①

+41.0 C

−41.0 C

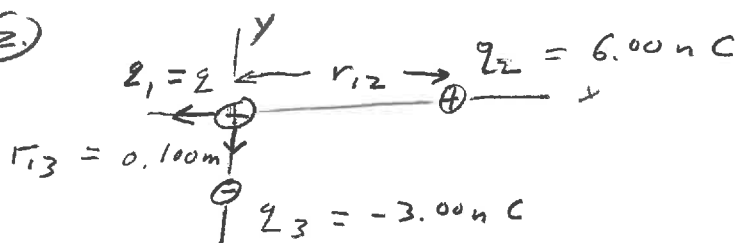$H_e = 9 \times 10^9 \; \dfrac{N m^2}{C^2}$

$r = 1.90 \; km = 1.90 \times 10^3 \, m$

Attractive Force on top close
(directed downward) with magnitude

$$F = \frac{H_e |q_1||q_2|}{r^2} = \frac{9 \times 10^9 \frac{N \cdot m^2}{C^2} \, (41 C)^2}{(1.9 \times 10^3 m)^2}$$

$$= \boxed{4.19 \times 10^6 \, N}$$

②

$q_1 = q \xleftarrow{\; r_{12} \;} \quad q_2 = 6.00 \, nC$

$r_{13} = 0.100 \, m$

$q_3 = -3.00 \, nC$

$q = 5.30 \, nC = 5.30 \times 10^9 \, C$

$r_{12} = 0.325 \, m$

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}}$$

$$= \frac{H_e |q_1||q_2|}{r_{12}^2} (-\hat{x}) + \frac{H_e |q_1||q_3|}{r_{13}^2} (-\hat{y})$$

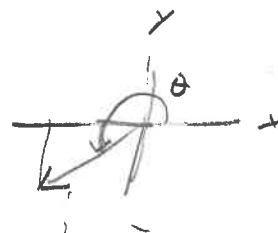$$= -2.71 \times 10^{-6} \, N \, \hat{x} - 1.43 \times 10^{-5} \, N \, \hat{y}$$

$$|\vec{F_1}| = \sqrt{F_{1,x}^2 + F_{1,y}^2}$$
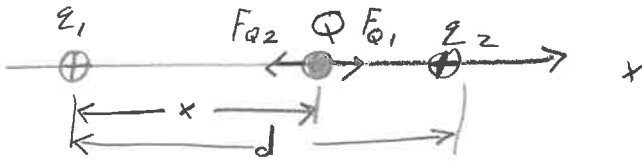
$$= \boxed{2.02 \times 10^{-5} \, N}$$

Direction:

$$\theta = \arctan\left(\frac{F_{1,y}}{F_{1,x}}\right)$$

$$= \arctan\left(\frac{-2.71 \times 10^{-6}}{-1.43 \times 10^{-5}}\right)$$

$$= 45° + 180° = \boxed{225°}$$

③.

$q_1 = 4q$

$q_2 = q$

$d = 1.5 \text{ m}$

Suppose Q has same charge as q (>0)

Equilibrium: $F_{Q1} = F_{Q2}$

$$\frac{k_e |Q||q_1|}{x^2} = \frac{k_e |Q||q_2|}{(d-x)^2}$$

$$\frac{|q_1|}{x^2} = \frac{|q_2|}{(d-x)^2}$$

$\rightarrow$ $\dfrac{4q}{x^2} = \dfrac{q}{(d-x)^2}$

$x^2 = 4(d-x)^2 = 4d^2 + 4x^2 - 8dx$

$0 = 3x^2 - 8dx + 4d^2$

$x = \dfrac{+8d \pm \sqrt{64d^2 - 4 \cdot 3 \cdot 4d^2}}{2 \cdot 3}$

$64 - 48 = 16$

$= \dfrac{8d \pm \sqrt{16d^2}}{6}$

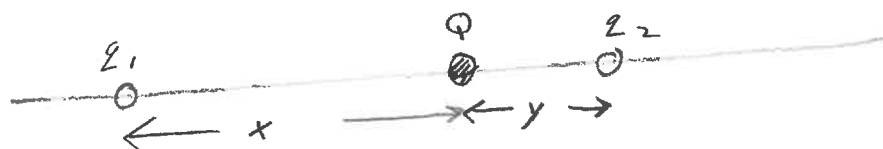$= \dfrac{8d \pm 4d}{6}$

$= 2d$ or $\boxed{\dfrac{2}{3}d}$ Only physically allowed value

Equilibrium is stable if $Q > 0$
    Unstable if $Q < 0$

Stable vs. unstable equilibrium

q_1 Q q_2

$\longleftarrow x$ $\leftarrow y \rightarrow$

Two cases: (i) $\vec{F}_{Q2} \longleftarrow\!\!\!\bullet\!\!\!\longrightarrow \vec{F}_{Q1}$ (repulsion by q_1, q_2)

(ii) $\longleftarrow\!\!\!\bullet\!\!\!\longrightarrow$ (attraction by q_1, q_2)
            $\vec{F}_{Q1}$ $\vec{F}_{Q2}$

In both cases, $|\vec{F}_{Q1}| = |\vec{F}_{Q2}| \equiv F_{equilibrium}$ at equilibrium position

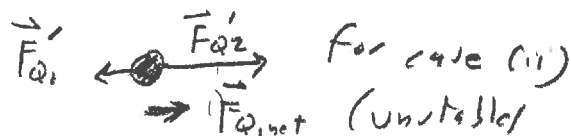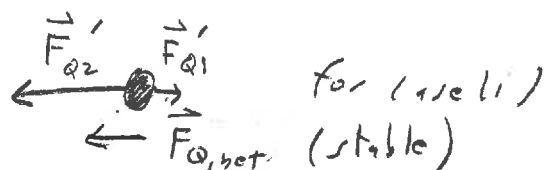① Suppose Q is moved to the right relative to equilibrium

Then: $x \to x + \epsilon \equiv x'$
$\qquad y \to y - \epsilon \equiv y'$

$F_{Q1} \to F'_{Q1} = \dfrac{k |Q||q_1|}{(x+\epsilon)^2} < F_{equilibrium}$

$F_{Q2} \to F'_{Q2} = \dfrac{k |Q||q_2|}{(y-\epsilon)^2} > F_{equilibrium}$

Thus $\vec{F}_{Q,net} = \vec{F}'_{Q1} + \vec{F}'_{Q2}$

$\vec{F}'_{Q2} \; \vec{F}'_{Q1}$
$\longleftarrow\!\!\!\bullet\!\!\!\longrightarrow$ for case (i)
$\longleftarrow \vec{F}_{Q,net}$ (stable)

$\vec{F}'_{Q1} \longleftarrow\!\!\!\bullet\!\!\!\xrightarrow{\ \vec{F}'_{Q2}\ }$ for case (ii)
$\longrightarrow \vec{F}_{Q,net}$ (unstable)

② Similarly, if Q is moved to the left

Then $\quad x \to x - \epsilon \equiv x'$
$\qquad\qquad y \to y + \epsilon \equiv y'$

$F_{Q1} \to F'_{Q1} > F_{equilibrium}$

$F_{Q2} \to F'_{Q2} < F_{equilibrium}$

$\vec{F}'_{Q2} \; \vec{F}'_{Q1}$
$\longleftarrow\!\!\!\bullet\!\!\!\longrightarrow$ for case (i)
$\longrightarrow \vec{F}_{Q,net}$ (stable)

$\vec{F}'_{Q1} \longleftarrow\!\!\!\circ\!\!\!\xrightarrow{\ \vec{F}'_{Q2}\ }$ for case (ii)
     (unstable)

$\longleftarrow$
$\vec{F}_{Q,net}$

(4.)

$\vec{E}_3$ $\vec{E}_2$

$q_1 = Aq$ a   q $\vec{E}_1$

a   a

$q_2 = Bq$ a $q_3 = Cq$
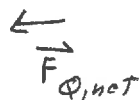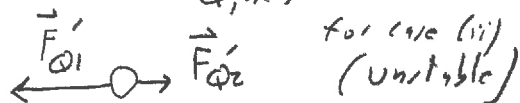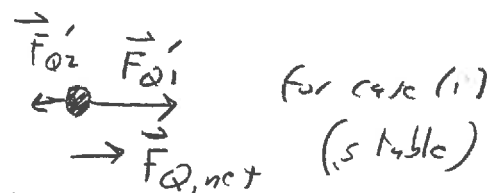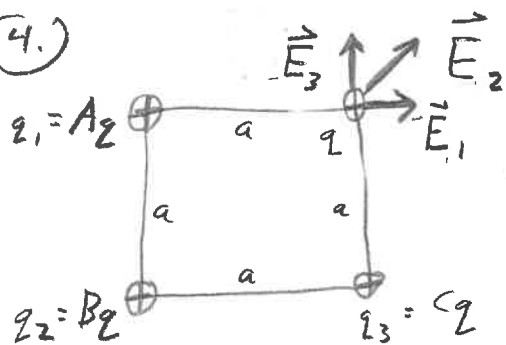
$\left(A = 4, \; B = 4, \; C = 8 \right)$

a) Electric field at q:

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{k A q}{a^2} \hat{x} + \frac{k B q}{\left(\sqrt{2} a \right)^2} \left(\cos\theta \, \hat{x} + \sin\theta \, \hat{y} \right)$$

$$+ \frac{k C q}{a^2} \hat{y} \qquad \left(\text{where } \theta = 45° \right)$$

$$\vec{E}_{tot} = \frac{k q}{a^2} \left[A\hat{x} + \frac{B}{2} \left(\cos\theta \, \hat{x} + \sin\theta \, \hat{y} \right) + C\hat{y} \right]$$

$\cos 45° = \sin 45°$
$= \frac{\sqrt{2}}{2}$

$$= \frac{k q}{a^2} \left[A\hat{x} + \frac{B\sqrt{2}}{4} \left(\hat{x} + \hat{y} \right) + C\hat{y} \right]$$

$$= \frac{k q}{a^2} \left[\left(A + \frac{B\sqrt{2}}{4} \right) \hat{x} + \left(C + \frac{B\sqrt{2}}{4} \right) \hat{y} \right]$$

b) Electric force on q:

$$\vec{F}_{tot} = q \vec{E}_{tot}$$

$$= \frac{k q^2}{a^2} \left[\left(A + \frac{B\sqrt{2}}{4} \right) \hat{x} + \left(C + \frac{B\sqrt{2}}{4} \right) \hat{y} \right]$$
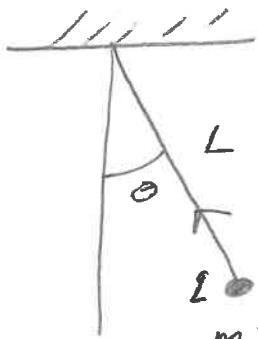
(5)

$m = 2.00 \, gm \qquad = 2 \times 10^{-3} \, kg$

$L = 24.1 \, cm \qquad = .241 \, m$

$\theta = 16.7°$

$\vec{E} = 1.00 \times 10^3 \, \dfrac{N}{C}$

$m = 2.00 \, g$

$y \, \llcorner_x$

$\vec{F_e} = q \vec{E} = q E \hat{x}$

$\vec{F_g} = mg(-\hat{y})$

$\vec{T} = tension = -T \sin\theta \, \hat{x} + T \cos\theta \, \hat{y}$

$\vec{F_{net}} = \vec{0} \rightarrow \vec{0} = \vec{F_e} + \vec{F_g} + \vec{T}$

$= q E \hat{x} - mg \hat{y} - T \sin\theta \, \hat{x} + T \cos\theta \, \hat{y}$

$= (q E - T \sin\theta) \, \hat{x} - (m - T \cos\theta) \, \hat{y}$

Thus,

$m - T \cos\theta = 0 \qquad \rightarrow T = m/\cos\theta$

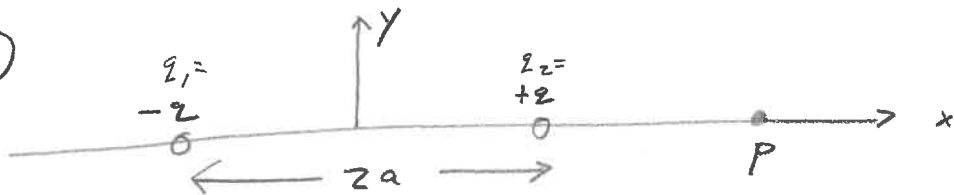$q E - T \sin\theta = 0 \qquad \rightarrow \boxed{q = \dfrac{T \sin\theta}{E}}$

$\boxed{= \dfrac{m}{E} \tan\theta}$

substitute in numbers $\rightarrow$ $q = \dfrac{(2 \times 10^{-3} \, kg) \, \tan(16.7°)}{\left(1. \times 10^3 \, \dfrac{N}{C}\right)}$

$= 0.6 \times 10^{-6} \, Coulombs$

$= \boxed{0.6 \, \mu C}$
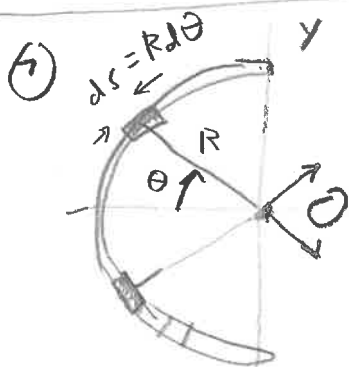
⑥

$q_1 = -q$ $\quad$ $q_2 = +q$

$\overleftrightarrow{2a}$

$$\vec{E}(P) = \frac{k\,|q_1|}{(x+a)^2}\,(-\hat{x}) + \frac{k\,|q_2|}{(x-a)^2}\,\hat{x}$$

$$= kq\,\hat{x}\left[-\frac{1}{(x+a)^2} + \frac{1}{(x-a)^2}\right]$$

$$= \frac{kq\,\hat{x}}{x^2}\left[-\frac{1}{(1+\frac{a}{x})^2} + \frac{1}{(1-\frac{a}{x})^2}\right]$$

for $x \gg a$

$$\simeq \frac{kq\,\hat{x}}{x^2}\left[-\left(1 - \frac{2a}{x}\right) + \left(1 + \frac{2a}{x}\right)\right]$$

$$= \frac{4kq\,a}{x^3}\,\hat{x}$$

---

⑦

$ds = Rd\theta$

$-7.5\,\mu$ $Q = q = $ total charge, $L = 15$ cm

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\pi R = L \rightarrow R = \frac{L}{\pi}$$

$$\lambda = \frac{Q}{L} \quad (\text{charge per unit length})$$

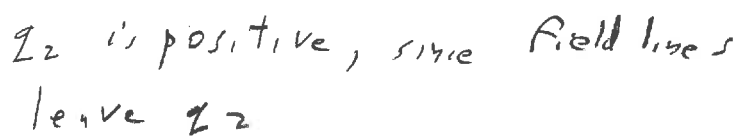$$dq = \lambda\,ds = \lambda R\,d\theta = \left(\frac{\lambda L}{\pi}\right)d\theta$$

(b) $\vec{E}(o)$ only has a component in the $+x$ direction (to the right) since y-component will cancel out from charge element symmetrically placed above/below the x-axis.

(a) $\displaystyle \vec{E}(o) = \hat{x}\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{k\,dq}{R^2}\cos\theta = \frac{\hat{x}\,k}{R^2}\,2\int_{0}^{\pi/2}\lambda R\,d\theta\cos\theta = \hat{x}\,\frac{2k\lambda}{R}\sin\theta\Big|_0^{\pi/2}$

$$= \hat{x}\,\frac{2k\lambda}{R} = \boxed{\hat{x}\,\frac{\lambda}{2\pi\epsilon_0 R}} \qquad \boxed{= -1.88\times 10^{-7}\,N}$$

(like ∞-line charge)

(when substituting in numerical values for Q, L, $\lambda = Q/L$, $R = L/\pi$, $\epsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$)

(8.)

q_2 is positive, since field lines leave q_2

q_1 is negative, since field lines terminate on q_1

Number of field lines surrounding $q_2 = 18$

" $q_1 = 6$

Magnitude of electric field is proportional to density of field lines passing thru a surface, so number/area.

For the same area of a sphere $(A = 4\pi R^2)$ centred at q_1 and then at q_2 we have $3\times$ as many lines for q_2 than for q_1
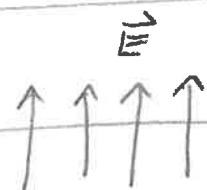
$$\rightarrow \boxed{q_2 = 3q_1}$$

(9.)

$(q = e)$ proton $\quad v = 3.80 \times 10^5 \text{ m/s}$

$(m = 1.67 \times 10^{-27} \text{ kg})$

$\vec{E}$

↑↑↑↑

uniform electric field

$E = 8.20 \times 10^3 \text{ N/C}$

a) $\Delta x = 4.50 \text{ cm} (= 0.0450 \text{ m})$

$\rightarrow \Delta t = \dfrac{\Delta x}{v} = \dfrac{0.0450 \text{ m}}{3.8 \times 10^5 \text{ m/s}} = \boxed{1.18 \times 10^{-7} \text{ s}}$

b) $F_y = qE = ma \quad \rightarrow \quad a = \dfrac{qE}{m}$ (uniform)

$\rightarrow \Delta y = \frac{1}{2} a \Delta t^2 = \frac{1}{2}\left(\dfrac{qE}{m}\right)(\Delta t)^2 = \frac{1}{2}\dfrac{(1.6 \times 10^{-19} \text{ C})(8.20 \times 10^3 \text{ N/C})(1.18 \times 10^{-7} \text{ s})^2}{1.67 \times 10^{-27} \text{ kg}}$

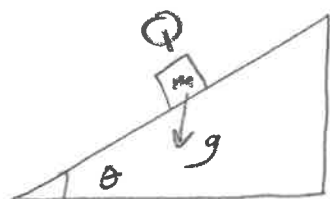$\Delta y = 0.0055 \text{ m} = \boxed{5.5 \text{ mm}}$

(c) $v_x = 3.80 \times 10^5 \text{ m/s}$ (since $\vec{F}$ is in the y-direction)

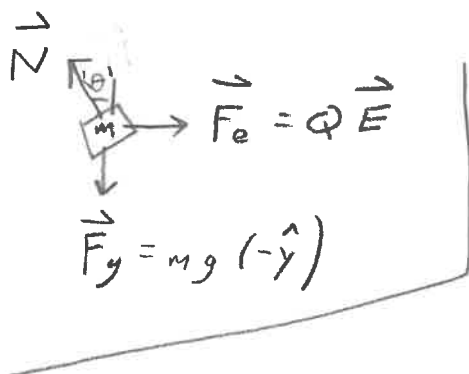$v_y = a \, \Delta t = \left(\dfrac{qE}{m}\right) \Delta t$

$= 9.32 \times 10^4 \text{ m/s}$

Thus, $\boxed{\vec{v} = 3.80 \times 10^5 \text{ m/s } \hat{x} + 9.32 \times 10^4 \text{ m/s } \hat{y}}$

⑩

Frictionless

a) In order for m to be at rest, $\vec{F}_{net} = \vec{0}$,

Thus, $\vec{0} = \vec{F}_g + \vec{F}_e + \vec{N}$

$= -mg\hat{y} + QE\hat{x}$

$\qquad + N\cos\theta \hat{y} - N\sin\theta \hat{x}$

$= (QE - N\sin\theta)\hat{x}$

$\qquad + (N\cos\theta - mg)\hat{y}$

$\vec{F}_e = Q\vec{E}$

$\vec{F}_g = mg(-\hat{y})$

This implies:

$QE - N\sin\theta = 0 \rightarrow E = \dfrac{N\sin\theta}{Q}$

$N\cos\theta - mg = 0 \rightarrow N = \dfrac{mg}{\cos\theta}$

$\Biggr\}$ $\boxed{E = \dfrac{mg}{\cos\theta}\dfrac{\sin\theta}{Q} = \dfrac{mg\tan\theta}{Q}}$

b) $\quad$ Take $\quad m = 5.06 \, g = 5.06 \times 10^{-3} \, kg$, $\quad q = -7.56 \, \mu C$, $\quad \theta = 24.9°$ $\quad$ ⑨

$\rightarrow \quad E = \dfrac{mg \tan \theta}{q} \quad = \quad \dfrac{(5.06 \times 10^{-3} \, kg)(9.8 \, m/s^2) \tan(24.9°)}{-7.56 \times 10^{-6} \, C}$

$$= \quad -3.04 \times 10^3 \, N/C$$

So $\quad \boxed{\begin{array}{l} |\vec{E}| = 3.04 \times 10^3 \, N/C \\ \text{direction } (\text{to the left}) \cdot \, -\hat{x} \end{array}}$

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⑪

$q_1 = 0.728 \, \mu C$

$q_2 = -0.54 \, \mu C$
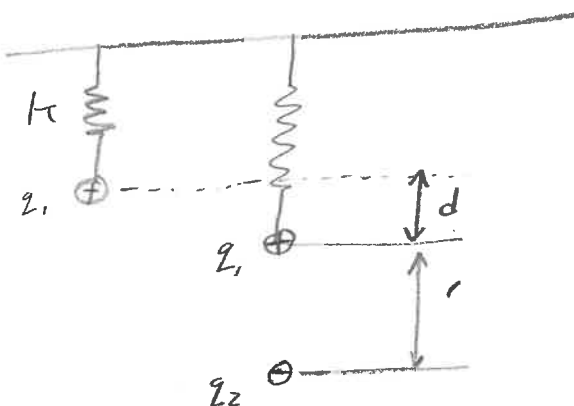
$d = 3.60 \, cm$

$r = 4.80 \, cm$

spring force $=$ electrostatic force

$$k d = \dfrac{k_e \, q_1 q_2}{r^2}$$

$\rightarrow k = \dfrac{k_e \, q_1 q_2}{r^2 d}$

$$= \boxed{42.7 \, \frac{N}{m}}$$

(12.) $\vec{E}$: magnitude $660 \, N/_c$, proton :

$\Delta V = 1.40 \, Mm/s = 1.4 \times 10^6 \, m/s$

$m = 1.67 \times 10^{-27} \, kg$

$q = 1.602 \times 10^{-19} \, C$

a) $F_e = qE = ma$

$a = \dfrac{qE}{m} = \dfrac{1.602 \times 10^{-19} C \cdot \left(660 \, N/_c\right)}{1.67 \times 10^{-27} \, kg} = \boxed{6.33 \times 10^{10} \, m/_{s^2}}$

b) $a = \dfrac{\Delta V}{\Delta t} \rightarrow \Delta t = \dfrac{\Delta V}{a} = \dfrac{1.40 \times 10^6 \, m/s}{6.33 \times 10^{10} \, m/s^2} = \boxed{2.21 \times 10^{-5} \, s}$

c) $\Delta x = \dfrac{1}{2} a \, \Delta t^2 = \left(\dfrac{1}{2}\left(6.33 \times 10^{10} \, \dfrac{m}{s^2}\right)\right)\left(2.21 \times 10^{-5} \, s\right)^2 = \boxed{15.5 \, m}$

$\underbrace{\qquad\qquad}$

const acceleration

d) $K = \dfrac{1}{2} m \Delta V^2 = \dfrac{1}{2}\left(1.67 \times 10^{-27} \, kg\right)\left(1.40 \times 10^{-3} \, \dfrac{m}{s}\right)^2$

$= \boxed{1.64 \times 10^{-15} \, J}$

1.4×10^6