PHYS1406: Physics of Sound and Music

Spring 2021

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Topics we'll cover this semester

- Preliminaries: Basic math, music, and physics terminology
- Physics of oscillations and waves
- Production of sound (instruments and voice)
- Perception of sound (hearing, loudness, pitch & timbre)
- Auditorium and room acoustics; electrical reproduction of sound
- Musical scales and tuning systems (standardization of musical notes)

Why are you in this class?

What questions about sound & music would you like to know the answer to?

What is sound? What differentiates speech, music, & noise?

- Sound is a **pressure wave** in air (or some other medium, which could be a liquid or solid).
- The pressure wave consists of alternating regions of **compression** and **expansion** of the air molecules.
- **Energy is transferred** from the source of sound to our ears, while the individual air molecules just oscillate back-and-forth in place.
- noise: chaotic, unorganized sound
- speech & music: organized sound
- **musical notes** have a **definite pitch** (low or high), while noise does not

Demos: sound measuring devices & musical instruments

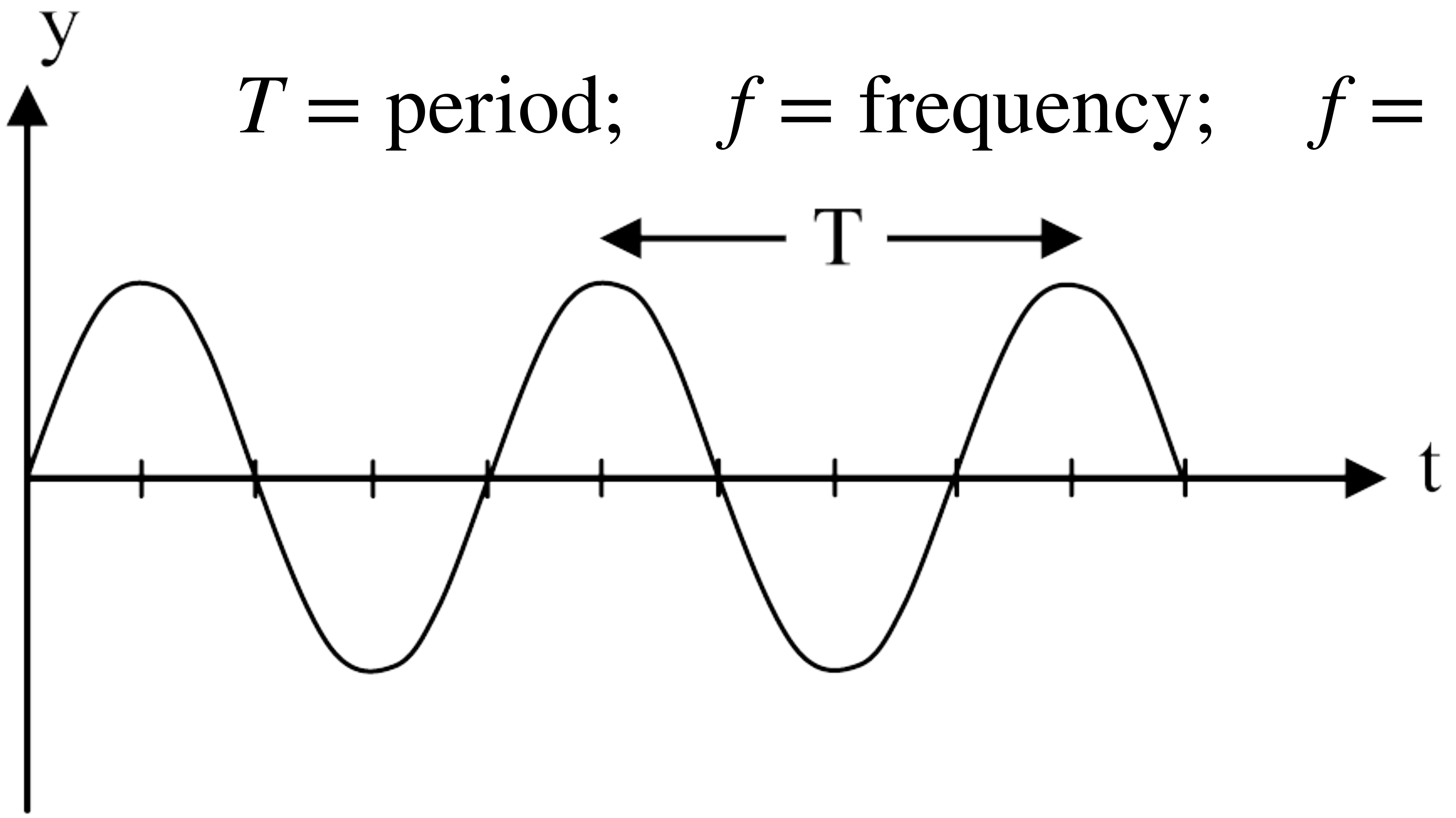
- Measuring devices
  - **oscilloscope**: shows how the sound pressure wave changes in time
  - **FFT analyzer**: shows how much sound energy is associated with different pitch components
  - **spectrogram**: shows how the pitch content of a sound changes in time
- Musical instruments and sound-making devices:
  - whistle, singing, speaking
  - penny whistle, recorder, funny plastic recorder, train whistle, other wind instruments
  - plucked guitar string, bowed violin string
  - bell, drum, shakers, marimba bar, other percussion instruments
  - ratchet, crumpled paper, applause

Range of human hearing

https://www.szynalski.com/tone-generator/

- Normal range: 20 Hz - 20,000 Hz
- What is frequency? Number of repetitions (oscillations, cycles, …) in a given time interval
- Example: Heart rate: 70 beats/1 minute = 1.14 beats/sec
- Hertz (Hz): 1 Hz = 1 cycle/sec

T = period; f = frequency; $f = 1/T$



1. Preliminaries

Basic math review

- Entering numbers on a calculator: What's the value of $1/2\pi$? **Ans:** $1 \div (2 \times \pi) = 0.16$ not $1 \div 2 \times \pi = 1.57$

- Fractions: What's the value of 2 divided by 3/2? **Ans:** $2 \div (3/2) = 2 \times (2/3) = 4/3 = 1.33$

- Powers (exponential notation): What's the value of 2^4? 10^3? 10^{-2}?

  Ans: $2^4 = 2 \times 2 \times 2 \times 2 = 16$; $\quad 10^3 = 100$; $\quad 10^{-2} = 1/10^2 = 0.01$

- Prefixes:

nano	micro	milli	centi	kilo	mega	giga	tera
10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{3}	10^{6}	10^{9}	10^{12}

- Comparing two numbers: Compare the heights of two people, one who is 5.5 ft tall versus another who is 72 inches all.

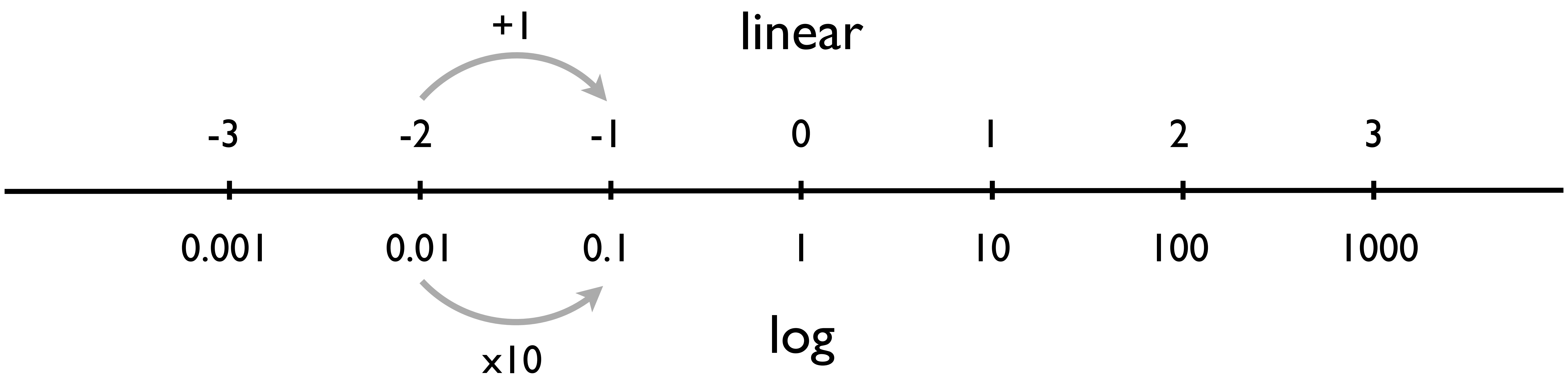
  Ans: First convert 5.5 ft to 66 inches. Then subtract ($72 \text{ in} - 66 \text{ in} = 6 \text{ in}$) or divide ($72 \text{ in}/66 \text{ in} = 1.09$) or calculate percent difference ($100 \times (72 - 66)/66 = 9\,\%$). For music applications, taking ratios or percent differences are most convenient and useful

- Converting units: The speed of sound in air at room temperature (25 celsius) is 346 m/s. What is its value in ft/s? miles/s?

  Ans: Using the conversion factors (1 m = 3.28 ft and 1 mi = 5280 ft), we find:

  $346 \ \frac{\text{m}}{\text{s}} \times \frac{3.28 \text{ ft}}{\text{m}} = 1135 \ \frac{\text{ft}}{\text{s}} \approx 1000 \ \frac{\text{ft}}{\text{s}}$ and $1135 \ \frac{\text{ft}}{\text{s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} = 0.21 \ \frac{\text{mi}}{s} \approx \frac{1 \text{ mi}}{5 \text{ s}}$

Linear vs logarithmic scales

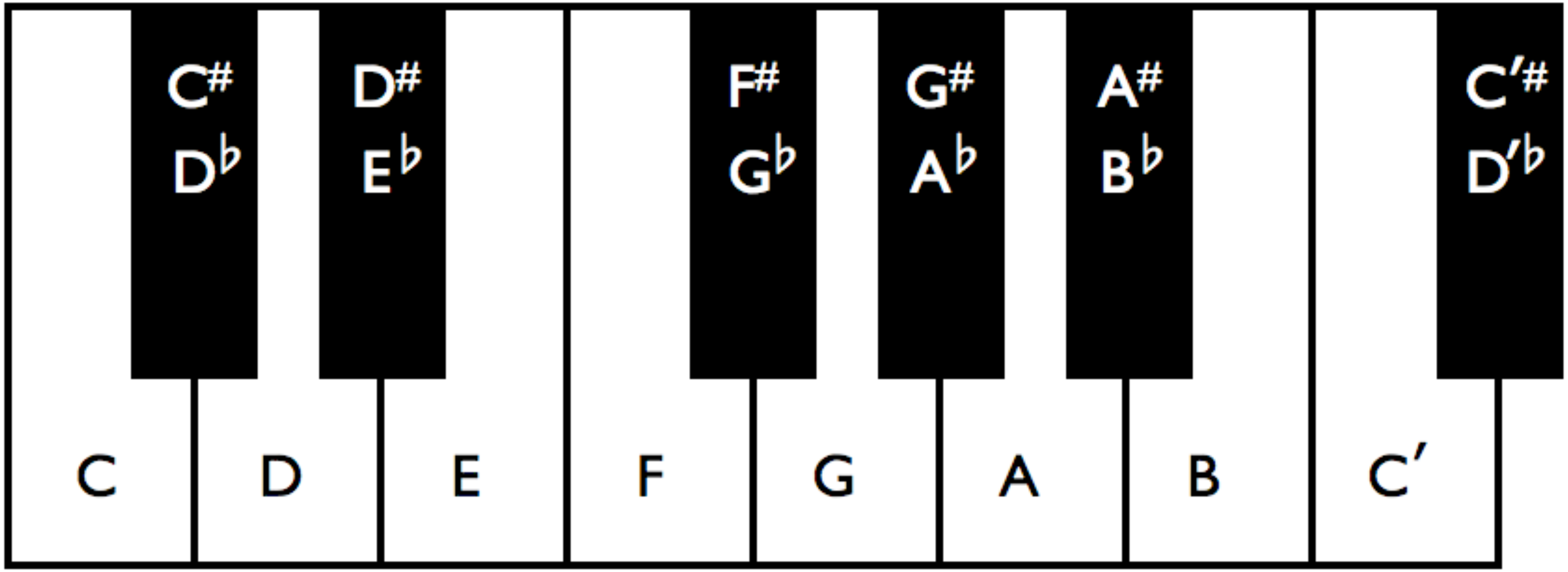


Music terminology

- Pitch: fundamental frequency
- Timbre: richness of a sound, associated with contributions from higher harmonics. It's what makes a guitar sound different from a flute, etc., even though they are all playing the same musical note.
- Octave: Factor of 2 in frequency (e.g., C3 to C4)
- Chromatic, diatonic, and pentatonic scales: divide the octave into 12, 7, and 5 pieces (intervals)
  - https://www.youtube.com/watch?v=jaMA8LWW3C0 (pentatonic scale; all black keys in C-major scale)
- Equal temperament: musical scale where all semitone intervals are equal to one another (6% higher in frequency)
- Musical intervals:
  - **fifth** (C to G; 7 semitones; frequency ratio = 3/2), **fourth** (C to F; 5 semitones; frequency ratio = 4/3), **major third** (C to E; 4 semitones; frequency ratio = 5/4), **minor third** (E to G; 3 semitones; frequency ratio = 6/5)
- Chord: Major chord C-E-G

Chromatic and diatonic scales

C - C# - D- Eb - E - F - F# - G - Ab - A- Bb - B - C′



do – re – mi – fa – sol – la – ti – do

Physics terminology

- Position, displacement: position = location of an object in space, specified by its distance from a reference point and its direction relative to reference axes; displacement = change in position Δx; units for both are m, in, ft, mi, …

- Time, duration: time = reading on a clock; duration = difference in times Δt; units for both are s, min, hr, year, …

- Velocity, speed, acceleration: velocity = displacement/duration; speed = distance traveled/duration; acceleration = change in velocity/duration; velocity and acceleration have both magnitude and direction; units of velocity and speed are m/s, mi/hr, …; units of acceleration are m/s^2, mph/s, …

    - (i) uniform circular motion: changing position, velocity, and acceleration even though speed = constant

    - (ii) mass on a spring: changing position, velocity, and acceleration; velocity = 0 at the turning points of the motion; acceleration = 0 at the equilibrium position where the speed is greatest

- Force, mass, Newton's 2nd law: force = that which produces an acceleration; mass = resistance that an object offers to changes in its state of motion; $a = F/m$ or $F = ma$ (units of force are Newtons or lbs; units of mass are grams or kilograms)

- Density, pressure, atmospheric pressure: density = mass/volume or mass/length; pressure = force/area; units of pressure are N/m^2 or lb/in^2 = psi; 1 atm = 10^5 N/m^2 = 14.5 psi

Exercise

- Calculate the pressure exerted by a 120 lb woman standing on the floor, wearing stilettos having approximately circular heels with radius 0.25 in. Compare to the pressure exerted by a 10,000 lb elephant whose four feet are approximately circles with radius 10 in.

- **Ans:** Recall that the area of a circle is πr^2, where r is the radius.

Woman: $P = F/A = 120 \text{ lb}/(2 \times \pi(0.25 \text{ in})^2) \approx 300 \text{ lb/in}^2$

Elephant: $P = F/A = 10000 \text{ lb}/(4 \times \pi(10 \text{ in})^2) \approx 8 \text{ lb/in}^2$

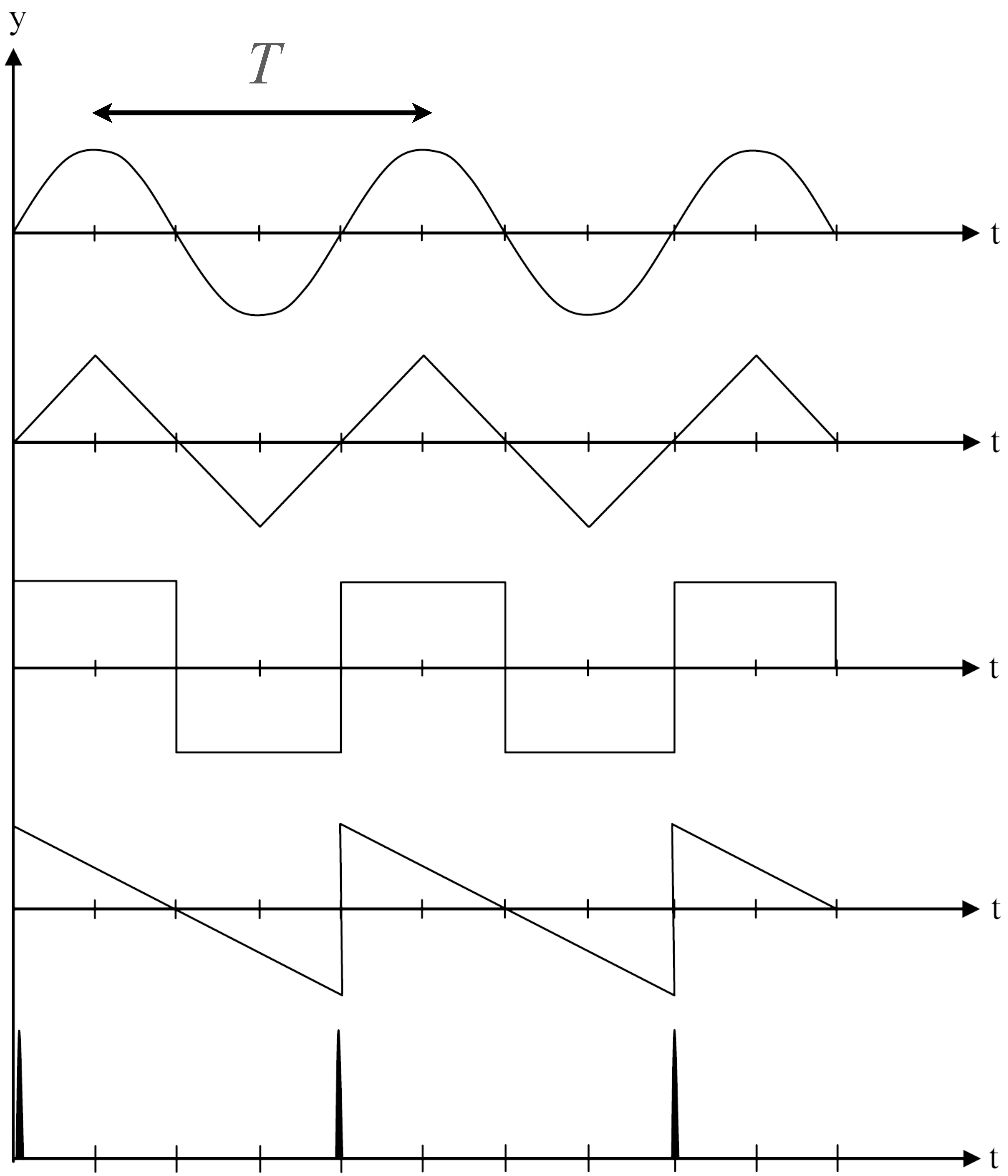
2. Oscillations

Periodic motion

- Oscillation: any motion that repeats
- Examples: vibrating guitar string; vibrating reed of a bassoon; swinging pendulum bob; mass on a spring; yearly orbital motion of Earth around the Sun; beating heart; …
- Period (T), frequency (f): period is the time for one complete oscillation; frequency is the number of oscillations in a given interval of time; $f = 1/T$ or $T = 1/f$. Pitch corresponds to the fundamental frequency of a musical note.
    - **Ques:** What are the periods of sound waves corresponding to the range of human hearing?
    - **Ans:** f = 20 Hz has T = 50 msec; f = 20,000 Hz has T = 50 microsec
- Amplitude: 1/2 the peak-to-peak displacement of an oscillation (related to the loudness of a sound)
- Waveform: the shape of a wave. Different waveforms having the same period (or frequency) sound differently. So the waveform of a sound corresponds to its timbre.

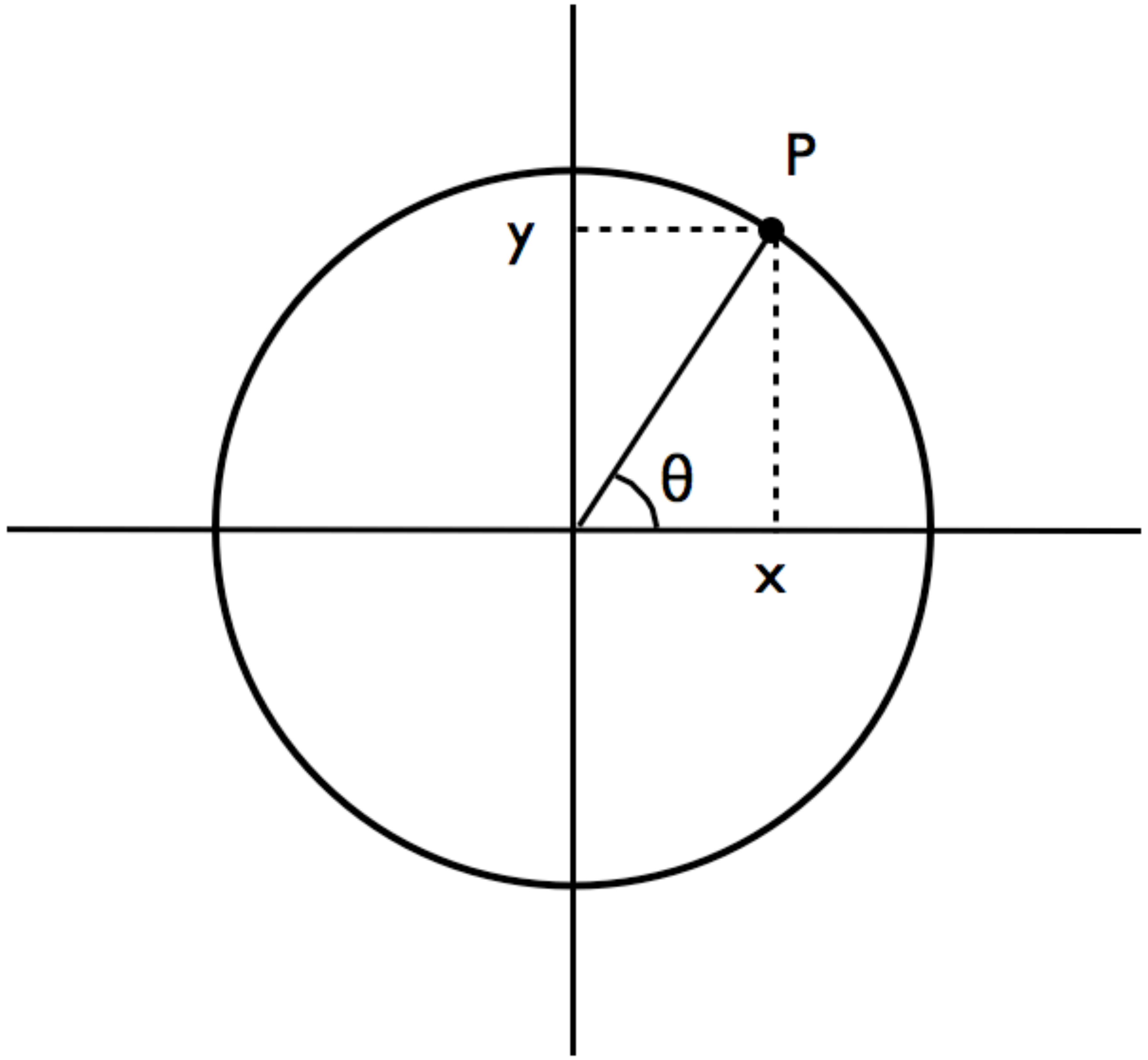
Different waveforms

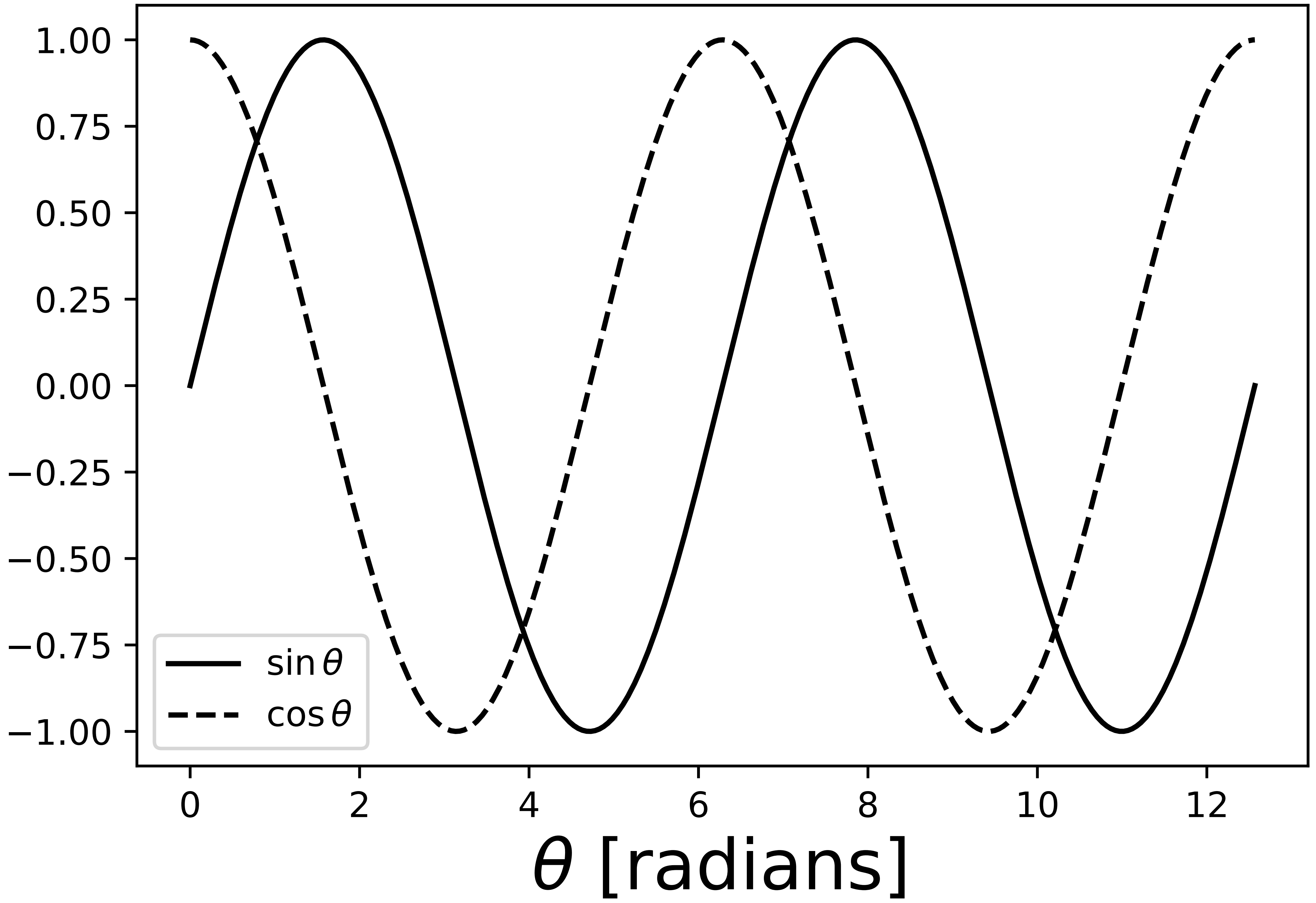
- Demo: Compare sounds
- Although the pitch is the same, the timbre (i.e., sound quality) is different



Simple harmonic motion (SHM)

- Produced whenever you have a **linear restoring force** acting on a system that has a **stable equilibrium**. (Linear means the restoring force is twice is great if the displacement from equilibrium is twice as large.)

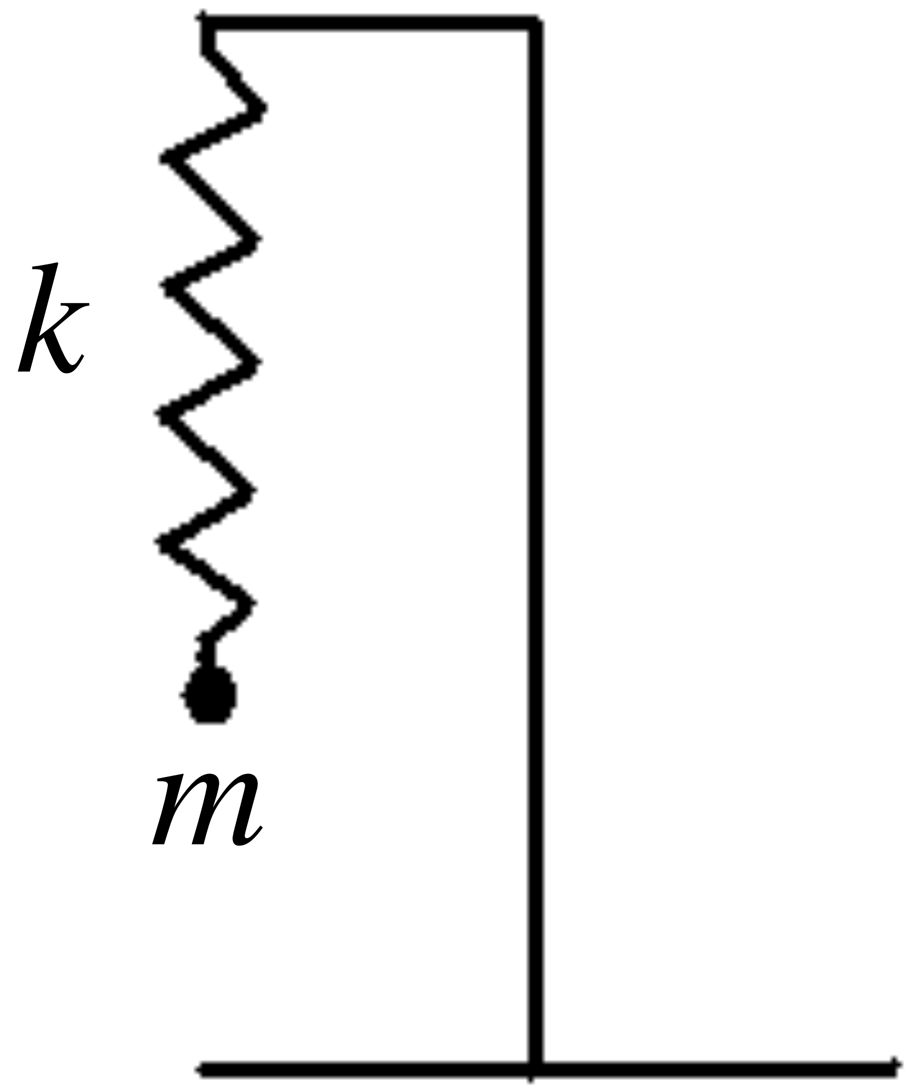




Examples of SHM

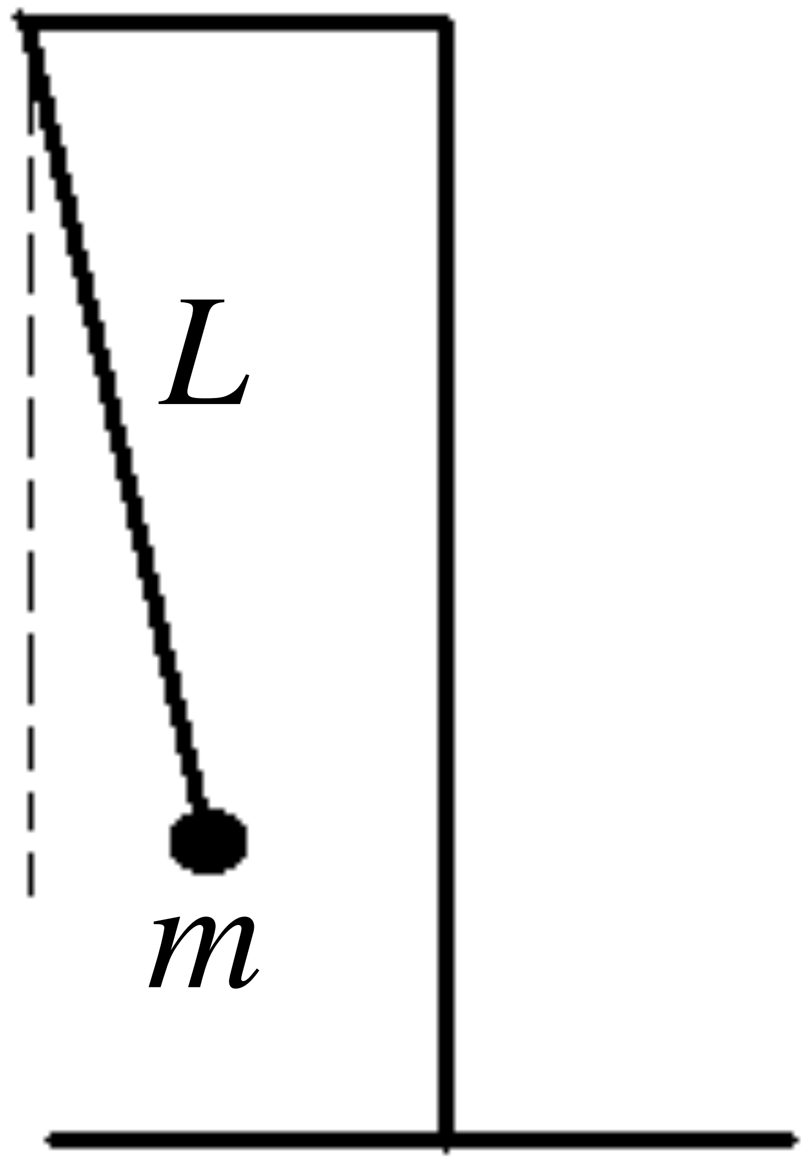
- **Mass on a spring**

$$T = 2\pi\sqrt{\frac{m}{k}}, \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



- **Swinging pendulum bob**

$$T = 2\pi\sqrt{\frac{L}{g}}, \qquad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$



Damping and resonance

- Demo with swinging pendulum
- Friction, air resistance, … cause oscillations to die out. Need to apply a **driving force** to keep them going.
- **Natural frequency** of a swinging pendulum: $f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$
- Compare to **driving frequency** f:
  - $f \ll f_0$: the pendulum bob follows the motion of the driving force
  - $f \gg f_0$: the pendulum bob oscillates back and forth, with a very small amplitude, 180 degrees out of phase with the driving motion
  - $f = f_0$: the amplitude of the swinging motion of the pendulum bob **increases** as the driving force is applied, even for small driving amplitudes. This is called **resonance.**

3. Waves & sound

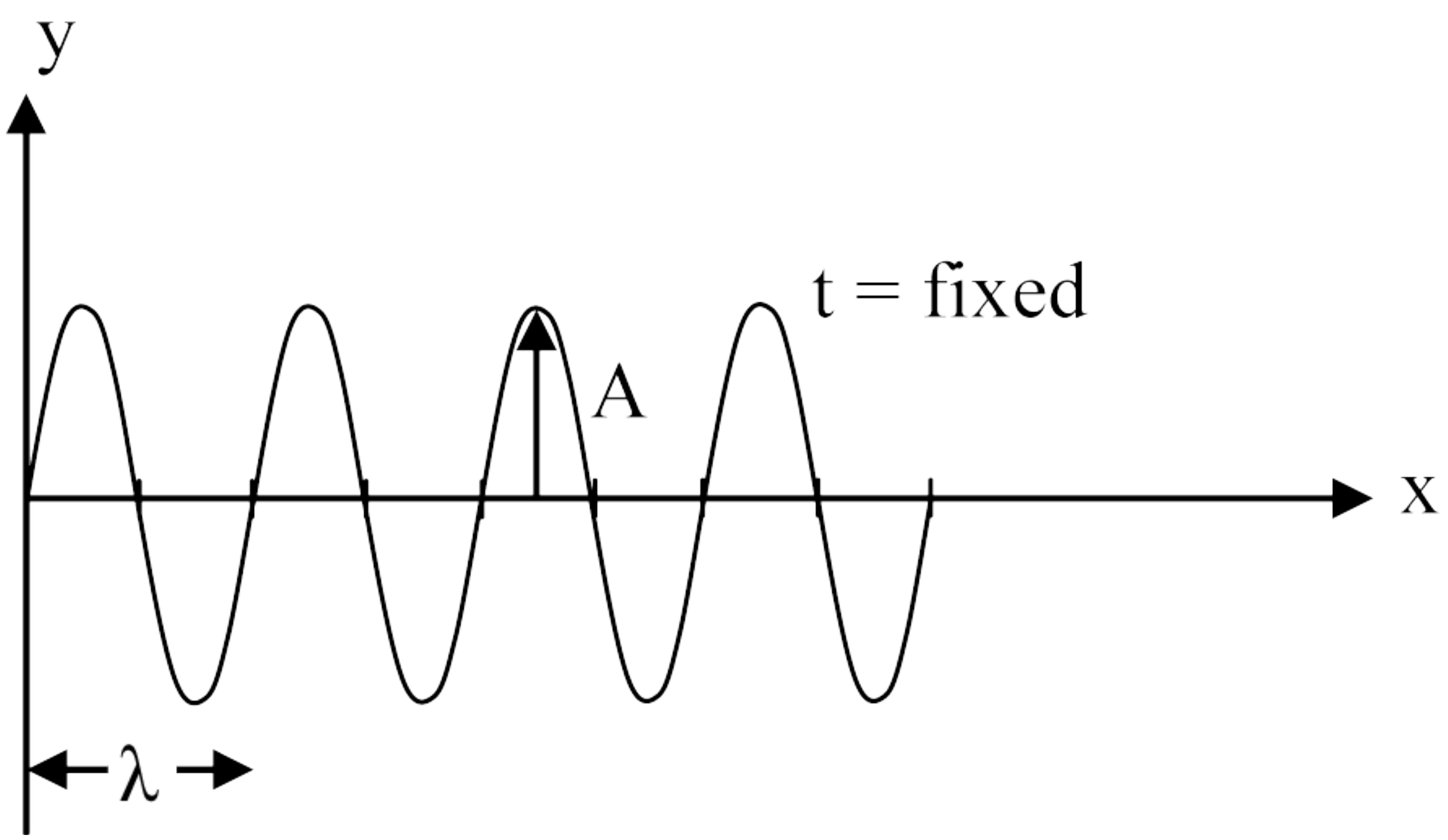
Wave motion, wave velocity

- **wave**: any ”disturbance” that transports energy from one location to another without the transport of matter
- Examples: sound waves in air; light (example of an electromagnetic wave, which can travel through empty space); water waves on the surface of a pond; “wave” at a football game
- **transverse waves**: the disturbance is perpendicular to the direction of wave propagation
- **longitudinal waves**: the disturbance is parallel to the direction of wave propagation
- **wave pulse** vs **periodic waves** traveling waves
- **wave velocity**: $v = \Delta x / \Delta t$
- $v = 346\ \mathrm{m/s}$ (speed of sound in air at room temp, 25 Celsius or 77 Farenheit)
- $v = 331\ \dfrac{\mathrm{m}}{\mathrm{s}}\sqrt{1 + \dfrac{T_C}{273.15}}$ (speed of sound in air increases with increasing temperature)

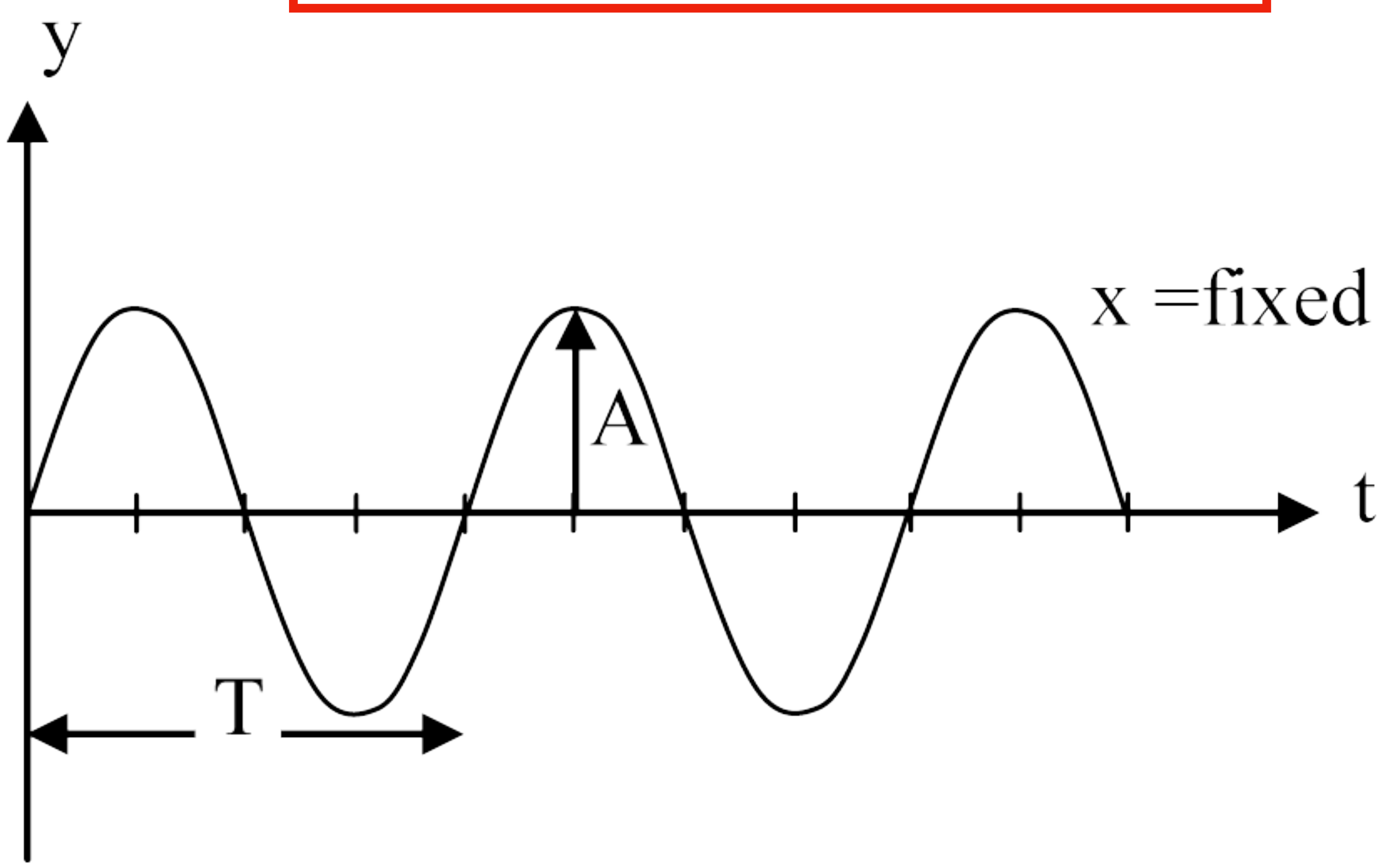
Periodic waves, wavelength

- Produced by **periodic sources**; λ is the **wavelength**

$$\boxed{v = \lambda/T = f\lambda}$$



"**snap shot**" at a fixed time, showing how the displacement varies with position



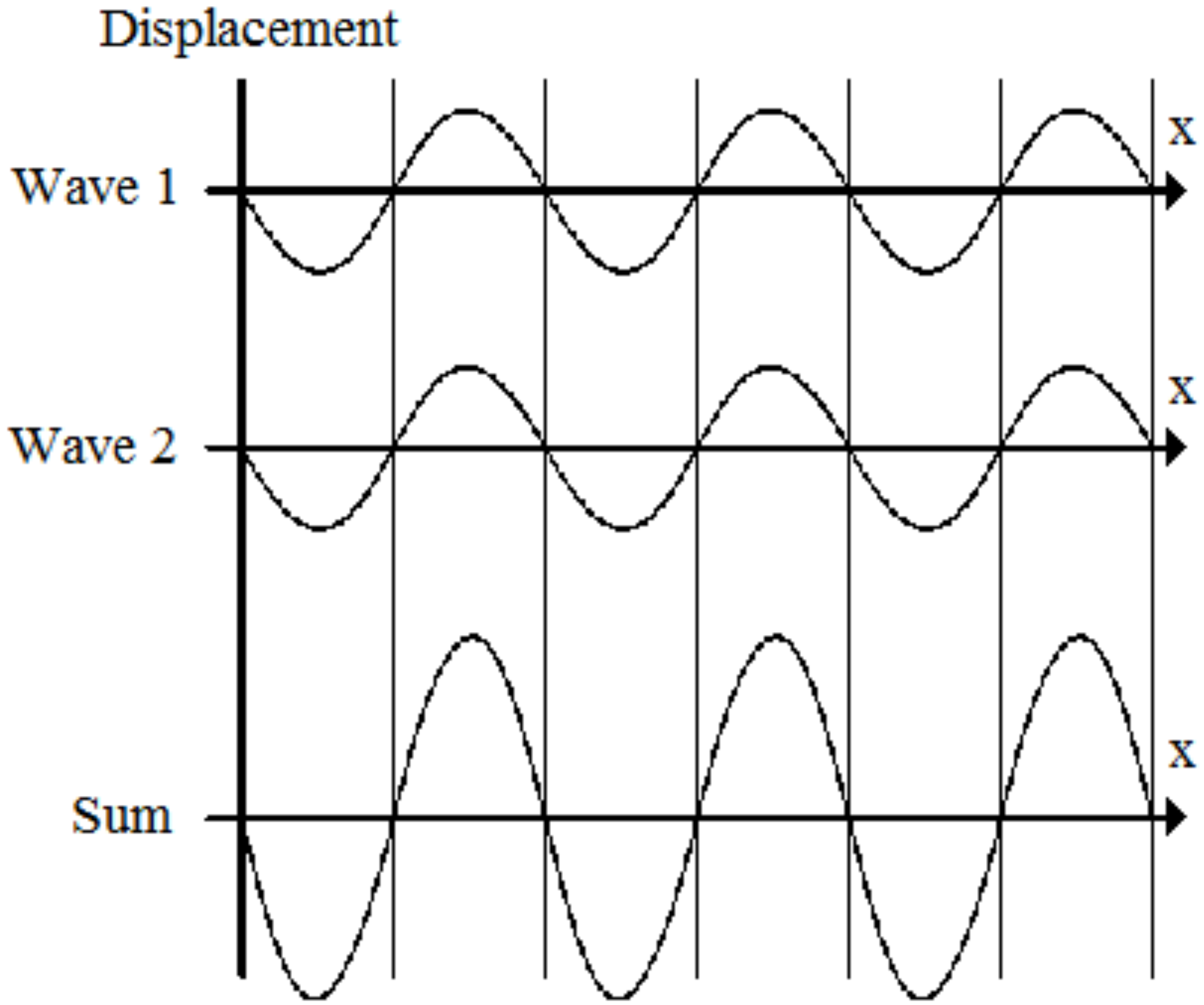
fixed location, showing how the displacement varies with time

Exercise

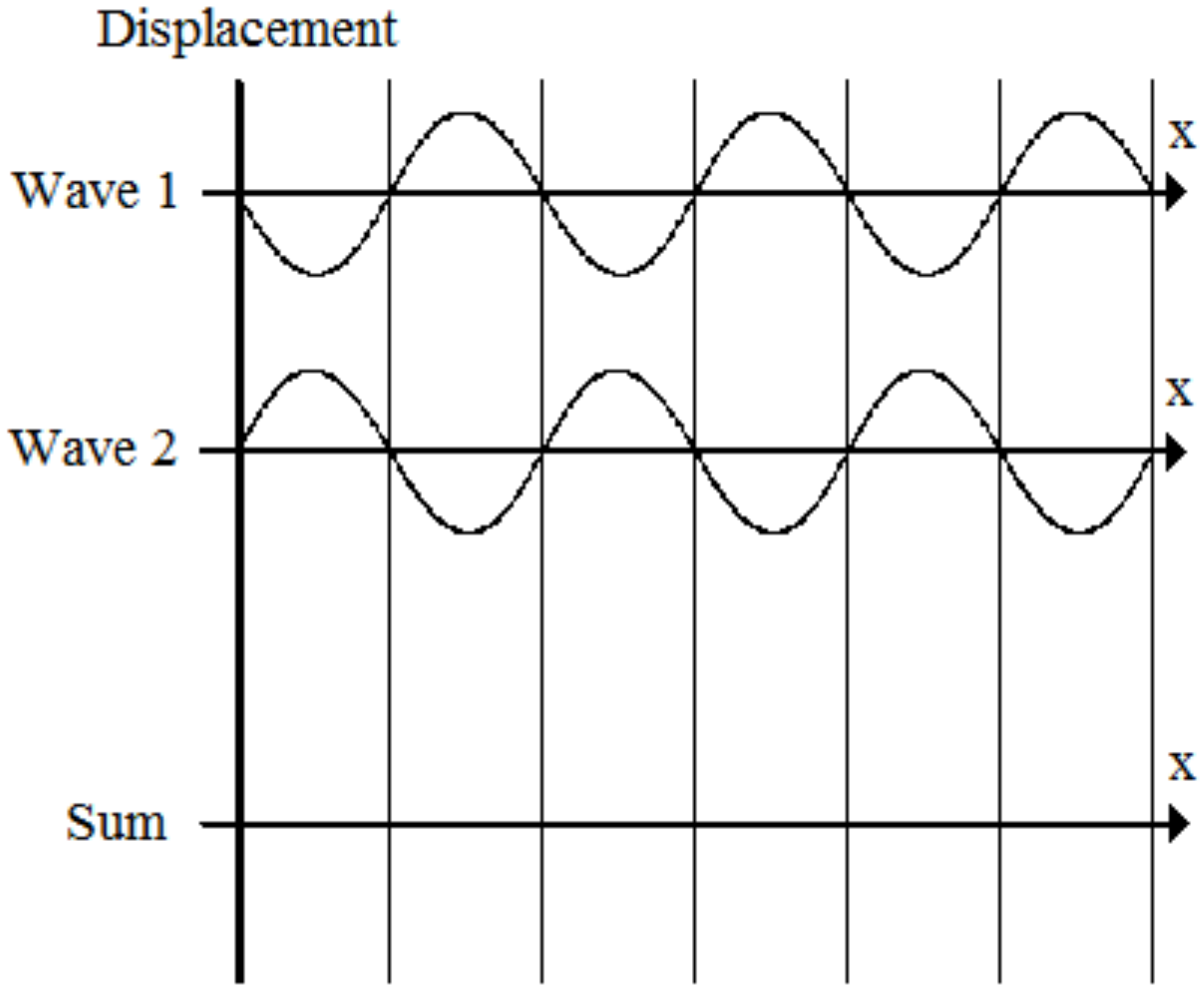
- Calculate the wavelengths of sound corresponding to the range of human hearing. Use $v = 346\ \mathrm{m/s}$ for the speed of sound in air at room temperature.
- **Ans:**
    - $f = 20\ \mathrm{Hz}$ has $\lambda \approx 17\ \mathrm{m}$
    - $f = 20{,}000\ \mathrm{Hz}$ has $\lambda \approx 1.7\ \mathrm{cm}$
    - Most musical sounds have wavelengths of order 1 meter

Superposition / interference

- the **combination of two waves** is another wave
- **constructive** & **destructive** interference depends on the **phase** difference (matlab demo)



constructive



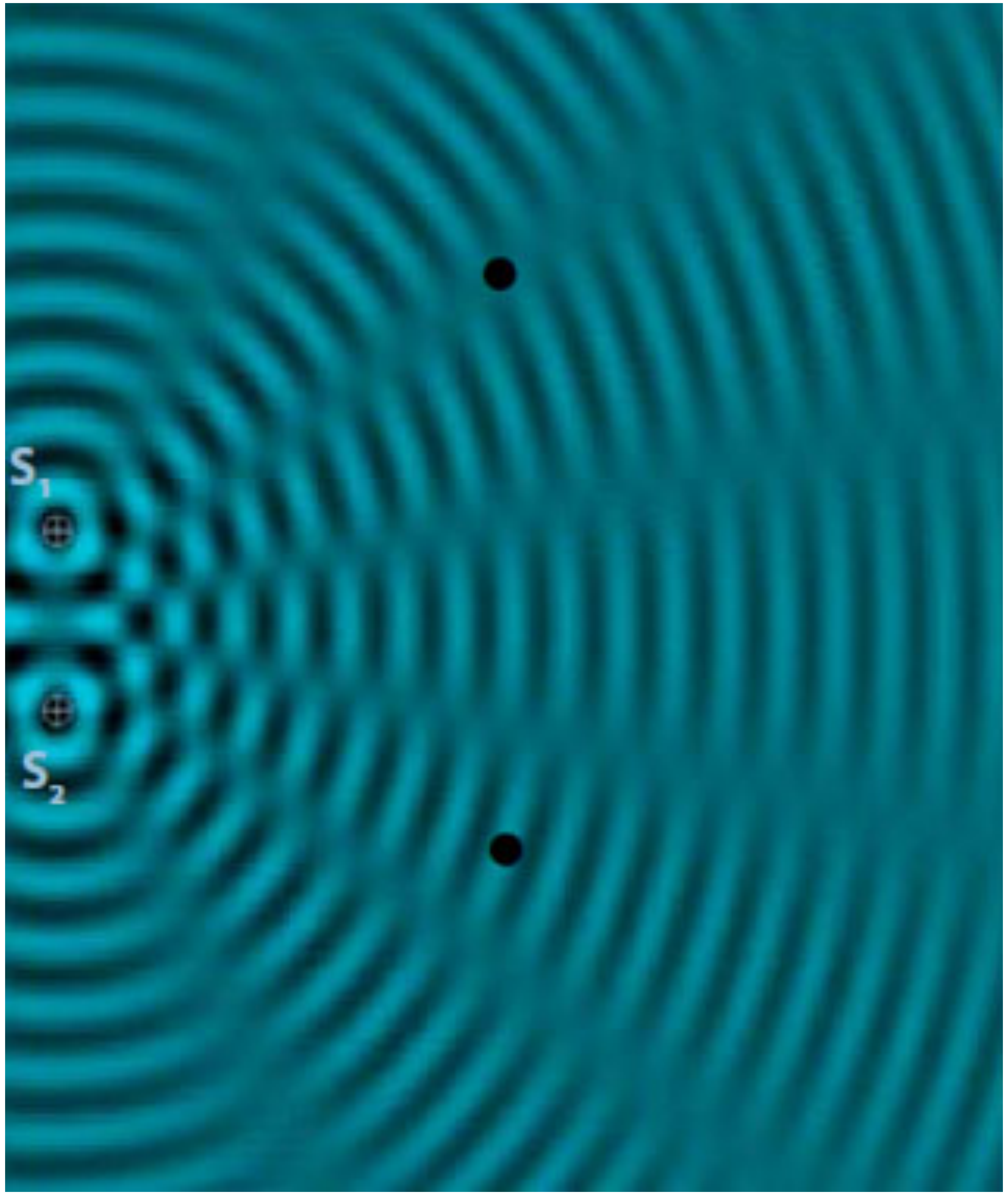
destructive

Examples

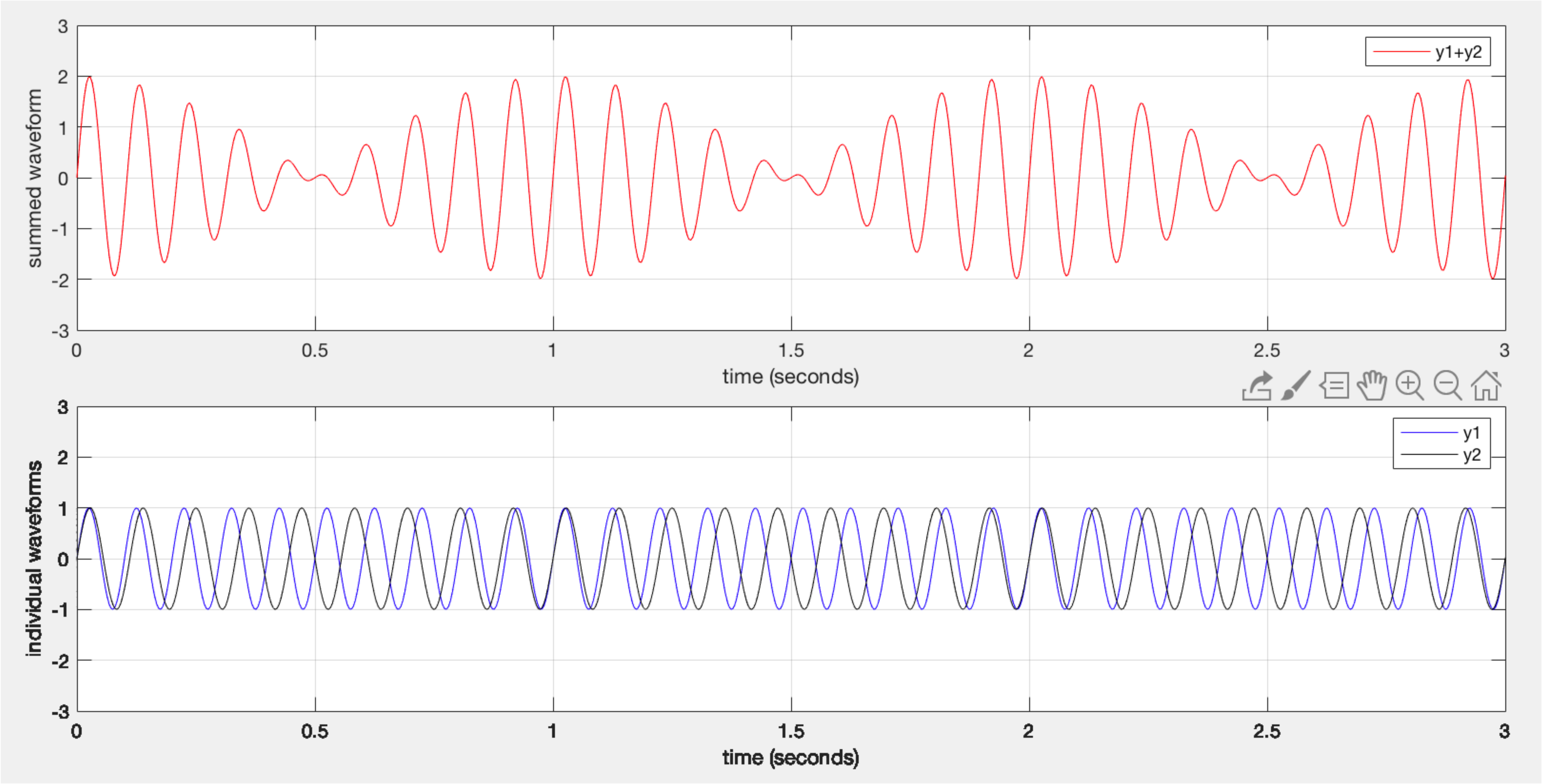
- Water waves in a ripple tank or sound waves produced by two speakers (interference in space)

- **Beats** (interference in time of two periodic waves having different frequencies; matlab demo, and using signal generators)

- Beat frequency: $f_{\text{beat}} = |f_1 - f_2|$

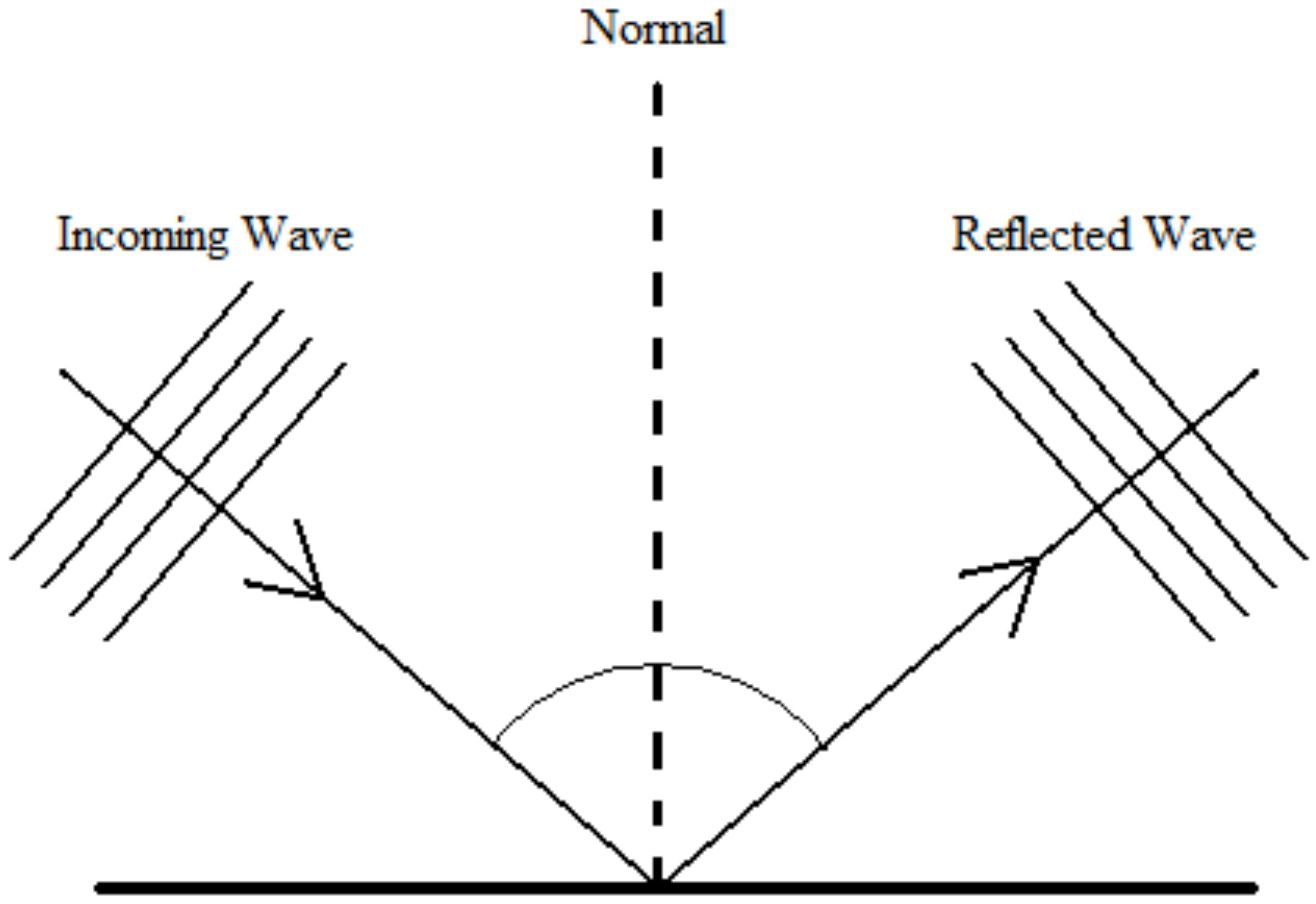


Demo showing beats

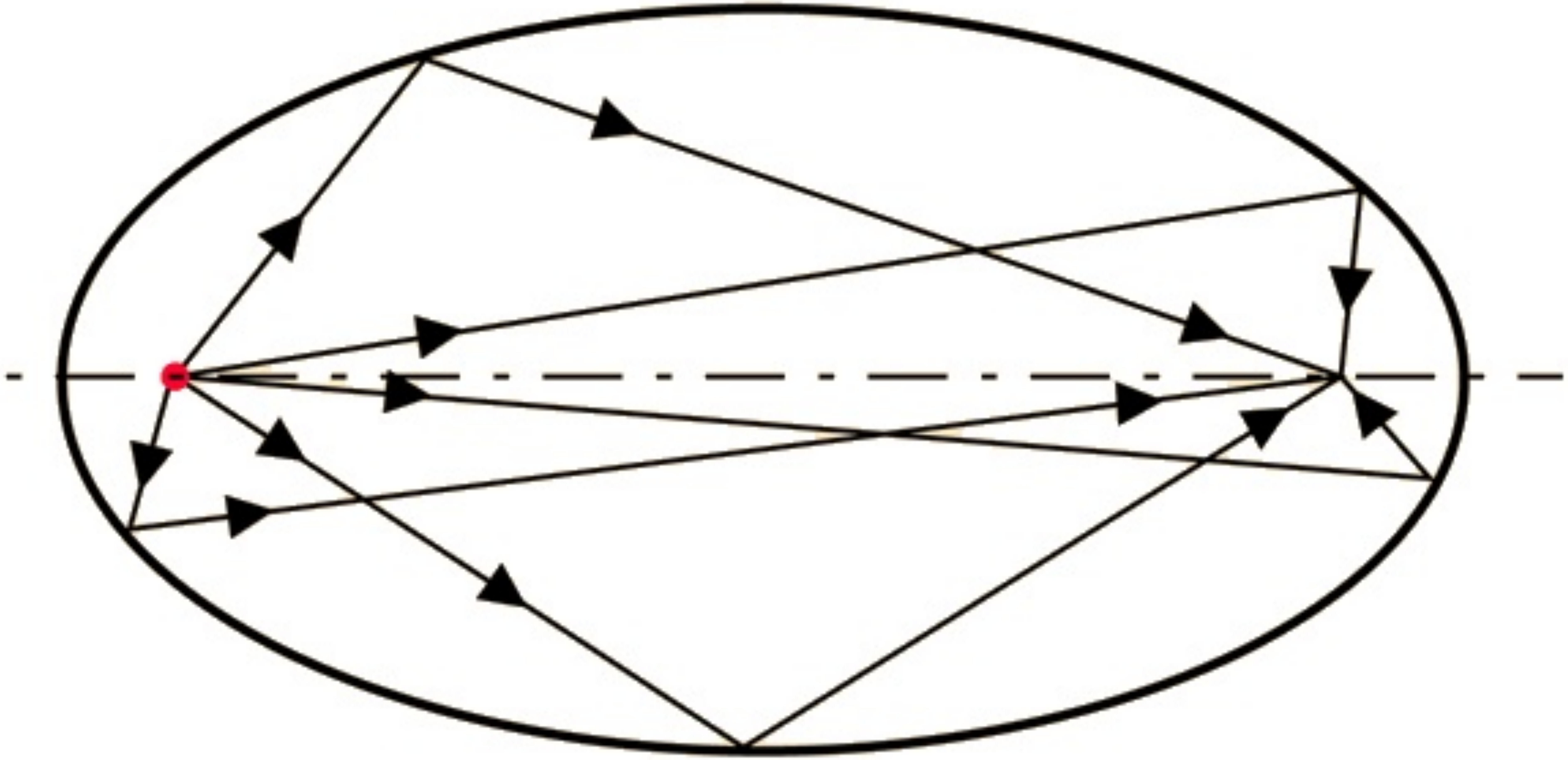


Reflection

- **Change in direction** of a wave when it encounters an interface between two media
- Demos with plane, concave, and convex mirrors

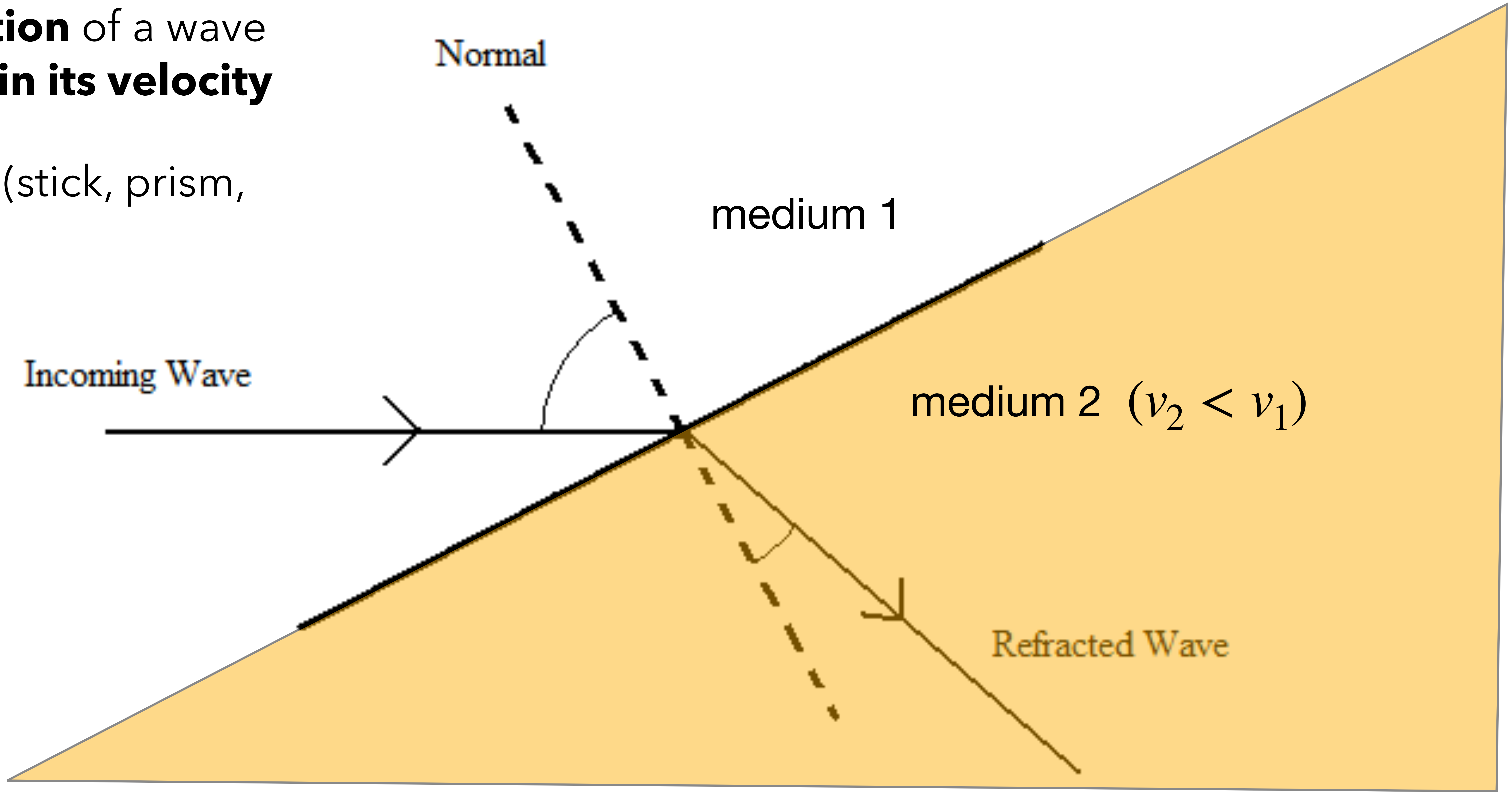


Reflections - whispering chamber

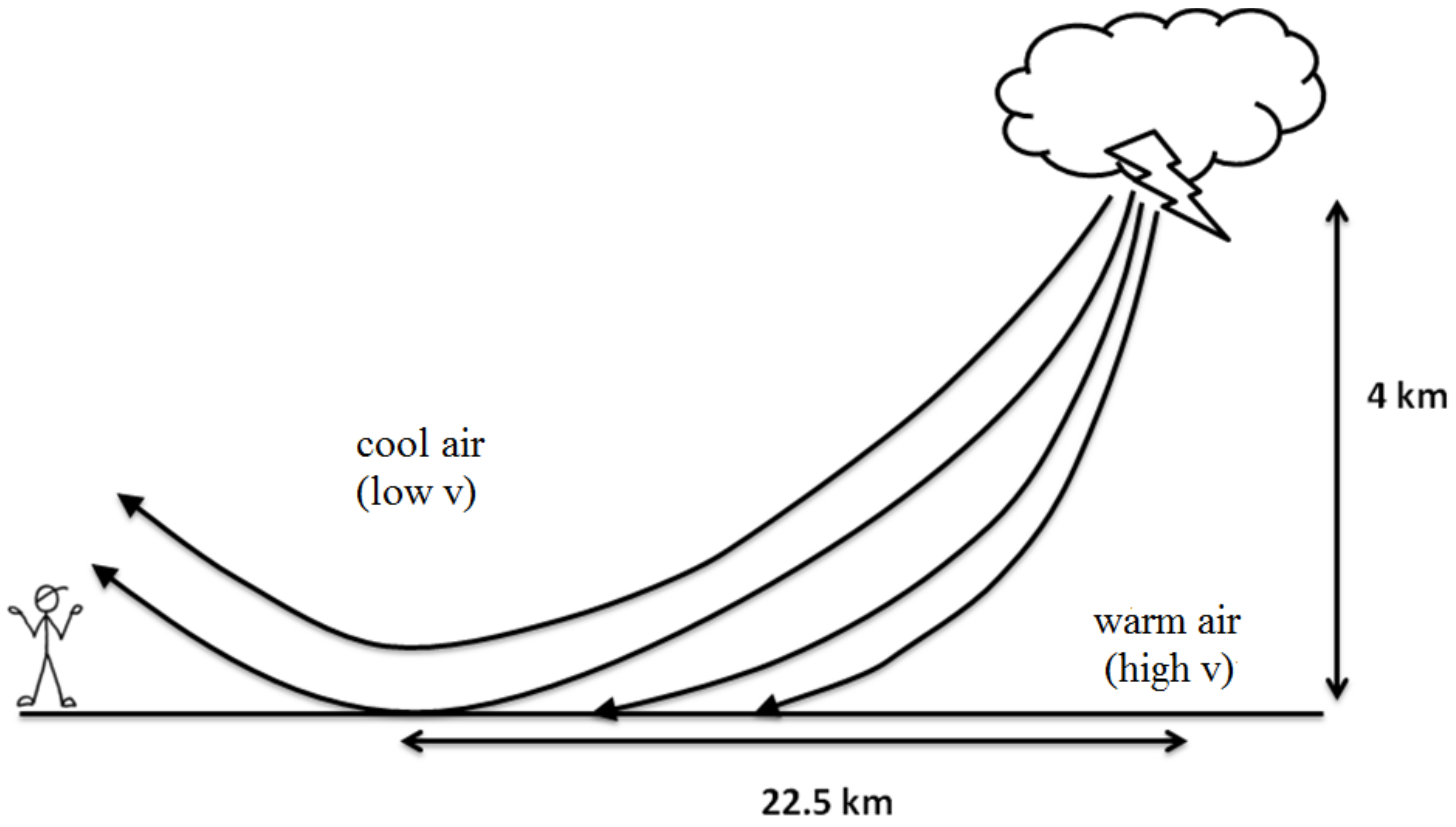


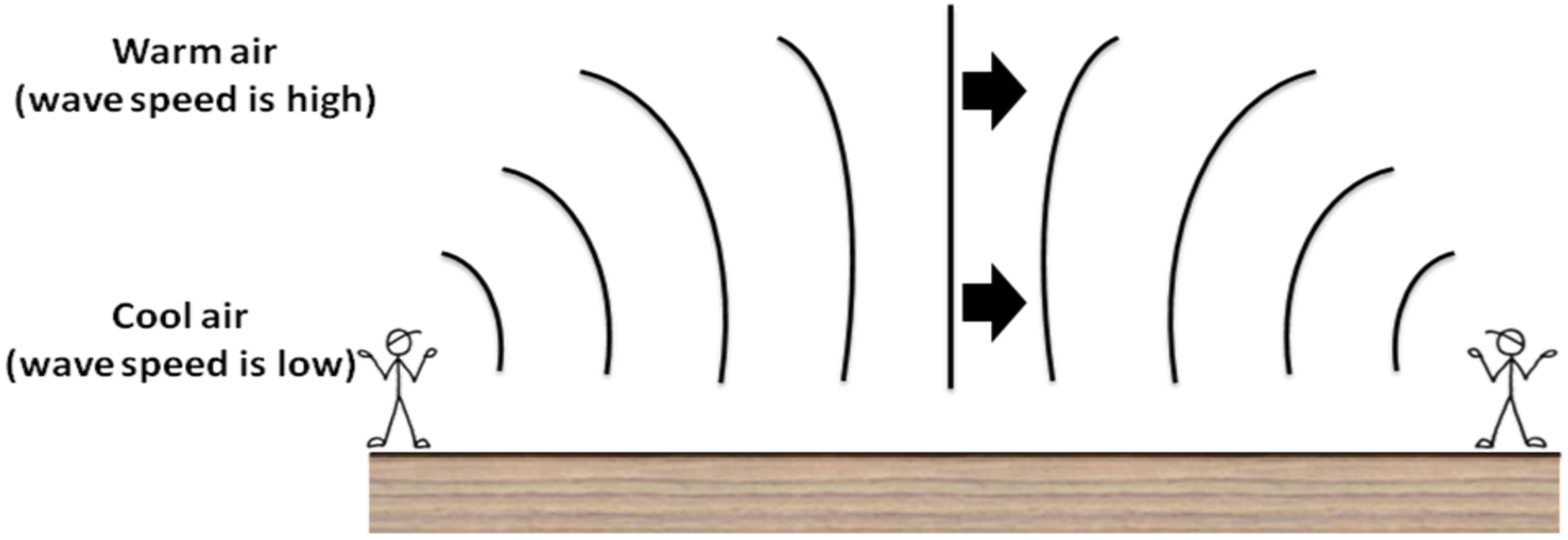
Refraction

- **Change in direction** of a wave due to a **change in its velocity**
- Demos with light (stick, prism, and laser)



Examples of refraction of sound

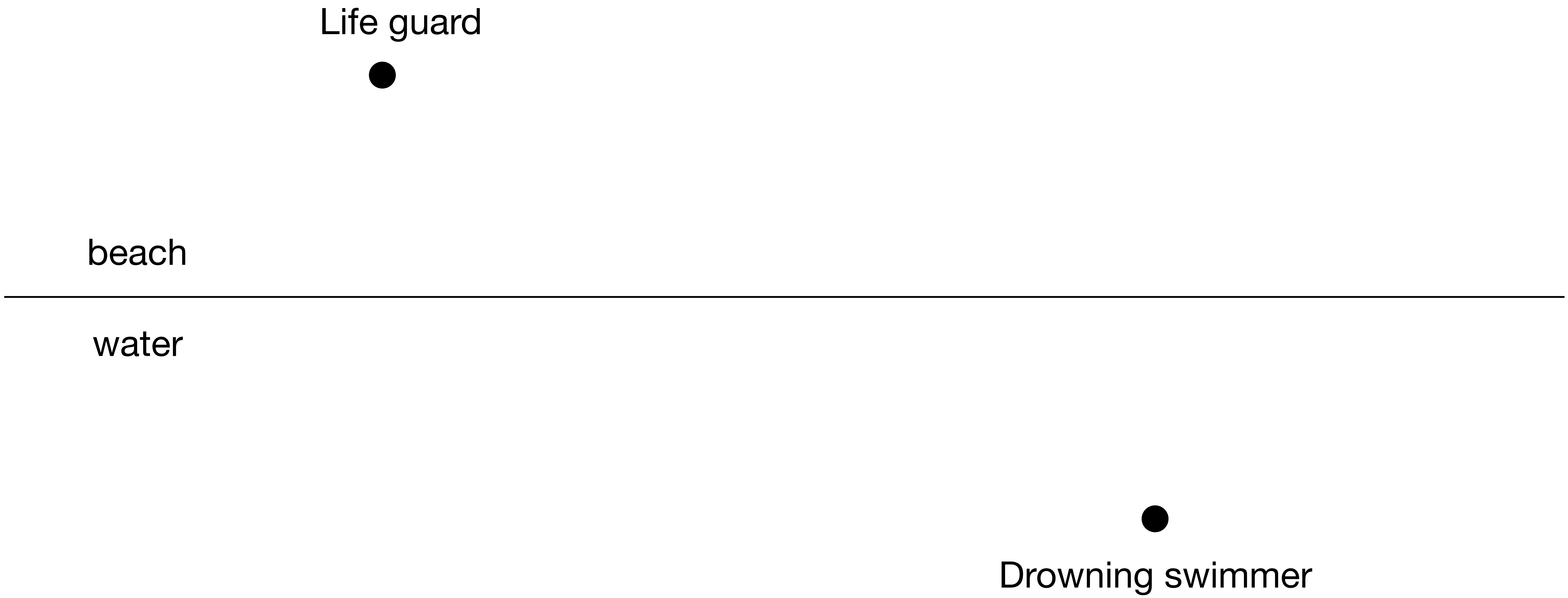




Usual temperature distribution

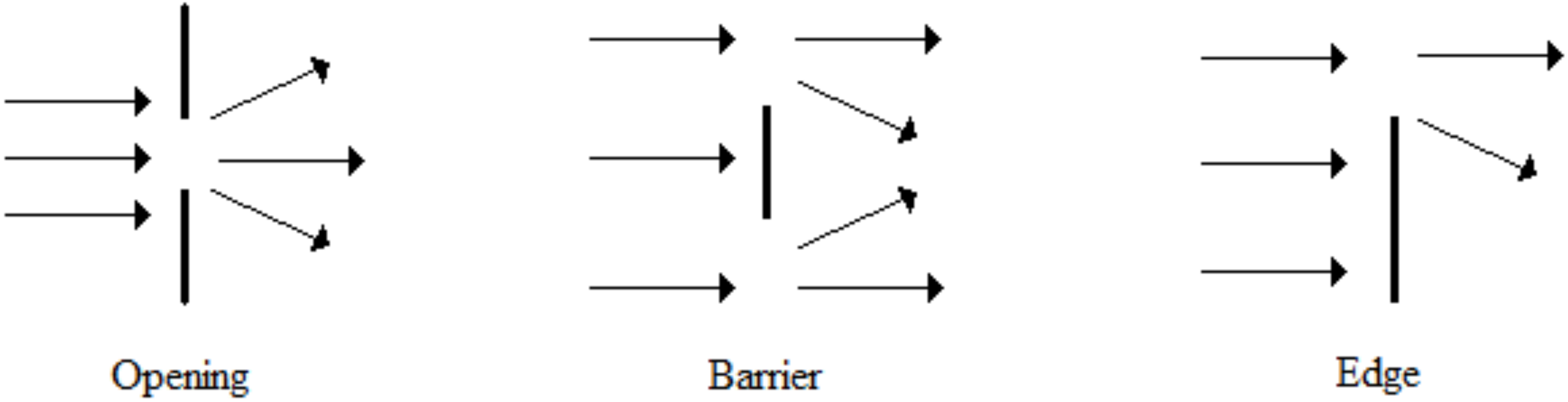
Temperature inversion

Understanding refraction



Diffraction

- **"spreading" of a wave** as it passes through openings or around barriers …
- amount of spreading depends on **relative size of wavelength and opening, barrier,** …
- **a lot of spreading** into the shadow regions if the **wavelength is greater than or comparable to** the size of opening, barrrier, …
- reason why you can hear somebody speaking in the hallway, but you can't see him / her

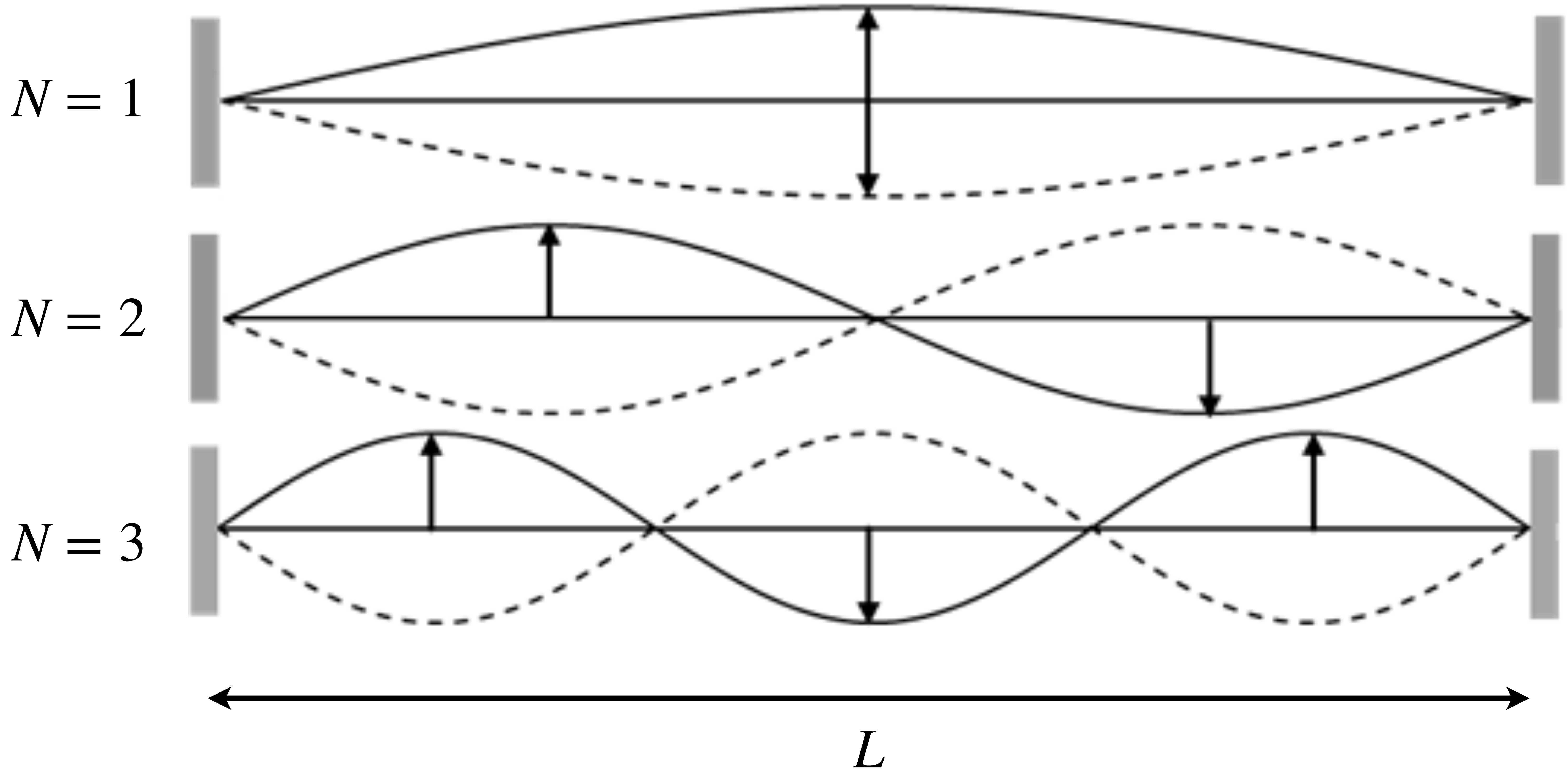


Doppler effect

- **Change in frequency** due to the motion of the observer or source (e.g., for sound, light, …)
- Examples: siren on an approaching / receding police car or ambulance; a train whistle; …
- Demo with nerf ball

Standing waves on a string

- Demo: superposition of right-moving and left-moving waves on a string
- occurs only for certain frequencies (**resonance** phenomenon; matlab demonstration)



$$v = \sqrt{\frac{\text{string tension}}{\text{mass density}}} = \sqrt{\frac{F}{\mu}}$$

$$\lambda_N = \frac{2L}{N}, \quad N = 1,2,\cdots$$

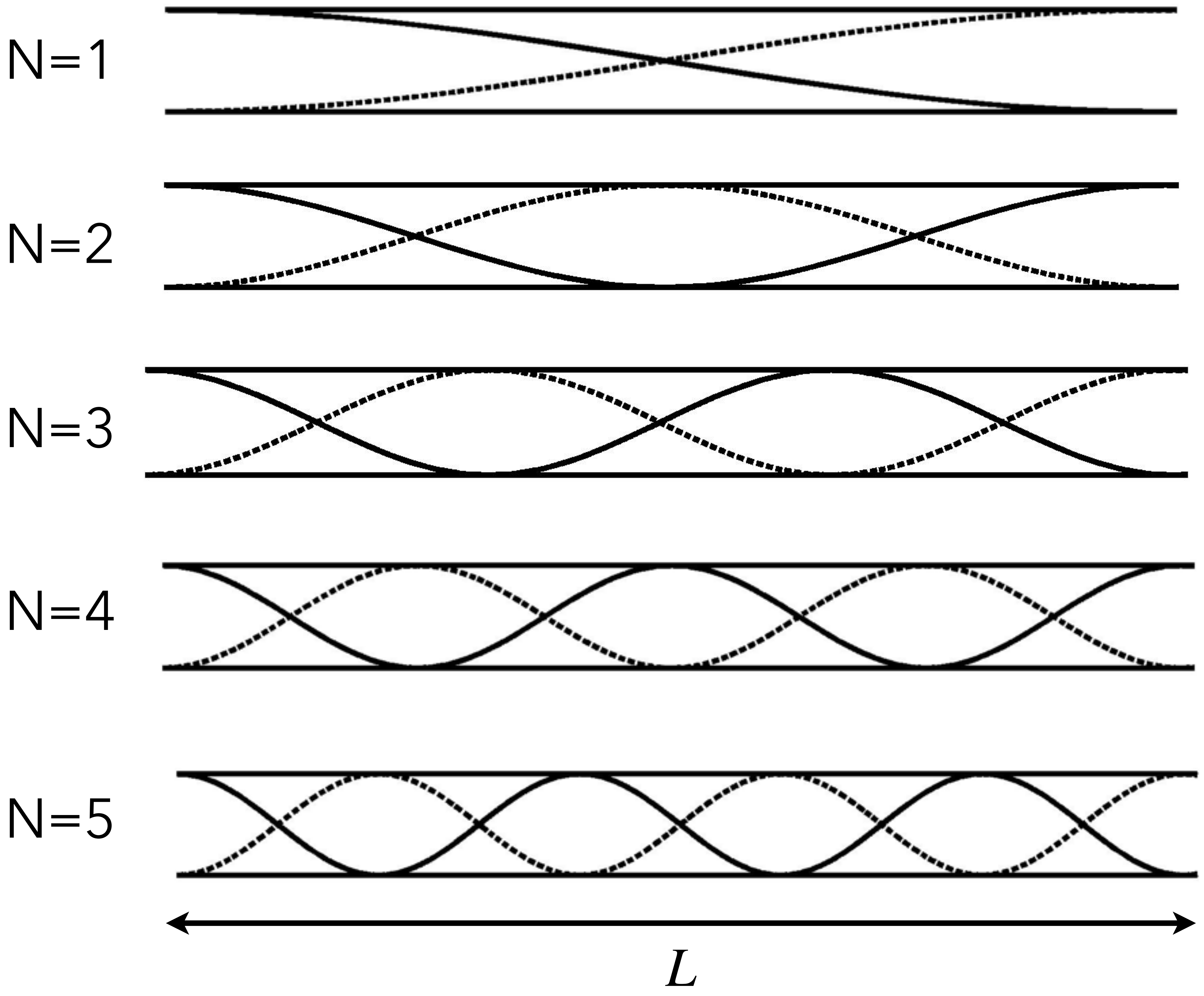
$$f_N = N f_1$$

$$f_1 = \frac{v}{2L} \qquad \text{(fundamental)}$$

Standing waves in a tube

(only odd harmonics!!)

open at both ends



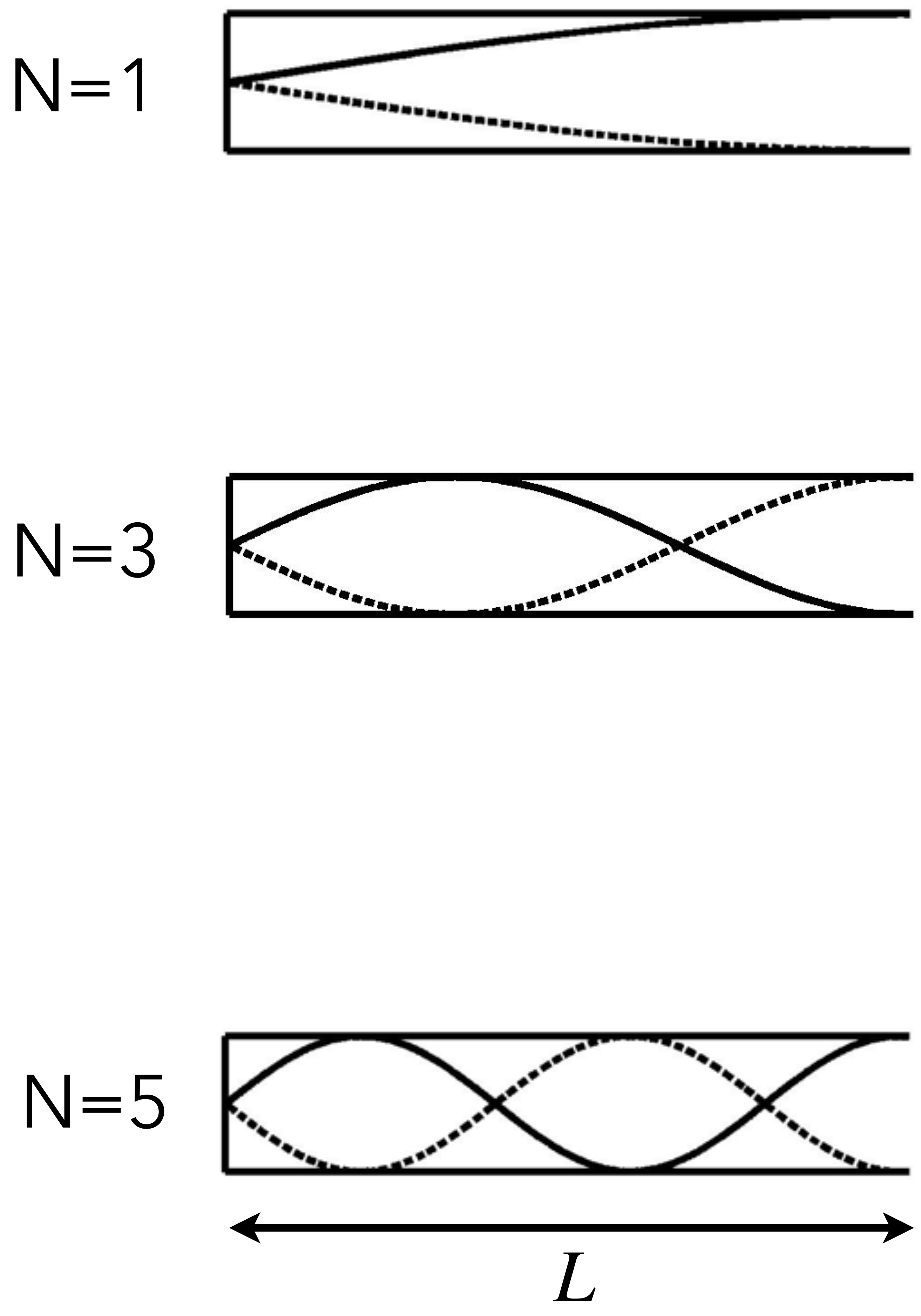
$$\lambda_N = \frac{2L}{N}$$

$$f_N = Nf_1$$

$$f_1 = \frac{v}{2L}$$

$$N = 1,2,\cdots$$

closed at one end



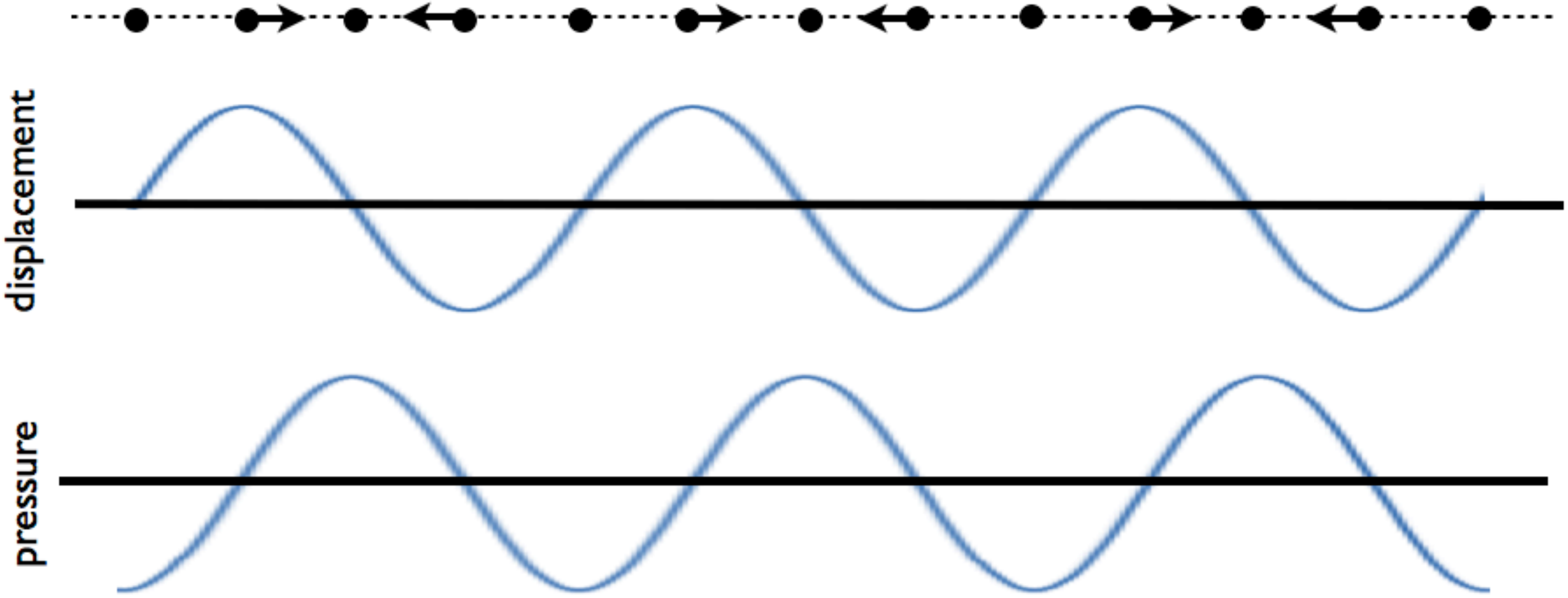
$$\lambda_N = \frac{4L}{N}$$

$$f_N = Nf_1$$

$$f_1 = \frac{v}{4L}$$

$$N = 1,3,5,\cdots$$

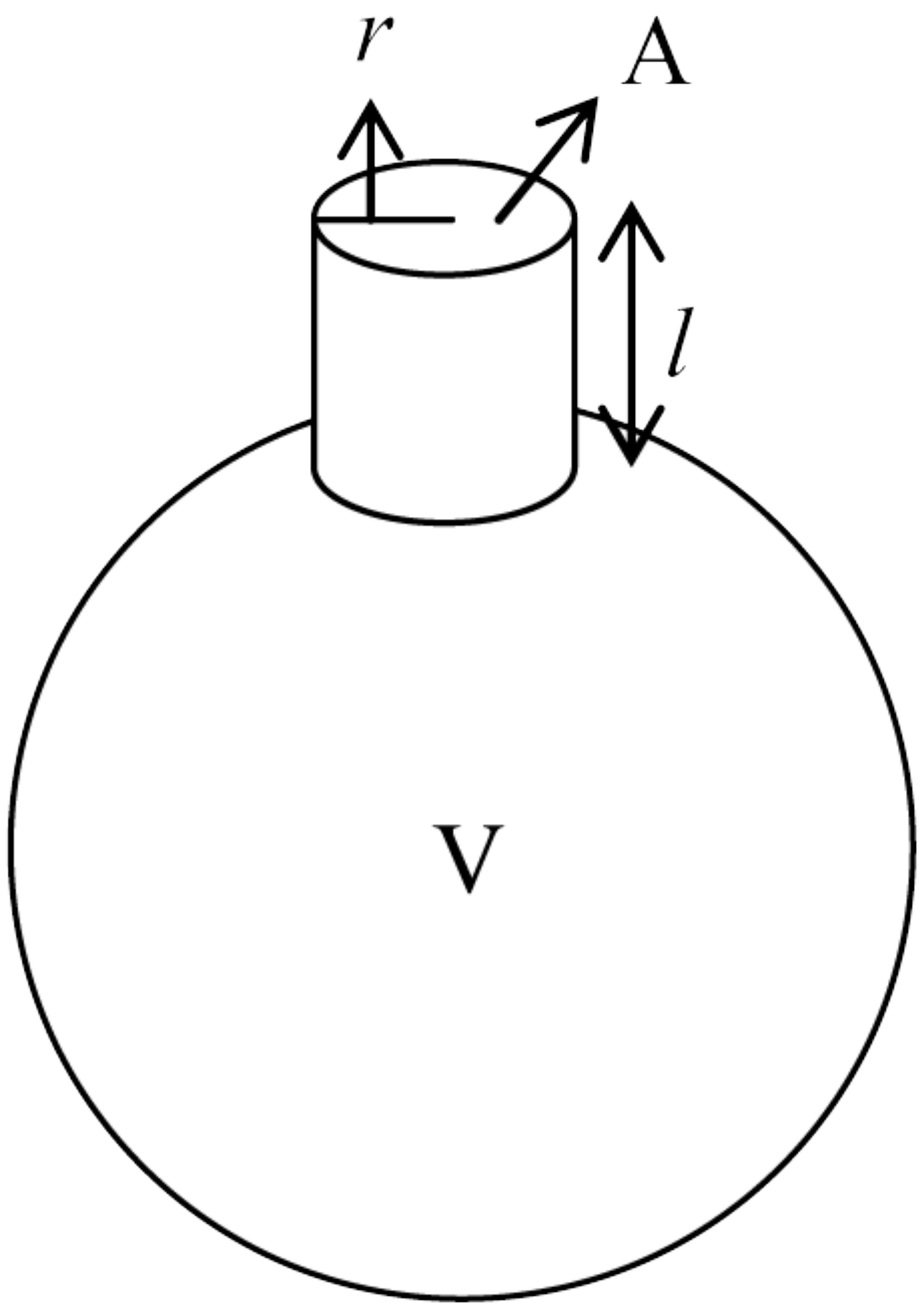
Air molecule displacement vs pressure deviation



Effective length, and “slap tube” determination of the speed of sound

- $L_{\text{eff, closed}} = L + 0.61r$ and $L_{\text{eff, open}} = L + 1.22r$ (where r is the radius of the tube)
- “slap tube” is closed at one end ($L = 38.5\ \text{cm},\ \text{diameter} = 2.5\ \text{cm}$)
- Determine fundamental frequency, then solve for wave velocity

Helmholtz resonator



$$f = \frac{v}{2\pi}\sqrt{\frac{A}{l_{\mathrm{eff}}V}}$$

- Example:

$r = 1\ \mathrm{cm}\,,\ l = 2.7\ \mathrm{cm}\,, V = 425\ \mathrm{mL}\,, v = 346\ \mathrm{m/s}$

$A = \pi r^2\,,\ 1\ \mathrm{mL} = 10^{-6}\ \mathrm{m}^3 \quad \Rightarrow \quad f = 239\ \mathrm{Hz}$

4. Fourier analysis & synthesis

Fourier's theorem

- **standing wave vibrations** are the **"building blocks"** for any complex vibration
- any complex periodic wave can be written as a **sum of harmonics**:

$$y(t) = A_1 \sin(2\pi f_1 t + \phi_1) + A_2 \sin(2\pi f_2 t + \phi_2) + \cdots$$

$$f_N = N f_1, \qquad N = 1,2,\cdots$$

- **Ohm's law of hearing**: Phases have little effect on the timbre of the sound
- **Fourier analysis**: decomposing a complex periodic wave into its contributing harmonics
- **Fourier synthesis**: constructing a complex periodic wave by combining harmonics

5. String instruments

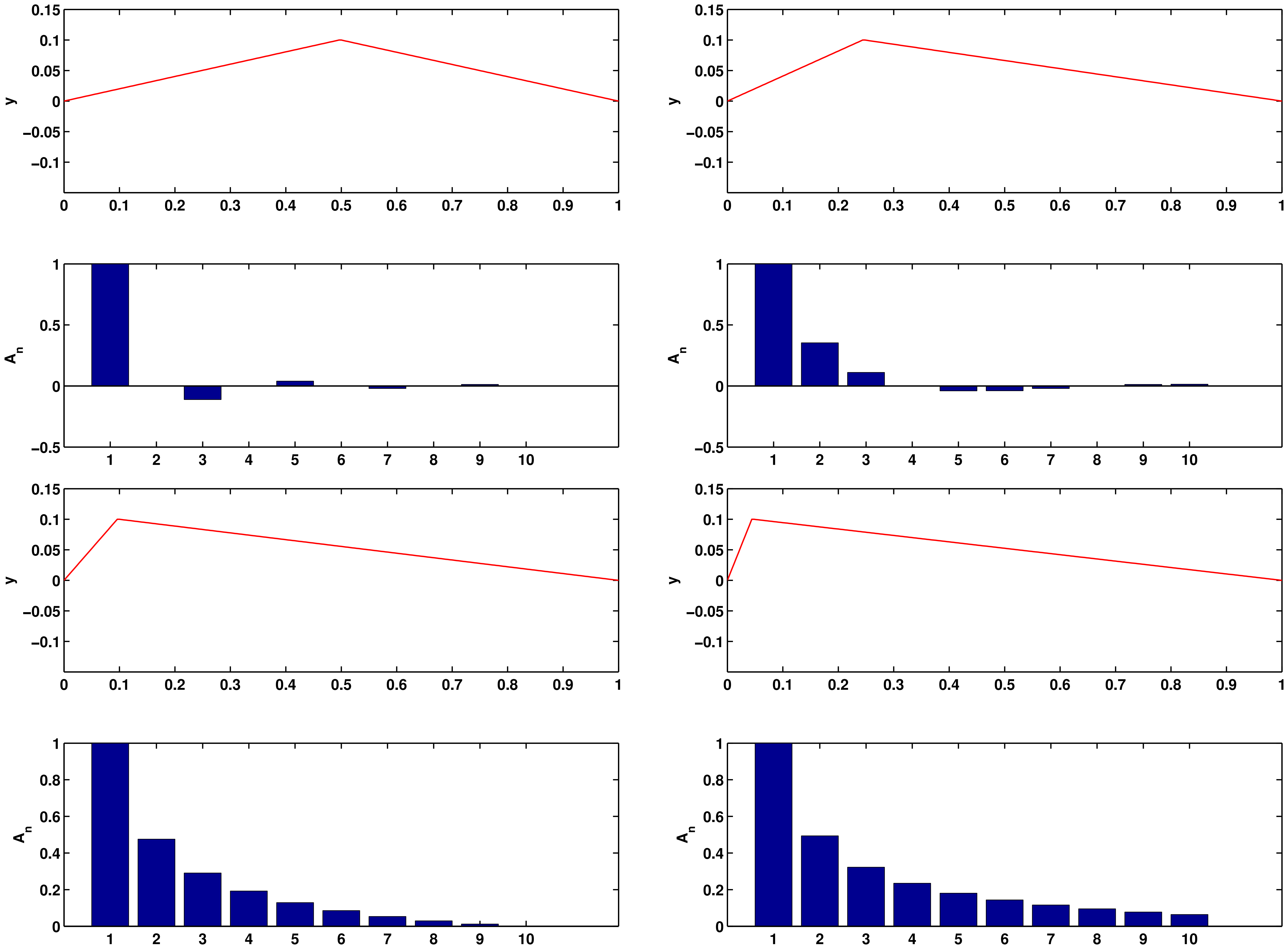
Plucked versus bowed strings

- Plucked string: https://www.youtube.com/watch?v=_X72on6CSL0
- Bowed string: https://www.youtube.com/watch?v=6JeyiM0YNo4
- iPhone guitar video: https://www.youtube.com/watch?v=TKF6nFzpHBU

- NOTE: the iPhone guitar video does not show the wave pulses on the strings as they really are. Rather one sees multiple images of the same pulse shape on the string due to the "rolling shutter" effect of the iPhone camera. The actual pulses on a guitar string behave as shown in the first video.

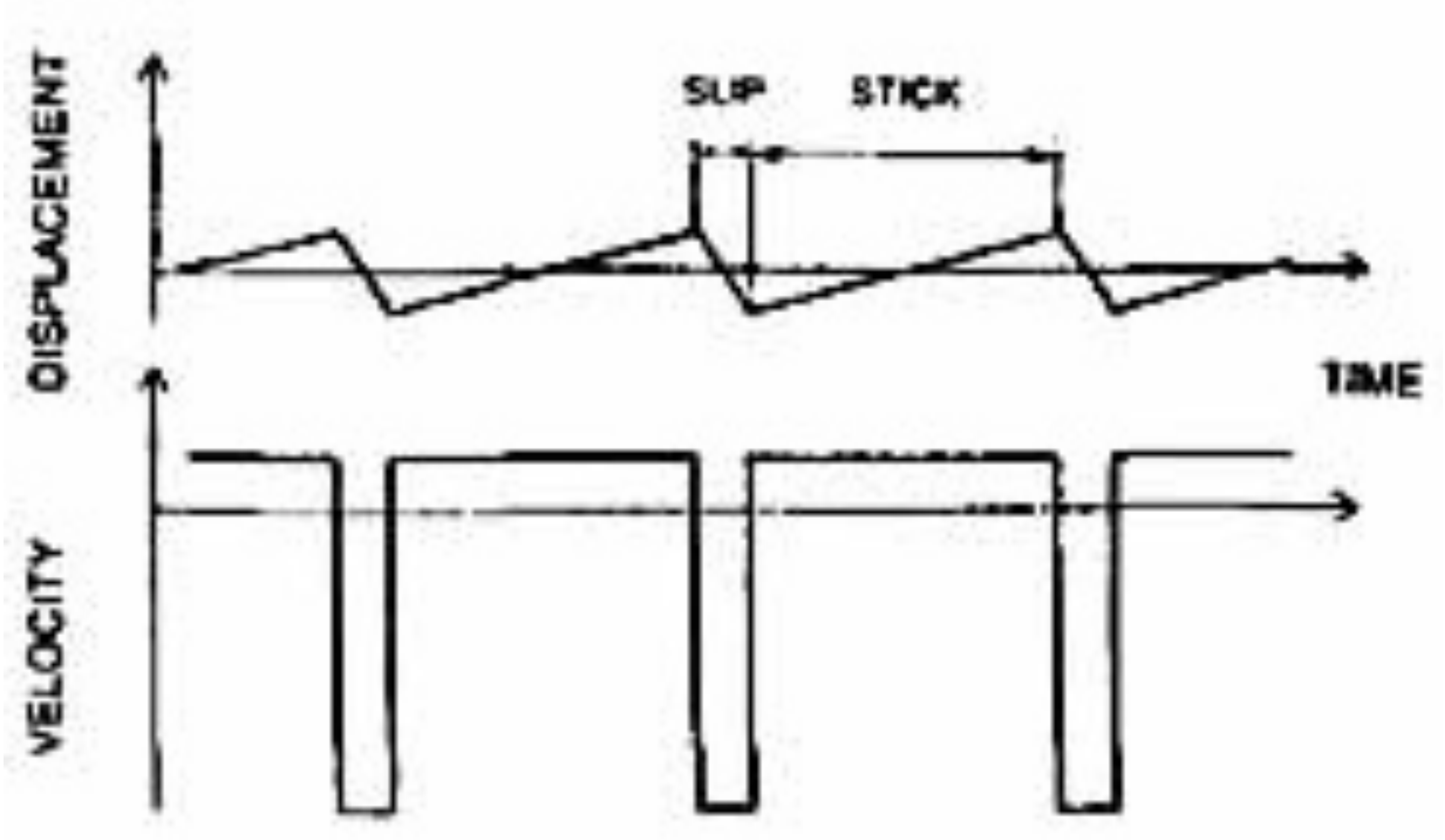
Fourier coefficients of a plucked string

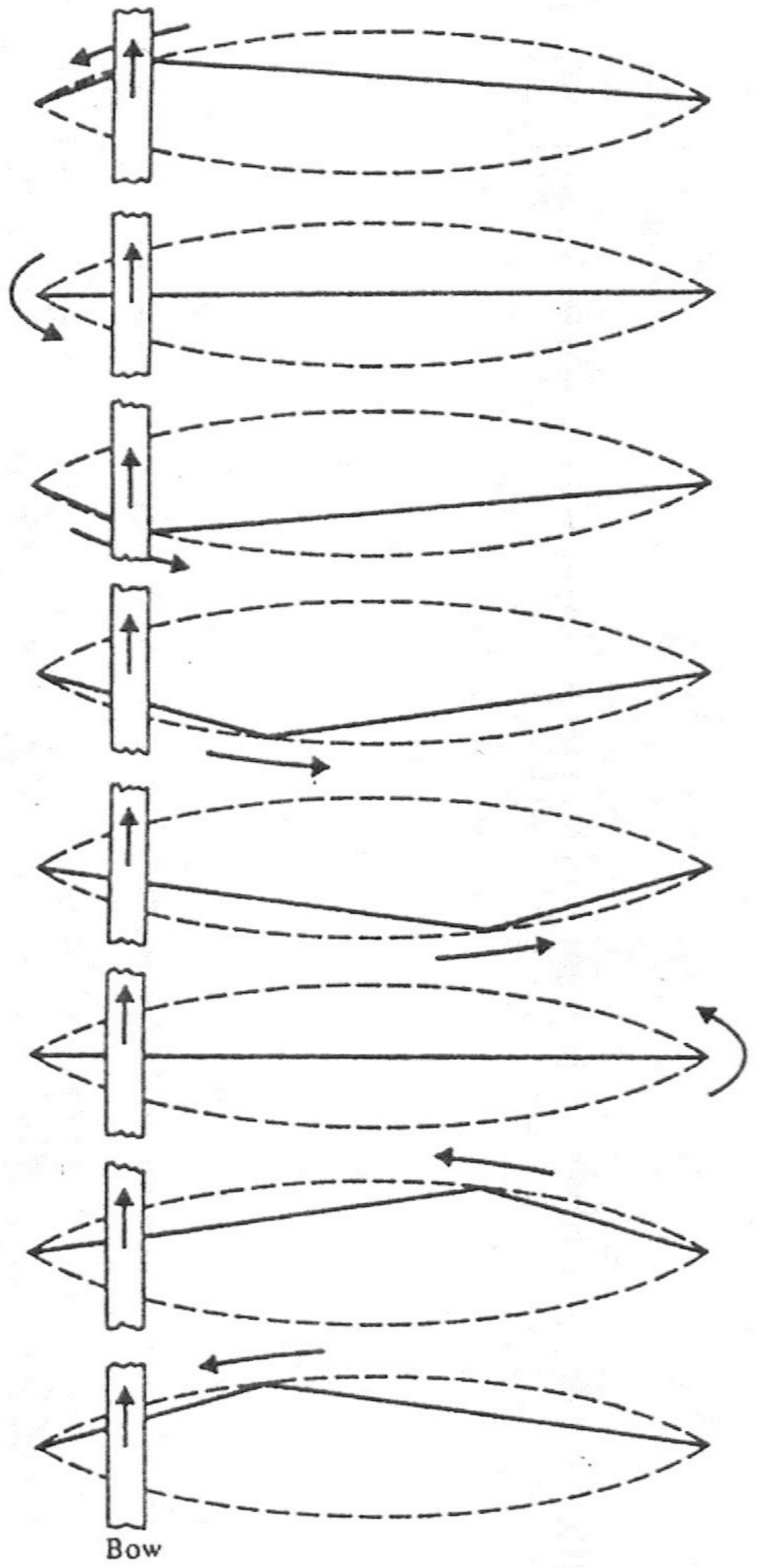
- Sounds are **richer** when the string is plucked **closer to the bridge**
- If the string is plucked in the **middle**, there at **no even harmonics**



Stick-slip motion of a bowed string

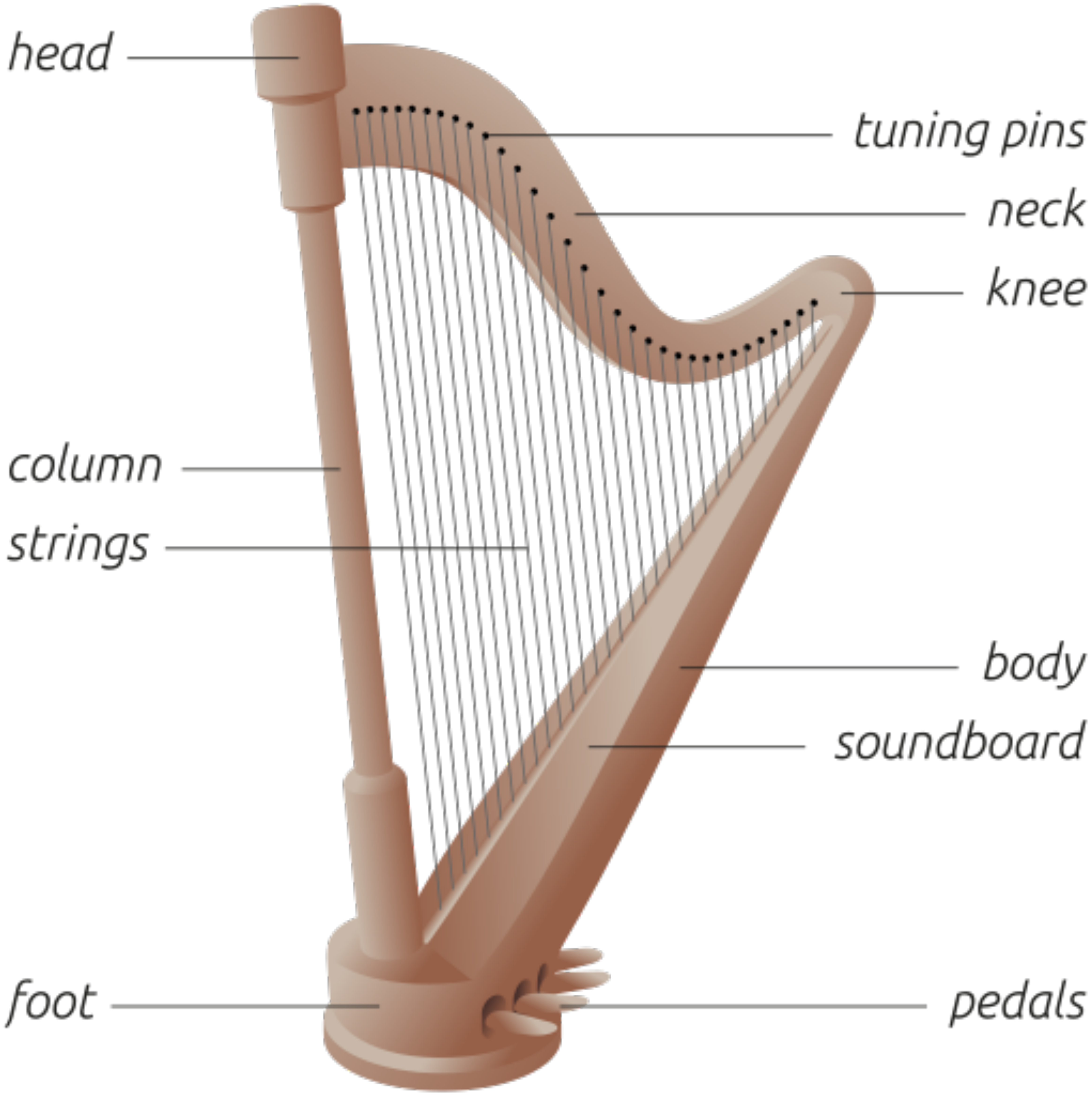
- The violin string alternately “sticks” and then “slips” against the bow hundreds of times per second





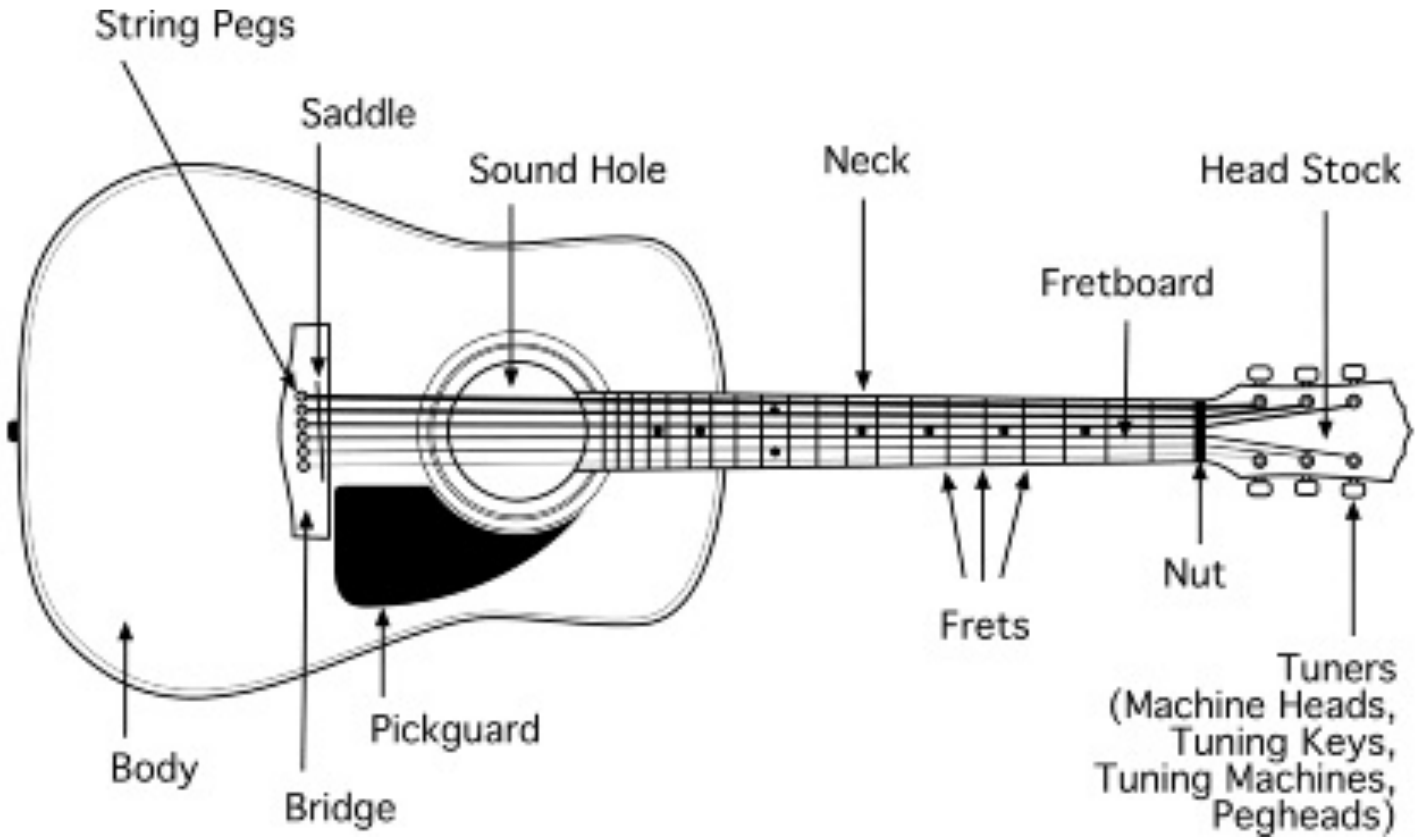
Harp

- the strings have **different fixed lengths** and are plucked
- only **one note** per string -> need lots of strings
- foot pedal can change the note, but only by **only a semitone**



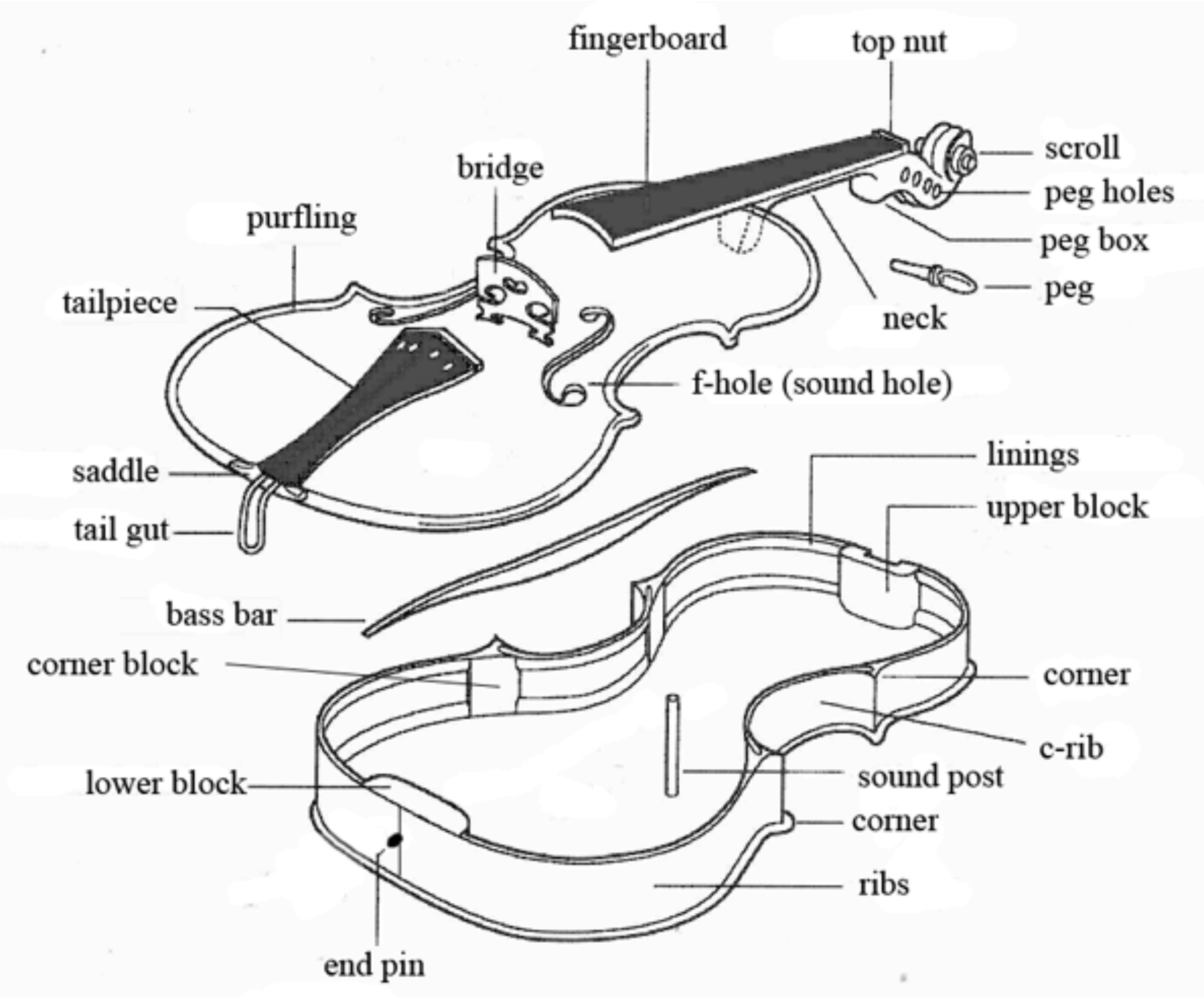
Guitar

- strings are all the **same length**, but are made of different materials and are under different tensions
- get **multiple notes** per string by pressing against a fret
- frets -> **fixed notes** (like a piano keyboard)



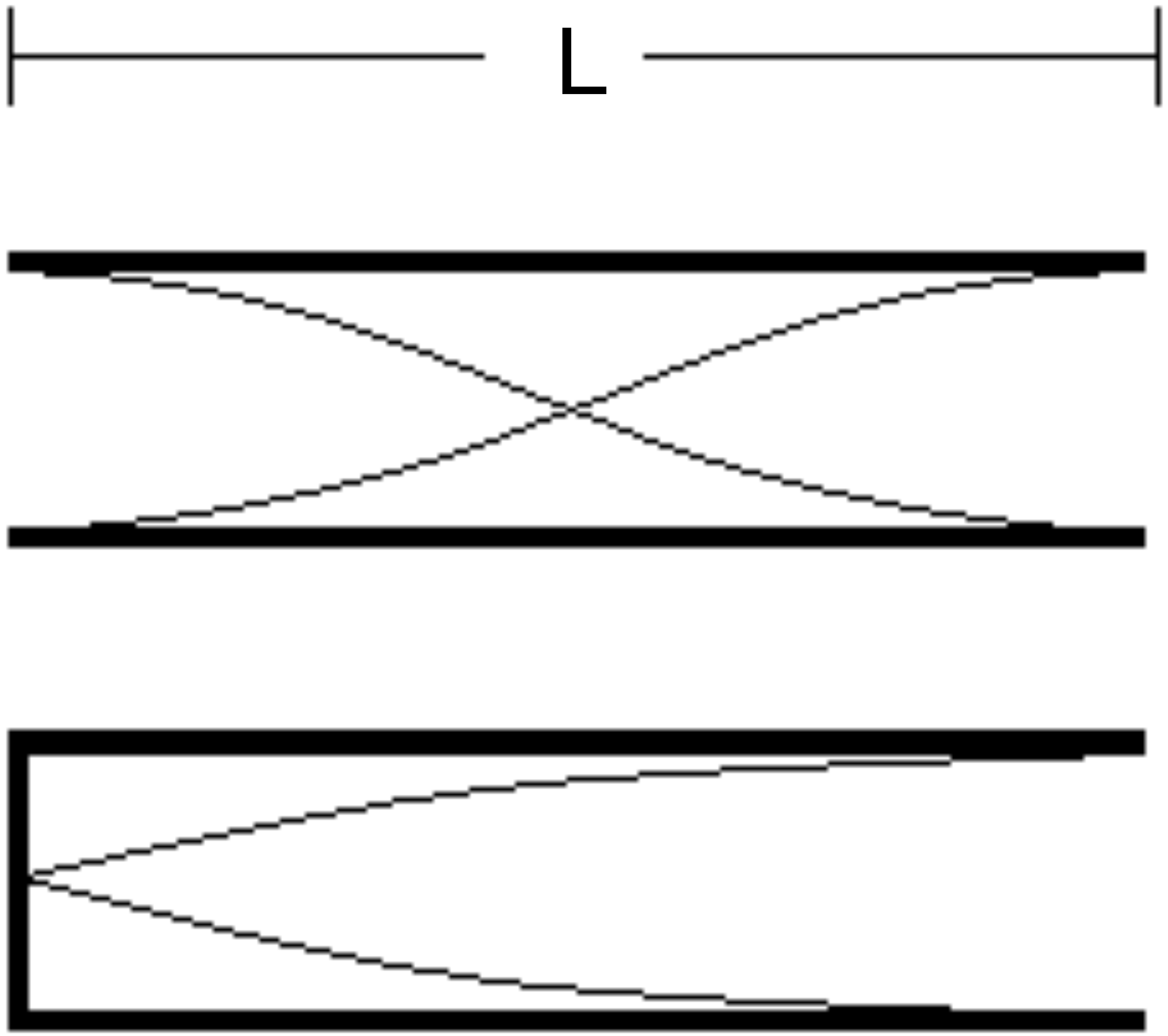
Violin

- strings are all the **same length**, but are made of different materials and are under different tensions
- get **multiple notes** per string by pressing against the neck
- no frets -> **no fixed notes**
- string vibrations are **quickly damped** if strings are plucked -> bowed instead
- can **vary tone quality** by adjusting the intensity of bowing



6. Wind instruments

Open and closed tubes (recall previous discussion)



$$\lambda_N = \frac{2L}{N} \qquad f_N = Nf_1 \qquad f_1 = \frac{v}{2L} \qquad N = 1,2,\cdots$$

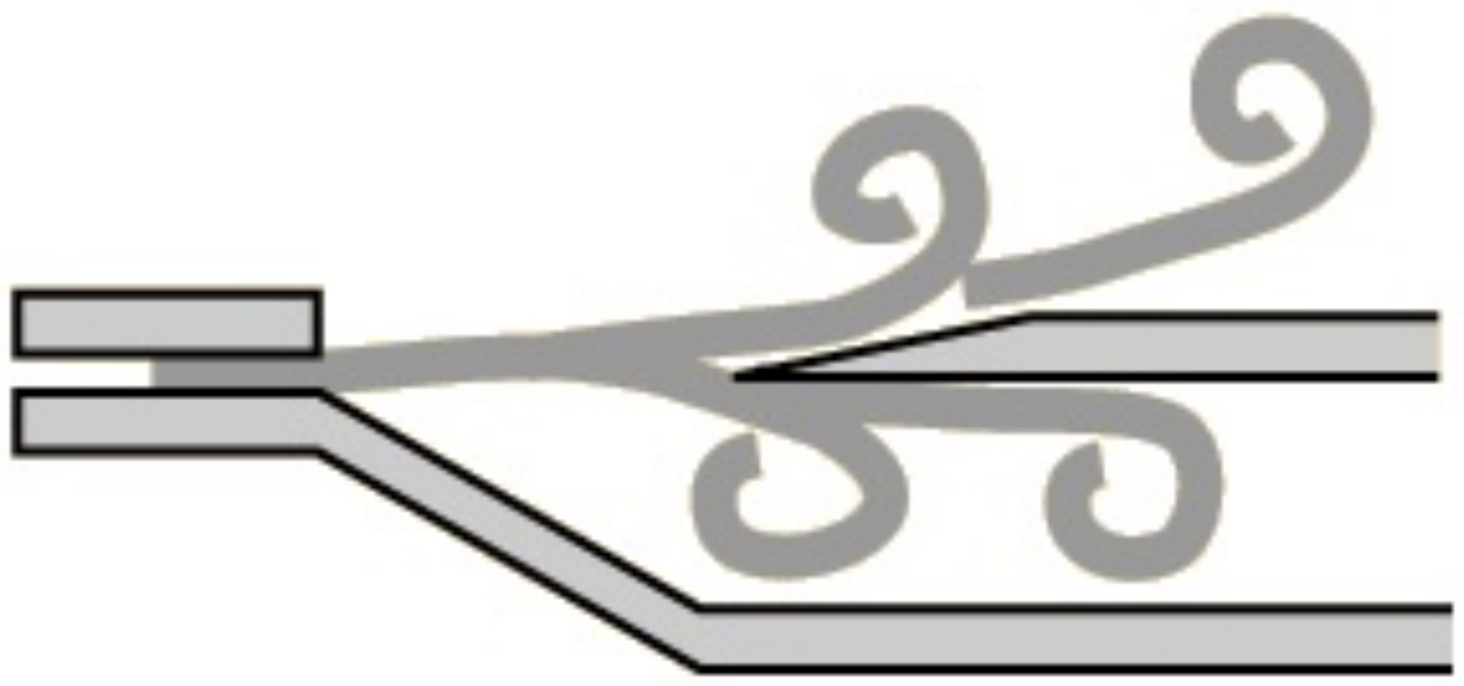
(both even and odd harmonics)

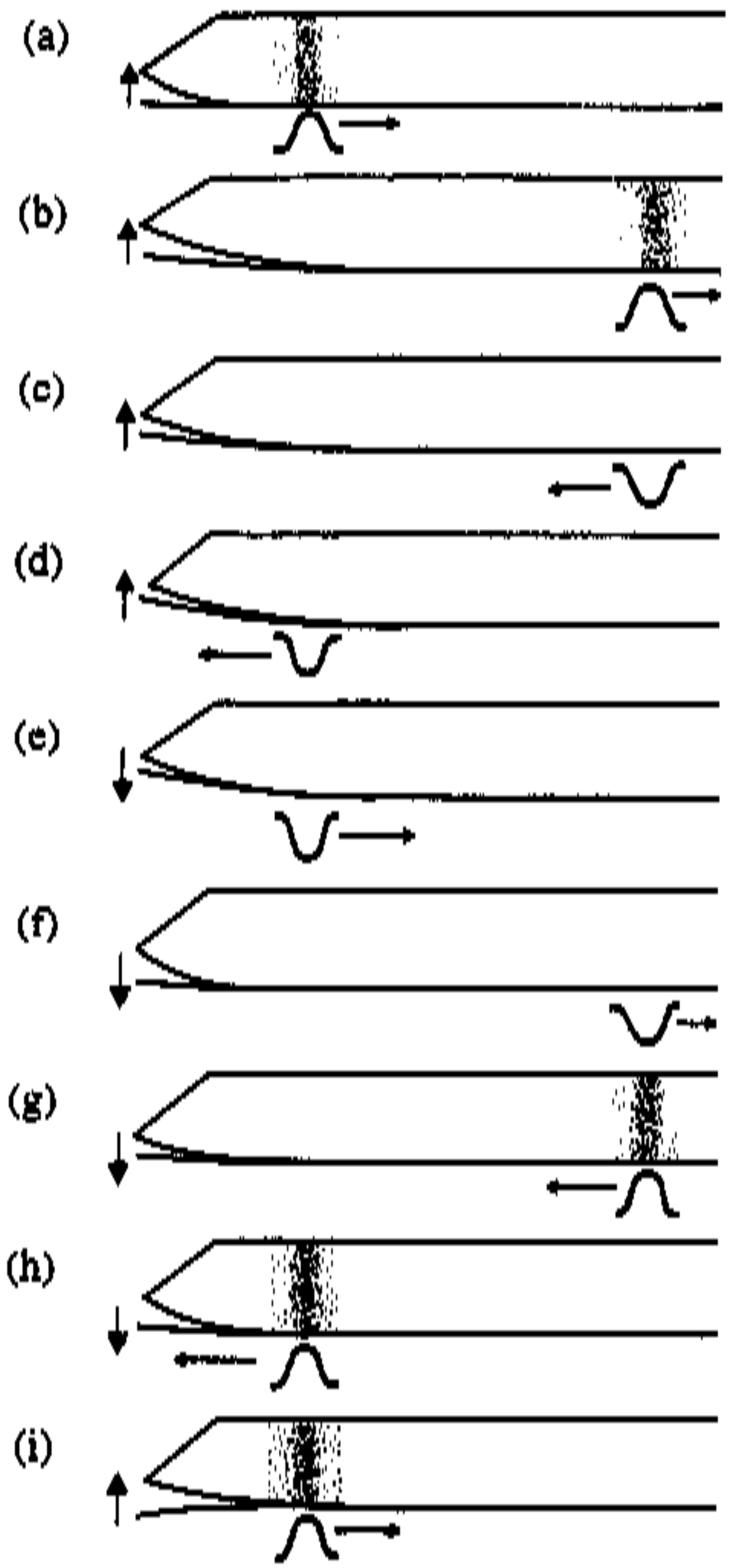
$$\lambda_N = \frac{4L}{N} \qquad f_N = Nf_1 \qquad f_1 = \frac{v}{4L} \qquad N = 1,3,5,\cdots$$

(only odd harmonics)

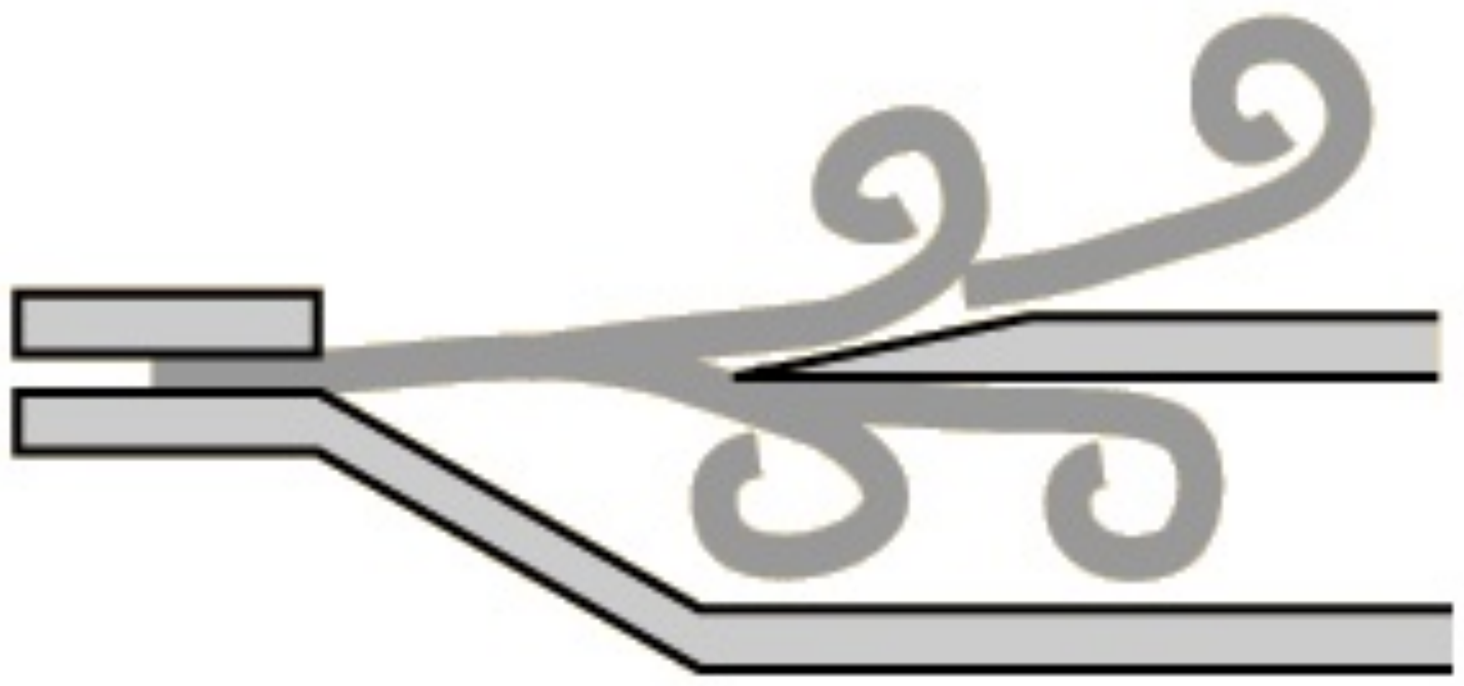
(air molecule displacements)

Excitations produced by an oscillating air stream or a vibrating reed





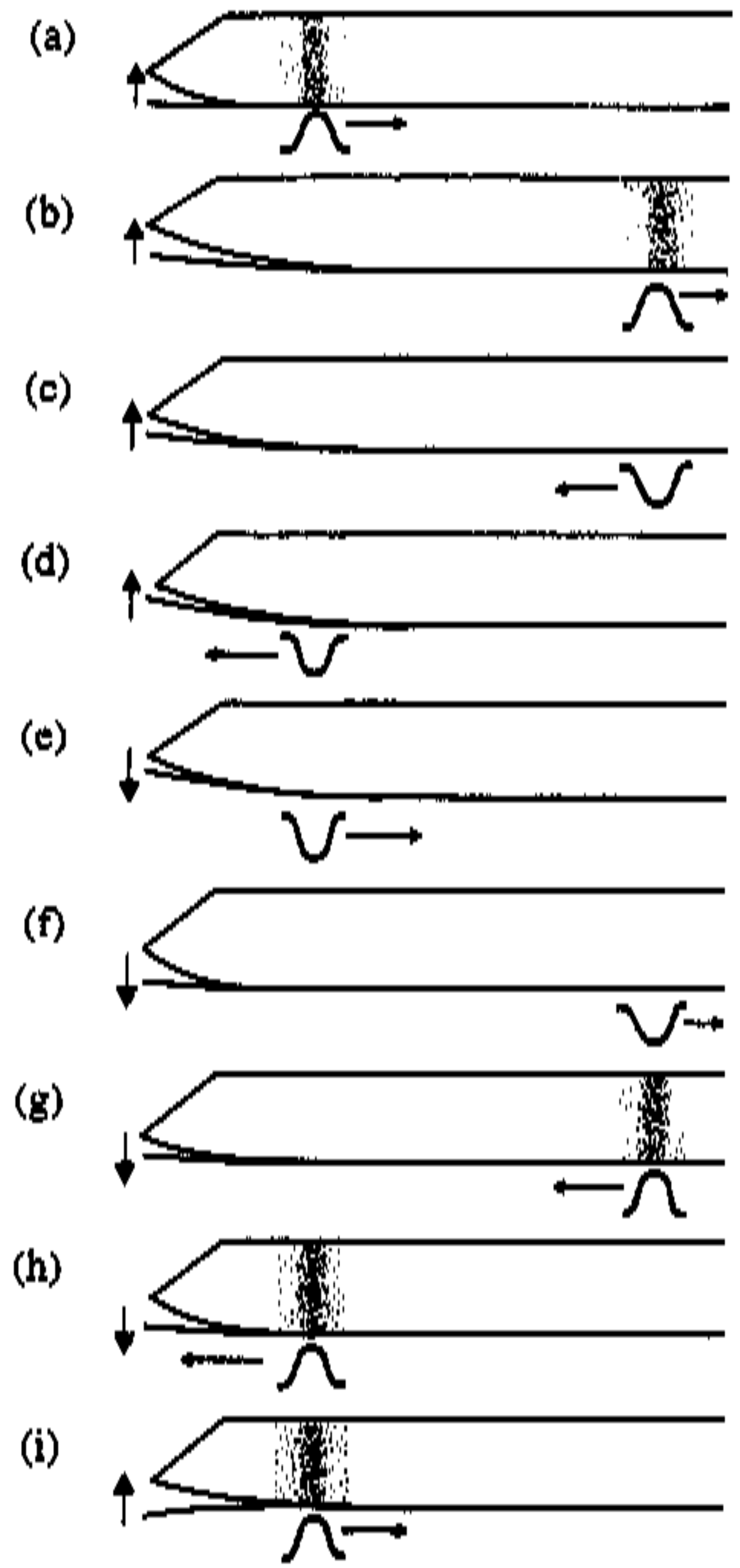
Oscillating air stream



- “**Flow-controlled**” excitation
- Used in recorders, flutes, penny whistles, etc.
- When the air stream hits the edge, it creates tiny **whirpools** (or “vortices”) of air, which **alternate** going either into or out of the tube
- The frequency of the alternating whirlpools is determined by the **natural frequencies** of the remainder of the tube (resonance phenomena)

Vibrating reed

- "**Pressure-controlled**" excitation
- Used in clarinets, saxophones, bassoons, etc. (i.e., any instrument with a reed)
- Frequency of the opening and closing of the reed is determined by the **natural frequencies** of the remainder of the tube (resonance phenomenon)
- Also applies to the didgeridoo and brass instruments where the musicians **lips** play the role of a reed

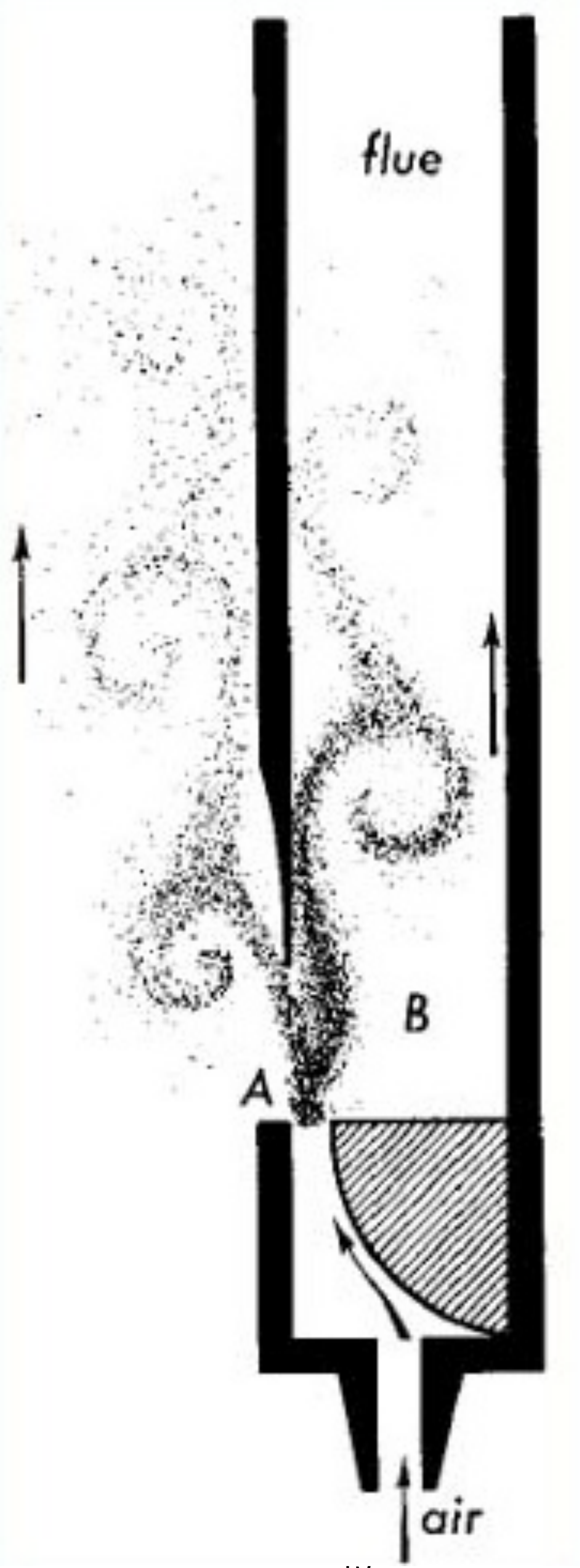


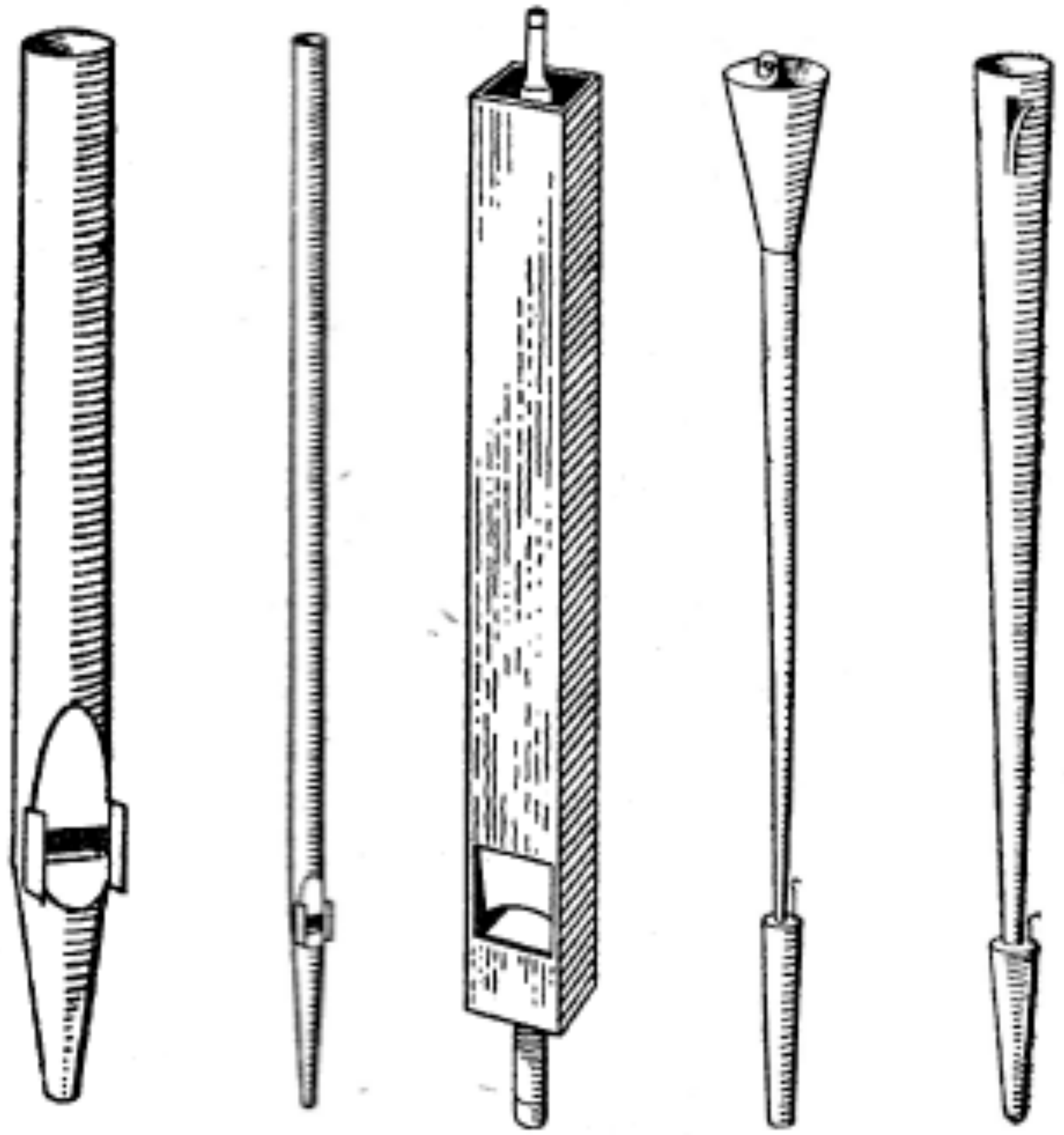
Flue-organ pipe

- Excitation can be either an oscillating air stream or a vibrating reed

- Pipe has **fixed length**

- **One note** per pipe -> need many pipes (e.g., church organ)

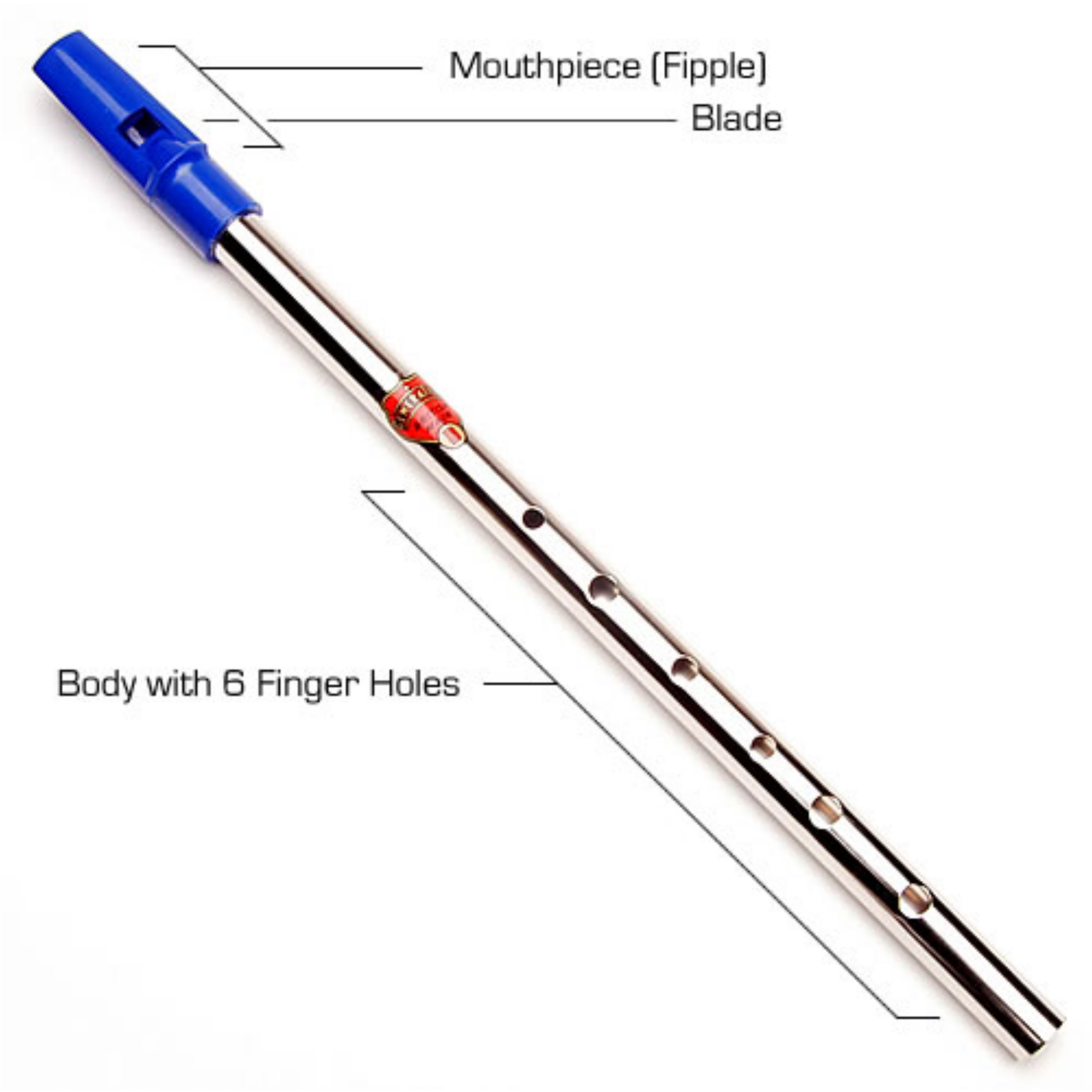




Different types of pipe
1. Open Diapason
2. Dulciana
3. Stopped Diapason
4. Oboe (reed pipe)
5. Trumpet (reed pipe)

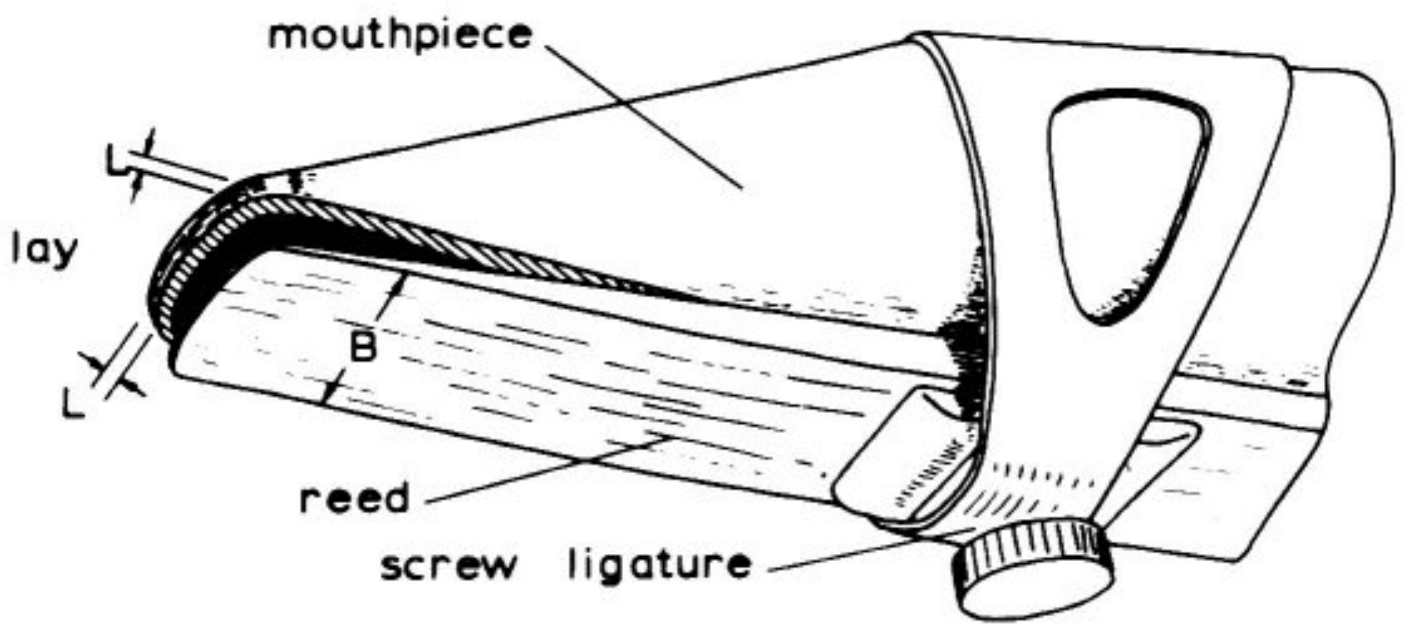
Penny whistle

- Oscillating air stream -> open at both ends
- **Tone holes** allow playing **multiple notes**



Clarinet

- Vibrating **reed** -> **closed** at one end
- Tone holes and register keys -> **multiple notes**
- Bell at end creates contribution from even harmonics



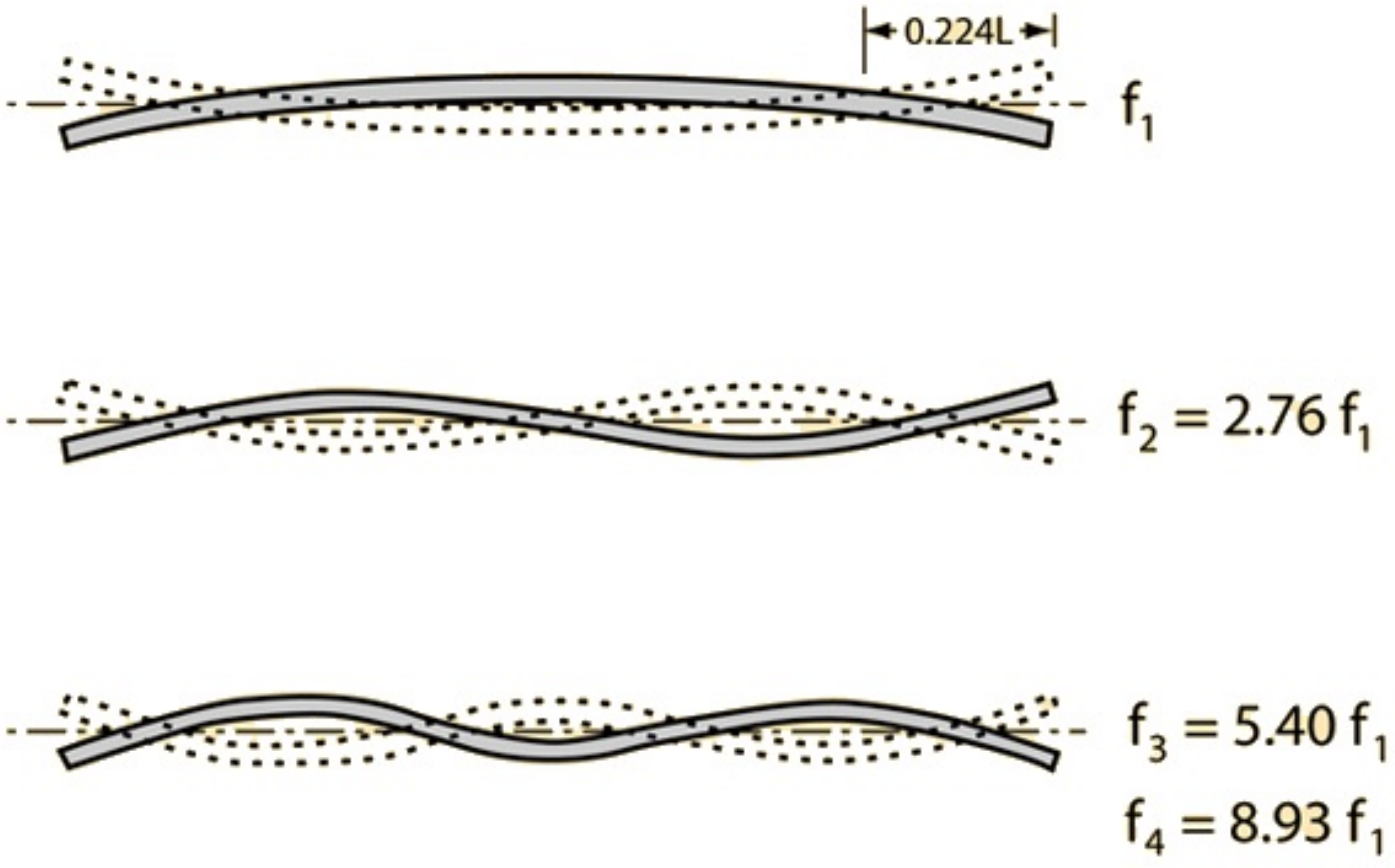
Brass instruments

https://www.youtube.com/watch?v=Bo7VRSQLpfY

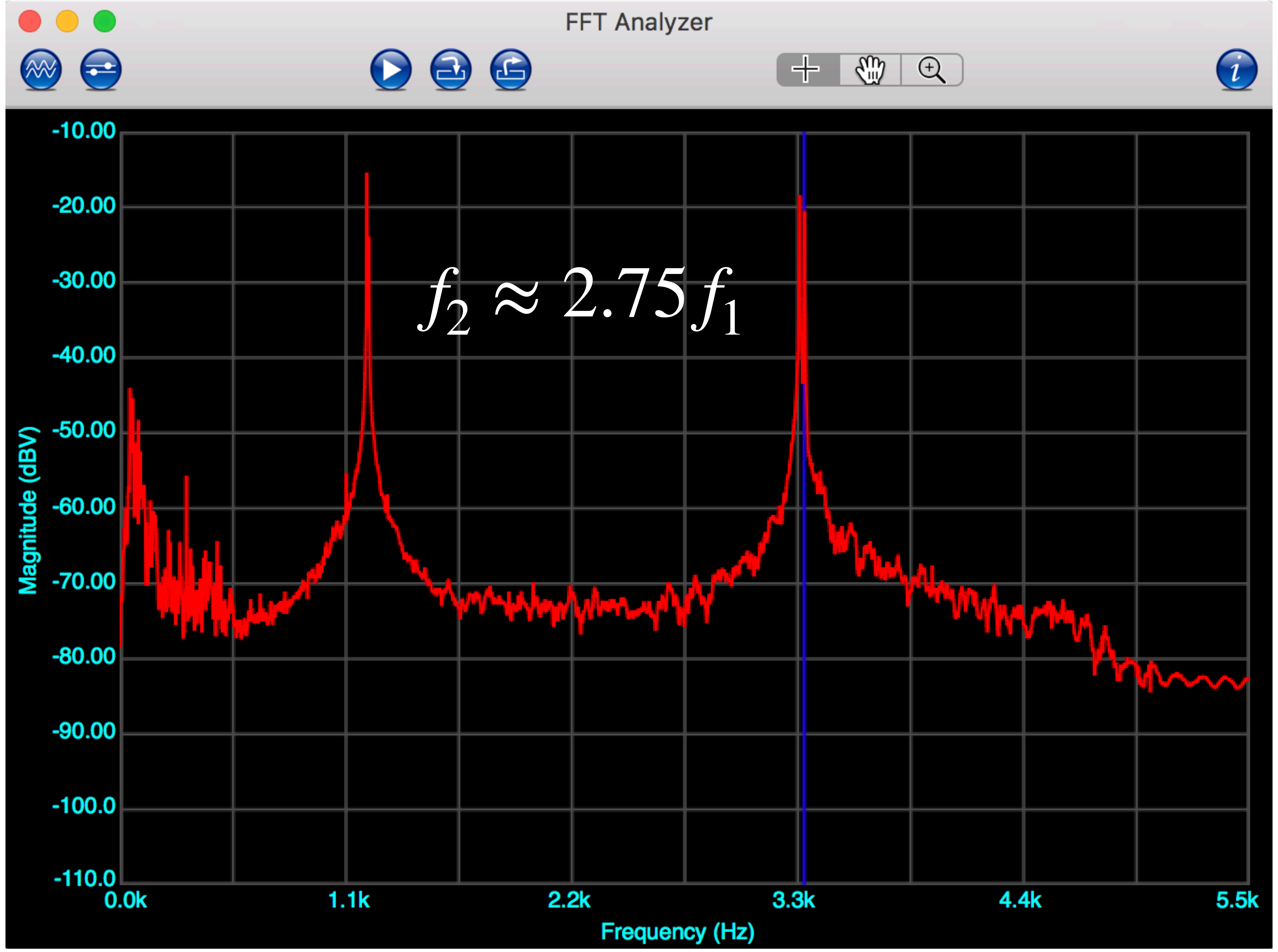
7. Percussion instruments (briefly)

Inharmonicity

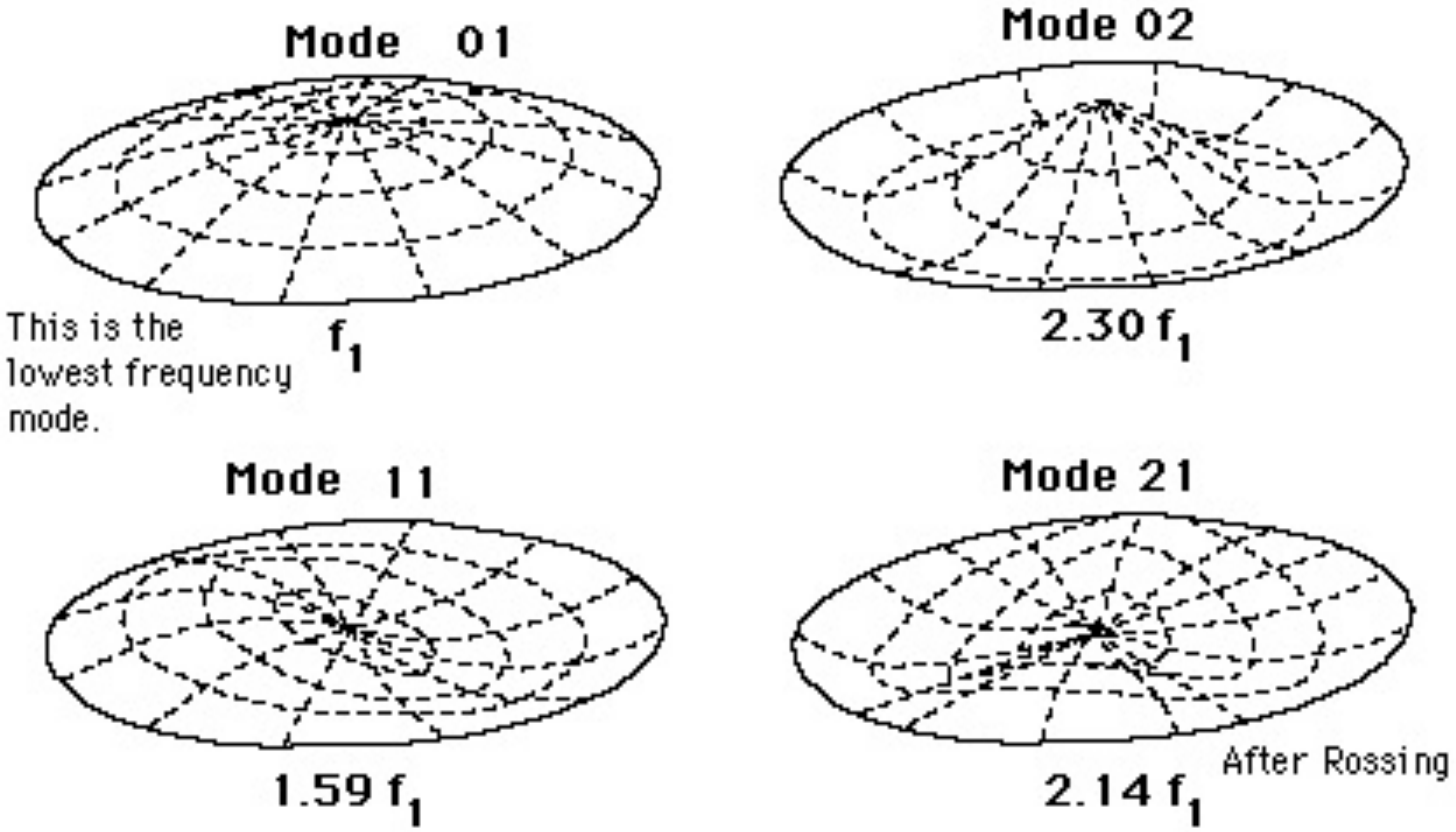
- Presence of **overtones** that are **not harmonically** related to the fundamental frequency



Example: Bell



Vibrating drum head (2-d standing waves)



mode = (# nodal diameters, # nodal circles)

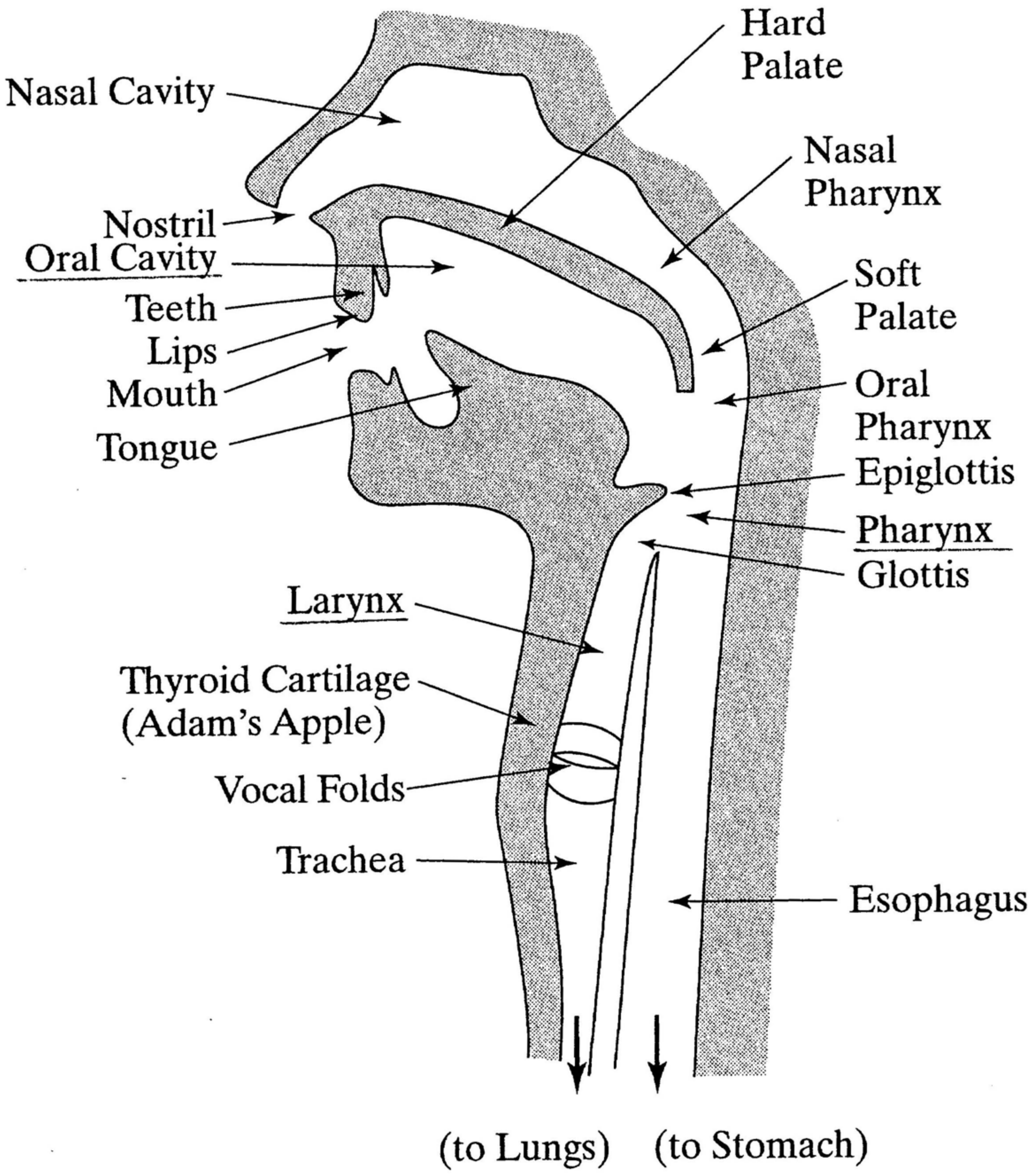
https://commons.wikimedia.org/wiki/Category:Drum_vibration_animations

https://josephromano.github.io/PHYS1406/labs/S2021/modified/Chladni_patterns.mov

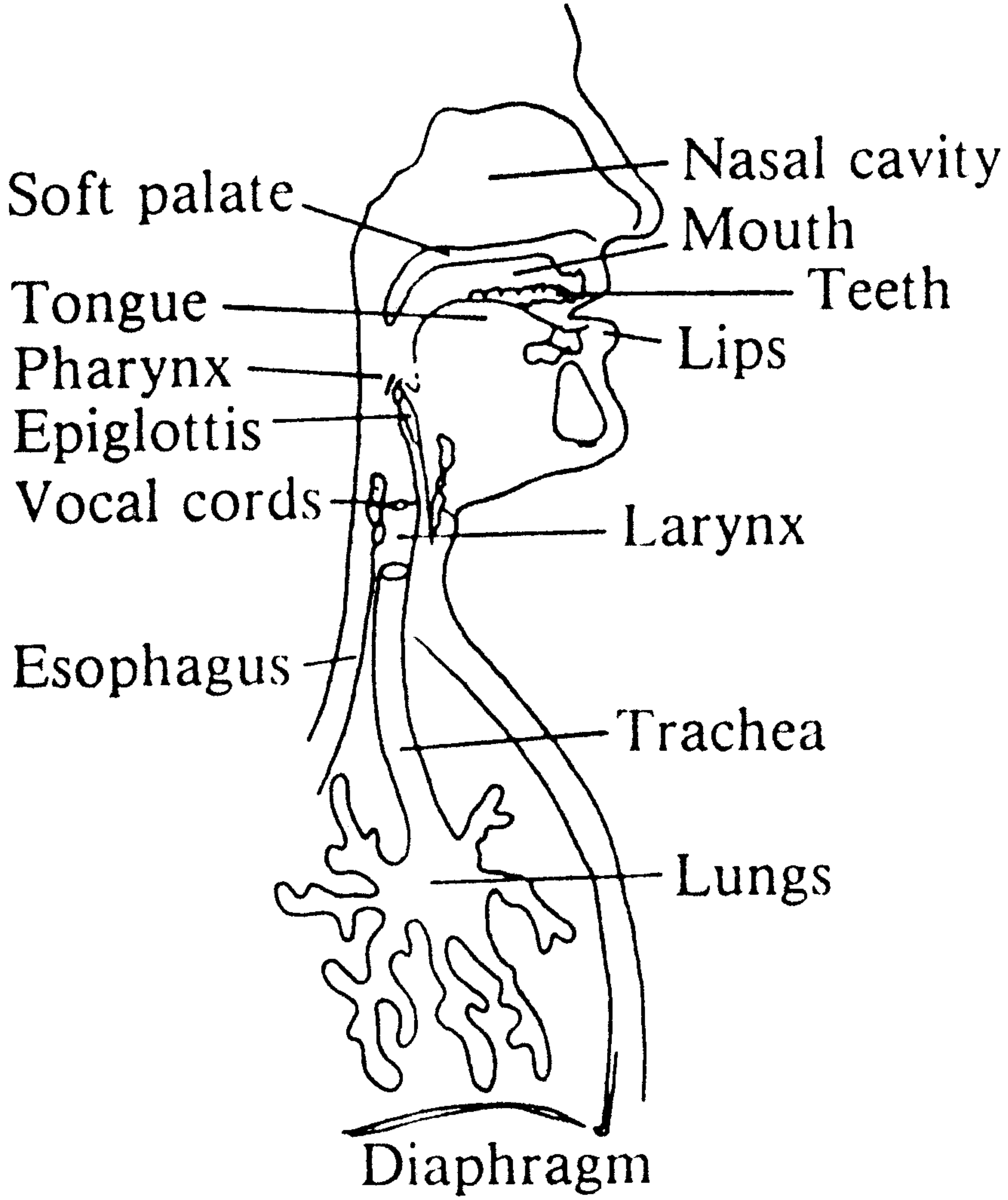
8. Voice

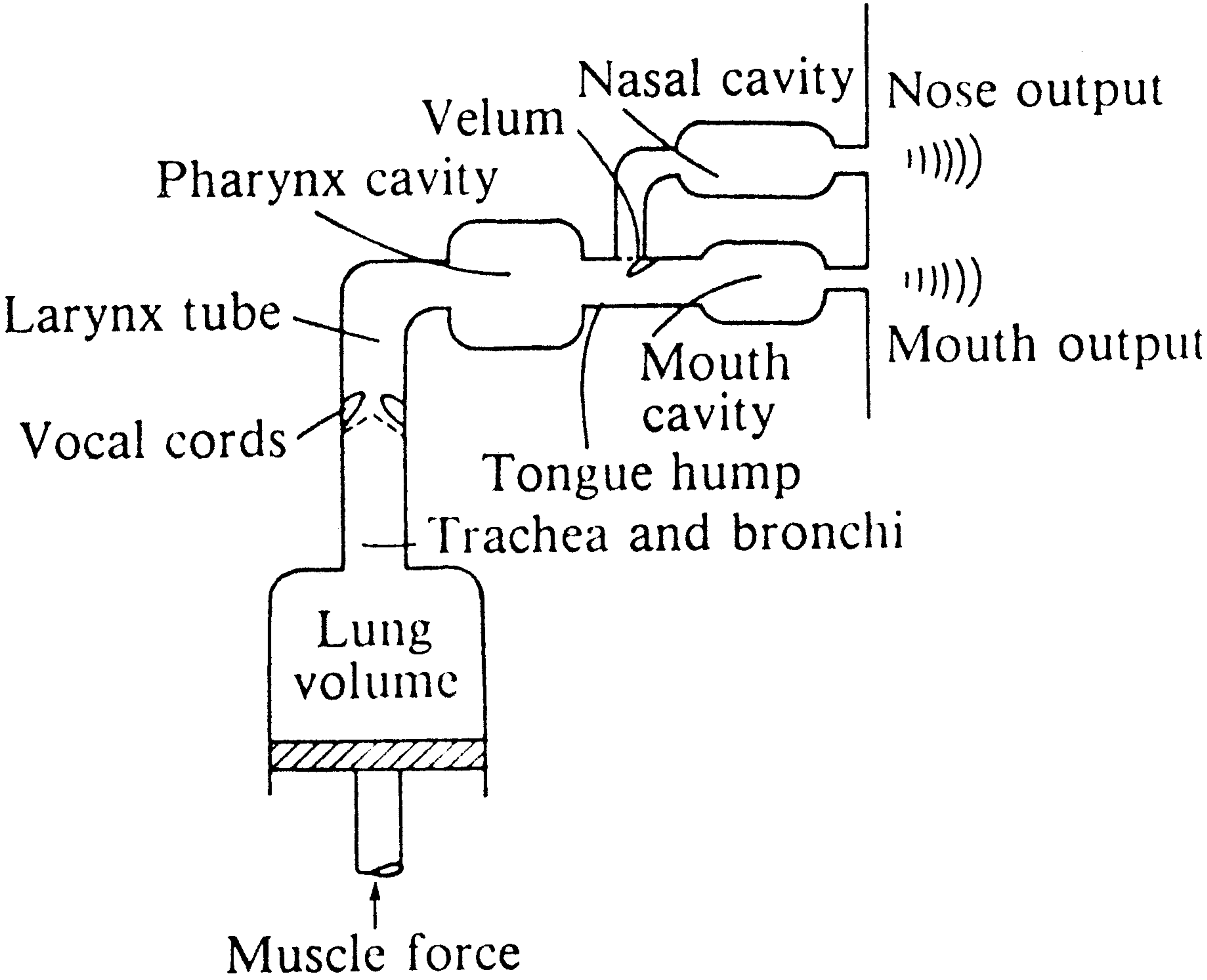
Vocal organs

- power supply: lungs
- generator/vibrator: vocal folds
- resonator: vocal tract (larynx, pharynx, oral and nasal cavities)
- radiator: mouth/lips and nostrils
- vocal folds:
    - women: ~10 mm, ~220 Hz
    - men: ~15-20 mm, ~110 Hz
    - forced open by air pressure, come together due to **Bernoulli effect** (demo)



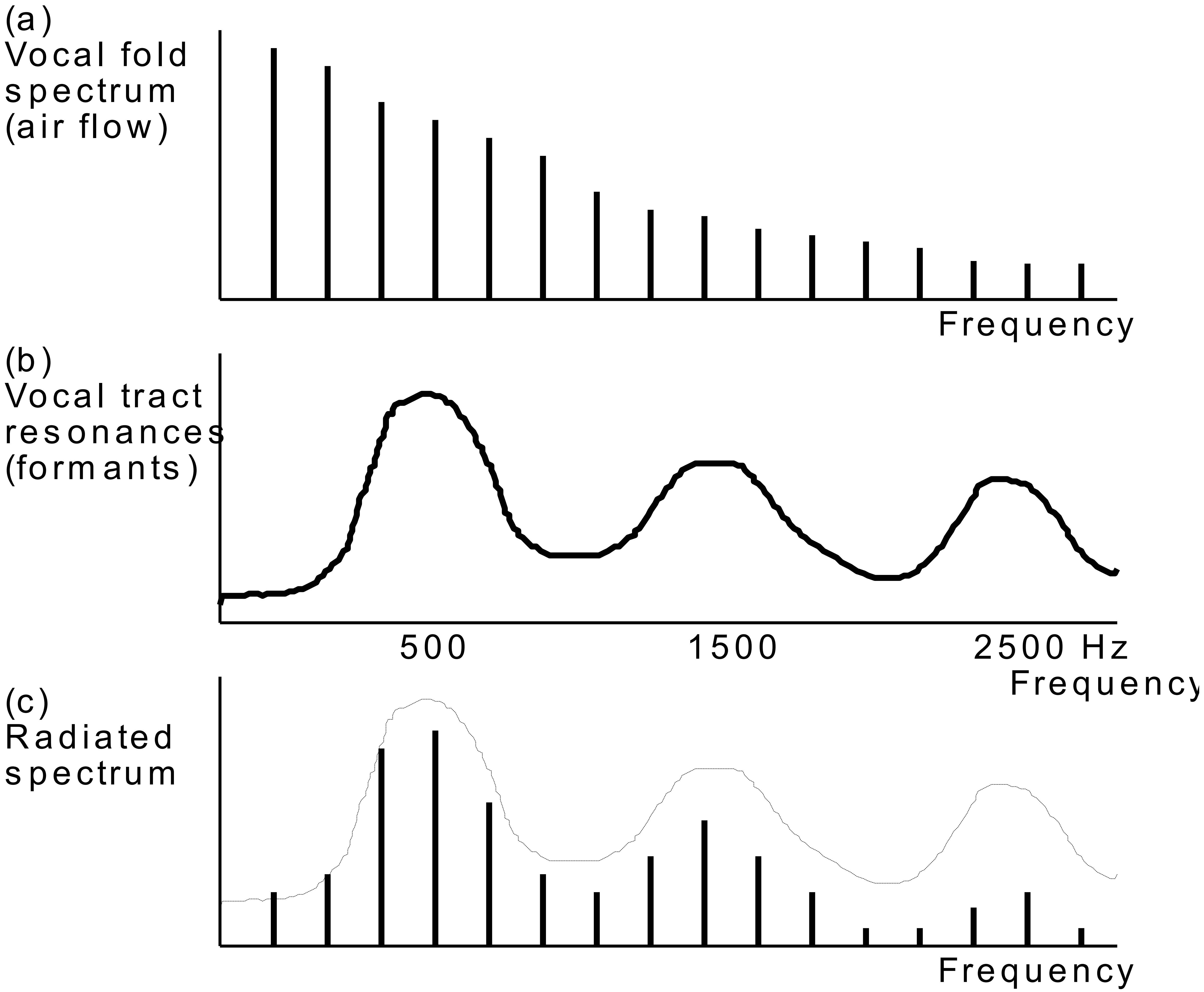
Physics / engineering model



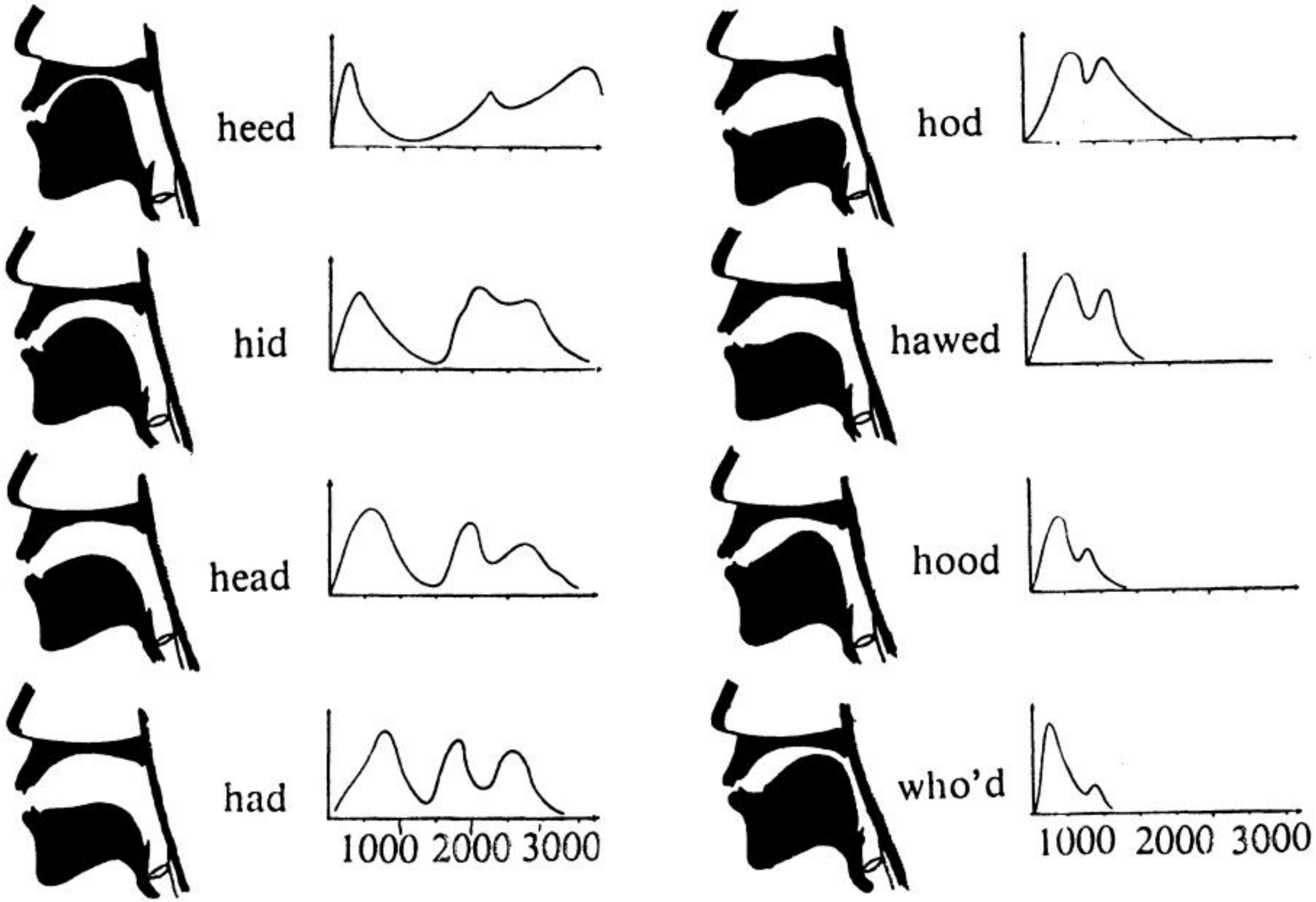


Formants

- Spectrum of vocal folds ~1/N^2
- Vocal tract acts as a **filter**
  - ~17cm cylindrical tube
  - $f_n = Nv/4L\,, \quad N = 1,3,\cdots$
  - ~500 Hz, 1500 Hz, 2500 Hz, …
  - "Donald duck" effect if one inhales helium

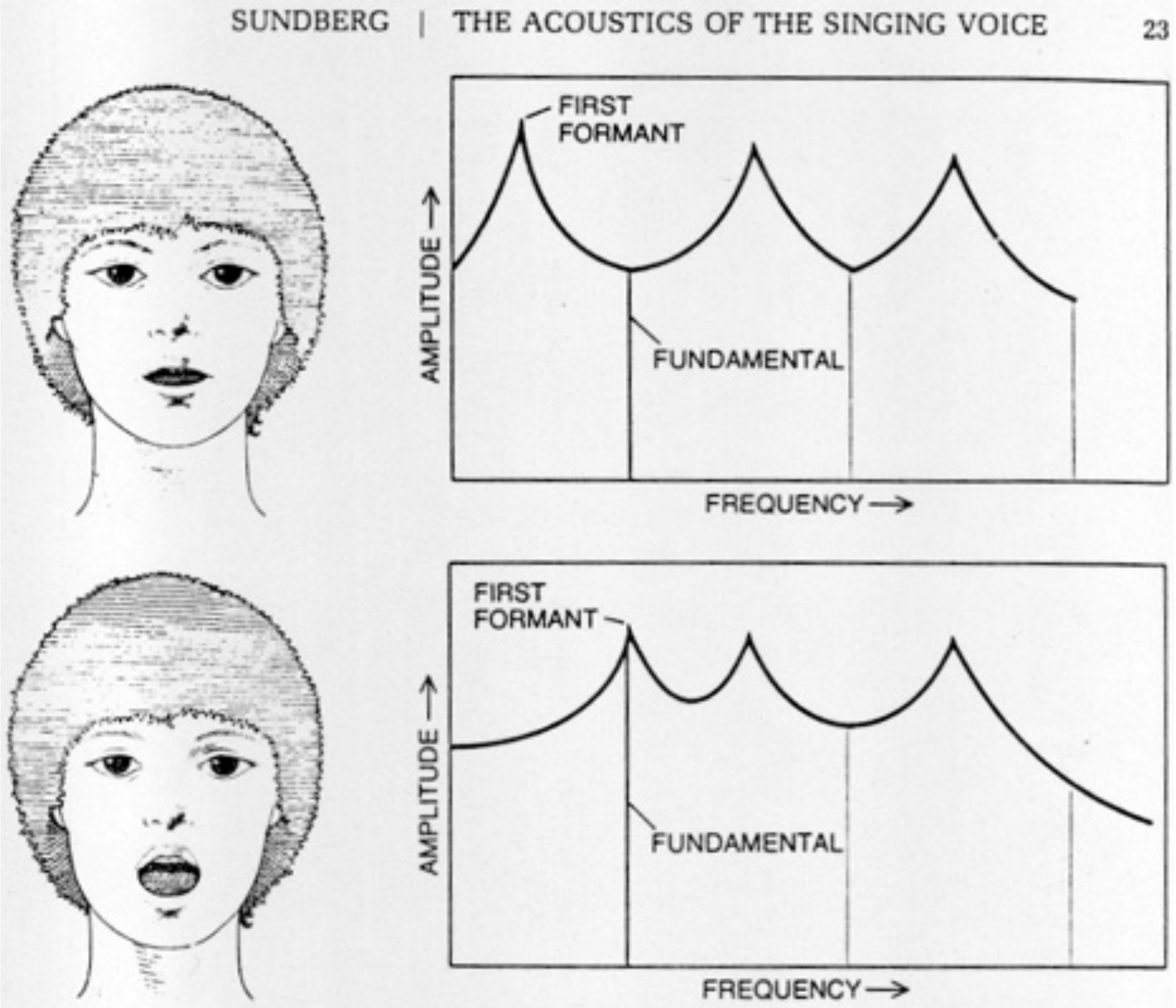


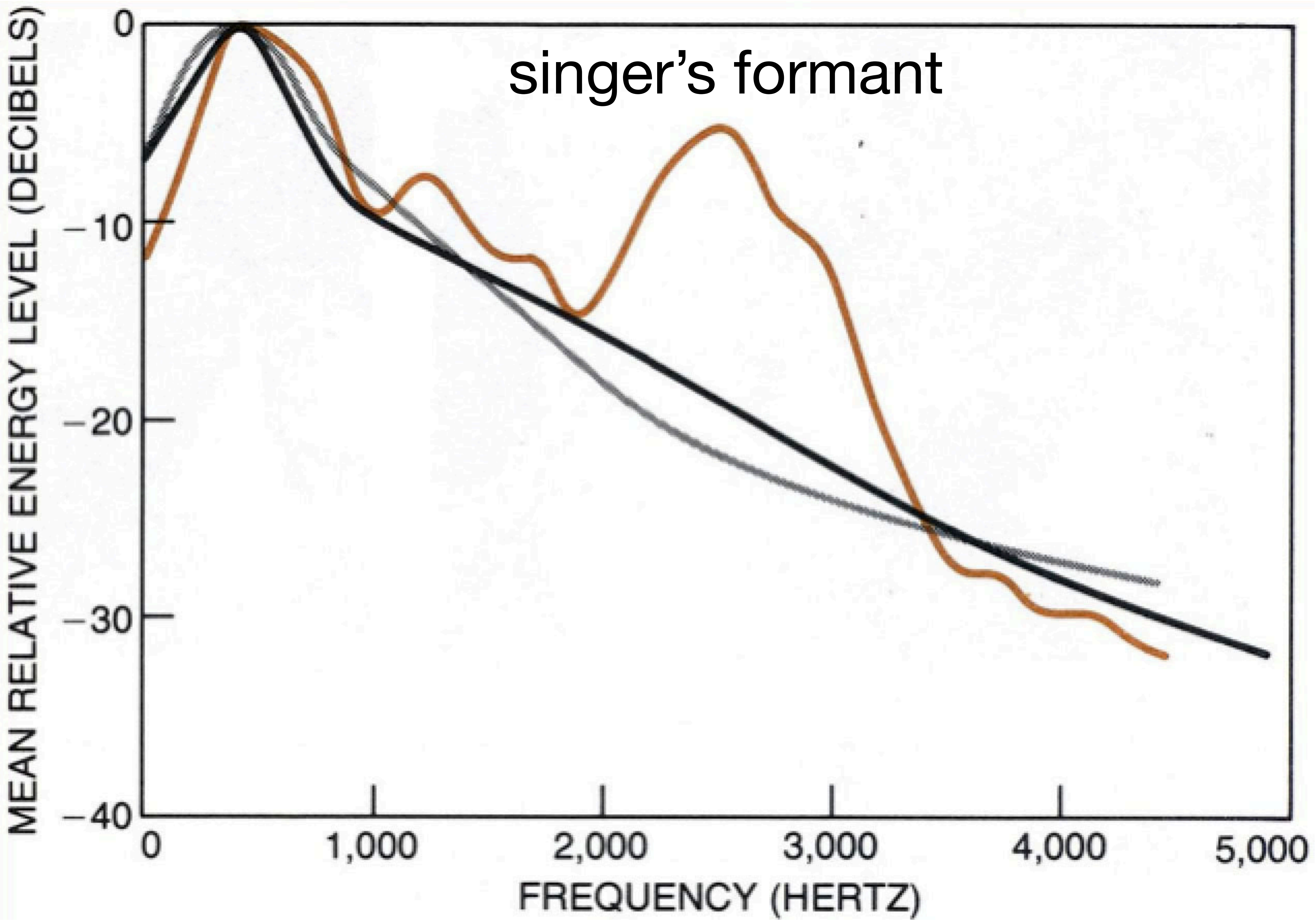
Formant regions for different vowel sounds



Demonstration: compare "who'd", "hod", and "heed" using spectrogram

Singing

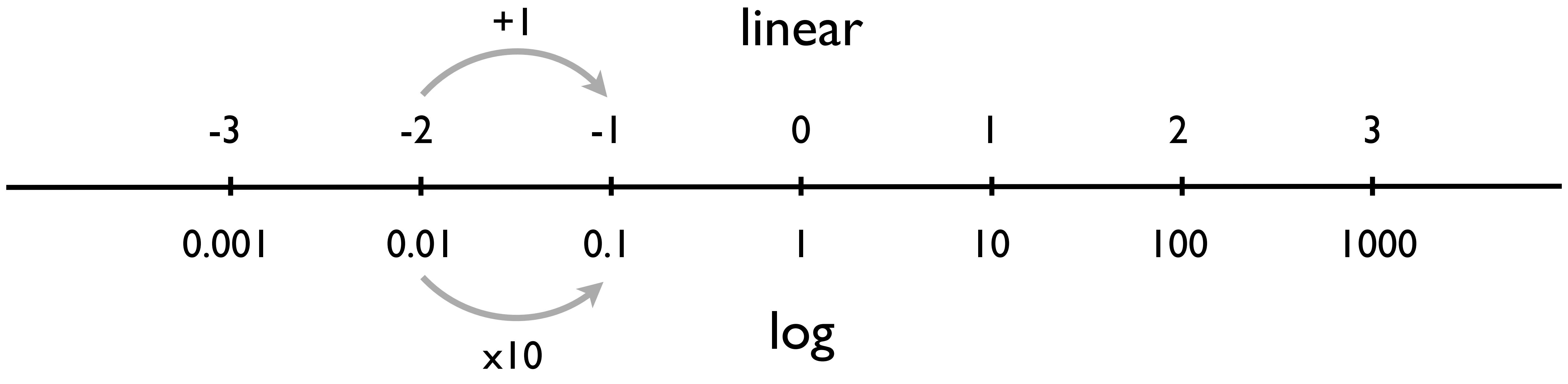

SUNDBERG | THE ACOUSTICS OF THE SINGING VOICE
23
FIRST FORMANT
AMPLITUDE →
FUNDAMENTAL
FREQUENCY →
FIRST FORMANT
AMPLITUDE →
FUNDAMENTAL
FREQUENCY →


singer's formant
MEAN RELATIVE ENERGY LEVEL (DECIBELS)
0
−10
−20
−30
−40
0
1,000
2,000
3,000
4,000
5,000
FREQUENCY (HERTZ)

9. Hearing

Fechner’s law and range of human hearing

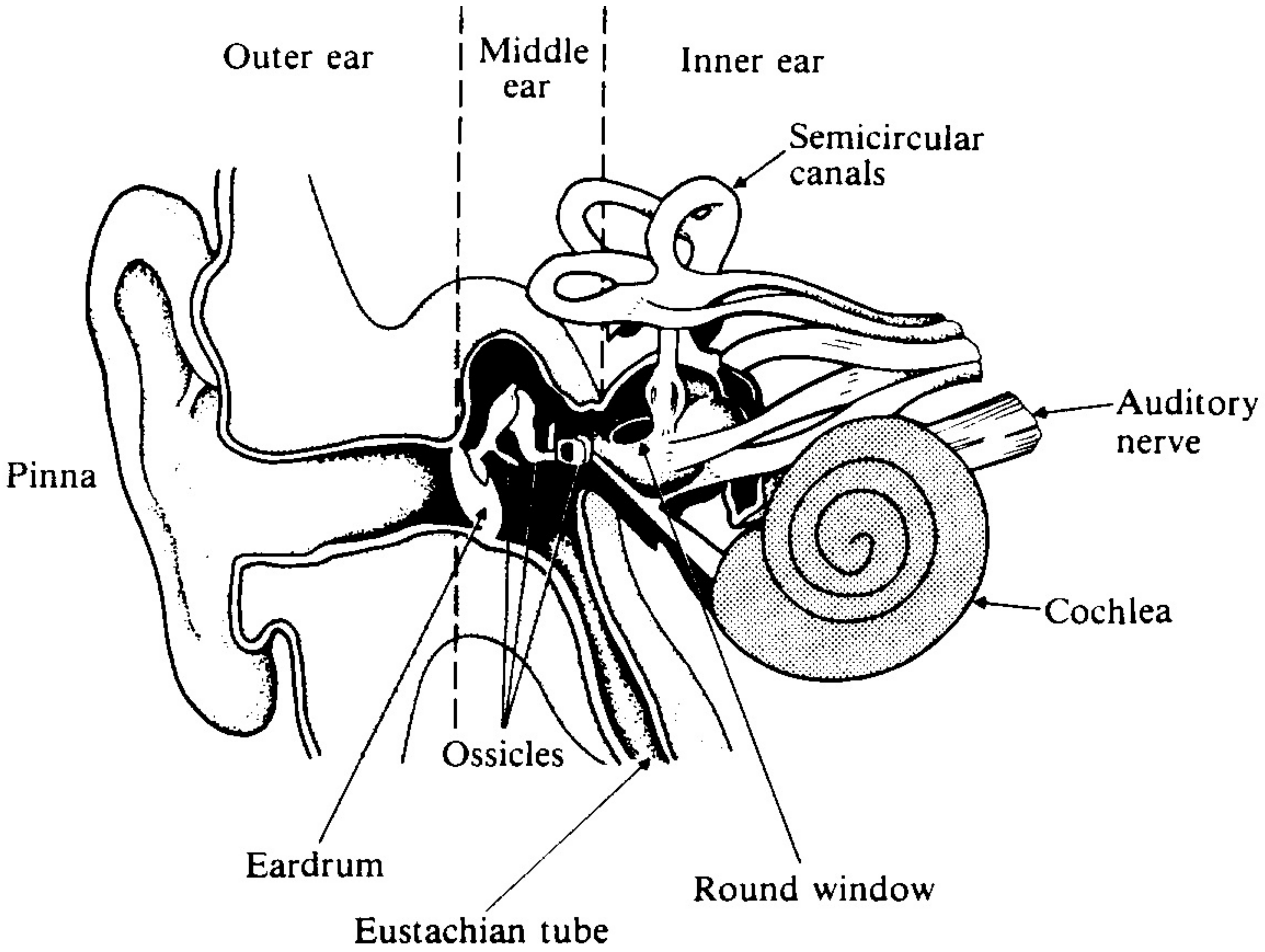
- Fechner’s law: “As **stimuli** are increased by **multiplication**, **sensation** increases by **addition**”



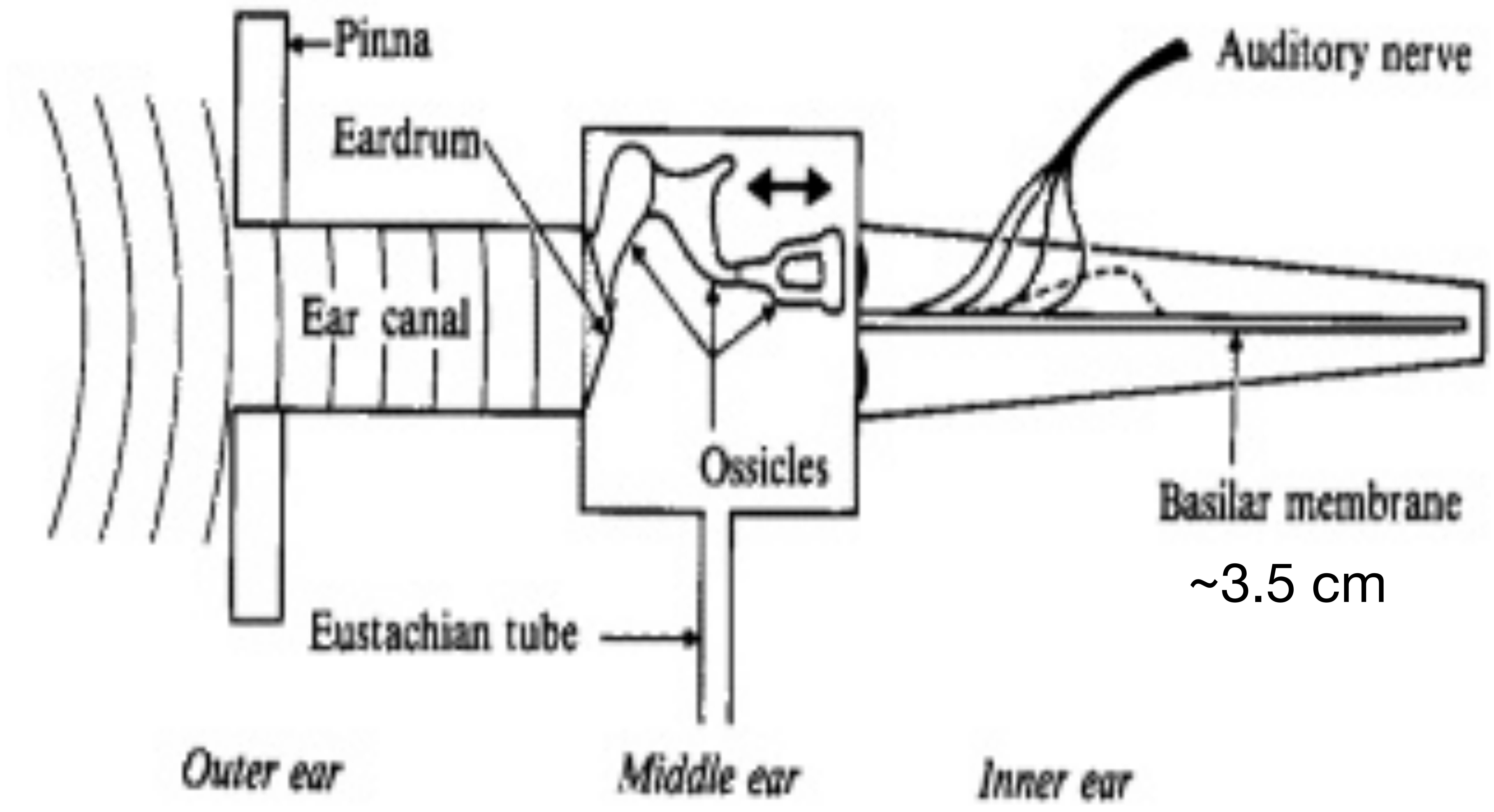
$$y = \log x \;\Leftrightarrow\; 10^y = x$$

- Range of hearing
  - **Pitch** (frequency): 20 – 20,000 Hz (~10 octaves)
  - **Loudness** (intensity): $10^{-12} - 1\ \mathrm{W/m^2}$ (12 orders of magnitude)
- Eye: sensitive to ~1 octave in color (frequency) and 5 orders of magnitude in brightness (intensity)

Anatomy of human ear



Anatomy of human ear - continued



~3.5 cm

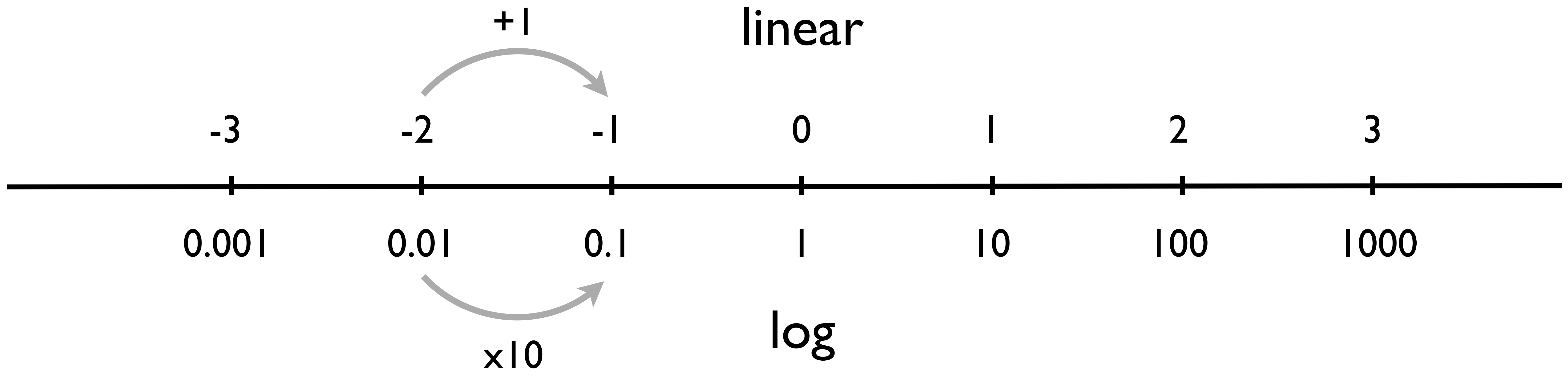
Place theory of pitch

- A pure tone excites a ~1.3 mm region of the basilar membrane (**critical band**)
- There are ~**24 critical bands** on the basilar membrane spanning 20-20,000 Hz
- Center frequencies of critical bands are **spaced logarithmically** on the basilar membrane like a piano keyboard (called **"place theory of pitch"**)
- Critical bands:
    - ~100 Hz for center frequencies below 500 Hz
    - ~3 semitones (1/4 octave) above 500 Hz

Hearing via air vs bone conduction and sound localization

- Air vs bone conduction:
    - Q: Why do you sound differently when you listen to a recording of your voice?
- Sound localization (binaural hearing)
    - **High frequency** sounds (> 4000 Hz): **intensity difference**
    - **Low frequency** sounds (< 1000 Hz): **time of arrival**

Logarithms



$$y = \log x \;\Leftrightarrow\; 10^y = x$$

$$\log(ab) = \log a + \log b$$

$$\log 2 \approx 0.3\,, \quad \log 3 \approx 0.5\,, \quad \log 4 \approx 0.6\,, \quad \log 5 \approx 0.7\,, \quad \log 10 = 1$$

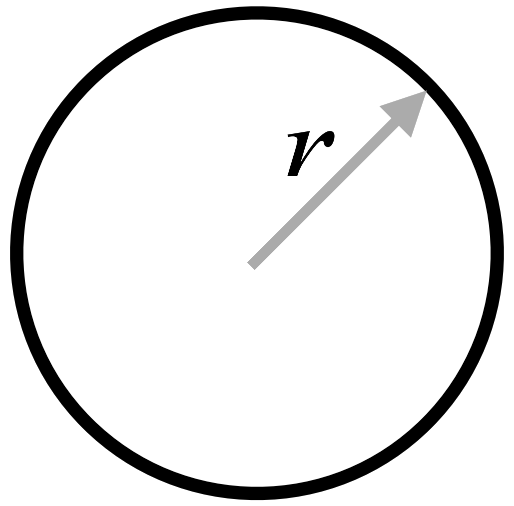
10. Loudness

Loudness — overview

- our perception of the **relative** strength of a sound (compare two sounds)
- depends on the **intensity** and **frequency** of the sound
- **logarithmic response** to intensity (consistent with Fechner's law)
- several different ways of quantifying loudness:
    - intensity
    - sound intensity level
    - sound loudness level
    - subjective loudness

Intensity

- Intensity is the power in a sound wave divided by the area it passes through (Watts/m^2)



$$I = \frac{P}{4\pi r^2}$$

(sound is less intense the farther you are away from the source of the sound)

- Range of intensities:
  - $I_0 = 10^{-12}\ \mathrm{W/m^2}$ (**threshold of hearing** at $f = 1000$ Hz)
  - $I = 1\ \mathrm{W/m^2}$ (**threshold of pain** at $f = 1000$ Hz)
- Intensity is proportional to the **square of the amplitude** of the sound wave: $I \propto (\Delta p)^2$
- Intensities add: (intensity of 2 violins = twice the intensity of 1 violin)

Sound Intensity Level (SIL, dB)

- Sound intensity level is the logarithm of the intensity compared to the threshold of hearing:

$$\boxed{\mathrm{SIL} = 10\ \log(I/I_0)\ \mathrm{dB}}$$

- Threshold of hearing: $I = I_0 = 10^{-12}\ \mathrm{W/m^2} \Rightarrow \mathrm{SIL} = 0\ \mathrm{dB}$

- Threshold of pain: $I = 1\ \mathrm{W/m^2} \Rightarrow \mathrm{SIL} = 120\ \mathrm{dB}$

$$\boxed{\Delta\mathrm{SIL} = 10\ \log(I_2/I_1)\ \mathrm{dB}}$$ (comparing two intensities)

- 2x intensity: $\Delta\mathrm{SIL} = 10\ \log(2)\ \mathrm{dB} = 3\ \mathrm{dB}$

- 10x intensity: $\Delta\mathrm{SIL} = 10\ \log(10)\ \mathrm{dB} = 10\ \mathrm{dB}$ (perceived as “twice as loud”)

- Just noticeable difference (JND): $\Delta\mathrm{SIL} = 1\ \mathrm{dB} \Leftrightarrow I_2 = 1.26\, I_1$

Sound Loudness Level (L_L, phon)

- Human ear **responds differently to different frequencies** (Fletcher-Munson curves)
- Most sensitive to frequencies **~4000 Hz**
- Equal loudness curves labeled by **phon** (not the same as dB)

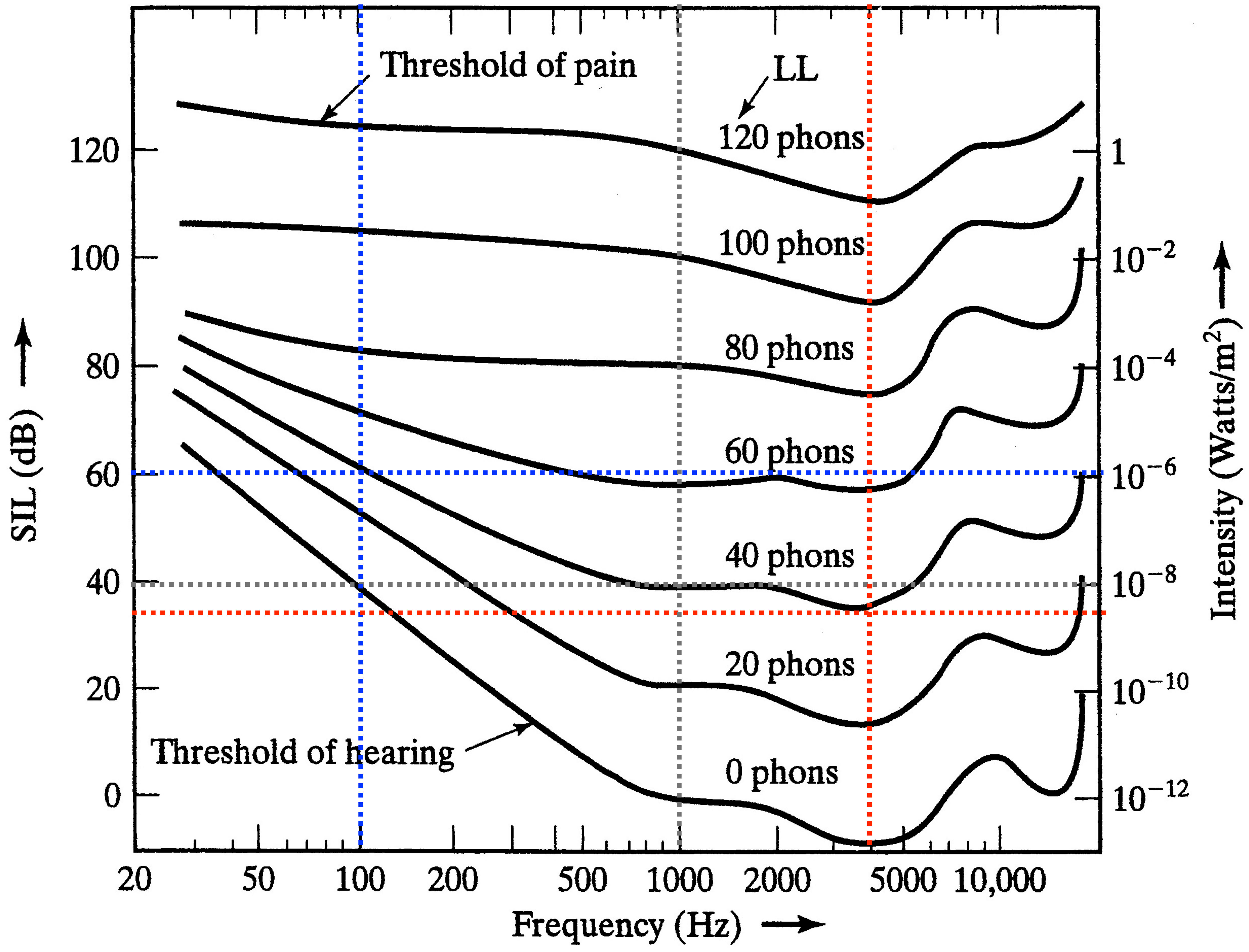
$$L_L \text{ (phon)} = \text{SIL (dB)} \quad \text{at} \quad f = 1000 \text{ Hz}$$

- Examples:

$\text{SIL}(40 \text{ phon}, 1000 \text{ Hz}) = 40 \text{ dB}$ (grey)

$\text{SIL}(40 \text{ phon}, 100 \text{ Hz}) = 60 \text{ dB}$ (blue)

$\text{SIL}(40 \text{ phon}, 4000 \text{ Hz}) = 35 \text{ dB}$ (red)



Subjective Loudness (S, sone)

- Measure of loudness where "**twice as loud**" corresponds to "**multiply by 2**"
- Most people perceive **10x increase in intensity** as "**twice as loud**"
  - Recall: 10x intensity –> $\Delta \mathrm{SIL} = 10 \text{ dB}$, $\Delta L_L = 10 \text{ phon}$
  - Some values:

$$
\begin{aligned}
L_L &= 40 \text{ phon} \quad \Leftrightarrow \quad S = 1 \text{ sone} \\
L_L &= 50 \text{ phon} \quad \Leftrightarrow \quad S = 2 \text{ sone} \\
L_L &= 60 \text{ phon} \quad \Leftrightarrow \quad S = 4 \text{ sone} \\
L_L &= 30 \text{ phon} \quad \Leftrightarrow \quad S = 1/2 \text{ sone}
\end{aligned}
$$

General formula:

$$S = 2^{(L_L - 40)/10} \text{ sone}$$

  - Questions:

$$
\begin{aligned}
L_L &= 70 \text{ phon} \quad \Leftrightarrow \quad S = ?? \text{ sone} \\
L_L &= 20 \text{ phon} \quad \Leftrightarrow \quad S = ?? \text{ sone}
\end{aligned}
$$

Different measures of loudness and safe noise levels

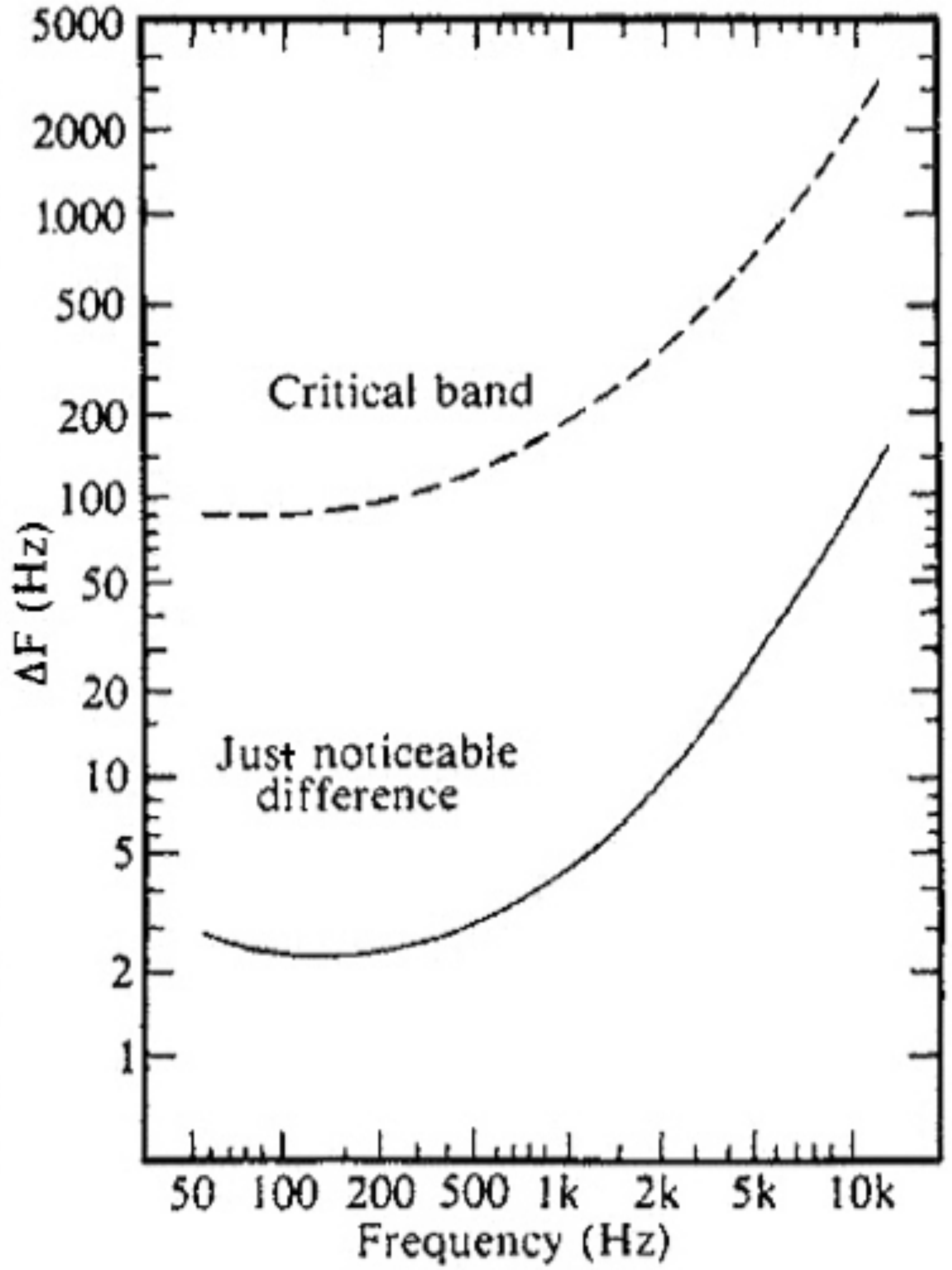
	SIL (dB) at 1000 Hz	L_L (phon)	S (sone)
Threshold of hearing	0	0	1/16
Recording studio	20	20	1/4
Quiet office	40	40	1
Ordinary conversation	60	60	4
Normal piano practice	80	80	16
Piano fortissimo	100	100	64
Threshold of pain	120	120	256

Duration per day, hours	Sound level dBA slow response
8	90
6	92
4	95
3	97
2	100
1 1/2	102
1	105
1/2	110
1/4 or less	115

11. Pitch & timbre

What distinguishes one musical note from another?

- **Pitch**, **timbre**, **duration**, **loudness** (intensity), **attack & decay transients**
- **Perfect pitch**: ability to determine **absolute pitch** without regard to a reference (only 1 out of ~10,000 people have it)
- **Pitch discrimination**: ability to distinguish two different pitches
    - depends on whether you play the two notes **sequentially** or **simultaneously**
    - **JND**: just noticeable difference (**sequential**; 0.5% of center frequency; 1/10th of a semitone)
    - **LFD**: limit of frequency discrimination (**simultaneous**; 10% of center frequency; 2 semitones)
    - Analogy with **sense of touch**: placing two pencil points on your arm



Where does pitch determination occur, in the ear or the brain?

- **Place theory**: pitch determined by the **location on the basilar membrane** excited by the sound wave

- **Periodicity theory**: pitch inferred by the brain from the **timing of electrical impulses** triggered by the period of the sound wave

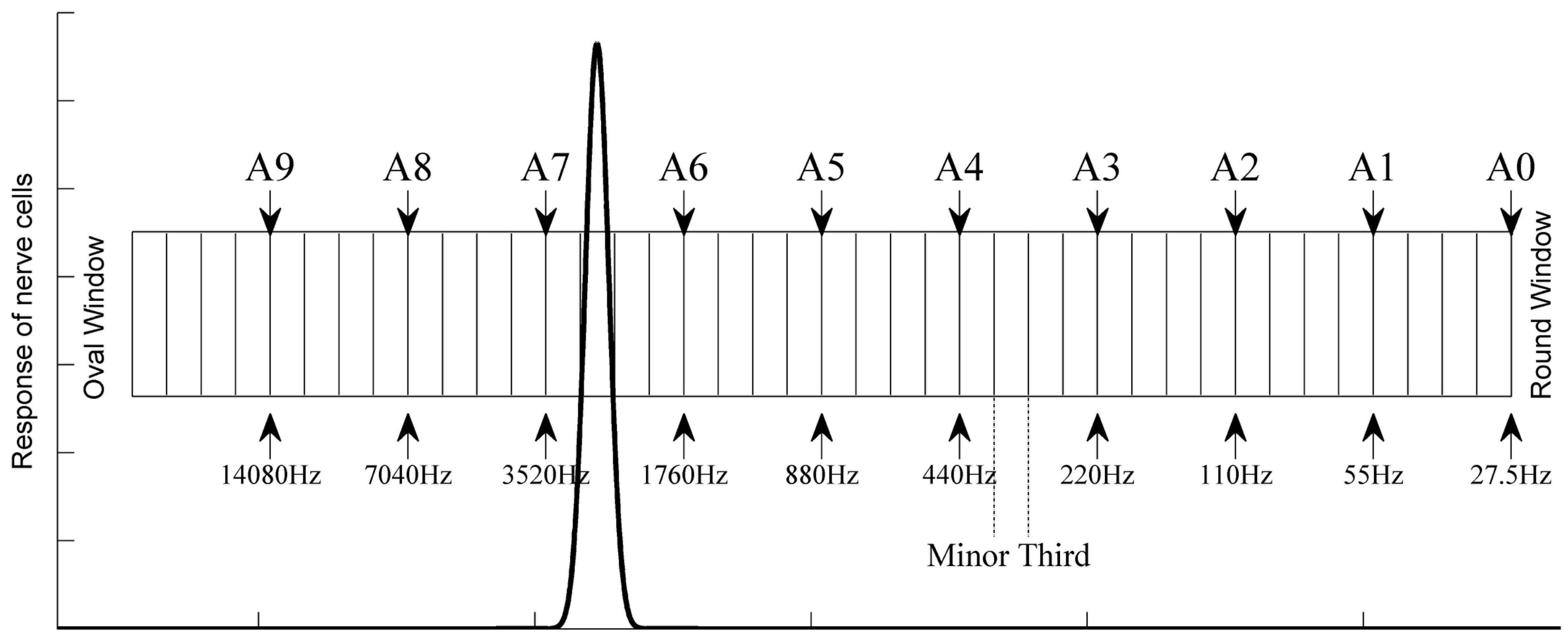
- **Missing fundamental** in support of periodicity theory:

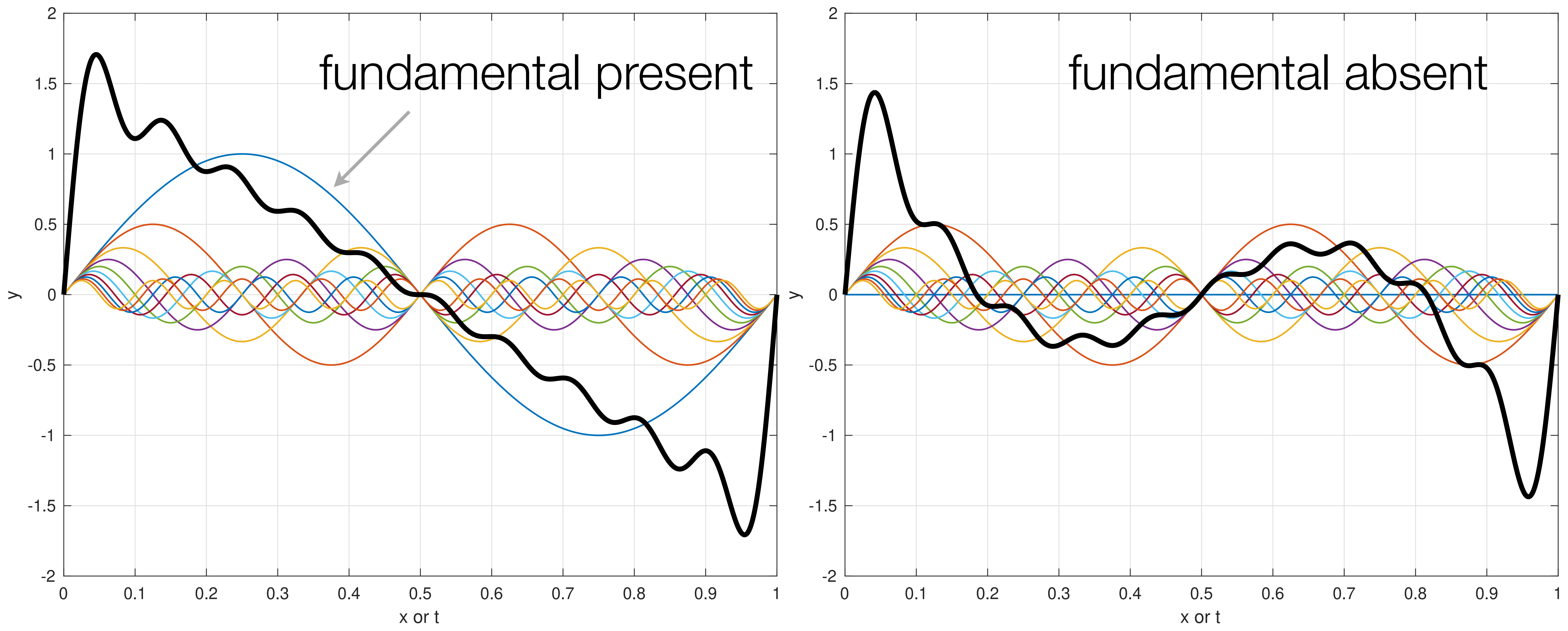
  - 200 Hz, 300 Hz, 400 Hz, …. –> hear 100 Hz

  - 300 Hz, 500 Hz, 700 Hz, …. –> hear ??? Hz

  - http://www.personal.psu.edu/meb26/INART50/psychoacoustics.html

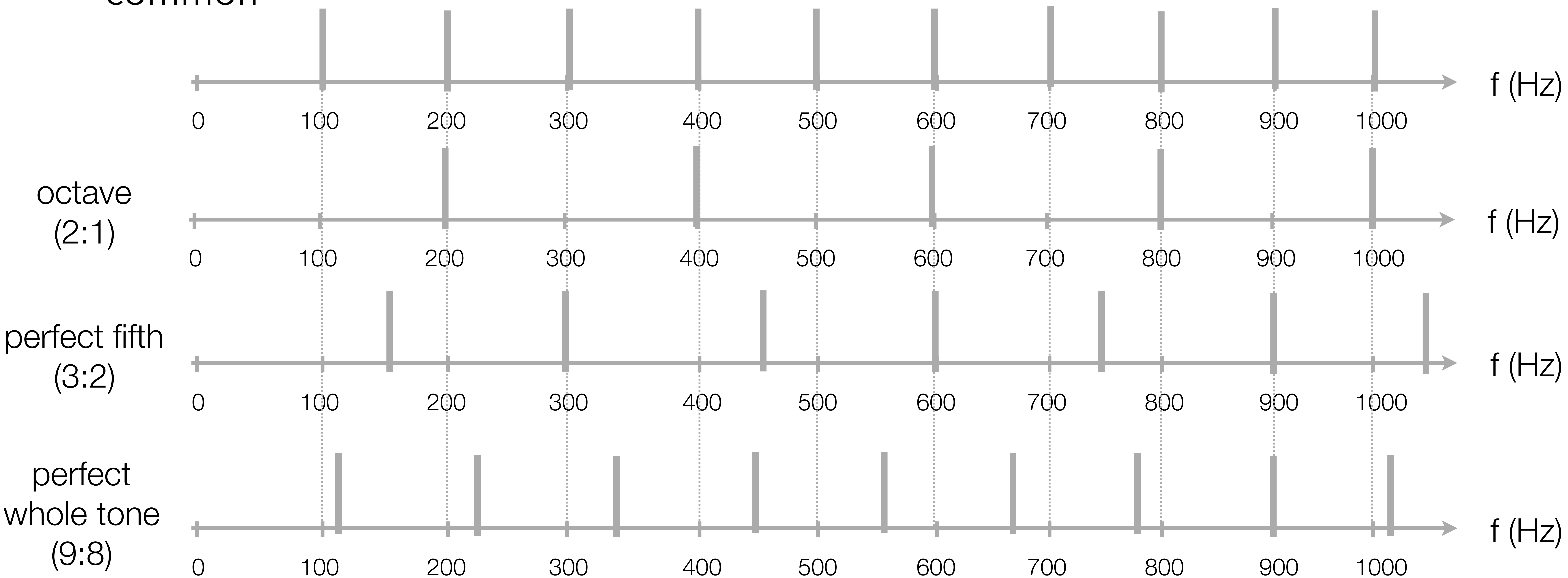
Basilar membrane (schematic)





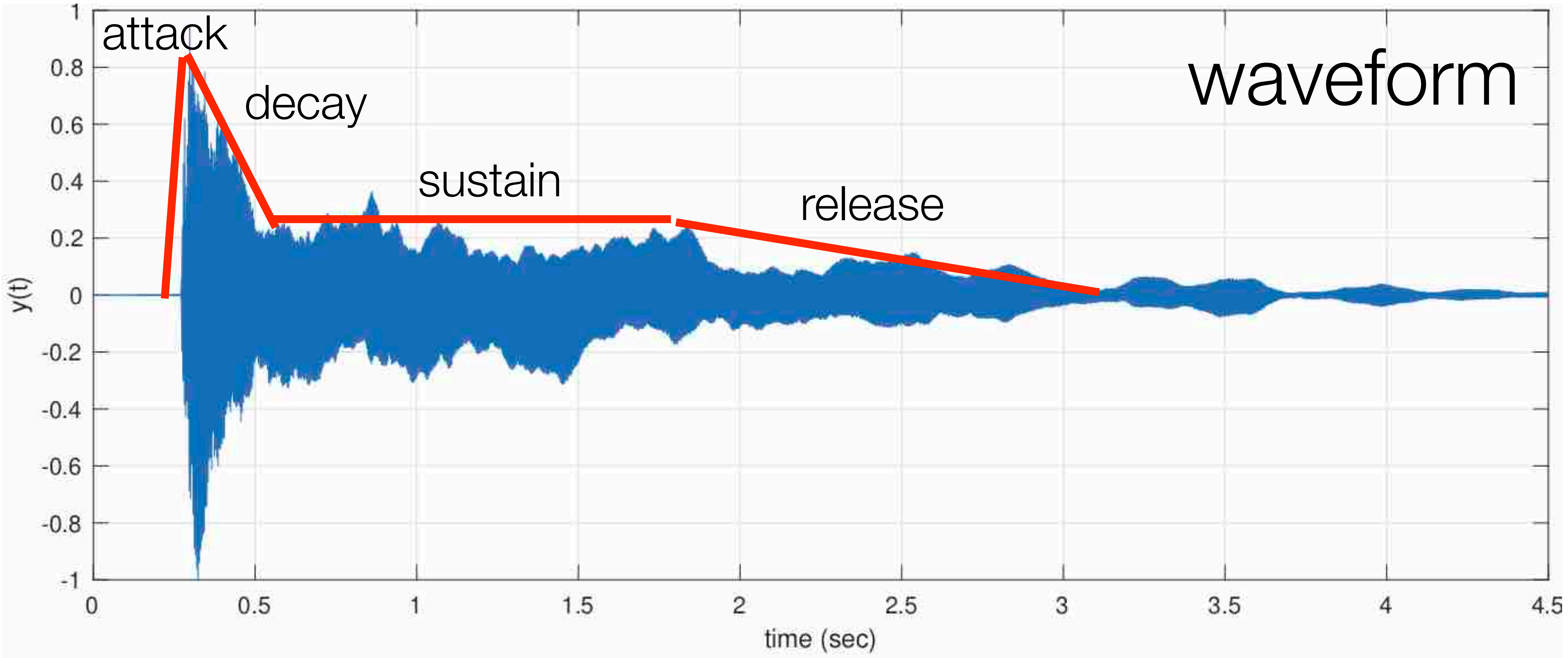
What makes two notes pleasing when they are played together?

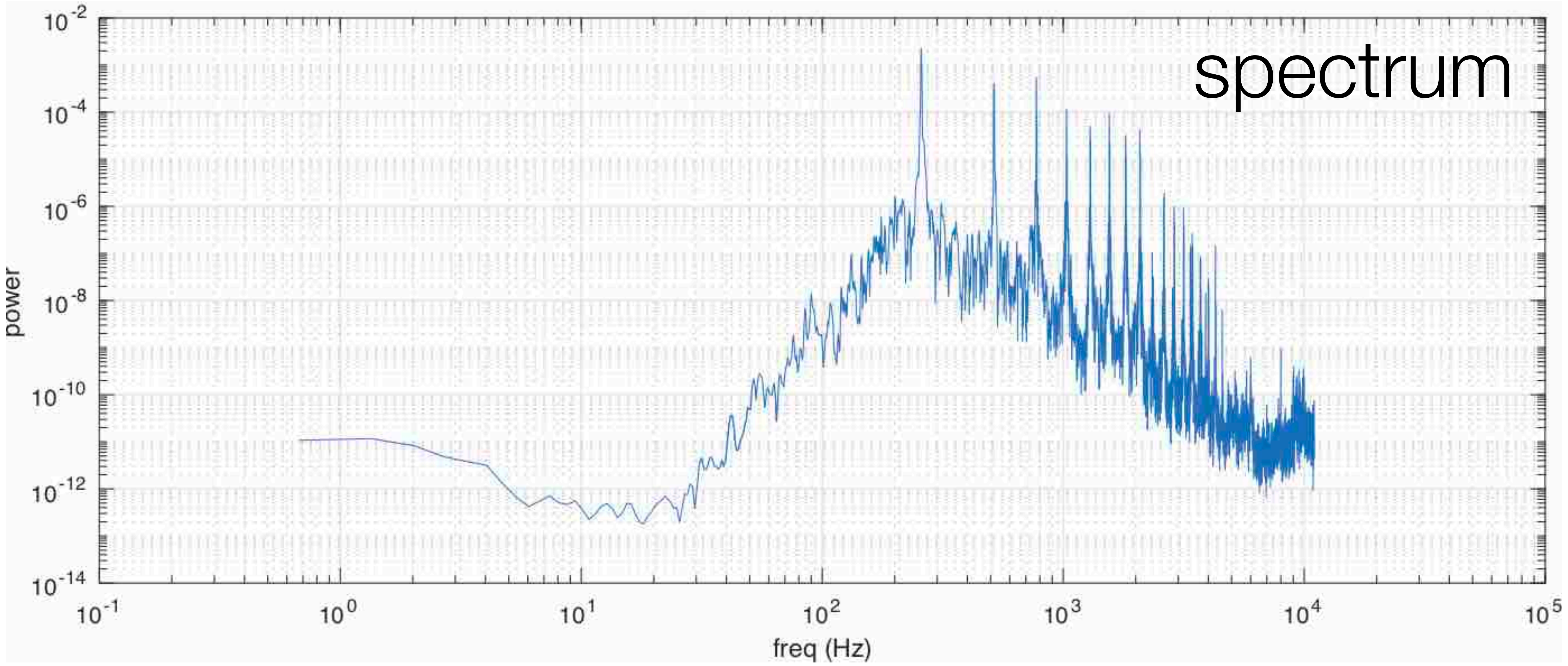
- Two notes are pleasing (**consonant**) when they have many harmonics in common
- Two notes clash with one another (**dissonant**) when they have very few harmonics in common



Attack and decay transients

- How a note starts and ends affects how it sounds
- Piano C4
- Piano C4 (reversed)
- Happy birthday
- Happy birthday backwards
- Happy birthday backwards (reversed)

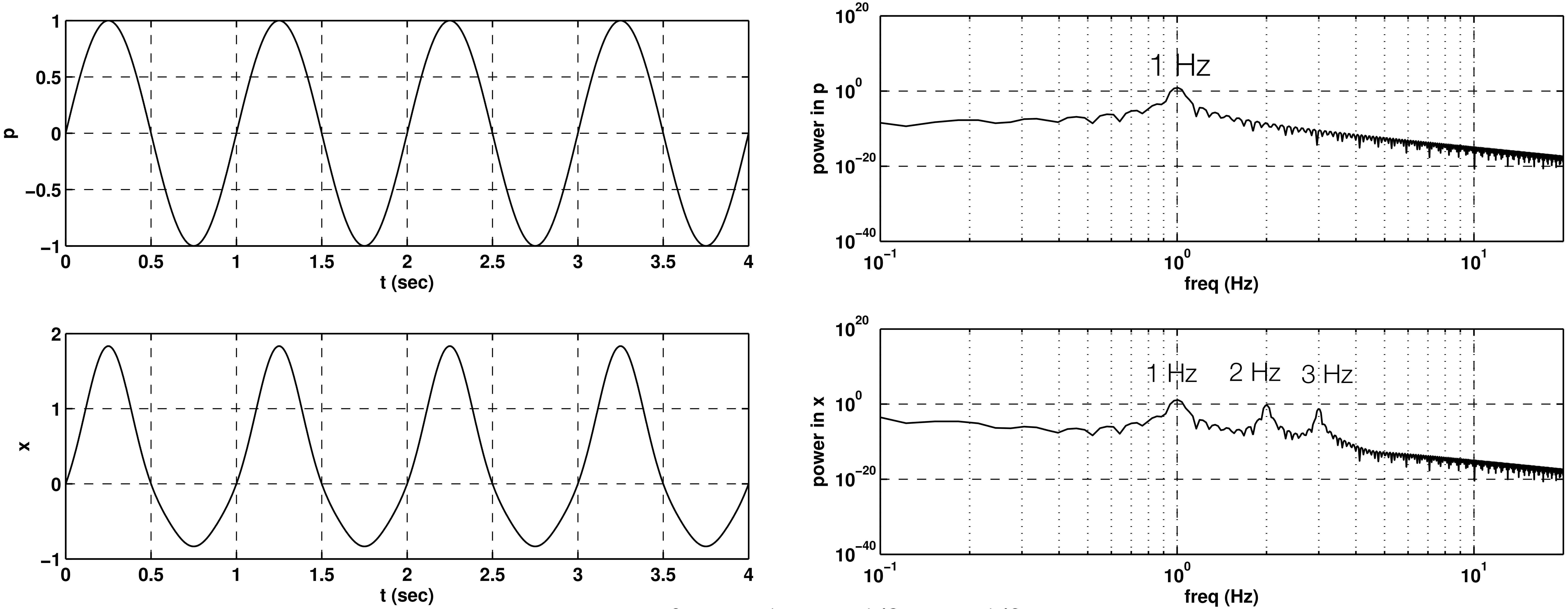




Aural harmonics – harmonics produced by the ear

- Ear introduces **distortions** which converts a pure tone to one having multiple harmonics

$$x(t) = a_0 + a_1\, p(t) + a_2\, p^2(t) + a_3\, p^3(t) + \cdots$$

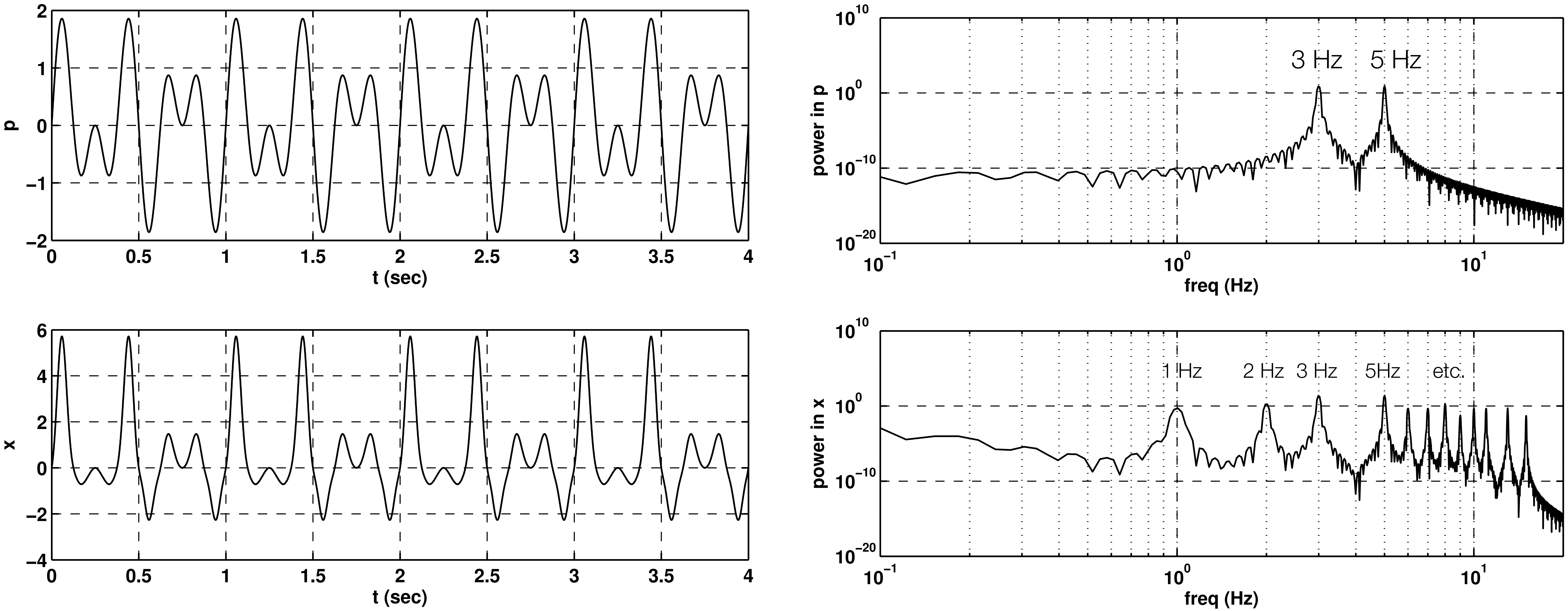


$a_0 = 0\,,\ a_1 = 1\,,\ a_2 = 1/2\,,\ a_3 = 1/3$

Aural combination tones — aural harmonics for complex tones

- If two pure tones f_1 and f_2 are played simultaneously and sufficiently loudly, one hears **sum and difference combination tones**

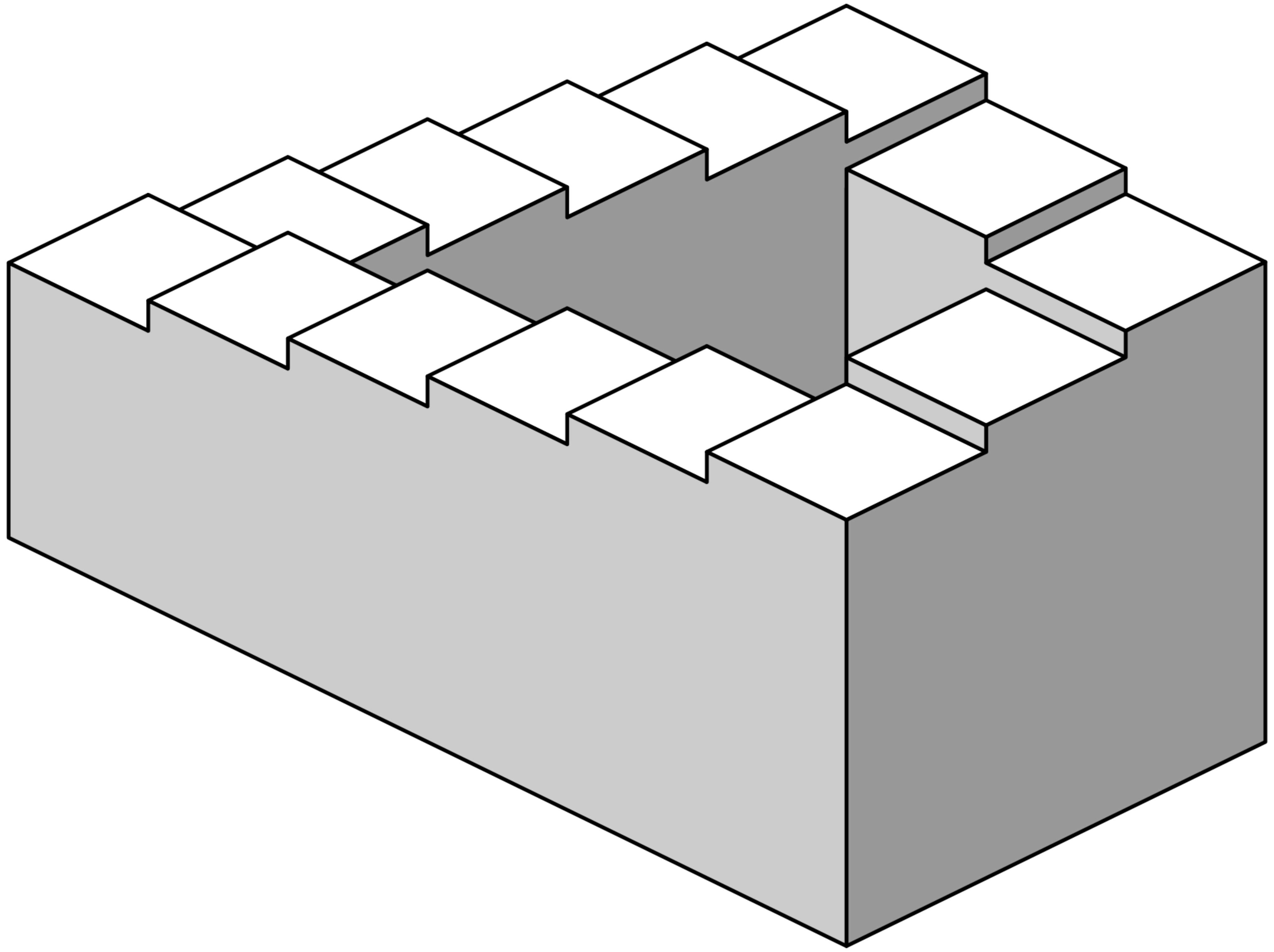
$$f_c = |mf_1 \pm nf_2| \quad \Rightarrow \quad |f_1 - f_2|,\ |2f_1 - f_2|,\ |3f_1 - f_2|, \cdots$$



$a_0 = 0\,,\ a_1 = 1\,,\ a_2 = 1/2\,,\ a_3 = 1/3$

Pitch paradox — the audio equivalent of an optical illusion

- **Shepard scale**: never-ending scale (pitch seems to increase indefinitely)
- YouTube videos:
    - http://www.youtube.com/watch?v=PCs1lckF5vI
    - http://vimeo.com/34749558



Never-ending staircase
(L. Penrose; M.C. Escher)

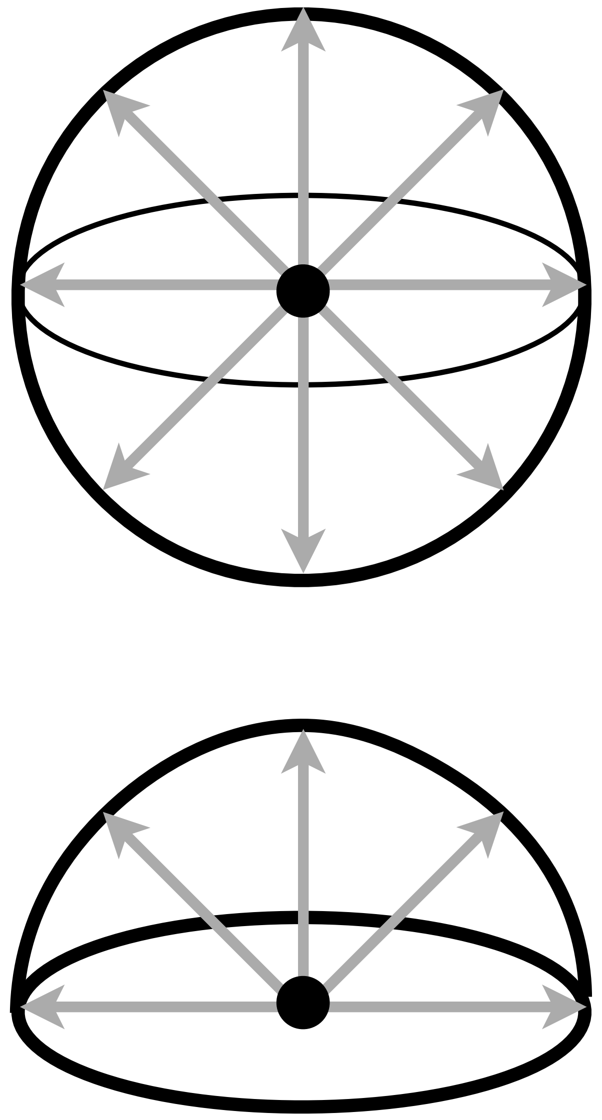
12. Auditorium & room acoustics

Auditorium and room acoustics — overview

- What makes for a good concert hall?
- Why do you sound good when you sing in a shower?
- Difference between “direct”, “reflected”, and “reverberant” sound
- Reverberation time is the most important characteristic of a room
- YouTube video / soundfile:
    - Anechoic chamber (https://www.youtube.com/watch?v=BYBSA9v8IRE)
    - “Sonic wonders” sound file (listen to -32:40)

Direct sound

- Sound received from a source in the absence of any reflections (e.g., anechoic chamber)
- Intensity: $I = \dfrac{P}{4\pi r^2}$ (omni-directional); $I = \dfrac{QP}{4\pi r^2}$ (directional source; Q is the directivity factor)



$Q = 1$

$Q = 2$

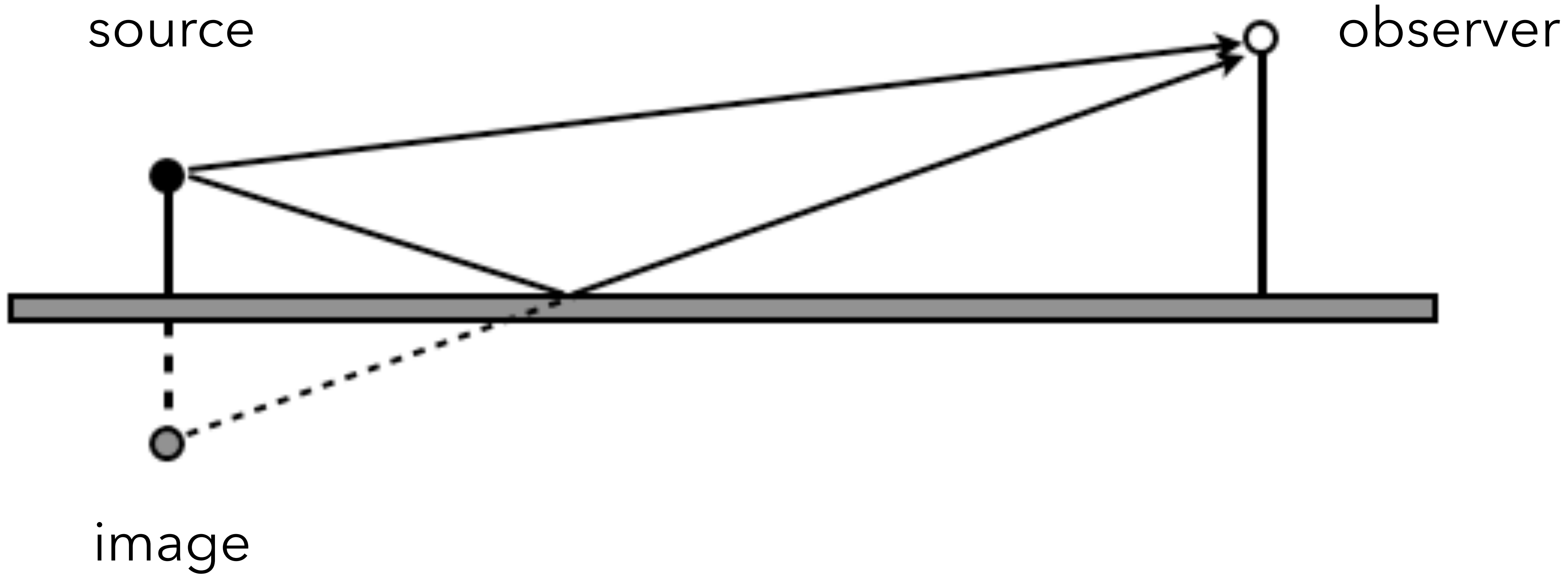
$Q = 0$

source

observer

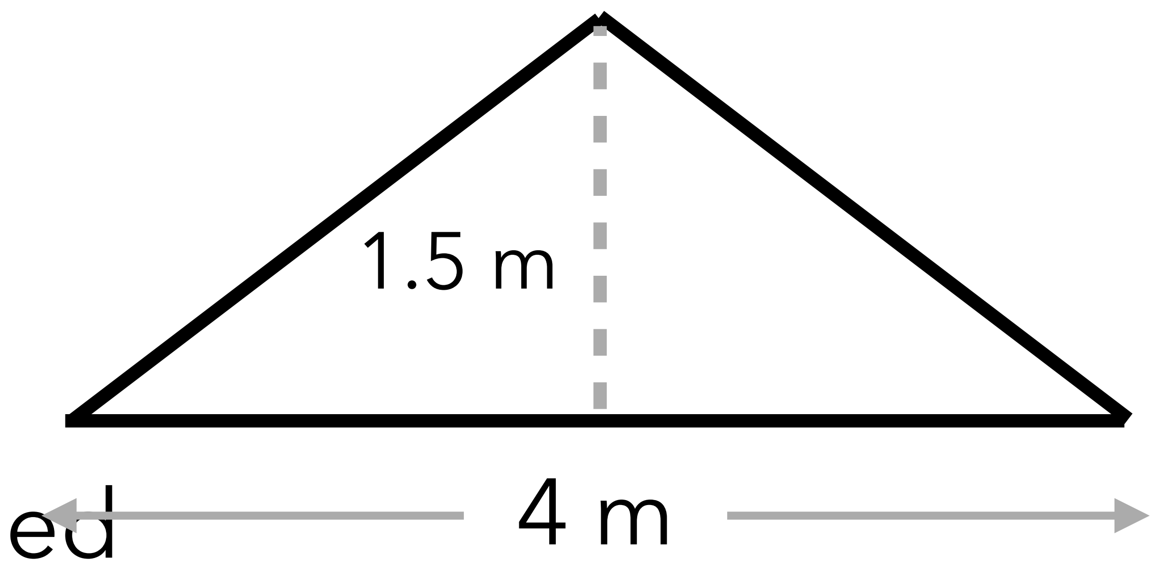
Reflected sound

- Hear an echo if the reflected sound is heard greater than 35 msec after the direct sound
- Recall: $v = 346 \text{ m/s} \approx 1000 \text{ ft/s} = 1 \text{ ft/msec}$
- $\text{SIL}_{\text{reflected}} < \text{SIL}_{\text{direct}}$ (reflected sound travels farther and can be partially absorbed)



Reflected sound - example

- A listener stands 4 m in front of an omni-directional loudspeaker that is 1.5 m from a reflecting wall.
- Calculate:
  - the time of arrival for both the direct and reflected sound
  - the decrease in SIL for the reflected sound due to the larger distance traveled
  - the decrease in SIL for the reflected sound assuming an absorption coefficient $a = 0.2$ for the wall
- Answer:



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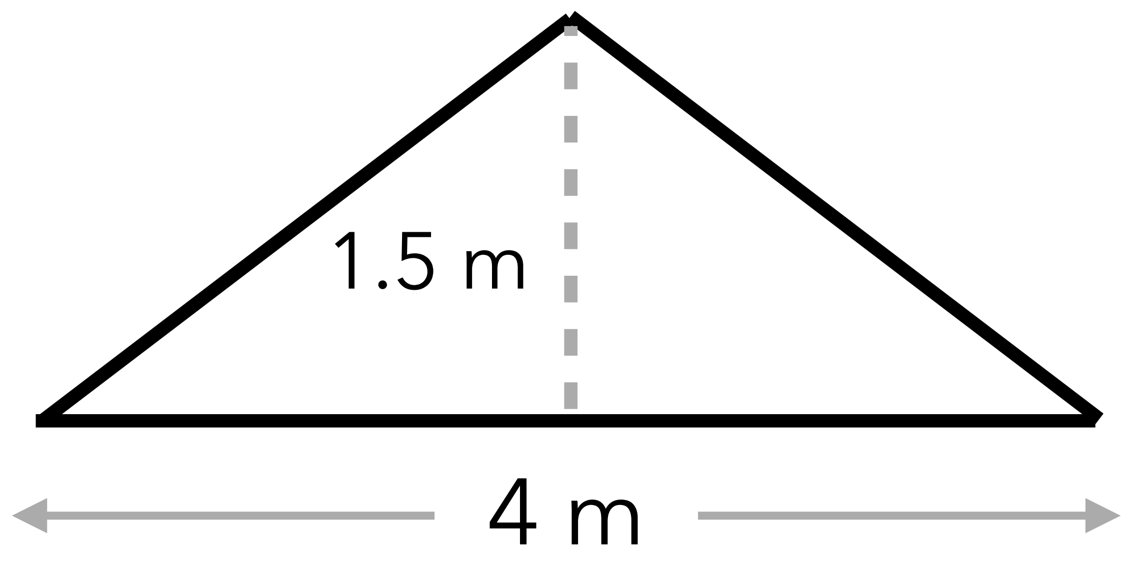
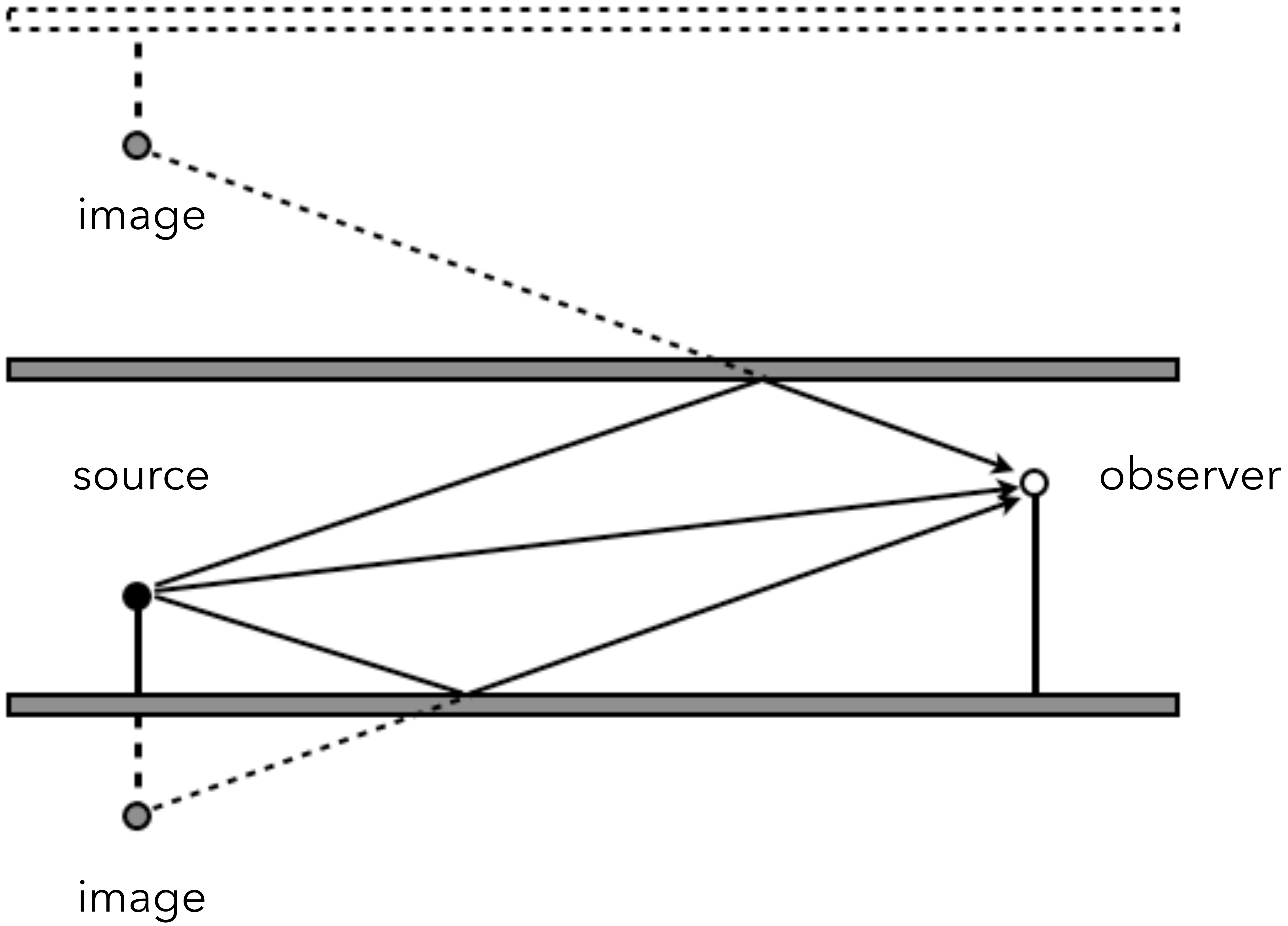
  - the decrease in SIL for the reflected sound due to the larger distance traveled
  - the decrease in SIL for the reflected sound assuming an absorption coefficient $a = 0.2$ for the wall

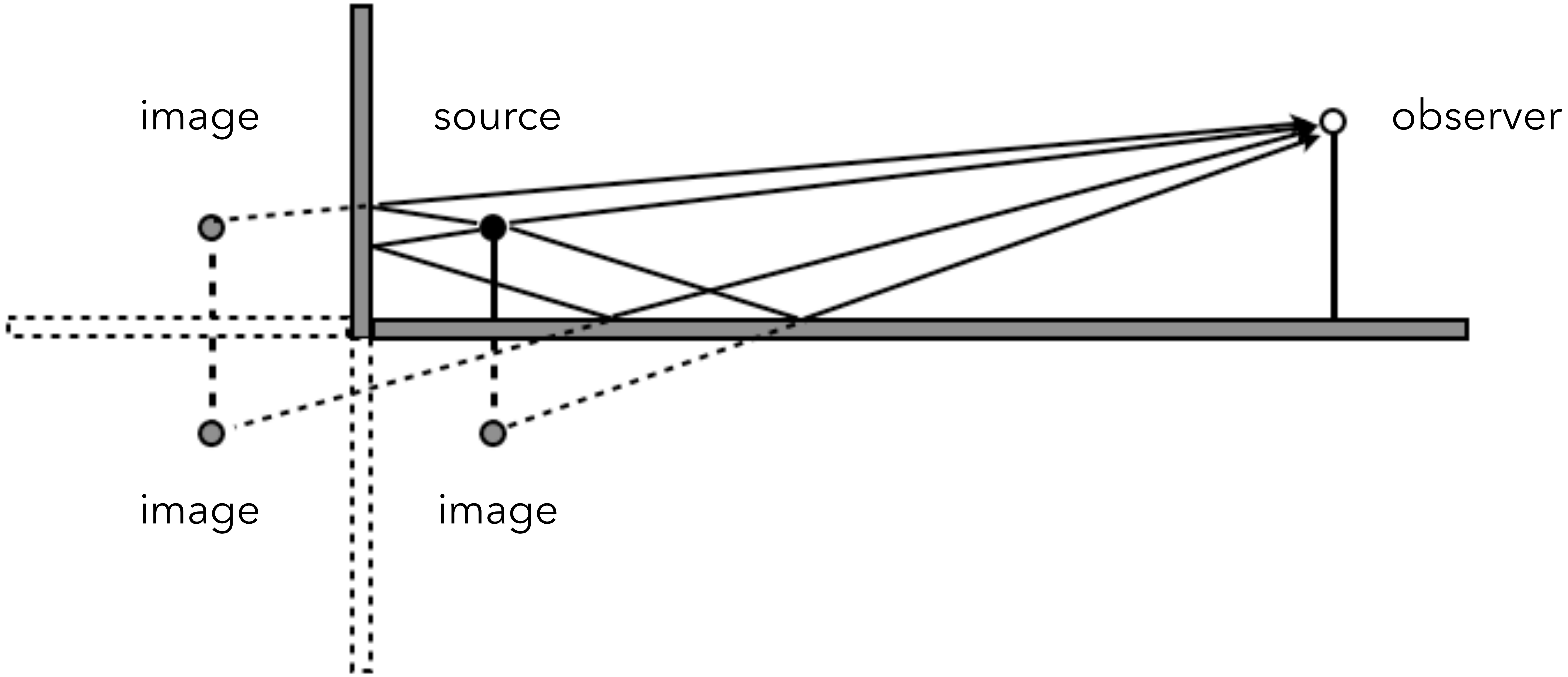


- Answer:
  - Reflected sound travels 5 meter: $t_{\text{direct}} = \dfrac{4 \text{ m}}{346 \text{ m/s}} = 12 \text{ msec}$, $t_{\text{reflected}} = \dfrac{5 \text{ m}}{346 \text{ m/s}} = 15 \text{ msec}$
  - $\Delta\text{SIL} = 10 \log \left[1/(r_{\text{reflected}}/r_{\text{direct}})^2\right] \text{ dB} = 10 \log \left[(4/5)^2\right] \text{ dB} = -2 \text{ db}$
  - $\Delta\text{SIL} = 10 \log(1 - a) \text{ dB} = 10 \log(1 - 0.2) \text{ dB} = -1 \text{ db}$

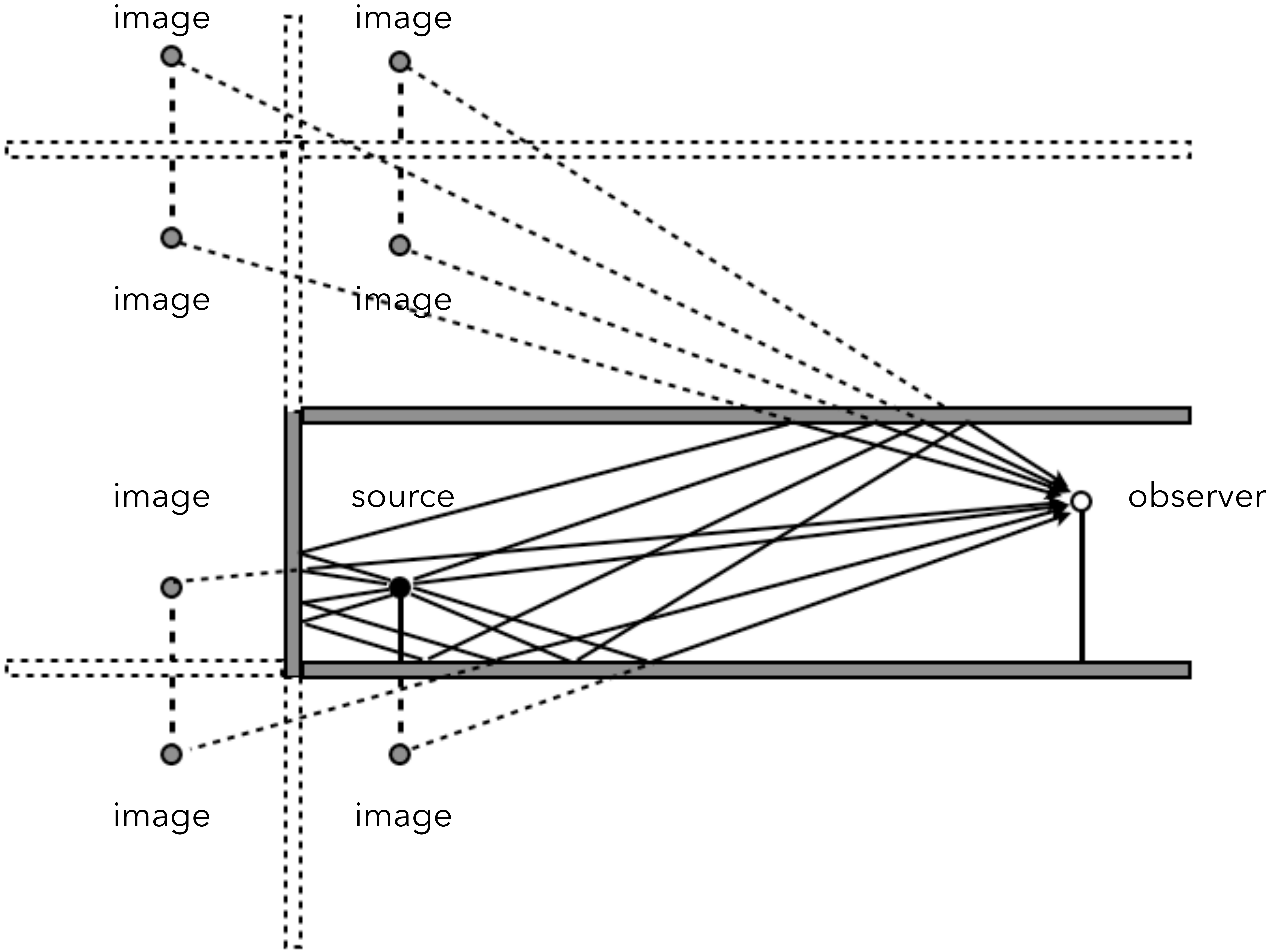
Multiple reflections — floor and ceiling



Multiple reflections – floor and back wall

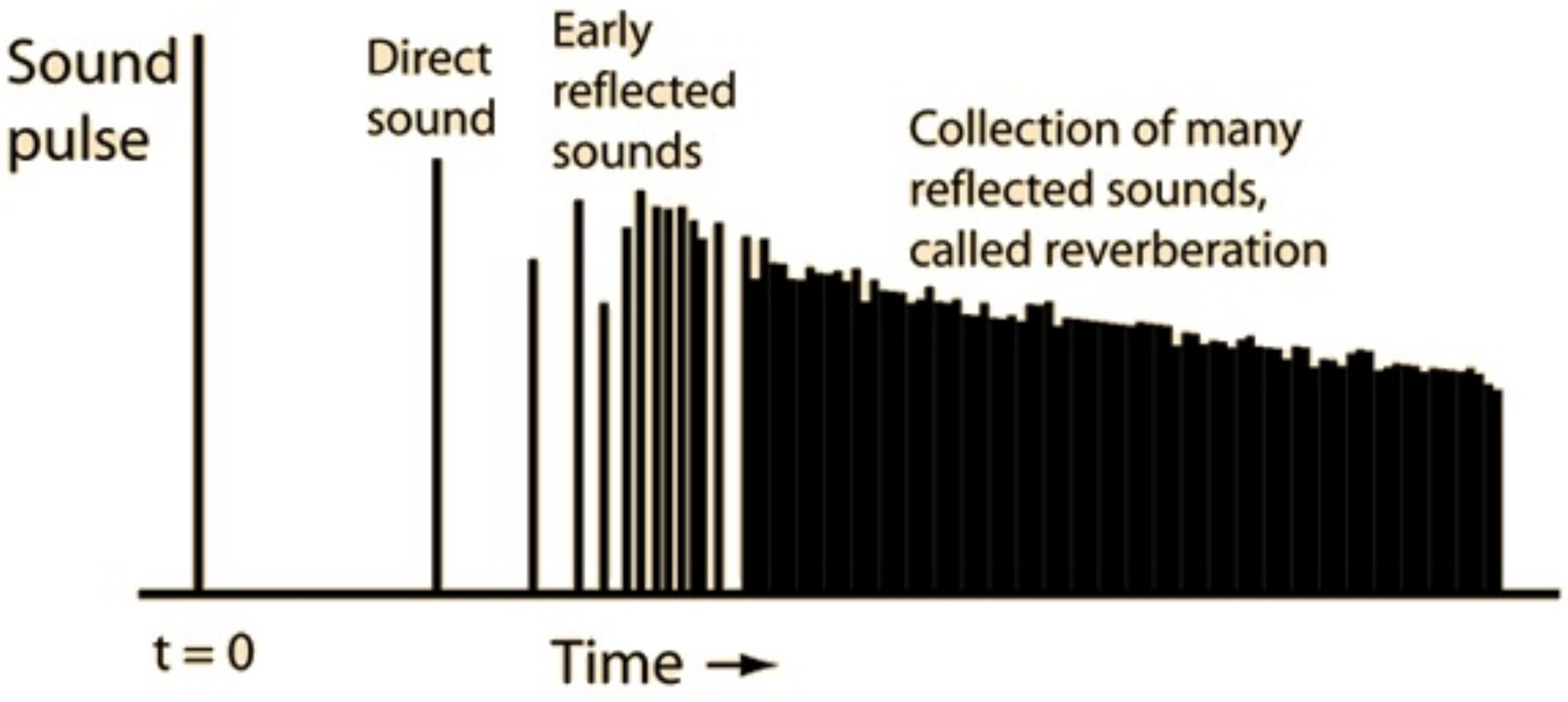


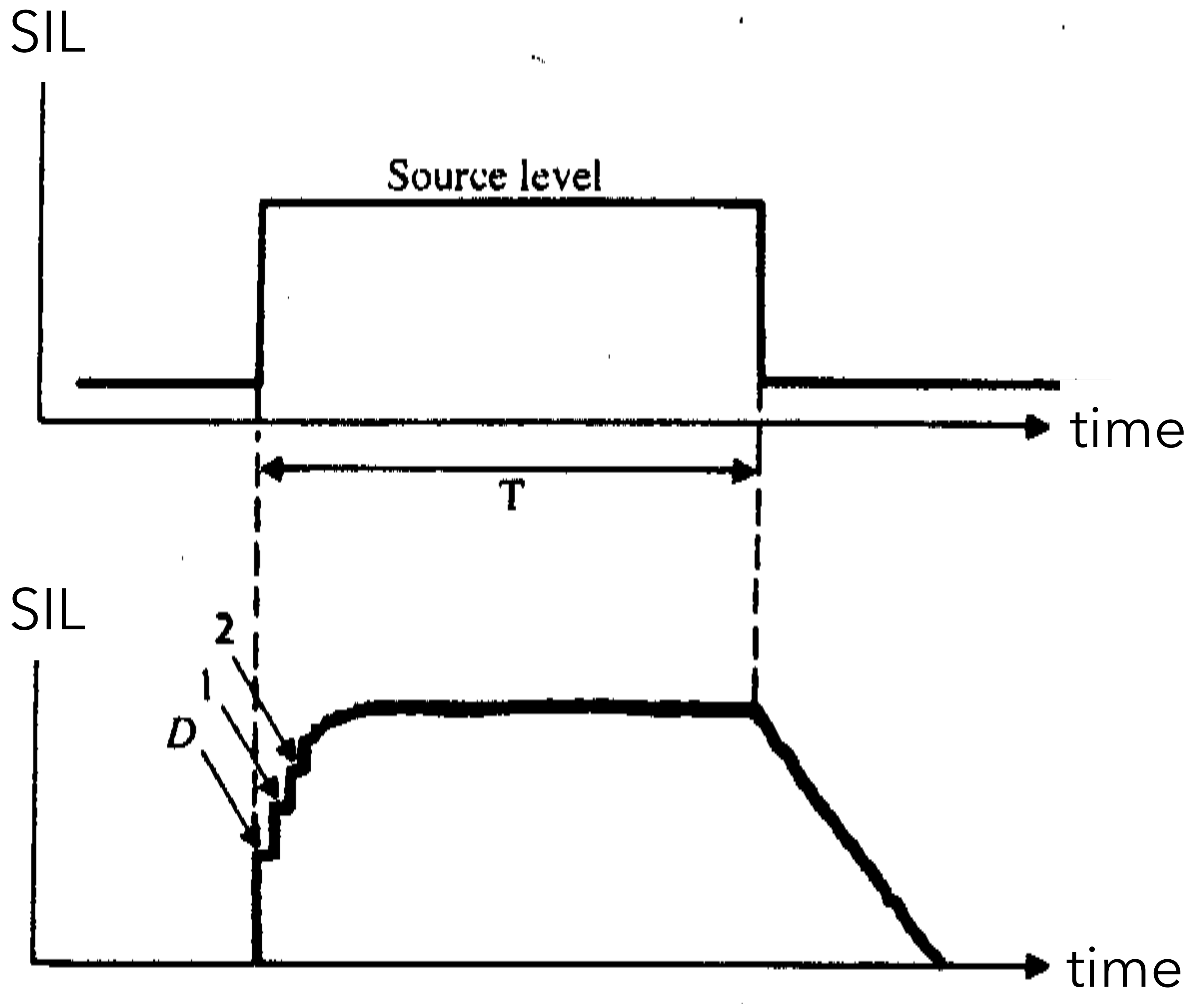
Multiple relfections — floor, back wall, and ceiling



Reverberant sound

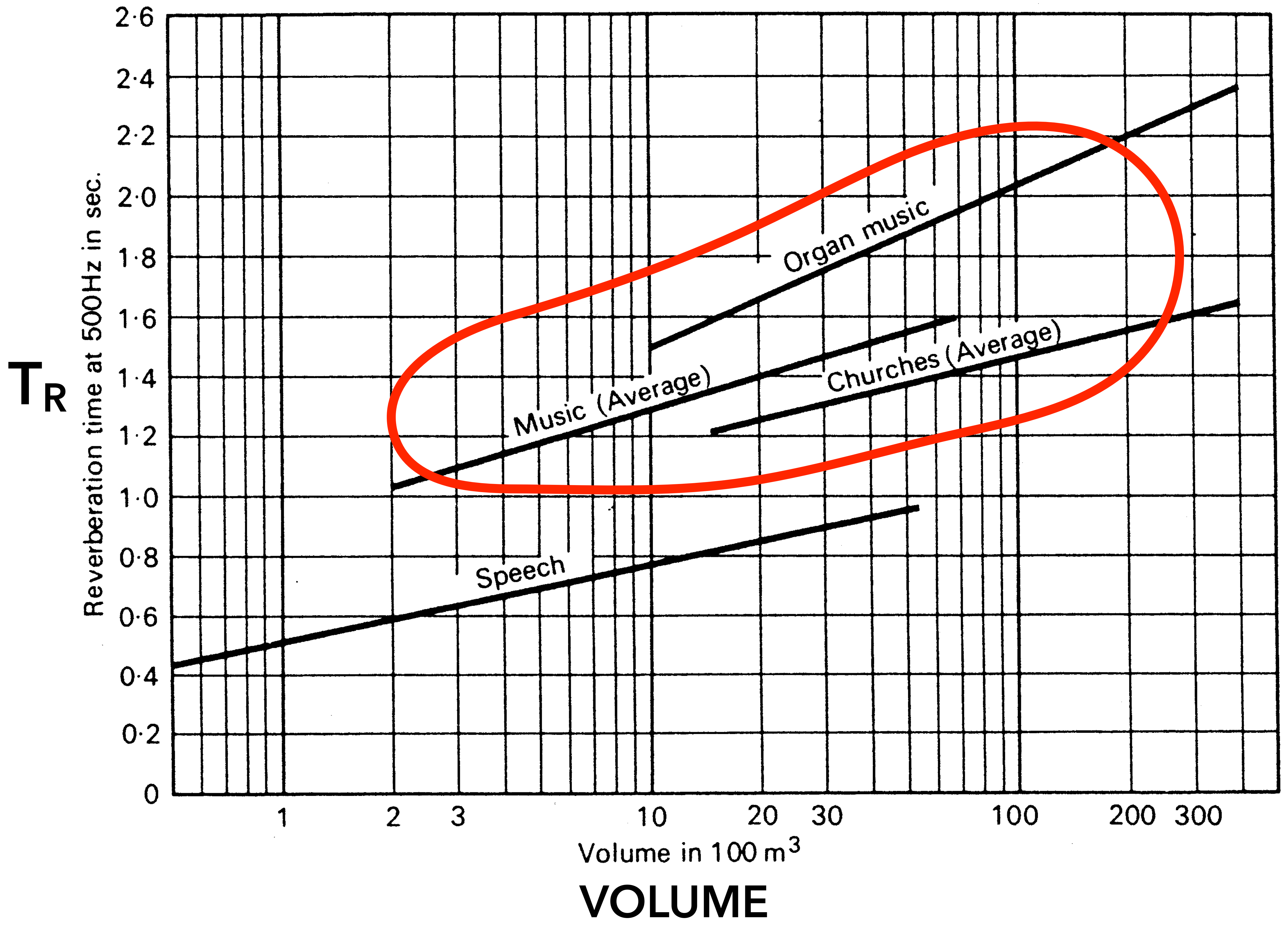
- sound formed from multiple reflections, coming from many different directions, and overlapping in time





Reverberation time

- time required for the reverberant SIL to decrease by 60 dB ($1/10^6$ in intensity)
- frequency dependent (low-frequency sounds typically have larger reverberation times)



Acoustical characteristics of various concert halls

	Year built	Volume (m^3)	Number of seats	Reverberation time (sec) Frequency (Hz) 125	500	2000
Teatro alla Scala, Milan	1778	11,245	2289		1.2	
Royal Opera House	1858	12,240	2180		1.1	
Royal Albert Hall	1871	86,600	6080	3.4	2.6	2.2
Carnegie Hall, New York	1891	24,250	2760	1.8	1.8	1.6
Symphony Hall, Boston	1900	18,740	2630	2.2	1.8	1.7
Royal Festival Hall	1951	22,000	3000	1.4	1.5	1.4
Philharmonic Hall, Berlin	1963	36,030	2200		2.0	
St. David's Hall, Cardiff	1983	22,000	2200	1.8	1.9	1.8

Calculating reverberation time

$$T_R = 0.05 \frac{V}{A_{\text{eff}}} \text{ s}$$

V : volume in (ft^3)

$$A_{\text{eff}} = A_1 a_1 + A_2 a_2 + \cdots + B_1 + B_2 + \cdots$$

- A_{eff}: total absorption in sabin (1 ft^2 of perfectly absorbing surface)
- $A_1, A_2, \cdots$: surface area of walls, etc. (in ft^2)
- $a_1, a_2, \cdots$: absorption coeffs (dimensionless, freq-dependent)
- $B_1, B_2, \cdots$: absorption for seats, people, etc. (in sabin)

absorption coefficients (dimensionless)

	Frequency (Hz)					
Material	125	250	500	1000	2000	4000
Concrete (painted)	0.10	0.05	0.06	0.07	0.09	0.08
Plywood panel	0.28	0.22	0.17	0.09	0.10	0.11
Plaster on lath	0.14	0.10	0.06	0.05	0.04	0.03
Gypsum board, 1/2 in.	0.29	0.10	0.05	0.04	0.07	0.09
Glass window	0.35	0.25	0.18	0.12	0.07	0.04
Curtains	0.14	0.35	0.55	0.72	0.70	0.65
Carpet (on concrete)	0.02	0.06	0.14	0.37	0.60	0.65
Carpet (on pad)	0.08	0.24	0.57	0.69	0.71	0.73
Acoustical tile, suspended	0.76	0.93	0.83	0.99	0.99	0.94

absorption (in m^2) **[multipy by 10.8 to convert to sabin]**

	Frequency (Hz)					
Material	125	250	500	1000	2000	4000
Wood or metal seat, unoccupied	0.014	0.018	0.020	0.036	0.035	0.028
Upholstered seat, unoccupied	0.13	0.26	0.39	0.46	0.43	0.41
Adult	0.23	0.32	0.39	0.43	0.46	—
Adult in an upholstered seat	0.27	0.40	0.56	0.65	0.64	0.56

Example

<u>Exercise</u>: Calculate the reverberation time at 500 Hz for a room with dimensions 20 m $\times$ 15 m $\times$ 8m (high). The walls are painted concrete, the ceiling is plaster, and the floor is carpet on pad. Also, assume that there are 200 upholstered seats, and that they are half-filled with people.

Example

<u>Exercise</u>: Calculate the reverberation time at 500 Hz for a room with dimensions 20 m × 15 m × 8m (high). The walls are painted concrete, the ceiling is plaster, and the floor is carpet on pad. Also, assume that there are 200 upholstered seats, and that they are half-filled with people.

$$L = 20\ \text{m} \times 3.28\ \text{ft/m} = 65.6\ \text{ft}$$
$$W = 15\ \text{m} \times 3.28\ \text{ft/m} = 49.2\ \text{ft}$$
$$H = 8\ \text{m} \times 3.28\ \text{ft/m} = 26.24\ \text{ft}$$

$$V = L \times W \times H = 2400\ \text{m}^3 = 8.47 \times 10^4\ \text{ft}^3$$

$$A_{\text{eff}} = \underbrace{0.06}_{\text{painted concrete}}\left[2(L \times H) + 2(W \times H)\right] + \underbrace{0.06}_{\text{plaster}}(L \times W) + \underbrace{0.57}_{\text{carpet on pad}}(L \times W) + 10.8(100 \times \underbrace{0.39}_{\text{empty upholstered}} + 100 \times \underbrace{0.56}_{\text{occupied}})$$
$$= 3.42 \times 10^3\ \text{sabin}$$

$$T_R = 0.05\ \frac{V}{A_{\text{eff}}}\ \text{s} = 1.2\ \text{s}$$ –> ideal for music (for V=2400 m^3)

Acoustical design

Criteria for good design

- Loudness
- Uniformity (no “live” or “dead” spots)
- Reverberance or liveness (feeling of being “bathed” in sound)
- Clarity (opposite of reverberance)

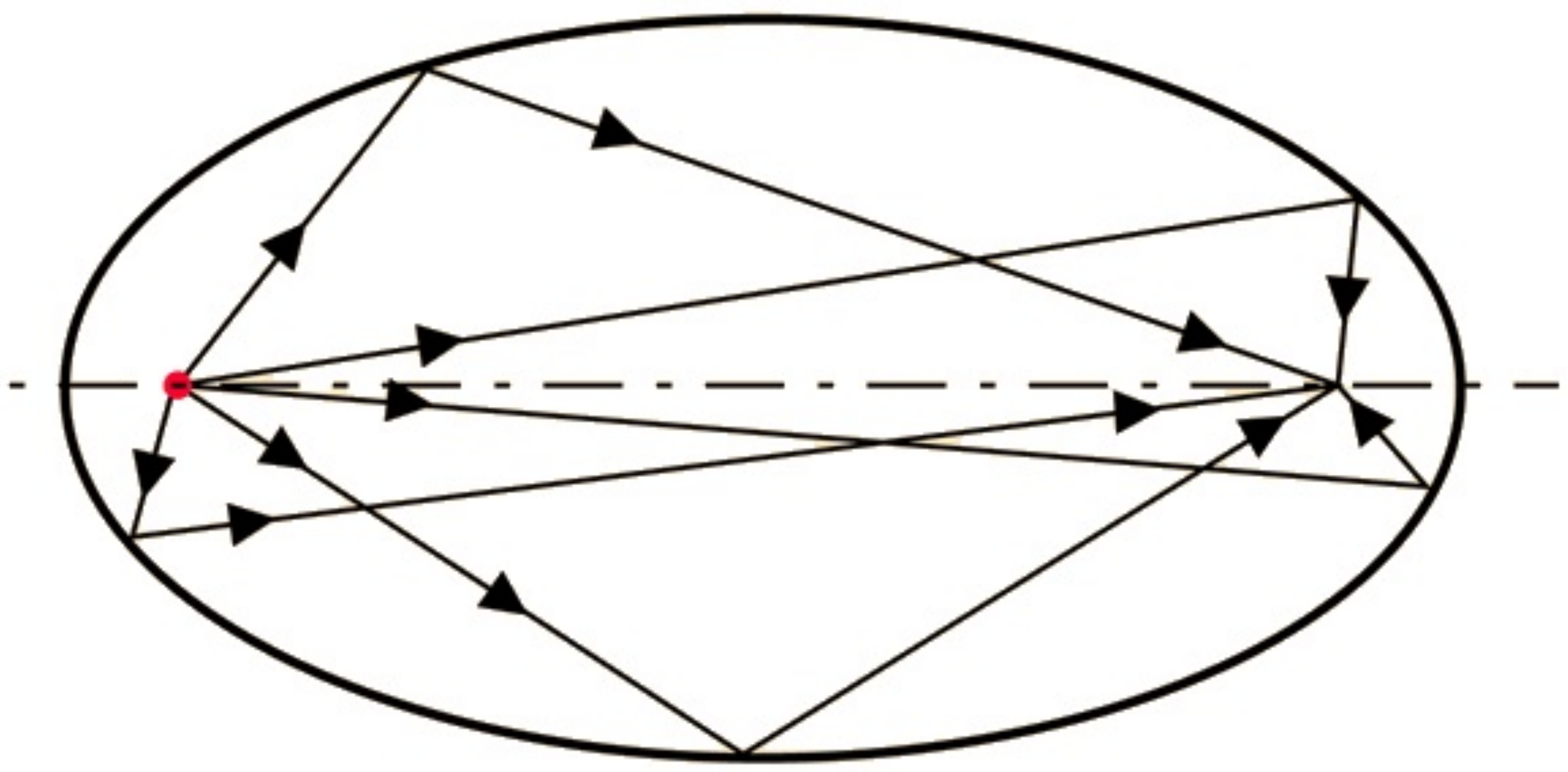
Problems to avoid

- Background noise (external noise due to heating, A/C, …)
- Shadow areas (produced by balconies, columns, …)
- Focusing of sound (“whispering room” effect)
- Echoes
- Room resonances (“shower stall” effect)

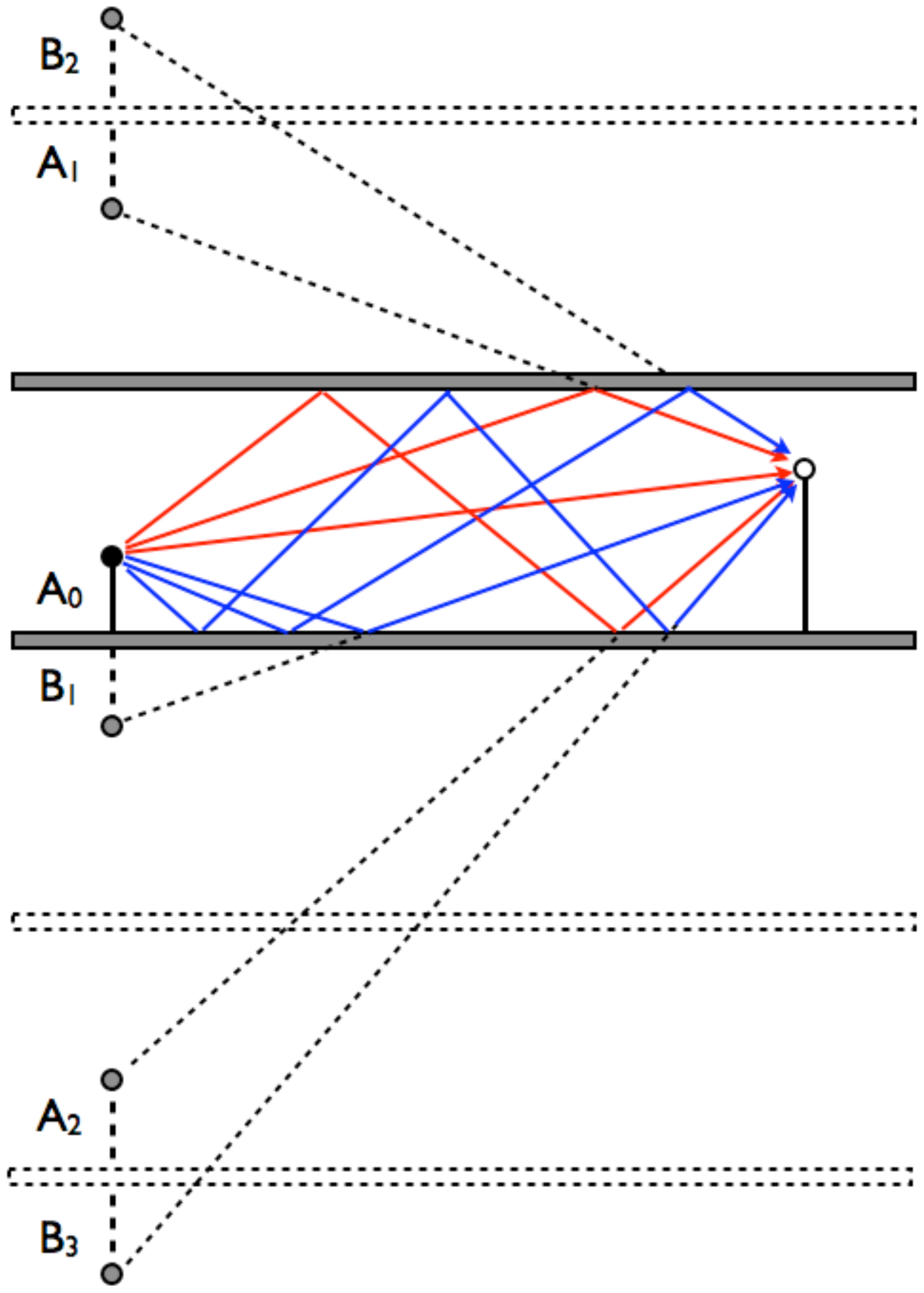
$$f_{lmn} = \frac{v}{2}\sqrt{\left(\frac{l}{L}\right)^2 + \left(\frac{m}{W}\right)^2 + \left(\frac{n}{H}\right)^2}$$

$$l, m, n = 0,1,2,\cdots$$

Problems to avoid



Whispering room effect



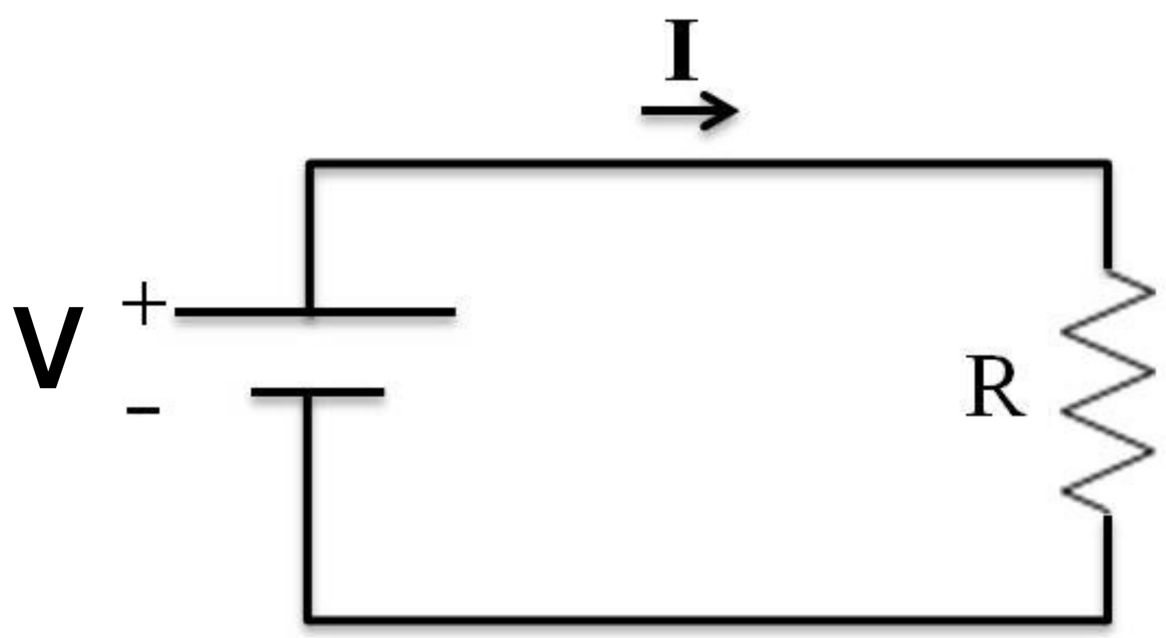
Flutter echoes

13. Electrical reproduction of sound

Electrical reproduction of sound — overview

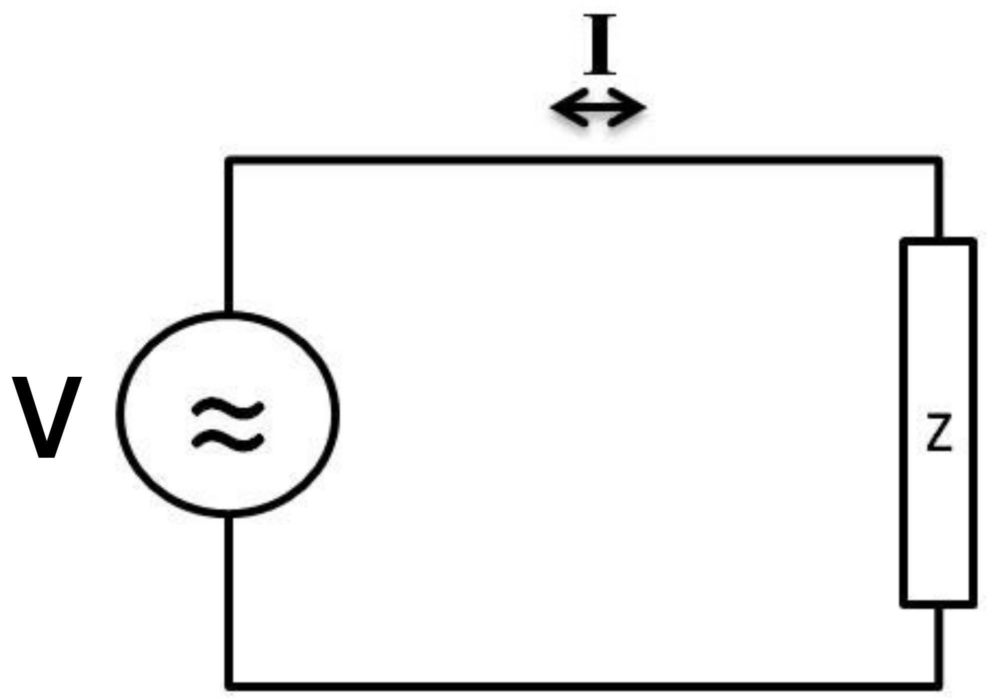
- Goal: Understand how microphones and loudspeakers work
- Need basic understanding of:
  - electricity and magnetism
  - Faraday's law of induction

Basic electricity



(DC circuit)

e.g., a battery connected
to a flashlight bulb



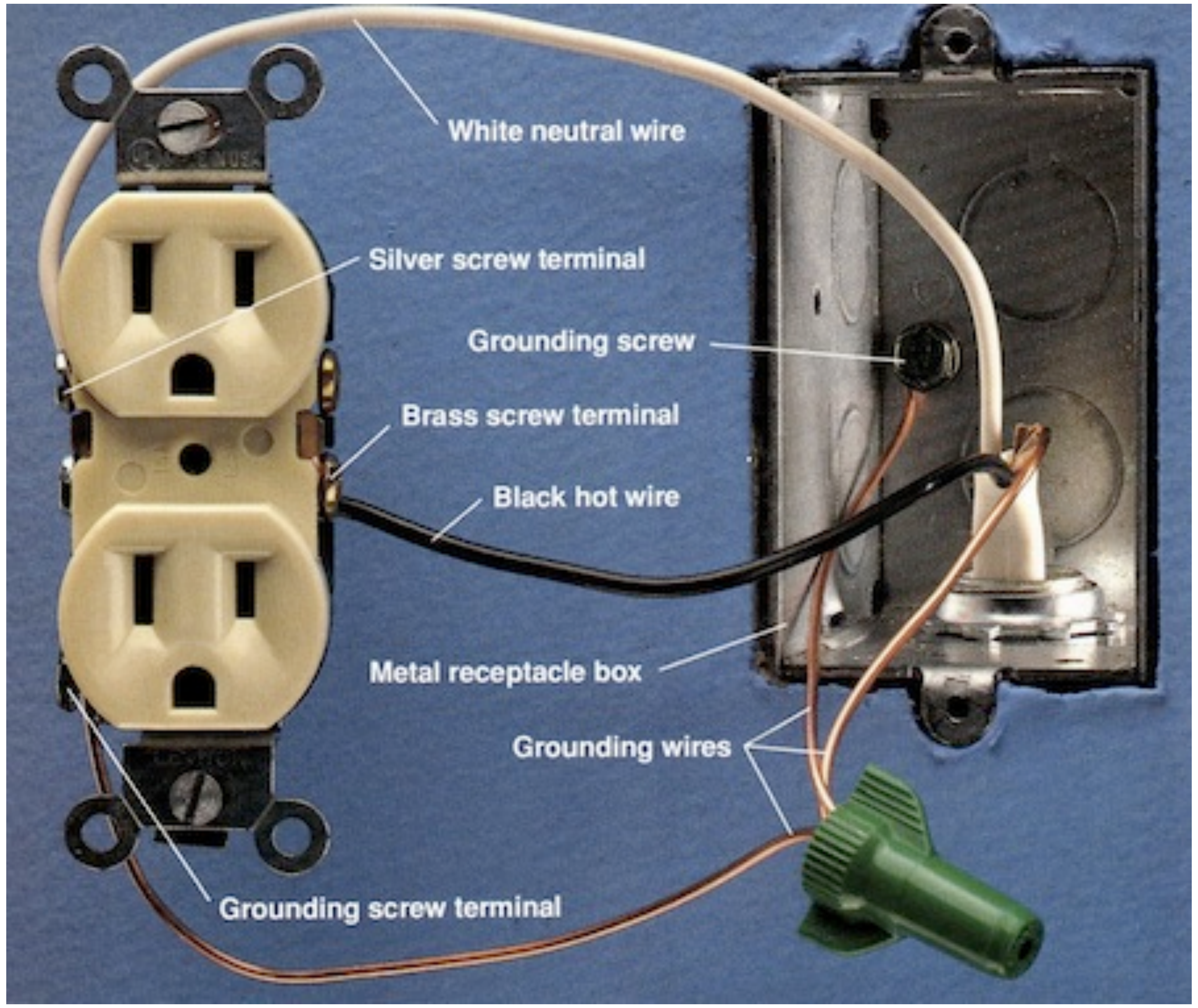
(AC circuit)

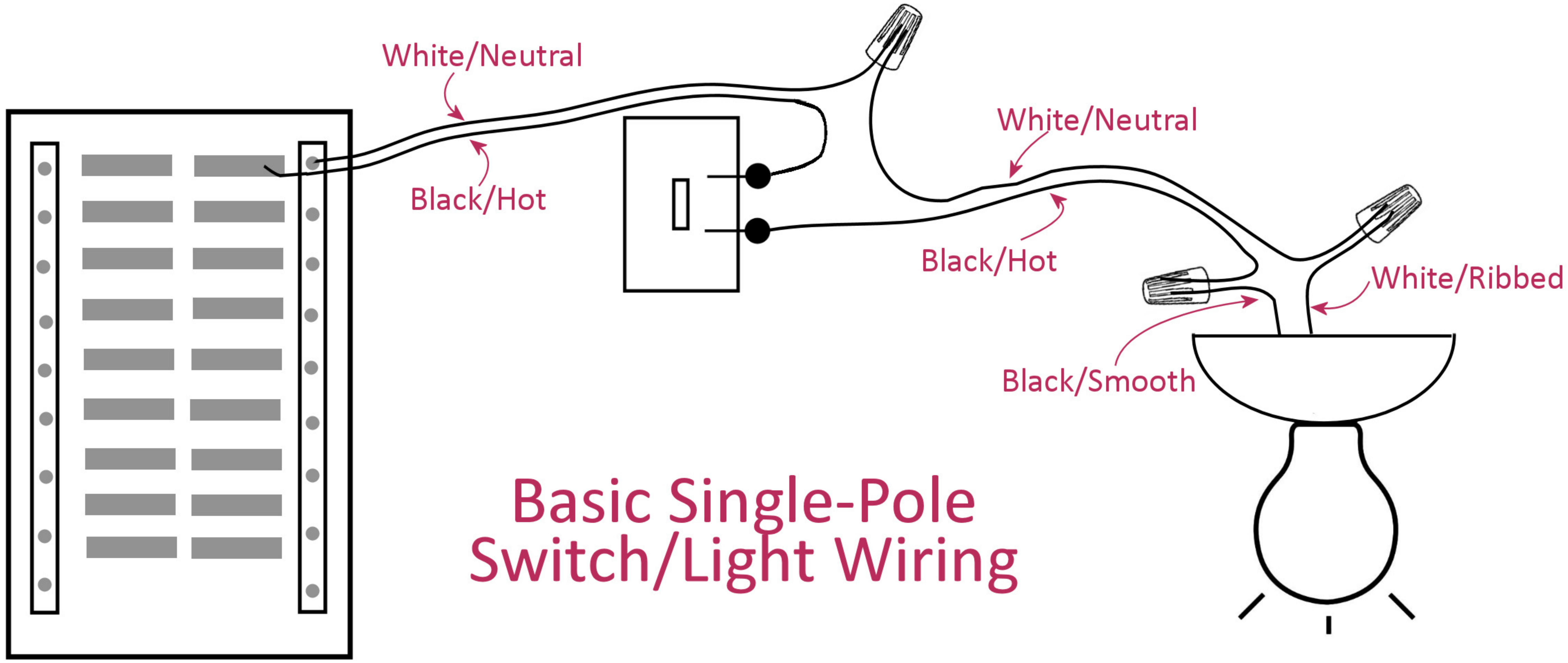
e.g., a household wall outlet
connected to a vacuum cleaner

- Voltage V (volts)
- Current I (amperes or amps)
- Resistance R or impedance Z (ohms, Ω)
- Direct current (DC) and alternating current (AC) circuits
- Ohm's law of electricity: $V = IR$
- Electrical power: $P = VI = I^2R$ (Watts)
- Relation to work or energy:

$$P = W/\Delta t \text{ (Watts)} \quad \text{or} \quad W = P\,\Delta t \text{ (Joules)}$$

Example – home wiring





https://gardnerbenderfaq.wordpress.com/tag/outlet/

https://www.addicted2decorating.com/how-to-wire-single-pole-light-switch.html

Example – kilowatt-hr and your electric bill

- A kilowatt-hr is a convenient unit of energy:

$$1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ hr} = 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J}$$

- Exercise: Suppose you paid \$100 for last month's electric bill at a cost of \$0.13/kWh.

  (a) How much energy (in kWh) did you use?

  (b) What was the average power consumption (in Watts) over the month (assume 30 days)?

- Answer:

  (a)

  (b)

Example – kilowatt-hr and your electric bill

- A kilowatt-hr is a convenient unit of energy:

$$1\ \text{kWh} = 1\ \text{kW} \times 1\ \text{hr} = 1000\ \text{W} \times 3600\ \text{s} = 3.6 \times 10^6\ \text{J}$$

- Exercise: Suppose you paid \$100 for last month's electric bill at a cost of \$0.13/kWh.

  (a) How much energy (in kWh) did you use?

  (b) What was the average power consumption (in Watts) over the month (assume 30 days)?

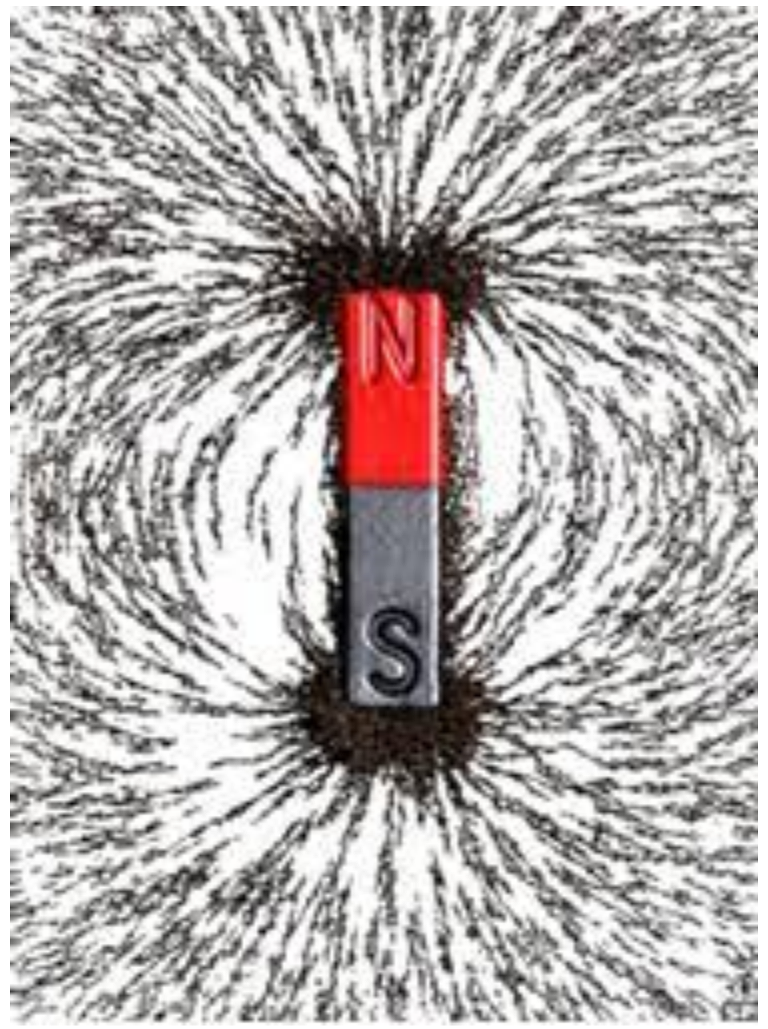
- Answer:

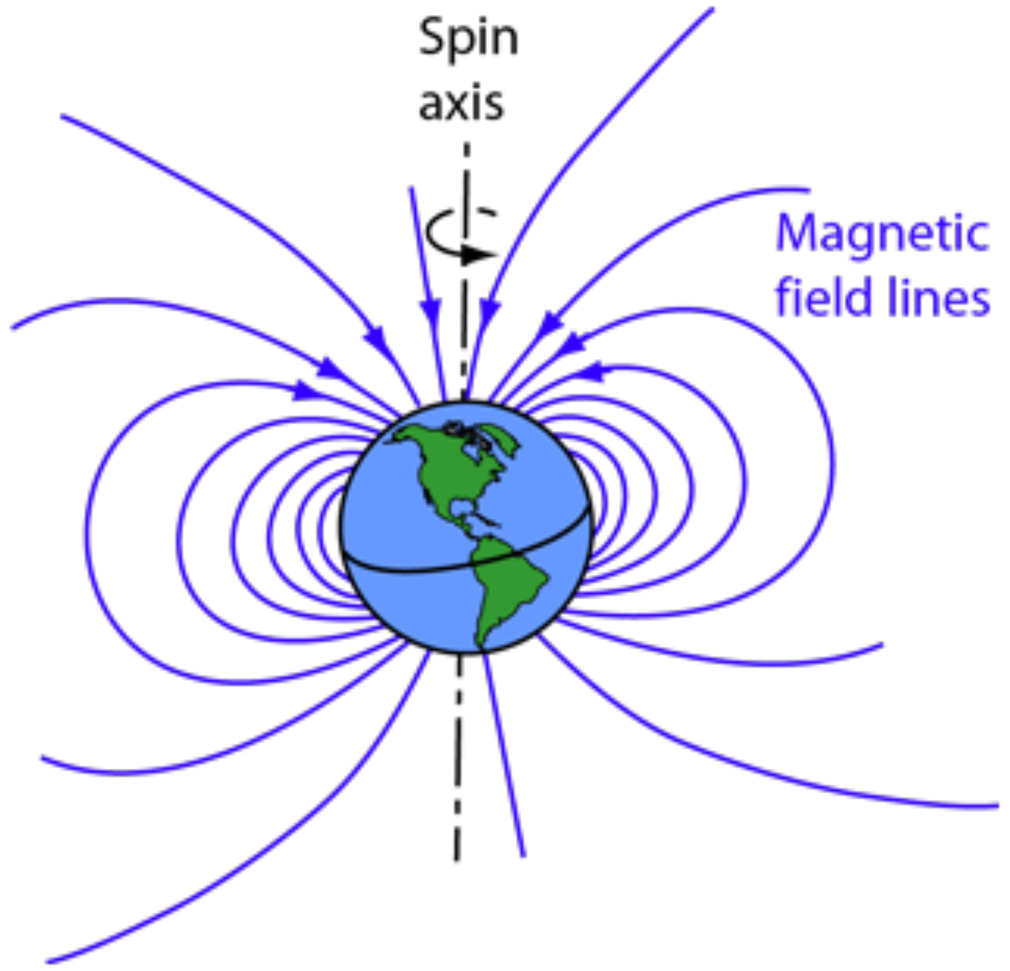
  (a) $W = \$100 \div \$0.13/\text{kWh} = 769\ \text{kWh}$

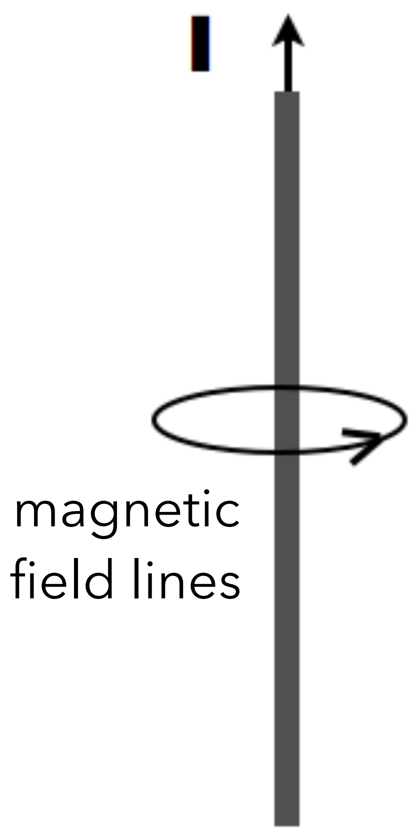
  (b) $P = \dfrac{W}{\Delta t} = \dfrac{769\ \text{kWh}}{30 \times 24\ \text{h}} = 1.1\ \text{kW} = 1{,}100\ \text{W}$ (eleven 100-Watt lightbulbs on continuously)

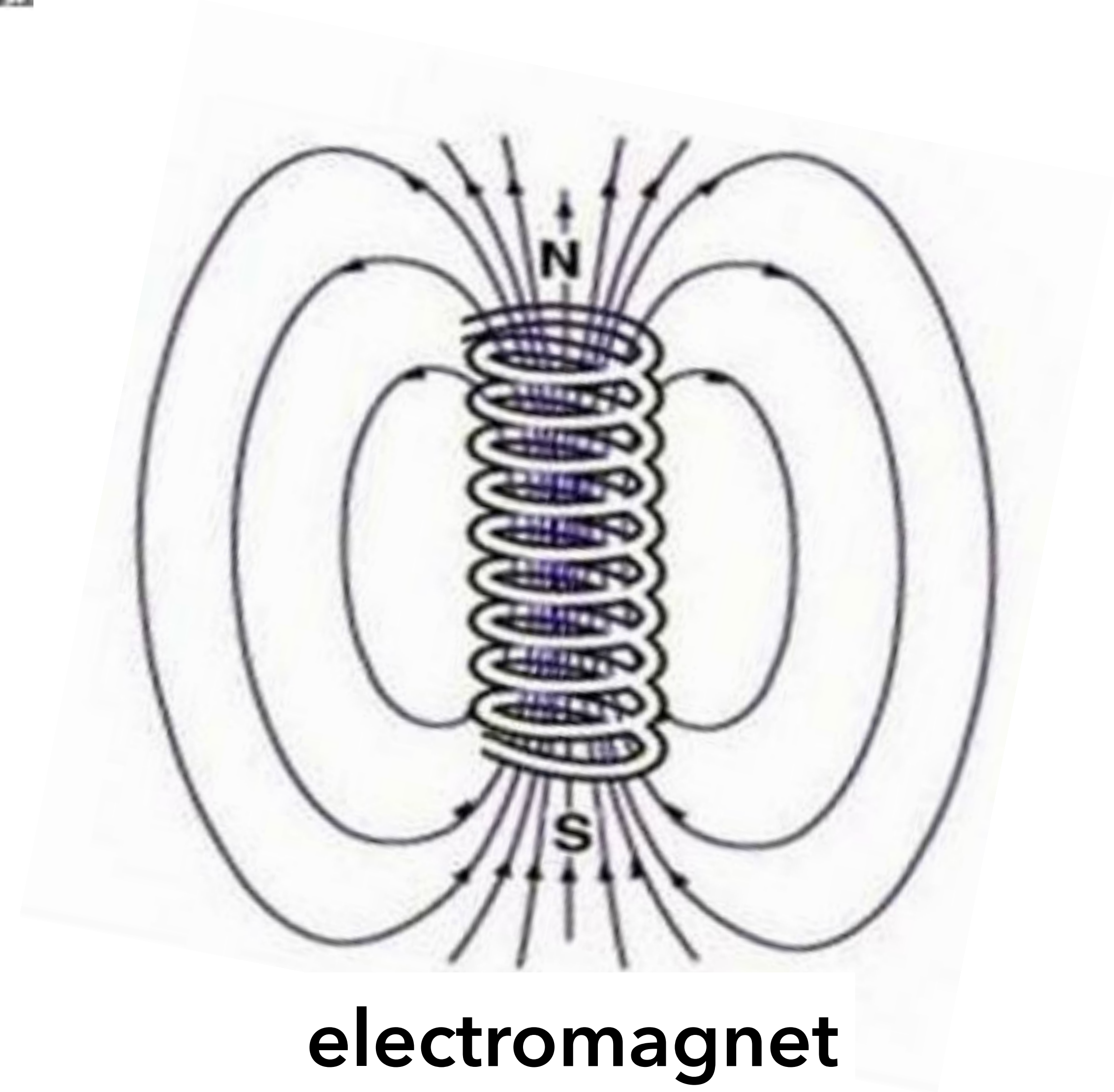
Basic magnetism

- A permanent magnet has **N** and **S poles** that attract pieces of iron
- **Like poles repel; unlike poles attract** (just like + and – electrical charges). But no isolated magnetic poles.
- A **compass needle is a tiny magnet** that is attracted to Earth's South magnetic pole.
- Oersted (1820): discovered that an **electric current produces a magnetic field**
- Can create an **electromagnet** by sending an electric current through a coil of wire









electromagnet

Faraday's law of induction (1831)

- A **change in magnetic flux** through a coil of wire induces a voltage in the coil:

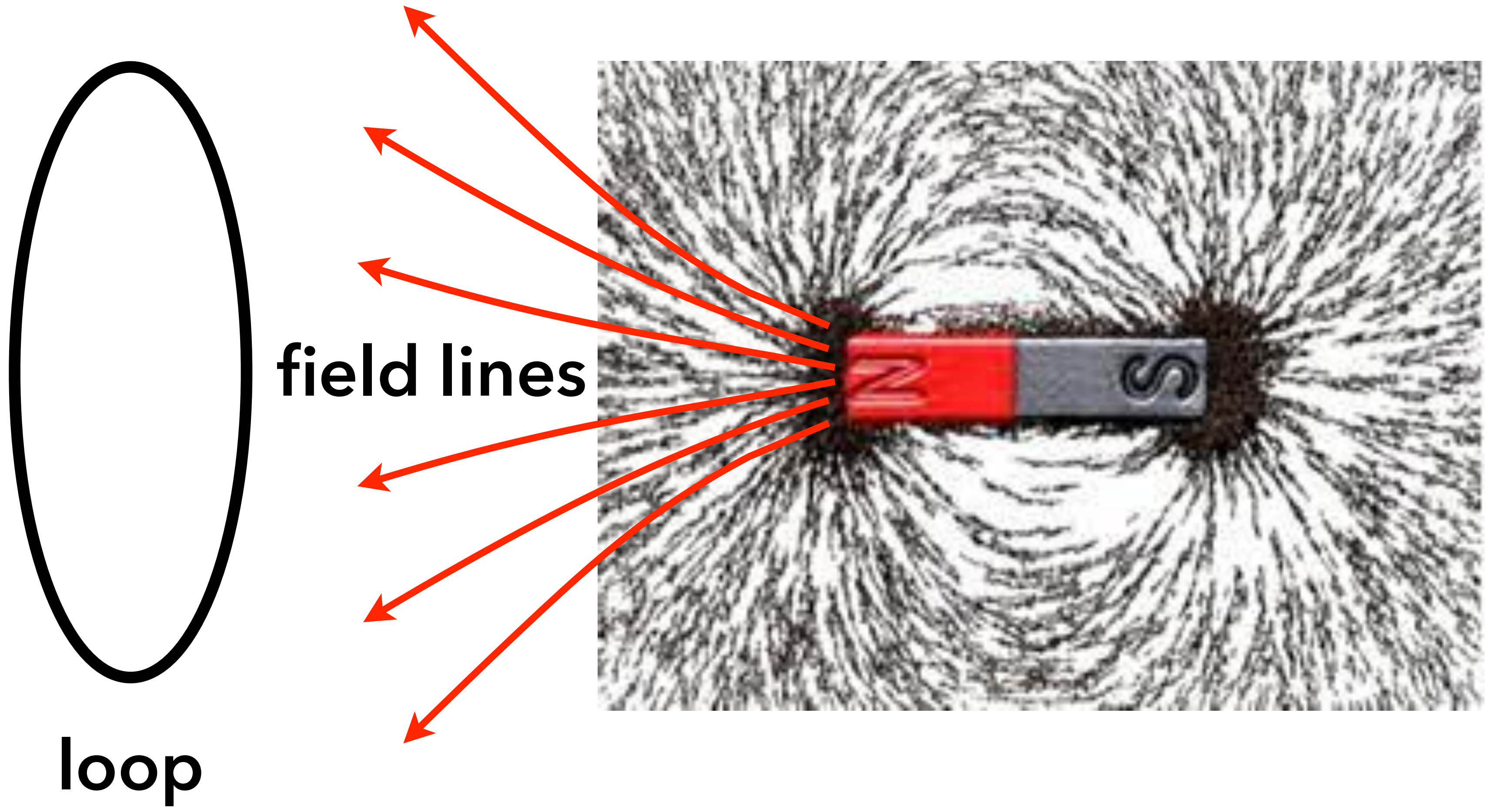
$$V = -N\frac{\Delta\Phi}{\Delta t}$$

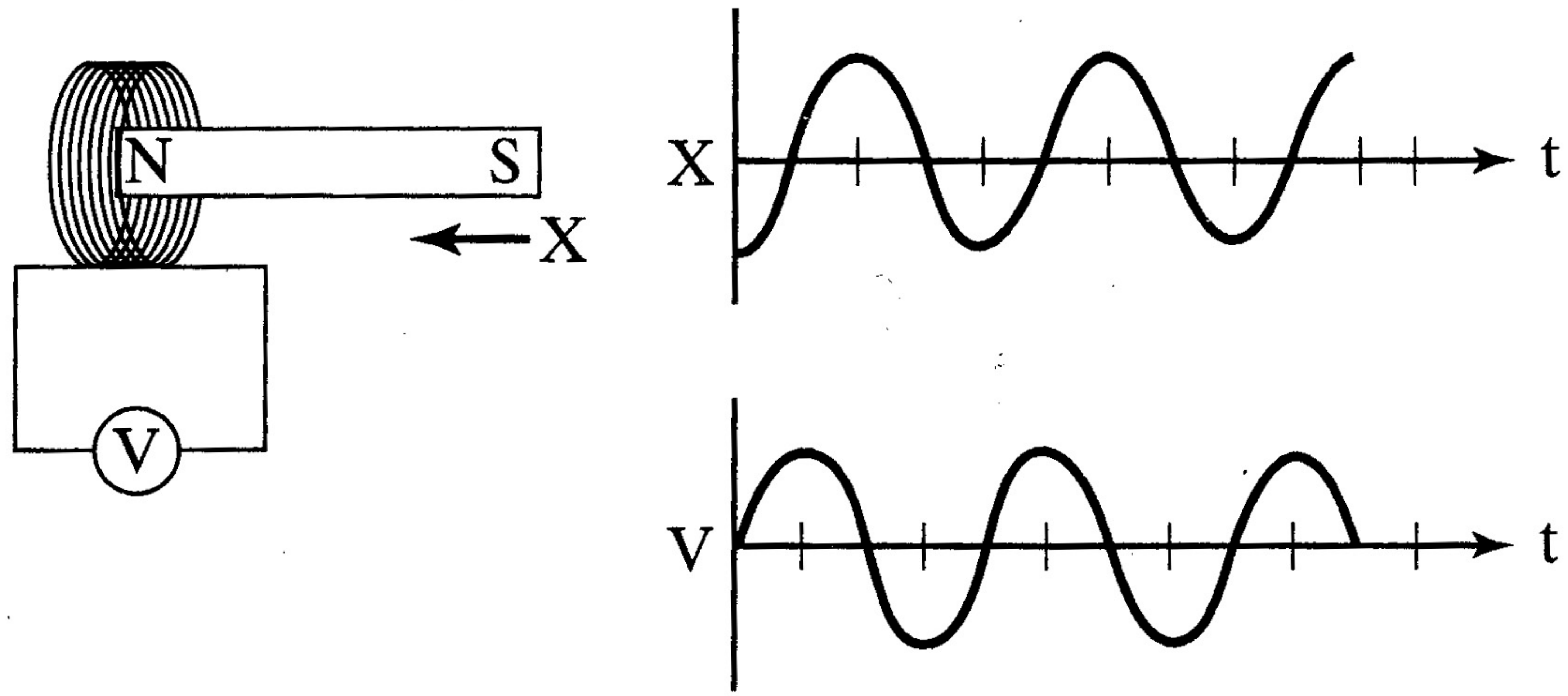
- Only **relative motion** is important

- Underlies the operation of **electric generators** and **electric motors**

- **Electric generator**: mechanical energy converted to electrical energy

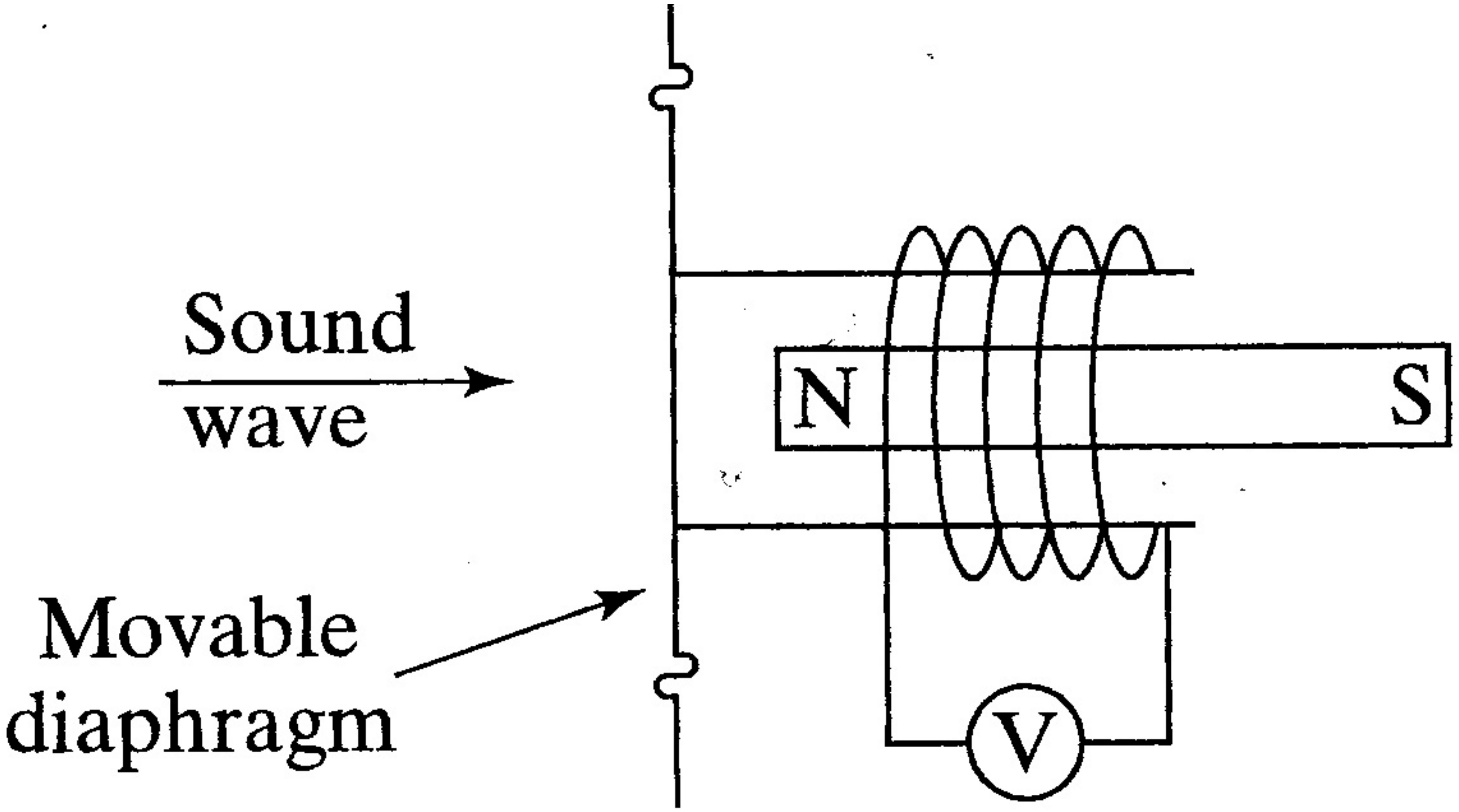
- **Electric motor**: electrical energy converted to mechanical energy





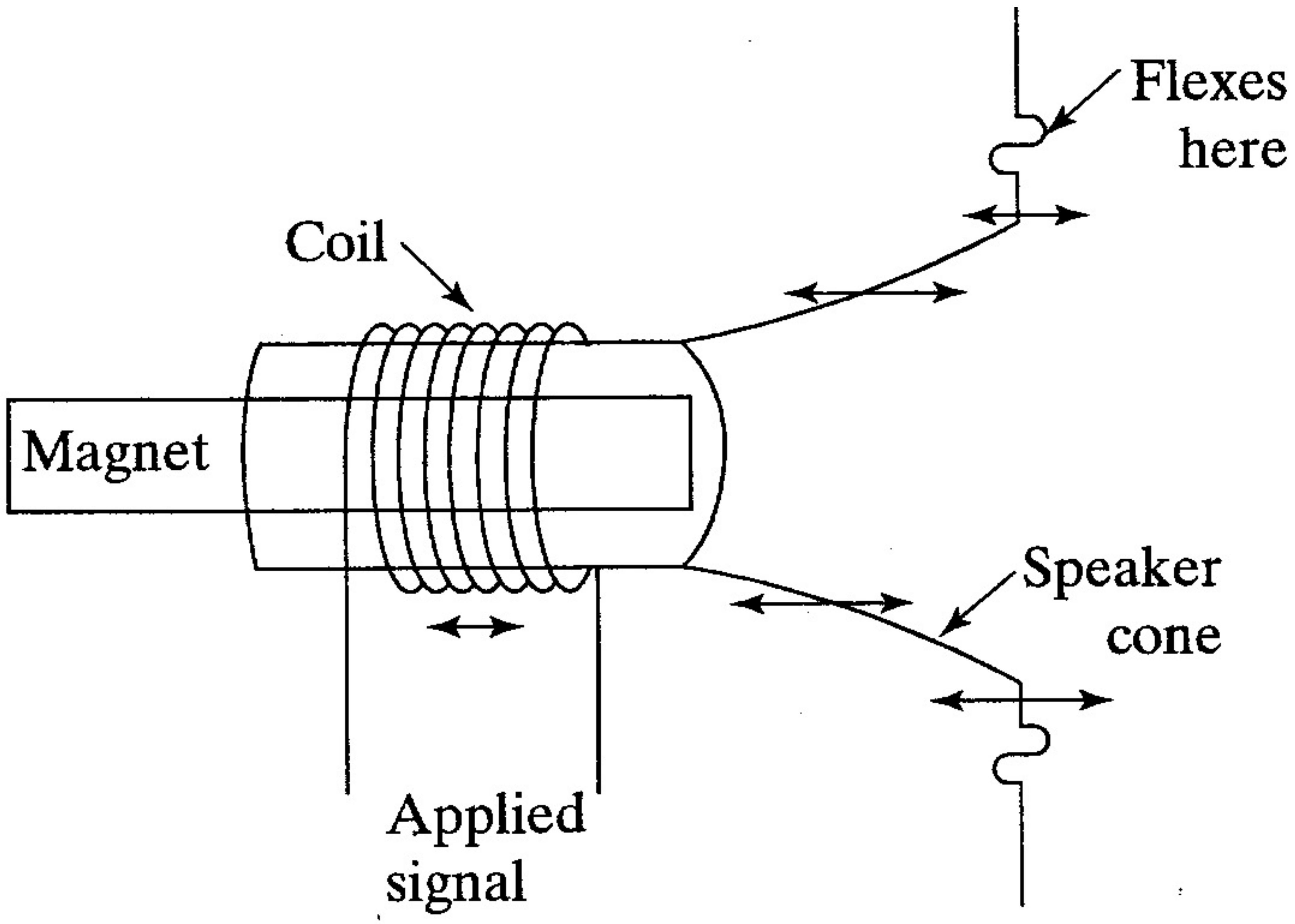
Application – microphones and loudspeakers

Dynamic microphone



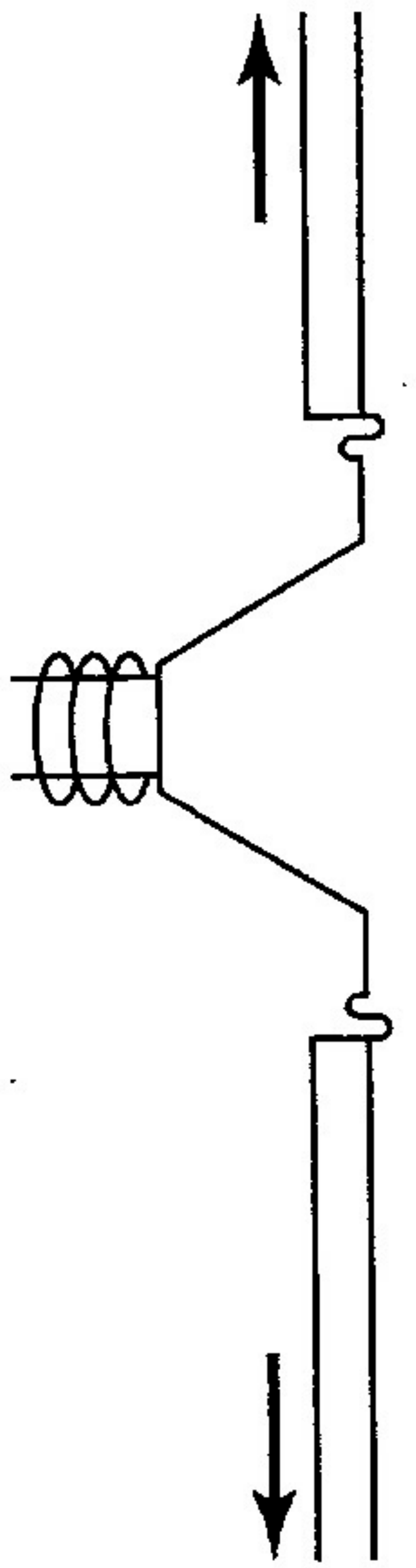
sound wave –> electrical signal

Loudspeaker

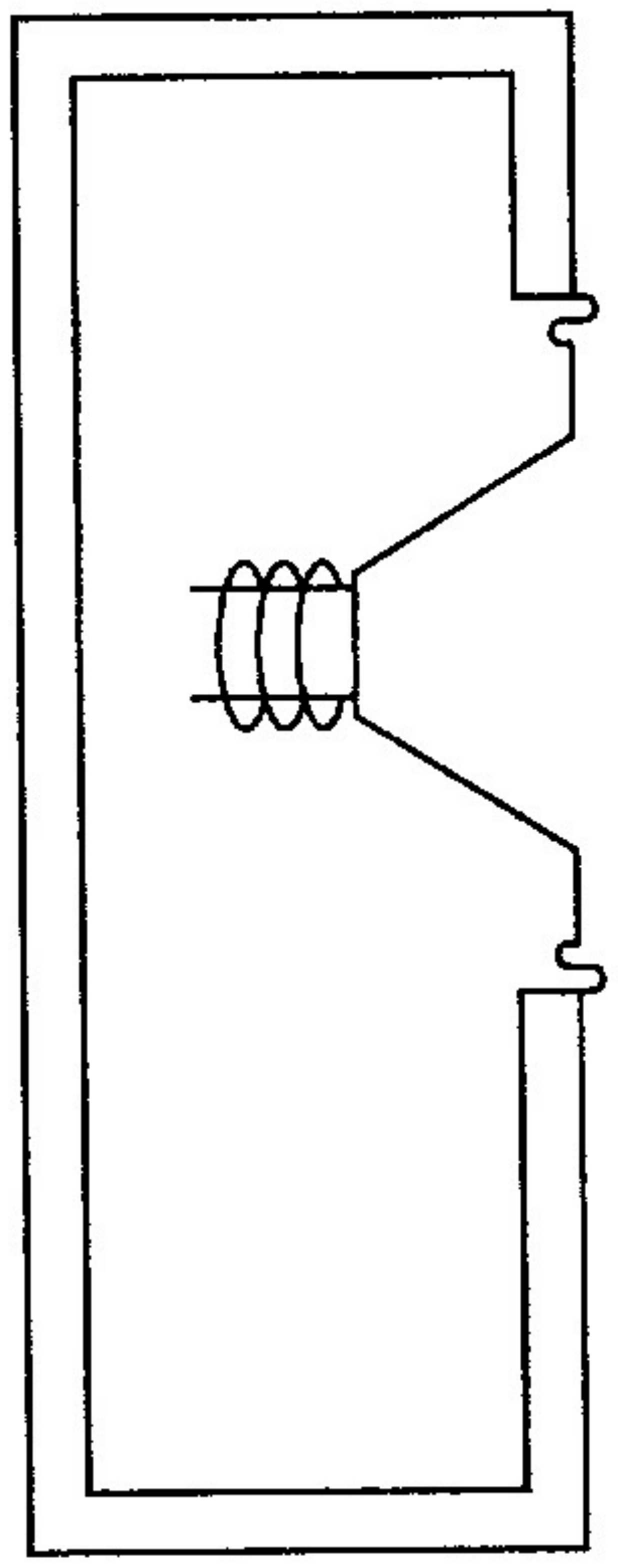


electrical signal –> sound wave

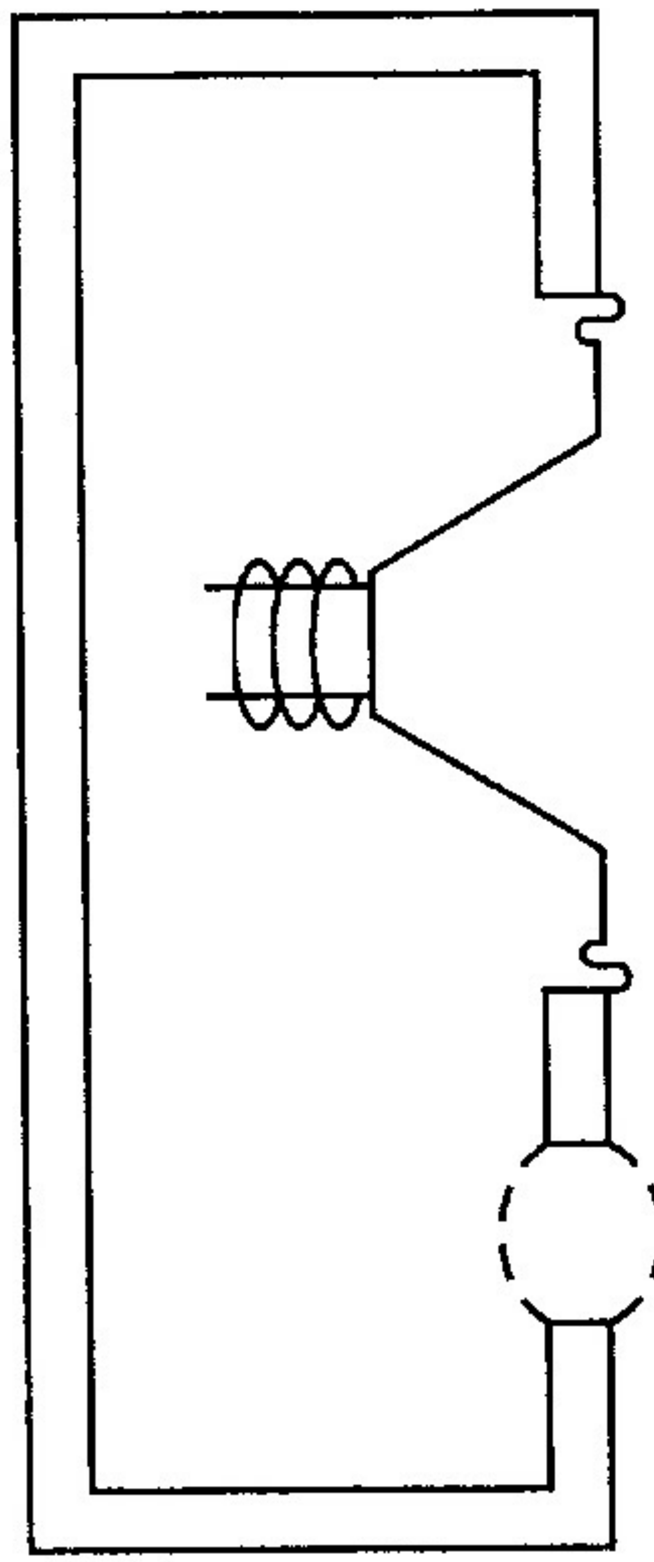
Loudspeakers



Infinite baffle

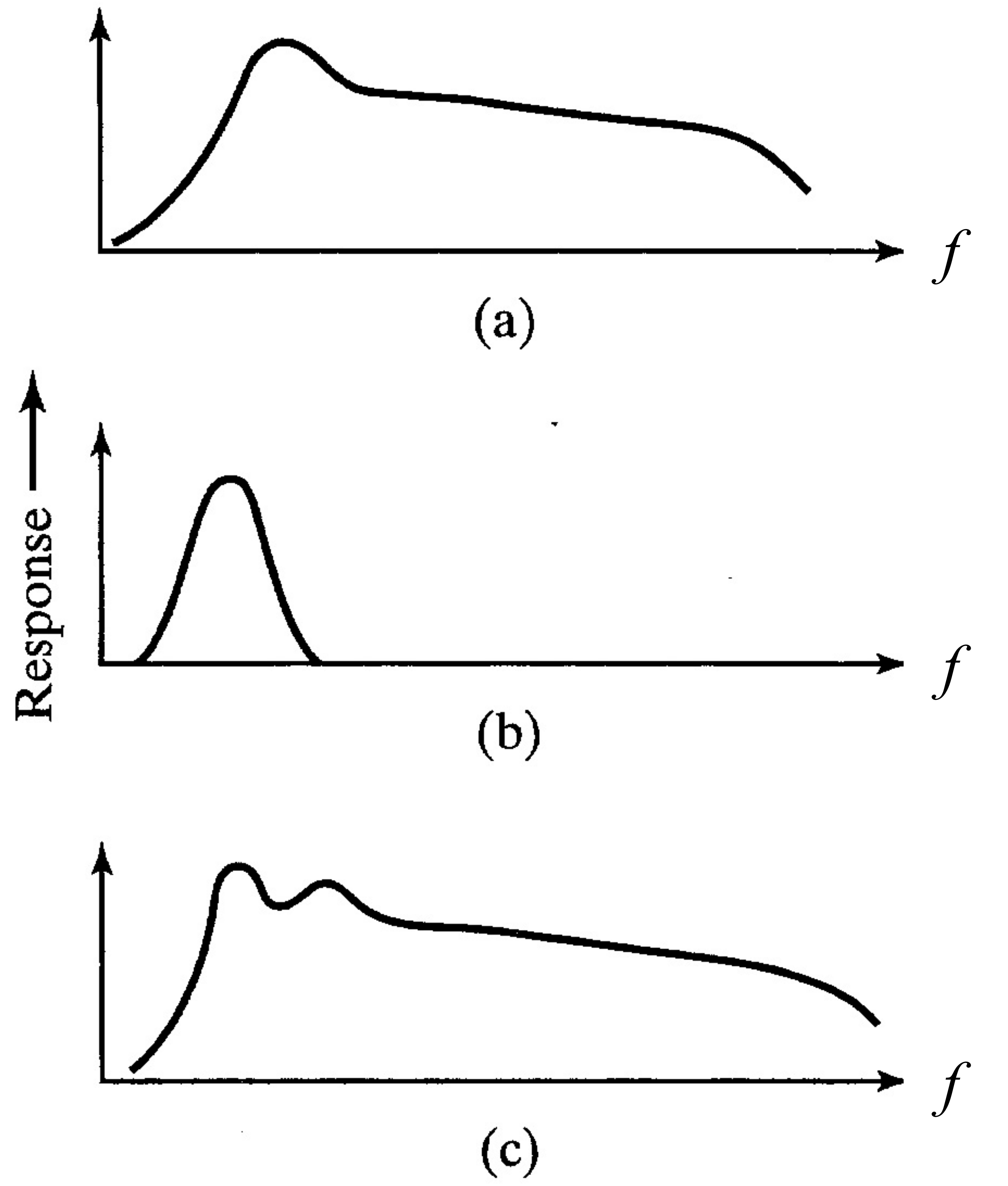


Acoustic suspension



Tuned port (bass reflex)

Frequency response curves (acoustic suspension, tuned port)



14. Elementary music theory

Music theory — the need to standardize musical notes

- A **tuning system** is an **assignment of precise frequencies** to all musical notes in an octave (reference note is **A4 = 440 Hz**; decided upon in 1939)
- Three standard tuning systems:
  - **Equal temperament**
  - **Pythagorean temperament**
  - **Just temperament**
- Each tuning system has its own **advantages and disadvantages**
- What tuning systems do real musicians use? (Richard, others??)

Musical scales — dividing up the octave into pieces

- **Chromatic scale**: 12 pieces (semitones)

  C - C# - D- Eb - E - F - F# - G - Ab - A- Bb - B - C′ (white and black keys on a piano)

- **Diatonic scale**: 7 pieces (semitones and whole tones)

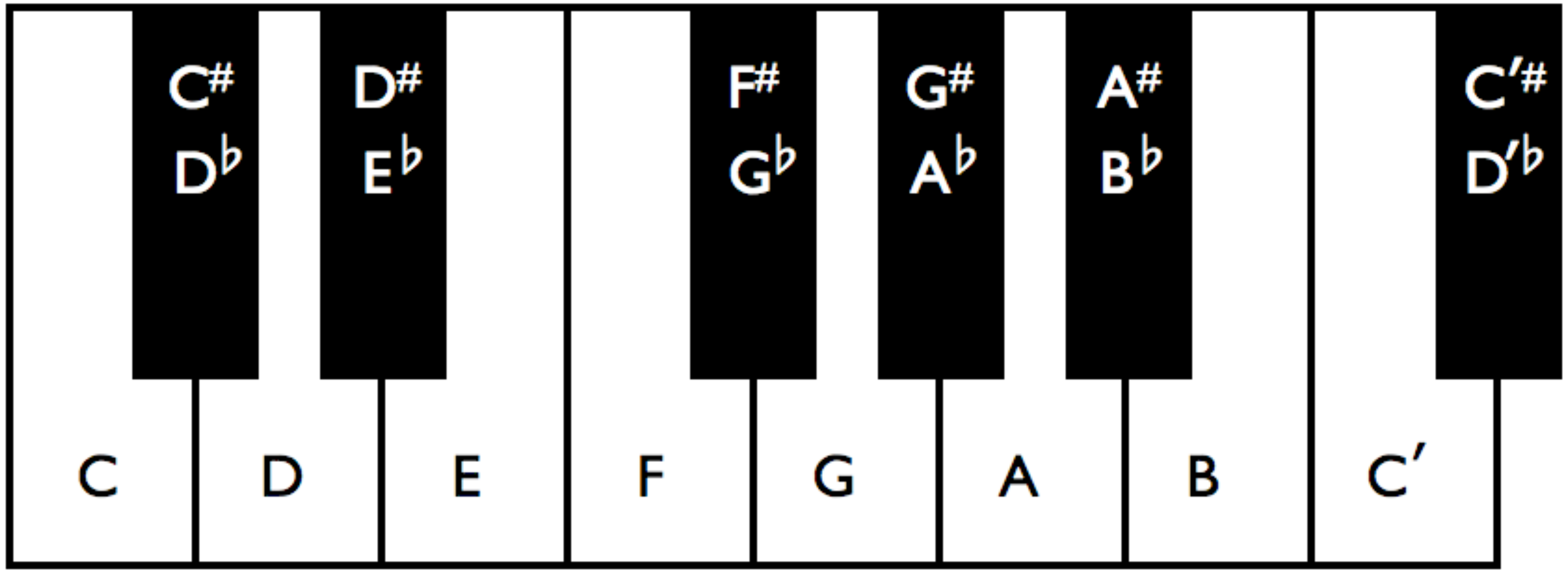
  T-T-S-T-T-T-S (do-re-mi-fa-sol-la-ti-do; white keys on a piano)

- **Pentatonic scale**: 5 pieces (whole tones and 3 semitones intervals)

  T-T-3-T-3 (F# - G# - A# - C# - D# - F#′; black keys on a piano)

Equal temperament

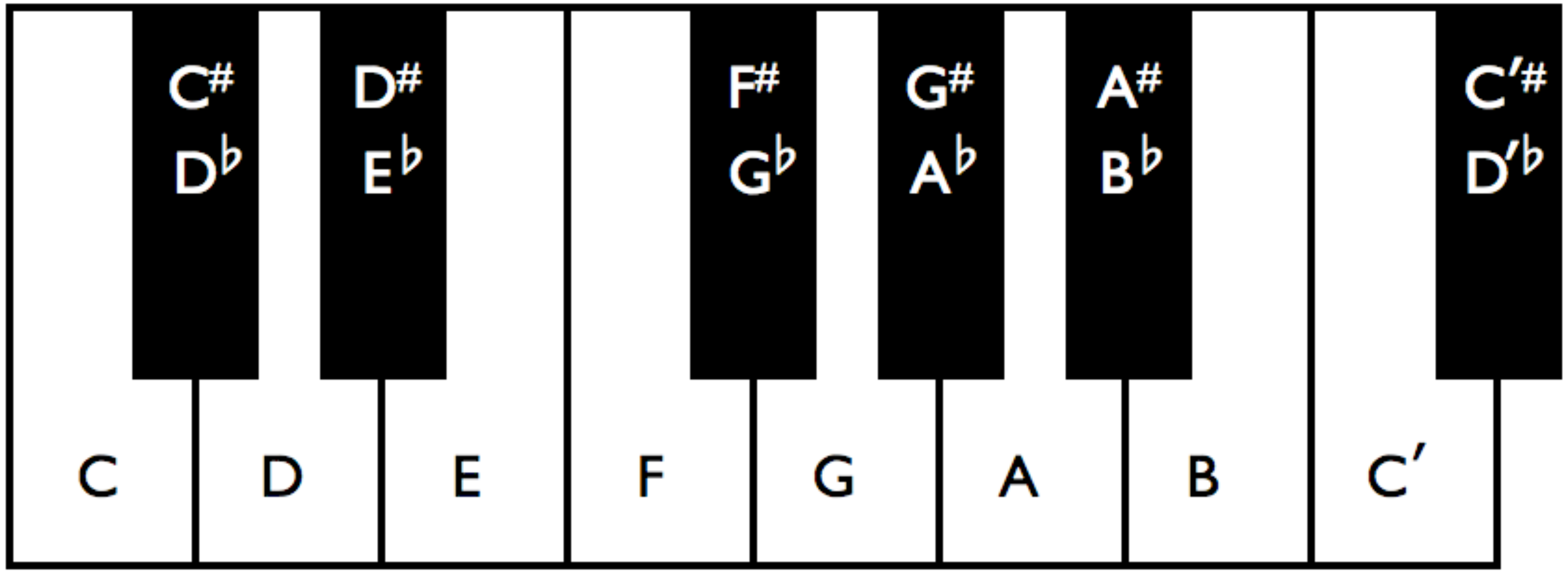
- **All semitones** intervals are **equal**: $2^{1/12} = 1.059$
- Cent (100 cents = semitone): $2^{1/1200} = 1.000578$ (JND: ~10 cents)
- **All sharps and flats** are **equal** to one another



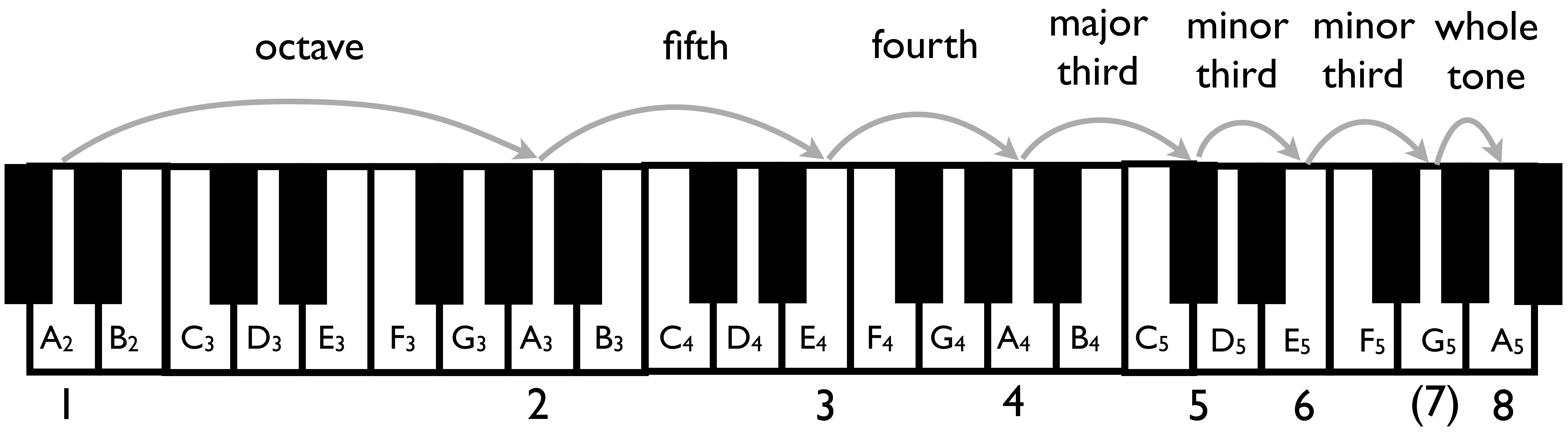
Note	ET freq ratio
C	$2^{0/12} = 1.000$
$C^\sharp/D^\flat$	$2^{1/12} = 1.059$
D	$2^{2/12} = 1.122$
$D^\sharp/E^\flat$	$2^{3/12} = 1.189$
E	$2^{4/12} = 1.260$
F	$2^{5/12} = 1.335$
$F^\sharp/G^\flat$	$2^{6/12} = 1.414$
G	$2^{7/12} = 1.498$
$G^\sharp/A^\flat$	$2^{8/12} = 1.587$
A	$2^{9/12} = 1.682$
$A^\sharp/B^\flat$	$2^{10/12} = 1.782$
B	$2^{11/12} = 1.888$
C'	$2^{12/12} = 2.000$

Musical intervals

Interval	# semitones	Just freq ratio	ET freq ratio	Difference (cents)	Example
Octave	12	$2:1=2.000$	2.000	0	C-C′
Fifth	7	$3:2=1.500$	1.498	2	C-G
Fourth	5	$4:3=1.333$	1.335	-2	C-F, G-C′
Major third	4	$5:4=1.250$	1.260	-14	C-E
Minor third	3	$6:5=1.200$	1.189	16	C-E$^\flat$, A-C′

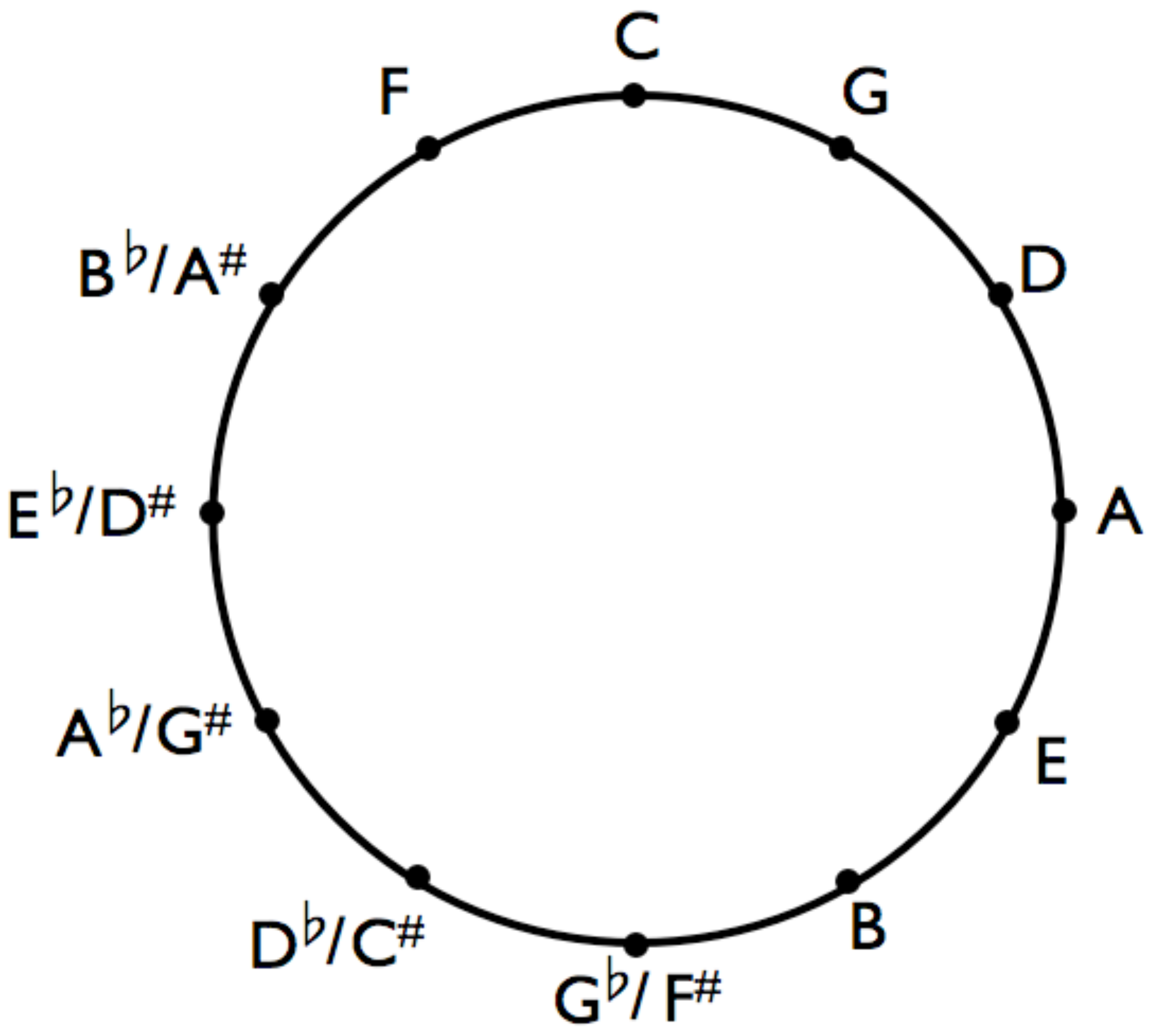


Harmonics

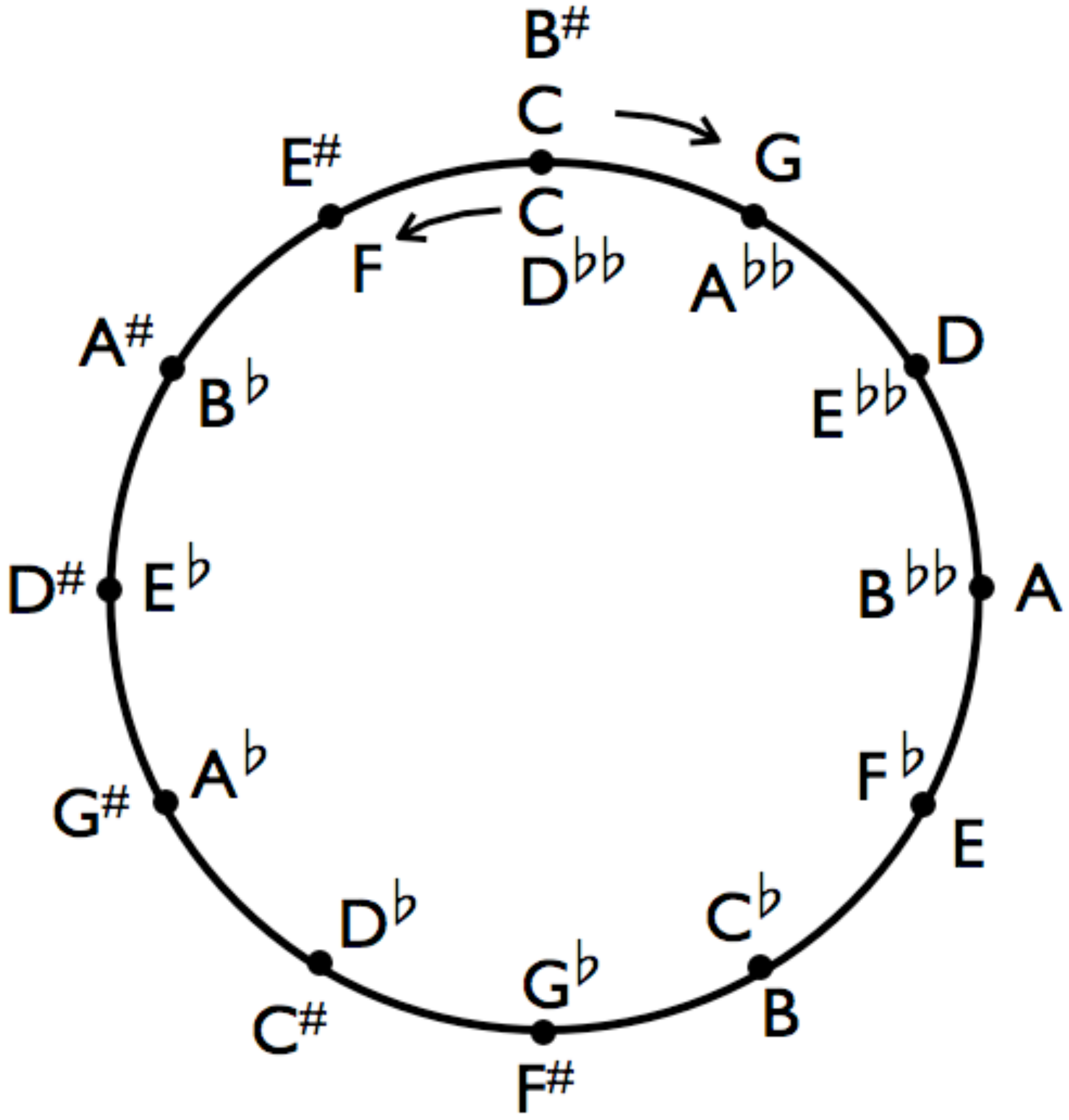


Harmonic	Exact freq (Hz)	Equal-tempered freq (Hz)	Difference (cents)	Piano note
1	110	110.00	0	A_2
2	220	220.00	0	A_3
3	330	329.63	2	E_4
4	440	440.00	0	A_4
5	550	554.37	-14	$C^\sharp_5$
6	660	659.26	2	E_5
7	770	783.99	-31	G_5
8	880	880.00	0	A_5

Circle of fifths



Equal Temperament



Other tuning systems

Pythagorean temperament

$$A^\flat \xleftarrow{2/3} E^\flat \xleftarrow{2/3} B^\flat \xleftarrow{2/3} F \xleftarrow{2/3} C \xrightarrow{3/2} G \xrightarrow{3/2} D \xrightarrow{3/2} A \xrightarrow{3/2} E \xrightarrow{3/2} B \xrightarrow{3/2} F^\# \xrightarrow{3/2} C^\#$$

- Constructed from **perfect fifth** and **octave** intervals
- For example:

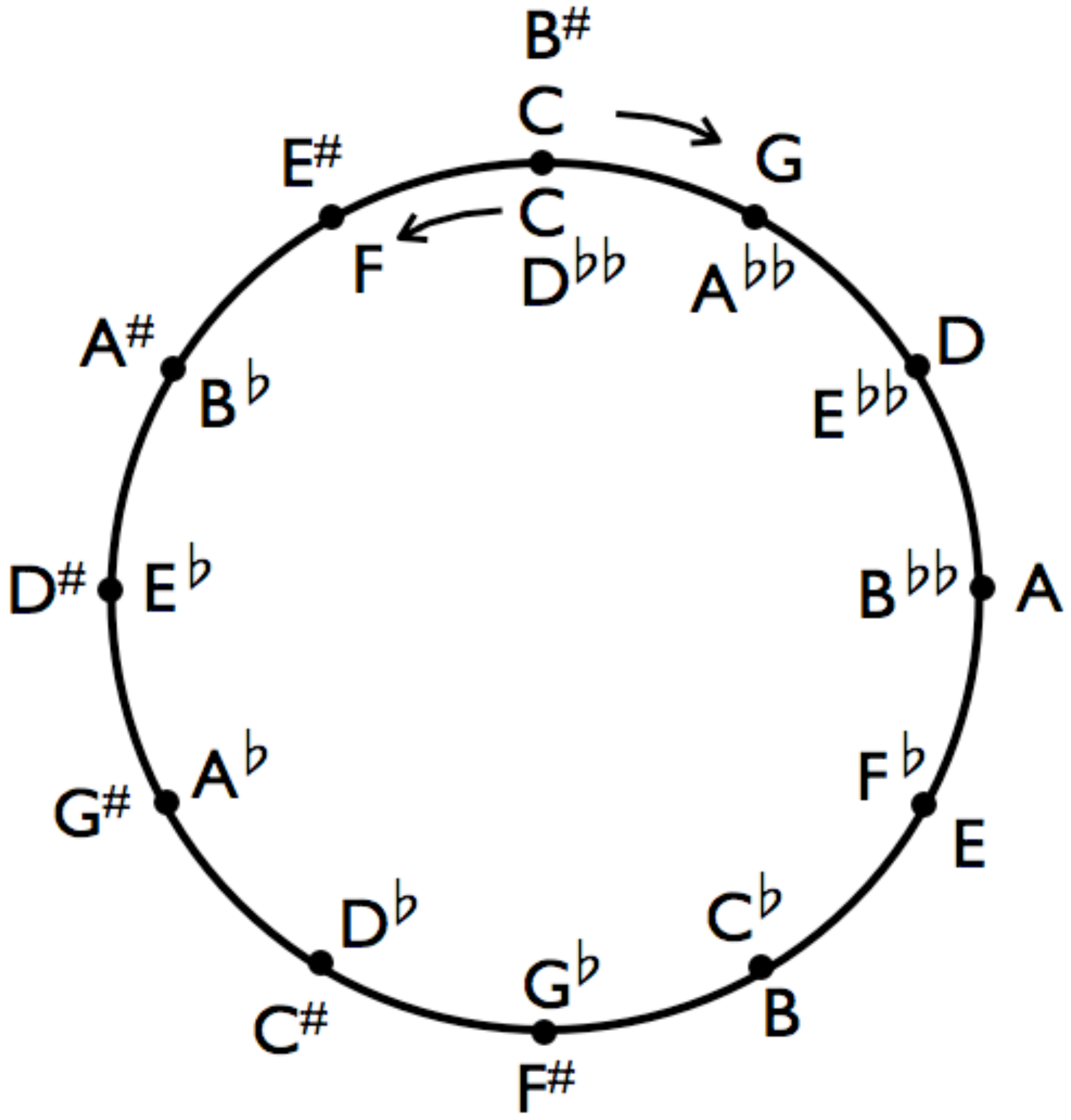
G: $3/2$

D: $(3/2)^2 \times (1/2) = 9/8$

A: $(3/2)^3 \times (1/2) = 27/16$

E: $(3/2)^4 \times (1/2)^2 = 81/64$

F: $(2/3) \times 2 = 4/3$



Just temperament

- Constructed from **perfect fifth**, **major third**, and **octave** intervals

- For example:

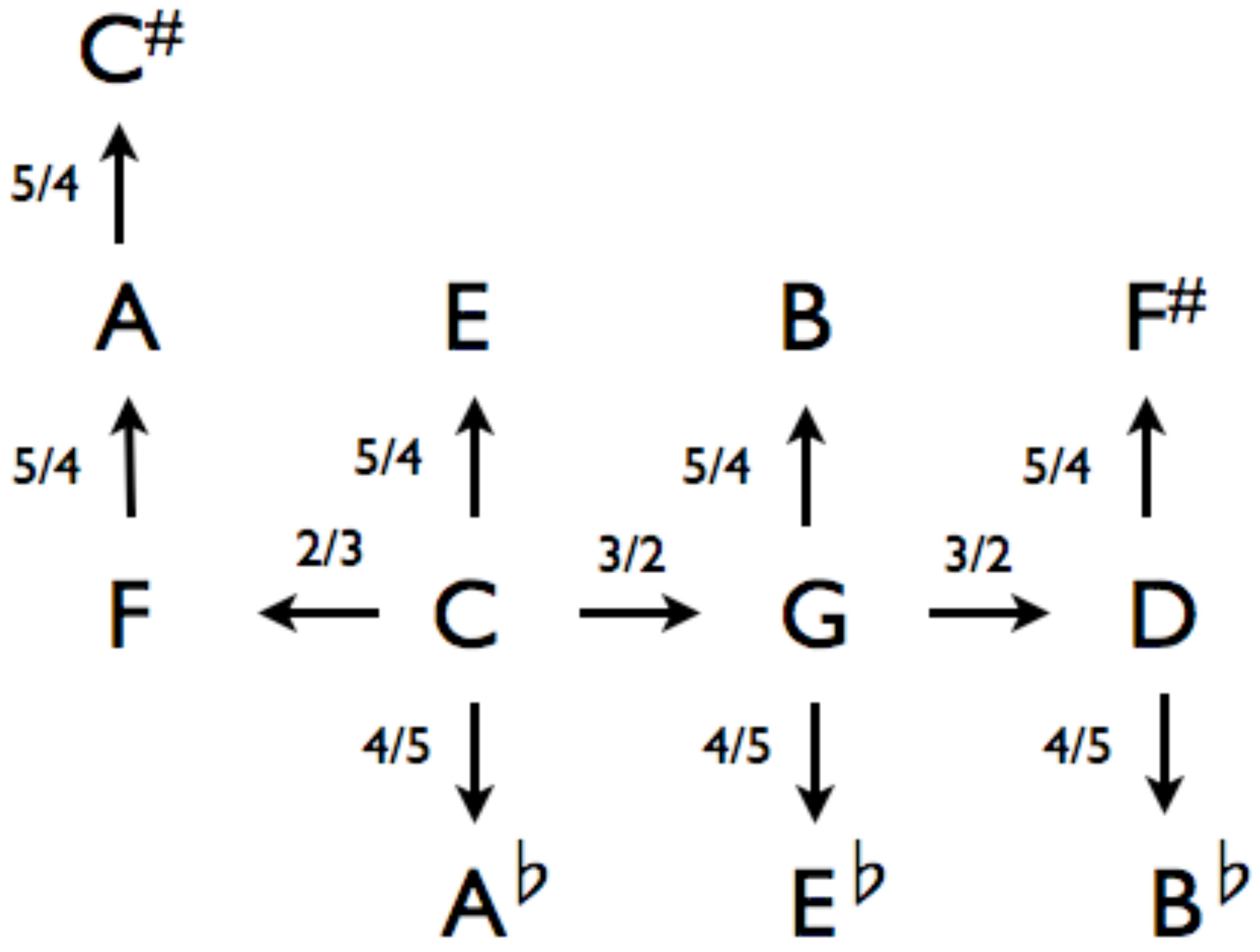
G: 3/2

D: $(3/2)^2 \times (1/2) = 9/8$

A: $(2/3) \times (5/4) \times 2 = 5/3$ (vs 27/16)

E: 5/4 (vs 81/64)

F: $(2/3) \times 2 = 4/3$



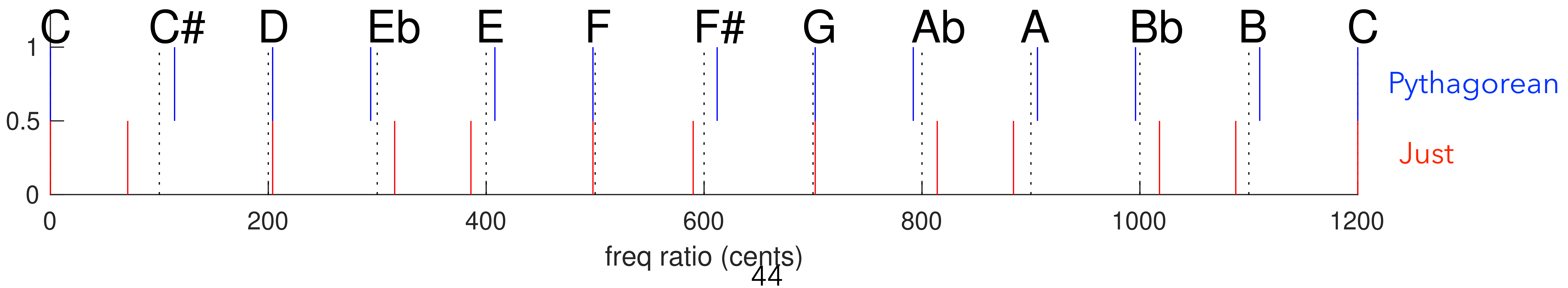
Comparing different tuning systems

Pythagorean vs Equal Temperament

Note	Pyth freq ratio	ET freq ratio	Difference (cents)
C	$1:1=1.000$	1.000	0
C$^\sharp$	$2187:2048=1.068$	1.059	14
D	$9:8=1.125$	1.122	4
E$^\flat$	$32:27=1.185$	1.189	-6
E	$81:64=1.266$	1.260	8
F	$4:3=1.333$	1.335	-2
F$^\sharp$	$729:512=1.424$	1.414	12
G	$3:2=1.500$	1.498	2
A$^\flat$	$128:81=1.580$	1.587	-8
A	$27:16=1.688$	1.682	6
B$^\flat$	$16:9=1.778$	1.782	-4
B	$243:128=1.898$	1.888	10
C$'$	$2:1=2.000$	2.000	0

Just vs Equal Temperament

Note	Just freq ratio	ET freq ratio	Difference (cents)
C	$1:1=1.000$	1.000	0
C$^\sharp$	$25:24=1.042$	1.059	-29
D	$9:8=1.125$	1.122	4
E$^\flat$	$6:5=1.200$	1.189	16
E	$5:4=1.250$	1.260	-14
F	$4:3=1.333$	1.335	-2
F$^\sharp$	$45:32=1.406$	1.414	-10
G	$3:2=1.500$	1.498	2
A$^\flat$	$8:5=1.600$	1.587	14
A	$5:3=1.667$	1.682	-16
B$^\flat$	$9:5=1.800$	1.782	18
B	$15:8=1.875$	1.888	-12
C$'$	$2:1=2.000$	2.000	0

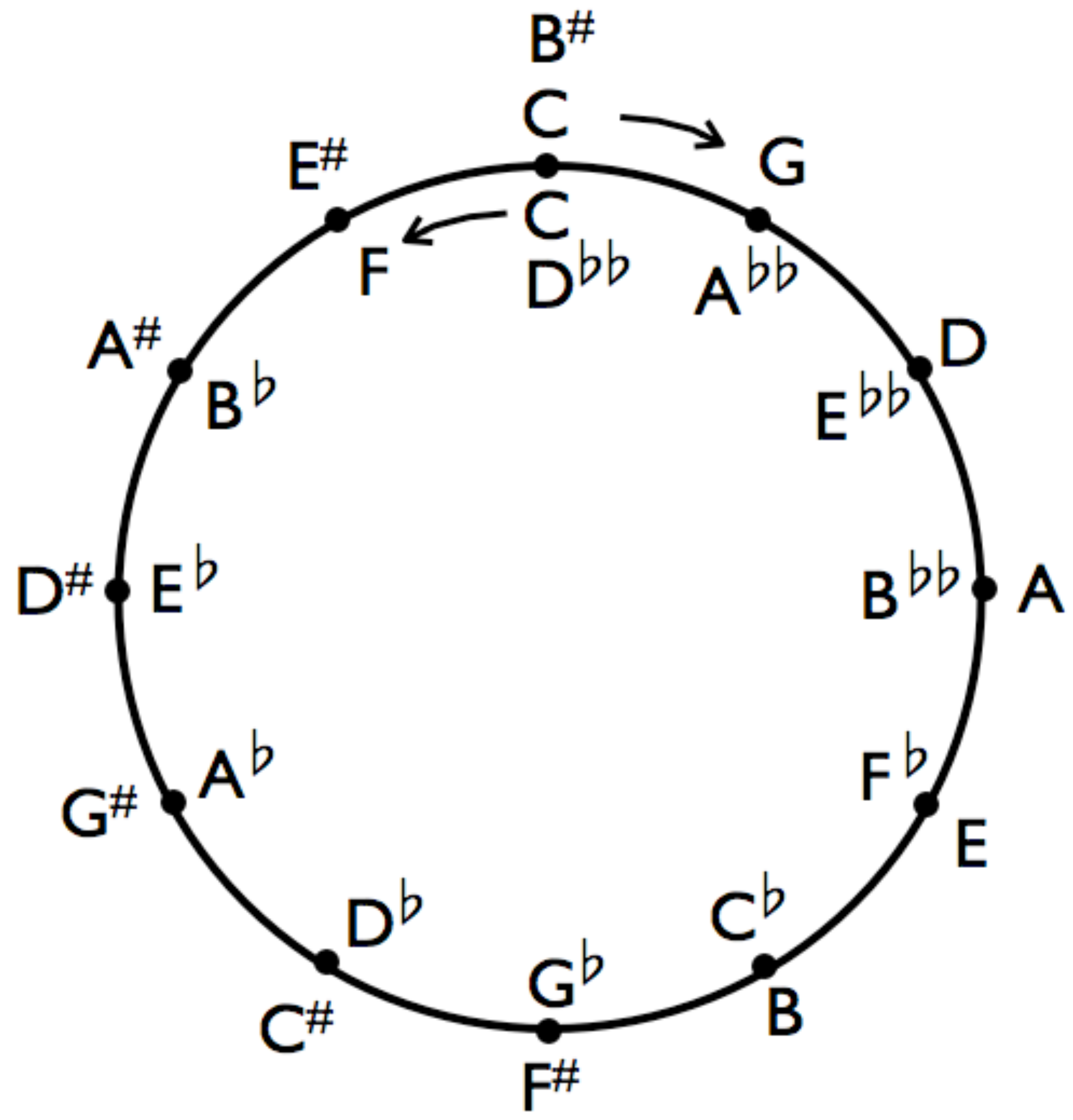


All tuning systems have problems!!

- **Equal-tempered** fifths, fourths, etc. are **never perfect** (only an octave)
- **Pythagorean circle of fifths doesn't close** (12 perfect fifths is not equal to 7 octaves)
- Pythagorean "**comma**":

$$\frac{B^\sharp}{C'} = \frac{(3/2)^{12}}{2^7} = 1.0136 \quad \text{(23 cents too large)}$$

- Fifth $C^\sharp$ to $A^\flat$ is too flat in Pythagorean temperament (**"wolf" fifth**) and too sharp in just temperament



Fifth	Temperament	Freq ratio	Difference (cents)
C-G	equal	1.498	-2
C-G	pyth	1.500	0
C-G	just	1.500	0
$C^\sharp$-$A^\flat$	equal	1.498	-2
$C^\sharp$-$A^\flat$	pyth	1.480	-23
$C^\sharp$-$A^\flat$	just	1.536	41

“The Well-Tempered Clavier”

- Written by Johannes Sebastian Bach in 1722
- Piece played in all 24 major and minor keys
    - Major interval order: T-T-S-T-T-T-S
    - Minor interval order: T-S-T-T-S-T-T
- Demonstrates the usefulness of equal temperament tuning (the piece sounds good only in the equal temperament tuning system)

https://www.youtube.com/watch?v=nPHIZw7HZq4

C Major - 846 0:00 / 2:41c minor - 847 4:39 / 6:26C# Major - 848 8:22 / 9:39c# minor - 849 12:13 / 15:12D Major - 850 17:51 / 19:25d minor - 851 21:13 / 22:51Eb Major - 852 24:59 / 28:32eb/d# minor - 853 30:21 / 34:19E Major - 854 38:14 / 39:58e minor - 855 41:20 / 43:20F Major - 856 44:50 / 45:54f minor - 867 47:22 / 50:14F# Major - 858 54:12 / 55:52f# minor - 859 57:58 / 59:03G Major - 860 1:02:19 / 1:03:14g minor - 861 1:05:50 / 1:08:03Ab Major - 862 1:09:58 / 1:11:30g# minor - 863 1:13:29 / 1:15:31A Major - 864 1:17:48 / 1:19:21a minor - 865 1:21:16 / 1:22:41Bb Major - 866 1:26:45 / 1:28:02bb minor - 867 1:29:59 / 1:32:26B Major - 868 1:35:44 / 1:36:43b minor - 869 1:38:33 / 1:41:01