Chpt 23

(1)

$k_e = 9 \times 10^9 \ \frac{N m^2}{C^2}$

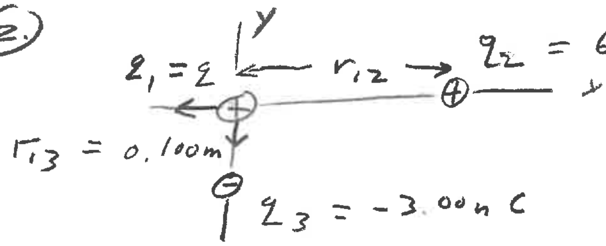
$r = 1.90 \text{ km} = 1.90 \times 10^3 \text{ m}$

Attractive Force on top charge (directed downward) with magnitude

$$F = \frac{k_e |q_1||q_2|}{r^2} = \frac{9 \times 10^9 \frac{N \cdot m^2}{C^2} (41 C)^2}{(1.9 \times 10^3 m)^2}$$

$$= \boxed{4.19 \times 10^6 N}$$

(2)



$q = 5.30 \text{ nC} = 5.30 \times 10^{9} C$

$r_{12} = 0.325 \text{ m}$

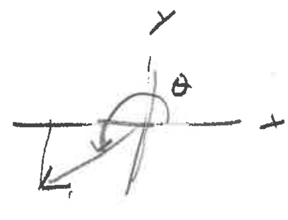
$$\vec{F_1} = \vec{F}_{12} + \vec{F}_{13}$$

$$= \frac{k_e |q_1||q_2|}{r_{12}^2}(-\hat{x}) + \frac{k_e |q_1||q_3|}{r_{13}^2}(-\hat{y})$$

$$= -2.71 \times 10^{-6} N \hat{x} - 1.43 \times 10^{-5} N \hat{y}$$

$$|\vec{F_1}| = \sqrt{F_{1,x}^2 + F_{1,y}^2}$$

$$= \boxed{2.02 \times 10^{-5} N}$$



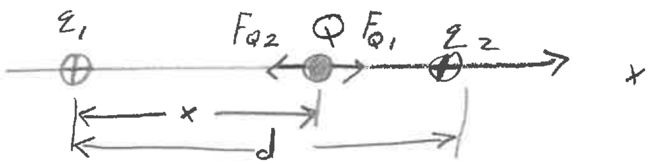
Direction:

$$\theta = \arctan\left(\frac{F_{1,y}}{F_{1,x}}\right)$$

$$= \arctan\left(\frac{-2.71 \times 10^{-6}}{-1.43 \times 10^{-5}}\right)$$

$$= 45^\circ + 180^\circ = \boxed{225^\circ}$$

3.



$q_1 = 4q$
$q_2 = q$
$d = 1.5\ m$

suppose Q has same charge as q (>0)

Equilibrium: $F_{Q1} = F_{Q2}$

$$\frac{k_e|Q||q_1|}{x^2} = \frac{k_e|Q||q_2|}{(d-x)^2}$$

$$\frac{|q_1|}{x^2} = \frac{|q_2|}{(d-x)^2}$$

$$\rightarrow \quad \frac{4\cancel{q}}{x^2} = \frac{\cancel{q}}{(d-x)^2}$$

$$x^2 = 4(d-x)^2 = 4d^2 + 4x^2 - 8dx$$

$$0 = 3x^2 - 8dx + 4d^2$$

$$x = \frac{+8d \pm \sqrt{64d^2 - 4 \cdot 3 \cdot 4d^2}}{2 \cdot 3}$$

$64 - 48 = 16$

$$= \frac{8d \pm \sqrt{16d^2}}{6}$$

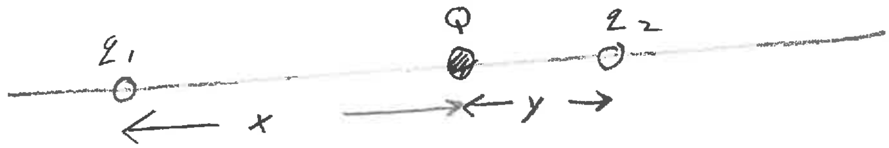
$$= \frac{8d \pm 4d}{6}$$

$$= 2d \quad \text{or} \quad \boxed{\frac{2}{3}d}$$

Only physically allowed value

Equilibrium is stable if $Q > 0$
unstable if $Q < 0$

Stable vs. unstable equilibrium



Two cases (i) $\vec{F}_{Q2} \leftarrow \bullet \rightarrow \vec{F}_{Q1}$ (repulsion by q_1, q_2)

(ii) $\vec{F}_{Q1} \leftarrow \bullet \rightarrow \vec{F}_{Q2}$ (attraction by q_1, q_2)

In both cases, $|\vec{F}_{Q1}| = |\vec{F}_{Q2}| \equiv F_{equilibrium}$ at equilibrium position

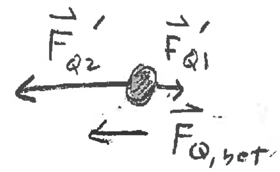
(1) suppose Q is moved to the right relative to equilibrium

Then: $x \to x+\epsilon \equiv x'$
$y \to y-\epsilon \equiv y'$

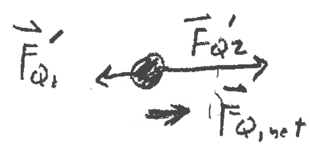
$$F_{Q1} \to F'_{Q1} = \frac{k|Q||q_1|}{(x+\epsilon)^2} < F_{equilibrium}$$

$$F_{Q2} \to F'_{Q2} = \frac{k|Q||q_2|}{(y-\epsilon)^2} > F_{equilibrium}$$

Thus $\vec{F}_{Q,net} = \vec{F}'_{Q1} + \vec{F}'_{Q2}$



for case (i) (stable)



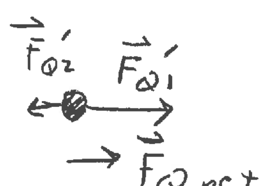
for case (ii) (unstable)

(2.) Similarly, if Q is moved to the left

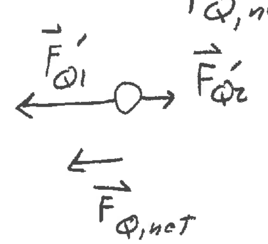
Then $x \to x-\epsilon \equiv x'$
$y \to y+\epsilon \equiv y'$

$F_{Q1} \to F'_{Q1} > F_{equilibrium}$

$F_{Q2} \to F'_{Q2} < F_{equilibrium}$

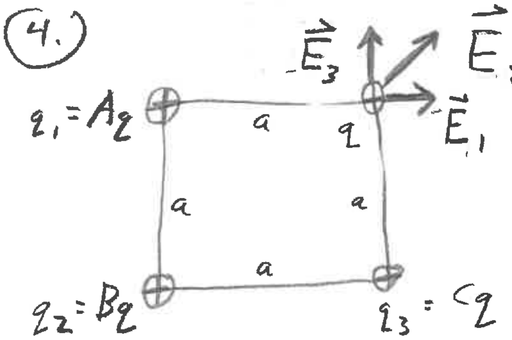


for case (i) (stable)



for case (ii) (unstable)

4.



$(A=4,\ B=4,\ C=8)$

a) Electric field at q:

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E} + \vec{E}_3$$

$$= \frac{kAq}{a^2}\hat{x} + \frac{kBq}{(\sqrt{2}a)^2}\left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right) + \frac{kCq}{a^2}\hat{y}$$

(where $\theta = 45^\circ$)

$$\vec{E}_{tot} = \frac{kq}{a^2}\left[A\hat{x} + \frac{B}{2}\left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right) + C\hat{y}\right]$$

$\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$$= \frac{kq}{a^2}\left[A\hat{x} + \frac{B\sqrt{2}}{4}\left(\hat{x} + \hat{y}\right) + C\hat{y}\right]$$

$$= \frac{kq}{a^2}\left[\left(A + \frac{B\sqrt{2}}{4}\right)\hat{x} + \left(C + \frac{B\sqrt{2}}{4}\right)\hat{y}\right]$$

b) Electric force on q:

$$\vec{F}_{tot} = q\vec{E}_{tot}$$

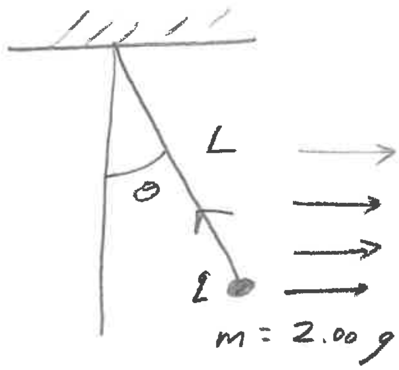
$$= \frac{kq^2}{a^2}\left[\left(A + \frac{B\sqrt{2}}{4}\right)\hat{x} + \left(C + \frac{B\sqrt{2}}{4}\right)\hat{y}\right]$$

(5.)

$m = 2.00 \text{ gm} = 2\times10^{-3}\text{ kg}$

$L = 24.1 \text{ cm} = .241 \text{ m}$

$\theta = 16.7^\circ$



$\vec{E} = 1.00\times10^3 \frac{N}{C}$

y x

$$\vec{F}_e = q\vec{E} = qE\hat{x}$$

$$\vec{F}_g = mg(-\hat{y})$$

$$\vec{T} = \text{tension} = -T\sin\theta\,\hat{x} + T\cos\theta\,\hat{y}$$

$$\vec{F}_{net} = \vec{0} \rightarrow \vec{0} = \vec{F}_e + \vec{F}_g + \vec{T}$$

$$= qE\hat{x} - mg\hat{y} - T\sin\theta\,\hat{x} + T\cos\theta\,\hat{y}$$

$$= (qE - T\sin\theta)\hat{x} - (m - T\cos\theta)\hat{y}$$

Thus,

$$m - T\cos\theta = 0 \rightarrow T = m/\cos\theta$$

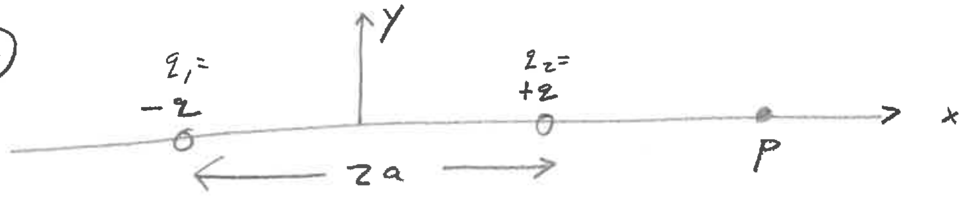
$$qE - T\sin\theta = 0 \rightarrow \boxed{q = \frac{T\sin\theta}{E} = \frac{m}{E}\tan\theta}$$

substitute in numbers $\rightarrow$ $q = \dfrac{(2\times10^{-3}\text{ kg})}{(1.\times10^3 \frac{N}{C})}\tan(16.7^\circ)$

$= 0.6\times10^{-6}$ Coulomb

$= \boxed{0.6\ \mu C}$

6.



$$\vec{E}(P) = \frac{k|q_1|}{(x+a)^2}(-\hat{x}) + \frac{k|q_2|}{(x-a)^2}\hat{x}$$

$$= kq\,\hat{x}\left[-\frac{1}{(x+a)^2} + \frac{1}{(x-a)^2}\right]$$

$$= \frac{kq\,\hat{x}}{x^2}\left[-\frac{1}{(1+\frac{a}{x})^2} + \frac{1}{(1-\frac{a}{x})^2}\right]$$

for $x >> a$

$$\simeq \frac{kq\,\hat{x}}{x^2}\left[-\left(\cancel{1} - \frac{2a}{x}\right) + \left(\cancel{1} + \frac{2a}{x}\right)\right]$$

$$= \frac{4kqa}{x^3}\,\hat{x}$$

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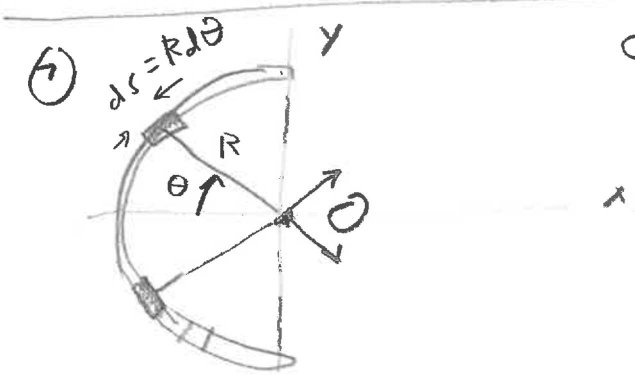
7)



Q = total charge, $L = 15$ cm

$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2})$

$\pi R = L \rightarrow R = \frac{L}{\pi}$

$\lambda = \frac{Q}{L}$ (charge per unit length

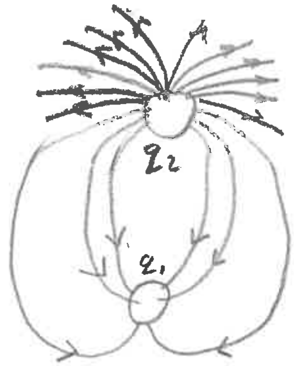
$dq = \lambda ds = \lambda R d\theta = \left(\frac{\lambda L}{\pi}\right) d\theta$

(b) $\vec{E}(0)$ only has a component in the $+x$ direction (to the right) since y-component will cancel out from charge element symmetrically placed above/below the x-axis

(a) $$\vec{E}(0) = \hat{x}\int_{\theta=-\frac{\pi}{2}}^{\pi/2} \frac{k\,dq}{R^2} = \frac{\hat{x}\,k}{R^2}\int_{\theta=0}^{\pi/2} 2\left(\frac{\lambda L}{\pi}\right) d\theta$$

$$= \hat{x}\,\frac{2k\lambda L}{\pi R^2}\,\theta\Big|_0^{\pi/2} = \boxed{\frac{k\lambda L}{R^2}\,\hat{x}}$$

8.



q_2 is positive, since field lines leave q_2

q_1 is negative, since field lines terminate on q_1

Number of field lines surrounding $q_2 = 18$

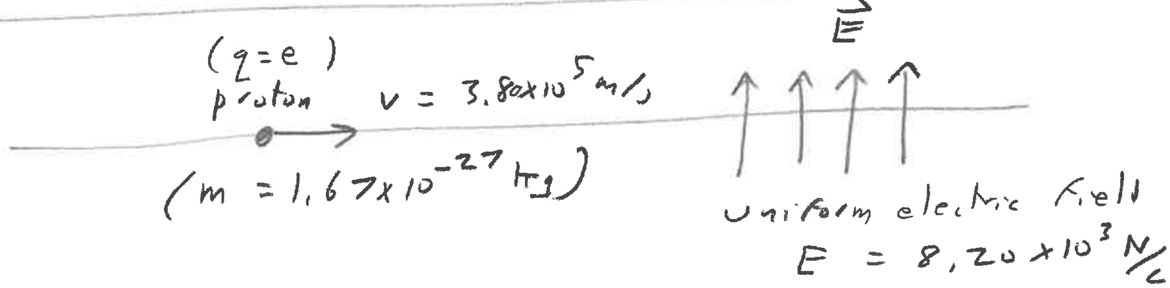
" " " $q_1 = 6$

~~3 lines ...~~ Magnitude of electric field is proportional to density of field lines passing thru a surface, so number/area.

For the same area of a sphere ($A = 4\pi R^2$) centred at q_1 and then at q_2 we have 3x as many lines for q_2 than for q_1

$\rightarrow \boxed{q_2 = 3q_1}$

9.



a) $\Delta x = 4.50$ cm $(= 0.0450$ m$)$

$\rightarrow \Delta t = \dfrac{\Delta x}{v} = \dfrac{0.0450\text{ m}}{3.8\times10^5\text{ m/s}} = \boxed{1.18\times10^{-7}\text{ s}}$

b) $F_y = qE = ma \longrightarrow a = \dfrac{qE}{m}$ (uniform)

$\rightarrow \Delta y = \dfrac{1}{2}a\,\Delta t^2 = \dfrac{1}{2}\left(\dfrac{qE}{m}\right)(\Delta t)^2 = \dfrac{1}{2}\dfrac{(1.6\times10^{-19}\text{ C})(8.20\times10^3\text{ N/C})}{1.67\times10^{-27}\text{ kg}}(1.18\times10^{-7}\text{ s})^2$

$$\Delta y = 0.0055 \text{ m} = \boxed{5.5 \text{ mm}}$$

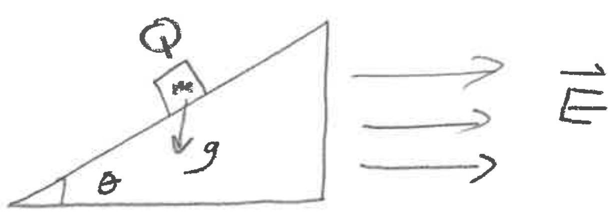
(c) $v_x = 3.80 \times 10^5$ m/s (since $\vec{F}$ is in the y-direction)

$$v_y = a\,\Delta t = \left(\frac{qE}{m}\right)\Delta t$$

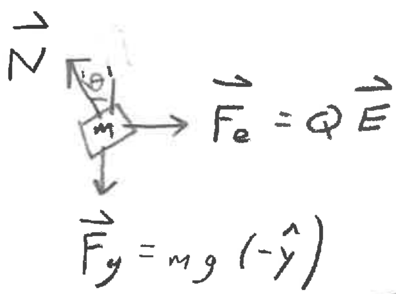
$$= 9.32 \times 10^4 \text{ m/s}$$

Thus, $\boxed{\vec{V} = 3.80 \times 10^5 \text{ m/s}\ \hat{x} + 9.32 \times 10^4 \text{ m/s}\ \hat{y}}$

(10.)



Frictionless



a) In order for m to be at rest, $\vec{F}_{net} = \vec{0}$,

Thus, $\vec{0} = \vec{F}_g + \vec{F}_e + \vec{N}$

$$= -mg\hat{y} + QE\hat{x} + N\cos\theta\,\hat{y} - N\sin\theta\,\hat{x}$$

$$= (QE - N\sin\theta)\,\hat{x} + (N\cos\theta - mg)\,\hat{y}$$

This implies

$$QE - N\sin\theta = 0 \rightarrow E = \frac{N\sin\theta}{Q}$$

$$N\cos\theta - mg = 0 \rightarrow N = \frac{mg}{\cos}$$

$$\boxed{E = \frac{mg}{\cos\theta}\frac{\sin\theta}{Q} = \frac{mg\tan\theta}{Q}}$$

b) In the $m = 5.06\,g = 5.06\times10^{-3}\,kg$, $Q = -7.56\,\mu C$, $\theta = 24.9°$ (9)

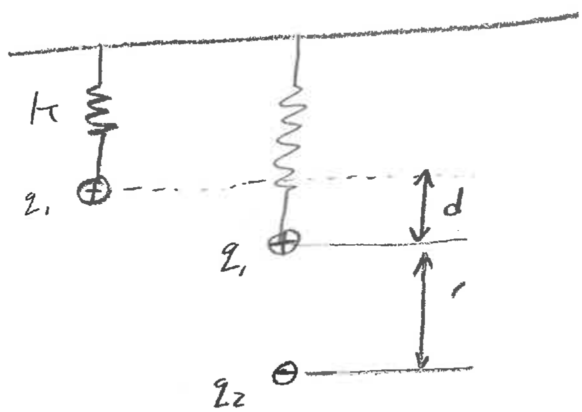
$$\rightarrow \quad E = \frac{mg\tan\theta}{Q} = \frac{(5.06\times10^{-3}\,kg)(9.8\,m/s^2)\tan(24.9°)}{-7.56\times10^{-6}\,C}$$

$$= -3.04\times10^{3}\,N/C$$

so $\boxed{|\vec{E}| = 3.04\times10^{3}\,N/C}$

direction (to the left) $-\hat{x}$

(11.)



Spring Force = electrostatic Force

$$kd = \frac{k_e q_1 q_2}{r^2}$$

$$\rightarrow k = \frac{k_e q_1 q_2}{r^2 d}$$

$$= \boxed{42.7\,\frac{N}{m}}$$

$q_1 = 0.728\,\mu C$

$q_2 = -0.54\,\mu C$

$d = 3.60\,cm$

$r = 4.80\,cm$

12. $\vec{E}$: magnitude $660\ N/C$, proton: 10

$\Delta v = 1.40\ Mm/s = 1.4 \times 10^6\ m/s$

$m = 1.67 \times 10^{-27}\ kg$

$q = 1.602 \times 10^{-19}\ C$

a) $F_e = qE = ma$

$$a = \frac{qE}{m} = \frac{1.602 \times 10^{-19} C \cdot (660\ N/C)}{1.67 \times 10^{-27}\ kg} = \boxed{6.33 \times 10^{10}\ m/s^2}$$

b) $a = \frac{\Delta v}{\Delta t} \rightarrow \Delta t = \frac{\Delta v}{a} = \frac{1.40 \times 10^6\ m/s}{6.33 \times 10^{10}\ m/s^2} = \boxed{2.21 \times 10^{-5}\ s}$

c) $\underbrace{\Delta x = \frac{1}{2} a \Delta t^2}_{\text{const acceleration}} = \left(\frac{1}{2}\left(6.33 \times 10^{10} \frac{m}{s^2}\right)\right)\left(2.21 \times 10^{-5}\ s\right)^2 = \boxed{15.5\ m}$

d) $K = \frac{1}{2} m \Delta v^2 = \frac{1}{2}\left(1.67 \times 10^{-27}\ kg\right)\left(1.40 \times 10^{-3} \frac{m}{s}\right)^2$

$= \boxed{1.64 \times 10^{-15}\ J}$

1.4×10^6