5. Fourier Analysis and Synthesis of Waveforms

PURPOSE AND BACKGROUND

The simplest sound is a pure sine wave with a single frequency and amplitude. Most sound sources and instruments do not produce such simple waves. Usually their sound contains many sine waves with higher frequencies, called *harmonics.* These act together according to the *superposition principle* to produce a complex tone. This addition of sine waves with suitable amplitudes and phases is called *Fourier synthesis of sound*. The opposite, the decomposition of sound into its sine-wave components, is called *Fourier analysis*. Periodic sound can be synthesized or analyzed with a sufficient number of sine waves. A *pure* tone is a sine wave with a single frequency. Many sine waves added together form a complex tone and waveform periodic in time. This laboratory is about the analysis and synthesis of sound and how electronic *synthesizers* can mimic real instruments.

I Fourier Synthesis of Waveforms

For our experiment, we will use the Fourier Series Applet, which is available online at `https://www.falstad.com/fourier/`. Listen to several available wave forms (e.g., sine, triangle, square, and sawtooth waves) at a fundamental frequency of $f_1 = 500$ Hz. To do so, you will need to adjust the "Playing Frequency" slider as close as you can to 500 Hz, and the check the "Sound" box to listen to the sounds.

Questions

1. Draw sketches of the four waveforms.
2. Which waveform most resembles a pure sine wave?
3. Which waveform least resembles a pure sine wave?
4. Which tone sounds least like the pure sine wave?

Complex waveforms are produced by adding sine waves of different frequencies and amplitudes. The tone heard in all four cases has the same *pitch* or fundamental frequency $f_1 = 500$ Hz. For a pure tone (sine wave), the fundamental is the only frequency present. For complex tones, sine waves with integer multiples of the fundamental frequency and suitable amplitudes are added together. For example, the next integer multiples of the fundamental $f_1 = 500$ Hz are $f_2 = 2f_1 = 1000$ Hz, $f_3 = 3f_1 = 1500$ Hz, and so on.

These higher frequencies are called *overtones* or *harmonics*. Just like the fundamental, each overtone has a single frequency. A complex waveform can be produced with the fundamental plus higher harmonics of suitable amplitudes. This process is called *superposition* of waves or, mathematically speaking, *Fourier synthesis* of waves. Conversely, you can take a complex waveform apart by decomposing it with a spectrum analyzer into its individual harmonics. This is called *Fourier analysis* of waves.

A Sawtooth Waveform

The harmonics of the sawtooth wave follow a simple pattern. All harmonics exist from $N = 1$ to $N = \infty$, with amplitudes given by $A_N = A_1/N$. Thus all integer multiples of the fundamental frequency contribute to the waveform. Since in practice we cannot add an infinite number of harmonics, we shall only use the first five or six harmonics and add them up.

Using the online app, start by clicking the “Sine” box. You should see a single white dot, sticking up above the rest, at a height corresponding to the amplitude of the first harmonic. The second harmonic $N = 2$, $f_2 = 1000$ Hz should have an amplitude $A_2 = A_1/2$ for a sawtooth wave. Add this harmonic to the fundamental by adjusting the height of the second white dot to half the height of the first white dot. Take a look at and listen to the waveform generated.

Questions

1. Find the frequencies of the next three higher harmonics and their relative amplitudes in percent. Complete the entries in Table 1.

N	f_N	A_N
1	500Hz	100%
2	1000Hz	50%

Table 1: Sawtooth waveform: Harmonic numbers, frequencies, and relative amplitudes.

2. What would be the frequency and amplitude of the $N = 10$ harmonic for a sawtooth waveform of fundamental frequency $f_1 = 500$ Hz.

Continue adding harmonics (3rd, 4th, 5th, etc.) by adjusting the heights of their white dots appropriately. Note the changes in the tone and the waveform. With each addition of a harmonic, the wave should look more and more like a sawtooth.

B Square Wave

A square or rectangular waveform is similar to the sawtooth in that the amplitudes of the harmonics follow the $A_N = A_1/N$ dependence. However, the major difference is that only the *odd* harmonics $N = 1$, $N = 3$, $N = 5$, etc., contribute.

Questions

1. Use this information and complete the entries in Table 2 for the square wave.

Synthesize a square wave using the online app by starting as before with just a “Sine”, and then successively adding the higher harmonics with amplitudes given in Table 2. Note the changes in tone and shape of the waveform as more harmonics are added.

N	f_N	A_N
1	500 Hz	100%
3	1500 Hz	33.33%

Table 2: Square wave: Harmonic numbers, frequencies, and relative amplitudes.

1.

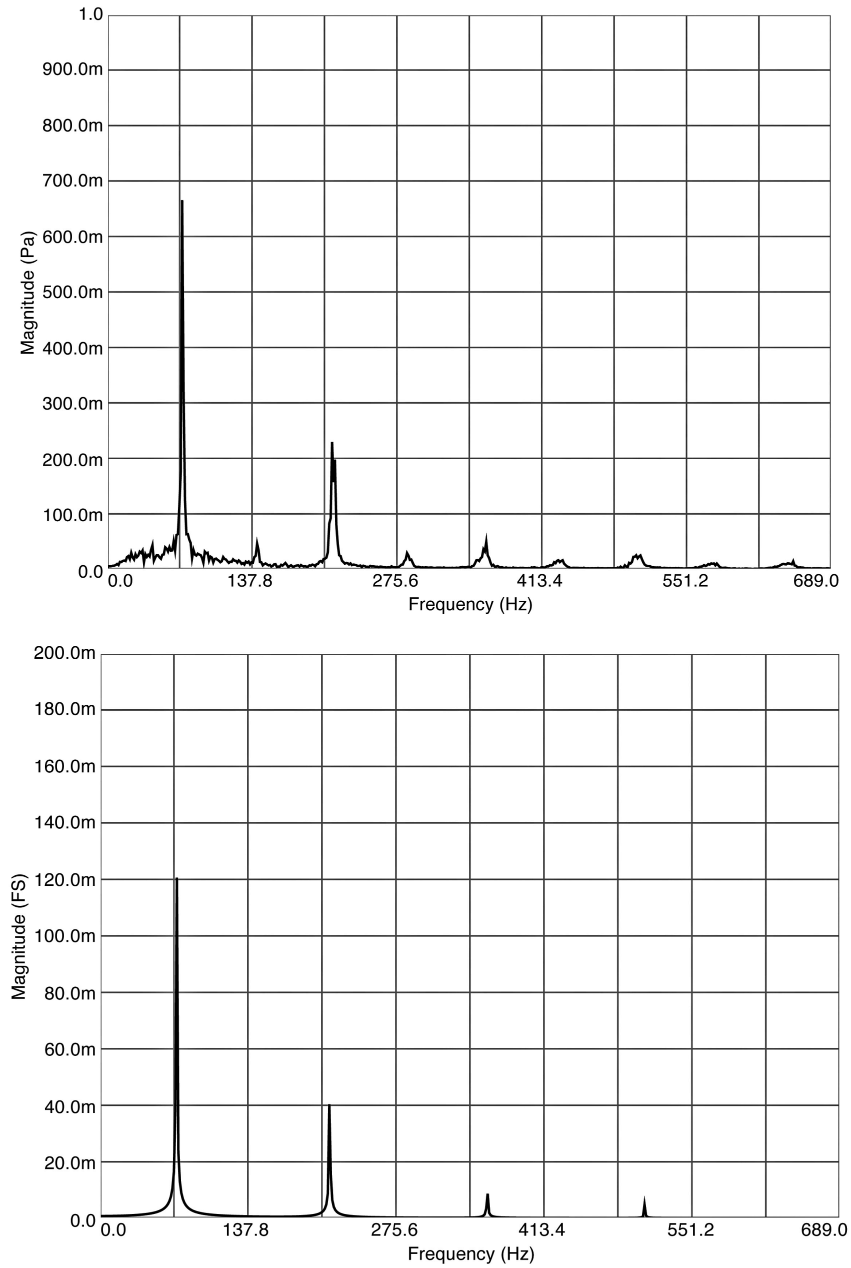
N	f_N	A_N
1	500 Hz	100%
3	1500 Hz	11.11%

Table 3: Triangular waveform: Harmonic numbers, frequencies, and relative amplitudes.

C Triangular Waveform

Questions

II Fourier Analysis of Waveforms



1.0
900.0m
800.0m
700.0m
600.0m
500.0m
400.0m
300.0m
200.0m
100.0m
0.0
Magnitude (Pa)
0.0
137.8
275.6
413.4
551.2
689.0
Frequency (Hz)
200.0m
180.0m
160.0m
140.0m
120.0m
100.0m
80.0m
60.0m
40.0m
20.0m
0.0
Magnitude (FS)
0.0
137.8
275.6
413.4
551.2
689.0
Frequency (Hz)

Figure 1: