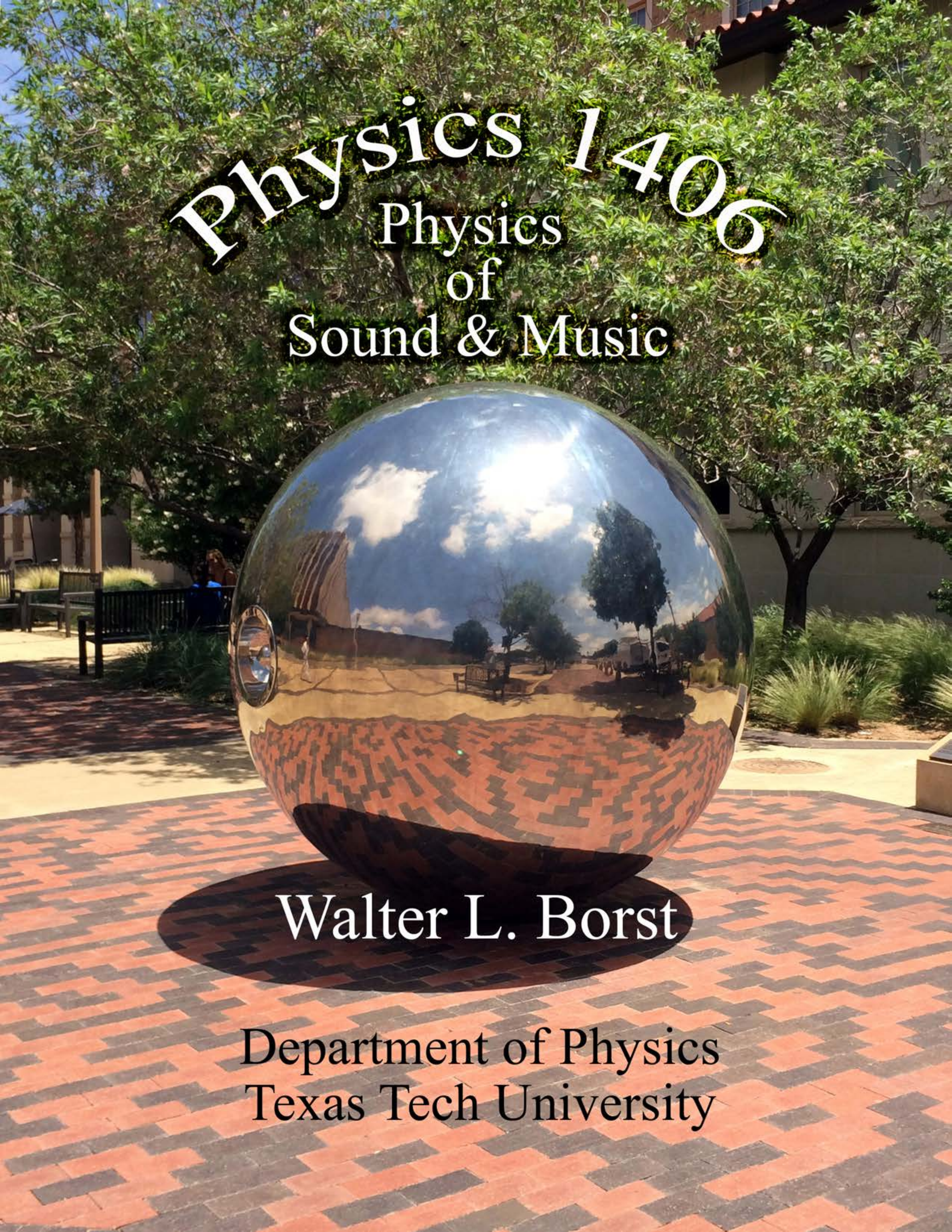
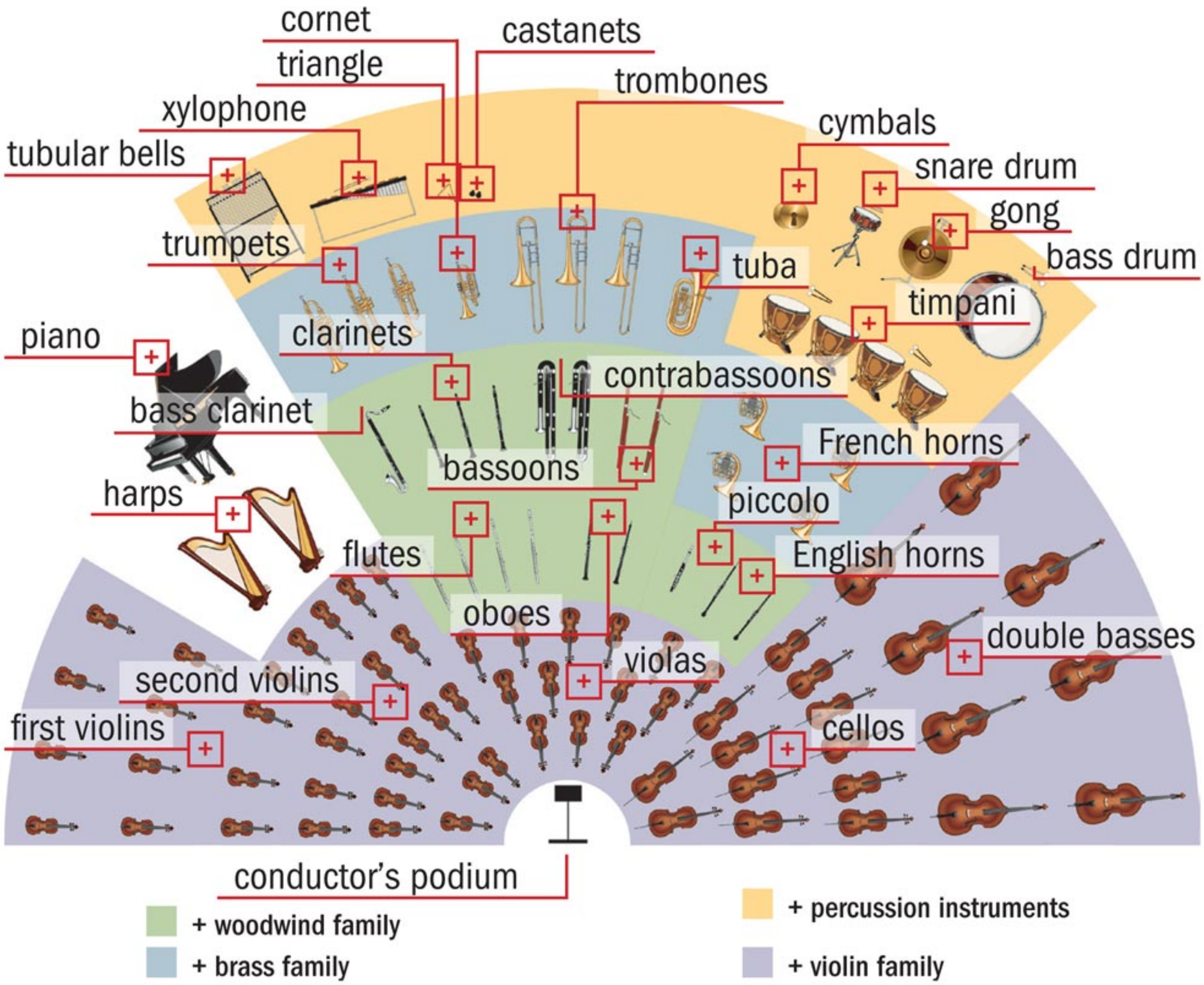


Physics 1406
Physics of Sound & Music

Walter L. Borst

Department of Physics
Texas Tech University



Typical Layout of a Classical Symphony Orchestra
(http://visualdictionaryonline.com/arts-architecture/music/symphony-orchestra.php)

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COURSE GUIDE and LABORATORY MANUAL

PHYSICS OF SOUND AND MUSIC

PHYS 1406

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Professor of Physics

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Texas Tech University, Lubbock

Spring 2021

Front outside cover:
Metal sculpture near the Library at Texas Tech University
Title: “Comma”, 2007
Artist: Po Shu Wang

When you push in one of the two pistons on this 7-foot spherical sculpture, a tone of 1782.8 Hertz can be heard that represents the “ringing of the sun”, although many octaves higher. Pushing in the other piston produces a tone with a slightly different pitch. Pushing in both pistons at the same time produces a combination tone with beats. The beat frequency is the small difference between the two pitches, called a “Comma”. This is the origin of the name of this sculpture.

About the Author
Walter Borst joined Texas Tech University as Professor of Physics and Department Chairman in 1984. He received his Ph.D. degree from the University of California at Berkeley in experimental atomic and molecular physics in 1968. Since his youth he has had an interest in sound and music. He has taught “Physics of Sound and Music” at Texas Tech University since Spring 2008 as a 4-hour course that satisfies the Natural Sciences Core Curriculum Competency requirement. The author wishes that this course may also be of general interest to students.

Acknowledgments
I appreciate the encouragement and support for this course and its laboratory by the chairman of the department of physics, Professor Sungwon Lee, and the former chairman, Professor Nural Akchurin. I also appreciate the technical support by Kim Zinsmeyer, Phil Cruzan, Chris Perez, Arnold Fernandez, and Sarah Stubbs. They have skillfully built equipment for the lectures and laboratory.

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Professor Richard Meek from the School of Music has been a loyal participant in this course since 2008. His musical contributions have enriched our class and provided a unique perspective. I am also grateful to Professor William Ballenger, Director of the School of Music at Texas Tech University, for his support.

I wish to thank the many members of the music faculty and graduate students at Texas Tech University, who have demonstrated their instruments and played great music in our class. They include Professors Amy Anderson (oboe), Quinn Patrick Ankrum (mezzo-soprano), Gregory Brookes (voice), Annie Chalex-Boyle (violin), Thomas Cimarusti (accordion, world instruments), James Decker (trombone), David Dees (Saxophone), Gerald Dolter (voice), Eric Fried (violin), John Gilbert (violin), Jeffrey Lastrapes (cello), Peter Martens (music theory), Richard Meek (bassoon), Mark Morton (double bass), Renee Skerik (viola), David Shea (clarinet), Kimberly Sparr (viola), Michael Stoune (flute), Will Strieder (trumpet), Kevin Wass (Tuba), Rebecca Wascoe-Hays (Soprano), and William Westney (piano).
Other performers include Dr. Nataliya Sukhina (piano), Dr. David Barrientos (clarinet), Dale Blevins (guitar maker), Dr. Liudmila Chise (piano), Dr. Luke Darville (Cello), Dr. Dominique Gagnon (flute), Brian Gum (violin, violin technology), Dr. Stuart Hinds (overtone singing), Dr. Ra Inta (physics of the guitar), Dr. Patrick McLaurin and Jordan Langehennig (bagpipe), Charles Olivier (bandoneon), and many students from class.

This Course Guide and Laboratory Manual was printed by CopyOutlet, Lubbock. They have done an excellent job for many years, and this is much appreciated.

To the Students

This **Course Guide and Laboratory Manual** for "Physics of Sound and Music" contains the majority of the course topics, as well as the homework, quizzes and examinations from previous semesters.

Please bring this book with you to class and to the laboratory. Add your notes.

Read about the experiment of the day in the Laboratory Manual before coming to the laboratory.
The quiz questions at the beginning of each lab are taken from this manual.
For your laboratory reports, answer the numbered questions in the manual. Your lab instructor will discuss them with you.
Always hand in your laboratory report at the next lab meeting.

Please read the Preface to the Laboratory Manual.
You must obtain a **minimum laboratory score of 75% to pass this course.**

I wish you success and fun this semester.
Walter L. Borst
Professor of Physics
Texas Tech University

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Part 3. Standing Waves and Overtone Series

Part 4. Waveform Analysis and Synthesis

Part 5. Hearing and Auditory Effects

Part 1

Introduction to PHYSICS OF SOUND AND MUSIC

Please always bring this Course Guide with you to class.
Add your own notes and mark in the Guide what we have discussed in class.

Survey of Course Topics
Basics of acoustics, sound, and music
Waves and harmonics
Harmonic analysis and frequencies in sound
Voice and hearing
Environmental sound and noise
Architectural acoustics
Some elementary music theory, intervals, scales, temperament
Microphones and loudspeakers
CDs and DVDs
Lecture demonstrations for sound, acoustics, and musical instruments.
Music performances by faculty from the TTU School of Music and by students in class.

Oldest Known Musical Instrument.
A stone age flute was found in 2008 at Hohle Fels cave in Germany that was made from a griffon vulture's thin radial bone 35,000-40,000 years ago.

The flute dates to a time when modern humans first migrated from Africa to Europe.
The flute here is held by the researcher Nicholas J. Conrad of the University of Tubingen.
Beautiful songs and melodies can be played on a reproduction of this flute by Friedrich Seeberger. (See CD "Klangwelten der Altsteinzeit, for instance the song "Spiel mir das Lied vom Knochen" or "Play for me the Song of the Bone".)
(For a related article about the discovery of the flutes, see Science News, page 13, July 18, 2009.)

What is Sound? What is Music?

Sound consists of pressure waves in air, more specifically, sound can be understood as organized vibrations of air molecules.
We can distinguish between general sound, noise, and musical sound.

"Without music, life would be a mistake" (Friedrich Nietzsche, 1844 – 1900).

Demonstrations
1. Produce sound with some musical instruments.
2. Listen to voice.
3. Produce your own sound.
4. Produce some noise.
5. Distinguish between periodic general sound and periodic musical sound.
6. When does a sound represent a musical sound?
7. Produce sound with a percussion instrument. Does it sound musical?

Questions
1. Describe the "quality" of sound from speech, musical sound, and noise.
2. Can you describe in physical terms the difference between sound from singing a tone and from a lawn mower, both being periodic and having the same pitch?

"Consonance" and "Dissonance" in Sensory Perceptions
Hearing: Harmony, melody, rhythm – noise.
Sight: Pleasing colors – color mismatch.
Taste: "Consonant" – "dissonant" food combinations.
Touch: Gentle touch – scratching.
Smell: Perfume, i.e. "consonance", and "dissonance": ______________________________

Perception of External Reality
See examples above for humans.
The acute sense of smell of dogs corresponds to our sense of hearing in terms of sensitivity. Some smell combinations can be "harmonious" (without a "melody" or" rhythm" of course).
Bats perceive external reality from echoes of textured objects (objects at night, insects).

Sense of Hearing versus Seeing
The human range of hearing extends over 10 octaves in frequency and 12 powers of ten in intensity. In contrast, our faculty of sight covers only about 1 octave in frequency (i.e. colors) and 5 powers of ten in intensity.

Pleasant Sound
The ancient Greek philosopher Pythagoras (570-495 B.C.) noticed pleasing sound when tones were played together whose pitches stood in simple ratios. He used a single string ("monochord") with a movable bridge. Pleasant musical intervals were heard when the two lengths of the divided string were in the ratios of 2:1 octave, 3:2 musical fifth, 4:3 musical fourth, 5:4 musical major third, etc. Such ratios were used to build an 8-note musical scale closely related to the diatonic scale in Western music today.

Demonstrations
1. "Play" a monochord and keyboard to demonstrate Pythagorean musical intervals.
2. Divide the string of a monochord into two sections with a wedge. Pluck the two sections together and listen to the prevalent dissonant and rare consonant musical intervals while you move the wedge under the string. Note the octave, fifth, fourth, third corresponding to ratios of the two string sections of 2:1, 3:2, 4:3, 5:4, respectively.
3. Listen to the consonance in these intervals, and dissonance in others where the lengths of the two string sections are not in such simple ratios.
4. Show Galilean pendulums and the corresponding simple musical intervals.
5. Play some notes and musical intervals on a Native American flute.
6. Show the vibrational modes on a vibrating vertical circular wire ring.

What is Music? Some Possible Answers
Music is organized sound.
Music is a sequence of notes that are varied to sound pleasant.
Music is speech without words, a universal form of communication.
"Music is above all wisdom" (Ludwig van Beethoven).
Music amplifies human emotions and may be the highest art form.

Elements of Music
Pitch is the fundamental frequency of a periodic sound. The sound repeats itself after a period T. The pitch is $f = 1/T$. For instance, "concert A4" has a pitch of 440 Hz.
Rhythm results when notes are played with different durations (see also body rhythms).
Melody results from a combination of notes with different pitches and rhythms.
Harmony results when different notes are played simultaneously to create chords.
Dynamics relate to the loudness of notes and how it changes over several notes.
Timbre is tonal quality and makes a piano sound different from a violin, etc.
Tempo is the speed of a musical piece, often similar to heart rate or breathing.
Meter is a pattern in which rhythmic pulses are organized, e.g. 3/4, 4/4, 6/8, etc.
(For more details, see: "Birth of the Beat", Science News, August 14, 2010, pp. 19.)

Range of Hearing

Demonstration on Range of Hearing
Use a frequency generator and loudspeaker. Start with a very low frequency and increase it. Ask students at which high frequency they no longer can hear the sound. This varies from student to student and depends on age and other factors.
Write down the range of hearing from class: ____________Hz to ___________kHz.

For a young person the range of hearing is about **20 to 20,000 Hz** or **20 Hz to 20 kHz.**

Question
How did this particular range of hearing evolve in humans?
A Possible Answer
Our hearing has evolved so that we are able to hear the frequencies of running and falling water and thus be able to find this commodity essential for life.
(See also Trevor Cox, "The Sound Book", p. 187).

Concept of frequency: f = number of cycles per second
The physical unit of frequency is Hertz (abbreviated Hz).

Example: A heart beats 60 times per minute or once per second.
Therefore the frequency is f = 1 Hz.

Example: The frequency of the household voltage in the USA is f = 60 Hz.

Example: The piano has one of the widest frequency ranges of all musical instruments. The frequency range of the 88 keys extends from key A0 = 27 Hz to key C8 = 4186 Hz. (See also the figure of piano keys, their names, and frequencies.)

The relation between frequency f and period of oscillation T is **f = 1/T**.

Question
Given a frequency of oscillation f = 50 Hz for a deep bass tone, what is the period of oscillation T?
Answer: T = 1/f = 1/50 Hz = 0.02 s = 20 millisecond = 20 ms
Note that we have used the notation 1 ms = 0.001 s = 10^{-3} s.

Demonstrations
1. When does a periodic sound start to sound "musical"?
Drive a mechanical vibrator with a frequency generator. Increase the frequency from 10 Hz to 100 Hz. When does it sound "musical"?
Answer: The approximate frequency is f = ____________Hz.
2. A rattle: Why does it not sound "musical" despite the fact that the sound has a pitch?
3. Maracas for rhythmic accompaniment produce "periodic" noise.
4. Play a percussion triangle: Does it sound "musical"?

Simple Harmonic Motion (SHM) and Applications

Simple Harmonic Motion versus General (Non-Simple) Periodic Motion
General Periodic Motion: The motion repeats itself after a period T.
Examples
Heartbeat T = about 1 second
Sounding the horn of an automobile

Special Case: Simple Harmonic Motion (SHM)
The motion follows a simple *sine- or cosine-curve* with period T.

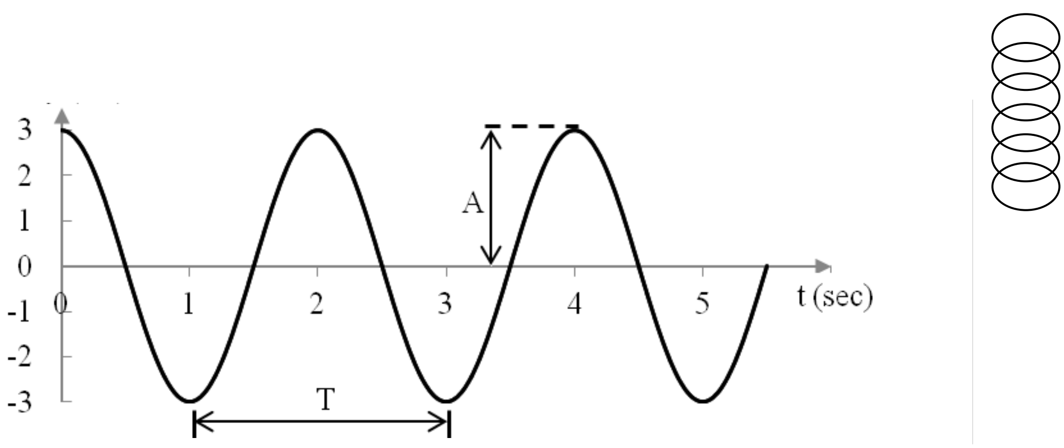


Figure. Simple harmonic motion (SHM) showing the displacement y (in cm) from equilibrium as a function of time t. The oscillating spring on the right would show such a displacement as a function of time.
Exercise: Read the amplitude A = _______cm and period T = _______s

Conditions for Simple Harmonic Motion
An equilibrium position must exist about which the medium oscillates.
A restoring force F exists that is proportional to the displacement y from equilibrium, i.e.
F ∝ y is expressed by **Hooke's Law F = - ky**.
(The minus sign indicates that F is in the opposite direction to y; k is the spring constant.)

Demonstrations
1. Hooke's law: Weights on a scale or postal scale. Read the force in Newton (N).
2. SHM: Spring with a weight suspended and oscillating up and down.
3. A simple pendulum swinging back and forth.
4. "Walking" a spring pendulum: Follow the sine wave traced by the oscillating weight.
5. A pure sine-tone is played on a keyboard synthesizer: Hear a sine wave.
6. A flute and recorder approximately produce sine tones, with air molecules moving back and forth about an equilibrium position while the wave travels.
7. Perpetual motion pendulum driven by a photovoltaic cell.
8. Chaotic double pendulum: Show SHM, non-SHM, and chaotic motion ("noise").

Comparison of Simple Harmonic Motion with General Periodic Motion

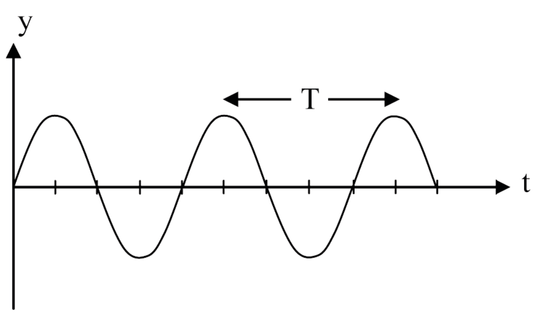
Simple Harmonic Motion

Periodic with period T

Single frequency (fundamental)

Pure sine wave
Angel singing

A sine wave show of period T:



Lecture Demonstration
Listen to a sine wave
(pure tone)

Mechanical Example:
Simple pendulum
Spring pendulum

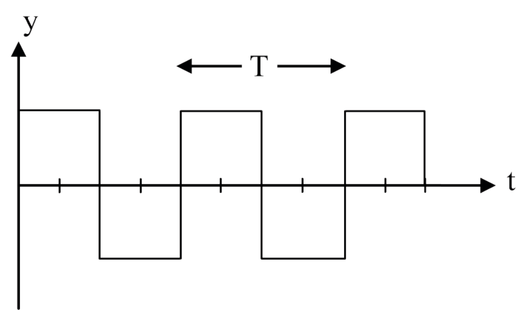
General Periodic Motion

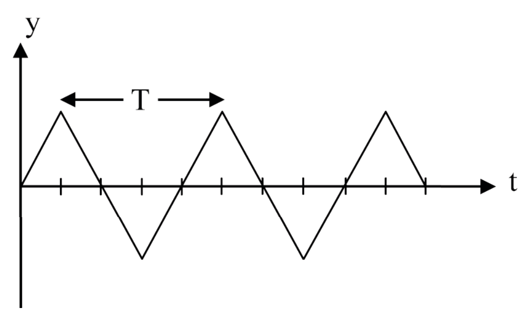
Periodic with period T

Fundamental plus overtones

Complex wave
Tone from a bassoon, violin

A square and triangular wave of period T:





Lecture Demonstration
Listen to a square wave and triangular wave (composite tone with overtones)

Mechanical Example:
Windshield wiper
Heartbeat
Skateboarding a “half-pipe”

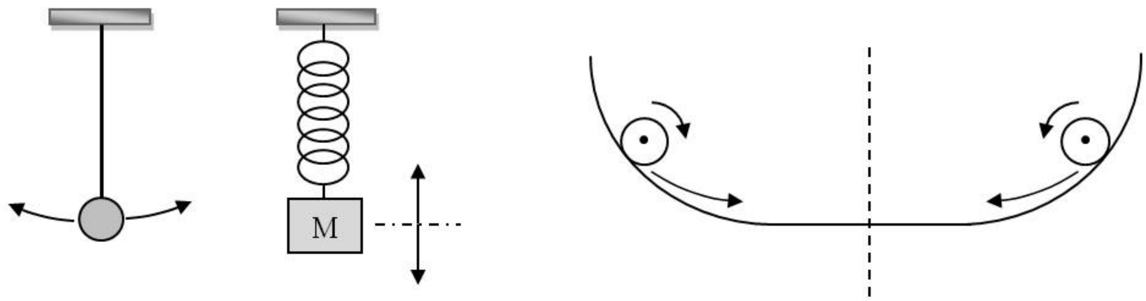


Figure. Left: Simple harmonic motion of a ballistic pendulum and a spring pendulum.
Right: Periodic (not simple) harmonic motion of a skateboarder in a “half pipe”.

Challenge question

a) What shape would the lowest part of a half-pipe have to be so that a skater moves with simple harmonic motion (SHM)? (Hint: Consider the motion of a simple pendulum where a ball is attached to a string.)

Answer: A circle for small amplitudes (and a so-called cycloid for large amplitudes)

b) For the skate boarder in the half pipe, is the motion simple harmonic? Is it periodic?

Answer: ______________________________ ______________________________

Demonstration

1. Show a cylinder or a ball bearing rolling up and down a “half pipe” of semi-circular shape.
2. Show a cylinder or ball bearing rolling up and down a “half pipe” having circular arcs on the sides and a flat bottom.
3. Let the cutout part from the semi-circular half pipe rock back and forth and observe the motion.
4. Show a double pendulum and its harmonic and inharmonic motions.
5. Show a physical pendulum driven by a solar cell and magnet.
6. Discuss in all the above cases whether the motion is harmonic, periodic, or inharmonic.

Various Wave Forms

A sine wave corresponds to simple harmonic motion (SHM). It is the most elementary wave as it contains only a single frequency, the so-called *fundamental frequency*. Many other waveforms exist. Some of these are shown here:

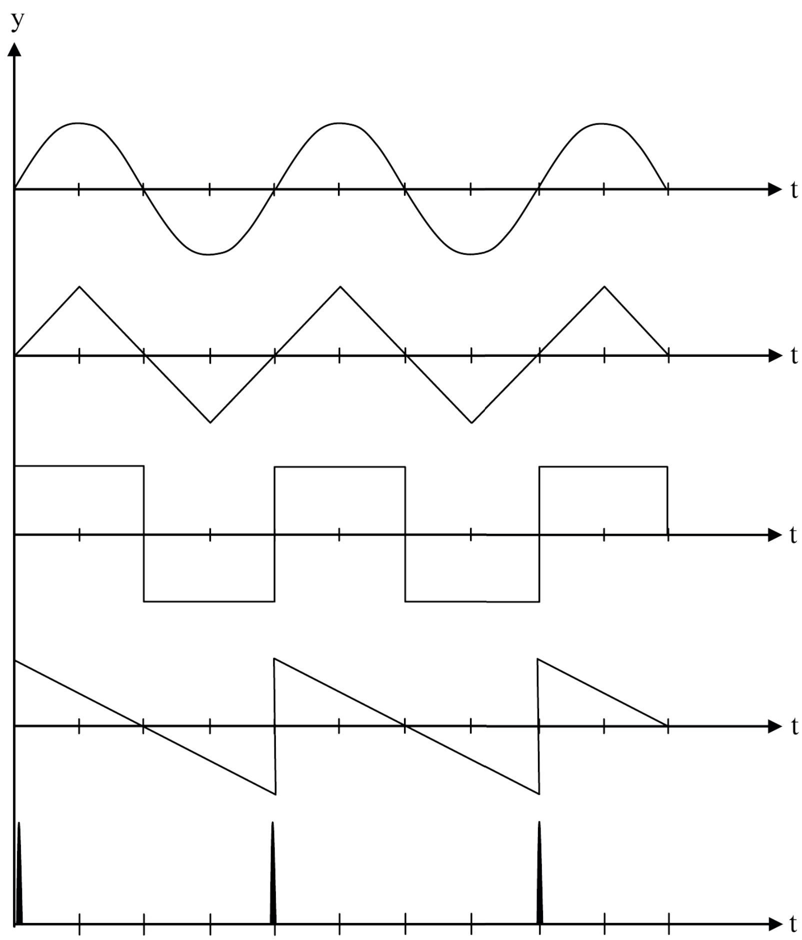


Figure. Sine wave (SHM), triangular wave, square wave, saw tooth wave or ramp, and pulse train. All waves have the same period, fundamental frequency, and amplitude. The waveforms shown are the most basic ones. They are for instance used for testing audio equipment.

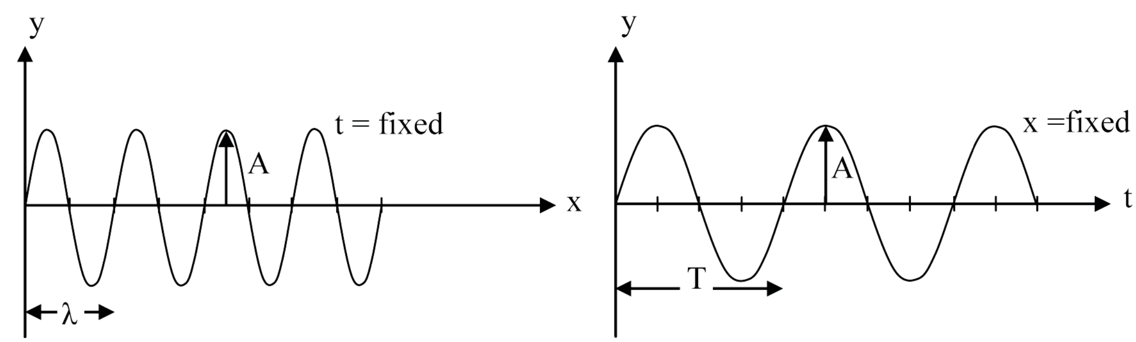
Question Concerning Harmonics and Overtones
Guess what causes the differences in the shape of these waveforms, even though they all have the same fundamental frequency.
Answer: ____

Demonstration
1. Listen to a sine wave and observe it on a monitor.
2. Listen to the different *quality or timbre* of the sound of various waveforms from a signal generator. Alternatively, listen to some synthesized waveforms from a keyboard.

Representation of a Sine Wave as a Function of Frequency and Distance

Draw a sine wave in the *spatial domain*. Show the wavelength λ:

Draw a sine wave in the *time domain.* Show the period T:



Speed of Sound

Speed or velocity is given by distance x traveled, divided by time t, or **v = x/t**.

Rewrite this as v = x/t = λ/T, where λ is the wavelength, and T the period of oscillation. We know that "period = inverse of frequency", T = 1/f, and so we have the result

$$\mathbf{v = \lambda f} \qquad \text{(Speed = wavelength times frequency)}$$

We shall use this formula on the following pages.
The speed of sound depends on the temperature.

Example: At typical class room temperatures of **25^{0}C (77^{0}F)** we have **v = 346.1 m/s**
Example: At the freezing point 0^{0}C (32^{0}F) of water we have = 325.2 m/s

We shall take the value of **v = 346 m/s at 25^{0}C (77^{0}F)** if not otherwise specified.

Determination of the Speed of Sound with a Plosive Aerophone ("Slap Tube")

Use a common 1-inch plastic pipe, slap it with the open hand or a rubber glove. Listen to the pitch of the emitted tone. Obtain the pitch (frequency) from the key on a keyboard closest in frequency to the sound from the tube. The effective length L_{eff} of the pipe gives us the wavelength $\boldsymbol{\lambda = 4 \cdot L_{eff}}$ of the sound. (See Figure below.) The speed of sound then follows from $\mathbf{v = \lambda f}$. This is a very simple and fun experiment!

Consider a plastic pipe (length L = 38.5 cm, radius R = 1.25 cm). After the hand slaps the tube, the latter is closed, as see on the left of the Figure below. For the fundamental air resonance in the tube, we see that $\mathbf{L = \lambda/4}$. Also shown is a wave representing the air movement or displacement in the tube. The air moves most vigorously at the "antinode" at the open end and least at the "node" at the closed end of the tube.

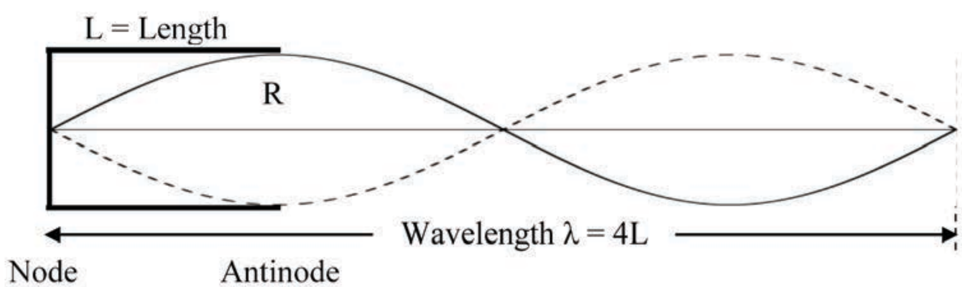


Figure. A tube closed at one end (node) and open at the other end (antinode).

The pitch of the sound from the pipe is close to the key A3 on the piano. Therefore the frequency is f = 220 Hz.
Now substitute the numbers: $L = \lambda/4 \quad \rightarrow \quad \lambda = 4 \times 0.385 \text{ m} = 1.54 \text{ m}$.
Then $v = \lambda f = 1.54 \text{ m} \times 220 \text{ Hz} = 339 \text{ m/s}$.
This is a good result.
But we can do even better by using the **effective length L_{eff}** of the tube, which includes a correction for the open end. The formula for this is:

$\mathbf{L_{eff} = L + 0.6 \cdot R}$, where L is the actual length and R the radius of the tube.

Use the values for L and R and obtain: $L_{eff} = 0.385 \text{ m} + 0.6 \times 0.0125 \text{ m} = 0.3925 \text{ m}$
The wavelength with the end correction then is $\lambda = 4\ L_{eff} = 1.57$ m.
From this follows the speed of sound as $v = \lambda f = 1.57 \text{ m} \times 220 \text{ Hz} =$ **345.4 m/s**.
This is very close to the speed of sound v = 346 m/s in air at 25^0C.

Demonstration
1. Students play a familiar tune on a set of "slap tubes".
2. When the hand is released quickly, the emitted sound is an octave higher. Hold a 2x220 = 440 Hz tuning fork at the open end and listen to the resonance.

Determination of the Speed of Sound with a Didgeridoo
Use a plastic model of an aboriginal Australian didgeridoo and play it.

The fundamental frequency of our didgeridoo is close to the note D2 ($f = 73.4$ Hz). A more exact determination for our tube yields $f = 71.5$ Hz.
The length of the didgeridoo is $L = 1.208$ m.

Finding the speed of sound in this case is very similar to the "slap tube". We again are dealing with a tube, closed at one end by the mouth ("closed" by the buzzing lips).

Here we use the actual length L for the effective length L_{eff} and ignore the end correction from the diameter of the tube, i.e. we ignore the extra length $\Delta L = 0.6 \cdot R$. We can justify this because the tube radius is very small compared to the tube length. We also ignore the flared shape of the open end of the didgeridoo as well as the slightly undulating shape over its entire length.

We then have $L = \lambda/4$ and $\lambda = 4L = 4 \cdot 1.208 \text{ m} = 4.832 \text{ m}$.
For the fundamental frequency (pitch) of the didgeridoo we take $f = 71.5$ Hz.

Then the speed of sound is $v = \lambda f = 4.832 \text{ m} \cdot 71.5 \text{ Hz} = 345.5 \text{ m/s}$.

This is a very good value for the speed of sound, considering the simplicity of the experiment. Unless otherwise specified, we shall always use **v = 346 m/s.**

Demonstrations
1. Play a second didgeridoo of the *same length* and listen to the pitch. Compare the pitch with the first didgeridoo. It should be very similar.
2. Use a straight plastic pipe of similar length and diameter and "play" it. Is the pitch similar, as it should be?

Question
The didgeridoos in the above demonstrations produce a very similar pitch. But do they sound the same? In other words, is the *quality of the sound or timbre* the same?
If not, what could cause the difference in the timbre?
Answers: ____

Demonstration - Determination of the Speed of Sound with a Cardboard Tube

We use a cardboard tube and excite standing waves of the air in it. Place a loudspeaker near one opening of the tube and tune the frequency to the lowest resonance. Write down the frequency for which the sound from the tube is loudest.
For the tube with *two* open ends, the wavelength for the lowest or fundamental vibrational mode is given by **$\lambda = 2L$**, where L is the effective length of the pipe.

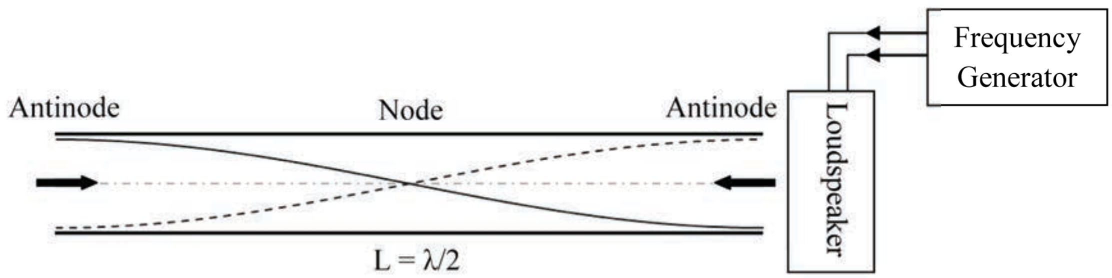


Figure. Sketch of the apparatus, including cylindrical open tube, loudspeaker, and frequency generator. At the ends of the tube we have maximum air movement (antinodes).
The longitudinal air displacement in the tube is shown in the vertical direction for clarity.

Data:
Measured length L = 1.32 m (see later for using the *effective length!*)

Wavelength from $\lambda = 2L$	$\lambda = 2.64$ m
Measured resonance frequency (fundamental mode):	f = 123 Hz
Calculate the speed from $v = \lambda f$ and verify	**v = 325 m/s**

Question
Compare the value for the speed from this experiment with the value of v = 346 m/s. Discuss possible causes in the discrepancies between the two values (e. g. temperature effects, diameter of tube, i.e. end correction).

Including the End Correction of the Tube
The diameter of the tube contributes a correction to the length of the tube at its open end (but not at its closed end):

$$\mathbf{\Delta L = 0.6 \cdot R}, \text{ where R = inside radius of tube}$$

For our tube with two open ends we have the effective length

$$\mathbf{L_{eff} = L + 2 \cdot \Delta L = L + 1.2 \cdot R}$$

The radius of the tube is R = 6.9 cm = 0.069 m.
This yields $L_{eff} = L + 1.2 \cdot R = 1.32 + 2 \cdot 0.6 \cdot 0.069 = 1.403$ m.
Hence $\lambda = 2L_{eff} = 2.806$ m.
The speed of sound then is $v = \lambda f = 2.806 \cdot 123 =$ **345 m/s.**
This is in excellent agreement with the value v = 346 m/s at a temperature of

Table. Values for the Speed of Sound

Speed of Sound in Air		Hydrogen	Helium	
T(^{0}C)	v_{vair} (m/s)	v (m/s)	v (m/s)	
-10	325.2			
-5	328.3			
0	331.3	1286	972	
5	334.3			
10	337.3			
15	340.3			
20	343.3			
25	**346.1**			
30	349.0			
35	351.9			
40	354.7			
Liquids, 25^{0}C	v (m/s)	Solids	v_{vair} (m/s)	
Glycerol	1904	Diamond	12000	
Seawater	1533	Pyrex glass	5640	
Water	1493	Iron, steel	5000-6000	
Mercury	1450	Steel rod	5000	
Kerosene	1324	Aluminum	5100	
Methyl-		Brass	4700	
alcohol	1143	Gold	3240	
Carbon-		Lucite	2680	
tetrachloride	926	Lead	1960	
		Rubber	1600	
The values given are for longitudinal waves in bulk materials.				
Speeds for longitudinal waves in thin rods are smaller.				

Temperature dependence: $v_{air} = 331.3\sqrt{1+\dfrac{T}{273.15}} \approx 331.3 + 0.606 \cdot T$

where v_{air} is in meter/second (m/s) and the temperature T in degree Celsius (^{0}C).

Example: At T = 25^{0}C (classroom temperature): v_{air} = 331.3 + 0.606 ·25 =346.1 m/s

Example: Where does the change of the speed of sound with temperature play a role?

Answer: In the pitch change of wind instruments with change in ambient temperature: Play a note A4 on a wind instrument such as a flute or bassoon with f = 440 Hz at 20^{0}C. Play it later at 30^{0}C. The relative frequency change is $f_{30}/f_{20} = v_{30}/v_{20}$ = 349.0/343.3 = 1.0166. The frequency has risen from f_{20} = 440 Hz to f_{30} = 1.0166 x 440 Hz =447 Hz, a noticeable change in pitch. You have to retune the instruments!

Basic Physics for Acoustics

Physical Quantities, Symbols, and Units

Quantity	**Symbol**	**Unit**	
Distance, length	x, L	meter	m
Time	t	second	s
Velocity, speed	v	meter/second	m/s
Acceleration	a	change of v/time	m/s^2
Mass	m	kilogram	kg
Force	F	Newton	N
Pressure	p	force/area	N/m^2, Pascal Pa

Distance, Length

1 meter = 1 m = 100 cm = 1000 mm
1 cm = 0.01 m = 10^{-2} m
1 mm = 0.001 m = 10^{-3} m
1 micrometer = 1 μm = 10^{-6} m
1 kilometer = 1 km = 1000 m = 10^{3} m

Conversions:
1 inch = 2.54 cm
1 foot = 12 inches = 30.5 cm
1 mile = 1.609 km = 1609 m

Note:
The physical quantities “distance”, “velocity”, “force”, and “acceleration” are vectors that include a value, physical unit, and direction, for instance 30 m northwest. The quantity “length” only includes a value and unit, irrespective of the direction, and similarly for “speed”. Nonetheless, we shall use distance and length, and also velocity and speed, interchangeably in this course.

Exercises
1. The height of a person is 6 ft. 2 in. Convert this to meters and give the answer with three (3) significant figures (for instance 1.73 m).
Answer: Height = ______m
2. The distance between your apartment to our lecture hall at TTU is 1.7 miles. Express this in kilometers.
Answer: x = ______km
3. The length of a cello string is 71 cm. Convert this to millimeters.
Answer: L = ______mm

Time and Frequency

Time
Use your own words to define the concept of "time".
Albert Einstein said: "Time is what a clock reads."

Unit of time: 1 second = **1 s**

1 year = 365 or 366 days
1 day = 24 hours = 24 hr
1 hr = 3600 s

Time units needed: (please review your knowledge of **"power-of-ten" notation!**)
1 second
1 millisecond = 1 ms = 0.001 s = 10^{-3} s
1 microsecond = 1 μs = 0.000001 s = 10^{-6} s

Exercises
1. How many seconds are in a year of 364 days? Express this as a plain number and also in power-of-ten notation. (Example for power-of ten notation: 1.23456×10^{6})
Answer: Plain notation 1 year = 31,449,600 s
Power-of-ten notation 1 year = $3.14496 \cdot 10^{7}$ s

2. How many milliseconds are in one minute?
Answer: 1 minute = 60,000 ms

Frequency
Frequency f of oscillation = number of cycles/second
Unit of frequency f: 1 Hertz = 1 Hz

Period of oscillation: Time T for one full cycle
Relation between frequency and period: $\mathbf{f = \frac{1}{T}}$

Exercises
3. A pure tone has a frequency f = 500 Hz. What is the period T in ms?
Answer: T = 2.00 ms

4. A tone near the upper limit of hearing has a frequency of 16,000 Hz. What is the period of oscillation in milliseconds and microseconds?
Answer: T = 0.0625 ms and T = 62.5 μs

5. The period of oscillation of a tone is T = 1.25 ms. What is the frequency in Hz?
Answer: f = 800 Hz

Speed and Velocity

Speed = distance/time: **$v = x/t$**

Alternatively: **$x = v \cdot t$** and **$t = x/v$**

Examples

1. The speed of sound at 25^0C is v = 346 m/s.

2. The speed of light in vacuum or air is v = 300,000 km/s.

Exercises

1. A car travels a distance of 390 miles from Lubbock to Austin in 7 hours. What is the average speed in m/s?
Answer: v = 24.9 m/s

2. What distance does sound travel in one minute? Use v = 346 m/s.
Answer: x = 20.8 km

3. A car travels at a speed of 26.8 m/s. Express this in kilometers and miles per hour.
Answer: v = 96.5 km/hr v = 60.0 mi/hr

4. The Earth-Moon distance is about 384,000 km. How long does it take a laser pulse or radio signal to travel to the moon and back? (This is the minimum time lag for communication with astronauts on the moon.) Use 300,000 km/s for the speed of light.
Answer: t = 2.56 s

Remark
Speed has a number and a unit, without indicating the direction, for instance 100 km/hr. *Velocity* is a vector whose magnitude is the speed, but it also includes the direction of travel, for instance 100 km/hr *north*, e.g. going from Lubbock to Amarillo.
We usually only need *speed*, but may use the concepts of *speed* and *velocity* interchangeably.

Acceleration

Acceleration = change of speed/time or **$a = \Delta v/\Delta t$**

Example
The acceleration of a freely falling body in Earth's gravitational field is $a = g = 9.8\ m/s^2$, neglecting air resistance. This means that the falling body experiences an increase in speed of 9.8 m/s per second. This results in the unit of m/s^2 for acceleration.

Example
A car accelerates from 0 to 60 mi/hr in 8 seconds. Express this result in units of m/s^2 and compare with the value of g.
Answer: Convert the increase in speed to m/s and show that

$$\Delta v = 60\ mi/hr = 26.8\ m/s$$

Then use $\Delta t = 8$ s and obtain the acceleration

$$a = \Delta v/\Delta t = 3.35\ m/s^2$$

Compare this with the acceleration $g = 9.8\ m/s^2$ in Earth's gravitational field and obtain the result as a fraction of g and percentage:

$$a/g = 0.34 = 34\%$$

Where do we need acceleration in this course?
Acceleration is of interest when we consider vibrating strings in string instruments or vibrating air columns in wind instruments. Vibrations with accompanying accelerations also occur in percussion instruments.

Comment about Sound Generation, Sound Radiation, and Detection
For sound to be radiated from a source (voice, musical instrument, loudspeaker) or be detected by a receiver (ear, microphone), it is necessary that the air molecules accelerate and not just move with constant speed. Any vibration with its back-and-forth motion is associated with an acceleration.

Mass and Mass Density

The mass m of an object is a measure of its amount of matter and the number of atoms.

Unit of mass: 1 kilogram = 1 kg
1 kg = 1000 gram = 1000 g

Conversion factors
1 kg = is 2.2046 pound mass
1 pound mass = 0.45359 kg
1 ounce = 28.35 gram = 0.002835 kg

Example
l liter of water at 4^0 C has a mass of 1 kg.

Mass Density
Density of a substance = mass/volume. The symbol for it is the Greek letter ρ "rho".
Hence we have the formulas

$$\boldsymbol{\rho = m/V}, \text{ hence also } \boldsymbol{m = \rho V}, \quad \boldsymbol{V = m/\rho}$$

The unit of density ρ is kg/m^3. The densities of some substances are:

Steel, Iron	$\rho =$ 7900 kg/m^3
Aluminum	2700
Copper	8930
Gold	19300
Earth average	5520
Concrete, brick, glass	~2000
Water	1000
Air at 1 atm, 0^0C	1.293

Exercises
1. A steel string has a length of 1 m and a diameter of 1 mm. What is its mass in kg and gram?
Answer: Mass $m = \rho\pi r^2 L$ = 0.00620 kg = 6.20 g

2. Find the volume in cubic centimeters of 1 kg of gold. (Note that 1 $cm^3 = 10^{-6}$ m^3.)
Answer: Volume V = 51.8 cm^3

Demonstration
Show 2 ounces (≈ 60 cm^3) of water in a beaker. What would this be worth in gold?
Answer: About $60,000. (Verify this!)

Note that we will need the concepts of *mass* and *density* when discussing the frequencies of vibrating strings and waves.

Force

Force is “push” or “pull”.

Unit of force = 1 Newton = **1 N**

Conversion factors: 1 N = 0.2250 pound force or 1 pound force = 4.445 N

For accelerating bodies we have Newton’s 2^{nd} law: $\mathbf{F = ma}$

Exercises

1. A car accelerates with an acceleration a = 2 m/s^2. The mass of the car is 1500 kg. What is the magnitude of the force that accelerates the car, friction neglected?
Answer: Force F = 3000 N

2. A person has a mass of 72.6 kg. What is his weight in Newton? (Use g = 9.8 m/s^2.)
Answer: W = mg = 711 N

3. What is the person’s weight in pounds?
Answer: W = 160 lb force

Weight of a Body
You are standing on a scale. What is your weight? The short answer is: W = mg.
But there are some subtle details in case you are interested:
Gravity pulls you down. The reaction force from a scale, the so-called “normal force”, pushes you up. The two forces are in equilibrium as long as you don’t accelerate. In this case the scale reads the reaction force with a compressed spring or its equivalent, and this is your weight! The absolute values of the gravitational force and the normal force are the same, but their directions are opposite.

Gravity pulls down with a force F = mg. The reaction force from the scale when standing still is (–mg) up. We call this the “weight”. Its magnitude is W = mg, but we cavalierly omitted the minus sign. Did you ever think of your weight in this way?
Note that a person standing on a scale and falling freely is *weightless*. What happened? There is no reaction force up from the scale, and so the scale reads nothing. You are “weightless”. The only force acting on the object is the gravitational force down - *not* the weight.

Demonstrations
1. Suspend a mass on a spring. See how gravity pulls the mass down and the reaction force from the spring pulls it up.
2. Use a spring scale and suspend a mass from it. Read the weight directly in Newton from the dial on the scale.

Pressure

Pressure = Force/Area: **p = F/A**

Unit of pressure = 1 N/m^2 = 1 Pascal = **1 Pa**

Examples

1. Average atmospheric pressure at sea level:
1 atmosphere = 1 atm = 101,325 Pa = 1013.25 millibar (mb)
1 atm = 14.70 lb/in^2 =14.7 PSI

2. The pressure in the tires of a car is p = 32 PSI = 32/14.70 = 2.18 atm.

Pressure Fluctuations in Sound Waves

Sound waves in air cause small pressure fluctuations about the average atmospheric pressure. We hear these fluctuations as sound.

Typical amplitudes of pressure fluctuations in sound waves in air are:

Smallest audible sound	$\Delta p \approx 0.00002$ Pa
Soft music	$\Delta p \approx 0.002$ Pa
Moderately loud music	$\Delta p \approx 0.02$ Pa
Threshold of pain	$\Delta p \approx 20$ Pa

Obviously the pressure fluctuations in sound waves are very small compared to the static atmospheric pressure of about 100,000 Pa.

Exercises

1. Calculate the ratio $\Delta p/p_{atm}$ for moderately loud music and interpret your result.
Answer: $\Delta p/p_{atm} = 2.0\cdot 10^{-7}$

2. An elephant and a woman with high-heeled shoes are standing on a beach. Who exerts a higher pressure on the ground? Estimate the pressure in both cases.
Hint: Calculate the weight from F = mg and use p = F/A. Estimate the mass m of the elephant and of the woman, and the total area A in contact with the ground. Don't forget that the elephant has four feet and the woman only two. You should find that the pressure on the ground from each of the woman's heels is on the order of magnitude p ~ 50 atm!
Answer (do it yourself)

Mass of the elephant m = ______kg,	mass of the woman m = ______kg
Weight of elephant W = mg = _______N	Weight of woman W = ______N
Total area A: Elephant A = ______m^2	Woman A = ______m^2
Pressure: Elephant p = ____________N/m^2	Woman p = ____________N/m^2

Vibrating Strings - Some Basics

"There is geometry in the humming of the strings. There is music in the spacing of the spheres." (Pythagoras, 570–495 B.C.)

A string under tension accelerates and radiates sound when bowed or plucked.
The pitch (fundamental frequency) of the sound depends on the tension T in the string, its length L, and linear mass density μ (mass/length, $\mu = m/L$ - see later).

String Length L

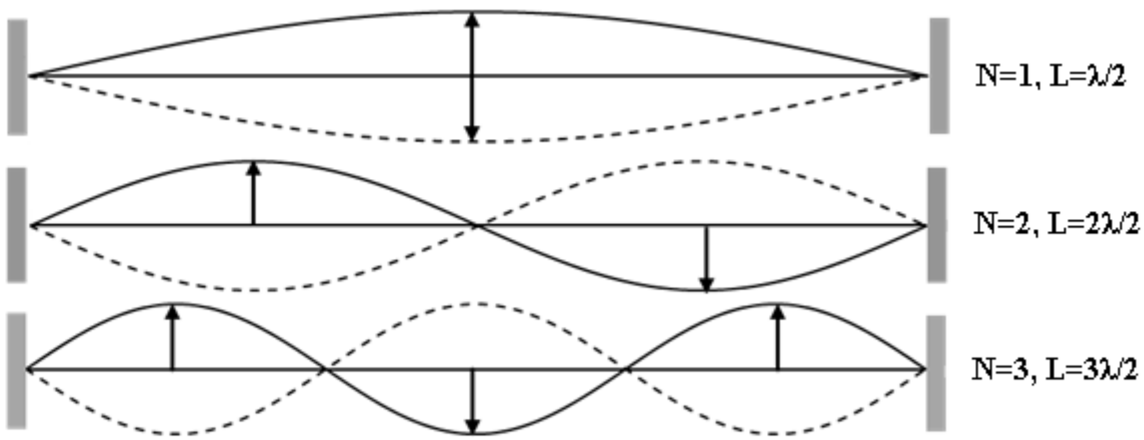


Figure. The lowest three vibrational modes of a stretched string fastened at its ends and the relationship between string length L and wavelength λ.

N = 1	first harmonic or fundamental mode,	$L = (1/2)\cdot\lambda_1$	or	$\lambda_1 = (2/1)\cdot L$
N = 2	second harmonic or first overtone,	$L = (2/2)\cdot\lambda_2$	or	$\lambda_2 = (2/2)\cdot L$
N = 3	third harmonic or second overtone,	$L = (3/2)\cdot\lambda_3$	or	$\lambda_3 = (2/3)\cdot L$

etc…. Generally we have:

$\boldsymbol{\lambda_N = 2L/N}$, where N = 1, 2, 3, 4, … is the so-called **harmonics number**.

For the harmonic frequencies of the vibrating string, we then have $\mathbf{f_N = v/\lambda_N = N(v/2L)}$, or $\mathbf{f_N = Nf_1}$, with $\mathbf{f_N = f_1, 2f_1, 3f_1, 4f_1, 5f_1, \ldots}$, where $\mathbf{f_1 = v/\lambda_N = (v/2L)}$ is the fundamental frequency (or "pitch").

Demonstrations

1. Drive a stretched cord with a vibrator and show the different vibrational modes.
2. Use a sonometer or monochord with a stretched string fastened at its two ends.

Vary the length or tension of the string and listen to the change in frequency.
Divide the string into two equal parts with a wedge and listen to the resulting *octave*.
Divide the string into two appropriate parts and listen to the musical intervals of a *fifth*, *fourth*, and *third*.

3. Show the strings on a violin. Bow and pluck them.
4. Show a "slinky" and simulate the transverse vibrations of a string.
5. With the slinky, also simulate the longitudinal vibrations in air.

Resonance

Resonance generally occurs when a driving force of frequency *f* excites vibrations in an object whose own natural frequency of vibration is the same as that of the driver, or

$$\mathbf{f_{driver} = f_{vibrator} = f_{resonator}}$$

The distinction between vibrator and resonator may not always be obvious.

Examples

1. A child is on a swing: His parent (driver) pushes on the swing with the same frequency as the natural frequency of the swing (resonator). There is no obvious vibrator.
2. You hit a metal bar with a mallet (driver). The bar is the vibrator and resonator.
3. You strike a marimba bar (vibrator) with a mallet (driver). The tube (resonator) below amplifies the sound from the vibrating bar
3. You slap a plastic tube with your hand (driver) on one of its open ends. The impulse from your hand contains a range of frequencies. The tube is the resonator and briefly resonates with its own natural frequency. The resonance frequency of the fundamental vibrational mode then is $f = v/\lambda = v/4L$.). There is no obvious vibrator (maybe the air).
4. The bow (driver) of a violin excites a string (vibrator). The vibrations are transferred to the body (resonator) of the violin. The body radiates the sound. Little sound comes from the vibrating strings themselves.

Demonstrations

1. Observe the different resonance frequencies of 5 washers suspended from a rod.
2. Move a spring pendulum up-and-down. Note the resonance when $f_{driver} = f_{resonator}$.
3. Place a vibrating A4 tuning fork in front of a 38.5 cm long plastic pipe (resonator) tuned to 440 Hz and note the increase in loudness caused by the resonance.
4. Excite the vibrations in a large cardboard packing tube by tapping it on the ground.
5. Place wooden bars or tile strips on top of a tuned closed tube and note the resonance.

Helmholtz Resonators (Hermann von Helmholtz, 1821–1894)

Enclosed cavities (resonators) such as boxes, cylinders, and bottles with small openings have characteristic resonances when excited by tapping, blowing, or striking with sticks or mallets (drivers).

Demonstrations

1. Blow across the opening of a bottle and listen to the resonance resembling a deep and “throaty” tone. Blowing produces a broad frequency spectrum. The air in the bottle responds by vibrating selectively at the frequency of the Helmholtz resonance.
2. A loudspeaker enclosure is a Helmholtz resonator that enhances the low frequencies.
3. Pluck or bow the strings of a sonometer and violin. Listen to the “amplified” sound from the openings in the wooden Helmholtz resonator.
4. A tuning fork (C4) on a wooden box with one end open sounds louder than without the box. Other tuning forks than C4 do not meet the resonance condition and sound less loud.
5. Helmholtz resonator boxes with vibrating bars on top (440 Hz, 441 Hz). Cover the hole of one box and note the diminished sound intensity. Remove the cover again and hear the sound come back louder again.

Summary of Wave Parameters

A sine wave is characterized by:
Frequency f
Amplitude A
Phase φ
Wavelength λ
Wave speed $v = \lambda f$

Mathematical Expression for a Traveling Sine Wave

For a simple sine wave, the displacement of the vibrating medium at a location x and at time t is given by

$$\mathbf{y(x,t) = A \cdot \sin[2\pi(x/\lambda - f \cdot t) - \varphi]},$$

where y(x,t) is the displacement of the vibrating medium from equilibrium at position x and time t, λ the wavelength, f the frequency, and φ the phase difference when y is not the maximum displacement (amplitude A) at position x = 0 and time t = 0.
The above expression is the mathematical representation of a sine wave traveling in the x-direction.

Transverse Wave

The medium vibrates perpendicular to the direction of wave propagation.
Example: A string under tension.

Longitudinal Wave (Pressure Wave)

Medium vibrates in the direction of motion. Example: sound waves in air.

Demonstrations

1. Transverse wave traveling along a rope or long spring.
2. Longitudinal waves on a slinky.
3. Longitudinal waves on a vertically oriented spring driven by a vibrator.

Remember the Formulas?

Wave speed $v = x/t$ = wavelength/period = λ/T $\rightarrow$ $\mathbf{v = \lambda/T}$
Frequency f = inverse period = $1/T$ $\rightarrow$ $\mathbf{v = \lambda f}$

Example

A wave travels on a stretched string. The period of oscillation is T = 5 ms = 0.005 s and the wavelength is λ = 1.2 m. What is the speed of the wave on the string?
Answer: Method 1: $v = \lambda/T$ = 1.2 m/0.005s = 240 m/s
Answer: Method 2: Convert to $f = 1/T$ = 1/0.005 s = 200 Hz
$v = \lambda f$ = 1.2 m x 200 Hz = 240 m/s, same as it must be.

Part 2

Wave Properties

1. Reflection

2. Refraction

3. Inverse Square Law for Sound Intensity

4. Huygens's Principle

5. Interference and Superposition of Waves

6. Diffraction and Superposition of Waves

7. Beats

8. Polarization

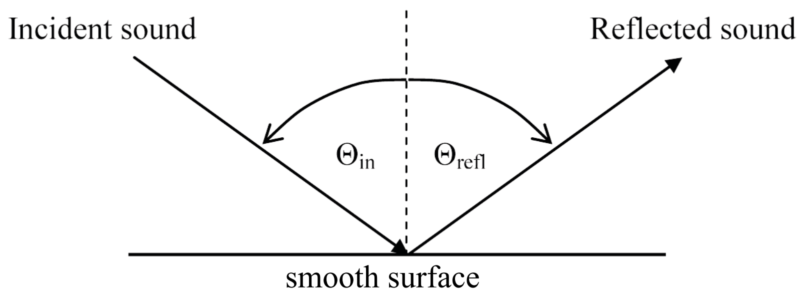
9. Transverse Waves

10. Longitudinal Waves

11. Doppler Effect

Law of Reflection
When a wave strikes a surface, the angle of incidence is equal to the angle of reflection with both angles counted from a line perpendicular to the surface.

$$\theta_{in} = \theta_{reflected}$$



Condition for reflection: The surface must be smooth compared to the wavelength.

Question: Where does reflection of sound occur?
Answer: On exterior walls (echo), interior walls of buildings, room walls, lecture halls, concert halls, theaters, churches.

Question: Where is reflection important?
Answer: It may improve the acoustic quality of a room or make it worse.

Reflection of sound is desirable when trying to keep the sound intensity in a room high and evenly distributed. Multiple reflections from surfaces cause longer reverberation times that may be favorable for the reproduction of some types of music.

Reflection of sound may be undesirable when concentrated on “hot spots” in a room. Multiple reflections from hard surfaces in large rooms give rise to too long reverberation times that make the sound linger. Speech in large churches constructed of stone may become unintelligible. But organ music does sound good!

Additional Examples for Reflection
1. Amphitheaters.
2. Parabolic reflector microphone for recording distant sounds, e.g. from birds.
3. “Whisper chamber” with ellipsoidal reflectors: You stand at one focus of the ellipsoid, your friend hears you clearly at the other focus, even when you are speaking softly. (The Science Spectrum in Lubbock has such a setup.)
4. Parabolic mirrors in terrestrial and astronomical telescopes for concentration of light.

Demonstrations
Show a parabolic “mirror” microphone.
Show an optical parabolic mirror inside a flashlight or camping lantern.

Refraction of Waves

Refraction occurs when a wave that travels in a medium enters another medium. The direction of propagation changes at the interface between the two media. This effect is called *refraction*. It happens for instance when a light wave enters glass. Eyeglasses and lenses work on the principle of refraction. The amount of bending of the wave at the interface between the two media is described by Snell's law (mathematical form not needed here). The wave bends towards the surface normal when it enters a medium where the wave velocity is lower than in the medium from what it came.

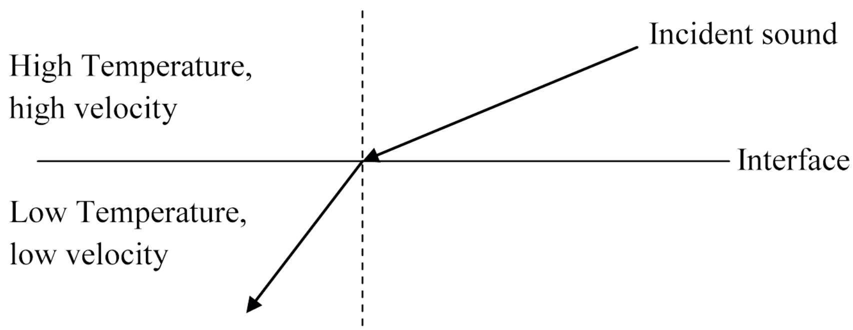


Figure. Refraction of a sound wave traveling from high to low temperature regions through a boundary layer (interface).

Refraction of sound waves occurs mostly outdoors. It is not very important for room acoustics. The following interesting natural phenomena are based on refraction.

1. **Temperature Inversion in the Atmosphere**
A temperature inversion may exist above a placid lake at night when the air is cooler near the water and warmer higher above. Sound waves are bent down as they travel from the source to the listener. This increases the sound intensity heard by the observer. This way you may hear campers from the far side of a lake at night. You may not hear them during the day when there is no temperature inversion and the air is warmer near the water and cooler higher up.

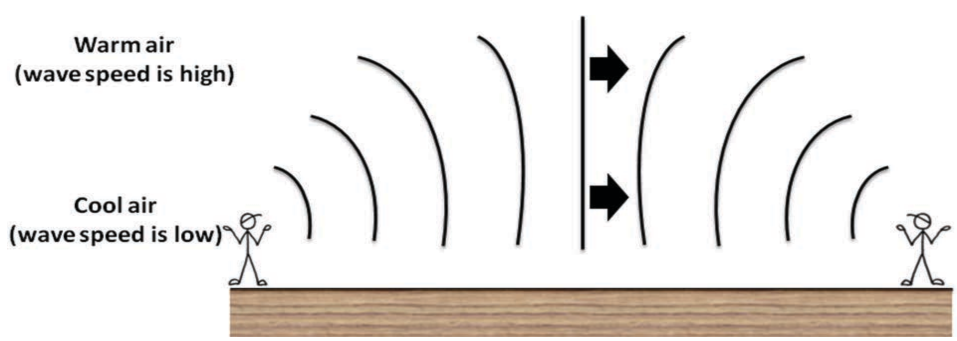


Figure. A temperature inversion occurring above a placid lake at night. Sound waves originally traveling upward are refracted back down and reach the observer.

2. **Maximum Distance for Hearing Thunder**
A normal temperature gradient without inversion usually exists at the beginning of a thunderstorm. The air is warmer near the surface and the speed of sound is higher there. Sound waves will then bend upward. When thunderheads typically are at an elevation of about 4 km, the maximum range for hearing thunder is about 22 km. Beyond that the sound is refracted back up and cannot be heard at larger distances.

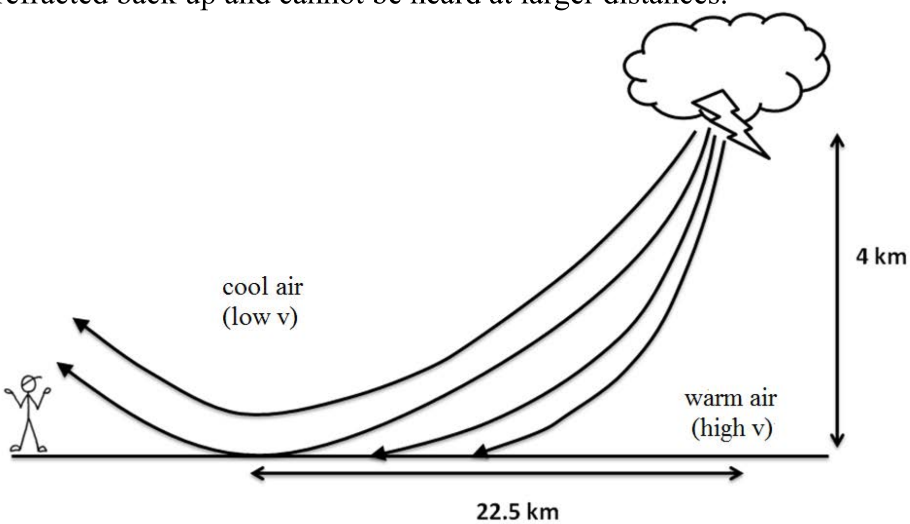


Figure. Upward bending of sound waves from a thunderhead in the normal atmosphere. The air near the ground is warmer and the wave speed larger than farther above.

3. **Ocean Waves Near a Beach**
Waves approaching a beach under an angle to the shore generally become more parallel to the shore. The reason is that the waves closer to the shore slow down due to the shallower water. The faster waves in the deeper water catch up and align their wave fronts parallel to the shore. The bending is gradual if the water depth decreases gradually.

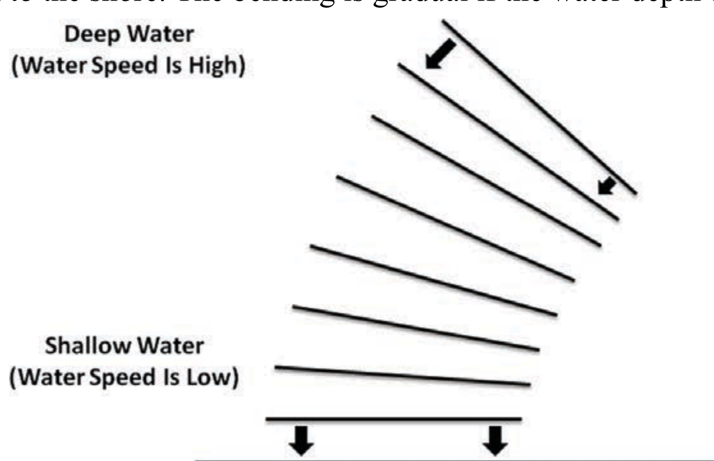


Figure. Water waves approaching a beach line up parallel to the shore. The wave in the shallower water is slower than in the deeper water farther. Refraction bends the wave.

4. A **Mirage** shows the “blue sky” on the “watery” surface of a dry highway.

Doppler Effect

The so-called Doppler effect occurs when the sound source or observer are moving with respect to each other. The frequency (pitch) of the sound changes.

Case 1
The source is moving towards the observer or the observer moving towards the source.
The wave fronts crowd together and the wavelength decreases.
The pitch of the sound increases.
Example
A police car is approaching. The siren sound has a higher pitch than at rest.

Case 2
The source or the observer are moving away from each other.
The wave fronts are farther apart and the wavelength increases.
The pitch decreases.
Example
A police car speeds away. The siren sounds at a lower pitch than when approaching.

Where does the Doppler effect play a role?
1. One can determine the speed of underwater objects (submarine, whales, etc.) with *sonar*. A sound wave is emitted in the direction of the object under study. The wave is reflected from the object and shows a shift in frequency. The formula for the frequency shift includes the speed of the object, and thus the speed can be calculated.
2. Bats navigate in dark caves and at night by emitting ultrasound (1 kHz – 150 kHz) and processing the reflected sound. They emit short pulses of ultrasound and detect the frequency shift and the time it takes for the sound to return. In this way, they find out how far an object is and how fast it is moving, even in which direction. Their auditory system is so sensitive that they can catch flying insects, their favorite meal. “Bats can see with their ears.”

Demonstrations
1. Launch a foam ball containing a high-pitched mine-speaker. Note the change in pitch due to the Doppler effect.
2. Use a tuning fork and move it toward or away from an observer. Can you hear the change in pitch?
3. Use three tuning forks sounding the major triad C4, E4, and G4. Have three students move the forks back and forth to produce small frequency changes. Listen to the rather pleasant vibrato in the major triad.

Inverse Square Law for Sound Intensity

When you move away from a sound source in an open space, the sound intensity decreases because the total power emitted by the source is spread over a larger surface.

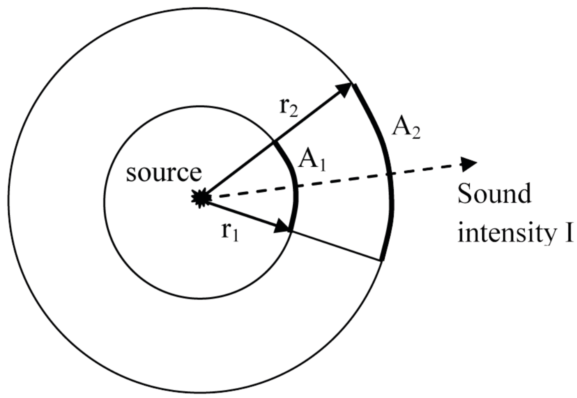


Figure. A sound wave from an unobstructed point source moves spherically outward.

Definition of Sound Intensity

Intensity = Power/Area	$I = P/A$
Area of a sphere	$A = 4\pi r^2$
Hence	$I = P/4\pi r^2 \propto 1/r^2$ or $\mathbf{I \propto 1/r^2}$

The total emitted power P is the same through any sphere at a distance *r* from the source. But the intensity decreases proportional to $1/r^2$. This is the *inverse square law.*

Example

You double the distance from a sound source from 10 m to 20 m. The sound intensity decreases 4-fold (not 2-fold!).

Question: Increase the distance 4-fold. By what factor does the sound intensity decrease?
Answer: It decreases _________fold.

In open-air theaters or concert stages, the sound decreases rapidly with distance from the stage if no remedies are used. The problem can be addressed with sound reflectors near the stage and seating on sloping surfaces such as in amphitheaters.

The ground and uneven terrain are obstacles that can alter the sound intensity and cause substantial deviations from the inverse square law. The law is of little practical importance for sound propagation and reproduction in *enclosed* spaces.
Challenge question. Do reflections from the ground at an open-air concert increase or decrease the sound intensity from what can be expected from the inverse square law?
Answer: Decrease _________ Increase _________ (Give your reasons.)

Wavelength λ, Frequency f, Period T, Amplitude A, Wave Speed v = λf

Displacement of the Medium in a Traveling Wave versus Time and Distance - A Thought Experiment

Imagine water waves traveling in the ocean.
1. Observe the displacement of the water in its up-and-down motion in the y-direction as a function of time ("movie") at a fixed position x.
2. Take a snapshot of the wave in which you see the displacement y of the water waves as a function of x in the direction of propagation, but with the time t now fixed ("snapshot"). Note the period T, the wavelength λ, and the speed v of the wave. Make two drawings.

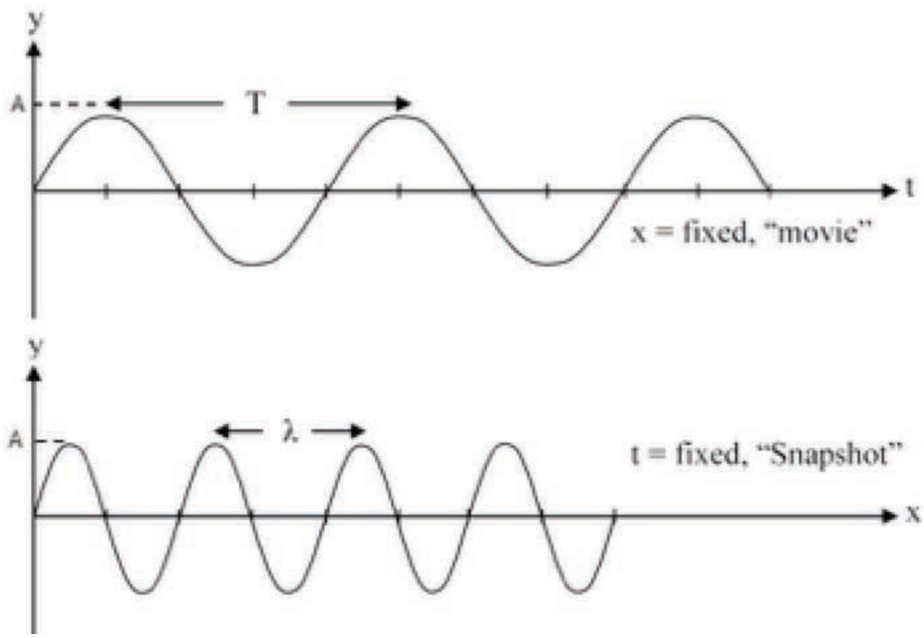


Figure. Upper graph: "Movie" of a water wave or sine wave as a function of time, at a fixed position x, with period T. Lower graph: "Snapshot" of a water wave as a function of location x, at a fixed time t, with wavelength λ. The amplitude A is shown in both cases.

Note that both graphs are sine curves, but the labels on the horizontal axes are different. It is the time *t* in the first drawing and the position *x* in the second drawing. The reason for the similarity is that the displacement *y* of the traveling transverse wave has a sinusoidal dependence on both variables (x,t). The displacement y(x,t) of the simplest traveling wave is a sine wave. Assuming that the wave has zero displacement at x = 0 and t = 0, i.e. no phase shift, it is given by (for the mathematically interested)

$$\mathbf{y(x,t) = A \cdot \sin[2\pi(x/\lambda - t/T)]}, \text{ where } A = \text{amplitude}, \lambda = \text{wavelength}, T = \text{period}.$$

Question
You bob up and down with an ocean wave by hanging onto a buoy. Then you surf the wave. Describe how the wave looks to you in either case. When do you see it move?

Huygens's Principle

Christians Huygens's (1629-1695) principle explains the wave phenomena of interference, diffraction, reflection, and refraction, more specifically the creation of new waves from existing waves. The principle can be stated as follows:
"All points on a wave act as sources of small circular (2-D) or small spherical (3-D) wavelets. The wave pattern at a later time is the superposition or sum of these wavelets. The leading edge of the individual wavelets forms the next wave front."

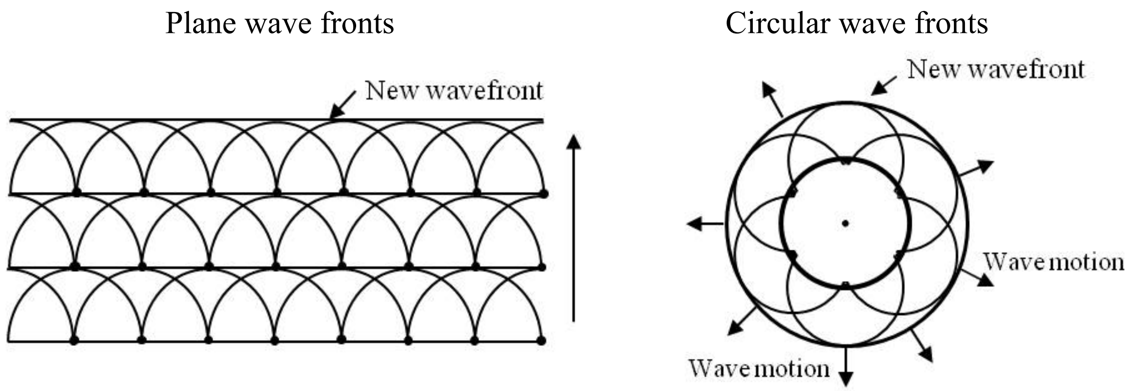


Figure. Huygens's principle applied to plane waves and circular (spherical) waves. Note that the direction of propagation of the wave is always perpendicular to the wave fronts.

Examples
1. Drop a stone into a pond. The expanding wave fronts are circles around the impact.
2. The sound of a firecracker high up in the air expands in spherical wave fronts.
3. Dip a long board periodically into water and observe the emitted plane waves. .

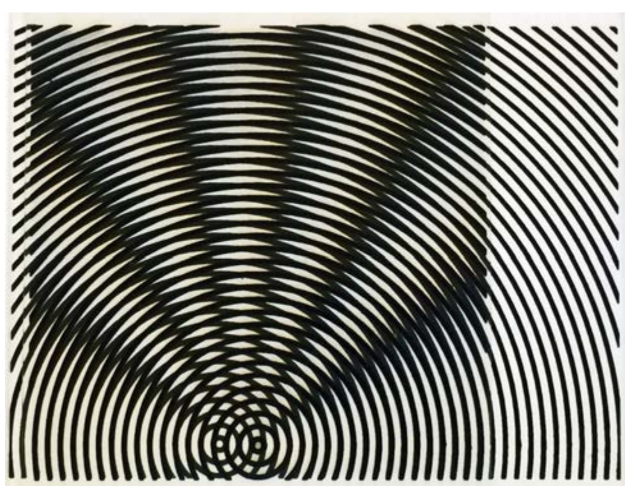


Figure. Simulated interference of waves from two point sources (located at the bottom of the picture, for instance two dippers periodically touching a water surface). The bright and dark regions show constructive and destructive interference, respectively. Two transparent plates were overlaid, each having black concentric circles for the "waves".

Interference of Waves and Superposition Principle

When two waves meet, they interfere with each other in the region of overlap. The resulting wave is the sum of the two superimposed waves (Superposition Principle).

Constructive and Destructive Interference

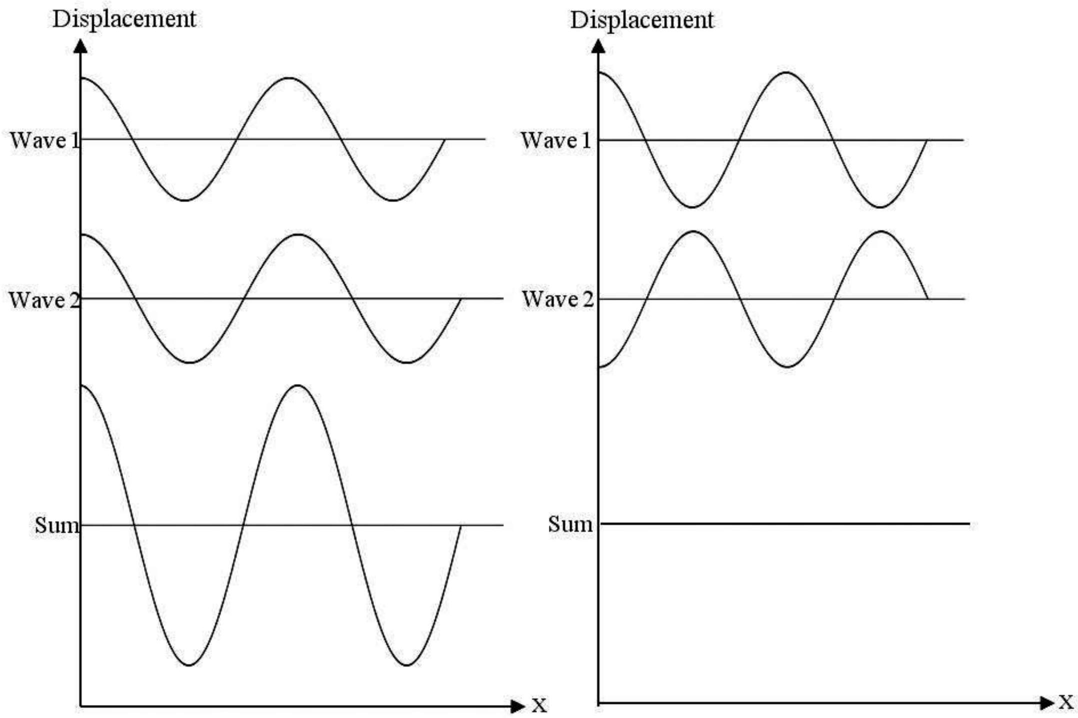


Figure. Left: Constructive interference of two waves of equal amplitude that are in phase. The amplitude of the resulting wave is twice as large. Right: Destructive interference of two waves of equal amplitude 180^0 out of phase. The resulting wave has zero amplitude.

Demonstrations

1. Play a sine function of 700 Hz from a single frequency generator into two loudspeakers. Pass in front of the speakers. Hear the interference maxima and minima.
2. Show two overlapping interference plates on the overhead projector. Observe the maxima and minima. The interference effect becomes pronounced as the separation *d* between the two sources becomes comparable to the wavelength λ, i.e. $\mathbf{d \approx \lambda}$.
3. Show the TTU mechanical wave maker on the overhead projector.
4. Throw two pebbles into a lake and observe the expanding circular wave fronts and how they merge. Note the interference maxima and minima.

Interference of sound and room acoustics

Interference of sound waves should be avoided in rooms where an even distribution of sound intensity is desired without "hotspots". The problem can be corrected with multiple reflections from walls and ceilings and the strategic placement of sound absorbers.

Diffraction and Superposition of Waves

Diffraction occurs around the edges of obstacles and narrow openings. Waves travel into "shadow regions" behind obstacles and sound can be heard "around corners".

Example

You are hiding from your mother behind a tree or below a window, but you hear her well because of the diffraction of the sound around the edges of the tree or window.

Demonstrations

1. Stand in the hallway to the side of the open classroom door. Say something. Students in the lecture room may hear you although they cannot see you.
2. Look at a CD or DVD and note the color spectrum from diffraction of the white light on the grooves of the disc.
3. Interference and diffraction of water waves are demonstrated effectively in our laboratory with a "ripple tank". What you see there is similar to what you hear in the case of sound waves.

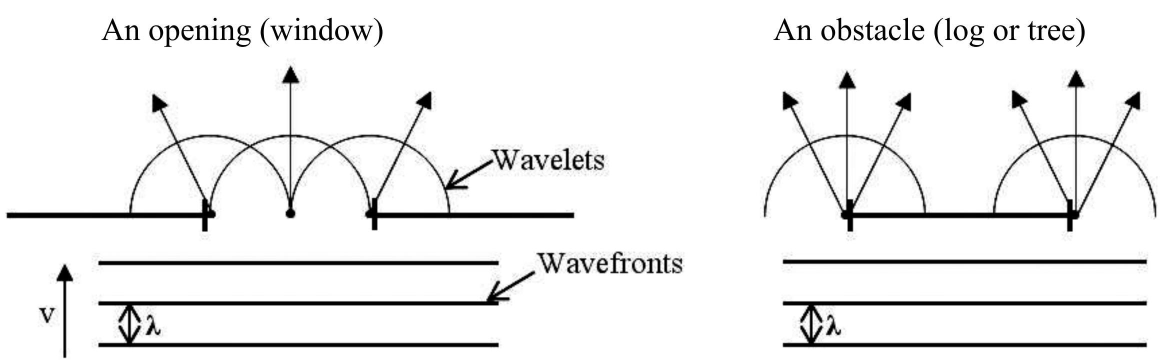


Figure. Application of Huygens's principle showing diffraction of sound waves into the "shadow region" behind an obstacle.

Condition for Diffraction

The size *s* of the obstacle (e.g. tree diameter, window opening) must be comparable to or not much larger than the wavelength λ of the wave, i.e. $\boldsymbol{\lambda \approx s}$**.** Otherwise the waves travel in straight paths without showing much diffraction into the "shadow regions".

Difference between Interference and Diffraction

Interference of waves occurs in free space without obstacles and around obstacles, while diffraction occurs only around obstacles.

Common Features of Interference and Diffraction

Waves resulting from interference and diffraction are explained by the Principle of Superposition of waves (Huygens's principle).

Diffraction and Interference in Room Acoustics

Diffraction of sound occurs around obstacles and openings in rooms, concert halls, auditoriums, and outdoors. Diffraction screens and reflectors at odd angles can be used to minimize diffraction and optimize the uniformity of sound. Interference of sound waves may occur without obstacles and produce areas of low and high intensities.

Beats

When two sine waves of similar frequencies and amplitudes interfere, so-called *beats* occur. You hear a swelling and decrease in sound intensity at the *beat frequency*, which is much lower than the frequency of either wave.
The amplitude of the beating wave varies between zero and the sum of the amplitudes of the component waves (assuming that the two waves have equal amplitude).

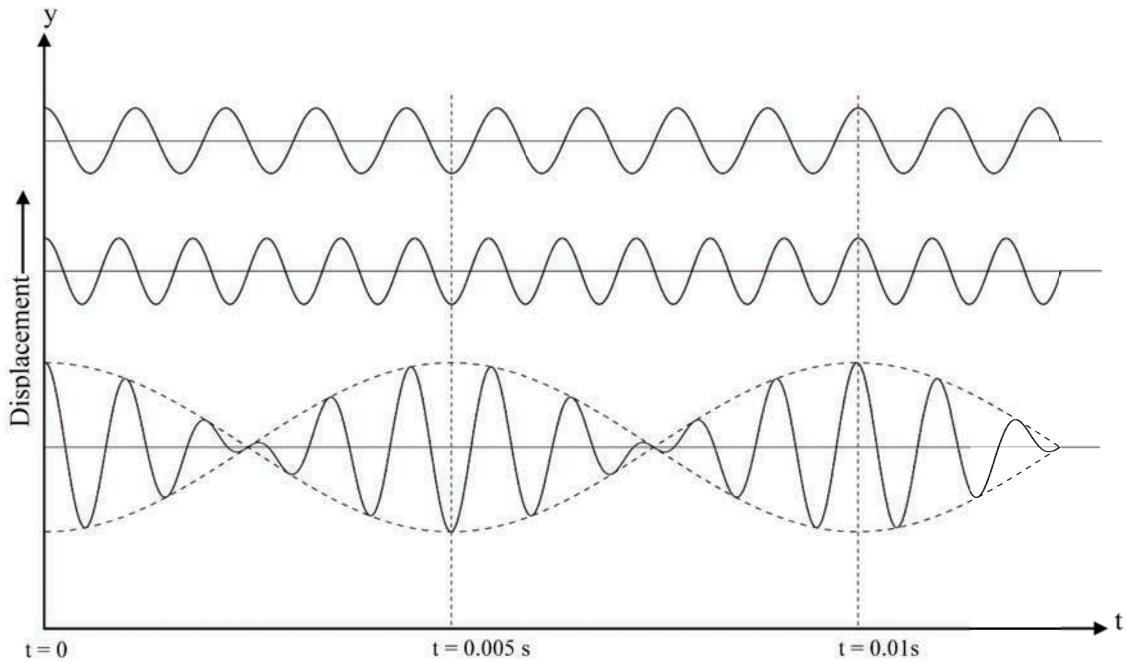


Figure. Beats calculated from the superposition of two sine waves having the same amplitude and frequencies of 900 Hz and 1100 Hz. The dashed line shows the "beating" of the resulting wave. The large frequency difference of 200 Hz was chosen for a better display and would not be realistic for actually hearing beats. (The two frequencies would be heard separately.)

Question
What is the maximum and minimum amplitude of the beating wave in the above figure, given equal amplitudes of the two original waves?
Answer: Maximum = ________ Minimum = _________

The beat frequency is the absolute value of the frequency difference

$$\boldsymbol{\Delta f_{beat} = |f_2 - f_1|}$$

It does not matter which of the two waves has the higher frequency.

Exercise
Calculate the beat frequency of 200 Hz from the above formula and also read it off directly from the graph.

The frequency of the resultant wave is the average of the two frequencies:

$$\mathbf{f_{resultant} = (f_2 + f_1)/2.}$$

Exercise
Calculate the frequency of the resultant wave from the two frequencies in the preceding graph and also read it off directly from the graph.

Condition for Beats
The frequencies of the two sine waves that give rise to beats should be within about 5% of each other. For larger differences you start hearing the two frequencies separately and a low frequency roughness, which is called a “difference tone” instead of beats.

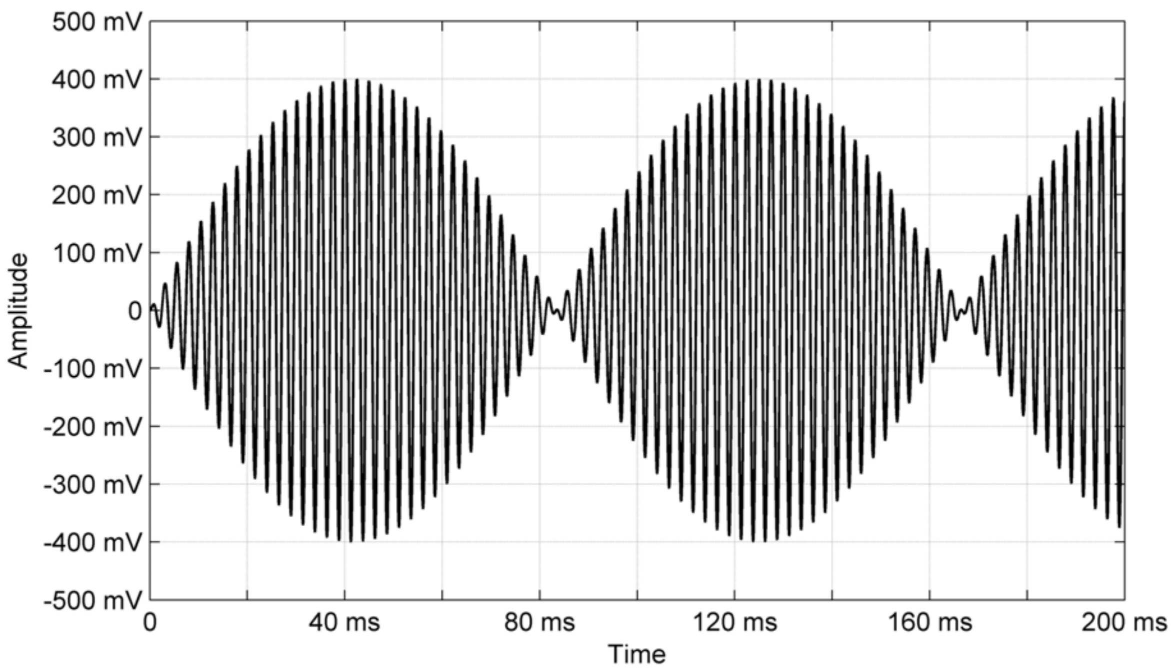


Figure. Beats recorded from the superposition of two sine waves with amplitudes of 200 mV and frequencies 400 Hz and 412 Hz, respectively. The beat frequency is 12 Hz and could be heard clearly. The frequency of the resultant sum wave as shown is 406 Hz.) (The two individual sine waves are not shown.)

Demonstration
Use two frequency generators and set them to f_1 = 400 Hz and f_2 = 412 Hz, respectively. Listen to the beat frequency of 12 Hz.

Demonstrations of Beats

1.Use two speakers, each driven with a sine-wave generator. Start with the same frequency. Then slowly change one frequency and hear the beats. Start for instance with $f_1 = f_2 = 500$ Hz. Then go to $f_2 = 510$ Hz. You should hear a beat frequency of $\Delta f_{beat} = 10$ Hz and a frequency of $f_{tone} = 505$ Hz of the resultant wave.

2. Increase f_2 further. The beats will disappear at a frequency difference of about 5 – 10% and you will start hearing the two individual frequencies together with a rough deep tone.

3. Use a keyboard and play two adjacent semitones together. These are separated by about 6% in frequency. Listen to the fast beats. This interval sounds dissonant to most people. Now play larger musical intervals 2, 3, or 4 semitones apart. The beats disappear at two semitones, but the interval still sounds rough. Increase the interval further and hear the two separate frequencies. Once you reach the intervals of a minor third (3 semitones), major third (4 semitones), fourth (5 semitones), and fifth (7 semitones), a pleasing sound can be heard. This is especially true for musical fifths.

Example

Where do beats play a role?

You can tune string instruments by beats. Any musical fifths between neighboring open strings on a violin, viola, or cello that are out of tune will produce beats, because certain overtones from the two strings will beat with each other. Adjust the string tension to make these beats go away and you have tuned the instrument to perfect fifths as desired.

Demonstration

Tune a violin. The open strings of the violin are G3, D4, A4, and E5. The intervals between them are musical fifths. Tune the A-string to "Concert A4 = 440 Hz" by comparing with the key A4 on the piano. Then tune the D4 and E5 strings to perfect fifths by listening to any beats. Eliminate these beats by adjusting the tension of the strings. Finally tune the lowest string G3 on the violin by eliminating beats. Most people have a natural tendency to discern perfect fifths very acutely.

More Details on the Tuning by Beats

The vibrating strings of string instruments produce overtones. Certain overtones from different strings have the same frequency and they beat with each other when out of tune. By eliminating these beats you are tuning overtones to the same frequencies. The overtones include the especially important musical fifths. Tuning these is accomplished by adjusting the string tension. For more, see the harmonic series later.

Example

Start with A4 = 440 Hz on the A-string. The overtones of A4 are **A5** - E6 - A6 - etc. The overtones on the D4 string are D5 - **A5** - D6 - etc. Hence the A5 overtone from the D4 string beats with the A5 overtone from the A4 string. Eliminate these beats and you have tuned the D4 string! Similarly, the E6 overtone from the D4 string beats with the E6 overtone from the A4 string.

Polarization

A phenomenon called polarization occurs with transverse (but not with longitudinal) waves. In a polarized wave, the oscillating medium such as a steel string vibrates in one direction, e.g. up and down, while the wave propagates in the horizontal direction.

Example
1. Ocean waves are polarized largely in the vertical direction.
2. The blue sky consists of partially polarized light and becomes a deeper blue when you wear polarizing sunglasses.
3. The reflected light from glass and water surfaces is partially polarized, and sunglasses can reduce the glare. This allows you to see better through store windows or into the water when fishing. Light is a transverse electromagnetic wave with electric and magnetic fields oscillating perpendicular to the direction of propagation. The wave is *linearly polarized* if the electric field oscillates only in one direction.

Challenge Question
Why can sound waves not be polarized?
Answer: ____

Demonstration
A slinky is stretched in the horizontal direction. Shake an end in the vertical direction and a wave propagates parallel to the ground. The wave is "vertically polarized."
Similarly, if you shake the wave horizontally (on a table) it is "horizontally polarized".
In both cases the direction of polarization is perpendicular to the direction of propagation.

More Examples
1. The waves on vibrating strings and the solid material of timpani, cymbals etc. may be polarized. But note that the sound waves radiated from them are not polarized. Why?
2. Earthquake waves in Earth's crust consist of longitudinal and transverse waves. The direction of polarization of the transverse waves can give information about the direction of movement of the underlying rock in the fault region.

Demonstrations with Polarized Light
Light is a transverse electromagnetic wave. Polarized light can be demonstrated with polarizers and polarizing sunglasses on an overhead projector.
a) Demonstrate how light can be extinguished with "crossed polarizers."
b) Show how glare can be reduced with polarizing sunglasses.

Transverse and Longitudinal Waves
For transverse waves, the medium vibrates perpendicular to the direction of wave propagation. The medium consists of an elastic, solid material such as steel, aluminum, etc. Transverse waves do not occur in liquids or gases.

Demonstrations – Transverse Waves
1. Send transverse waves down a rope or slinky.
2. Demonstrate waves on the string of a monochord (sonometer) or violin.
3. Show transverse vibrations on a string. One end of the string is fastened to a vibrator on the table. The other end runs over a pulley at the edge of the table with a weight attached. The vibrator excites transverse standing waves on the string. The frequency can be adjusted to see various vibrational modes (harmonics).
4.Show transverse and longitudinal waves with a hand-cranked mechanical wave machine.

Longitudinal Waves
The medium oscillates back and forth along the propagation direction.
Such longitudinal or pressure waves occur in gases, liquids, and solids.

Examples
Sound waves in air are longitudinal waves.
Sound waves emitted by whales or submarines in the ocean are longitudinal waves.
Earthquake waves produce both longitudinal and transverse waves.

Demonstrations - Longitudinal Waves
1. Strike the coils of a slinky in the direction of propagation. This sends compressions and rarefactions down the spring and creates longitudinal waves.
2. Suspend a spring from a stand. Attach a vibrator to the bottom end of the spring. Vary the frequency of the vibrator. At certain discrete frequencies, standing longitudinal waves can be seen from the expansion and compression of the coils of the spring.
3. Rotate an upright vibrating tuning fork about its axis. When the tines vibrate toward and away from a listener, the sound is much louder than when they vibrate transversally with respect to the listener.

What Travels in a Wave?
A wave is a displacement of the vibrating medium from an equilibrium position. This displacement - not the medium - travels with the wave. The medium itself only oscillates back and forth about the equilibrium position.
The displacement of the medium in a wave contains *energy* that travels with the wave. This energy can do useful work.

Examples
1. A sound wave does work by moving your eardrum or the diaphragm of a microphone.
2. Ocean waves can move a buoy up and down and do work. A generator connected to such a “wave motor” can produce electricity.

Standing Waves and Overtone Series

Transverse Standing Waves

Take two sine waves of the same frequency and amplitude traveling in *opposite* directions and interfering with each other. By the principle of superposition you add the two waves to obtain the resultant wave. What is the result? It is a **standing wave** with its crests and troughs not moving in the direction of the original waves. Instead, the medium oscillates up and down about an equilibrium position. The displacement does not keep moving in the direction of the original traveling waves. The two traveling waves have become a standing wave or stationary wave. It has twice the amplitude of the component waves and oscillates with the same frequency. The points of maximum and minimum displacement in the standing wave are called **antinodes** and **nodes**, respectively.

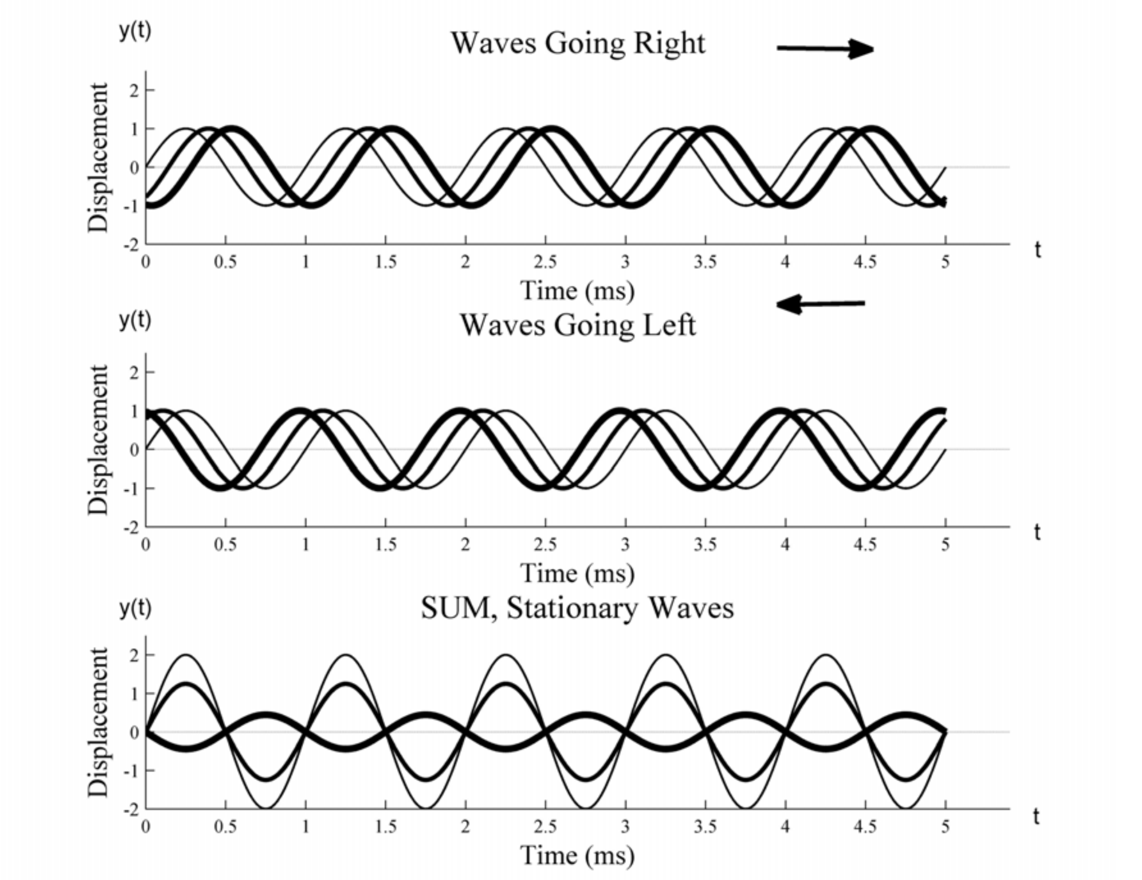


Figure. Two sinusoidal waves on a string have equal amplitude and travel in opposite directions. Dashed, light solid, and dark solid lines are for three progressing times. The standing wave at the bottom is the addition (superposition) of the two traveling waves at the three different times.

Demonstrations of Transverse Traveling and Standing Waves

1. Send a transverse traveling wave down a slinky, which at the other end is held by a person or fastened to the wall. Observe the reflected wave coming back. Keep sending waves down the slinky. Soon you will see a standing wave, provided that you hit the right frequency. (Otherwise you will get a scramble and no discernible standing wave.) By starting at low frequencies, you first will produce the fundamental mode of the wave, designated as N = 1. At integer multiples of this frequency, you will see the higher vibrational modes or harmonics. It should be fairly easy to make the slinky vibrate to about the 5th vibrational mode N = 5.
2. Demonstrate waves on the overhead projector with the little mechanical "wave machine" from the early days of TTU. Turn the shafts and see traveling waves, standing waves, and different amplitudes and phases.

Demonstration of Standing Waves on a Vibrating Rope or String

Use a stretched rope or string and show the standing wave modes, starting with the fundamental mode. Fasten one end of the wire to a vibrator, guide the other end over a pulley and add a weight. Vary the vibrator frequency to show the first few modes.

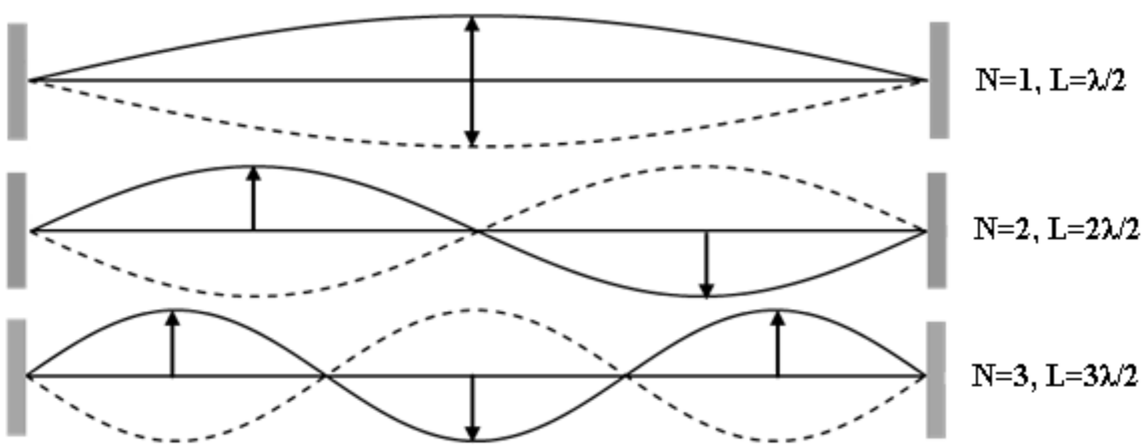


Figure. The first three standing wave modes on a stretched rope or wire.

The fundamental mode has harmonic number N = 1.
The 2nd harmonic or 1st overtone has N = 2. The harmonic overtones are N = 2, 3, 4, 5,…
For the fundamental mode we have $L = \lambda_1/2$, where λ_1 is the wavelength.
For the Nth mode we have $\mathbf{L = N\lambda_N/2}$ or $\mathbf{\lambda_N = 2L/N}$
The frequencies are:
$\mathbf{f_N = v/\lambda_N = (v/2L)N}$ or $\mathbf{f_N = Nf_1}$ or $\mathbf{f_N = f_1, 2f_1, 3f_1, 4f_1, 5f_1, \ldots}$
where $\mathbf{f_1 = v/2L}$ is the fundamental frequency and $\mathbf{v}$ the wave speed on the string.

Exercise

Continue with the above drawing and sketch the next two vibrational modes N = 4 and 5.

Overtone Series, Stretched Strings, Open and Closed Tubes

A sustained tone can be represented by its *amplitude-frequency spectrum* or simply *spectrum*. A spectrum shows the amplitude A on the y-axis versus the frequency f on the x-axis, beginning with the fundamental frequency ("pitch"), followed by the overtones.

Demonstrations

1. Play the overtone series with a PASCO frequency generator for the harmonics N = 1, 2, 3, 4, 5, 6, 7, 8. Acquire a spectrum of the harmonics on the computer.
2. Play a note with a violin or wind instrument. Acquire the *amplitude-frequency spectrum* in real time with a computer. Note the fundamental frequency and overtones.
3. Pluck the string of an Indian string instrument or violin in the middle and see the odd harmonics N =1, 3, 5... Now pluck the string very close to its fastened end and see the odd harmonics as well as even harmonics in the sound spectrum. (To understand this, draw the first 6 or so vibrational modes and imagine plucking the string in the middle or near its end.)

Harmonic Numbers, Wavelengths, and Frequencies of a Vibrating String

From the preceding drawing of the vibrational modes of a string we can see the relationship between the harmonic number and the wavelength. From the wavelength follows the frequency with the formula f = v/λ, where v is the wave speed on the string (not the speed of sound in air!).

Table. Harmonic number, wavelength, and frequency for the first six standing wave modes on a string of length L, with *v* the wave speed on the string.

Harmonic number N	Wavelength λ	Frequency $\left(f = \frac{v}{\lambda}\right)$
1	$\lambda_1 = 2L$	$f_1 = \frac{v}{2L}$
2	$\lambda_2 = L = \frac{1}{2}\lambda_1$	$f_2 = \frac{v}{L} = 2f_1$
3	$\lambda_3 = \frac{2}{3}L = \frac{1}{3}\lambda_1$	$f_3 = \frac{v}{\frac{2}{3}L} = 3f_1$
4	$\lambda_4 = \frac{1}{2}L = \frac{1}{4}\lambda_1$	$f_4 = \frac{v}{\frac{1}{2}L} = 4f_1$
5	$\lambda_5 = \frac{2}{5}L = \frac{1}{5}\lambda_1$	$f_5 = \frac{v}{\frac{2}{5}L} = 5f_1$
6	$\lambda_6 = \frac{1}{3}L = \frac{1}{6}\lambda_1$	$f_6 = \frac{v}{\frac{1}{3}L} = 6f_1$

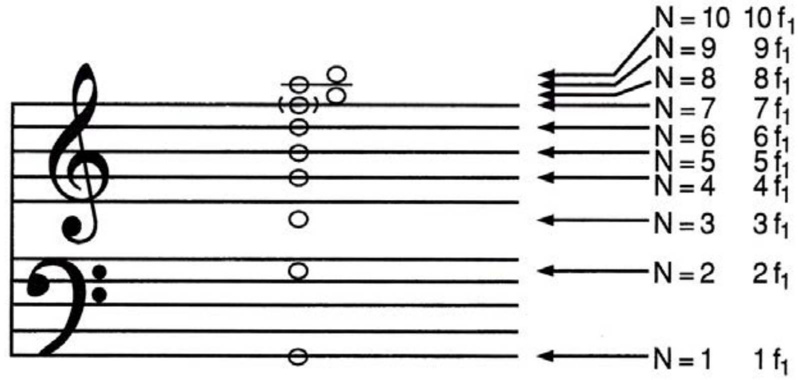


Figure. The first 10 harmonics in the overtone series, starting with the note G2 on the musical stave system. The first 6 notes correspond to the lowest 6 modes of a string vibrating at the fundamental G2 (i.e. N = 1). (From Berg & Stork, 3rd edition, Fig. 3-12, p. 76.) The symbols on the staves are the "treble clef" (top) and "bass clef" (bottom).

Harmonic Series, Frequencies, Musical Intervals and Notes

N	f	Interval	Note	Frequency Hz
1	f_1	Unison	G2	98.0
2	$2f_1$	One octave	G3	196.0
3	$3f_1$	One octave + perfect fifth	D4	294.0
4	$4f_1$	Two octaves	G4	392.0
5	$5f_1$	Two octaves + major third	B4	490.0
6	$6f_1$	Two octaves + perfect fifth	D5	588.0
7	$7f_1$	Two octaves + minor seventh	F5	686.0
8	$8f_1$	Three octaves	G5	784.0

Challenge Question for the Musically Interested
How can you get a musical third (ratio 5:4) from the column labeled "Interval" in the above table without making explicit use of the first two columns?
Answer: ____

Demonstrations
1. Play the overtone series on the keyboard, using the note G2 as the fundamental. Call out the notes (see above Table).
2. Start with C2, play the overtone series, and call out the notes.
3. Use a function generator to play the harmonic series 100, 200, 300, 400, 500, 600, 700, 800 Hz.
4. Use additional function generators to play the musical intervals 2:1, 3:2, 4:3, 5:4, 6:5.

Exercises

1. Construct a table similar to the earlier one for G2, but this time with C2 as the fundamental frequency, for which f_{C2} = 65.406 Hz. Consult the piano keyboard.
2. Draw two musical staffs, treble clef above bass clef. Insert the first 8 notes of the C2 overtone series.
3. Take the frequency ratio for the N = 3 and N = 2 entries in the above table. What is the resulting value and musical interval? Can you also obtain the name of this interval from the column labeled "Interval"?

Construction of Musical Intervals

Example: Start with the tonic C4 ("middle C")

N =	1		2		3		4		5		6
	f_1		$2f_1$		$3f_1$		$4f_1$		$5f_1$		$6f_1$
	C4		C5		G5		C6		E6		G6
		C5/C4		G5/C5		C6/G5		E6/C6		G6/E6	
		2/1		3/2		4/3		5/4		6/5	
		octave		fifth		fourth		major third		minor third	

Demonstration

Visualize the intervals on a keyboard:
Start with C4, count 8 white keys, including C4, and arrive at C5 an octave above C4.
From there, count 5 white keys, including C5, and arrive at G5, a fifth above C5.
From there, count 4 white keys, including G5, and arrive at C6, a fourth above G5.
From there, count 3 white keys, including C6, and arrive at E6, a third above C6.

Question

Name the musical intervals C4/C3, G4/C4, C5/G4, E5/C5, G5/E5.

Another Way to Memorize the Names of Musical Intervals

Example: Start with the note C (e.g. C4). Count the first 5 white keys on the piano keyboard, including C4. You land at G4 and you are a musical fifth up from C4!
Similarly:

Third:	do, re, **mi** — 1, 2, **3**
Fourth:	do, re, mi, **fa** — 1, 2, 3, **4**
Fifth:	do, re, mi, fa, **sol** — 1, 2, 3, 4, **5**
Octave:	do, re, mi, fa, sol, la, ti, **do** — 1, 2, 3, 4, 5, 6, 7, **8**

Vibrating Strings - Mersenne's Law

The frequency of the fundamental mode of a vibrating stretched string of length L is given by Mersenne's law (Marin Mersenne 1588-1648)

$$f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

where F is the tension in Newton (N) and μ the linear mass density of the string. We have $\mu = m/L$ (kg/meter), where m is the mass of the string and L its length.

Demonstrations

1. Fasten a string at one end. Guide the other end over a pulley and hang a weight on the string. Pluck the string and listen to the tone produced.
2. Change the tension on the string with different weights. Note the change in pitch. Doubling the weight raises the pitch by $\sqrt{2}$ in accordance with Mersenne's law.
3. Alternatively, use a sonometer with several strings under tension. Change the tension or the effective length. Listen to the change in pitch.

Sample Calculation of the Fundamental Frequency of a Stretched String

Calculate the fundamental frequency f_1 (N = 1) for the G2 string of a cello.
The effective length of the cello string between the nut and bridge is L = 0.70 m.
The string is made of steel wire with a mass density $\rho = 7900$ kg/m^3.
The wire diameter of the G2 string is D = 0.97 mm = 0.97×10^{-3} m.
A weight of mass of 11.2 kg provides the tension in the string.
First calculate the linear mass density of the string: Use $\mu = m/L = \rho V/L$, where the cylindrical volume V of the string is $V = \pi r^2 L$, or $V = \pi D^2 L/4$.
Answer: $\mu = \rho\pi D^2/4 = 7900\ (\text{kg/m}^3) \times \pi(0.97 \times 10^{-3}\ \text{m})^2/4 =$ **5.84×10^{-3} kg/m**.
Tension $F = 9.8\ \text{m/s}^2 \times 11.2\ \text{kg} =$ **110 N**.
Put the values for L, μ, and F into Mersenne's formula. Verify that **f = 98.0 Hz** (be sure to do that!). This is the pitch or fundamental frequency of the musical note G2.

The tension in the strings of a cello, violin, guitar, and other string instruments can be quite high, i.e. on the order of 100 N. The above is a typical example for a cello.

Exercise

The lowest note on the cello is C2 with f = 65.41 Hz. A C2-string has a linear mass density μ = 18 g/m. Calculate the tension in the string. For the length of the string between nut and bridge use L = 0.70 m. (P.S.: The diameter of the string is D = 1.8 mm, not needed here.)
Answer: Verify that <u>F = 151 N</u>.

Exercise for Guitar Players (optional)

The strings of a guitar are E3, A3, D4, G4, B4, E5. Find their thicknesses, lengths, and material of which they are made. Calculate their linear mass densities. Calculate the string tensions.

Wave Speed on a String

Consider the fundamental frequency of a vibrating string: $f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

From this we can obtain the speed of the wave on a stretched string.
Remember that for the fundamental mode we have $\lambda = 2L$. For the wave speed we have seen earlier that $v = \lambda f$. Therefore the wave speed is

$$v = \sqrt{\frac{F}{\mu}}$$

Example

Consider the G2 string of a cello. Assume a tension of F = 110 N and a linear mass density $\mu = 5.84 \times 10^{-3}$ kg/m. Substituting these values in the above formula yields v = 137 m/s = 307 mph. This is the speed with which waves travel down the wire, are reflected back and forth at the ends, and form standing waves on the string.

How do the String Parameters Affect the Fundamental Frequency or Pitch?
Exercise

Rewrite Mersenne's law and show that $f = \frac{1}{\sqrt{\pi}LD}\sqrt{\frac{F}{\rho}}$,

where L is the length of the string, D the diameter, and ρ the usual mass density in kg/m^3 (*not* linear mass density μ) of the string. For instance ρ = 7900 kg/m^3 for steel wire.

We see that the frequency increases with increasing string tension F and decreases with increasing length L, diameter D, and density ρ. For a given string instrument such as a violin, the length is given and the mass density depends on the chosen string material. The adjustable parameters then are the tension F in the wire and its diameter D. Properly choosing the values for F and D is the task of s*tring scaling* so that one arrives at the needed pitch and quality of sound.

Piano Strings

The piano poses special challenges for "string scaling" because of its wide frequency range of more than 7 octaves. Ideally, the tension in the strings should be rather constant across the entire piano to avoid warping of the frame. For the high notes, short strings with low linear mass density are used. For the low notes, the strings are made of thicker and longer wire. But that is not sufficient. Therefore one wraps these strings with additional wire to make them more massive without greatly increasing their rigidity.

Demonstrations

1. Show strings for string instruments and piano (wrapped and unwrapped strings).
2. Show a violin or guitar.
3. Suspend a lead block from a string. Guide the string horizontally over a pulley and fasten the string at its end. Pluck the string, listen to the sound and demonstrate the dependence of the frequency f_1 on the quantities F, L, and D in Mersenne's. Do so by varying the weight F, the length L, and using another string with a different diameter D.

Overtone Series, Standing Waves in Air, Open and Closed Tubes

Below are shown standing sound waves in the air column of a tube. They are the longitudinal analog to standing transverse waves on a string. The standing waves are a superposition of waves traveling in opposite directions in the tube as a result of reflections at the ends.

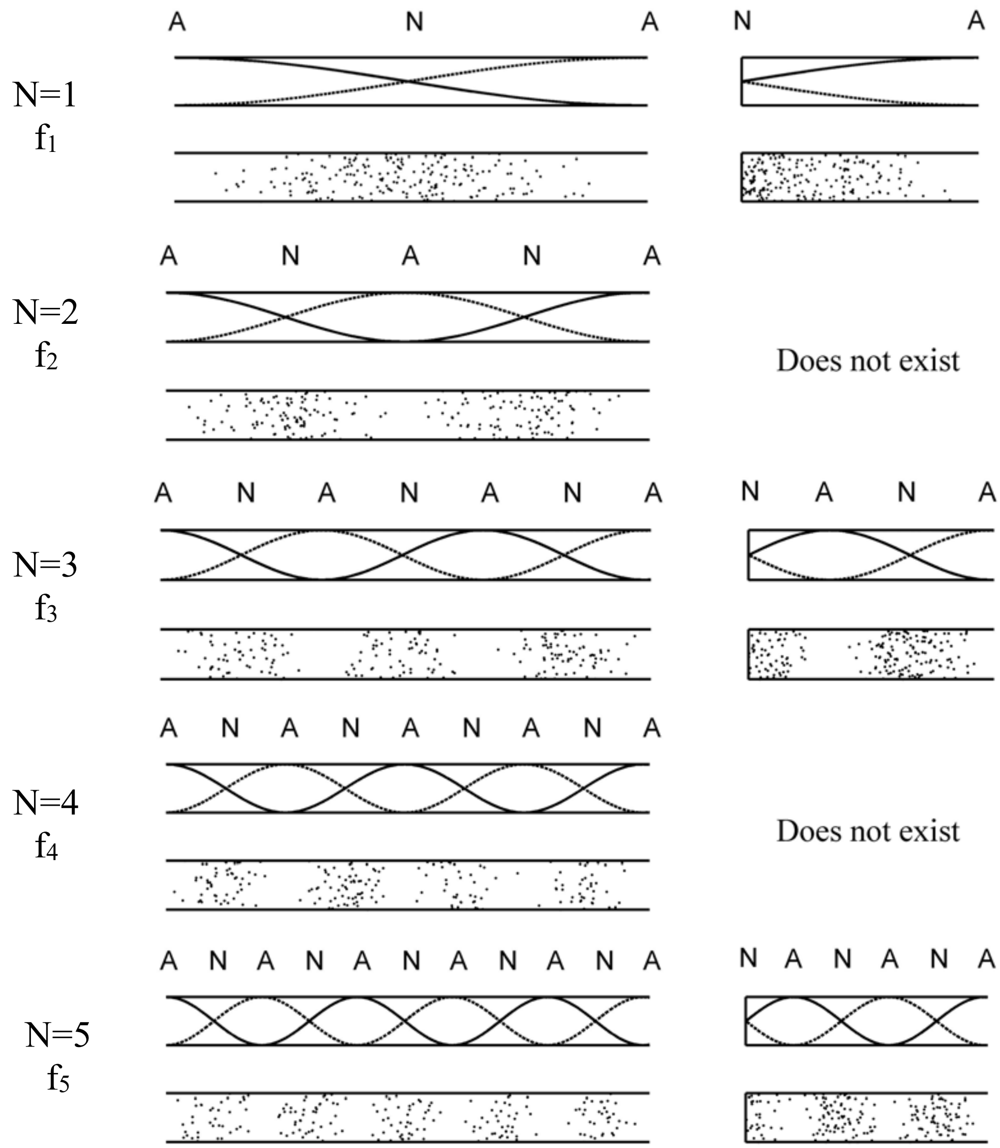


Figure. Standing longitudinal waves inside a cylindrical tube. “N” are the nodes and “A” the antinodes of the motion of air molecules. Left: First 5 resonance modes (harmonics) in a tube open at both ends. Right: The first 3 existing resonance modes in a tube with one end open and the other end closed (stopped). Note that only the odd harmonics N = 1, 3, 5 … occur in this case. The dotted scatter plots show where sand particles or small Styrofoam pellets would accumulate.

For each tube drawn in the figure, the air displacement (or velocity) is drawn on the vertical axis as a function of position along the tube, which is the horizontal axis. Note the close analogy and differences with a vibrating string discussed earlier!
Understand what displacement "nodes" and "antinodes" mean and that the air *pressure fluctuations* in the tube are largest at the displacement nodes N and smallest at the antinodes A.

At the **nodes N**: The displacement and the velocity of the air molecules is a minimum. The pressure fluctuation Δp is a maximum, i.e. there is a pressure antinode. There is no phase change in the pressure Δp upon reflection.
At the **antinodes A**: The displacement or velocity is a maximum.
The pressure fluctuation Δp (not shown above) is a minimum, i.e. there is a pressure node. There also is a phase change of 180^0 in Δp upon reflection.

Open and Closed (Stopped) Tubes
At the open end of a tube there will be a displacement antinode. In contrast, at a closed end, there will be a displacement node for the air molecules (no motion).

Wavelength and Frequencies of the Longitudinal Modes in Tubes
For a cylindrical tube of effective length L and the preceding figure, we have the following for the wavelengths λ_N, frequencies f_N, and harmonic number N of the N-th vibrational mode (using $\lambda f = v$ for the speed of sound again):

Tube open at both ends **("open tube")**. All harmonics are allowed (same as for strings):
$\boldsymbol{\lambda_N = 2L/N}$, N = 1, 2, 3, 4… (all integers) and $\mathbf{f_N = v/\lambda_N = Nv/2L}$
For the first vibrational mode N = 1 in the open tube, we have
$\lambda_1 = 2L$ (from the Figure we can see that $L = \lambda/2$) and $\mathbf{f_1 = v/2L}$ (fundamental frequency).
Examples: Harmonica, flute, Indian flute, recorder, open organ pipes.

Tube closed at one end **("stopped tube")**. Strictly, only the odd harmonics are allowed:
$\boldsymbol{\lambda_N = 4L/N}$, N = 1, 3, 5, 7… (odd integers) and $\mathbf{f_N = v/\lambda_N = Nv/4L}$
For the first vibrational mode N = 1 in the closed tube, we have (different from strings)
$\lambda_1 = 4L$ (from the Figure we can see that $L = \lambda/4$) and $\mathbf{f_1 = v/4L}$ (fundamental frequency).
Examples: Clarinet, didgeridoo, closed organ pipes, other closed pipes.

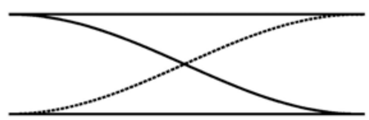
Exercise
1. Make a table of the harmonic frequencies of a pipe, open at both ends, for the first five harmonics N = 1, 2, 3, 4, 5 (see also the table for a vibrating string on p. 3-3). Add an additional column of the allowed harmonic frequencies for N = 1, 3, 5 of a pipe with one end closed and one end open.
2. Show that closed and open cylindrical tubes of the same length L have no common harmonics. (Neglect the end corrections for the tubes.)
Answer: Use the two expressions for f_N above and substitute the values for N. We obtain **1**, 3, 5, 7…•(v/4L) and f_N = **2**, 4, 6, 8,…•(v/4L), respectively.

We see that the two sets have no common frequencies between them.

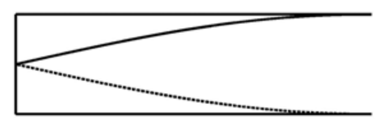
Exercise on Standing Waves in Open and Closed Tubes

Practice by drawing plots of the displacement of air molecules and the pressure variation in the tube. Note that a plot for velocity of the air molecules would look the same as for displacement and no separate drawing is required. Make drawings of the fundamental vibrational mode of air molecules in an open and closed tube**.** Also make corresponding drawings of the pressure variations in the tubes.

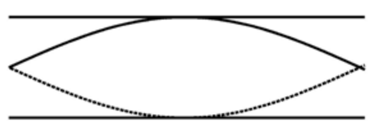
Open Tube:
Air displacement or velocity in the tube



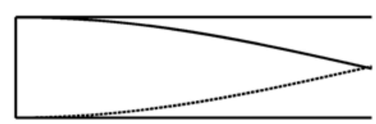
Closed Tube: (Closed at one end)
Air displacement or velocity in the tube



Open Tube:
Pressure variation in the tube



Closed Tube: (Closed at one end)
Pressure variation in the tube



Exercise
Indicate in the top two drawings the displacement nodes and antinodes and in the bottom two drawings the pressure nodes and antinodes. Be sure to understand the differences and relationships between displacement and pressure nodes and antinodes.

Questions
1. Why must the closed end of a tube be a node for the displacement of air molecules?
Answer: ____
2. Consider an "open tube" of length L. For the fundamental mode with the longest wavelength λ and lowest frequency f, what is the relationship between L and λ?
Answer: $L = \lambda/2$
3. Consider a "closed tube" of length L. For the fundamental mode with the longest wavelength λ and lowest frequency f, what is the relationship between L and λ?
Answer: $L = \lambda/4$

Exercise
Show that, for an open tube of length L and a closed tube of length L/2, the fundamental frequencies are the same.

Example of Air Resonances in a 3-Inch Iron Pipe

We investigated the resonances in a vertical pipe made of thick-walled iron. The pipe was set in concrete in the ground. It formerly had been used as the mast of a satellite dish antenna. After decommissioning the antenna the pipe was used for acoustics experiments. It could be closed off with a removable well-fitting plastic cap at the top. When the cap was pulled off rapidly, sound rushed into the pipe through the now open end. The resulting booming sound had a resonance spectrum shown in the figure below.

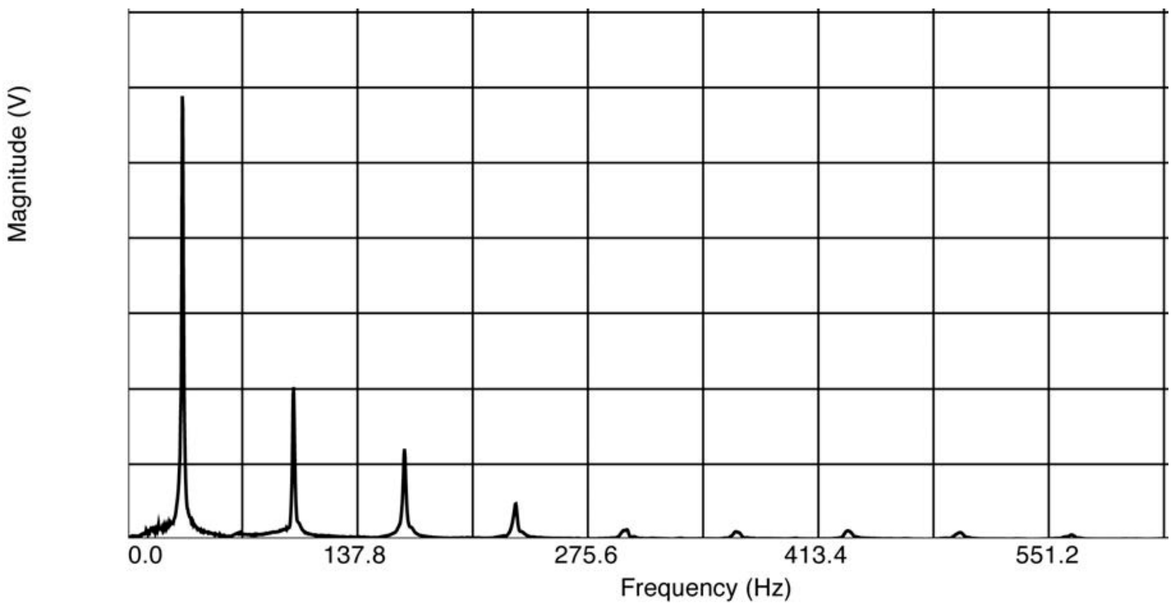


Figure. Example of the resonances in a vertical iron pipe with the top open and the bottom end closed. Only odd harmonics occurred, as expected, whose amplitudes decreased with increasing harmonic number. Nine odd harmonics can be seen. (P.S.: The decrease in the amplitudes of the first 3 odd harmonics resembles that of a square wave.)

Exercise

The inside diameter of the pipe is D = 7.6 cm (Radius R= 3.8 cm).
The measured frequencies of the first 4 odd harmonics are:
$f_1 =$ 33.2 Hz
$f_3 =$ 99.6 Hz
$f_5 =$ 166 Hz
$f_7 =$ 233 Hz
For this pipe with one end closed and the other end open, we have for the effective length $L_{eff} = L + 0.6 \cdot R$, and furthermore $f_1 = v/\lambda = v/(4 \cdot L_{eff})$ or $L_{eff} = v/4f_1$ (use v = 346 m/s).
Calculate the effective length L_{eff} of the pipe from the measured frequencies f_1, f_2, or f_3.
From this calculated effective length L_{eff} calculate the actual length L and compare with the measured length of 2.56 m.
Answer: Calculated: L = 2.58 m, measured: L = 2.56.

Figure. Top: Satellite mast of 3-inch diameter iron pipe. Bottom: Plug and cap in the top of the mast. Pulling the plug out rapidly produced deep-bass air resonances in the pipe. The lower end of the pipe was deep in the ground and acted acoustically as a closed end.

Demonstrations with Longitudinal Waves

1. With a “longitudinal wave spring”, show simulated standing wave modes in a tube. Mount the spring vertically and drive it with a vibrator from the lower end. At certain discrete frequencies, longitudinal/compressional standing waves can be seen.

2. Use a plastic tube (“plosive aerophone”) and slap one end with your palm (“closed tube”). Listen to the pitch. Acquire the sound spectrum and note the odd harmonics. Next, slap the tube and immediately remove your hand. The tube acts as a “closed” tube first and “open tube” next in quick succession. Now see odd and even harmonics in the sound spectrum. Also, the pitch from the open tube is about an octave higher. Explain!

3. Eight students play a set of 8 slap tubes tuned to C3, D3, E3, F3, G3, A3, B3, C4.

4. Hold a corrugated plastic tube (“boom whacker”) at one end and whirl the other end around in a circle. Listen to the harmonics as you change the speed of rotation. Only discrete harmonics are heard. Note that the fundamental N=1 (approximately A3) is missing. The mode N=2 (A4) is weak. The harmonics N = 3, 4, 5 (E5, A5, C6#) are stronger. By changing the speed of rotation (or swirling around two of these tubes) we can hear musical fifths 3:2, fourths 4:3, and major thirds 5:4.
Question: The harmonic frequencies do not change with the speed of rotation? Why?
Answer: ____

Question: Calculate the fundamental frequency of the plastic tubes for a length of 76 cm and radius of the two openings 1.9 cm and 1.2 cm, respectively. (Use v = 346 m/s.)
Answer: The effective length is $L_{eff} = 76 + 0.6(1.9 + 1.2) = 77.9$ cm = 0.779 m.
The fundamental frequency is $f_1 = v/(2L_{eff}) = 346/(2 \times 0.779) = 222$ Hz ≈ A3.

5. Demonstrate the harmonics from a Native Indian flute, train whistle, organ pipes, etc.

6. Play a didgeridoo. Acquire the sound spectrum and note the odd harmonics. Play a different didgeridoo of the same length and compare the spectrum.
Question: Why do the two didgeridoos not sound quite the same?
Answer: ____

7. Show a custom-built “string-tube device” with a string stretched over a wooden bridge that rests on a plug closing one end of a 4-inch plastic pipe. The bridge divides the string into two segments in the ratio 3:1. The pipe length is tuned to F3 (N = 1). The first existing overtone is C5 (N = 3). When the string is tuned to F3, the sound becomes louder. The odd string harmonics are amplified by the tube resonances.

8. Cut a balloon in half and stretch it over a large plastic pipe. Pinch the mouthpiece and blow into it. Does the sound have a pitch? Is it a simple sound?

9. Show longitudinal waves by cranking the shaft of a mechanical wave maker.

Part 4

Waveform Analysis and Synthesis

Pure Tones and Complex Tones

Pure tones or sine waves have a single frequency. The sound from musical instruments, voice, and "noise", on the other hand, consists of complex waves. There may be a discernible *pitch* of frequency f, but the sound no longer is that of a simple sine wave.

A sustained complex tone contains many *partial waves*, also called *overtones or harmonics*. (These three designations are synonymous for us at present). We shall use mostly the term *harmonics*. Harmonics are sine waves with frequencies from the *harmonic series* f, 2f, 3f, 4f, 5f, 6f, etc. Here *f* is the so-called *fundamental frequency* that corresponds to the pitch of the tone. The fundamental frequency also is called the first harmonic with harmonic number N = 1. Successive harmonics have the integer *harmonic numbers* N = 1, 2, 3, 4, 5, etc. Each harmonic N is a sine wave of frequency $f_N = N \cdot f$. Each of these sine waves (partial waves) has its own *amplitude*. The relative heights of the amplitudes determine the *quality of sound* or *timbre.*

A sound or tone in air arises from a *displacement* or oscillation of the air molecules about their equilibrium position. The molecules move back and forth about this position while the sound travels through the air. (The sound travels, not the air as a whole. There is no wind blowing as you hear music!)

For those interested in more details:
For a sustained tone in air you can represent the displacement y(t) of the molecules as a function of time. The sound wave then is composed of a sum of harmonics or sine waves, given at a fixed position by

$$\mathbf{y(t) = \sum_N A_N \sin(2\pi Nft - \varphi_N)}$$

The sum is taken over all harmonics of harmonic number N =1, 2, 3, 4….
The quantity A_N is the amplitude of N-th harmonic, $f_N = Nf_1$ the frequency, and φ_N the phase difference between the harmonics. Note that f in this expression is the fundamental frequency f_1 with N = 1.
It is a very remarkable mathematical fact that a periodic tone can be described completely by a sum of sine waves with uniquely determined amplitudes A_N, discrete frequencies f_N, and phases φ_N.
The phases φ_N are counted relative to the first harmonic or fundamental N = 1. Most of the time we are unable to hear phase differences and therefore will not discuss them much in this course. (See also "Ohm's Law of Hearing" later).

Fourier's Theorem

"Any periodic wave of fundamental frequency (pitch) f can be assembled or synthesized as a sum of sine waves, also called partial waves or harmonics, of frequency $f_N = Nf$, where N is the harmonic number N = 1, 2, 3, 4, 5, 6, ... Conversely, any periodic wave can be analyzed with sine waves of fundamental frequency f and harmonics $f_N = Nf$. The amplitudes and phases of the harmonics are uniquely defined by the shape of the wave."

Waveforms and Fourier Spectra

A *waveform* shows the displacement, for instance of air molecules, as a function of time. Its *Fourier spectrum* shows the harmonic amplitues as a function of frequency.

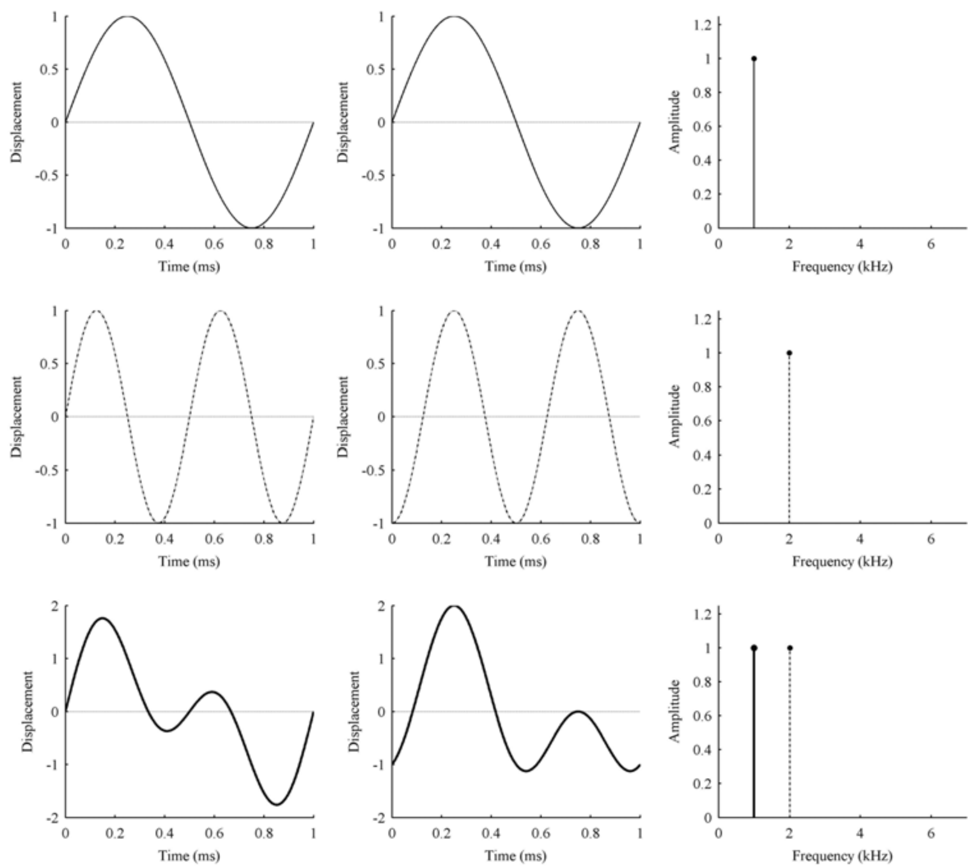


Figure. Waveforms and their Fourier spectra from the combination of two sine waves. Top: A sine wave corresponding to the 1^{st} harmonic or fundamental (N = 1). The Fourier spectrum is a single spike.
Middle: Two sine waves corresponding to the 2^{nd} harmonic or 1^{st} overtone (N = 2), but 90^0 out-of-phase with respect to each other. The Fourier spectrum is a single spike at twice the frequency of the fundamental.
Bottom: Sum of the two sine waves corresponding to the 1^{st} and 2^{nd} harmonics. The resulting complex waveforms look different, but their Fourier spectra are the same and they also "sound" the same (in accordance with Ohm's law of hearing – see later).

A Complex Waveform

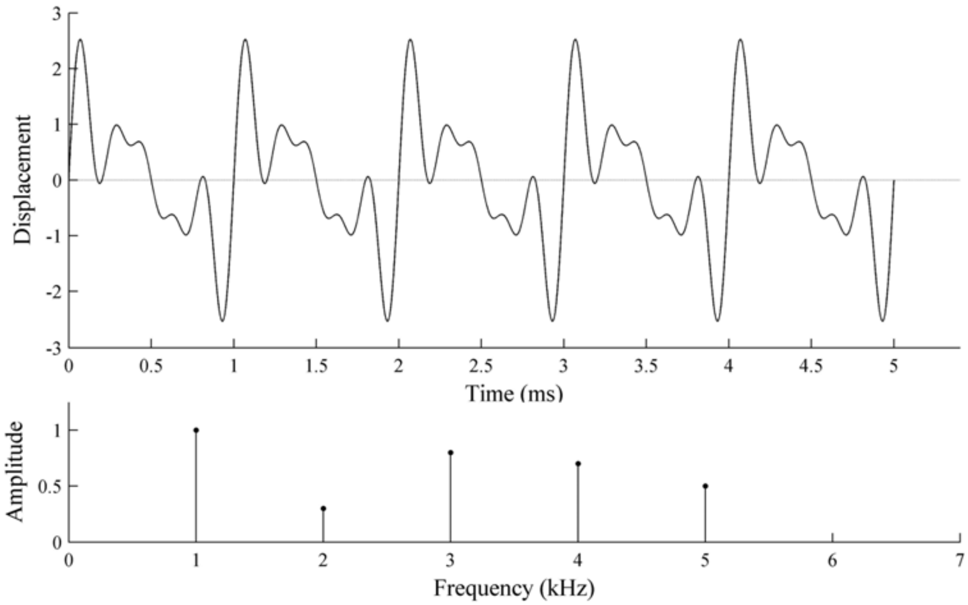


Figure. Top: A complex waveform consisting of the sum of 5 sine waves. Bottom: The Fourier spectrum of the waveform with the relative amplitudes of the 5 harmonics.

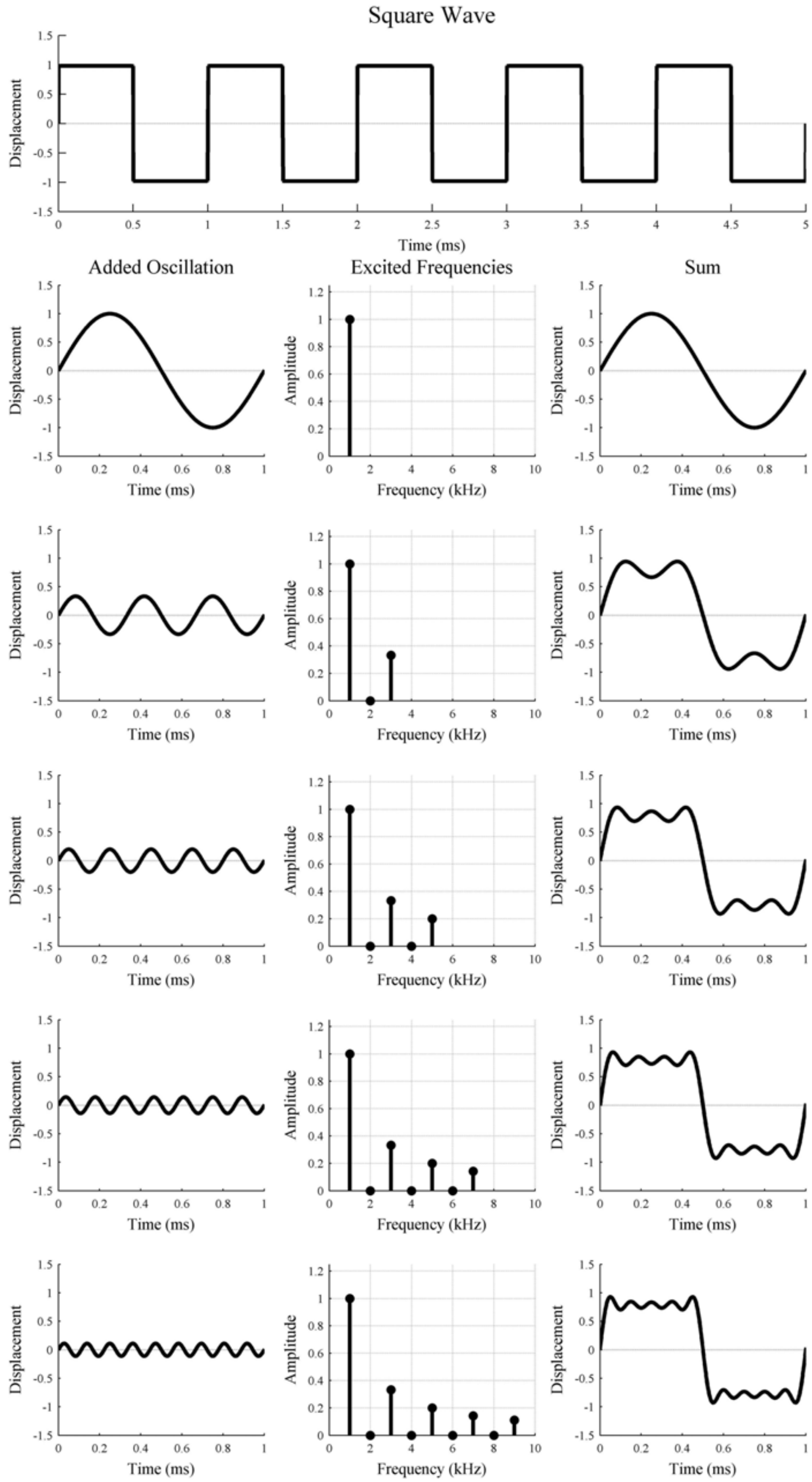
Standard Waveforms

The following pages show a **square wave**, **triangular wave**, and **sawtooth wave**.

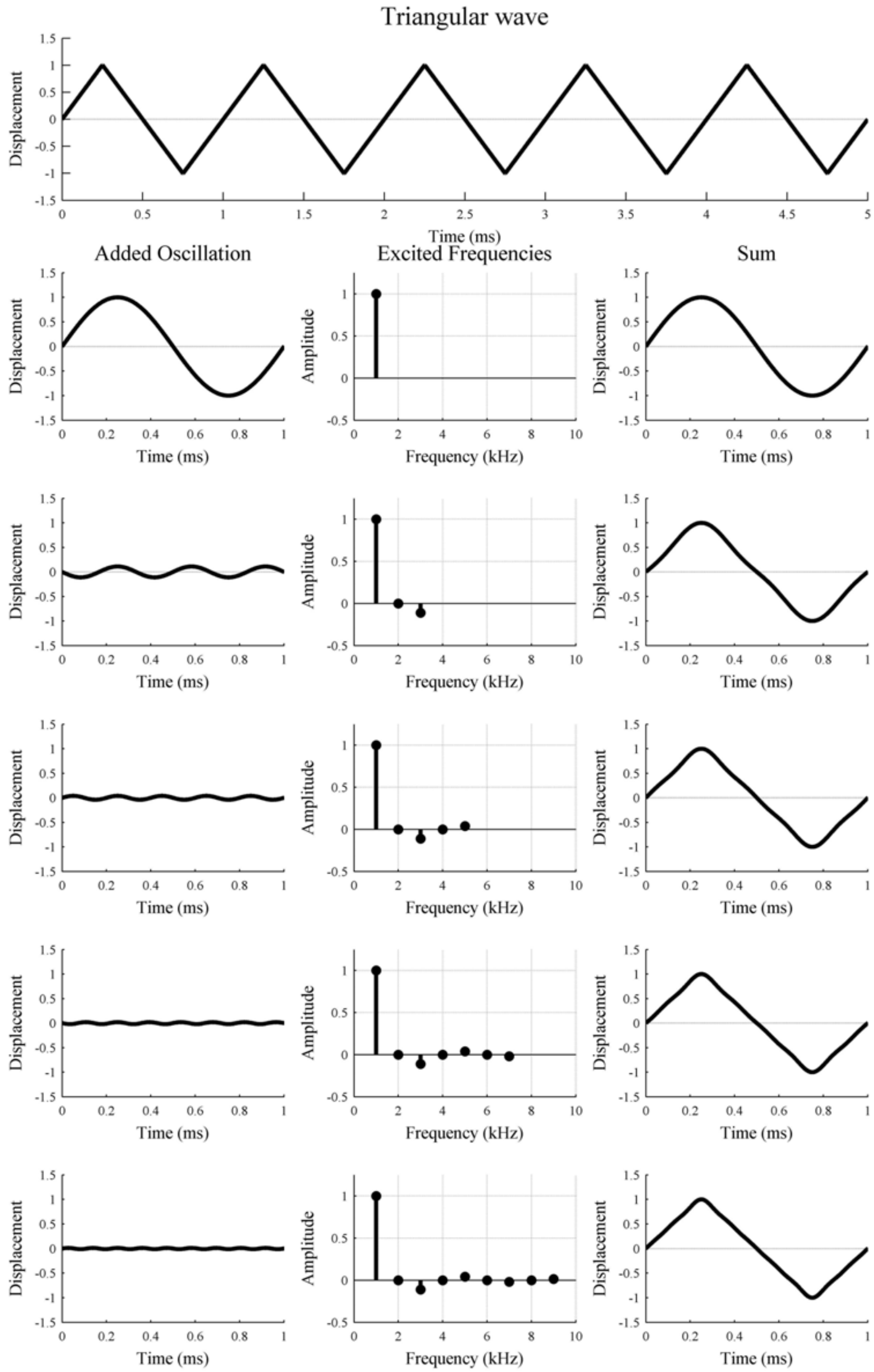
Explanation of the figures:
The Fourier synthesis of the waves is displayed below the original waveforms.
The rows from top to bottom represent the successive inclusion of additional harmonics.
The columns on the left show the waveforms of the individual harmonics.
The columns in the middle show the Fourier spectra up to the N-th harmonic.
The columns on the right show the resulting waveforms up to the N-th harmonic.
It can be seen that, as one progresses down in the right column from top to bottom, the synthesized waveforms resemble more and more the original waveform.
The Fourier synthesis with up to the first 9 harmonics is shown. Of these, only the odd harmonics N = 1, 3, 5, 7, 9 are non-zero.
P. S.: The negative amplitudes A3 and A7 for the triangular wave mean that the harmonics N = 3 and N= 7 start phase-shifted by 180^0 at time t = 0.

For a perfect synthesis of the original waveform, one theoretically would have to include an infinite number of harmonics with decreasing amplitudes.

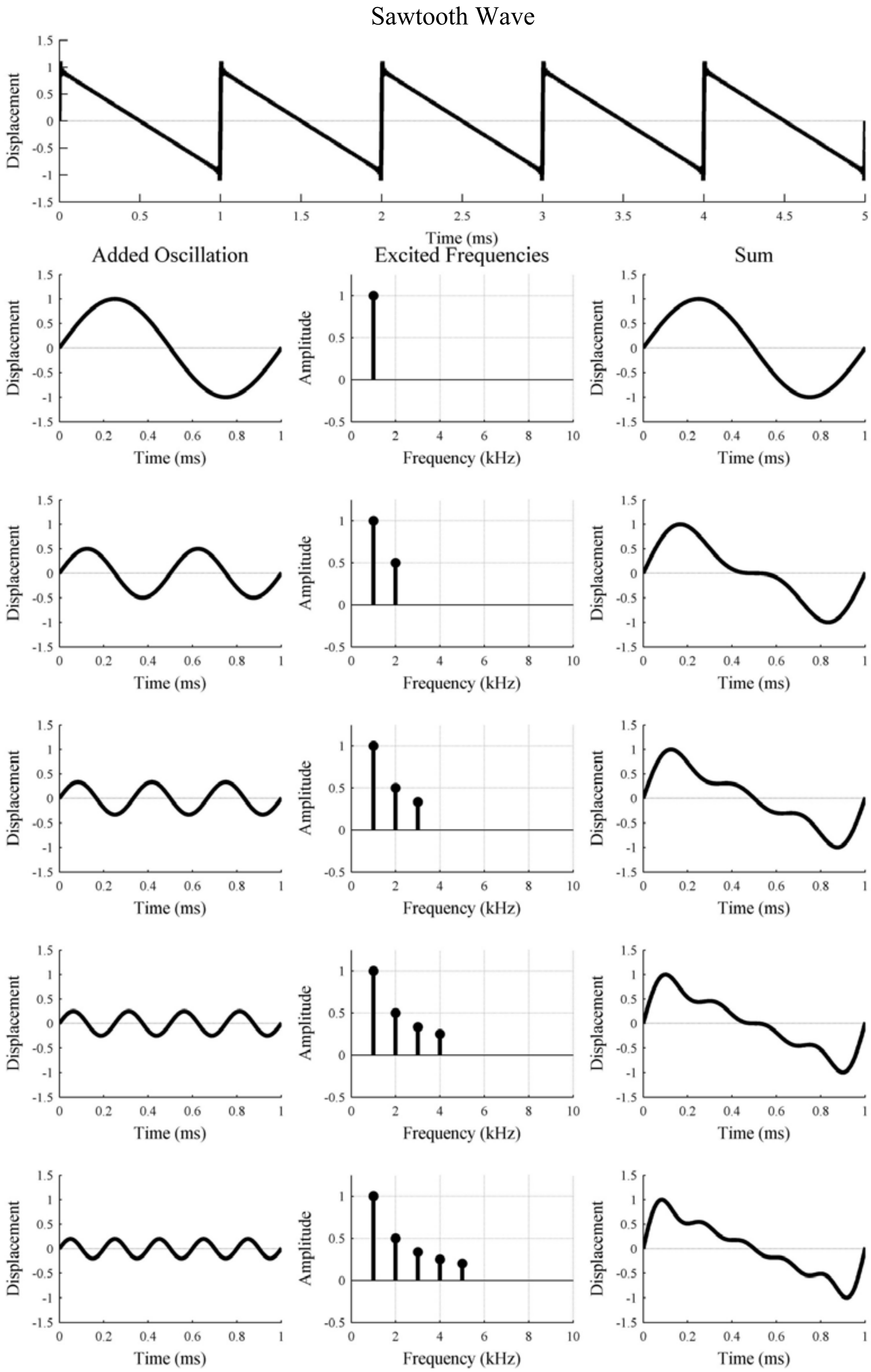
Square Wave and Fourier Synthesis



Triangular Wave and Fourier Synthesis



Sawtooth Wave and Fourier Synthesis



Standard Waveforms continued

Fourier spectrum of a triangular wave — Fourier spectrum of a square wave

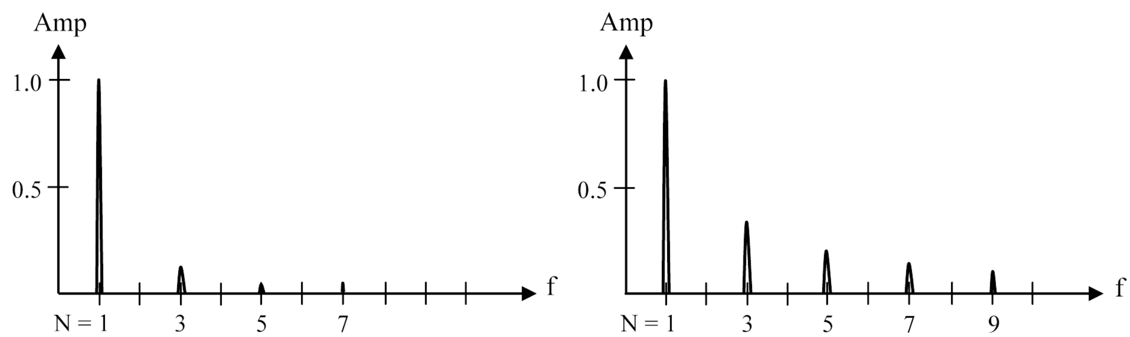


Figure. Relative amplitudes (Fourier spectra) of a triangular wave and a square wave, revisited. Only the odd harmonics contribute in these two cases.

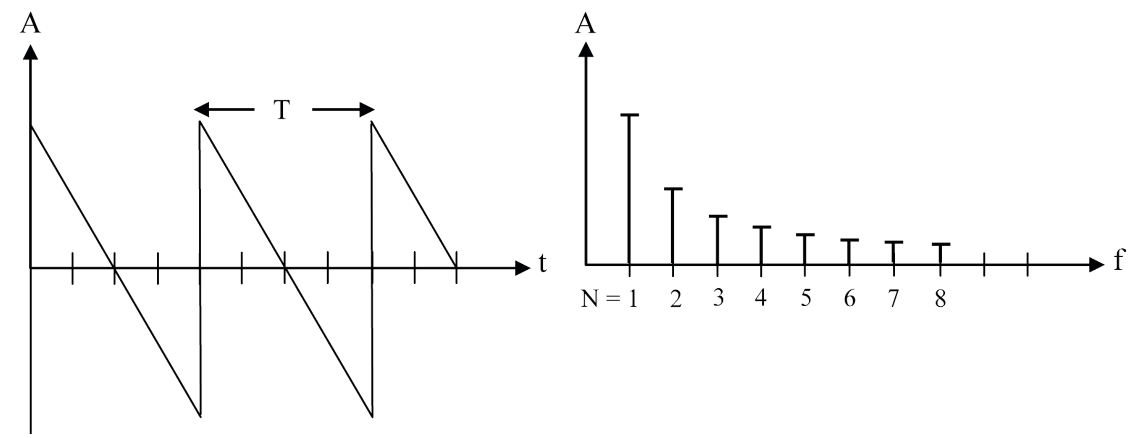


Figure. The sawtooth wave revisited. Left: Waveform. Right: Amplitude spectrum. The amplitudes decrease according to $A_N = A_1/N$, see also table below.

Table. Relative amplitudes of the first 10 harmonics of five standard waveforms. The amplitude of the first harmonic in all cases, i.e. fundamental N = 1, is set to $A_1 = 1$.

Wave Type	Amplitudes											
	N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
Sine		1	0	0	0	0	0	0	0	0	0	Only N=1
Triangle		1	0	$-\frac{1}{9}$	0	$\frac{1}{25}$	0	$-\frac{1}{49}$	0	$\frac{1}{81}$	0	Odd N
Square		1	0	$\frac{1}{3}$	0	$\frac{1}{5}$	0	$\frac{1}{7}$	0	$\frac{1}{9}$	0	Odd N
Sawtooth		1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	All N
Pulse Train		1	1	1	1	1	1	1	1	1	1	All N

Demonstrations of Waveforms and Fourier Spectra

Show some standard waveforms and their Fourier spectra (amplitude spectra).

1. Listen to some standard waveforms from a frequency generator or keyboard.
Sine wave
Triangular wave
Square wave
Saw tooth wave (ramp)
Pulse train

2. Show some of the waveforms.
Remember: A *waveform* represents the displacement **y(t)** of air molecules from their equilibrium position as a function of time t.

3. Observe the spectra of some of the waveforms.
Remember: The *Fourier or amplitude spectrum* of a waveform shows the amplitude **A(f)** as a function of frequency f.

4. Show *waveforms* and *amplitude spectra* of the sound from some musical instruments. Use actual instruments or use the keyboard to simulate their sound.

Question
Which of the two displays, waveform y(t) or amplitude spectrum A(f), do you find more informative?
Answer: ____

5. Play an instrument such as a didgeridoo, Indian flute, violin, trombone etc. Observe the amplitude spectra. Synthesize one of the spectra (e.g. for a didgeridoo) with a software-based sound generator or with sine wave generators. Select the frequencies of the harmonics as multiples of the fundamental frequency. Adjust the amplitudes until they look close to the recorded spectrum. Is the synthesized sound similar to the one from the instrument?

6. Use 5 frequency generators and set them to the first five harmonics 100, 200, 300, 400, 500 Hz. Vary the amplitudes and listen to the change in the quality of the sound (change in timbre). Do the resulting sounds resemble any instrument?

Fourier Spectra of a Clarinet and Viola

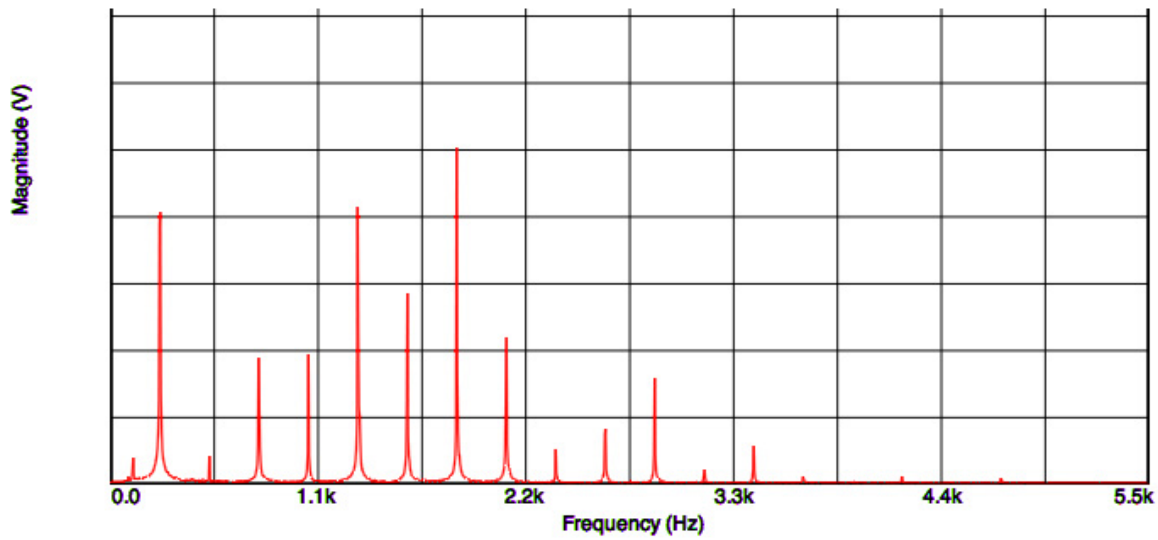


Figure. Harmonic spectrum from a clarinet playing middle C4 = 261.6 Hz.
(Played by Professor David Shea, School of Music, Texas Tech University.)

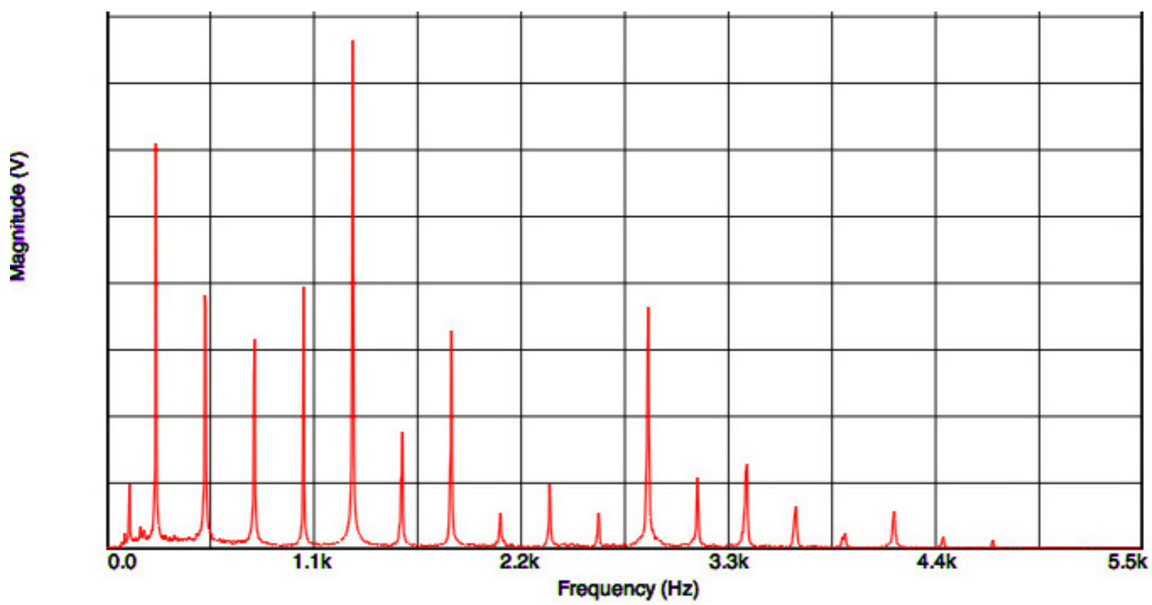


Figure. Harmonic spectrum from a viola playing middle C4 = 261.6 Hz.
(Played by Professor Renee Skerik, School of Music, Texas Tech University.)

Exercise
Highlight the dominant frequency regions (formants) in both figures.
Question
Why do the clarinet and the viola sound different?
Answer: ____

Fourier Analysis of Sound continued, Harmonic and Inharmonic Spectra

Remember that the Fourier spectrum of a complex tone in a Cartesian coordinate system shows the amplitudes of the harmonics on the ordinate (the y-axis) as a function of the frequency of the harmonics on the abscissa (the x-axis).

Demonstrations
1. Show additional Fourier spectra for a variety of sounds.
2. Play "middle C4" on some instruments or simulate them on the keyboard synthesizer.
3. Listen to the *quality of sound* or *timbre* and observe the Fourier spectra.

Questions
1. Can you see a relationship between *timbre* and the Fourier spectrum?
Answer: Pronounced harmonics in a spectrum and their groupings of them called "formants", determine timbre.
2.Which instruments sound "plain"? Which produce a rich sound? How is this related to the number of harmonics present?
Answer: ____

Types of Sound Spectra
a) Harmonic frequency spectra
A sustained tone from a string or wind instrument has a nearly perfect *harmonic* frequency spectrum. The vibrating medium is highly elastic and satisfies Hooke's law. The spectrum is *discrete* and shows the harmonics as sharp spikes. The harmonics are equally spaced at the frequencies f_1, $2f_1$, $3f_1$, $4f_1$, $5f_1$, …, where f_1 is the fundamental frequency that determines the pitch of a tone.

b) Inharmonic frequency spectra
The frequencies in the sound spectra from chimes, rods, marimba, cymbals, etc. are not evenly spaced and are not harmonics of a fundamental. The vibrating medium is stiff and rigid. Inharmonicities also occur with drums, where the medium is fairly elastic but stretches in two dimensions. We still may observe a *discrete* frequency spectrum but with non-equidistant separations between the frequency spikes. The complex sound is still made up of a series of individual waves, which, however, are not harmonics of the fundamental. We call these waves *partial waves* or *overtones* in distinction to *harmonics.* Non-equidistant frequency spacings are a sign of *inharmonicities* in the sound.

For a recording of an inharmonic spectrum see following page. It shows the partials from a brass rod struck at one end with a hammer. As it is clearly seen, the frequencies of the partial waves are not equally spaced and are not harmonics.

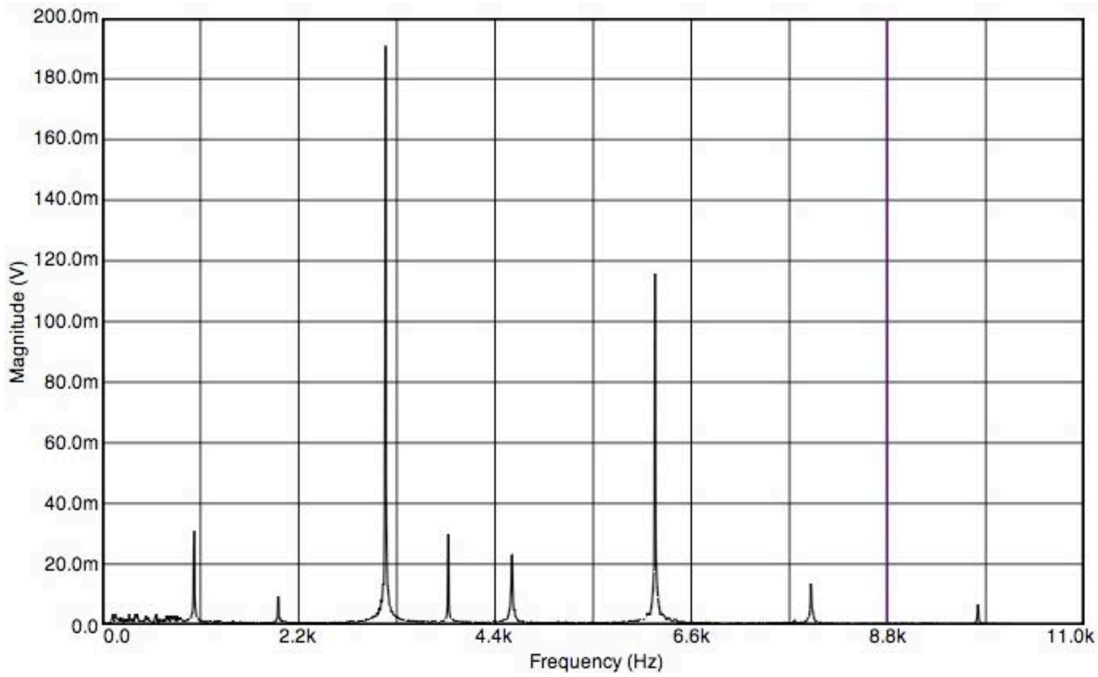


Figure. An inharmonic amplitude spectrum from a brass rod containing 8 non-equidistant partials, which no longer are harmonics. (Length of rod 46.0 cm, diameter 2.54 cm.)

Question:
What causes such inharmonicities in the vibrating medium? (Remember Hooke's law.)
Answer: For vibrations to be harmonics, the restoring force has to be proportional to the displacement of the medium (Hooke's law). This condition is not satisfied for the rigid medium of bars, bells, cymbals, etc.

c) Quasi-continuous frequency spectrum (make some noise)
A spectrum may have so many closely spaced frequencies that it looks almost continuous. The resulting sound generally is "noisy" and not "musical".

Demonstrations
1. Observe the sound spectra when striking an aluminum or brass rod.
2. Marimba bars
3. Chromatic bell set, Glockenspiel
4. Cowbell, other bells
5. Show piano wires and note their stiffness.
6. Djembe (a membrane drum)
7. Strike the brass rod gently with a hammer, place the rod on an open styrofoam box. Notice the low fundamental frequency that was inaudible before. Hear the higher overtones with the rod on and off the box.
8. Triangle
9. Cymbals

Factors Affecting Tone Quality (Timbre)

Consider a *sustained* tone. Its timbre is determined by the harmonic content of the tone, or in other words: the relative amplitudes in the spectrum determine timbre.

(a) Amplitude spectrum
More overtones produce a richer tone. Compare a violin with a flute.
(b) Attack and decay transients
Compare a harpsichord with a church organ. A harpsichord has a short attack and slow decay transient, an organ has a long attack transient and a short decay transient.
(c) Inharmonicities
Deviations from the harmonic series f, 2f, 3f, 4f, 5f, … result in inharmonicities and may allow identification of the instrument, for instance percussion instruments.
(d) Formants
These are frequency regions where harmonics are pronounced. An example is a bassoon as compared with a flute. A flute has only very few harmonics while the bassoon has harmonics grouped in *formant regions.* The human voice has a rich and unique tone quality that originates in the resonant cavities of the vocal tract (vocal folds, larynx, pharynx, mouth, nasal cavity). These account for several vocal formant regions.
(e) Vibrato and tremolo
Vibrato means slight changes in *frequency*, *tremolo* means changes in *amplitude*.
You can produce vibrato with string instruments, but not with the plucked strings of a harp. Singers can produce tremolo and vibrato separately or simultaneously. A trombone exhibits pitch vibrato, a flute tremolo, the latter also called "diaphragm vibrato".
(f) Chorus effect
The chorus effect results from several instruments of the same kind playing together. The phase differences and slight frequency differences result in a characteristic orchestral timbre that differs from a single instrument played at the same loudness level. This allows one to distinguish the solo violin from the violin section in a violin concerto.

Transient Sound
The amplitude spectrum here is of limited use at the first instant when a note is struck, for instance when a piano key is pressed or a violin bow touches the strings.
Then the **attack transients** and **decay transients** in the overall sound play a significant role in defining the timbre.
Demonstrations
Listen to attack and decay transients and observe them on sonograms, which display frequencies as a function of time, with the intensity of the sound in color:
1. Strike a piano (keyboard) key.
2. Strike a percussion instrument.
3. Pluck a string on a violin and listen to the attack and decay transients.

Question
When does a tone sound "musical" in the presence of attack and decay transients?
Answer: When the tone is much longer in duration than the attack and decay transients.

Timbre From Contrabasses With Gut Strings and Steel Strings

The two figures below show the harmonics of the note A1 from two contrabasses, one with gut strings (top figure) and the other with steel strings (bottom figure). The gut strings gave a softer and more subdued timbre, while the steel strings with their more pronounced higher harmonics sounded more brilliant.
Both spectra show missing fundamentals and strong formants below 270 Hz and between 270 and 550 Hz.

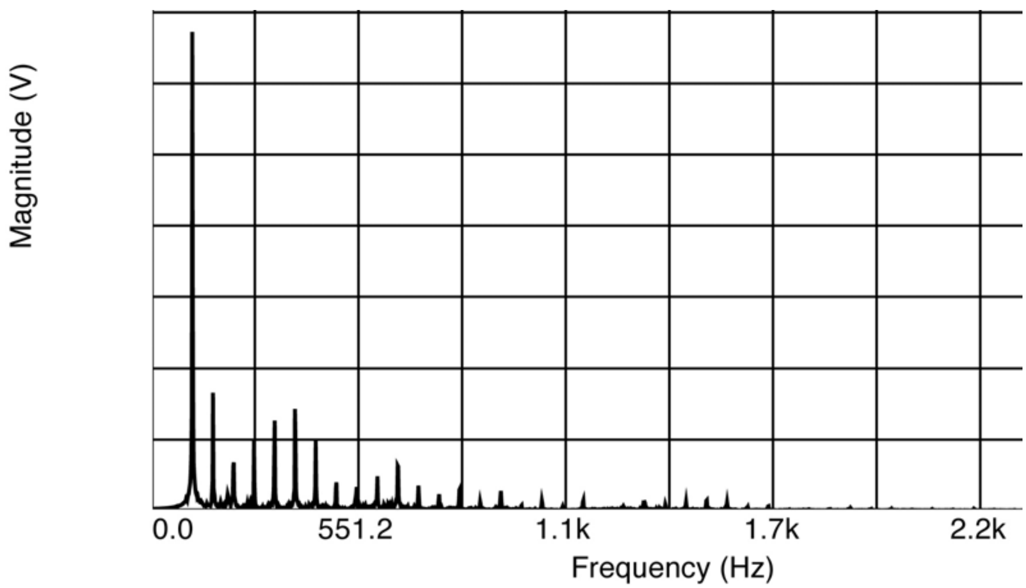


Figure. 18th century contrabass strung with gut strings, played by Mark Morton.

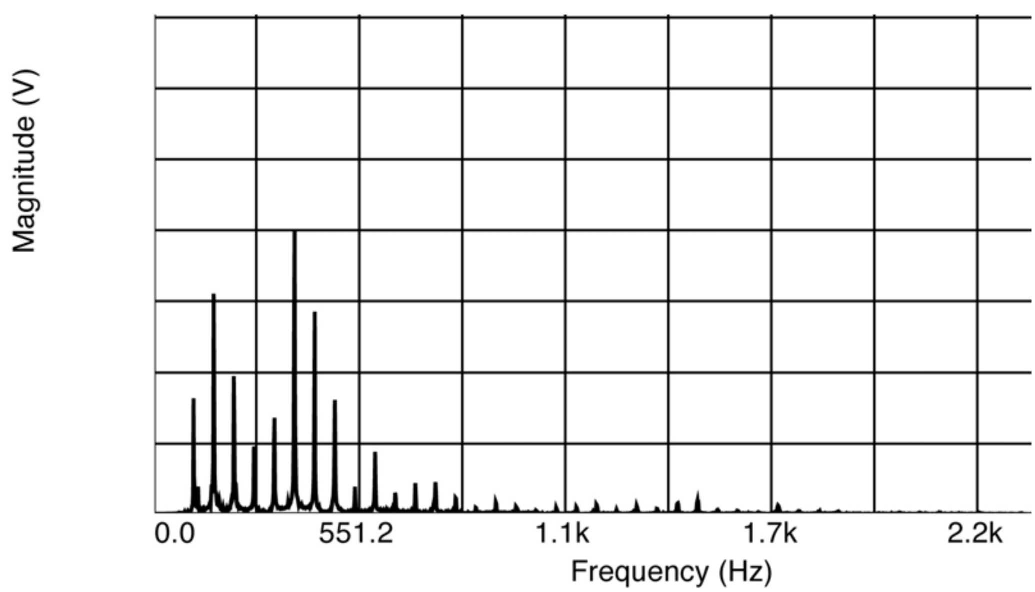


Figure. Sound spectrum from steel strings on a reproduction of the contrabass above.

The Piano and Inharmonicities

The inherent stiffness of the piano wires gives rise to inharmonicities in the sound. The stiffness produces a region with no motion on the string near the nodes of the harmonics, while for an elastic string the nodes are point-like. This effect shortens the effective string length for the higher harmonics and increases their frequency. Thus, the higher the vibrational mode, the higher the frequency deviation from true harmonics. The overtones become progressively sharper resulting in the "spicy" sound of the piano.

Exercise

Draw a piano string vibrating in its first two near-harmonic modes. Show the shortening of the effective string length near the node of the 2nd harmonic due to stiffness of the string.

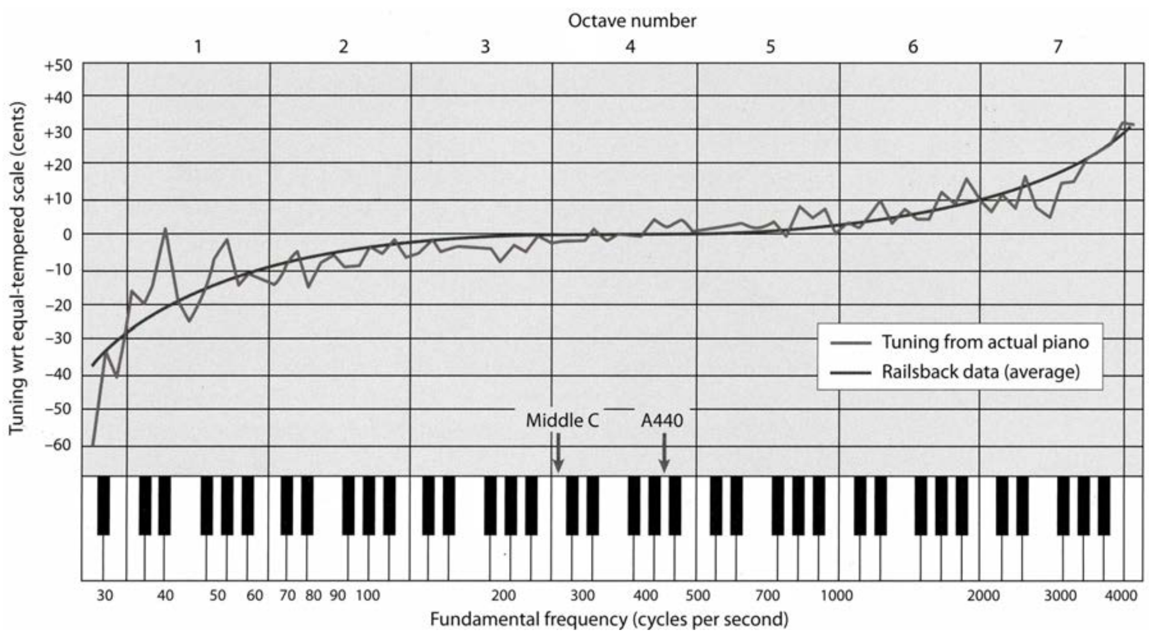


Figure. The Railsback curve showing the difference in cents between the usual piano tuning and equal temperament (100 cents = 1 semitone or half-step). The smooth curve is an average from many pianos, the jagged curve is for an individual piano. Without inharmonicities, the curve would be a horizontal line at 0 cents. (From: Eric J.Heller, Why You Hear What You Hear, Fig. 19.3, 395, Princeton University Press, 2013.)

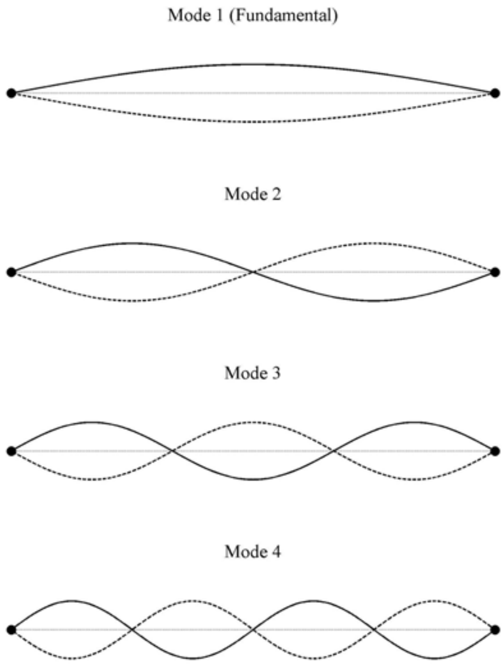
The piano sound is a complex combination of:
1. Inharmonicities
2. Attack transients
3. Decay transients
4. Tuning, where the high notes are tuned slightly high and the low notes slightly low.

Challenge Question

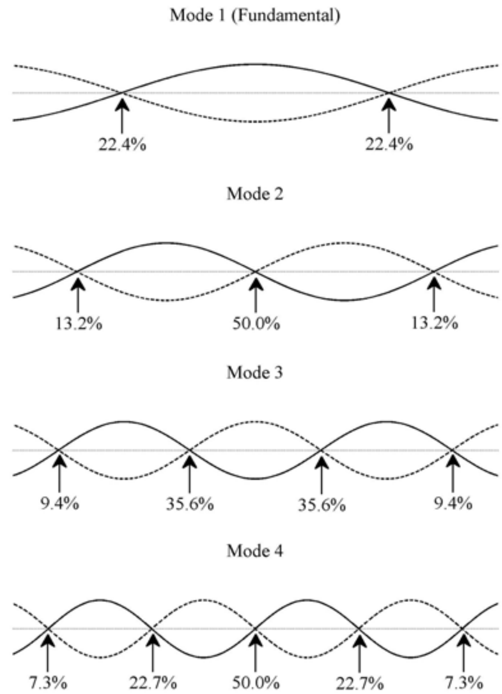
Why do the usual string instruments such as a violin produce harmonic spectra and not inharmonic spectra?

Vibrating Strings and Bars

Harmonic Vibrational Modes



Inharmonic Vibrational Modes



Left figure. The first 4 vibrational modes of a stretched string fastened at its ends.
Right figure. The first 4 transverse vibrational modes of a bar, rod, or rigid tube, free to move at its un-clamped ends.

Mode number	Frequency	Interval
1	f_1	unison
2	$2f_1$	octave
3	$3f_1$	twelfth
4	$4f_1$	2 octaves

Mode number	Frequency	Interval
1	f_1	unison
2	$2.756\ f_1$	1st partial
3	$5.404\ f_1$	2nd partial
4	$8.933\ f_1$	3rd partial

Demonstrations

1. Show an aluminum tubing and demonstrate its first three vibrational modes this way: Strike the tubing and let it glide down between your fingers to the nodal points. Listen to the individual modes as they become alive. You will still hear a mix of the modes, but you should hear a dominant one.
2. Play chimes from the "TTU Physics Chime Set" and observe the amplitude spectrum. Note the non-equidistant spacing of the partials, representative of inharmonicities.
3. Excite longitudinal (and transverse) modes in a long aluminum rod or a brass rod with a mallet, slide the rod vertically between your fingers. Note the vibrational modes.

Ohm's Law of Hearing

The quality or timbre of a tone is not affected much even by large changes in the relative *phases* of the harmonics. What matters primarily are the *amplitudes* of the harmonics.

We have seen in several demonstrations that the timbre of the sound from an instrument is given by the relative amplitudes of the harmonics. We have not spoken yet about the phases of the harmonics. We have done this for good reason, which can be summarized in the empirical *"Ohm's law of hearing"*, said here in colloquial form:

What you hear is the *amplitude spectrum* of a tone, *not the phases* between the harmonics.

For the mathematically interested, remember that you can write a complex wave as the function

$$\mathbf{y(t) = \sum_N A_N \sin(2\pi N f t - \varphi_N)}$$

where y(t) is the displacement from equilibrium of the air molecules (or parts of a string) as a function of time t at a given position. The sum extends over all harmonic numbers N. The angle φ_N is the relative phase angle of the Nth harmonic with respect top the phase of the fundamental.

Ohm's law of hearing then says that you hear the same no matter what the relative phases are between the harmonics. You can even leave all phases off ($\varphi_N = 0$) and you hear the same, with the sound wave now represented by

$$\mathbf{y(t) = \sum_N A_N \sin(2\pi N f t)}$$

However, you do need the phases φ_N for a mathematically correct representation of the waveform, but you do not need them for correctly hearing the sound. We more directly sense or "hear" the amplitude spectrum of a sound rather than its waveform. Our auditory system thus is a spectrum analyzer (with a few exotic exceptions).

Demonstrations

1. Start two sine waves of different frequencies, one after the other at arbitrary times, i.e. arbitrary phases between them. Note that it does not matter at what time the second sine wave is turned on. It always sounds the same, and the timbre is the same.
2. Add two sine waves with sound processing software. Shift the phase of one of them with respect to the other. Can you hear any changes in timbre? If you can't, your auditory system is obeying Ohm's law.

Resonance and Noise

When you play an instrument, resonances are set up in the air, strings, solid material, or membranes of the instrument. Resonators, such as the body of string instruments, drums, or loudspeakers, enhance certain frequency regions of the emitted sound. In order to excite these resonances, some sort of broadband noise with many frequencies is required. The resonator then responds favorably with its own resonance frequencies to that noise. It selects the frequencies which it "likes best" and "resonates" preferably at these frequencies.

Examples
1. The bow of string instruments produces a broadband spectrum through its "slip and stop" motion on the string. The string then responds and vibrates with its characteristic harmonic frequencies.
2. The reed of a bassoon produces a broadband frequency spectrum or "white noise". The air column in the instrument responds selectively by sounding its resonance frequencies. As a result, the sound from the instrument is "musical". An interplay exists between the instrument and its reed. The player notices how the instrument reacts with its resonance frequencies to the buzz from the reed.

Resonance Curves
A resonance curve of a musical instrument is a summary of its acoustical properties. It tells us about the possible frequencies and overtones that could be present and in what strength and therefore is a physical description of the resonating system.

Fourier Spectrum
In distinction to the resonance curve of an acoustical system, the Fourier spectrum of an emitted tone gives us a description of the amplitude/frequency structure of that particular tone.

Types of Noise

When the spectrum consists of many densely spaced spikes, we have a quasi-continuous spectrum. An important feature of such spectra is that they have no discernible periodicity. The sound is aperiodic and not musical.

White Noise

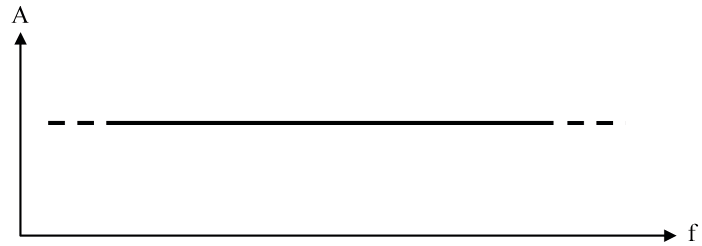


Figure. White noise. The spectrum has equal intensity at all frequencies and is nearly continuous.

Colored or Filtered Noise

An example is the howling wind with a low frequency maximum. The maximum may be perceived as a low rumble. A microphone is sensitive to this wind noise. To protect against it, a bulky windscreen of fuzzy material can be placed over the microphone.

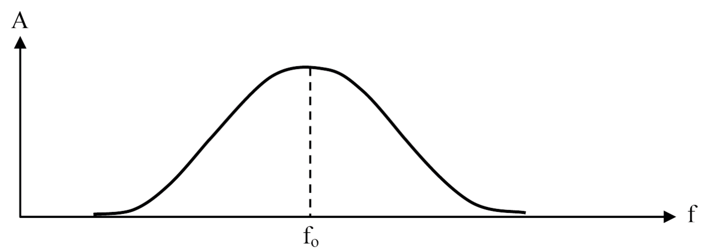


Figure. The frequency spectrum is broadband with a maximum at some frequency f_0.

Pink Noise

This is a special case where the sound intensity decreases uniformly with increasing frequency over the audible range. Specifically, the intensity decreases by a factor of 2, i.e. 3 dB per octave. Pink noise is called "1/f-noise" and is used for testing audio equipment. A 1/f-decrease in sound intensity closely approximates the time-averaged energy distribution of the sound in orchestral music. The time-averaged spectrum is a noise spectrum. (However, the instantaneous sound may contain discrete frequencies.)

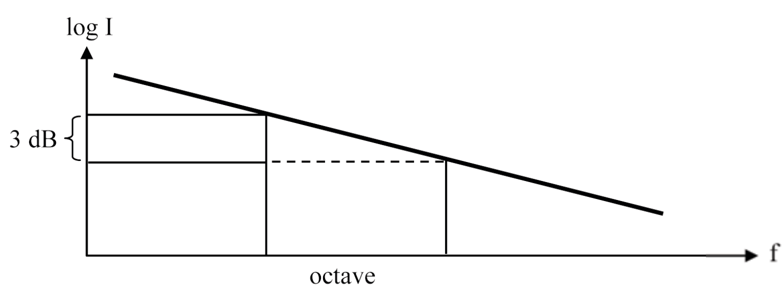


Figure. Pink noise with a 3 dB decrease per octave.

Helmholtz Resonators

Hermann Ludwig Ferdinand von Helmholtz (1821–1894) used a series of spherical resonators of different sizes to perform frequency analyses of musical instruments.

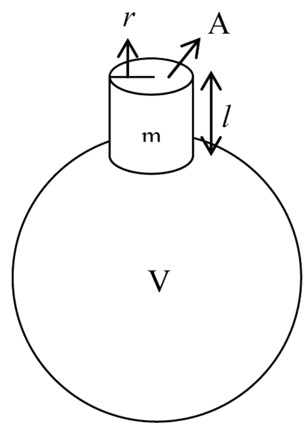


Figure. Prototype Helmholtz resonator. The neck has area $A = \pi r^2$, where r is the neck radius, m is the mass of the vibrating air column, l the length of the bottleneck, and V the volume of air in the resonator. (The volume does not have to be of spherical shape.)

The resonator acts as a harmonic oscillator, where the air mass m in the neck is the "weight" that moves harmonically a small distance in and out. The air in the volume V acts as the "spring". One can derive a spring constant for this system, and from that the resonance frequency f. The result is given by

$$f = \frac{v}{2\pi}\sqrt{\frac{A}{\ell_{\text{eff}} V}}$$

In this formula, v is the speed of sound, A area of the neck opening, l_{eff} the length of the neck after adding the corrections at the open ends, and V is the volume of the resonator**.**

Numerical Example – Glass Flask
Calculate the resonance frequency for a spherical glass flask having the following dimensions: volume $V = 500\ \text{cm}^3$, average neck radius = 0.65 cm, neck length $l = 3.0$ cm.
Answer: $l_{\text{eff}} = l + 2\cdot0.6\cdot r = 3.0 + 1.2\cdot r = 3.0 + 1.2\cdot0.65 = 3.78$ cm = 0.0378 m.
Frequency $f = (v/2\pi)(A/l_{\text{eff}}V)^{1/2} = (346/2\pi)[\pi\cdot0.0065^2/(0.0378\cdot0.0005)]^{1/2} = 145.9$ Hz.

The following figure shows the Helmholtz resonance of this glass flask excited by blowing air across the neck opening. Note that the measured resonance frequency is f = 146.7 Hz. This is in very good agreement with the calculated frequency above.

P.S.: More strictly, the neck of a flask or bottle is an unbaffled opening on the outside, with a length correction of $0.61\cdot r$, and a baffled opening to the inside, with a correction of $0.85\cdot r$. This results in an effective length $l_{\text{eff}} = l + 0.61\cdot r + 0.85\cdot r = l + 1.46\cdot r = 0.0395$ m
However, this gives no improvement in the calculated resonance frequency.

The resonance curve of a Helmholtz resonator has a prominent peak.

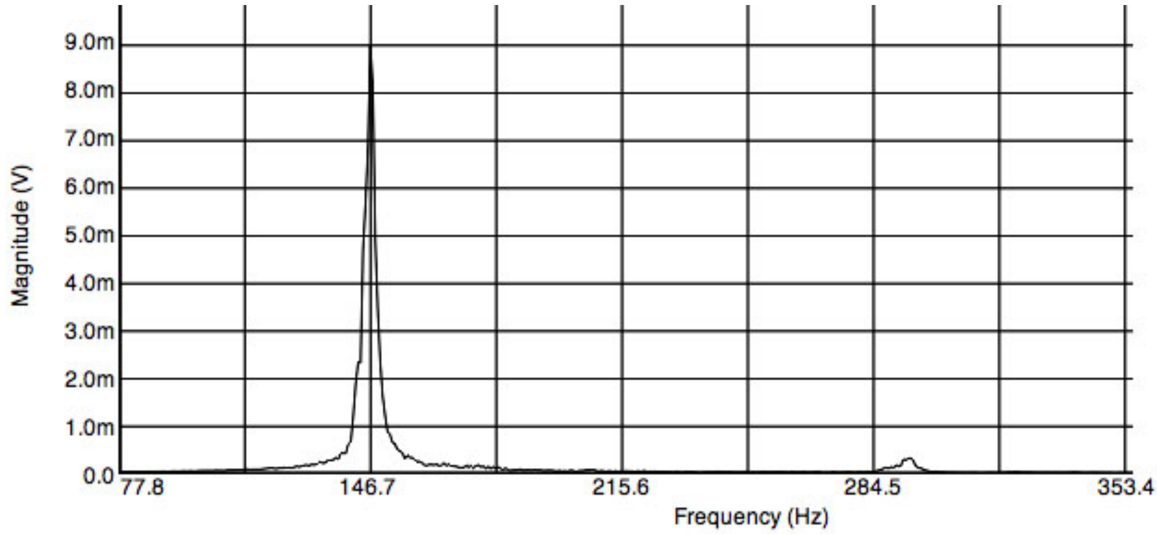


Figure. Resonance curve of a spherical Helmholtz resonator consisting of 500 cm^3 spherical glass flask. The measured peak is at f = 146.7 Hz.

The assumption in the formula for the resonance frequency is that f is low (or wavelength λ large) so that there are no significant nodes or antinodes in the neck and resonator volume. This means that $l_{eff} << \lambda$, with $\lambda = v/f$. This condition generally is satisfied. For our example of the spherical flask we had f = 146 Hz $\rightarrow \lambda = 346/146 = 2.37$ m, while l_{eff} = 0.0378 m is much smaller than this, and so is the diameter of the flask (D = 0.0985 m).

Further Numerical Examples and Demonstrations

1. Wine bottle. Volume V = 1.5 liter, diameter of opening d = 1.87 cm, length of bottle neck l = 7.7 cm, i.e. effective length $l_{eff} = l + 2 \cdot 0.6 \cdot R$ = 8.822 cm = 0.08822 m.
Calculate resonance frequency from formula and verify: f = 79.3 Hz ± 5.0%
Resonance frequency measured with spectrum analyzer: f = 81.6 Hz ± 0.5%
This is very good agreement between theory and experiment (better than expected).

2. Metal bar mounted on a rectangular resonator box of volume V = 677.4 cm^3.
We treat this as a Helmholtz resonator with a hole but without an explicit neck.
The opening has a thickness l = 0.60 cm of the surrounding wood.
The opening here is an oval of area of area A = 25.9 cm^2.
We approximate this oval with a circular opening of the same area A having an effective diameter $d_{eff} = 2(A/\pi)^{1/2}$ = 5.80 cm.
For the length l_{eff} in the formula for the frequency we take $l_{eff} = l + 0.85 \cdot d_{eff}$ = 5.53 cm.
The correction factor here of 0.85 is for an opening baffled on both sides. This replaces the factor of 0.61 for a non-baffled opening such as an open cylindrical pipe.
Then the calculated resonance frequency is f = 456 Hz.
Compare this with the frequency of f = 440 Hz of the bar tuned to “concert A”.
The agreement is quite good considering the assumption of a circular opening.

Exercise
Verify the resonance frequencies calculated in the two preceding examples. Fill in the blanks with your answers:
Answer: Wine bottle f = __________Hz   Marimba bar f = ___________Hz

More Examples for Helmholtz Resonators
1. Loudspeakers with ducted ports.
2. String instruments. The air volume in the body is a complex Helmholtz resonator. The tone quality and loudness for low notes are improved. (Note that the so-called “wood” or body resonances are not Helmholtz resonances but come from the vibrating solid material)

Demonstrations With Helmholtz Resonators
1. White Zinfandel wine bottle, V = 1.5 liter, and other bottles. Blow over the top and listen to the Helmholtz resonance.
2. Hold a metal bar over a resonator box, with and without a cover over its opening.
3. Hold a seashell, empty can, or drinking glass close to your ear and listen to the sound. Where does the perceived “sound of the ocean” come from?
4. Move a can towards and away from the ear. Does the “pitch” change?
5. Make a cup with one hand and place it over your ear. Listen to the sound.
6. Make a larger cup with both hands close together and hold them over an ear. How does the frequency change compared to the smaller “cup”?
7. Take two cans and hold one over each ear for a “stereo effect”.
8. Play a “cajon”. This is a rhythm box acting as a Helmholtz resonator.
9. Play a “djembe”, an African drum. Point the opening towards the audience.
10. Have students applaud in the classroom. Place a coffee can over a microphone. Observe the filtered noise due to the Helmholtz resonance in the can.

Questions
1. Describe how the bottle sound is excited. Compare the sound with a simple sine wave.
2. Does the frequency or pitch increase or decrease as you move a can farther away from your ear?
3. Qualitatively explain this frequency change with Helmholtz’s formula. Hint: Discuss how the resonance frequency depends on the area A, length l_{eff}, and volume V of the resonator.

Remark
Hermann von Helmholtz used a series of spherical resonators of different sizes to determine the Fourier spectra of musical instruments, all done in the second half of the 19th century. This was an ingenious early way to analyze the timbre of instruments before the advent of electronic spectrum analyzers.

Indian Rudra Vina. Other Helmholtz Resonators

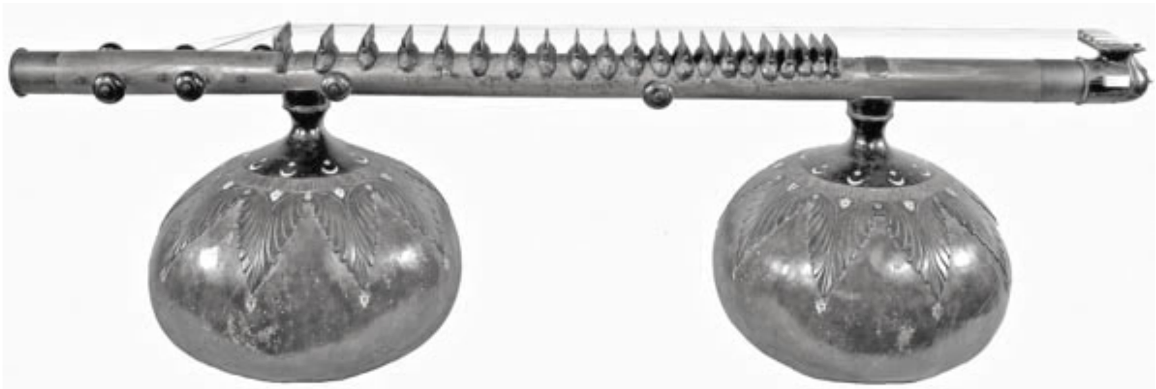


Figure. A rudra vina string instrument from India with two Helmholtz resonators. The resonators are made of gourds and connected to a bridge holding the strings.

Figure. Helmholtz resonators. Father John A. Zahm, Professor of Physics of Notre Dame University, Indiana, bought this set of seven Helmholtz resonators and gave them to St. Mary's College for Women. This set is one of many at St. Mary's.

Figure. A cylindrical Helmholtz resonator made of two tubes for changing the volume and thus changing the resonance frequency. The frequencies and corresponding notes can be engraved on the sides of the cylinder. This is one of the tunable Helmholtz resonators at the University of Vermont.

More About Musical Instruments

WIND INSTRUMENTS
One end is an "open end", the other end can be open or closed.

Instruments with "both ends open"
Standing waves of all harmonics are allowed.
The harmonics are f_1, $2f_1$, $3f_1$, $4f_1$, $5f_1$, $6f_1$, $7f_1$, $8f_1$,……
Examples
Orchestral flute: Two open ends, including the embouchure hole (mouth piece).
Recorder family: Two open ends, including the wedge-like mouthpiece (fipple).
Question
When you "overblow" a flute or recorder, what is the first existing overtone? What is the musical interval between it and the fundamental?
Answer: N = 2, with frequency $2f_1$. The interval is an "octave" with ratio 2:1.

Instruments with "one end closed"
Only standing waves of odd harmonics theoretically are allowed.
The harmonics are f_1, $3f_1$, $5f_1$, $7f_1$, …
In practice, this is true approximately for the lowest harmonics of instruments having a *cylindrical* bore.
Examples
Reed instruments such as the clarinet with a straight *cylindrical* bore.
The reed acts as a closed end, because it is "slapped" closed much of the time.
Question
When you "overblow" a clarinet, what is the first existing overtone? What is the musical interval between it and the fundamental?
Answer: N = 3, with frequency $3f_1$. The interval is an "octave + fifth" with ratio 3:1.

Many reed instruments have conical bores and curved tubes of varying diameter. These admit all harmonics, even and odd, whether one end of the instrument can be considered "closed" or "open".
Examples
Bassoon, oboe, saxophone

BRASS INSTRUMENTS
The player buzzes his lips directly into the mouthpiece (e.g. without a reed).
Brass instruments also have a tube of varying diameter and a bell shape at the end.
Question
Do you expect all harmonics or only odd harmonics from a brass instrument such as a trumpet or tuba?
Answer: All harmonics because of the complex non-cylindrical shape of the instrument.

Demonstrations
Recorder, American Indian flute, didgeridoo, clarinet, trumpet, trombone, harmonica.

Harmonic and Non-Harmonic Vibrations. Pitch, Timbre, and Noise

Vibration Type	Pitch and Timbre	Examples and Demonstrations
Sine wave, single frequency (fundamental only)	Definite pitch, boring, bland tone	Sine wave from synthesizer, whistle, short narrow tubing
Fundamental and a few harmonic overtones	Definite pitch, timbre depends on the relative amplitudes of harmonics	Some woodwind instruments, recorders, Indian flute, tuned (carved out) marimba bars, Helmholtz resonators, bottles
Fundamental plus many harmonics	Definite pitch, full and clear tone, spiciness of tone determined by strong higher harmonics	Well-made string instruments woodwinds and brass, bassoon, krummhorn, harmonica
Fundamental plus poorly tuned harmonics	Discernible pitch, but may sound slightly off the fundamental	Poorly made instruments, strings stiff, piano, inadequate tension
Fundamental with inharmonic widely spaced partials	Defined pitch, spicy timbre	Marimba bars (not tuned), bells, chimes, tubings, glockenspiel
Closely spaced non-harmonic frequencies	Different pitches may be heard, chordal, muddy, or jangly sound	Irregular strings, some gongs and triangles, scrap metal
Many closely spaced non-harmonic frequencies	No dominant pitch, tone quality may vary widely	Cymbals, some gongs and triangles, long thin metal rods struck longitudinally
Distinct fundamental with many inharmonic partials and noise	Rough pitch, discernible timbre depending on mix of partials	Snare drums, djembe, membranes, rattle, electric razor, kitchen blender, lawn mower, scraping sound
Disordered vibrational pattern, quasi-continuous	Noise, sound frequencies over a broad band, some wide peaks, no isolated strong frequencies	Many people randomly clapping hands, radio static, rushing wind, rain, maracas

(Some parts adapted from Bart Hopkin, Musical Instrument Design, 6th printing, page 4, Sharp press, Tucson, 2007.)

Sachs-Hornbostel Categorization of Musical Instruments (1914)

According to Curt Sachs and Erich von Hornbostel there are 4 broad instrument classifications according to what the vibrating medium is. There are 9 additional sub-classifications (not shown here).
The categories 5) and 6) below have been named more recently.

1) **Idiophones**
A s*olid material* vibrates with its own stiffness without being stretched.
Free-bar instruments: Marimbas, chimes
Rods fixed at one end: Kalimbas, tongue drums
Tuning forks
Bells, cymbals, gongs

2) **Membranophones**
A s*tretched membrane* vibrates.
Drums, djembe

3) **Chordophones**
The initial vibrator is a *stretched string*.
String instruments: Violin, viola, cello, double bass, guitar
Harp, zither, lyre, lute

4) **Aerophones**
The initial vibrator is *air*, enclosed in a chamber, or free.
Wind instruments: Flute, clarinet, oboe, bassoon, recorder, panpipe
Brass instruments: Trumpet, trombone, French horn, tuba, euphonium
Plosive Aerophone
Siren

5) **Electrophones**
Electrons vibrate in a wire or circuit.
Synthesizer
Electric guitar
Theremin

6) **Hydrophones**
The initial vibrator is a *liquid*.
Droplets falling into water can make a musical sound.

(Partly taken from Bart Hopkin, Musical Instrument Design, 6th printing, Sidebar 4-1, p. 30, See Sharp Press, Tucson, 2007.)
See also “Musical Instrument Categorization Systems” in the Appendix of this Course Guide.

Part 5. Hearing and Auditory Effects

Auditory System Parts

Outer Ear
The *pinna* collects and concentrates the sound from the outside into the *auditory canal*.

Eardrum
The eardrum separates the outer from the middle ear, so that pressure fluctuations in the sound will not equalize rapidly. Instead, the eardrum vibrates with these fluctuations. (N.B.: Slow pressure changes such as those occurring in an elevator are equalized by the Eustachian tube.)

Middle Ear
The middle contains the three ossicles called *hammer (malleus), anvil (incus), and stirrup (stapes)*. These three tiny bones amplify the vibrations of the eardrum and apply them to the *oval window* between middle and inner ear.

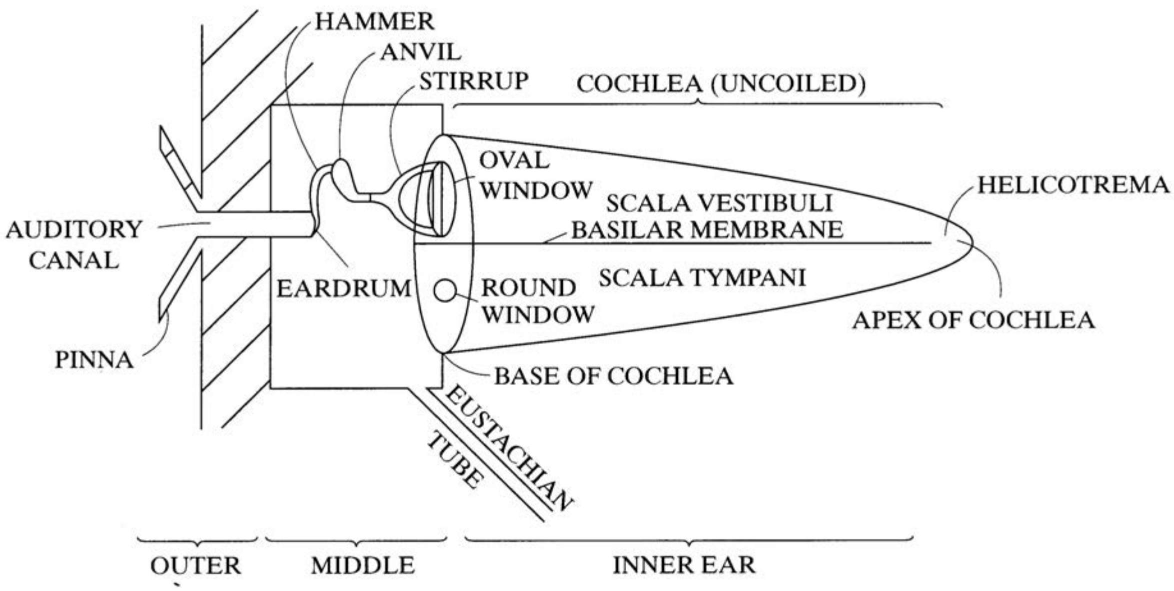
Inner Ear
The principal hearing organ of the inner ear is the coil-like *cochlea.* Vibrations from the stirrup of the middle ear are transmitted at the *oval window* to the fluid in the cochlea.
Traveling waves move through this fluid in the *scala vestibuli.* The cochlea has an opening at the end of the scala vestibuli, called the *helicotrema.* The waves travel through there and on through the *scala timpani* to the *round window*. This is a flexible membrane vibrating with opposite phase to the oval window. The elasticity of the round window allows the fluid in the cochlea to move. This in turn lets the "*hair cells"*, actually nerve cells, of the *basilar membrane* move and be stimulated. The round window also dampens and absorbs the oscillations and minimizes reflections.

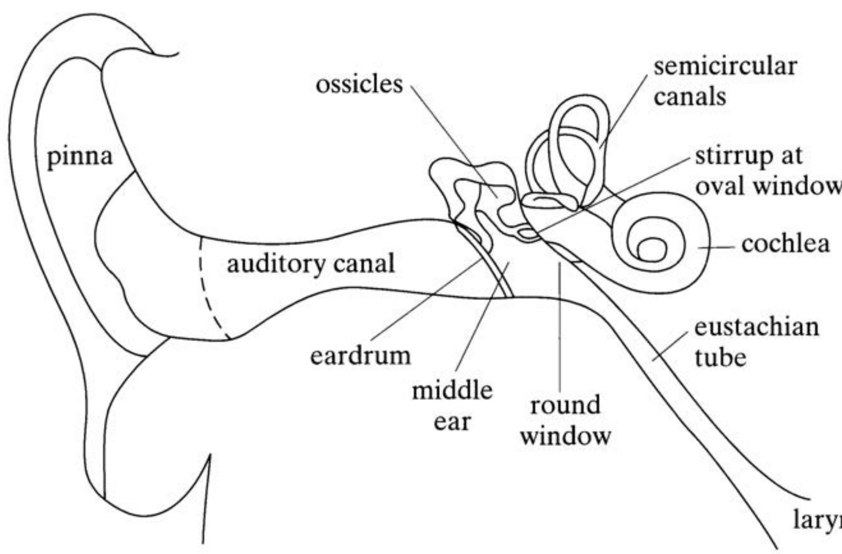
Question
Why does the membrane of the round window have to be flexible?
Answer
If the round window were rigid, the stapes would push against an incompressible fluid, which then would not move, and therefore hearing of sound would be impossible. (A case of hearing loss of this kind actually exists.)

Basilar Membrane
The basilar membrane separates the scala vestibuli and scala tympani. The *organ of Corti* is the region along the basilar membrane that contains the nerve endings or "hair cells". These generate electrical signals from the traveling waves. The signals are collected by the *auditory nerve* and conducted to the brain.

The organ of Corti is about 35 mm long and contains 30,000 "hair cells".
The brain compares the composite electrical signal from the auditory nerve to sound patterns previously stored. Speech, music, and other sounds are then recognized in this way.

Cross Section - Auditory System





Lower Figure: Ear and peripheral auditory system.
Upper Figure: Cross section and schematic of the peripheral auditory system.
(From Berg & Stork, 3rd edition, Fig. 6-1, p.146.)

Exercises

1. In the lower figure, clearly mark in color the main three parts of the peripheral auditory system.
2. Add to the lower figure the basilar membrane, even though it is hidden.

Schematic of Ear

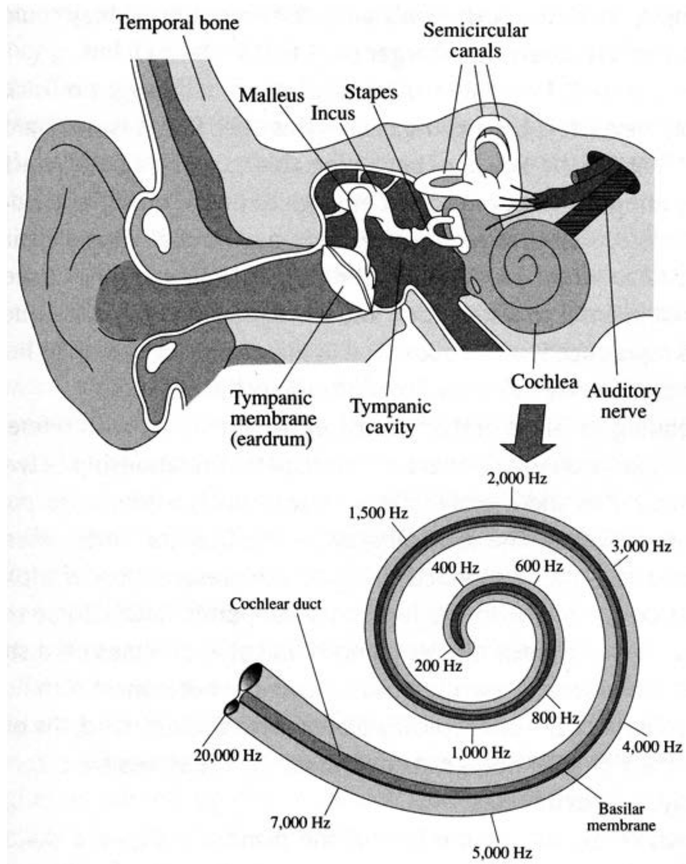


Figure. Schematic diagram of the ear and the distribution of frequencies along the basilar membrane. "Hair cells" (not shown) protrude above the surface of the membrane. Vibrations in the fluid of the cochlea make the hairs move. This opens small pores in the cell walls into which charged ions from the fluid can enter. In this way the charge state of the cells is altered and generates signals in the nerves that lead to the brain.
(From Philip Ball, The Music Instinct, Fig. 3.2, p. 37, Oxford University Press, 2010.)

Engineering Schematic of the Ear. Frequency Response

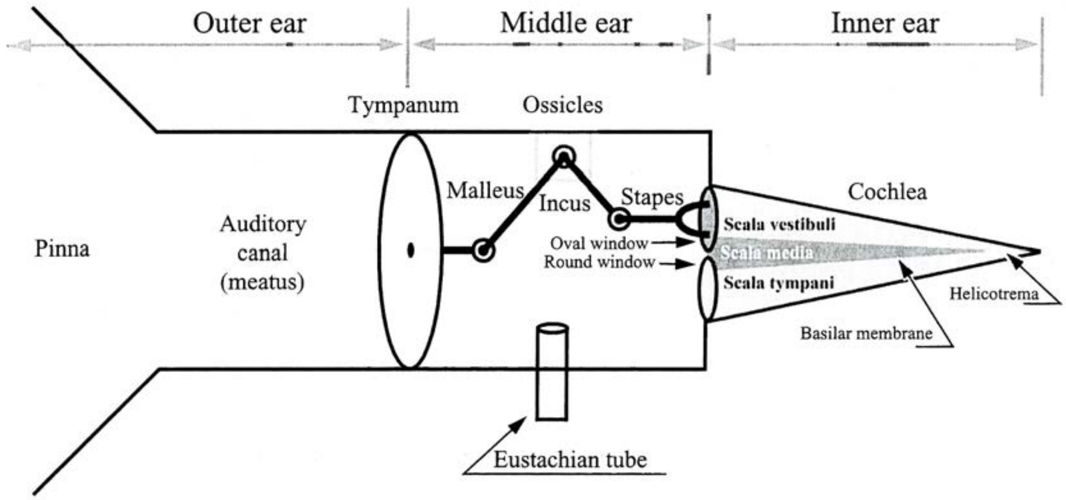


Figure. “Mechanics” of the human ear. Note the levers (ossicles) of the middle ear and the cochlea with the basilar membrane of the inner ear.
(From: Gareth Loy, Musimathics, volume 1, Fig. 6.1, p. 151, The MIT Press, 2006.)

[redacted]

← Distance in mm from base (oval window) to apex oval window

Figure. Response of the basilar membrane for various frequencies. Note the peak response for excitation frequencies of 25, 100, 400, and 1600 Hz. The base is at the oval window where the high frequencies are detected. The apex is farthest away from it and responds to the lowest frequencies.

Basilar Membrane and Place Theory of Hearing

A direct correlation exists between the frequency of a pure tone and the location of the maximum nerve cell response on the basilar membrane.

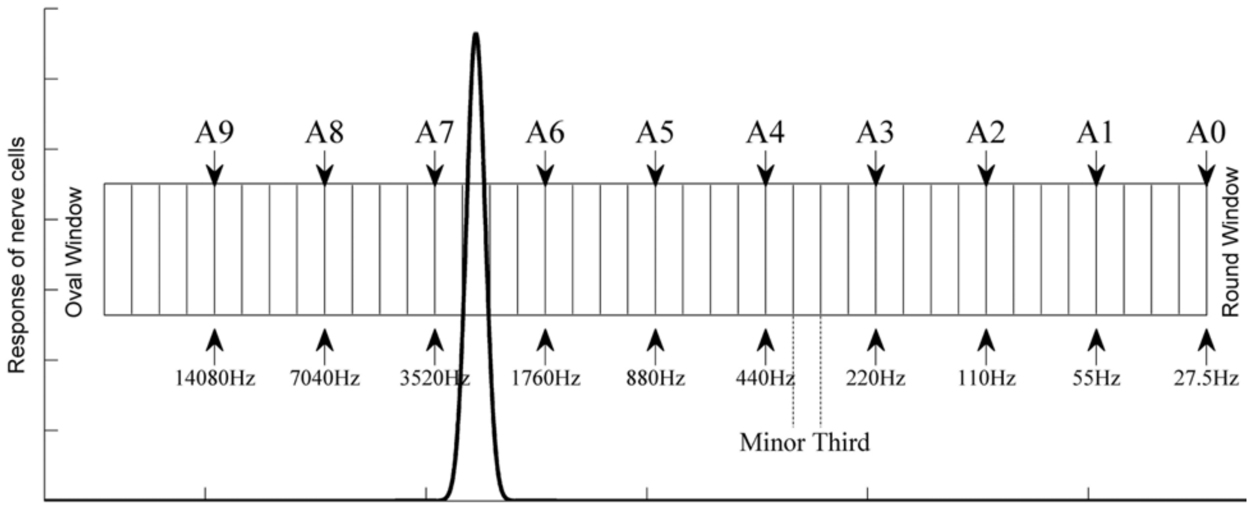


Figure. Schematic diagram of the uncoiled basilar membrane of the cochlea in the inner ear. The bell shaped curve is a simplified schematic of the *"critical band"* and shows the region of nerve cells that respond to a pure sinusoidal sound. The width of the critical band at half the height of the maximum is about a minor third over much of the audible range in the middle.
Note the logarithmic frequency scale in the figure, which corresponds to the arrangements of frequencies on the basilar membrane.
(Adapted from Ian Johnston, Measured Tones, 3rd edition, p. 238-9, CRC Press.)

Acoustic vibrations from the stirrup of the middle ear are transferred to the oval window on the left in the above figure and travel along the basilar membrane. The audible range extends logarithmically over the 35 mm long basilar membrane and contains about 30000 hair cells ("resonators") over the audible range of 10 octaves. The region over which the nerve endings show a large response to a pure tone (sine wave) is the *critical band*. The notes A1 to A9 are spaced octaves apart.

According to the *place theory of hearing,* the maximum response for a given note occurs at a specific location on the basilar membrane. This is shown in the figure by arrows for the maxima of the notes "A", where A4 = 440 Hz is the "concert A". The range of hearing spans almost 10 octaves. The highest frequencies are sensed near the oval window, the lowest frequencies farthest away from it near the end of the basilar membrane (apex of the cochlea). Given the nearly logarithmic frequency response of the cochlea, the spacing between adjacent octaves (or any corresponding musical intervals) is the same, although the frequency increases exponentially.

Critical Band, Auditory Sharpening, Musical Intervals

Critical Band and Place Theory of Hearing

According to the *place theory of hearing*, each frequency is sensed at a certain place on the basilar membrane. This is the place where a wave of given frequency "breaks" or "crashes" in the cochlear fluid, similar to ocean waves nearing a beach. Octaves occupy equal lengths along the basilar membrane. The spacing of octaves and other frequency intervals is nearly *logarithmic*. Each octave covers a length of 3 to 4 mm or about 3000 hair cells.

With 12 semitones to an octave, there are about 250 cells to a semitone. There are 3 semitones to a minor third and hence 750 hair cells. One octave covers about 3.5 mm of the basilar membrane. Therefore, one semitone spacing is about 1/12 of this or about 0.3 mm = 300 μm.

The *critical band* on the basilar membrane is defined as the number of hair cells that respond to a *pure sinusoidal tone*. The critical band is about a *musical minor third* (frequency interval 6/5), or 3 musical half steps over most of the audible range. This corresponds to about 1 mm on the basilar membrane, or close to 750 hair cells. On the other hand, at frequencies below 200 Hz the critical band is almost an octave wide or about 12 semitones. For low notes such as G2 on the piano, minor thirds therefore sound very rough, and composers rarely use small musical intervals at very low frequencies.

Demonstrations

1. Select the "sine tone" mode on the keyboard. Play minor thirds in the middle of the keyboard to demonstrate the width of the critical band.
2.Use sine waves and play the minor thirds G4-B^b4 and G2-B^b2. Note that the critical band at low frequencies is wider. Where does the minor third sound less coarse and more pleasing?
3. Select "grand piano" or "square wave" mode on the keyboard. Repeat the above. There is less coarseness in the sound now for the minor thirds. This is due to the *sharpening effect.* Sharpening is more acute for complex waves because of the additional frequency information from the harmonics. Tones less than one semitone apart may thus be resolved.
4. Play musical fifths, fourths, and thirds at various locations on the keyboard and listen whether they "sound similar".

Auditory Sharpening

How can we hear a *pure tone* or sine wave with one single sharp frequency if the critical band is a minor third wide? This is a major puzzle. Apparently our neural system and brain narrow down a large range of frequencies to a tone that is perceived with single sharp pitch! This process is called *auditory sharpening*.

Research Paper

Write a research paper on the mechanism of auditory sharpening.

Hearing on a Log-Frequency Scale, Musical Intervals

Equal musical intervals such as octaves, minor thirds etc., occupy the same length on the basilar membrane. For a musical third this distance is about 1 mm. This shows that our "hearing" takes place on a compressed, logarithmic frequency scale on the basilar membrane. Musical intervals sound the same over much of the basilar membrane, independent of pitch.

Piano Keyboard Analogy

The arrangement of hair cells on the basilar membrane is analogous to the arrangements of keys on the piano. An octave occupies 3.5 mm on the basilar membrane and 160 mm on the keyboard.

Demonstration of Musical Intervals on a Log-Frequency Scale

1. Show on the keyboard that a musical interval "sounds the same" irrespective of where the interval is played. Listen to octaves, fifths, fourths, thirds at different locations on the keyboard.
2. Select a musical interval such as a major third E4-C4 with two frequency generators. Select *"log-scale"* on the frequency axis of the computer display. Note the spacing E4-C4 in the spectrum. Next select the major third E5-C5. Again note the spacing on the logarithmic frequency scale. They are the same! While the frequency differences E4-C4 and E5-C5 are different on a linear scale, the ratios E4/C4 and E5/C5 are the same and have the same separation between them on the logarithmic scale. The log-scale for frequency in this demonstration simulates how we hear musical intervals. Mathematically speaking we have:

$f_{E4}/f_{C4} = f_{E5}/f_{C5} \rightarrow \log(f_{E4}/f_{C4}) = \log(f_{E5}/f_{C5})$ and, therefore, on the log-scale

$\log f_{E4} - \log f_{C4} = \log f_{E5} - \log f_{C5}$, while on a linear frequency scale we have $f_{E4} - f_{C4} \neq f_{E5} - f_{C5}$.

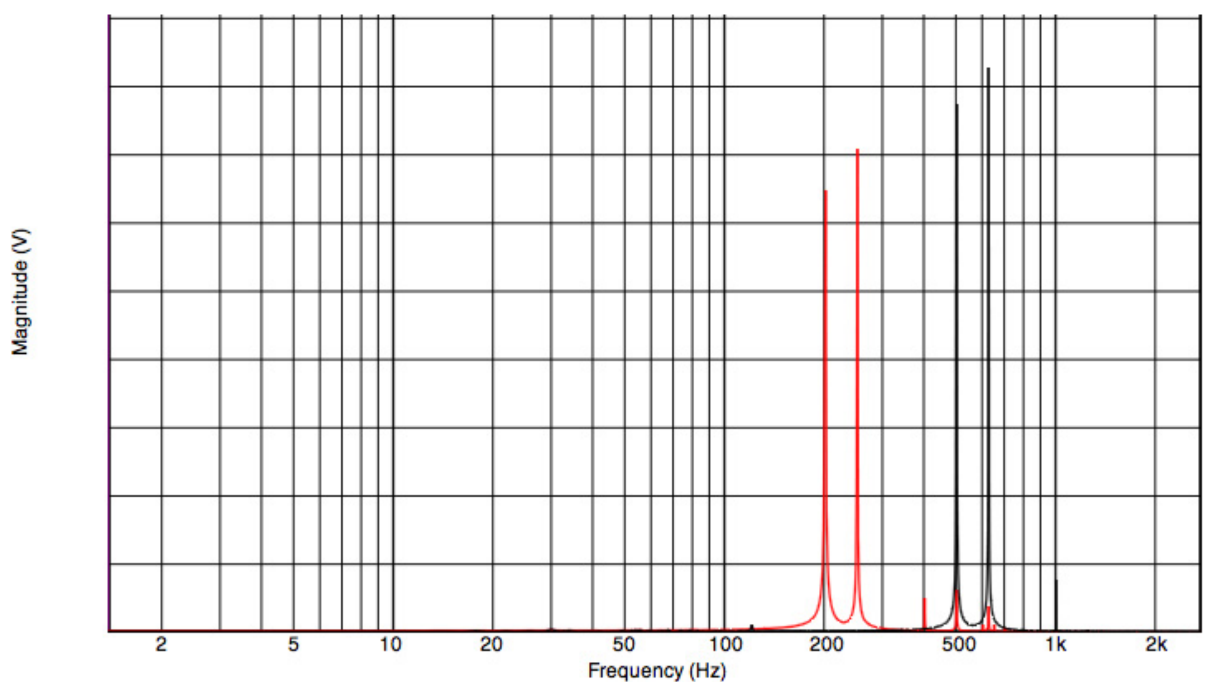


Figure. Two musical major thirds on a logarithmic frequency scale, namely 250Hz/200Hz = 5/4 and 625Hz/500Hz = 5/4. The width of the intervals on the logarithmic scale is the same, and this is the way we hear them (in contrast to a linear frequency scale).

Frequency Discriminations by the Ear (Pure Tones)

Critical Bandwidth (CB)	Limit of Frequency Discrimination (LFD)	Just Noticeable Difference (Frequency JND)
Single pure tone	Two simultaneous pure tones	Two sequential pure tones
15% at high f to 100% at low f	approximately 10%	0.5% at high f to 3% at low f
(minor third to octave)	(Semitone to full tone)	(0.1 to 0.5 semitone)

Example: **f = 2000 Hz**

Critical bandwidth	Limit of Frequency Discrimination	Just noticeable difference
300 Hz	200 Hz	10 Hz

Example: **f = 200 Hz**

Critical bandwidth	Limit of Frequency Discrimination	Just noticeable difference
100 Hz	25 Hz	3 Hz

Demonstrations

1. **Critical bandwidth (CB)**

Play simultaneously two sine waves of the same frequency and amplitude. Gradually increase one frequency. We first hear the average frequency as the pitch as well as beats with the difference frequency. When we increase one of the frequencies further, the beats become more rapid and both frequencies become distinguishable. A low frequency rumble may also be heard. When this disappears with an increasing frequency difference, we have reached the *critical band*.

Critical bandwidth measured: _______ Hz, or _____%, at f = ________ Hz

2. **Limit of frequency discrimination (LFD)**

Related to the above, play *simultaneously* two pure tones of different frequencies. Can you hear two tones? Bring one of the frequencies closer to the other until beating starts. The difference between the two frequencies where this occurs is the *limit of frequency discrimination.*

Limit of frequency discrimination: ______Hz, or _____ %, at f = ________ Hz

3. **Just noticeable difference in frequency (frequency JND)**

Play two pure tones *sequentially*. Start with the same frequency. Increase one frequency slightly and keep alternating between the two frequencies. Note when you hear the *just noticeable difference* in frequency.

Measured just noticeable difference: _______ Hz, or _____%, at f = ________ Hz

4. **Touch analogy for the JND and LFD**

Touch the forearm with two pencil tips as close together as possible. Keep moving away with one tip while raising and lowering it. Note when you can feel both pencil tips separately. You have reached the *"just noticeable difference"* (JND) for touch! Now touch the arm with the two pencil tips simultaneously far enough apart so that you can feel them individually. Move closer with one tip towards the other while still touching the skin. Stop when you no longer can feel the tips individually. The distance between them could be called the *"limit of touch discrimination",* corresponding to the LFD above. Note that JND < LFD, as for hearing!

Logarithms, Exponents, Bel and Decibel (dB), Hearing on a Log-Intensity Scale

We have seen that the ear has a logarithmic response to frequency. It also has a logarithmic response to sound intensity. It is time for some practice with logarithms.

Logarithms to the base of 10
Let $y = 10^x \rightarrow \log y = \log 10^x = x \log 10 = x \cdot 1 \rightarrow x = \log y$

Examples
1. Let $y = 1 = 10^0$ $\rightarrow$ $x = 0$, $\log y = \log 1 = 0$
2. Let $y = 100 = 10^2$ $\rightarrow$ $x = 2$, $\log y = 2$
3. Let $y = 1{,}000{,}000 = 10^6$ $\rightarrow$ $x = 6$, $\log y = 6$
4. Let $y = 0.001 = 10^{-3}$ $\rightarrow$ $x = -3$, $\log y = -3$

The advantage of logarithms is that a large range of numbers is compressed into a small range of logarithms. This is important for understanding the response of our auditory system.

Example
Consider the range of numbers from 1 to 1,000,000.
The logarithms of this range are 0 to 6.
Obviously the number 6 is more easily written than 1,000,000.

The Bel and Decibel
Consider an increase in the intensity of a sound from 1 to 10, i.e. a 10-fold increase.
Take the log: $\log(10/1) = \log 10 - \log 1 = 1 - 0 = 1$
This increase is called **1 Bel** in honor of Alexander Graham Bell.
A smaller unit, used more often, is the **decibel (dB)**, with **1 bel = 10 dB** or **1 dB = 0.1 Bel**
A 10-fold increase in the value of a quantity corresponds to an increase of 10 dB.

Examples
1. A quantity increases 100-fold.
The increase is $\log(100/1) = \log 10^2 = 2{\cdot}\log 10 = 2$ Bel $= 20$ dB
2. A quantity increases 2-fold.
The increase is $\log(2/1) = \log 2 = 0.301$ Bel $= 3.01$ dB ≈ 3 dB increase.
3. A certain quantity increases by 5 dB.
In other words, the quantity has increased by a factor of $10^{5/10} = 3.19$ times larger.
4. A quantity does not change, i.e. 0 dB.
This means $10^{0/10} = 1$, or in other words, no change.

Table of Some Logarithms (remember approximate values!)

log 1	log 2	log 3	log 4	log 5	log 6	log 7	log 8	log 9	log 10
0.000	0.301	0.477	0.602	0.699	0.778	0.845	0.903	0.954	1.000 Bel
0.00	3.01	4.77	6.02	6.99	7.78	8.45	9.03	9.54	10.00 dB

Equal Loudness Curves (Fletcher-Munson Curves)

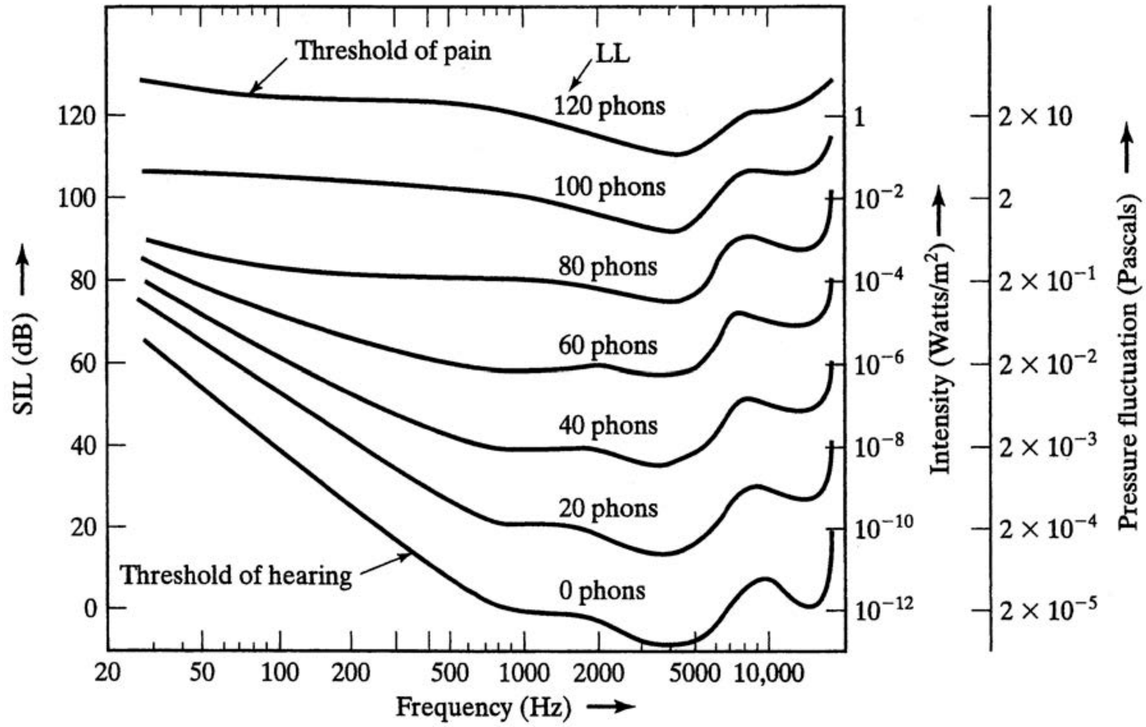


Figure. Fletcher-Munson curves, or Equal Loudness Curves, showing the response of the human ear to sound intensity as a function of frequency. For each given curve the ear perceives the same loudness over the entire frequency range. Note the logarithmic scales for the response of the human ear to frequency and intensity. (From Berg & Stork, Fig. 6-4, p. 152.)

The psychoacoustic *unit of loudness* is the *phon*. The physical *unit of sound intensity* is power in Watt per square meter or *W/m²*. The threshold of hearing is 10^{-12} W/m^2 at 1000 Hz, while threshold of pain is 1 W/m^2 at 1000 Hz for the "average person". The *sound intensity level SIL* is a logarithmic scale for sound intensity and is expressed in *decibel (dB)*. The sound intensity varies over 12 orders of magnitude from 10^{-12} W/m^2 to 1 W/m^2. Accordingly, the SIL varies from 0 to 120 dB between the threshold of hearing and the threshold of pain.

The loudness level in *phon* is chosen to have the same value as the sound intensity level SIL in dB at f = 1000 Hz. Note that for a given equal-loudness curve, for instance 60 phon, the SIL in dB varies greatly, namely from 85 dB at 30 Hz, to a minimum of 60 dB at 4000 Hz, and to 80 dB at 20000 Hz. This means that the ear is most sensitive around 4000 Hz, and much less sensitive at the lowest and highest frequencies. At all the different SIL values for the 60-phon curve, the ear perceives the same loudness, namely 60 phon. The difference in SIL of about 85 dB – 60 dB = 25 dB means that the sound intensity has to be about 300 times higher at 30 Hz than at 4000 Hz in order to be heard with equal loudness.
The intensity range of 120 dB corresponds to an amplitude range of 6 orders of magnitude in the sound pressure waves. The highest amplitude of 20 Pa at the threshold of pain still is 5000-times smaller than the static atmospheric pressure of 100,000 Pa. The ear is a sensitive organ.

Hearing best at the Minima in the Equal Loudness Curves

Two local minima can be seen in each of the Fletcher-Munson equal loudness curves. The ear is more sensitive and hears best around these minima. The more important minimum for all curves occurs around 4000 Hz and the other near 13,000 Hz. These frequencies correspond to the first two existing resonances in the auditory canal.

Exercise
Estimate the resonance frequencies for the two minima in the equal loudness curves. Assume an effective length L = 2 cm for the ear canal.
Answer
Consider the ear canal as a cylindrical tube, closed at the ear drum.
We know for the fundamental resonance of such a tube that $L = \lambda/4$ and $f_1 = v/\lambda$.
Hence $\lambda = 4L = 8$cm = 0.08 m, and $f_1 = 346/0.08 = 4330$ Hz.
This is in good agreement with the observed first minimum in the equal loudness curves.
The first resonance N = 1 in the ear canal makes our hearing most acute around 4000 Hz.
The next existing resonance is N = 3 with $f_3 = 3f_1 = 3\cdot 4330$ Hz = 12990 Hz ≈ 13000 Hz.
This is in good agreement with the second minimum in the equal loudness curves. Sound near this frequency will also be heard loudly (by those who can still hear such high frequencies).

Exercise
From the Fletcher-Munson curves determine the sound intensity level (SIL) for a loudness of 50 phon at f = 500 Hz. Interpolate between the 40 phon and 60 phon curves. **Answer:** ________dB

Vocal Tract Analogy
The resonance frequencies of the vocal tract (see also later) can be calculated in a similar way as for the ear canal. Very roughly, the overall length of the vocal tract can be considered a closed tube of effective length L = 17.3 cm.
Exercise
Verify that this length yields the resonances at $f_1 = 500$, $f_3 = 1500$, $f_5 = 2500$, $f_7 = 3500$ Hz etc. These are the center frequencies of the so-called *formant regions* of the human voice. The resonances actually are not sharp, but are broadened because of the soft tissues of the oral tract.

Demonstrations
1. Set two signal generators to 100 Hz and 4000 Hz sine waves. With a sound level meter, adjust the volume of the two tones to the same sound intensity level, e.g. SIL = 75 dB. Switch between the two tones. The 4000 Hz tone should sound louder according to the Fletcher-Munson curves.
2. Conversely, adjust the two signal generators to the same perceived loudness.
Read the SIL values for the two tones. The 4000 Hz tone should show a lower SIL reading according to the Fletcher-Munson curves.
3. Strike a brass rod longitudinally producing a 3000 to 4000 Hz sound. Note the SIL, or more accurately the increase in the SIL from the background. Produce a 70- Hz tone with a loudspeaker set at the same SIL. Compare your perceived loudness in both cases. The brass rod should sound much louder than the speaker.
4. Blow into an emergency whistle at about 3000 Hz. Note how loud and shrill it sounds.

Amplitude-Frequency Spectra in Linear-Linear and Log-Log Displays

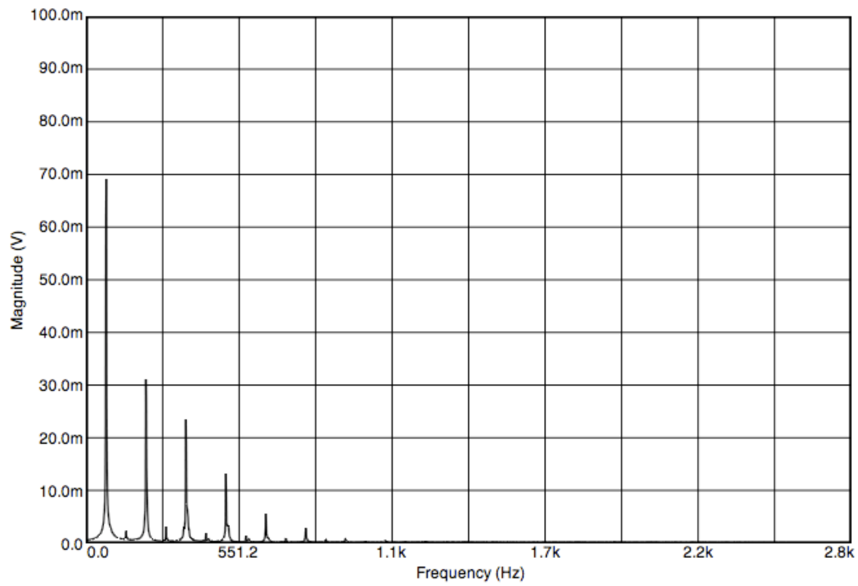


Figure. Amplitude-frequency spectrum from a didgeridoo in a double-linear display. The note played is D2. (The odd harmonics dominate for the tube "closed" by the lips at one end.) This type of display is tidy. However, it does not take into account the human ear response.

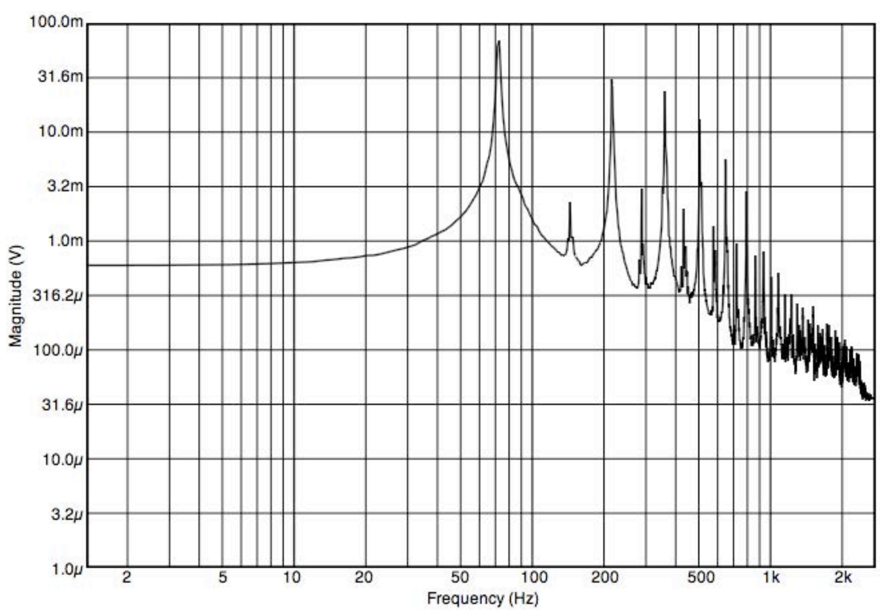


Figure. The amplitude–frequency spectrum from above, now shown in a double-logarithmic display. This representation more closely approximates the human ear response. For instance, the log-amplitudes (perceived loudness) of the first 4 harmonics are more similar in height than on the linear display above. The log-spectrum also is richer in the number of noticeable harmonics. Finally, we hear better at higher frequencies. We therefore perceive the higher harmonics even louder than shown on the log-scale in the figure.

Sound Intensity Level (SIL) and Sound Intensity (I)

The *sound intensity I* is defined as the acoustic power passing through an area of one square meter. It is measured in units of Watt per square meter (W/m^2). The *sound intensity level SIL* is the logarithm (in dB) of the *sound intensity I.* The relationship between *SIL and I* is

$$\mathbf{SIL = SIL_0 + 10\log(I/I_0)},$$

where SIL_0 is the sound intensity level of a reference intensity I_0.

Examples

1. At the threshold of hearing $I_0 = 1\mathrm{x}10^{-12}\ W/m^2$ (see Fletcher-Munson curves)
2. At the threshold of pain: $I = 1\ W/m^2$ (see Fletcher-Munson curves)
3. For the threshold of hearing, we have $I = I_0$ and $SIL = SIL_0 + 10\log(I_0/I_0) = SIL_0 + 0$.
One can choose $SIL_0 = 0$ dB at the threshold of hearing at a frequency f = 1000 Hz.
4. For the threshold of pain, we have $I = 1\ W/m^2$.
Then $SIL = 0 + 10\log(1/10^{-12}) = 10\log10^{12} = 10\cdot 12 = 120$ dB, as we already know.

Important Note

The above equation for the sound intensity level SIL holds for any levels I_0 and SIL_0, not just the threshold of hearing. It holds for instance when a noise increases from an initial reference level $SIL_0 = 50$ dB to higher sound intensity level SIL = 90 dB.

Demonstration

Three students in class applaud the instructor. We measure $SIL_0 = 60$ dB.
Now 30 students applaud, each equally loud as the three students before.
Question: What is the SIL, and what is the increase in SIL?
Answer:
$SIL = SIL_0 + 10\log(I/I_0) = 60 + 10\log(30/3) = 60 + 10\log10 = 60 + 10 = 70$ dB
The increase in SIL is 10 dB, from 60 dB to 70 dB.

Exercise

If a sound source such as a fan produces an $SIL_0 = 50$ dB, how many identical fans will produce an SIL = 70 dB?
Answer: Use $SIL = SIL_0 + 10\log(I/I_0)$. Then $70 = 50 + 10\log(I/I_0)$, or $\log(I/I_0) = 20$, or $I/I_0 = 100$. This means 100 fans.

Demonstration - Logarithmic Response of the Ear to Intensity Changes

Increase the SIL of a sound in equal steps, e.g. 60→70→80→90 dB. For each step, the perceived increase in loudness sounds similar. For such a 10 dB increase, many listeners sense this as “twice as loud”. (P.S.: Others may already sense a 6 dB increase as “twice as loud”.)

Fechner’s Law

An empirical observation says that our senses respond logarithmically to the intensity of an external stimulus. This is ***Fechner’s law***.

Exercise: Give some examples where our other senses respond logarithmically.

Change in Sound Intensity Level

We saw earlier that the sound intensity level (SIL) for a sound *intensity I* is defined as

$$\mathbf{SIL = SIL_0 + 10\,log(I/I_0)}$$

SIL is the sound intensity level in dB and *I* the corresponding sound intensity *I* in W/m^2.
SIL_0 is the sound intensity level of a reference intensity I_0.

Change in Sound Intensity Level
Let the sound intensity change from I_0 to I.
Question: What is the change ΔSIL in the sound intensity level?
Answer: The change is

$$\mathbf{\Delta SIL = SIL - SIL_0 = 10\,log(I/I_0)}$$

Example
You increase the sound intensity from a loudspeaker from I_0 to an intensity 5-times greater, i.e. $I = 5I_0$.
Question: What is the corresponding change in the sound level intensity (in dB)?
Answer: $\Delta SIL = 10\log(I/I_0) = 10\log(5I_0/I_0) = 10\log 5 = 10 \cdot 0.699 \approx 7$ dB.

On the other hand, if we are given the change ΔSIL, we invert this equation and obtain for the intensity ratio

$$\mathbf{I/I_0 = 10^{\Delta SIL/10}}$$

Example
When 3 students applaud the instructor in class, we measure $SIL_0 = 85$ dB.
When 24 students applaud, we measure SIL = 94 dB.
Question: How many times has the sound intensity changed?
Answer: ΔSIL = 94 dB – 85 dB = 9 dB. Thus $I/I_0 = 10^{9/10} = 10^{0.9} = 7.9$
The intensity has increased approximately 8-fold.
Many people perceive such an increase as about "twice as loud".

Challenge Question
Why did the sound intensity not quite increase 10-fold, as was to be expected, when 30 instead of 3 students where applauding?
Answer: ____

A Challenging Example: Absolute Sound Intensity I
Question: What is the *absolute* sound intensity (in W/m^2) when 3 students applaud with a sound intensity level SIL = 85 dB?
Answer: We must use a reference level such as the threshold of hearing, i.e. use $SIL_0 = 0$ dB.
Then $SIL - SIL_0 = SIL – 0 = 10 \log(I/I_0) = 85$ dB or $I/I_0 = 8.5$ Bel $= 10^{8.5} = 3.16 \cdot 10^8$
At the threshold of hearing $I_0 = 10^{-12}$ W/m^2.
Therefore $I = 10^{-12}$ W/m$^2 \cdot 3.16 \cdot 10^8 = 3.1 \cdot 10^{-4}$ W/m^2 = 0.000316 W/m^2 = 0.316 mW/m^2.

Sound Intensity *I*, Sound Intensity Level *SIL*, Fletcher-Munson Curves

Questions
1. What is the combined SIL of two tones, each having an SIL of 50 dB?
Answer:
The intensity of both tones played together is twice the intensity of each: $I = 2I_0$
Hence $SIL = SIL_0 + 10\log(2I_0/I_0) = 50 + 10\log2 = 50 + 3 = 53$ dB.

2. Similar to above, but take three tones with 50 dB each.
Answer: $SIL = 50 + \log(3I_0/I_0) = 50 + 10\log3 = 50 + 5 = 55$ dB.

3. **Challenge Question:**
Find the combined SIL for two tones, one with $SIL_1 = 50$ db, the other with $SIL_2 = 60$ db.
Answer: $(I_1 + I_2)/I_1 = (10^5 + 10^6)/10^5 = 11$.
Then $SIL = 50 + 10\log11 = 50 + 10.4 = 60.4$ dB.

Demonstrations
1. Set two sine generators to 500 Hz and the same SIL for each. (This means equal loudness as the frequency is the same.) First play one tone, then both tones together. The measured increase in the SIL should be 3 dB.
Question
How much louder subjectively do the two tones sound together?
They will not sound "twice as loud". This is consistent with the logarithmic response of the ear to sound intensity.

2. Play some sound tracks from the CD "Auditory Demonstrations" by the Acoustical Society of America, for instance the track "Decibel Scale and Intensity".

Fletcher-Munson Curves (Equal Loudness Curves) Revisited
Understand the Fletcher-Munson curves and answer the corresponding questions in the homework!

Example:
From the equal loudness curve of 40 phon, what is the SIL at f = 100 Hz?
Answer: SIL = 60 dB.

Environmental Sound Intensity Levels

Table. Approximate sound intensity levels encountered in various environments, corresponding sound intensities, and human reaction. These are typical values; individual cases might give readings 10 dB higher or lower.

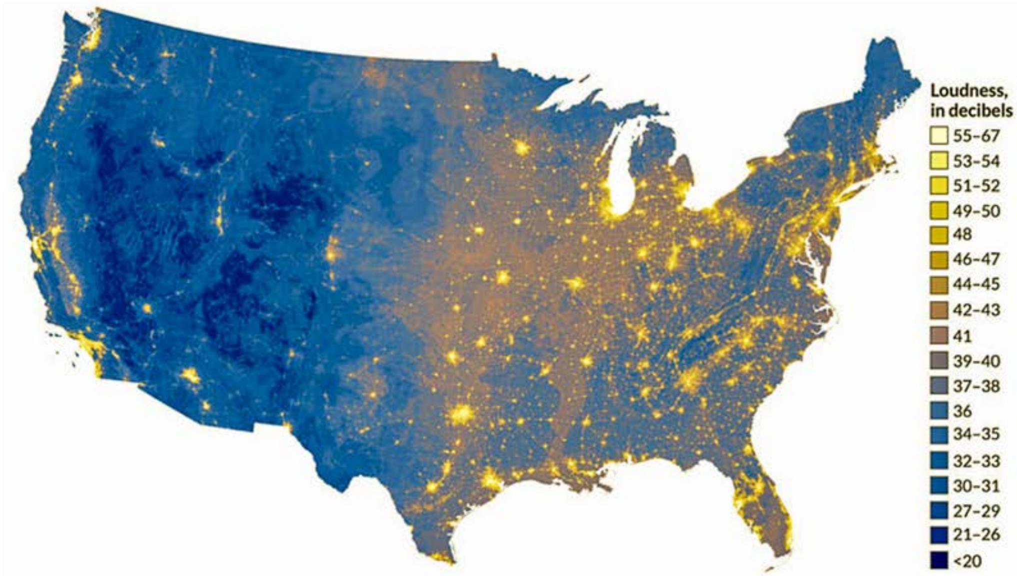
Sound Source	Sound Level (dB)	Intensity (W/m^2)	Human Reaction
Jet Engine at 10 m	150	10^3	Serious damage
	140		
	130		
SST takeoff at 500 m	120	1	Very painful
Amplified Rock Music	110		
Machine Shop	100		
Subway Train	90	10^{-3}	
Factory	80		
City Traffic	70		
Quiet Conversation	60	10^{-6}	Musically Useful
Quiet Auto Interior	50		
Library	40		
Empty Auditorium	30	10^{-9}	
Whisper at 1 m	20		
Falling pin	10		
	0	10^{-12}	Inaudible

Table. United States limitations on permissible daily occupational noise exposure and the more conservative limits suggested by OSHA for avoidable, non-occupational exposure. Some countries allow only a 3 dB level increase for each halving of the exposure time.

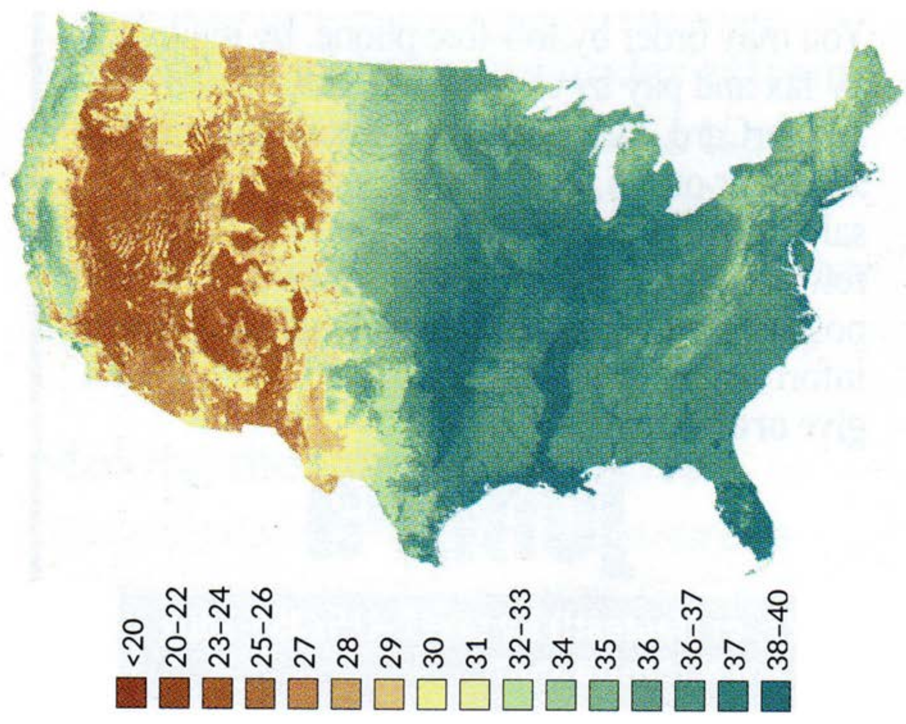
	Maximum 24 Hour Exposure	
Sound Level (dBA)	Occupational	Non-occupational
80		4 hr
85		2 hr
90	8 hr	1 hr
95	4 hr	30 min
100	2 hr	15 min
105	1 hr	8 min
110	30 min	4 min
115	15 min	2 min
120	0 min	0 min

P.S.: Collapse of Atmospheric Pressure
Suppose the entire atmospheric pressure of 100,000 Pa were to collapse suddenly to a vacuum. What would the dB-level of the resulting pressure wave be? As 120 dB corresponds to a pressure amplitude of 20 Pa, an amplitude of 100,000 Pa would result in an SIL = 194 dB (Exercise)!

Sound Map USA



Loudness levels of a typical summer day, including people and machinery.
The decibel levels apply to loudness that is exceeded half the time at given spots.



Loudness levels of the natural environment on a summer day, excluding people.
(Reference: Science News, February 21, 2015, p. 32.)
Machinery and other man-made noises are also exluded from the lower map.

Sound Intensity Level and Human Annoyance Curve

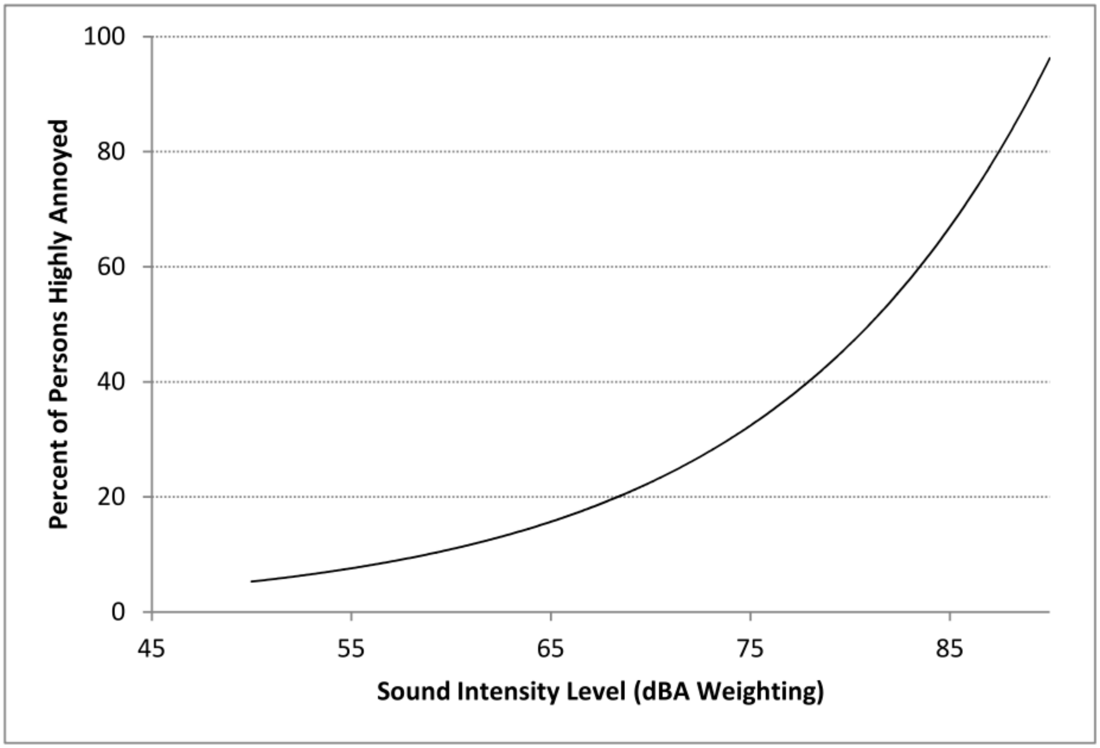


Figure: The degree of human annoyance as a function of average environmental noise intensity level. The graph is an average of the results of numerous surveys, as reported by T.J. Schultz, JASA, 64, 377 (1978). Nighttime levels as low as 55 dB can cause serious disturbance of sleep.

Just Noticeable Difference (JND) in Sound Intensity Level

Let us consider a change in sound intensity of a sine tone and a more complex tone.
The question is:
How much does the intensity have to change to become “just noticeable” to us?
This difference is called the just noticeable intensity difference or **intensity JND**.
Tests show that the difference lies in the range $\Delta SIL = 0.5$ to 1.5 dB.

Demonstrations
1. Play a 500 Hz sine tone through a loudspeaker. Measure the sound intensity level SIL with a sound level meter. Slowly increase the intensity and stop when you hear a change. (The same can be done with a keyboard set to the “sine wave” mode.)
Note the just noticeable increase ΔSIL:
Answer: ΔSIL = _______dB

2. Set the piano keyboard to “sine wave”. Keep tapping a key near 500 Hz while increasing the SIL. Note the just noticeable difference ΔSIL:
Answer: ΔSIL = _______dB

Question
From what you have found from the two demonstrations above, is the ear more sensitive to abrupt or gradual changes in intensity?
Answer: ____

3. Set the piano keyboard to “grand piano”. Then tap again as in the preceding demonstration. What is the just noticeable difference this time?
Answer: ΔSIL = _______dB

Question
Is the ear more sensitive to intensity changes in pure tones (sine waves) or complex tones containing several overtones?
Answer: ____

Question
Assume that the just noticeable difference in sound intensity level is 1 dB.
Find the percentage change in the sound intensity *I* (not in sound intensity level *SIL*) that corresponds to this ΔSIL = 1 dB.
Answer: $\Delta SIL = 10\log(I/I_0) = 1$ dB = 0.1 Bel, or $I/I_0 = 10^{0.1} = 1.26$
The sound intensity in this case has to change by the rather large amount of 26% to become “just noticeable”.

Demonstration
Use a sound level meter and record the SIL of several students applauding. How many more students have to applaud for you to hear a noticeable change in intensity?
Data and Answer

Initial number of students applauding	____________
Number of students applauding for a "just noticeable difference"	____________
Percentage change in the number of students applauding	____________%
Decibel change in the relative number of students	____________dB
Measured change in the SIL	____________dB
Compare the two dB-values. Are they similar, as they should be?	____________

Comparison between Intensity JND and Frequency JND

Intensity JND:	0.5dB – 1.5 dB or 12% - 41% change	(Exercise)
Frequency JND:	0.5% – 3% or change of 0.022 dB – 0.13 dB	(Exercise)

Exercise
Verify the above percentage ranges in the intensity JND from the dB-ranges given.
Exercise
Verify the above dB-ranges in the frequency JND ranges given.

Comment on the Sensitivity of the Ear to Frequency and Intensity Changes
From the numbers obtained we see that the ear is about 10-times more sensitive to frequency changes than to intensity changes. The ear can determine much more acutely whether two tones are out of tune compared to when they differ in intensity.

Dissonance and Consonance

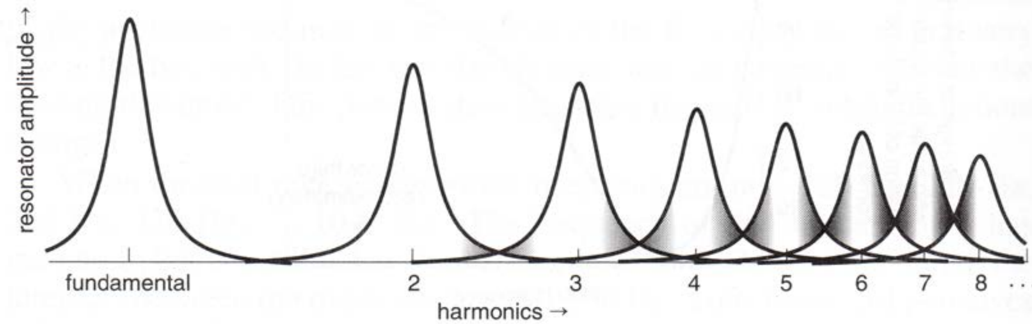


Figure. The frequency spectrum of a single complex tone with harmonics displayed along the basilar membrane on a logarithmic scale. Increasing overlap of the critical bands occurs for the higher harmonics, but the sound is still consonant.

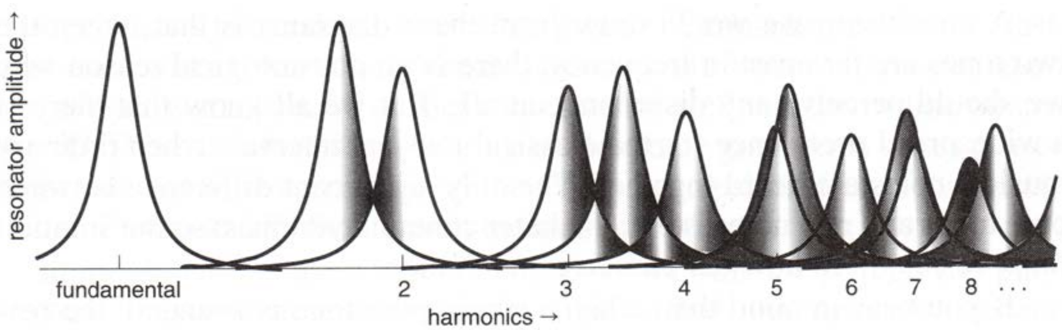


Figure. Two randomly chosen tones separated by more than a minor third and played together. Significant overlap occurs for the higher harmonics. The sound is dissonant and rough.

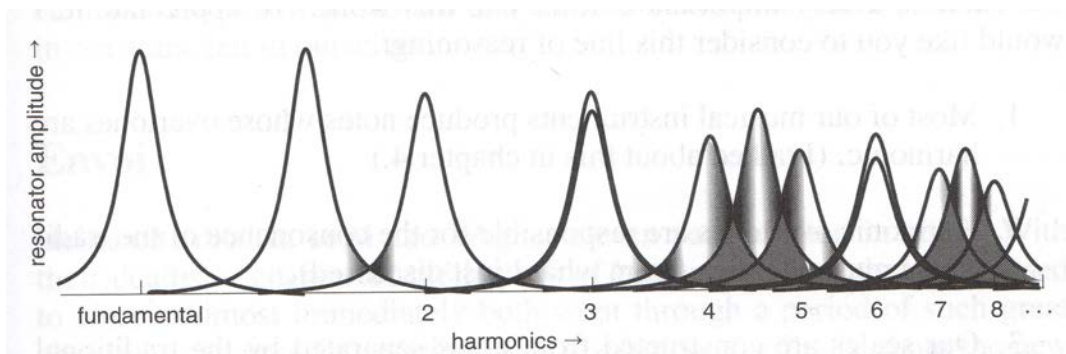


Figure. Two complex tones a perfect fifth (ratio 3/2) apart. Every 2nd peak of the higher tone coincides with every 3rd peak of the lower tone. The total overlap is reduced and the sound is rather consonant, but less consonant than for the harmonics of the single complex tone above. (From: "Measured Tones" by Ian Johnston, 3rd edition, CRC Press, 2009, pp. 248, 249.)

Challenge question: For which musical interval (i.e. pair of notes) do you expect to hear the least roughness in the sound?

Answer: ____

A Dissonance Curve

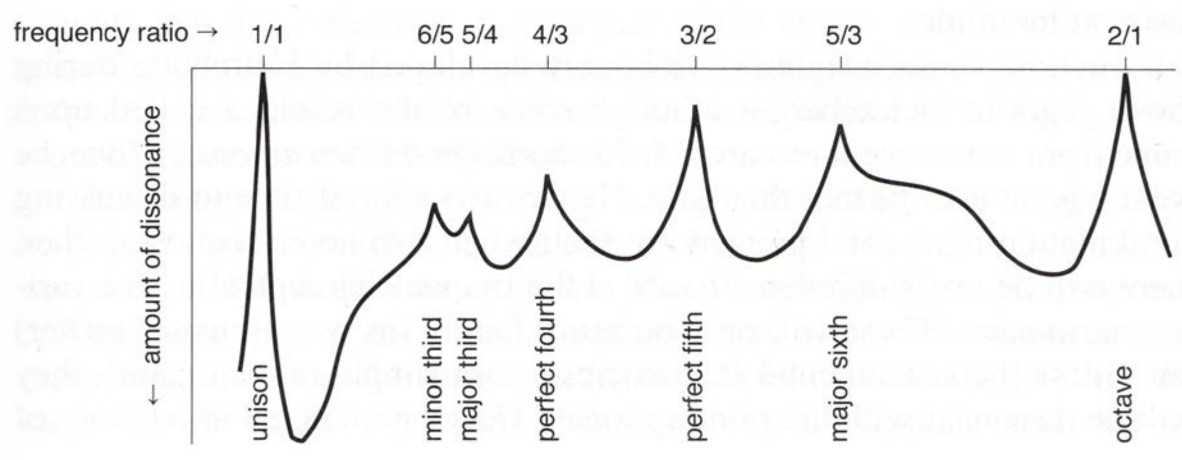


Figure. A dissonance curve calculated by Plomp and Levelt from the overlap of the critical bands of pairs of notes. The historic musical intervals are relatively free of dissonance, especially fourth, fifth, major sixth, and octave. A half step and full step immediately to the right of the unison show the largest dip and thus the greatest dissonance.
(From: Ian Johnston, "Measured Tones", 3rd edition, DRC Press, 2009, p. 249.)

The overlap of critical bands on the basilar membrane of the ear thus may explain the perception of dissonance. Conversely, we may say that

Consonance = Absence of dissonance

Some Related Comments
Consonance is highest near the just musical intervals.
Musical scales are constructed from traditional intervals.
Harmonic overtones are responsible for the consonance of traditional musical intervals.
Many musical instruments produce notes whose overtones are harmonic.

Our scales are what they are because of the kind of instruments we play.
Is this a musical anthropic principle?

Compare with **Cosmological Anthropic Principle** (Carter):
The Universe and the fundamental parameters on which it depends must be such as to admit the creation of observers…. (Descartes: *Cogito ergo mundus talis est*.)

Listening Interludes - When is Sound "Musical"?

For a sound to be perceived as "musical", several conditions must be met:
a) The sound must be of sufficient duration, e.g. several periods long (~10T).
b) The spectrum must show distinct frequency spikes.
c) The spikes have narrow widths, i.e. they are "sharp" and not resembling "pink noise".
d) Play a minor third such as G2 – B2b with sine waves and with piano sound. The interval sounds more musical with piano sound, because of narrower critical bands of the overtones and more sensitivity of the ear to higher frequencies than to the low-frequency fundamental.

Demonstrations
1. Play a percussion instrument, for instance a djembe (drum) or cymbal. Observe the Fourier spectrum. Play the instrument to sound "musical". Then play it to sound more "percussive". Compare the frequency spectra.
Question: What are the common and different features in the two spectra?
Answer: ____

2. Play a wind instrument, for instance a recorder or whistle.
Compare the frequency spectrum with the percussion instrument. Describe the pronounced differences in the spectra.
Answer: ____

3. Make some vocal sounds and observe the frequency spectra.
a) Vowel sounds
b) Buzzing sounds
c) Sibilant or hissing sounds
What changes do you see in the spectra when changing from "musical" to "non-musical" sound?
Answer: ____

4. Play Sabian and Wuhan cymbals.
Display the rich Fourier spectra. Many individual densely packed spikes can be seen over almost the entire audible frequency range.
Question: Do cymbals sound musical? Which of the two cymbals sounds more "musical". Why?
Answer: ____

5. Play a snare drum.
Does it sound "noisier" than the Sabian Cymbals? Explain by inspecting the frequency spectra.

Sound Tracks of Auditory Effects
Listen to some CD tracks from "Auditory Demonstrations" by the American Acoustical Society:
a) Decibel scale and intensity
b) Cancelled harmonics
c) Filtered noise
d) Logarithmic and linear frequency scales
e) Effect of spectrum on timbre

Part 6

More Auditory Effects, Voice and Vocal Tract, Vocal and Instrumental Formants

Missing Fundamental and Periodicity Pitch

Consider a sustained complex tone containing several harmonics. If you leave out the fundamental N = 1 and play only higher harmonics from the overtone series, you may hear the "missing fundamental". The missing fundamental is composed in the brain from nerve impulses.

Example

N = 1	f = 200 Hz	Fundamental - not played (missing fundamental)	
N = 2	f = 400 Hz	Octave 400/200	Difference 200 Hz
N = 3	f = 600 Hz	Fifth 600/400	Difference 200 Hz
N = 4	f = 800 Hz	Fourth 800/600	Difference 200 Hz, etc.

The frequency differences between successive harmonics are 200 Hz in this example. If you hear a tone with a pitch of 200 Hz, you are "hearing" the missing fundamental. This effect also is called *periodicity pitch* because the pitch and period of the tone are obtained from the harmonics.

Question

Two successive frequencies in a harmonic series are 1000 Hz and 1200 Hz. What is the frequency of the fundamental?
Answer: f = ______Hz

Challenge Question

Can two frequencies of 300 Hz and 500 Hz be *successive* harmonics in an overtone series?
Answer: Yes, for a closed tube having only the odd harmonics 100 Hz, 300Hz, 500 Hz etc.

Demonstrations

1. Play some CD tracks demonstrating missing fundamentals.
2. Bow the lowest empty C2 string on a cello. The fundamental (C2 = 65.4 Hz) is very weak or non-existent in the Fourier spectrum. Nonetheless, the note C2 sounds loud because of the missing fundamental frequency being put together in the brain from the harmonics.
3. Play Beethoven's "Ode to Joy" on the piano to become familiar with the melody. Then play "Ode to Joy" from a recording where the first four (!) harmonics in the melody are missing. Most people nonetheless can "hear" the melody by tracking the fundamental.
4. Play C2 on the keyboard. The fundamental may be missing from the spectrum, but the higher harmonics are visible. As a result of *periodicity pitch*, the note C2 is "heard" with its pitch.

Fundamental Tracking

In the preceding demonstrations, especially in "Ode to Joy", we hear the missing fundamental and how it is *tracked* as the music plays. This is called *fundamental tracking.*

The fundamental of the lowest notes on the piano is very weak because the soundboard responds poorly below 50 Hz. Nonetheless, you "hear" the notes because the missing fundamental is tracked from the harmonics.

Missing Fundamentals from a Bassoon and Bass Tuba

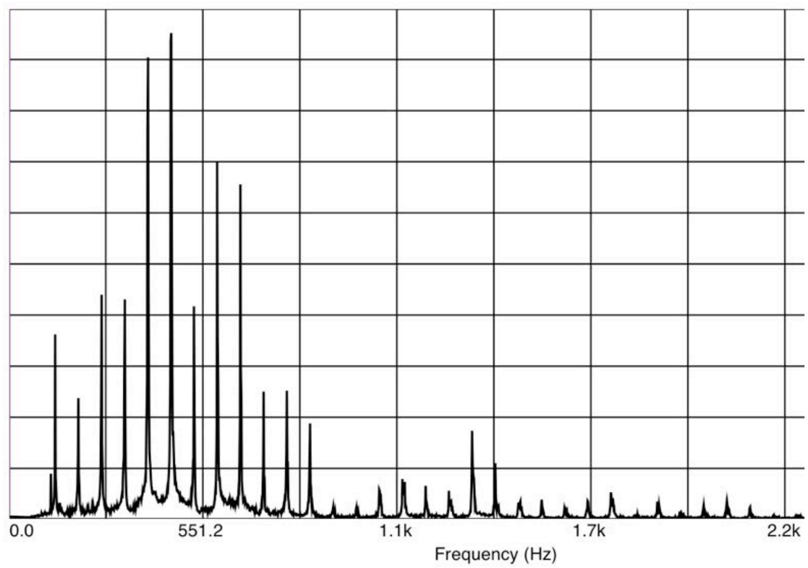


Figure. Sound spectrum of the note C2 = 65.4 Hz from a bassoon played by Professor Richard Meek, Texas Tech University. The fundamental at 65.4 Hz (harmonic number N=1) is missing. The lowest harmonic is C3 (N=2) an octave above C2.

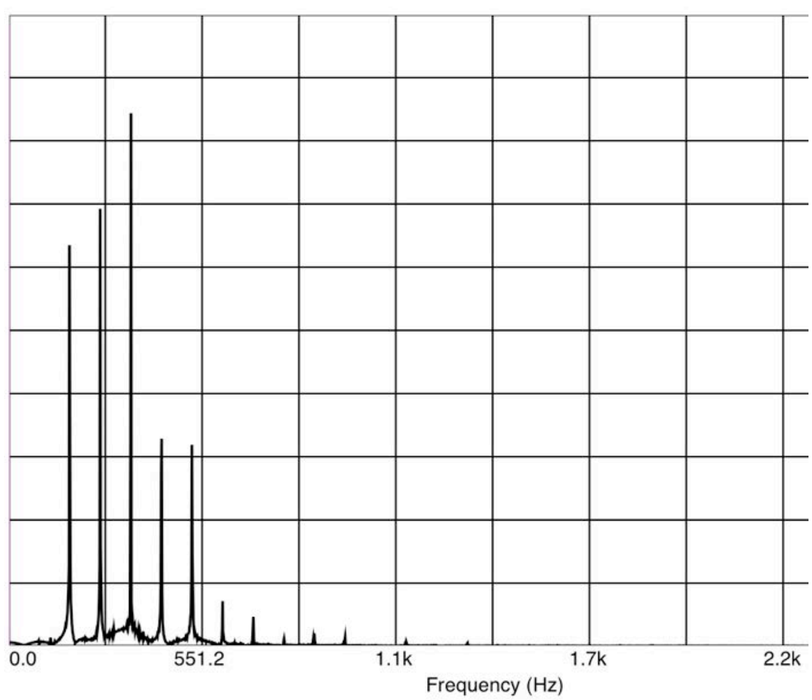


Figure. Sound spectrum of F2 = 87.3 Hz from a bass tuba played by Professor Kevin Wass, Texas Tech University. The fundamental at 87.3 Hz (harmonic number N=1) is missing. The lowest harmonic is F3 (N=2) an octave above F2.

Aural Harmonics

The response of the ear to sound intensity is *non-linear*; it is in fact approximately logarithmic. A *loud* sine wave of frequency *f* will thus sound distorted. The distorted wave "heard" still has the same fundamental frequency f, but it is now a complex waveform. According to *Fourier's Theorem*, it consists of a fundamental *f* and harmonics *2f, 3f, 4f...* These additional harmonics are called *aural harmonics*. They become more and more pronounced with increasing intensity of the fundamental. The tone still has the same pitch, but it sounds "spicier" or more "brilliant".

Question
Why does the timbre change?
Answer: ____

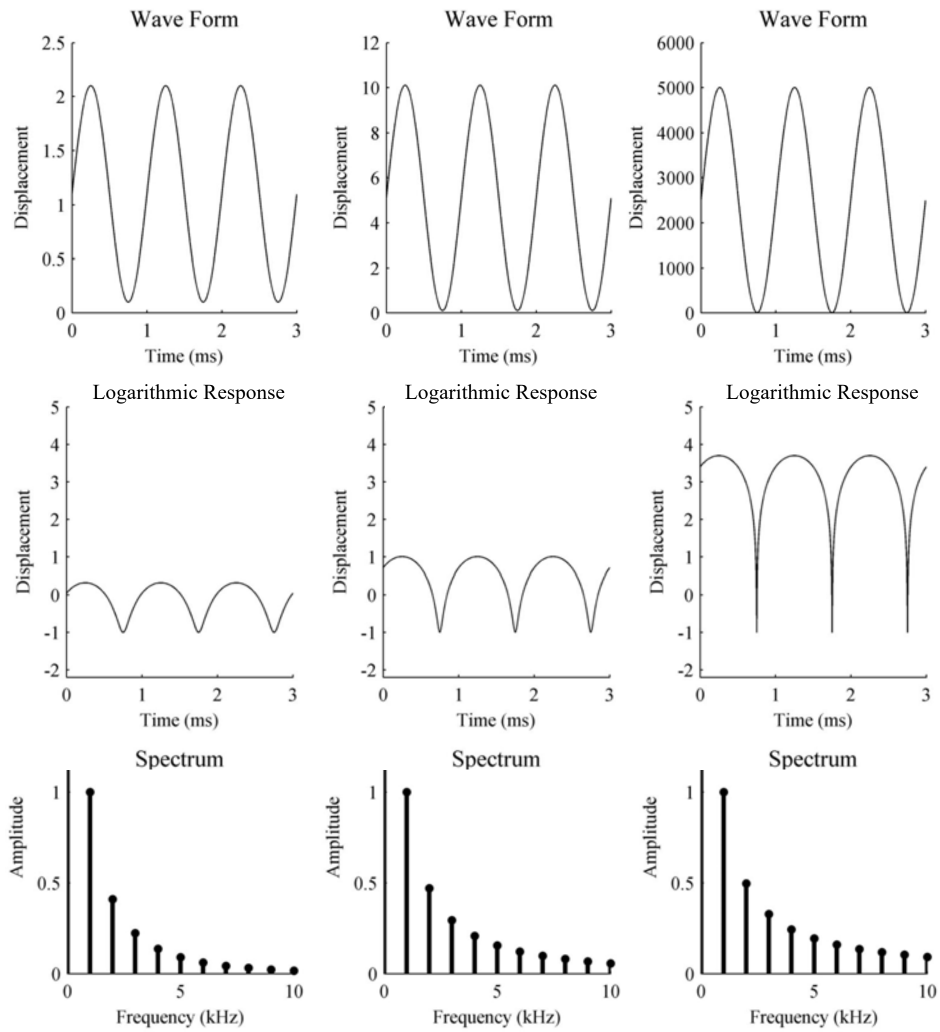
Demonstration
Play a pure sine wave, first softly, then increasingly louder. Listen to the change in timbre arising from more and more higher harmonics "sneaking" in.

Intensity Increase of the Aural Harmonics
For a 1 dB increase in the intensity of the fundamental of frequency *f*, we have the following intensity increases in the aural harmonics:
2 db increase in the 2nd harmonic 2f (N = 2)
3 db increase in the 3rd harmonic 3f, (N = 3)
4 db increase in the 4th harmonic 4f, (N = 4)
……………………………………………….etc.

Question
For a 2 dB increase in the fundamental, what are the dB-increases of the 2nd and 3rd harmonics?
Answer: ____

On the next page we show a simulation of the increasing presence of higher aural harmonics as the amplitude, and hence the intensity, of the sinusoidal wave is increased. As a consequence, the timbre of the tone, as "heard" by a listener, has changed from a pure sine tone to a complex tone with higher harmonics. Of course, the tone arriving at the ear still is a pure sine wave!

Simulation of Stronger Aural Harmonics with Increasing Intensity



Top Figure. Three sine waves of the form **y(t) = Asin(2πft) + B** are shown, where the amplitude A of the waves increases from left to right, and B is a constant chosen to provide non-zero signal bias of the basilar membrane (so that a logarithm may be taken).
The three sine waves have a frequency f = 1000 Hz = 1 kHz (fundamental frequency).
The pairs of constants A and B chosen for the sine waves are, from left to right:
A = 1.0 and B = 1.1 A = 5.0 and B = 5.1 A = 2500 and B = 2500.1
Middle Figure. The logarithm curves of the three sine waves from the top row are shown. This is intended to simulate the logarithmic, non-linear response of the cochlea of the inner ear to sound intensity.
Bottom Figure. The Fourier spectra of the three log-response curves from the middle figure are shown. In addition to the fundamental frequency, with an assumed amplitude 1, higher harmonics or simulated *aural harmonics* now appear. It is seen that with increasing sound intensity, higher harmonics become more and more prominent. The tone quality or timbre thus becomes increasingly “spicier” or more “brilliant”.

Aural Combination Tones, Difference Tones, Sum Tones

When two tones of fundamental frequency f_1 and fundamental frequency f_2 are played simultaneously with comparable intensity, additional tones, called *combination tones*, are produced. These are a result of *aural harmonics* caused by the non-linear response of the ear to an external auditory stimulus. The origin of this non-linearity originally was thought to be in the outer and middle ear. Hermann Helmholtz ascribed the non-linearity to the curved eardrum (tympanum) connected to the hammer of the middle ear. However, more recent experiments on animals indicate that the frequencies of combination tones are not present at the oval window. Instead, they come from distortions of the original waveform in the cochlea of the inner ear. The combination tones have frequencies that are a combination of the two frequencies f_1 and f_2 and their aural harmonics, i.e. $2f_1, 3f_1, 4f_1$…. and $2f_2, 3f_2, 4f_2$…. as follows:

$$\mathbf{f = \left| nf_1 \pm mf_2 \right|},$$

where n and m are the integers 1, 2, 3, … The *minus* sign produces *difference tones* and the *plus* sign *sum tones.* The sum tones usually are very difficult to hear, and may not have been heard at all so far, because they have higher frequencies, are masked by tones of lower frequency, and have smaller amplitudes. On the other hand, difference tones can be heard more easily. Guiseppe Tartini (1692-1770), Italian composer and violin virtuoso, used difference tones for tuning double stops on his violin while performing violin concertos.
The most easily heard difference tones have the frequencies

$$\mathbf{\Delta f = f_2 - f_1 \qquad \Delta f = 2f_1 - f_2 \qquad \Delta f = 3f_1 - f_2}$$

where $f_2 > f_1$. The 2nd and 3rd difference tones here are best heard when $1.1f_1 < f_2 < 1.3f_1$.

Demonstrations – Missing Fundamentals and Difference Tones

1. We can demonstrate difference tones with the same experimental setup used for beats. Play two pure loud tones of similar amplitude and frequency. We hear beats if the two frequencies are within about 10% of each other. If you increase the frequency separation further, the sound becomes rough. Eventually the two frequencies can be distinguished, and the coarseness between them becomes a low frequency tone in the audible range. This tone is the *difference tone.*
Start with $f_1 = f_2 = 700$ Hz. Increase f_2 until you hear the coarse difference tone.
2. Use a third frequency generator and play a sine tone of the difference frequency. Does the sine tone have the same frequency as the coarse difference tone in the preceding demonstration?
3. Consider the pure harmonics in the harmonic series

A3	A4	E5	A5	C$^{\#}$6	E6
220	440	660	880	1100	1320 Hz

Use two frequency generators and play two loud sine tones with $f_1 = 1100$ Hz and $f_2 = 1320$ Hz. Can you hear a tone with $\Delta f = f_2 - f_1 = 1320 - 1100 = 220$ Hz = A3?
The frequency Δf in this case is both a difference tone and missing fundamental.
P.S.: On a piano the difference Δf will not be exactly equal to the frequency of the missing fundamental because the piano is tuned to "equal temperament" rather than to pure harmonics. On the piano we have E6 - C$^{\#}$6 = $\Delta f = f_2 - f_1 = 1318.5 - 1108.7 = 209.8$ Hz, but A3 = 220 Hz.

4. Use a third frequency generator to notice the difference tone more clearly. Set the frequency to the difference $\Delta f = 220$ Hz. Listen and decide whether this is the pitch of the difference tone from the preceding demonstration. If you are having trouble hearing the difference tone, slightly vary the frequency of the third audio generator and listen for beats between its frequency and the 220 Hz of the difference tone A3. If you hear beats you are "hearing" the difference tone.

5. Use available sound files and play the intervals B^b6-G6, B6-G6, C7-G6, D^b7-G6, D7-G6.
a) Can you "hear" missing fundamentals from these intervals?
b) What are the fundamentals corresponding to these frequency differences?
Partial answers: B^b6 – G6 = E^b4, D7 – G6 = G5.

Question for Musicians
Consider pure harmonics and show that the difference between B^b6 and G6 is the note E^b4. From the piano keyboard we know that E^b4 = 311 Hz but for pure harmonics B^b6 – G6 = 297 Hz. Why does this not equal 311 Hz?
Answer: From "equal temperament tuning" of the piano we have 311 Hz, from exact Pythagorean intervals (pure harmonics) we have 297 Hz.

6. **Difference Tones from a Contrabass**
Professor Mark Morton from the School of Music at Texas Tech University played his contrabass in class. When he played with high loudness the notes $C^\#$5 (550 Hz) and E4 (330 Hz) together, some could hear the difference tone (here also missing fundamental) A3 = 220 Hz.

Distinction between "Missing Fundamental" and "Difference Tone"
Difference (and sum tones if they ever can be "heard") are caused by the non-linear response of the ear to sound intensity. They are produced by the waves in the cochlear fluid and are physically real. On the other hand, "hearing" a missing fundamental comes from the frequency differences between harmonics. The resulting missing fundamental seems to be created in the brain from electrical nerve impulses. It does not originate as a physical phenomenon in the ear.

Difference Tones, Fundamental Tracking, Small Loudspeakers and Cell Phones
Bass notes may be "heard" from small speakers of transistor radios and cell telephones in spite of the fact that such low frequencies cannot possibly be reproduced by these speakers.
Two phenomena act together and provide an explanation:
1. *Difference tones* produce a low frequency due to the non-linear response of the ear.
2. The *fundamental tracking mechanism* in the brain analyzes intervals of harmonics and adds the missing fundamental.

Note on Sum Tones and Research Project
We have only been able to "hear" difference tones in class, but not so-called "sum tones" from two sine waves. An interesting research project would be finding beats between an anticipated sum tone and a nearby frequency from a signal generator. Hearing such beats would be a clear indication for the presence of sum tones.

A Test for Difference Tones and Missing Fundamentals

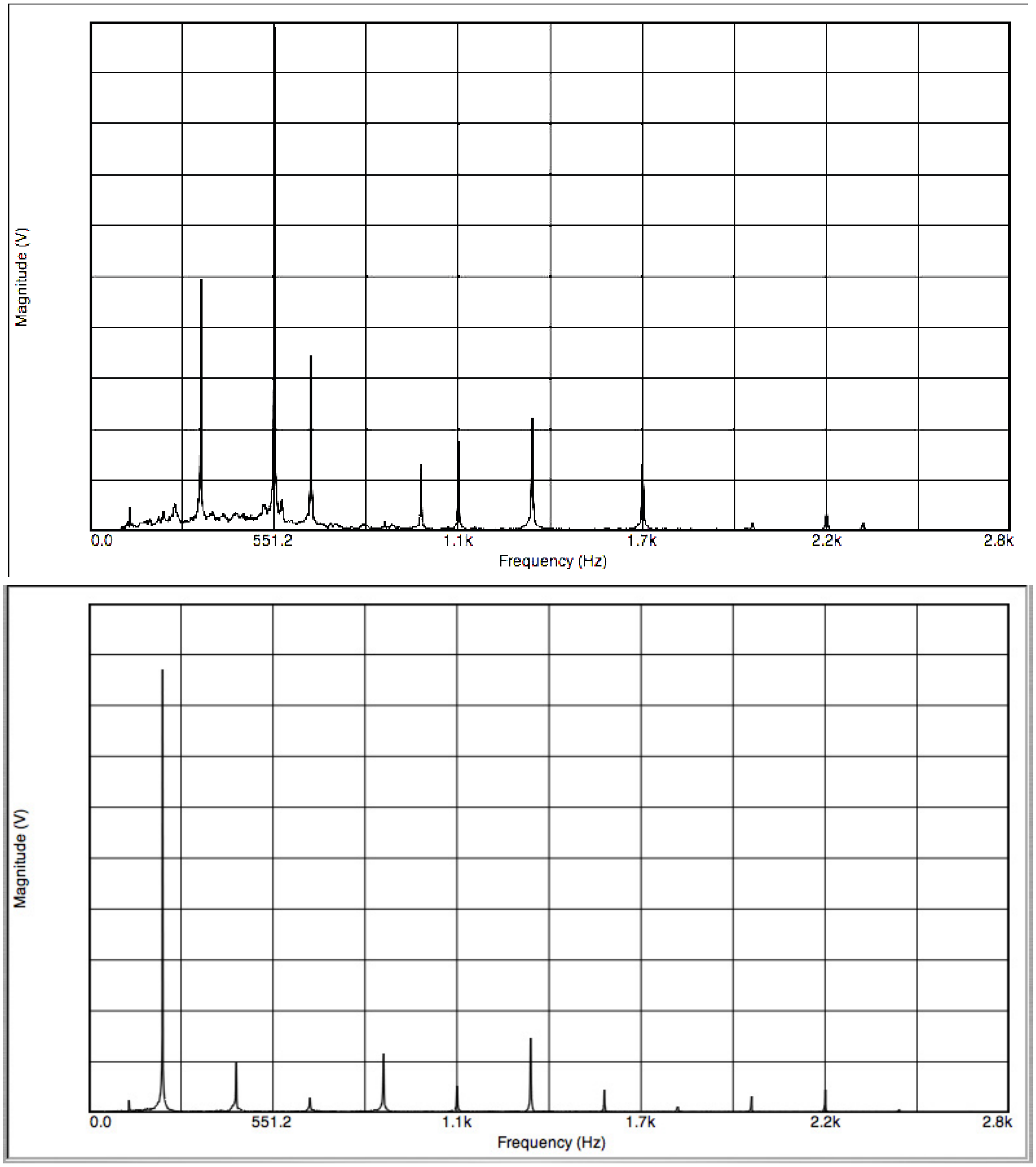


Figure. Top panel: Notes E4 (330 Hz) and C5# (550 Hz) were played simultaneously at high intensity on a double bass by Professor Mark Morton, Texas Tech University. The fundamentals and harmonics of the two notes are seen. The difference frequency 550 Hz – 330 Hz = 220 Hz (A3) is absent in the recording but was "heard" by several students.
Bottom panel: The note A3 (220 Hz) was played separately to show where the difference tone would have been. It is absent in the recording but was produced by the auditory system for those who could hear it.

Ohm's Law of Hearing, First and Second Order Beats

Ohm's Law of Hearing : "The timbre of a complex tone is given solely by the amplitude spectrum of the harmonics. The relative phases between the harmonics are unimportant."

First Order Beats or Primary Beats
These beats are consistent with Ohm's law. The phases between the two beating tones do not matter in what you hear. It makes no difference when you start the tones. They interfere with each other and produce a tone with a slow amplitude modulation that varies with the beat frequency $\Delta f = | f_2 - f_1|$. The frequency we hear is the average $f = (f_2 + f_1)/2$ of the two frequencies. The timbre or sound quality of the beating tone does not change with time.

Second Order Beats or Quality Beats
When the frequencies of two pure tones are about one octave apart, so-called second order or quality beats may be heard at the beat frequency $\mathbf{\Delta f = |f_2 - 2f_1|}$, where now $\mathbf{f_2 \approx 2f_1}$. For good audibility $f_1 < 1500$ Hz is needed. *The phase of the resultant waveform changes continually*. The ear can detect changes in the waveform and thus in the phases as the sound quality changes with time. The amplitude does not change significantly. Ohm's law no longer applies to this rather subtle effect. Second order beats can be heard when playing soft tones where non-linearities in the ear are unimportant. Hence these beats are not created from aural harmonics that are caused by non-linearities in the ear. Rather, quality beats appear to be created directly in the brain.

Calculated Air Displacement

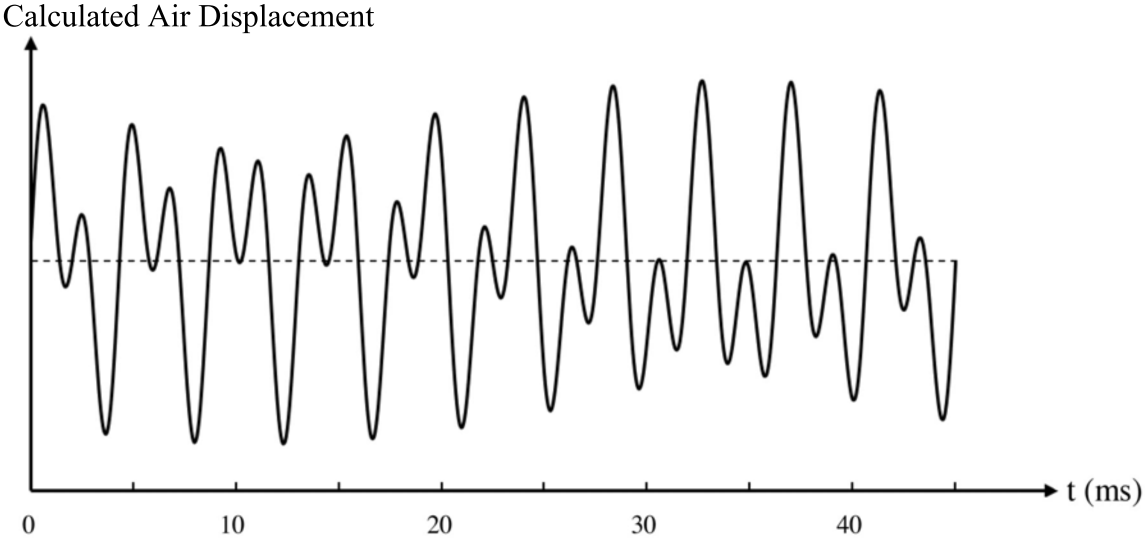


Figure. Calculated second order or quality beats. The waveform shown is the combination of a 222 Hz sine wave and its mistuned octave 466 Hz. The amplitude of the two sine waves is the same. The beat frequency is $\Delta f = |f_2 - 2f_1| = 466 - 444 = 22$ Hz.
Shown is one full cycle of second order beats with a period of about 45 ms.

Question
Find the frequency of the 2nd order beats from the graph. Compare with the calculated result.
Answer: From graph: $\Delta f =$ ________ Hz, calculated: $\Delta f =$ ________ Hz

First and Second Order Beats (continued)

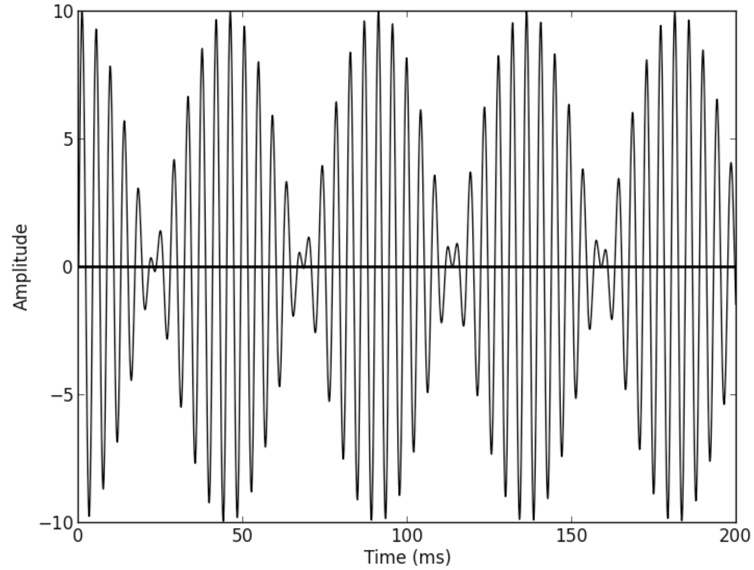


Figure. Waveform of first order beats between two sine waves of equal amplitude, with $f_1 = 222$ Hz, $f_2 = 244$ Hz, $\Delta f = f_2 - f_1 = 22$ Hz.

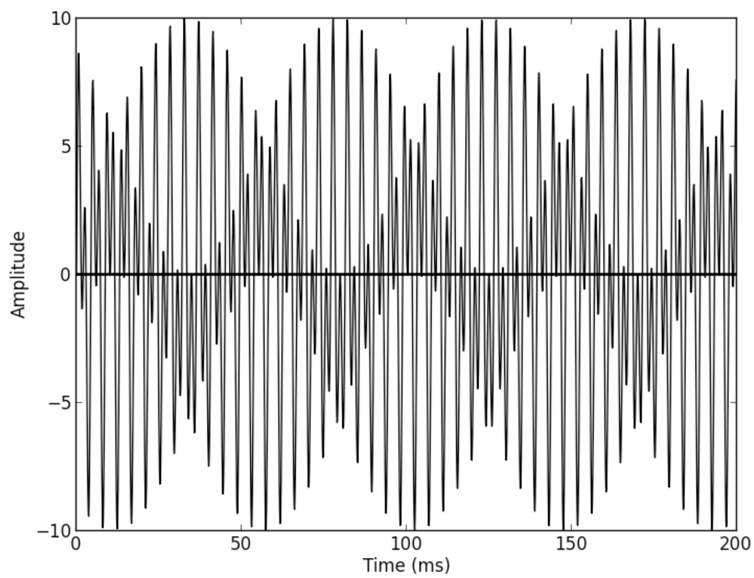
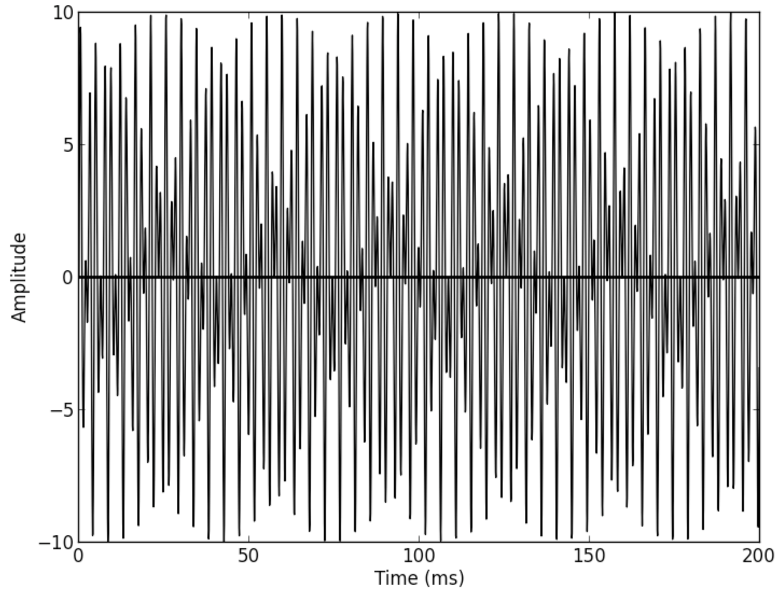
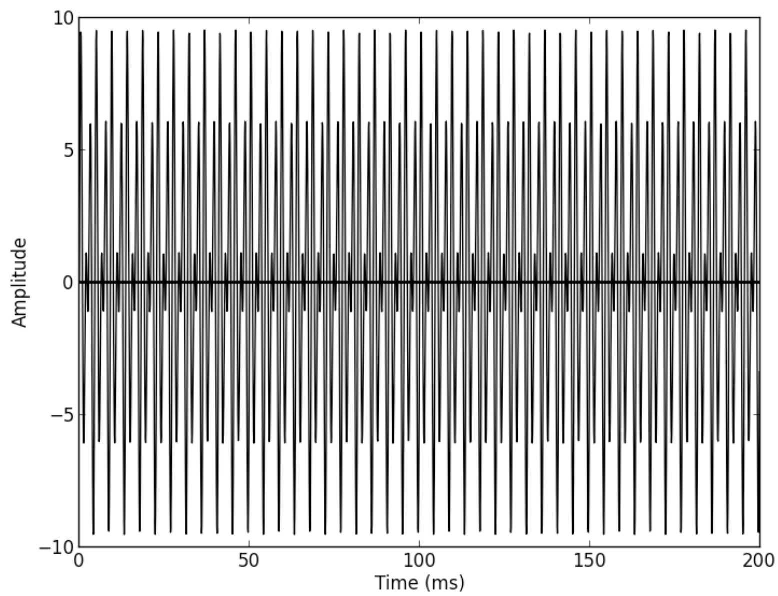


Figure. Waveform of mistuned octave. Second order beats are shown between two sine waves of equal amplitude, $f_1 = 222$ Hz, $f_2 = 466$ Hz, and $\Delta f = f_2 - 2f_1 = 466 - 444 = 22$ Hz.

Second Order Beats (Quality Beats) and Mistuned Consonances
Quality beats not only occur between mistuned octaves as described above, but more generally between mistuned harmonics. As an example of mistuned fifths we consider the open violin strings A4 (f_1 = 440 Hz) and E5 (f_2 = 660 Hz). The first figure below shows the waveform for a highly mistuned E5 string at f_2 = 675 Hz, the second figure shows an exactly tuned fifth.





Top Figure. Simulation of the waveform of a mistuned fifth and resulting second order beats from a violin. Two sine waves of equal amplitude are assumed, with f_1 = 440 Hz (A4 string), f_2 = 675 Hz (mistuned E5 string), and $\Delta f = 2f_2 - 3f_1 = 1350 - 1320 = 30$ Hz.
Bottom Figure. The waveform of the exactly tuned fifth shows no beats (Δf = 0 Hz).

Demonstrations

1. Use two frequency generators with $f_1 = 360$ Hz, the other at $f_2 = 362$ Hz, with loudness about the same. Listen to the first order beats having a beat frequency of $\Delta f = 2$ Hz. Note the intensity variations without a change in timbre.
2. Use two frequency generators, one at $f_1 = 360$ Hz and the other at a mistuned octave of $f_2 = 720 + 2 = 722$ Hz. Listen to the quality beats with a beat frequency of $\Delta f = f_2 - 2f_1 = 722 - 720 = 2$ Hz. Note the second order beats without much change in intensity. Choose the intensity of the higher frequency about equal to half the intensity of the lower-frequency tone. Furthermore, choose $f_1 < 1500$ Hz.
3. Listen to tracks demonstrating beats on the CD "Auditory Demonstrations" by the American Acoustical Society (Track 62, track 63).
4. Set two sine wave generators to the frequencies 222 Hz and 466 Hz as in the preceding figure and show the resulting waveform on the computer monitor. Can you hear the 2nd order beats?

The Importance of Ohm's Law in Sound Reproduction

Sound recording and reproduction systems change the relative phases of harmonics in many ways. Luckily, Ohm's law still applies and one can hear music sounding like the original, because we are not sensing the phase distortions.

Question

Good recordings do not sound distorted in spite of the many phase distortions in the electronics. Why is this so? (Hint: Which quantities in Fourier's theorem generally do we not "hear"?)

Answer: ____

Mistuned Consonances

Second order beats also may be heard between *mistuned consonances* such as mistuned fifths and fourths. These can be used in tuning pianos and string instruments. Good tuning is achieved once second order beats have been eliminated.

Demonstrations

1. Mistuned fifths: Play two tones with $f_1 = 360$ Hz, and $f_2 = 540 + 2 = 542$ Hz. Do you hear beats? What is the beat frequency? Does this work with sine waves or only complex waves?

Answer: The beat frequency is $\Delta f = 4$ Hz, not 2 Hz as we might think! Explanation: Consider the harmonics and take $2f_2 - 3f_1 = 1084 - 1080 = 4$ Hz

2. Play two tones with $f_1 = 200$ Hz, and $f_2 = 301$ Hz. The beat frequency is $\Delta f = 2$ Hz!
3. Play two tones with $f_1 = 201$ Hz, and $f_2 = 300$ Hz. The beat frequency is $\Delta f = 3$ Hz!

Explain these two puzzling results similar to the explanation given for the first demonstration.

Binaural Beats

Use earphones to play a soft tone into one ear and another one of similar frequency and loudness into the other ear. The resulting beats move "around the head" and are called *binaural beats*. No physical mixing of the two waves can occur in this case, the only mixing occurs in the brain. Binaural beats have been used for therapeutic purposes, specifically in "music therapy".

Demonstration

Listen to CD track 71, binaural beats, from "Auditory Demonstrations". Unfortunately we cannot do this in class, because earphones need to be used for this effect.

Auditory Masking

Auditory masking occurs in the presence of two or more tones, when one tone “drowns” out or *masks* the other.

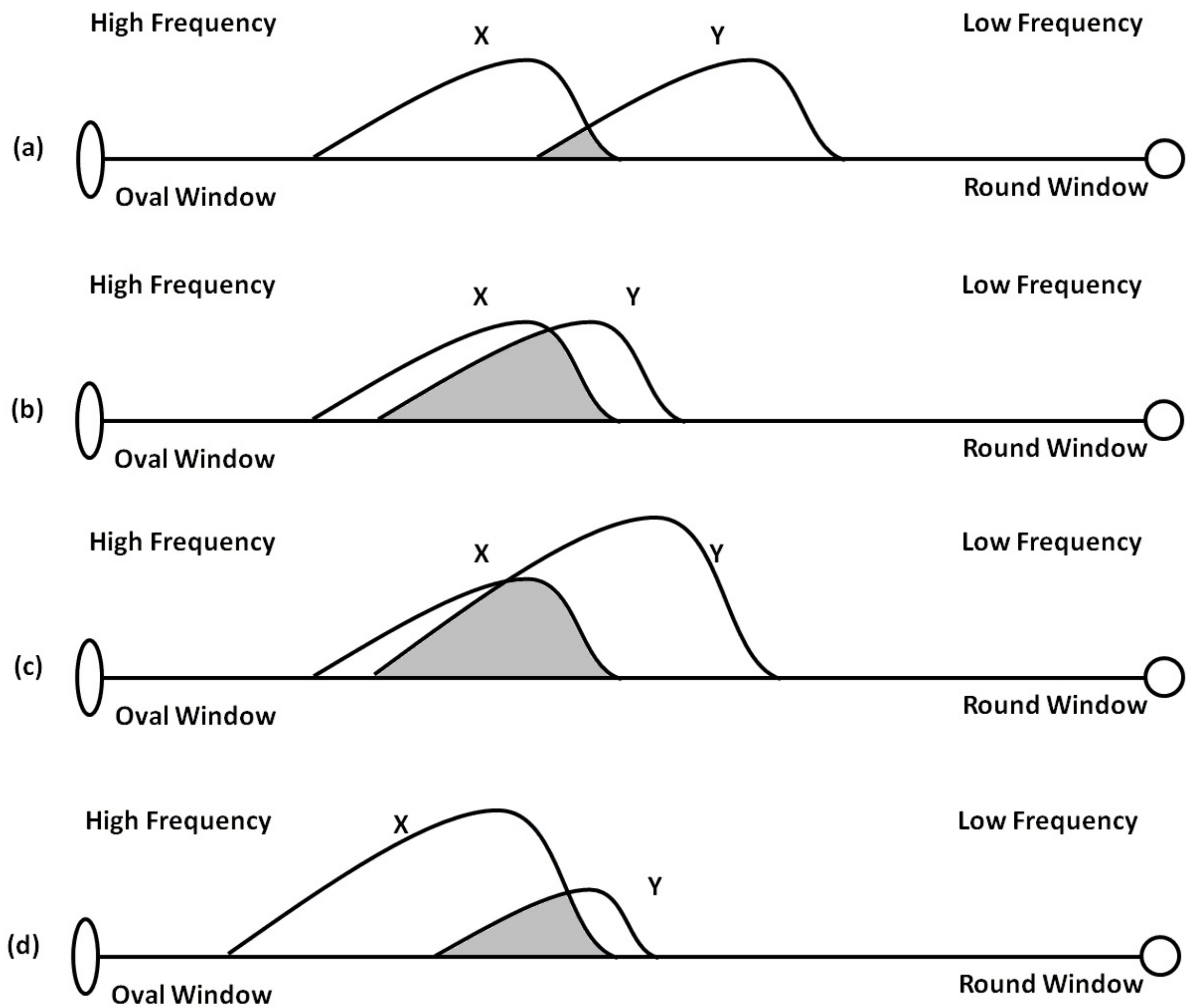


Figure. Masking explained schematically with the overlap of critical bands for two pure tones X and Y on the basilar membrane.
(a) The two critical bands hardly overlap. Little masking occurs.
(b) The two curves have appreciable overlap. The lower frequency tone Y masks the higher frequency tone X somewhat more than X masks Y.
(c) The lower frequency tone Y is more intense than the higher tone X and almost completely masks X.
(d) The higher frequency tone X is more intense, but does not completely mask the lower tone Y.
(Figures partly adapted from Thomas D. Rossing, The Science of Sound, 3rd edition Fig. 6.10, p. 114, Addison-Wesley, 2002.)

Exercise
Shade the regions of overlap between tones A and B in the figure to understand better the masking effect in the four cases (a), (b), (c), and (d).

Masking and Masked Tone
The *masking tone* is the tone drowning the *masked tone.*

Masking Level
The masking level is the minimum sound intensity level (SIL in dB) to which the masked tone has to be raised to become audible. Masking strongly depends on frequency.

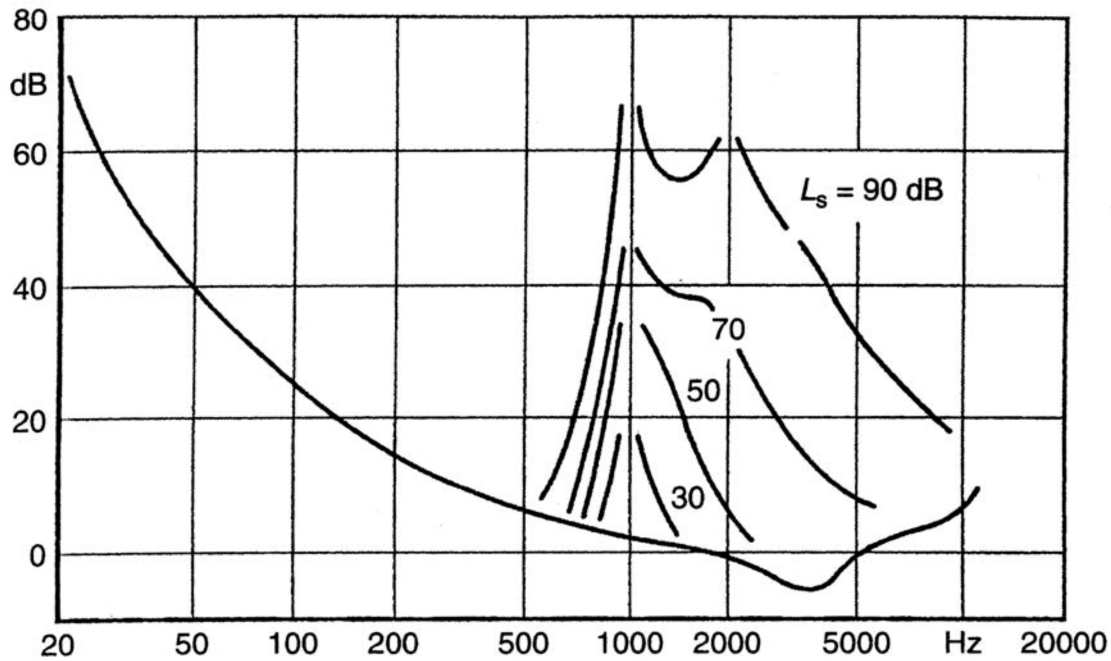


Figure. Masking by a 1000 Hz masking tone at sound intensity levels of 30, 50, 70, and 90 dB. The SIL on the ordinate and frequency on the abscissa are for the masked tone. In the absence of the masking tone, the threshold of hearing is given by the lowest curve. This threshold is raised in the presence of a masking tone as shown by the four labeled masking curves. The asymmetry in these curves, with a steeper rise at lower frequencies, indicates that low frequency tones below the given masking frequency of 1000 Hz are masked less than above. A tone is inaudible below the masking curve and becomes audible only above it. For instance, for the 1000 Hz masking tone at 70 dB, a masked tone at about 1500 Hz becomes audible only if it exceeds 40 dB. (From: Jürgen Meyer, Acoustics and the Performance of Music, 5th ed,. p. 11, Springer, 2009.)

Case 1. Special case: The frequencies of the masked tone and masking tone are the *same*. Masking is very effective. We know that the intensity JND (Just Noticeable Difference) is about 1 dB = 26%. Hence for a masking tone having an SIL = 70 dB, the masked tone must be raised to 69 dB to become audible. This masking level of 69 dB is high.

Case 2. The masked tone has a lower frequency (800 Hz) than the masking tone (1000 Hz). Masking is not very effective. For example, if a 1000 Hz masking tone has an SIL = 70 dB, the level to which an 800 Hz masked tone has to be raised and heard is only about 25 dB. The masking level is 25 dB. In other words, it takes a high intensity of the higher frequency masking tone to mask a low frequency tone. Low frequency tones easily mask high frequency tones.
Case 3. This is the opposite case to Case 2. The masked tone has a higher frequency (e.g. 1500 Hz) than the masking tone (1000 Hz). Masking is more effective. For example, if the 1000 Hz masking tone again has an SIL = 70 dB, the level to which a 1500 Hz masked tone has to be raised to become audible is about 40 dB. Thus the masking level is 40 dB. This is 15 dB higher than the masking level in Case 2.

Demonstrations on Auditory Masking
Case 1. Special case: Take $f_{masked} = f_{masking} = 1000$ Hz. Play the masking tone at moderate intensity while raising the intensity of the masked tone until you hear it. Now play both tones separately. They sound almost equally loud and are separated by about 1 dB, which as we know, is the intensity JND. Thus the masking level for the masked tone is almost as high as the intensity of the masking tone.
Case 2. Use two signal generators and select 800 Hz $= f_{masked} < f_{masking} = 1000$ Hz. Raise the level of the masked tone until it becomes audible. Take a mental note of the intensity of the masked tone and the masking tone (by ear is sufficient).
Case 3. Select 1200 Hz $= f_{masked} > f_{masking} = 1000$ Hz on the two signal generators. Raise the level of the masked tone until it becomes audible. Take a mental note of the intensity of the masked tone and the masking tone (by ear is sufficient).
Question
In which case, Case 2 or Case 3, did we have to raise the masking level of the masked tone more in order for this tone to become audible?
Answer: ____

Other Demonstrations
Play sound tracks of "Masking Differences" from the CD entitled "Auditory Demonstrations" by the Acoustical Society of America (ASA).

The Significance of Masking
1. Polyphonic Music
Johann Sebastian Bach's polyphonic fugues sound more transparent when played softly, because the masking effect decreases with decreasing loudness. Furthermore, when the melodic lines in polyphonic music are close in frequency, a single melody with rich harmonies may be heard. When the melodic lines move farther apart, the individual melodies can be heard.
2. MP3 Sound Files
Motion Picture Experts Group, Audio Layer 3 (MP3) files are compressed audio files. In the compression process, data are discarded that would be inaudible due to efficient masking of closely spaced frequencies. This results in a reduced file size of about one tenth of the original. However, some of the nuances in the music may be lost and this may be noticeable to a connoisseur.

Subjective Loudness. What is "Twice as Loud"? Phons and Sones

When you increase the loudness level of a tone 10-fold, for instance for a 1000 Hz tone from 40 phon to 50 phon, the perceived loudness is not 10-times higher. Instead, many people will say that the sound is about "twice as loud".
In order to express loudness quantitatively, the unit of *sone* was created.
By definition, **1 sone corresponds to 40 phons for a 1000 Hz sine tone.**
For a loudness level of 50 phon, the loudness is 2 sones, for 60 phon it is 4 sones, for 70 phons it is 8 sones, for 80 phons it is 16 sones, etc. The sone number thus doubles for every 10 phon increase in loudness level.

Figure. Subjective loudness in *sones* versus loudness level in *phons* for a 1000 Hz tone.

Questions
1. What is the perceived loudness in sones for a 1000 Hz tone at 70 phons?
Answer: This is a 30 phons increase from 40 phons, so the loudness is $2^3 = 8$ sones

2. The loudness of a motor increases from 16 to 32 sones. What is the loudness level and the change in loudness level in phons?
Answer
The loudness of 16 sones corresponds to 80 phon, 32 sones correspond to 90 phons, i.e. the loudness level increases by 10 phon.

Example
A quiet bathroom fan may have a noise rating of 1 sone, a rather loud one may produce 4 sones.

An improvement to the straight line dependence between loudness and loudness level in the preceding figure shows that the loudness depends significantly on the frequency for a given sound intensity level (see following Figure).

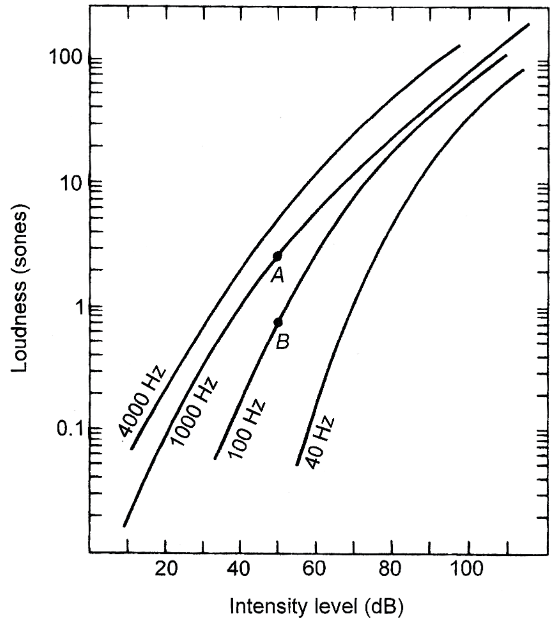


Figure. Subjective or perceived loudness in sones versus sound intensity level in dB for sine waves of different frequencies. A doubling in the number of sones means "twice as loud". (From Donald E. Hall, Musical Acoustics, 3rd edition, Fig. 6.10, p.102, Brook/Cole, 2002.)

We have for a 1000 Hz sine tone at 40 phons a loudness of 1 sone and SIL = 40 dB by definition. At point "A" in the Figure, the loudness of a 1000 Hz tone with SIL = 50 dB is about 2.5 sones. "Twice as loud" in this case (i.e. 5 sones) means SIL = 60 dB, which is the 10 dB increase stated earlier. For the 100 Hz curve we have 0.7 sones at 50 dB. "Twice as loud" or 1.4 sones is reached at about 56 dB, corresponding to an increase of only 6 dB. All four curves shown become parallel above 90 dB, and "twice as loud" means a 12 dB increase from then on.

The curves shown are an improvement over the older definition where a 10 dB-increase in the SIL was said to be "twice as loud" at all frequencies. We now see that the dB-increase for "twice as loud" depends on both frequency and SIL.

Exercise
Show from the 1000 Hz curve that "twice as loud" means an increase of about 6 dB at low intensity levels, and about 10 dB at high levels.

Demonstration
Increase the intensity of a tone and ask at what point it sounds "twice as loud".
Is your answer closer to an increase of 6 dB or 10 dB?

Challenge Question
Show that the intensity of a sound decreases by a factor of 4 with a doubling of the distance from the source according to the inverse square law. Show that this means a 6 dB decrease in the SIL and thus a sound about "half as loud".
Answer: ____

The Human Vocal Tract and Voice

The human vocal tract can be considered as a group of Helmholtz resonators acting together:
Larynx

Oral pharynx	Nasal pharynx
Oral cavity	Nasal Cavity

Other anatomical parts of the vocal tract playing a role in sound production are the lungs, trachea, vocal folds/cords, epiglottis, tongue, mouth, lips, teeth, soft palate, hard palate, and nostrils.

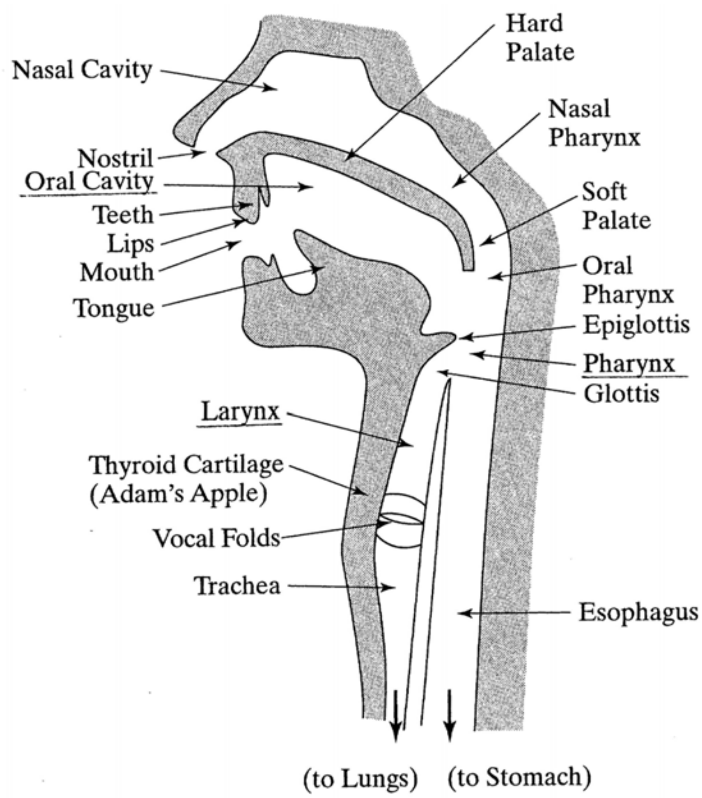


Figure. Schematic diagram of the human vocal tract. (From Berg & Stork, 3rd edition, Fig. 6-8, p. 167, Pearson/Prentice Hall, 2005.)

As the air pressure from the lungs builds up, the vocal folds open. A buzzing sound ("noise") is produced by the air rushing through the folds. This is similar to the functioning of the reeds in woodwind instruments. After a short time, the pressure decreases and the folds close. Then the cycle repeats itself. The cavities of the vocal tract act as a filtering system to the broadband noise from the "buzz". The overall response of the vocal tract causes the sound we hear. It may be musical (singing) or non-musical (speaking).

Bernoulli Effect

The closing and opening of the vocal folds when the air rushes through can be explained with the *Bernoulli Effect*. When the air velocity is high (low), the pressure is low (high) in the flow, and the vocal folds close (open). After the air pressure from the lungs rises again, the vocal folds open. This cycle repeats and produces a noisy buzz that enters the larynx and excites the air resonances in the cavities of the vocal tract.

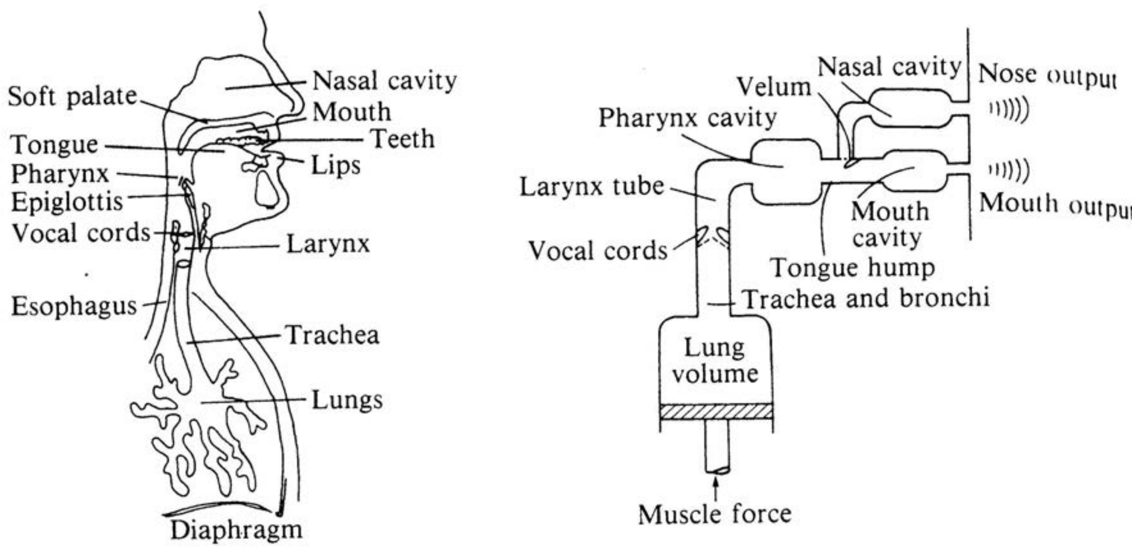


Figure. Human vocal tract and simplified schematic of the acoustical parts.
(From Thomas D. Rossing, The Science of Sound, 3rd edition, Fig. 15.1, p. 337, Addison Wesley, 2002.)

Lecture Demonstration – Bernoulli Effect
Hold two sheets of paper close together with a small gap between them. Now blow air between the sheets. They will move closer together rather than apart, because the air pressure is lower at the higher flow velocity that exists between the sheets compared to outside. When the sheets finally touch, the pressure increases and the gap opens again. As a result a buzzing sound is produced while air is blown. This simple experiment simulates the action of the vocal folds.

Vocal Formants and Singing Formant

The human vocal tract, with the oral cavity the major part, can be approximated as a tube of length L = 17 cm, with an "open" end at the lips and a "closed" end at the vocal folds.

Exercise

Calculate the first four existing resonance frequencies for a closed tube of length 17 cm.
Answer: Use the formula $f_1 = v/\lambda = v/4L$ and find the frequencies $f_1 = 500$ Hz, $f_3 = 1500$ Hz, $f_5 = 2500$ Hz, $f_7 = 3500$ Hz. These frequencies define the peaks of the so-called *formant regions.*

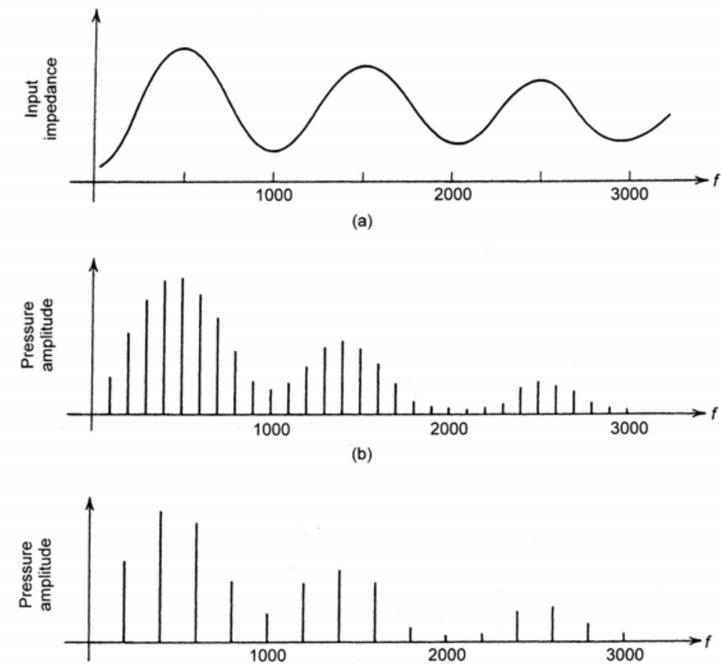


Figure. Top: Response curve of an idealized cylindrical "vocal tract" with three formant regions. Middle: Harmonics from the vocal folds with fundamental f = 100 Hz after passing through a cylindrical "vocal tract".
Bottom: Harmonics with a fundamental frequency of 200 Hz. (From Donald E. Hall, Musical Acoustics, 3rd edition, Fig. 14.8, p. 303, Brooks/Cole, 2002.)

Formants, Pitch and Harmonics

Note that the formants of the vocal tract define the overall frequency response of the resonating cavities. The formants are not sharp spikes at 500, 1500, 2500 Hz, as we would expect from a 17 cm long closed tube with rigid walls. In reality the resonances are weak and broadened because of the elasticity and softness of the cavity walls. The soft walls absorb much of the vibrational energy. This causes a broadening and lowering of the peak amplitudes. The broad maxima giving the overall response of the vocal tract are called *formants or formant regions*.

The *harmonics* in the formant regions come from the vocal folds and not from the oral cavities. The frequency with which they vibrate is controlled by muscle tension in the larynx. For the male voice the fundamental frequency of the vocal cord for speech ranges from 70 to 140 Hz, with integer multiples for the harmonics. For the female voice the range is 140 to 400 Hz plus harmonics. Good singers can extend these ranges by more than an octave when singing.

The vocal folds act as hard reeds whose frequency and sound output are adjusted by the cord tension, mass, and separation, not by the cavities of the vocal tract. When a male sings the note G2 at about 100 Hz, the vocal cords vibrate at this fundamental frequency and harmonics. When this sound from the vocal folds enters the vocal tract, the latter acts as a filter. This modifies the amplitudes of the individual harmonics (but not their frequencies) according to the shape of the formants. The broad formants can be adjusted in height, width, and frequency of the broad maxima by varying the geometry of the vocal tract.

Summary - Speaking and Singing

1. Harmonics from the vocal cords enter the vocal tract.
2. Broad resonances in the cavities of the vocal tract define the formant regions. The harmonics from the vocal tract excite the formants. The formants act as filters for the harmonics. Each formant region may contain several of the closely spaced harmonics.
3. Adjustment of the pitch of a tone is accomplished primarily by adjusting the tension in the vocal cords. The formants themselves are adjusted by varying the geometry of the cavities in the vocal tract.

Singing Formant

Accomplished singers have strong vocal and singing formants that can be louder than the accompanying orchestra. This applies, for instance, to operatic tenors with their strong singing formant around 2500 Hz. (The 2nd and 4th formants at 1500 Hz and 3500 Hz may be weaker.)

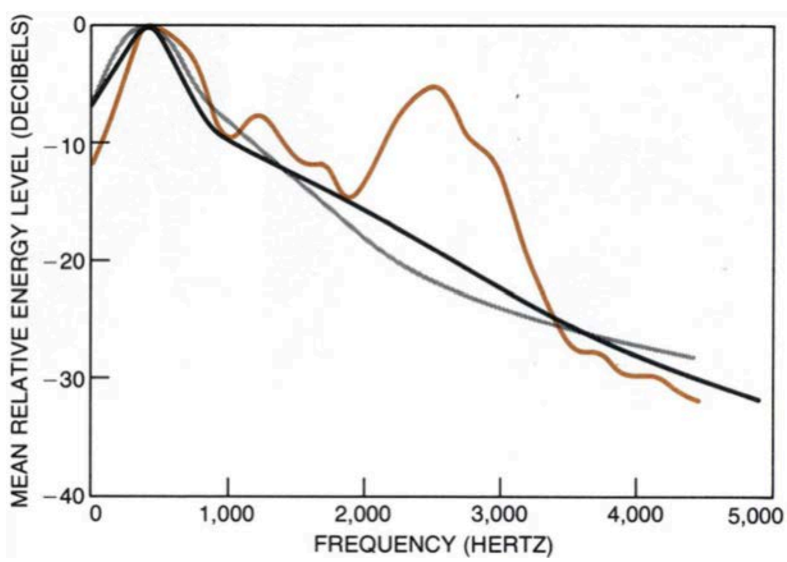


Figure. Intensity spectra in sound are shown from a good singer (brown curve), ordinary speech (light curve), and orchestral accompaniment (dark curve). The curves are normalized to 0 dB at about 500 Hz, which is the peak of the 1st singing formant. The pronounced 3rd singing formant around 2500 Hz enables the singer to be heard well above the orchestra.
(Reference: Johan Sundberg; The Acoustics of the Singing Voice; Sci. Amer. March 1977.)

Research Project

Compare the singing formants of male and female singers and write a report.

Singing Formants

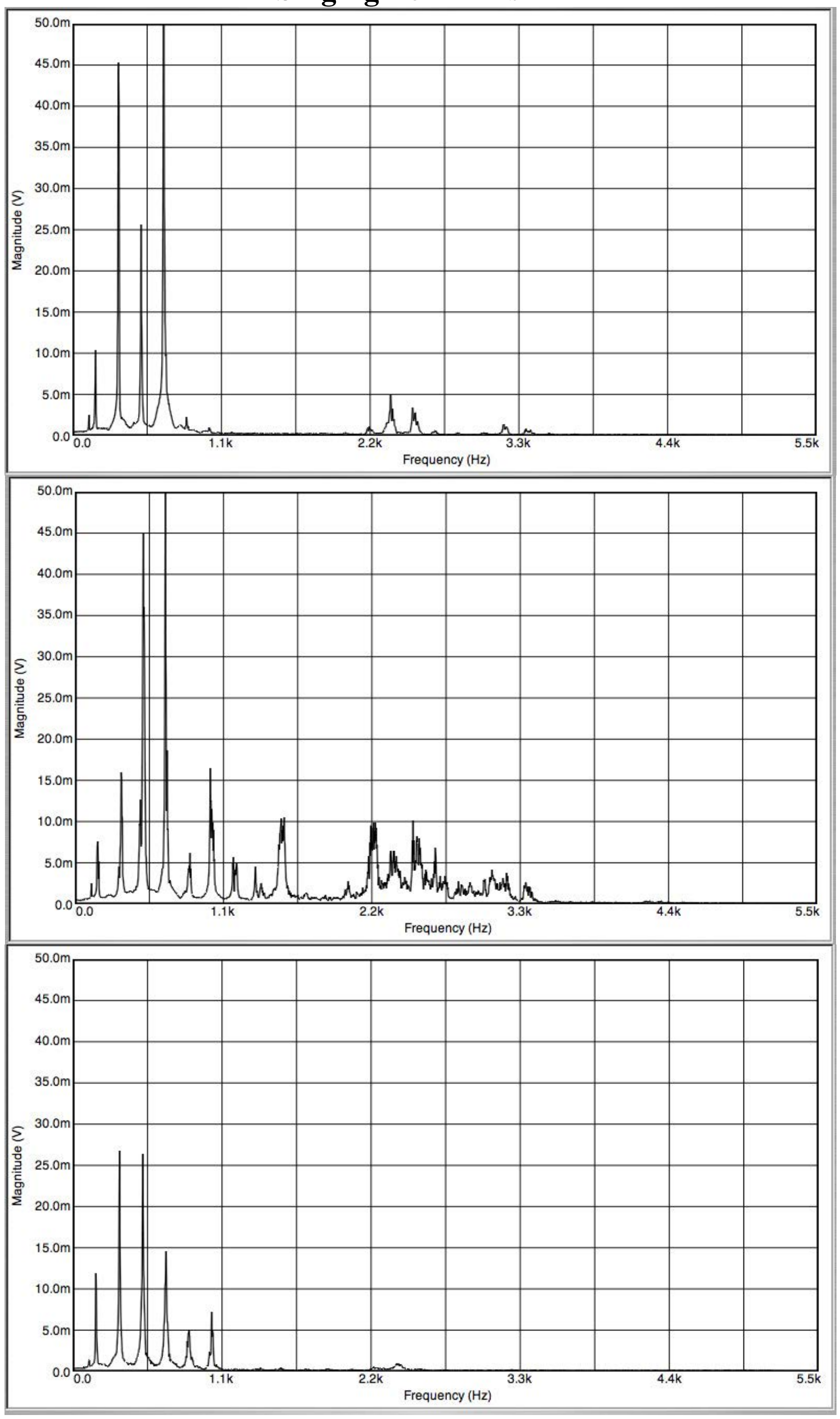
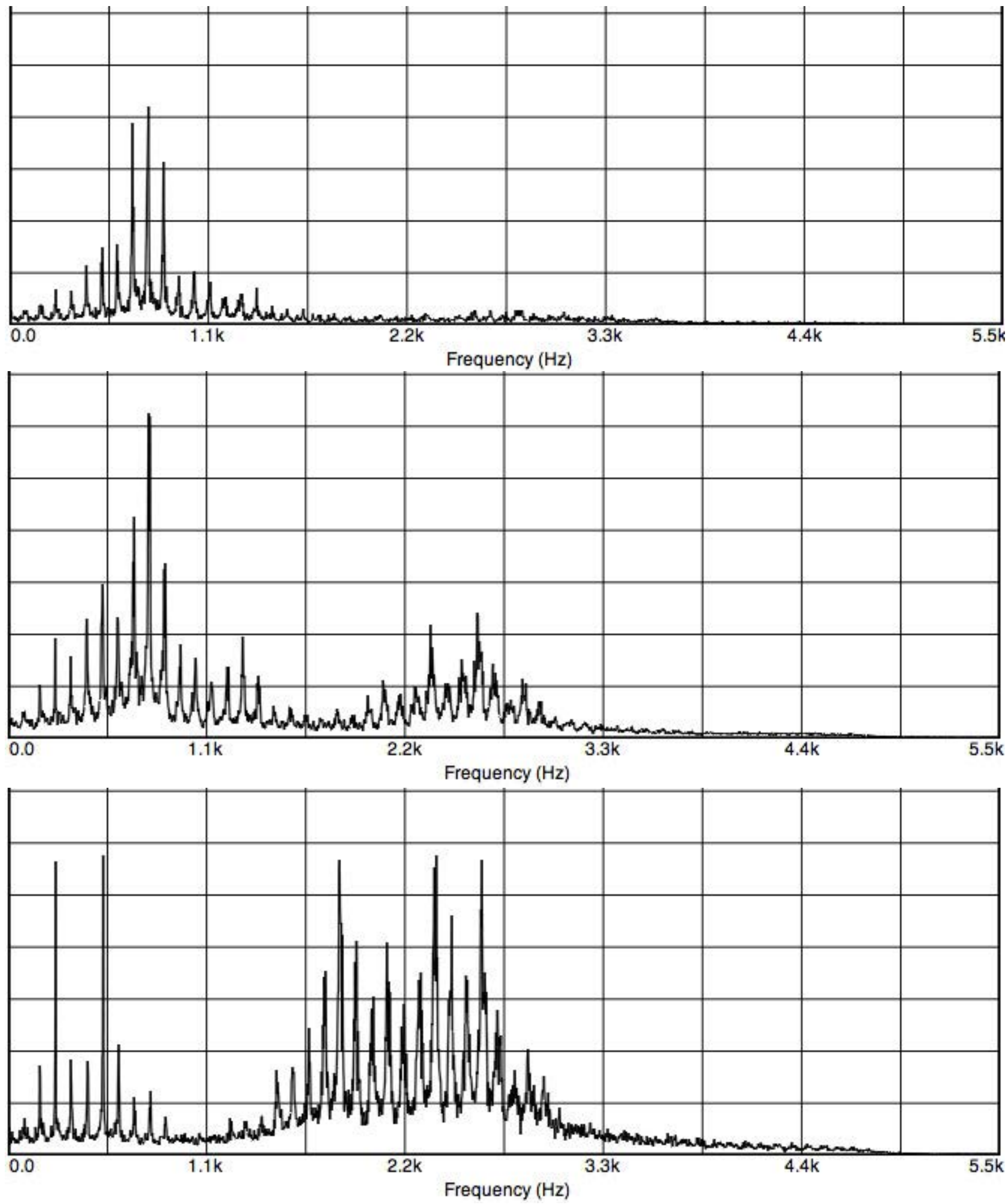


Figure. Vowel sounds at pitch a F3 = 175 Hz. Top: “oh”. Middle: “ah”, both sung by Professor Gerald Dolter, School of Music, Texas Tech University. Three formant regions are seen around 500, 1500, and 2500 Hz. Bottom: Vowel “ah” from an untrained voice. Only the formant region around 500 Hz is seen, no singing formant.

Singing Formants (continued)



Singing formants of a bass-baritone (Professor Gerald Dolter, Texas Tech University),
Note sung: F2 = 87 Hz
Top: Vowel sound “ah”, controlled shouting, without singing formant.
Middle: Vowel sound “ah”, singing formant between 2 and 3 kHz.
Bottom: Vowel sound “eh”, very strong singing formants between 1.5 and 3 kHz.

Overtone Singing

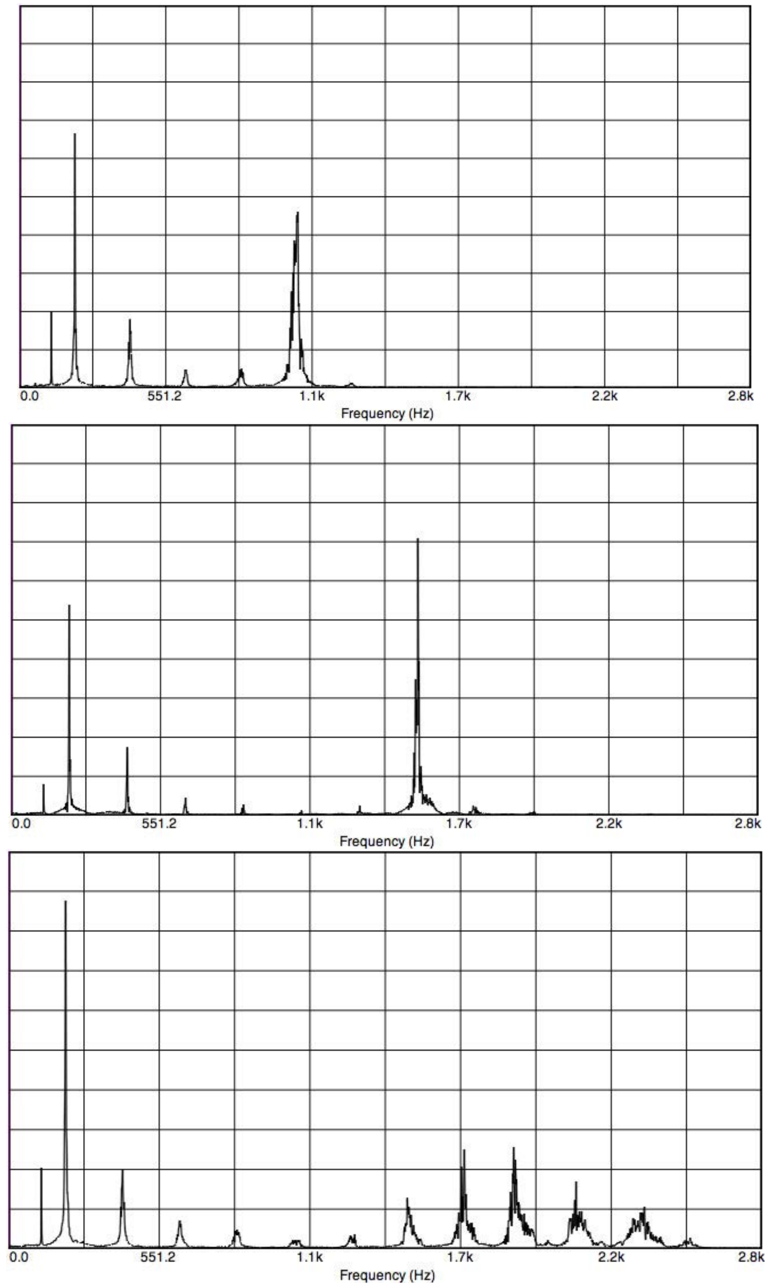


Figure. Harmonic or overtone singing by Dr. Stuart Hinds. Fundamental frequency f = 208.6 Hz or about $G3^{\#}$. (The lowest spike at 120 Hz is ambient noise from a fan.)
Top: Harmonic N = 5 stressed.
Middle: Harmonic N = 7 stressed.
Bottom: Harmonics N = 7, 8, 9, 10, 11 stressed.

Formants of Musical Instruments

Formants also exist in musical instruments. They show frequency groupings where the overtones are strong.

Formants of a Krummhorn

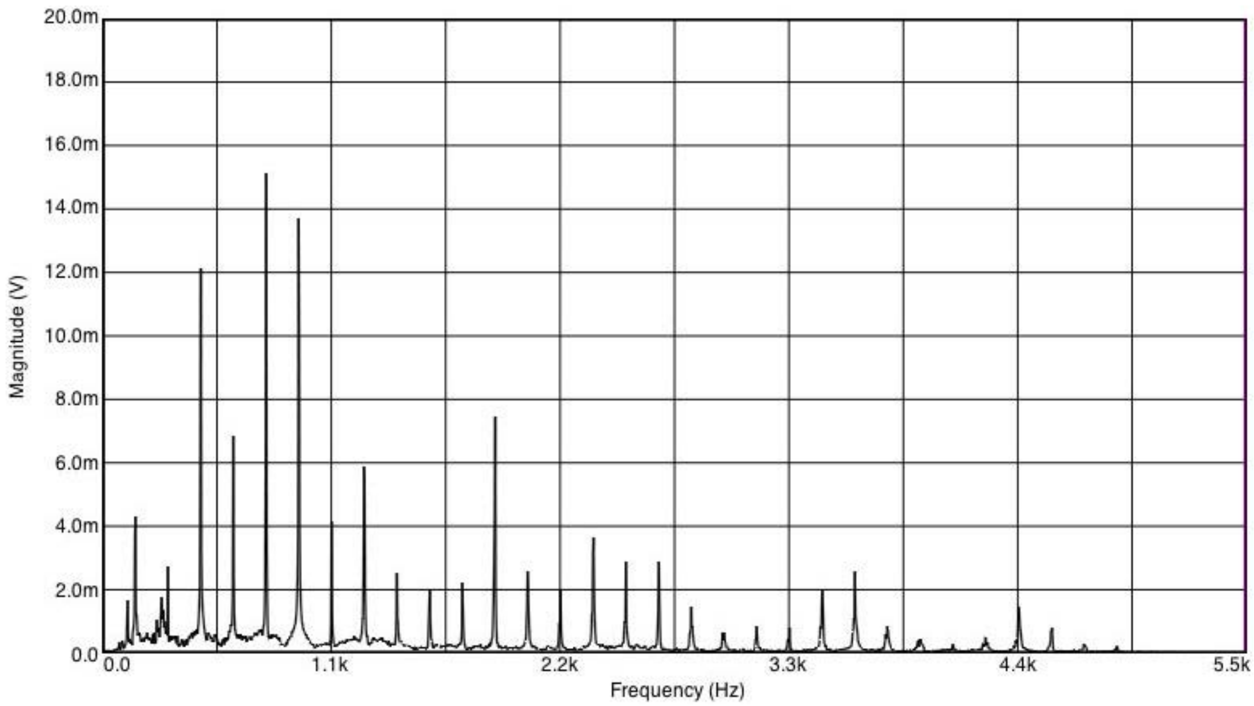


Figure. Fourier spectrum of the note E3 = 164.8 Hz from a Krummhorn (played by Professor Richard Meek, School of Music, Texas Tech University). Three formant groups of harmonics can be seen, centered around 600 Hz, 2400 Hz, and 3600 Hz.

Exercises

1. Draw a curve over the Fourier spectrum of the Krummhorn to highlight the formants. Read off the peak frequencies of the broad formant maxima.

Answer: Formant maxima at: _____ Hz, ______Hz, _____Hz, _____Hz, _____Hz

2. How many harmonics do you see in the overtone spectrum of the Krummhorn?

Answer: Number of harmonics: _________

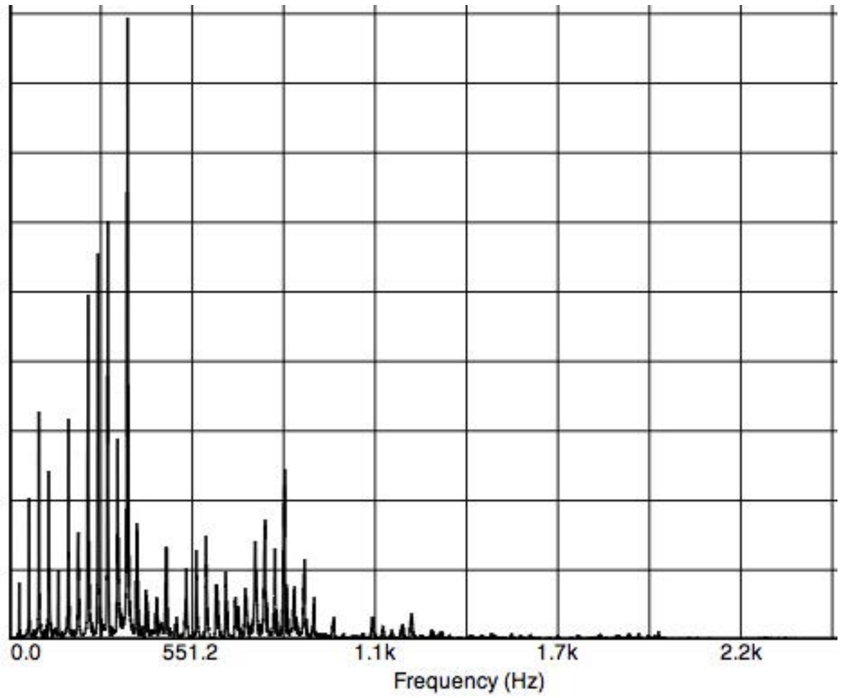
Demonstrations

Observe some formant regions in the Fourier spectra of instruments:

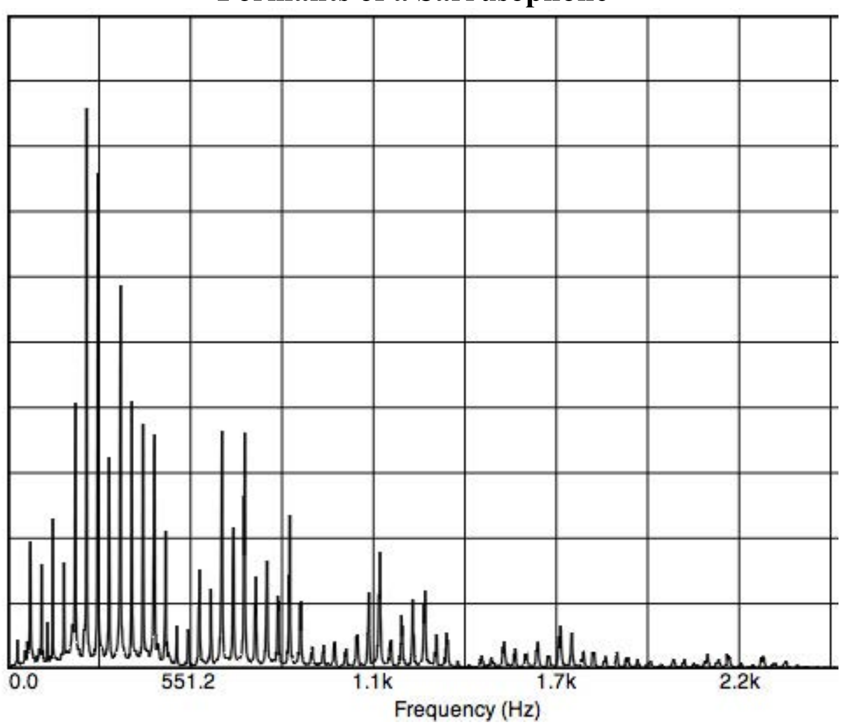
1. Krummhorn
2. Harmonica
3. Bassoon
4. Clarinet
5. Singing voice

and other available instruments.

Formants of a Contrabassoon



Formants of a Sarrusophone



Top figure: Two pronounced formant regions from a contrabassoon for the note $B0^b$ (29.1 Hz). Bottom figure: Four pronounced formant regions for the note $C1^\#$ (34.0 Hz) from a sarrusophone, a saxophone-like instrument. The amplitudes of the harmonics are shown on a linear scale. (Instruments played by Professor R. Meek, Texas Tech University.)

Intensity-Frequency Boundaries of Hearing, Music, and Speech. Presbycusis

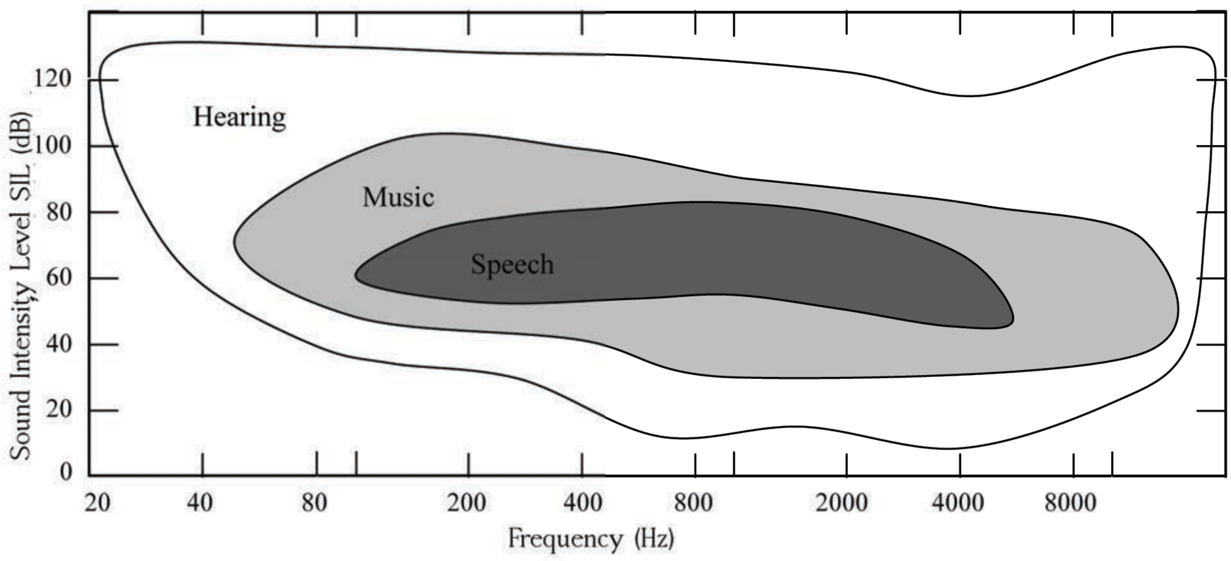


Figure. Intensity boundaries versus frequency boundaries for normal hearing, orchestral music, and speech. (Adapted from Irving P. Herman, Physics of the Human Body, Springer Verlag Berlin, 2007, Fig. 10.30, p. 594.)

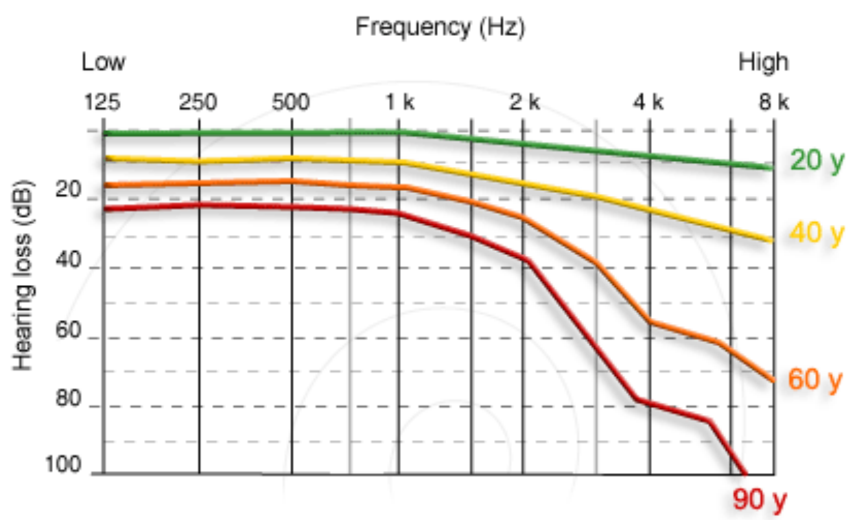


Figure. Age-related hearing loss (presbycusis) in decibel (dB) as a function of frequency, with age as a parameter on the colored curves. The loss can be severe beyond age 60 (or earlier).

Human Pain and Infrasonic Vibrational Frequencies

The human body may suffer certain pains and discomforts in the infrasonic range (below 20 Hz):

Chest pain	5-7 Hz
Jaw pain	6-8 Hz
Abdominal pain	5-10 Hz
Back pain, e.g. lumbar region	8-12 Hz
Headache	13-20 Hz

(From: Irving P. Herman, Physics of the Human Body, Springer Verlag Berlin, 2007, p.617.).)

Room and Auditorium Acoustics

Room Characteristics and Design

We can apply many concepts from this course to room acoustics. These include:
Frequency, sound intensity, Fourier spectra, reflection of waves, interference and diffraction, resonance.
Some important additional concepts in the following are:
Absorption of sound, reverberation time, room characteristics and design.

Topics and Key Words
Reverberation time
Sound reflection and absorption in enclosed spaces

Acoustical Design Criteria
Liveness
Intimacy
Fullness
Clarity
Warmth
Brilliance
Texture
Blend
Ensemble

Acoustical Design Problems
External noise
Double-valued reverberation time
Focusing of sound
Echoes and flutter echoes
Shadows
Diffraction of Sound
Resonances

Demonstrations
1. Play some music from CDs and judge the acoustical qualities of the lecture room for speech and different types of music.
2. Tap a big cylindrical cardboard packing tube on the floor. Listen as the resonance decays with time. This is only a simulation of the real reverberation time of a room.

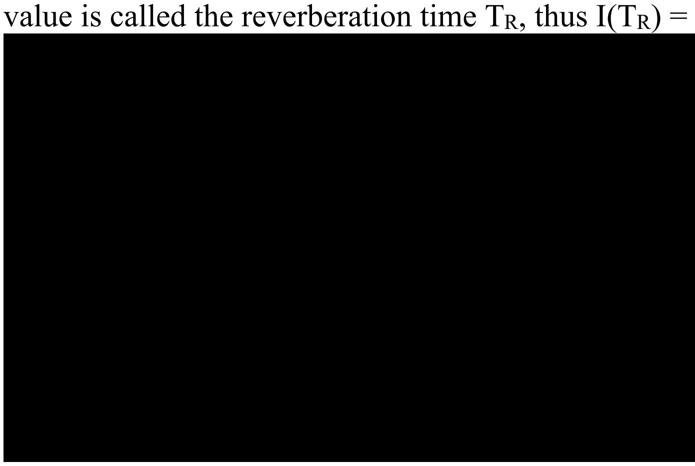
Reverberation Time T_R

The reverberation time of a room, auditorium, or concert hall is one of the most important acoustical characteristics of an enclosed space. Sound does not immediately disappear when turned off. It reflects from walls, floor, and ceiling in multiple paths. With every reflection, the intensity decreases due to absorption by the material of the surfaces.

Figure. Sound paths in an enclosed room. Note the direct sound from the source “S” to the listener “L” and the reflected sound from the walls.

Definition of the Reverberation Time

The time it takes a sound to decay from an initial intensity I_0 to one-millionth of this value is called the reverberation time T_R, thus $I(T_R) = 10^{-6} \cdot I_0$, i.e. a decrease of 60 dB.



Change in sound intensity level:
$\Delta SIL = 10 \cdot \log(I/I_0)$ or
$\Delta SIL = 10 \cdot \log 10^{-6} = -60$ dB

Figure. Definition of the reverberation time T_R. The intensity in the room builds up to a plateau after the first few distinguishable reflections. After turning the sound off, the intensity decays exponentially by 60 db during the reverberation time T_R.

Questions

The sound intensity in a room is I_0 at time $t = 0$. At what times has the original sound intensity I_0 decreased by 10 dB, 20 dB, 30 dB, 40 dB, 50 dB, 60 dB?
Answer: $T = T_R/6$, $T_R/3$, $T_R/$,…, T_R. Can you figure out the missing two values?

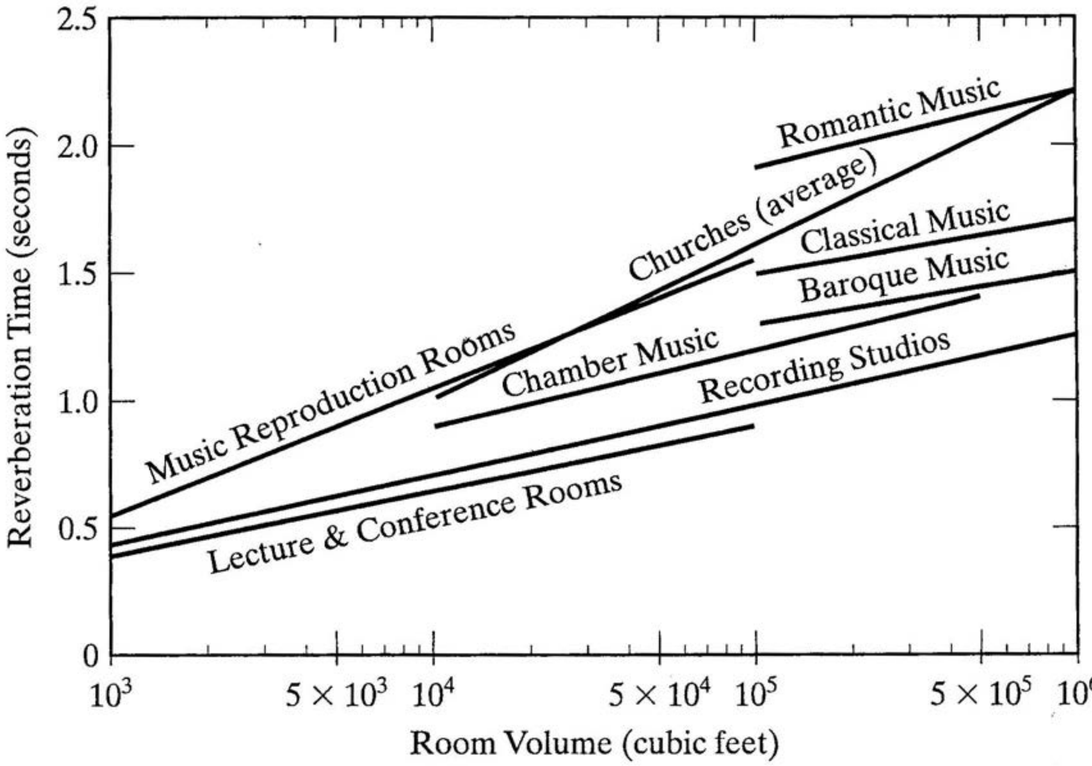


Figure. Ideal reverberation times for several room types and music styles. (From Berg & Stork, Fig. 8-4, p. 218.)

Demonstrations

1. Measure the SIL of the background noise in the lecture room with a sound level meter. Then clap your hands and note the SIL. Abruptly stop clapping. Guess the time it took for the sound to decay to the background level. Take the difference between the two dB readings. (This probably will be much less than the 60 dB decrease for one entire T_R.) Calculate the reverberation time T_R from this difference and compare with your guess.

Answer: T_R = _____________s for the lecture room.

2. Make sound pulses by clapping, gently striking the desk with a hammer, or repetitively pressing a key on the keyboard. Observe the pulses on a sonogram as a function of time. Note the rising attack and falling decay transients. You may be able to see a 30 dB decrease. Double this time to obtain the reverberation time.

Answer: T_R = _____________s for the lecture room.

Exercise

Estimate the volume of the lecture room in cubic feet. Read the reverberation time from the line in the above figure labeled "Lecture & Conference Rooms".
Compare this with the values obtained in the demonstrations 1 and 2 above.

Answer: Read from the graph: T_R = ______s

Demonstration 1: T_R = _____s Demonstration 2: T_R = _____s

3. Tap a large cylindrical cardboard packing tube on the floor and record the waveform of the pulses. Estimate the reverberation time T_R from the exponential decay of the waveform.

Reverberation Time Formula and Acoustical Materials

The reverberation time is the time for the sound intensity to decrease to one millionth of the original intensity, i.e. by a factor of 10^6 or 60 dB, after the sound is turned off.

The formula for calculating the reverberation time in seconds is

$$T_R = 55.2 \frac{V}{v \cdot A_{sabin}} \quad \text{(metric units)}$$

Here V is the room volume, v the sound velocity, and A_{sabin} the total effective absorbing area of all surfaces combined, expressed in units of *sabin*.

After substituting v = 346 m/s or 1136 ft/s (at 25^0C) in the above formula, we obtain two alternative expressions, one in SI-metric units, the other in British units:

$$T_R = 0.160 \frac{V}{A_{sabin}} \quad \text{(dimensions in meter)} \quad \text{and} \quad T_R = 0.0486 \frac{V}{A_{sabin}} \quad \text{(dimensions in feet)}$$

In the first formula, the effective area in *metric sabine*, in the second formula it is in *British sabine* or simply *sabin*.

We shall use British units of ft, ft^2, and ft^3 for calculating the reverberation time because of their greater familiarity when dealing with room dimensions.

The area A_{sabin} takes into account the degree of sound absorption by materials. If all sound were absorbed 100% by a surface, the effective area would be the actual area in ft^2, let's say 1 ft^2. If only 30% of the sound intensity is absorbed, then the effective area is only 0.3 ft^2. This effective area then is called 0.3 *sabin* to distinguish it from the actual area. (The unit *sabin* was chosen in honor of the American physicist William C. Sabine (1868-1919), a pioneer in acoustics.)

The total absorption by all room surfaces is given by the sum

$$A_{sabin} = a_1A_1 + a_2A_2 + a_3A_3 + a_4A_4 + \ldots,$$

where the coefficients a_1, a_2, a_3, etc. are the *absorption coefficients* of the various surfaces in the room. The areas A_1, A_2, A_3, etc. are the actual surface areas. For values of the absorption coefficients a_i, see the following Table.

Sound Absorption Data

Sound Absorption Coefficients of Building Materials at Octave Intervals

	Frequency (Hz)					
Material	125	250	500	1000	2000	4000
Concrete, bricks	0.01	0.01	0.02	0.02	0.02	0.03
Glass	0.19	0.08	0.06	0.04	0.03	0.02
Plasterboard	0.20	0.15	0.10	0.08	0.04	0.02
Plywood	0.45	0.25	0.13	0.11	0.10	0.09
Carpet	0.10	0.20	0.30	0.35	0.50	0.60
Curtains	0.05	0.12	0.25	0.35	0.40	0.45
Acoustical board	0.25	0.45	0.80	0.90	0.90	0.90

Note: These are dimensionless *absorption coefficients*. (They give the percentage absorption when multiplied by 100, e.g. 13% for plywood at 500 Hz.)

Sound Absorption by Persons and Seats in Sabin

	Frequency (Hz)				
	125	250	500	1000	2000
Unupholstered seat	0.15	0.22	0.25	0.28	0.50
Upholstered seat	3.0	3.1	3.1	3.2	3.4
Adult person	2.5	3.5	4.2	4.6	5.0
Adult in upholstered seat	3.0	3.8	4.5	5.0	5.2

Note: These are the *total absorption values* in sabin (effective areas). *Total absorption* is not to be confused with the dimensionless *absorption coefficients* in the upper table.

(Values in the two tables taken from Berg & Stork, Table 8-3 and Table 8-4, p. 227.)

Timbre and Reverberation

From the frequency dependence of the sound absorption coefficients we see that the timbre of a tone depends on the particular room in which it is played. For instance, in an enclosure made of glass or plasterboard, a given note will sound bright or brilliant, because the high frequencies are absorbed less than the low frequencies. Conversely, the same note will sound warmer in a room covered with carpet, curtains, and acoustical board.

Calculation of the Reverberation Time of a Lecture Room

As an exercise we calculate the reverberation time T_R of our lecture room.
Width of room: W = 24 ft
Length: L = 29 ft
Average height between ceiling and sloping floor: H = 9.5 ft

Verify that for the four walls the total area is $A_{walls} = (2W + 2L)H = 1007\ ft^2$,
$A_{ceiling} = W \times L = 696\ ft^2$, $A_{floor} \approx A_{ceiling} = 696\ ft^2$.
Materials: Concrete walls $a_{concrete} = 0.02$, acoustical tile ceiling $a_{acoustical\ tile} = 0.90$,
wood floor $a_{floor} = 0.15$.
The numbers are approximate and are for frequencies in the middle range in the Table, around 500 Hz.

Verify that the total absorption in sabin is
A_{sabin} = 20 sabin (walls) + 626 sabin (ceiling) + 104 sabin (floor) = 750 sabin

Add to this the absorption from occupied and unoccupied chairs:
Unoccupied unupholstered chair: A = 0.25 sabin around 500 Hz
Occupied unupholsered chair: A = 4.3 sabin around 500 Hz
Assume 30 chairs unoccupied: A = 30 x 0.25 = 7.5 sabin ≈ 8 sabin
Assume 20 chairs occupied: A = 20 x 4.3 = 86 sabin

The the total absorption in sabin is
A_{total} = 750 sabin (room) + 86 sabin (people in chairs) + 8 sabin (unoccupied chairs)
A_{total} = 844 sabin

The volume of the room is V = W x L x H = 24 x 29 x 9.5 ≈ 6600 ft^3
Final Answer
The reverberation time then is obtained by substituting the values in the formula

$$T_R = 0.050\frac{V}{A_{sabin}} \quad \text{or} \quad T_R = 0.050\frac{6600}{844} = 0.39\ \text{s or } \mathbf{T_R \approx 0.4\ s}$$

This is in very good agreement with a value of about 0.5 s in the Figure for the reverberation time labeled “Lecture & Conference Rooms”.

See the next page for the measurement of the reverberation time of our lecture room.

Question
If the reverberation time is 0.4 second for a 60 dB decrease in sound intensity, what is the time for the intensity to decrease by only 30 dB, as is more likely to be achieved in the lecture room because of background noise?
Answer: 0.2 second

Sonogram of the Reverberation Time of a Lecture Room

The reverberation time of our lecture room was measured by clapping and recording a sonogram of the pulses.

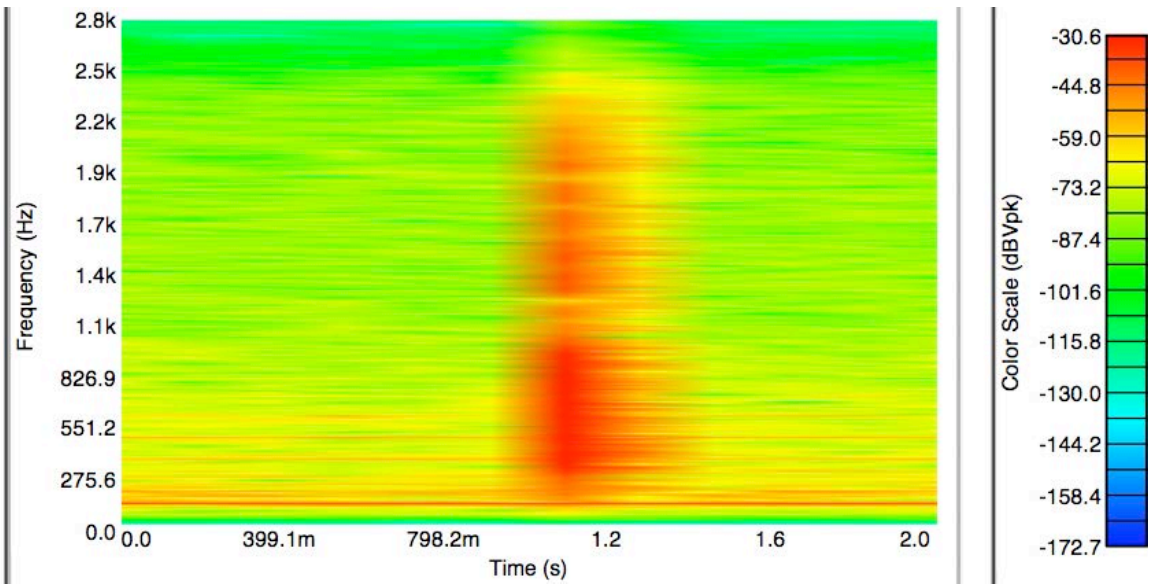


Figure. Sonogram of a rapid clap as recorded in the lecture room. Shown are the frequency on the ordinate and progressing time on the abscissa. Color indicates the sound intensity on the dB-scale (see also the color bar on the right of the figure).

We see that the signal from the clap is a noise spectrum ranging from below 200 Hz to about 2.5 kHz. The signal starts at the red area. The intensity builds up (attack transient) and reaches the deepest red in about 0.15 s. After an additional 0.1 s the intensity decreases again.

The reverberation time for a 60 dB decrease is reached when the color has changed from deep red to light green. This happens within about 0.25 s in the range 1 to 2 kHz and in 0.4 s in the range 300 Hz to 1 kHz . The longer reverberation at these lower frequencies means that our lecture room exhibits more "warmth" than "brilliance".

Note that the reverberation time of 0.4 s agrees well with the time of $T_R \approx 0.4$ s calculated earlier for frequencies around 500 Hz.

Reverberation Time in the Southwest Collections Library at Texas Tech University

A sonogram of the reveration of sound was recorded in the entry hall to the Southwest Collections Library. The sound was produced with a clap of the hand.

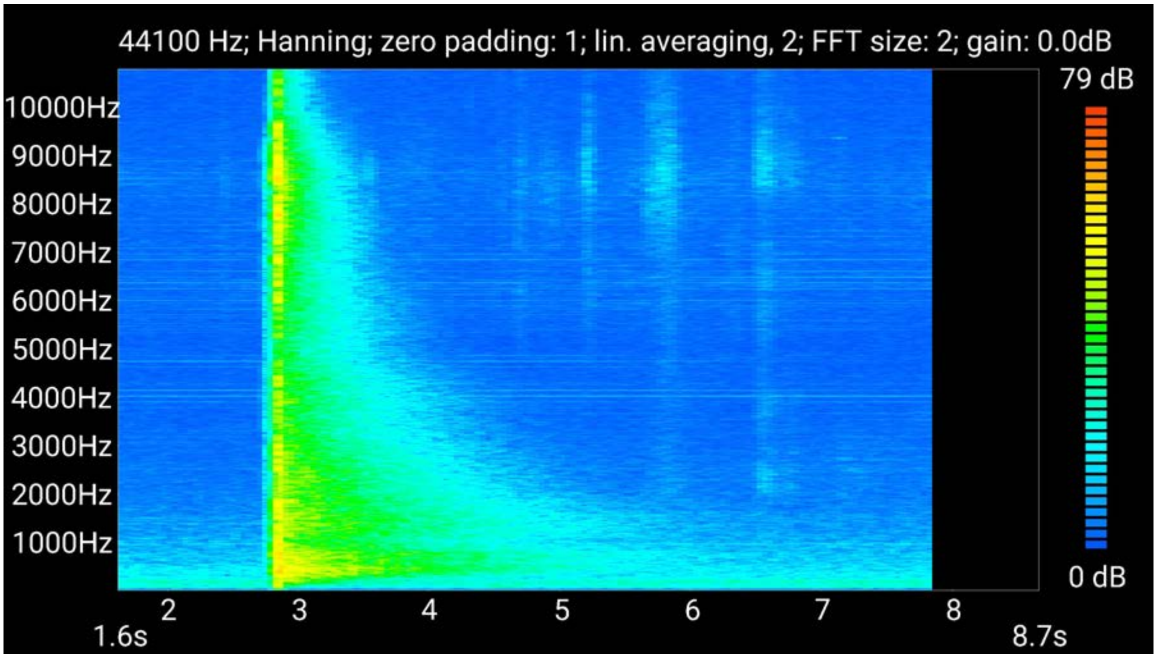


Figure. Sonogram from a single clap of the hand in the Southwest Collection Library at Texas Tech University. Its entry hall is big, the surfaces absorb little sound, and consequently the reverberation times are rather long. At the lowest frequencies the reverberation time is about 3 seconds, at the highest about 1 second. This means that the hall exhibits acoustical “warmth” rather than “brilliance”. (Recording taken by Binod Rajbhandari.)

Project

Go to the Southwest Collections Library and clap your hand in the entry hall. Estimate the reverberation time. Note how the quality of the sound changes during the reverberation. Does the sound become warmer or more brilliant as it decays?

Acoustical Design Criteria

Liveness
Liveness is a qualitative term for the reverberation time T_R. A room is “live” if it has a long reverberation time. The desired “liveness” depends on the type of music played (see figure of reverberation times).

Intimacy
A room is said to have *intimacy* when the first reflected sound from surfaces reaches the listener within 20 ms after the original direct sound. This generally is the case for smaller lecture halls. For large auditoriums or halls one can use canopies above the speaker or the performers to reduce the time to the desired 20 ms. The canopies may be inclined to reflect the sound to the listeners.
Exercise
How far does sound travel in 20 ms? What, therefore, is a “small” or “large” hall?
Answer: Travel distance ≈ 7 m $\rightarrow$ “small hall” = 5 to 10 m, “large hall” > 10 m.

[redacted]

Figure. A canopy above a singer in a large hall to reflect the sound to the listener and reduce the travel time for better room intimacy.

Fullness
An auditorium has *fullness* if the intensity of the reflected sound is higher than the direct sound. Greater fullness means a longer reverberation time. Fullness is desired for romantic music played by a large orchestra and less so for chamber and baroque music.
Clarity
Clarity is the opposite of fullness. The intensity of the direct sound is higher than the reflected sound. Acoustical clarity is needed for speech and early orchestral music. The reverberation time is short (see figure of reverberation times).
Warmth
For a room having *warmth,* the reverberation time should be about 1.5-times longer at low frequencies up to about 500 Hz than for higher frequencies. Above 500 Hz the reverberation time should be approximately constant. Such a frequency-dependent reverberation time can be obtained by the suitable choice of materials for walls, ceilings, and floor. The room will lack clarity if the reverberation time becomes too long at the lowest frequencies.
Brilliance
Brilliance is the opposite of warmth. The reverberation time is longer for high frequencies than for low frequencies. If it becomes too long at high frequencies the room may be “ringing” at a high pitch.

Texture

Texture addresses the time pattern of how the reflections reach the listener. For good texture the first reflection should arrive within 20 ms after the direct sound in order to have intimacy. The next four or five reflections should arrive within 60 ms after the direct sound. The overall intensity from the reflections should decrease monotonously with time. Echoes and focusing of sound result in poor texture and produce intensity spikes during the overall decrease of the sound intensity with time.

Blend

Blend means a proper mixing of sound from all instruments at the location of the listener. It is best to mix the sound first at its origin. This can be accomplished with reflecting surfaces surrounding the stage. Individual instrument sections should not be singled out and the sound should be diffused before it reaches the audience.

Ensemble

A stage has good *ensemble* if the performers can hear each other, enabling them to play well together. For this it is necessary that fast notes in the music should not be delayed too much by reflections. This limits the size of the stage to less than 20 m.

Exercise

Suppose that the temporal separation between the fastest notes is 50 ms in a piece of music. Estimate the maximum distance between the sides of the stage.

Answer: Size of stage $x = v \cdot t = (346 \text{ m/s}) \cdot (0.050 \text{ s}) = \underline{17.3 \text{ m}}$.

Ensemble and Opera

The *ensemble* between opera singers on the stage and the musicians in the orchestra pit may be poor. Reflectors directing sound from the pit to the singers may help, but then the audience does not get much direct sound. As an alternative, small loudspeakers may be used to aim the sound from the orchestra to the singers.

Ensemble and Marching Band

A lack of ensemble is likely to exist in a marching band on a football field. The band is spread out and the time delays between the various band sections can be large compared to the time between successive notes in the music. Watching a band director signaling the rhythm may minimize the problem, but then the players from the far ends of the band should not listen to each other! People in the stands listening to the music still will not hear the instruments quite synchronized because of different time delays from the band.

Exercise

1. Take a typical size of a marching band and estimate the differences in the arrival times of the music from the different band sections at a listener in the stands.

Answer: Take $d = 70$ m for the size of the band. Then the maximum time difference is given by $\Delta t = d/v = 70/346 = 0.2 \text{ s} = \underline{200 \text{ ms}}$.

2. Guess what time delay might still be tolerable for most people. Or how much could the sound from different band sections be out of sync and still be agreeable?

Answer: A guess for tolerable time delays: _______s.

Acoustical Design Problems

External Noise

An auditorium may have background noise from traffic, air blowers, air conditioners, etc. Noise has a wide frequency spectrum. Its low frequencies can mask the higher frequencies in music and speech. Means to address the problem include better sound insulation and constructing the room as a "box in a box". This was done at the John F. Kennedy Center for the Performing Arts in Washington, D.C. Environmental noise levels should not exceed the values in the following table. Actual noise levels often are much higher.

Table. Acceptable background Noise Levels. (Add 5 to 10 dB for more typical values.)

Recording studios	25 dB
Auditoriums and theaters	30 dB
Lecture halls	30 dB
Hospitals	30 dB
Homes	40 dB
Offices	45 dB
Restaurants	50 dB

Demonstrations: Does our classroom meet these requirements? Measure the sound intensity level (SIL) and compare with the table.

Answer: Measured SIL in classroom: SIL = _______ dB

From table: SIL = _______ dB

Conclusion: ____

Double-Valued Reverberation Time

For recorded speech or music, the reverberation times of the recording and listening rooms most likely are different. When the listening room has a long reverberation time, the recording may sound unpleasant and speech may become unintelligible. Smaller listening rooms or wearing headphones can partly address the problem.

Focusing of Sound

Sound may be focused in a room by reflection from curved surfaces such as spherical surfaces, ellipsoids, paraboloids, or flat panels improperly arranged. If some musicians in an orchestra are at one focal point of a curved surface, the listeners at the other focal point will hear these players louder than others.

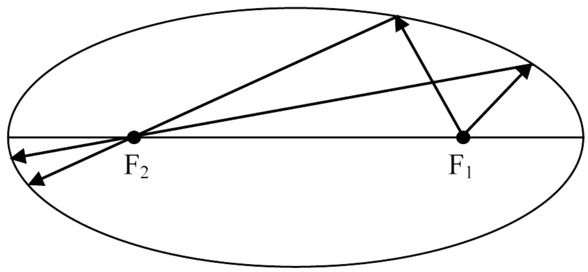


Figure. Sound from one focal point of an ellipsoid is focused into the other focal point. You may find this effect in "whispering chambers" in science museums. It is to be avoided in concert halls and auditoriums.

Echoes and Flutter Echoes

Echoes generally are undesirable and adversely affect the sound texture of a room. They originate from concave curved surfaces. This happens when musicians are at the focus of a curved surface behind them. The direct sound to the audience is then followed by the echo waves from the curved surface, giving the illusion of being in a cave.
Sound reflecting back and forth between two parallel walls is called a flutter echo.

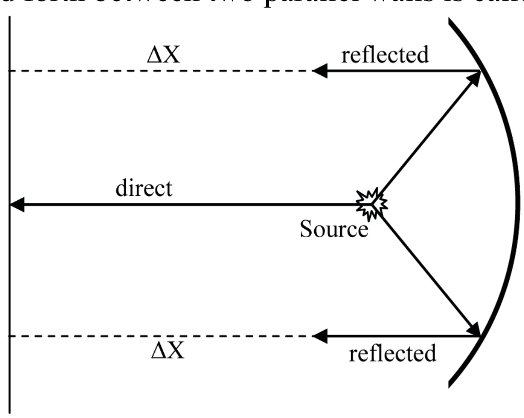


Figure. Parabolic reflecting surface with a sound source at its focus. The direct sound arrives earlier at the audience than the reflected sound. The path difference Δx causes an echo-like sound.

A Project

Clap your hands at the center of the Memorial Circle at Texas Tech University. Do you hear echoes, reverberation, or resonances? Step off the center and note the changes.

Shadows

Acoustical shadow regions may exist in parts of an auditorium that protrude into the space near the listener. In order to minimize the effect, an open space in front of the listener should exist with a sufficiently large angle between the obstruction and the stage.

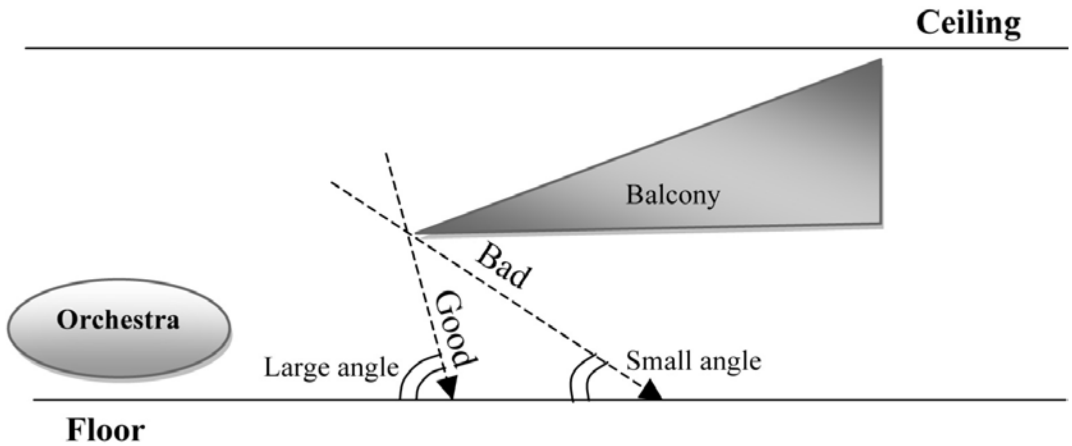


Figure. A listener (arrow tips) below a balcony in a concert hall. The balcony casts a sound shadow impairing the sound quality. A large subtended angle with respect to the front of the balcony is good, a small angle is bad.

Diffraction of Sound

Diffraction of sound occurs when the wavelength approaches the size of an obstacle. The resulting intensity variations behind the obstacle depend on the location of the observer and the wavelength of the sound. Therefore, the timbre of the direct sound from an orchestra may change behind the obstacle. However, reflections from other surfaces will diminish this effect.

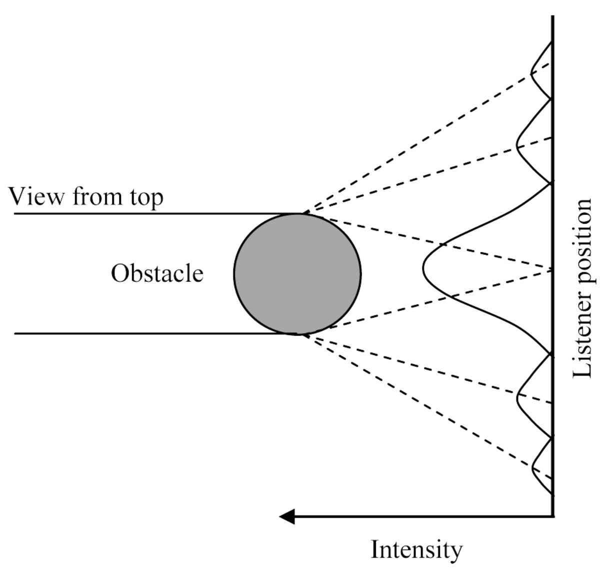


Figure. Sound diffraction by a column (as viewed from above). Intensity minima and maxima occur behind the column and the sound color or timbre may change.

Room Resonances

Sound resonances in rooms generally are undesirable. They adversely affect the sound texture, but are an interesting phenomenon and so we treat them in some detail here. A close analogy exists between the air resonances in a room and the resonances of vibrating strings. Strings are fastened tightly at two “closed ends”. Similarly, resonating air columns encounter “closed ends” at opposing rigid walls. We can make good use of what we know from vibrating strings, because the vibration patterns look the same.

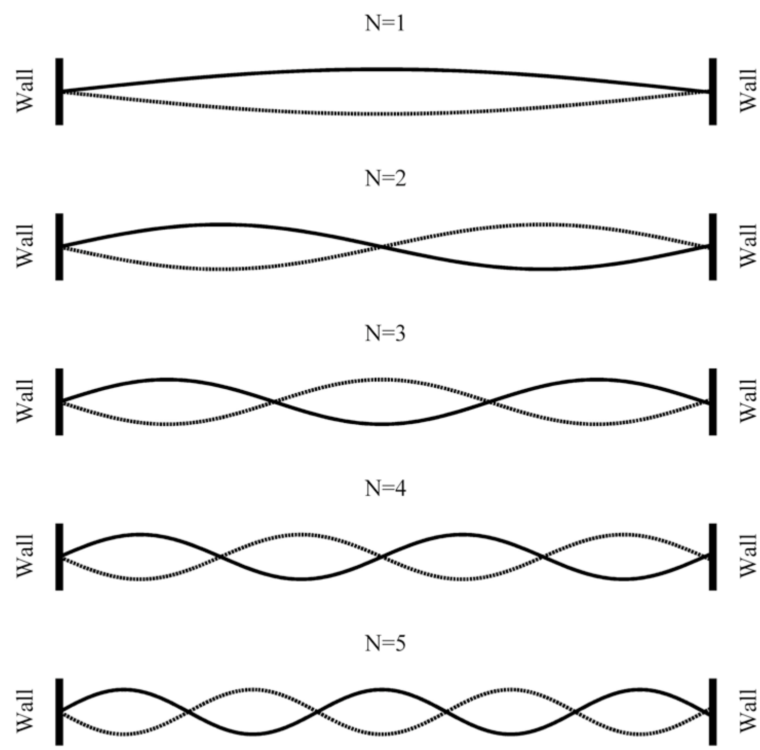


Figure. The first five air resonances between the parallel walls of a room. The ordinate shows the longitudinal displacement of the air molecules between the walls, but plotted here in the vertical direction for clarity.

For the fundamental mode we have $x = \lambda/2$ or $\lambda = 2x$.
The fundamental frequency is $f_1 = v/\lambda_1 = v/2x$, where $v = 346$ m/s is the speed of sound. For the higher vibrational modes, we have the integer multiples $f_N = Nf_1 = N(v/2x)$, where N = 1, 2, 3, 4, …… are the harmonic numbers.

Exercise
You are singing in a shower stall of length x = 1.2 m. What are the lowest three resonance frequencies between two parallel walls separated by the distance x?
Answer: Verify that $f_1 = 144$ Hz, $f_2 = 288$ Hz, $f_3 = 432$ Hz.

For a room we actually have the three dimensions length *x*, width *y*, and height *z*. All three contribute to room resonances, and so we must include them. We consider only the simplest case of an empty box-like room. For the vibrational modes in the individual x-, y-, and z-directions we have, respectively,

$$f_x = N_x(v/2x) \qquad f_y = N_y(v/2y) \qquad f_z = N_z(v/2z)$$

where N_x, N_y, N_z are the integers 1, 2, 3, 4, ….

The question arises how these three sets of frequencies are combined for vibrations in three dimesnions. What are the resultant frequencies?The answer is

$$f_{xyz} = \sqrt{f_x^2 + f_y^2 + f_z^2} = \frac{v}{2}\sqrt{\left(\frac{N_x}{x}\right)^2 + \left(\frac{N_y}{y}\right)^2 + \left(\frac{N_z}{z}\right)^2}$$

Example
A tornado shelter has the dimensions x = 3.46 m, y = 2.77 m, z = 1.73 m. Assume for the speed of sound v = 346 m/s. Calculate the frequencies of some of the lowest resonance modes (N_x, N_y, N_z) in the shelter.
Verify the following values for the first few resonance frequencies:

Modes		Mode Frequency
(1, 0, 0)	→	f_{100} = 50.0 Hz
(0, 1, 0)	→	f_{010} = 62.5 Hz
(0, 0, 1)	→	f_{001} = 100.0 Hz
(1, 1, 0)	→	f_{110} = 80.0 Hz
(1, 0, 1)	→	f_{101} = 111.8 Hz
(0, 1, 1)	→	f_{011} = 117.9 Hz
(1, 1, 1)	→	f_{111} = 128.1 Hz
(2, 0, 0)	→	f_{200} = 100.0 Hz
(0, 2, 0)	→	f_{020} = 125.0 Hz
(0, 0, 2)	→	f_{002} = 200.0 Hz etc.

We can arrange these frequencies in order of ascending values:
50, 62.5, 80.0, 100, 100, 111.8, 117.9, 125.0, 128.1, 200.0 Hz.
Some of these frequencies are closely spaced. Generally, the frequency spectrum becomes more crowded for the higher resonances.

Question: What measures can you take to minimize undesirable room resonances?
Answer: ____

Resonances in a Cubical Model "Room"

In the laboratory of this course we use a wooden box as a "model room" and observe some of its "room resonances". (They will have higher values than for the tornado shelter because of the smaller size of the box.) The simplest box is a cube. Resonances for such a cube are shown in the following two figures:

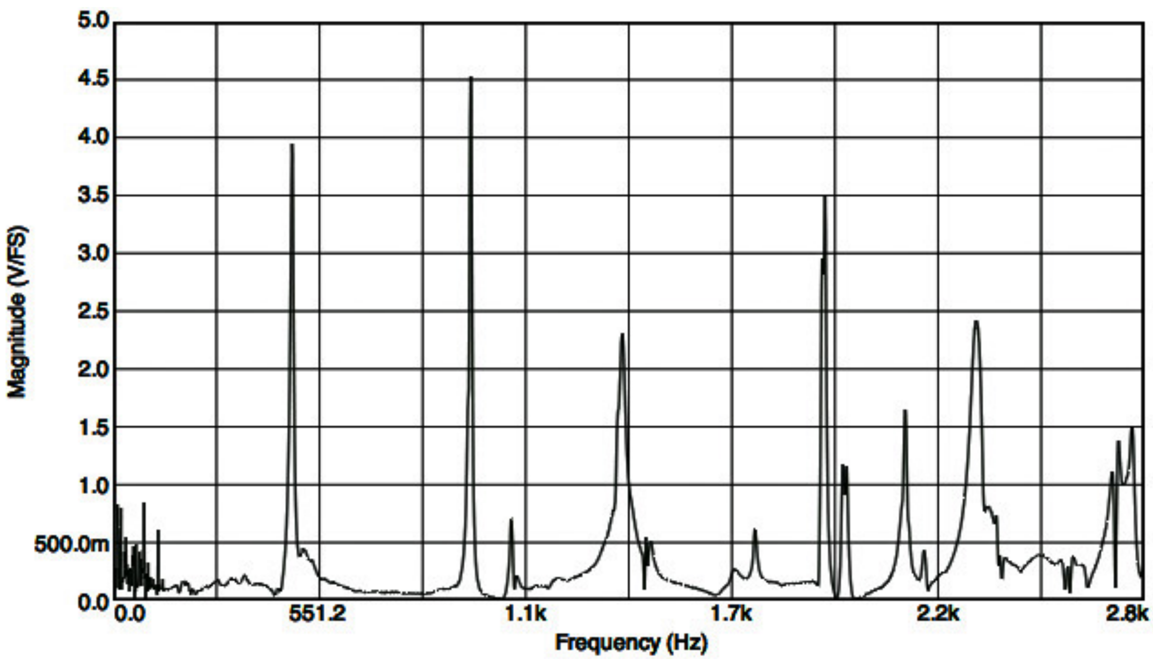


Figure. Frequency spectrum of a cubical plywood box with interior dimensions $x = y = z = 361.5$ mm (25.5 ^{0}C, sound velocity $v = 346.3$ m/s). The lowest resonance is the (1, 0, 0) mode with a frequency of $f_{100} = 479$ Hz.

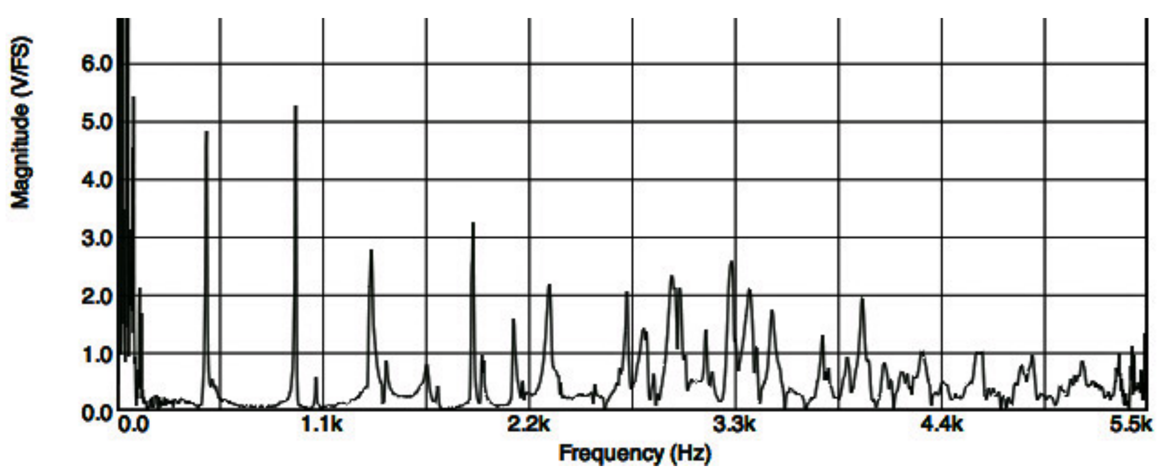


Figure. Compressed frequency scale for the same cubical box showing the ever more densely packed resonances at higher frequencies.

One sees that the lowest room resonances are well separated. They become more densely packed at higher frequencies. Some "formant regions" of the room may be seen, especially beween 2.4 to 3.7 kHz. The amplitudes of the resonances depend on microphone and speaker placement inside the "room". Some possible frequencies may not be seen at all.

We had for the frequencies of a box-like room with dimensions x, y, z the formula

$$f_{xyz} = \sqrt{f_x^2 + f_y^2 + f_z^2} = \frac{v}{2}\sqrt{\left(\frac{N_x}{x}\right)^2 + \left(\frac{N_y}{y}\right)^2 + \left(\frac{N_z}{z}\right)^2}$$

where N_x, N_y, N_z are the harmonic mode numbers 1, 2, 3, 4, ….that correspond to the directions *x*, *y*, *z*, respectively. For our cubical box this formula simplifies considerably, as all three sides are the same: x = y = z = 361.5 mm. This simplifies considerably the above formula:

$$f_{xyz} = \frac{v}{2x}\sqrt{(N_x)^2 + (N_y)^2 + (N_z)^2}$$

Exercise
Read the frequencies of the strongest 5 resonances from the upper of the two preceding figures. For good accuracy, use a ruler on the frequency scale and a calculator.
Answer: The strongest 5 measured frequencies are

f = _____Hz, f = ______Hz, f = ______Hz, f = ______Hz, f = ______Hz

Exercise
Calculate the five lowest resonance frequencies for the cubical box. Take v = 346.3 m/s and x = y = z = 361.5 mm = 0.3615 m. Label the vibrating modes, for instance (2, 1, 0). Compare your calculated frequencies with those from the preceding exercise.
Answer: Write down the measured frequencies and mode labels, for instance (1, 0, 1). Write the vibrational mode numbers (N_x, N_y, N_z) below each frequency this way:

f = ______Hz, f = ______Hz, f = ______Hz, f = ______Hz, f = ______Hz
(, ,) (, ,) (, ,) (, ,) (, ,)

Home Rooms

Usually we cannot build our own studio for home listening. But a few design features are under our control:
1. Materials of the room surfaces (walls, ceiling, floor).
2. Placement of reflecting panels or furniture.
3. Carpets, rugs, flooring material.
4. Audio systems with equalizers.
5. Loudspeaker placement.

Audio Systems
Perform some experiments in your room by placing the speakers strategically for best sound, especially for multi-speaker systems. For example, a 5.1 audio system has one subwoofer and 4 speakers. Place two speakers in front and two in back. The subwoofer can be anywhere. An optimal place for speakers is above ground and away from walls or corners. The listening area should be between the speakers and not far from them.

Great amplifier power such as 500 Watt of *electrical* output generally is not needed in a living room. However, a relatively high electric power is needed because of the very low efficiency of less than 1% for converting electric to acoustic power. Short sound bursts also require a higher electrical power from the amplifier. Normally less than 1 Watt of *acoustical* power is quite sufficient. Taking all this into account, a 100 or 200 Watt amplifier should be adequate for most home audio systems.

Room Resonances, Pink Noise, and Room Tuning

Acoustic resonances in small rooms are undesirable. The problem can be addressed by "tuning" the room with *equalizers* in the audio system. Strong resonances that are "booming" at certain frequencies can be minimized this way. Conversely, frequencies that are absorbed strongly can be boosted with an equalizer.

Room resonances also can be minimized by design elements positioned under an angle with respect to the walls. Absorbent wall hangings are an example.

The acoustical power in typical music, per unit frequency interval, drops off with about 3 dB/octave. A steady decrease like this is called *pink noise.* In order to achieve such a 1/f response in a room, try the following:
Play pink noise from a test CD through your audio system. Use a sound-level meter at different locations in the room. Ideally the sound intensity level (SIL) should be fairly constant throughout the room. If it is higher at some locations, you are having room resonances or speaker resonances. Adjust the equalizer of the audio system, change the room decor, or change the speaker placement to cut down these intensity maxima. This may take some experimentation, but the room probably will sound better.

Auditorium Design

Open Air Auditoriums

On a flat unobstructed field the sound intensity *I* drops off with distance *r* from the source according the *inverse square law* $I \propto 1/r^2$. For every doubling of the distance, the intensity decreases by a factor of four, where 4 = 2x2 or 3 dB + 3 dB = 6 dB. But the actual attenuation can be higher than this if the source is close to the ground and there are sound absorbing obstacles in the path to the listener such as other people, grass, etc.. This may result in a nearly 12 dB decrease in intensity for each doubling of the distance. In order to minimize the absorption by the ground the stage can be raised. On the other hand, the stage was lowest in ancient Greek and Roman amphitheaters where the spectators sat on an upward sloping surface. This contained the sound well within the theater. Small valleys also are suitable as natural amphitheaters.

Steps in Constructing an Indoor Auditorium

We can start with an "open-air auditorium" and cover the sides and top to build an indoor auditorium or concert hall.

1. Add a shell above the performers for reflecting the sound and achieving fullness.
2. Add side panels for more fullness and texture.
3. Keep these additions close enough for intimacy.
4. Diffuse the sound with side panels that should be flat to avoid focusing.
5. Provide better ensemble for the performers with panels on the sides.
6. Enclose the audience with a ceiling and walls.
7. Let the sides of the hall fan out from the stage.
8. Cover the sides and rear wall with absorbers to avoid standing waves and flutter echo.

Additional Auditorium Components

1. Balconies

Balconies should be steeply sloped. They should not protrude far into the hall so that diffraction of waves and shadows in the opening under the balcony are minimized. Pillars and columns under a balcony should be avoided.

2. Movable Elements and Reverberation Time

The reverberation time can be adjusted with retractable absorbers and choice of materials.
For speech, use sound absorbing panels for shorter reverberation times.
For small groups of performers, use small shells.
For chamber music, use movable ceiling sections and lower them.

3. Electronic Enhancements

Loudspeakers and amplifiers can be part of the reverberation system. The amplified sound should not arrive before the direct sound. Otherwise the listeners may get the impression that the orchestra is playing from the ceiling! This happens in the so-called *precedence effect*, where a listener places the sound source from where he hears the sound first.

Exercise: Identify some auditoriums on campus with good sound characteristics.
Answer: ____

Sound Concentration with Curved Surfaces Outdoors

Figure. Acoustic mirror at Denge, Great Britain, built in 1928 to listen to approaching airplanes from continental Europe in the 1930s. A microphone was attached to the metal rod at the focus of the parabolic mirror to collect the sound. (Reference: Thomas Cogley.)

Figure. Open-air stage at Oberstdorf (Allgäu), Germany. The curved surface behind the stage reflects the sound towards the listeners thus increasing the loudness.

Acoustics at the Campus Circle of Texas Tech University

Figure. The Center of the Campus Circle at Tech Tech University (Pfluger Fountain). If you clap your hands once at the center, you will hear an echo due to focusing of sound from the surrounding circular low brick walls. If you step away from the center, the echo becomes much weaker because of de-focusing of the sound.

Question

Besides an echo, do you also hear sound reverberation and resonances?
Whether or not you hear all three phenomena, describe the differences between them.

Project

Do this experiment at the Campus Circle and write a brief report.

Part 8

Electricity and Magnetism, Sound Recording and Reproduction

Brief History of Sound Recording and Reproduction
1853 Leon Scott de Martinville: *Phonoautograph* paper recorder
1877 Thomas Edison invents sound reproduction on tin foil *Phonograph*
1885 Bell and Tainter introduce *wax cylinder*
1887 Emile Berliner invents the disc *Gramophone*
1925 Western Electric *Orthophonic* electrical system
1920s, 1930s Wire recorders
1929 Edison production ends. Introduction of *lacquer disc*
1947 *Magnetic tape* in production use, e.g. Ampex 200A
1948 *33.33 rpm long playing (LP)* record introduced
1958 *Stereophonic LP* on sale
1963 Magnetic *tape cassettes*
1980 Phillips and Sony: *Compact Disc* (CD), alternative to vinyl disc and audiocassette
1995 Toshiba and Time Warner: *Digital Versatile Disc* or *Digital Video Disc* (DVD)
1999 USB *flash drive* invented, 2000 first commercial *flash drives* sold
2001 Apple *iPod*

Milestones of Video Production (for comparison)
1964 Videocassette for consumers developed
1968 Sony Portapak first consumer 2-piece video recorder
1969 Bell Labs develops the first Charged Couple Device (CCD)
1975 Betamax decks by Sony
1976 VHS decks by JVC
1980 First consumer camcorders by Sony and JVC
1981 IBM introduces the PC
1984 Apple introduces the Macintosh
1985 Sony introduces the Video8 format
1985 VHS-C developed
1987 S-VHS introduced
1988 Hi8 introduced
1990 First non-linear video editing system introduced by Newtek
1991 First CD burner
1992 Sharp introduces the first LCD screen for camcorders
1992 First smartphone introduced by IBM
1995 Panasonic introduces the Mini DV
1996 First DVD-ROM players
1997 D-VHS introduced
1999 Digital8 introduced
2000 Hitachi introduces first DVD-RAM camcorder
2001 First DVD burner
2003 High definition video (HDV) standardized
2003 First HDV camcorder by JCV
2004 Panasonic and Sanyo release first flash memory camcorders
2005 Samsung introduces the DuoCam, a still camera and video camera combined
2007 Steve Jobs introduces the iPhone
2008 Nikon releases D90, the first DSLR to shoot video
2010 Apple releases the iPad
2011 Apple releases the iPad 2, with TV
2011 Canon announces the first 4K consumer camcorder
(From: The History of Video, Videomaker, April 2012, p.62.)

Basic Concepts of Electricity and Magnetism

Voltage V (Volt, V)

Current I (Ampere, A)

Power P (Watt, W)

Work and energy W (Joule, J)

Resistance R (Ohm, Ω)

Impedance Z (Ohm, Ω)

Ohm's law of electricity: $V = RI$ (only for simple DC circuits in this form)

Faraday's law of electromagnetic induction

Electric generator
Microphone

Electric motor
Loudspeaker

Amplifiers

Compact Disc (CD)

Cell telephone, electromagnetic waves

DC and AC circuits, Ohm's Law of Electricity

When a constant voltage V such as that from a battery is applied to a load resistance R, the result is a constant, *direct current I* (DC).
When an alternating, time-varying voltage such as from a household outlet or an audio amplifier is applied to a load, the result is an *alternating current I* (AC). In this case the reaction by the load to the applied voltage is called the *impedance Z*.
For pure resistances such as electric heaters and incandescent light bulbs, Z = R. But if the circuit contains other elements such as *inductances* or *capacitances*, the impedance Z and resistance R of the circuit are different. We shall ignore this difference.

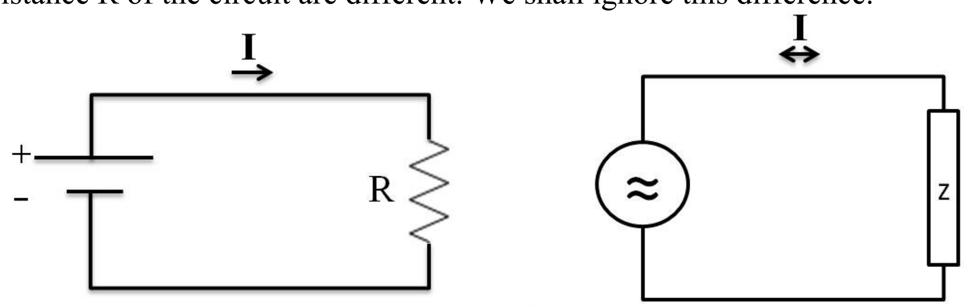


Figure. Left: DC circuit. The voltage source is a battery, photovoltaic cell etc. The load resistance is R, and the current I flows in one direction. Right: AC circuit. The voltage source is an audio amplifier, generator etc. The load is called the impedance Z. The current changes direction between clockwise and counterclockwise in the loop.

Ohm's Law of Electricity

For a DC circuit, the relation between voltage V (in Volt V), resistance R (in Ohm Ω), and current I (in Ampere A) is given by *Ohm's law of electricity* (not to be confused with Ohm's law of acoustics!):

$$\mathbf{V = RI}$$

Example 1

You connect an 8 Ω loudspeaker to an audio amplifier. The amplifier supplies a voltage of 12 V to the speaker. What is the current flowing at that instant?

Answer: I = V/R = 12V/8Ω = 1.5 A (P.S.: This would give a rather loud sound.)

Example 2

A current of I = 0.5 A flows through an incandescent old-style light bulb. The resistance of the light bulb when hot is R = 240 Ω.
What is the voltage across the contacts of the light bulb?

Answer: V = RI = 240Ω x 0.5A = 120 V (effective household voltage in the USA)

Demonstrations

1. Show a flashlight circuit with two 1.55 V batteries in series, 3.1 V total, I = 0.29 A. Measure V and I. The resistance is R = 3.10/0.29 = 10.7 Ω.
2. Show a circuit with three light bulbs: 60W old-style incandescent, 14 W compact fluorescent (CFL), and 6 W light emitting diode (LED). Read the currents through the light bulbs. The power consumptions are very different, but the light outputs are similar.

Electric Power, Work and Energy

When a voltage V is applied to a load such as a loudspeaker or light bulb, a current I flows and electrical power P is delivered.
The power can be calculated from the formula: $\mathbf{P = V \cdot I}$
The physical unit of power is Volt·Ampere (V·A) or **Watt** (**W**)

Exercise: Show that alternative expressions are: $\mathbf{P = I^2R = V^2/R}$
Answer: Use Ohm's law V = RI in the formula P = VI and eliminate either V or I.

Example and Demonstration
A loudspeaker with an impedance Z = 8 Ω receives a voltage of 12 V from an audio amplifier at a certain instant. What is the value of the instantaneous electrical power to the speaker? (Assume that the values of impedance Z and resistance R are the same.)
Answer: The current is I = V/R = 12V/8Ω = 1.5 A.
The power is P = VI = 12V x 1.5 A = 18 W.
Also get the same answer from $P = V^2/R = I^2R = 18$ W.
This is the *electrical power* delivered to the loudspeaker. The *acoustical power* emitted by the speaker is much lower, for example 0.1% of the electrical power.

Note: **Energy is the ability to do work, power is the rate at which energy is used.**
Energy (Work) = Power·time or **W = P·t. Power = Work/time** or **P = W/t.**
The physical unit of work and energy is 1 Watt·second = **1 W·s = 1 Joule = 1 J**.

Question: An incandescent 60 W light bulb is on for 20 minutes. What is the electrical work done or the energy delivered?
Answer: W = Pt = 60W·1200s = 72000 J.

Question: A current of 0.5 A flows through a standard incandescent light bulb. What is the power rating of the light bulb? (Use 120 V for the household voltage.)
Answer: P = IV = 0.5·120 (V·A) = 60 W (a standard light bulb).

Question: Where does the electric energy go in a standard incandescent light bulb?
Answer: Most of it goes into heat. About 5% of it is converted into useful light.

We can do better than this by using more efficient compact fluorescent light bulbs (CFLs) that produce 4 to 5-times more light output (i.e. 20-25%) for the same electrical input. Therefore about 5-times less electricity is used. CFLs may cost twice as much, but they last 10-times longer (10,000 hours). Not only is energy saved, money is saved as well.

Demonstrations
1. Apply power from a signal generator at 500 Hz to a PASCO loudspeaker. Measure the electrical power with a General Radio Power Meter (e.g. 0.5 W). Does it sound loud?
2. Touch the contacts of a loudspeaker (Z = 8 Ω) with a 1.5 V battery. See how the speaker cone moves. Calculate the current I and the electric power P.

Difference between Energy and Power

Energy is a total amount that can be used. One gallon of gasoline contains energy (work potential W). Power is the amount of energy used in a certain time. An automobile engine delivers power from one gallon of gasoline (energy) in 30 minutes (work/time).

Question

a) When do you pay for energy? b) When do you pay for power?

Answer: a) We pay for energy (W) in the form of gasoline, natural gas, or electricity.

Answer: b) We pay for power when we buy equipment with the desired power output. A 500 W audio amplifier costs more than a 100 W amplifier. When the equipment is on, power P is delivered (P = W/t).

Understanding Your Electric Bill

When we get our monthly electric bill, we pay for the amount of electricity used in kilowatt·hours (kWh). Note that 1 kWh = 1000W·3600s = 3,600,000 W·s = 3.6×10^6 J.

Example 1. The electric bill for a 30-day month shows an amount of 700 kWh of electricity used. What is the average power delivered to the house or apartment during these 30 days?

Answer: Use P = W/t

We first have to convert to proper physical units of *Joule* and *second*:

700 kWh = $700 \times 3.6 \times 10^6$ J = 2.52×10^9 J

30 days = 30 x 24 x 3600 s = 2.59×10^6 s

Therefore the average power is $P = W/t = 2.52\text{x}10^9\ \text{J}/2.59\text{x}10^6\ \text{s} = 972\ \text{W} \approx 1\ \text{kW}$.

(This power fluctuates greatly during the day. It could be as high as 10,000 W when the air conditioner, electric dryer, and stove are on. Or it could be as low as 60 W when only a light bulb is on or some electronic equipment on standby.)

Example 2 The above electric bill for 700 kWh is $77.00. What is the cost of electricity in cents/kWh? ____________

Incandescent, Compact Fluorescent (CFL), and Light Emitting Diode LED Light Bulbs

When you use a 7 W LED, it delivers about the same light output as a 60 W incandescent bulb. But LEDs use only about tenth the electricity. We pay correspondingly less for the same light output.

Question

A 7 W LED costs $2.00. A 60 W incandescent light bulb costs $0.60. The LED lasts10-times longer than the incandescent light bulb. The light bulbs are turned on for 200 hours in a 30-day month. The cost of electricity is 13 cents/kWh.

a) How long does it take to recover the higher cost of the LED?

b) How much money do you save over the life of 10,000 hours of the LED?

Answer: a) Time to recover the cost = ________months. b) Money saved = $________

A Fun Question About Weightlifting

Estimate how many repetitions you have to do for 1 kWh of work when you raise a weight of 20 kg through by 1 meter. Answer: 18,000 repetitions! (Any conclusions?)

Audio Systems

The possible signal inputs into an audio system are:
Microphone
CD/DVD Player
AM/FM tuner
Cassette player, record player – almost obsolete
The signals from these components go into a preamplifier and amplifier.

The preamplifier, amplifier, and AM/FM tuner found in Hi-Fi systems today genereally are combined into a *receiver.* Some receivers may have a frequency *equalizer* with sliders or knobs, one per octave, for "tuning" a room for optimal frequency response.

Impedance Matching
The output impedance of a component should be matched to the input impedance of the next component for maximum signal transmission (minimal signal reflection). For instance, the output impedance of a microphone should match the input impedance of the preamplifier or receiver. Even more importantly, the output impedance of the receiver or amplifier should match the impedance of the loudspeaker to which it is connected.

The *preamplifier* is the heart of an audio system. It accepts low-level signals in the μV to mV range from audio sources such as a CD player or microphone. It then produces a *line level signal* output of about 1.5 Volt, applied to the power amplifier. The preamplifier also contains the controls for volume, left-right balance, and possibly an equalizer.

The *amplifier* has an input impedance of about 50 kΩ and accepts line level signals from the preamplifier at about 1.5 Volt with power levels in the milliwatt range. The electric power to the speakers is in the range 1 to 100 W or sometimes even higher. Most of this ends up as heat and very little as acoustic power. A loud sound in a living room may require 0.2 W of acoustical power. For an assumed speaker efficiency of 0.1%, the electrical output from the amplifier hence needs to be around 200W.
The output impedance of the amplifier should match the input impedance of the speakers, e.g. 4 Ω, 8 Ω, or 16 Ω, with 8 Ω being the most common.

System Linearity
For the faithful reproduction of sound, all components of an audio system should have a *linear response.* That means that the shape of the input waveform should not be distorted at any loudness level. Preamplifiers and amplifiers are highly linear to better than ±0.5 dB. Good microphones are linear to within ±1 dB over most of the audible range. Loudspeakers have the least linear response, especially in the bass region. A frequency *equalizer* can restore some of the linearity. A power amplifier working at full output during loud sound bursts may show increased distortion and non-linearity. Maintaining system linearity is a primary justification for buying a power amplifier with a higher output than normally needed.

Microphones

Demonstrations. Show various types of microphones

Condenser Microphones
A fixed charge is placed on a capacitor resembling a system of two parallel plates. One of the plates acts as a diaphragm moving with the arriving sound waves. This changes the plate separation and produces a changing voltage between the capacitor plates.

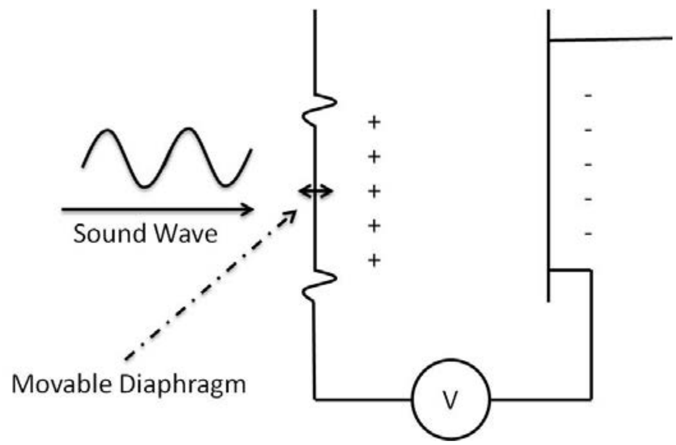


Figure. Schematic of a condenser or electrostatic microphone. The condenser plates are pre-charged with a phantom voltage from an outside source. As the distance between the plates changes, the voltage changes in tune with the arriving waveforms. The voltage changes are amplified and ultimately reach the loudspeakers. (From Berg and Stork, Fig. 7-5, p. 189.)

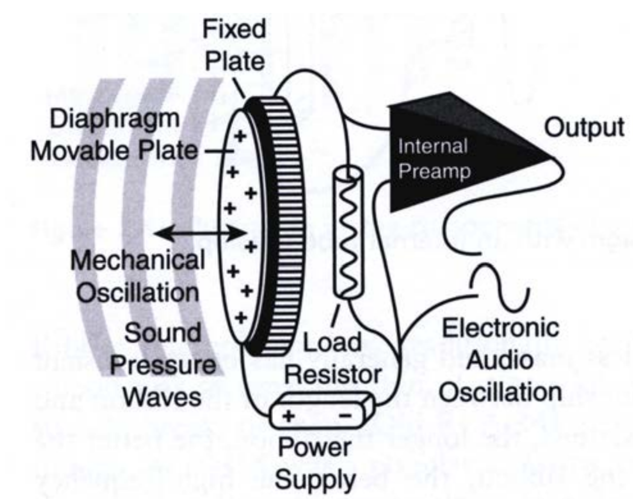


Figure. A more detailed view of the components of a condenser microphone.
(From Tom Lubin, The Microphone Book, Course Technology – Cengage Learning 2010, page 20.)

Faraday's Law of Electromagnetic Induction

A voltage is induced by a changing magnetic flux through a coil of wire

$$\textbf{Induced voltage } \varepsilon = -N\Delta\Phi/\Delta t$$

The induced voltage ε is equal to the change of magnetic flux ΔΦ with time Δt through a wire coil of N turns. Note that the magnetic flux through a wire coil has to change with time to get an output voltage.

Important Applications
Dynamic microphone
Electric generator
Electric transformer
Electromagnetic waves: cell phones, radio, television, radar

Dynamic Microphones
Dynamic microphones work on the principle of Faraday's law of electromagnetic induction, published by Michael Faraday in England in 1831.

When a sound wave arrives at a dynamic microphone, the pressure fluctuations move a diaphragm fastened to a light coil of wire. The coil moves inside a magnet and the magnetic field in the coil changes. This induces a time-varying output voltage between the ends of the coil. The voltage tracks the arriving waveform and reproduces it as an electric output. The time-varying voltage is amplified and applied to a loudspeaker.

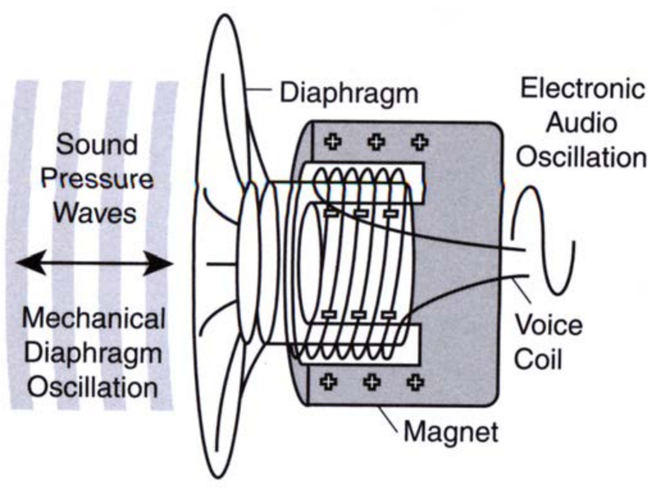


Figure. Dynamic microphone and its major components. Note the movable coil attached to the microphone cone. The coil oscillates inside a permanent magnet and produces an induced voltage at its output according to Faraday's law of electromagnetic induction. (From Tom Lubin, The Microphone Book, Course Technology – Cengage Learning 2010, page 16.)

Demonstrations of Faraday's Law of Electromagnetic Induction

1. A bar magnet moves in and out of a solenoid coil (or the coil moves with respect to the magnet; only the relative motion counts). While the magnet moves, a voltage is induced between the ends of the coil. When this is applied to a galvanometer, a small current flows in tune with the applied voltage. This demonstrates the operating principle of a *dynamic microphone*.

2. Show a small model electric generator. Crank the shaft and see a flickering light.

3. Play a *Theremin*. The device is the only musical instrument which the musician does not touch when playing it. It is based on Faraday's law of electromagnetic induction and the transmission of electromagnetic waves through space.

4. Faraday's law "in reverse". Apply a voltage to a wire coil. Hold the coil near a magnet. Observe how the coil moves as a changing voltage is applied. This is the reverse of the above demonstration and shows the principle of *an electric motor and loudspeaker*.
See the Figure in the next page of the basic components of a dynamic microphone.

5. Show a "toy loudspeaker" made from a plastic cup. A coil is taped to the bottom of the cup. Connect the earphone output from a radio or the output from a signal generator to the coil. As you approach the coil with a strong magnet the music starts playing!
Question
Why is there no sound when the magnet is far away from the speaker?
Answer: ____

6. Use a dynamic loudspeaker with the grill removed so that you can see the cone. Apply a slowly varying voltage (e.g. 5 Hz) to the speaker and see the movement of the cone. Touch the speaker terminals with a battery and see the speaker move and hear the clicks when you touch. Reverse the battery polarity. Such a loudspeaker works on the same principle (i.e. Faraday's law in reverse) as an electromagnetic motor.

Sample Calculation of the Induced Voltage from Faraday's Law (not required)
Use a coil with N = 100 turns, a cylindrical toy magnet with a maximum magnetic flux of $\Delta\Phi = 10^{-6}$ units (V·s) through a 1 cm^2 opening of the coil, and change this flux in Δt = 0.01 s by rapidly inserting the magnet into the coil. By moving the magnet in and out of the coil an AC voltage is induced as can be seen on the galvanometer. Calculate the maximum induced voltage.
Answer: Use $\varepsilon = - N\Delta\Phi/\Delta t$ or $\varepsilon = - 100\cdot10^{-6}/0.01 = - 0.01$ V = - 10 mV.
This would be considered a large signal from a dynamic microphone.
(The minus sign in Faraday's law is unimportant here. It signifies the validity of the principle of conservation of energy.)

Principle of Electric Generator (Microphone) and Electric Motor (Loudspeaker)

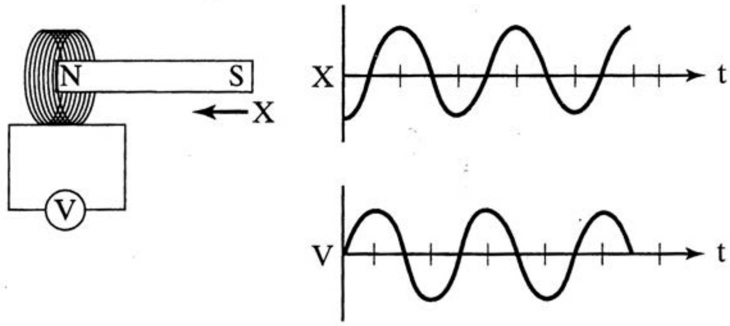


Figure. Electric generator based on Faraday's law of a changing magnetic field through a wire coil. A voltage V is induced by electromagnetic induction. It does not matter whether the magnet or the coil moves. (From Berg & Stork, Fig. 7-3, p. 188.)

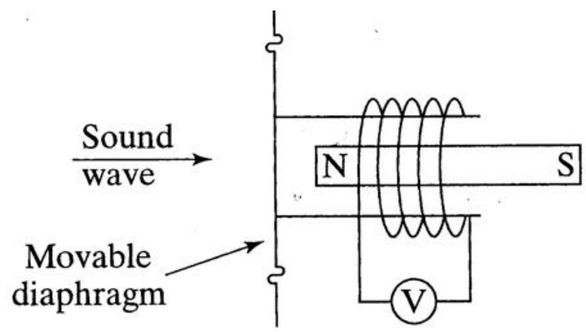


Figure. Dynamic microphone. A wire coil moves with respect to a stationary magnet. A voltage is induced between the ends of the coil. The coil is fastened to a diaphragm that moves with the sound waves. The induced voltage tracks the arriving waveform. (From Berg & Stork, Fig. 7-4, p. 188.)

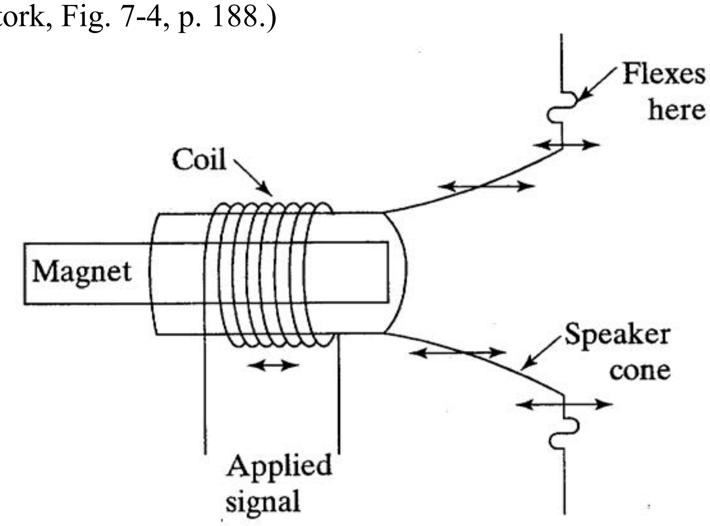


Figure. Dynamic loudspeaker. A time-varying voltage is applied to a moving wire coil attached to the loudspeaker cone. The magnet inside the coil is fixed. (This also is the basic principle of an electromagnetic motor, which is the reverse of an electric generator.) (From Berg & Stork, Fig. 7-10, p. 192.)

Directional Characteristics and Frequency Response of a Microphone

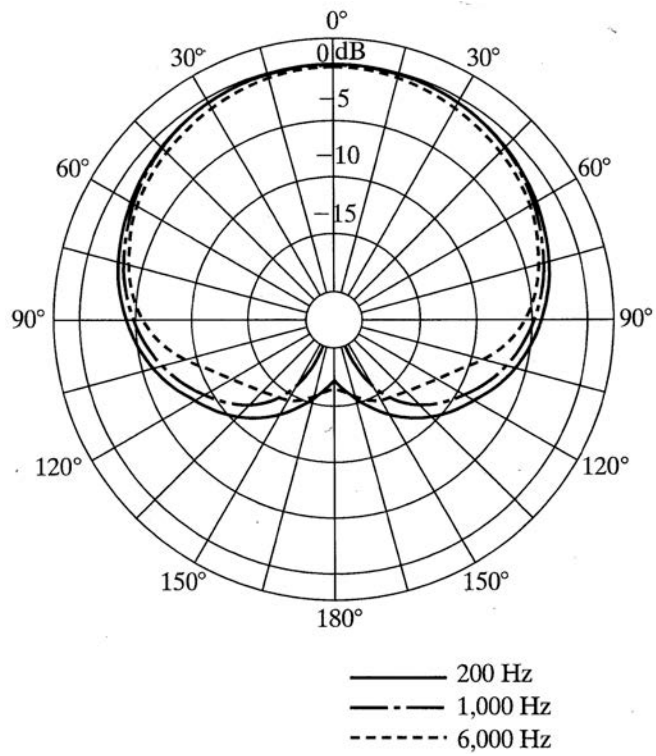


Figure. Directional response of an electret condenser microphone. Shown is a cardioid response resembling the shape of a heart. An angle of 0^0 indicates the forward direction toward the sound source. (From Berg & Stork, 7-6. p. 189.)

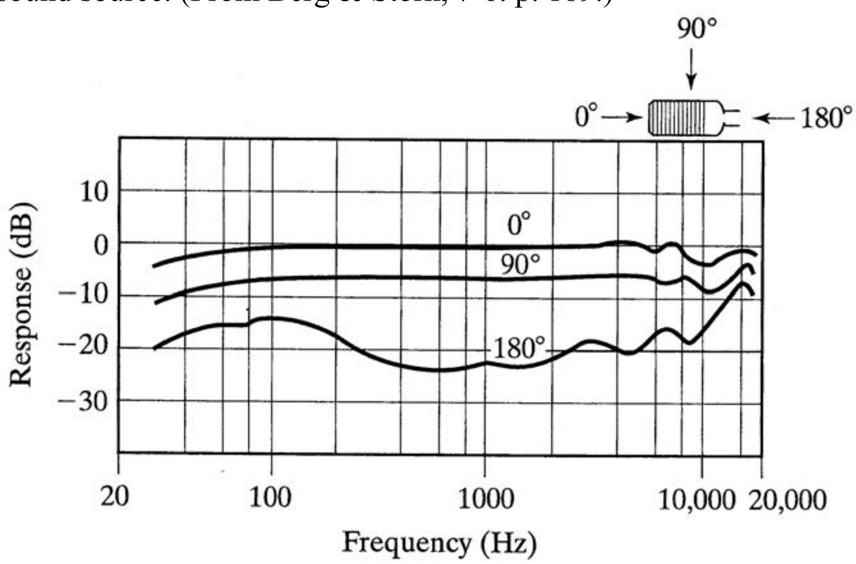
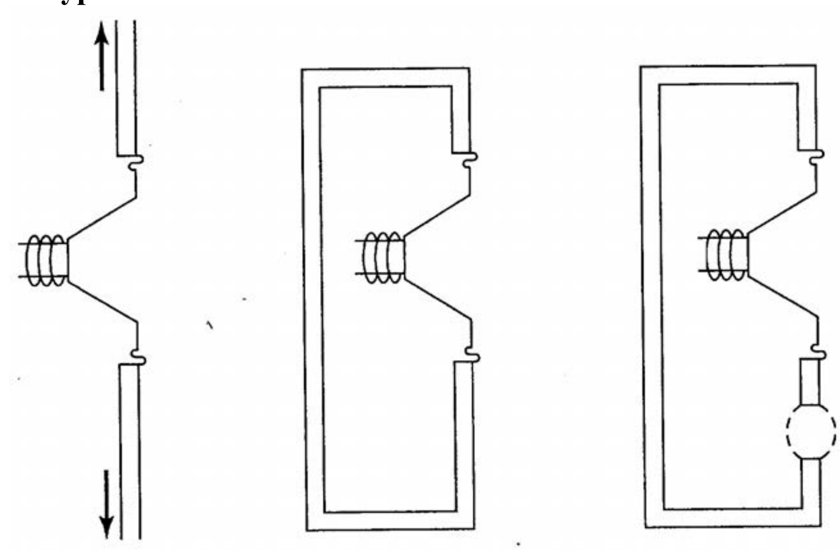


Figure. Frequency response of an electret condenser microphone. The response is uniform in the forward direction to within 1 dB between 100 and 8000 Hz. It is similarly uniform at right angles of 90^0. In the backward direction, for which the microphone is not intended, the response is lower and non-uniform.
(From Berg & Stork, Fig. 7-7, p. 191.)

Loudspeaker Types



Infinite baffle Acoustic suspension Tuned port (bass reflex)

Figure. Three loudspeakers types. The infinite baffle can be a wall or ceiling. The acoustic suspension speaker is in an airtight box. Waves cannot get out from the back and interfere with waves from the front. For a tuned port speaker the box is a Helmholtz resonator and extends the bass response. (From Berg & Stork, Fig. 7-11, p. 193.)

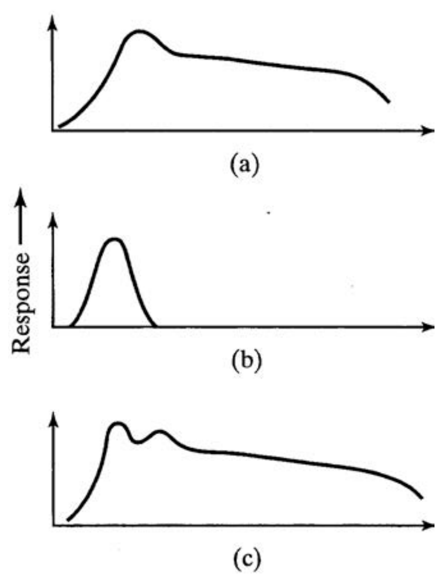


Figure. (a) Frequency response of an acoustic suspension speaker. The bump at low frequencies is due to speaker resonance (at about 100 Hz). (b) Frequency response of an empty box having a tuned port that acts as a Helmholtz resonator. (c) Overall response with the speaker in the box. The dimensions of the box and port are tuned to extend the frequency range farther into the bass region. (From Berg & Stork, Fig. 7-12, p. 193.)

Demonstration

Compare the bass response of an open speaker and a similar one in a bass reflex box.

Efficiency of a Loudspeaker and Double Bass

Loudspeaker
We did an experiment where we applied an electric power of $P_{electric}$ = 1.5 W from a sine wave generator to a loudspeaker at 1000 Hz. The measured sound intensity level SIL at a distance r = 1 m in front of the speaker was SIL = 93 dB. The corresponding sound intensity *I* is obtained from the Fletcher-Munson curves or the formula $I/I_0 = 10^{SIL/10}$.
With $I_0 = 1x10^{-12}$ W/m^2 and SIL = 93 dB, we obtain I = $2.0x10^{-3}$ W/m^2.
In order to obtain the loudspeaker efficiency, we assumed that the sound intensity was emitted uniformly into thehalf-sphere in front of the speaker.
The area of this surface at a distance r = 1 m is A = πr^2 = 3.14 m^2.
Hence the total emitted acoustical power is $P_{acoustic} = I \cdot A = 2x10^{-3}x3.14$ m$^2 \approx 0.0063$ W.
Designating the loudspeaker efficiency by the Greek letter η, we finally have
$\eta = P_{acoustic}/P_{electric}$ = 0.0063/1.5 = 0.0042 or 0.42%.

Double Bass
Professor Mark Morton from the TTU School of Music visited our class in Spring semester 2017 and played the double bass. We performed an experiment to determine roughly the efficiency for converting the mechanical power from bowing a string to the acoustical power emitted.
The mechanical power was given by $P_{mech} = F \cdot v$, where F is the force from the bow on the string and v the speed of bow across the string in (open A-String).
On a spring scale, we simulated the force of the bow and found approximately F = 5 N.
We also estimated the speed of the bow across the bow as v = 0.5 m/s.
Hence the mechanical power was $P_{mechanical}$ = 5x0.5 = 2.5 Watt.
At a distance of 1 m from the string, we measured the sound intensity level SIL = 90 dB.
The corresponding sound intensity I, with the same formula $I/I_0 = 10^{SIL/10}$ as above, was then I = $1x10^{-3}$ W/m^2.
Assuming that the sound intensity was emitted uniformly into half of a hemisphere at a distance of =1 m in front of the double bass, as in the above example, we obtained for the acoustical power
$P_{acoustical} = I \cdot A = I \cdot \pi r^2 = 1x10^{-3}x3.14 \approx 0.0031$Watt.
The efficiency then was given by $\eta = P_{acoustic}/P_{mechanical}$ = 0.0031/2.5 = 0.0012 or 0.12%.

This efficiency is small again and of the same order of magnitude as for the loudspeaker above. In both cases, the sound was quite loud because of the high sensitivity of the ear.

There is an extra energy conversion in the case of the loudspeaker from electrical to mechanical energy of the vibrating speaker cone, with an efficiency less than 100%. In our two examples here, the efficiency for converting mechanical into acoustical energy is larger for the loudspeaker than for the double bass.
(P.S.: In both cases more than 99% of the input energy is converted into heat and very little sound energy is emitted, but the latter is the precious energy form that counts.)

The Compact Disc (CD)

1980-81 CD standards established
1980-85 CD development

CD Characteristics

High fidelity
Low noise
Large dynamic range > 90 dB
Frequency range ≥ 22 kHz
Data Storage ≈ 700 MB

Rotation with constant linear velocity of 1.2 m/s
Width of pits 0.5 μm
Depth of pits 0.2 μm
Row spacing center-to-center 1.6 μm (limited by optical diffraction of laser beam)
CD readout by red laser
In distinction to a CD, a DVD. It uses a blue or ultra-violet laser of shorter wavelength, together with multiple layers, and has a much greater storage capacity of 4.7 GB.

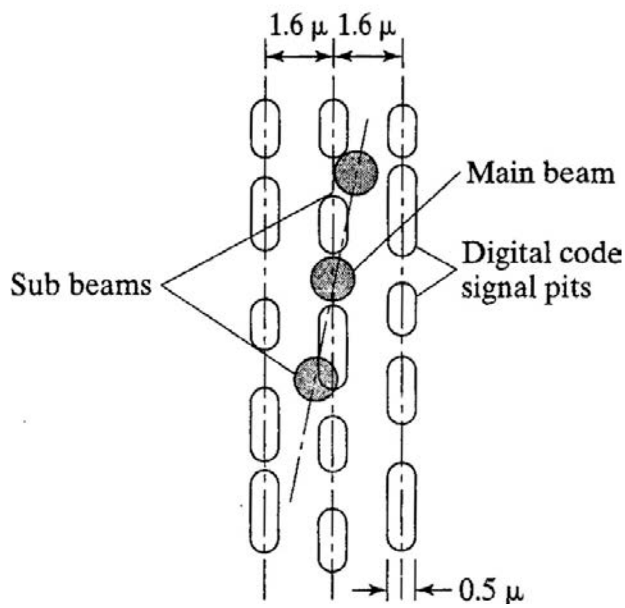


Figure. Tracking of the pits on a CD by a laser. Three laser beams are used. The main beam scans the signal pits, the two sub-beams guide the main beam along the pits. The pit length is not to scale. (From Berg & Stork, Fig. 7-21, p. 208.)

Demonstration
Show a CD and DVD and observe the different colors resulting from diffraction of light.

Calculation of Compact Disc Parameters

A CD has pit tracks placed between radii of 2.3 cm and 5.7 cm.
The spacing between tracks is 1.6 μm.

Questions

a) How many tracks are on a CD?
Answer: The region on the CD occupied by the tracks is 5.7 cm – 2.3 cm = 3.4 cm
Number of tracks N = $0.034\text{m}/1.6\text{x}10^{-6}\text{m}$ = **21250 tracks**

b) What is the total length of the spiral track on the CD?
Answer: The average radius of the tracks is r_{ave} = (5.7 cm + 2.3 cm)/2 = 4.0 cm
The total length of the spiral track is L = $2\pi r_{ave}N$ = 2π x 0.04 m x 21000 m = **5278 m**
= 5.278 km = 3.28 miles.

c) The total playing time of the CD is 72 minutes and 6 seconds.
What is the speed with which the laser reads the recording?
Answer: v = L/t = 5278m/4326s = **1.22 m/s**.
(Note that this linear track speed is constant. Therefore the angular velocity changes from high on the inner tracks to lower on the outer tracks.)

d) The data points on the disc consist of 16-bit binary numbers, read at a rate of 44000 numbers/second for each of the two stereo channels.
How many binary bits per second are read?
Answer: Number of bits/s = 16 bits 44000/s x 2 (stereo) = **$1.41\text{x}10^6$ bits/s**

e) What is the average length for one binary bit along the track?
Answer: l = (1.22m/s)/($1.4112\text{x}10^6$ bits/s) = 0.86 x 10^{-6} m = **0.86 μm**
About half, or **0.43 μm**, is occupied by the burnt pit of a bit, the other half is empty space between successive pits.

f) What is the storage capacity of the CD?
Answer: Use the data from parts c) and d):
Storage = 1.4112 x10^6 bits/s x 4326 s = 6.106 x 10^9 bits = $6.106\text{x}10^6$ kilobit
Convert to Bytes: Use 8192 kilobit = 1 megabyte (8 bits = 1 byte)
The storage capacity is $6.106\text{x}10^6/8192$ = 745 Megabyte ≈ **700 MB.**
This is the familiar "700 MB" storage capacity of a CD.

Dynamic Range of a CD

A CD can cover a wide amplitude range in electrical signal of 2^{15}:1.
This corresponds to a range in dB of $10\cdot\log(2^{15}/1) = 150\cdot\log2 = 45.2$ dB.
Sound intensity is proportional to amplitude squared.
Hence the dynamic range of a CD, expressed as a range in sound intensity level, is
SIL = $20\cdot\log(2^{15}/1) = 90.4$ dB ≈ 90 dB. This is a very large range compared to older means of sound reproduction such as tapes or vinyl records.

Elementary Music Theory, Musical Scales, Intervals, Cents

Piano Keyboard

For those interested in musical scales and intervals, the following is some cursory music theory. Perhaps you are interested in musical intervals such as octaves, fifths, fourths, etc., or how to translate “do, re, mi, fa, sol, la, ti, do” into a musical scale.
Also, it may be useful to know the basics behind the so-called “equal tempered scale” that has dominated Western music for about two centuries.
The musical stave system and notation are given for completeness.
Note that “concert-A” in the United States is A4 = 440 Hz.
Recognize some of the systematic features in the layout of the piano keys.

Piano Keyboard

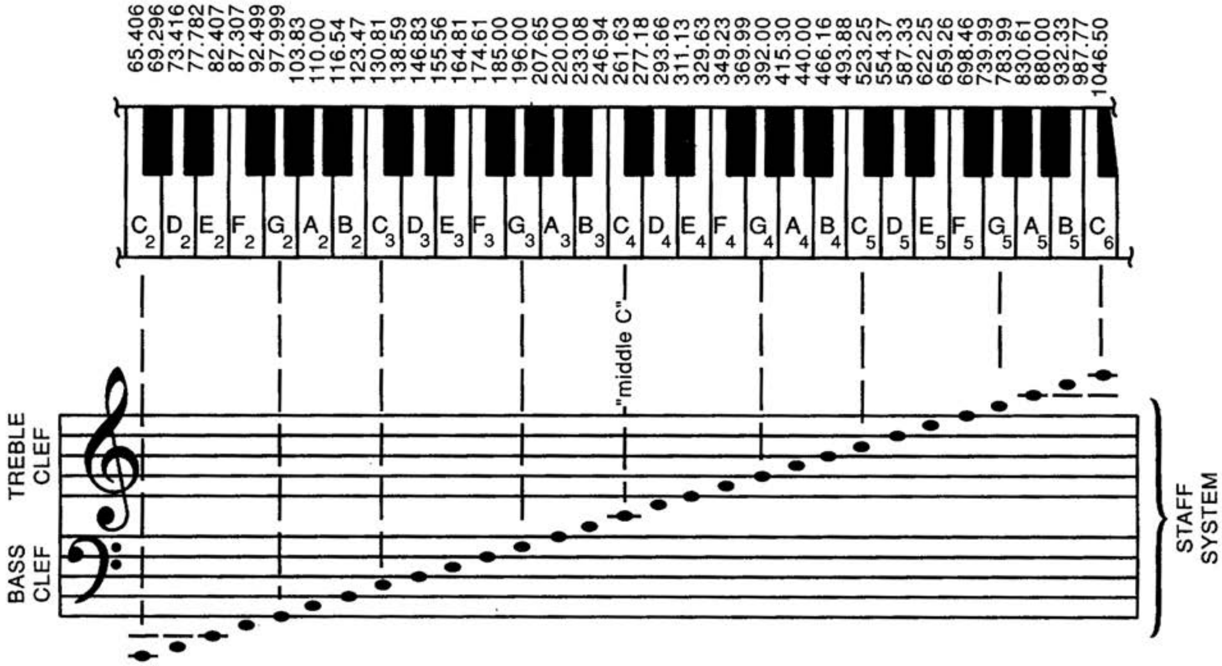


Figure. Middle section of the piano keyboard, frequencies, and notes on the musical staves. (From Berg & Stork, Fig. A-1, p. 368.)

Circle of Fifths, Pythagorean Comma, Equal Tempered Scale

Start with any musical note on the keyboard, e.g. C1, move up 12 successive fifths, i.e.
C1 - G1 - D2 - A2 - E3 - B3 - F4# - C5# - G5# - D6# - A6# - F7 - C8.
If we take pure fifths, the frequency ratio C8/C1 between the ending and starting note is $(3/2)^{12}$ = 531,441/4,096 = **129.7463**.
Now start with C1 again and move up 7 octaves, i.e.
C1 - C2 - C3 - C4 - C5 - C6 - C7 - C8
The ratio C8/C1 this time is 2^7 = **128.0000**.
We see that the 12 pure fifths result in a slightly higher frequency than the 7 octaves. The ratio of the two numbers is slightly different from unity:

$$\mathbf{(3/2)^{12}/(2^7) = 129.7463/128.0000 = 1.013643}\ldots$$, called the **“**Comma of Pythagoras”

Demonstration
Choose f = 700 Hz (“7 octaves”) and f = 700·1.013643 = 709.6 Hz (“12 fifths”) on two signal generators. Listen to the beats. You are “hearing” the Pythagorean comma.

The *Pythagorean comma* is the difference of 1.3643% between 12 pure fifths and 7 octaves. The comma is clearly audible. It corresponds to a difference of approximately 1/4 of a semitone, which is something to be avoided in music.
The so-called *“Circle of Fifths”* of 12 pure fifths does not close exactly into 7 octaves. But we ended with the same note (C8) on the keyboard! This forced agreement is a compromise resulting from the so-called “tempering” of the fifths.

The problem of the Pythagorean comma existed for several hundred years. Finally and ingeniously, the discrepancy was spread equally over all 12 fifths by lowering or “tempering” them all equally. This led to the ***Equal-Tempered Scale*** **(ET)** as follows. Let us designate the tempered fifth by the symbol *y*. It is no longer equal to 3/2. On the other hand, we do wish to preserve the exact ratio of 2/1 for the octave. We thus require 12 “tempered” fifths to equal exactly 7 octaves. If we call the tempered fifth “y”, then

$$\mathbf{y^{12} = 2^7 \quad or \quad y = 2^{7/12} = 1.498307\ldots.}$$

The tempered fifth in equal temperament (ET) is smaller than 3/2 by 0.113%.

The 12-tone chromatic scale has 12 half-steps (semitones) in an octave. Therefore the frequency ratio between two notes a semitone apart is given by

$$\mathbf{2^{(7/12)/7} = 2^{1/12} = 1.059464\ldots}$$ = semitone spacing in the *equal-tempered (ET) scale*.

If f_0 is the frequency of a note, then the frequencies of the 12 notes n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the chromatic scale are

$$\mathbf{f_n = 2^{n/12} f_0}, \quad \text{with } f_0 = \text{tonic and } f_{12} = 2^{12/12} f_0 = 2f_0 \text{ (octave).}$$

Revisiting Musical Fifths, Octaves, Pythagorean Scale, and Wolf Fifth

We rephrase our preceding discussion and ask:
Can one build a 12-tone scale from musical fifths and *octaves* alone? The answer is "yes", but we must use modified, not Pythagorean, fifths.
Let m and n be two positive integers. We then ask for a solution to the equation

$$(3/2)^m = 2^n \quad \text{or} \quad \frac{m}{n} = \frac{\log 2}{\log 3/2} = 1.709511.....$$

But no exact solution exists for any pair of integers (m, n). Hence one cannot pack exactly an integer number of Pythagorean fifths into an integer number of octaves, and so the circle of fifths does not quite close. We ask instead:
For which set of *small* integers (m, n) do we come close to the ratio of 1.70951…?
The following table shows the ratios m/n of the exponents for the fifths and octaves.

Number m of Fifths and Number of n of Octaves and Corresponding Ratios m/n
(Table computed by Professor Igor Volobouev, Texas Tech University)

m\n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	0.5	0.333333	0.25	0.2	0.166667	0.142857	0.125	0.111111	0.1	0.0909091	0.0833333	0.0769231	0.0714286
2	2	1	0.666667	0.5	0.4	0.333333	0.285714	0.25	0.222222	0.2	0.181818	0.166667	0.153846	0.142857
3	3	1.5	1	0.75	0.6	0.5	0.428571	0.375	0.333333	0.3	0.272727	0.25	0.230769	0.214286
4	4	2	1.33333	1	0.8	0.666667	0.571429	0.5	0.444444	0.4	0.363636	0.333333	0.307692	0.285714
5	5	2.5	1.66667	1.25	1	0.833333	0.714286	0.625	0.555556	0.5	0.454545	0.416667	0.384615	0.357143
6	6	3	2	1.5	1.2	1	0.857143	0.75	0.666667	0.6	0.545455	0.5	0.461538	0.428571
7	7	3.5	2.33333	1.75	1.4	1.16667	1	0.875	0.777778	0.7	0.636364	0.583333	0.538462	0.5
8	8	4	2.66667	2	1.6	1.33333	1.14286	1	0.888889	0.8	0.727273	0.666667	0.615385	0.571429
9	9	4.5	3	2.25	1.8	1.5	1.28571	1.125	1	0.9	0.818182	0.75	0.692308	0.642857
10	10	5	3.33333	2.5	2	1.66667	1.42857	1.25	1.11111	1	0.909091	0.833333	0.769231	0.714286
11	11	5.5	3.66667	2.75	2.2	1.83333	1.57143	1.375	1.22222	1.1	1	0.916667	0.846154	0.785714
12	12	6	4	3	2.4	2	1.71429	1.5	1.33333	1.2	1.09091	1	0.923077	0.857143
13	13	6.5	4.33333	3.25	2.6	2.16667	1.85714	1.625	1.44444	1.3	1.18182	1.08333	1	0.928571
14	14	7	4.66667	3.5	2.8	2.33333	2	1.75	1.55556	1.4	1.27273	1.16667	1.07692	1
15	15	7.5	5	3.75	3	2.5	2.14286	1.875	1.66667	1.5	1.36364	1.25	1.15385	1.07143
16	16	8	5.33333	4	3.2	2.66667	2.28571	2	1.77778	1.6	1.45455	1.33333	1.23077	1.14286
17	17	8.5	5.66667	4.25	3.4	2.83333	2.42857	2.125	1.88889	1.7	1.54545	1.41667	1.30769	1.21429
18	18	9	6	4.5	3.6	3	2.57143	2.25	2	1.8	1.63636	1.5	1.38462	1.28571
19	19	9.5	6.33333	4.75	3.8	3.16667	2.71429	2.375	2.11111	1.9	1.72727	1.58333	1.46154	1.35714
20	20	10	6.66667	5	4	3.33333	2.85714	2.5	2.22222	2	1.81818	1.66667	1.53846	1.42857
21	21	10.5	7	5.25	4.2	3.5	3	2.625	2.33333	2.1	1.90909	1.75	1.61538	1.5
22	22	11	7.33333	5.5	4.4	3.66667	3.14286	2.75	2.44444	2.2	2	1.83333	1.69231	1.57143
23	23	11.5	7.66667	5.75	4.6	3.83333	3.28571	2.875	2.55556	2.3	2.09091	1.91667	1.76923	1.64286
24	24	12	8	6	4.8	4	3.42857	3	2.66667	2.4	2.18182	2	1.84615	1.71429

In this table the m-numbers are shown in the left-most column and the n-numbers in the top row. The other columns and rows show the corresponding ratios m/n.

We see that for **m = 12 and n = 7** we have

$$\frac{m}{n} = \frac{12}{7} = 1.714286.....$$ This is closest to the number 1.709511…

The 10 ratios closest to the ideal value m/n = 1.709511 are:

1. m = 12, n = 7, ratio = 1.71429
2. m = 24, n = 14, ratio = 1.71429
3. m = 17, n = 10, ratio = 1.70000
4. m = 22, n = 13, ratio = 1.69231
5. m = 19, n = 11, ratio = 1.72727
6. m = 7, n = 4, ratio = 1.75000
7. m = 14, n = 8, ratio = 1.75000
8. m = 21, n = 12, ratio = 1.75000
9. m = 5, n = 3, ratio = 1.66667
10. m = 10, n = 6, ratio = 1.66667

For all other sets (m, n) for m from 1 to 24 and n from 1 to 14, the ratio m/n is larger than for 12/7 = 1.714286, although other ratios, especially 17/10 and 19/11, are also close the the ideal value of 1.709511.

The number m = 12 yields the 12 degrees in the octave of the *Chromatic Scale* in Western music, with all notes constructed solely from fifths and octaves.

In order to construct the chromatic scale, we move 12 fifths up (or down) and scale them down (or up) by octaves, so that we always land within the original octave. This yields the 12 notes (degrees) in that octave. (See next page for the detailed construction.)

How can we close the "circle of fifths" so that 12 Pythagorean fifths exactly equal 7 octaves?

We show two possibilities:

1. In the Pythagorean 12-tone scale, all fifths have values of 3/2 except the last one. This bad fifth or **"wolf fifth"** is obtained by moving up 7 octaves and 11 Pythagorean fifths of **3/2** from the same starting note and taking the ratio $2^7/(3/2)^{11}$ = **1.479810**… **≠ 3/2**.
2. In the equal tempered scale, all fifths "*y*" are modified or tempered in the same way according to $y^{12} = 2^7$, or **y = 1.498307…**, as already mentioned.

Exercise

Show that the bad "wolf fifth" deviates from a pure fifth by the comma of Pythagoras.
Answer: Take the ratio 1.500000/1.479810 = 1.01364…, which is the "Comma of Pythagoras".

The "wolf fifth" caused serious problems in musical modulation and transposition.
The problem was solved after many years by the introduction of the equal tempered scale around the time of Johann Sebastian Bach. The harmony in the pure Pythagorean intervals was slightly damaged as a consequence, but this was outweighed by getting rid of the "wolf fifth" and the ability of composing in many different musical keys.

Construction of Musical Scales from Musical Fifths and Octaves

The 12-degree (12-tone) Pythagorean chromatic scale can be constructed entirely from pure fifth (ratio 3/2 or 2/3) and octaves (2/1 or 1/2). This is done by moving up in frequency by ratios of 3/2 and dividing the result by octave ratios of 1/2 to bring the final result to within one octave from the starting frequency. A number of up to 12 steps of fifths is needed for this procedure in order to obtain the 12 notes in the chromatic scale. The final result is obtained by ordering the 12 results in ascending numerical values. The last note will then be an octave above the starting note.

Example
Construct the note $C^{\#}$ a half step above C.
Answer
Go up 7 fifths, divide by 4 octaves, and get
$C^{\#}/C = (3/2)^7/2^4 = 218700/204800 = 1.06787$.
Reminder: In equal temperament the ratio is $C^{\#}/C = 2^{1/12} = 1.05946$.

The construction of the other 11 notes in the 12-tone scale is left as an exercise for those who are more deeply interested in music theory. However, see later in this chapter for the explicit construction of the 7-note Pythagorean scale, which is a subset of the 12-note scale.

Wolf Fifth
The construction of the 12-note Pythagorean scale and using it in different musical keys is hampered by the Wolf fifth. Hence again, equal temperament (ET) is the preferred tuning in most of Western music.

Demonstrations
1. Compare a "wolf fifth" of 1.479810 with a pure fifth of 3/2 = 1.500000:
Set two signal generators to $f_1 = 400$ Hz and $f_2 = 400 \times 1.500000 = 600.0$ Hz.
Listen to the pure fifth $f_2/f_1 = 600/400 = 1.500000$.
Set a third frequency generator to $f_3 = 400 \times 1.479810 = 591.9$ Hz.
Listen to the "wolf fifth" $f_3/f_1 = 591.9/400 = 1.479810$.
This probably sounds bad to most people.
The "wolf" beats with a frequency of $\Delta f = f_2 - f_4 = 600 - 591.9 = 8.1$ Hz.

2. Compare an equally tempered fifth of 1.498307 with a pure fifth of 1.500000:
Set a fourth frequency generator to the fifth in equal temperament,
i.e. $f_4 = 400 \times 1.498307\ldots = 599.32$ Hz.
Play f_2 and f_4 together and hear very slow beats at $\Delta f = f_2 - f_4 = 0.68$ Hz.
Listen to the fifth in equal temperament, $f_2/f_1 = 599.32/400 = 1.498307$, and alternate with the pure fifth.
Many people will not be able to hear the difference.
Equal temperament has now become the standard in Western music.

Killing the Wolf, Equal Temperament and Ruining Harmony

Killing the "wolf" and "sweeping the Pythagorean comma under the rug" became ever more important as music progressed and became more complex. Changes were necessary for musical transposition and modulation, and the tuning of different instruments.

In equal temperament (ET), the discrepancy of the Pythagorean comma is spread *equally* over the 12 notes of the chromatic scale.
All pure intervals are forsaken, except the octave, whose interval remains a ratio of 2:1. We already have seen that we require 12 modified fifths to equal exactly 7 octaves, yielding a frequency ratio of $2^{(7/12)} = 1.498307\ldots$ for the fifth in equal temperament.

The modified fifth is smaller than the ratio 3/2 for a pure fifth, but only by very little. Summarizing, this fifth then finally led to the *Equal-Tempered Scale* (ET).

In the 12-tone chromatic scale, a musical fifth contains 7 half steps or semitones, for instance from C4 to G4. "Tempering" all the 7 semitone intervals to the same size means

$$2^{(7/12)/7} = \mathbf{2^{1/12} = 1.059464\ldots}$$

The semitone interval therefore is given by a frequency ratio of 1.059464, or a frequency difference of **5.9464 %.**
This interval can be divided further (useful for the tuning of instruments) into so-called **musical cents**, where $\mathbf{2^{1/12} = 100}$ **cents**.

The frequencies of the 12 successive notes of the equal-tempered chromatic scale, starting with any note (e.g. f = C4 = 261.63 Hz), are given by

$$f,\ 2^{1/12}f,\ 2^{2/12}f,\ 2^{3/12}f,\ 2^{4/12}f,\ 2^{5/12}f,\ 2^{6/12}f,\ 2^{7/12}f,\ 2^{8/12}f,\ 2^{9/12}f,\ 2^{10/12}f,\ 2^{11/12}f,\ 2^{12/12}f = 2f$$

Example: The C-major *Equal-Tempered* Diatonic Scale
It is based on A4= 440.00 Hz, which yields C4 = 261.63 Hz, and is given by:

C4	D4	E4	F4	G4	A4	B4	C5
$2^{0/12}$	$2^{2/12}$	$2^{4/12}$	$2^{5/12}$	$2^{7/12}$	$2^{9/12}$	$2^{11/12}$	$2^{12/12}$
1	1.1225	1.2599	1.3348	1.4983	1.6818	1.8877	2.0000
261.63	293.66	329.63	349.23	392.00	440.00	493.88	523.25 Hz

More than a millennium had passed before the *Equal Tempered Scale* was finalized at the beginning of the 19th century. In retrospect, it does not look that complicated, neither musically nor mathematically, and it is simpler than using earlier scales. The latter do give purer harmonies that can be played by string instruments, trombone etc., but not by the piano with its fixed notes.

Construction of the *Pythagorean Diatonic Scale*

For constructing this historical scale, only musical fifths **3:2** and octaves **2:1** are needed. Start with C4 and move either up or down. We obtain

F3		C4		G4		D5		A5		E6		B6
	2/3		3/2		3/2		3/2		3/2		3/2	

Now rearrange all notes so that they fall within one single octave. For example, raise F3 by an octave to obtain F4, i.e. multiply the frequency by a factor of 2. Similarly, lower D5 to D4 by an octave and A5 to A4 by an octave, i.e. divide by a factor of 2. Next, lower E6 to E4 and B6 to B4, i.e. lower the frequency by 2 octaves or divide by $2^2 = 4$. Finally, add to this the note C5 one octave above C4. We now have arrived at all notes in the Pythagorean diatonic scale, summarized as follows:

C4 from C4	1	=	**1**
D4 from D5	3/2 x 3/2 x 1/2	=	**9/8**
E4 from E6	3/2 x 3/2 x 3/2 x 3/2 x 1/2 x 1/2	=	**81/64**
F4 from F3	2/3 x 2	=	**4/3**
G4 from G4	**3/2 given**	=	**3/2**
A4 from A5	3/2 x 3/2 x 3/2 x 1/2	=	**27/16**
B4 from B6	3/2 x 3/2 x 3/2 x 3/2 x 3/2 x 1/2 x 1/2	=	**243/128**
C5 from C4	**2/1 given**	=	**2**

The resulting frequencies, based on C4 = 261.63 Hz (not on A4 = 440 Hz) are:

C4	D4	E4	F4	G4	A4	B4	C5
do	re	mi	fa	sol	la	ti	do
1	9/8	81/64	4/3	3/2	27/16	243/128	2
261.63	294.33	331.11	348.83	392.44	441.49	496.67	523.25 Hz

Exercise
Musical fifths such as G_4/C_4 can also be obtained from harmonics (in this case from C_4), without resorting a priori to the ratio 3/2. Derive this ratio instead.
Answer: The first 3 harmonics (N = 1, 2, 3) in the harmonic series of C_4 are C_4, C_5, G_5. We see that $G_5 = 3C_4$. We also know that G_5 is an octave above G_4.
Hence $G_5 = 2G_4$ and $G_4 = G_5/2 = 3C_4/2$ and finally $G_4/C_4 = 3/2$.

Demonstration
Play the major triad C - E - G in the
Pythagorean scale 261.63 - 331.11 - 392.44 Hz, and compare with
equal temperament 261.63 - 329.63 - 392.00 Hz.
First, play the triads separately in the two tunings. Do you hear the difference?
Then play them together. Listen to the beats.

Construction of the *Pentatonic* and *Syntonic Diatonic Scales*

Pentatonic Scale

The Pythagorean Diatonic Scale contains the five notes of the *pentatonic scale* as a subset, resulting in one of the simplest and oldest musical scales.

C4	D4	F4	G4	A4	C5
1	9/8	4/3	3/2	27/16	2
261.63	294.33	348.83	392.44	441.49	523.25 Hz

Exercise

Construct the pentatonic scale C D F G A C yourself from fifths and octaves without copying the preceding information.

An example for the pentatonic scale in Western music is the beautiful melodic main theme in Dvorak's "New World Symphony", which uses only these 5 notes.

Syntonic Diatonic Scale

We now construct the so-called C-major syntonic diatonic scale in analogy to the Pythagorean scale, but this time with pure major thirds **5/4 (**according to Claudius Ptolemy) in addition to Pythagorean fifths **3/2** and octaves **2/1.** Since we are given these 3 intervals to start with and thus already have 3 notes in the scale, we construct the remaining 5 notes with the simplest possible rational numbers. The most complicated rational number of these will be 15/8 for the note B4. All other ratios will only contain single-digit numbers. Starting again with C4, the procedure is this:

C4 from C4	1	=	**1**
D4 from D5	3/2 x 3/2 x 1/2	=	**9/8**
E4 from E6	**5/4 given**	**=**	**5/4**
F4 from F3	2/3 x 2	=	**4/3**
G4 from G4	**3/2 given**	=	**3/2**
A4 from F4	4/3 x 5/4	=	**5/3**
B4 from G4	3/2 x 5/4	=	**15/8**
C5 from C4	**2/1 given**	=	**2**

Summary (with frequencies based on C4 = 261.63 Hz)

C4	D4	E4	F4	G4	A4	B4	C5
do	re	mi	fa	sol	la	ti	do
1	9/8	5/4	4/3	3/2	5/3	15/8	2
261.63	294.33	327.03	348.83	392.44	436.05	490.55	523.25 Hz

The *syntonic diatonic scale* is also called a *just major scale*.
It goes back to Claudius Ptolemy (Roman citizen in Egypt, mathematician, astronomer and astrologer, 90-168 A.D.)

Frequencies of Three Diatonic Musical Scales. Musical Cents

All frequencies are based on C4 = 261.63 Hz.

do	re	mi	fa	sol	la	ti	do
Equal Temperament							
C4	D4	E4	F4	G4	A4	B4	C5
$2^{0/12}=1$	$2^{2/12}$	$2^{4/12}$	$2^{5/12}$	$2^{7/12}$	$2^{9/12}$	$2^{11/12}$	$2^{12/12}=2$
1	1.1225	1.2599	1.3348	1.4983	1.6818	1.8877	2.0000
261.63	293.66	329.63	349.23	392.00	440.00	493.88	523.25 Hz
Pythagorean Scale							
C4	D4	E4	F4	G4	A4	B4	C5
1	9/8	81/64	4/3	3/2	27/16	243/128	2
261.63	294.33	331.11	348.83	392.44	441.49	496.67	523.25 Hz
Just Major (Syntonic Diatonic) Scale							
C4	D4	E4	F4	G4	A4	B4	C5
1	9/8	5/4	4/3	3/2	5/3	15/8	2
261.63	294.33	327.03	348.83	392.44	436.05	490.55	523.25 Hz

We see that the frequency discrepancies between the three scales generally are within ±1% or less. The greatest discrepancies are in the notes E4, A4, B4, the smallest in D4, F4, G4. These differences are audible when comparing the same notes from different scales. One easily can hear beats between the same notes (demonstration). Equal temperament contains compromises by equalizing all semitone intervals. We have become quite used to this in spite of the slight destruction of harmony and the surrender of pure intervals.

Musical Cents

The smallest musical interval in Western music is the spacing between discrete semitones or half steps in equal temperament, see for instance the keys on the piano. However, deviations from this occur in mistuned instruments and in different temperaments. In order to quantify such small variations, the *musical cent scale* is used. It is defined by

$$\mathbf{c = 1200 \frac{\log f_2/f_1}{\log 2}},$$

where "*c*" is the number of musical "cents" for the interval f_2/f_1 between two frequencies f_2 and f_1. Obviously, one octave with $f_2/f_1 = 2$ is 1200 cents.
Intervals of 10 cents or even less at mid-frequencies are audible to many listeners.

Exercise: Show that a half step $2^{1/12}$ in equal temperament is 100 cents.
Proof: $f_2/f_1 = 2^{1/12} \rightarrow c = 1200\cdot\log(2^{1/12})/\log 2 = 1200\cdot(1/12)\cdot\log2/\log2 = 100$ cents.
Demonstration: Play *pure* thirds or fifths C4-E4 or C4-G4 and compare with *equal-tempered* thirds and fifths. Note the resulting beats!

Demonstration of Musical Fifths and Thirds with a 4-Chime Set

A set of 4 chimes was custom built by *Music of the Spheres* in Austin, Texas, in order to demonstrate some musical intervals. The chimes were made of aluminum tubing with an outside diameter of 6.34 cm and inside diameter 5.37 cm.

The intervals that can be played with these chimes are the Ptolemaic major third A3-C$^{\#}$4 (interval of 5/4), the third A3-C$^{\#}$4 in equal temperament (ratio $2^{4/12}$), and a pure Pythagorean fifth A3-E4 (ratio 3/2).

Table. Musical notes, tuning, frequencies, and lengths for the 4-chime set.

A3		220.00 Hz	chime length 129.65 cm
C$^{\#}$4 just Ptolemaic	220.00 x 5/4	= 275.00 Hz	115.65 cm
C$^{\#}$4 equal temperament	220.00 x $2^{4/12}$	= 277.18 Hz	115.20 cm
E4 Pythagorean	220.00 x 3/2	= 330.00 Hz	105.45 cm

Question

What other musical interval(s) can you play with these chimes besides the three intervals mentioned above?

Demonstrations

1. Strike the chimes C$^{\#}$4 "pure" and C$^{\#}$4 "ET" with a soft beater at their center to excite the fundamental vibrational mode. Listen to the beat frequency of about 2.2 Hz (14 cents or about 1/7 of a semitone).
2. Strike the above two chimes near their ends to excite the 2nd vibrational mode. (Higher modes may be excited at the same time.) Listen to the beats again. You should predominantly hear beats with a 2.756 higher frequency than before, i.e. about 6.0 Hz. This is due to the fact that the 2nd mode is inharmonic with a 2.756-times higher frequency than the fundamental mode. In addition, you may still hear the beats at 2.2 Hz from the fundamental mode as before.
3. Strike the three chimes A3, C$^{\#}$4 "*pure*", and E4 "pure" together and listen to the major triad.
4. Strike the three chimes A3, C$^{\#}$4 "*ET*", and E4 "pure" together and listen to the major triad. Do you hear a difference?
5. Strike all 4 chimes together and listen to the major triad, but now containing the beats between C$^{\#}$4 "*pure*" and C$^{\#}$4 "*ET*".

The Comma of Didymus

How much do the musical major thirds of Pythagoras (81/16) and Claudius Ptolemy (5/4) differ?
The answer is that they differ by the so-called **Comma of Didymus**, which we can obtain numerically in the following way.

We start with the diatonic scale

do	re	mi	fa	sol	la	ti	do
C4	D4	E4	F4	G4	A4	B4	C5

We have for the intervals of the diatonic scale of Pythagoras and its major third E4/C4:

1/1	9/8	**81/64**	4/3	**3/2**	27/16	243/128	**2/1**

For the syntonic diatonic scale of Claudius Ptolemy we have:

1/1	9/8	**5/4**	4/3	**3/2**	5/3	15/8	**2/1**

From this follow for the ratio of the major thirds of Pythagoras and Ptolemy:

$$\frac{81/64}{5/4} = \frac{81}{80} = 1.0125.$$

This is the *"Comma of Didymus"*, also called the *"syntonic comma"*.

We see that the Pythagorean major third is "sharp" with respect to the Ptolemaic major third by

$$c = 1200 \frac{\log f_2/f_1}{\log 2} = 1200 \frac{\log 81/80}{\log 2} = 21.5 \text{ cents}.$$

The difference of 21.5 cents is about 1/5 of a semitone, with easily audible beats between the two intervals.

A Musical Comma at Texas Tech University

There is a work of art, a big shiny metal sphere, between the Student Union Building and Library at Texas Tech University, called “Comma”. Push in, at the same time, the two pistons on opposite sides of the sphere. Two slightly different high-pitched tones are generated. You should hear a beat frequency between them, called the “Comma” here.

The plaque “Comma” next to the sculpture does not say which musical comma is meant. It probably does not mean a ratio between musical intervals as we have discussed in this chapter. More likely, it is the smallness of the beat frequency compared to the frequencies of the two generating tones that is the “Comma” here. (The high-pitched tones represent the ringing of the sun, moved up many octaves.)
It is rather amazing how this sculpture at TTU by Po Shu Wang combines elements from art, music, and science!

Physics 1406, "The Physics of Sound and Music"
Homework 1

Regularly work these problems between now and the due date.
Consult the Course Guide and your own notes for useful information.
Be ready for the quiz. **Quiz grade = homework grade**.

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1. Consider the *audible range* from 20 Hz to 20 kHz.
a) What are the periods of oscillation associated with the two frequencies?
b) Use v = 346m/s for the speed of sound and calculate the wavelengths corresponding to these two frequencies.

2. a) Give an example of simple harmonic motion (SHM).
b) Give an example of motion that is periodic, but is not SHM.
c) Give an example of non-periodic motion.

3. Draw a y-t graph, i.e. displacement y versus time t, of simple pendulum motion. Draw 3 full periods T, starting from the equilibrium position x = 0 at t = 0.

4. Draw a y-t graph of periodic motion where the amplitude decreases to zero over time.

5. Consult the figure of the piano keys as well as their names and frequencies.
a) Which key is a musical interval of a *fifth* above the key C_4?
b) Which key is a *fifth* below C_4?
c) Which key is a *fourth* above C_4?
d) Which key is a *third* above C_4?
e) From this, why are these musical intervals called a *third, fourth,* and *fifth*?

6. Consider the 4 pendulums as in class and the laboratory. The longest pendulum has a length of L_1 = 50 cm. The lengths of the pendulums were chosen to correspond to the musical notes C, E, G, and C one octave higher. The three pendulums C, E, G are a "visualization" of a musical *triad*.
Calculate the length of the 3 pendulums E, G, and C.
Hint: Use the fact that the frequency of a pendulum goes like the inverse square root of the length, i.e. $f \propto 1/L^{1/2}$. Furthermore, if we give the frequency of the longest pendulum the value 1, then the frequencies of the other pendulums are 5:4 = 1.25 *(third)*, 3:2 = 1.50 *(fifth)*, 2:1 = 2.00 (*octave)*. For example, if the string is 9-times longer, the frequency will be 3-times lower.

7. You are given a thin stretched string of length L = 100 cm, fastened at both ends, as in a string instrument. Divide the string into two parts so that you hear a musical *fifth* when you pluck both parts simultaneously. What are the lengths of the two parts?

8. Determine the velocity of sound from the following information: An organ pipe with both ends open has a fundamental frequency $C_4 = 261.6$ Hz. The pipe has an effective length 0.66 m. What is the velocity of sound in air?

9. Practice the relation between speed v and distance x traveled in time t, i.e. $x = vt$.
a) How many seconds does it take for sound to travel a distance of 1200 m? (Use $v = 346$ m/s.)
b) You drive a distance of 390 miles from Lubbock to Austin in 5 hours and 45 minutes. Calculate the average speed in meter per second.

10. Based on what you have learned so far, answer this question with a few words: "What is sound"?

Physics 1406, "The Physics of Sound and Music"
Homework 2

Regularly work these problems between now and the due date.
Consult the Course Guide and your own notes for useful information.
Be ready for the quiz. **Quiz grade = homework grade**.

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1. You see a flash of lightning and its thunder 3 seconds later. How far are you from the lightning? (Use v = 346 m/s)

2. A car accelerates from 0 to 60 miles/hr in 10 seconds.
a) Express this in proper units of m/s^2.
b) Compare your answer with Earth's acceleration g = 9.8 m/s^2.

3. Consider a pure sine wave.
What characterizes such a wave? Use words and symbols in your answer.

4. What is a necessary condition for an object to execute *simple harmonic motion* (SHM)?

5. A person weighs 150 lbs.
a) What is the weight in N (Newton)?
b) What is the mass of the person?

6. A very loud tone at the threshold of pain reaches your ear with an amplitude of 20 Pa. What is the fraction of this amplitude compared to the static atmospheric pressure?

7. a) Look up the speed of sound at 25 and 40 ^{0}C.
b) A wind instrument plays a 500 Hz note at 25 ^{0}C. The instrument is warming up. What will the frequency of the same note be at 40 ^{0}C? (Assume that the length of the instrument does not change and hence neither does the wavelength. Use $v = \lambda f$.)
c) Do you expect this change in pitch to be audible?

8. A plastic pipe is open at both ends and has a fundamental frequency of f_1 = 115 Hz. What are the resonance frequencies of the next three harmonics?

9. We listened in class to a pure sine wave and a square wave of the same pitch.
a) Describe the difference in sound quality or "timbre" of the two waves.
b) Make sketches of the two waves, lined up on top of each other.

10. List the types of waves discussed in class, for instance "traveling wave", "standing wave" etc. Add 5 more designations to your list.

11. a) When do sound beats occur?
b) What is the beat frequency between a sine wave of 995 Hz and another at 1005 Hz?

12. We used a "slap tube" (plosive aerophone) in class to determine the speed of sound. Assume an actual length of the tube of $L = 40.0$ cm and an inner radius of 3.0 cm.
a) What is the effective length of this tube? (Keep in mind that it is closed at the end where you slap it.)
b) What is the wavelength of the fundamental mode of the vibrating air in this tube?

13. How did Einstein define *"time"*?

14. The period of oscillation of a tone is $T = 2000$ μs (microsecond).
a) What is the frequency of the tone?
b) Is the tone in the audible range?

15. A cello string is made of steel wire with the density of $\rho = 7900$ kg/m^3. The wire diameter is $d = 0.97$ mm and the length is $L = 71$ cm. Calculate the mass in gram (g) of this string. (Hint: Find the cylindrical volume of the string and multiply by the density.)

16. Consider an atmospheric pressure of 100,000 Pa. What is the force in Newton (N) exerted on 1 cm^2 of surface?

17. Describe an example of *resonance.*

18. A deep water wave in the ocean has a wavelength of 6.24 m and period of 2 s. What is the wave speed in m/s and km/hr?

Physics 1406, “The Physics of Sound and Music”
Homework 3

Regularly work these problems between now and the due date.
Consult the Course Guide and your own notes for useful information.
Be ready for the quiz. **Quiz grade = homework grade**.

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1. Draw the first four resonant or vibrational modes of a stretched string of length L. Label the nodes as “N” and the antinodes as “A”.
Write down the harmonic number next to each mode and the frequency as a multiple of the fundamental frequency f_1 of the string.

2. A cylindrical tube is open at both ends. Draw the first four resonant modes in the tube. On the y-axis, draw the displacement of air for each mode as a function of distance x along the tube. Label the nodes as “N” and the antinodes as “A”.
Label each mode by its harmonic number and the frequency as a multiple of the fundamental frequency f_1 of the open tube.

3. Show that a tube open at both ends can have a node at its center. Also show that this is impossible for a tube closed at one end.

4. Show mathematically that a closed tube of length L has the same fundamental frequency as an open tube of length 2L. The diameter of the two tubes is the same.

5. Practice the harmonic series: Consult the piano keyboard. Start with C2 and write down the notes for the first 8 harmonics, including the key C2.

6. Write out the notes of the overtone series of G2 for the first six harmonic numbers N = 1, 2, 3, 4, 5, 6. Label each note with its harmonic number and write down its frequency. (Hint: Consult the layout of the piano keys and use f_1 = 98.0 Hz for the fundamental frequency of G2.)

7. a) What is the effective length of a closed tube having the fundamental f_1 = 98 Hz?
b) What is the harmonic number and frequency of the next existing mode?

8. A stretched string has a fundamental frequency of 150 Hz. By what factor must each of the following quantities be changed individually to lower the fundamental frequency to 100 Hz?
a) Length
b) Tension
c) Mass per unit length (linear mass density)
d) Diameter

9. The C2-string of a cello (C2 = 65.41 Hz) has a linear mass density $\mu = 0.018$ kg/m. The effective length of the cello strings between nut and bridge is 0.71 m. What is the tension in the string? (Hint: Use Mersenne's law and solve for the tension F.)

10. The C2-string (lowest note) of a cello has a length L=71cm and diameter D=1.40mm. The string is made of steel wire having a density $\rho = 7900$ kg/m^3. Calculate the linear mass density μ of the string.(Hint: Assume that the wire is a cylinder with a circular cross section. Then we have $\mu = m/L = \rho V/L = \rho A = \rho\pi D^2/4$.)

11. A guitar string has the following specifications:
Diameter D = 0.406 mm
Effective length of the vibrating part of the string L = 648 mm.
The wire material is steel with a density $\rho = 7800$ kg/m^3.
The string tension from a suspended mass of 10.57 kg is
F = mg = 10.57 kg x 9.8 m/s^2 =103.6 N.
a) Calculate the resonance frequency of the fundamental mode of the string.
(Hint: Use Mersenne's law $f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$)
b) Which string on an acoustic guitar is this?
Hint: The empty strings of a guitar are tuned to the notes E2, A2, D3, G3, B3, E4.

12. Consider the harmonic series of G2 and its first 6 harmonics.
a) Which pair of notes in this overtone series of G2 are spaced intervals of a "fifth", "fourth", and "major third" apart?
b) What are the corresponding harmonic numbers for the corresponding pairs of notes?

13. Two different instruments play the same sustained notes. The Fourier spectra of the two notes are identical.
a) Do the two notes sound the same?
b) Why do the waveforms of the two notes probably not look the same?

Physics 1406, "The Physics of Sound and Music"
Homework 4

Regularly work these problems between now and the due date.
Consult the Course Guide and your own notes for useful information.
Be ready for the quiz. **Quiz grade = homework grade**.

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1. A tube is open at both ends. It has an effective length of 60 cm. Calculate the first 4 *existing* resonance frequencies. (Use v = 346 m/s.)

2. A tube is open at one end and closed at the other end. It has an effective length of 60 cm. Calculate the first four *existing* resonance frequencies. (Use v = 346 m/s.)

3.a) Describe briefly what a "formant" is.
b) Simulate the human vocal tract as a tube of effective length 17 cm, closed at one end and open at the other end. Calculate the peak frequencies of the first two formants. (Use v = 346 m/s.)

4. A saw tooth wave has an amplitude of A_1 = 12.00 Volt of the fundamental. Write down the amplitudes and harmonic numbers of the next four *existing* harmonics.

5. A square wave has an amplitude of A_1 = 12.00 Volt of the fundamental. Write down the amplitudes and harmonic numbers of the next four *existing* harmonics.

6. A clarinet is a reed instrument with a cylindrical bore.
a) Which of the lowest harmonics do to you expect to be strong?
b) Which "standard wave" does the sound spectrum of a clarinet resemble?

7.a) Approximately how many harmonics do you expect in the lower notes from a flute?
b) Qualitatively describe the sound of a flute or recorder.

8. Professor Meek played a Krummhorn in class and we recorded the sound spectrum.
a) Approximately how many harmonics could we see clearly?
b) Describe the sound of the Krummhorn as compared to a flute.

9. A plosive aerophone ("slap tube") shown in class had a length L = 249.5 mm and an inside diameter D = 40 mm.
a) Calculate the effective length L_{eff} of the tube.
b) Calculate the fundamental frequency f_1. (Use v = 346 m/s.)

10. You slap a plosive aerophone ("slap tube") with your hand and let go immediately. By how much approximately do the two resulting tones differ in pitch?

11. Give an example of a musical instrument in each of the following instrument classes of the Hornbostel-Sachs classification system:
a) idiophones
b) chordophones
c) aerophones
d) membranophones
e) electrophones
f) hydrophones

12. A chime is made of electric conduit and tuned to a pitch of C5 in the inharmonic resonance mode 2. What is the frequency of the fundamental (i.e. mode 1)?
Hint: Consult the information for vibrating bars in the Course Guide.

13. A wine bottle, considered a Helmholtz resonator, has the following dimensions: Volume V = 1.5 liter = 0.0015 m^3, inside diameter of the neck opening D = 18.7 mm, length of the neck l = 77 mm. Use v = 346 m/s.

Calculate the Helmholtz resonance frequency from the formula $f = \frac{v}{2\pi}\sqrt{\frac{A}{l_{eff}V}}$.

Compare your result with the measured resonance frequency of 81.6 Hz.

14. Consider two plastic "Club Soda" bottles of volumes of 2 liter and 1 liter, respectively. The two bottles have the same bottleneck.
a) What is the ratio of the two resonance frequencies?
b) The bigger bottle gives a pitch of approximately G2. What is the frequency?
c) What is the pitch of the smaller bottle?
(For the musically interested: The two bottles produce a "tritone" when played together.)

15. Describe when a musical tone sounds "plain" and when it sounds "rich". Use your knowledge of harmonics and Fourier spectra.

16. When does a sound still sound "musical" when strong attack and decay transients are present?

17. Why do string instruments generally produce harmonic and not inharmonic spectra? (Hint: Remember our discussion of thin strings as compared to heavy strings and vibrating bars.)

18. Cup one hand over your ear or use an empty tin can or seashell. As you move these objects away from the ear, does the pitch increase or decrease? Qualitatively explain this with the formula for the resonance frequency of the Helmholtz resonator.

Physics 1406, "The Physics of Sound and Music"
Homework 5

Regularly work these problems between now and the due date.
Consult the Course Guide and your own notes for useful information.
Be ready for the quiz. **Quiz grade = homework grade**.

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1. a) Briefly explain the "place theory of hearing".
b) Where, with respect to the oval window, are the high and low frequencies placed on the basilar membrane?

2. Briefly explain the following:
a) Critical Band
b) Auditory sharpening
c) Just noticeable frequency difference (JND)
d) Limit of frequency discrimination

3. For concert A4 = 440 Hz, what is the distance in millimeter from the oval window for the peak response on the basilar membrane? Assume a length of 35 mm of the basilar membrane.

4. What is the octave spacing in millimeter on the basilar membrane for mid-range frequencies?

5. What is the effective length in millimeter on the basilar membrane for a "minor third" (i.e. the width of the critical band) over much of the audible range?

6. Consider a sine tone of frequency f = 800 Hz. Obtain values for:
a) Critical Band
b) Limit of frequency discrimination
c) Just noticeable frequency difference (JND)

7. From the Fletcher-Munson curves of equal loudness obtain the following:
a) The loudness level in phon and the SIL in dB at the threshold of hearing at 100 Hz.
b) The loudness level in phon and the SIL in dB at 1000 Hz.
c) The loudness level in phon and the SIL in dB at the threshold of pain at 10 kHz.

8. a) From the Fletcher-Munson equal-loudness curve of 40 phon, read off the SIL in dB at a frequency of 3 kHz.
b) From the Fletcher-Munson equal-loudness curve of 40 phon, what is the SIL in dB at a frequency of 100 Hz?

9. From the Fletcher-Munson curves, find the sound intensity level SIL in dB for a 5 kHz tone whose intensity is $I = 10^{-6}$ W/m^2. (Note that there is a difference between *SIL* and *I*.)

10. Consult the 60 phon equal-loudness curve at a frequency of 300 Hz.
For what other frequency or frequencies will the perceived loudness be the same?

11. Model the ear canal as a tube of 2 cm length, with the eardrum at the closed end.
a) What are the first two resonant frequencies of the ear canal? Compare the lower of the two frequencies with the frequency where our hearing is most acute.
b) Two minima can be seen in the equal-loudness curves. For the 60-phon curve, read off the corresponding two frequencies.
c) Compare the two frequencies calculated in part a) with those on the 60 phon curve.

12. A flute plays a note with an SIL of 60 dB. How many times must the intensity be increased for an SIL of 68 dB?

13. A single violin softly plays a note at an SIL of 50 dB. The violin section of an orchestra contains 10 players. At what SIL does each player have to play so that the section produces a combined SIL of 60 dB?

14. Five students are applauding the instructor in class and produce an SIL = 60 dB. How many students should applaud for an SIL = 66 dB? (The assumption is that all students applaud equally loud.)

15. a) Describe what vocal formants are.
b) What is a *singing formant*? Why is this important for an operatic singer?

16. A 110 Hz *square* wave and a 336 HZ *sine* wave are sounded simultaneously.
a) Why can they beat with each other?
b) What is the beat frequency?
c) To what frequency should you tune the square wave so that the beats disappear?

Physics 1406, "The Physics of Sound and Music"
Homework 6

Regularly work these problems between now and the due date.
Consult the Course Guide and your own notes for useful information.
Be ready for the quiz. **Quiz grade = homework grade**.

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1. a) Explain the auditory phenomena "periodicity pitch" and "fundamental tracking".
b) Give an example for each.

2. a) Explain how second-order beats or quality beats differ from first-order beats. Also explain how binaural beats differ from first order beats.
b) How are each of these beats produced?

3. Explain the functioning of the vocal folds with the aid of Bernoulli's principle.

4. Consult the graph about auditory masking by a 1000 Hz masking tone at different sound intensity levels (see the figure in this Course Guide referring to Jürgen Meyer, "Acoustics and the Performance of Music").
a) For the masking tone at 1000 Hz and SIL of 70 dB, read off the masking levels for masked tones at 600 Hz and 1500 Hz. (You will have to do some interpolating of the numbers on the x- and y-axes.)
b) Based on your values from part a), is masking more effective for low frequencies below the masking frequency or high frequencies above the masking frequency?

5. Consult the graph of the subjective loudness in sones (see the figure in this Course Guide referring to Donald E. Hall, "Musical Acoustics").
a) Read off the loudness in *sones* at 100 Hz, 1000 Hz, and 4000 Hz, at SIL= 60 dB.
b) Why are the sone-values so different at the same SIL?

6. Model the human vocal tract with a cylindrical tube of effective length L = 17.5 cm. Calculate the first four existing resonance frequencies (use v = 346 m/s). These roughly correspond to the peaks of the broad formant regions of the human vocal tract.

7. Where do the closely spaced frequencies within a given formant region come from?

8. Describe the following problems arising from improper acoustical design, and discuss how these problems can be corrected or even avoided.
a) Flutter echo
b) Excessive liveness
c) Poor texture
d) Problems relating to the precedence effect
e) Focusing

9. "Intimacy" of a room means that the first reflected sound arrives at a listener in less than 20 ms after the direct sound.
a) What is the extra distance that the reflected wave travels for this delay?
b) Describe which of the following spaces could have intimate acoustics:
cathedral, lecture hall, concert hall.

10. a) How can you improve the acoustic quality of your dormitory room, apartment, or home music-listening area?
b) How can you reduce sound penetration into your neighbor's space?

11. The small practice rooms in some music schools have non-conventional shapes, for instance walls at odd angles and inclined ceilings. Give reasons for these designs.

12. a) Show that the inverse square law means an intensity decrease of 6 dB for each 2-fold increase in distance from the sound source.
b) What is the change in loudness?

13. a) Calculate the reverberation time T_R at a frequency of 500 Hz for our lecture room. Use the following data:
Width of room W = 24 ft
Length L = 29 ft
Average ceiling height H = 9.5 ft.
Concrete wall with $a_{concrete} = 0.02$, acoustical tile ceiling with $a_{acoustical\ tile} = 0.90$,
wood floor with $a_{floor} = 0.15$.
The room has 50 un-upholstered chairs, of which 20 are occupied, and 30 unoccupied.
b) Compare your result with the desirable reverberation time for "Lecture & Conference Rooms" shown in the pertinent figure in the Course Guide, i.e. read off the reverberation time at the volume of our classroom.

14. Consider a "model room" represented by a cubical box, with all sides having a length of 362 mm. See also the laboratory experiment on "Room Acoustics".
a) Calculate the resonance frequencies for the first five modes in order of ascending frequency, i.e. take the modes (1,0,0), (1,1,0), (1,1,1), (2,0,0), and (2,1,0). For the velocity of sound use 346 m/s.
b) Such resonances in an actual small room may be undesirable. How can you minimize these resonances? Consider different geometries and materials.

Physics 1406, "The Physics of Sound and Music"
Homework 7

Some questions from this homework will be on the final examination. Study the Course Guide and answer the questions with the information provided there.

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1. a) Explain Ohm's law of electricity, including the physical quantities and their units.
b) An 8 Ω loudspeaker is connected across the terminals of a stereo amplifier whose output at a certain instant of time is 12 V. What is the electrical current?
c) What is the electrical power dissipated in the loudspeaker in part b)?
d) If 0.2% of the electrical power is converted into sound, what is the acoustic power?
e) Into which energy form is most of the total electrical power converted?

2. a) Explain Faraday's laws of electromagnetic induction in your own words.
b) Relate this fundamental law of physics to the operation of dynamic microphones and electric generators.
c) Faraday's law is the principle behind which commodity in everyday life?

3. Your electric bill in a 30-day month is $60.00. A kilowatt-hour (kwh) costs 13 cents.
a) What is the average electrical power delivered during the 30-day period?
b) If all the power were used for 15 W compact fluorescent light bulbs, how many light bulbs would be on continuously?
c) In practice, you only have a few light bulbs turned on for a few hours each day. Where is most of the energy in a typical household used?

4. Describe the three types of microphones, i.e. omni-directional, cardioid, and shotgun. Which microphone would you use for recording of:
a) a large crowd scene.
b) an orchestral concert.
c) a birdcall from the roof of a tall building.

5. Describe the following loudspeaker systems:
a) Infinite baffle
b) Acoustic suspension
c) Tuned port

6. a) Calculate the power from a microphone to a preamplifier if the microphone output voltage is 3 mV and the input impedance of the preamplifier is 600 Ω.
b) Calculate the power from a preamplifier to a power amplifier if the voltage output of the preamplifier is 1.5 V and the input impedance of the preamplifier is 50,000 Ω.
c) Calculate the power from a power amplifier to a loudspeaker if the amplifier output voltage is 10 V and the speaker impedance is 8 Ω.

7. Suppose that the smallest amplitude on a CD is $A_0 = 1$ binary bit and the largest amplitude is $A = 2^{15}$ bits (32,768-times the smallest amplitude). What is the corresponding range in the sound intensity level SIL in dB, also called the dynamic range of the CD? Hint: Assume that the SIL corresponding to the smallest amplitude is $SIL_0 = 0$ dB, and use the formula $SIL = SIL_0 + 20\log(A/A_0)$. (The factor of 20 is no misprint!)

8. A CD has continuous pit tracks between the radii 2.3 and 5.7 cm.
a) The spacing between tracks is 1.6 microns (μm). How many tracks are on the CD?
b) Determine the average radius of the tracks and the total length of the spiral CD track.
c) The total playing time of the CD is 72 min 6 sec. What length of track does the laser of the CD player read each second?
d) The data on the disc consist of 16-bit binary numbers, read at a rate of 44,100 numbers per second for each of the two stereo channels. How many binary bits per second are being read? Assume that an additional equal number of bits is required for bookkeeping purposes and error checks.
e) Use the result of from parts c) and d) and determine the average length of one binary bit on the track.

Musical Scales Do the following three questions together
9. a) Construct the 8-degree C-major diatonic scale C4-D4-E4-F4-G4-A4-B4-C5 exclusively with octave ratios of 2/1 and Pythagorean fifth ratios of 3/2. Organize your results in a Table showing the 8 notes, the ratio for each with respect to C4, and calculate the corresponding frequencies based on C4 = 261.63 Hz.
(Example: D4 = C4 · 3/2 · 3/2 · 1/2 = C4 · 9/8 = 261.63 Hz · 9/8 = 294.33 Hz.)
b) What is the name of this musical scale?

10. a) Construct the C4-major diatonic scale similar to above, but this time using the major third of 5/4 according to Claudius Ptolemy in addition to octaves and fifths,**.** Use these three intervals and construct the remaining 5 notes (or "degrees") of the diatonic scale with the simplest possible rational numbers. (The most complicated rational number will be 15/8 for the note B4.) All other ratios will have single-digit integers in the numerator and denominator. Organize your results in a table.
b) What is the name of this musical scale?

11. a) Calculate the frequencies of C4-D4-E4-F4-G4-A4-B4-C5 in equal temperament. Start with C4 = 261.63 Hz.
b) Make a side-by-side comparison of the three scales thus constructed and comment on the frequency differences of corresponding notes (such as E4 from all three scales).
c) Which of the notes show the greatest differences in the three scales?
d) Which of the notes show the smallest differences in the three scales?

PHYSICS 1406 YOUR NAME ______________________________

"Physics of Sound and Music" Please print – no abbreviations

Quiz 1 (10 points for each question)

Remarks:

a. Your own notes are allowed you prepared are allowed, but NOT the Course Guide.
b. Use a simple calculator and a ruler for graphs.
c. Show your work to receive full credit.
d. Give all numerical answers with 3 significant figures.

---

1. Give two characteristics that distinguish music from general sound:

______________________________ ______________________________

2. Example of "consonance" for a sense other than hearing ________________________

3. Give an example of general periodic motion (not SHM) ________________________

4. The two didgeridoos shown in class had the same pitch but sounded different.
Using a musical term, what was different? ______________________________

5. Draw a graph of displacement versus time of the motion of a spring pendulum. Start at the equilibrium y = 0 at t = 0. Draw 2 periods, and label the amplitude A and period T.

Displacement y

time t

6. A tone without overtones is what kind of wave? ______________________________

7. The note A4 has a pitch of = 440 Hz. What is its period in millisecond (ms)?
Show your work:

Period T = ____________ ms

8. The frequency range on a smartphone is about ____________ Hz to __________ Hz

9. Name an item we used to determine the speed of sound in class:

____

10. For a tone with a fundamental frequency f = 86.5 Hz, calculate the wavelength in air (take v = 346 m/s).
First, write down the formula you use for the wavelength: λ = _______________
Then work out the answer with a 3-digit accuracy:
Answer: λ = ____________m

11. You are given a thin stretched string of length L = 95 cm. Divide the string into two segments so that you hear a musical fifth (frequency ratio 3:2) when plucked together. What are the lengths of the two string segments? Show your work:
L_1 = __________cm (higher pitch) L_2 = __________cm (lower pitch)

12. A didgeridoo has an effective length of 1.18 m. Calculate the fundamental frequency (pitch) of the sound it emits. (Use v = 346 m/s.) Show your work:
Frequency f = _______________Hz

13. Draw the fourth vibrational mode on a slinky as shown in class. Show the length L of the slinky and write down the relationship between L and the wavelength λ.
Drawing:
Relationship: λ = ______L

14. Given the fixed quantities A, λ, f, T for a traveling sound wave, on which variables does the air displacement in the wave depend?

Name and symbol of variable 1 _________________ variable 2 _____________________

15. A firecracker explodes high in the air. Person 1 is a distance of 50 m away from it. Another person 2 is 150 m away. How much less is the sound intensity at the location of person 2 as compared to person 1? (Hint: Use the inverse square law.)
Shown your work:

Answer: The sound intensity is ___________-times less.

16. A main point made by Professor Meek when demonstrating his bassoon:
____

17. A "slap tube" has an effective length of 0.500 m. What is the frequency of the emitted sound when "slapped"? (Use v = 346 m/s.) Show your work:
Answer: f = ________________Hz

18. What characterizes SHM? ____

Bonus Question (5 points):
How did you learn about this course?____

PHYSICS 1406 YOUR NAME ______________________________
"Physics of Sound and Music" Please print – no abbreviations

Quiz 2 (10 points for each question)

Remarks:

a. Your own notes on are allowed, but NOT the Course Guide.
b. Use a calculator for the calculations and a ruler for graphs where appropriate.
c. Show your work for full credit.
d. Give all numerical answers with 3 significant figures.

1.. Do the problem assigned from class concerning the fundamental frequency of the longest pipe of the Nidaros organ in Tronheim, Norway. Show your work:

Fundamental frequency = ___________________ Hz

2. Consider the harmonic numbers N = 1 2 3 4 5 6 7 8 9 10.
Circle with solid lines a pair of numbers that gives a musical fifth. Circle with dashed lines another pair of numbers that gives a musical third.

3. Which display on the computer monitor did we find more useful in class? Check one:

a) Waveform ______ b) Spectrum ______

4. Name three "Factors Affecting Tone Quality" (timbre) from the Course Guide:

___________________ ___________________ ___________________

5. A cylindrical pipe is open at one end and closed at the other end. Its fundamental frequency is 100 Hz. What are the frequencies of the first 4 existing harmonics?

______ ______ ______ ______ Hz

6. A cello and a violin play the same note. What most likely is different according to Fourier's theorem? (Do not write "overtones" or "phases".)

The following is different: ________________________________

7. What would be a problem for us if Ohm's law of hearing did not hold?

---

8. Make a graph of a harmonic and an inharmonic frequency spectrum in the same Cartesian coordinate system. Both spectra have the same pitch. Assume all amplitudes to be the same. Draw the fundamental and the next three partials in both cases. Use solid lines for the harmonics and dashed lines for the inharmonic partials. Mark the fundamental. Label the axes. (Use a ruler.)

Drawing:

9. Consider the two bottles shown in class as examples of Helmholtz resonators. Both bottles have the same bottleneck. The volumes of the bottles are 0.75 liter and 1.5 liters, respectively. The larger bottle has a Helmholtz resonance frequency of 75 Hz. Calculate the resonance frequency of the smaller bottle. Show your work.
Calculation:

Larger bottle f = ______________ Hz

10. Give an example for a ________________________ (chordophone) __________________________ (membranophone)

11. Hearing "ocean surf" from a sea shell is an example of __________________________

12. Why would the sound from an orchestra generally not be the best in an open field?

____

13. What is a "formant": ____

____

14. A saw tooth wave has an amplitude of $A_1 = 60$ millivolt for the fundamental. Write down the amplitudes of the next four harmonics in millivolt:

_____________ _____________ _____________ _____________

15. A bottle has the following dimensions: volume V = 1.5 liter = 0.0015 m^3, inside diameter of the neck D = 18.7 mm, length of the neck l = 77 mm. (Use v = 346 m/s.) Calculate the Helmholtz resonance frequency. First write down the formula needed and only then substitute the numbers. Show your work:

Answer: Frequency = ___________ Hz

16. When does a musical tone sounds "plain" and when does it sound "rich"?

____

17. Cup one hand over an ear. As you move the hand a little farther away from the ear, the "pitch" (mark one): _____________ (increases) _____________ (decreases) _____________ (stays the same)

Bonus question (5 points)
Consider two pipes of the same length L and radius R One pipe is open at both ends, the other pipe is open at one end and closed at the other end. Show that these pipes have no common harmonics. Ignore the end corrections.
Mathematical proof:

PHYSICS 1406 YOUR NAME ______________________________
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Quiz 3 (10 points for each question)

Remarks:

a. Your own notes on separate paper are allowed, but NOT the Course Guide.
b. Use a simple calculator for calculations and a ruler for graphs where appropriate.
c. Show your work for full credit.
d. Give all numerical answers with 3 significant figures.

---

1. Give the name of an auditory effect that lets you "hear" bass sound on your smartphone in spite of its tiny speaker: ____

2. Bernoulli's principle helps explain the functioning of the ______________________

3. What does the unit of "sone" measure? ______________________________________

4. Why are the formant regions of the human voice broad bands and not sharp peaks, as you might expect naively in analogy to a plastic pipe?
____

5. Model the human vocal tract with a cylindrical tube of effective length L = 17.0 cm. Calculate the first four existing resonance frequencies (use v = 346 m/s). These roughly correspond to the peaks of the broad formant regions of the human vocal tract. (Hint: Use the formula for a closed tube.) Calculation:

Resonance frequencies: ________ ________ ________ ________Hz

6. Why is there not a big difference between the vocal formant regions of a female and male singer? ____

7. Where are the vocal harmonics produced? ______________________________

8. Where do the closely spaced frequencies within a vocal formant region come from?
____

9. Solve $x = 7\log 10^y$ for y. Show your work:

Answer: y = _____________

10. What kind of beats do you hear between mistuned violin strings? ________________

11. Consider two sine tones of different frequencies. Which tone masks the other tone more easily? ____

12. Write down the most important room characteristic by name and letter symbol:
Name ________________________________ Symbol __________________

13. A room has negligible background noise. A sound has an SIL = 100 dB. It is turned off abruptly. What is the SIL right after the reverberation time has elapsed?
Show your work:

SIL= ____________dB

14. Mention three acoustic design criteria for rooms and concert halls:

____________________ ____________________ ____________________

15. Mention three acoustical design problems for rooms or concert halls:

____________________ ____________________ ____________________

16. A box-shaped tornado shelter has the dimensions 7 x 7 x 5 ft. It is made of concrete with a sound absorption coefficient a = 0.020 at 500 Hz. Calculate the volume and the effective area of the unoccupied shelter in *sabin*. Show your work:

Volume V = ____________________

A_{sabin} = ______________ sabin

17. Calculate the reverberation time of the unoccupied shelter above. Show your work:

Reverberation time = ____________ s

18. Three persons now occupy the above shelter. Each one of them contributes an effective area of 4.0 sabin. Calculate the reverberation time of the occupied shelter in this case. Show your work:

T_R = ______________ s

19. Calculate the lowest resonance frequency in the tornado shelter above.
(Use the conversion 1 ft = 0.3048 m, and v = 346 m/s for the speed of sound.)

Formula for the resonance frequency: f = ________________

Calculation:

Answer f = ____________ Hz

20. The small practice rooms in some music schools have unconventional shapes, for instance walls at odd angles and inclined ceilings. Give reasons for these designs.

____

21. What does "fullness" of a room mean? ____________________________

____

22. What does "blend" of a room mean? ______________________________

____

23. A typical value of T_R for our lecture room: T_R =______________s

24. A typical value of T_R for a large church: T_R =______________s

25. "Intimacy" of a room means that the first reflected sound arrives at a listener in less than 20 ms after the direct sound. What is the distance that the reflected wave travels during this delay? Show your work:

Distance = ______________m

26. Assume that an increase in SIL of 6 dB means "twice as loud". How many times as loud is an increase of 18 dB? Show your work:

Answer: The increase is _____- times as loud

27. What is "fundamental tracking"? _______________________________

____

28. Write down the names of the physical quantities (coordinates) in a sonogram:
On the x-axis (horizontal axis): Name ______________________
On the y-axis (vertical axis): Name ______________________
The color of the sonogram tracks means: Name ______________________

Bonus Questions (5 points each)

1. A main point made by Professor Gregory Brookes during his presentation:

____

2. A main point made by Professor James Decker during his presentation:

____

Examination 1 **Your Name:** ________________________________

PHYSICS 1406, "Physics of Sound and Music"

Please put your answers in the space provided. Show your work for possible partial credit. Your own notes on separate paper may be used, but NOT the Course Guide.
Use a calculator for numerical answers and a ruler for drawings.
Give all numerical answers with three (3) significant figures, e.g. L = 3.65 m, 0.0246 m.
Credit: 10 points for each question.

---

1. Do the problem assigned in class for this examination concerning a vertical string put under tension with a weight. Calculate the fundamental frequency of the string.

Formula needed: f = ____________

Show your work:

Calculated frequency f = _______ Hz

2. A bassoon and a didgeridoo sound the same note D2. Concerning their harmonics, what will be the same and what different (answers in plural)?
same __________________________ different ______________________________

3. You divide a string into two parts and pluck them together. According to Pythagoras, pleasant musical intervals can be heard when the two lengths are in certain simple ratios. Give the names and values of three such ratios:

__________________ __________________ __________________

4. How did we demonstrate longitudinal waves in class?

____

5. Calculate the pressure ratio $\Delta p/p_{atm}$ for an extremely loud tone, where Δp = 20 Pa.
First, write down the value for standard atmospheric pressure p_{atm} = ___________ Pa
Then obtain the pressure ratio (in power-of-ten-notation):
$\Delta p/p_{atm}$ = ________________

6. Give an example from a demonstration in class where resonance occurred satisfying the resonance condition:

____

7. Consider the Galilean pendulums in the laboratory. If the length of one pendulum is 80 cm, what must be the length of another pendulum that oscillates an octave higher?
Show your work:

Length of pendulum L = __________ cm

8. The period of a sine tone is T = 60 millisecond.
What is the corresponding frequency?

Frequency f = __________Hz

This frequency is in the audible range: True______ or False ______

9. The speed of sound at 20 ^{0}C and 30^{0}C is 343.3 m/s and 349.0 m/s, respectively. A bassoon plays a 261 Hz note ("middle C") at 20 ^{0}C. The instrument warms up. What is the frequency of this note at 30 ^{0}C? (Hint: Use $f = v/\lambda$ and assume that the length of the instrument does not change.)
Show your work:

Frequency at 30 ^{0}C: f = ____________Hz

10. A cylindrical tube of length L is open at both ends. It has a fundamental frequency of 75 Hz. The tube is cut in half. What is the fundamental frequency of the resulting halves? (Ignore the end corrections.)
Show your work:

f_{open} = ___________Hz

11. Give an example for diffraction of sound waves:

____

12. Two sine waves have frequencies (pitches) of 530 Hz and 540 Hz, respectively.
What is the pitch of the beating wave? Pitch =_______________Hz

13. What is a harmonic?

____

14. The fundamental frequency of a stretched string is 110 Hz. What is the frequency of the 2nd overtone?

f = _______Hz

15. The number of sine waves in the 3rd harmonic of a periodic tone is _____________

16. We showed the INDIVIDUAL harmonics on a string in class. Why can many harmonics be present simultaneously on the string of a violin?

____

17. What travels in a sound wave from source to listener wave? ______________________

18. What is another good word for "color" in a musical tone? ________________________

19. Give an example for a longitudinal wave ______________________________________

20. A plastic pipe is closed at one end and open at the other end. The fundamental frequency is f_1 = 70 Hz. The frequencies of the next three existing harmonics are:

_____________ Hz _____________ Hz _____________ Hz

21. Give an example for refraction of sound waves: ________________________________

22. What is the role of noise in producing musical sound?

23. How did we show vibrations on a string?

24. Why must the closed end of a tube be a node for the displacement of air molecules?

25. How do standing waves arise from traveling waves?

26. Why are harmonic numbers integer numbers? (You may confine yourself to a vibrating string or air resonating in a tube.)

27. The relative amplitudes of a saw tooth wave are 1, 1/2, 1/3, 1/4, 1/5 for the harmonic numbers N = 1, 2, 3, 4, 5, respectively.
CAREFULLY Plot this spectrum TO SCALE in a coordinate system using a ruler. Label the fundamental frequency and the axes of the coordinate system.
Graph:

28. Consider a pure sine wave. What characterizes such a wave?

29. An ocean wave has a wavelength of 6.24 m and period of 2 s. What is the speed of the wave in kilometer/hour?

v = __________ km/hr

30. What is the effective length of a closed tube having the fundamental f_1 = 98 Hz? Show your work:

L_{eff} = ____________ m

31. A tube is open at one end and closed at the other end. It has an effective length of 60 cm. Calculate the first three *existing* resonance frequencies. (Use v = 346 m/s.) Show your work:

Existing frequencies: ______________ ______________ ____________ Hz

32. Two different instruments play the same sustained notes. The Fourier spectra of the two notes are identical. The instruments sound the same: T_______ F_____

33. A stretched string has a fundamental frequency of 100 Hz. How many times must each of the following quantities be changed individually to raise the fundamental frequency to 150 Hz? Show your work:

The length L must be changed to ____________ times L
The tension F must be changed to ____________ times F

34. You slap a plosive aerophone ("slap tube") with your hand and let go immediately. By approximately how much do the two resulting tones differ in fundamental frequency or pitch?
Choose one of the following: Fifth__________ Octave __________

35. A plosive aerophone ("slap tube") shown in class had a length L = 249.5 mm and an inside diameter D = 40 mm. Calculate the fundamental frequency f_1. (Include the end correction and use v = 346 m/s.)
Show your work:

Fundamental frequency = ____________ Hz

Bonus Question (10 points)
Show mathematically that a closed tube of length L has the same fundamental frequency as an open tube of length 2L. The diameter of the two tubes is the same. Ignore the end corrections.
Mathematical proof:

Examination 2 **Your Name:** ____________________________________

PHYSICS 1406, "Physics of Sound and Music"

Please put your answers in the space provided. Show your work for possible partial credit. Your own handwritten notes may be used, but NOT the Course Guide.
Use a calculator for numerical answers and a ruler for drawings.
Give all numerical answers with three (3) significant figures, e.g. L = 3.65 m.
Credit: 10 points for each question.

---

1. Assigned in class: Compare a vibrating string and vibrating bar having the same fundamental f_1 = 200 Hz. Obtain the frequency f_2 for the bar. Show your steps.

f_2 = ____________ Hz

2. A repeated question from Exam 1: The plosive aerophone ("slap tube") shown in class had a length L = 249.5 mm and an inside diameter D = 40 mm. Calculate the fundamental frequency f_1. (Include the end correction and use v = 346 m/s.)
Show your work:

Fundamental frequency = ____________ Hz

3. Show that the range of hearing from 20 Hz to 20,000 Hz corresponds to very nearly to 10 octaves. Proof:

Range = ________________ octaves

4. The 1 cm marks on a 10 cm ruler correspond to ________________________ on the basilar membrane of the ear.

5. A bottle has these dimensions: inside radius of cylindrical bottleneck 0.0083 m, effective length of bottle neck l_{eff} = 0.090 m, bottle volume V = 1.50 liter = 0.00150 m^3. Calculate the resonance frequency. (Use v = 346 m/s.)
(Hint: The Helmholtz resonance frequency is given by the formula $f = \frac{v}{2\pi}\sqrt{\frac{A}{l_{eff}V}}$.)

Show your work:

Calculated frequency f = ____________ Hz

6. Generally, we cannot hear the phases of the harmonics in a tone.
This is consistent with what law? ______________________________________

7. The arrangement of frequencies on the piano keys is in close analogy to the arrangement of the
_________________________ on the ____________________________ of the ear.

8. Draw the waveform and corresponding spectrum of a pure sine wave in two separate side-by-side coordinate systems. For the amplitude, take A = 1, for the frequency f = 100 Hz. Label all axes with appropriate symbols and numbers. Show the time t, period T, and amplitude A in the left coordinate system, and the frequency f and amplitude A in the right coordinate system. (Use a ruler!)

Waveform: Spectrum:

9. The ear has a logarithmic response to ________________ and _____________________

10. Describe a key demonstration for the logarithmic frequency response of the ear from class. ____

11. Musical intervals such as fifths sound “the same”, independent of the pitch. Why?

____

12. What is the function of the oval window of the ear?

____

13. The ear is less sensitive to intensity changes than frequency changes ______ ______
T F

14. The Fletcher Munson curves are not horizontal because _________________________

____

15. The minima in the Fletcher-Munson curves can be explained with
_______________________________ in the _____________________________________

16. At what frequency is the value of the phon number equal to the SIL number?

Frequency = __________ Hz

17. When 5 students applaud in class, we measure SIL = 75 dB. When more students applaud, we measure SIL = 81 dB. Calculate the number of students applauding in this latter case. Show your work and give your answer as an integer number.

Number of students = _____________

18. List three auditory effects discussed in class:

___________________ ___________________ ___________________

19. Write the next 4 successive harmonics from a harmonic series that has a missing fundamental.

_________ _________ _________ __________Hz

20. a) Approximate frequency range of hearing: _____________to____________Hz
    Approximate SIL range of hearing: _____________to____________dB
    b) Approximate frequency range of music: _____________to____________Hz
    Approximate SIL range of music: _____________to____________dB
    c) Approximate frequency range of speech: _____________to____________Hz
    Approximate SIL range of speech: _____________to____________dB

21. On any given Fletcher-Munson curve, the __________________________is the same

22. What is the "critical band": ____

____

23. A violin is playing a note at an SIL = 70 dB. Two more violins are added, playing the same note equally loud. What is the combined SIL for all three violins?
Show your work:

Combined SIL = __________dB

24. What is the most consonant musical interval (excluding the unison)?

____

25. State Fechner's law: ____

____

26. A violin plays with an SIL = 70 dB, another one with SIL = 80 dB.
What is the SIL when the two violins play together?
Calculation:

Combined SIL = ________dB

27. A typical frequency JND is ___________%

28. A typical JND in the SIL is ___________%

29. A typical SIL with the instructor speaking in class is SIL = ____________dB

30. What is the effective length of a closed tube having the fundamental f_1 = 98 Hz?
Show your work:

Leff = _____________m

31. Model the ear canal as a closed cylindrical tube of 2 cm length, with the eardrum at the closed end. Ignore the end correction. Calculate the first two existing resonant frequencies of the ear canal. Show your work

Answers: f_1 = ________________ Hz f_3 = ____________________ Hz

32. What does the place theory of hearing say? ____________________________________

____

33. (Assigned for this exam from HWK 5, Question 8): From the Fletcher-Munson of 40 phon, read off the SIL at frequencies of 100 Hz and 3 kHz:

SIL at 100 Hz = _________ dB
SIL at 3 kHz = __________ dB

34. Show with the aid of the inverse square law that the sound intensity decreases by 6 dB for a doubling of the distance from an unobstructed sound source.
Proof:

The decrease is __________ dB

35. What is consonance according to our discussion in class and the book?
Consonance is the ____

36. Mention a criterion when a sound sounds "musical":

____

37. Over how many powers of 10 does the range of hearing sound intensity extend? _____

38. Main point made by Mr. Jordan Langehennnig: _________________________________

____

Bonus questions (5 points each)

1. A main point made by Professor Annie Chalex-Boyle: ___________________________

____

2. A main point made by Dr. Stuart Hinds (overtone singing): ______________________

____

YOUR NAME: ________________________________

Final Examination PHYSICS 1406, "Physics of Sound and Music"

Please put your answers in the space provided.
Show your work for possible partial credit.
Your own notes on separate paper may be used, but NOT the Course Guide.
Use a calculator for numerical answers and a ruler for drawings.
Give numerical answers with 3 significant figures (e.g. L = 3.65 m), except where noted otherwise.
Each question counts 10 points.

---

1. What is sound?

---

2. What is music?

---

3. What is physics?

---

4. What is "Physics of Sound and Music"? ________________________________

5. Musical sound generally contains these elements:

________________ ________________ ________________

6. What travels in a sound wave? ________________________________

7. The human ear responds ______________________ to ________________
(give a mathematical term) (physical quantity)
and ________________
(physical quantity)

8. The period of a sine wave is T = 10.0 ms (milliseconds).
What is the frequency?

Frequency f = __________ Hz

9. A sine wave of wavelength $\lambda = 2.00$ m travels a distance of 300 m in 0.88 s. What is the Period T of this wave in millisecond (ms)?
Formula(s) used: ________________ ________________
Show your work:

Period = ____________ ms

10. What kind of vibrating medium produces inharmonic tones?

____

11. What replaces the word “harmonics” in an inharmonic sound spectrum?____________

12. A complex periodic tone is completely defined by

1. ____________________________________
2. ____________________________________
3. ____________________________________

13. A cello and tuba play the same note, but they sound different. What is the same according to Fourier’s theorem? (Do not write “pitch” or “fundamental”.)

Answer with words in plural: ____

14. The number of harmonics in the 5th harmonic of a musical tone is ____________

15. Two sine waves with frequencies of 442 Hz and 880 Hz are sounded together. What is the beat frequency of the resulting second order beats?

Show your work:

Beat frequency = ______________ Hz

16. A string has a length L = 120 cm. Divide it into two parts so that you hear a musical fifth when you pluck them together. What are the two lengths?

Show your work:

For the lower note: L_1 = ___________ cm, for the higher note: L_2 = ___________ cm

17. We showed individual harmonics on a string in class. Why can all these harmonics be present on the string of a violin at the same time?

____

18. The arrangement of the keys on a piano is in close analogy to the arrangement of the _________________________ on the _____________________________ of the ear.

19. Musical intervals such as fifths from two sine waves sound “the same” independent of the pitch. Why?____

____

20. a) Approximate frequency range of hearing: ____________to___________ Hz

b) Approximate SIL range of hearing: _____________to___________ dB

21. What is a typical just noticeable difference (JND) in the sound intensity level?

JND = __________ dB

22. A room has negligible background noise. A sound is turned on with SIL = 100 dB. After a while it is turned off abruptly. What is the SIL after the time $T_R/2$ has passed?
Show your work:

SIL = __________ dB

23. An empty box-shaped tornado shelter has the dimensions 1.5m x 3.0 m x 2.0 m. Calculate the *lowest* resonance frequency. Use v = 346 m/s for the speed of sound.
Formula for the resonance frequency: f = ________________
Calculation:

Answer f = ____________ Hz

24. You are at a distance of 20 m from an unobstructed sound source. You then move to a new location 60 m away from the source. Ignoring reflections from the ground, how much less loud does the sound appear to you? (Hint: First determine the dB decrease, then how much less loud this will sound.)
Show your work:

The sound will sound approximately __________ as loud.

25. How can you create acoustical "brilliance" in a concert hall?
____
____

26. The frequency range from 400 to 800 Hz occupies a length of 3.00 mm on the basilar membrane of the inner ear. What length on the basilar membrane does the frequency range 800 to 1600 Hz occupy?
Show your work:

The length is = ____________ mm

27. The ear is more sensitive to intensity changes than to frequency changes ____ T ____ F

28. The minima in the Fletcher-Munson curves can be explained with ______________________________ in the ______________________________

29. What changes when you move from one Fletcher-Munson curve to another? _______

30. Which display mode on the computer monitor did we use most often in class when analyzing sound? a) Waveform _______ b) Spectrum ______ c) Sonogram _______

31. Give a reason why you may "hear" bass sound on your smartphone in spite of its tiny speaker (do not write "difference tones"): ____________________________________

32. What is a harmonic? ____
____

33. What is a formant? ______________________________

34. Name three "Factors Affecting Tone Quality" (Timbre) according to our book"

____________ ____________ ____________

35. An 8 Ω loudspeaker is connected across the terminals of a stereo amplifier whose output at a certain instant of time is 9.0 V.
What is the electrical power dissipated in the loudspeaker at that instant?
Show your work:

$P_{electric}$ = ________ W

36. A 150-Watt hi-fi power amplifier is turned on for 2 hours a day. The cost of electricity is 14 cents per kWh. What is the electric bill in a year (365 days)?
Show the calculation:

Electric bill = $ ________

37. Write down a necessary and sufficient condition for inducing a voltage according to Faraday's law: ______________________________

____

38. Two practical applications of Faraday's law: ______________________________

____

39. The common household voltage in the USA is ________ Volt

40. The common frequency of the household voltage in the USA is ________ Hz

41. A dynamic loudspeaker operating in reverse could be called a ____________

42. Professor Meek played which instrument in class? ______________________________

43. Approximate audible frequency range of a smartphone: ______ Hz to ______ Hz

44. A 130 Hz *square* wave and a 396 HZ *sine* wave are sounded together.
What is the beat frequency? Beat frequency =______ Hz
To what frequency should you tune the sine wave for the beats to disappear? ______ Hz

45. The Pythagorean musical scale can be built entirely from ________and ________

46. What is "equally tempered" in equal temperament? (Do not write "Wolf fifth"):

____

47. How was the musical "wolf" killed? (Writing "ET" is insufficient.) _______________

____

48. Construct the musical third f(E4)/f(C4) in the Pythagorean diatonic scale from octaves and Pythagorean fifths only, as assigned in class. First show the answer as a fraction and then as a number with 5 significant figures.
Construction:

f(E4)/f(C4) = ____________________ = ________________________
show as a fraction 5 significant figures

49. Calculate the frequency ratio f(E4)/f(C4) in equal temperament (ET). Hint: Use the fact that E4 is 4 half steps or semitones above C4. Give your answer with 5 significant figures. (It should be similar, but not the same, as in the preceding problem.)
Calculation:

f(E4)/f(C4) = ____________________
5 significant figures

50. Take the frequency f(C4) = 261.63 Hz and obtain the frequency of the note one half-step above with 5 significant figures. Show your work:

f(C4#) =________________Hz
5 significant figures

51. What did you find, acoustically speaking, at the Campus Memorial Circle?

____

What does the "Anthropic Principle" say? ____

____

52. What is the big shiny metal sphere between the TTU Library and the SUB called, and what does it signify acoustically? ____

____

Bonus Questions (5 points each).

a. A symphony orchestra generally does not include the following common instruments:

____________________ ____________________ ____________________

b. Name and frequency of the classical radio station at TTU:___________ _________MHz
name frequency

d. Michel Faraday published his law in the year ____________________

Appendix

Music, Sound, Noise

Sound
Longitudinal pressure waves in air
Fluctuations about mean atmospheric pressure
Arrow of time, time flowing forward

Music
Superposition of waves
Non-equilibrium transitions
Finite function space with <100 discrete harmonic elements
88 pitches for piano, symphony orchestra
Melody
Harmony
Rhythm
"Speech without words"

Noise
Absence of consonance
Overlap of harmonics in auditory system
Non-discreteness

Transition between Music and Noise
Attack and decay transients
Inharmonicities
Percussive, non-melodic sound

The Many Roles of Music

(Excerpts from “Birth of the Beat”, Science News, August 14, 2010, pp. 18-23.)

Bonding
National anthems, alma mater, battle hymns, marching bands

Creative Expression
Jazz, improvisation, dance accompaniment, movies

Declarations of Love
Songs, ballads, opera

Entertainment
Lightens mood, passing time, pop concerts

Learning
Simple children’s rhymes, learning cultural rules

Meditation
Get in touch with spiritual world

Mourning
Funeral dirges, jazz funeral

Mythology
Songs telling stories about the journeys of ancestors

Relaxation
Mothers singing lullabies to put babies to sleep

Revolt
Raves, emergence of subcultures, all-night dance parties

Social Interactions
African participatory music

Social Pressure
Chanting to motivate fighting or call people to brotherhood

Work
Synchronizing repetitive movements in work

Worship
Traditional church music, gospel music, muezzin chanting

Jean-Philippe Rameau and Jean-Jacques Rousseau

(Excerpts from "Temperament" – The Idea that Solved Music's Greatest Riddle - by Stuart Isacoff, 2001, pp. 204-209.)

Jean-Philippe Rameau (1683-1764)
Harmony, not melody or language, is the root of musical expression.

The *sonorous body,* a universal natural phenomenon, governs harmony.
A vibrating object radiates a fundamental tone and overtones (octave, third, fifth).

The sonorous body is an ARCHETYPE from which the rules of music should be derived (e.g. musical scales, tuning, etc.). Nature loves harmony.

Jean-Jacques Rousseau (1712-1778)
"Back to Nature", "Noble Savage".
Music is man's most powerful medium for expressing thoughts and feelings.

Melody was born of man's untamed impulses craving release.
"… melody has a hundred times more energy than speech..."

The raw intensity of music and its primitive power has been stifled by conventions and civilization.

French as the language of reason produces only reasonable things. Singing in French is inherently unmusical.
Italian: Its dulcet, liquid tones are better suited for expressing passion.

French composers: Focusing on artificial harmony
Italian composers: Focusing on natural melody

Rameau versus Rousseau: Harmony versus Melody.

Rameau's tuning of keyboard instruments to equal temperament ("Rameau's Tuning") allows musical transposition and modulation – key elements of classical and romantic music.

Tubular Chime Set

A chime set made of electrical conduit tubing was built for this class.
The mode frequencies of the transverse vibrations of the chimes are given by

$$f_n = \frac{\pi K}{8L^2}\sqrt{\frac{Y}{\rho}}\, m^2 \quad \text{where} \quad m^2 = 3.0112^2, 5^2, 7^2, 9^2, 11^2, \ldots\ldots (2n+1)^2 ,$$

where Y = Young's modulus, ρ = density, L = length of tubular "bar" and

$$K = \frac{\sqrt{\left(a_i^{\,2} + a_o^{\,2}\right)}}{2}$$ where a_i = inner radius, a_o = outer radius of tubing

Material: Commercial ½-inch electric conduit

$$f_n = const.(1.000, \quad 2.757, \; 5.404, \; 8.933, \; 13.345, \ldots\ldots)$$
$$f_n = const.(0.3627, \; 1.000, \; 1.960, \; 3.240, \; 4.840, \ldots.)$$

Mode 1 **2** 3 4 5 …..

Eleven (11) chimes were tuned in resonating mode 2 to the notes
G4 – A4 – B4 – C5 – D5 – E5 – F5 – G5 – A5 - B5 - C6

The chimes are mounted at the nodes of Mode 2.
The anharmonic partials from the higher modes determine the tone quality.

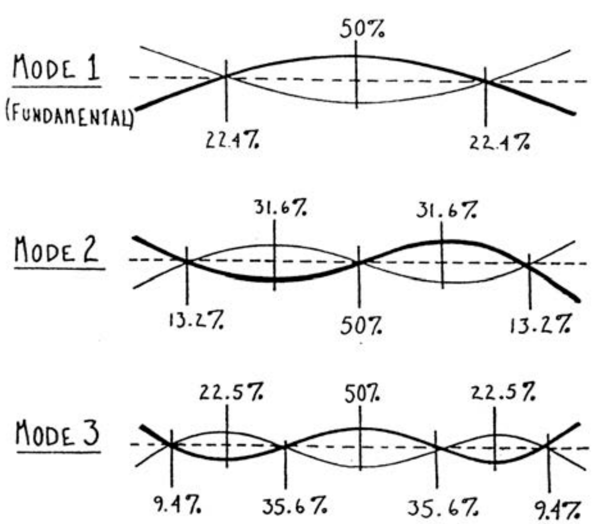


Figure. Modes, nodes, and anti-nodes for a transversely vibrating bar.
(From Bart Hopkin, Musical Instrument Design, Fig. 4-1, p. 31.)

Derivation of the Helmholtz Resonator Formula

The following is a derivation of the frequency of a Helmholtz resonator.
You will only need the end result, which we also have used earlier in this Guide and in the Laboratory. The derivation is intended for those who have taken a calculus-based physics course and are interested in more details of a Helmholtz resonator. We use concepts from classical mechanics (harmonic oscillator) and thermodynamics.

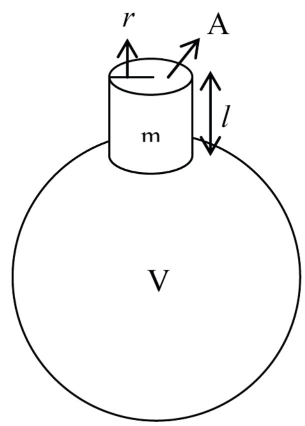


Figure. Schematic of a Helmholtz resonator. The cylindrical neck ("bottle neck") has a radius r, opening $A = \pi r^2$, and length *l.* The resonator volume V can be spherical, cylindrical ("wine bottle"), or box-like. The oscillating air plug in the neck has a mass m.

When you blow across the opening, the air plug m in the neck oscillates up and down.
It experiences a restoring force from the air in the resonator, which acts as the "spring" of a harmonic oscillator that causes the plug in the neck to oscillate up and down.

We derive the "spring constant" k for this system, and from that the resonance frequency.
For a harmonic oscillator satisfying Hooke's law F = - kx, the frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (1)$$

When the plug in the neck moves down (or up) a distance x from equilibrium, the pressure in the resonator increases (decreases) by Δp.
Therefore we have a restoring force on the plug, given by

$$F = -\Delta p A. \qquad (2)$$

The pressure changes in the resonator volume V from the oscillating mass m happen so fast that we must assume this to be an adiabatic process, where there is no time for the temperature to equilibrate with the environment outside the resonator. For an adiabatic process in an ideal gas we have pV^{κ} = constant, where $\kappa = c_p/c_v$ is the is the ratio of the specific heat at constant pressure to that at constant volume.

(Helmholtz Resonator continued)
We express the pressure change Δp in terms of the volume change ΔV in the resonator:
From pV^κ = constant we have $\Delta(pV^\kappa) = 0$. Differentiating this by the product rule yields

$$V^\kappa \Delta p + p\kappa V^{\kappa-1}\Delta V = 0 \quad \text{or} \quad \Delta p = -\,p\kappa\Delta V/V \tag{3}$$

But for the volume change we have $\Delta V = -\,A\cdot x$. Therefore $\Delta p = (p\kappa A/V)\cdot x$ (4)
(We are assuming here that the positive x-direction is *into* the volume V.)
The restoring force on the air plug is

$$F = -\,\Delta pA = -\,(p\kappa A^2/V)\cdot x \tag{5}$$

We see that this force is proportional to the displacement x.
Hence Hooke's law $F = -kx$ is satisfied, and the resulting motion is harmonic.
Comparing $F = -\,(p\kappa A^2/V)\cdot x$ with $F = -kx$ we find the "spring constant"

$$k = p\kappa A^2/V \tag{6}$$

With the mass density ρ and $m = \rho\Delta V = \rho A l$, we have $k/m = p\kappa A^2/mV = (\kappa p/\rho)\cdot A/lV$.
Substituting this into equ.(1) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{\kappa p A}{\rho \ell V}} \tag{7}$$

From the ideal gas law of thermodynamics we know that $pV = NRT$. Therefore we can write $\kappa p/\rho = \kappa NRT/\rho V = \kappa NRT/M$, where N is the total number of moles and M the total mass of the air in the resonator. We rewrite this as $\kappa p/\rho = \kappa N N_0 kT/(N N_0 m_{molec}) = \kappa kT/m_{molec}$, where N_0 is Avogadro's number und m_{molec} the mass of an air molecule.

We also know that $\kappa kT/m_{molec} = v^2$, hence $\kappa p/\rho = v^2$, where v is the speed of sound in air.
This finally yields the frequency of the Helmholtz resonator:

$$f = \frac{v}{2\pi}\sqrt{\frac{A}{\ell_{eff} V}} \tag{8}$$

In this formula, v is the speed of sound, A the area of the neck opening, l_{eff} the effective length of the neck including the two end corrections, and V the volume of the resonator**.**
We use this important result several times in our course.
Note that formula (8) applies to a Helmholtz resonator whose size is small compared to the wavelength of the emitted sound. For example, a bottle resonator may be 0.2 m tall and its neck 0.08 m long. At a frequency of 100 Hz, the wavelength is about $\lambda \approx 3$ m.
Note also that the volume of the oscillating air mass in the neck should be much smaller than the volume of the resonator.

Decibel Levels of Various Sound Sources

From: Reliability Direct, League City, Texas, www.reliabilitydirectstore.com

Sound Level Decibel Loudness Comparison Chart

Environmental Noise	
Weakest sound heard	0dB
Whisper Quiet Library	30dB
Normal conversation (3-5')	60-70dB
Telephone dial tone	80dB
City Traffic (inside car)	85dB
Train whistle at 500', Truck Traffic	90dB
Subway train at 200'	95dB
Level at which sustained exposure may result in hearing loss	*90 - 95dB*
Power mower at 3'	107dB
Snowmobile, Motorcycle	100dB
Power saw at 3'	110dB
Sandblasting, Loud Rock Concert	115dB
Pain begins	*125dB*
Pneumatic riveter at 4'	125dB
Even short term exposure can cause permanent damage - Loudest recommended exposure WITH hearing protection	*140dB*
Jet engine at 100', Gun Blast	140dB
Death of hearing tissue	180dB
Loudest sound possible	194dB
OSHA Daily Permissible Noise Level Exposure	
Hours per day	Sound level
8	90dB
6	92dB
4	95dB
3	97dB
2	100dB
1.5	102dB
1	105dB
.5	110dB
.25 or less	115dB

(continued next page)

Perceptions of Increases in Decibel Level	
Imperceptible Change	1dB
Barely Perceptible Change	3dB
Clearly Noticeable Change	5dB
About Twice as Loud	10dB
About Four Times as Loud	20dB
Sound Levels of Music	
Normal piano practice	60 -70dB
Fortissimo Singer, 3'	70dB
Chamber music, small auditorium	75 - 85dB
Piano Fortissimo	84 - 103dB
Violin	82 - 92dB
Cello	85 -111dB
Oboe	95-112dB
Flute	92 -103dB
Piccolo	90 -106dB
Clarinet	85 - 114dB
French horn	90 - 106dB
Trombone	85 - 114dB
Tympani & bass drum	106dB
Walkman on 5/10	94dB
Symphonic music peak	120 - 137dB
Amplifier rock, 4-6'	120dB
Rock music peak	150dB

NOTES:

- One-third of the total power of a 75-piece orchestra comes from the bass drum.
- High frequency sounds of 2-4,000 Hz are the most damaging. The uppermost octave of the piccolo is 2,048-4,096 Hz.
- Aging causes gradual hearing loss, mostly in the high frequencies.
- Speech reception is not seriously impaired until there is about 30 dB loss; by that time severe damage may have occurred.
- Hypertension and various psychological difficulties can be related to noise exposure.
- The incidence of hearing loss in classical musicians has been estimated at 4-43%, in rock musicians 13-30%.

Statistics for the Decibel (Loudness) Comparison Chart were taken from a study by Marshall Chasin , M.Sc., Aud(C), FAAA, Centre for Human Performance & Health, Ontario, Canada. There were some conflicting readings and, in many cases, authors did not specify at what distance the readings were taken or what the musician was actually playing. In general, when there were several readings, the higher one was chosen.

From: Reliability Direct, League City, Texas, www.reliabilitydirectstore.com

Physics, Psychophysics and Perception, Fechner's Law

Physical Acoustics	**Psychoacoustics**
Physical properties of sound	Psychological attributes of sound
Objective terms, outer world (spectrum)	Subjective terms, inner world (consonance, dissonance)
Intensity (SIL, dB)	Loudness (phon)
Fundamental frequency (Hz)	Melodic pitch
Frequency spectrum	Timbre, quality of sound

Fechner's Law and Physical Stimuli
Most of our sense organs show a logarithmic response to stimuli.

"As stimuli are increased by multiplication, sensations increase by addition."

Example
Sound Intensity Level (SIL)

$SIL_1 = SIL_0 + 10\log(I_1/I_0)$

Let the increase in a sensory stimulus be 100-fold, i.e. $I_1/I_0 = 100 = \mathbf{10^2} \rightarrow$
$SIL_1 = SIL_0 + 10\log 10^2 = SIL_0 + \mathbf{20\ db}$

Let the stimulus increase be $I_1/I_0 = 100{,}000 = \rightarrow$
$SIL_1 = SIL_0 + 10\log(10^2 x 10^3) = SIL_0 + 10\log 10^2 + 10\log 10^3$ or
$SIL_1 = SIL_0 + \mathbf{20\ db + 30\ db} = SIL_0 + \mathbf{50\ db}$

The sensation increased additively from 20 db to 50 db.
The increase is 30 db, while the stimulus increased by a factor of 1000.

Gustav Fechner (1801-1887) is the inventor of Psycho-Physics.
Fechner's ideas were used more successfully by Hermann Helmholtz, rather than by Fechner himself, in Helmholtz's book entitled "On the Sensations of Tone" (1885).
(See for instance Ian Johnston, Measured Tones, 3rd ed., CRC Press, 2009, p. 244)

Musical Scale Conjectures, Harmonic Overtone Series

Musical scales are free creations of the human mind.
They do not exist in nature.

Inductive Scale Conjecture
The prime faculty of the ear is the perception of small intervals
1:1 2:1 3:2 4:3 5:4 6:5 ……

Musical scales are *Platonic archetypes*.
Musical instruments are realizations of the archetypes.

Harmonic Overtone Series
Fourier series of periodic wave
1f 2f 3f 4f 5f 6f 7f 8f ……

Deductive Scale Conjecture
Musical scales are deduced from the overtone series
1f 2f 3f 4f 5f 6f 7f ……

Super-particular musical intervals
1:1 2:1 3:2 4:3 5:4 6:5 ……

Nomenclature

2:1	octave (diapason)	8th degree in diatonic scale
3:2	fifth (diapente)	5th degree in diatonic scale
4/3	fourth (diatesseron)	4th degree in diatonic scale
5/4	major third	3rd degree in diatonic scale
1:1	unison	1st degree in diatonic scale

The octave 2:1 is universal in most cultures

Number of degrees (notes) in an octave

5	Pentatonic scale
7	Western diatonic scale
12	Western chromatic scale
17	Common Arabic scale
22	Sruti, Hindustani scale, India
43	Harry Parch scale

Frequencies and Frequency Ratios for Five Musical Scales

Table. Frequency ratios for five musical scales (Pythagorean, Just Major, Quarter-Comma Meantone, Werckmeiser No. 1, and Equal-Tempered) for the 12-note chromatic scale from C4 and C5 (middle C to octave above).

	Pythagorean scale		Just major scale		Mean-tone scale		Werckmeister scale		Equal tempered scale
C	2.0000	C	2.0000	C	2.0000	C	2.0000	C	2.0000
B	1.8984	B	1.8750	B	1.8692	B	1.8793	B	1.8877
B^b	1.7778	B^b	1.8000	B^b	1.7889	$B^b/A^\sharp$	1.7778	$B^b/A^\sharp$	1.7818
A	1.6875	A	1.6667	A	1.6719	A	1.6705	A	1.6818
A^b	1.5802	A^b	1.6000	A^b	1.6000	$A^b/G^\sharp$	1.5802	$A^b/G^\sharp$	1.5874
G	1.5000	G	1.5000	G	1.4953	G	1.4951	G	1.4983
$F^\sharp$	1.4238	$F^\sharp$	1.4063	$F^\sharp$	1.3975	$G^b/F^\sharp$	1.4047	$G^b/F^\sharp$	1.4142
F	1.3333	F	1.3333	F	1.3375	F	1.3333	F	1.3348
E	1.2656	E	1.2500	E	1.2500	E	1.2528	E	1.2599
E^b	1.1852	E^b	1.2000	E^b	1.1963	$E^b/D^\sharp$	1.1852	$E^b/D^\sharp$	1.1892
D	1.1250	D	1.1250	D	1.1180	D	1.1175	D	1.1225
$C^\sharp$	1.0679	$C^\sharp$	1.0417	$C^\sharp$	1.0449	$D^b/C^\sharp$	1.0535	$D^b/C^\sharp$	1.0595
C	1.0000	C	1.0000	C	1.0000	C	1.0000	C	1.0000

(From Berg & Stork, Table 9-1, p. 241.)

Table. Frequencies of the notes between C4 (middle C) and C5. The scales are the same as in the above table. The frequency for middle C4 is 261.626 Hz for all scales.

	Pythagorean scale		Just major scale		Mean-tone scale		Werckmeister scale		Equal-tempered scale
C	523.25	C	523.25	C	523.25	C	523.25	C	523.25
B	496.67	B	490.55	B	489.03	B	491.67	B	493.88
B^b	465.12	B^b	470.93	B^b	468.02	$B^b/A^\sharp$	465.12	$B^b/A^\sharp$	466.16
A	441.49	A	436.05	A	437.41	A	437.05	A	440.00
A^b	413.42	A^b	418.60	A^b	418.60	$A^b/G^\sharp$	413.42	$A^b/G^\sharp$	415.30
G	392.44	G	392.44	G	391.21	G	391.16	G	392.00
$F^\sharp$	372.50	$F^\sharp$	367.92	$F^\sharp$	365.62	$G^b/F^\sharp$	367.51	$G^b/F^\sharp$	369.99
F	348.83	F	348.83	F	349.92	F	348.83	F	349.23
E	331.11	E	327.03	E	327.03	E	327.76	E	329.63
E^b	310.08	E^b	313.96	E^b	312.98	$E^b/D^\sharp$	310.08	$E^b/D^\sharp$	311.13
D	294.33	D	294.33	D	292.50	D	292.37	D	293.66
$C^\sharp$	279.39	$C^\sharp$	272.54	$C^\sharp$	273.37	$D^b/C^\sharp$	275.62	$D^b/C^\sharp$	277.18
C	261.63	C	261.63	C	261.63	C	261.63	C	261.63

(From Berg & Stork, Table 9-2, p. 241.)

Ranges of Some String Instruments

Instrument	Open strings	Highest note (approximately)
Violin	G3 D4 A4 E5	E7 (A7)*
Viola	C3 G3 D4 A4	A6 (D7)*
Cello	C2 G2 D3 A3	E5 (G6)*
Double bass[#]	E1 A1 D2 G2	D4 (F5)*
Guitar	E2 A2 D3 G3 B3 E4	A5

[#]Double bass with extender: The lowest note is C1.
*means using harmonics for the highest notes

Wave Packets and Musical Uncertainty Principle

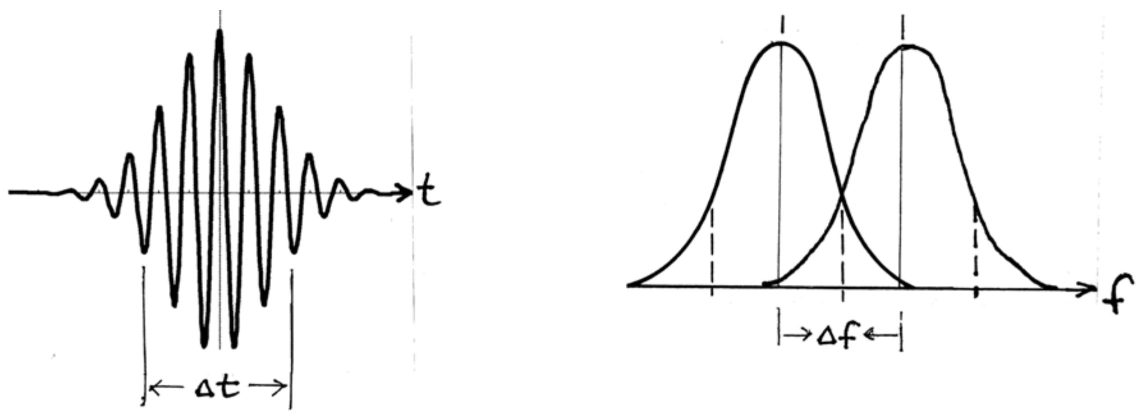


Figure. A minimum duration Δt of a wave packet is required in order to discern the pitch of a tone within a frequency uncertainty Δf.

For example, we wish to distinguish the pitch of two adjacent semitones (half-steps). Consider the pitch difference $\Delta f = B4^b – A4 = 440.0$ Hz – 466.2 Hz = 26.2 Hz.

From mathematics we have a minimum uncertainty product for Δf and Δt that is given by

$$\Delta f\ \Delta t \geq 1,$$

where Δf is the frequency range and Δt the minimum duration of the wave packet.

In our case we have $\Delta f = 26.2$ Hz and hence $\Delta t \geq 1/\Delta f = 1/26.2 = 0.038$ s = 38 ms. Furthermore, the period of oscillation is $T = 440 = 0.0023$ s = 2.3 ms.

Therefore the number of oscillations in the wave packet needed for a discernible, albeit imprecise, pitch is

$$T/\Delta t = 38\text{ms}/2.3\text{ms} \approx 17 \text{ oscillations}$$

Fewer oscillations result in a poor perception of “tone” and the sound is “percussive”.

Let’s summarize this:
a) If $\Delta t \approx T$, the duration of the transient is comparable to the period → **“percussive sound”** (percussion instruments with highly damped vibrations).
b) If $\Delta t >> T$, the tone contains many oscillations → **“melodic sound”** (majority of instruments playing melody).

We see that when the duration Δt of a tone is long, its frequency uncertainty Δf is small. Then the ratio Δf/f is very small and the pitch accuracy is very high.

Twice as Loud and x-Times as Loud

Judging the loudness of a sound is highly subjective.
The determination what sounds "twice as loud" is equally subjective. An increase in the sound intensity level of ΔSIL = 10 dB conventionally has been taken as sounding "twice as loud" by many persons. However, the value does not only depend on the person making the assessment, but also on the frequency, the intensity, and the type of sound. Giving a range 6 to 10 dB has been an improvement over the single old value of 10 dB.

If ΔSIL(2) is the value for an increase in SIL to sound "twice as loud", the increase ΔSIL(x) for a sound "**x-times as loud**" is given by

$$\Delta\mathrm{SIL(x)} = \Delta\mathrm{SIL(2)}\log_2 x\,,$$

where $\log_2$ is the logarithm to the base of 2.

Example: Let ΔSIL(2) = 10 dB be the increase for "twice as loud".
What is the increase in SIL for "4-times as loud"?
Answer: We have x = 4. Hence $\Delta\mathrm{SIL}(4) = 10\log_2(4) = 20\log_2(2) = 20$ dB.

It may be inconvenient to do the calculations with a logarithm to the base 2. We therefore rewrite the above formula in terms of the logarithm to the base 10. The calculation can be done more easily this way with a calculator. We then have

$$\Delta\mathrm{SIL(x)} = \Delta\mathrm{SIL(2)}\frac{\log \mathrm{x}}{\log 2}$$

Example: With ΔSIL(2) = 10 dB and x = 4, we again obtain ΔSIL(4) = 20 dB.

Example: Let ΔSIL(2) = 10 dB again. What is ΔSIL for "**5-times as loud**"?
Answer: We have x = 5. Hence $\Delta\mathrm{SIL}(5) = 10\frac{\log 5}{\log 2} = 23.2$ dB.

Example: Let ΔSIL(2) = 6 dB for "twice as loud". What th ΔSIL for "**5-times as loud**"?
Answer: We have x = 5. Hence $\Delta\mathrm{SIL}(5) = 6\frac{\log 5}{\log 2} = 13.9$ dB.

The first formula above can be inverted to yield

$$\mathrm{x} = 2^{\Delta\mathrm{SIL(x)}/\Delta\mathrm{SIL(2)}}$$

Example: If an increase in sound intensity level is ΔSIL(x) = 15 dB and ΔSIL(2) = 6 dB, how many times louder is the sound?
Answer: $\mathrm{x} = 2^{15/6} = 5.7$ or about 6-times louder.

Electronic Music and Synthesizers

Many years ago, electronic *analog* synthesizers were used for producing music. They are practically obsolete now. Modern digital synthesizers have replaced them. Today complex waveforms are either synthesized from sine waves or, better yet, from real instrument sounds stored in large memories.

Digital Synthesizers and Modern Electronic Keyboards

When a key is pressed, a digital waveform stored previously is selected for that particular key. Such *emulation of piano sound* is well advanced. Other sounds or “voices” are also available on the keyboard, for instance organ, string instruments, wind instruments, percussion. Furthermore one can select

Vibrato
Tremolo
Glissando
Reverberation
Percussion and rhythm accompaniment

Older digital and analog synthesizers used waveform synthesis based on Fourier’s principles. This is no longer done extensively on the more advanced keyboard synthesizers. Instead, waveforms from actual instruments are sampled in *digital waveform tables* and recalled. This requires very large computer memory. The recall of stored waveforms is more an assembly of building blocks than true Fourier synthesis with sine waves. This required far less memory. But assembling sound from wave tables yields more realistic sound than Fourier synthesis.

Synthesizers also are called wave station, music work station, performance synthesizer, composition synthesizers. All are operated from a keyboard.

MIDI

The “Musical Instrument Digital Interface” facilitates the use of digital synthesizers and allows interaction with computers for purposes of composition. One can connect an audio system to the computer or to the synthesizer directly.

The Synthesizer Keyboard as a Piano

A full 88-key electronic keyboard is far lighter than a piano and more portable, especially for performances of live music. It costs less, is more reliable, and requires less maintenance and hardly any tuning. Each stored note is a sample of real piano tones at various dynamic levels. The keyboard is sensitive to touch and recalls the sound sample at the appropriate loudness. The samples also include the attack and decay transients for the notes. In spite of these advantages of synthesizers, a grand piano still is unmatched in the performance of classical music.

Entropy in Music and Art; Pythagoras and Aristotle

Good art strikes a balance between extremes:

Redundancy	←→	Entropy
Order	←→	Disorder
Predictability	←→	Too many surprises and meanings
Repetition in music	←→	Random notes
Repetition in paintings (e.g. wall paper)	←→	Random paint splashes

Quotes From Pythagoras and Aristotle

Pythagoras (ca. 570 – 495 B. C.)
"There is geometry in the humming of the strings. There is music in the spacing of the spheres."
"Virtue is harmony."
"Numbers rule the universe."
"A thought is an idea in transit."

Aristotle (384 - 322 B.C.)
"A low musical tone on a string contains the tone of an octave above it."
"Learning is not child's play; we cannot learn without pain."
"Education is the best provision for old age."

Steinmeyer Organ in the Nidaros Cathedral at Trondheim, Norway

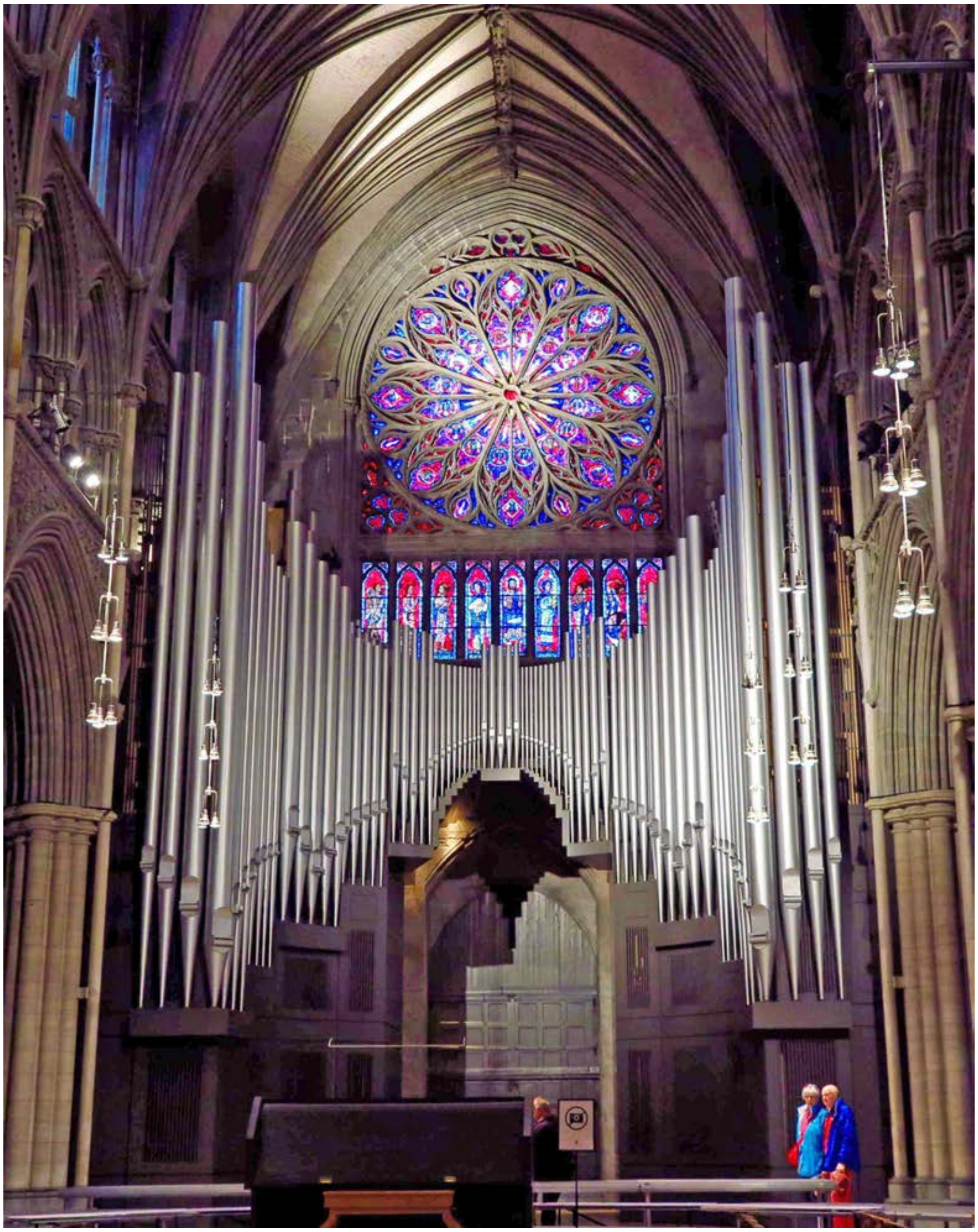


Figure and Exercise
Note the two longest pipes of the organ and obtain their effective length. Calculate the fundamental frequency. The pipes are open at both ends. The overall length is 32 feet. Estimate the length of its tapered lower end. Subtract that to get the effective length of the cylindrical part of the pipe. (Ignore the end corrections due to the radius of the pipes.) Verify that the fundamental frequency is in the range of hearing.

History of the Guitar

MEDIEVAL TO METAL

THE ART & EVOLUTION OF THE GUITAR

Guitars and other modern stringed instruments evolved from two ancient types of musical instruments. The origin of these devices goes back to at least 3000 BC. The first were harp-like instruments such as the lyre, where many strings were tied over an open space like a gourd bowl or a tortoise shell, or strung from a bowl up to a crossbar. The second type was a stick to which a few strings were attached at the top and bottom. Small gourds were attached to the sticks to increase the volume and improve the sound.

The difference between the two is significant. Instruments with necks allowed musicians to create different notes by pressing the strings down at different points on the neck. But the width of the stick-like necks limited the number of strings; sometimes to only a single string but rarely more than three or four. The lyre had four or more strings, allowing for a combination of notes to be plucked or strummed as chords. Their large bowls produced a vibrant sound. But because they were strung across open space, there was no place to press down on the strings and change their notes.

Today's stringed instruments are descended from one or both of these instruments. Instrument makers combined the best of both styles to create instruments suitable for their time. Gourds eventually gave way to carved wooden bowls, sticks became wide wooden necks with many strings, and the instruments we know today began to take shape.

The guitar evolved from European and Asian instruments during the Middle Ages (primarily the oud and lute). From bowls to flat surfaces to slightly curved lines, guitar makers experimented with hundreds of different shapes looking for the perfect blend of beauty, physics, and sound.

The first instrument to feature an hourglass shape was the vihuela. It was short-lived and gave way to an instrument known by a variety of names, including *kithara* (Greek), *cithara* (Latin), *qitar* (Arabic), *gittern* (English), *gitarre* (German), *guitare* (French), *chitarra* (Italian), and *guitarra* (Spanish). European luthiers, stringed instrument makers, who emigrated to the United States in the 1800s changed the guitar's structure to make it louder and sturdier. This included making the bodies bigger to project more sound.

The guitar was the first instrument that allowed singers and performers to accompany themselves, and was easy to travel with. The traveling guitar gave rise in the early 1900s to the blues in the Deep South and country western music in the expanding American west. At the same time, technicians, luthiers, and musicians attempted to make guitars louder for band members who could not hear their guitars above drummers and horn players. In the 1930s, they began to use electricity to amplify the guitars. Since that time, the development of the guitar has focused on making guitars louder and more beautiful.

Wide-scale production of the electric guitar started in the 1950s and has continued to the present. Inventors and designers like George Beauchamp, Guy Hart, Leo Fender, Les Paul, Ted McCarty, and Paul Bigsby introduced innovations and new body shapes from the 1930s to the 1950s.

Modern instruments incorporate an ever-increasing array of designs, materials, and technology, yet the relationship to their centuries-old predecessors is undeniable.

New Mexico Museum of Art
nmartmuseum.org

Guitar Models

The Evolution of the Guitar

Oud (Ud)

3000 BC, design unknown

The oud is one of the oldest stringed instruments, with references going back to 3,000 B.C. It is feasible the oud originated in the Middle East, perhaps in Mesopotamia. It was widely used in Northern Africa and was brought into Europe in 711 AD. This instrument is played with a long wooden pick.

Theorbo

1580s–1800s, designed in Italy

Being a modified lute, the Theorbo was created in Italy towards the end of the 16th century. Its most notable feature is an extended neck, which allowed for much deeper bass notes than traditional lutes. With its elongated neck, a theorbo can range up to six feet in length and feature more than a dozen courses, or sets, of strings. The theorbo had a harp-like set of strings running alongside the main neck. These were "drone" strings that were left open and never fretted. Instead, they were plucked like harp strings and allowed to resonate while the other strings were being fretted.

Romantic Guitar "Goldklang"

1700s, design unknown, after Stauffer manufactured by F. & R. Enders, Markneukirchen, Germany

In the late 1700s, many players ultimately determined that using six single strings, instead of six courses, created an instrument with excellent nuance. The Romantic guitar made by Goldklang, circa 1890, is notable for the "six on a side" tuning peg arrangement developed in Vienna during the 1820s by Johann Stauffer; the branding of the Goldklang logo onto the outside of the body on the headstock; and the unusual rosette, a spider web with a spider at its center, which has no effect on the guitar's sound.

Spanish Guitar

1930, design by Antonio de Torres Jurado, luthier Jose Ramirez III

Antonio de Torres Jurado discovered that the shape and depth of the guitar contributed primarily to its tonal qualities. Using this knowledge, he created internal bracing and an overall guitar design that has since been used as the foundation for Spanish guitars, now commonly referred to as "classical guitars." In the 20th century, luthier Jose Ramirez III built on the Torres model to create an instrument that would be suitable for concert performances. Louder bass, better sound projection, and comfort were all elements that performers wanted as more concert halls hired guitarists to present solo shows.

Rickenbacker Silver Hawaiian Lapsteel

1937–1943, design by Adolph Rickenbacker, manufactured by Rickenbacker International Corporation

In the early 1930s, guitarist George Beauchamp created a magnetic device that "picked up" vibrations directly from the metal strings of a guitar and sent them to an amplifier. Working with manufacturer Adolph Rickenbacker in 1931, he fitted the "pickup" onto a Hawaiian guitar, creating a solidbody electric guitar known as the "frying pan." Rickenbacker followed that with the N.S. Silver Hawaiian, using the same pickup wrapped over the top of the strings with one knob for tone and one for volume. The hollow body was made of stamped sheet metal and then chrome-plated. Like all lapsteel guitars it was played face up, using a piece of glass or steel to slide over the squared-off neck.

Fender Telecaster

1949, designed by Leo Fender, manufactured by Fender Musical Instruments

Leo Fender created the first mass-produced solidbody wood guitar. Naming it first the Esquire and then Broadcaster, Fender came up with the name Telecaster in 1951 due to his fondness for technology and television. This guitar has been known by that name ever since, and it is the longest continuously produced electric guitar in the world. Unlike other guitars of the time, it was designed to be manufactured as easily as possible and was not hand-carved. It was one of the first electric guitars to come in "custom" paint finishes—achieved using automotive industry paints.

Gibson Flying V

1958, designed by Ted McCarthy, et al., manufactured by Gibson

Prior to 1958, the Gibson company had crafted guitars, both acoustic and electric, that relied on the shape of the classical guitar. This meant rounded upper and lower bouts joined with incurved sides. On several models, including the Les Paul, the company had cut away part of one side so that players' fingers could reach higher on the fingerboard or neck. Breaking furthest from the standard shape of the classical guitar was the Flying V. Straightening the bouts and incurved sides into straight lines, and joining them at the center of the body, the company came up with a shape almost devoid of curves.

Eko 700

1965, designed and manufactured by EKO, Recanti, Italy

Oliviero Pigini founded EKO in 1959 to produce acoustic guitars. EKO manufactured other companies' guitars on a large scale, and was also able to sell some of them in Italy under its own name. As the company grew, it entered the electric guitar market with its own designs. Beginning with inexpensive versions of popular models, the company then designed the 700 Series. A visually impressive instrument, the 700 features dramatic curves, bold sparkle paint, large swaths of chrome, and more buttons than American guitars. The company still produces a wide range of guitars with distinctive styling, including the 700.

Giannini Craviola

1969, design by Paulinho Nogueira, manufactured by Giannini, São Paulo, Brazil

Noted Brazilian guitarist Paulinho Nogueira approached Giannini in 1968 with drawings he had made for a new guitar shape. He reimagined the upper bouts, where the body meets the neck, to dramatically slope away from the neck. His idea also included a D-shaped sound hole instead of the traditional circular hole. Nogueira stated he was surprised by the sound produced by the guitar, saying it sounded a bit like a harpsichord and also like the ten-string viola caipira, a type of Brazilian folk guitar. The name thus came from "*cravo*" (Portuguese for harpsichord) and "viola."

Di Donato

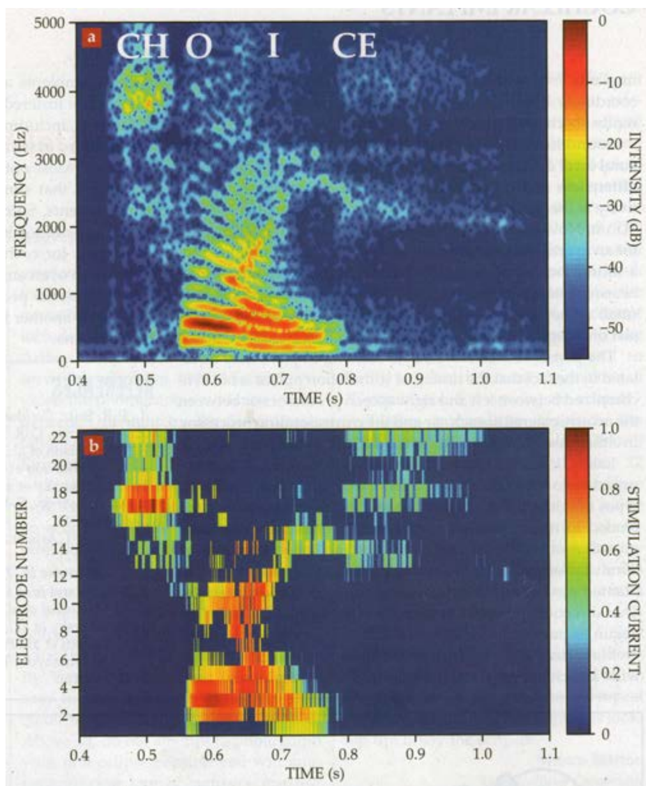
2012, design by Edoardo and Enrico Di Donato, manufactured by Di Donato Guitars, Venice, Italy

Modern times, technology, and materials have convinced a new generation of luthiers that stripping the guitar down to its essential parts is perhaps a better way to create a guitar that sounds good and feels nice to play. This approach dispenses with everything that isn't essential to the production of a desirable tone. The Di Donato is reduced to its component parts, with there being as much empty space in the guitar's design as there are physical elements.

Cochlear Implant

Severely hearing-impaired persons can be helped with so-called cochlear implants since the mid 1980s. The 3-part implant consists of an external *microphone and sound processor* mounted behind the earlobe. Second, a radio *frequency receiver* implanted in the temporal bone receives an electrical stimulation pattern from the processor. Third, the receiver transmits electric pulses to electrodes in a *thin silicone tube* implanted into the cochlea.

The figure shows the sonogram of the word “CHOICE” (upper figure). The lower figure shows the synthetic sonogram transmitted to 22 electrodes implanted in the cochlea. The two figures show similar patterns but also significant differences. For instance, the harmonics for the “OI” part of “CHOICE” are no longer distinct and sound different. Nonetheless, the word can be “heard” and understood reasonably well with some training. The discrete array of 22 electrodes along the cochlea mimics a continuous frequency distribution along the basilar membrane (see also “place theory of hearing”).



Upper Figure. Sonogram of the spoken word “CHOICE” from about 100 to 5000 Hz over a time period 0.45 to 1.05 seconds. The sound of “OI” consists of harmonics, whereas the “CH” and “CE” parts come from noisy hissing sounds in the word caused by turbulence.

Lower Figure. Synthetic sonogram from processing the original sonogram of “CHOICE” into 22 frequency bands. The frequency-intensity-temporal information is transmitted to the 22 electrodes implanted in the cochlea. From there, the still functioning nerve cells send the information to the brain.

(Courtesy of American Institute of Physics, Mario Svirsky, “Cochlear Implants and Electronic Hearing”, Physics Today, August 2017, Vol. 70, Nr. 8, pp. 52 – 58.)

LABORATORY MANUAL

PHYSICS OF SOUND AND MUSIC

PHYS 1406

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PREFACE TO THE LABORATORY MANUAL FOR PHYS 1406

This laboratory is an integral part of "Physics of Sound and Music", PHYS 1406.
This course fulfills the Natural Sciences Core Curriculum Competency requirement.

The experiments cover harmonic motion, waves, resonance, analysis and synthesis of sound, hearing and voice, room acoustics, electrical and acoustical energy, musical instruments, and very elementary music theory.

The manual provides the background for the experiments. Some experiments are single sets and are done in a group.
Participate!

Laboratory Reports
The manual includes the questions for the laboratory reports.
Follow the directions of your instructor for the reports.
The reports are always due at the next lab meeting.

Required
Attendance, participation, answering the quizzes, and submitting reports.
Notify your instructor about any absence in case of an emergency.

Please note:
You need a laboratory score of 75% or higher to pass the course "Physics of Sound and Music".

Acknowledgments
I appreciate the support for this course and laboratory by our department chairman Professor Sung-Won Lee and former chairmen Professor Nural Akchurin and Professor Roger Lichti.
I also acknowledge the technical support by Kim Zinsmeyer, Phil Cruzan, Chris Perez, Arnold Fernandez, and Sarah Stubbs, all in the Department of Physics. They built and improved equipment for the laboratory and lectures.

I am grateful for the earlier work by Professor Lynn Hatfield, who taught the laboratory and lectures for many years and provided many good ideas.

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1. Simple Harmonic Motion (SHM)

EQUIPMENT

Sonometer, dynamic microphone, Mac mini, string, pendulum stand, spring, aluminum and lead bobs, timers, metronome, stopwatch, multi-pendulum setup for visualizing and tuning of musical intervals.

PURPOSE AND BACKGROUND

In order to understand sound and music, we need to understand periodic motion and how it gives rise to sound. Periodic motion is any sort of movement that repeats itself after an amount of time called the *period*. For example, a violin string or the reed of a bassoon exhibit periodic motion when playing a sustained tone. A grandfather clock exhibits periodic motion as the pendulum swings back and forth, and so does a Ferris wheel that rotates at a constant speed.

Simple harmonic motion *(SHM)* is the purest form of periodic motion. Two conditions have to be met:

1. There exists a stable *equilibrium position*. If the system is at rest it will stay at rest there. It will tend to return to that position if displaced from it.

2. There exists a *restoring force* towards the equilibrium position. This force is proportional to the amount of displacement from equilibrium. For example, if a mass hanging from a spring originally is at rest and then pulled down a small distance, the mass will oscillate up and down with SHM when let go. The spring provides a restoring force to bring the mass back to the equilibrium position. If the mass instead is pulled twice as far, the spring provides twice the force to bring it back. In this manner, the system is *linear* and it is said to obey ***Hooke's Law***.

Much of music and sound is generated from periodic vibrations of the air or solid material in musical instruments. Examples are the vibrating strings of a violin and the reeds of woodwind instruments. In practice, however, very few musical tones come from pure SHM. That sound actually would be rather boring. Instead, musical tones consist of a combination of harmonics - see a tone from a plucked violin string in Figure 1. The lowest frequency corresponding to the first peak is called the *fundamental frequency*. This is the only frequency present in SHM. The peaks at the higher frequencies in Figure 1 are the higher harmonics or *overtones* that make up the tone. We shall discuss this in more detail in later laboratories.

QUESTIONS

1. Give your own example of simple harmonic motion and describe how it meets the two required conditions.

2. Give an example of *periodic* motion that is *not simple* harmonic. Give reasons why it is periodic but not simple harmonic.

3. Give an example of motion that is neither periodic nor simple harmonic. What would you call this type of "sound"?

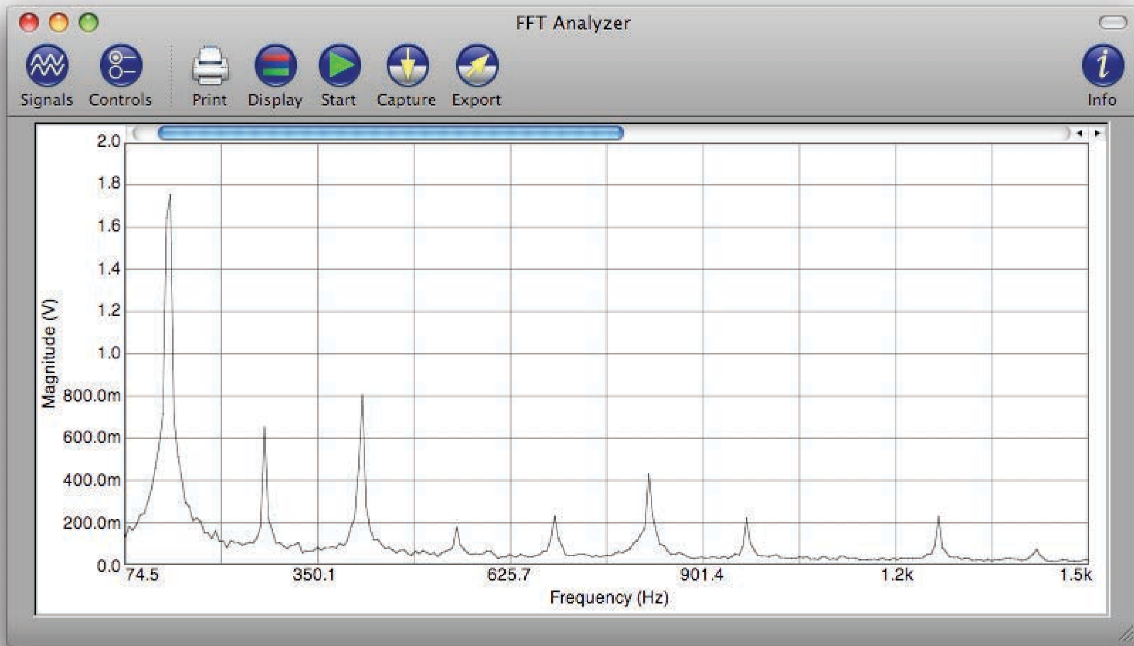


Figure 1. Frequency spectrum of a plucked string showing the fundamental frequency and higher harmonics (overtones).

THEORY AND EXPERIMENT

A basic property of simple harmonic motion and any periodic motion is the *period* T and directly related to it the *frequency* f. The period is the time for one complete cycle of motion. The frequency is the number of cycles during that time. These two quantities are inversely related. For example, if it takes a mass on the spring two seconds to complete a cycle, then $T = period = 2s$, and the frequency is one cycle per two seconds. As a formula we can write

$$f = \frac{1}{T} = \frac{1}{2s} = 0.5Hz \qquad (1)$$

The unit of frequency is *Hertz*, abbreviated Hz, and is the number of cycles per second. A cycle can be one revolution, a completion of a periodic process, or one oscillation.

The Pendulum

Make a pendulum using a string and either a lead (Pb) or aluminum (Al) ball as the mass (see Figure 2).

4. Which mass is heavier? If the length of the string is the same for both masses, which one do you believe will have the longer period?

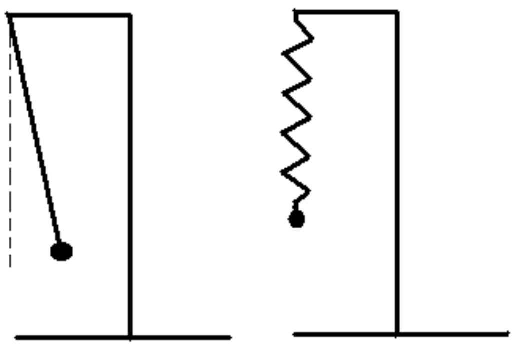


Figure 2. Pendulum and spring

The length of the string should be L = 20.0 cm from the support to the center of the mass ball. Obtain the time it takes for one period of oscillation using a small displacement. For accuracy, find the time for ten oscillations, then divide the total by 10 to get the average time for one period. Repeat this process three times and record the average time. Repeat the whole process for the other mass using the same length of L = 20.0 cm, then repeat for the lead and aluminum balls using L=80.0 cm.

5. Put your results in Table 1:

Pendulum Periods T (second)

	L=20.0 cm		L=80.0 cm	
Trial	Pb	Al	Pb	Al
#1				
#2				
#3				

6. Compare your results for the different masses and lengths. Which variable had an effect on the period? Which had no effect? (This might puzzle you and is different from the spring below.)

7. Note that the long pendulum was four times longer than the shorter one. Compare the periods of the longer and shorter pendulums.

We know from basic mechanics that the period T is proportional to the square root of the length of the pendulum according to the formula

$$T = 2\pi\sqrt{L/g} \propto \sqrt{L} \qquad (2)$$

So, if the long pendulum is four times as long as the short one, the period T is only twice as long. (The quantity g is the acceleration in Earth's gravitational field, given by $g = 980\ \text{cm/s}^2$.)

8. If the length of the long pendulum were 9x longer (i.e. L=180 cm) than the short pendulum, what would be the period?

Springs

For springs the formula for the period of oscillation is

$$T = 2\pi\sqrt{m/k}\,, \tag{3}$$

where m is the mass suspended from the spring and k is the so-called spring constant.

Attach a 50 g mass to the spring. Pull slightly down on the spring, let go, and record the time for ten oscillations, dividing by 10 again to obtain the average period. Repeat with a mass of 200 g. (Choose your own masses that work best.)

9. Complete Table 2

Spring Periods (seconds)

Trial	Mass m	Period T
#1		
#2		
#3		

10. How does the period T of a spring depend on the mass suspended from it? Write a simple proportionality to describe your observation.

Strings

We study the simple harmonic motion of a vibrating string. Guitars and other string instruments have strings under tension. We use a so-called sonometer, which is an apparatus with strings whose tension can be adjusted. A string is fastened at one end to a tension meter and led over a bridge near the other end (see Figure 3).

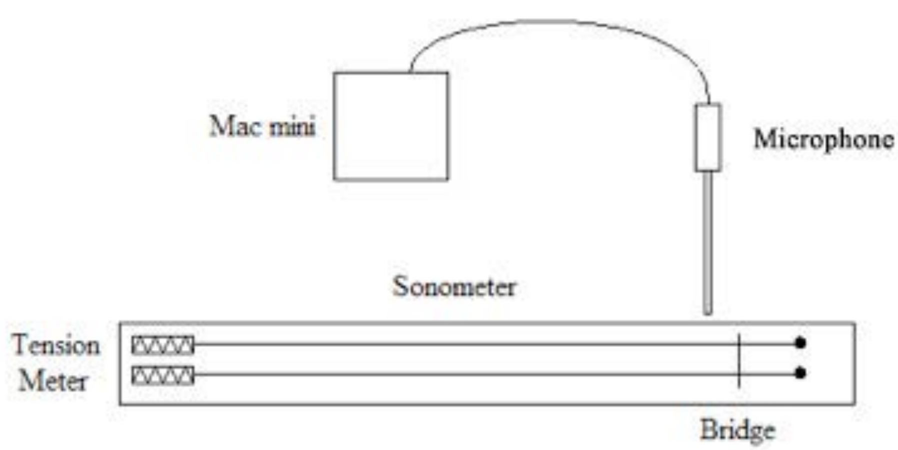


Figure 3. Sonometer setup with two vibrating strings (only one of the strings is needed).

The tension meter on the Sonometer measures the equivalent of "mass". For instance, when the tension scale reads 6 kg, it is the equivalent of having the string attached to a 6 kg mass hanging over the edge of the table.

Drive a loudspeaker with a signal generator and tune to the same frequency f of the vibrating string. Get the period from T = 1/f. Also display the waveform on the computer and obtain T.

Use the microphone connected to the Mac mini-computer and record the frequency spectrum of the vibrating string with the spectrum analyzer in our "Electroacoustics Toolbox" software. See Figure 1 again, which shows the fundamental frequency but also the frequency spectrum from the vibrating string. (P.S.: In the figure, the sound intensity from the microphone is displayed on the y-axis, versus the frequency on the x-axis.) Several frequencies are present when a string is plucked, namely the fundamental and the higher harmonics.
Compare the fundamental frequencies: a) From the signal generator readout. b) From the "spectrum analyzer" mode on the computer. c) From the "oscilloscope" mode on the computer.

11. Complete the following Table 3: (The mass is from the tension meter on the sonometer. The fundamental frequency from the spectrum analysis. Compare the calculated period T = 1/f from the spectrum analysis and signal generator with T as read directly from the waveform.)

String Period (seconds)

Trial	Mass m	Frequency f	Period T
#1			
#2			

Pluck the string gently in the middle and observe whether the spectrum becomes simpler or stays the same. Repeat by plucking the string at different locations and observe any changes in the frequency spectrum.

12. How does the period change with increasing mass? Is this similar or different from the spring?

13. How does the fundamental frequency (pitch) change as the mass increases?

Multi-pendulum Setup, Musical Intervals, and Harmony

Use the 4 pendulums mounted on a horizontal bar and held by a stand. Adjust the lengths of the strings so that the frequencies and periods of oscillation correspond to simple musical intervals. For instance, take a length L_1 = 50 cm for the longest string. Call its period of oscillation T_1, and the corresponding "fundamental" frequency f_1. The lengths of the other 3 pendulums are adjusted to simulate the important musical intervals of a *third, fifth* and *octave.* The frequencies associated with these intervals are simple fractions of the fundamental f_1 as shown in Table 4.

14. Call T_1 the period of oscillation of the *fundamental* oscillation of the L_1-string. What are the other periods? Hint: use f = 1/T. Insert these periods in Table 4 as multiples of T_1.

15. Calculate the lengths of the 3 shorter pendulums with the aid of Formula (2). Hint: Use L_1 = 50.0 cm, and from that get the other lengths with your results for the periods T in Table 4. For example, the length of the string of the musical *third* is given by $L = (4/5)^2 L_1 = 32.0$ cm, and so forth for the other pendulums.

Table 4. Musical Intervals Visualized With Pendulums

Musical Interval Name	C-major scale analog	Frequency f	Period T calculated	Period T measured	Pendulum length L	Oscillations for synchronization
Fundamental	C	f_1		T_1	50 cm	NA
Third	E	$5/4\ f_1$				
Fourth	F	$4/3\ f_1$				Not on set, but calculate it
Fifth	G	$3/2\ f_1$				
Octave	C	$2\ f_1$				

16. Start the longest pendulum and the next shorter one (musical *third)* at the same time so that they are synchronized at the beginning. Will they ever return together to their original position and be synchronized again? In other words, are the pendulums *in tune?* (P.S.: The simple frequency ratios in Table 4 correspond to so-called *Pythagorean temperament*.)

17. Count the smallest number of full periods for each of the two pendulums when they are back at the starting position again. Insert this pair of numbers in the last column of Table 4, with the smaller number *X* of oscillations for the longer string first, followed by the number *Y* of oscillations of the shorter string. Write this down as a pair *X – Y.*

18. Carry out the same procedure for the remaining two pendulums for the *fifth and octave.* Insert the pairs of numbers in Table 4 to complete the last column.

19. Observe how long the four pendulums stay synchronized and remain “in tune” with the fundamental frequency. Do not change the length of the pendulums! Only your instructor may do so if for some unfortunate reason the pendulums have been de-synchronized or “detuned”.

20. Start the pendulums for the *fundamental, third, and fifth* at the same time. Observe the oscillations and their regular behavior, assuming *they are well tuned.* What are you visualizing here, musically speaking? (Answer: A *major triad*, with its pleasing consonance!) Music students: Play a major triad on the keyboard in the laboratory, for instance C-E-G. Does it sound pleasing?

21. Start all 4 pendulums together. Do they ever come back together again at the starting position? For the music students: Play the corresponding notes, e.g. C4 –E4 – G4 – C5, on the keyboard. Does it sound pleasing? Any ideas why?

22. Challenge question:
Consider the pendulums for the intervals of the *third and fifth.* If you start them at the same time, how many oscillations will it take for each to find themselves back together again at the starting position? Figure out your answer with help from Table 4, and verify it experimentally.

Pythagorean Intervals and String Division

Use a sonometer and divide its strings with a wedge.

23. Move the wedge under the string. Pluck the two sections of the string and listen when the two resulting tones sound consonant. Do this for three different string divisions. Write down the lengths of the two string sections and take their ratio. Show the ration as a decimal fraction with 3 significant figures.

Example:

L_2/L_1 = 60.6 cm/ 39.4 cm = __1.54 _ (close to 3/2, i.e. musical fifth)

Your 3 measurements:

L_2/L_1 = ________cm/________cm = _______ (close to ________, i.e. ______________________)
ratio musical interval

L_2/L_1 = ________cm/________cm = _______ (close to ________, i.e. ______________________)
ratio musical interval

L_2/L_1 = ________cm/________cm = _______ (close to ________, i.e. ______________________)
ratio musical interval

The Metronome

We have an old metronome in the laboratory that exhibits periodic (but not simple harmonic) motion. This essentially is an inverted rigid pendulum. As the pendulum stick swings, a spring pulls the bob back towards the equilibrium position in the center. The effective length of the pendulum and thus the frequency can be adjusted by moving the bob up and down.

24. Read the frequency range of the metronome on its pendulum stick and compare it with the measurement on a stopwatch.

25. Describe the purpose of a metronome.

2. Wave Phenomena in Water and Air

PURPOSE AND BACKGROUND

Wave motion is responsible for the propagation of sound. In this laboratory we study various wave characteristics and how they are related to the production and propagation of sound. We will take a look at *reflection*, *refraction*, *interference*, and *diffraction*. A “ripple tank” with water waves is used to simulate the properties of sound waves. A light source at the ripple tank illuminates the waves so that they are visible. For actual sound waves we use a two-speaker system to demonstrate interference and diffraction.

EQUIPMENT

PASCO WA-9897 Ripple Tank, ripple tank wave generator, stroboscope light source, protractor for measuring angles, stop watch, Mac mini, two loudspeakers.

Caution: Please inform the instructor if you feel uncomfortable from the light flashes of the stroboscope or if you have photosensitive epilepsy.

THEORY AND EXPERIMENT

A wave is a periodic disturbance or fluctuation in a medium about its equilibrium position. We use water waves as a good example. Waves transport energy and can do work. A simple *sine* wave can be used to demonstrate important properties of waves. Figure 1 shows the *displacement* of the vibrating medium (e. g. air, water, or a string) as a function of time. The horizontal axis is the time, and the vertical axis is the displacement. The equilibrium position is at x=0. The period T is the time for one complete cycle, in other words the time for a system to return to its initial position.

The *frequency* of oscillation is defined as the inverse of the period, $f = \frac{1}{T}$.

The physical unit of the frequency is *Hertz*, abbreviated Hz. The oscillation in Figure 1 has a period of T = 0.10 s and a frequency of f = 10 Hz. For water, the molecules move up and down (transversely), while the wave itself travels in a direction perpendicular to the up and down motion. This kind of wave is called a *transverse traveling wave*. The *wavelength* λ is the distance the wave travels during one “up and down” cycle. It is the distance from any one crest to the next nearest crest, or from wave trough to next trough, or between any two corresponding points having the same *phase* - see Figure 2. If we call v the wave speed, then we have

$$v = \frac{\lambda}{T} = \lambda f \qquad (1)$$

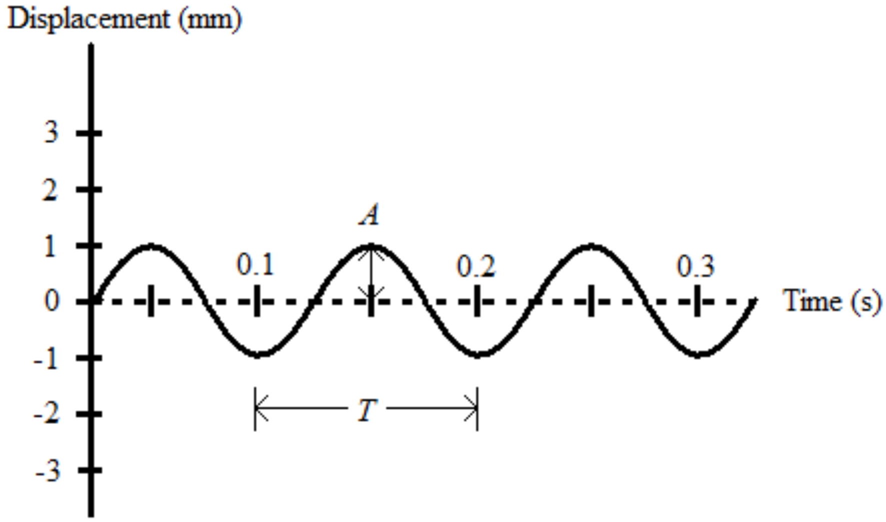


Figure 1. Displacement of the medium (e.g. water) of a wave from equilibrium as a function of time, for a fixed point of observation.

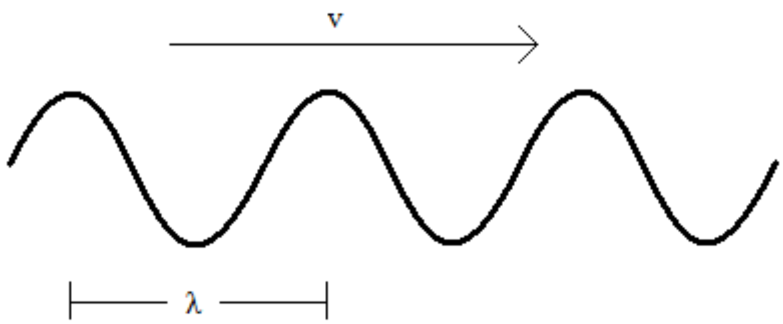


Figure 2. A transverse wave traveling with velocity v and wavelength λ. We see a snapshot at a fixed time with vertical direction showing the displacement of the medium and horizontal direction the position of the wave.

Use the PASCO ripple tank and produce plane waves in the "strobe light" setting.

1. Trace three or four plane waves on a sheet of paper located on the screen of the ripple tank. Draw a line showing the direction of the traveling waves.

2. Measure the length of 5 consecutive crests and determine the wavelength λ by dividing that distance by 5. For instance, a frequency setting to give you a wavelength around 6 cm may be suitable. The frequency f is displayed on the PASCO wave generator itself. What is the velocity v of the waves according to equation (1)? Answer: v = _________m/s

The wave speed can also be obtained from the measured time t the wave takes to travel a distance d, according to the formula $v = \frac{d}{t}$. With a stopwatch, measure the time t it takes the wave to travel a given distance d, without the strobe light turned on.

3. Take three time measurements. Record the average time: t =________s.
4. Now calculate the wave speed from v = d/t: v =________m/s.
5. Compare your values for the velocity from questions 2 and 4 and discuss possible reasons for any discrepancy.

6. For the velocity for shallow water waves we have $v = (gd)^{1/2}$, where $g = 9.8\ m/s^2$ and d is the depth of the water. Obtain the value for the velocity: v =________m/s.

Reflection

When sound waves hit a barrier such as a wall, some of the sound is reflected (with the rest absorbed by the wall). Waves obey *law of reflection*. A line drawn perpendicular to a point on the wall is called the "surface normal" - see Figure 3. The angle that the incoming wave makes with the normal is called the *angle of incidence*. The law of reflection states that for a wave approaching a barrier, the wave will be reflected from the surface at an angle equal to the angle of incidence. For the law of reflection to hold, the surface roughness must be small compared to the size of the wavelength. In other words, we need a smooth, or optically speaking, a "mirror-like" surface. This is the case in our experiments with the ripple tank.

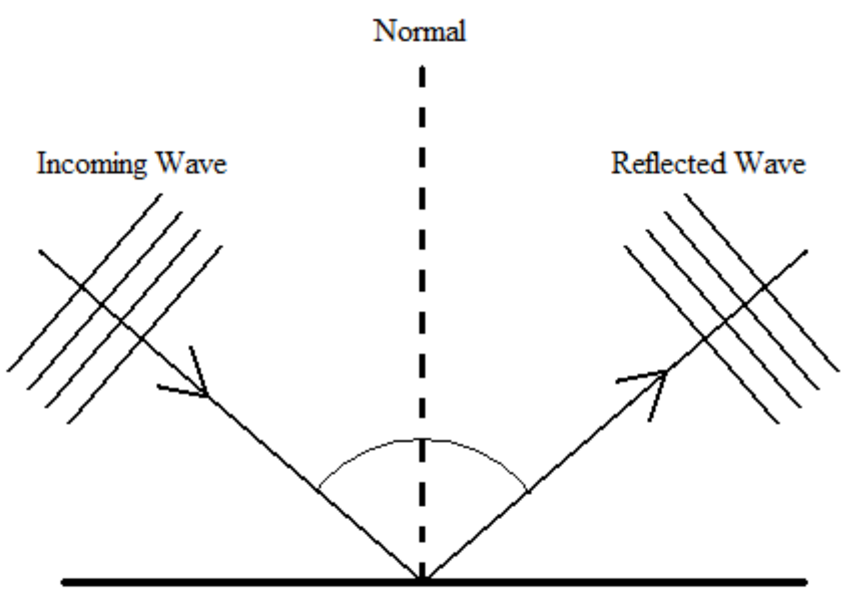


Figure 3. Incoming wave reflected off a smooth surface

Produce a *plane wave* with the PASCO ripple tank by placing one of the longer straight barriers in the center of the tank. Observe how the waves are reflected. The light and the wave generator are synchronized in STROBE mode so that when the generator is producing waves with f =20Hz, the light will strobe at the same frequency. This gives the appearance of non-moving "frozen" waves and thus facilitates observation. By changing the strobe frequency of the light slightly, the waves can be made to look as if they were traveling slowly. This is for clearly seeing the direction of the traveling waves.

7. Trace the wave barrier on a sheet of paper. Trace the incoming and reflected waves and draw two lines representing the direction of travel for each of them.

8. Use a protractor to measure the angles of both the incoming and reflected waves. Are the two angles the same in accordance with the law of reflection?

Note that an *interference* pattern also is created in this experiment (see *interference* below).

Place the concave plastic piece in the water to act as a "mirror" for the water waves. Observe focusing of the waves in analogy to an optical mirror.

Refraction

Refraction means a change in the direction in which a wave travels (see Figure 4). This happens for instance in water where the depth changes, and the wave speed changes as a consequence. Although refraction has only limited applications to sound propagation in enclosed rooms such as our laboratory, it accounts for some interesting atmospheric phenomena (see below). Refraction also occurs with light waves, where it accounts for the action of optical lenses. In all cases where refraction occurs, the wave speed and direction of propagation change.

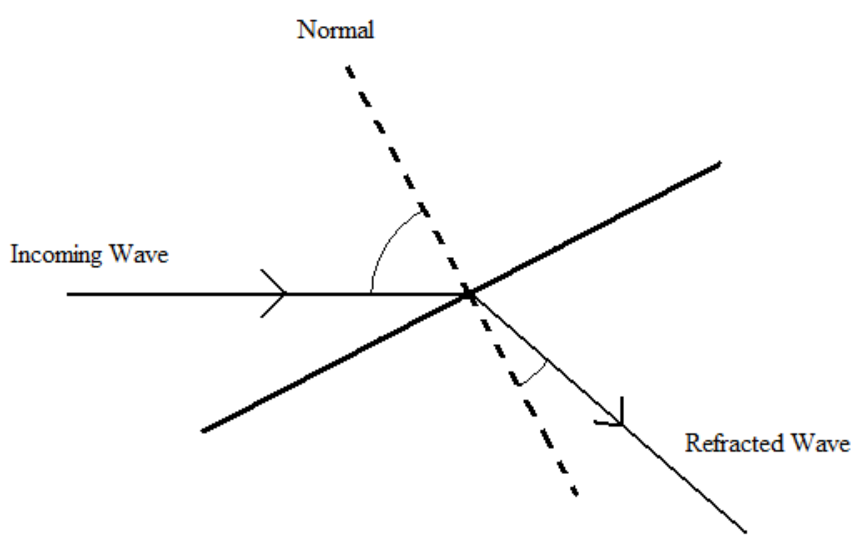


Figure 4. Refraction of wave on a barrier such as in water.

In our experiment, place the shaped trapezoidal plastic plate into the ripple tank. Produce plane waves that move toward the plate. You should see that the wave speed and wavelength in the deeper water are greater than in the shallower water on top of the plastic plate. Note that, whereas the wavelength changes, the frequency does not change in the two regions!

9. Draw the shape of the plastic plate on a sheet of paper. Draw a line representing the direction of the incoming waves and a line for the direction of the refracted waves. Clearly show that the two lines are not parallel, but have a kink instead, meaning refraction.

10. Where is the wave speed slower and what do you think causes the decrease in speed?

Place the plastic lens in the water to act as a lens for the water waves. Observe focusing of the waves in analogy to an optical lens.

A Remark on Refraction of Sound Waves in the Atmosphere

The speed of sound depends on the temperature of the air. Cooler, denser air will transmit sound more slowly than warmer air. Under normal conditions, the air near the ground is warmer than the air above it. This is the reason why you may not always hear the thunder from a lightning strike several miles away: The sound traveling through the cold air higher up travels more slowly than through the warm air closer to the ground. The sound therefore is refracted upwards and may not reach you. In contrast, a *temperature inversion*, where the air is cooler closer to the ground, produces the opposite effect. A cool lake at night and in the morning hours can cause such a temperature inversion: Sound is refracted downwards towards the listener, effectively amplifying the direct sound across the lake. This makes the sound, for instance from people on the opposite shore, sound louder and closer than it actually is.

Interference of Waves

When two or more wave trains move through the same region of space, the waves interfere with each other at any given spot. *Constructive interference* occurs when two waves with the same phase, such as two wave crests, align at the same location. The two amplitudes add together to create a "hotspot" of twice the amplitude and thus a maximum in intensity - see Figure 5.

On the other hand, if the wave crest of one wave meets with a wave trough of another wave, the two waves are completely out-of-phase and suffer *destructive interference*. The resultant amplitude is nearly zero, and so is the wave intensity.

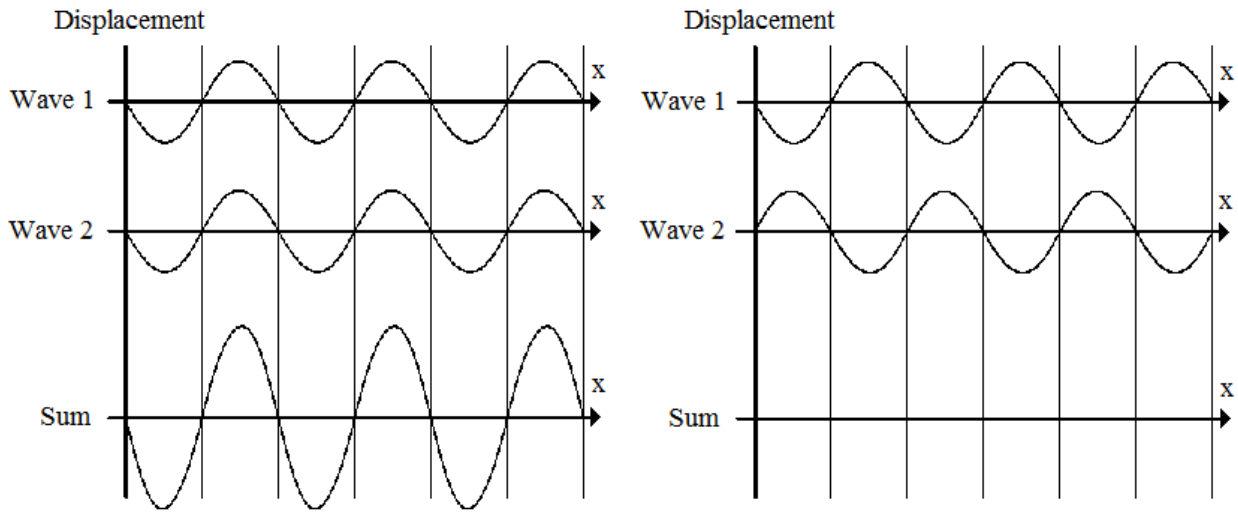


Figure 5. Interference and superposition of two waves. The diagram on the left shows *constructive* interference and on the right *destructive* interference.

Interference of Water Waves

Study the interference maxima and minima with water waves in the "ripple tank". Use the wave generator with two small point-like dippers, each producing circular waves. Space the plungers a few wavelengths apart. Choose a frequency between 15 and 20 Hz. Observe the interference pattern.

11. Change the frequency of the dippers. Describe how the interference pattern changes.

Set up the long barriers side by side so that they make a barrier parallel to the plane waves. In the gap between the two long barriers place a small barrier so that two small gaps or "slits" are created. Use the wave generator and produce plane waves with a frequency of about 10 Hz moving towards the "slits".

The two slits can be considered new "point sources" by themselves for the emission of waves. (Our experiment is a direct analog to the interference in the famous *double slit experiment* in optics, where light was shown for the first time to be a wave.) After passing through the slits, the emerging waves interact and create an interference pattern according to the *principle of superposition*. You should be able to see bright spots where the waves add constructively and dark spots where they add destructively. Record the frequency shown on the wave generator. Trace the barriers and the double slits on a sheet of paper. Mark the lines of constructive and destructive interference.

12. Increase the frequency of the wave generator. What happens to the interference pattern?

Diffraction

Diffraction is a wave phenomenon with direct applications to sound propagation when there is a barrier, opening, or corner. The effect is pronounced when the wavelength is comparable to the size of the obstacle. In such cases diffraction enables one to hear sound "around corners" - see Figure 6. We all have heard sound from a door opening when we were outside a room but not in the line-of-sight of the sound source inside.

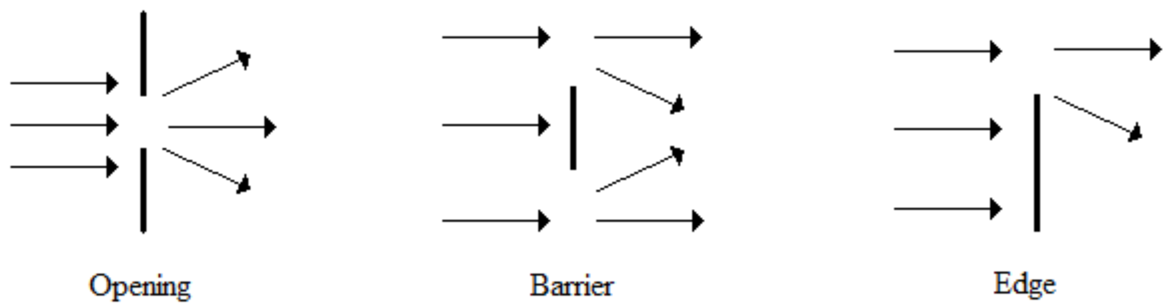


Figure 6. Diffraction of sound waves after passing through an opening, around a barrier, and around an edge.

Create a small opening ("single slit") of about 2 cm between two long barriers in the ripple tank, the barriers being in line with each other and parallel to the approaching plane waves. Produce plane waves of frequency 20 Hz with the wave generator.

13. Trace the opening in the barrier on a sheet of paper. Trace the incoming and diffracted waves.

14. If the incoming waves were circular instead of planar, what would the diffracted waves look like? Confirm your guess by removing the plane wave generator and adding a single dipper in its place, which now produces circular waves.

Set up a barrier for the waves in the ripple tank instead of the opening. Let a wave approach a barrier with width large compared to the wavelength. You should see a "shadow" region without waves behind the barrier, as expected. If, however, you make the wavelength comparable to the barrier width, the "shadow" region behind the barrier becomes small and waves travel into the region. Verify this by using a small barrier or by decreasing the frequency of the wave generator (this increases the wavelength).

Finally, use one of the long barriers for an "edge" or "corner" in the tank and block about half of the incoming wave. Note again that the wave passing the edge is diffracted into the "shadow" region behind the barrier (Figure 6).

P.S.: Water waves are an example of *transverse waves*, whereas sound waves are *longitudinal waves*. In the first case the medium oscillates *transversely* to the direction of wave propagation, in the second case (air) the oscillations are *longitudinal* along and against the propagation direction. These differences do not affect the basic study of wave behavior in this laboratory.

Interference of Sound waves

Sound waves from two speakers behave exactly like the interfering water waves in the "double slit experiment" in the ripple tank above. For interference from the speakers to be clearly audible, the wavelength should be somewhat smaller than the distance between the speakers.

Use a signal generator, set the frequency between 600-2000 Hz, and play the same signal through two loudspeakers. Walk rather quickly in front of the speakers. Can you hear the interference maxima (*constructive interference)* and minima (*destructive interference)* as you walk?

15. At the midpoint in front of the speakers. i.e. at the same distance from each speaker, do you hear constructive or destructive interference? Why?

16. Does the number of audible maxima and minima increase or decrease if the frequency is increased?

17. Is interference of sound waves desirable or undesirable in rooms and concert halls? Why? How would you address such problems?

18. Give examples for diffraction of sound from openings, barriers and edges. Discuss this in the context of desirable or undesirable room acoustics.

3. String Resonance

PURPOSE AND BACKGROUND

Standing waves on stretched strings and in pipes offer a convenient way to study vibrations, including the fundamental frequency and harmonics (overtones). For strings in particular, the frequency depends on the tension, the mass of the string per meter (*linear mass density*), and the total length. In wind instruments, with air as the vibrating medium, the frequency is defined by the *speed of sound* and the effective length of the pipe. Once the fundamental frequency is known the higher harmonics are found as simple integer multiples of that frequency.

EQUIPMENT

PASCO Sonometer Model WA-9611 with Driver/Detector Coils, weights, function generator, loudspeaker, violin, Faber Electroacoustics Toolbox software, Mac mini.

Resonances and Modes

When a string is plucked, a *transverse* standing wave is created on the string - see Figure 1. In the simplest case, we have only one *anti-node* with maximum movement in the center. The points at the two ends of the string do not move and are called *nodes*. The standing waves result from two waves traveling in *opposite* directions along the string. The superposition of the two waves yields a standing wave, provided that the *resonance conditions* are met.

The first 3 vibrational modes of a string are shown in Figure 1. For the *fundamental mode* (harmonic number N = 1), the wavelength is $\lambda = 2L$, where L is the length of the string. For the next higher mode, the *first overtone* or *second harmonic* (N = 2), the wavelength $\lambda = L$.

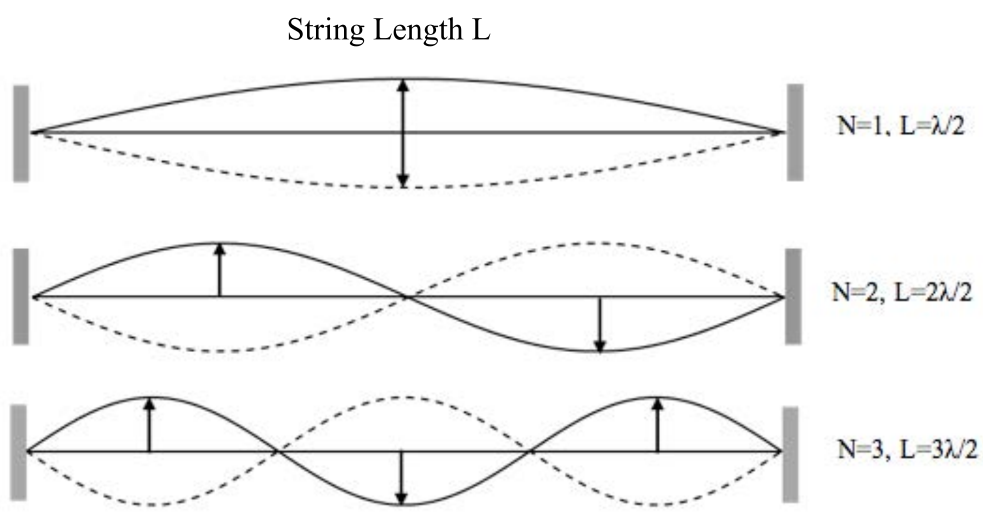


Figure 1. Vibrations of a string. The wavelengths of the standing wave resonance modes are $\lambda_N = 2L/N$ and the frequencies are $f_N = v/\lambda_N = Nv/2L = Nf_1$, where N is the harmonic number, v the velocity of the wave along the string and f_1, and the fundamental frequency.

The velocity v of the wave *on the string* (not the speed of sound in air!) is given by

$$v = \sqrt{\frac{F}{\mu}}, \qquad (1)$$

where F is the *tension* on the string and μ its *linear mass* density (mass per unit length or kg/m).

For example, a typical metal guitar string has a mass per unit length of $\mu = 6.3\times10^{-3}$ kg/m. For a tension F =73.3 N, the velocity of the wave along the string is

$$v = \sqrt{\frac{73.3\text{N}}{6.3\times10^{-3}\,\frac{\text{kg}}{\text{m}}}} = 108.0\,\frac{\text{m}}{\text{s}}$$

The *fundamental frequency f* is given by

$$f = \frac{v}{\lambda} = \frac{v}{2L}, \text{ and hence } f = \frac{1}{2L}\sqrt{\frac{F}{\mu}} \quad \text{(Mersenne's Law)} \quad (2)$$

Note that this frequency also defines the *pitch* of the sound from the string. Strings on a classical guitar have a length of L= 0.65m. With the velocity known, the fundamental frequency is

f = 108.0/(2x0.65) = 83.0 Hz. This is close to the frequency of the E-string of a guitar.

Question: Discuss how the tension, mass, material, diameter, and length of the string affect the fundamental frequency and the wave velocity on the string.

String Vibration Experiments

Stretch a horizontal string made of flexible fabric over a pulley. Place a suitable weight on the vertical end of the string. Fasten the horizontal end of the string to a vibrator. Connect the vibrator to a frequency generator. Tune the frequency to the fundamental vibrational mode and fundamental frequency (1st harmonic) of the string. Increase the frequency until you get the 2nd vibrational mode (2nd harmonic). Keep increasing the frequency and note the appearance of successively higher harmonics. How many harmonics are you able to produce?

Change the weight on the string and note the change in the fundamental frequency.

Change the length of the string and again note the change in the fundamental frequency.

Sonometer Experiments

Use a *sonometer* **(**PASCO Model WA 9611) – see Figure 2. This allows us to study the resonance modes and frequencies of a stretched string and to determine the wave velocity.

Sonometer setup instructions: The tensioning lever for the weights must hang level. The bridges, over which the strings are stretched, can be placed at any location and define the vibrating part of the string length L. Hang a mass of approximately 1 kg from the tensioning lever to produce the desired tension. Adjust the string adjustment screw so that the lever is *level* – See Figure 2.

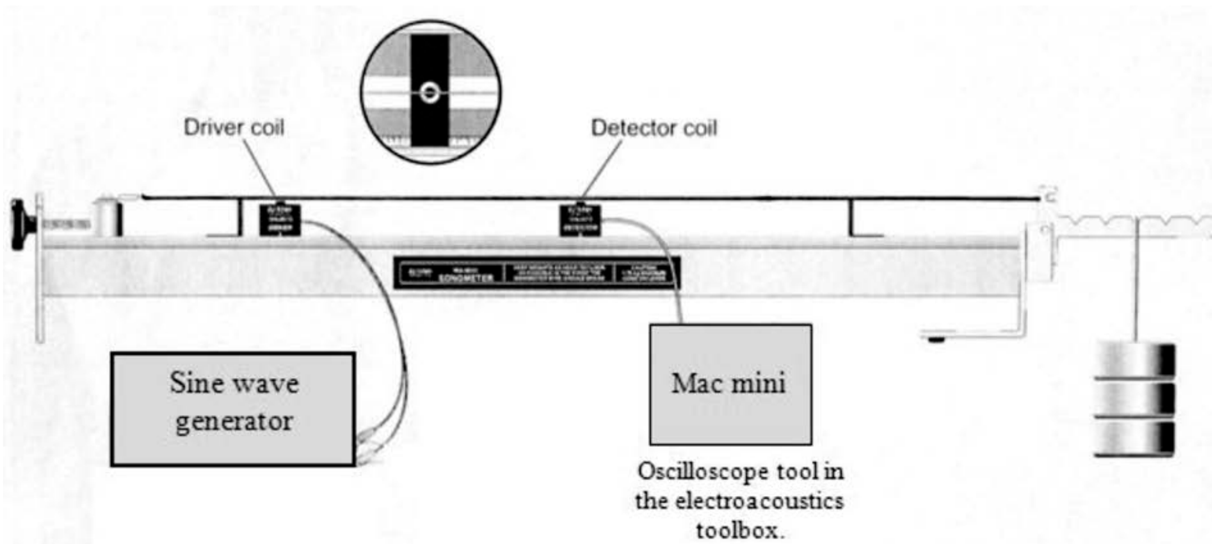


Figure 2. PASCO Sonometer Model WA-9611 for studying vibrating strings. The string is excited with a Driver Coil and the vibrational modes are analyzed with a Detector Coil. The sine wave generator activates the Driver Coil. The vibrating string induces a voltage in the Detector Coil. The latter is connected to the Mac computer. Open an "Oscilloscope Tool" there in the Electroacoustics Toolbox to observe the signal.

The tension is determined as follows: For a mass M in slot 1 of the lever, the tension is F = Mg, where $g = 9.8\ m/s^2$. If you hang the mass from slot 2, the tension is 2 Mg, and so on.

Some qualitative experiments: Pluck the string. Vary the tension, length, and linear mass density of the string, one at a time. Listen the effect on the pitch. Observe the change in pitch (fundamental frequency) with the spectrum analyzer on the computer.

1. Vary the tension by hanging the mass M from different slots in the tensioning lever. Keep the lever level. How does the pitch (fundamental frequency) change with tension?
2. Vary the length L of the string by adjusting the distance between the bridges. How does the pitch change? How can you also infer this from equation (2)?
3. Change strings to vary the linear mass density. How does the pitch change, as heard and also seen on the computer? How can you see this from equations (1) and (2)?

Table 1. Linear Mass Density of Guitar Strings

String diameter	Linear Mass Density μ (g/m)
0.010in (0.254mm)	0.39 g/m
0.014in (0.356mm)	0.78 g/m
0.017in (0.432mm)	1.12 g/m
0.020in (0.508mm)	1.50 g/m
0.022in (0.559mm)	1.84 g/m

Sonometer Experiments with the Driver Coil and Detector Coil

Connect the Driver Coil to a Pasco Signal Generator instead of a function generator as shown in Figure 2. Connect the detector coil directly to the Mac computer and open an oscilloscope tool in Faber Electroacoustics Toolbox.

Position the driver coil approximately 5 cm from one of the bridges. More generally, the driver will drive the string best if placed at an anti-node of the wave pattern. However, if you place the driver near one of the bridges, it will work reasonably well for most frequencies.
Position the detector midway between the bridges initially. You may experiment with this for optimal signal. It works best when positioned near an anti-node of the wave pattern.

Choose a frequency between 100 and 200 Hz. Increase the amplitude. Slowly vary the frequency. When you reach a resonant frequency, you should see a vibration of the string and the sound produced should be loudest. The wave pattern seen on the oscilloscope should become a clean sine wave. You may need to vary the amplitude on the Pasco Signal generator slightly for best results.
Keep the detector coil at least 10 cm away from the driver coil. This minimizes the interference between driver and detector.
Important: The frequency observed on the wire usually is *twice* the driver frequency. The reason is that the electromagnet of the driver exerts a force on the wire *twice* during each cycle. Also try a violin bow as the "driver" (this does not double the frequency).

An example of a frequency spectrum from the sonometer is shown in Figure 3.

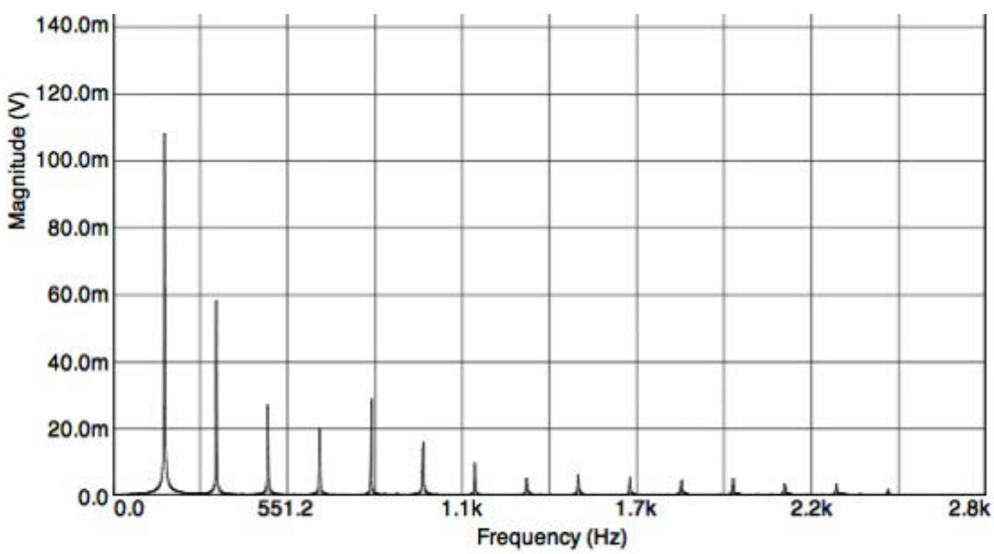


Figure 3. Sonometer string excited with a driver coil placed near one of the two bridges and harmonics recorded with an electromagnetic detector coil.

Some quantitative Experiments and Calculations

4. Determine the first 4 lowest harmonics on the string by varying the frequency of the signal generator. Observe in the "oscilloscope" mode when the vibrations are largest, or observe this with the spectrum analyzer. Read the individual frequencies directly from the function generator while you choose them, or get all at the same time from the spectrum analyzer. Note that the harmonic frequency chosen gives the strongest signal.

f_1 = ________Hz f_2 = ________Hz f_3 = ________Hz f_4 = ________Hz

Add these frequencies to Table 2.

5. Calculate the velocity of the waves on the string from equation (1) by using your values of the tension F and the linear mass density μ of the chosen string given in Table 1.

Answer : v = _____________m/s

6. Calculate the fundamental frequency f_1 from equation (2) by using the velocity and your value for L. Obtain the next three higher harmonics as integer multiples of f_1. Show all 4 calculated frequencies in Table 2. Also add to the table the location of the driver coil as measured from one of the two bridges. Add the number of nodes and antinodes for each of these harmonics.
7. For which harmonics would you have detected very little signal if you had placed the detector at the center of the string?

Table 2. Standing Waves on a String

	Calculated *f*	Observed *f*	Location of Detector Coil	Location of Driver Coil	Number of Nodes	Number of Antinodes
Fundamental						
1st Overtone						
2nd Overtone						
3rd Overtone						

Disconnect the detector coil from the Mac. Use a microphone connected to the Mac and record frequency spectra of a violin.

8. *Pluck* the string. Record the frequency spectrum. How many harmonics can you see? Note the relative amplitudes of the harmonics and the overall shape of the spectrum (Figure 4, bottom). Listen to the quality of the sound.

9. *Bow* the string. Record the frequency spectrum. Discuss the similarities and differences in the spectra of the plucked and bowed string. Listen to the quality of the sound (Figure 4, top).

10. Think of reasons why the spectra from the plucked and bowed string are different.

11. Compare the timbre or quality of sound from the bowed string with the plucked string.

12. List some other string instruments, including some "exotic" ones.

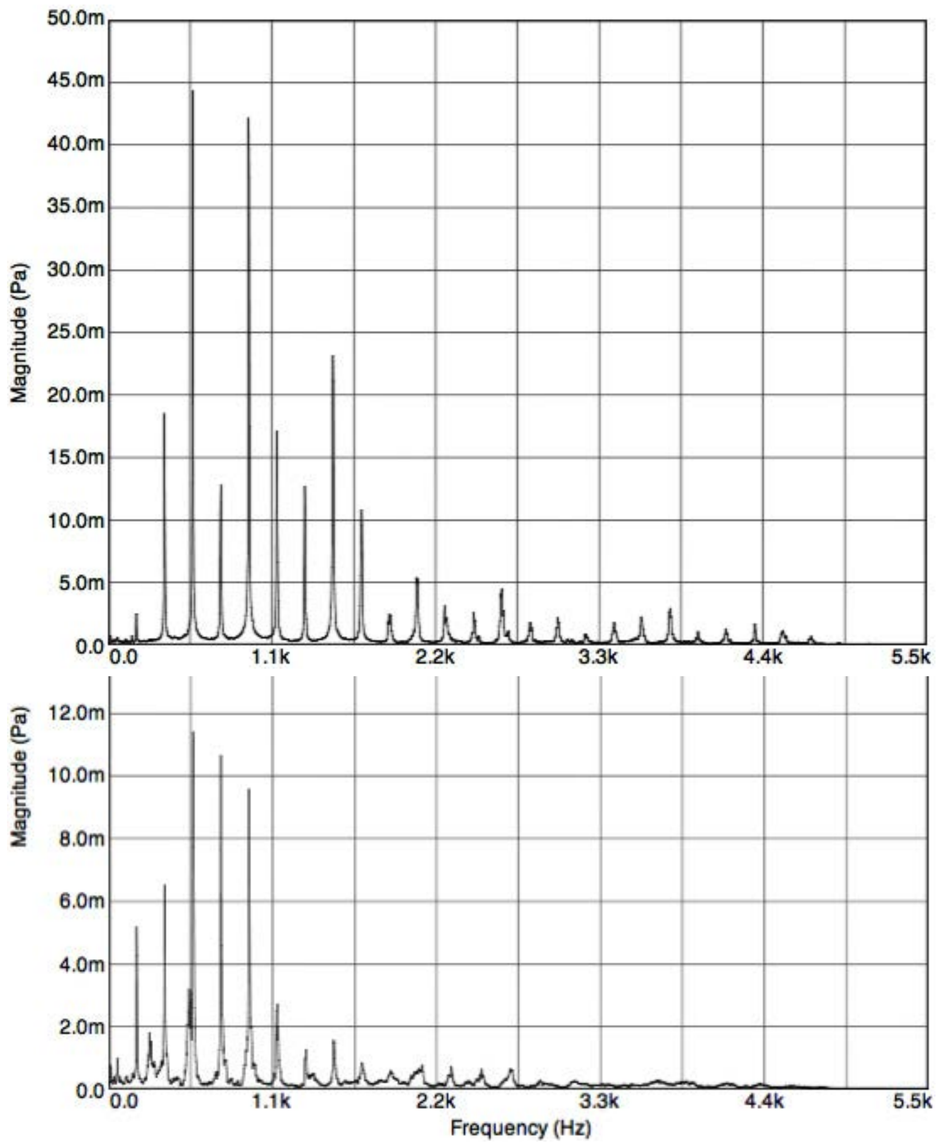


Figure 4. Sound spectrum form the G3 open string of a violin. Top figure: Spectrum from bowed string. Bottom figure: Spectrum from plucked string. Note that the bowed string has more pronounced higher harmonics resulting in a richer sound.

Illumination of a Vibrating String and Spring with a Stroboscope
Use a stroboscope and set it to the resonance frequency of a vibrating string. Observe the "frozen" standing wave modes. Do this for the first few resonance modes of the string. Similarly, use a vertical spring (not string!) fastened at the bottom end to a mechanical vibrator. Observe the nodes and anti-nodes of the spring, without and with the stroboscope.

4. Air Resonance

EQUIPMENT

PASCO Resonance Tube of variable length, large cardboard packing tube, speaker, large spherical Helmholtz resonator, didgeridoo, wine bottle, Faber Electroacoustics Toolbox (FEaT) software, Mac mini, microphone, organ pipes.

PURPOSE AND BACKGROUND

The concept of *resonance* in a pipe is similar to that of a string. The waves in pipes consist of compressions and rarefactions of the air, with back-and-forth motion of the air molecules in the direction of propagation or against it. The waves in air thus are *longitudinal waves*. In this laboratory we study standing waves in a pipe. They are the result of two waves traveling in opposite directions inside the pipe, with each wave being reflected at the ends of the pipe. In this way the superposition of two waves yields a standing wave, provided that in addition the *resonance conditions* are met.

For a pipe with both ends open, resonance at the *lowest* frequency (*fundamental* frequency or *first harmonic*) occurs when there are anti-nodes of the air motion at the ends – and only there, with a single velocity node at the center, see Figure 1. The motion of air molecules is highest at the anti-nodes and lowest at the nodes.

For a pipe with one end closed and one end open, resonance at the lowest frequency occurs when we have a velocity node at the closed end and an anti-node at the open end. Plotted in Figure 1 is the displacement or velocity of air molecules as a function of position along the pipe. The two curves for each pipe in Figure 1 are one-half period of oscillation apart.

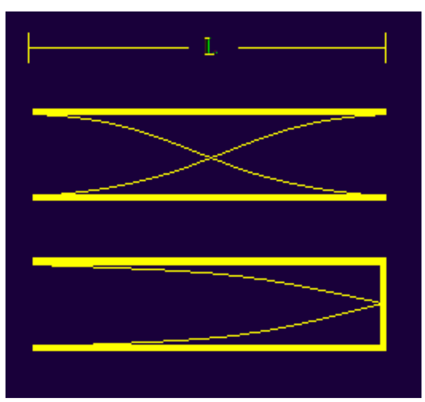


Figure 1. Open and closed pipe

For the pipe with both ends open, we have L = λ/2 according to Figure 1. For the closed pipe we have L = λ/4. The fundamental frequency is given by

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} \text{ (both ends open)} \qquad f_1 = \frac{v}{\lambda} = \frac{v}{4L} \text{ (one end closed)} \qquad (1)$$

where v is the velocity of sound.

1. For a pipe with both ends open, what are the formulas for the fundamental and the frequencies f_2, f_3, f_4 of the next three overtones or harmonics)? (Hint: Higher harmonics have frequencies that are integer multiples of the fundamental, and all integers are allowed for a pipe with both ends open.)

   f_1 = _________ f_2 = _________ f_3 = _________ f_4 = _________

2. For the pipe with one end closed and one end open, we have L = λ/4 according to Figure 3. Write down the *equations* for the fundamental frequency and for the first three existing overtones. (Hint: Only *odd integers are allowed* according to class, or as you can see by extending the drawings in Figure 1 to higher harmonics.)

   f_1 = _________ f_3 = _________ f_5 = _________ f_7 = _________

3. Choose a suitable value for the length L of the PASCO Resonance Tube. Take v = 346 m/s as the velocity of sound at a room temperature of 25 ^{0}C. Calculate the fundamental frequencies of the open pipe and the closed pipe. Record the values under *Calculated f* in Table 1 and Table 2. Add the *overtone frequencies* or *higher harmonics* as integer multiples of the fundamental frequency. (Caution with the closed pipe!)

Table 1. Open Ended Pipe

	Harmonic Number N	Calculated *f*	Observed *f*	Corrected *f*	Number of Nodes	Number of Antinodes
Fundamental	1					
2^{nd} Harmonic	2					
3^{rd} Harmonic	3					
4^{th} Harmonic	4					

Table 2. Closed End Pipe

	Harmonic Number N	Calculated *f*	Observed *f*	Corrected *f*	Number of Nodes	Number of Antinodes
Fundamental	1					
3^{rd} Harmonic	3					
5^{th} Harmonic	5					
7^{th} Harmonic	7					

Experimental Procedure

Prop the PASCO Resonator Tube in front of the loudspeaker, with the microphone at the other end of the tube - see Figure 2.

Method 1. Connect the speaker to the Mac mini. Select *white noise* from the frequency generator in the Faber Acoustic Toolbox. Take a frequency spectrum.

Method 2. Connect the speaker to the Mac mini. Select a *frequency sweep* from the frequency generator in the Faber Acoustic Toolbox. Take a frequency spectrum.

Note the large increase in sound intensity from the tube at the fundamental frequency. Figure 3 shows a frequency spectrum from Method 1 with the fundamental frequency and harmonics (tube open at both ends). Record the fundamental frequency and next three harmonics in Table 1 table under *Observed f*. Compare the calculated and observed frequencies.

Repeat this procedure for the closed pipe. In this case the closed pipe must have the microphone and speaker on the same side of the tube. Record the lowest four frequencies in Table 2 under *Observed f*. Compare the calculated and observed frequencies.

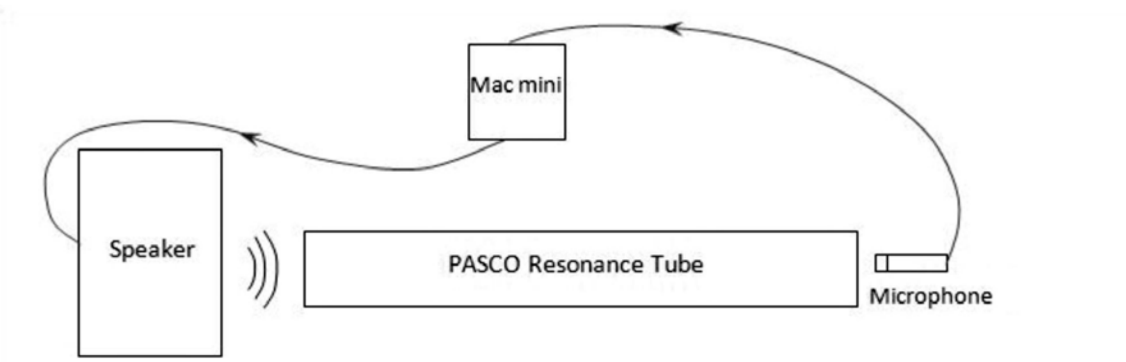


Figure 2. Set up of the resonance tube in the "open tube" configuration. White noise or a sine-sweep from the Mac mini is applied to the speaker. The sound enters the tube on the left and excites the resonances. The microphone on the right records them for display in the computer. In the "closed tube" configuration the speaker and microphone must be on the same (right) side.

Pipe Length Correction

Note that the calculated and observed fundamental frequencies may not agree well. This has to do with the fact that in pipes, waves reflect from the ends of the tube by sticking out a little bit. There is an end correction that increases the wavelength. This correction is proportional to the radius of the tube. Therefore, the larger the tube radius, the more the wave will "stick out" and cause an increase in wavelength. The correction results in an extra length ΔL, given from theory by ΔL = 0.61r for each open end, where *r* is the radius of the pipe. Thus for a closed pipe and open pipe of length L and radius R, the effective lengths are, respectively,

$$L_{effective} = L + 0.61r \text{ (closed)} \qquad L_{effective} = L + 1.22r \text{ (open)} \qquad (2)$$

4. Calculate the resonance frequencies for the *corrected* pipe lengths. Add your results in the column entitled *Corrected f* in Tables 1 and 2.

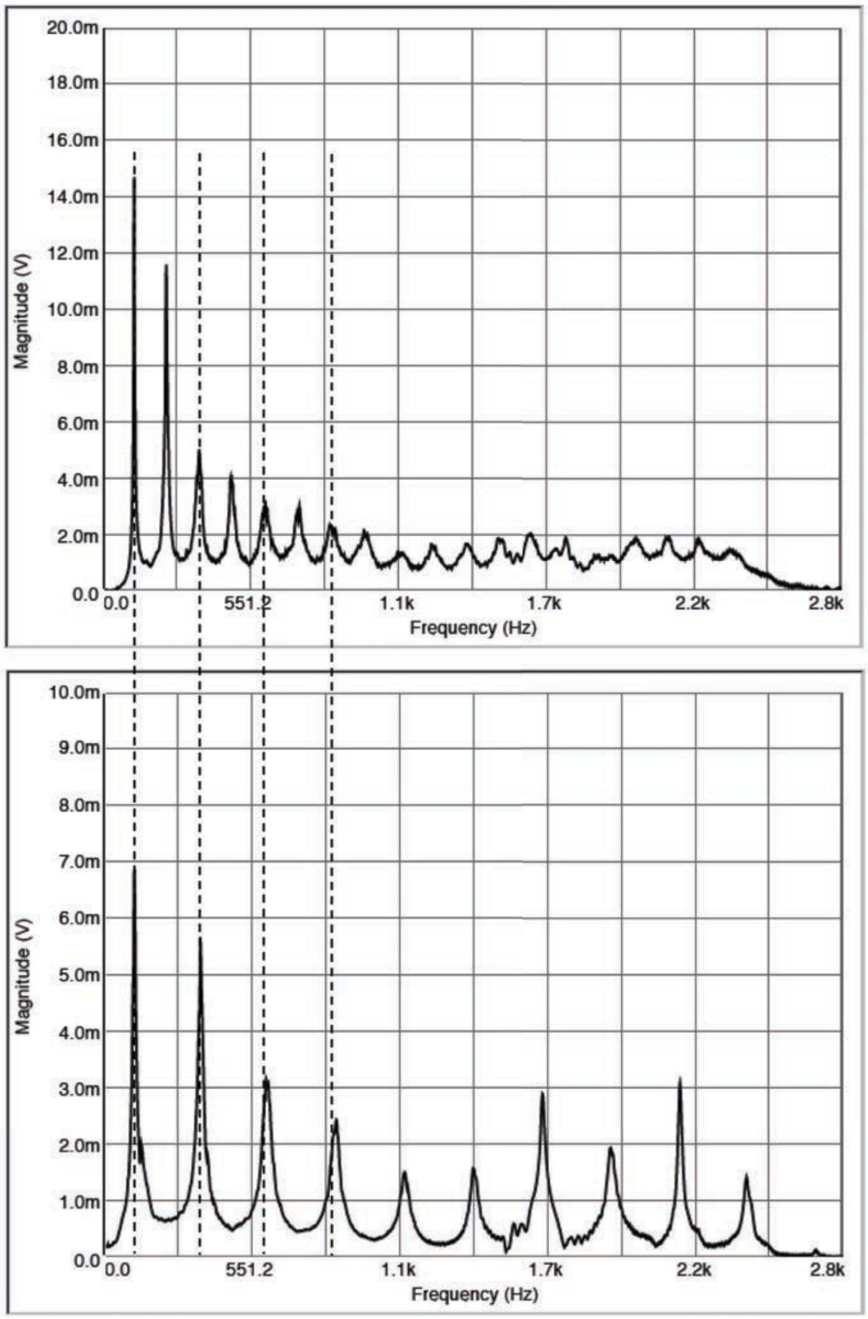


Figure 3. Resonances of a PASCO resonance tube excited with a frequency sweep. Upper figure: Tube open at both ends with an effective length $L_{eff,\ open}$ = 1.18 m. Lower figure: Tube closed at one end with an effective length $L_{eff,\ closed} = L_{eff,\ open}/2 \approx 0.59$ m (with a plug in the tube to shorten its length). The fundamental frequency for both tubes is f_1 = 146 Hz, but only the odd harmonics are observed in the closed tube.

Determination of the Sound Velocity

Determine experimentally the velocity of sound with the resonance tube. Use the *observed* value of the fundamental frequency f_1 together with the corrected pipe length L_{eff} in equation (1) for a pipe with two open ends or one end closed.

5. Answer: f_1 = _______Hz, L = ______m , L_{eff} = _______, v = ________m/s

How does your value compare with the value of 346 m/s assumed earlier? If there is a discrepancy, what might be the reasons?

Experiments with the Large Cardboard PASCO Packing Tube

Do some other experiments with a large brown cardboard packing tube from PASCO in order to study resonance and the decay of sound intensity. Close one end with a plug. Ask a partner to hold the microphone near the top of the tube. On the computer you should see a peak in the acquired FFT frequency spectrum. This peak corresponds to the resonating fundamental, which gets excited just from the broadband background noise in the room. The tube acts as a resonator that picks out its resonance frequency from the ambient noise and responds much less to the other frequencies in noise.

Open an Oscilloscope Tool in the Electroacoustics Toolbox. Tap the tube with its closed end on the floor to excite the tube resonances more strongly than just from the ambient noise. Listen to the resulting resonance and record the frequency spectrum with the microphone and the FFT mode in the Electroacoustics Toolbox as usual. An example of such a resonance curve is shown in Figure 4.

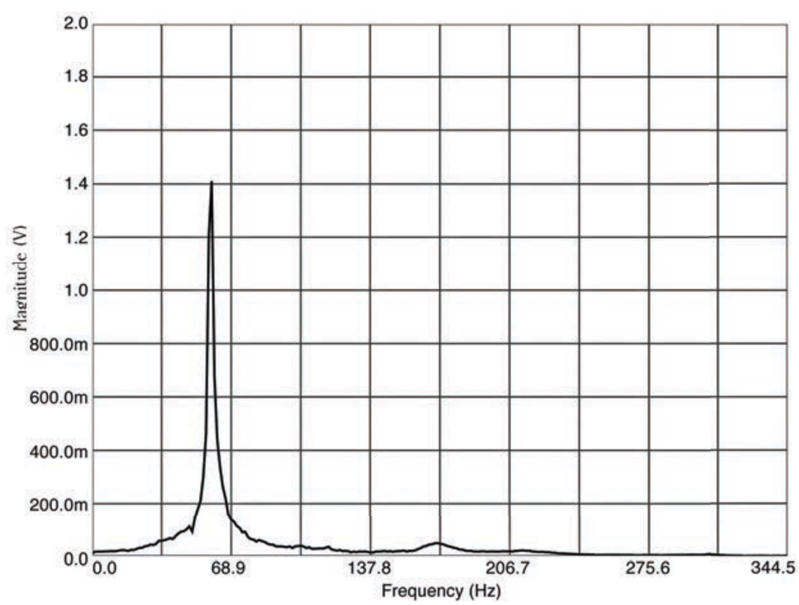


Figure 4. Frequency spectrum from a cylindrical PASCO packing tube, tapped on the floor with the closed end. The large peak is the N =1 harmonic, the small peak is the N = 3 harmonic. The N= 2 harmonic is missing, as is to be expected for a tube closed at one end.

Next, record the *waveform* of the damped oscillation. Use a measurement window of 200 ms per division. Figure 5 shows the decay of the signal as a function of time after tapping the tube on the floor at time t = 400 ms. The waveform closely resembles a *damped sine wave*.

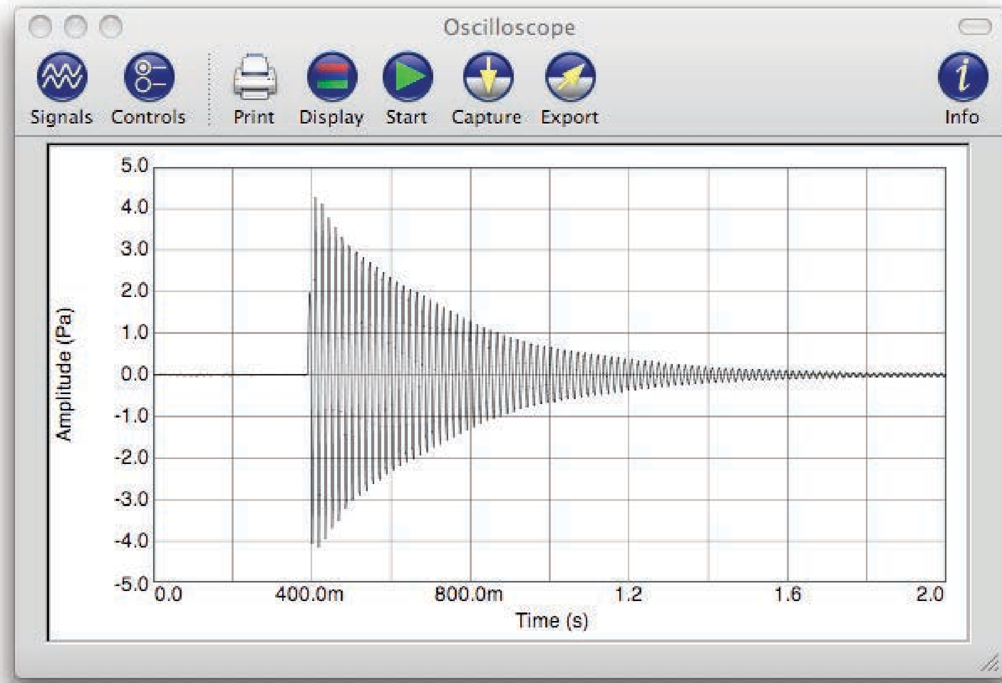


Figure 5. Decaying waveform of the packing tube.

6. Look at the observed waveform in Figure 5 and estimate the so-called *exponential decay time*. This is the time it takes for the signal to decrease to 37% of its original value.

7. Produce sound with some of the organ pipes in the laboratory. Describe the *quality* or *timbre* of the sound. Acquire a frequency spectrum and discuss its relationship to timbre.

8. Play a model aboriginal *didgeridoo***.** Measure the length, record the sound spectrum, and read the fundamental frequency. Calculate the frequency. Compare both frequencies.

$f_{measured}$ = __________Hz $f_{calculated}$ = ___________Hz

Helmholtz Resonator

Experiment with a simple spherical cavity called a *Helmholtz Resonator.* We have a large hollow metal sphere, with a tube protruding from one side for admitting *white noise* or a *frequency sweep* from a computer speaker. It has another smaller tube on the opposite side for listening to the resonance frequency or for recording the frequency spectrum with a shotgun microphone on a long shaft that can be inserted into this tubing.

First listen to the sound from the Helmholtz resonator when exposed to ambient room noise and note the deep rumbling tone, which is the resonance. The resonance is excited from the broad noise spectrum in the room. Hermann Helmholtz (1821-1894) used a series of such "Helmholtz Resonators" of different sizes to analyze the frequency spectrum of sounds and musical instruments, all before the advent of electronic tools!

Next use the setup for the Helmholtz resonator shown in Figure 6. Connect the speaker directly to the Mac mini (you can do away with the stereo receiver). Acquire a resonance spectrum from the Helmholtz resonator by either using white noise from the signal generator or a frequency sweep in the software. A resonance curve is shown in Figure 7. It has one prominent peak.

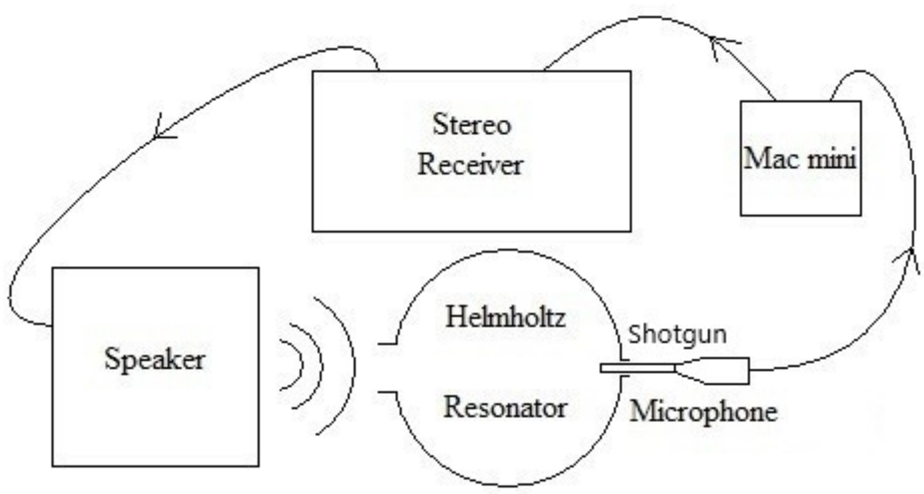


Figure 6. Experimental setup for spherical Helmholtz resonator.

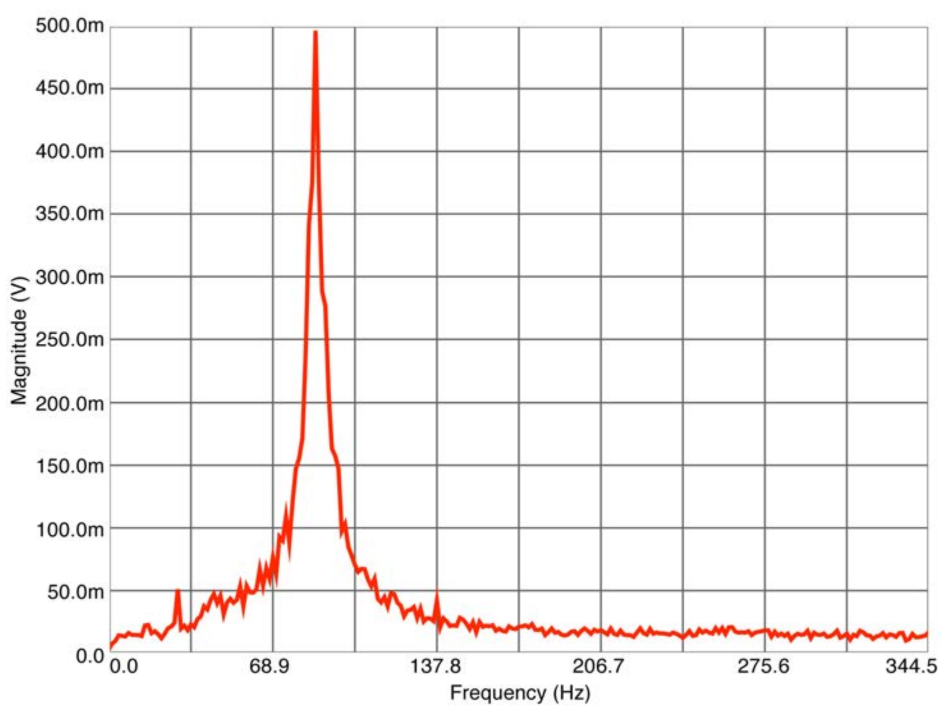


Figure 7. Helmholtz resonance curve from a large aluminum sphere. The measured and calculated values of the resonance frequency at the peak are 92 Hz and 93 Hz, respectively.

9. On the computer monitor, set the cursor to the peak of the resonance curve for the large metal sphere. Read off the value of the resonance frequency. Answer: f = ____________Hz

10. Use a ruler together with a calculator and determine the frequency at the peak of the resonance curve of the Helmholtz resonator in Figure 7 from a previous experiment.
Answer: f = ____________Hz.

Comment on the agreement/disagreement of the two values of the resonance frequency obtained here and in the preceding part.

Calculation of the Resonance Frequency of Helmholtz Resonator
The resonance frequency of a Helmholtz resonator is given by the formula

$$f = \frac{v}{2\pi}\sqrt{\frac{A}{L_{eff}V}} \qquad (3)$$

where v is the velocity of sound, A the area of the opening of the resonator, L_{eff} the effective length of the cylindrical neck, and V the volume. This formula is quite general and can by used for spheres, bottles, etc.

P.S.: If you have a Helmholtz resonator such as a box with *just a hole* in it, rather than a “bottle neck”, you can still use formula (3). For the actual length we have L = 0. But L_{eff} is not zero. The hole is open at both ends. So use L = 0 in equ.(2) and obtain $L_{effective}$ = 1.22r for the hole.

11. Back now to our large spherical metal Helmholtz resonator: Measure the radius R of the sphere, the length L of the “bottle neck”, and its inner radius r. Calculate the values for A, V, and $L_{effective}$ = L + 1.22r needed in equ.(3). Then calculate the resonance frequency.

R = 0.150 m L = 0.080 m r = 0.041 m

V = ________ m^3 L_{eff} = ______ m A = ________ m^2

Calculated resonance frequency: f = _________Hz

Compare your measured resonance frequency from Question 9 with your calculated frequency from Question 10.

12. **Helmholtz Resonance in Bottles**
Distinct Helmholtz resonances can be obtained by blowing gently across the opening of bottles. Practice this with various bottles.
Record the Fourier spectrum and resonance frequency for a wine bottle.
Measure the inside diameter of the bottle neck.
Calculate the area A of the opening needed in equ.(3).
Measure the length L of the bottle neck and calculate its effective length. (Use the formula for an open tube, i.e. $L_{effective}$ = L + 1.22r according to equ.(2)).
Find the volume V from the label on the bottle.
Calculate the resonance frequency from equ.(3) and compare with the measured frequency:
Answers: $f_{calculated}$ = ________Hz $f_{measured}$ = ________Hz

An example of an earlier measurement for a 0.75 liter wine bottle is shown in Figure 8.

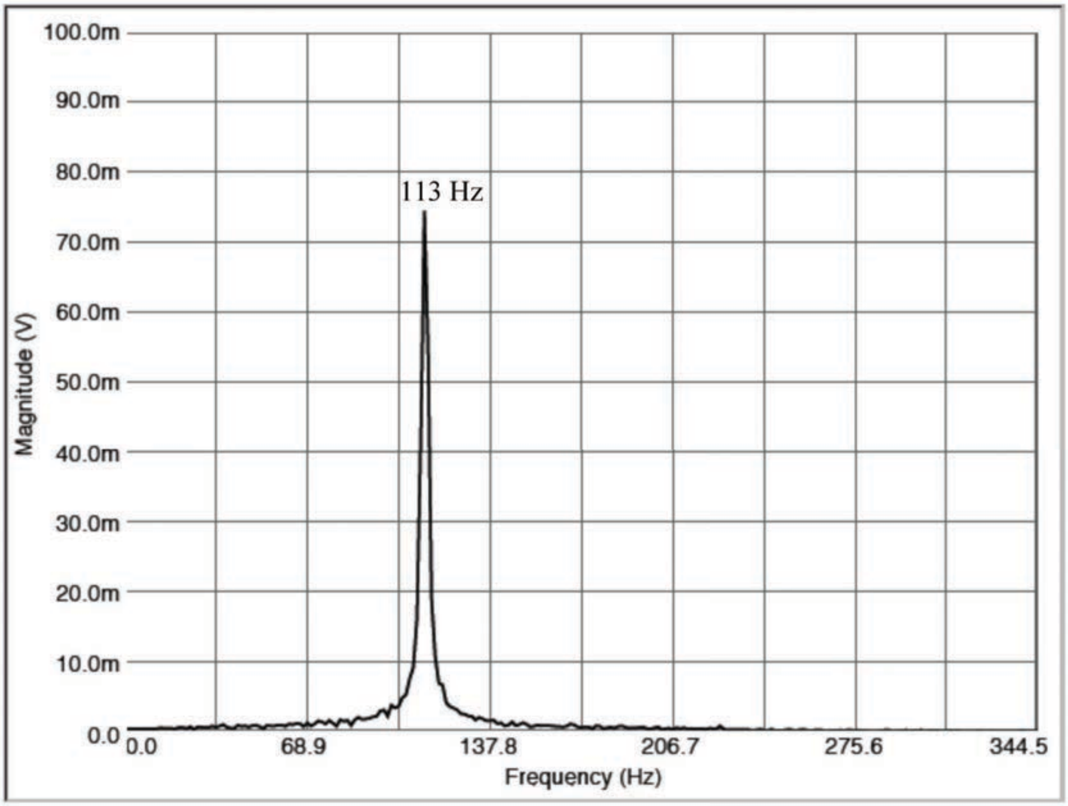


Figure 8. Helmholtz resonance from a 0.75 liter wine bottle.

Data for Figure 8:
Measured peak frequency: f = 113.0 Hz
Volume V = 0.75 Liter = $0.75 \cdot 10^{-3}$ m^3
Average radius at middle of bottleneck: r_{avg} = 10.53 mm = 0.01053 m
Actual bottle neck length: L = 8.5 cm = 0.085 m
For the length correction at the un-baffled outside opening at top of the bottleneck we take $0.61 \cdot r_{avg}$, and for the baffled opening inside the bottle we take $0.85 \cdot r_{avg}$ (from physics theory).
Thus the effective length of bottle neck is given by
$L_{eff} = L_0 + (0.61 + 0.85) \cdot r_{avg} = 0.085 + 1.46 \cdot 0.01053 = 0.1004$ m.
Then the calculated frequency from equ.(3) is f = 118.4 Hz ± 5%.
The uncertainty of 5% arises primarily from the dimensions of the bottleneck (try to verify this).

13. Use 1 liter and 2 liter plastic soda bottles having the same dimensions of the bottle necks. Measure their Helmholtz resonance frequencies. Derive the frequency ratio and compare with your measurement.
Answer: Calculated frequency ratio f_{1liter}/f_{2liter} = __________, measured f_{1liter}/f_{2liter} = ___________
P.S.: This frequency interval is the so-called “tritone”, also called the “devil’s tone”.

5. Fourier Analysis and Synthesis of Waveforms

PURPOSE AND BACKGROUND

The simplest sound is a pure sine wave with a single frequency and amplitude. Most sound sources and instruments do not produce such simple waves. Usually their sound contains many sine waves with higher frequencies, called *harmonics*. These act together according to the *superposition principle* to produce a complex tone. This addition of sine waves with suitable amplitudes and phases is called *Fourier synthesis of sound.* The opposite, the decomposition of sound into its sine-wave components, is called *Fourier analysis.* Periodic sound can be synthesized or analyzed with a sufficient number of sine waves. A *pure* tone is a sine wave with a single frequency. Many sine waves added together form a complex tone and waveform periodic in time. This laboratory is about the analysis and synthesis of sound and how electronic *synthesizers* can mimic real instruments.

EQUIPMENT

Mac mini, speakers, function generator.

THEORY AND EXPERIMENT

Fourier Synthesis of Waveforms

For our experiment, use the FEaT (Faber Electroacoustics Toolbox) software, open a Signal Generator tool. Listen to the four available wave forms (sine, triangle, square, and saw tooth) at a fundamental frequency of f_1 = 500 Hz (pitch).

1. Which tone sounds most like the pure sine wave? (Do not look at the computer screen.)
2. Which tone sounds the least like the pure sine wave?

Open an Oscilloscope tool and choose the Built-In-Output under the Device drop-down menu. The oscilloscope measures the amplitude of the sound wave, which the speaker produces as a function of time. Adjust the amplitude to 100mV full-scale (FS). Listen again to the four different tones while observing their waveforms. All four tones should have the same output of 100mV FS.

3. Draw sketches of the four waveforms.
4. Which waveform most resembles a pure sine wave?
5. Which waveform looks the least like the sine wave?
6. Do some of the waveforms sound louder than others, even though they all have the same amplitude of the fundamental? Why or why not?

Complex waveforms are produced by adding sine waves of different frequencies and amplitudes. The tone heard in all four cases has the same *pitch* or fundamental frequency f_1= 500 Hz. For a pure tone (sine wave), the fundamental is the only frequency present. For complex tones, sine waves with integer multiples of the fundamental frequency and suitable amplitudes are added together. For example, the next integer multiples of the fundamental f_1 = 500Hz are $f_2 = 2f_1$= 1000 Hz, $f_3 = 3f_1$ = 1500 Hz, and so on.

These higher frequencies are called *overtones or harmonics*. Just like the fundamental, each overtone has a single frequency. A complex waveform can be produced with the fundamental plus higher harmonics of suitable amplitudes. This process is called *superposition of waves* or, mathematically speaking, *Fourier synthesis* of waves. Conversely, you can take a complex waveform apart by decomposing it with our software into its individual harmonics. This is called *Fourier analysis* of waves.

In our experiment, and with the superposition principle in mind, several sine waves are added up to generate a complex waveform. Open five different Signal Generator tools in the software. Set Sig 1 (Signal Generator 1) to be the fundamental f = 500 Hz. For the four tones used, the fundamental frequency has the highest amplitude. (In musical instruments, some overtones actually may have higher amplitudes. Even then the pitch of the complex tone you hear comes from the lowest fundamental.)

Set the Master Volume of the fundamental to 100%. The other four Signal Generators, Sig2 to Sig5, are the first four overtones. For the different waveforms we use, i.e. the triangle, square wave, saw tooth, the relative amplitudes of the harmonics are different. Since our FEaT software is limited to a finite number of sine waves, we only use five of them to imitate a theoretically infinite sum of waves.

Sig 2 will be the *first overtone* or *second harmonic*. It is convenient to use "harmonic number" for the words "first overtone", "second overtone", etc. The fundamental is called the first harmonic (N=1), the first overtone is called the second harmonic (N=2), the second overtone is called the third harmonic (N=3), and so on.

Saw-Tooth Waveform

The harmonics of the saw tooth wave follow a simple pattern. All harmonics exist from N=1 to N=∞. Thus all integer multiples of the fundamental frequency contribute to the waveform. Since in practice we cannot add an infinite number of harmonics, we shall only use the first five and add them up.

For the first harmonic N=1, f_1=500Hz, set the amplitude to A_1=100% on the Master Volume slider of the Sig 1 tool. The second harmonic N=2, f_2=1000Hz, has an amplitude $A_2 = A_1/2$. Continuing this trend, the amplitudes of the harmonics of the saw tooth waveform decrease according to A_1/N. Set them in this way on the Master Volume slider of each Signal Generator tool.

7. Find the frequencies of the next three higher harmonics and their relative amplitudes in %. Complete the entries in Table 1

Table 1. Saw tooth: Harmonic numbers, frequencies, and relative amplitudes.

N	f_N	A_N
1	500Hz	100%
2	1000Hz	50%

Synthesize a sawtooth by adding the first two harmonics according to the information in Table 1. Take a look at and listen to the waveform generated. Continue adding successive harmonics N = 2, 3, 4, 5, 6 and note the changes in tone and waveform. With each addition of a harmonic, the wave should look more and more like a saw tooth. If this is not the case because of an electronic artifact, turn off all harmonics, then turn them on again all at once "Universal ON" in the FEaT software. The waveform may now look more like a saw tooth. Note it sounds practically the same as before, no matter how it looks. The reason for this is that the human ear is not sensitive to the phase differences between individual harmonics, but only to the amplitudes. (This is called "Ohm's Law of Hearing").

In order for the summed harmonics to *look* like a saw tooth on the screen, they must all begin at the same time. But that does not matter for the ear to *hear* a saw tooth. The ear primarily senses the frequencies and amplitudes of the harmonics, not the relative phase differences between them, and thus you keep hearing a "saw tooth".

8. What would be the frequency and amplitude of the N = 10 harmonic for a saw tooth waveform of fundamental frequency f_1 = 500 Hz?

   F_{10} = ______________Hz A_{10} = ______________%

Rectangular Waveform

A square or rectangular waveform is similar to the saw tooth in that the amplitudes of the harmonics follow the $\frac{A_1}{N}$ dependence. However, the major difference is that only the *odd* harmonics N=1, N=3, N=5 etc. contribute.

9. Use this information and complete the entries in Table for the square wave.

Table 2. Square wave: Harmonic numbers, frequencies, and relative amplitudes.

N	f_N	A_N
1	500 Hz	100%
3	1500 Hz	33.33%

In our experiment, reset all five Signal Generators to the configuration for the square wave. Listen to the combination of the first two harmonics, then add the higher harmonics successively. Note the changes in tone and waveform. Again, starting all five frequencies at once may give a better looking square wave on the screen, but what you hear is unaffected by how it looks.

Triangular Waveform

The triangular wave is similar to the square wave in that it too consists of odd harmonics only. However, the amplitudes no longer follow the $\frac{A_1}{N}$ dependence, but rather a $\frac{A_1}{N^2}$ dependence. For instance, given an amplitude of the first harmonic of 100%, the amplitude of the third harmonic now is $\frac{A_1}{3^2} = \frac{100\%}{9} = 11.11\%$.

10. Complete the entries in Table 3 for the triangular waveform.

Table 3. Triangular waveform: Harmonic numbers, frequencies, and relative amplitudes.

N	f_N	A_N
1	500 Hz	100%
3	1500 Hz	11.11%

Use the completed Table 3 to reset the five Signal Generators for a triangle waveform and listen to the result.

11. Of the three waveforms, which had the least noticeable contributions from its overtones to the overall form and tone?

12. Which of the three waveforms had the most noticeable contributions from its overtones?

13. How could you get sharper "edges" on the square and saw tooth waves than those seen on the screen?

14. Question: Why can you hear a 1 Hz square wave?

Fourier Analysis of Waveforms

Select the signal generator tool in the FEaT software. Open a FFT tool and select the Built-In Input from the drop-down menu. FFT is the abbreviation for *Fast Fourier Transform*. It is a tool that analyzes an incoming signal with a mathematical operation to identify the different frequencies in the signal. The display is a *frequency spectrum.* Select a sine wave on the function generator. The FFT tool will show a frequency spectrum with one peak for the only frequency present, with the amplitude being the height of the peak. For non-sinusoidal periodic waveforms you will see many peaks.

Change the waveform from *sine wave* to *square wave* on the function generator. The FFT signal will change to show only odd harmonics. Adjust the amplitude of the fundamental frequency to a simple value such as 100 mV. Then verify that the amplitudes of the first few harmonics follow the theoretical values discussed above. Finally, select a triangular waveform and inspect its harmonics. Compare the frequency spectra of the three selected waveforms.

Musical Synthesizers

Modern keyboards are capable of simulating sounds from real instruments quite well. They work on the basis of *Fourier analysis and synthesis*. Every tone from a given instrument has its own timbre and Fourier spectrum. The fundamental frequency determines the pitch of the tone. Often the fundamental does not have the highest amplitude. Some higher harmonics may be stronger. Nonetheless, the ear discerns the frequency of the fundamental as the *pitch* of the tone. Musical instruments produce sound with complex Fourier spectra. These change with every note. For example, the Fourier spectra of "middle C" (f = 261.63 Hz) from a violin and a viola or bassoon look quite different.

Play "middle C" of some synthesized tones on the keyboard, such as violin, trombone, saxophone, guitar etc. Also play non-sustained tones from percussion instruments such as drums, cymbals etc. Listen to the attack and decay transients. Observe some corresponding spectra.

Play some real instruments such violin, trumpet, saxophone, etc. and compare with the synthesized sound from the keyboard.

15. Overlay the spectra on the same display and comment on the similarities and differences.

16. What do *Fourier analysis* and *Fourier synthesis* of sound have in common?

Real and Synthesized Sound of a Didgeridoo

17. Play a didgeridoo (pitch D2) and record the sound spectrum. Use the signal generator in the Electroacoustic Toolbox and synthesize the sound with the 4 lowest odd harmonics. Compare the synthesized sound with the real sound.

An example of the spectrum from a didgeridoo and the corresponding synthesized tone is shown in Figure 1 below. The synthesized tone sounds similar to the actual one, but not quite the same.

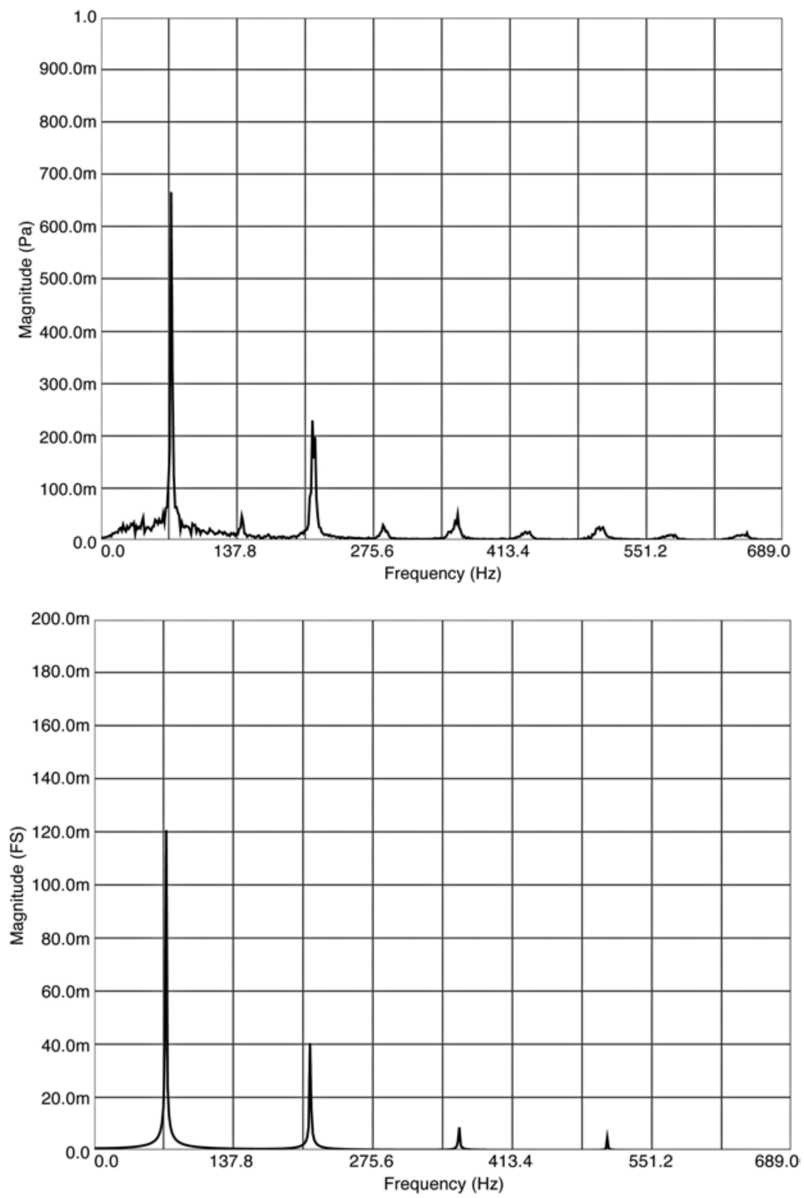


Figure 1. Top: Actual sound spectrum of the note D2 from a didgeridoo. The odd harmonics dominate, as expected for a “closed tube”. Bottom: Synthesized sound spectrum, using only the first four odd harmonics N = 1, 3, 5, 7.

18. Why does the synthesized tone not sound exactly like the real tone from the didgeridoo?

6. Spectrum Analysis of Instruments and Voice

PURPOSE AND BACKGROUND

The frequency spectrum of sound is analyzed with the Faber Electroacoustics Toolbox (FEaT) software. We continue with our discussion of harmonics. All musical instruments produce tones that are unique and have a characteristic *timbre* or quality of sound. The frequency spectra tell the harmonic content of a tone and how it can be synthesized. Electronic keyboards make use of this to reproduce sounds. We analyze the sound from a variety of instruments, from human voice, noise, and pulse trains.

EQUIPMENT

Musical instruments from the lab or from students, for instance violin, guitar, bassoon, organ pipes, recorder, saxophone, trumpet, didgeridoo, corrugated plastic tubes. Yamaha Keyboard, microphone, Mac mini with ElectroacousticsToolbox software.

THEORY AND EXPERIMENT

String Instruments

All musical instruments use a driving force to set an oscillator into motion. Stringed instruments use a bow or plucking for exciting the vibrations. Figure 1 shows an example of a frequency spectrum from the open G3 of a violin. The first 6 harmonics are seen. Placing a finger down on the fingerboard reduces the effective length of the string and increases the pitch.

1. The strings of a violin are tuned to the notes G3, D4, A4, and E5, for the guitar they are E2, A2, D3, G3, B3 and E4. You see that the note G3 is common to both instruments. Play G3, the lowest note on the violin, by bowing and by plucking. Observe the different frequency spectra in the FFT (Fast Fourier Transform) mode of the Electroacoustics Toolbox. How do the spectra from the bowed and plucked string differ? Then play G3, the fourth lowest note on the guitar, and compare the sound spectrum and timbre with the violin.

Figure 1 shows an example of a frequency spectrum from the G3 strings of a violin and guitar.

Placing a finger down on the fingerboard reduces the effective length of the string and increases the pitch.

2. Repeat by bowing and plucking the violin string at a higher pitch. How do the bowed and plucked tones differ? What is still similar?

3. What differences do you see in the frequency spectra of the two bowed tones?

4. Obtain the frequency spectra of all four bowed empty strings of the violin with the FFT tool.

5. Have the instructor or a violinist play one of the highest notes on the violin and estimate the frequency range of a violin with the FFT.

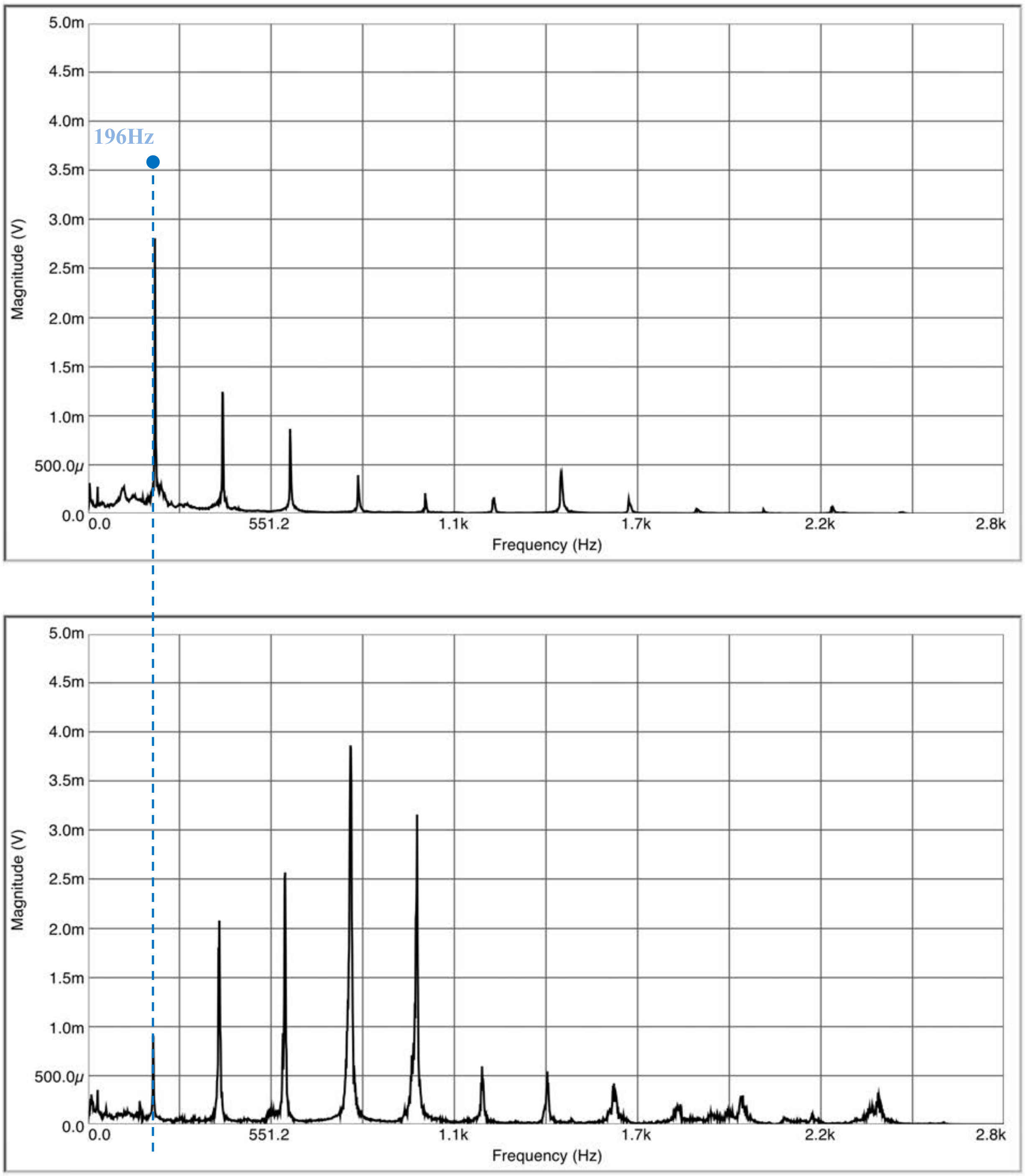


Figure 1. Upper figure: Frequency spectra of the plucked open G3 strings of a guitar (upper figure) and violin (lower figure). The fundamental frequency (pitch) and frequencies of the harmonics are the same. But the timbre (quality of sound) is very different because of the different relative amplitudes of the harmonics.

Wind Instruments

Wind instruments use air as the vibrating medium. Brass instruments have closed pipes, with the closed end near the mouth. Woodwinds such as the clarinet, oboe and bassoon also are closed pipes, with a reed at the closed end. The lowest harmonics of these instruments are primarily the odd harmonics, as is expected for closed pipes. This applies especially to the clarinet due to its straight cylindrical bore. For other wind instruments, all harmonics are present without a special dominance of odd or even harmonics. Flutes, piccolos, recorders and, more exotically, an ocarina, are open pipes with even and odd harmonics.

Use a slide whistle and obtain the frequency spectrum of its lowest note, or use another available wind instrument such as a recorder. The simple, almost purely sinusoidal frequency spectrum from a slide whistle is shown in Figure 2. Try to over-blow the lowest note and note the next harmonic f_3 of the closed pipe. Blowing harder may produce f_5 and even f_7.

6. Determine the frequency range of the slide whistle by moving its piston.

7. How does the frequency spectrum of the slide whistle compare to that of the violin?

Play and record the sound spectra from a flexible corrugated plastic tube (“whirly”) by swirling it around in a circle. Record spectra from a trumpet and trombone if possible.

8. How do these spectra compare with that from the slide whistle?

9. Determine the harmonic numbers N that are active in the spectrum of the corrugated plastic tube. Note that the fundamental most likely does not show. What are the musical intervals between the harmonics that are present (e.g. octave, fifth, fourth, third)?

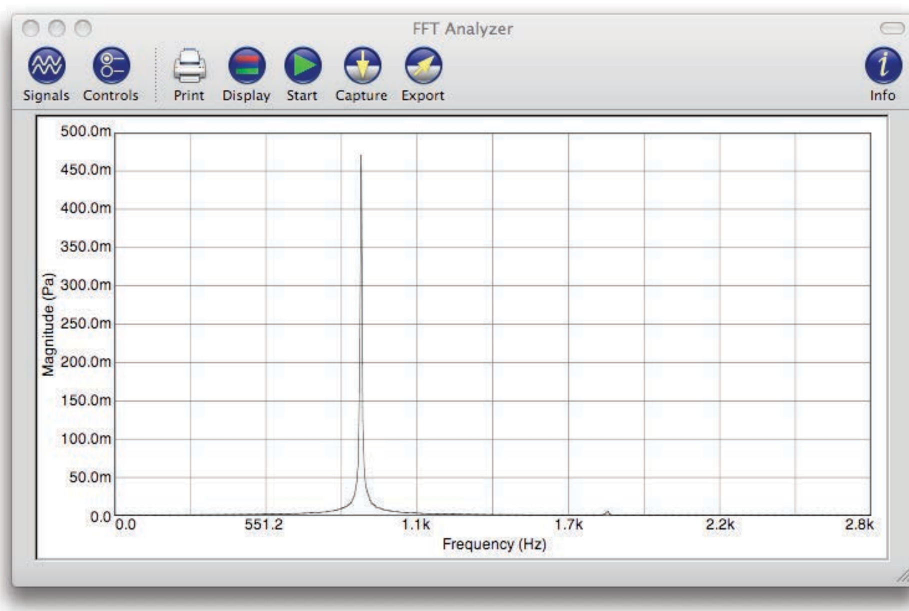


Figure 2. Frequency spectrum of a slide whistle with f = 880Hz.

Voice

The human vocal tract is an intricate system for producing sound. The voice of each person is unique. The sound is produced, and its quality determined, by the vocal tract consisting of throat, nasal cavity, and mouth. Each of these components acts as a resonator with characteristic resonant frequencies. The different vowel sounds come from different regions of the vocal tract. This allows for a large variety of sounds, but some general characteristics exist.

10. The human throat has a typical overall length of 17 cm. Consider it as a simple pipe, with one end closed at the vocal folds and the other open at the mouth. What is the fundamental resonance frequency?

Have a male and female student sing the vowels "oo" or "ah" into the microphone. Observe the resulting frequency spectrum with the FFT tool. Figure 3 shows such a spectrum for a male voice singing "ah" with a pitch of 220 Hz.

The frequency regions where several neighboring harmonics have high amplitudes are called v*ocal formants*. Some persons may have similar formants because of similar size and shape of their vocal tract. The individual resonators of the tract produce the different formants. They can be adjusted by a change in size and shape of the throat, nasal cavity and mouth. How this is done distinguishes a great singer from a bad one. Vocal formants are what we listen to in order to recognize persons. Adjusting the cavities of the vocal tract changes the formant regions. Adjusting the tension in the vocal cords changes the pitch and associated harmonics.

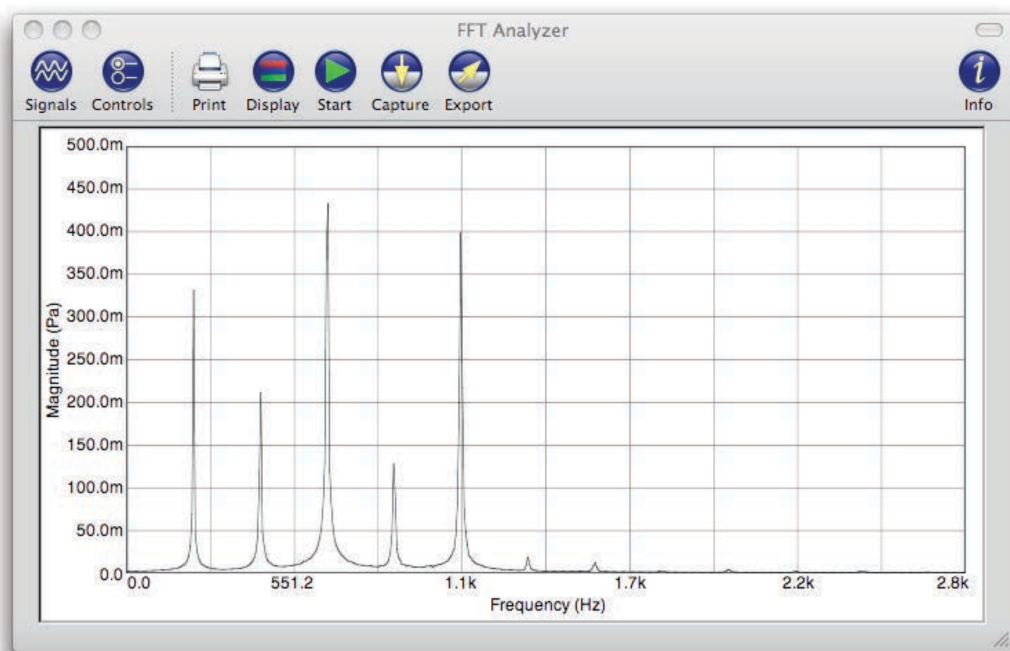


Figure 3. Male voice singing an "ah" sound at $f_1 = 220$ Hz

11. Have a male and female student sing a vowel sound “oo”. What is the frequency range of the first formant region for these voices? Why can the formants be different?

12. Have two or more students with noticeably different voices sing the vowel sounds “oo”, “ah”, and “ee” into the microphone. Acquire the frequency spectra. Compare the formant regions of the students. In Table 1, record the first and second formant region for one of the students. An example for the vowel sound “ee” from a male and female student is shown in Figure 4 and from the sound “eh” in Figure 5.

Table 1: Vowel sounds and corresponding vocal formant regions.

Sound	1^{st} Formant region (Hz)	2^{nd} Formant region (Hz)
oo		
ah		
ee		

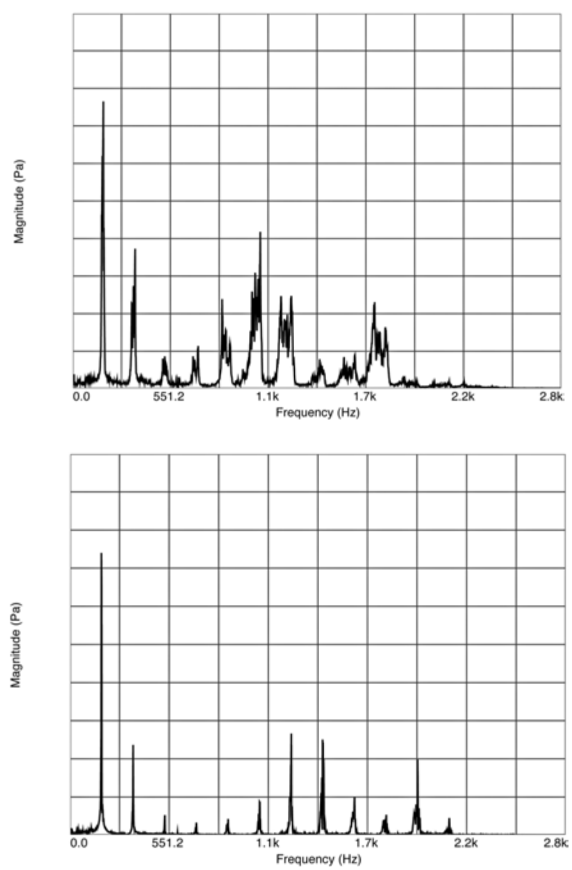


Figure 4. Top: Vowel sound “ee” from male voice. Bottom: Vowel “ee” from female voice. The female voice has purer harmonics. Note the formant regions.

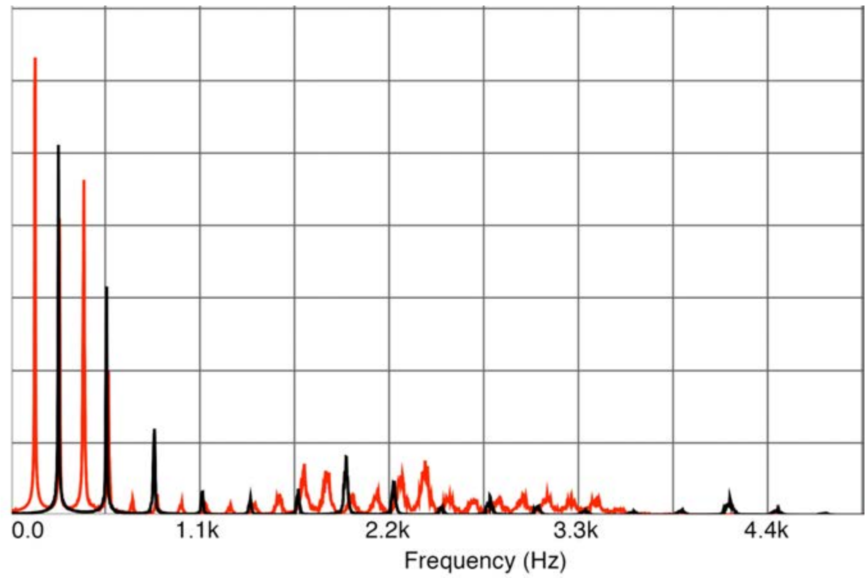


Figure 5. Vocal "eh" sound from a male (red) and female voice (black). The pitch from the female voice is an octave higher. The formant regions cover similar frequency ranges, as is to be expected because of the similar sizes of the vocal tracts.

13. Identify any formant regions in the male and female in Figure 4 where the amplitudes are pronounced.

14. How can you effectively change the resonant frequencies of the vocal tract (formant regions) and of the vocal folds?

15. Telephones transmit frequencies only in the approximate range 300-3000 Hz. Why is this frequency range sufficient for most purposes?

The human vocal tract produces various types of sound. *Continuant sounds* are consonant sounds such as "m" and "n" that have a soft continuous tone. *Sibilant sounds* are consonants such as "s" and "z" that can be continuous and sound rather harsh. *Plosive sounds* are short and explosive like "p" and "t". Observe some of these sounds and their frequency spectra.

16. Which vocal sounds sound more "musical"? Hint: Which sounds have a *discrete* frequency spectrum as compared to a more *continuous* spectrum with many closely spaced frequencies characteristic of noise?

17. Which sounds are not "musical" and rather "noisy" and have a rather *continuous* frequency spectrum?

Voice and Audio Spectrograms

Select the spectrogram mode in the Electracoustics Tool box software. You will see frequency on the y-axis versus progressing time on the x-axis, with the intensity of the sound in color. This 3-parameter display of sound may be the most informative one in certain situations, for example voice analysis.

18. Observe a *voice sonogram* (*not* a frequency spectrum) from a student singing a vowel. Do you see any discrete bands in the *spectrogram* indicating vocal formants?

19. Have a group of female and male students separately sing the same vowel sound. Record and compare both the sound spectra and voice spectrograms with respect to any formants.

20. Play instruments producing simple sounds, e.g. an organ pipe or didgeridoo. Interpret their *sound spectrograms* in terms of the small number of harmonics present.

21. Play an instrument that produces a complex sound, e.g. a saxophone, krummhorn, or harmonica. Interpret their sound spectrograms and compare with the pipe or didgeridoo.

Keyboard

Connect the keyboard to the Mac mini and simulate some of the real instruments played in this laboratory. Select different "voices" on the keyboard. For example, use Voice #056 for "Violin". Play the same note on the keyboard "violin" that you played on the real violin. Take a frequency spectrum of the synthesized tone from the keyboard and compare with the spectrum of real violin. Play the synthesized sound of some other instruments such as "guitar" on the keyboard. Listen and compare with the real instrument.

22. Which synthesized keyboard sound most closely resembles the actual instrument? (Try the trumpet!)

23. What features does synthesized sound generally lack compared to the real sound? How does this affect the tonal quality or *timbre* of the sound?

24. Which instrument would be the hardest to synthesize, and why?

25. Which instrument would be the easiest to synthesize, and why?

Percussion Instruments, Noise, and Pulse Trains

Play a real snare drum and a "snare drum" synthesized on the keyboard. Analyze their frequency spectra. Note that the spectra are largely continuous. This is a general feature of non-periodic, non-sustained sound.

26. Describe the similarities and differences in the spectra of the real and synthesized snare drum.

27. How could you change the frequency spectrum of the synthesized snare drum to make it sound more like the real one?

28. Undo the latch on the snare drum so that it becomes a tom drum. Analyze the frequency spectrum of the tom. Does it sound more musical? Describe the difference between the frequency spectra of the snare drum and the tom drum.

29. Emit a pulse train from your lips by producing a buzzing sound. Record a frequency spectrum. Is the spectrum *continuous* or *discrete*? Is it noise? Is the sound periodic? Why does it not sound musical? (Hint: Discuss the structure and width of the frequency spikes.)

7. Sound Intensity, Hearing, Just Noticeable Difference (JND)

PURPOSE AND BACKGROUND

We can hear a wide range of sound intensities and frequencies. The *intensity* between the thresholds of hearing and the threshold of pain varies by a factor of 10^{12}, i.e. by 12 orders of magnitude or 120 decibel. The corresponding range *in air pressure amplitudes* is a factor of 10^6. In view of this extreme range in sound intensity level (*SIL*), numbers are most conveniently expressed in power-of-ten notation and with a decibel or dB-scale.

We study in the present laboratory sound intensity levels (SIL) and the frequency response of the human ear. We also discuss “just noticeable differences” (JND) in intensity and frequency that the ear can discern.

The ear is sensitive to a range in frequencies from about 20 Hz to near 20 kHz. This *audible range* thus covers a factor of 10^3 in frequency, which is not nearly as large as the *intensity range* of 10^{12}. In order to cover these large ranges, the ear response is compressed or *logarithmic* with respect to both frequency and sound intensity.

EQUIPMENT

Microphone, calibrated sound level meter, speakers, Mac mini, 2 stand-alone signal generators.

THEORY AND EXPERIMENT

The *amplitude* of a sound wave corresponds to air pressure fluctuations or *compressions* and *rarefactions* of the air in a longitudinal wave.

The *threshold of hearing* is a sound intensity at the ear of $I_0 = 1 \times 10^{-12} \frac{W}{m^2}$ at f = 1000 Hz.

This is the reference intensity for measurements. The *sound intensity level (SIL)* is defined by comparing any intensity *I* to the threshold of hearing I_0 according to

$$\mathrm{SIL} = 10\mathrm{dB} \cdot \log_{10} \frac{\mathrm{I}}{\mathrm{I}_0} \quad (1) \qquad \text{Inverse equation: } \mathrm{I} = \mathrm{I}_0 \cdot 10^{\frac{\mathrm{SIL}}{10\mathrm{dB}}} \quad (2),$$

where the logarithm taken to the base 10.

The SIL is measured in *decibel* or dB.

For example, let the sound intensity in a room be $\mathrm{I} = 1 \times 10^{-6} \frac{\mathrm{W}}{\mathrm{m}^2}$. The SIL then is

$$\text{SIL} = 10\text{dB} \cdot \log_{10}\left(\frac{1\times10^{-6}}{1\times10^{-12}}\right) = 10\text{dB} \cdot \log_{10}10^{6} = 10 \text{ dB} \cdot 6 = 60 \text{ dB}$$

The SIL also can be used to express a *change in intensity* from one value to another, without referring to the threshold of hearing I_0. We are then dealing with a *change in SIL* and not the SIL itself. For instance if the intensity *I* doubles to *2I*, we have a

$$\text{Change in SIL} = 10\text{dB} \cdot \log_{10}\frac{2\text{I}}{\text{I}} = 10\text{dB} \cdot \log_{10}2 = 10\text{dB} \cdot 0.3 = 3 \text{ dB} \qquad (3)$$

Therefore, a doubling in intensity corresponds to an increase of 3 dB in the SIL.

1. Use a sound level meter and find the SIL of the background noise in the room. There always is ambient noise from air conditioners, computer fans etc. The sound intensity level in a typical environment generally is much higher than the threshold of hearing. What is the measured SIL of the background noise in our laboratory?

SIL =__________dB

2. What is the sound intensity *I* of this background noise, expressed in units of W/m^2? Hint: Use equation (1) and solve for *I*. Ask your instructor for help if needed.

1. I =__________ $\frac{W}{m^2}$

3. Use the FEaT Sound Level Meter software and record the sound intensity level of one student clapping.

SIL_1 =__________dB

4. Calculate the theoretical increase in sound intensity level, if the intensity I_{10} for ten students clapping is ten times the intensity I_1 for one:

SIL_{10} - SIL_1=__________dB

5. Make an educated guess of the sound intensity level of ten students clapping together, each equally loud as the student before. Measure the actual value and record it here:

SIL_{10} =__________dB

A sound of frequency f =1000 Hz and an intensity of $I = 1\frac{W}{m^2}$ becomes quite painful to the ear.

6. What is the sound intensity level SIL in dB of a 1000Hz sinusoidal tone at the threshold of pain?

SIL =__________dB

Frequency Response of the Ear

The ear can hear sound over a wide frequency range from about 20 Hz to 20 kHz. However, the *perceived* intensity varies quite dramatically with frequency. The so-called *Fletcher-Munson curves* in Figure 1 show lines of *equal perceived loudness*. The curve at the bottom marked “0 phons” represents the threshold of hearing, and the line marked “120 phons” the threshold of pain. Each curve has a “phon” designation and indicates equal perceived loudness as a function of frequency. The “decibel” and “phon” scales agree by convention at a frequency of 1000 Hz (see Figure 1). For example, if a loudspeaker produces a 1000 Hz tone with SIL = 60 dB at your location, you perceive this as SIL = 60 dB and loudness of 60 phon. If on the other hand the speaker produces a tone at 100 Hz with the same SIL = 60 dB, you hear this less loud than the 1000 Hz tone. In order for the two frequencies to sound equally loud, the speaker must produce the 100 Hz tone at about SIL = 70 dB instead. Verify this on the curve labeled “60 phons”!

You can see from Figure 1 that the human ear is most sensitive to sound around 4000 Hz, where the Fletcher-Munson curves dip lowest. Therefore, if you follow a Fletcher-Munson curve from 4000 Hz to lower frequencies, the sound intensity must be raised to be perceived as equally loud. The same applies to higher frequencies above 4000 Hz.

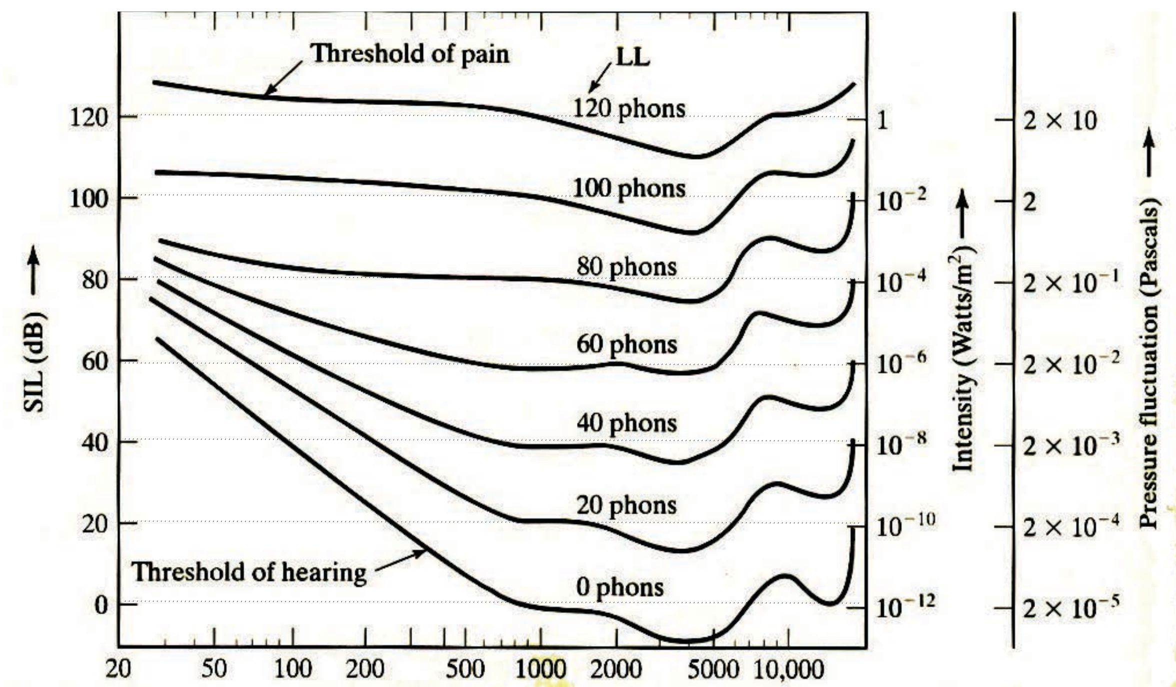


Figure 1. Fletcher-Munson curves of equal loudness. (“Physics of Sound” by R. A. Berg and D. G. Stork.)

Open three Signal Generator tools in the FEaT software. Set them to frequencies of 100, 1000, 4000 Hz. Set the Master Volume of all three tools to 20% maximum. Use the volume knob on the stereo receiver to adjust the f =1000 Hz tone to 60 dB on a *calibrated Sound Level Meter*.

Adjust the Master Volume of the other two tools for a perceived loudness equal to that of the 1000 Hz tone.

7. What is the measured SIL of the 100Hz tone?

8. What is the measured SIL of the 4000 Hz tone?

9. From the Fletcher-Munson curve labeled "60 phons" in Figure 1, read the SIL at 100 Hz and 4000 Hz. How close are your measurements to the dB-values on the 60-phon curve?

Just Noticeable Difference (JND) in Intensity

The just noticeable difference in intensity (JND) is the smallest change in SIL the ear can discern. Usually a 25% or 1 dB change in intensity is detected. This depends somewhat on sound intensity and frequency as can be seen in Figure 2. As the intensity or frequency decreases, the ear becomes less sensitive to changes.

10. Express a 25% change in intensity I as a change in dB. Answer: Change in SIL = ______dB

(Hint: Use Change in SIL = $10\text{dB} \cdot \log_{10} \frac{I_2}{I_1}$, where $I_2 = 1.25 I_1$.)

Use an external function generator (without the computer) that produces sine waves and square waves. Play the sound through a loudspeaker. Use a portable sound level meter to read the sound intensity level in the room. Play a *sine wave*. Adjust the SIL on the signal generator so that it reads 80 dB. Increase the intensity *slowly* until you hear a change in intensity.

11. What is your measured JND from the sound level meter readings for a *sine wave?*
JND (80 dB, 1000 Hz) = ________________dB
What is the value for the JND in Figure 2 on the 1000 Hz curve at 80 dB?
JND (80 dB, 1000 Hz) = ________________dB

Use two signal generators at 1000 Hz and switch *quickly* between them. Keep switching between the generators while you change the SIL on one of them.

12. What is your JND when changing the intensity *quickly?* Compare with a slow change.
JND (80 dB, 1000 Hz) = __________dB

13. Obtain the JND at 1000 Hz from Figure 2. Compare your values for this from questions 11 and 12 with the value from Figure 2. Your answers: ___________________
From Figure 2:___________________

14. Compare the JND of a *square wave at* f = 1000 Hz with that of a sine wave. Alternate quickly between the two types of waves. For which do you get a smaller JND, i.e. for which can you hear smaller differences in SIL? Can you give a reason for this? Check one answer:

JND is smaller with square wave ___ JND is smaller with sine wave ______

Give a reason for your answer. (Hint: Consider the harmonics in the square wave.)

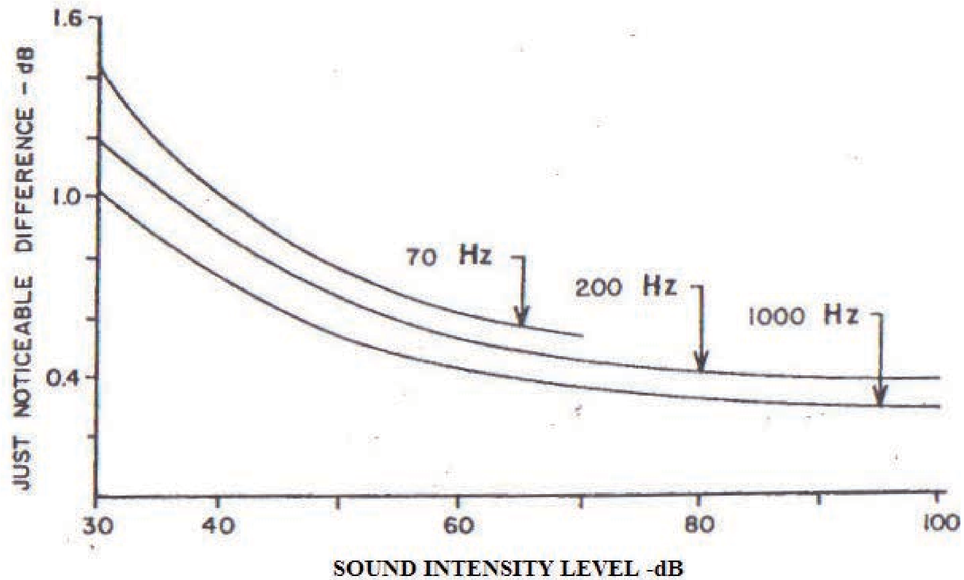


Figure 2. Just noticeable difference (JND) curves for 70 Hz, 200 Hz, and 1000 Hz sine waves. (From: "Physics of Sound" by R. A. Berg and D. G. Stock.)

Just Noticeable Difference (JND) in Frequency

In addition to discern changes in sound intensity, we have even a better ability to notice changes in frequency. Let us do the following simple experiment:

15. Play two pure tones *sequentially*. Start with the same frequency. Increase one frequency slightly and keep playing both tones one after another. When do you hear the *just noticeable difference* in frequency? Do this at frequencies of 200 Hz and 800 Hz.

Just noticeable difference = _______ Hz, or _____%, at a frequency f = 200 Hz.

Just noticeable difference = _______ Hz, or _____%, at a frequency f = 800 Hz.

Loudness in Sones

The dB-values above are based on objective measurements of the sound intensity. There also exists a subjective *sone scale* that tells what sounds "twice as loud" to many persons. Such a "twice as loud curve" is shown as a straight line in Figure 3. Note that both the ordinate and abscissa scales are logarithmic. On the sone scale, 1 sone corresponds to a loudness level of 40 phon for a pure sine wave with f = 1000 Hz. For the special case of a sine tone of frequency f = 1000 Hz, the number of phon is the same as the number of dB.

Figure 3 shows that, in order for sound to be perceived as twice as loud, the sound intensity level is higher by 10 dB. (Some persons do perceive a 6 dB increase as twice as loud.) For example, for an increase in loudness from 1 sone to 2 sone, the intensity increases by 10 dB from 40 dB to 50 dB. Generally, for every increase in intensity by 10 dB, the sone number doubles. Example: For a doubling in loudness from 4 to 8 sone, the sound intensity increases from 60 to 70 dB.

16. Start with a 1000 Hz sine tone at SIL = 60 dB and increase the intensity without looking at the sound level meter until you perceive the sound as twice as loud. By how many dB did the SIL increase?

Measured increase in SIL: ____________dB

Expected increase according to the above: ____________dB

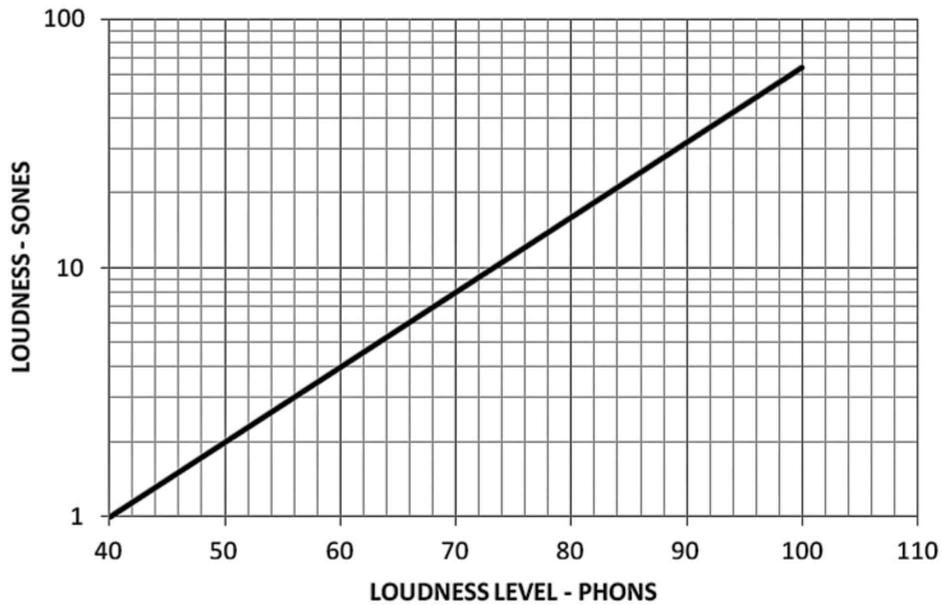


Figure 3. Sone scale, with "twice as loud" meaning a doubling in the sone number. The reference is 1 sone at a loudness level of 40 phon. The phon scale is the same as the dB scale for a pure tone of 1000 Hz.

17. According to Figure 3, what is the increase in phon for a doubling in loudness from 10 to 20 sone?

18. How many times louder does a 90 phon tone sound than a 60 phon tone?

The sone scale is used for specifying the loudness of fans and appliances. For instance, quiet bathroom fans have a rating of 1 to 2 sones, louder ones 3 to 4 sones or more.

Sound Intensity Level versus Pressure Amplitude

The sound intensity is proportional to the square of the wave amplitude. Our range of hearing corresponds to a factor of 10^{12} in the change of intensity. This corresponds to a range of 10^6 in the pressure amplitude. According to Figure 1, the pressure amplitude of sound at the threshold of hearing is 2×10^{-5} Pa. It is 20 Pa at the threshold of pain. These are minute values compared with the static atmospheric pressure. Our ear is very sensitive to pressure changes in the audible frequency range. Very small amplitudes suffice to hear well.

8. Room Acoustics

PURPOSE AND BACKGROUND

An enclosed space has characteristic resonance frequencies for standing waves between walls and other surfaces. We already have discussed the much simpler case of resonating air columns in pipes. The acoustics of a room depends on the volume of the room, the surface area of the walls, the sound absorption properties of the materials, and persons and furniture in the room. When you sing in a shower, you will notice that certain frequencies are enhanced. This is due to resonances in the box-shaped volume of the shower. Resonances may also play an undesirable role in concert halls. These resonances may cause feedback noise if electronic sound amplification is used. In such cases the resonances can be removed from the frequency spectrum with electronic equalizers.

A study of room acoustics includes the frequency analysis and time analysis of sound. The reverberation time is the time it takes for the sound intensity to decay by 60 dB or factor of 10^6. Large concert halls and churches have reverberation times of up to a few seconds. Our music laboratory has a reverberation time of about 1 second or less. Sound travels with v = 346 m/s at 25^0C. A large concert hall has large distances for sound to travel and consequently a long decay time. The materials from which sound is reflected also affect the reverberation time. Sound absorbing materials such as cloth, egg crating, acoustical boards, greatly absorb sound and have a short reverberation time. Highly reflective materials such as concrete walls and tile floors reflect sound with little absorption and result in long reverberation times. The sound absorption of a material depends on frequency and hence so does the reverberation time. One can "tune" the reverberation time of a room for best acoustics by the choice of materials and their placement.

EQUIPMENT

Dynamic microphone, Mac mini, loudspeaker, wooden acoustics cube ("model room").

THEORY AND EXPERIMENT

Room Resonance

For the simplest case of a box-like room, with all surfaces constructed of the same material, the resonant frequencies are given by the formula

$$f_{xyz} = \frac{v}{2}\sqrt{\left(\frac{N_x}{x}\right)^2 + \left(\frac{N_y}{y}\right)^2 + \left(\frac{N_z}{z}\right)^2}\ , \qquad (1)$$

where N_x, N_y, N_z are integer harmonic numbers, *x, y, z* are the dimensions of the room, and v is the speed of sound in air. For example, the lowest resonance frequency (fundamental) for the x-direction is $f_{100} = v/2x$, with $N_x=1$, and $N_y=N_z=0$. This is the same as for the fundamental frequency of a vibrating string, where the wavelength was twice the length of the string. Now the wavelength is twice the *x*-dimension of the box. The *y* and *z* dimensions have their resonance frequencies as well, calculated in the same way. Many higher resonance frequencies exist for the standing waves in each direction and also when waves get reflected at an angle with respect to the walls of the box. These frequencies are obtained from equation (1), with more than one of the harmonic numbers N_x, N_y and N_z equal to 1 or larger.

Our music laboratory has a complicated geometry. It is not a "box" and therefore it has a much more complex resonance spectrum than that from equation (1). The room contains furniture, equipment, and people that change the resonances. Nevertheless, we shall assume in a grand simplification that the room is box-like and calculate the lowest resonances.

Measure the length x, width y, and average height z of the music laboratory.

$x =$___________m $y =$___________m $z =$___________m

1. Calculate the three fundamental resonance frequencies of the room from the formula

$$f = \frac{v}{2L} \tag{2}$$

where L is any of the lengths x, y, z, and v = 346 m/s at an assumed room temperature of 25^0C.

$f_{100} =$ ________Hz $f_{010} =$ ________Hz $f_{001} =$ ________Hz

2. Calculate the first overtones (2nd harmonics) of each of the three fundamentals by doubling the frequencies from the preceding question.

$f_{200} =$ ________Hz $f_{020} =$ ________Hz $f_{002} =$ ________Hz

3. Write down the range of these first 6 frequencies: ______Hz to ______Hz

Acoustics Box.

Instead of studying our laboratory in more detail, we use a cubical box or "model room" with identical dimensions x = y = z = L = 362 mm. See Figure 1 for a similar but non-cubical box. The frequency spectrum for this cubical "room" is much simpler than for a real room with different dimensions, surfaces, furniture etc. Formula 1 above simplifies to

$$f_{1,2,3} = \frac{v}{2L}\sqrt{N_1^{\ 2} + N_2^{\ 2} + N_3^{\ 2}} \tag{3}$$

The integers N_1, N_2, N_3 in the formula are the harmonic or mode numbers. For example, the lowest mode with an air resonance in only the x-direction is (N_1, N_2, N_3) = (1, 0, 0). The corresponding resonance frequency is

$$f_{1,0,0} = \frac{v}{2L} \tag{4}$$

For a cubical box, we obviously have the same resonance frequency $f_{1,0,0}$ for the three modes (1,0,0), (0,1,0), (0,0,1).

Figure 1. Acoustics box (as shown or use an available wooden cube) to simulate the acoustics of a room (built by Arnold Fernandez). The small speaker at the top excites box resonances with a sine sweep or white noise. The microphone inserted on the right records the resonances.

Acquire some resonance spectra with the “FFT Analyzer” in the FEaT software. Example of “room spectra” from the cubical box is shown in Figure 2 and Figure 3. Excite the resonances with white noise or sine sweep and determine which yields the better spectra.

4. Calculate and read off the frequencies of the lowest resonances from Figure 2. For example, the lowest frequency in Figure 2 and Figure 3 is $f_{1,0,0}$ = 478 Hz. Read the next four higher frequencies and assign the corresponding modes to them:

Frequencies (calculated)	Hz	______Hz	______Hz	______Hz
Frequencies (observed)	______Hz	______Hz	______Hz	______Hz
Mode numbers	(, ,)	(, ,)	(, ,)	(, ,)

Calculation of the Reverberation Time

The reverberation time is one of the most important characteristics of a room. Just as in the case of the resonant frequencies, the reverberation time depends on the geometry of the room, on choice of sound absorbing materials, and on persons and furniture in the room. The reverberation time can be estimated from the formula

$$T_{reverb} = 0.050\frac{V}{A_{sabin}} \text{(in seconds)}, \qquad \text{where} \quad A_{sabin} = aA \qquad (5)$$

where V is the room volume in ft^3, A_{sabin} the effective room area called “absorption” in units of *sabin*, a the absorption coefficient of the wall material, and A the physical area square feet.

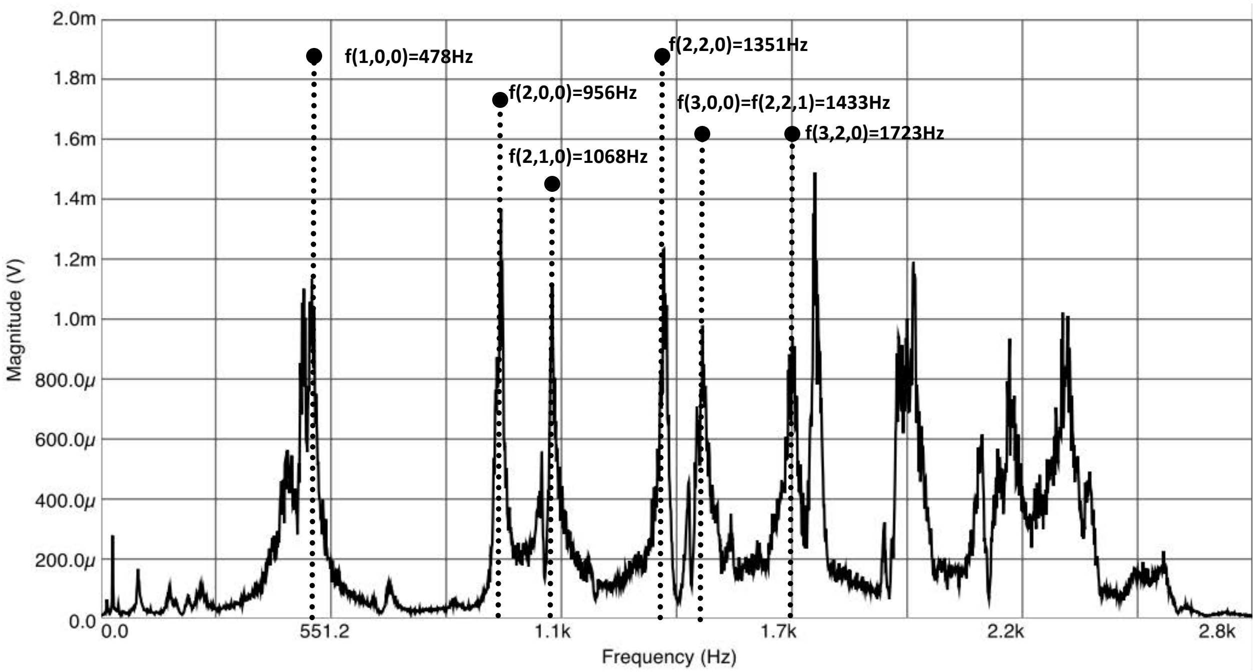


Figure 2. Resonances in a cubical plywood box of inside dimension L = 362 mm. The lowest resonances can be clearly seen and the vibrational mode numbers identified. Some of the modes are f(1,0,0)=477.901Hz, f(2,0,0)=955.801Hz, f(2,1,0)=1068Hz, f(2,2,0)=1351Hz, f(2,2,1)=f(3,0,0)=1433Hz. The resonances were excited with broadband white noise.

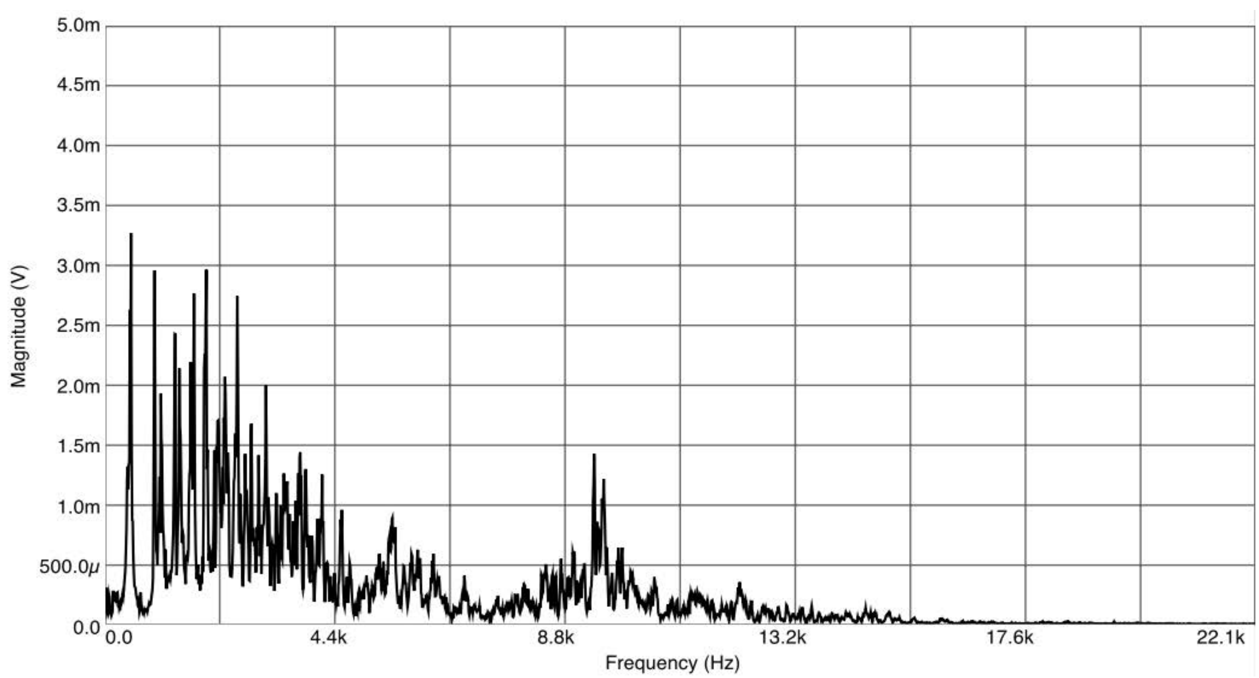


Figure 3. Resonances of the cubical box with white noise excitation on an extended frequency scale including the higher modes. The modes become denser with increasing frequency.

The unit *sabin* is named after Wallace C. Sabine, founder of architectural acoustics. One *sabin* is equal to a square foot of perfectly absorbing material. For instance, a 3 ft^2 hole in a wall is a perfect sound absorber. It reflects no sound and corresponds to an effective area A_{esabin}= 3 sabin. Table 1 shows the absorption coefficients of several common materials. For example, a piece of carpet with an area of 3 ft^2 at a sound frequency of 500 Hz has an absorption coefficient a = 0.3 and an effective area $A_{sabin} = aA = 0.3\cdot 3 = 0.90$ sabin.

Table 1. Absorption coefficients *a* of various materials. (Values from Richard E. Berg and David G. Stork, "The Physics of Sound")

	Frequency (Hz)					
Material	125	250	500	1000	2000	4000
Concrete, bricks	0.01	0.01	0.02	0.02	0.02	0.03
Carpet	0.10	0.20	0.30	0.35	0.50	0.60
Curtains	0.05	0.12	0.25	0.35	0.40	0.45
Acoustical Board	0.25	0.45	0.80	0.90	0.90	0.90
Glass	0.19	0.08	0.06	0.04	0.03	0.02
Plasterboard	0.20	0.15	0.10	0.08	0.04	0.02
Plywood	0.45	0.25	0.13	0.11	0.10	0.09

For an adult person, use $A_{esabin}= aA = 4.2$ sabin.

In order to find the total effective area entering in equation (3), each surface area A of a room in ft^2 is multiplied by its absorption coefficient *a*, resulting in the product a·A for each surface. The total effective area then is the sum over all areas

$$A_{sabin} = a_1A_1 + a_2A_2 + a_2A_2 \qquad (6)$$

5. Calculate the reverberation time of our lecture room (not the laboratory room) for which the width is x = W = 24 ft, length y = L = 29 ft, average height z = H = 9.5 ft. Use the absorption coefficients from Table 1 which you find most appropriate for the materials in the lecture room. Look up the values at a frequency of 500 Hz in Table 1. Calculate the effective area A_{sabin} from equation (6). Calculate the volume from $V = xyz$. Use your values for A_{sabin} and V in equation (5) to obtain the reverberation time.

Answer: A_{sabin} = ________________sabin, T_{reverb} = _________________s

6. Compare your values with those in this Course Guide (see chapter on "Room Acoustics"):

A_{sabin} = ________________sabin, T_{reverb} = _________________s

Calculation of the Reverberation Time of the Cubical Acoustics Box

7. Consider the much simpler case of our cubical box and calculate its absorption A_{sabin} and reverberation time T_{reverb}. Assume a length L = 362 mm for all three dimensions of the box. Use the value for the absorption coefficient for plywood at 500 Hz in Table 1.

Answer: A_{sabin} = ________________sabin, T_{reverb} = _________________s

8. Look up the least and most sound absorbing materials in Table 1 at 500 Hz and calculate the reverberation time of the cubical box for those materials instead of plywood.

Material: ___________ Largest absorption coefficient a = _______ T_{reverb} = ______s

Material: ___________ Smallest absorption coefficient a = ______ T_{reverb} = ______s

9. From your results for the shortest and longest reverberation times, describe how you can adjust this characteristic time with the choice of proper materials.

Reverberation Time of The Laboratory Room

Use the signal generator tool and select the *Spectrogram Mode* in the software. Set the signal generator to 500 Hz or use "white noise". Record a colored *spectrogram*, also called a *sonogram*, where the vertical axis on the display is the frequency, the horizontal axis is time, and the color indicates sound intensity. Run the spectrogram for a few seconds, then turn off the signal generator to stop recording. Observe the time within which the 500 Hz line or white noise fade out. This happens within about one second. This gives a good indication of the reverberation time. You will have to guess on the spectrogram display when the reverberating sound has faded into the background. You may also do this experiment by clapping instead of using a frequency generator or white noise.

10. Write down the reverberation time for our laboratory room as obtained from the sonogram:

Answer: T_{reverb} = ____________s

11. Compare this reverberation time with the value you calculated for our classroom (see Questions 5 and 6). Does the classroom or the laboratory room has a shorter reverberation time? Give reasons for the difference.

Reasons: ____

Reverberation Time of The Hallway

12. Finally, obtain an estimate of the reverberation time T_R for the much simpler geometry of the hallway outside the laboratory. Ask a student to clap once with his hands in order to produce an impulsive sound. Acquire a sonogram from the time before until after the clap. Read the reverberation time from the sonogram.

Answer: T_{reverb} = ____________s

Resonances in the Hallway

Place a loudspeaker in the hallway and direct white noise or a sine sweep to a wall near the laboratory door. Acquire some resonance spectra with the "FFT Analyzer" in the FEaT software. Note the first few resonance frequencies and compare with the frequencies you can calculate from the width of the hallway.

Reverberation Time in the TTU Southwest Collection/Special Collections Library

Go to the entry hall of the Southwest Collection Library at TTU. Note its size and building materials. Clap once with your hand and listen to the long decay of the sound. Acquire a sonogram. The lower frequencies around 500 Hz decay more slowly ($T_{reverb} \approx 2.5$ s) than the higher ones at 4000 Hz ($T_{reverb} \approx 1.5$ s).

13. Acoustically, does the hall have more "warmth" or more "brilliance"?

14. Use $V = 17000$ ft^3 and $A_{esabin} = 325$ sabin for the hall. Calculate the reverberation time T_R.

Figure 4. Entry Hall of the TTU Southwest Collection/Special Collections Library.

Focused Sound and Echoes at the Campus Circle at Texas Tech University
Take a field trip to the Campus Circle and the Pfluger Fountain at the center of TTU. Stand in the center and clap with your hands. You will hear an echo of focused sound from the low walls of the circle. Move away from the center. The echo will be less because of a lack of focus.

15. Do you hear an echo, reverberating sound, or resonances? Discuss the differences.

Figure 5. Campus Circle at Texas Tech University.

9. Electric Energy and Work, Acoustical Power

PURPOSE AND BACKGROUND

Electricity is one of the most important energy forms. In this laboratory we study electric energy, work, power, voltage, current, and resistance. We measure the power consumption of a conventional incandescent light bulb and compare it with the more efficient compact fluorescent light bulb (CFL) and the yet more efficient light emitting diode (LED). We will determine the energy and dollar savings with a CFL compared to an incandescent light bulb. In a second part of this laboratory we investigate the acoustic power radiated by a loudspeaker by finding the sound intensity level (SIL) in front of the speaker. The speaker efficiency then follows from the acoustic power divided by the electric power. We judge how loud a speaker sounds for a given acoustic power and find out how acute our sense of hearing is.

EQUIPMENT

Loudspeaker, light bulb fixture with incandescent light bulb, compact fluorescent light bulb (CFL), light emitting diode (LED), “Watts UP” power meter for light bulbs, power meter for loudspeaker (General Radio Output Power Meter 1840-A), two multi-meters for current and voltage measurements, PASCO Sine Wave Generator, EXTECH Digital Sound Level Meter.

Some Theory Concerning Power, Energy, Work, and Electricity

Energy is the ability to do work.

Example: 1 gallon of gasoline contains energy to do work. An automobile engine does work and moves a car 30 miles with this energy.

Unit of energy and work: **1 Joule (J)**

Power is the rate at which work is done   Power = Work/Time or $\mathbf{P = W/t}$

Work   Work = Power·Time $\mathbf{W = P \cdot t}$

The unit of power is Joule/second = Watt   **1 J/s = 1 Watt (W)**

Ohm’s law of electricity $\mathbf{V = I \cdot R}$, where

V = voltage across a load in **volt (V)**, for instance a light bulb or loudspeaker
I = current through the load in **Ampere (A)**
R = resistance of the load in **Ohm (Ω)**

Electric power $\mathbf{P = V \cdot I}$, or equivalently $\mathbf{P = I^2R}$ and $\mathbf{P = V^2/R}$

Common unit of energy: **kilowatt·hour (kWh)**

Conversion: 1 kWh = 1000 W·3,600 s = 3,600,000 W· s = 3,600,000 J = $\mathbf{3.6 \times 10^6}$ **J**

Example: A sedentary person consumes 2000 kcal (kilocalories) of food energy per day. The conversion is 1 kcal = 4185 Joule. Therefore 2000 kcal = 8,370,000 J. This amount of energy is consumed in a time t = 24 hours = 86,400 s.
Hence the rate of energy consumption is $P = W/t = 8{,}370{,}000\ \text{J}/86{,}400\ \text{s} = 97\ \text{J/s} = 97\ \text{W}$ or about 100 W.
This is typical for the resting metabolic rate of a person. You may know this rate as "2000 Kcal per day" rather than 100 Watt. When we sit around doing little, we burn food energy at the rate of 100 W, i.e. about the same as an old-style 100 W incandescent light bulb consumes in the form of electricity.

EXPERIMENTS

Power and Energy Consumption in Light Bulbs

Use the triple light bulb fixture. Note the brightness from the conventional incandescent light bulb, the compact fluorescent light bulb (CFL), and the light emitting diode (LED) light bulb.

1. After warming up, are the three light bulbs about equally bright? ___________

2. Write down the power rating of the three light bulbs, 40 W, 9 W, 6W, on the next line:

Incandescent light bulb P = ________W CFL P = ________W LED P = ________W

3. Plug the "Watts Up" Power Meter into a household outlet. Plug the triple light bulb fixture into the "Watts Up" meter. Read the power consumed by each light bulb. Compare with their nominal ratings above:

Incandescent light bulb P = ________W CFL P = ________W LED P = ________W

4. Read the current in ampere on the meter of the light bulb fixture. Assume a household voltage of V = 120 Volt. Calculate the power from the formula $P = V \cdot I$.

Incandescent light bulb P = ________W CFL P = ________W LED P = ________W

5. How well do the values for power in questions 2, 3, 4 agree? Express the differences in percent. Explain why the readings from experiment 4 above for the CFL and LED may differ from experiments 2 and 3.
Ask your instructor! (Hint: The AC power for a CFL and LED is not simply $P = V \cdot I$.)

6. Suppose you turn the light bulbs on for 5 hours each day. Calculate the energy used in a 30-day month. Express your answer in Joule and then convert to kWh.

Incandescent light bulb: Energy = _____________J = ______________kWh
CFL light bulb: Energy = _____________J = ______________kWh
LED light bulb: Energy = _____________J = ______________kWh

7. Electricity costs about 13 cents/kWh. What is the electric bill for the light bulbs in a month?

Incandescent light bulb	Electric bill = $____________
Compact fluorescent light bulb (CFL)	Electric bill = $____________
Light emitting diode (LED)	Electric bill = $____________

8. An incandescent light bulb costs $0.75 (if still available), a CFL $1.50, and an LED $5.00. How long does it take to amortize the extra cost of the CFL and LED over the incandescent light bulb?

Amortization time: CFL ________months   LED ________months

9. How much money is saved over the lifetime of 10,000 hours of a CFL and 20,000 hours of an LED, compared to the 2000 hours for an incandescent light bulb?
Include in your calculation the number of incandescent light bulbs you would need during the lifetime of a CFL and LED.

Money saved: CFL $ _____________   LED $ _____________

10. What are the energy savings in percent when using a CFL and LED instead of an incandescent light bulb? (Hint: Compare the wattages of the three bulbs.)

Energy savings in percent when using a   CFL = ______% or LED = ________%

Electric Power to a Loudspeaker

Connect a signal generator (e.g. PASCO WA 9867 sine wave generator) to a loudspeaker (not the dedicated computer speakers). Do not plug the signal generator into the household outlet yet. Connect a multi-meter, set to the Ammeter mode, in-line between the loudspeaker and the signal generator. Connect a multi-meter, set to the Voltmeter mode, in parallel to the speaker inputs.

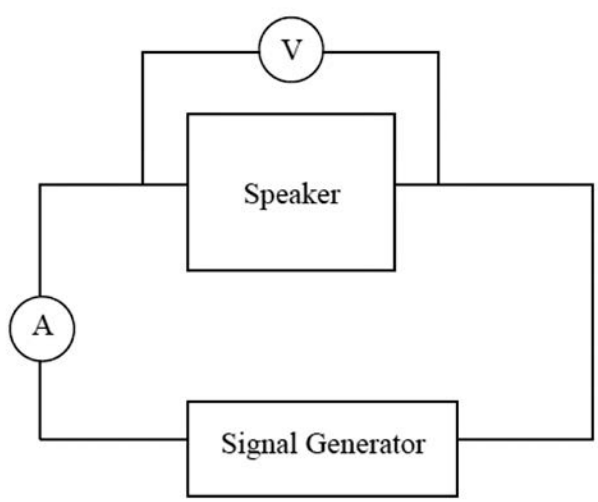


Figure 1. Schematic of the speaker connections to a signal generator, voltmeter, and ammeter.

Select *AC alternating current settings* (not DC settings!) on the ammeter and voltmeter. Choose an initially high range for Amperes and Volts on the meters. Turn down the amplitude on the signal generator. Now plug the signal generator into the household outlet and choose a frequency in the mid-range, i.e. 500 Hz. Observe the voltage and current on the meters. Play with the amplitude and frequency settings and listen to the loudness and pitch of the sound.

11. At a comfortable loudness from the speaker, note the current through the speaker and the voltage across its terminals.

Current I = _______Ampere Voltage V = ________Volt

12. Calculate the power to the loudspeaker from the formula $P = I \cdot V$.

Power $P =$ ________W

13. Loudspeakers of hi-fi systems often are rated at 100 W or higher.
How does your answer for our loudspeaker compare with such ratings?

14. Do you think a power of several hundred Watt is necessary? Why or why not?

Resistance or Impedance of a Loudspeaker, Power Continued

The reaction of a loudspeaker to an applied AC voltage is called *impedance*, labeled with the letter Z. Impedance is not the same as resistance because it also includes capacitance and inductance. We ignore this here and use resistance for impedance. Use Ohm's law $V = IR$ and calculate the resistance from $R = V/I$. Use the value of R to get an estimate for the impedance.

15. Obtain the impedance of the loudspeaker from your measured voltage and current. Compare this with the specification on the loudspeaker enclosure.

Impedance $Z = V/I =$ ______/______ = ______Ω
Written on enclosure $Z =$ ______Ω

16. Obtain the power to the loudspeaker from the expression $P = I^2R$.

Power $P =$ (______)$^2\cdot$(______) = _______W

Compare your answer with the result from question 12: $P =$ _________W

Loudspeaker Power Measured Directly with a Power Meter

Do not change the amplitude and frequency settings on the signal generator. Feed the output from the signal generator directly into the *"General Radio Output Power Meter"* without the loudspeaker, voltmeter, and ammeter in the circuit. Set the impedance dial on the power meter to the same value as for the impedance found in question 15.

17. What is the reading on the power meter? Compare with your calculated results from questions 12 and 16.

Power read from power meter:	a) P = ______W
Power from question 12:	b) P = ______W
Power from question 16:	c) P = ______W

Acoustical Power and Loudspeaker Efficiency

Keep a note of the power read from the "General Radio Output Power Meter". Keep the amplitude and frequency settings on the signal generator. Remove the power meter from the signal generator and replace it with the loudspeaker.

Use a sound level meter, e.g. the EXTECH Digital Sound Level Meter. Choose setting "A" on it corresponding to the human ear response. Measure the sound intensity level (SIL) at various locations in front of and close to the speaker. For instance, measure the SIL at a distance between 0.5 m and 1 m from the speaker. Move the sound level meter around the speaker in a circular arc, left to right, and up and down, always at the same distance from the center of the speaker. Note the reading on the sound level meter.

18. Write down the SIL readings from the sound level meter.

SIL_{center} = ____dB, SIL_{left} = ____dB, SIL_{right} = ____dB, SIL_{up} = ____dB, SIL_{down} = ____dB

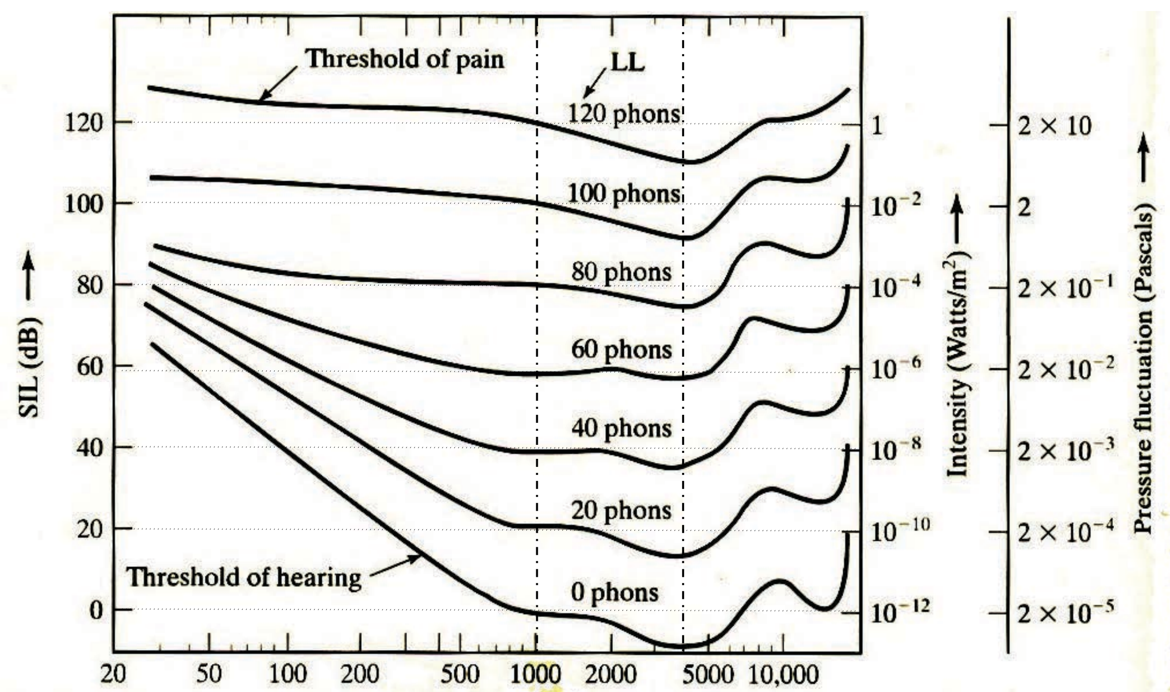


Figure 2: Fletcher-Munson curves of equal loudness. ("Physics of Sound" by R. A. Berg and D.G. Stork.)

19. Take the average of these sound intensity level values.

$SIL_{average}$ =______dB

20. Consult the Fletcher-Munson curves in Figure 2 and write down the sound *intensities* (not the sound intensity levels!) in Watt/m^2 corresponding to your SIL values.
Example: For SIL = 80 dB we have a sound intensity I = 0.0001 Watt/m^2 = $1\cdot10^{-4}$ Watt/m^2.

I_{center} = _________, I_{left} = __________ , I_{right} = __________ , I_{up} = __________ , I_{down} = _________

Average of these intensity values: I_{ave} = _______$\cdot 10^{-4}$ W/m^2

21. Calculate the total acoustical power radiated by the loudspeaker. Assume that the acoustical power is radiated into a cone in front of the speaker. For the area of the base of the cone at a distance *r* in front of the speaker take $A = 0.6\pi r^2$. Write down the distance where you took the measurements with the SIL meter and calculate the area through which the effective acoustical power went.

Distance r = _____m Area A = _____m^2

22. Multiply your value for the average intensity $I_{average}$ from question 20 by the area A. This is a good estimate for the total radiated acoustical power.

Acoustical power $P_{acoustical} = I_{average}\cdot A$ = _______W/m^2 $\cdot$ _______m^2 = ________Watt
You can now compare the radiated acoustical power with the electric power to the speaker (see result from 17a) and obtain the *loudspeaker efficiency.*

Speaker efficiency = $P_{acoustical}/P_{electric}$ x100 = _______%

23. Use your answer for the speaker efficiency to comment on the conversion of electrical to acoustical power.

24. Compare the acoustical power output from a speaker with the light output from a compact fluorescent light bulb. Assume that the same electrical power (e.g. 10 W) goes into the speaker and the CFL. For the CFL assume a conversion efficiency of 20% from electric power to light. Write down the acoustical output from the speaker and the light output from the CFL.

Speaker output $P_{acoustical}$ = ______W CFL output P_{light} = ______W

Your result should indicate that that the emitted acoustical power is much lower than the light output from a CFL.

10. Frequency Response of a Stringed Instrument

PURPOSE AND BACKGROUND

In this laboratory we study the frequency response of a violin when a sine sweep is applied to the bridge or the instrument is tapped at the front and back. Such measurements give information on the quality of an instrument. We also simulate the resonances of the instrument with so-called Chladni figures on a metal plate that is shaped like a violin body.

EQUIPMENT

Violin, Mac mini, microphone, violin support beam, vibrator, stereo receiver, fine dry sand, Chladni plate.

THEORY AND EXPERIMENT

A violin is played by bowing or plucking its strings. The string vibrations are transferred to a bridge mounted on the top plate, and from there to the sound post placed under pressure between the top plate and back plate of the violin body. All this couples the string vibrations to the instrument. As a result, the violin body resonates over a rather wide frequency range. The cavity of the violin acts as a Helmholtz resonator. The wood and the air of the cavity resonate to create the characteristic rich tone of a violin. The quality of the sound is affected by the materials, the way the wood is shaped, the glue for joining the components, the varnish, and the skills of the instrument maker.

1. The four strings of a violin are tuned in musical fifths to the notes G_3, D_4, A_4, and E_5. Look up the frequency range of the violin from G_3 to C_7 (with C_7 played on the E_5-string). Frequency of G_3 = __________Hz Frequency of C_7 = __________Hz

Chladni Figures

We use a so-called Chladni plate to simulate the vibrational patterns on the violin body. The Chladni plate is made of sheet metal and shaped in the form of the violin back plate. This is a very rough approximation of a violin body, where in reality wood is used and the plates are curved. Nonetheless, we produce resonance patterns with some resemblance to a real violin.

Place the metal plate horizontally on a vibrator that is driven by a frequency generator. Place a large sheet of paper under this setup and sprinkle some sand evenly on the plate. You need the paper to collect the sand and not mess up the lab.

Set the amplitude on the function generator to about halfway on the dial. Adjust the frequency until you can see clear vibrational patterns of the jumping sand particles on the plate. The resonances start at frequencies well below G_3 of a real violin. Take a quick look at the patterns from these low frequencies. Then begin at about 180 Hz and slowly increase the frequency until audible and visible resonances occur. The sand jumps around and forms patterns. The places where the sand collects are the vibrational *nodes* with minimum movement of the plate. (This is a 2-dimensional analogy to the 1-dimensional nodes of a vibrating string.) The places where no sand is left are the *anti-nodes* where the Chladni plate vibrates the most. The sand moves away from these anti-nodal regions towards the nodal areas. You will see many beautiful and strange looking patterns as you increase the frequency. Figure 1 shows an example of a Chladni figure.

Figure 1. Chladni figures from a metal sheet simulating the back plate of a violin. The resonances are excited with a vibrating shaft mounted from below the center.

Our Chladni figures are not really the vibrational modes of a violin. However, the wooden plates of a violin do show some qualitatively similar patterns. Find pictures of some real vibrational mode patterns in books or on the Internet! The plates of a good violin exhibit one or two major *wood resonances* and *air resonances* in the volume of the body. The cavity of the body acts as a Helmholtz resonator.

2. Write down 10 to15 frequencies of major resonances of the Chladni plate in the range 100 to 1000 Hz.

3. Use the camera on your cell phone or any camera and take some pictures of good-looking Chladni figures. Add the resonance frequency to each figure.

4. How does the complexity of the Chladni figures change as the frequency increases?

An interesting effect is seen when the resonance frequencies of the Chladni plate are plotted versus the resonance number N. The first visible resonance (N = 1) seen in the sand occurs at about f = 100 Hz. When the excitation frequency is increased slowly on the sine wave generator, the first 12 resonances are found to be in the range 100 to 800 Hz. Plotting these frequencies as a function of resonance number N reveals a nearly linear relationship, as seen in Figure 2. Shown also along with the resonance frequencies are the pictures of the corresponding Chladni figures.

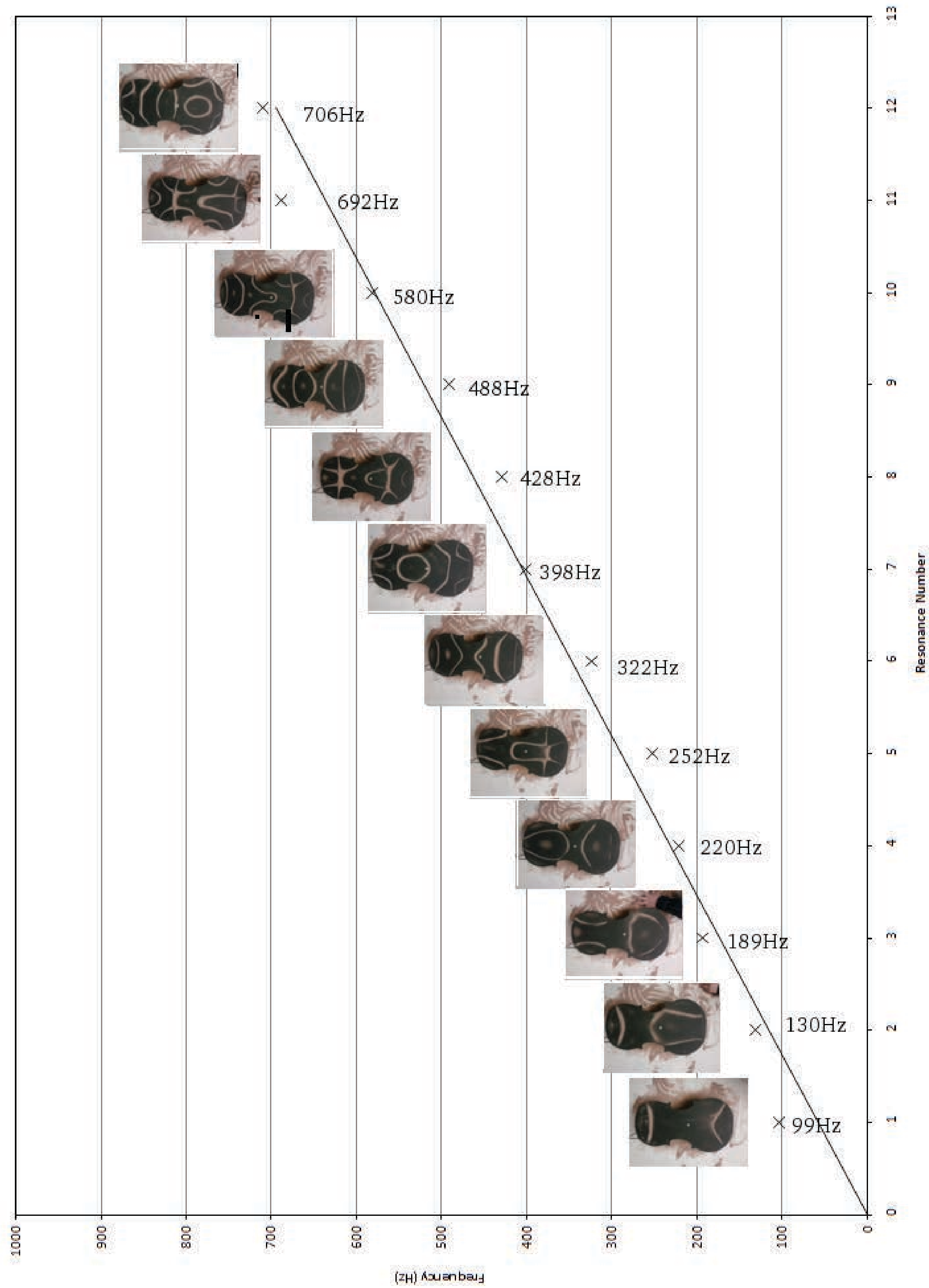


Figure 2. The first 12 resonance patterns (Chladni figures) in the sand of a vibrating metal plate. The relationship between the resonance frequency and the resonance number is nearly linear. The photographs show the Chladni figures for each resonance.

Response Curve of a Violin

The study of the vibrational modes of the violin is much more difficult than the simulation of its wood resonances with Chladni figures. The wood has two major resonances that greatly affect the tone and quality of a violin. Lower tones may be reinforced by a wood resonance called *wood prime resonance W'*, see Figure 3. On a good violin, the lower notes are given a boost by the *W' resonance* that contributes to a rich deep tone. An additional *wood resonance W* may boost the higher frequencies. If a note is played near the frequency regions of the wood resonances, the violin becomes louder in intensity. Consequently, any higher harmonics that fall within these regions increase in intensity as well, adding to the tone quality or timbre of the instrument. The *air resonance* in the cavity of the violin body (Helmholtz resonator) also increases the intensity and quality of the sound. This resonance is determined by the volume and shape of the violin, including the f-holes. The air resonance from Stradivarius and Guarneri violins is shown in Figure 3 by the open circle. It is seen that only the Stradivarius clearly exhibits the wood resonances W and W', and thus is superior to the Guarneri. (A Guarneri generally is an excellent violin, too!)

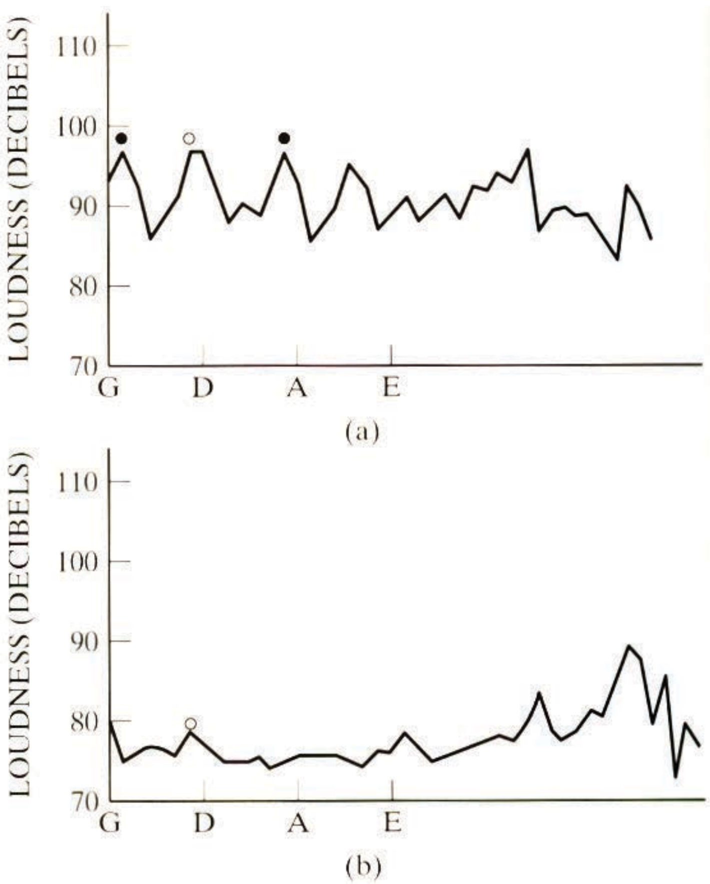


Figure 3. Response curves for (a) good Stradivarius violin, (b) "poorer" Guarneri violin. Only the Stradivarius clearly shows the *wood prime resonance W'* (left dark dot) and the *wood resonance W* (right dark dot). Both violins show the *air resonance* (open circle). (From C. Hutchins "The Physics of Music", Scientific American, 1962.)

Experiment

Use a vibrator and control it with the FEaT software in the Mac mini via the output from the stereo receiver, see Figure 4. Couple the vibrator to the bridge of the violin with a clip. The bridge directs the vibration of the violin strings to the sound post in the cavity and thus to the violin as a whole. Photographs of the violin setup are shown in Figure 5. The violin bridge rocks right and left, not straight up and down. Therefore, the coupling from the vibrator *must be off-center* in order to produce a good sound, see the clip mounting in Figure 5.

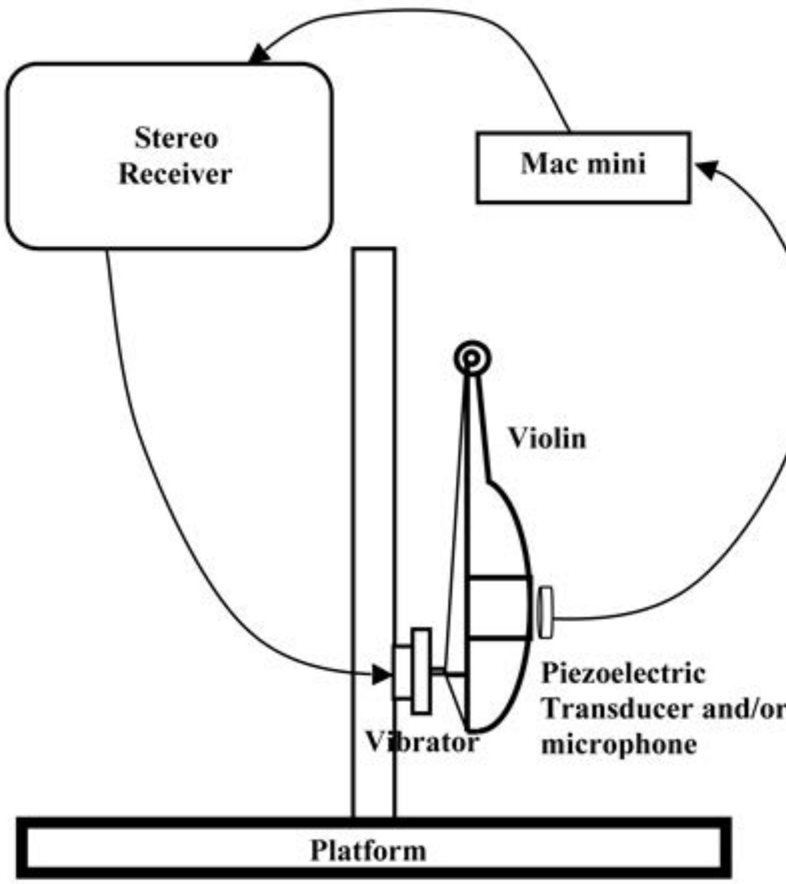


Figure 4. Violin setup for acquiring the response curve with the FEaT software. A microphone and a piezoelectric transducer can be used for signal acquisition. Measurements can also be taken by moving the two sensors to the front.

The piezoelectric transducer for sensing the plate vibrations can be taped to the back or front plate (be careful not to damage the varnish!). Similarly, the microphone can be placed near the front or back plate. For the front plate, position the microphone close to the *f*-holes of the violin. Set the FEat software to a higher resolution (e.g. 4096 lines) and the frequency scale to 2.8 kHz. Sweep a sine wave excitation into the violin via the vibrator-bridge connection and record the response curve as the sine frequency is ramped up. Record a loudness response curve (spectrum) on a *logarithmic* scale and *linear* amplitude scale.

Figure 5. Mounting of the violin for recording the response from the front plate. The driving rod of the vibrator is fastened off-center to the bridge of the violin with a clip. Note the microphone positions in both pictures. (A piezoelectric transducer can be taped to the back (or front) to complement the microphone measurements.)

An example of the frequency response measured near the violin front and back plates is shown in Figure 6 (on a linear scale). Several resonances are visible in the figure.

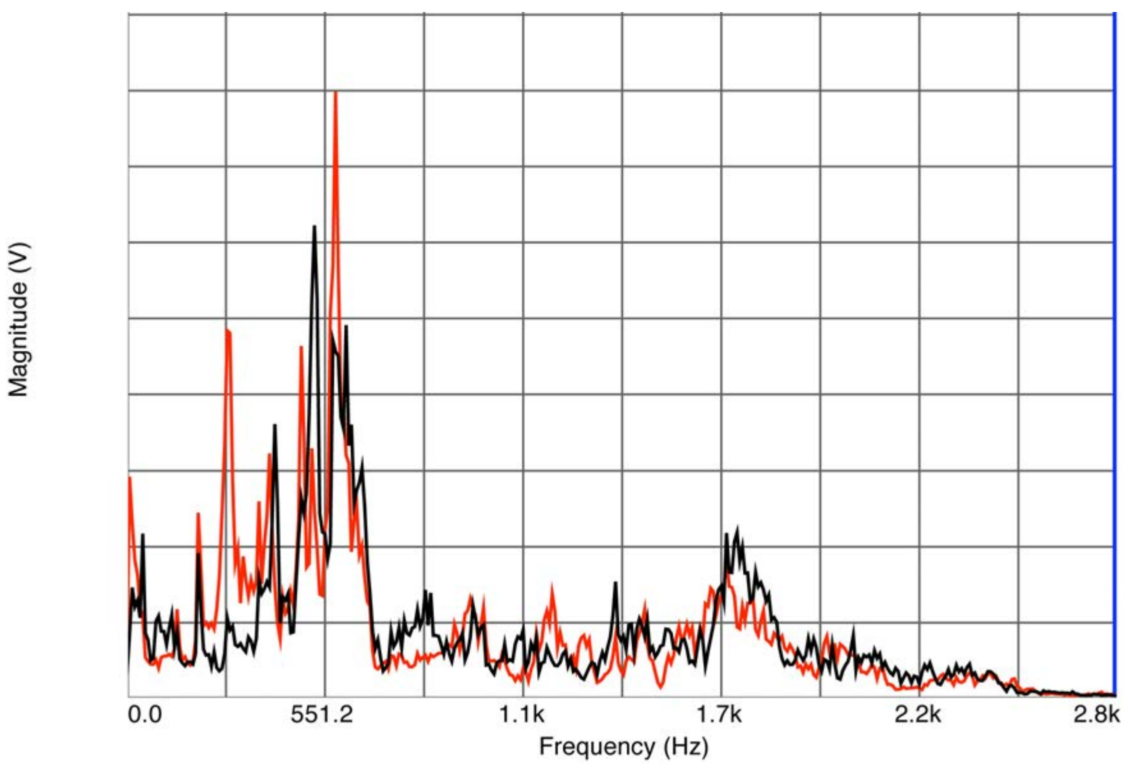


Figure 6. Response curve of a violin on a linear amplitude scale, excited with white noise by a vibrator on the bridge of the violin. Red curve: Microphone near front plate. Black curve: Microphone near back plate. (Ordinate scale: The red resonance at 280 Hz is about 500 mV.)

We see that the two curves in Figure 6 are qualitatively similar. Pronounced resonances are common around 550 Hz and 1800 Hz to both curves. The big difference is the sharp resonance at about 280 Hz in the red curve. This is the air resonance from inside the violin body that, of course, is only noticeable at the open f-holes of the front plate.

Alternative Excitation of the Violin with Tap Tones

An easy way for exciting the violin vibrations is tapping the back plate. The result is not the same as exciting the front plate with a vibrator. But it offers additional information. Tapping the back plate of the body excites the resonances similar to applying a noise spectrum. Figure 7 shows a response curve obtained this way.

5. Look at the response curves in Figure 6 and Figure 7. Can you detect the air resonance and wood resonances W' and W? Hint: See Figure 3 for the approximate locations of the resonances from two excellent violins. The frequencies of the open strings of the violin are G3 = 196.00 Hz, D4 = 293.66 Hz, A4 = 440.00 Hz, E5 = 659.26 Hz.

6. Can you definitively say which peak in Figure 6 is the air resonance? How can you be sure? What is the frequency of the air resonance? Answer: _________ Hz.

7. Compare Figure 7 for our violin with the response curves of the two violins in Figure 3. How would you rate the quality of our violin?

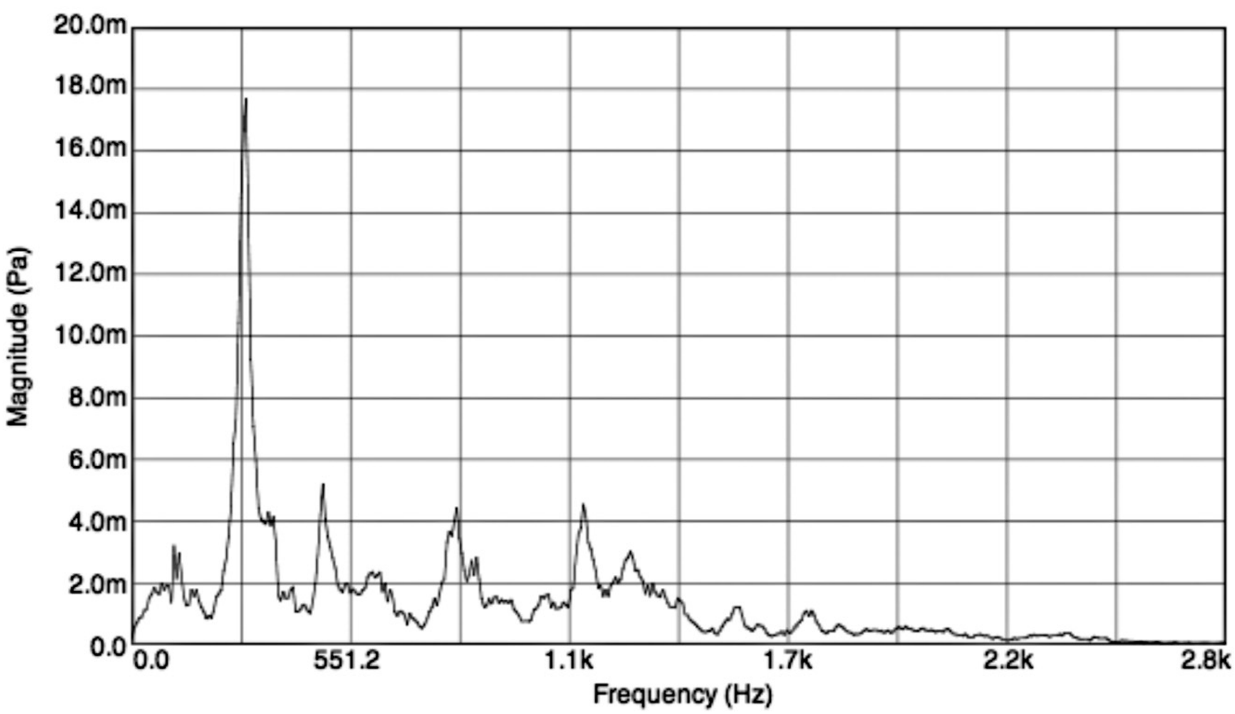


Figure 7.Violin response from tapping the back plate, with the microphone near the front plate. The pronounced peak near 280 Hz most likely is from the air resonance inside the violin body.

8. Remove the violin from the stand. Ask a violinist among the students to bow the four open strings with equal pressure. Which string(s) would you expect to sound louder than others,

based on your measured response curves? Specifically, play the open G3-string and then the open D4-string. Listen to the loudness of the notes G3 and D4 and compare. Which note sounds louder? Is this what you would expect from Figure 7?

Measurements With a Piezoelectric Transducer

Shown in Figures 6 and 7 are measurements obtained with a *microphone*. Take some recordings with a *piezoelectric transducer* mounted at the front and back plates of the violin. Compare with the microphone results.

a) Excitation with vibrator at bridge, piezoelectric transducer at front plate.

b) Excitation with vibrator at bridge, piezoelectric transducer at back plate.

Helmholtz Resonance From a Guitar

Excite the resonances in a guitar with a speaker placed above the guitar opening. Apply white noise to the speaker or do a sine-sweep. Listen to resonances of the sound from the opening. Place a microphone near the opening and acquire a sound spectrum. Can you detect a Helmholtz resonance (air resonance) in the spectrum?

You can also try exciting the resonances with a vibrator on the top plate of the guitar and a microphone near the opening. Again, analyze the spectrum for a Helmholtz resonance. Listen if you can hear it directly.

11. Musical Scales, Temperament, Elementary Music Theory

PURPOSE AND BACKGROUND

The arrangement of musical notes today is the result of centuries of changes in both musical style and taste. As music became more complex, the tuning of instruments, for instance the piano, took various forms. The older styles of tuning are primarily of historical interest, but it is still worthwhile to understand how music has arrived at the modern *equal temperament*. In tuning any instrument, a starting pitch must be chosen. For a violin, this pitch is *concert* A_4 (f = 440 Hz). The rest of the strings are tuned to this pitch. The manner in which they are tuned is called the *musical temperament.* A violin has only four strings and therefore tuning is not difficult. But how is a piano tuned with so many keys? What should be the starting pitch, and how do you tune the other keys? These questions arose from the need for standardization. This laboratory will discuss briefly some features of older temperaments, their benefits and drawbacks. The *equal temperament* will be treated in detail.

EQUIPMENT

Keyboard, violin, Mac mini

THEORY AND EXPERIMENT

Pythagorean Temperament

When music started using multiple parts, chords became integral objects of melodic structure. Initially *perfect fifths* were used to tune all notes on a keyboard. A perfect fifth is rather easy to discern, even by the musically challenged. The term *fifth* refers to the fifth note in a major scale. For example, in the C-major scale C, D, E, F, G, A, B, the note "G" is the fifth. The frequency ratio between the perfect fifth (G) and the *tonic* (C) is 3:2.

The *fifth* was the first ratio to be used in tuning. Early piano tuners would first tune middle C_4, then the notes G_4, D_5, A_5, E_6, B_6, $F^{\#}_7$ all in perfect fifths above C_4. They also tuned F_3, B^b_2, E^b_2, A^b_1, D^b_1 in perfect fifths below C_4. $F^{\#}_6$ is then tuned by a perfect octave down from $F^{\#}_7$, and $C^{\#}_6$ is tuned a perfect fifth above $F^{\#}_6$. In this way, the twelve different keys in the *chromatic scale* were determined. All other keys were tuned from these keys by *octaves* with frequencies in the ratio 2:1. The resulting temperament was known as the *Pythagorean temperament*.

1. *Middle C* is commonly taken as C_4 = 261.63Hz. What is the frequency of a perfect fifth above middle C, G_4 = __________Hz, and below middle C, F_3 = __________Hz?

2. Consider the notes D^b_1 and $C^{\#}_8$ tuned by fifths according to the Pythagorean temperament. Since all 12 keys in the chromatic scale are determined in this manner, there are 12 jumps of 3/2 in frequency, or twelve perfect fifths, from D^b_1 to $C^{\#}_8$. Calculate the frequency ratio

$$C^{\#}_8/D^b_1 = \_\_\_\_\_\_\_\_\_\_$$

3. D^b and $C^\#$ are the same key on the piano. We thus would expect D^b_8 and $C^\#_8$ to have the same pitch. (Note that $C^\#_8$ is one half step above the highest key on the piano, but answer this question anyway.) Going from D^b_1 to D^b_8 in octaves spans 7 octaves. Each octave increases the frequency by a factor of 2. Calculate the frequency ratio

$$D^b_8/D^b_1 = \_\_\_\_\_\_\_\_\_\_$$

4. Questions 2 and 3 arrive at the same notes by two different means, one by 12 fifths, the other by 7 octaves. How much do the frequency ratios in questions 2 and 3 differ?

The discrepancy is the *Pythagorean comma*. Octaves are retained as being perfect, with a ratio of 2:1. But the fifths must be re-tuned for 12 fifths to equal 7 octaves. In the *Pythagorean temperament,* all fifths are tuned true but the last one. The final fifth sounds very bad and is known as the *"wolf fifth"* because of its growling sound and beats. This became unacceptable as music evolved.

Just Temperament and Mean Tone Temperament.

When music began to have more differentiated harmonies, the inclusion of a *perfect third* with a frequency ratio 5:4 became increasingly important. The Pythagorean temperament had particularly bad thirds and so its use was slowly discarded and replaced by other temperaments.

The *just temperament* tuned a perfect fifth above and below middle C_4, resulting in G_4 and F_3, respectively. It also tuned perfect thirds above F_3 and above middle C_4. In this way, *major triads*, composed of the *tonic*, *third*, and *fifth* notes on a scale, sounded perfectly in tune. Tuning the other eight keys in just temperament is slightly more complicated than in the Pythagorean temperament.

5. Calculate the frequency of a perfect third above middle C_4: $E_4 =$__________Hz, and
below middle C_4: $A^b_3 =$__________Hz

As music expanded into *minor keys*, one of the serious weaknesses of just temperament became painfully apparent. Minor triads sound especially bad in this temperament. The error again is all concentrated in one area. In this case, above E^b, the pair $F^\#$ and D^b should be a perfect fifth, but each note is reached by two different routes. Therefore this fifth is off by a significant amount. The *mean tone temperament* was created to compensate for the error by spreading it over all the fifths, not just one. This too, however, caused problems when musical keys farther from the key C where used.

Equal Temperament

All these problems eventually led to a resolution with the introduction of e*qual temperament* or the *equal-tempered scale*. Here the error is spread *equally across all twelve notes* in the chromatic sequence. All keys then sound the same. However, there are no true fifths or thirds or any other chords, except the octave. The twelve keys in equal temperament are spaced by equal frequency ratios. Since an octave must have a 2:1 ratio, the interval between keys must be multiplied 12-times in order to give a value of two. This interval therefore is the 12th *root of two*, namely $\sqrt[12]{2}$. Thus the frequency of each note is multiplied by this number to give the next note one half step or semitone higher.

6. Use a calculator and compute the value of $\sqrt[12]{2}$ = _________. Calculate the frequencies of the remaining 11 notes of the chromatic scale, starting with middle C_4. Insert results in Table 1.

Table 1. Frequencies of the notes of the chromatic scale in equal temperament, starting with C_4.

Note	Frequency	Note	Frequency
C_4	261.63Hz	$F^{\#}_4/G^b_4$	
$C^{\#}_4/D^b_4$		G_4	
D_4		$G^{\#}_4/A^b_4$	
$D^{\#}_4/E^b_4$		A_4	
E_4		$A^{\#}_4/B^b_4$	
F_4		B_4	

Compare your calculated values in Table 1 with the frequencies shown in Figure 1 for the piano keyboard (see end of this chapter). This is the way all pianos are tuned today, including the keyboard in our laboratory.

Verify that the keyboard in the laboratory is correctly tuned to *equal temperament* by opening a FFT tool in FEaT. Make the frequency span range from 0 to 2756.2 Hz and adjust the number of spectral lines to 22050. Use the sine-wave-voice on the keyboard (#352). Press and hold a key until a definite peak is observed in the frequency spectrum on the computer screen. Read the frequency for the peak. Do this for three notes of your choice.

7. Record the values of the measured frequencies and compare them to the actual frequencies of the notes in Figure 1. Collect your data in Table 2.

Table 2. Three notes on the keyboard.

Note	Actual Frequency	Measured Frequency

8. How well is our keyboard tuned to equal temperament?

9. Compare the frequencies of the C major triad in equal temperament and just temperament. For just temperament, take the answer for the perfect fifth G_4 from Question 1, and perfect third E_4 from Question 4. For equal temperament, use values from Table 1. Insert all values in Table 3.

Table 3. Frequencies of the major triad based on C_4, in just and equal temperament.

Just		Equal	
Note	Frequency	Note	Frequency
C_4	261.63 Hz	C_4	261.63 Hz
E_4		E_4	
G_4		G_4	

10. Open three Sound Generator tools in FEaT on the computer and set the sine frequencies for the C-major triad in just temperament. Play and listen to this triad from the computer. Then set the keyboard to "sine-wave-voice #352" frequencies. Play the major triad in equal temperament on the keyboard. Listen to both triads simultaneously and compare.

11. Which triad sounds "better", and why?

12. Can you hear any *beats* between the two triads? If so, explain which notes they come from.

Tuning a Violin and Beats

String instruments in chamber music often are tuned to Pythagorean temperament. The A-string of a violin is first tuned *beatless* to concert $A_4 = 440$ Hz by comparing for instance with a tuning fork. The term beatless refers to the absence of "beats" that would otherwise be heard if a note were slightly out of tune with another. This can be observed on the computer with two sine wave notes a half-step apart, as follows:

Open an Oscilloscope tool in FEaT and play two sine notes on the keyboard (voice #352) one half-step apart, first one note at a time. Then play the two notes simultaneously. The two individual notes are sine waves, but when played together they produce a sine wave "within a sine wave envelope". You can hear two things: A tone with a frequency close to the individual frequencies f_1 and f_2, and a slow amplitude variation "beating" with the difference frequency $\Delta f = |f_2 - f_1|$. This difference frequency is the *beat frequency.*

13. Measure the beat frequency by finding the time interval from one node to the next of the wave form on the computer screen. Take the inverse of this time and find the beat frequency $\Delta f =$__________Hz. How is the beat frequency related to $\sqrt[12]{2} = 1.059463$? Does your measurement show this?

Once the A_4 string on the violin is tuned to 440 Hz, the E_5 string, a fifth higher than A_4, is tuned by beats: The 3rd harmonic of A_4, i.e. $E_6 = 1320$Hz, is tuned beatless with the 2nd harmonic of the E_5 string, which again is $E_6 = 1320$ Hz. In the same manner, the D_4 string, which is a fifth down from the A_4 string, is tuned beatless with the A_4 string. Finally, the G_3 string, which is a fifth down from the D_4 string, is tuned beatless with the D_4 string. An experienced string instrument player can easily hear the beats between two strings that are out of tune and thus tune them to be beatless. All strings are tuned by perfect fifths in this way according to the *Pythagorean temperament*. The resulting sound from a string ensemble can be very clean and pleasing. But slight dissonances may arise when string instruments tuned to *Pythagorean temperament* and an *equally-tempered* piano play together.

14. What are the frequencies of the four strings on the violin tuned to Pythagorean temperament, starting with $A_4 = 440$ Hz? Put your entries in Table 4.

Table 4. Frequencies of the four open violin strings in Pythagorean temperament and comparison with equal temperament.

String	Violin Tuned f	Equal Temp. f
G_3		
D_4		
A_4		
E_5		

15. Start with middle $C_4 = 261.63$ Hz and calculate the frequency of E_4 in Pythagorean temperament. (Answer: $E_4 = C_4 \bullet 81/64 = 331.13$ Hz). Select a sine wave on the signal generator tool at this frequency and play it. Then play $E_4 = 329.63$ Hz on the synthesizer keyboard in equal temperament. Do you hear beats between the two notes? What is the expected beat frequency? Use a stop watch, measure the beat frequency, and compare:

$\Delta f_{\text{calculted}} =$ __________ Hz $\Delta f_{\text{measured}} =$ __________ Hz

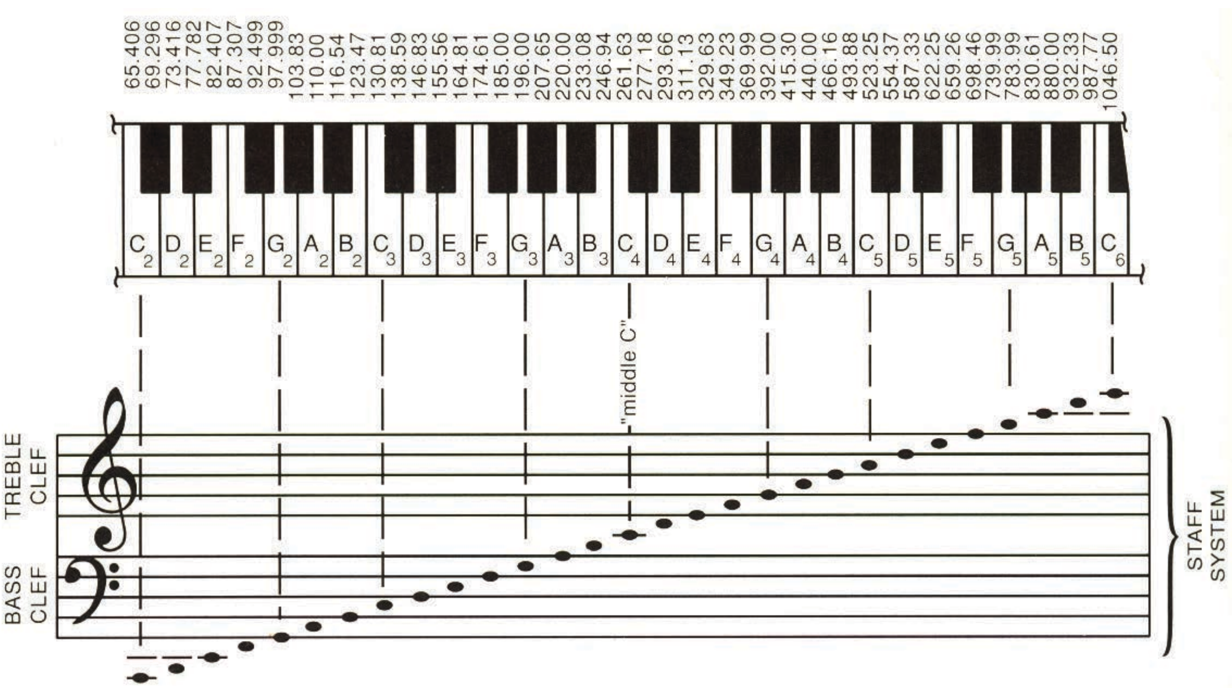
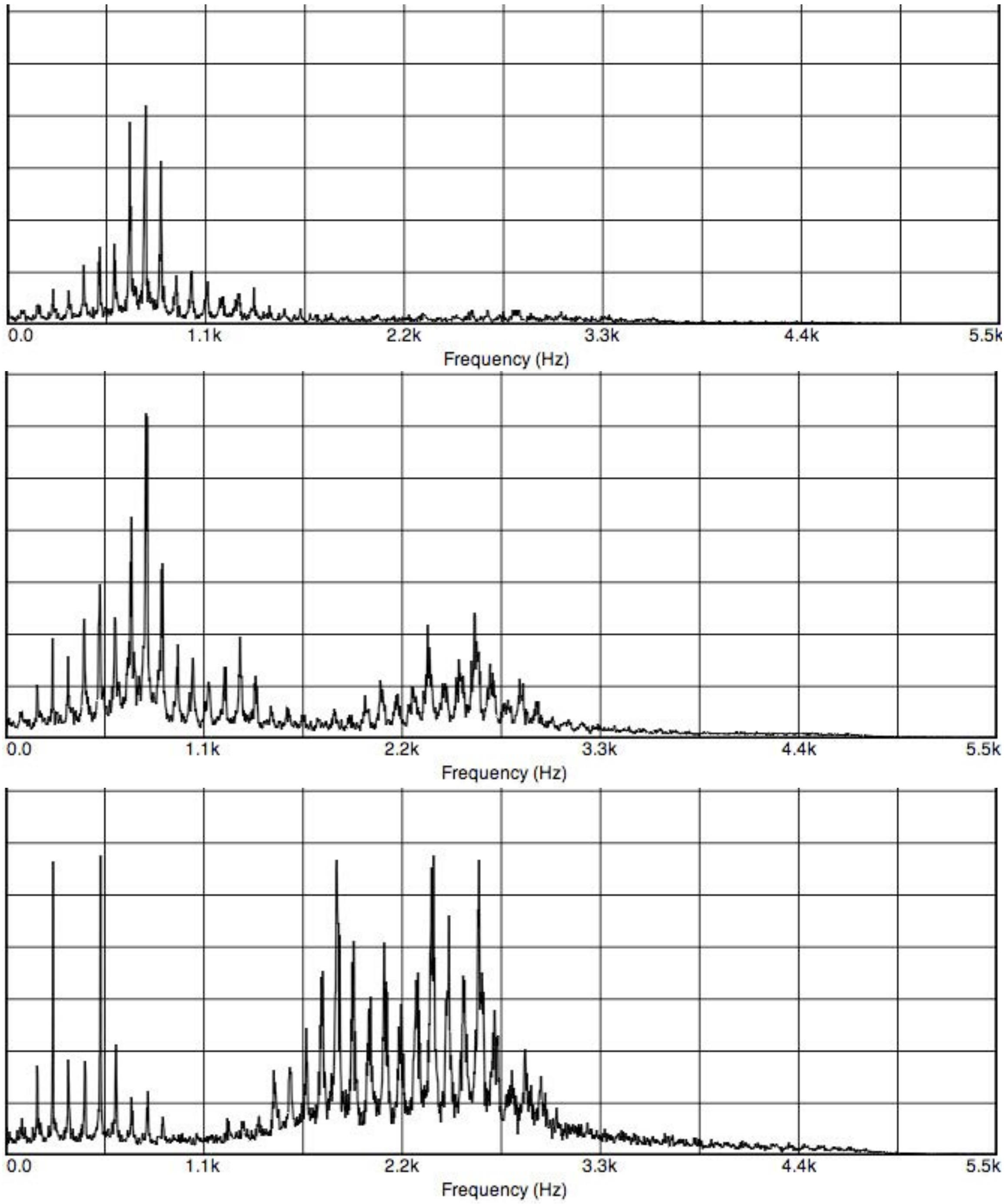
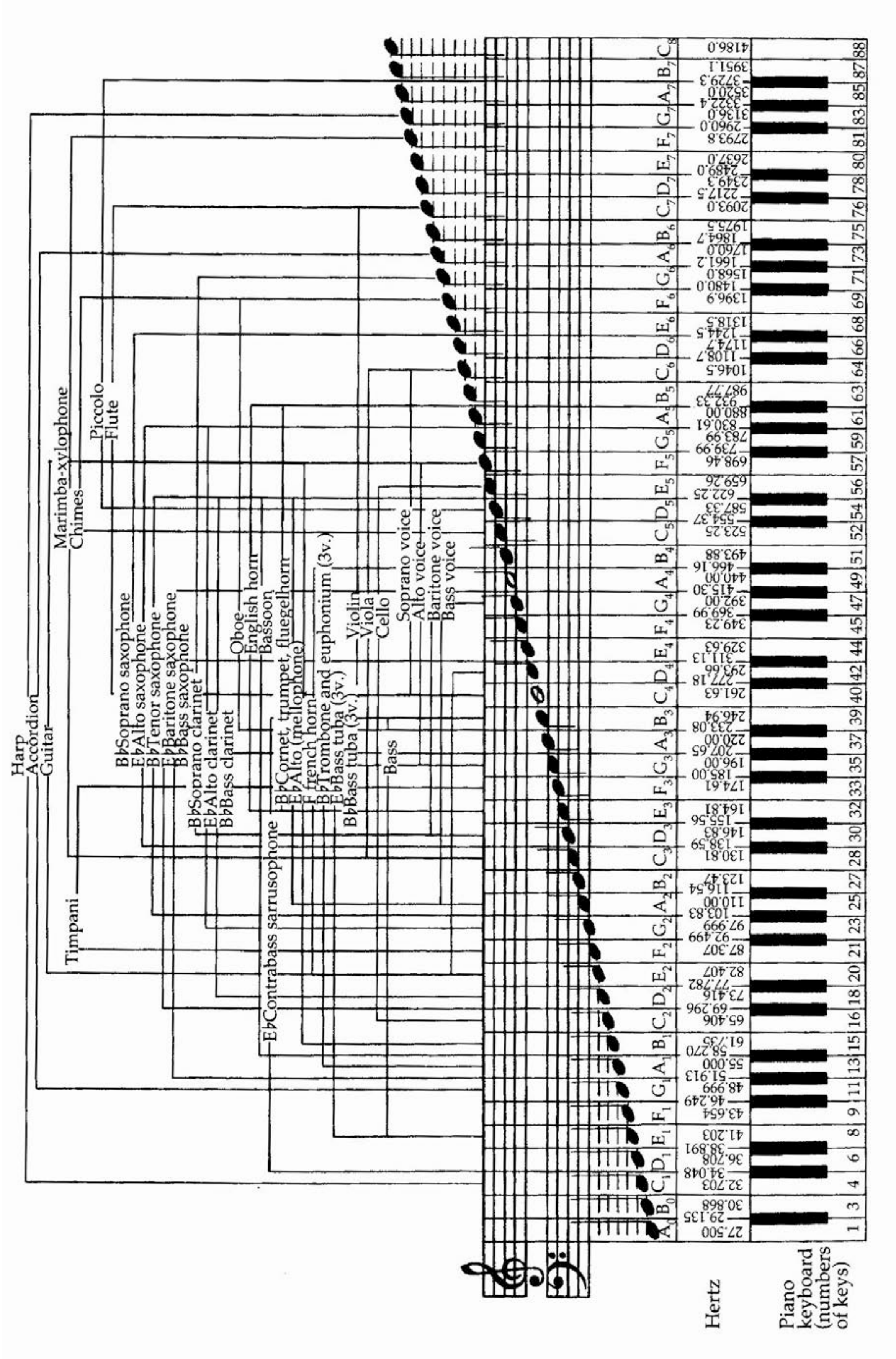


Figure 1. Frequencies of the equal temperament scale (Physics of Sound, Richard E. Berg and David G. Stork)

Sound Spectra and Singing Formants of a Bass-Baritone (Note F2 = 87 Hz)



Voice recorded from Professor Gerald Dolter, Texas Tech University, School of Music.
Top: Vowel sound "ah", controlled shouting, without singing formant.
Middle: Vowel sound "ah", singing formant between 2 and 3 kHz.
Bottom: Vowel sound "eh", very strong singing formants between 1.5 and 3 kHz.



The audible frequency range of various musical instruments and voices. Only the fundamental tones are included; the partials (not shown) extend much higher. Also not shown are the many high-frequency incidental noises produced.

(From: F. Alton Everest & Ken C, Pohlman, Master Handbook of Acoustics, 5th edition, McGraw Hill, 2009, Figure 5-10, p.81.)