft}$$

$$V = L \times W \times H = 2400 \text{ m}^3 = 8.47 \times 10^4 \text{ ft}^3$$

↓                      ↓                      ↓                      ↓                      ↓  
painted concrete              plaster              carpet on pad              empty upholstered      occupied  
$$A_{\text{eff}} = 0.06 [2(L \times H) + 2(W \times H)] + 0.06(L \times W) + 0.57(L \times W) + 10.8(100 \times 0.39 + 100 \times 0.56)$$
$$= 3.42 \times 10^3 \text{ sabin}$$

$$T_R = 0.05 \frac{V}{A_{\text{eff}}} \text{ s} = 1.2 \text{ s} \quad \rightarrow \text{ideal for music (for } V=2400 \text{ m}^3\text{)}$$

# Acoustical design

## Criteria for good design

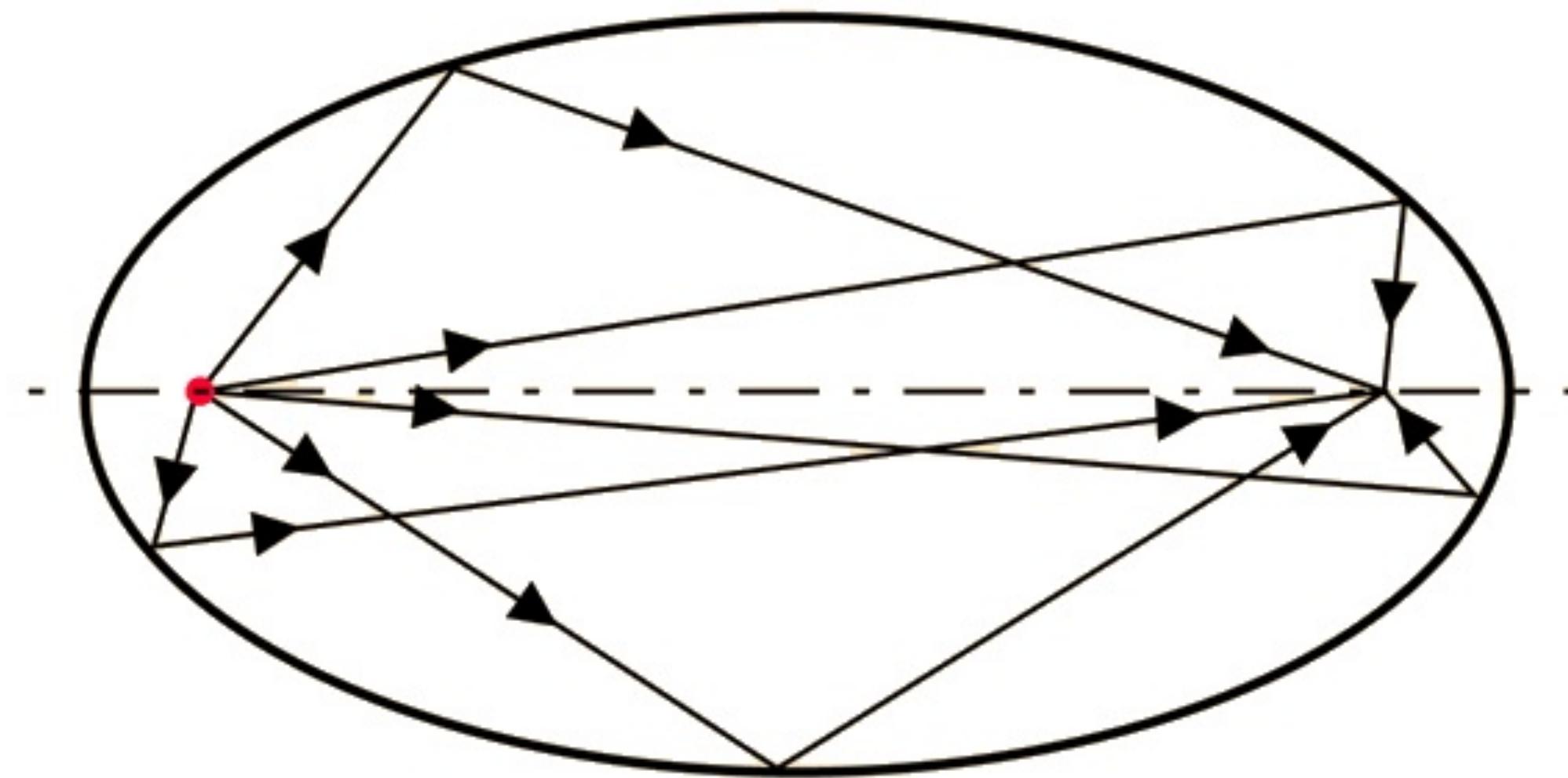
- Loudness
- Uniformity (no “live” or “dead” spots)
- Reverberance or liveness (feeling of being “bathed” in sound)
- Clarity (opposite of reverberance)

## Problems to avoid

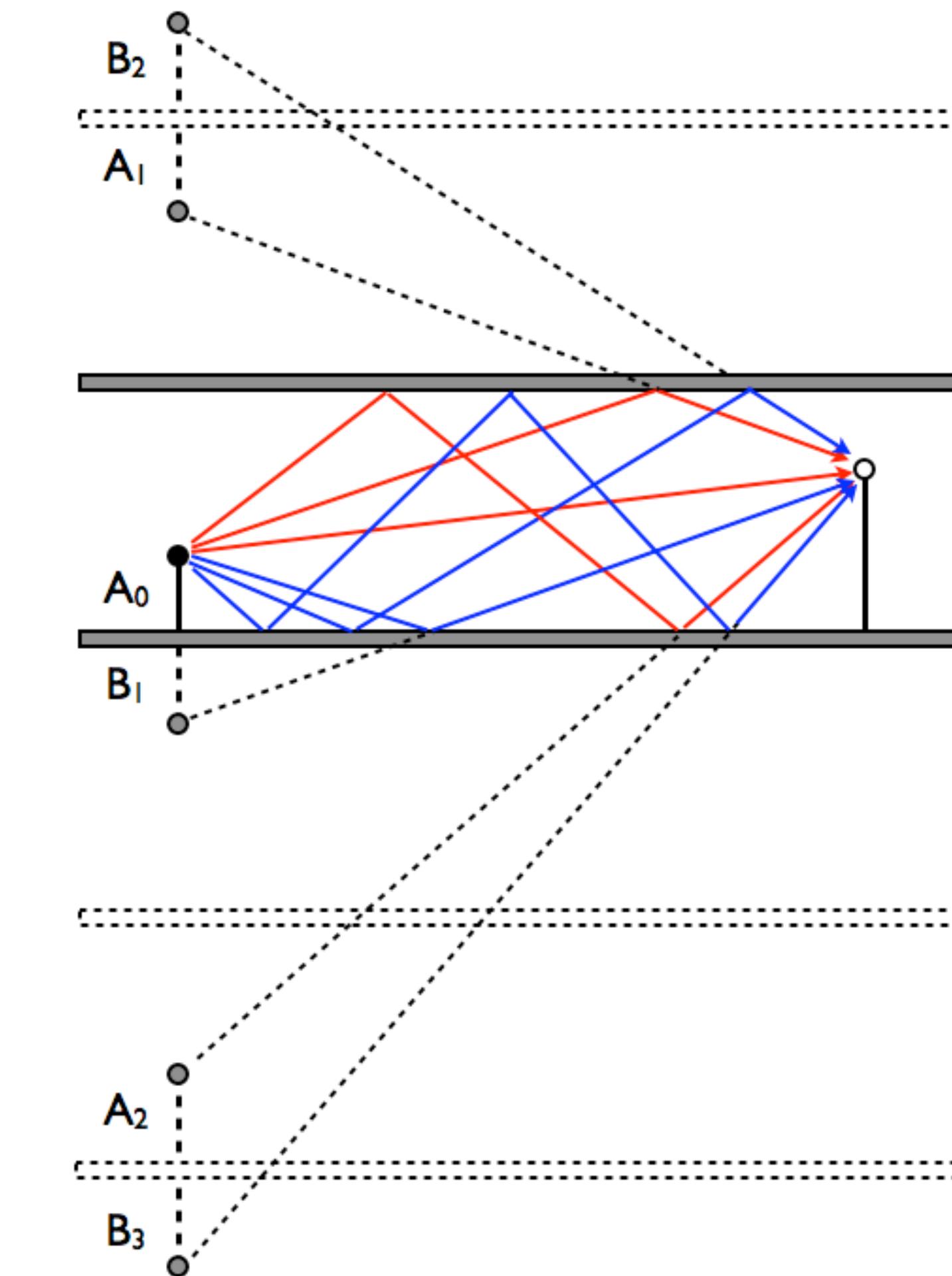
- Background noise (external noise due to heating, A/C, ...)
- Shadow areas (produced by balconies, columns, ...)
- Focusing of sound (“whispering room” effect)
- Echoes
- Room resonances (“shower stall” effect)

$$f_{lmn} = \frac{\nu}{2} \sqrt{\left(\frac{l}{L}\right)^2 + \left(\frac{m}{W}\right)^2 + \left(\frac{n}{H}\right)^2}$$
$$l, m, n = 0, 1, 2, \dots$$

# Problems to avoid



Whispering room effect



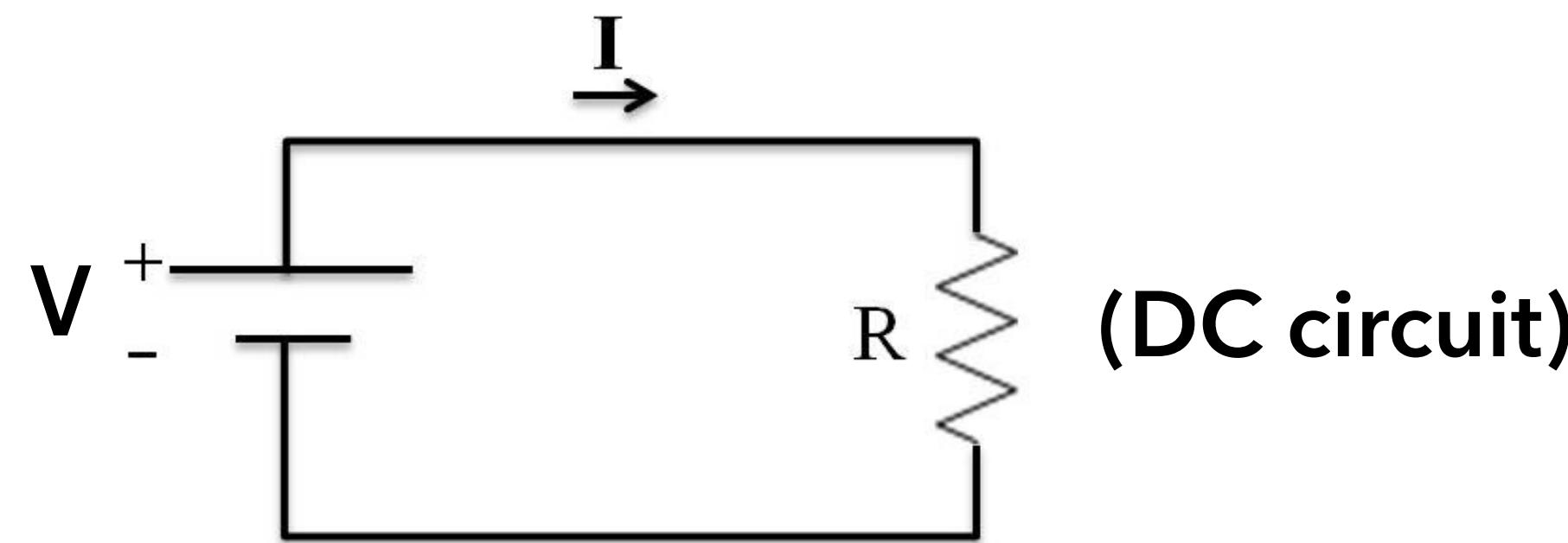
Flutter echoes

# **13. Electrical reproduction of sound**

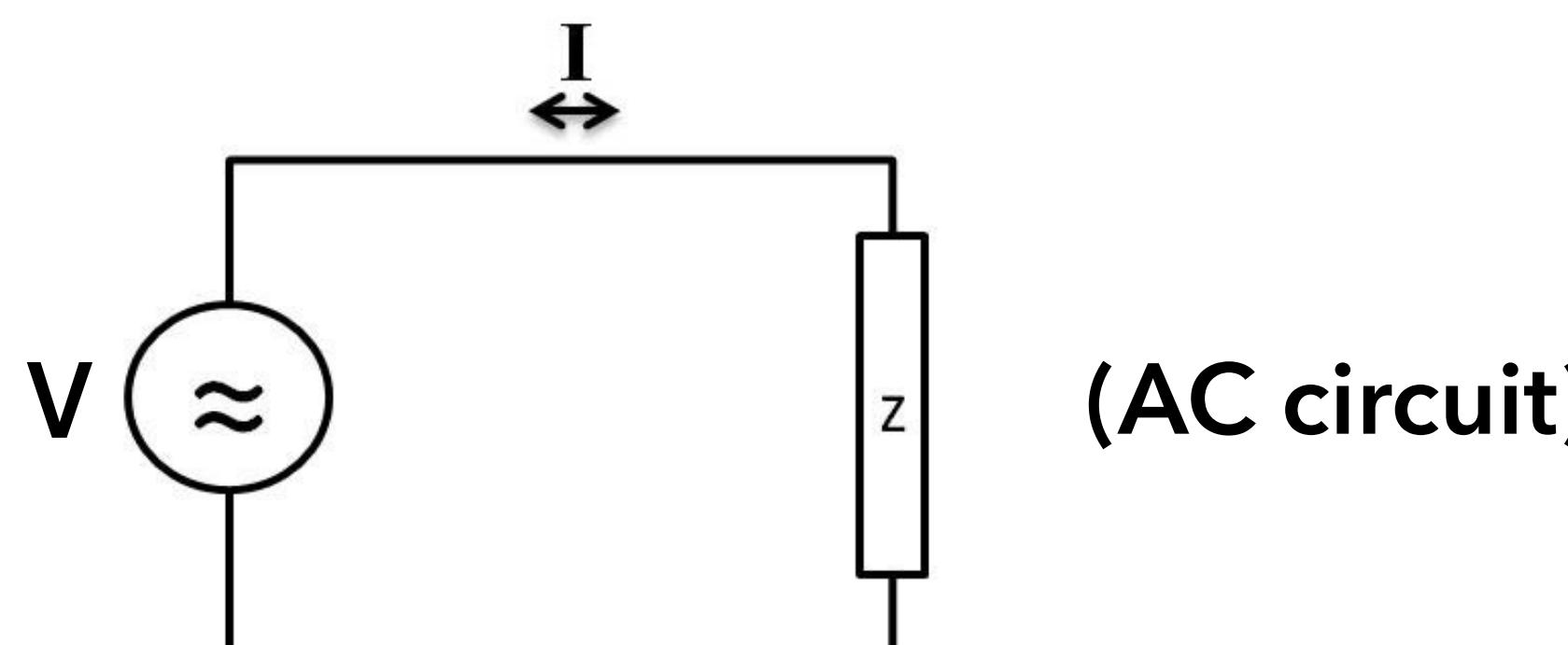
# Electrical reproduction of sound – overview

- Goal: Understand how microphones and loudspeakers work
- Need basic understanding of:
  - electricity and magnetism
  - Faraday's law of induction

# Basic electricity



e.g., a battery connected  
to a flashlight bulb

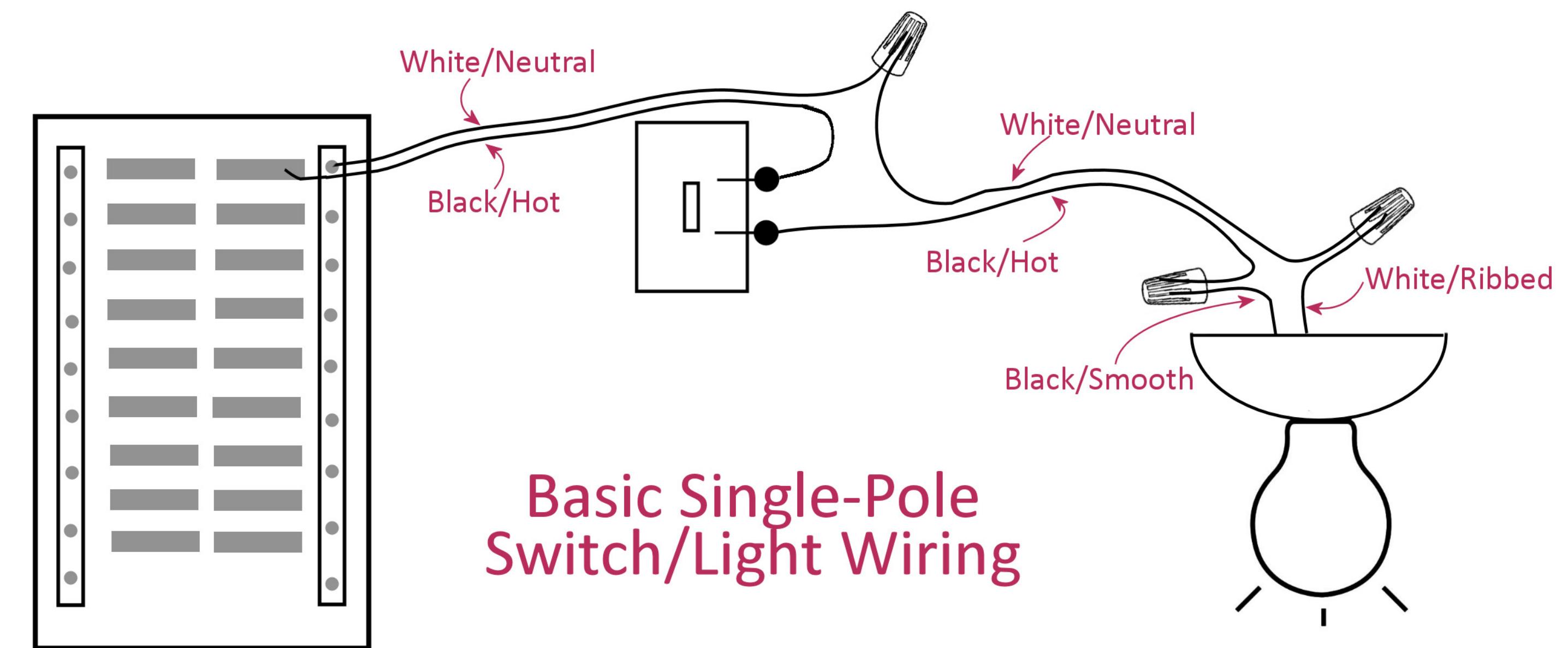
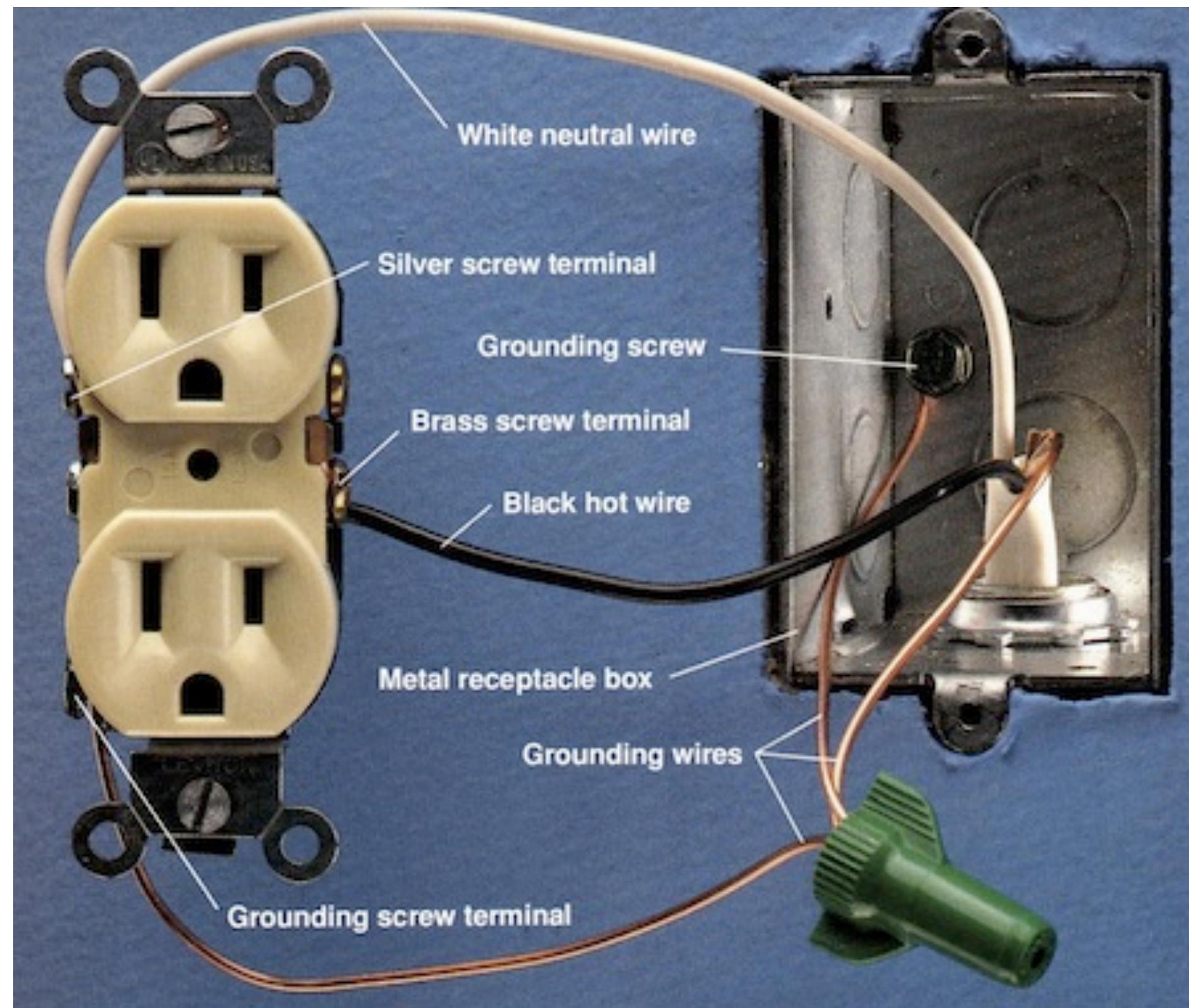


e.g., a household wall outlet  
connected to a vacuum cleaner

- Voltage  $V$  (volts)
- Current  $I$  (amperes or amps)
- Resistance  $R$  or impedance  $Z$  (ohms,  $\Omega$ )
- Direct current (DC) and alternating current (AC) circuits
- Ohm's law of electricity:  $V = IR$
- Electrical power:  $P = VI = I^2R$  (Watts)
- Relation to work or energy:

$$P = W/\Delta t \text{ (Watts)} \quad \text{or} \quad W = P \Delta t \text{ (Joules)}$$

# Example – home wiring



<https://gardnerbenderfaq.wordpress.com/tag/outlet/>

<https://www.addicted2decorating.com/how-to-wire-single-pole-light-switch.html>

# Example – kilowatt-hr and your electric bill

- A kilowatt-hr is a convenient unit of energy:

$$1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ hr} = 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J}$$

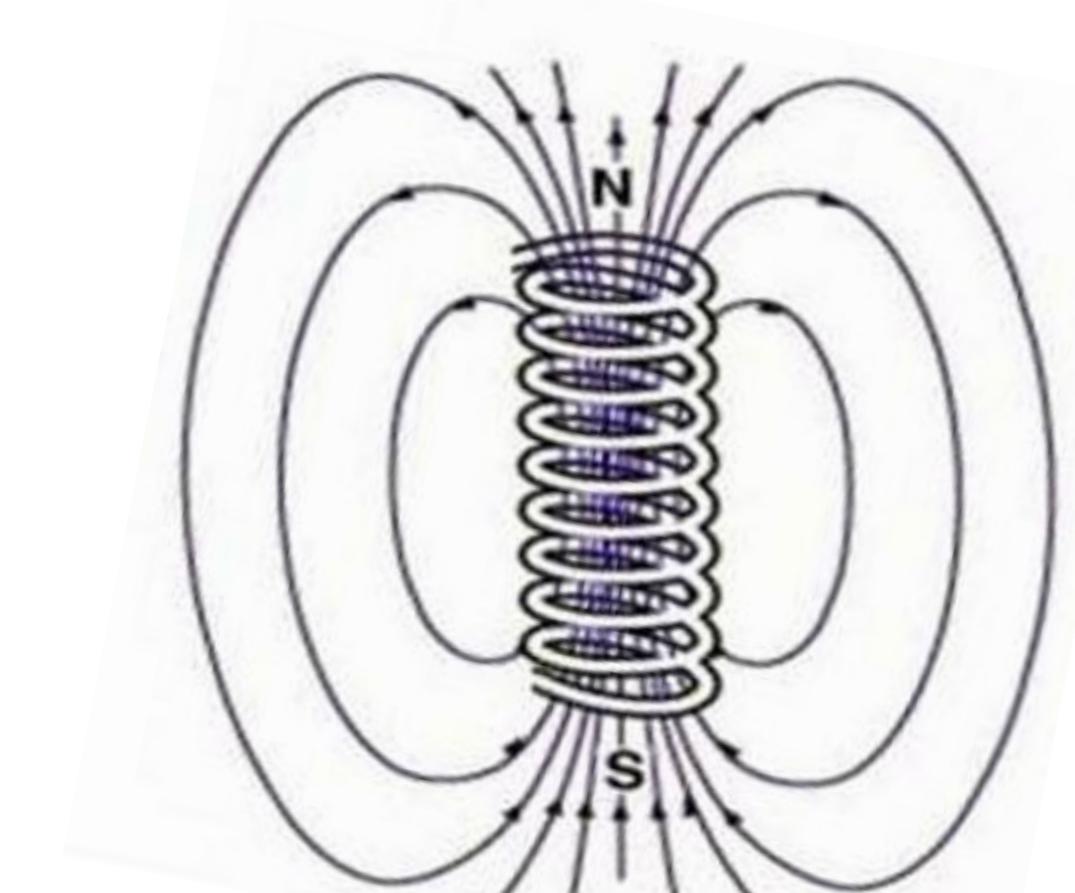
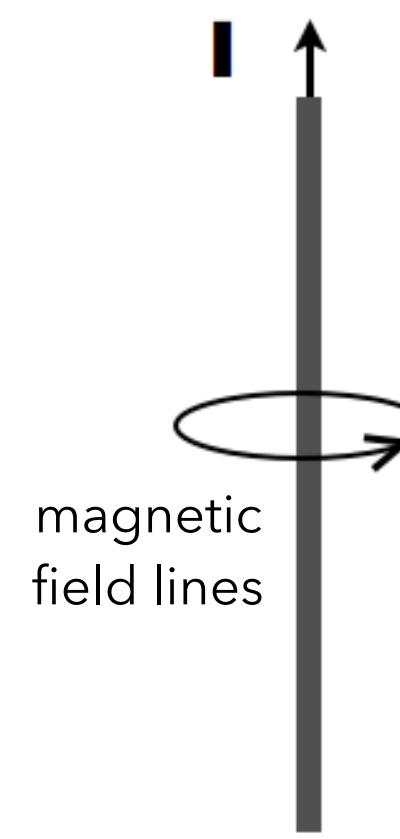
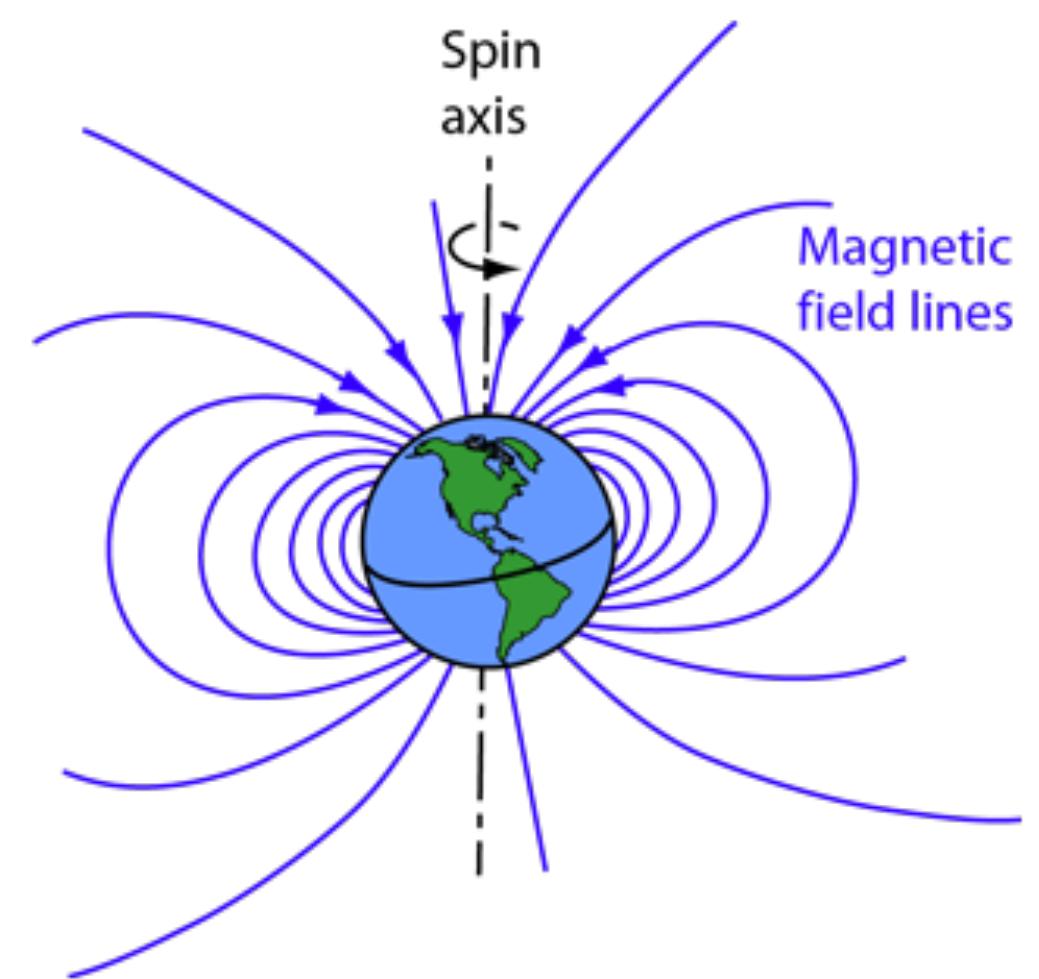
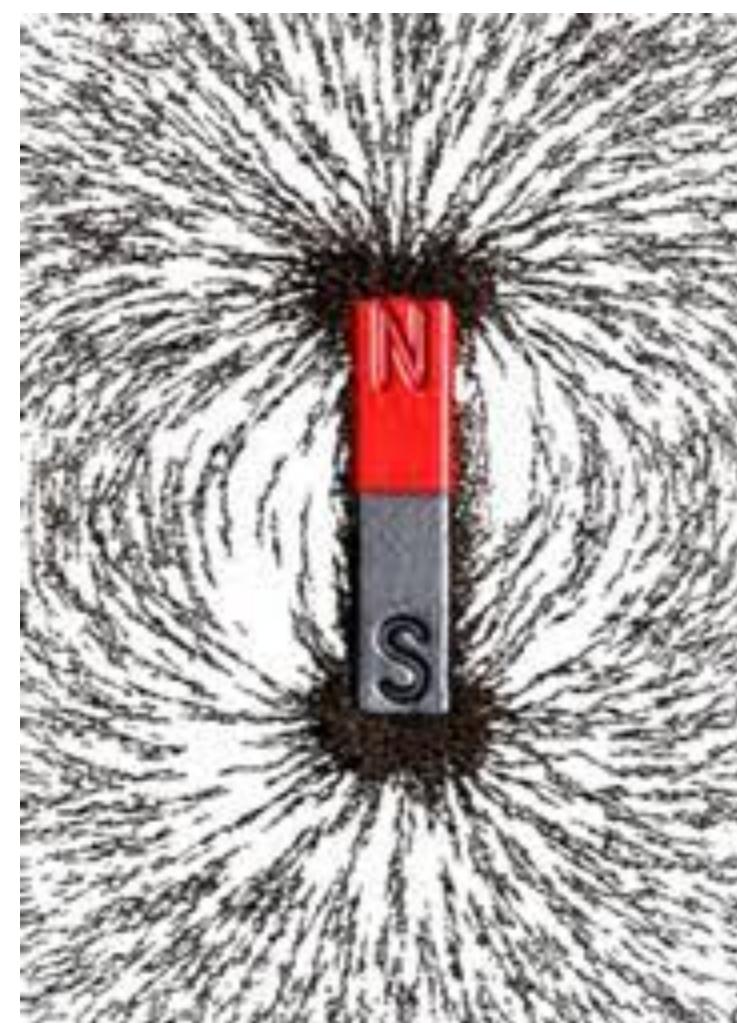
- Exercise: Suppose you paid \$100 for last month's electric bill at a cost of \$0.13/kWh.
  - (a) How much energy (in kWh) did you use?
  - (b) What was the average power consumption (in Watts) over the month (assume 30 days)?
- Answer:
  - (a)  $W = \$100 \div \$0.13/\text{kWh} = 769 \text{ kWh}$

$$(b) P = \frac{W}{\Delta t} = \frac{769 \text{ kWh}}{30 \times 24 \text{ h}} = 1.1 \text{ kW} = 1,100 \text{ W}$$

(eleven 100-Watt lightbulbs on continuously)

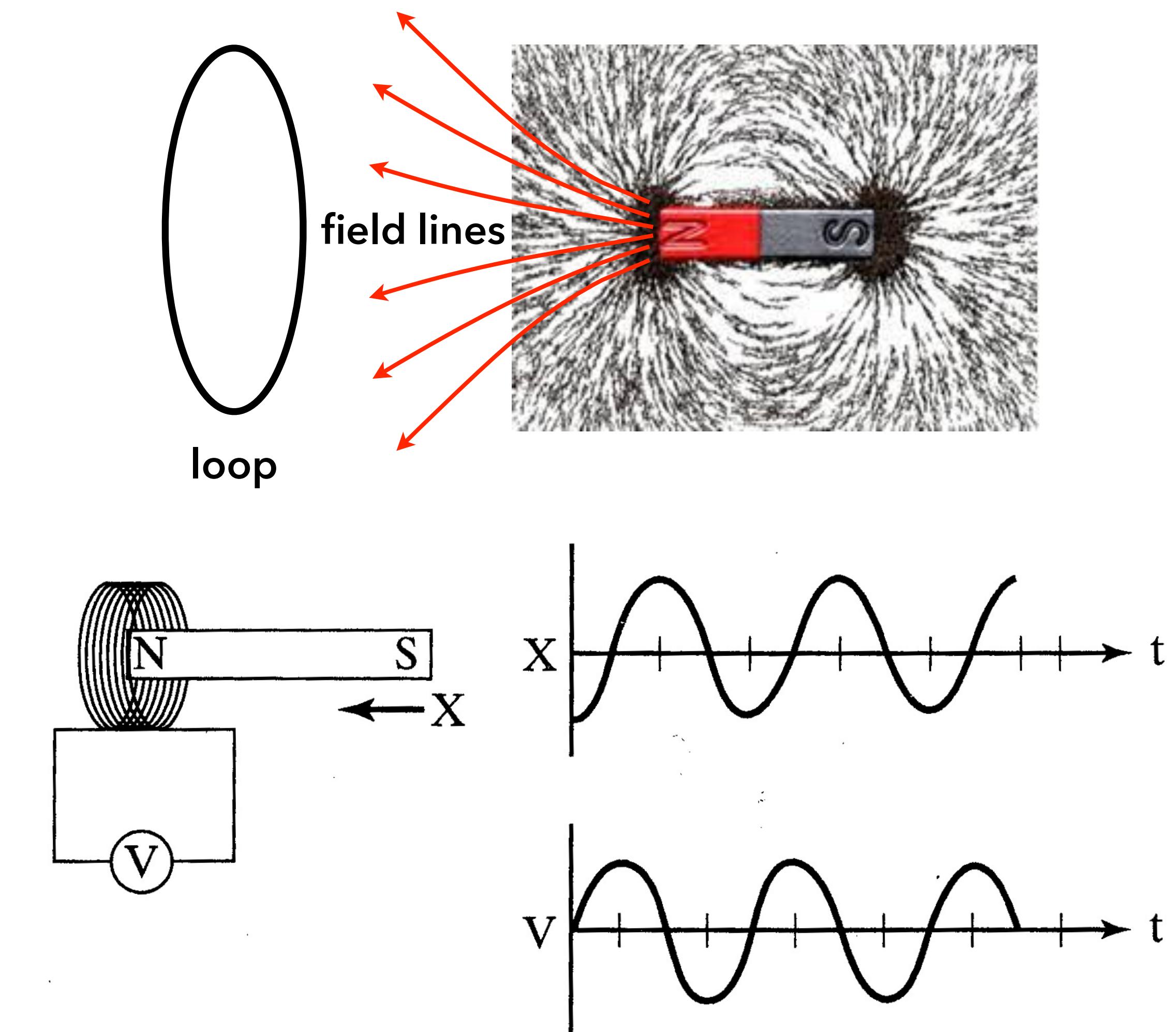
# Basic magnetism

- A permanent magnet has N and S poles that attract pieces of iron
- Like poles repel; unlike poles attract (just like + and – electrical charges). But no isolated magnetic poles.
- A compass needle is a tiny magnet that is attracted to Earth's South magnetic pole.
- Oersted (1820): discovered that an electric current produces a magnetic field
- Can create an electromagnet by sending an electric current through a coil of wire



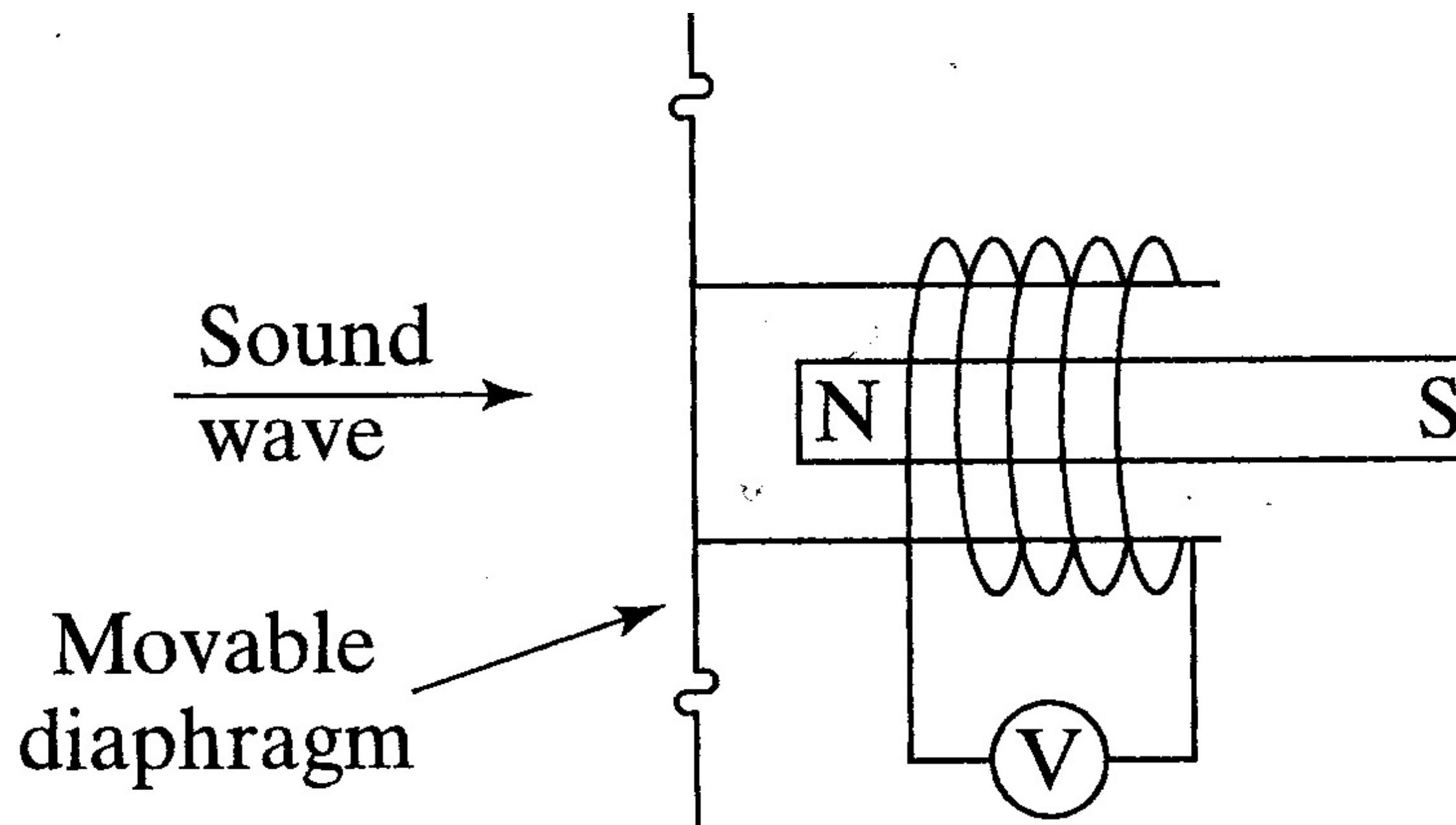
# Faraday's law of induction (1831)

- A change in magnetic flux through a coil of wire induces a voltage in the coil:
$$V = -N \frac{\Delta\Phi}{\Delta t}$$
- Only relative motion is important
- Underlies the operation of electric generators and electric motors
- Electric generator: mechanical energy converted to electrical energy
- Electric motor: electrical energy converted to mechanical energy
- 4 YouTube videos (linked from "Slide Presentations")



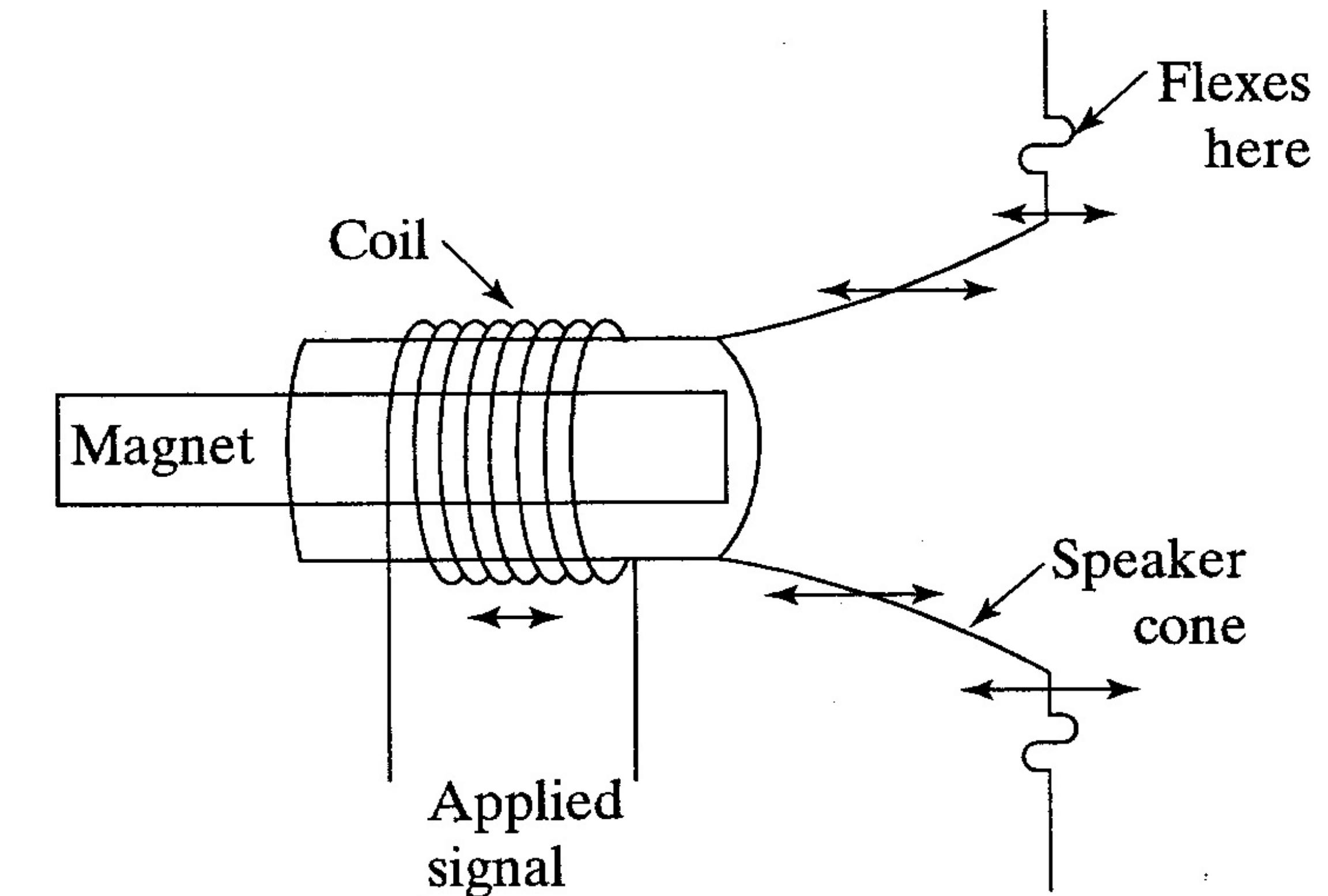
# Application – microphones and loudspeakers

Dynamic microphone



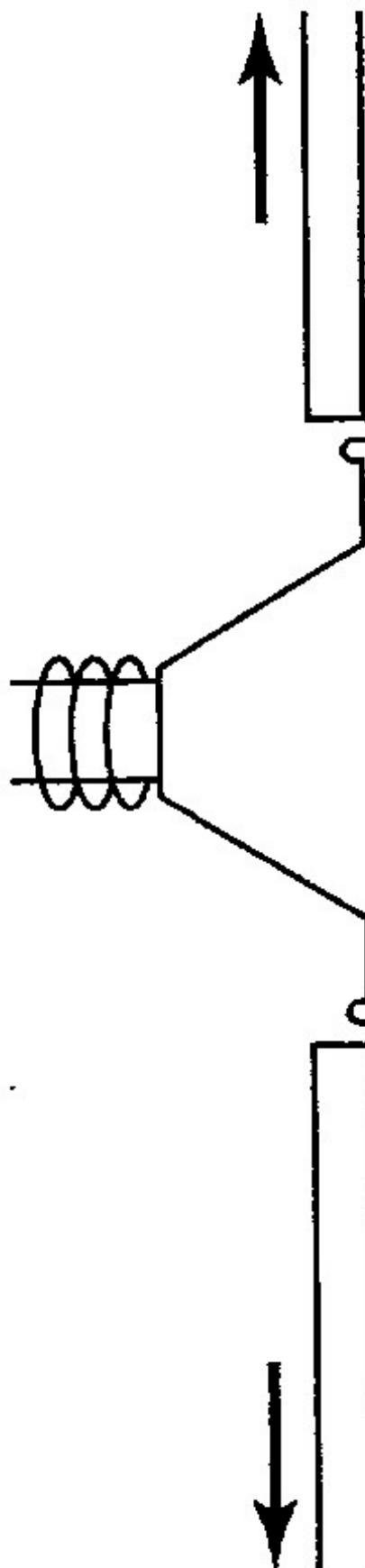
sound wave → electrical signal

Loudspeaker

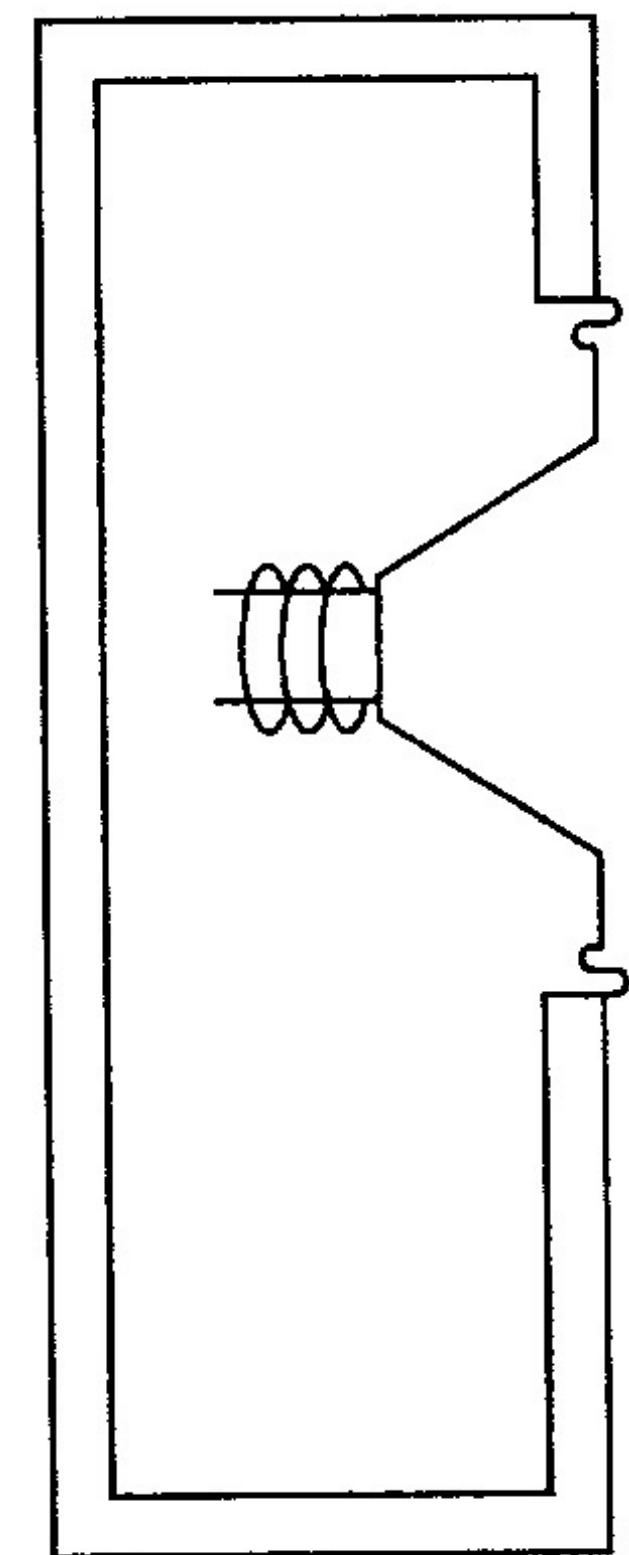


electrical signal → sound wave

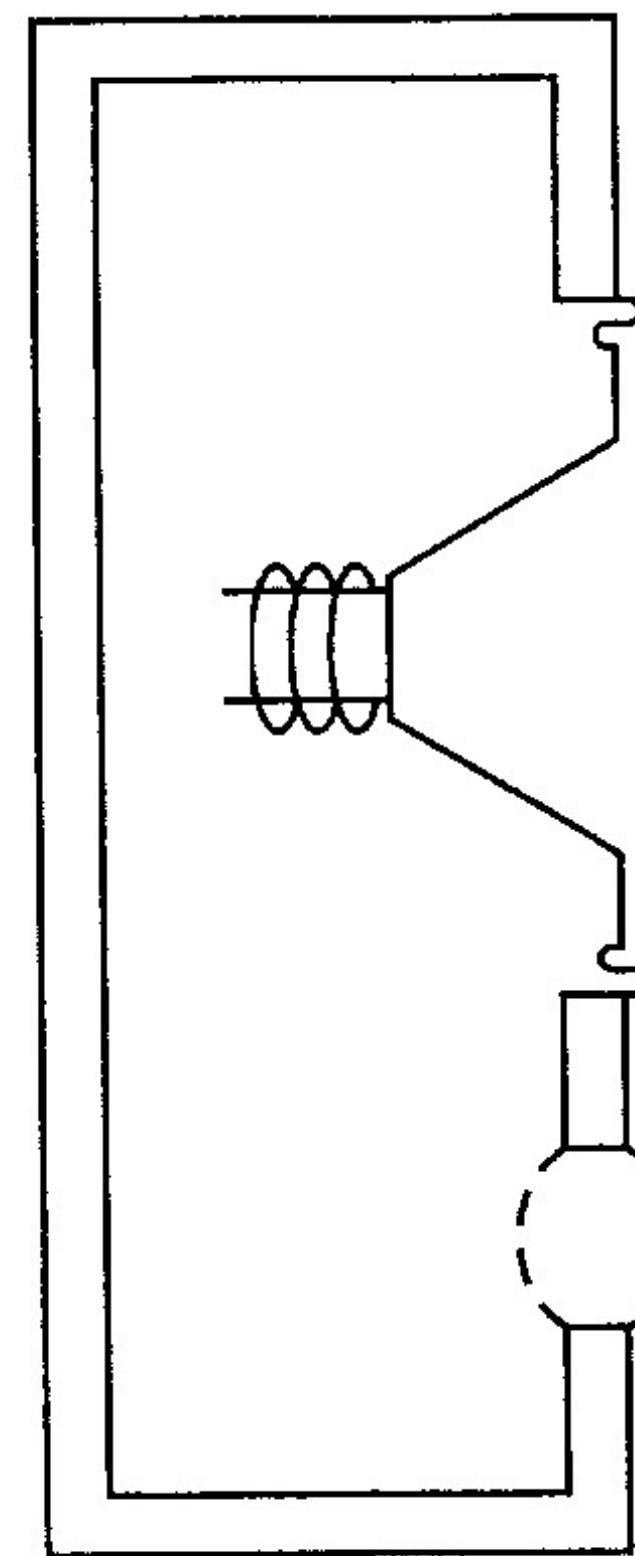
# Loudspeakers



Infinite baffle

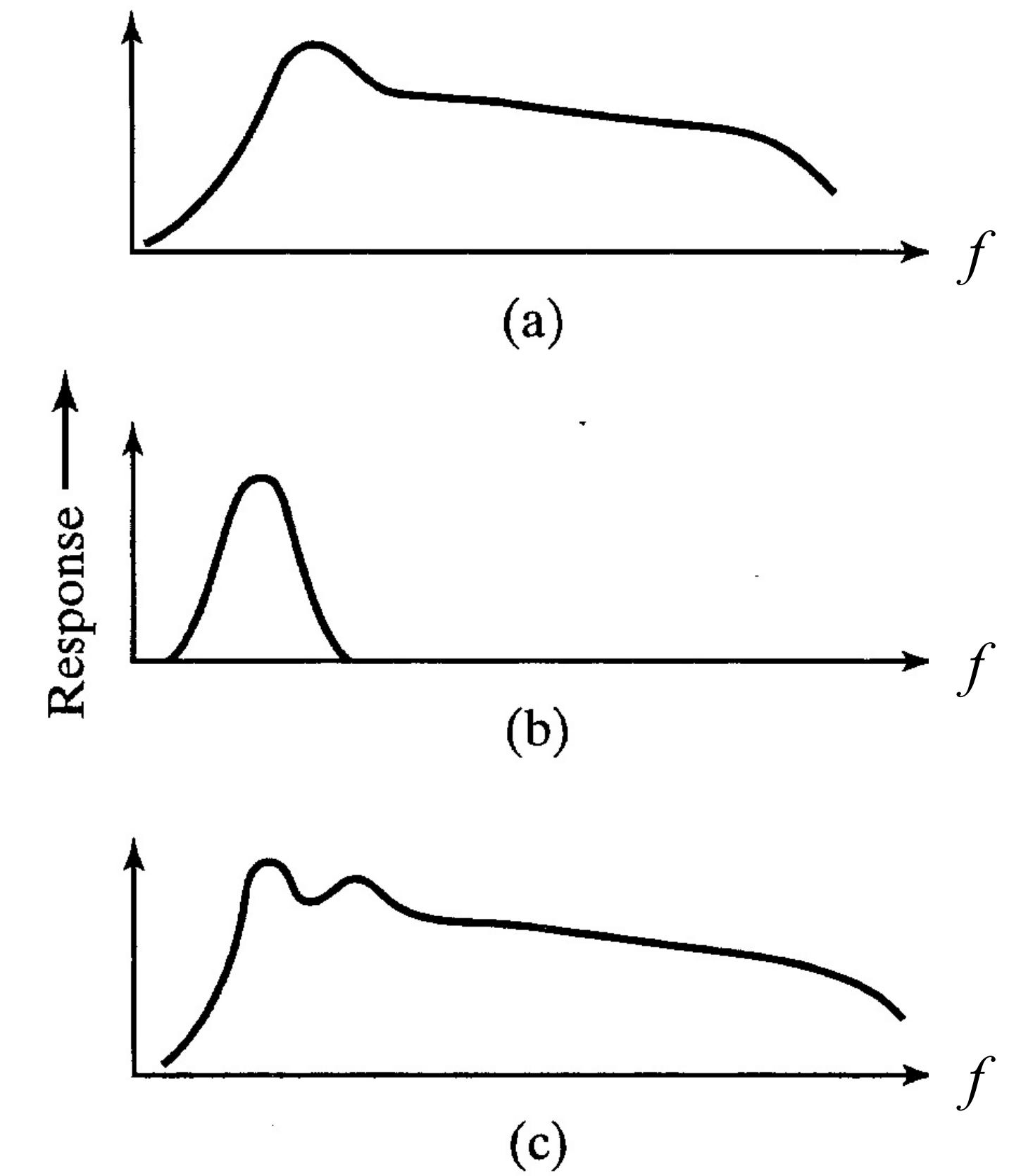


Acoustic suspension



Tuned port (bass reflex)

Frequency response curves  
(acoustic suspension, tuned port)



# **14. Elementary music theory**

# Music theory – the need to standardize musical notes

- A tuning system is an assignment of precise frequencies to all musical notes in an octave (reference note is A4 = 440 Hz; decided upon in 1939)
- Three standard tuning systems:
  - Equal temperament
  - Pythagorean temperament
  - Just temperament
- Each tuning system has its own advantages and disadvantages
- What tuning systems do real musicians use? (Diego??)

# Musical scales – dividing up the octave into pieces

- Chromatic scale: 12 pieces (semitones)

C - C# - D - Eb - E - F - F# - G - Ab - A - Bb - B - C' (white and black keys on a piano)

- Diatonic scale: 7 pieces (semitones and whole tones)

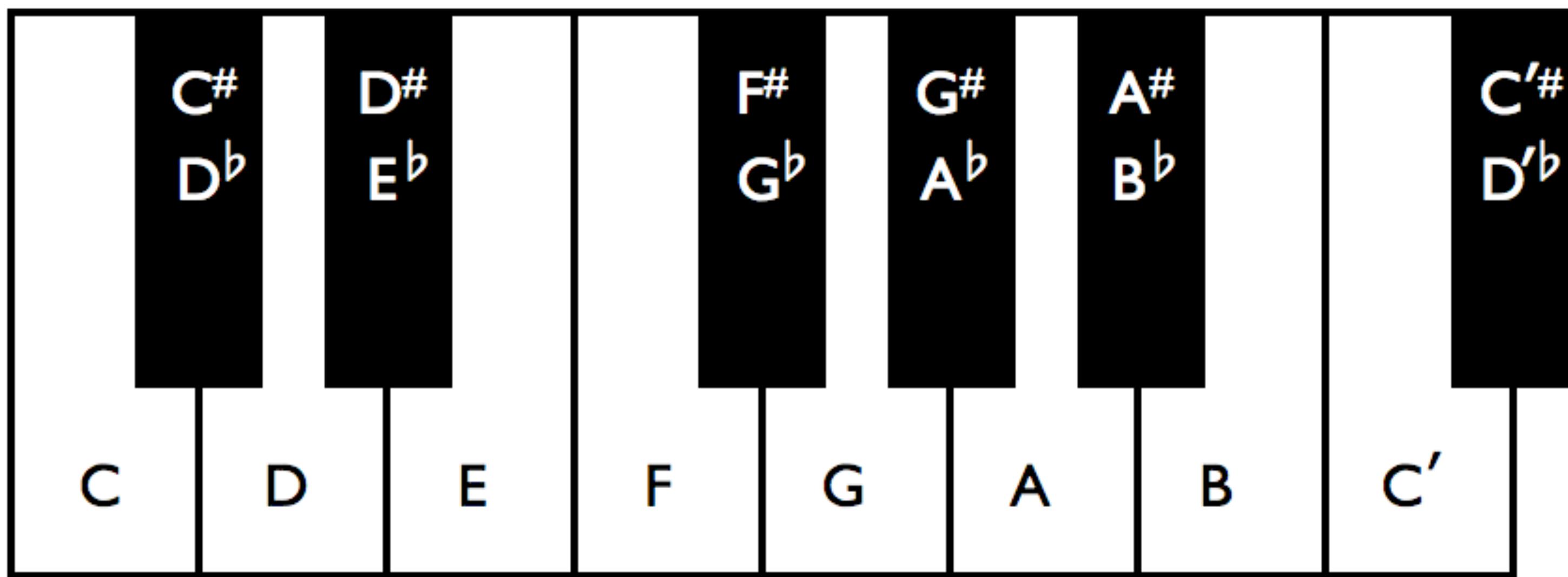
T-T-S-T-T-T-S (do-re-mi-fa-sol-la-ti-do; white keys on a piano)

- Pentatonic scale: 5 pieces (whole tones and 3 semitones intervals)

T-T-3-T-3 (F# - G# - A# - C# - D# - F#'; black keys on a piano)

# Equal temperament

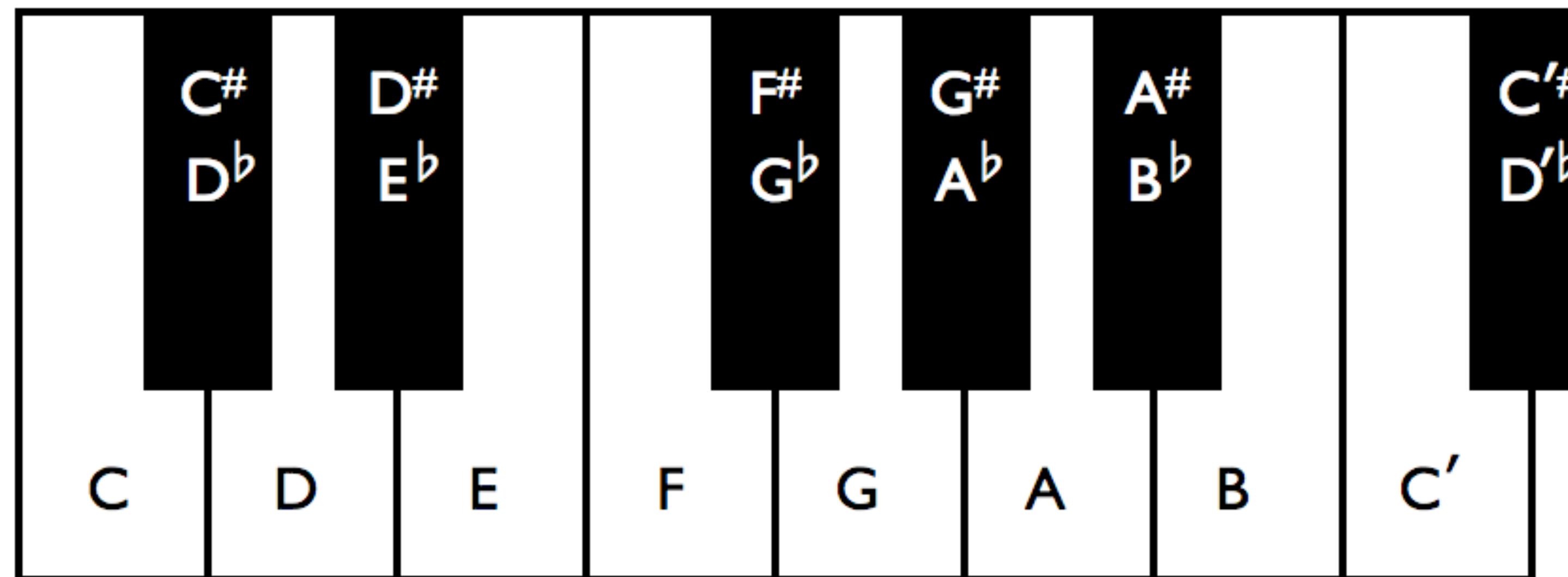
- All semitones intervals are equal:  $2^{1/12} = 1.059$
- Cent (100 cents = semitone):  $2^{1/1200} = 1.000578$  (JND: ~10 cents)
- All sharps and flats are equal to one another



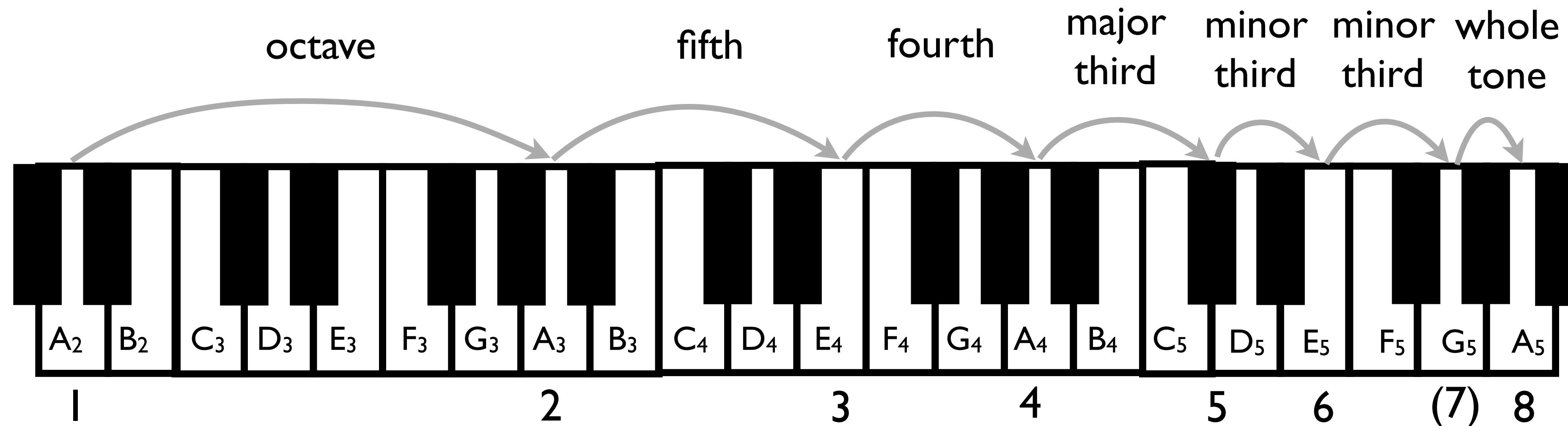
Note	ET freq ratio
C	$2^{0/12} = 1.000$
C♯/D♭	$2^{1/12} = 1.059$
D	$2^{2/12} = 1.122$
D♯/E♭	$2^{3/12} = 1.189$
E	$2^{4/12} = 1.260$
F	$2^{5/12} = 1.335$
F♯/G♭	$2^{6/12} = 1.414$
G	$2^{7/12} = 1.498$
G♯/A♭	$2^{8/12} = 1.587$
A	$2^{9/12} = 1.682$
A♯/B♭	$2^{10/12} = 1.782$
B	$2^{11/12} = 1.888$
C'	$2^{12/12} = 2.000$

# Musical intervals

Interval	# semitones	Just freq ratio	ET freq ratio	Difference (cents)	Example
Octave	12	$2 : 1 = 2.000$	2.000	0	C-C'
Fifth	7	$3 : 2 = 1.500$	1.498	2	C-G
Fourth	5	$4 : 3 = 1.333$	1.335	-2	C-F, G-C'
Major third	4	$5 : 4 = 1.250$	1.260	-14	C-E
Minor third	3	$6 : 5 = 1.200$	1.189	16	C-E <sup>b</sup> , A-C'

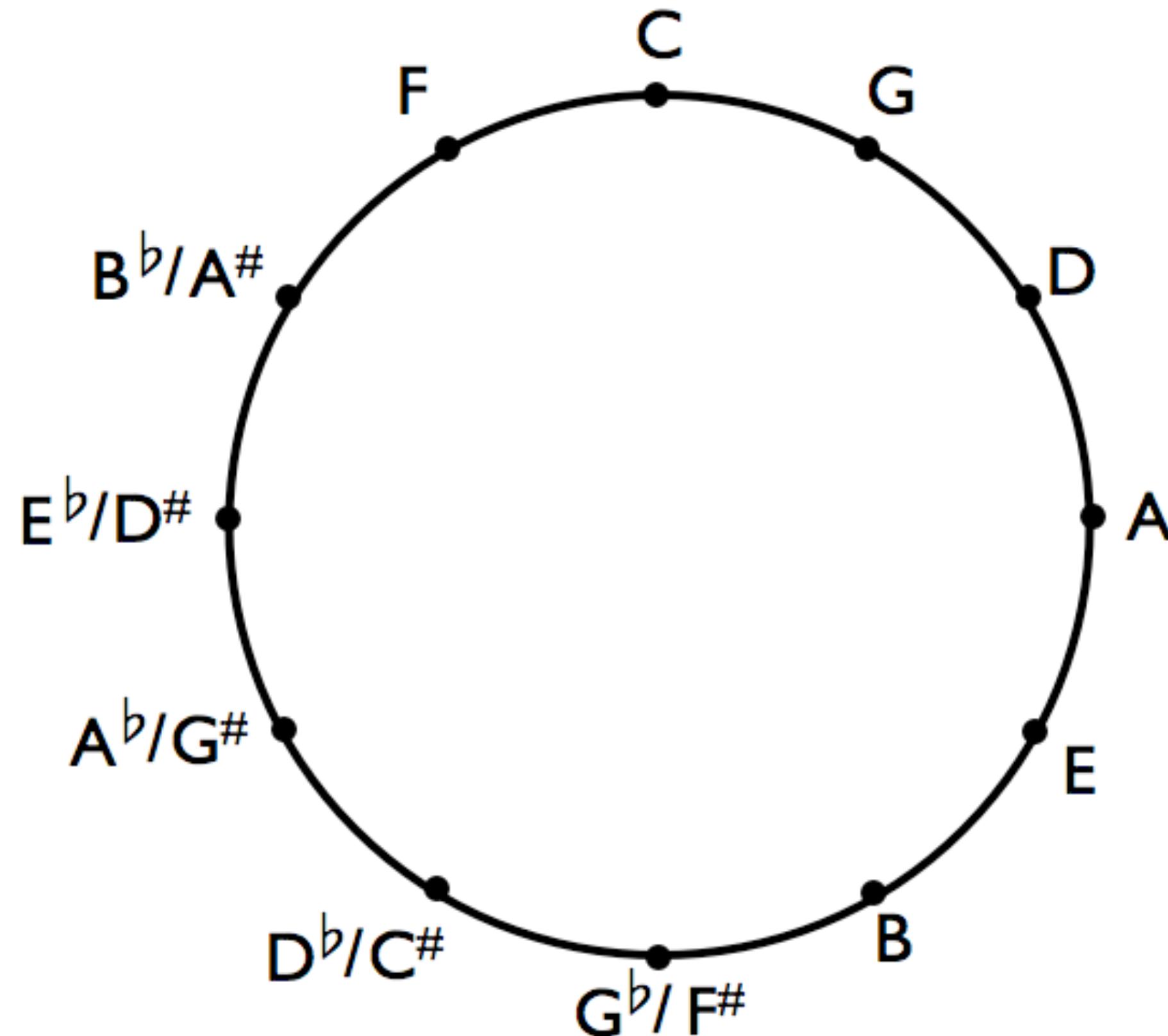


# Harmonics

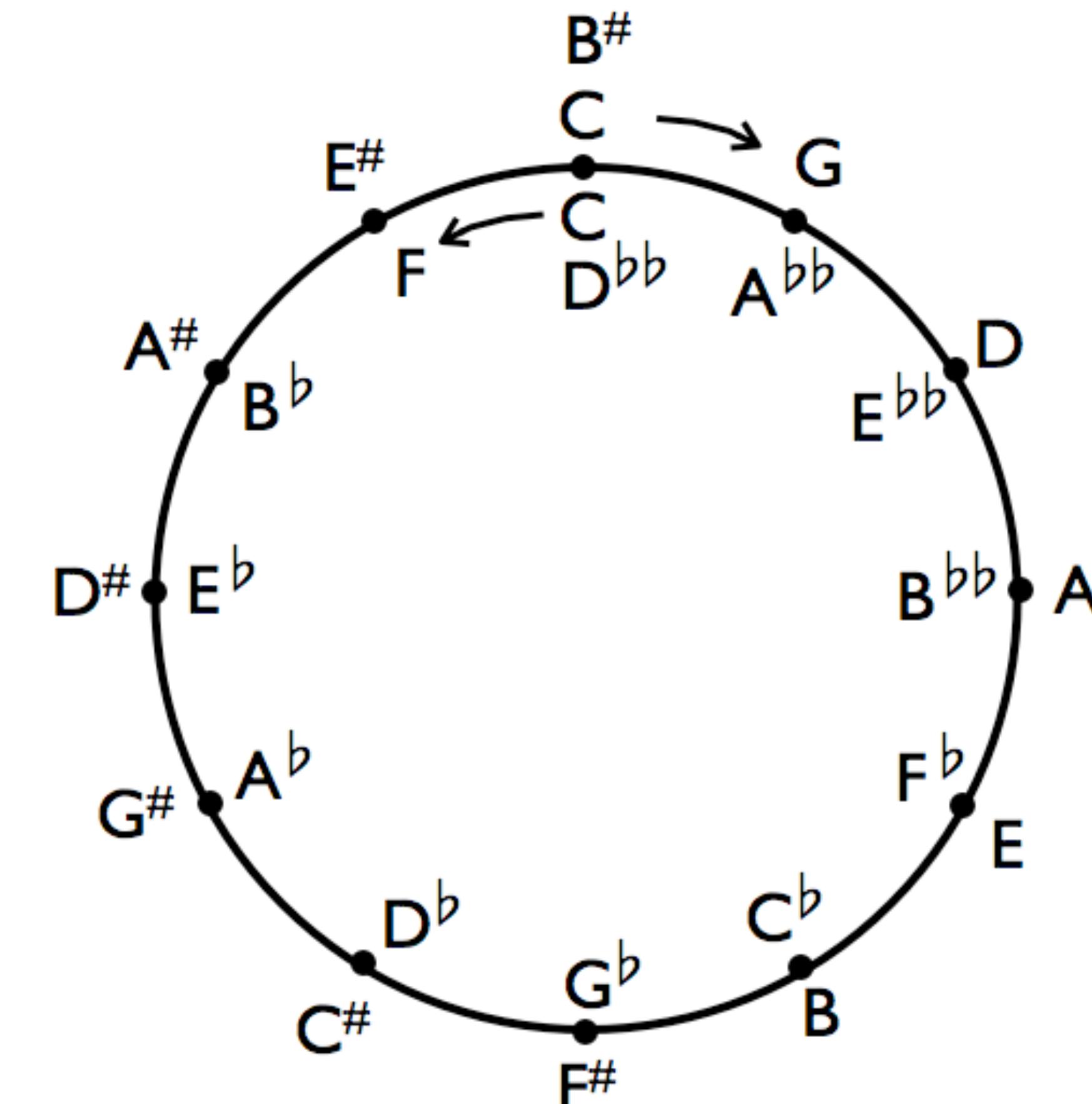


Harmonic	Exact freq (Hz)	Equal-tempered freq (Hz)	Difference (cents)	Piano note
1	110	110.00	0	A <sub>2</sub>
2	220	220.00	0	A <sub>3</sub>
3	330	329.63	2	E <sub>4</sub>
4	440	440.00	0	A <sub>4</sub>
5	550	554.37	-14	C <sup>#</sup> <sub>5</sub>
6	660	659.26	2	E <sub>5</sub>
7	770	783.99	-31	G <sub>5</sub>
8	880	880.00	0	A <sub>5</sub>

# Circle of fifths

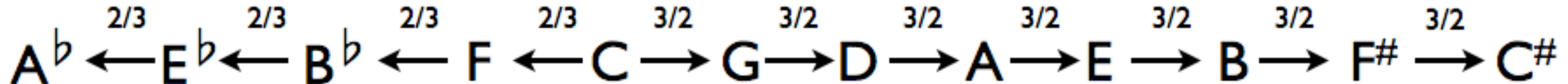


Equal Temperament



Other tuning systems

# Pythagorean temperament



- Constructed from perfect fifth and octave intervals
- For example:

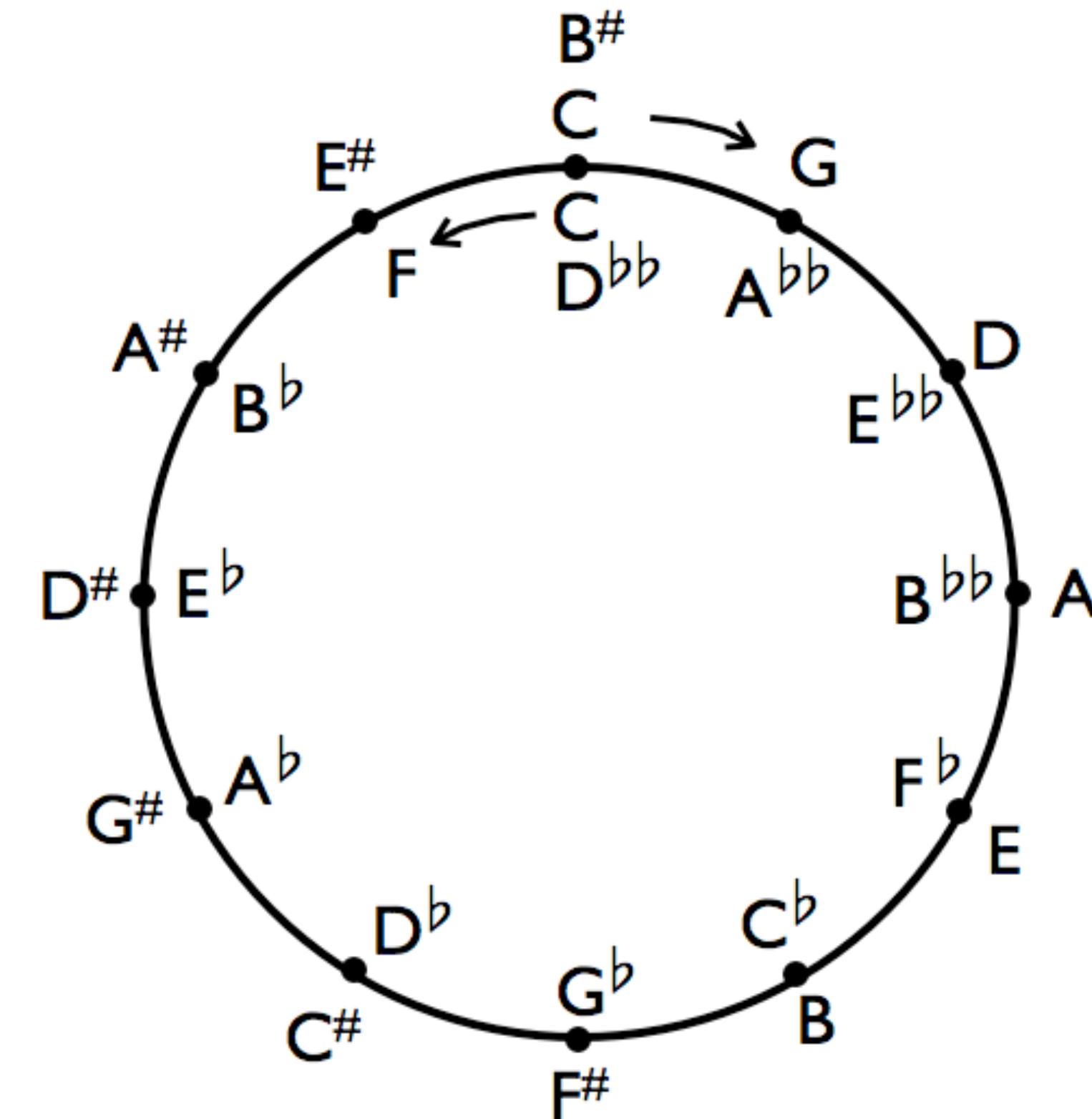
$G: 3/2$

$D: (3/2)2 \times (1/2) = 9/8$

$A: (3/2)3 \times (1/2) = 27/16$

$E: (3/2)4 \times (1/2)2 = 81/64$

$F: (2/3) \times 2 = 4/3$



# Just temperament

- Constructed from perfect fifth, major third, and octave intervals
- For example:

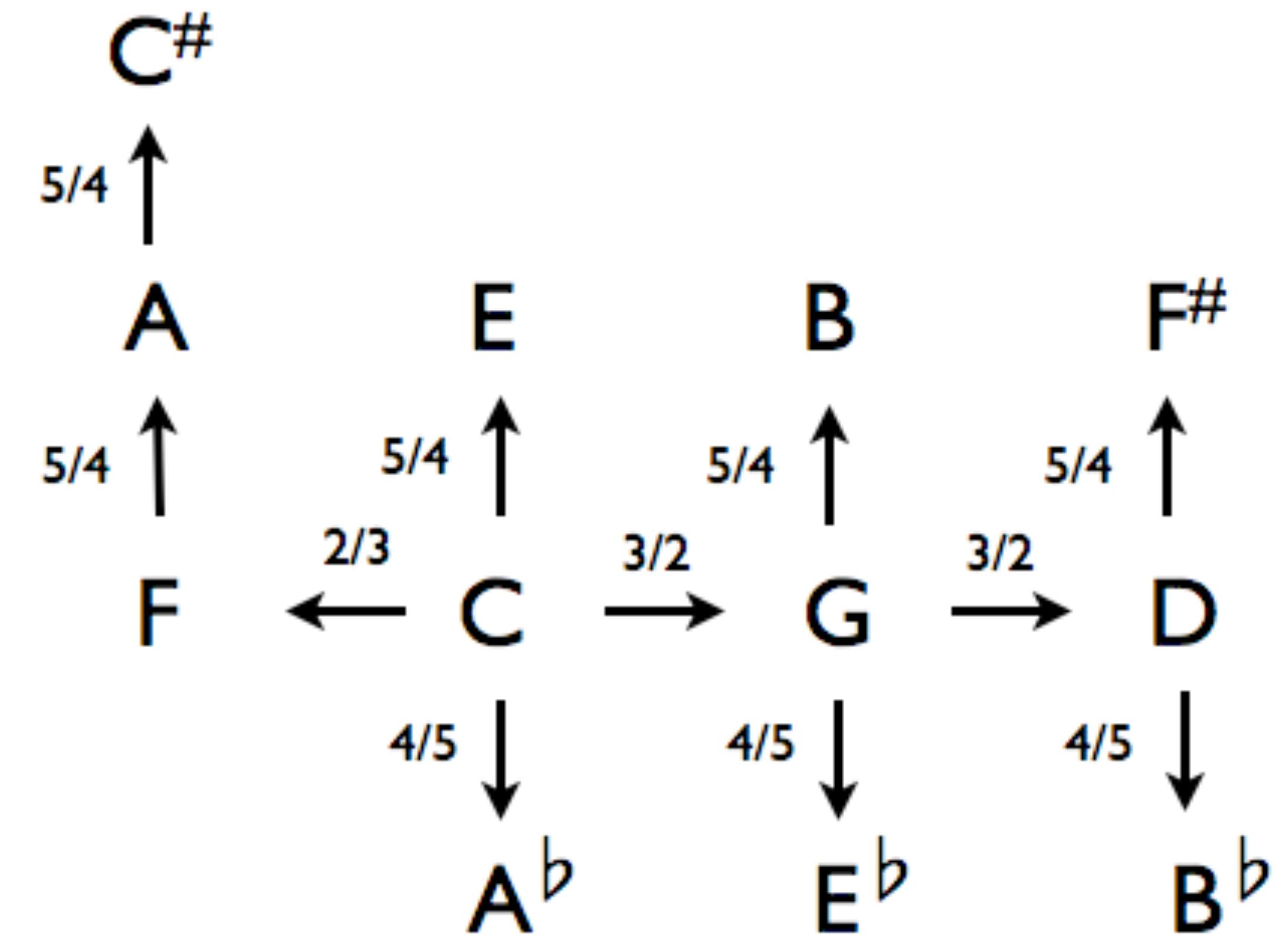
G: 3/2

D:  $(3/2)2 \times (1/2) = 9/8$

A:  $(2/3) \times (5/4) \times 2 = 5/3$  (vs 27/16)

E: 5/4 (vs 81/64)

F:  $(2/3) \times 2 = 4/3$



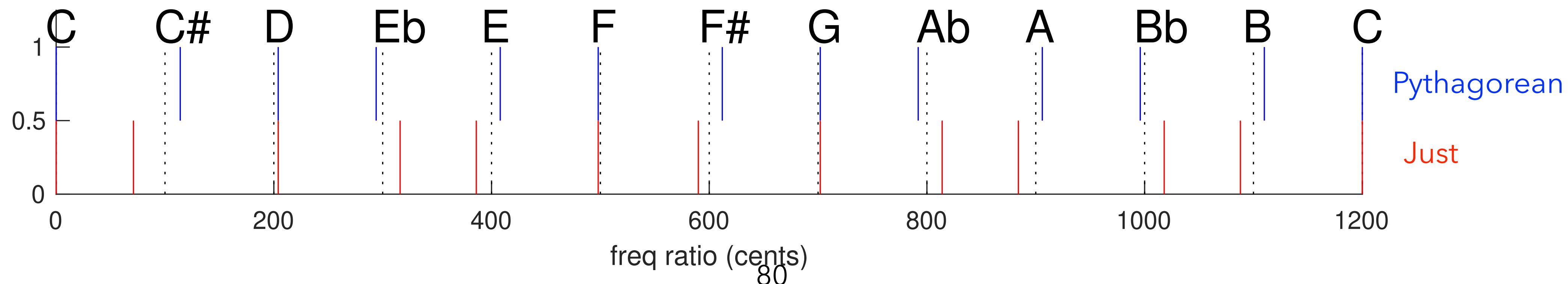
# Comparing different tuning systems

Pythagorean vs Equal Temperament

Note	Pyth freq ratio	ET freq ratio	Difference (cents)
C	$1 : 1 = 1.000$	1.000	0
C $\sharp$	$2187 : 2048 = 1.068$	1.059	14
D	$9 : 8 = 1.125$	1.122	4
E $\flat$	$32 : 27 = 1.185$	1.189	-6
E	$81 : 64 = 1.266$	1.260	8
F	$4 : 3 = 1.333$	1.335	-2
F $\sharp$	$729 : 512 = 1.424$	1.414	12
G	$3 : 2 = 1.500$	1.498	2
A $\flat$	$128 : 81 = 1.580$	1.587	-8
A	$27 : 16 = 1.688$	1.682	6
B $\flat$	$16 : 9 = 1.778$	1.782	-4
B	$243 : 128 = 1.898$	1.888	10
C'	$2 : 1 = 2.000$	2.000	0

Just vs Equal Temperament

Note	Just freq ratio	ET freq ratio	Difference (cents)
C	$1 : 1 = 1.000$	1.000	0
C $\sharp$	$25 : 24 = 1.042$	1.059	-29
D	$9 : 8 = 1.125$	1.122	4
E $\flat$	$6 : 5 = 1.200$	1.189	16
E	$5 : 4 = 1.250$	1.260	-14
F	$4 : 3 = 1.333$	1.335	-2
F $\sharp$	$45 : 32 = 1.406$	1.414	-10
G	$3 : 2 = 1.500$	1.498	2
A $\flat$	$8 : 5 = 1.600$	1.587	14
A	$5 : 3 = 1.667$	1.682	-16
B $\flat$	$9 : 5 = 1.800$	1.782	18
B	$15 : 8 = 1.875$	1.888	-12
C'	$2 : 1 = 2.000$	2.000	0



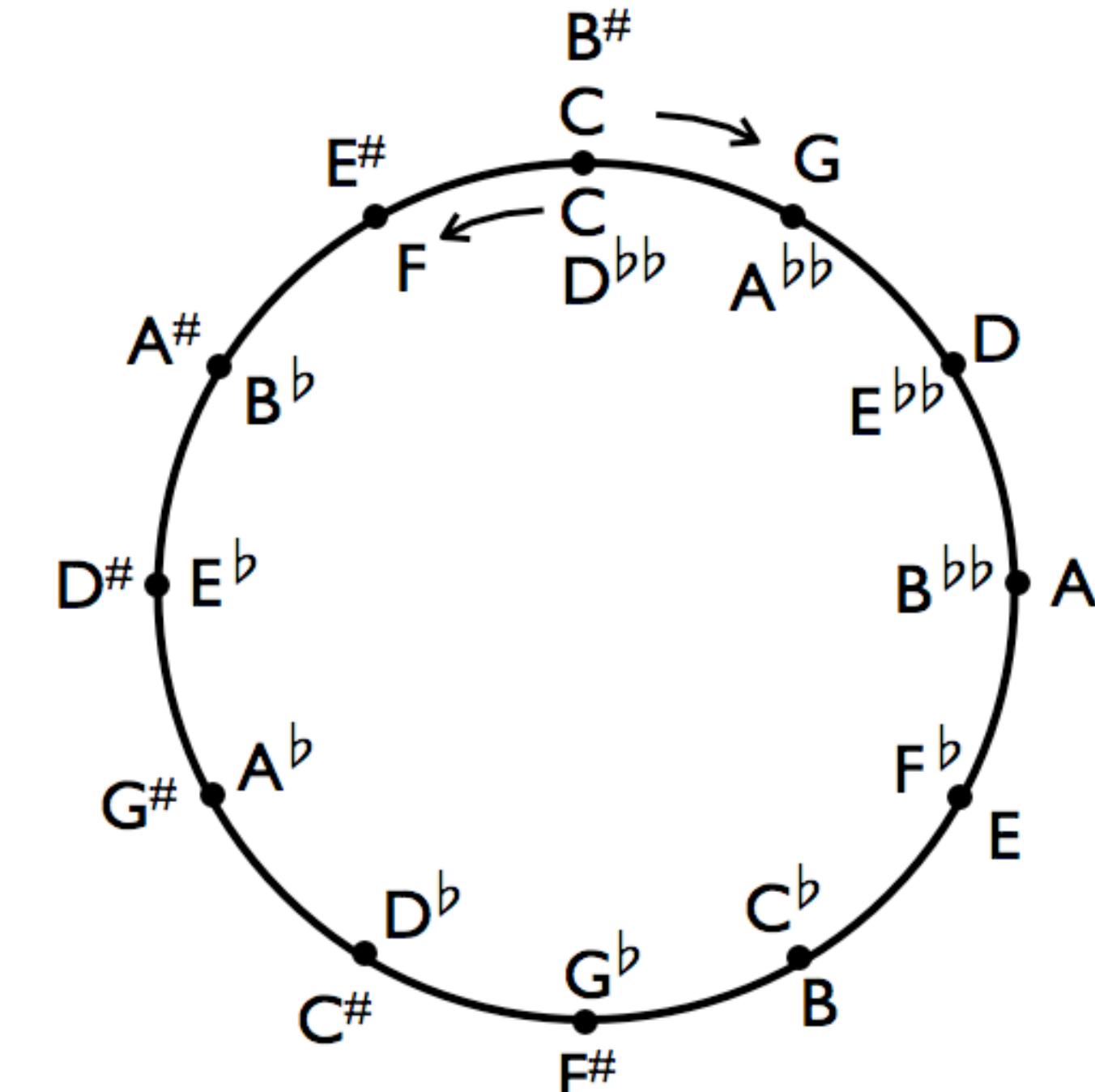
Pythagorean  
Just

# All tuning systems have problems!!

- Equal-tempered fifths, fourths, etc. are never perfect (only an octave)
- Pythagorean circle of fifths doesn't close (12 perfect fifths is not equal to 7 octaves)
- Pythagorean "comma":

$$\frac{B^\#}{C'} = \frac{(3/2)^{12}}{2^7} = 1.0136 \text{ (23 cents too large)}$$

- Fifth  $C^\#$  to  $A^\flat$  is too flat in Pythagorean temperament ("wolf" fifth) and too sharp in just temperament



Fifth	Temperament	Freq ratio	Difference (cents)
C-G	equal	1.498	-2
C-G	pyth	1.500	0
C-G	just	1.500	0
$C^\#-A^\flat$	equal	1.498	-2
$C^\#-A^\flat$	pyth	1.480	-23
$C^\#-A^\flat$	just	1.536	41