

PHYS1406: Physics of Sound and Music

Spring 2021

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Topics we'll cover this semester

- Preliminaries: Basic math, music, and physics terminology
- Physics of oscillations and waves
- Production of sound (instruments and voice)
- Perception of sound (hearing, loudness, pitch & timbre)
- Auditorium and room acoustics; electrical reproduction of sound
- Musical scales and tuning systems (standardization of musical notes)

Why are you in this class?

What questions about sound & music would you like to know the answer to?

What is sound? What differentiates speech, music, & noise?

- Sound is a **pressure wave** in air (or some other medium, which could be a liquid or solid).
- The pressure wave consists of alternating regions of **compression** and **expansion** of the air molecules.
- **Energy is transferred** from the source of sound to our ears, while the individual air molecules just oscillate back-and-forth in place.
- noise: chaotic, unorganized sound
- speech & music: organized sound
- **musical notes** have a **definite pitch** (low or high), while noise does not

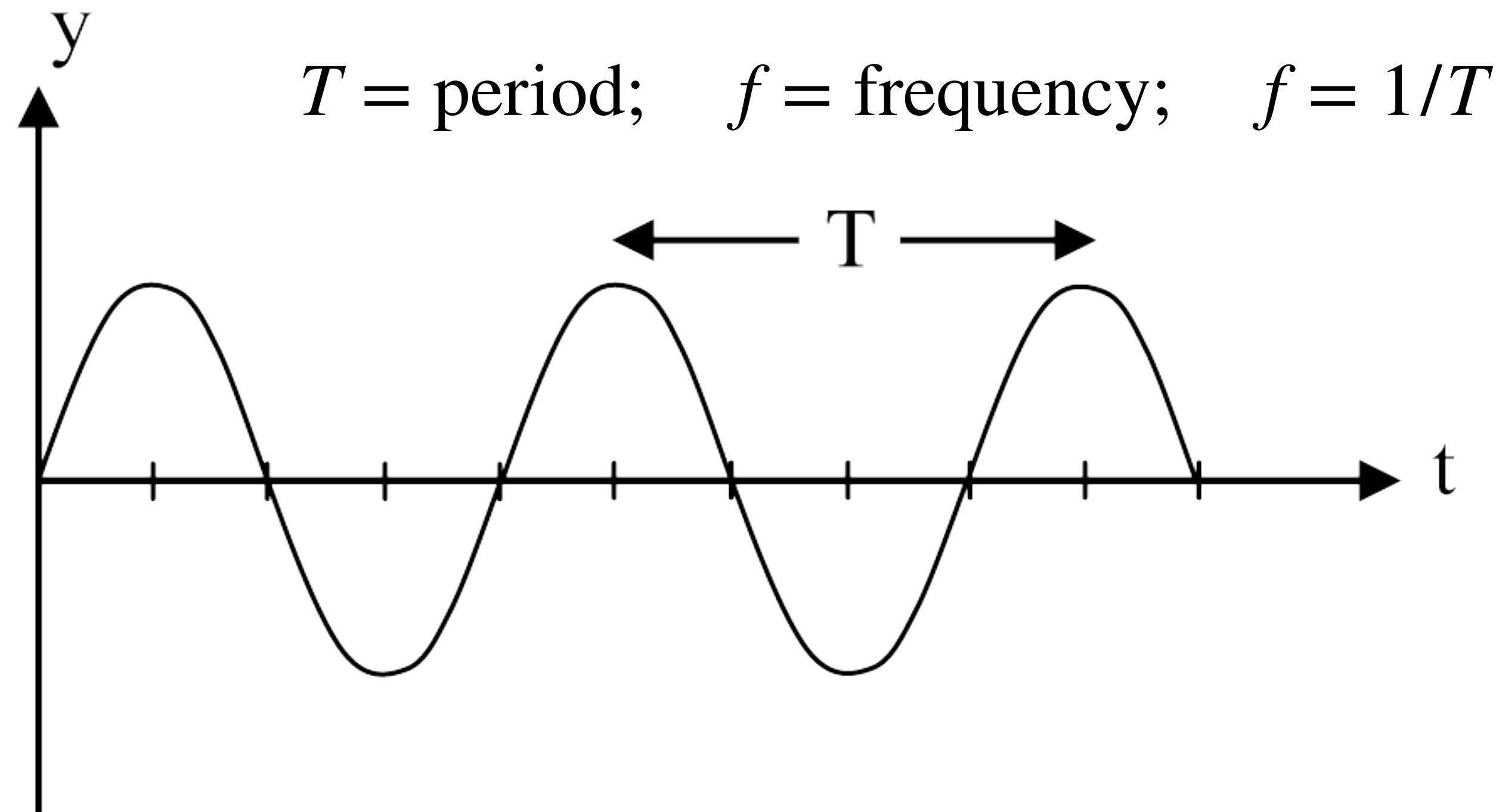
Demos: sound measuring devices & musical instruments

- Measuring devices
 - oscilloscope: shows how the sound pressure wave changes in time
 - FFT analyzer: shows how much sound energy is associated with different pitch components
 - spectrogram: shows how the pitch content of a sound changes in time
- Musical instruments and sound-making devices:
 - whistle, singing, speaking
 - recorder, plastic recorder, train whistle, other wind instruments
 - plucked guitar string, bowed violin
 - bell, drum, shakers, other percussion instruments
 - ratchet, crumpled paper, applause

Range of human hearing

<https://www.szynalski.com/tone-generator/>

- Normal range: 20 Hz - 20,000 Hz
- What is frequency? Examples?
- Hertz (Hz): 1 Hz = 1 cycle/sec



1. Preliminaries

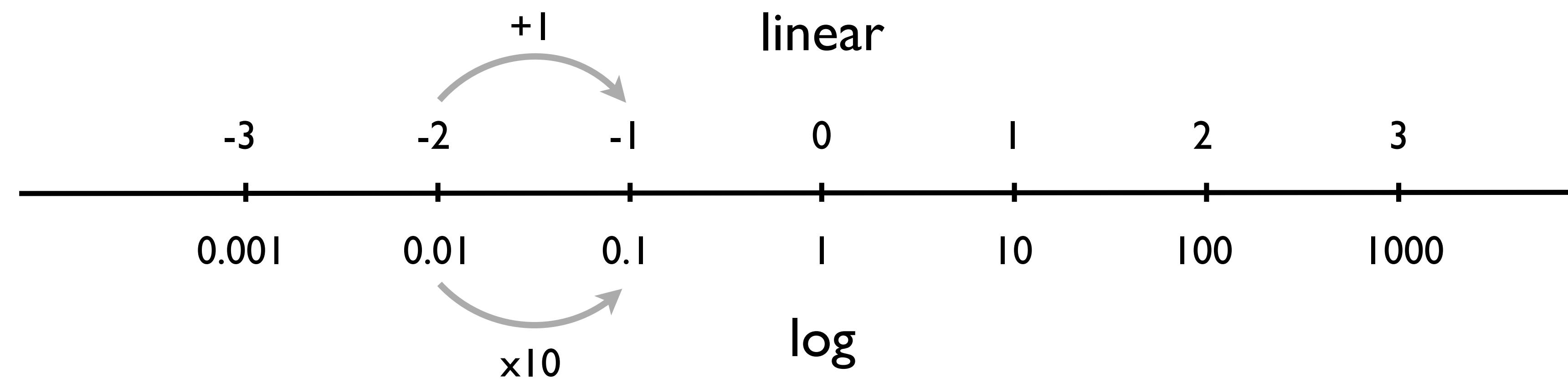
Basic math review

- Entering numbers on a calculator: What's the value of $1/2\pi$?
- Fractions: What's the value of 2 divided by $3/2$?
- Powers (exponential notation): What's the value of 2^4 ? 10^3 ? 10^{-2} ?
- Prefixes:

nano	micro	milli	centi	kilo	mega	giga	tera
10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^3	10^6	10^9	10^{12}

- Comparing two numbers: Compare the heights of two people, one who is 5.5 ft tall versus another who is 72 inches all.
- Converting units: The speed of sound in air at room temperature (25 celsius) is 346 m/s. What is its value in ft/s? miles/s?

Linear vs logarithmic scales

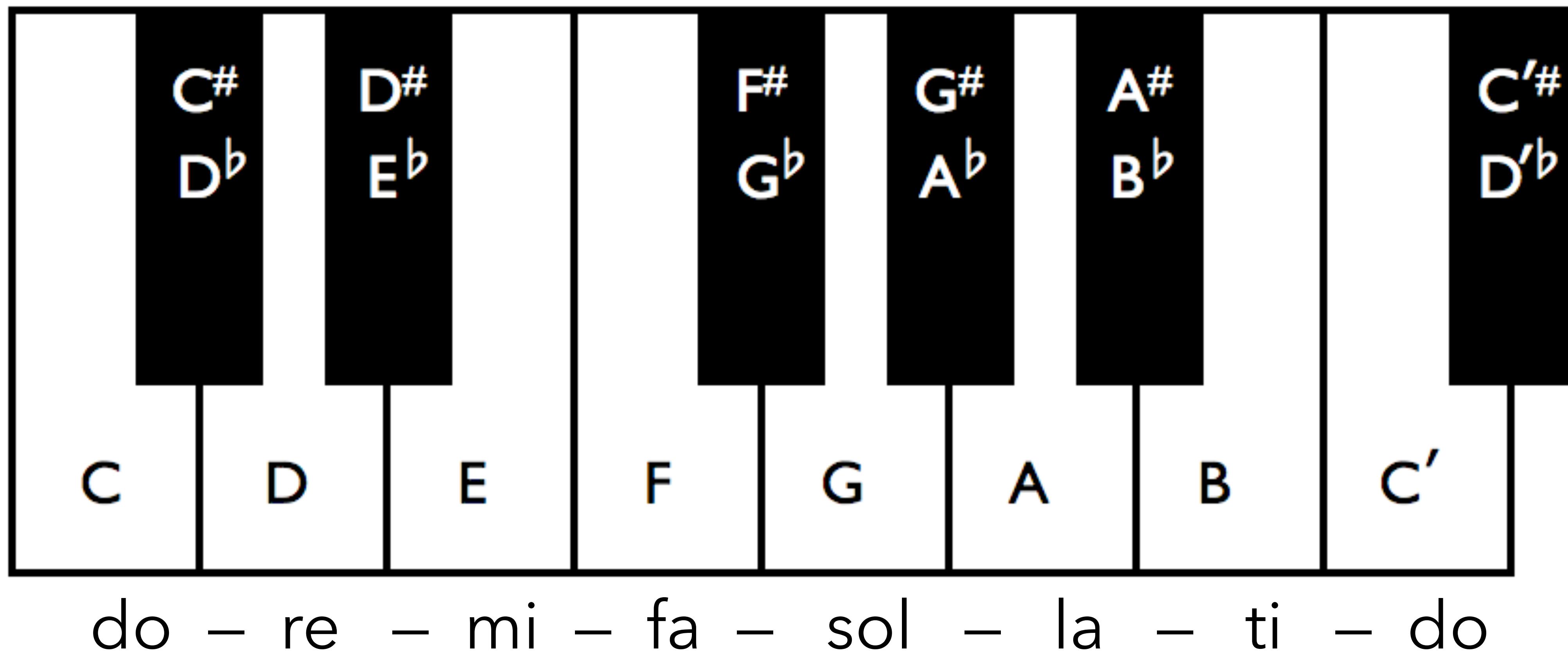


Music terminology

- Pitch:
- Timbre:
- Octave:
- Chromatic, diatonic, and pentatonic scales:
 - <https://www.youtube.com/watch?v=jaMA8LWW3C0> (pentatonic scale)
- Equal temperament:
- Musical intervals:
 - fifth, fourth, major third, minor third
- Chord:

Chromatic and diatonic scales

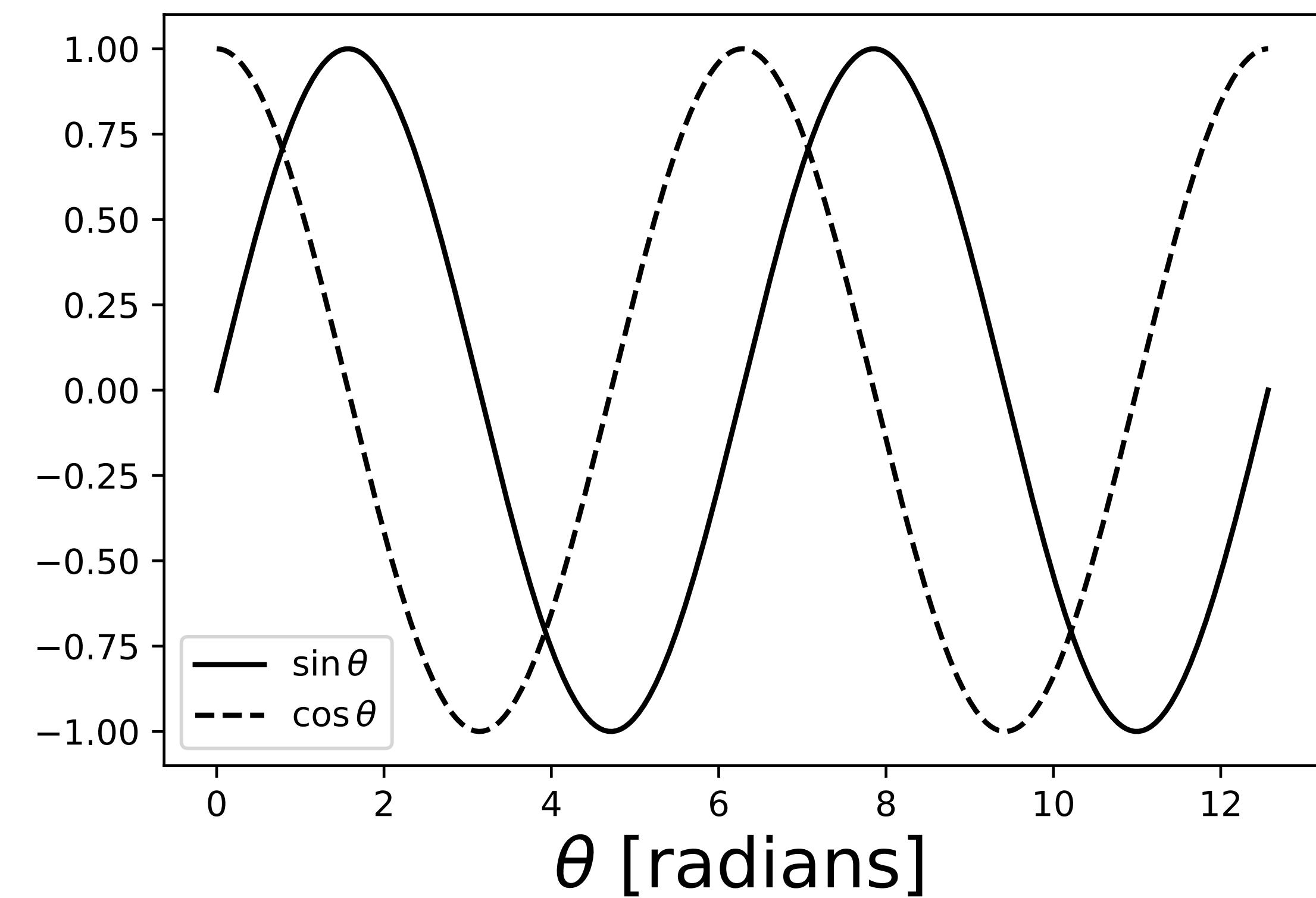
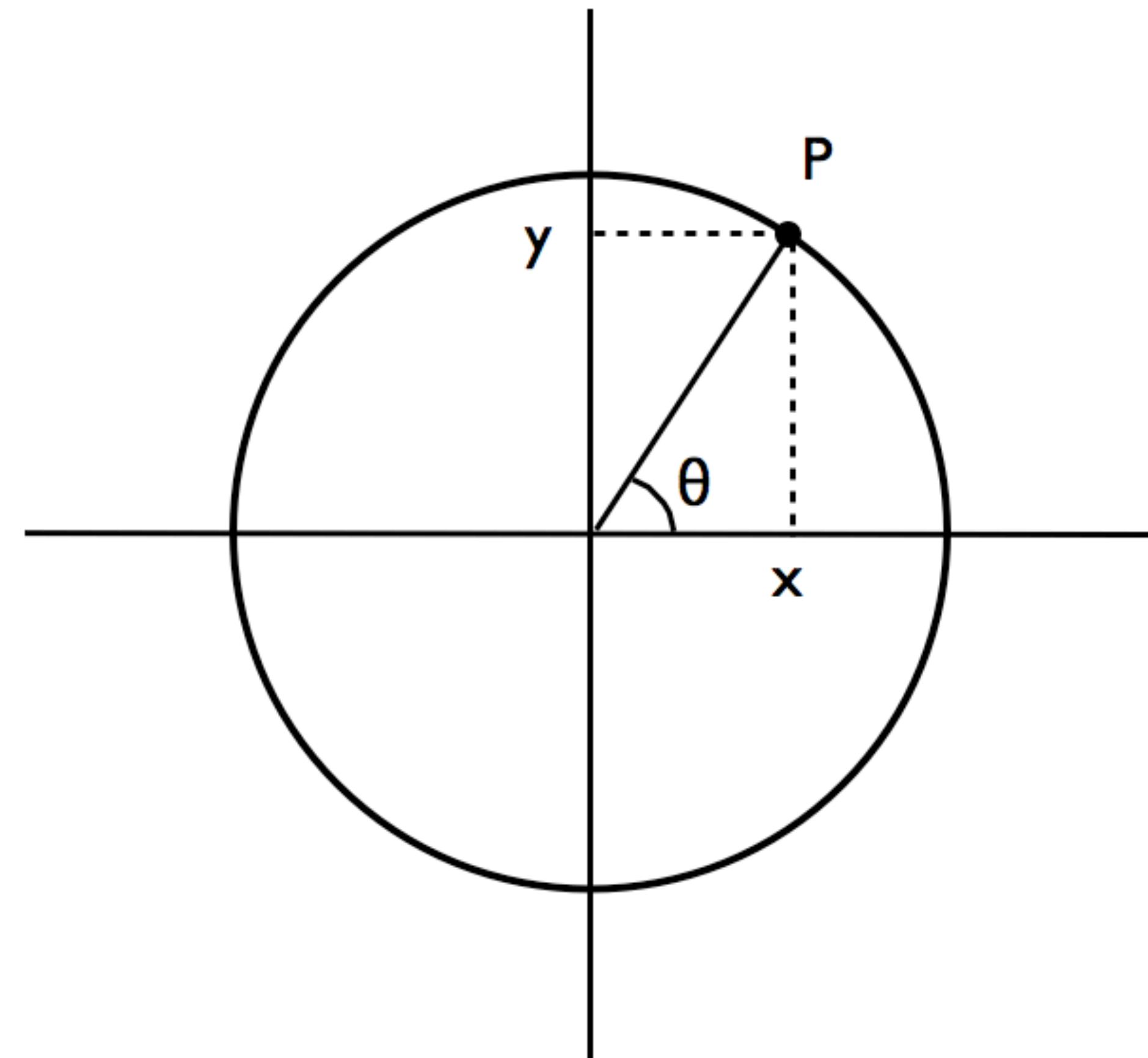
C - C# - D - Eb - E - F - F# - G - Ab - A - Bb - B - C'



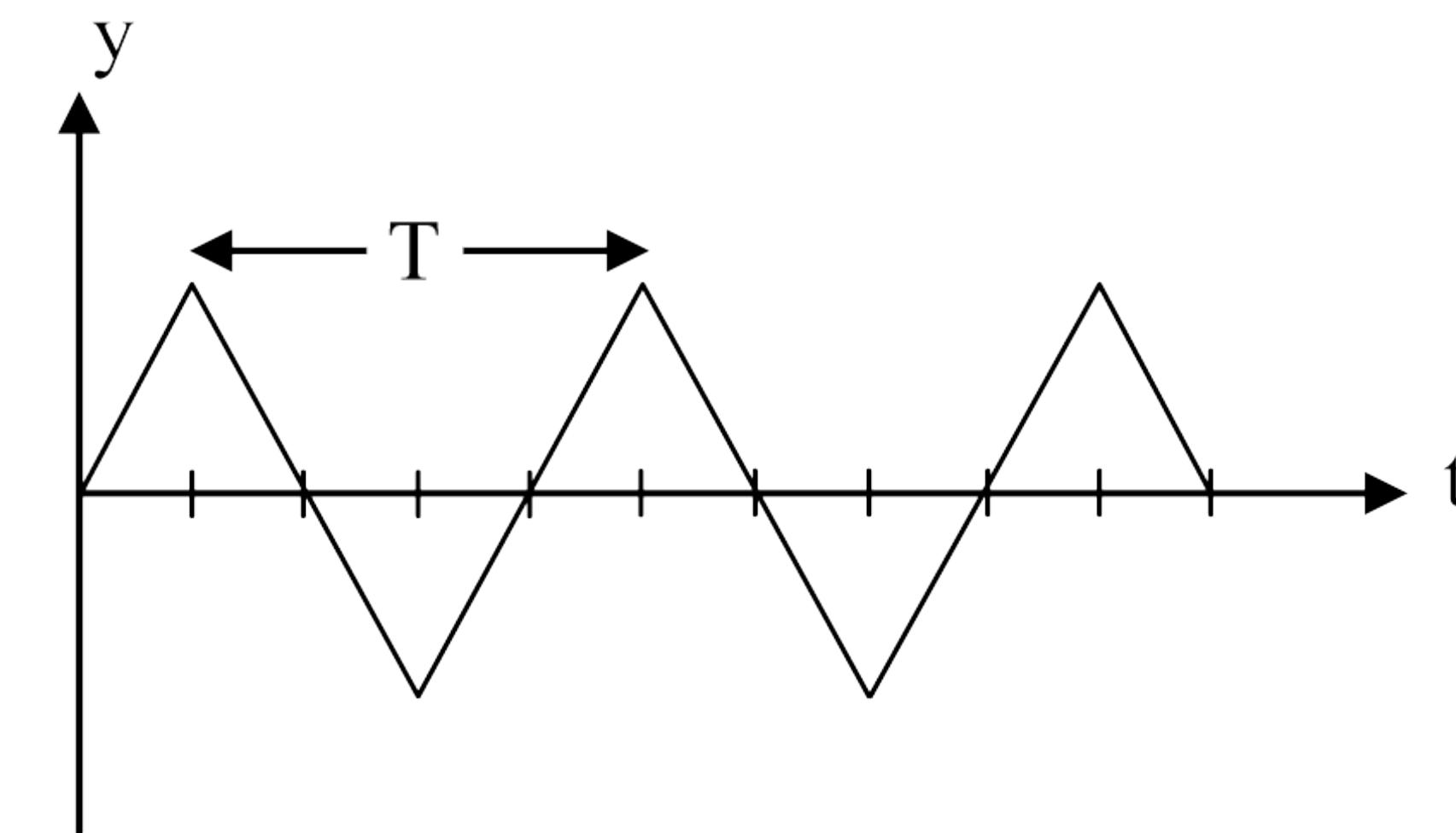
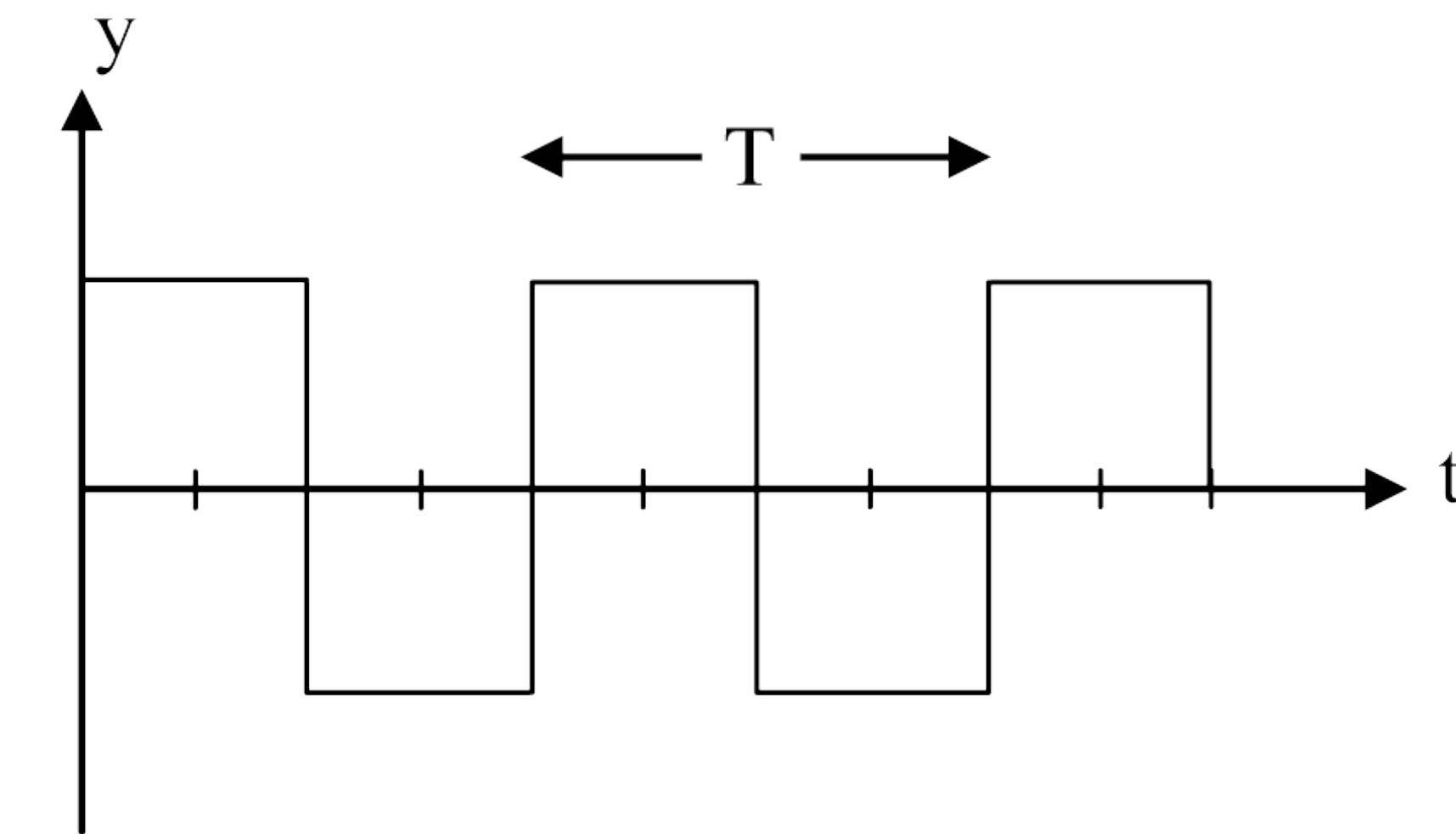
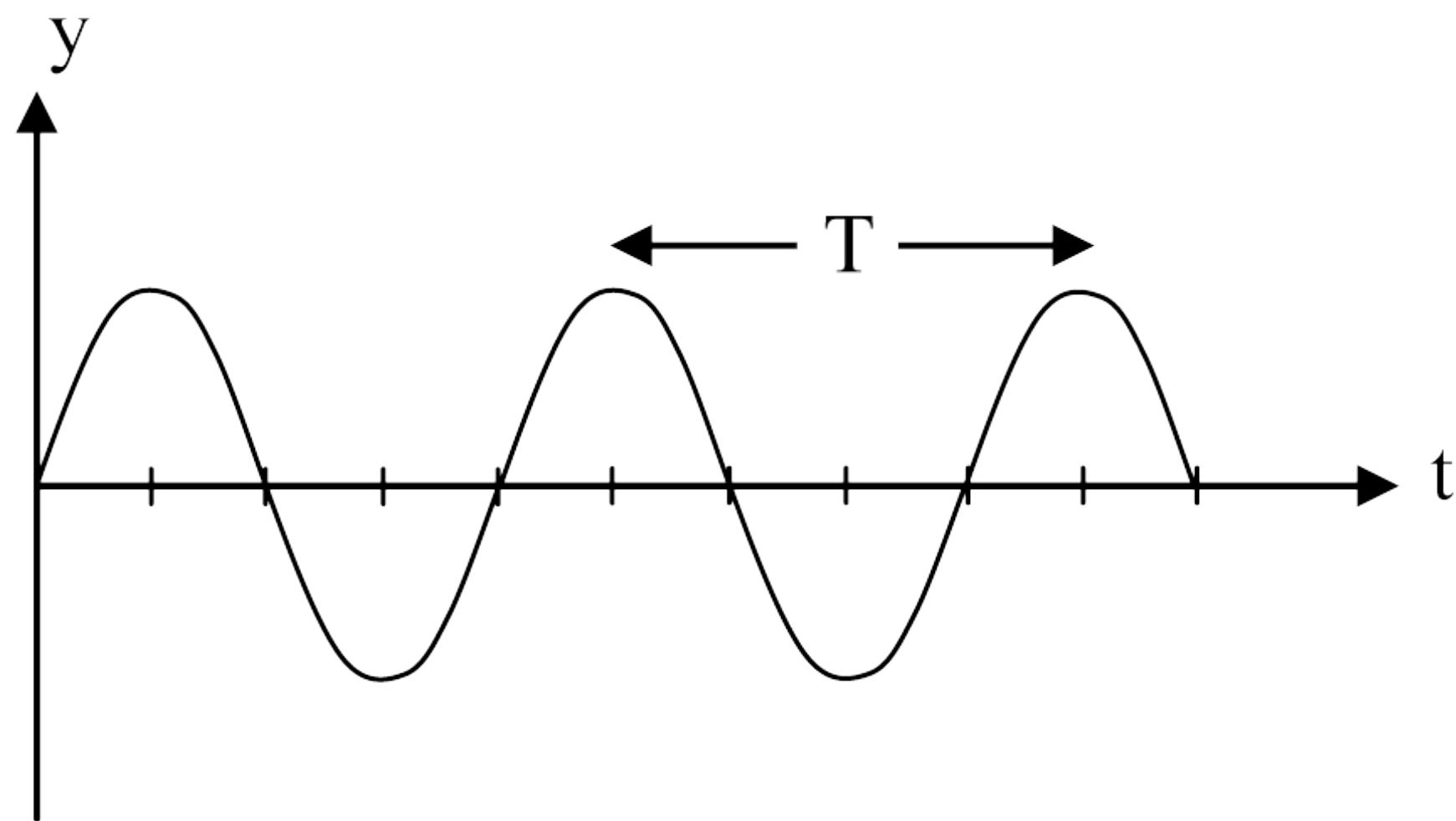
Physics terminology

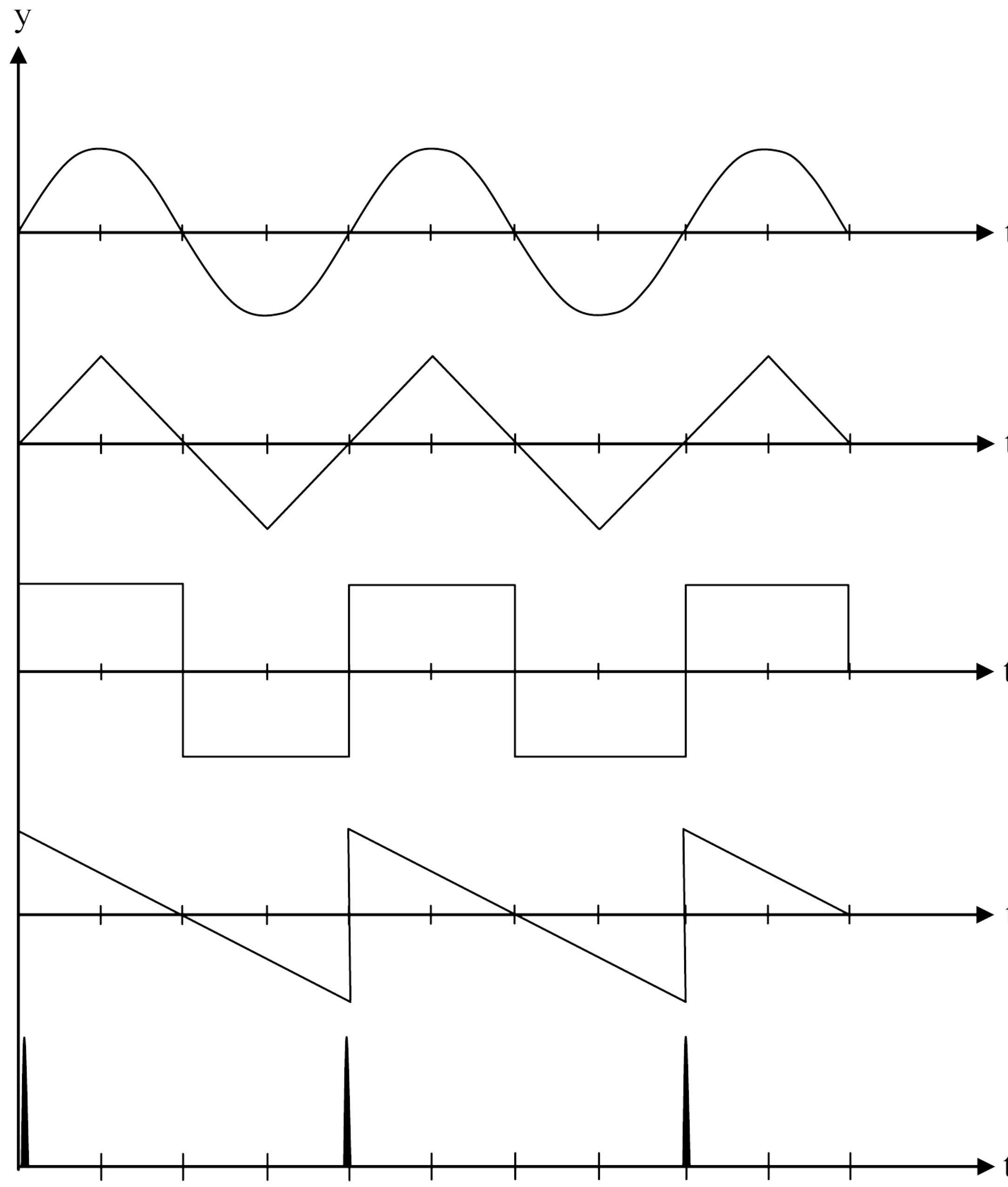
2. Oscillations

Simple harmonic motion (SHM)



SHM vs general periodic motion





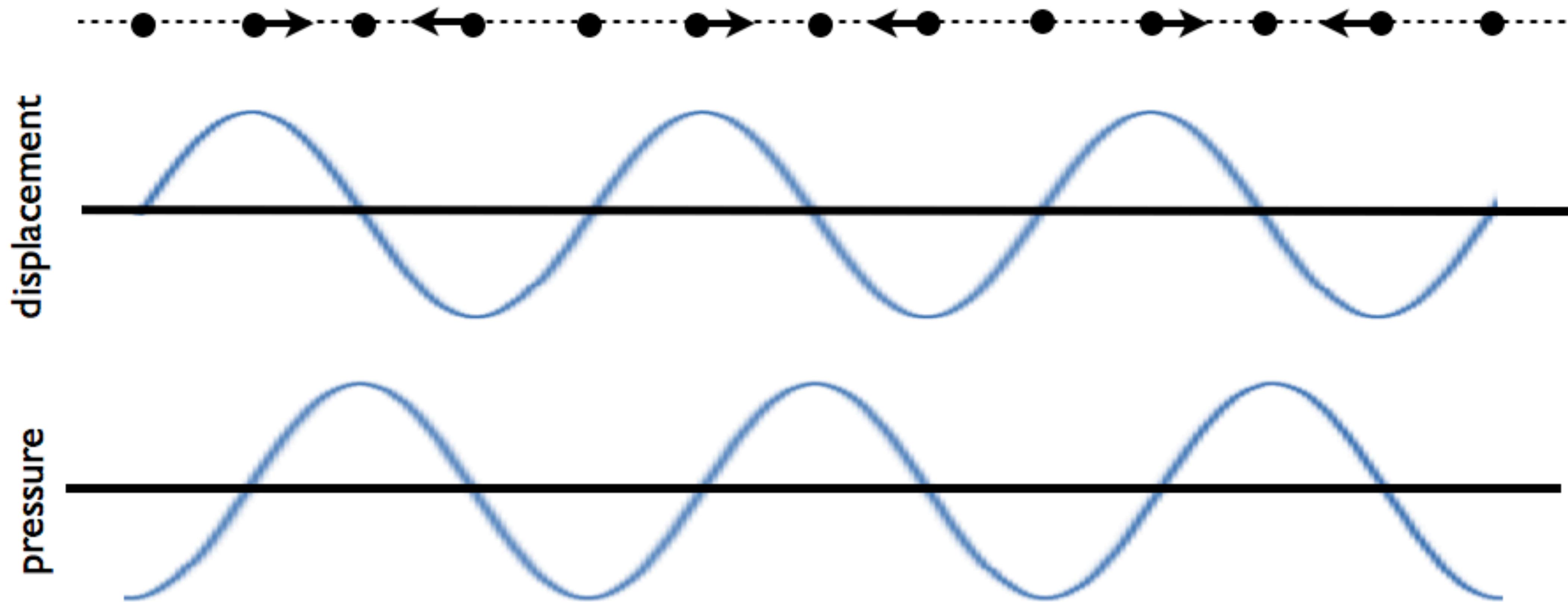
3. Waves & sound

Wave velocity

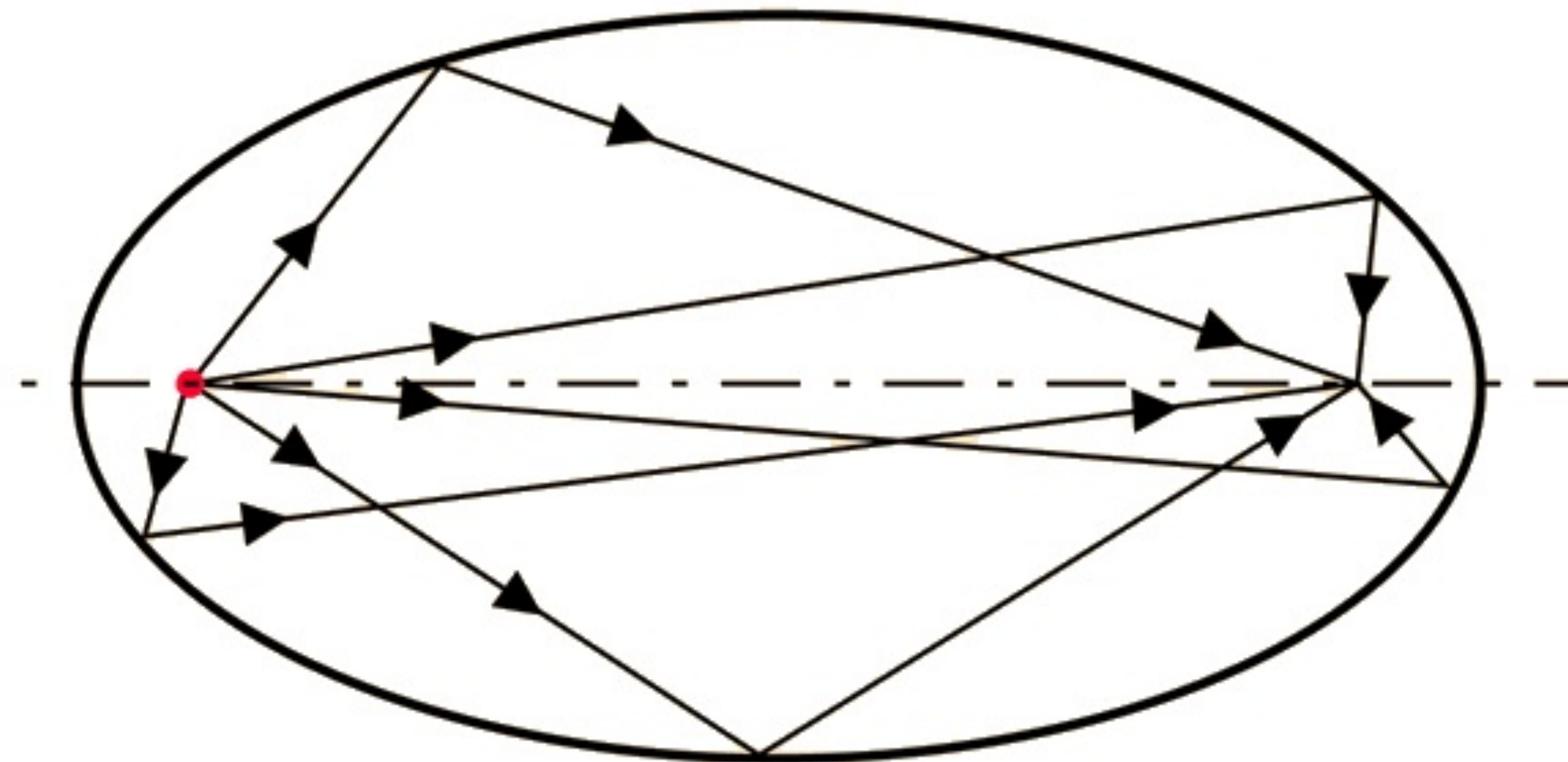
Superposition

Standing waves

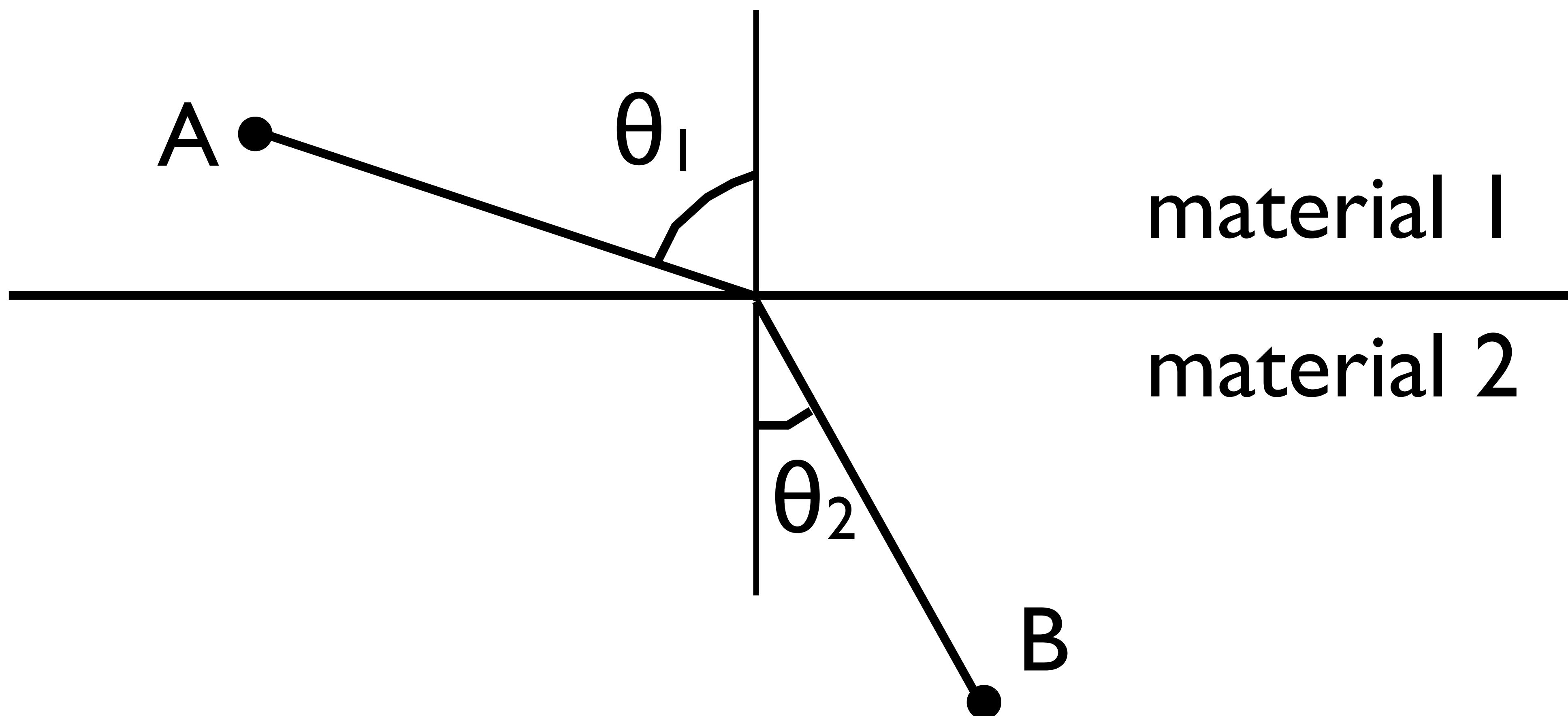
Air molecule displacement vs pressure deviation



Reflections - whispering chamber



Refraction



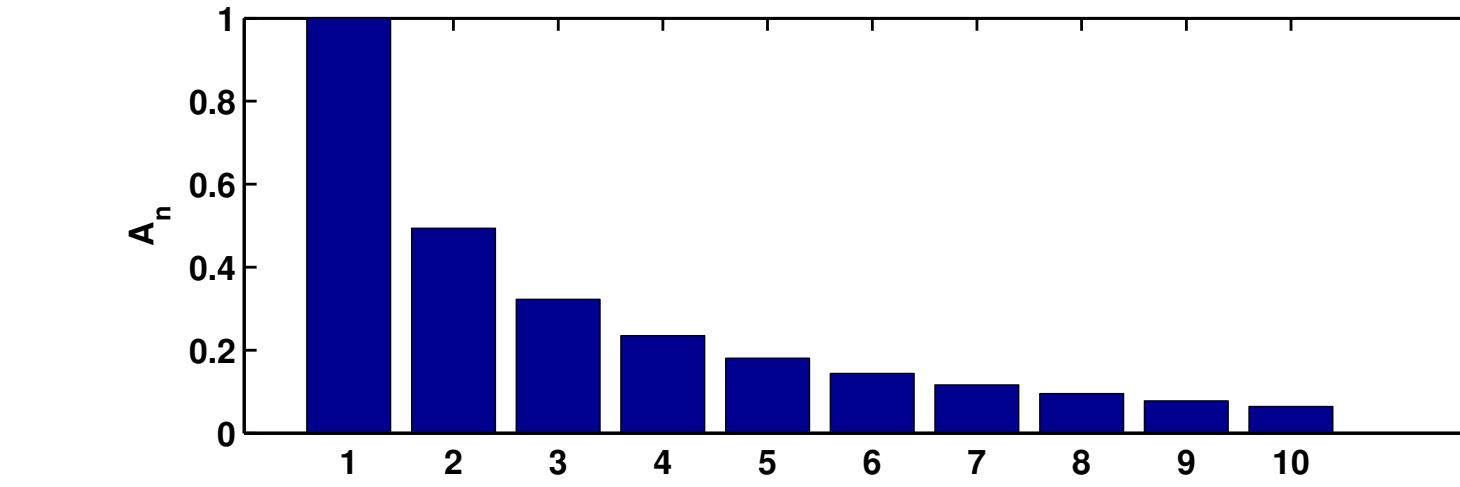
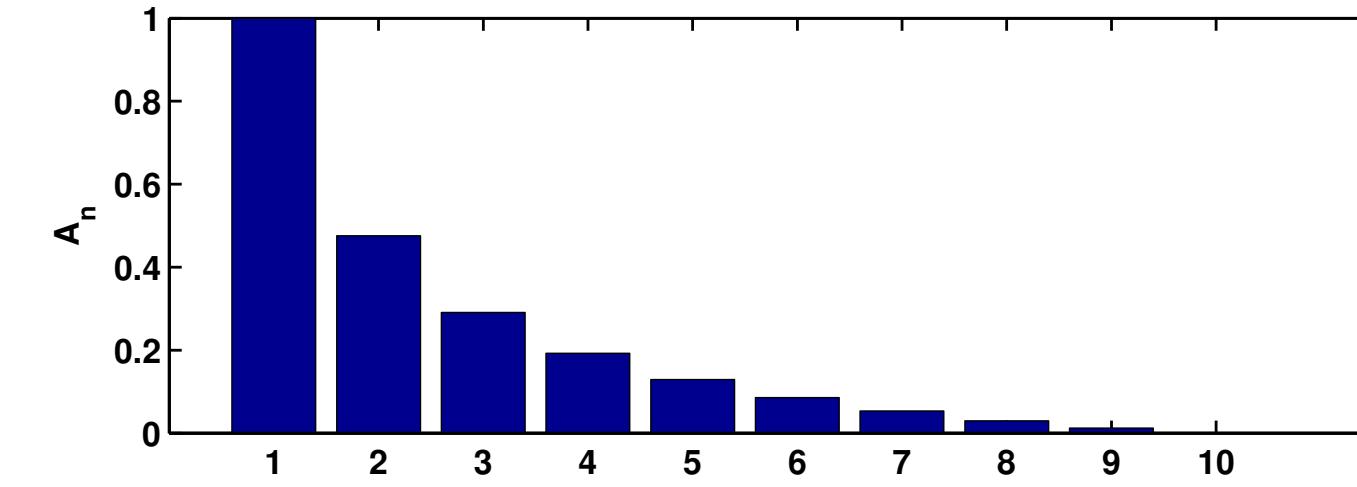
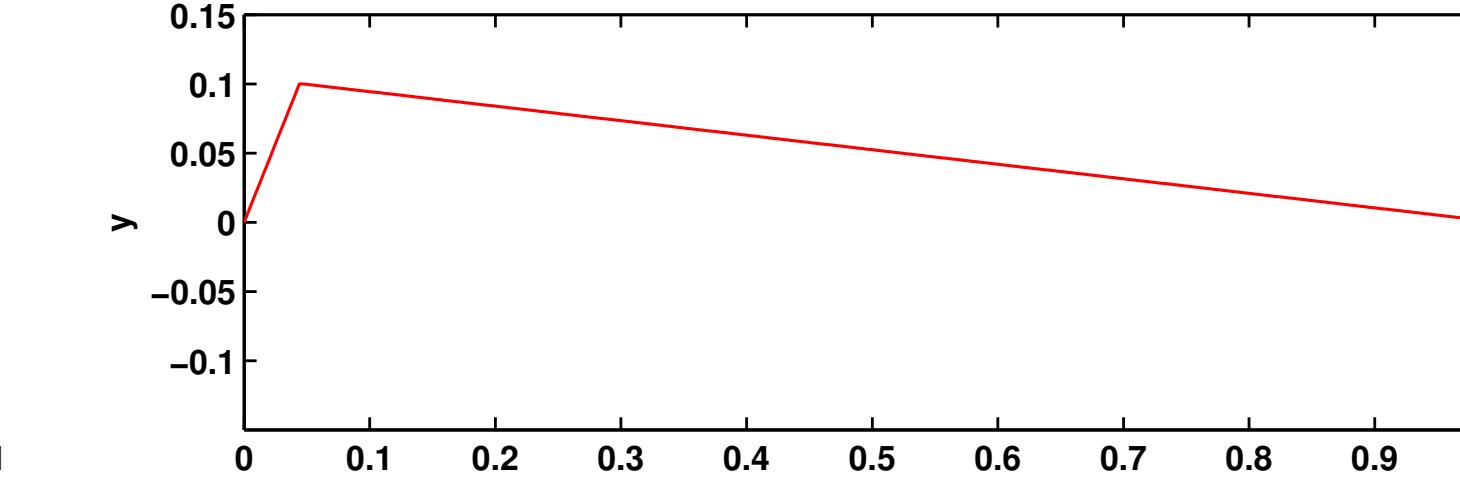
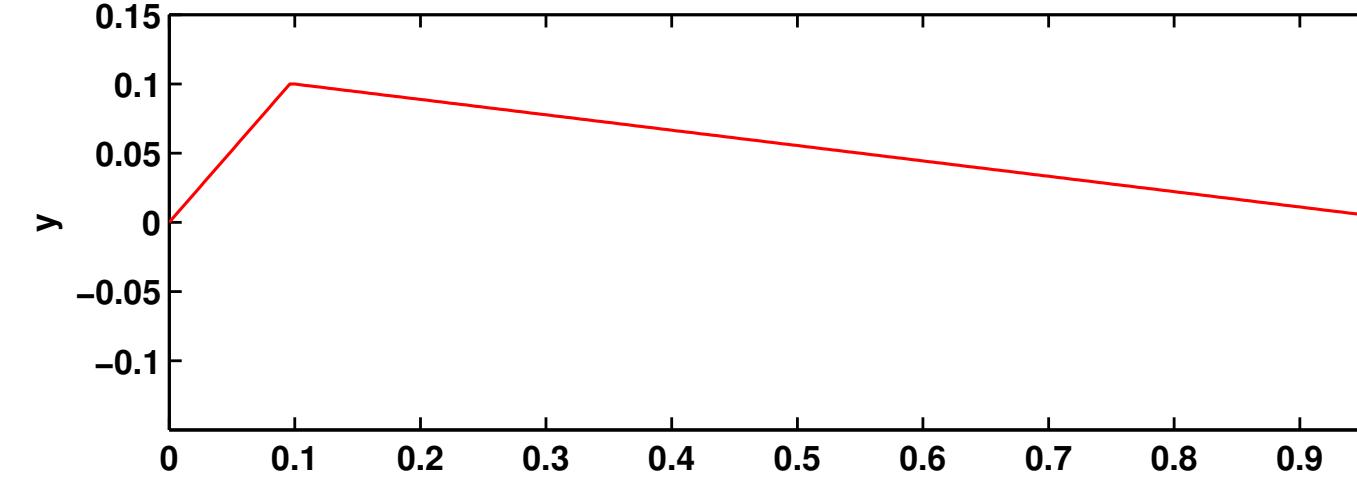
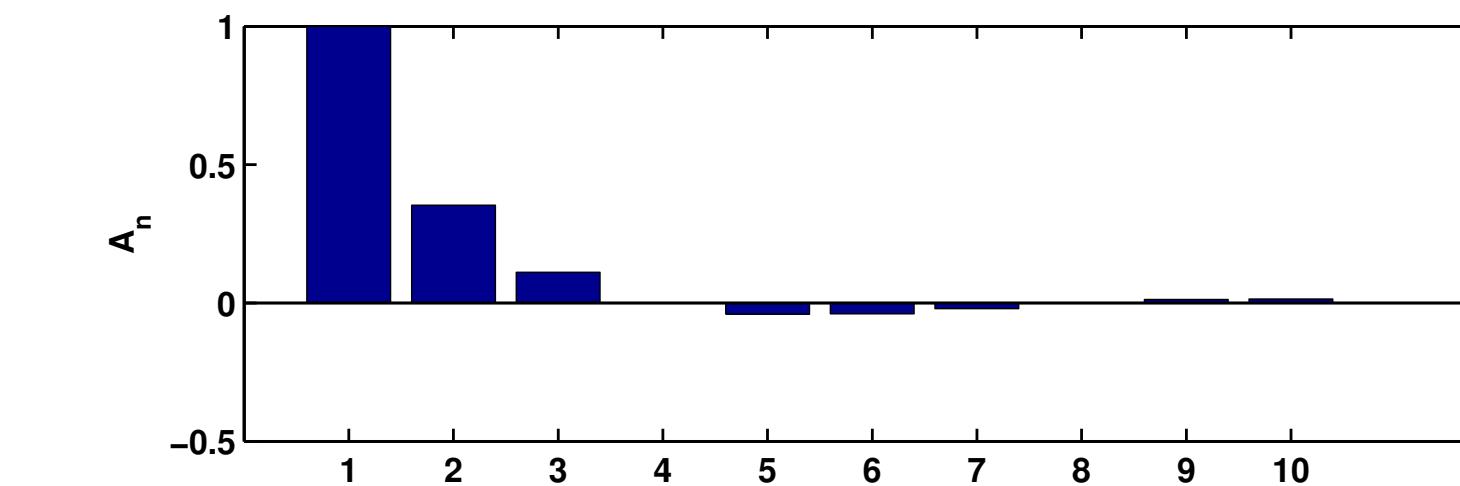
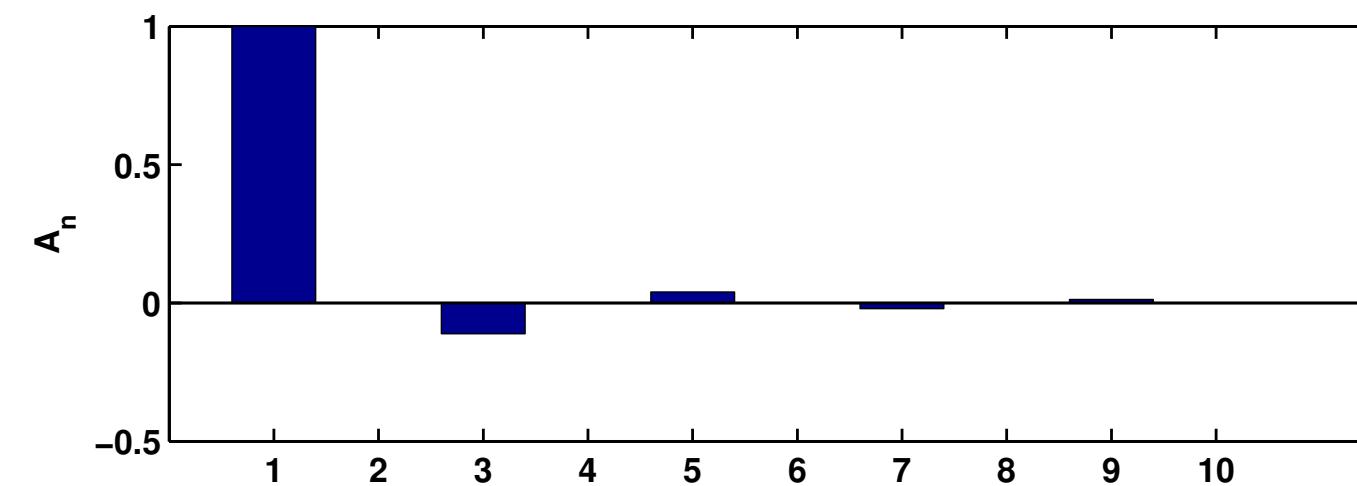
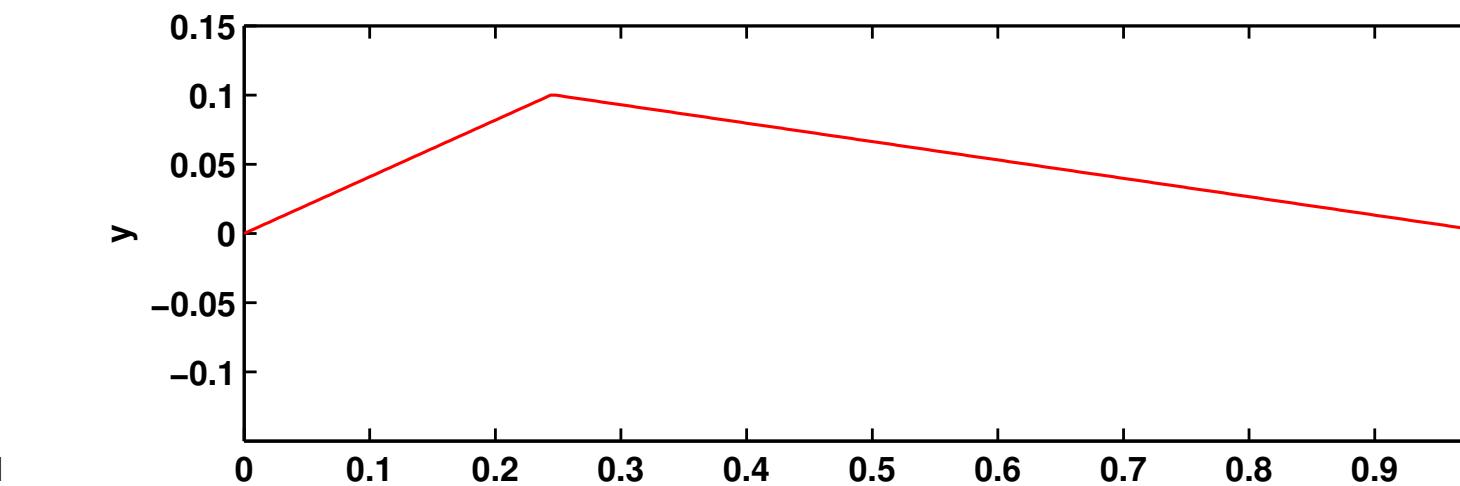
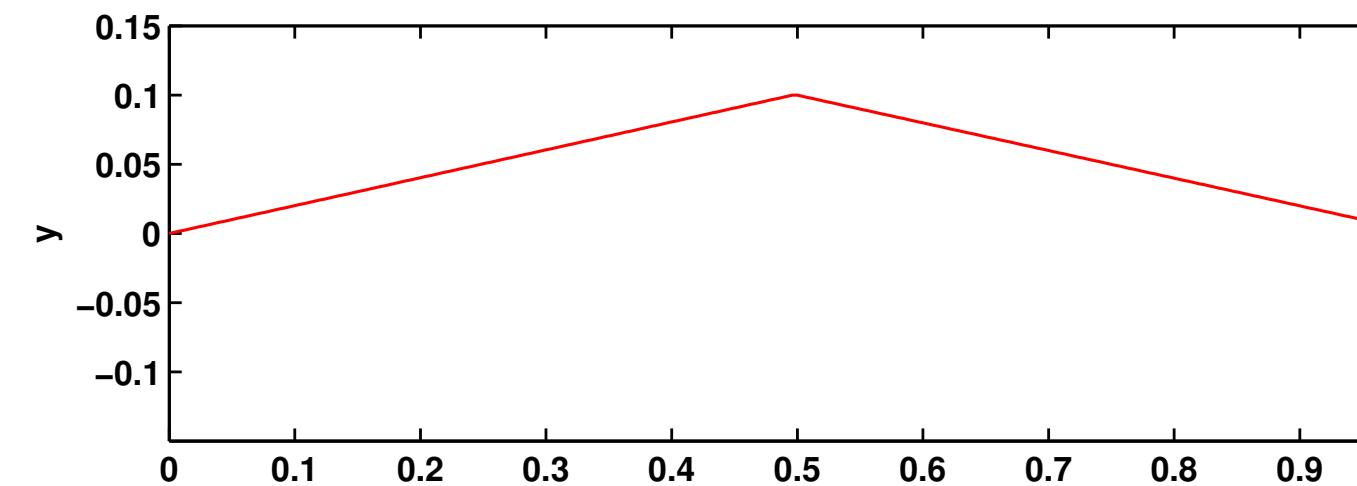
4. Fourier analysis & synthesis

Fourier's theorem

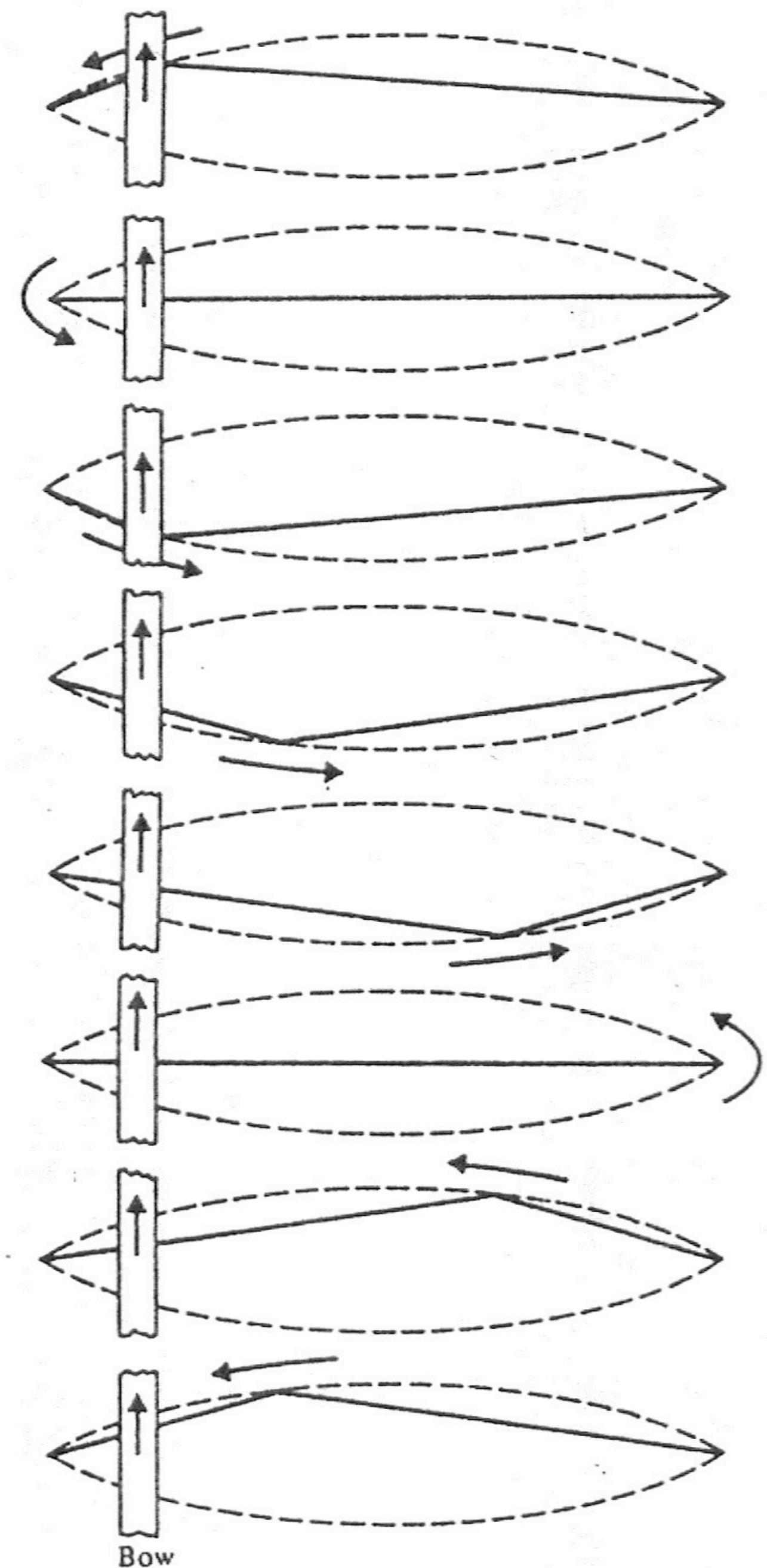
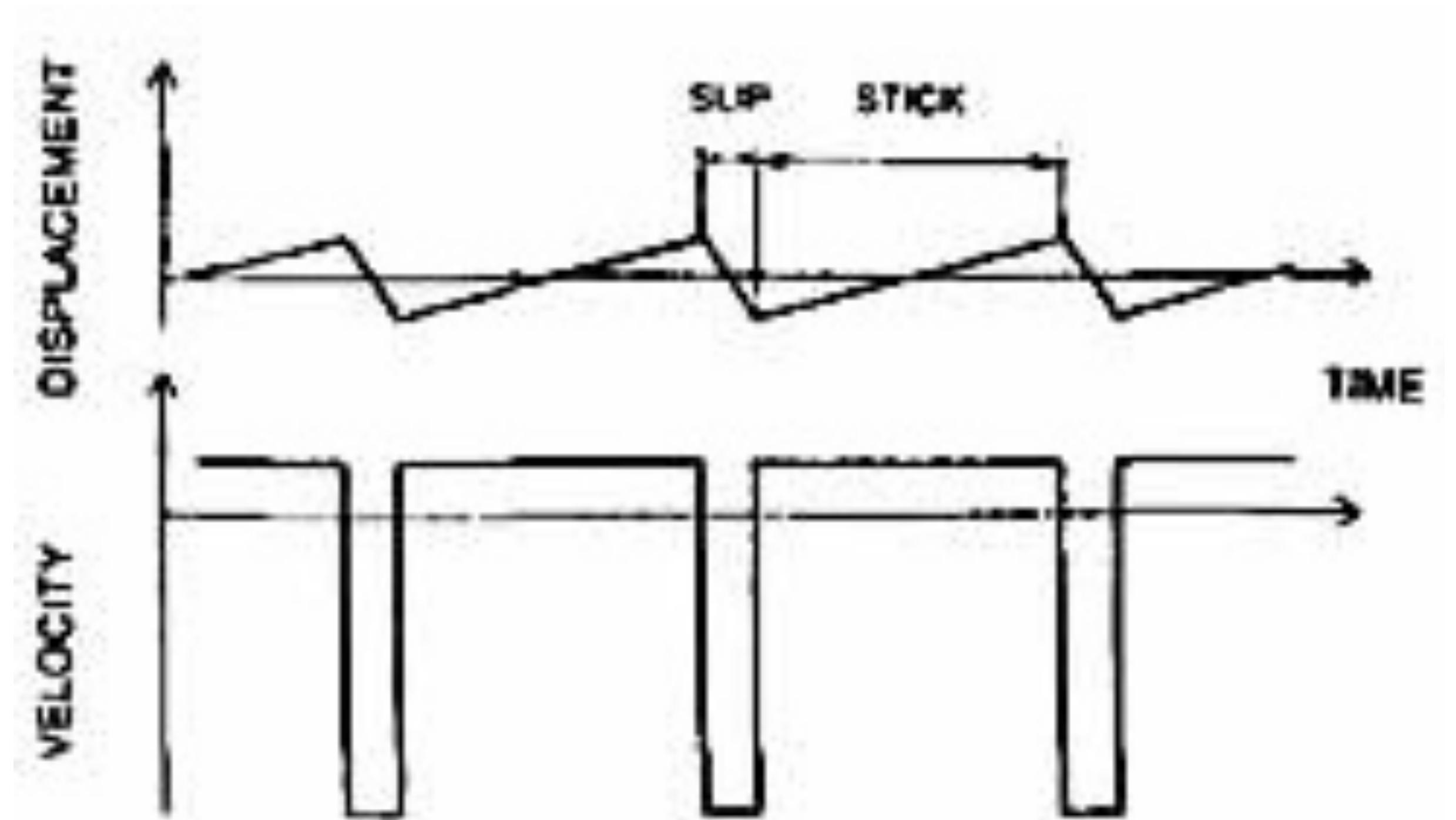
5. String instruments

Plucked versus bowed strings

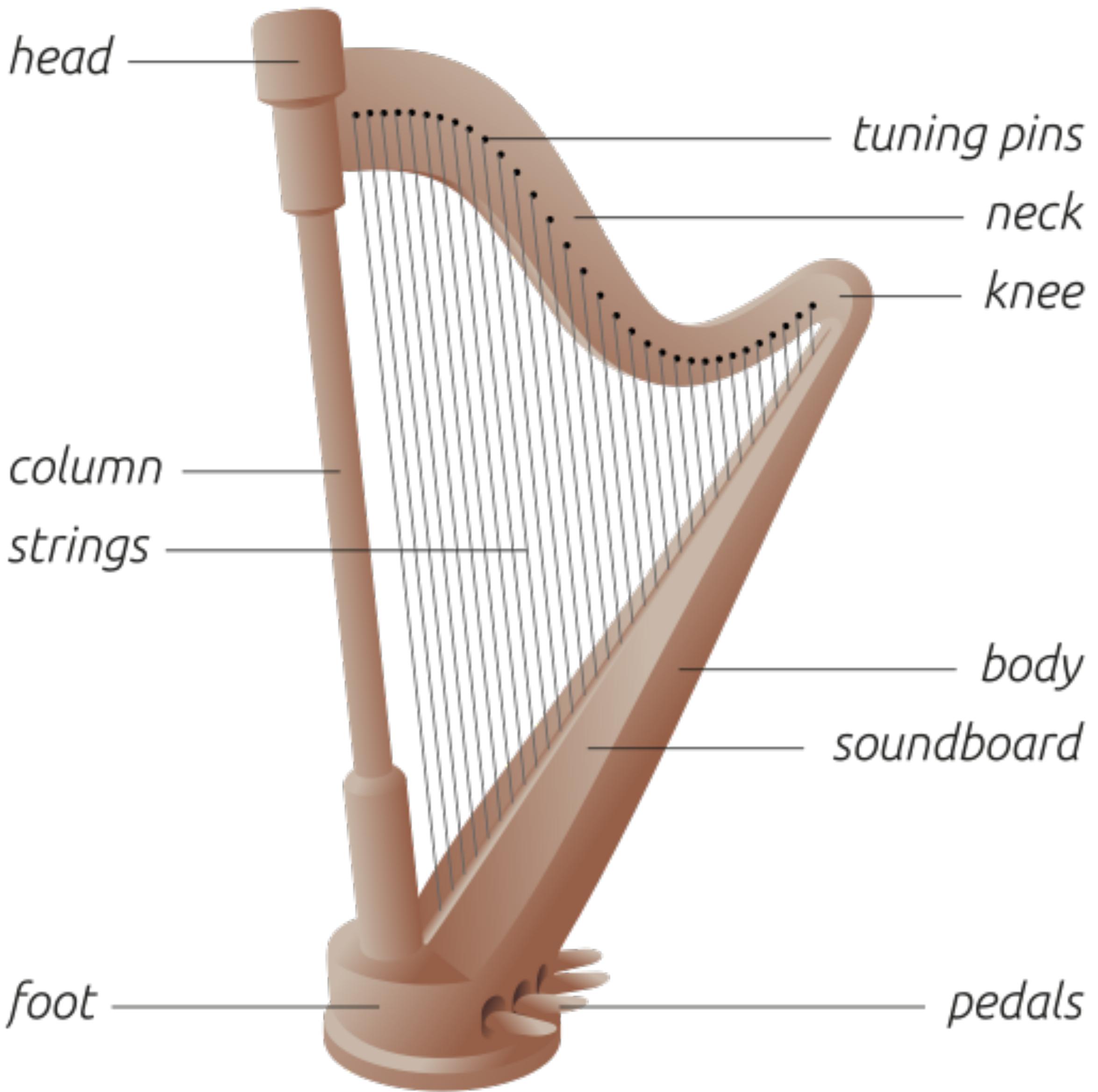
Fourier coefficients of a plucked string



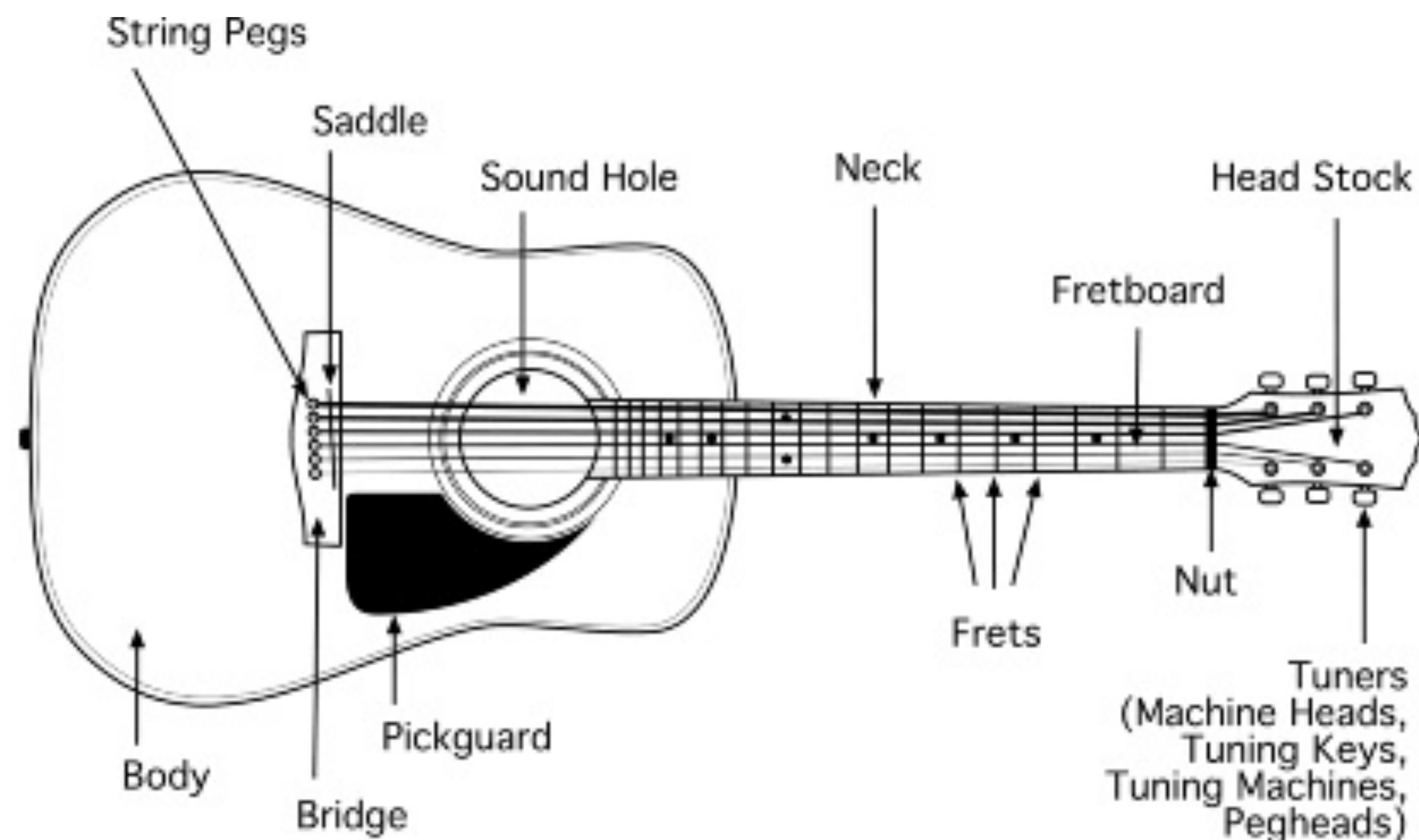
Stick-slip motion of a bowed string



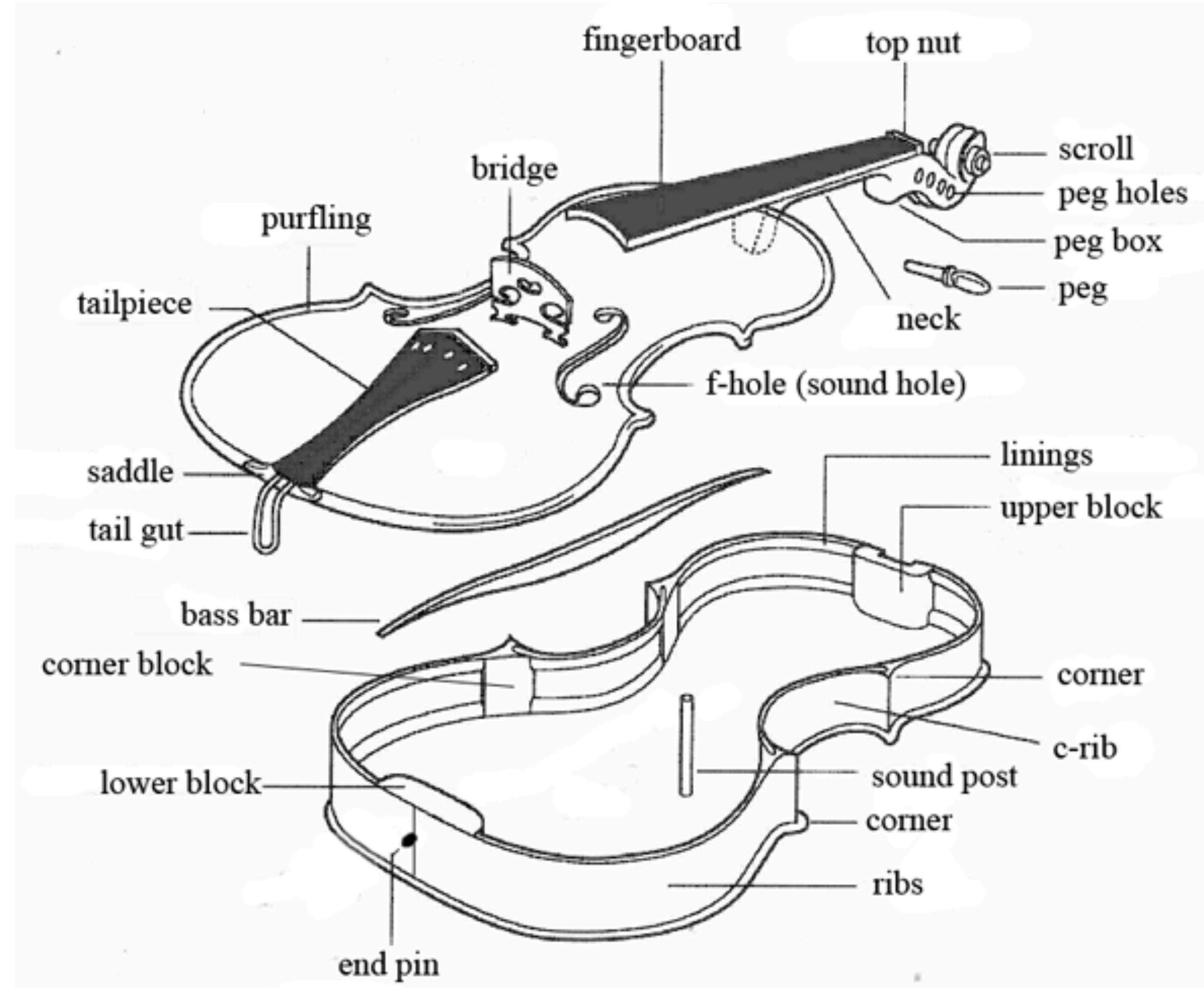
Harp



Guitar



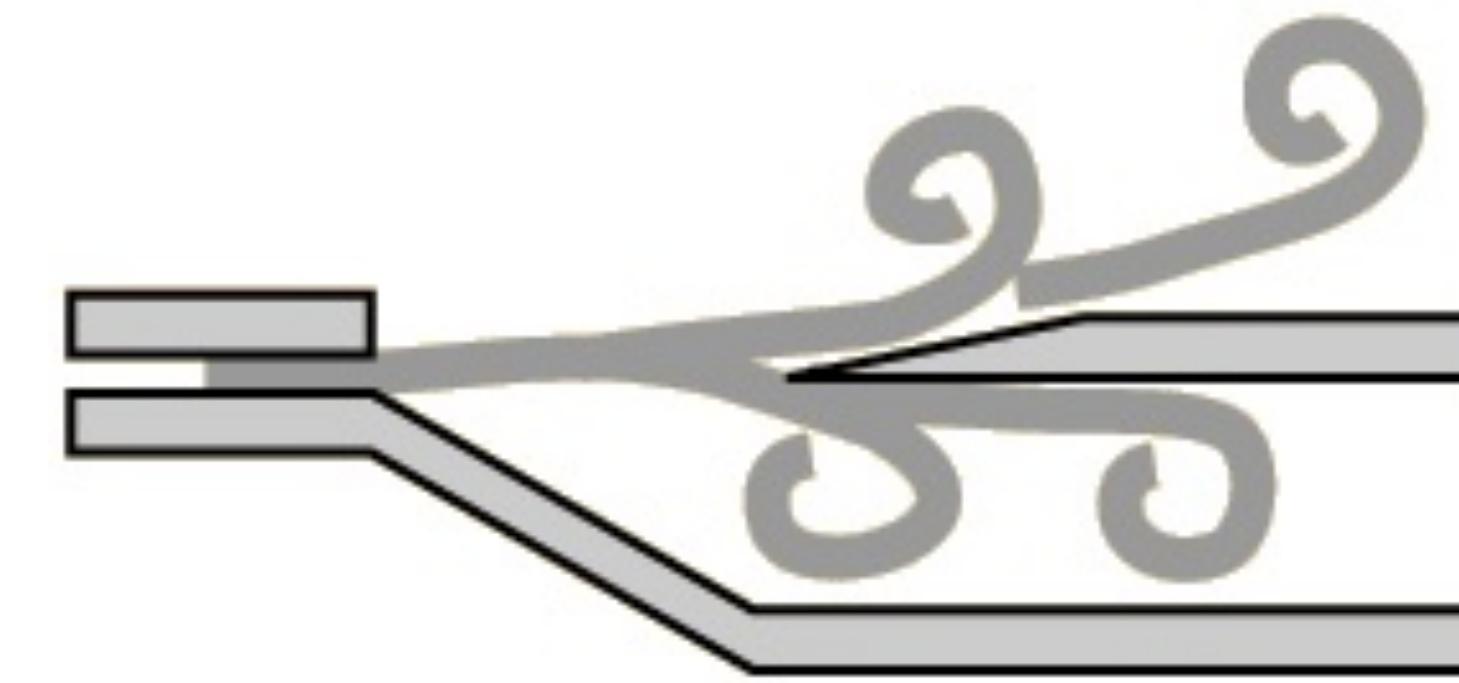
Violin

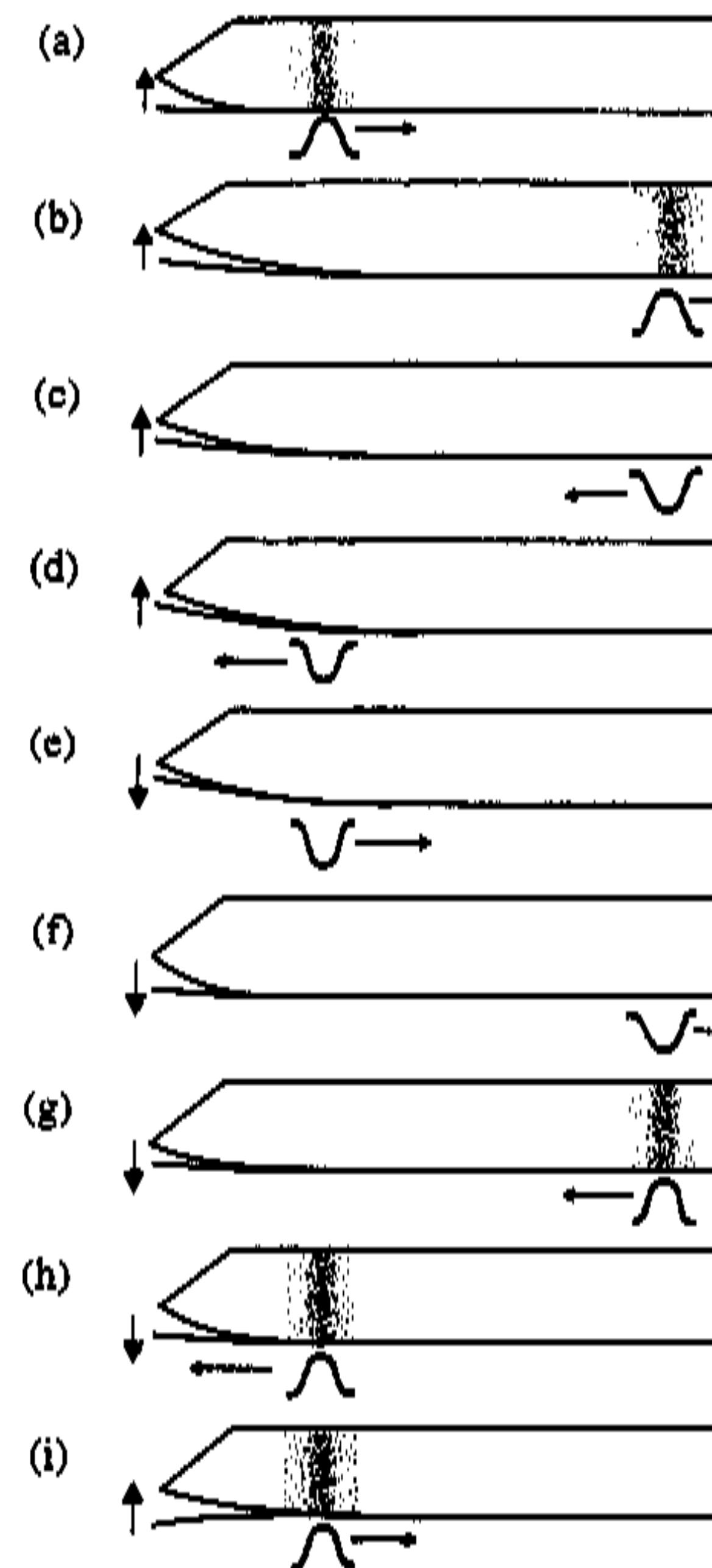


6. Wind instruments

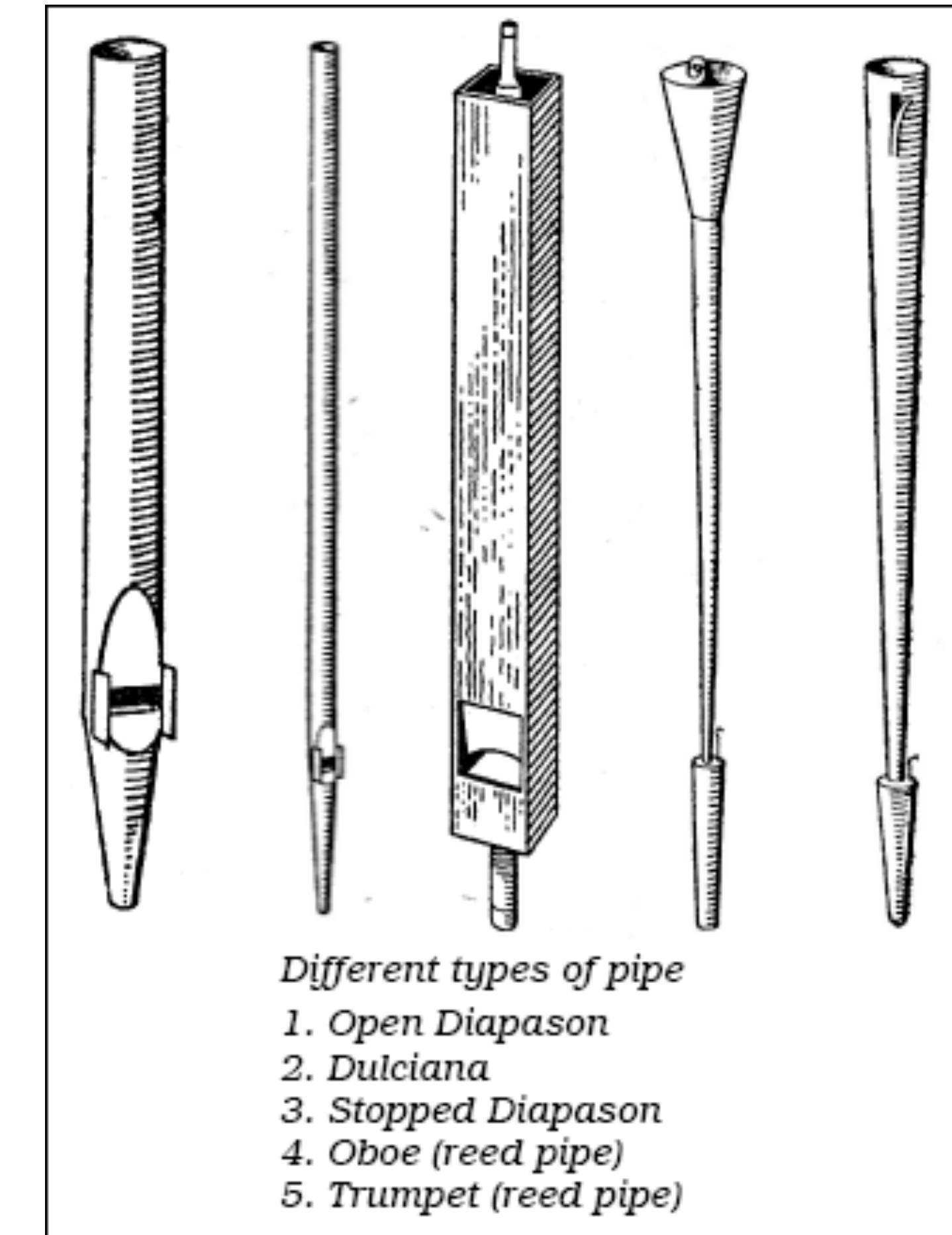
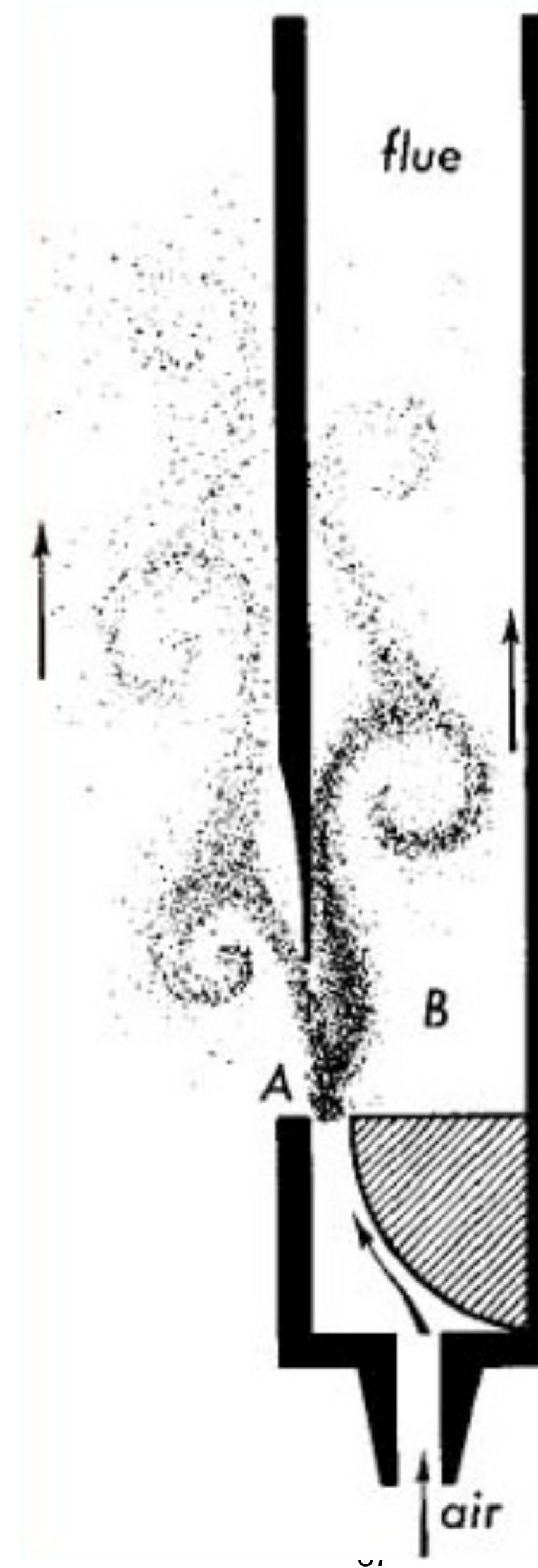
Open and closed tubes

Oscillating air streams vs vibrating reeds

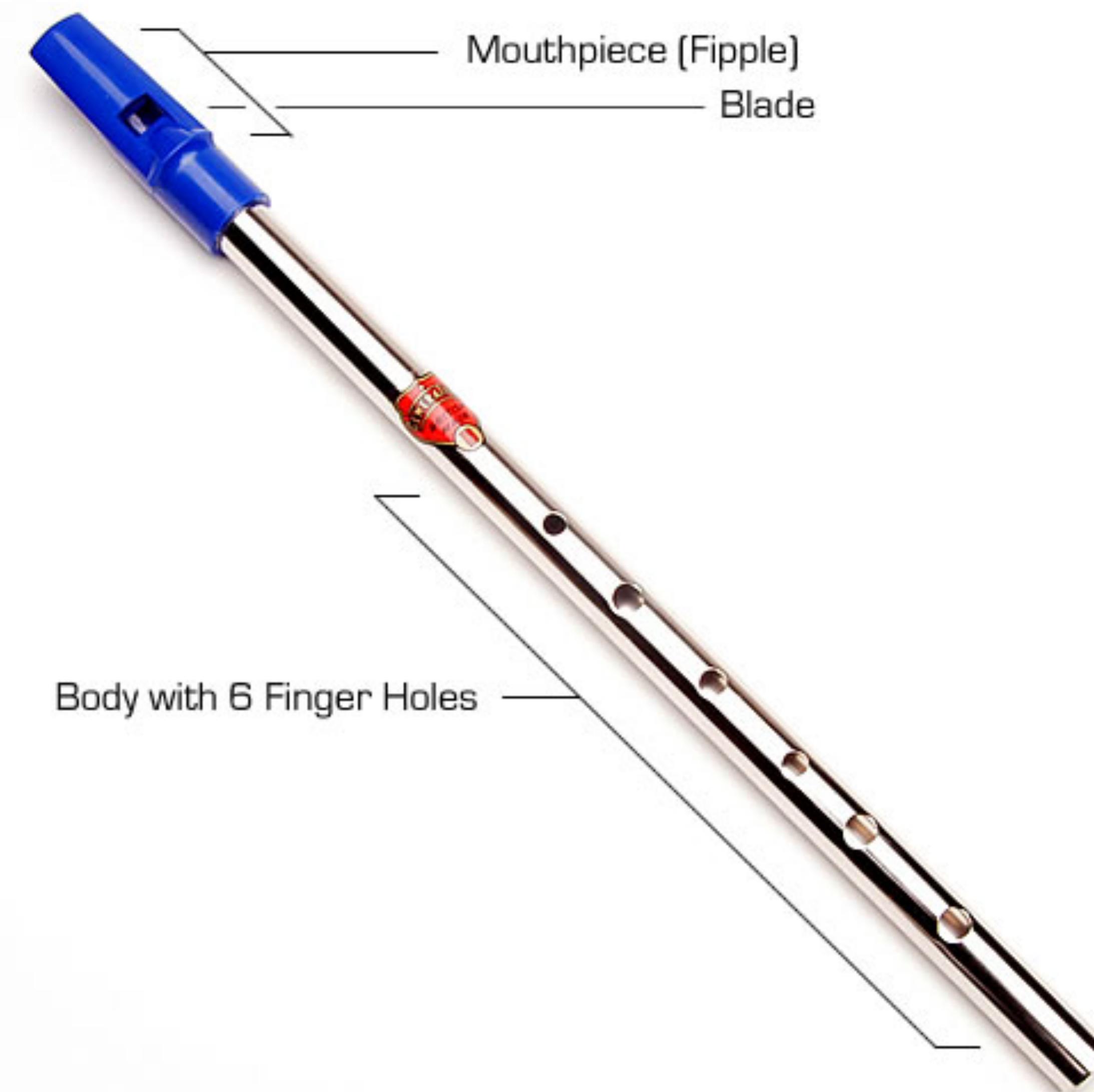




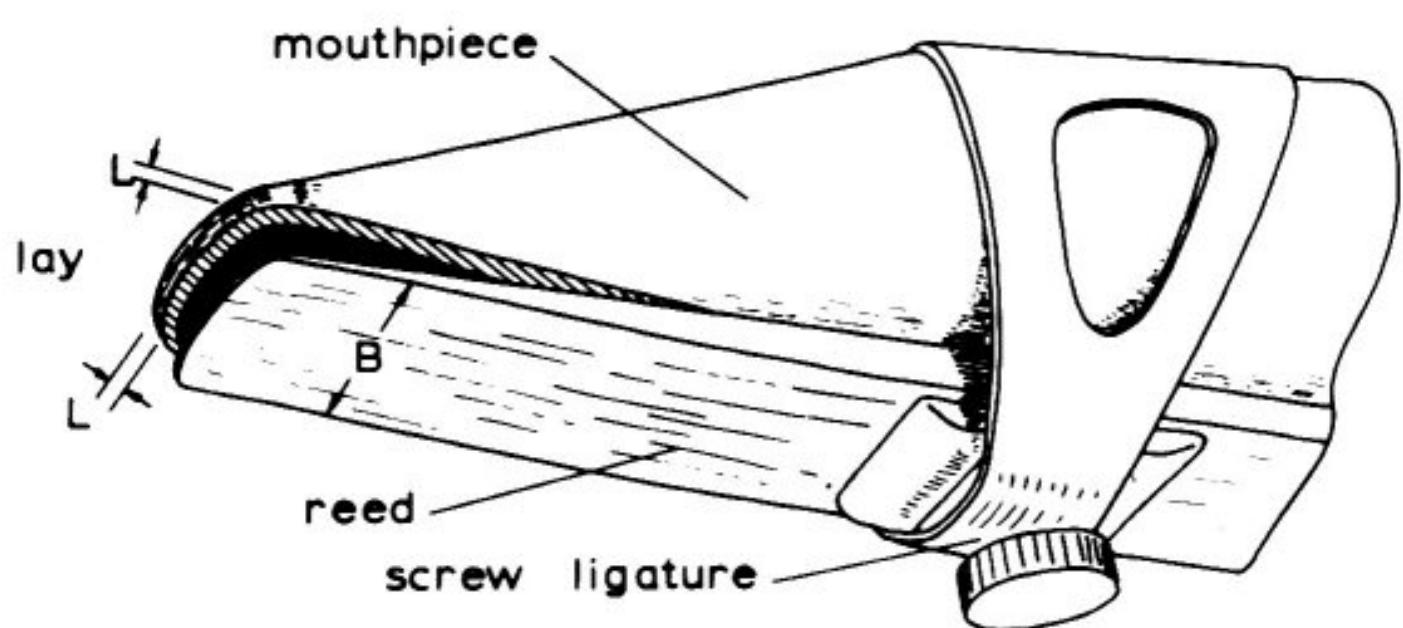
Flue-organ pipe



Penny whistle

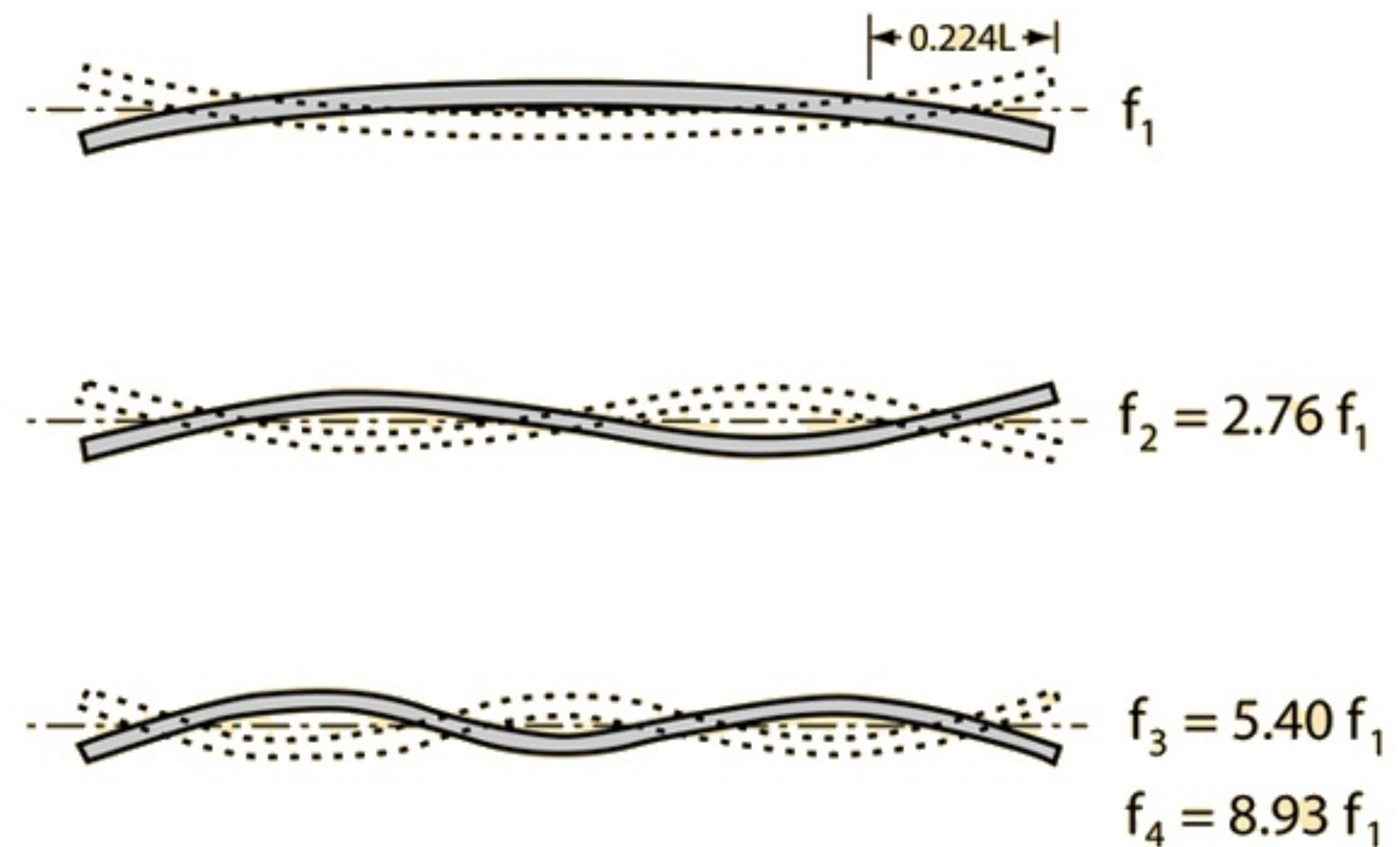


Clarinet



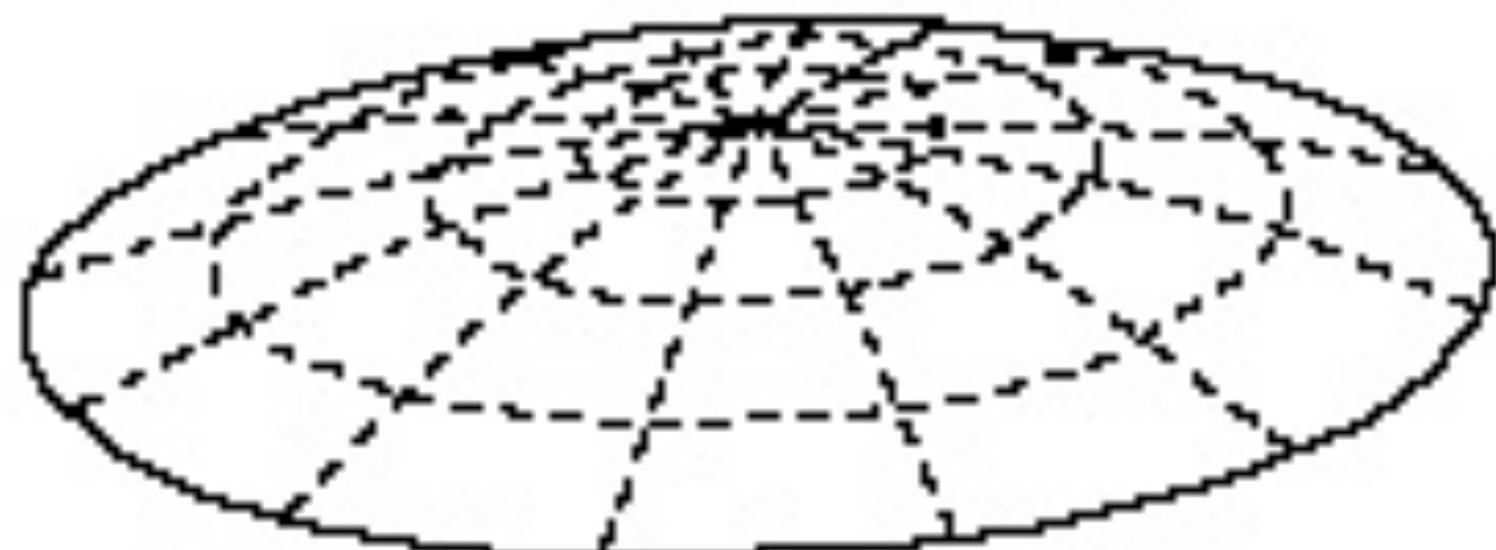
7. Percussion instruments

Inharmonicity



Vibrating drum head (2-d standing waves)

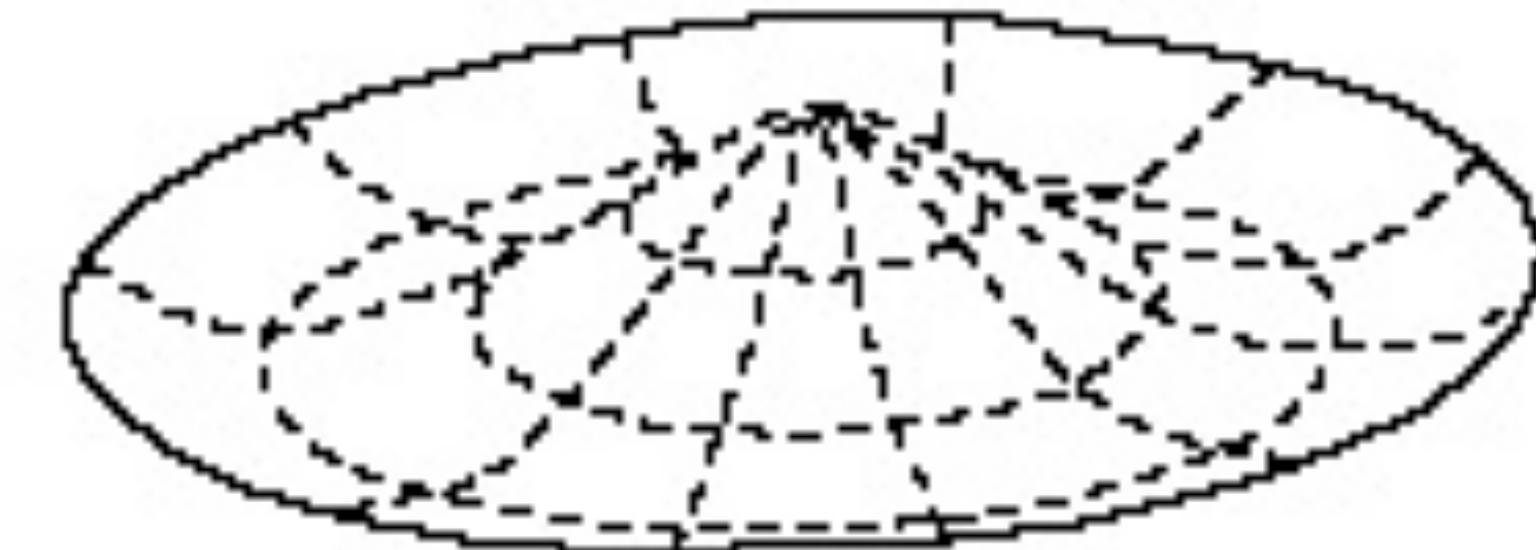
Mode 01



This is the
lowest frequency
mode.

$$f_1$$

Mode 02



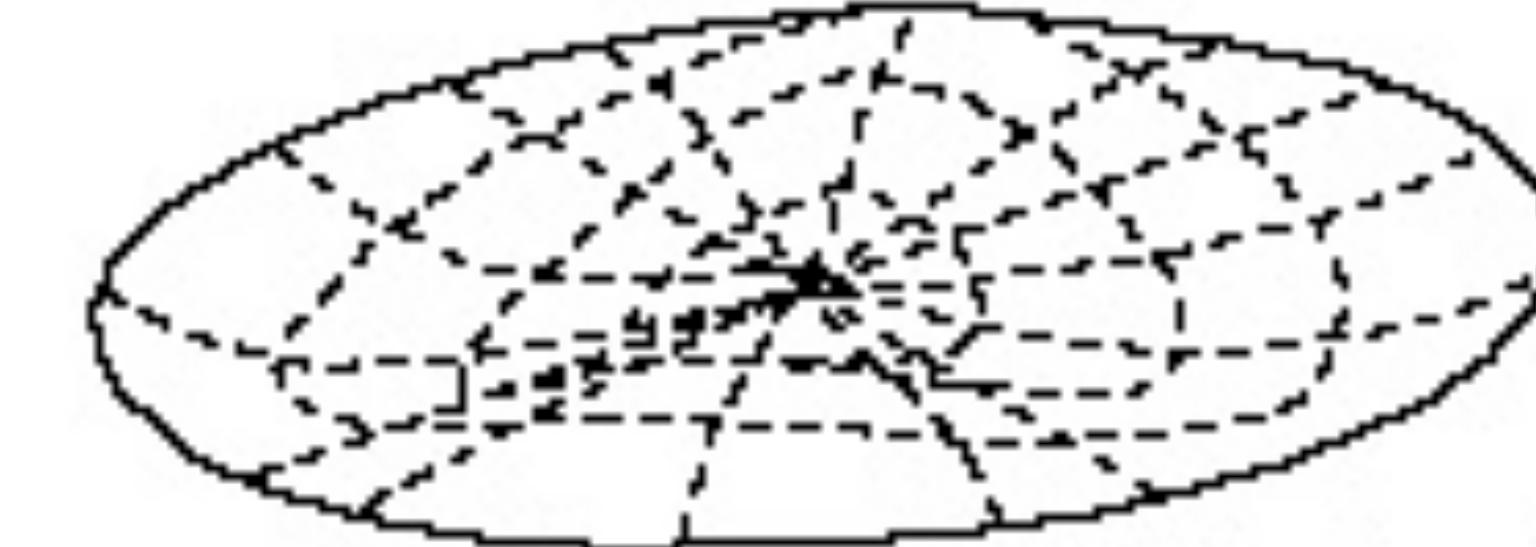
$$2.30 f_1$$

Mode 11



$$1.59 f_1$$

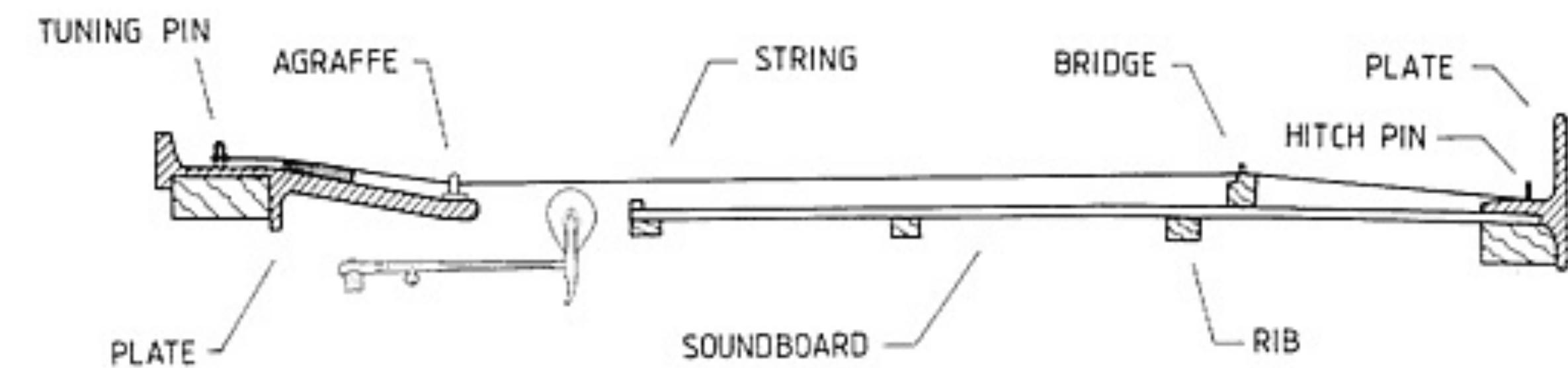
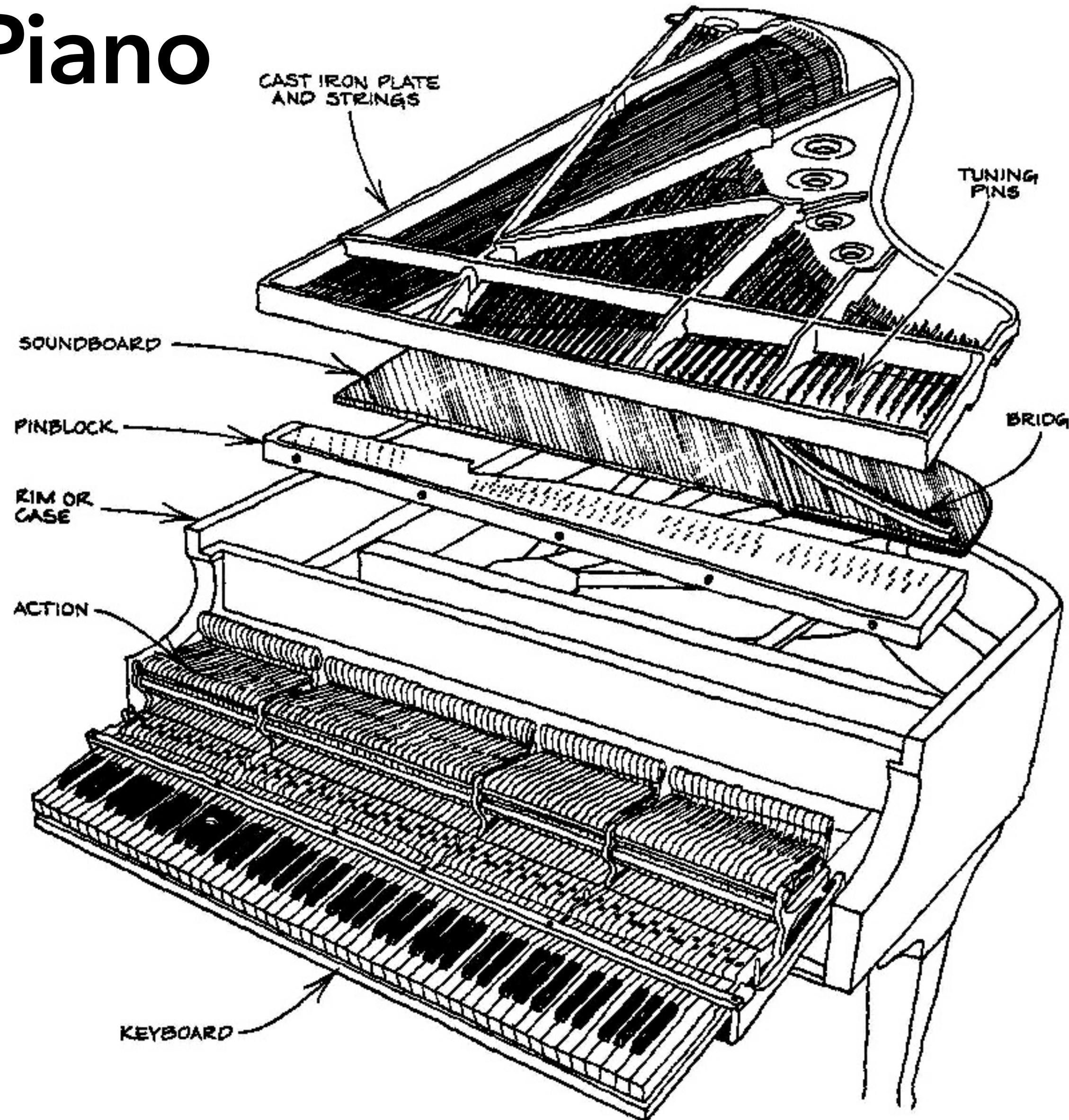
Mode 21



$$2.14 f_1$$

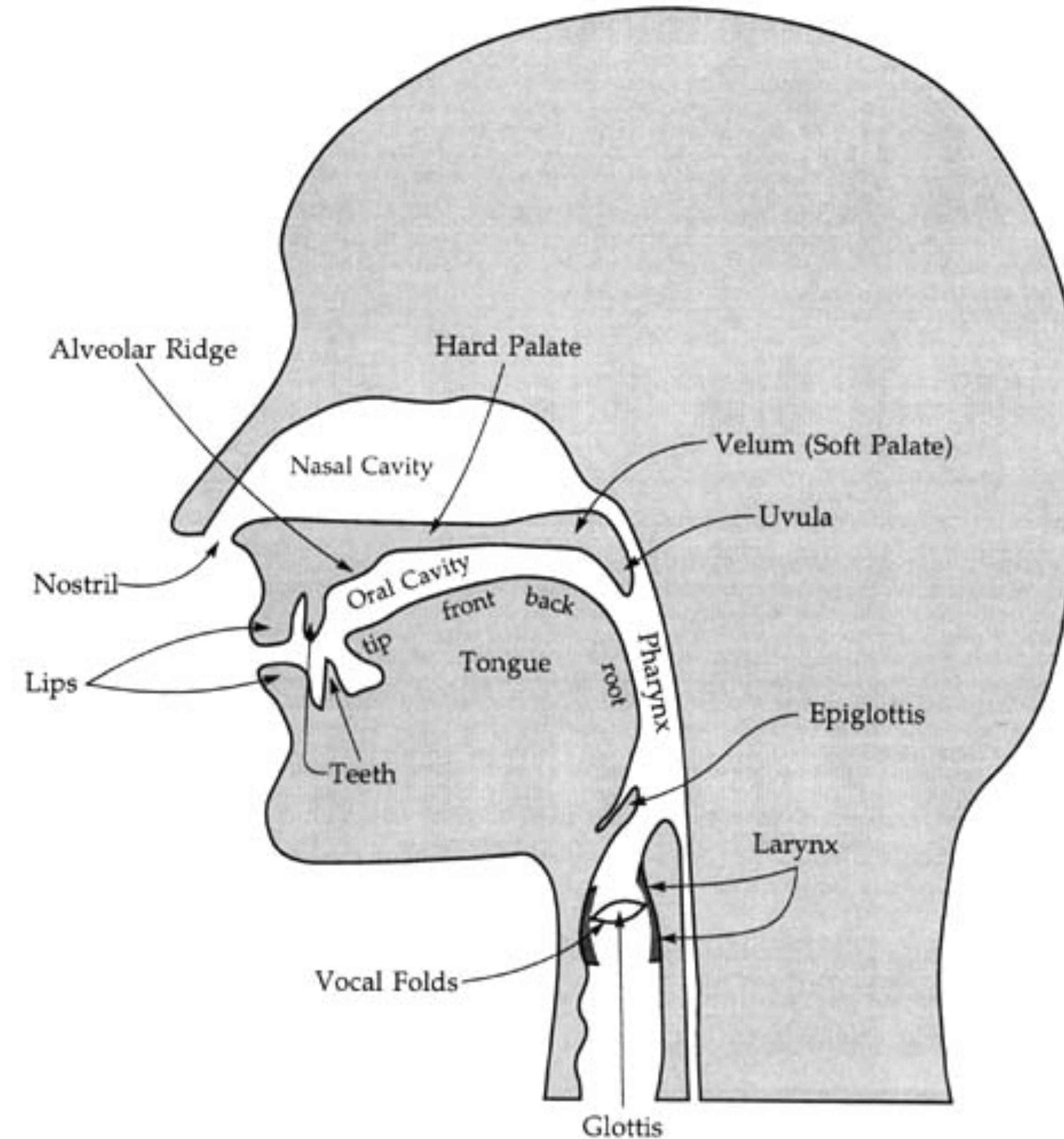
After Rossing

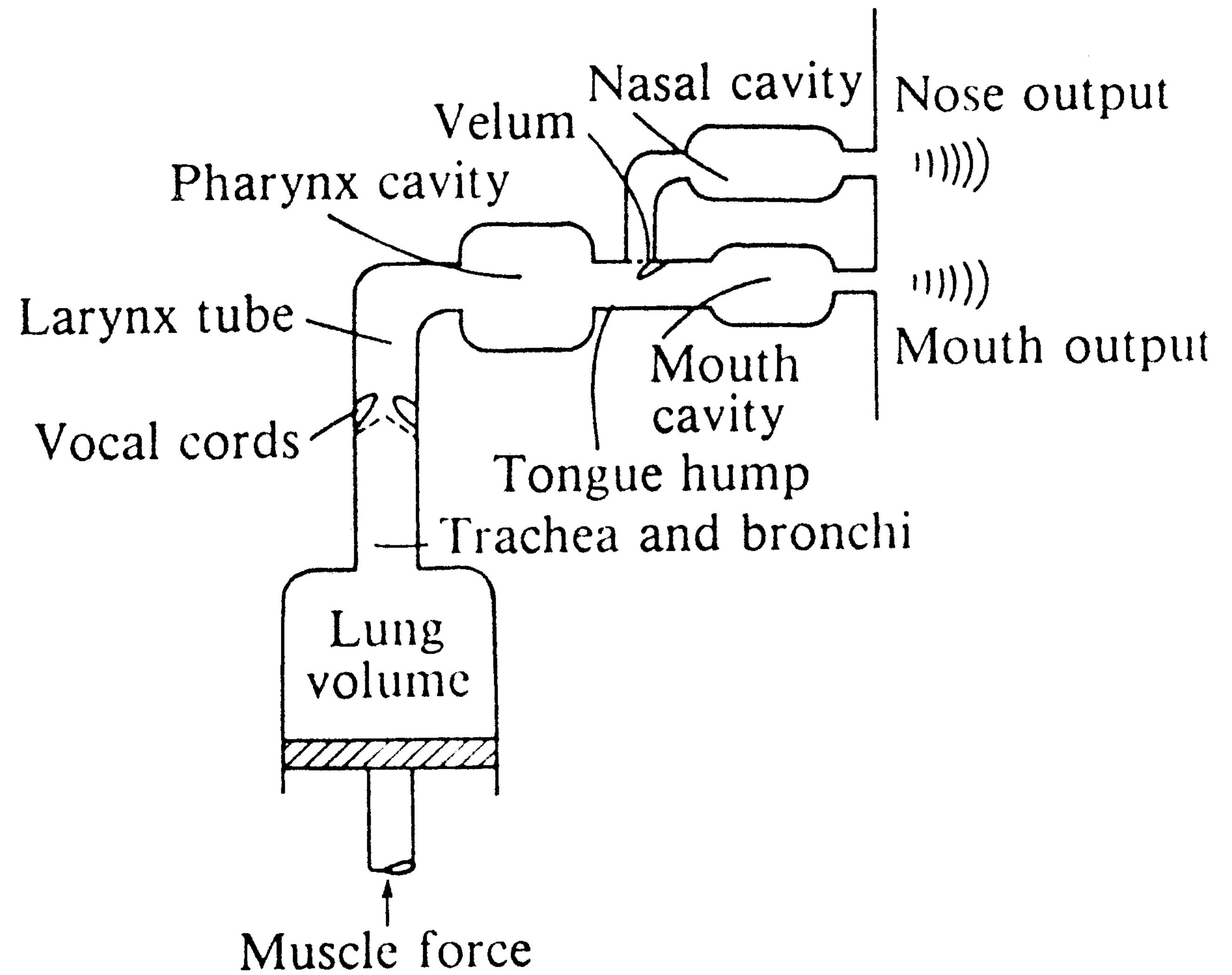
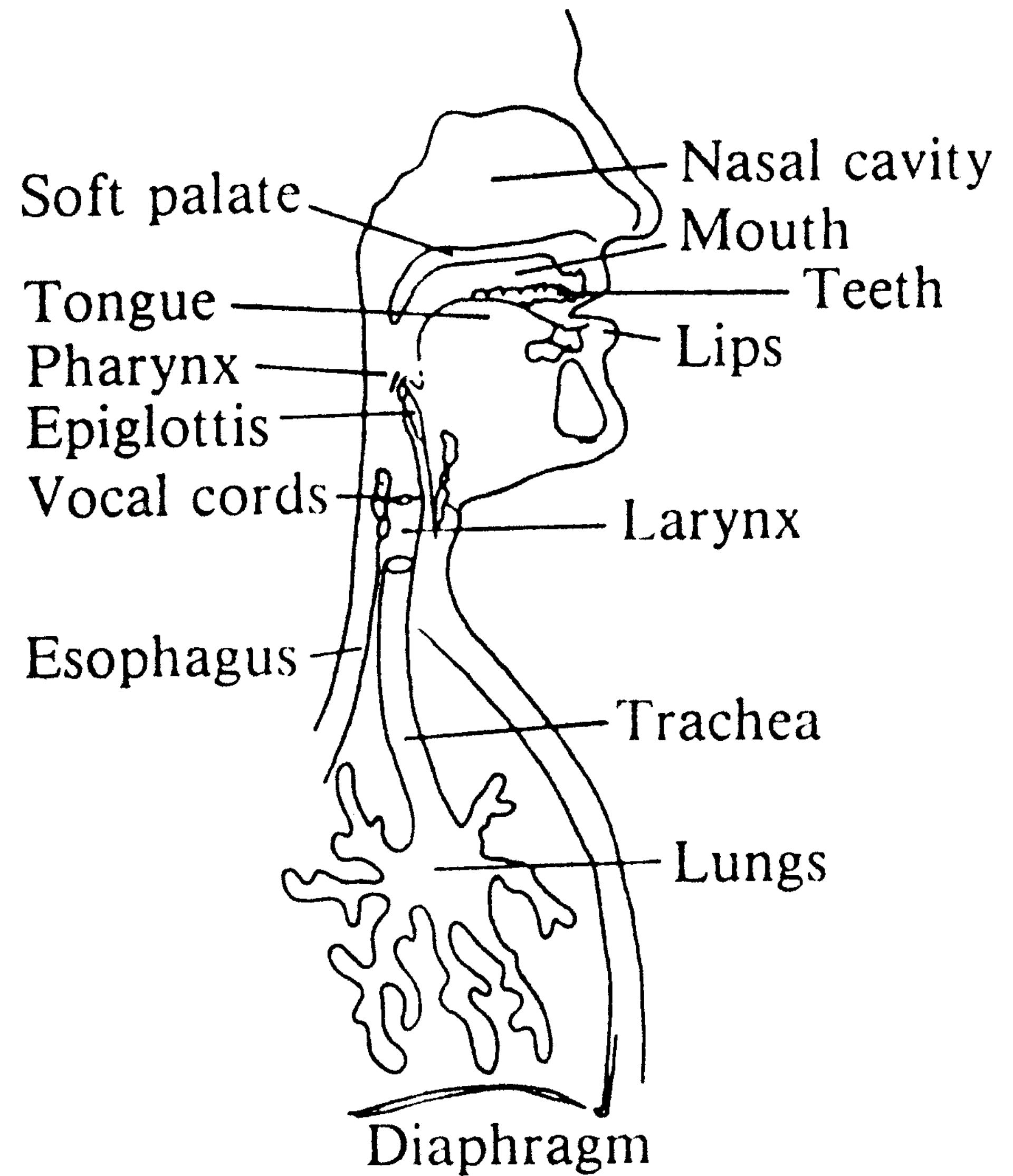
Piano



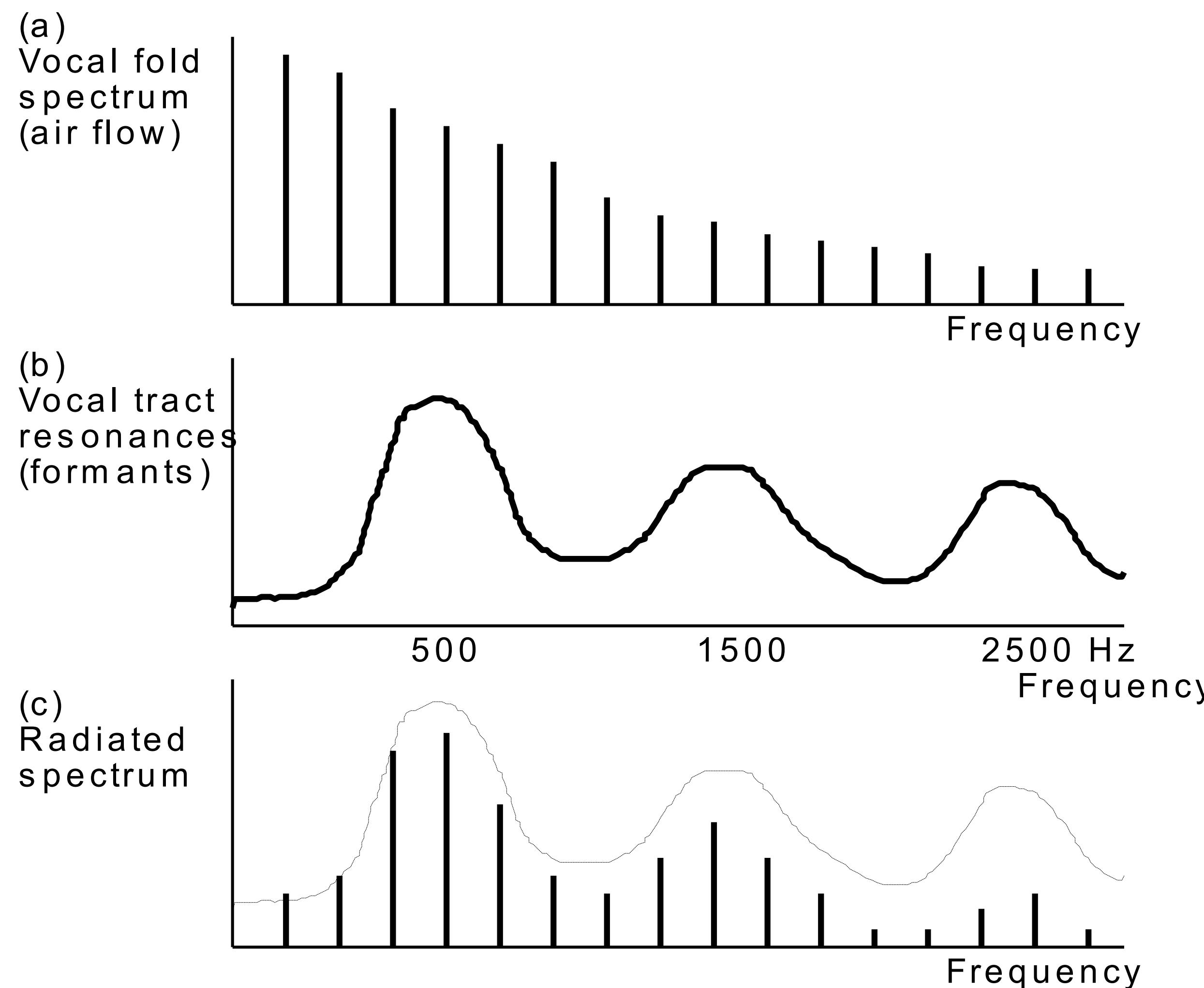
8. Voice

Vocal organs





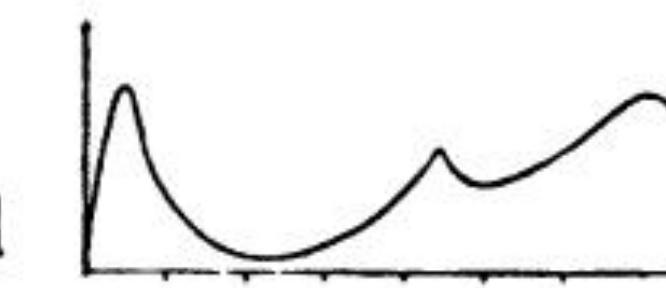
Formants



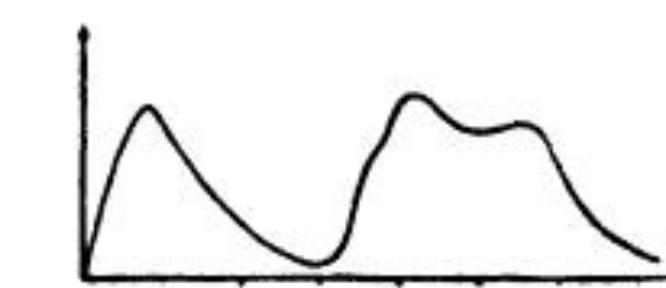
Formant regions for different vowel sounds



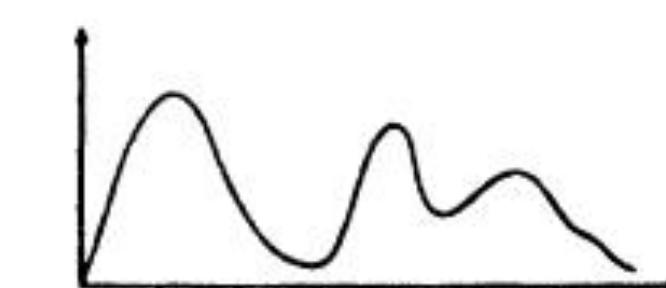
heed



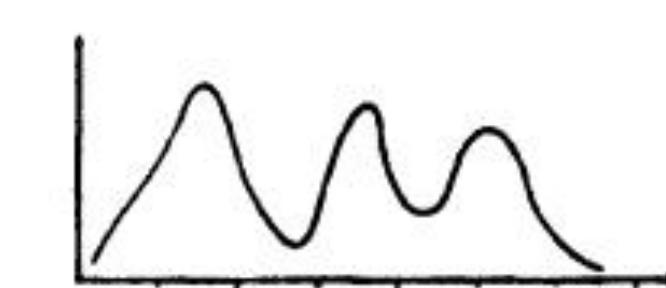
hid



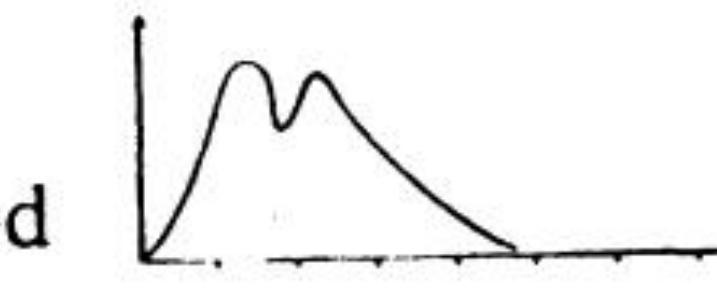
head



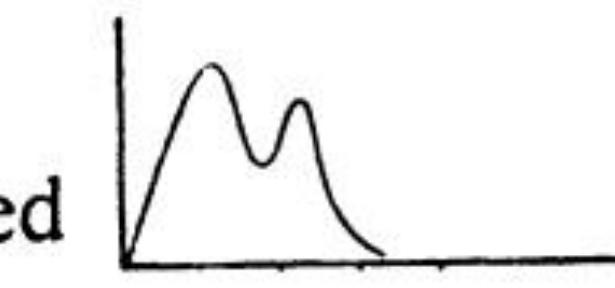
had



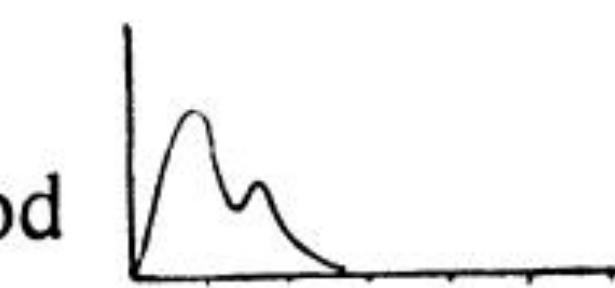
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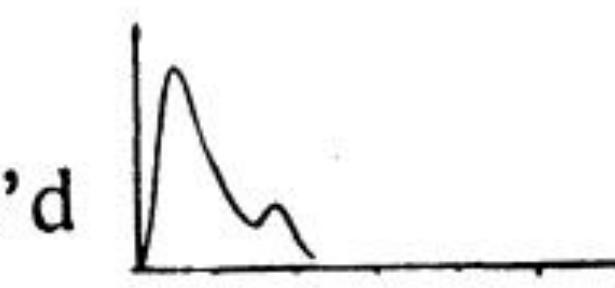
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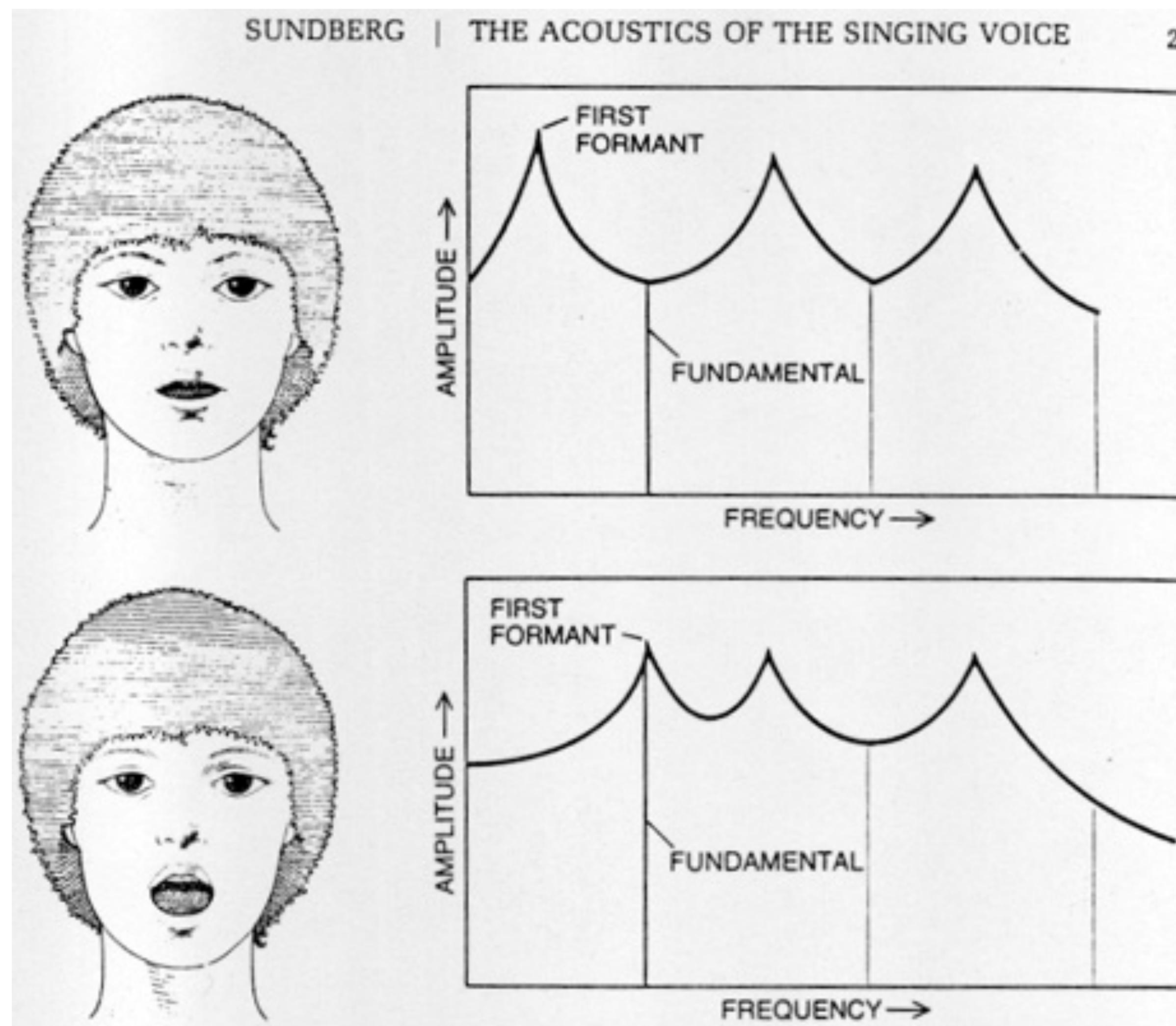
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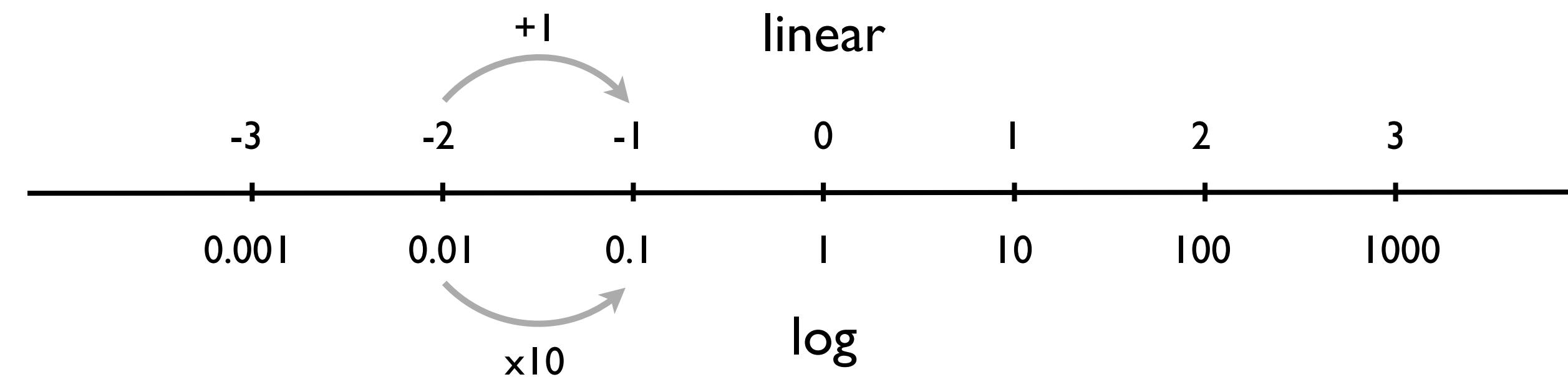
Singing



9. Hearing

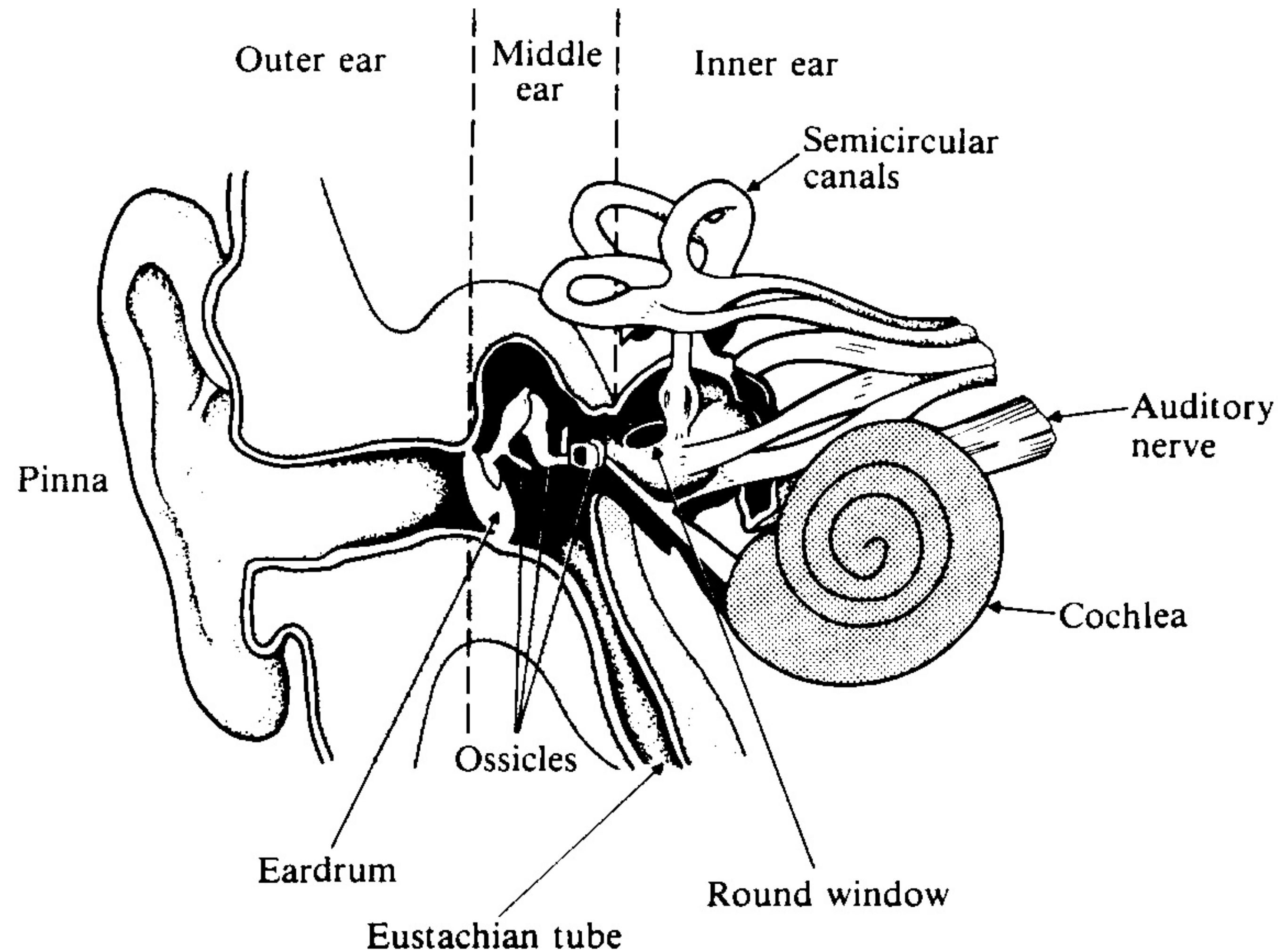
Fechner's law and range of human hearing

- Fechner's law: Our senses respond logarithmically to external stimuli.

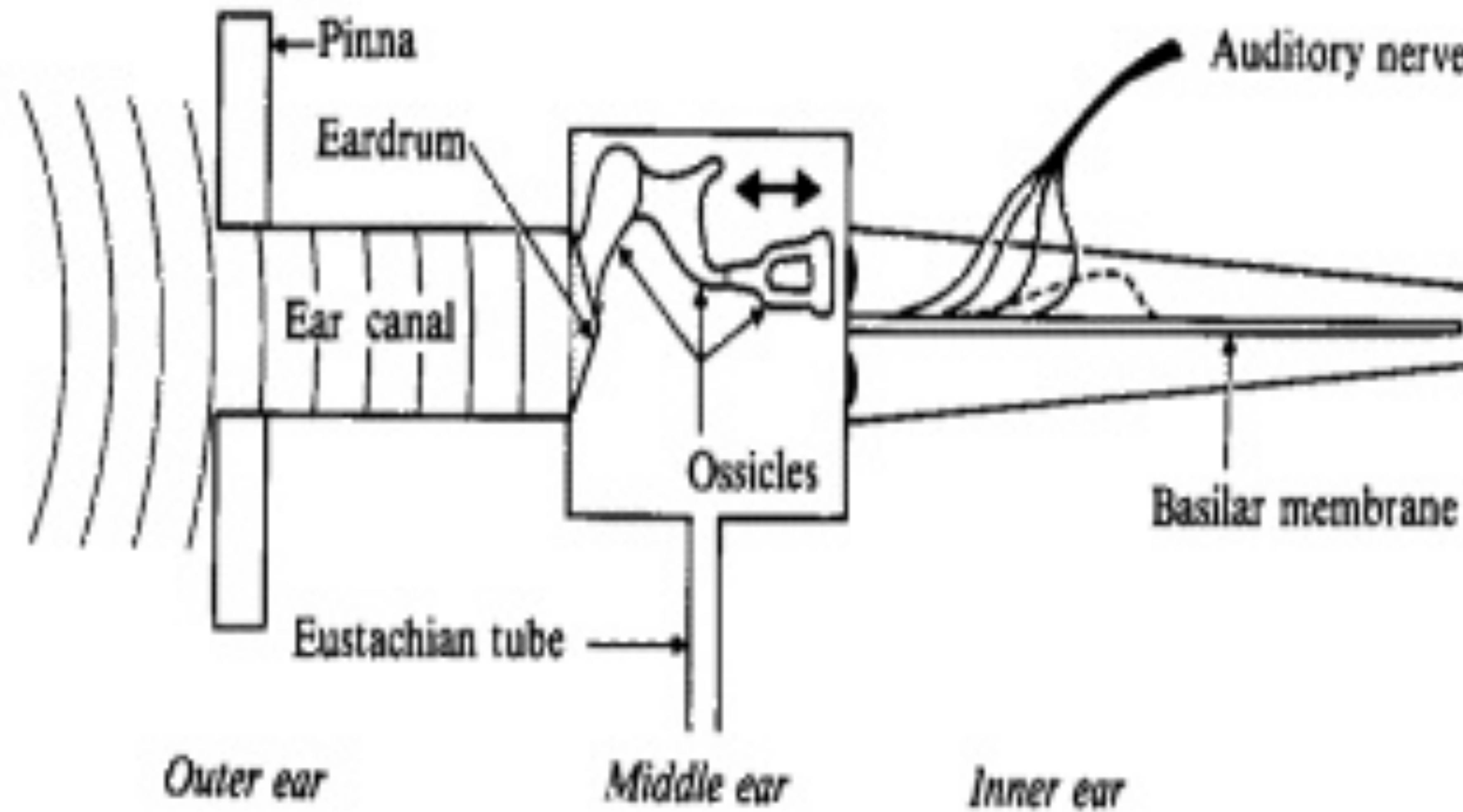


- Range of hearing
 - Pitch (frequency): 20 – 20,000 Hz (~10 octaves)
 - Loudness (intensity): 10^{-12} – 1 W/m^2 (12 orders of magnitude)
- Eye is sensitive to ~1 octave in colors (frequency) and 5 orders of magnitude in brightness (intensity)
- Q: So which do you think is more sensitive—our sense of sight or our sense of hearing?

Anatomy of human ear



Anatomy of human ear - continued



Sound localization – binaural hearing

- High frequency (> 4000 Hz) sounds – intensity difference
- Low frequency (< 4000 Hz) sounds – time of arrival

10. Loudness

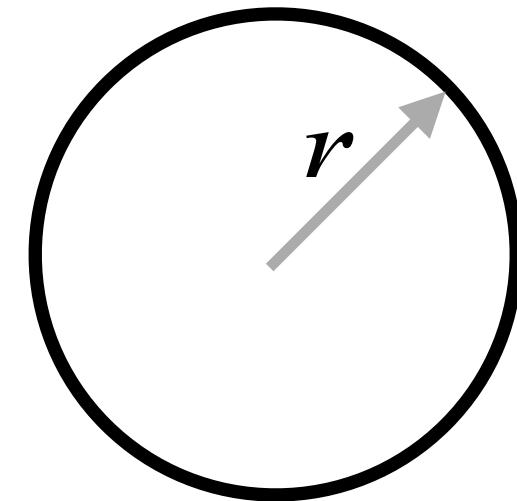
Loudness – overview

(compare two sounds, or compare one sound to the threshold of hearing)

- our perception of the relative strength of a sound
- depends on the intensity and frequency of the sound
- logarithmic response to intensity (consistent with Fechner's law)
- several different ways of quantifying loudness:
 - intensity
 - sound intensity level
 - sound loudness level
 - subjective loudness

Intensity

- Intensity is the power in a sound wave divided by the area it passes through (Watts/m²)



$$I = \frac{P}{4\pi r^2}$$

(sound is less intense the farther you are away from the source of the sound)

- Range of intensities:
 - $I_0 = 10^{-12} \text{ W/m}^2$ (**threshold of hearing** at $f = 1000 \text{ Hz}$)
 - $I = 1 \text{ W/m}^2$ (**threshold of pain** at $f = 1000 \text{ Hz}$)
- Intensity is proportional to the square of the amplitude of the sound wave: $I \propto (\Delta p)^2$
- Intensities add: (intensity of 2 violins = twice the intensity of 1 violin)

Sound Intensity Level (SIL, dB)

- Sound intensity level is the logarithm of the intensity compared to the threshold of hearing:

$$\text{SIL} = 10 \log(I/I_0) \text{ dB}$$

- Threshold of hearing: $I = I_0 = 10^{-12} \text{ W/m}^2 \Rightarrow \text{SIL} = 0 \text{ dB}$

- Threshold of pain: $I = 1 \text{ W/m}^2 \Rightarrow \text{SIL} = 120 \text{ dB}$

$$\Delta\text{SIL} = 10 \log(I_2/I_1) \text{ dB}$$

(comparing two intensities)

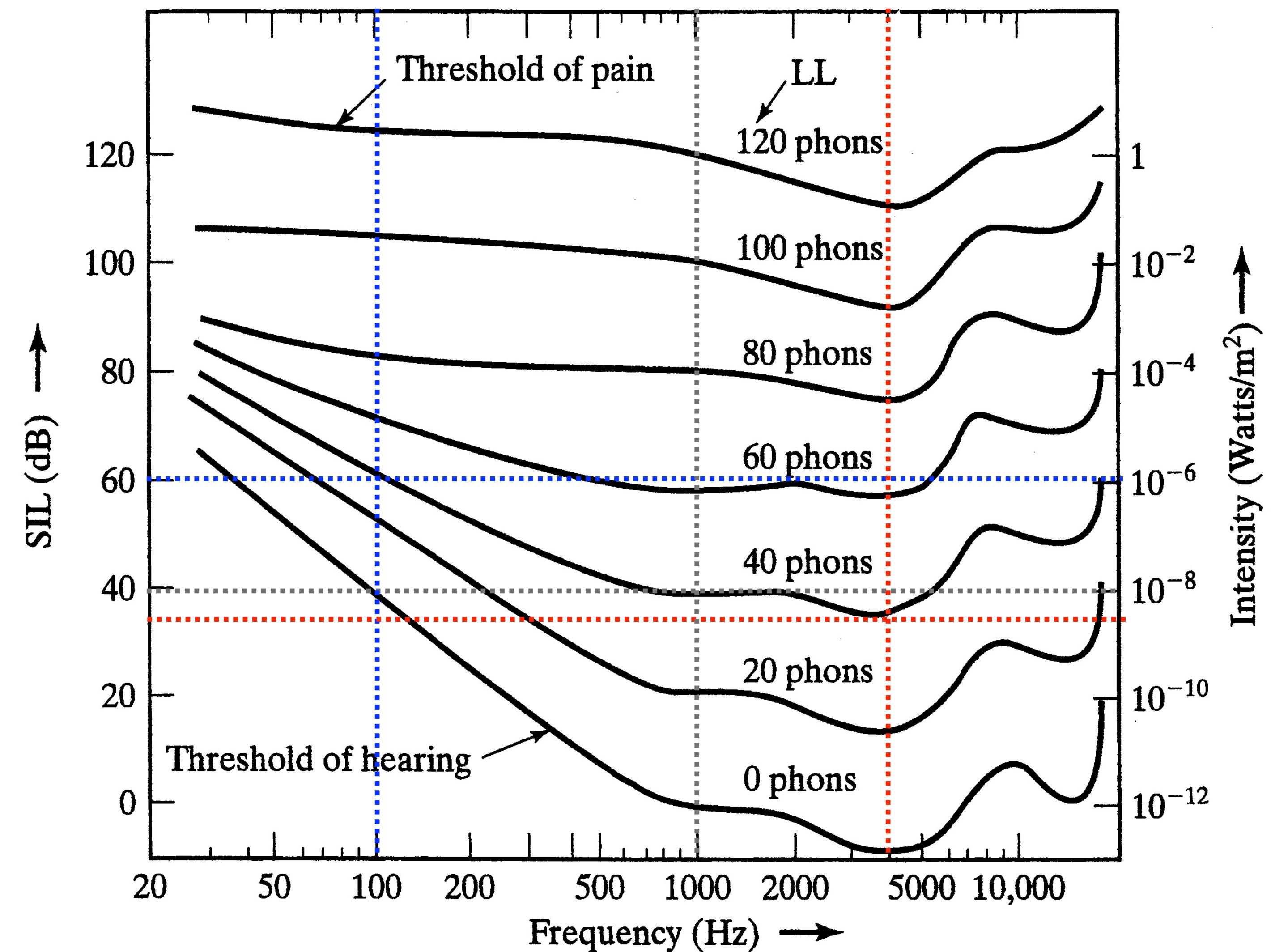
- 2x intensity: $\Delta\text{SIL} = 10 \log(2) \text{ dB} = 3 \text{ dB}$

- 10x intensity: $\Delta\text{SIL} = 10 \log(10) \text{ dB} = 10 \text{ dB}$ (perceived as "twice as loud")

- Just noticeable difference (JND): $\Delta\text{SIL} = 1 \text{ dB} \Leftrightarrow I_2 = 1.26 I_1$

Sound Loudness Level (L_L , phon)

- Human ear responds differently to different frequencies (Fletcher-Munson curves)
- Most sensitive to frequencies ~ 4000 Hz
- Equal loudness curves labeled by phon (not the same as dB)
 L_L (phon) = SIL (dB) at $f = 1000$ Hz
- Example:
 $SIL(40 \text{ phon}, 1000 \text{ Hz}) = 40 \text{ dB}$ (grey)
 $SIL(40 \text{ phon}, 100 \text{ Hz}) = 60 \text{ dB}$ (blue)
 $SIL(40 \text{ phon}, 4000 \text{ Hz}) = 35 \text{ dB}$ (red)



Subjective Loudness (S , sone)

- Measure of loudness where “twice as loud” corresponds to “multiply by 2”
- Most people perceive 10x increase in intensity as “twice as loud”
 - Recall: 10x intensity $\rightarrow \Delta \text{SIL} = 10 \text{ dB}$, $\Delta L_L = 10 \text{ phon}$
 - Some values:

$L_L = 40 \text{ phon}$	\Leftrightarrow	$S = 1 \text{ sone}$
$L_L = 50 \text{ phon}$	\Leftrightarrow	$S = 2 \text{ sone}$
$L_L = 60 \text{ phon}$	\Leftrightarrow	$S = 4 \text{ sone}$
$L_L = 30 \text{ phon}$	\Leftrightarrow	$S = 1/2 \text{ sone}$

General formula:
$$S = 2^{(L_L - 40)/10} \text{ sone}$$
 - Questions:

$L_L = 70 \text{ phon}$	\Leftrightarrow	$S = ?? \text{ sone}$
$L_L = 20 \text{ phon}$	\Leftrightarrow	$S = ?? \text{ sone}$

Different measures of loudness and safe noise levels

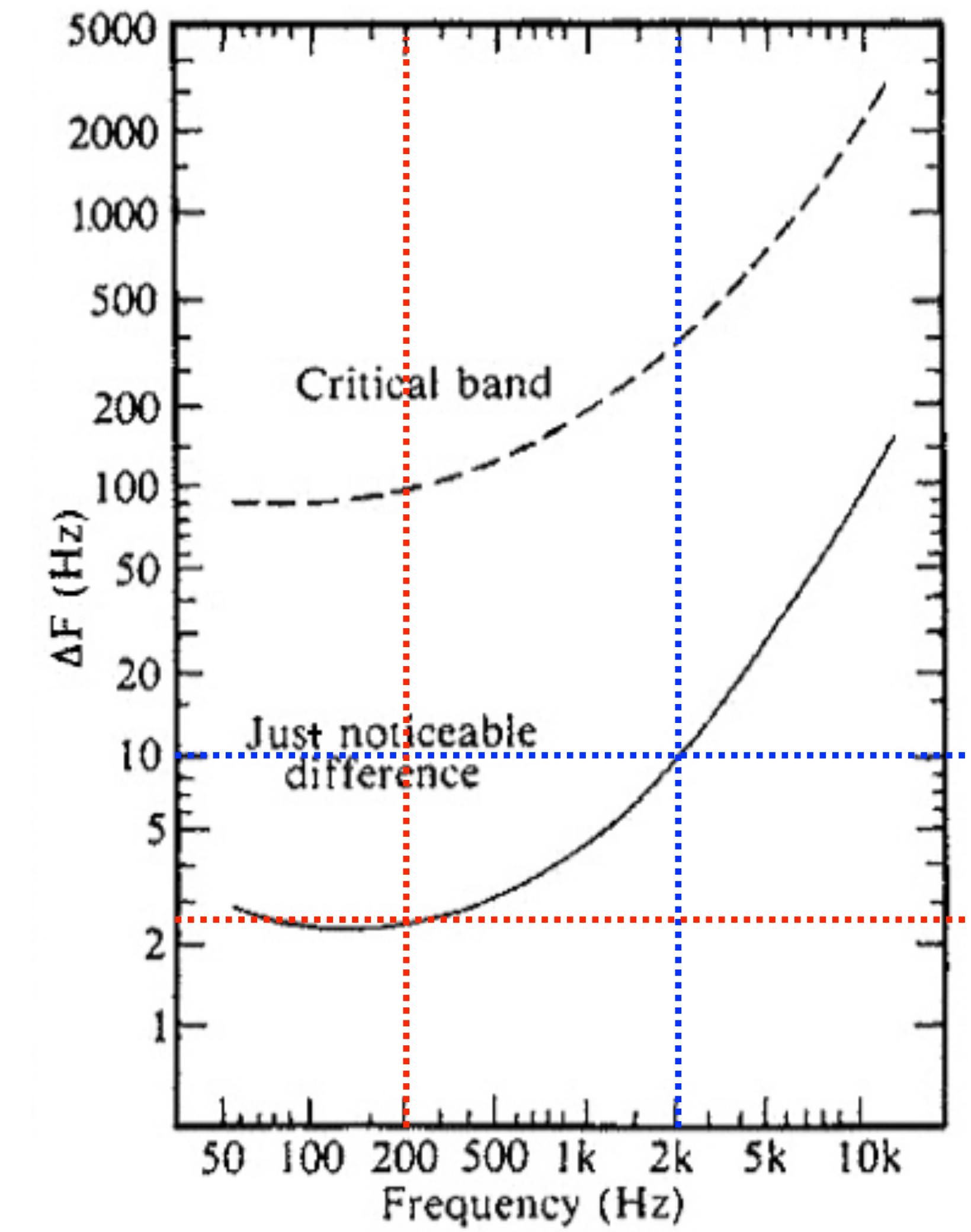
	SIL (dB) at 1000 Hz	L_L (phon)	S (sone)
Threshold of hearing	0	0	1/16
Recording studio	20	20	1/4
Quiet office	40	40	1
Ordinary conversation	60	60	4
Normal piano practice	80	80	16
Piano fortissimo	100	100	64
Threshold of pain	120	120	256

Duration per day, hours	Sound level dBA slow response
8.....	90
6.....	92
4.....	95
3.....	97
2.....	100
1 1/2	102
1.....	105
1/2	110
1/4 or less.....	115

11. Pitch & timbre

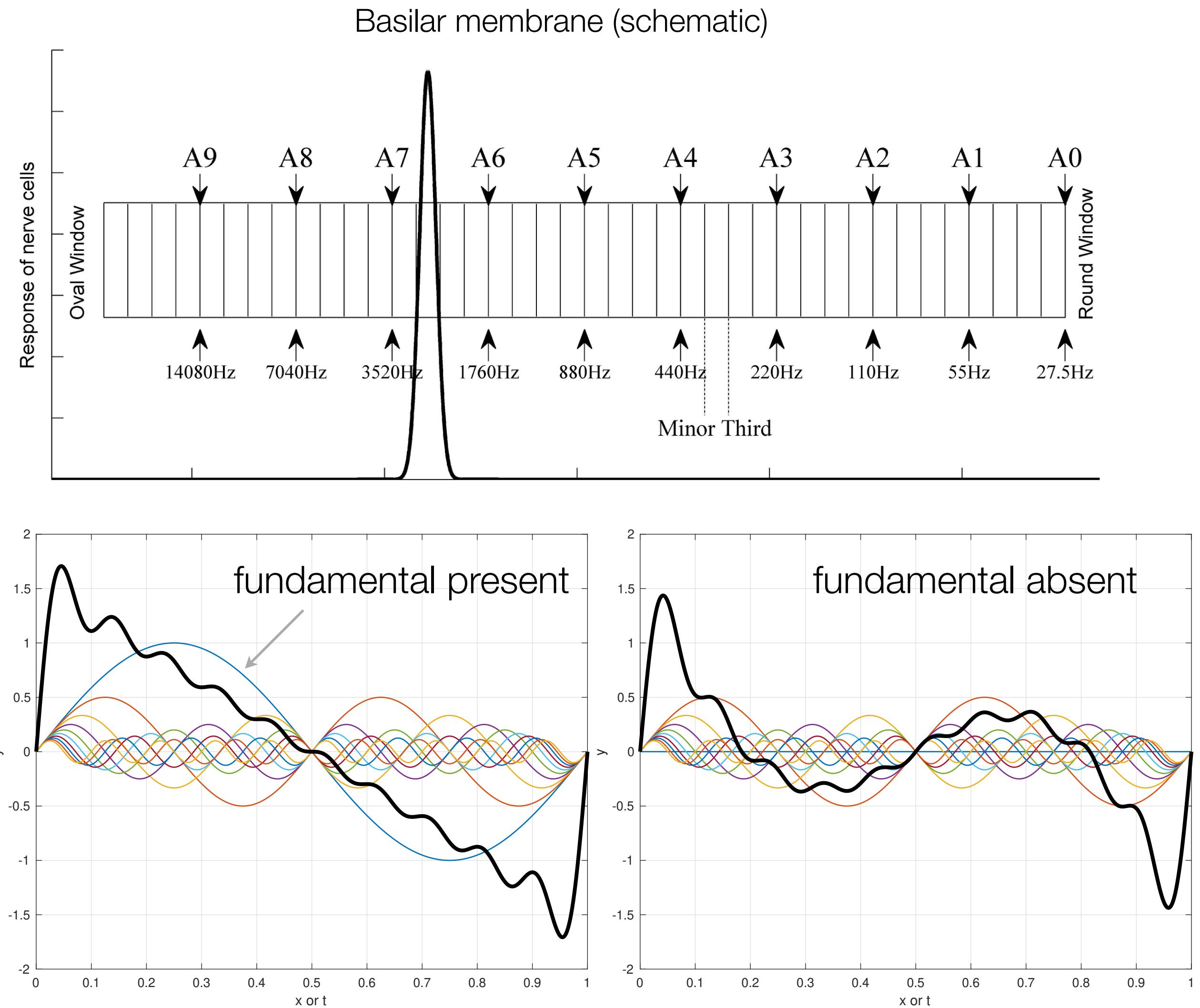
What distinguishes one musical note from another?

- Pitch, timbre, duration, loudness (intensity), attack & decay transients
- Perfect pitch: ability to determine absolute pitch without regard to a reference (only 1 out of ~10,000 people have it)
- Pitch discrimination: ability to distinguish two different pitches
 - depends on whether you play the two notes sequentially or simultaneously
 - JND: just noticeable difference (sequential; 0.5% of center frequency; 1/10th of a semitone)
 - LFD: limit of frequency discrimination (simultaneous; 10% of center frequency; 2 semitones)
 - Analogy with sense of touch: placing two pencil points on your arm



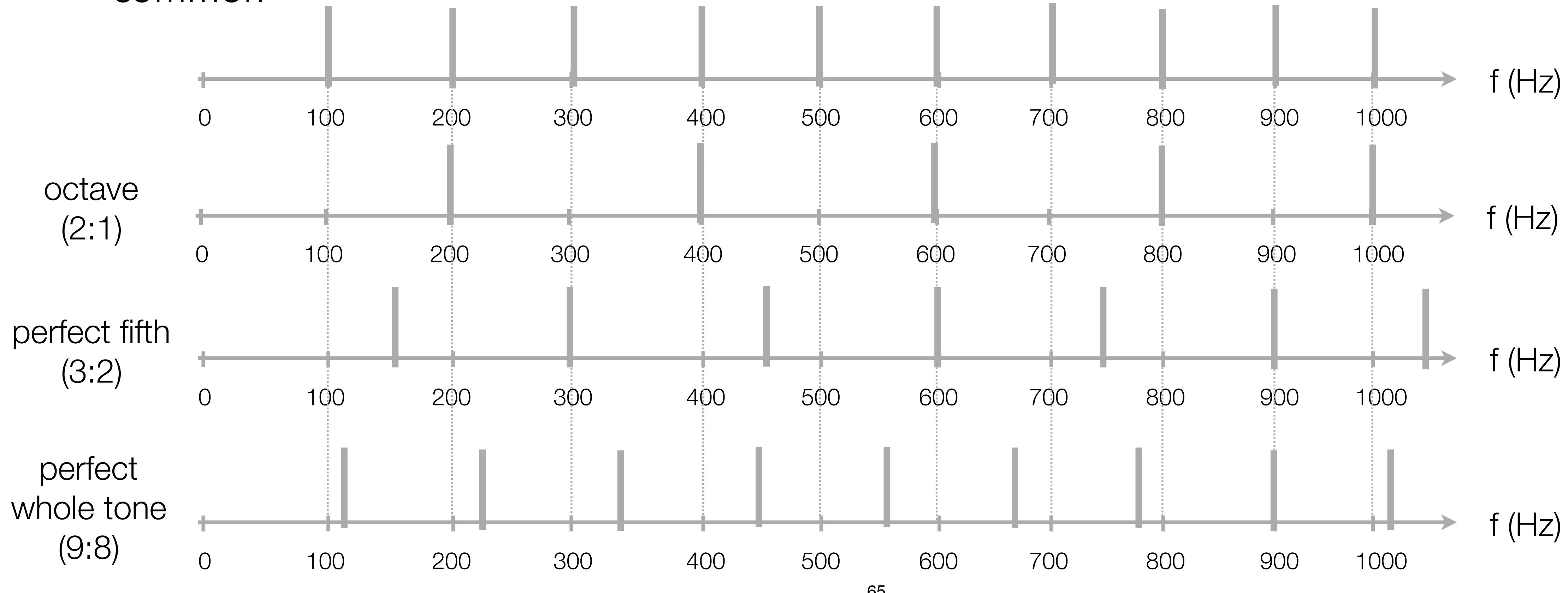
Where does pitch determination occur, in the ear or the brain?

- Place theory: pitch determined by the location on the basilar membrane excited by the sound wave
- Periodicity theory: pitch inferred by the brain from the timing of electrical impulses triggered by the period of the sound wave
- Missing fundamental in support of periodicity theory:
 - 200 Hz, 300 Hz, 400 Hz, → hear 100 Hz
 - 300 Hz, 500 Hz, 700 Hz, → hear ??? Hz
 - <http://www.personal.psu.edu/meb26/INART50/psychoacoustics.html>



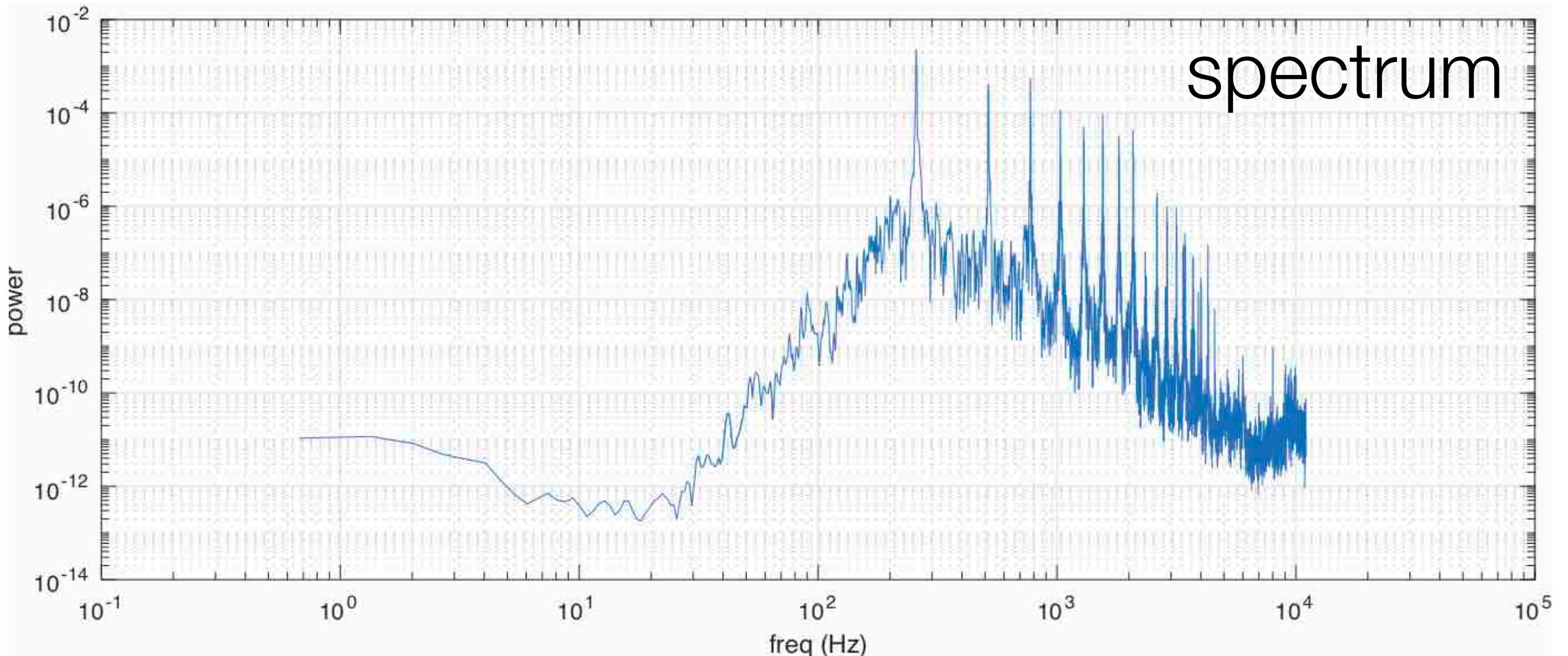
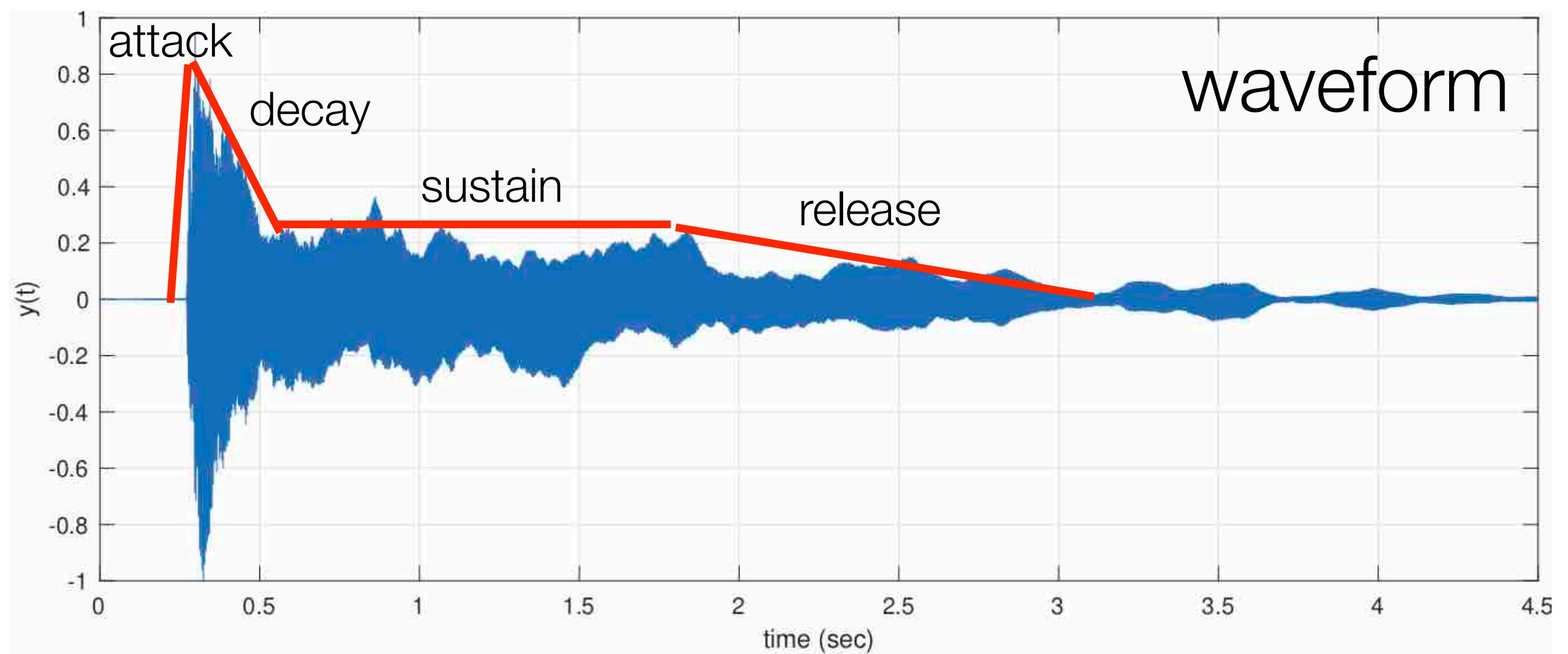
What makes two notes pleasing when they are played together?

- Two notes are pleasing (consonant) when they have many harmonics in common
- Two notes clash with one another (dissonant) when they have very few harmonics in common



Attack and decay transients

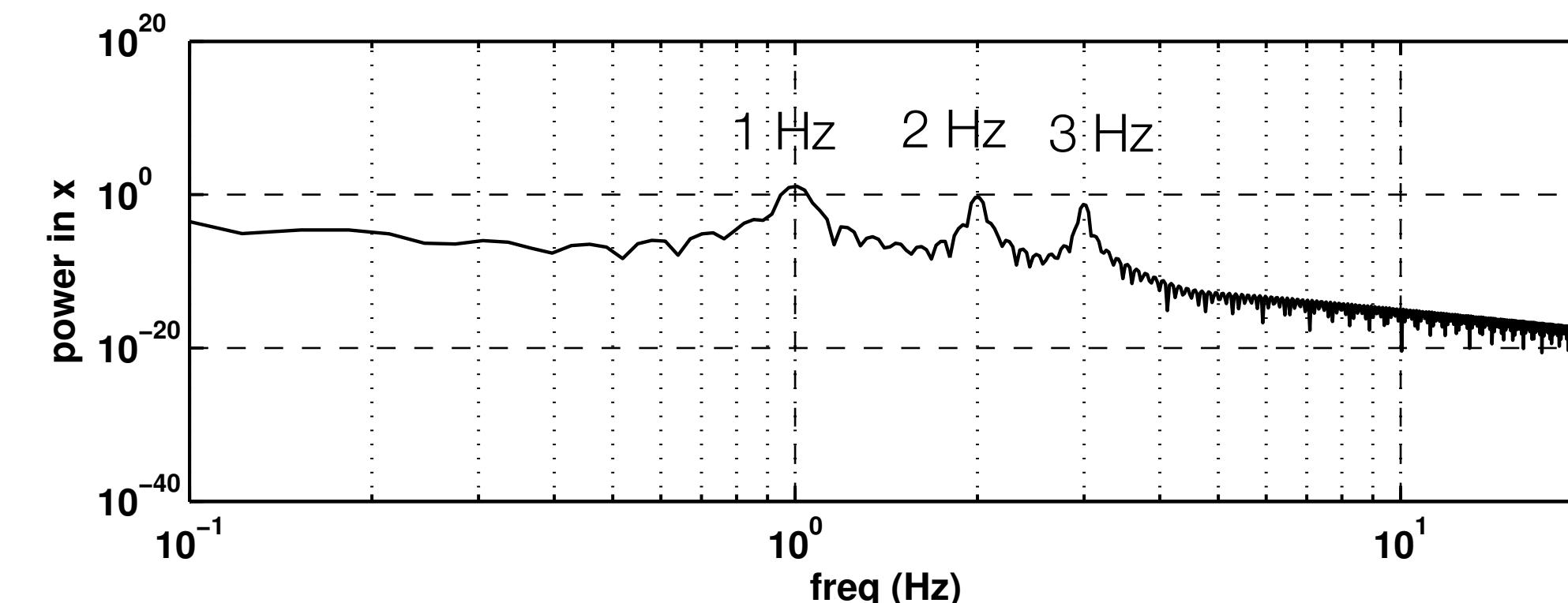
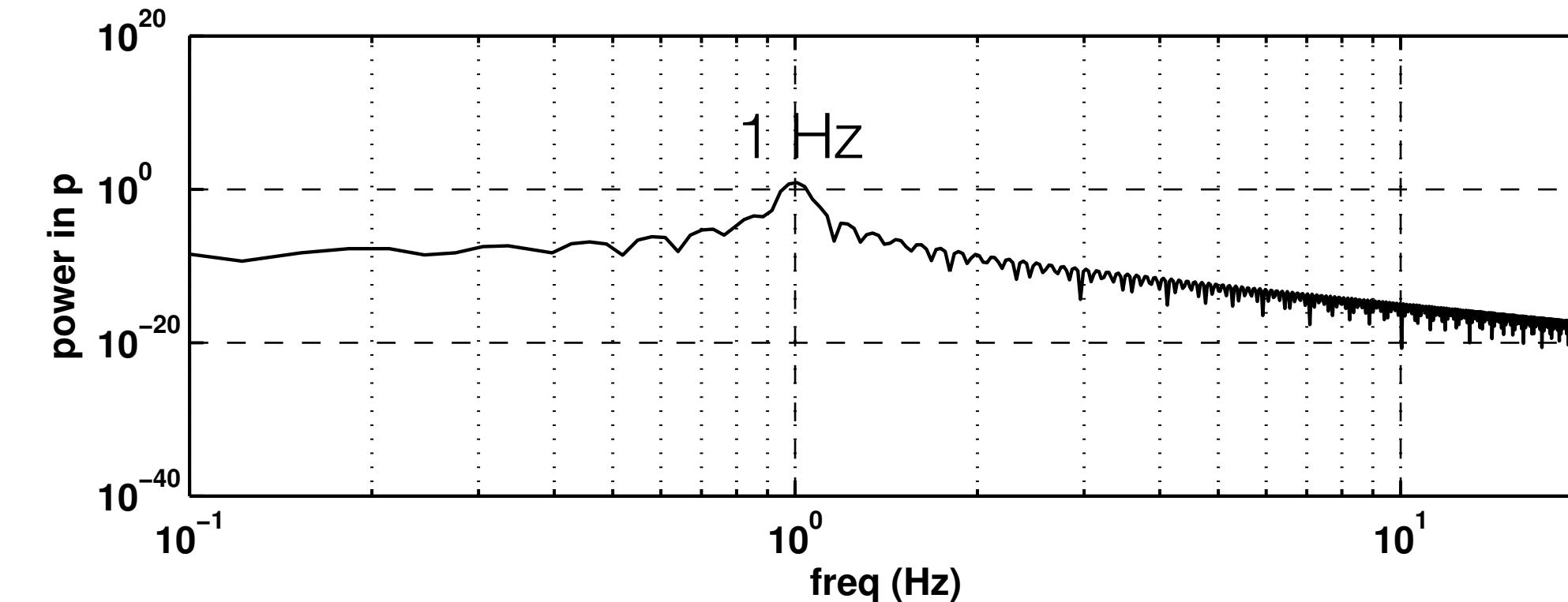
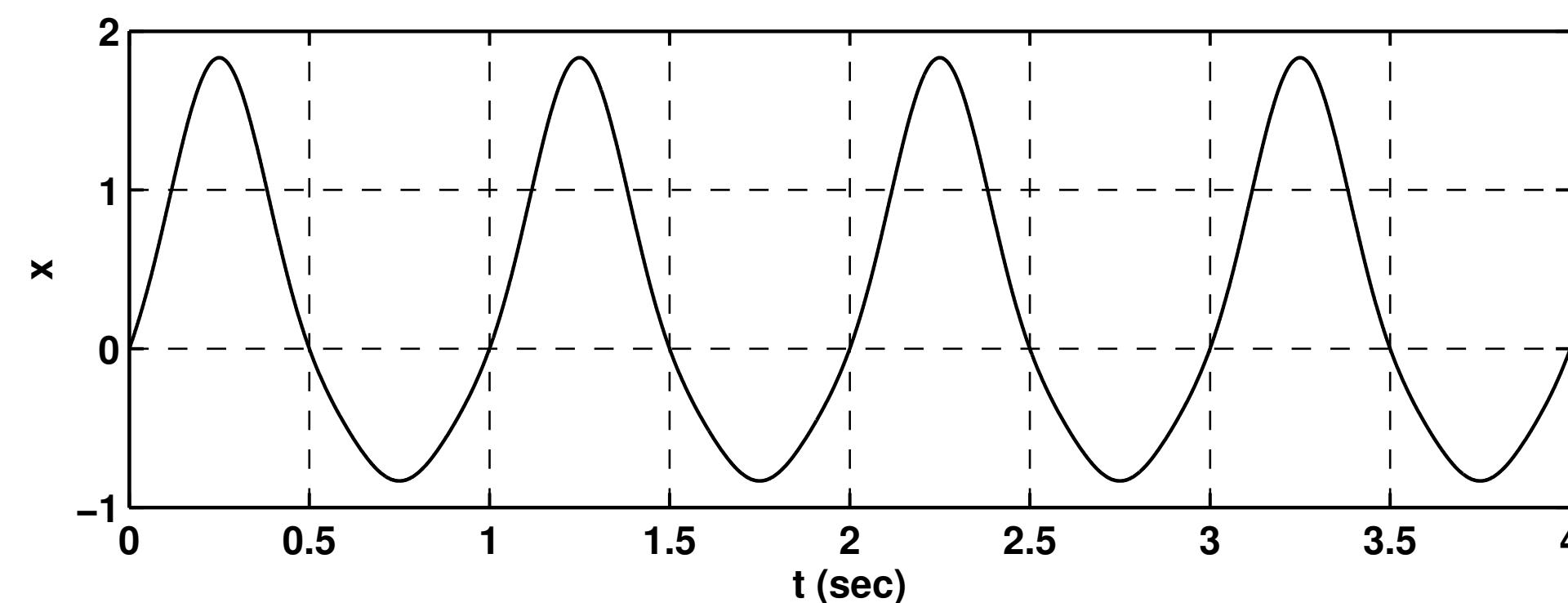
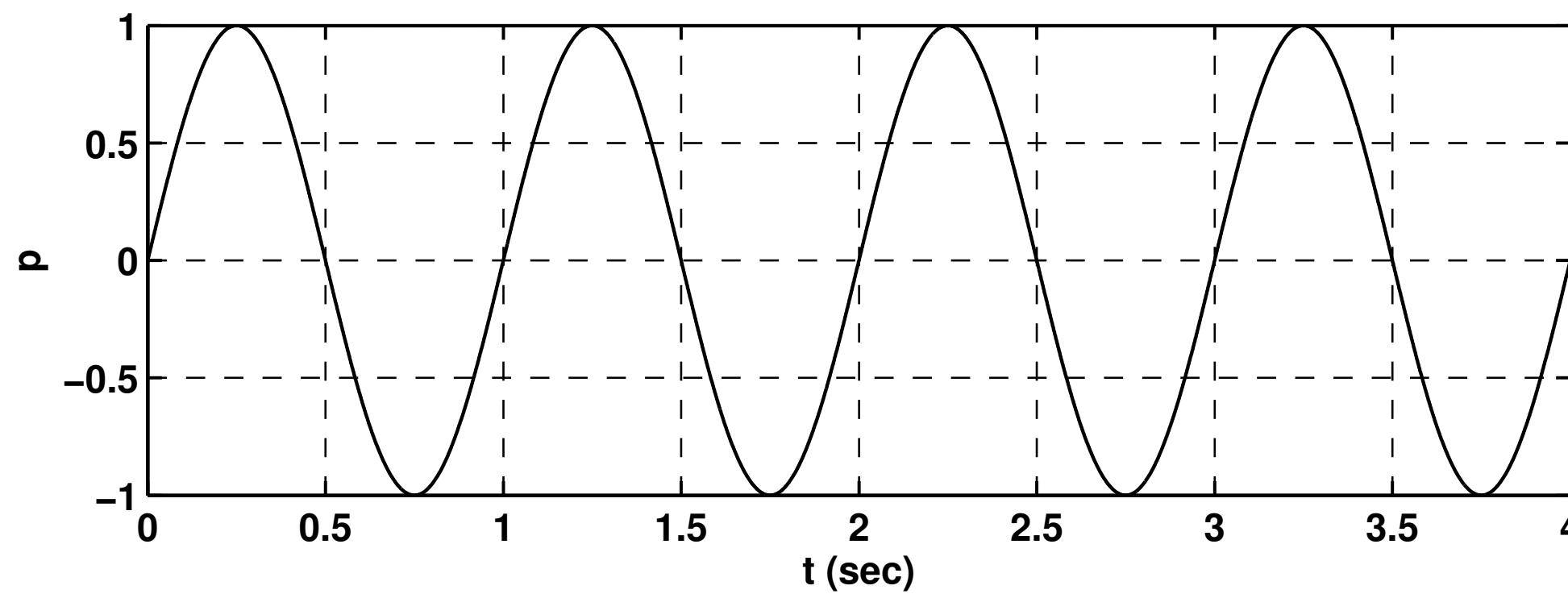
- How a note starts and ends affects how it sounds
- Piano C4
- Piano C4 (reversed)
- Happy birthday
- Happy birthday backwards
- Happy birthday backwards (reversed)



Aural harmonics – harmonics produced by the ear

- Ear introduces distortions which converts a pure tone to one having multiple harmonics

$$x(t) = a_0 + a_1 p(t) + a_2 p^2(t) + a_3 p^3(t) + \dots$$

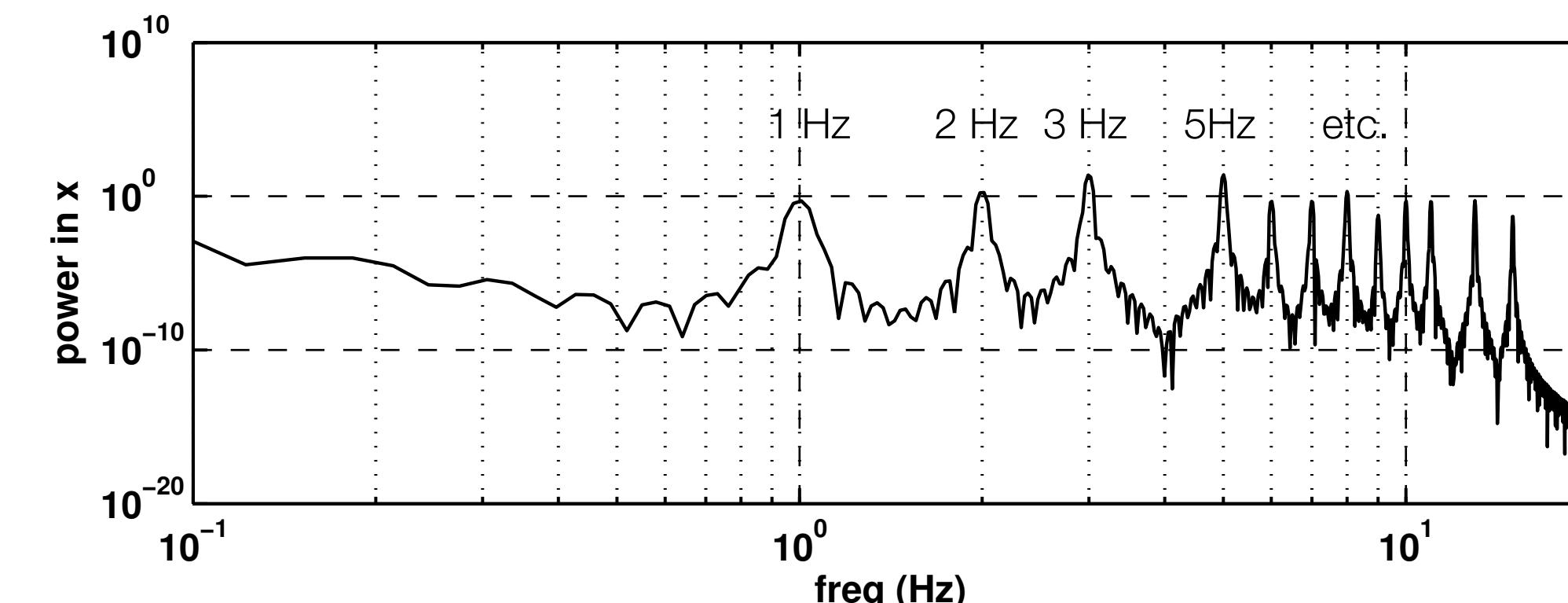
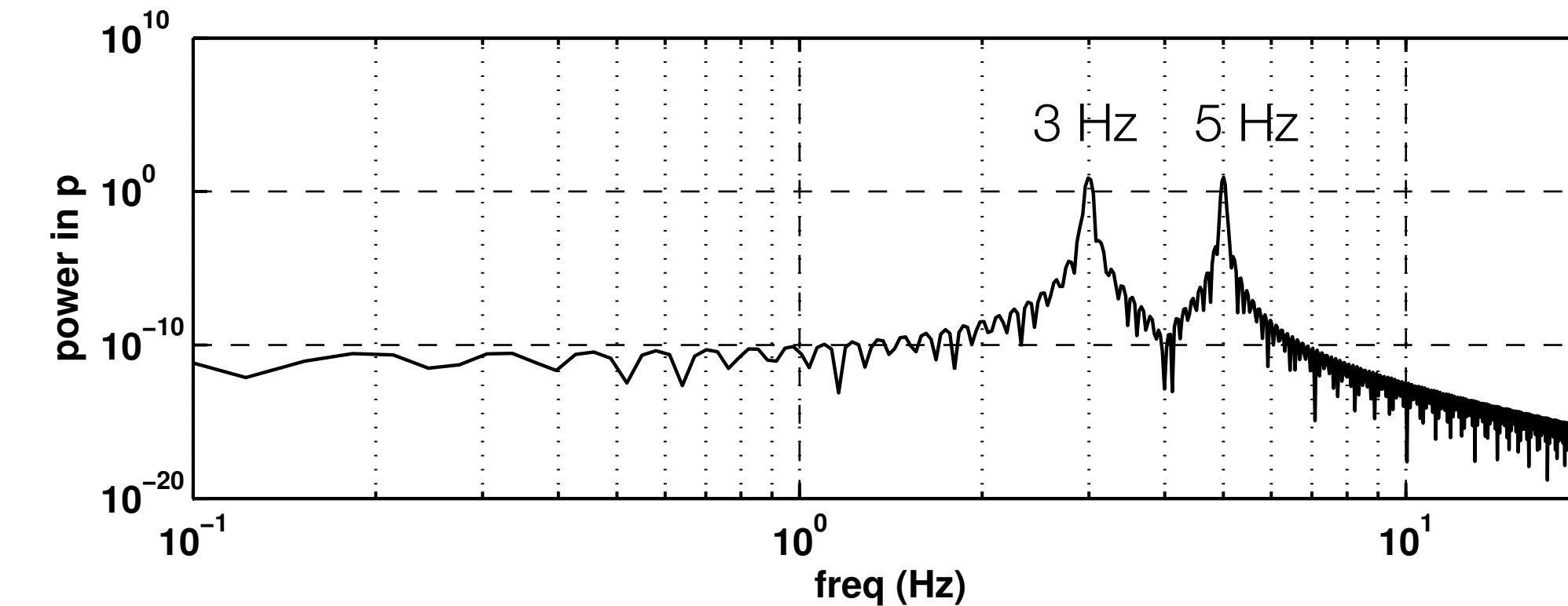
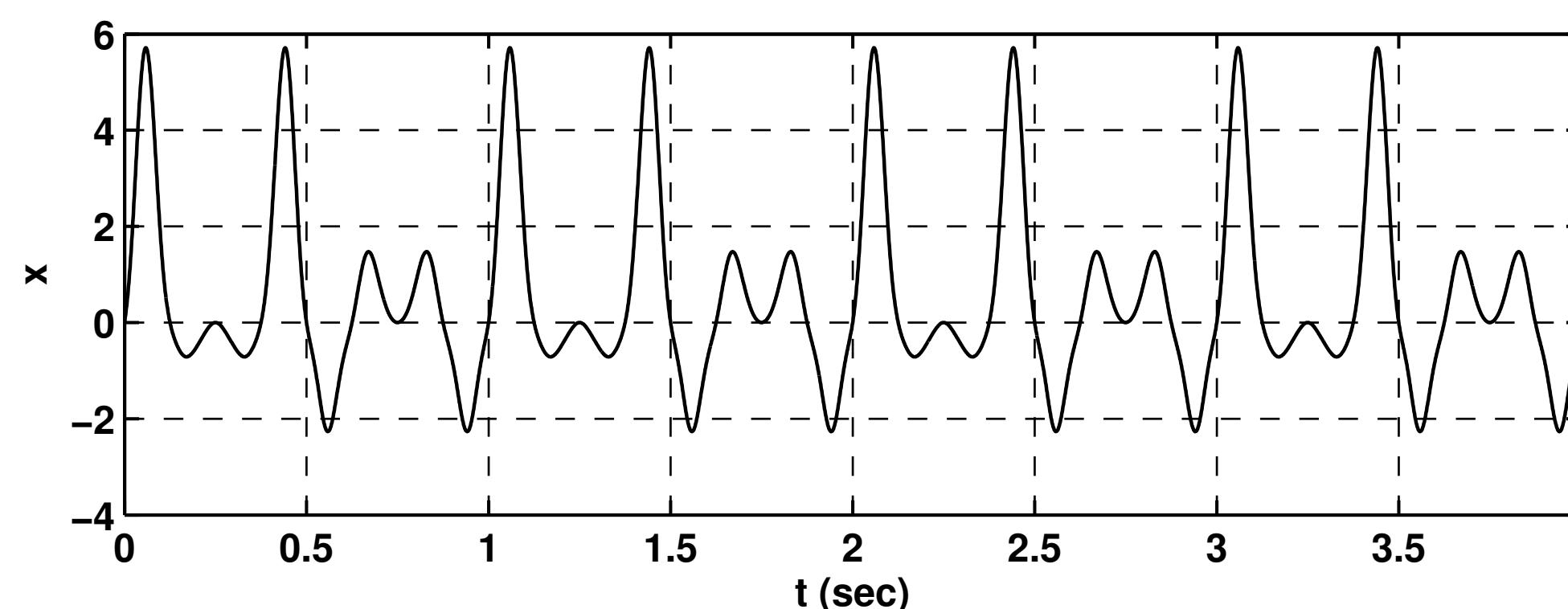
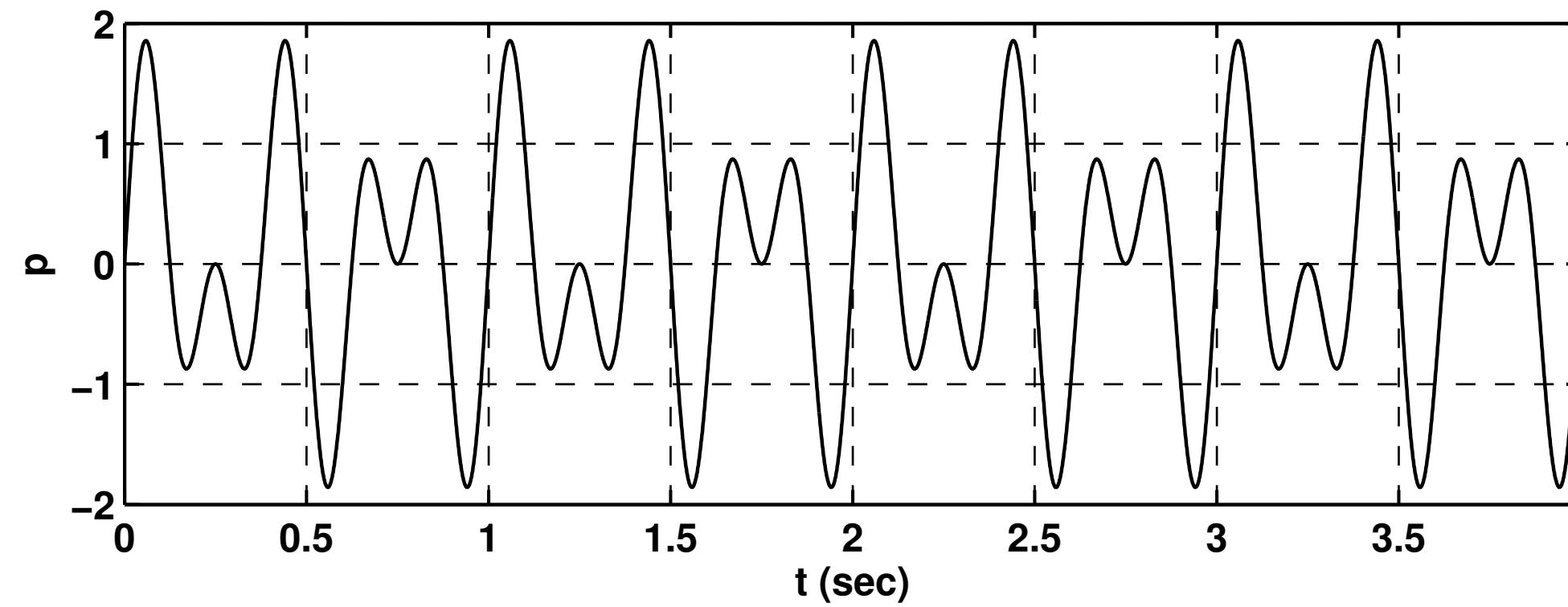


$$a_0 = 0, a_1 = 1, a_2 = 1/2, a_3 = 1/3$$

Aural combination tones – aural harmonics for complex tones

- If two pure tones f_1 and f_2 are played simultaneously and sufficiently loudly, one hears sum and difference tones

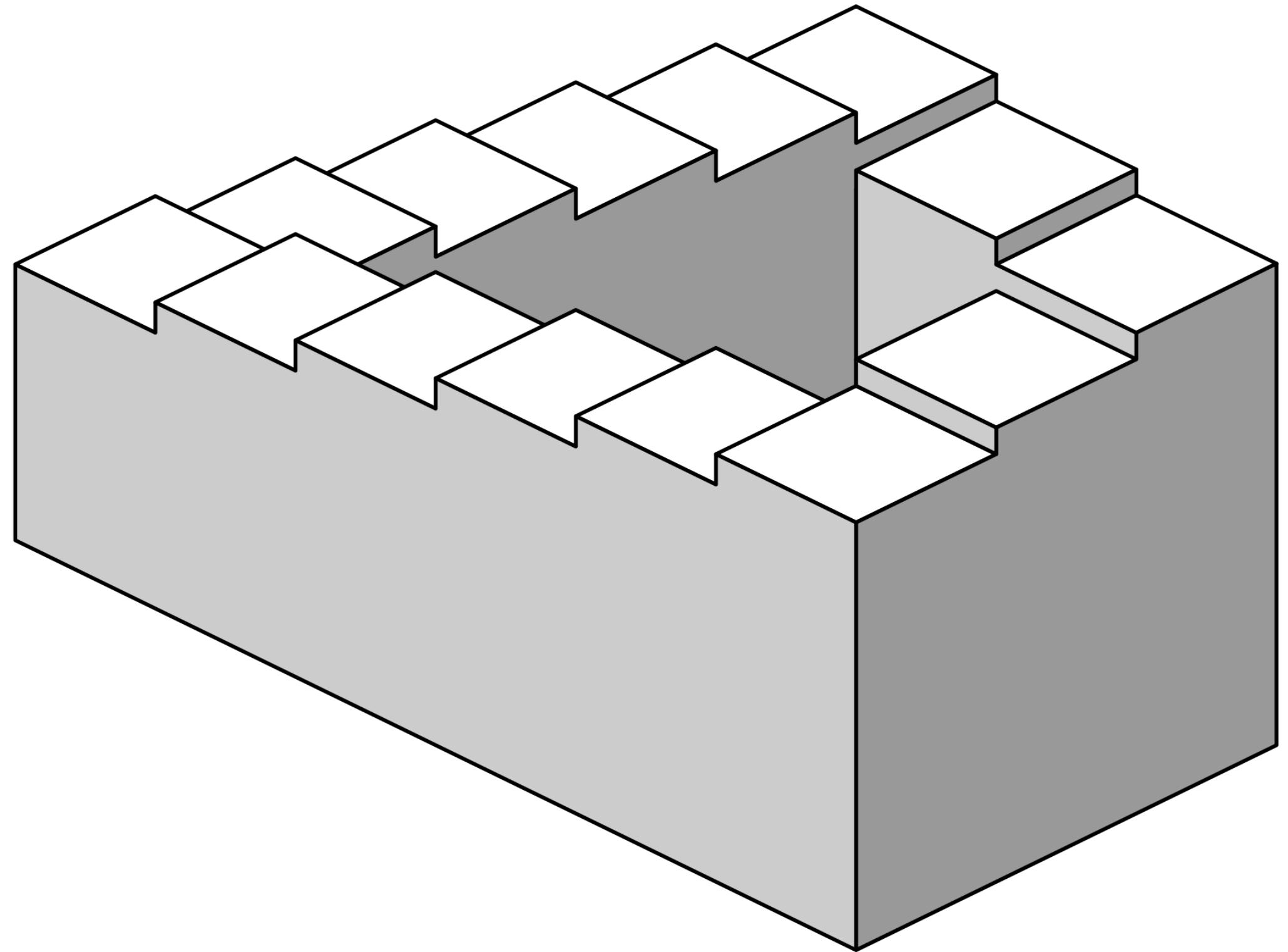
$$f_c = |mf_1 \pm nf_2| \Rightarrow |f_1 - f_2|, |2f_1 - f_2|, |3f_1 - f_2|, \dots$$



$$a_0 = 0, a_1 = 1, a_2 = 1/2, a_3 = 1/3$$

Pitch paradox – the audio equivalent of an optical illusion

- Shepard scale: never-ending scale (pitch seems to increase indefinitely)
- YouTube videos:
 - <http://www.youtube.com/watch?v=PCs1lckF5vl>
 - <http://vimeo.com/34749558>



Never-ending staircase
(L. Penrose; M.C. Escher)

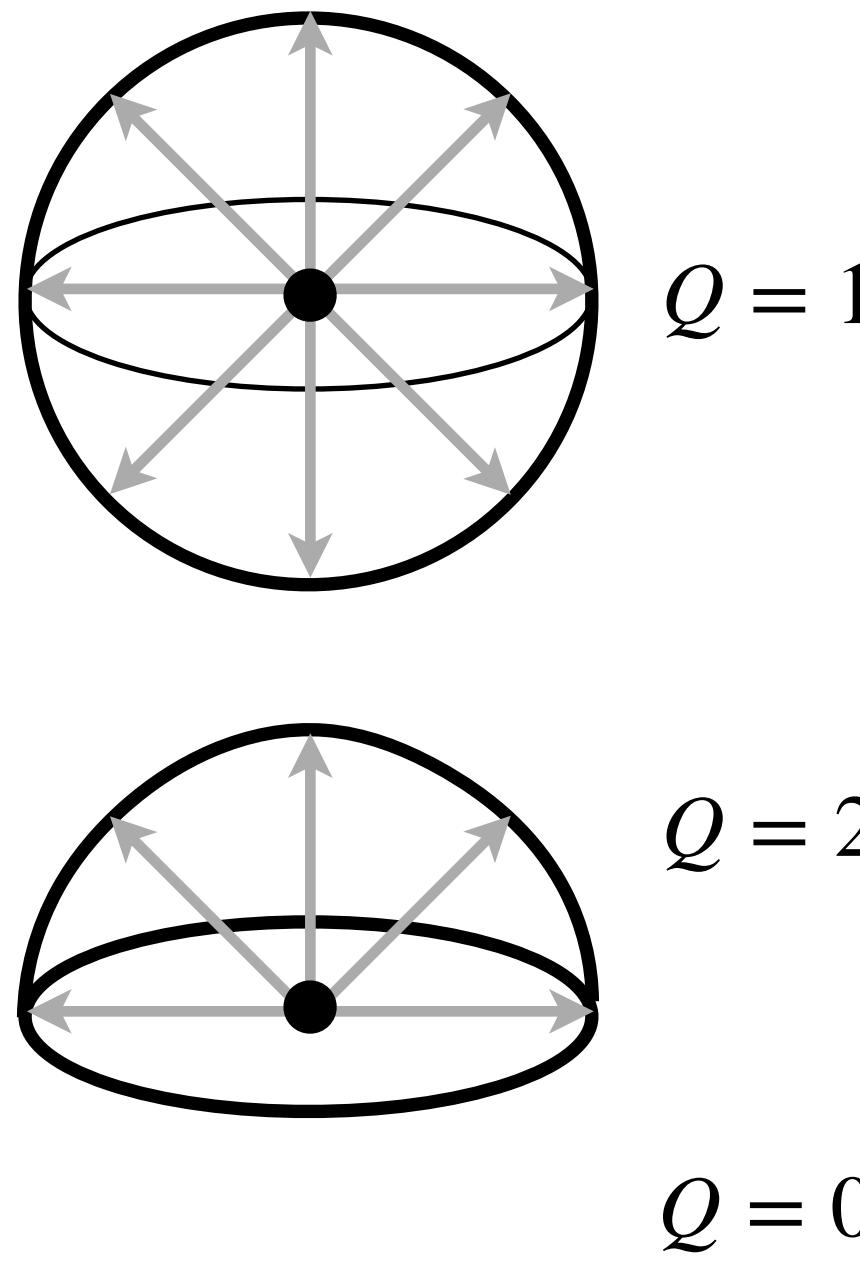
12. Auditorium & room acoustics

Auditorium and room acoustics – overview

- What makes for a good concert hall?
- Why do you sound good when you sing in a shower?
- Difference between “direct”, “reflected”, and “reverberant” sound
- Reverberation time is the most important characteristic of a room
- YouTube video / soundfile:
 - Anechoic chamber (<https://www.youtube.com/watch?v=BYBSA9v8IRE>)
 - “Sonic wonders” sound file (listen to -32:40)

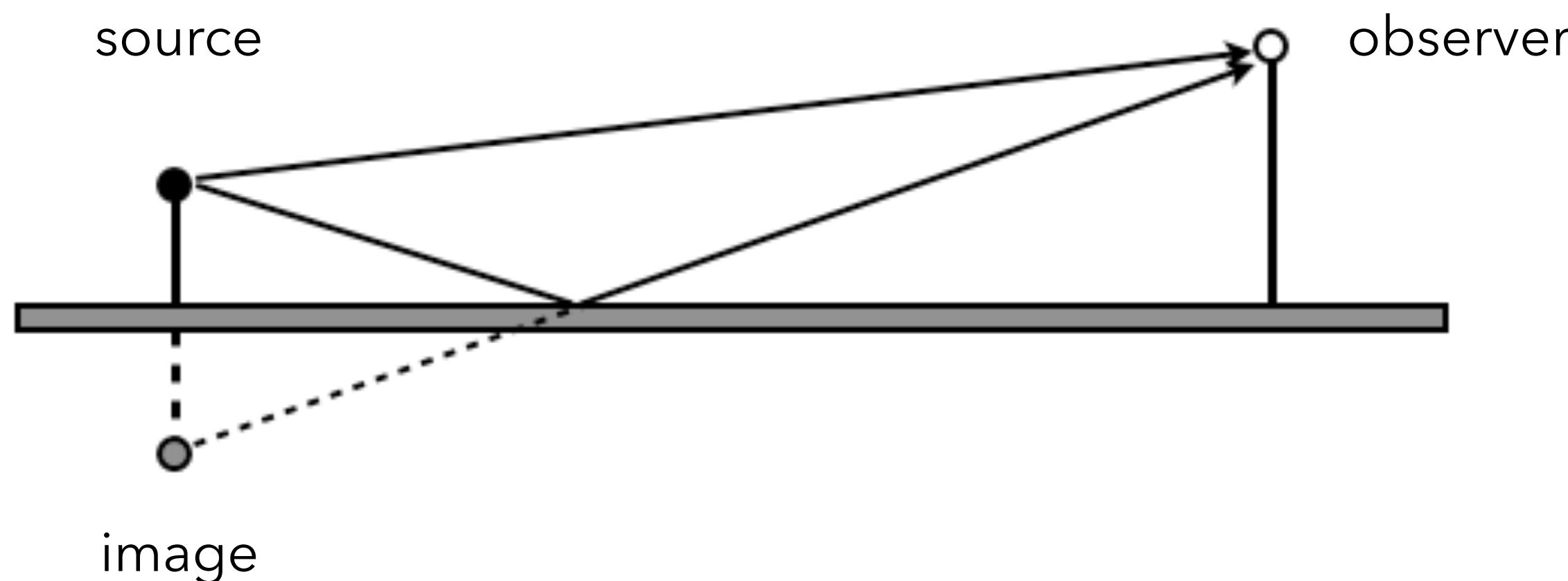
Direct sound

- Sound received from a source in the absence of any reflections (e.g., anechoic chamber)
- Intensity: $I = \frac{P}{4\pi r^2}$ (omni-directional); $I = \frac{QP}{4\pi r^2}$ (directional source; Q is the directivity factor)



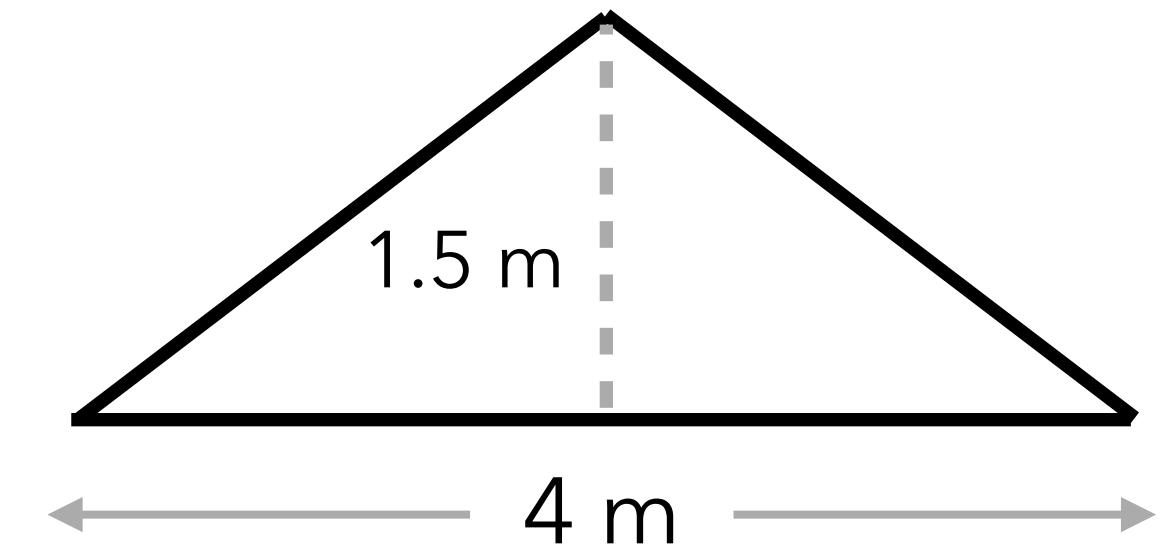
Reflected sound

- Hear an echo if the reflected sound is heard greater than 35 msec after the direct sound
- Recall: $v = 346 \text{ m/s} \approx 1000 \text{ ft/s} = 1 \text{ ft/msec}$
- $\text{SIL}_{\text{reflected}} < \text{SIL}_{\text{direct}}$ (reflected sound travels farther and can be partially absorbed)

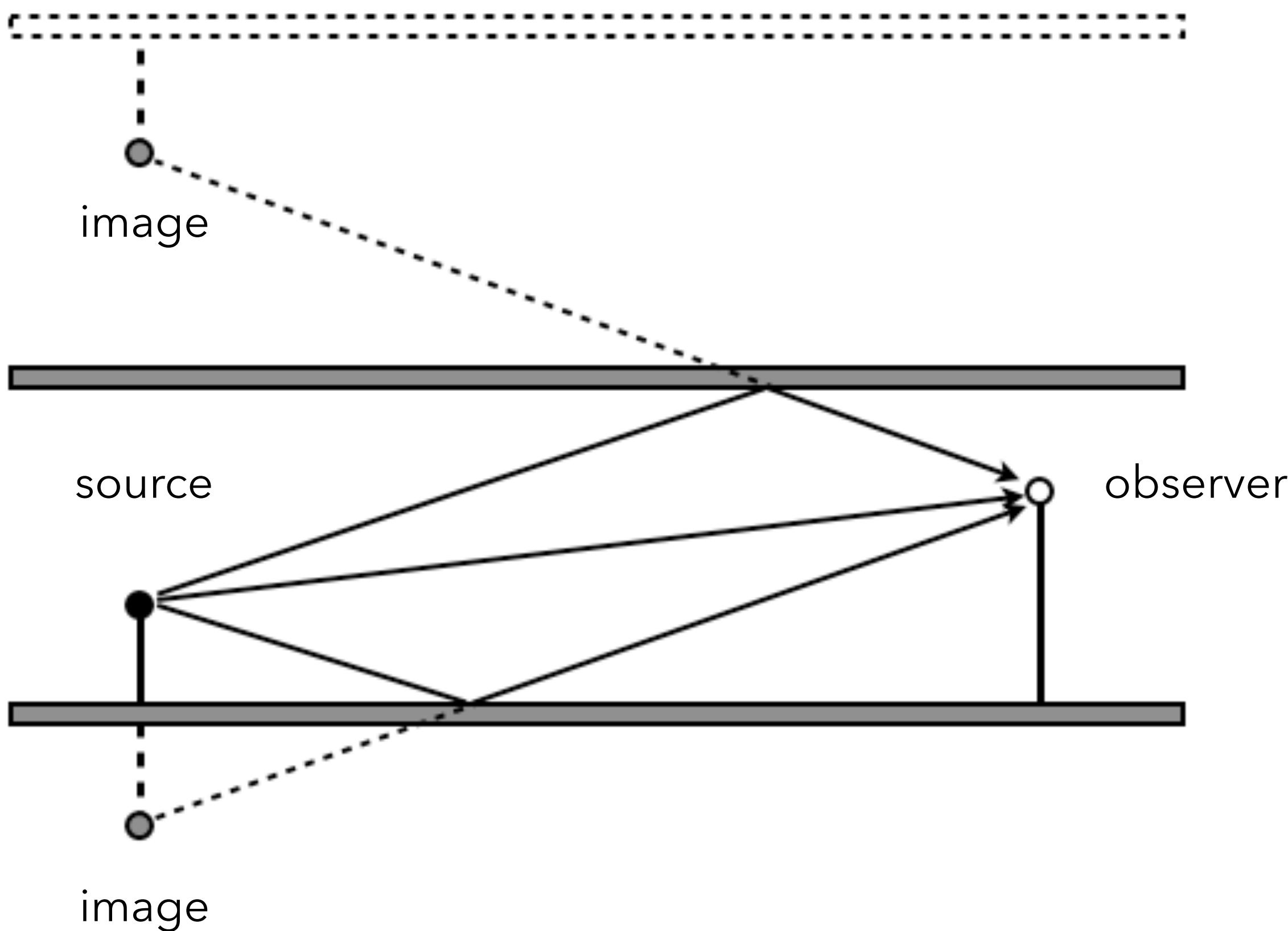


Reflected sound - example

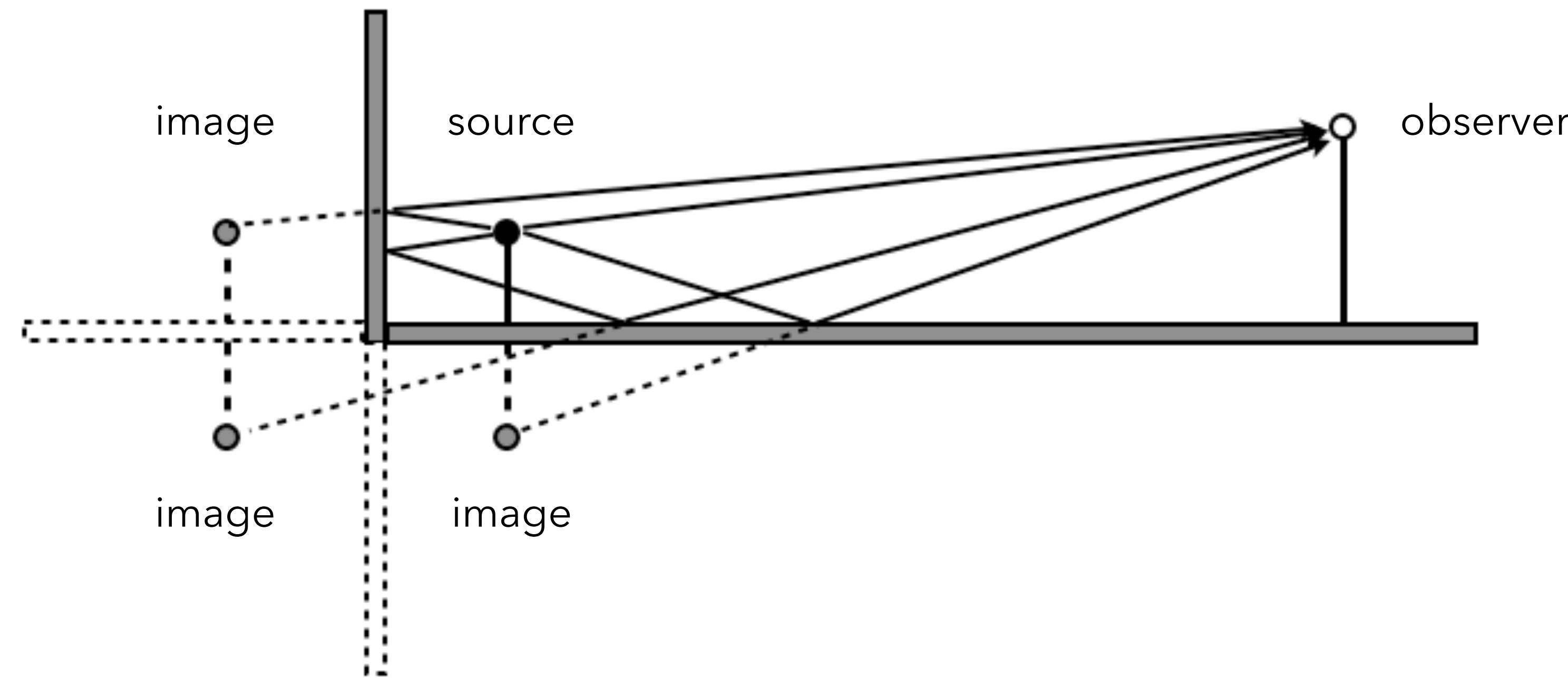
- A listener stands 4 m in front of an omni-directional loudspeaker that is 1.5 m from a reflecting wall.
- Calculate:
 - the time of arrival for both the direct and reflected sound
 - the decrease in SIL for the reflected sound due to the larger distance traveled
 - the decrease in SIL for the reflected sound assuming an absorption coefficient $a = 0.2$ for the wall
- Answer:
 - Reflected sound travels 5 meter: $t_{\text{direct}} = \frac{4 \text{ m}}{346 \text{ m/s}} = 12 \text{ msec}$, $t_{\text{reflected}} = \frac{5 \text{ m}}{346 \text{ m/s}} = 15 \text{ msec}$
 - $\Delta \text{SIL} = 10 \log \left[1/(r_{\text{reflected}}/r_{\text{direct}})^2 \right] \text{ dB} = 10 \log \left[(4/5)^2 \right] \text{ dB} = -2 \text{ dB}$
 - $\Delta \text{SIL} = 10 \log(1 - a) \text{ dB} = 10 \log(1 - 0.2) \text{ dB} = -1 \text{ dB}$



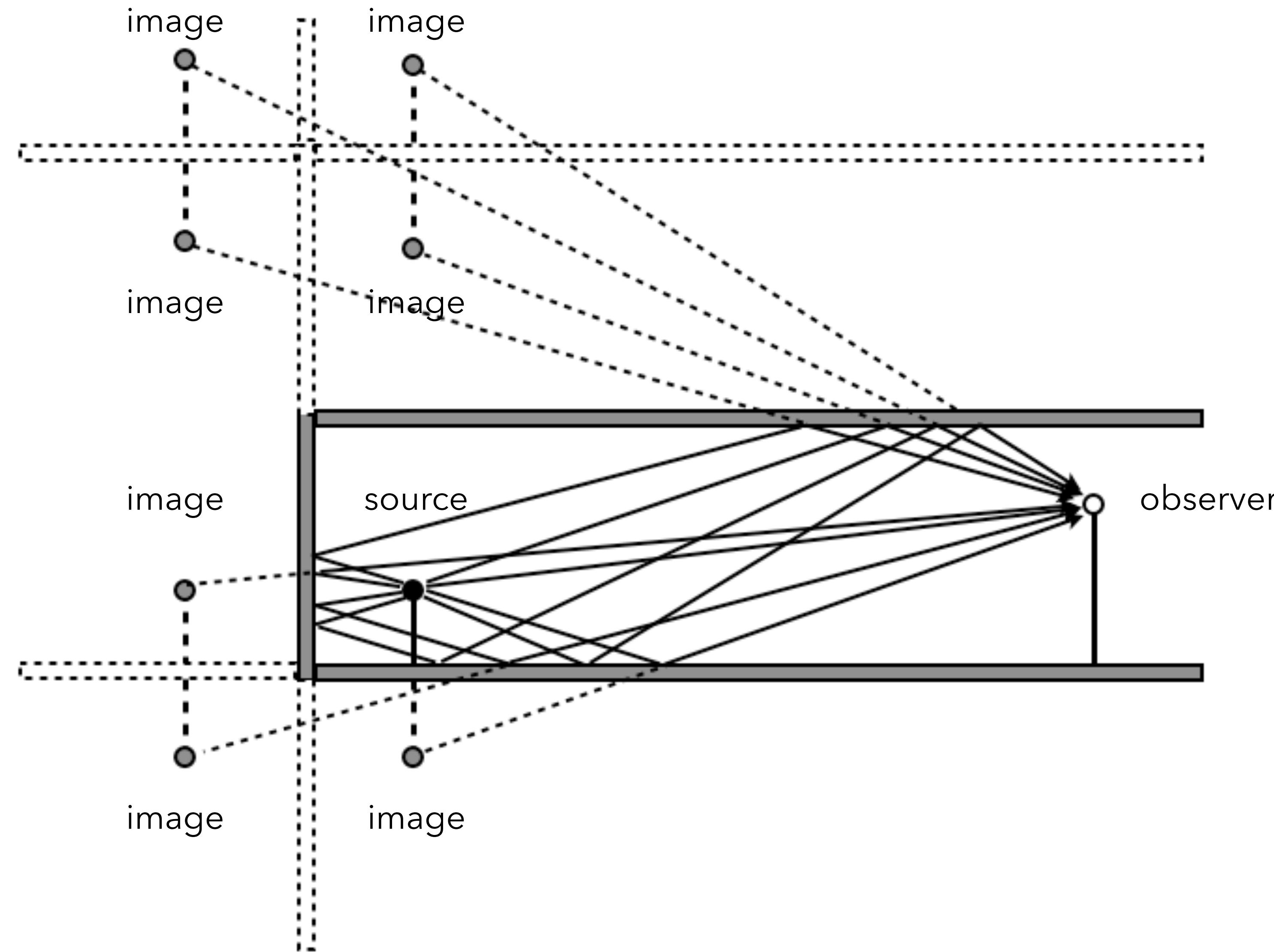
Multiple reflections – floor and ceiling



Multiple reflections – floor and back wall

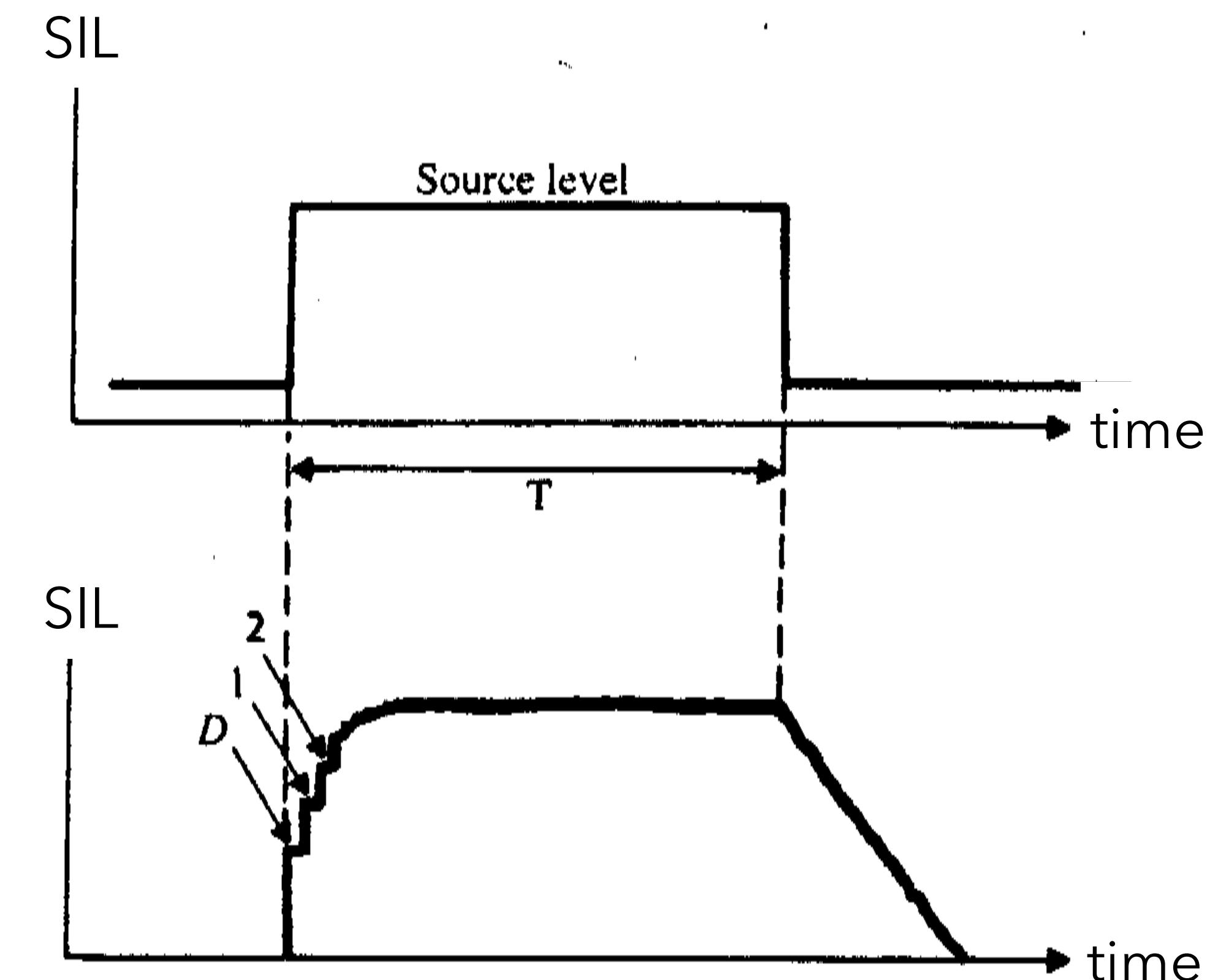
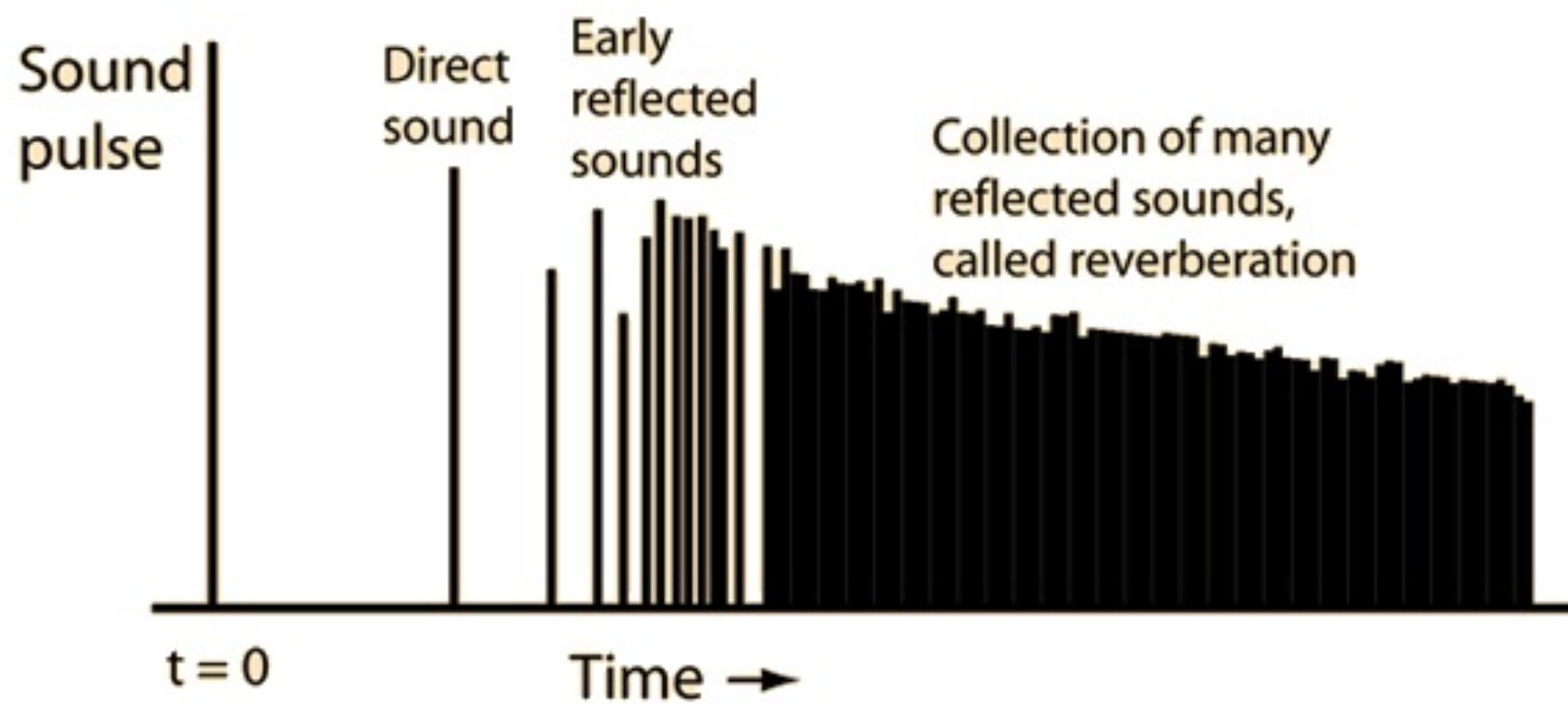


Multiple reflections – floor, back wall, and ceiling



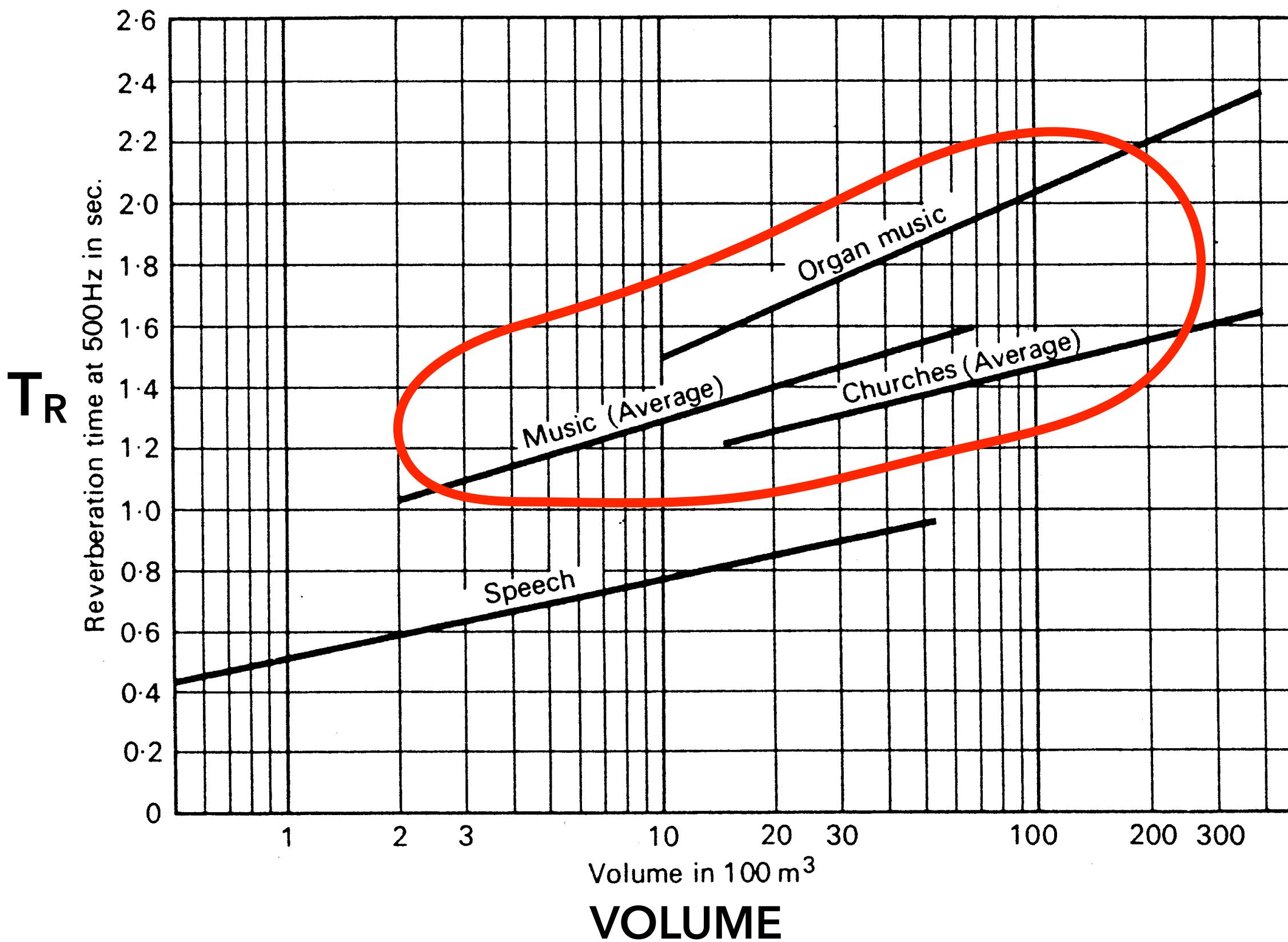
Reverberant sound

- sound formed from multiple reflections, coming from many different directions, and overlapping in time



Reverberation time

- time required for the reverberant SIL to decrease by 60 dB (1/106 in intensity)
- frequency dependent (low-frequency sounds typically have larger reverberation times)



Acoustical characteristics of various concert halls

	Year built	Volume (m^3)	Number of seats	Reverberation time (sec)		
				125 Hz	500 Hz	2000 Hz
Teatro alla Scala, Milan	1778	11,245	2289		1.2	
Royal Opera House	1858	12,240	2180		1.1	
Royal Albert Hall	1871	86,600	6080	3.4	2.6	2.2
Carnegie Hall, New York	1891	24,250	2760	1.8	1.8	1.6
Symphony Hall, Boston	1900	18,740	2630	2.2	1.8	1.7
Royal Festival Hall	1951	22,000	3000	1.4	1.5	1.4
Philharmonic Hall, Berlin	1963	36,030	2200		2.0	
St. David's Hall, Cardiff	1983	22,000	2200	1.8	1.9	1.8

Calculating reverberation time

$$T_R = 0.05 \frac{V}{A_{\text{eff}}} \text{ s}$$

V : volume in (ft^3)

$$A_{\text{eff}} = A_1 a_1 + A_2 a_2 + \dots + B_1 + B_2 + \dots$$

- A_{eff} : total absorption in sabin (1 ft^2 of perfectly absorbing surface)

- A_1, A_2, \dots : surface area of walls, etc. (in ft^2)

- a_1, a_2, \dots : absorption coeffs (dimensionless, freq-dependent)

- B_1, B_2, \dots : absorption for seats, people, etc. (in sabin)

absorption coefficients (dimensionless)

Material	Frequency (Hz)					
	125	250	500	1000	2000	4000
Concrete (painted)	0.10	0.05	0.06	0.07	0.09	0.08
Plywood panel	0.28	0.22	0.17	0.09	0.10	0.11
Plaster on lath	0.14	0.10	0.06	0.05	0.04	0.03
Gypsum board, 1/2 in.	0.29	0.10	0.05	0.04	0.07	0.09
Glass window	0.35	0.25	0.18	0.12	0.07	0.04
Curtains	0.14	0.35	0.55	0.72	0.70	0.65
Carpet (on concrete)	0.02	0.06	0.14	0.37	0.60	0.65
Carpet (on pad)	0.08	0.24	0.57	0.69	0.71	0.73
Acoustical tile, suspended	0.76	0.93	0.83	0.99	0.99	0.94

absorption (in m^2) [multiply by 10.8 to convert to sabin]

Material	Frequency (Hz)					
	125	250	500	1000	2000	4000
Wood or metal seat, unoccupied	0.014	0.018	0.020	0.036	0.035	0.028
Upholstered seat, unoccupied	0.13	0.26	0.39	0.46	0.43	0.41
Adult	0.23	0.32	0.39	0.43	0.46	—
Adult in an upholstered seat	0.27	0.40	0.56	0.65	0.64	0.56

Example

Exercise: Calculate the reverberation time at 500 Hz for a room with dimensions $20 \text{ m} \times 15 \text{ m} \times 8\text{m}$ (high). The walls are painted concrete, the ceiling is plaster, and the floor is carpet on pad. Also, assume that there are 200 upholstered seats, and that they are half-filled with people.

$$L = 20 \text{ m} \times 3.28 \text{ ft/m} = 65.6 \text{ ft}$$

$$W = 15 \text{ m} \times 3.28 \text{ ft/m} = 49.2 \text{ ft}$$

$$H = 8 \text{ m} \times 3.28 \text{ ft/m} = 26.24 \text{ ft}$$

$$V = L \times W \times H = 2400 \text{ m}^3 = 8.47 \times 10^4 \text{ ft}^3$$

↓ ↓ ↓ ↓ ↓
painted concrete plaster carpet on pad empty upholstered occupied
$$A_{\text{eff}} = 0.06 [2(L \times H) + 2(W \times H)] + 0.06(L \times W) + 0.57(L \times W) + 10.8(100 \times 0.39 + 100 \times 0.56)$$
$$= 3.42 \times 10^3 \text{ sabin}$$

$$T_R = 0.05 \frac{V}{A_{\text{eff}}} \text{ s} = 1.2 \text{ s} \quad \rightarrow \text{ideal for music (for } V=2400 \text{ m}^3\text{)}$$

Acoustical design

Criteria for good design

- Loudness
- Uniformity (no “live” or “dead” spots)
- Reverberance or liveness (feeling of being “bathed” in sound)
- Clarity (opposite of reverberance)

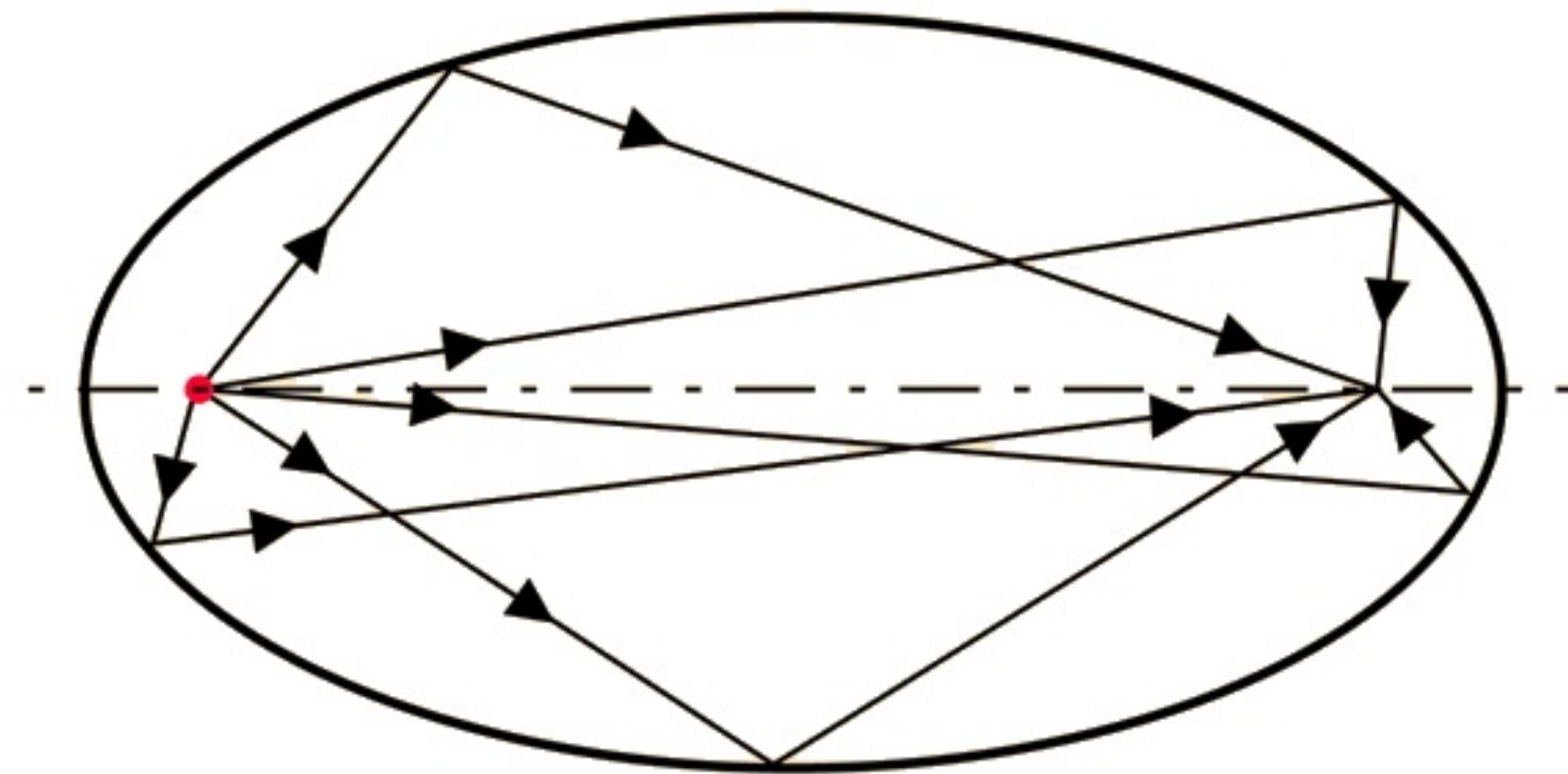
Problems to avoid

- Background noise (external noise due to heating, A/C, ...)
- Shadow areas (produced by balconies, columns, ...)
- Focusing of sound (“whispering room” effect)
- Echoes
- Room resonances (“shower stall” effect)

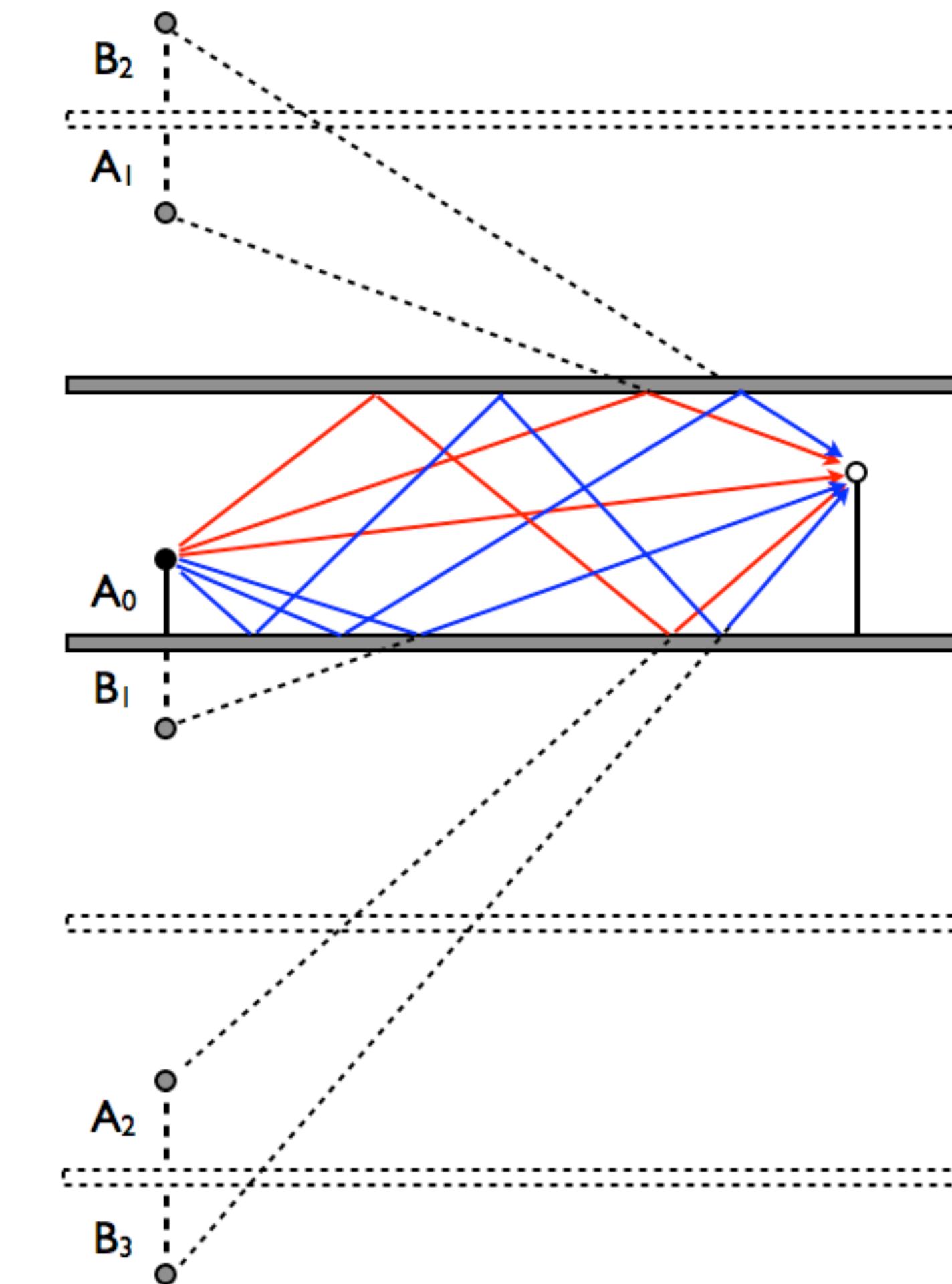
$$f_{lmn} = \frac{\nu}{2} \sqrt{\left(\frac{l}{L}\right)^2 + \left(\frac{m}{W}\right)^2 + \left(\frac{n}{H}\right)^2}$$

$$l, m, n = 0, 1, 2, \dots$$

Problems to avoid



Whispering room effect



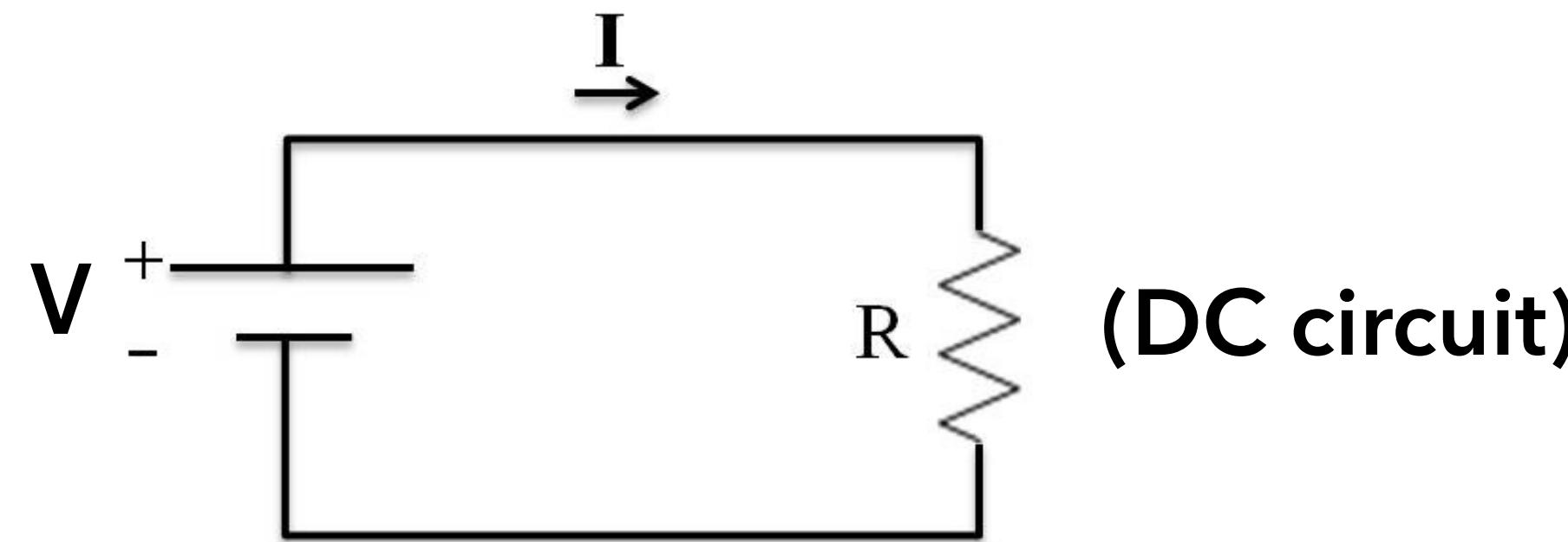
Flutter echoes

13. Electrical reproduction of sound

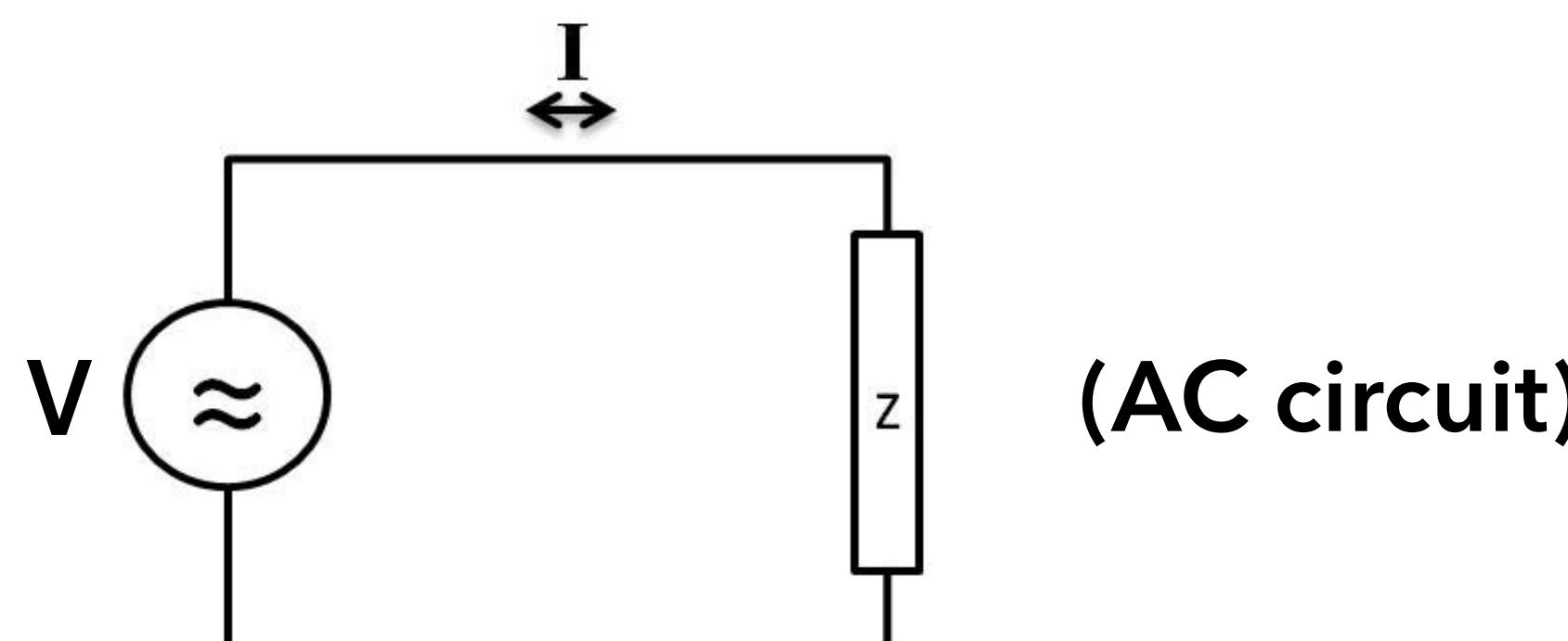
Electrical reproduction of sound – overview

- Goal: Understand how microphones and loudspeakers work
- Need basic understanding of:
 - electricity and magnetism
 - Faraday's law of induction

Basic electricity



e.g., a battery connected
to a flashlight bulb

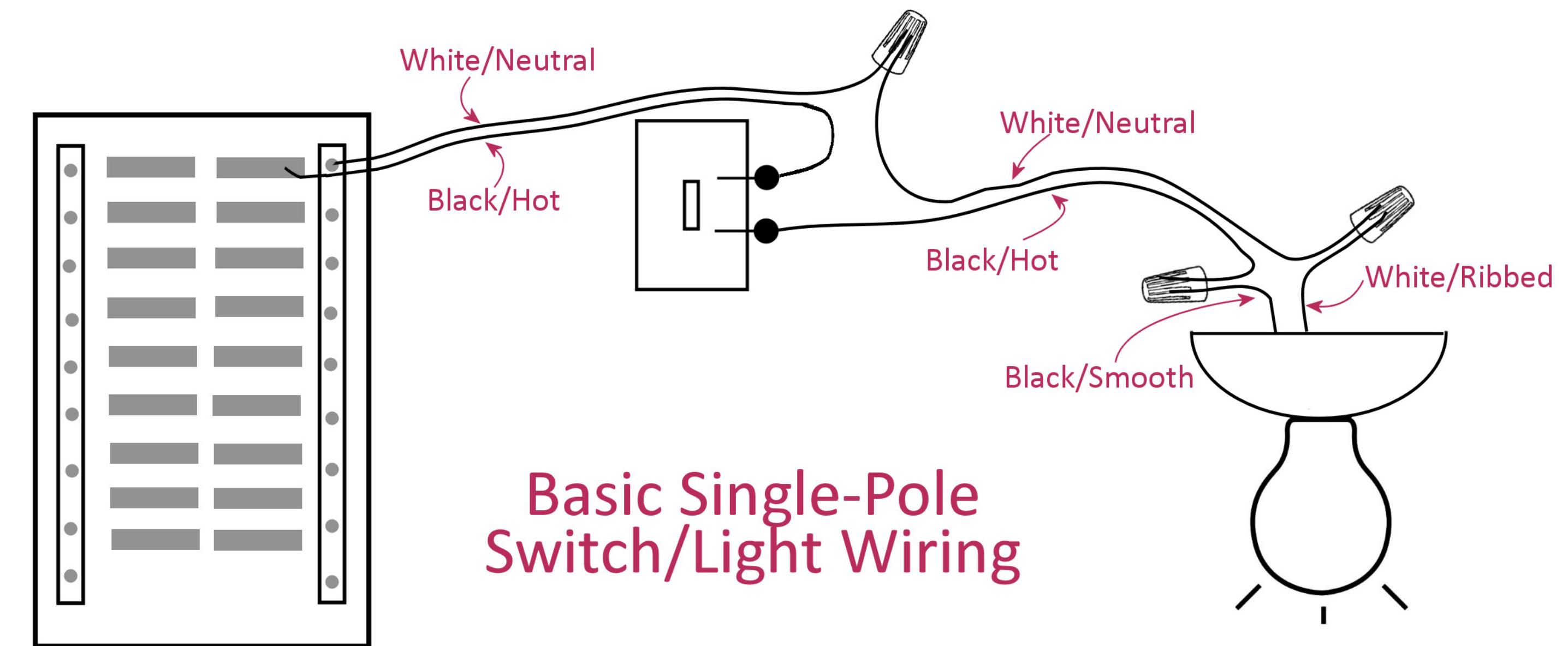
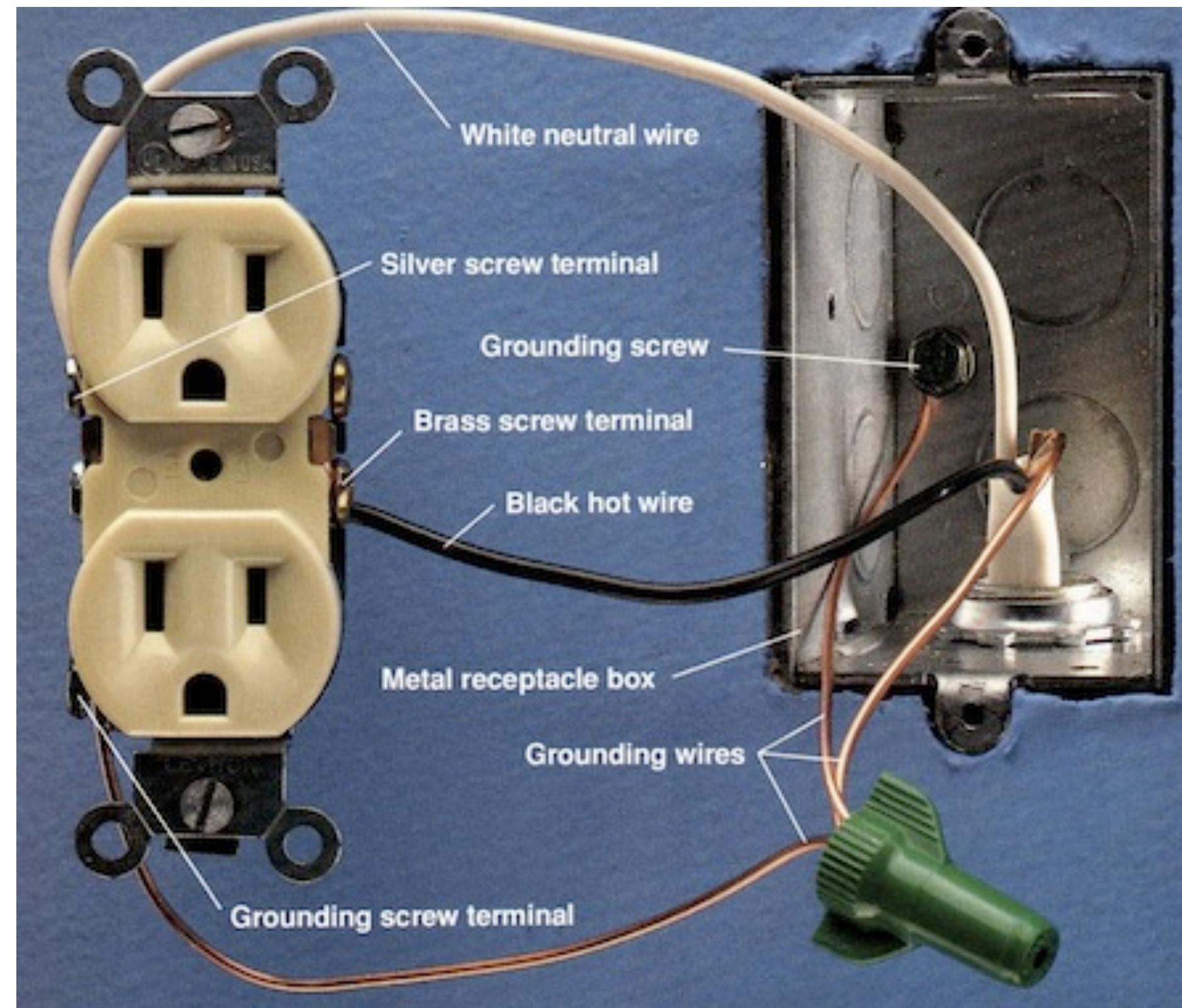


e.g., a household wall outlet
connected to a vacuum cleaner

- Voltage V (volts)
- Current I (amperes or amps)
- Resistance R or impedance Z (ohms, Ω)
- Direct current (DC) and alternating current (AC) circuits
- Ohm's law of electricity: $V = IR$
- Electrical power: $P = VI = I^2R$ (Watts)
- Relation to work or energy:

$$P = W/\Delta t \text{ (Watts)} \quad \text{or} \quad W = P \Delta t \text{ (Joules)}$$

Example – home wiring



<https://gardnerbenderfaq.wordpress.com/tag/outlet/>

<https://www.addicted2decorating.com/how-to-wire-single-pole-light-switch.html>

Example – kilowatt-hr and your electric bill

- A kilowatt-hr is a convenient unit of energy:

$$1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ hr} = 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J}$$

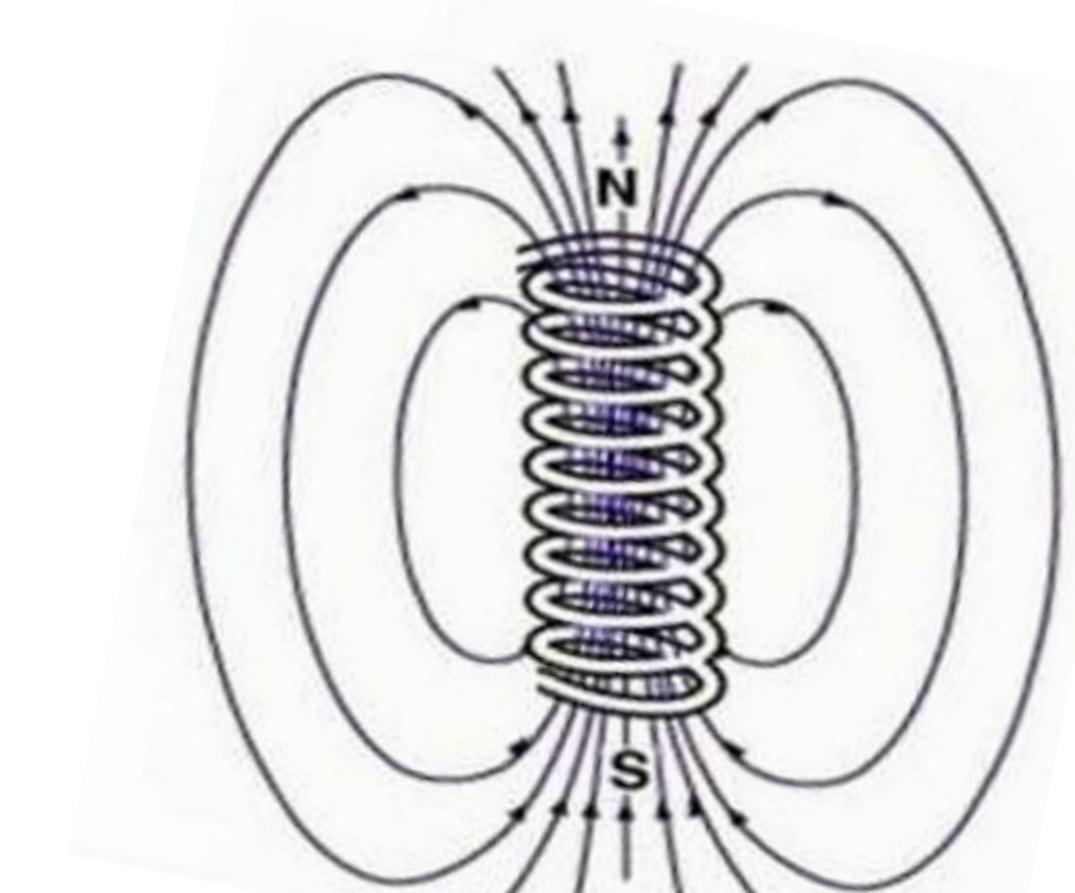
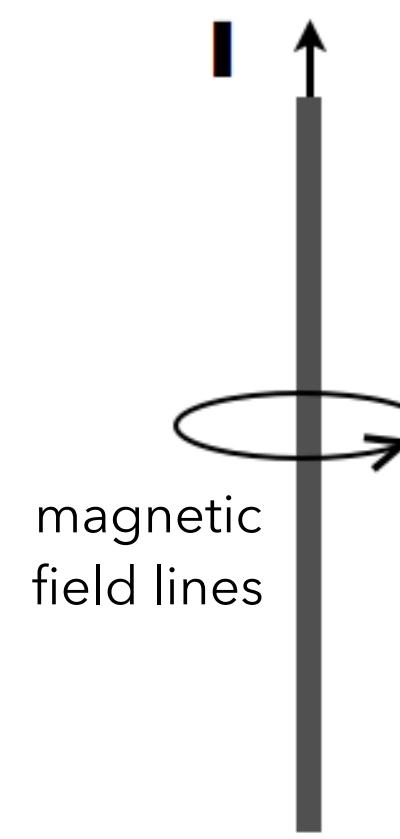
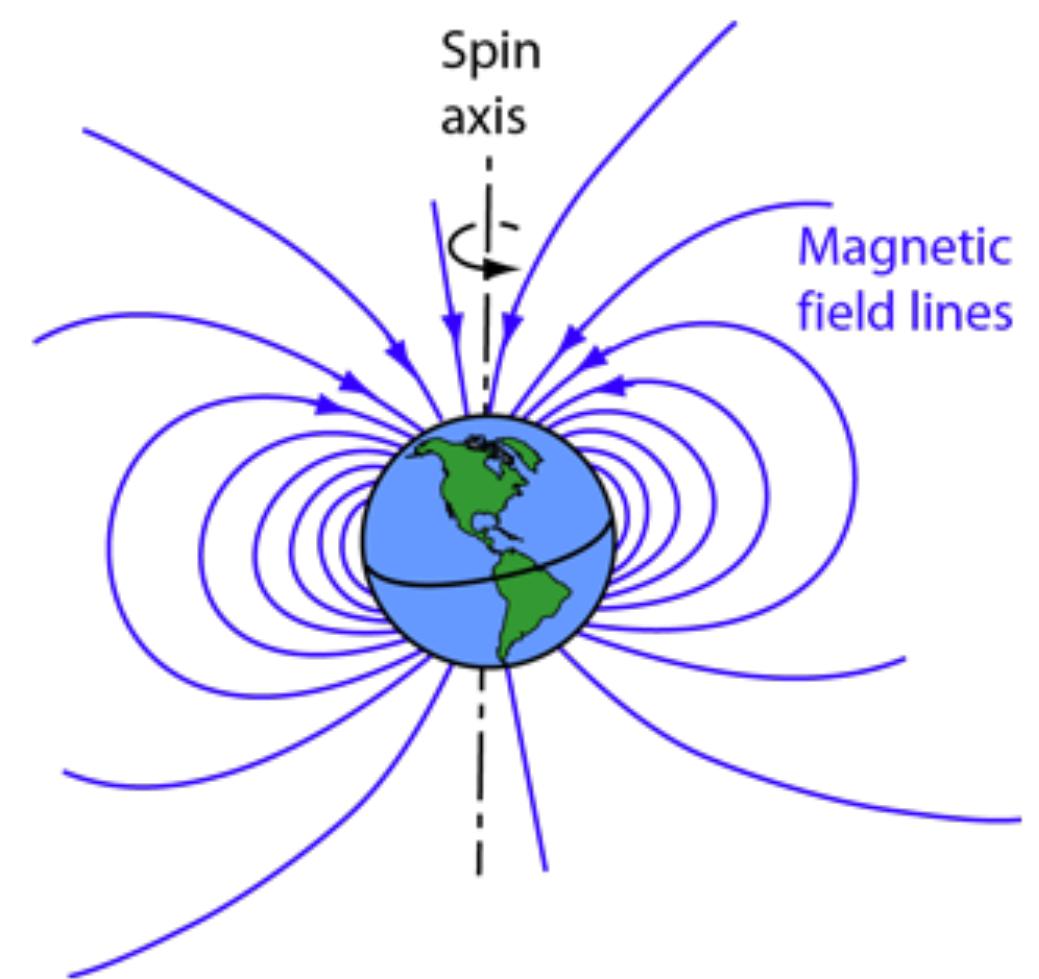
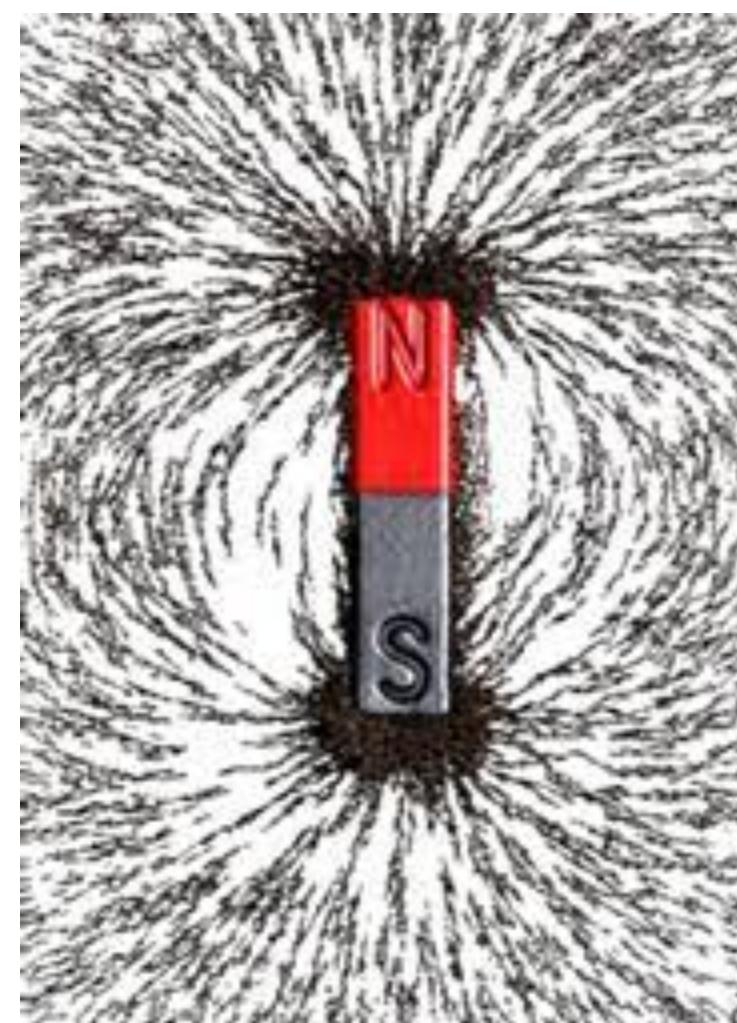
- Exercise: Suppose you paid \$100 for last month's electric bill at a cost of \$0.13/kWh.
 - (a) How much energy (in kWh) did you use?
 - (b) What was the average power consumption (in Watts) over the month (assume 30 days)?
- Answer:
 - (a) $W = \$100 \div \$0.13/\text{kWh} = 769 \text{ kWh}$

$$(b) P = \frac{W}{\Delta t} = \frac{769 \text{ kWh}}{30 \times 24 \text{ h}} = 1.1 \text{ kW} = 1,100 \text{ W}$$

(eleven 100-Watt lightbulbs on continuously)

Basic magnetism

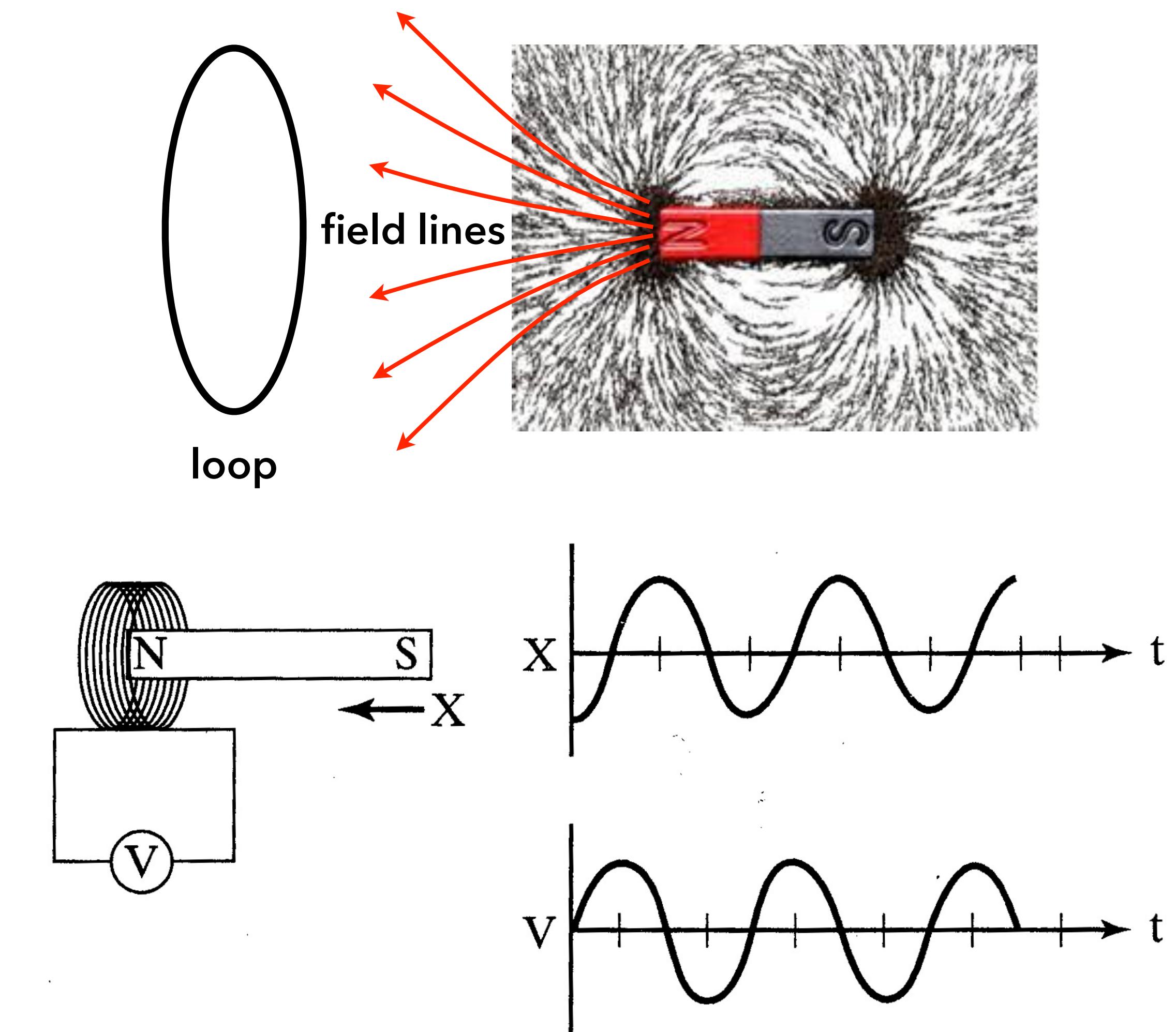
- A permanent magnet has N and S poles that attract pieces of iron
- Like poles repel; unlike poles attract (just like + and – electrical charges). But no isolated magnetic poles.
- A compass needle is a tiny magnet that is attracted to Earth's South magnetic pole.
- Oersted (1820): discovered that an electric current produces a magnetic field
- Can create an electromagnet by sending an electric current through a coil of wire



electromagnet

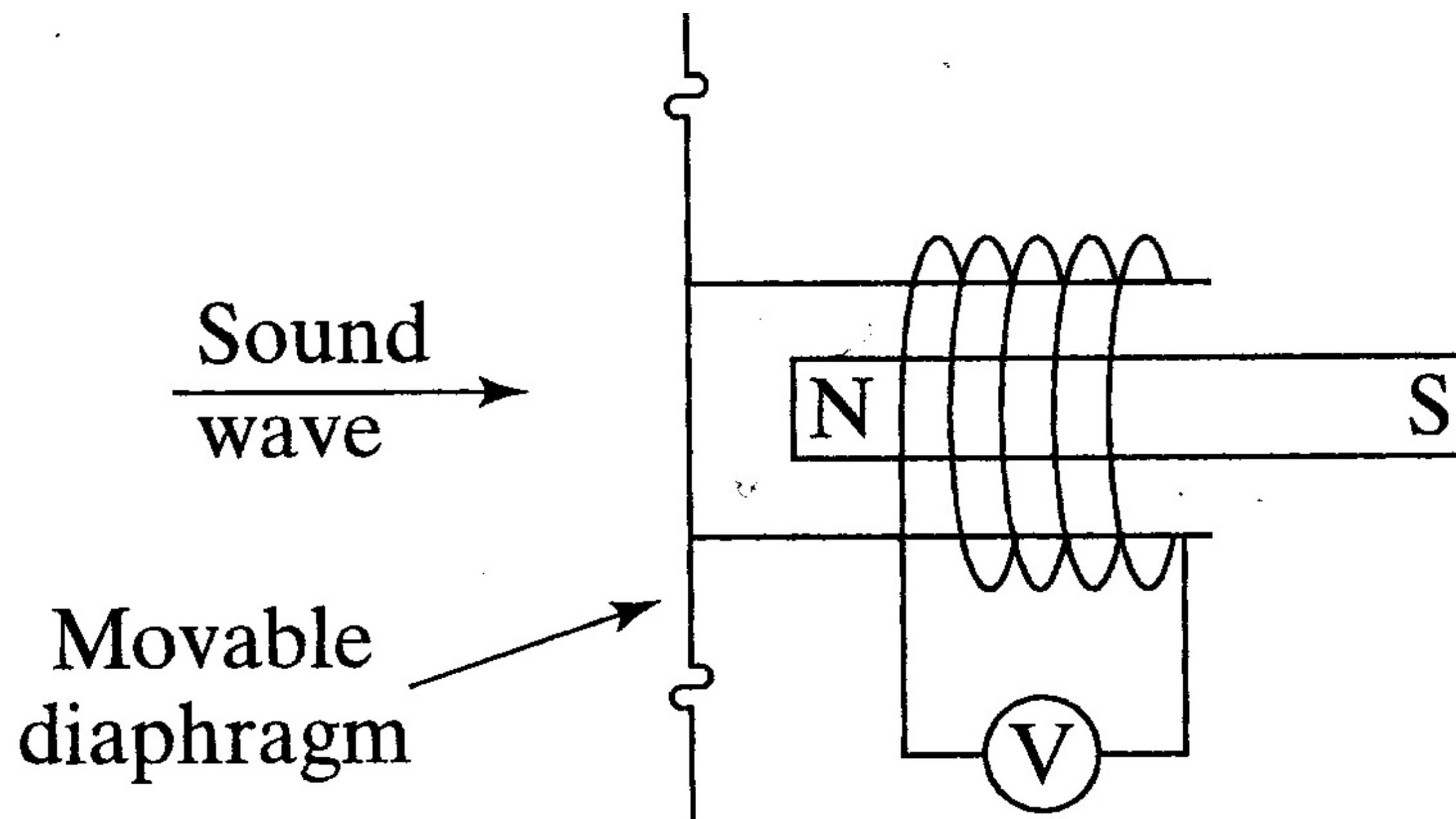
Faraday's law of induction (1831)

- A change in magnetic flux through a coil of wire induces a voltage in the coil:
$$V = -N \frac{\Delta\Phi}{\Delta t}$$
- Only relative motion is important
- Underlies the operation of electric generators and electric motors
- Electric generator: mechanical energy converted to electrical energy
- Electric motor: electrical energy converted to mechanical energy
- 4 YouTube videos (linked from "Slide Presentations")



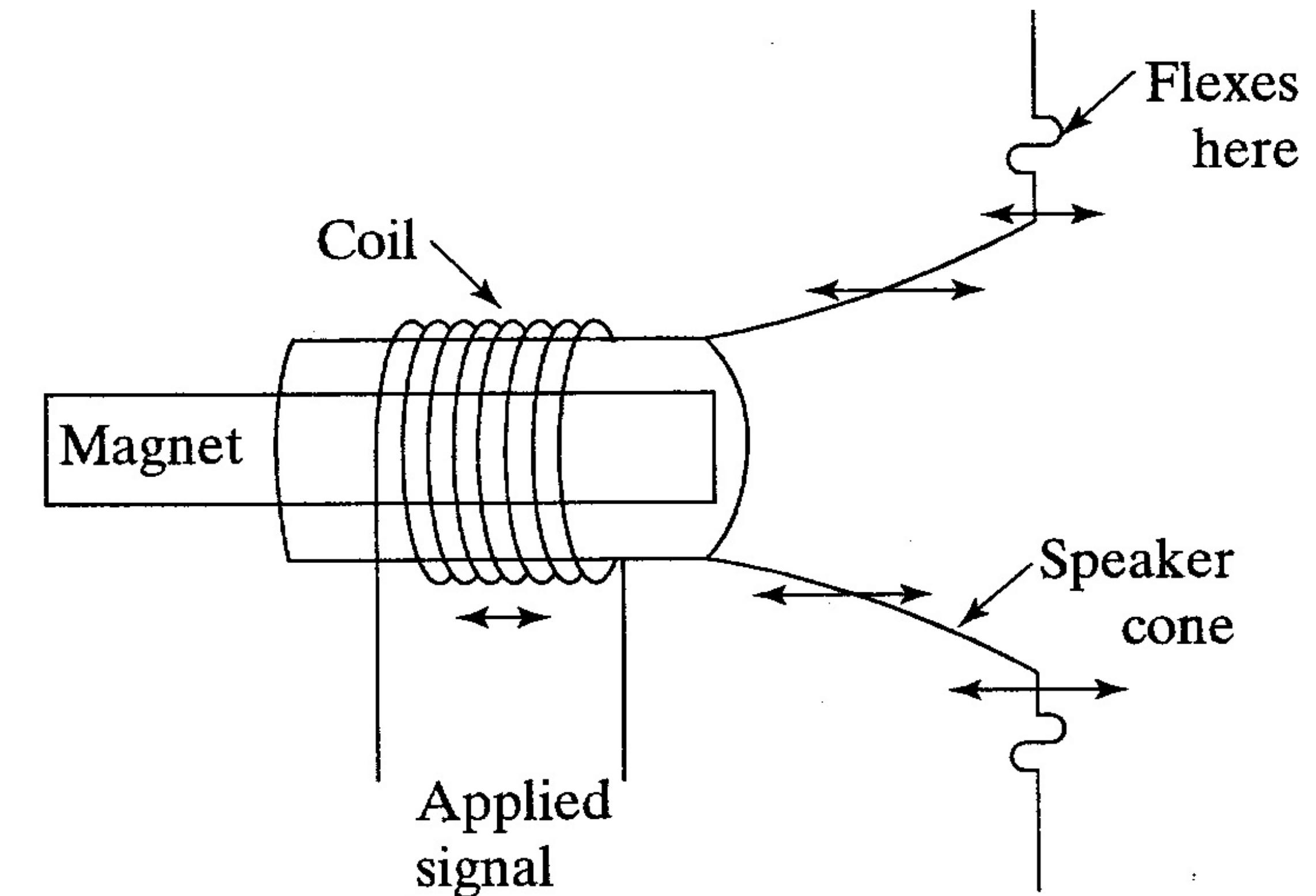
Application – microphones and loudspeakers

Dynamic microphone



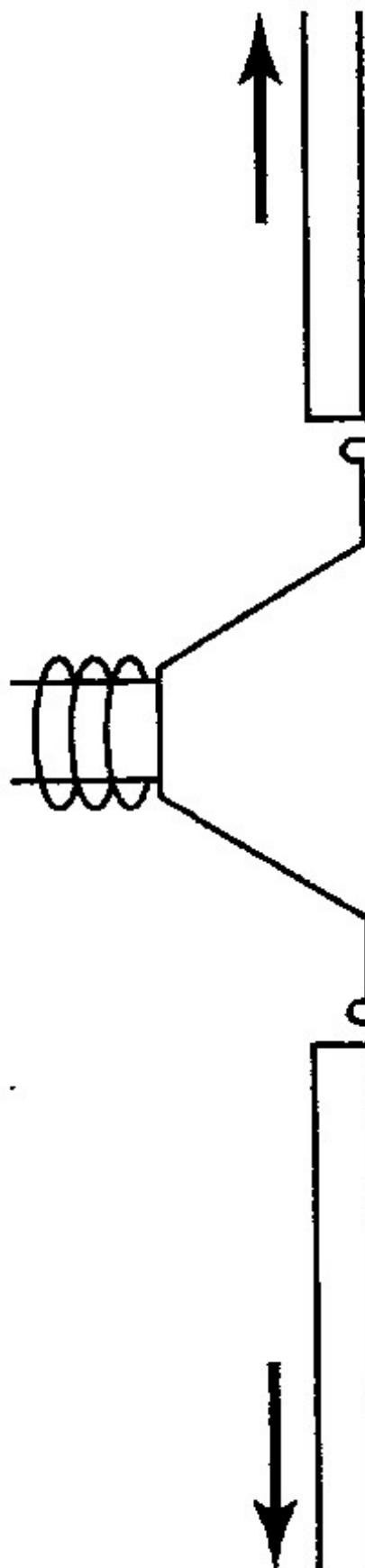
sound wave → electrical signal

Loudspeaker

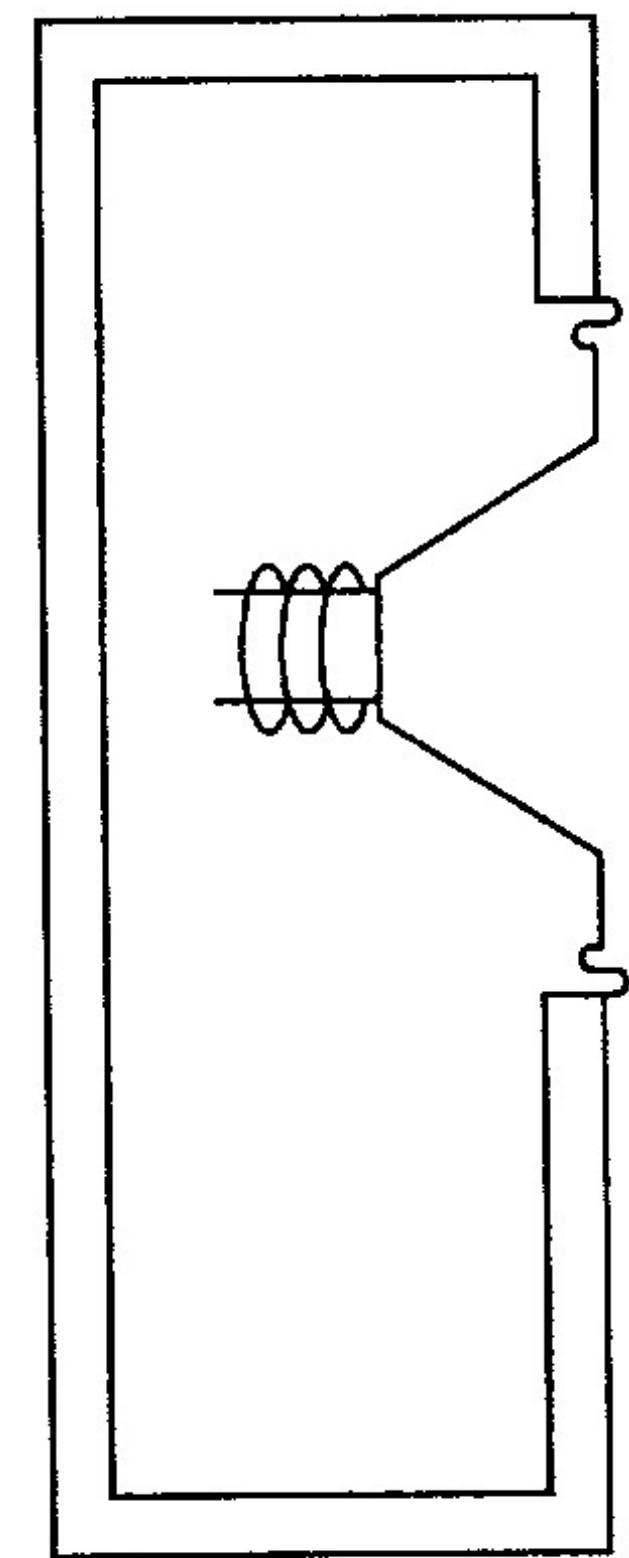


electrical signal → sound wave

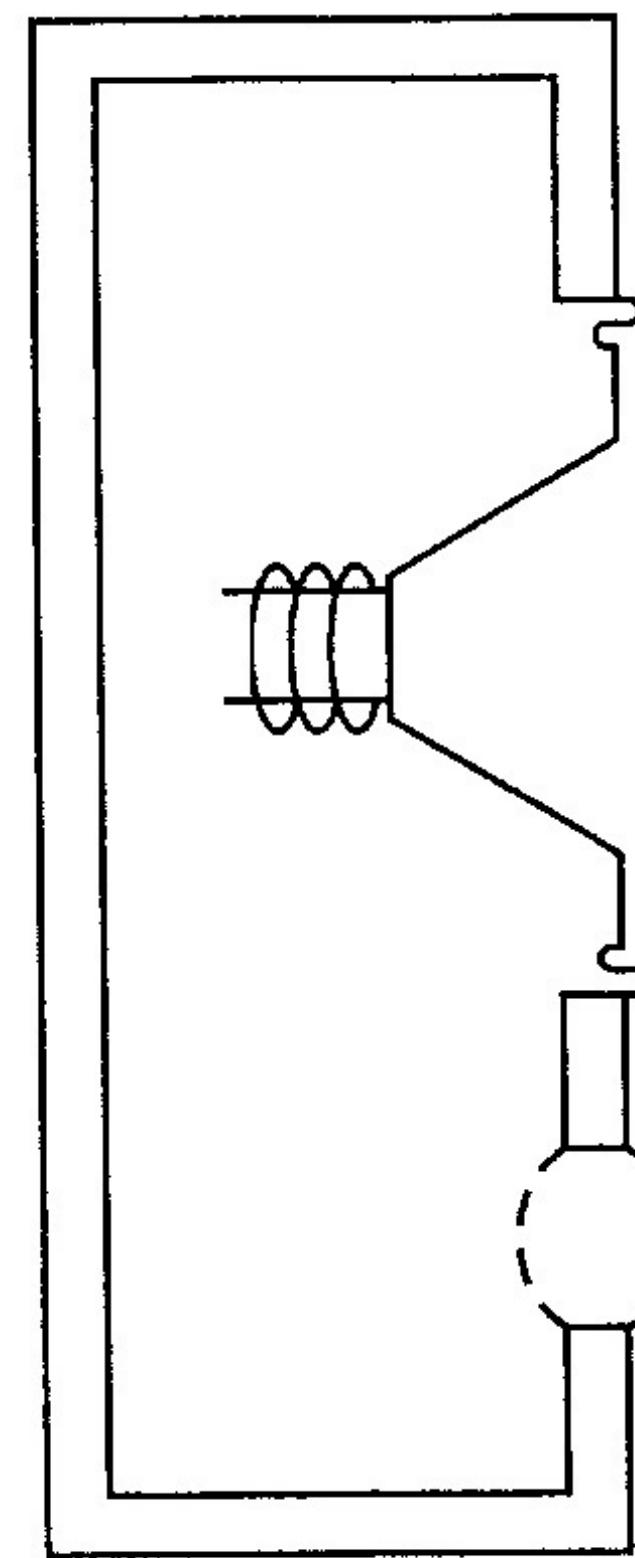
Loudspeakers



Infinite baffle

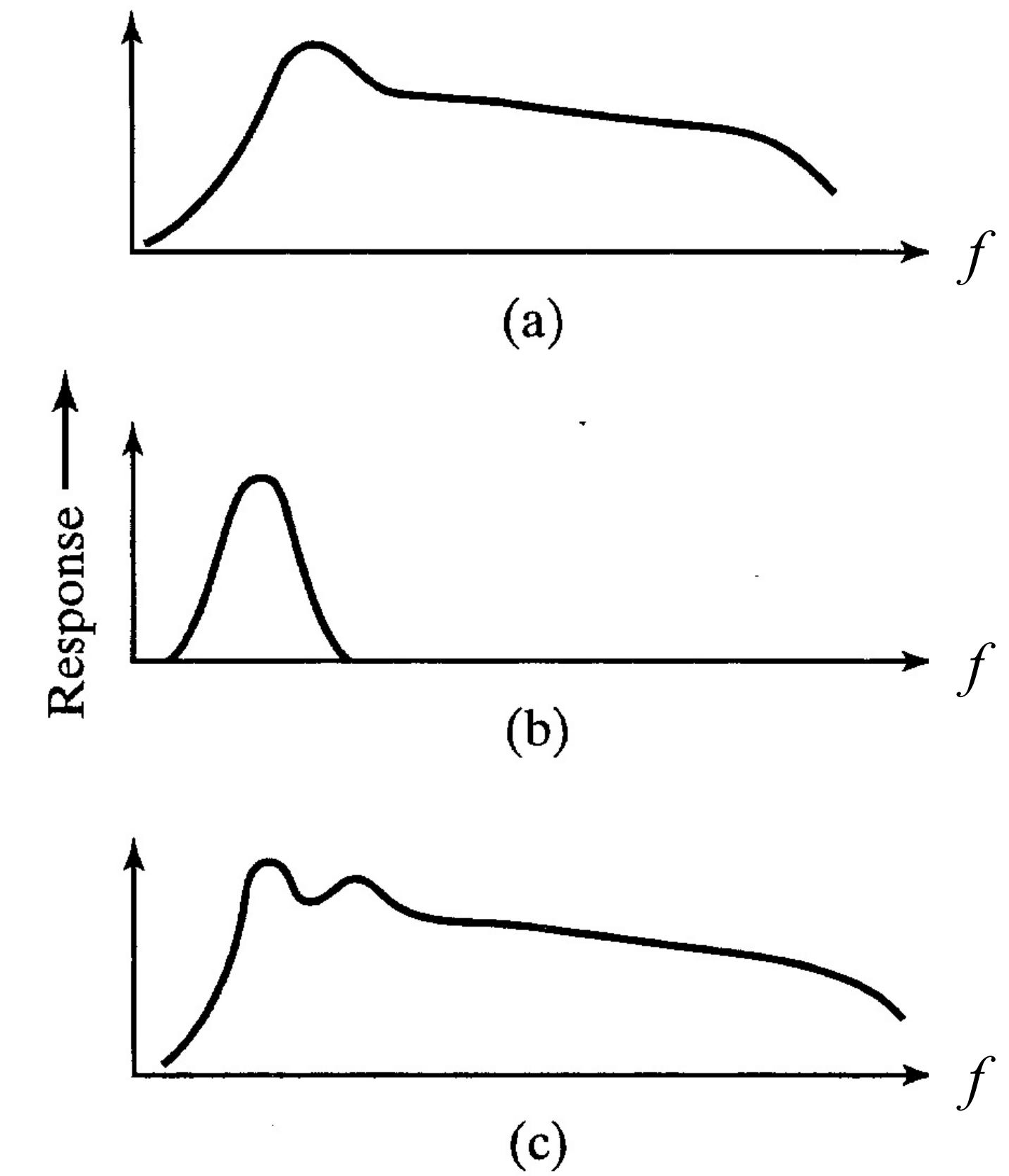


Acoustic suspension



Tuned port (bass reflex)

Frequency response curves
(acoustic suspension, tuned port)



14. Elementary music theory

Music theory – the need to standardize musical notes

- A tuning system is an assignment of precise frequencies to all musical notes in an octave (reference note is A4 = 440 Hz; decided upon in 1939)
- Three standard tuning systems:
 - Equal temperament
 - Pythagorean temperament
 - Just temperament
- Each tuning system has its own advantages and disadvantages
- What tuning systems do real musicians use? (Diego??)

Musical scales – dividing up the octave into pieces

- Chromatic scale: 12 pieces (semitones)

C - C# - D - Eb - E - F - F# - G - Ab - A - Bb - B - C' (white and black keys on a piano)

- Diatonic scale: 7 pieces (semitones and whole tones)

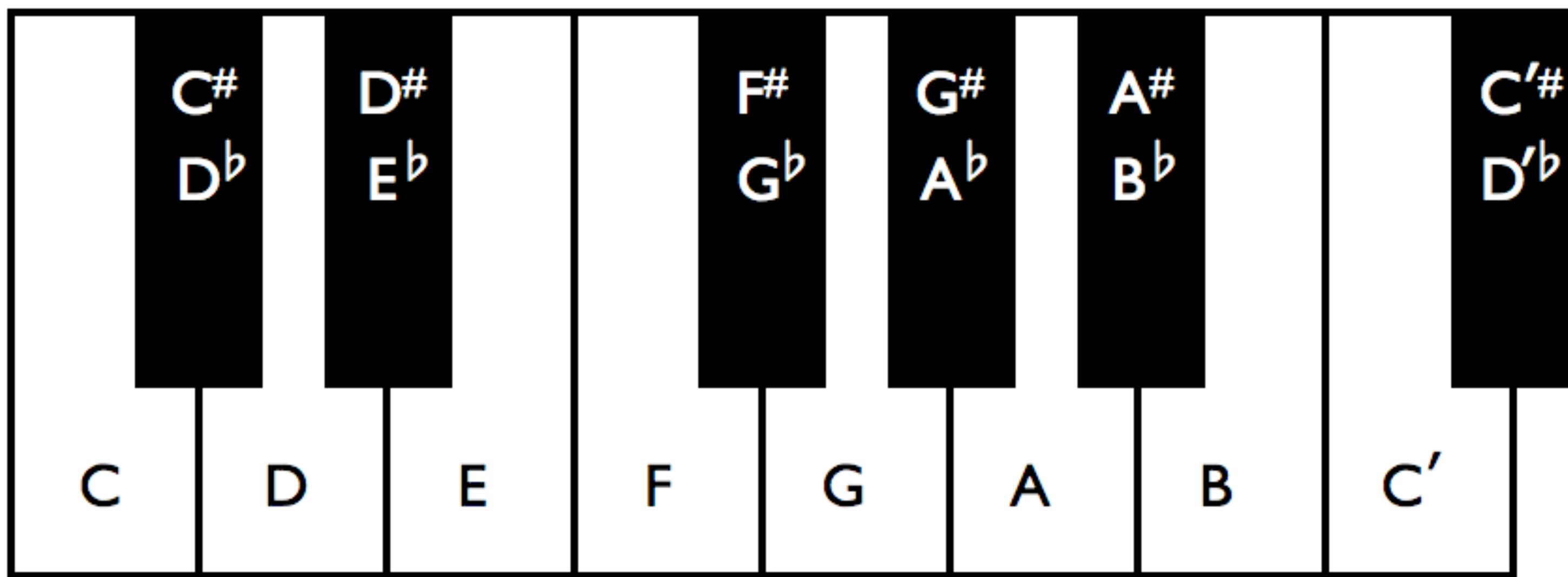
T-T-S-T-T-T-S (do-re-mi-fa-sol-la-ti-do; white keys on a piano)

- Pentatonic scale: 5 pieces (whole tones and 3 semitones intervals)

T-T-3-T-3 (F# - G# - A# - C# - D# - F#'; black keys on a piano)

Equal temperament

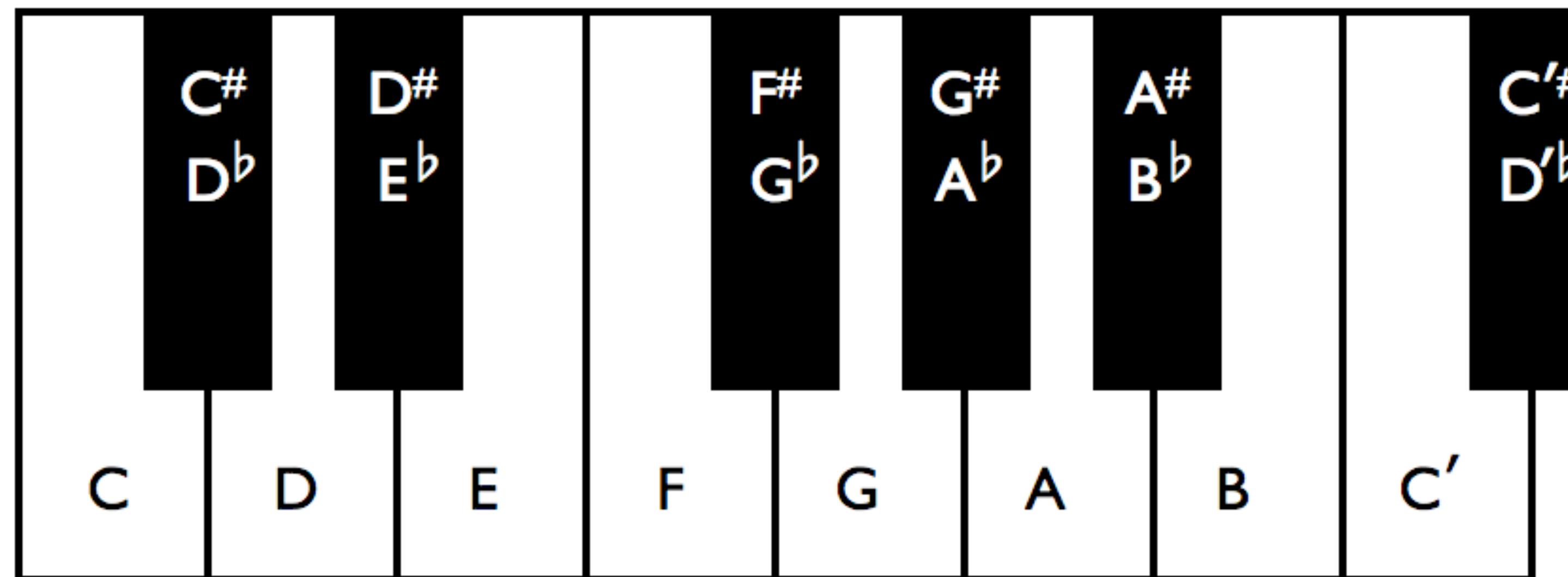
- All semitones intervals are equal: $2^{1/12} = 1.059$
- Cent (100 cents = semitone): $2^{1/1200} = 1.000578$ (JND: ~10 cents)
- All sharps and flats are equal to one another



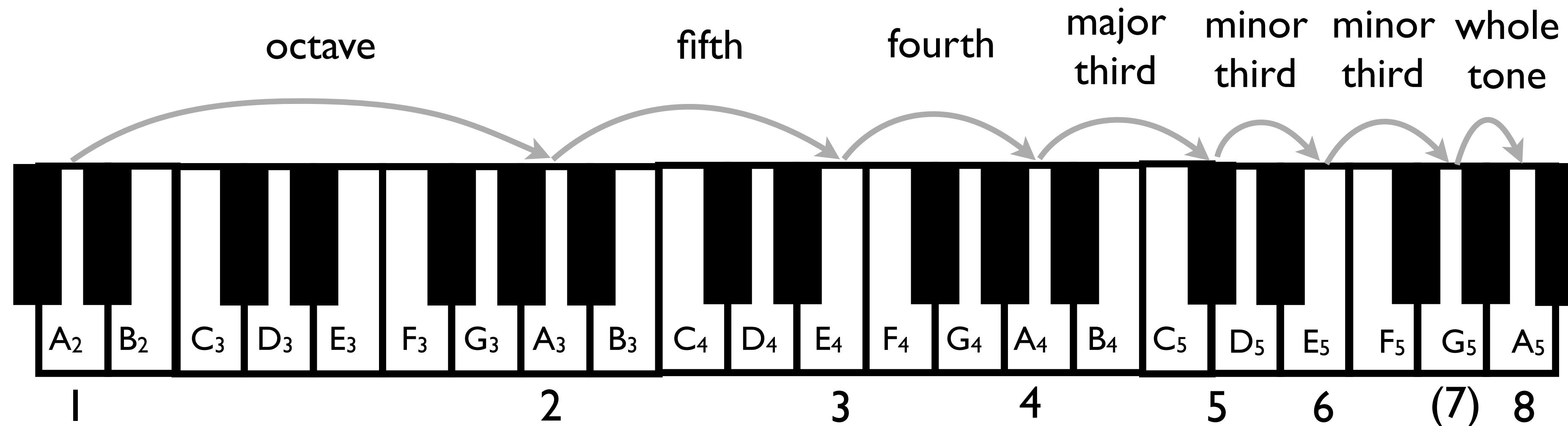
Note	ET freq ratio
C	$2^{0/12} = 1.000$
C [#] /D ^b	$2^{1/12} = 1.059$
D	$2^{2/12} = 1.122$
D [#] /E ^b	$2^{3/12} = 1.189$
E	$2^{4/12} = 1.260$
F	$2^{5/12} = 1.335$
F [#] /G ^b	$2^{6/12} = 1.414$
G	$2^{7/12} = 1.498$
G [#] /A ^b	$2^{8/12} = 1.587$
A	$2^{9/12} = 1.682$
A [#] /B ^b	$2^{10/12} = 1.782$
B	$2^{11/12} = 1.888$
C'	$2^{12/12} = 2.000$

Musical intervals

Interval	# semitones	Just freq ratio	ET freq ratio	Difference (cents)	Example
Octave	12	$2 : 1 = 2.000$	2.000	0	C-C'
Fifth	7	$3 : 2 = 1.500$	1.498	2	C-G
Fourth	5	$4 : 3 = 1.333$	1.335	-2	C-F, G-C'
Major third	4	$5 : 4 = 1.250$	1.260	-14	C-E
Minor third	3	$6 : 5 = 1.200$	1.189	16	C-E ^b , A-C'

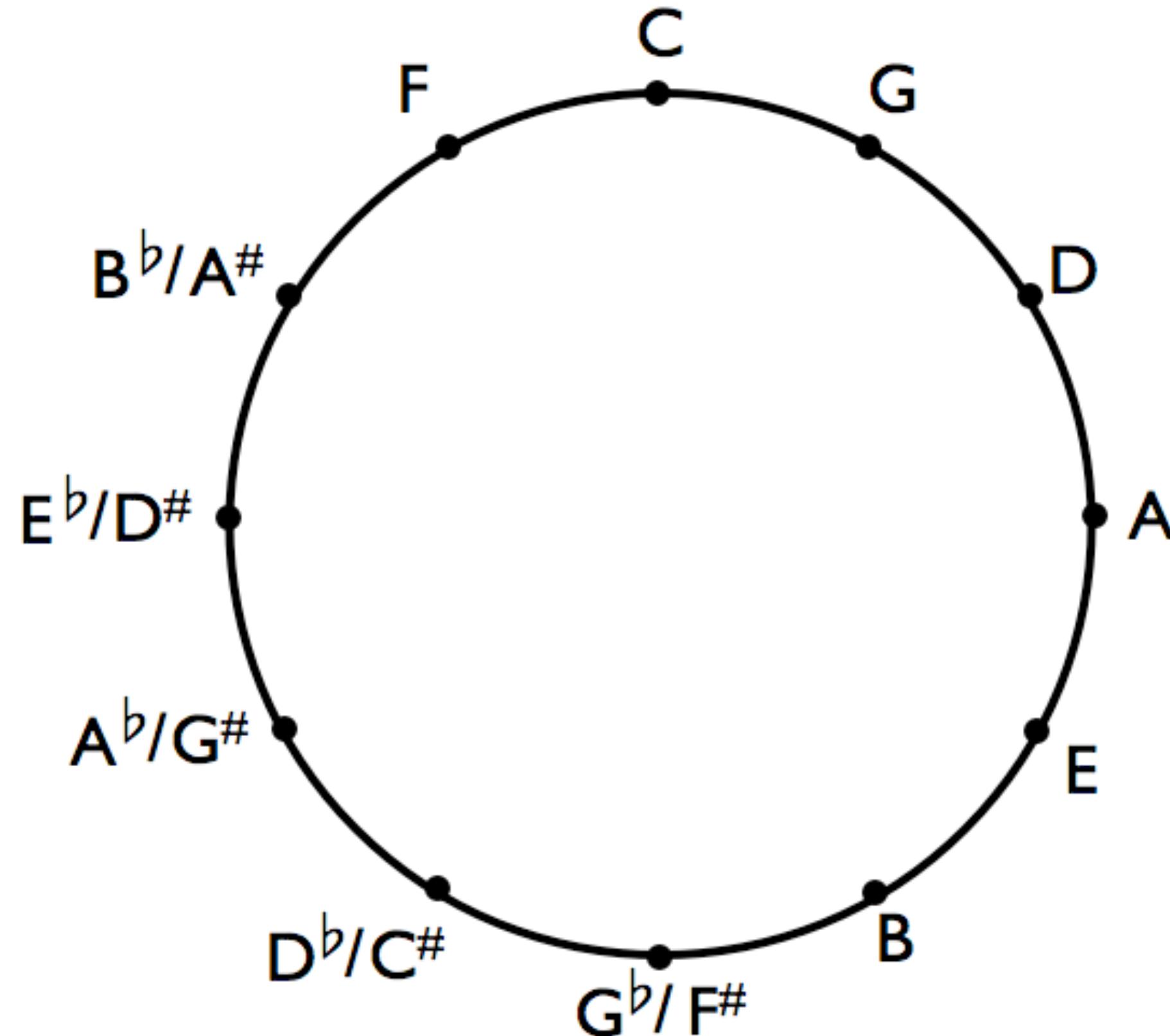


Harmonics

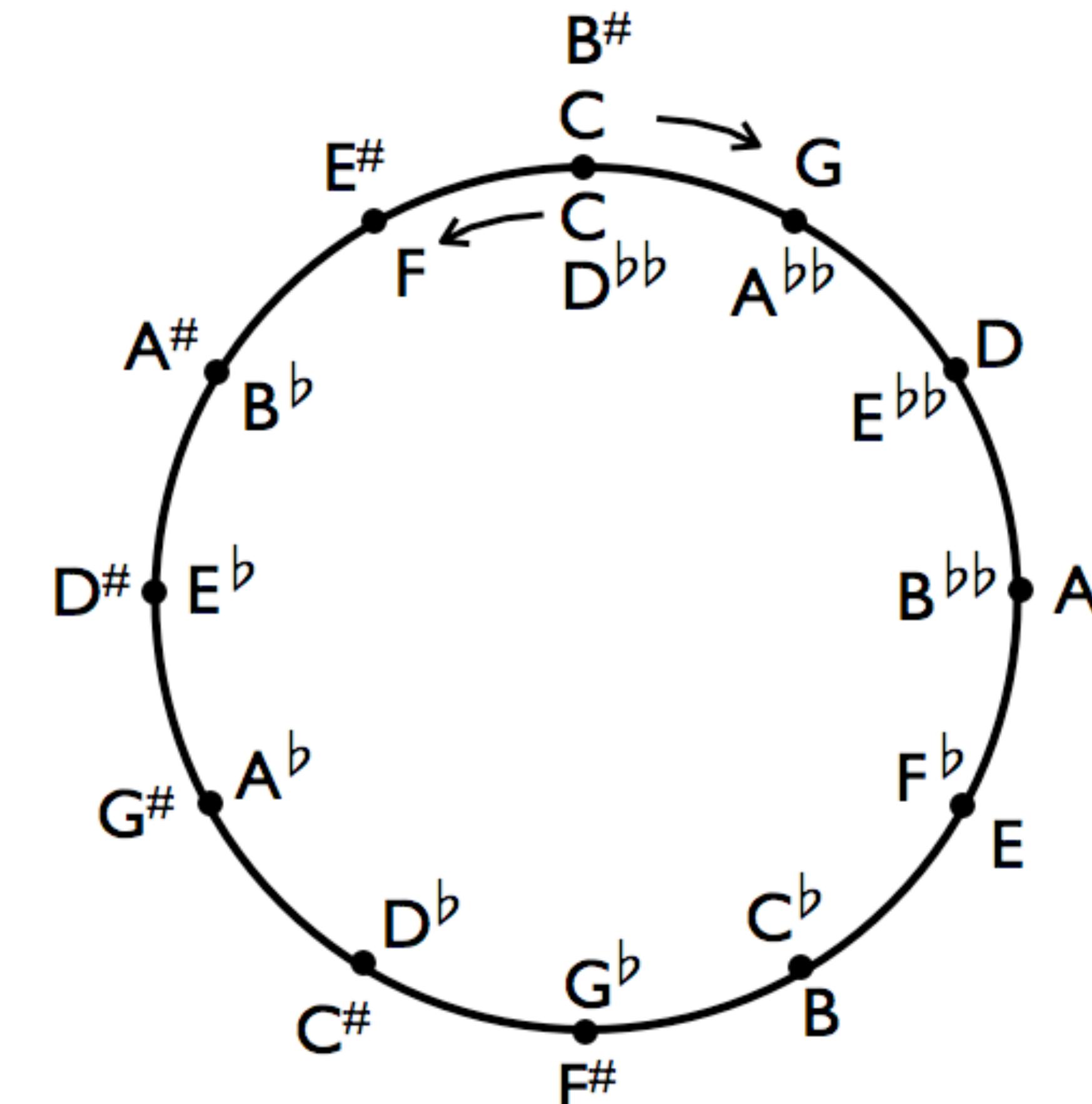


Harmonic	Exact freq (Hz)	Equal-tempered freq (Hz)	Difference (cents)	Piano note
1	110	110.00	0	A ₂
2	220	220.00	0	A ₃
3	330	329.63	2	E ₄
4	440	440.00	0	A ₄
5	550	554.37	-14	C ₅ [#]
6	660	659.26	2	E ₅
7	770	783.99	-31	G ₅
8	880	880.00	0	A ₅

Circle of fifths

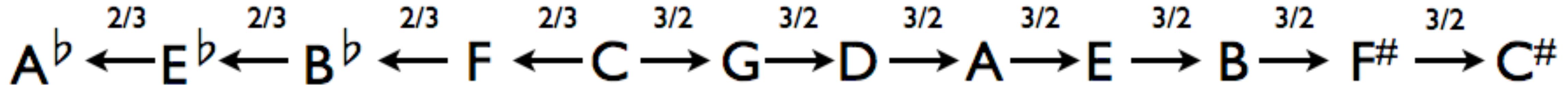


Equal Temperament



Other tuning systems

Pythagorean temperament



- Constructed from perfect fifth and octave intervals
- For example:

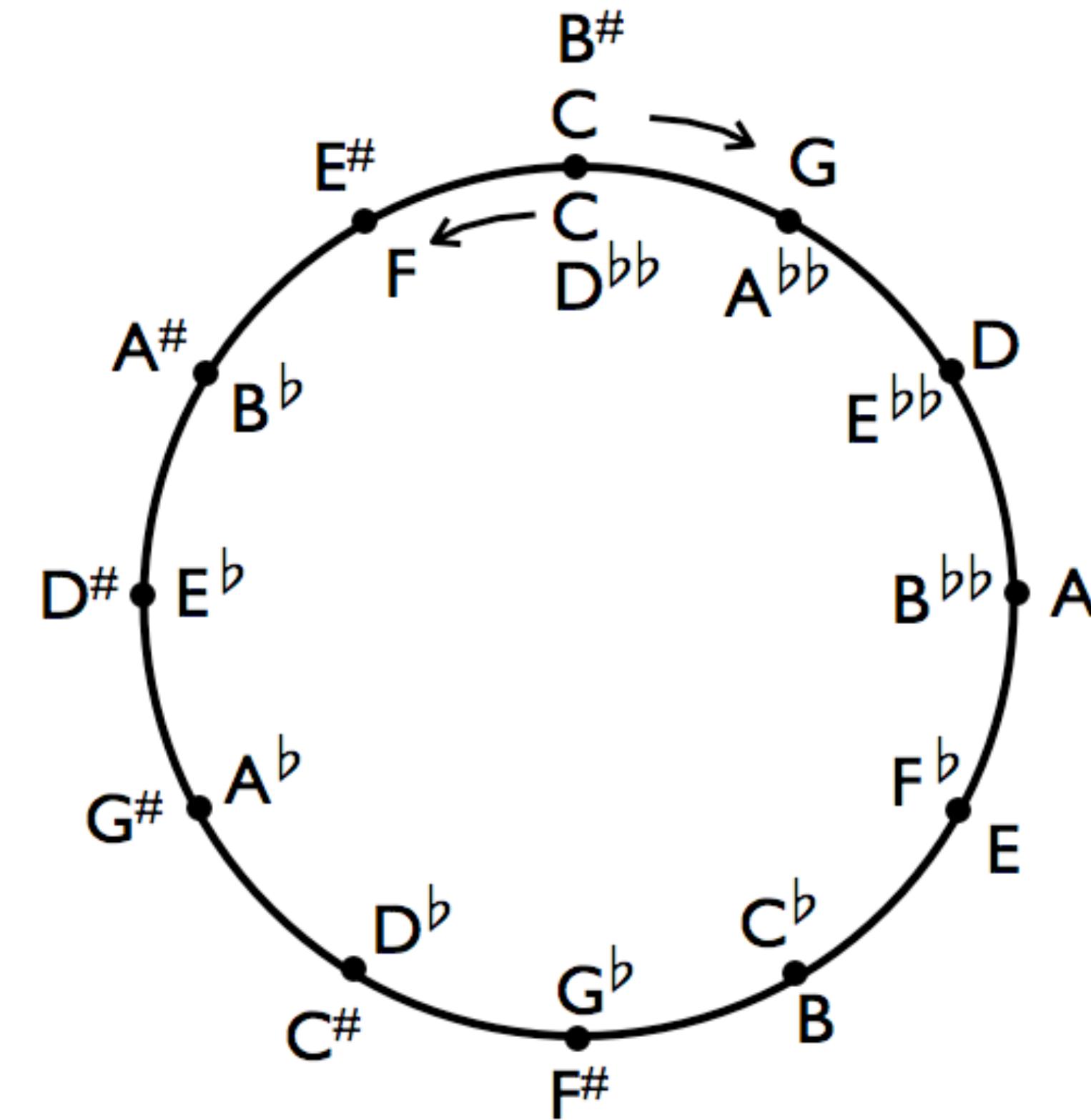
$G: 3/2$

$D: (3/2)2 \times (1/2) = 9/8$

$A: (3/2)3 \times (1/2) = 27/16$

$E: (3/2)4 \times (1/2)2 = 81/64$

$F: (2/3) \times 2 = 4/3$



Just temperament

- Constructed from perfect fifth, major third, and octave intervals
- For example:

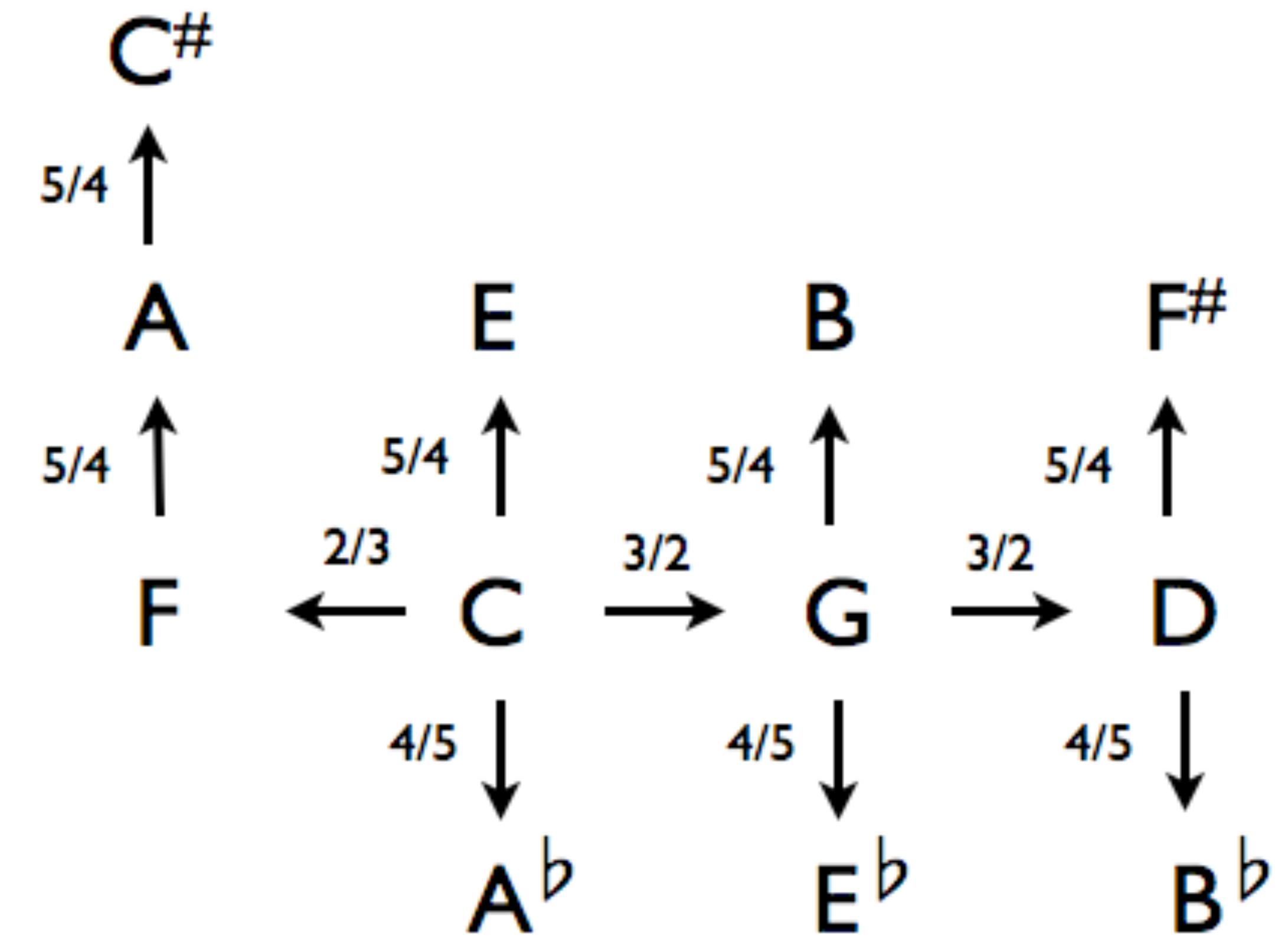
G: 3/2

D: $(3/2)2 \times (1/2) = 9/8$

A: $(2/3) \times (5/4) \times 2 = 5/3$ (vs 27/16)

E: 5/4 (vs 81/64)

F: $(2/3) \times 2 = 4/3$



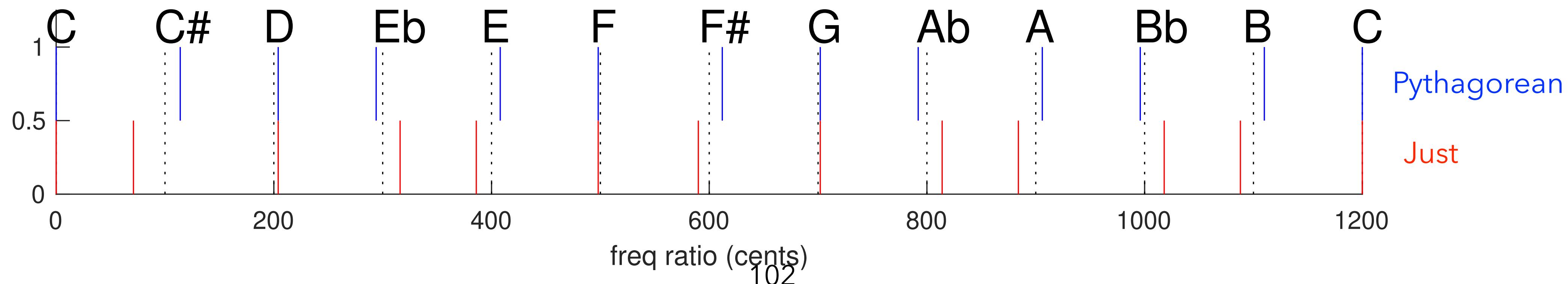
Comparing different tuning systems

Pythagorean vs Equal Temperament

Note	Pyth freq ratio	ET freq ratio	Difference (cents)
C	$1 : 1 = 1.000$	1.000	0
C \sharp	$2187 : 2048 = 1.068$	1.059	14
D	$9 : 8 = 1.125$	1.122	4
E \flat	$32 : 27 = 1.185$	1.189	-6
E	$81 : 64 = 1.266$	1.260	8
F	$4 : 3 = 1.333$	1.335	-2
F \sharp	$729 : 512 = 1.424$	1.414	12
G	$3 : 2 = 1.500$	1.498	2
A \flat	$128 : 81 = 1.580$	1.587	-8
A	$27 : 16 = 1.688$	1.682	6
B \flat	$16 : 9 = 1.778$	1.782	-4
B	$243 : 128 = 1.898$	1.888	10
C'	$2 : 1 = 2.000$	2.000	0

Just vs Equal Temperament

Note	Just freq ratio	ET freq ratio	Difference (cents)
C	$1 : 1 = 1.000$	1.000	0
C \sharp	$25 : 24 = 1.042$	1.059	-29
D	$9 : 8 = 1.125$	1.122	4
E \flat	$6 : 5 = 1.200$	1.189	16
E	$5 : 4 = 1.250$	1.260	-14
F	$4 : 3 = 1.333$	1.335	-2
F \sharp	$45 : 32 = 1.406$	1.414	-10
G	$3 : 2 = 1.500$	1.498	2
A \flat	$8 : 5 = 1.600$	1.587	14
A	$5 : 3 = 1.667$	1.682	-16
B \flat	$9 : 5 = 1.800$	1.782	18
B	$15 : 8 = 1.875$	1.888	-12
C'	$2 : 1 = 2.000$	2.000	0

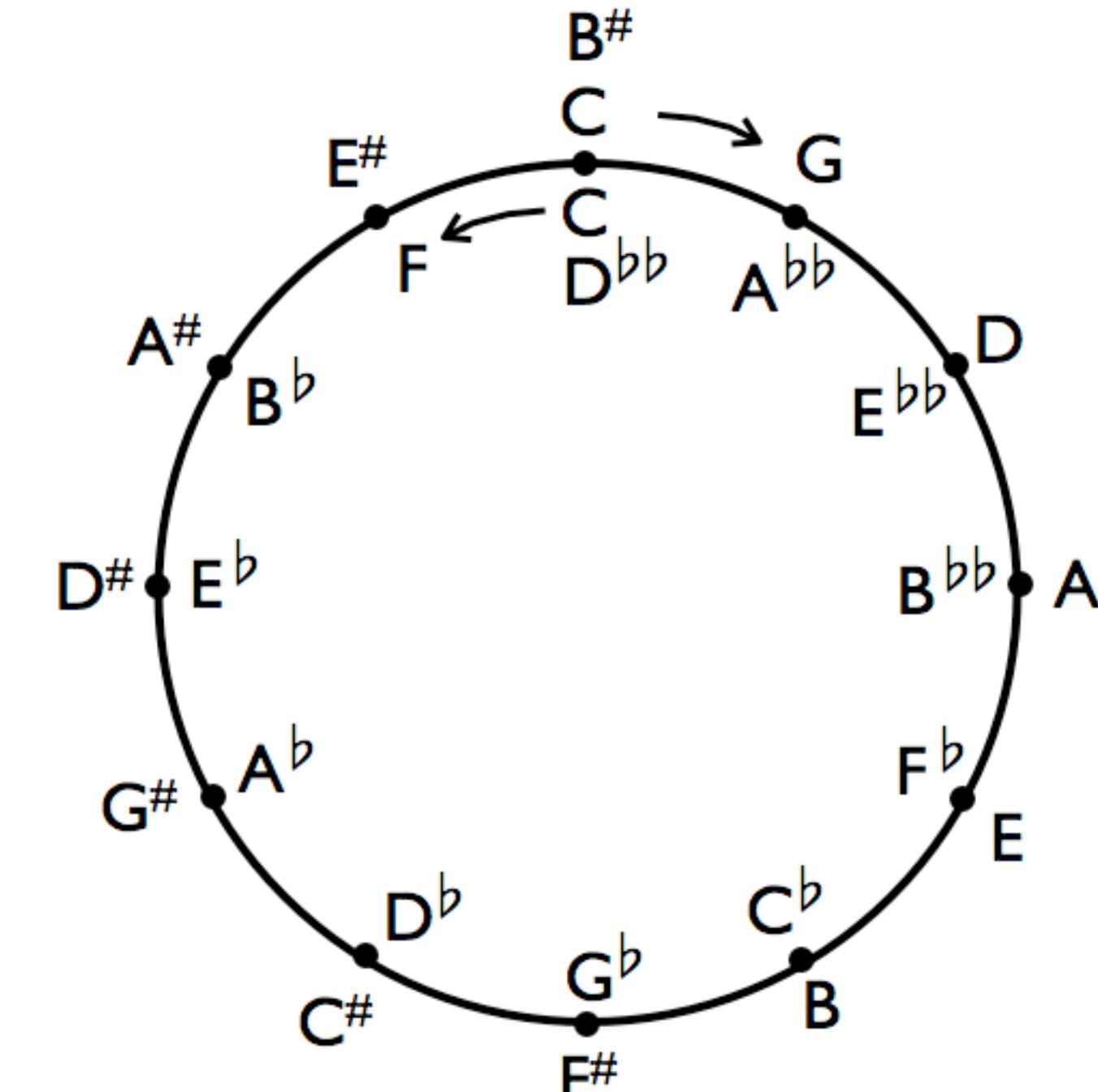


All tuning systems have problems!!

- Equal-tempered fifths, fourths, etc. are never perfect (only an octave)
- Pythagorean circle of fifths doesn't close (12 perfect fifths is not equal to 7 octaves)
- Pythagorean "comma":

$$\frac{B^\#}{C'} = \frac{(3/2)^{12}}{2^7} = 1.0136 \text{ (23 cents too large)}$$

- Fifth $C^\#$ to A^\flat is too flat in Pythagorean temperament ("wolf" fifth) and too sharp in just temperament



Fifth	Temperament	Freq ratio	Difference (cents)
C-G	equal	1.498	-2
C-G	pyth	1.500	0
C-G	just	1.500	0
$C^\#-A^\flat$	equal	1.498	-2
$C^\#-A^\flat$	pyth	1.480	-23
$C^\#-A^\flat$	just	1.536	41