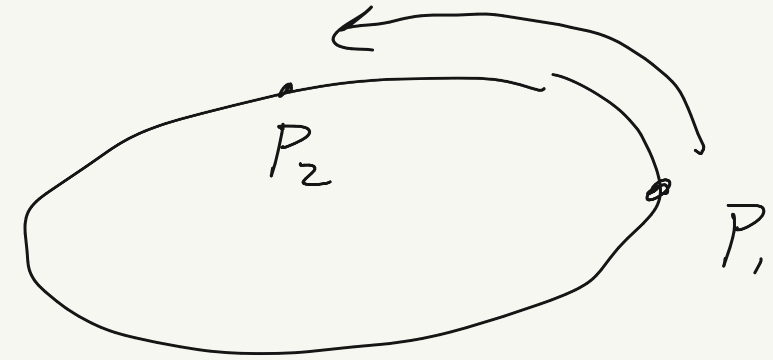
Lecture #1: Aug 24th

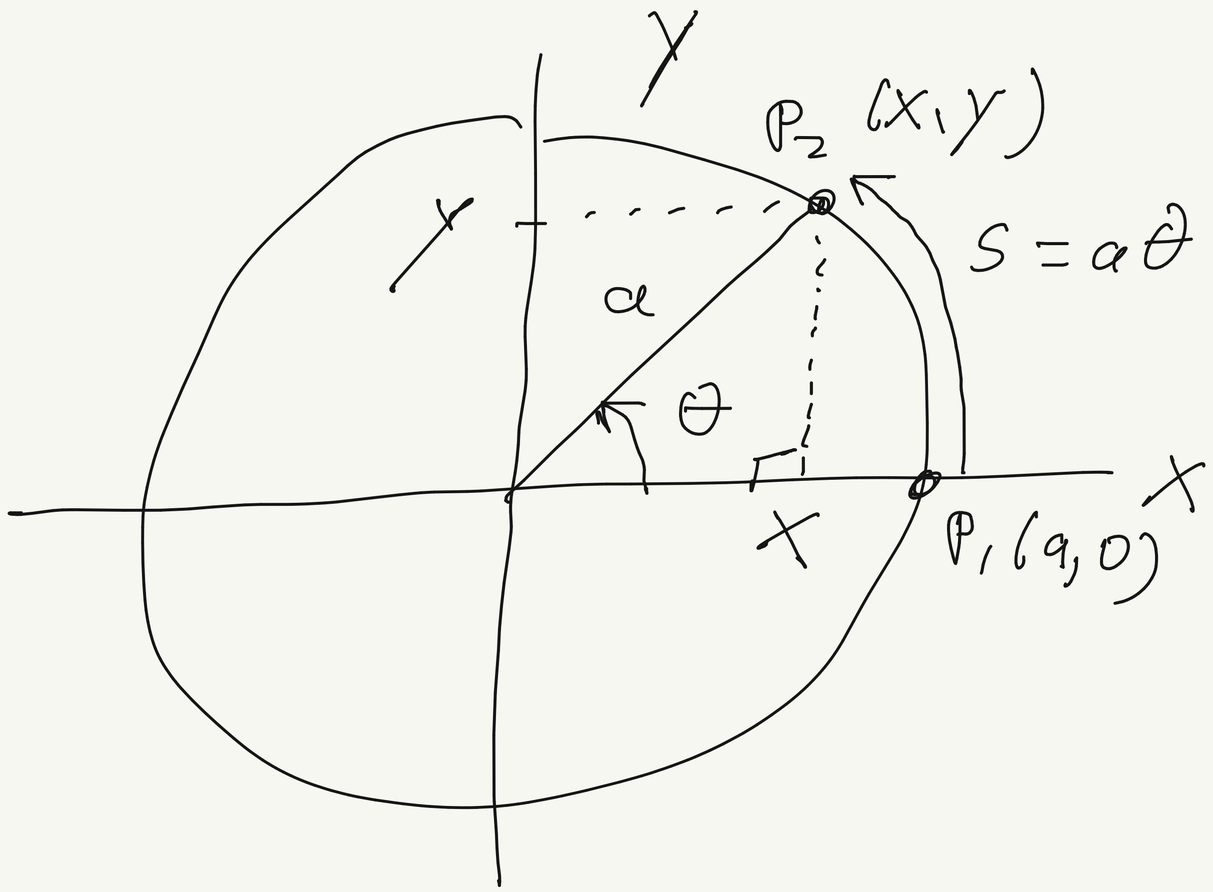
Elliptic functions / integrals:

i) circumference of an ellipse



ii) period of a simple pendulum beyond the small-angle approximation

Circular functions: sines, cosines,



a = radius

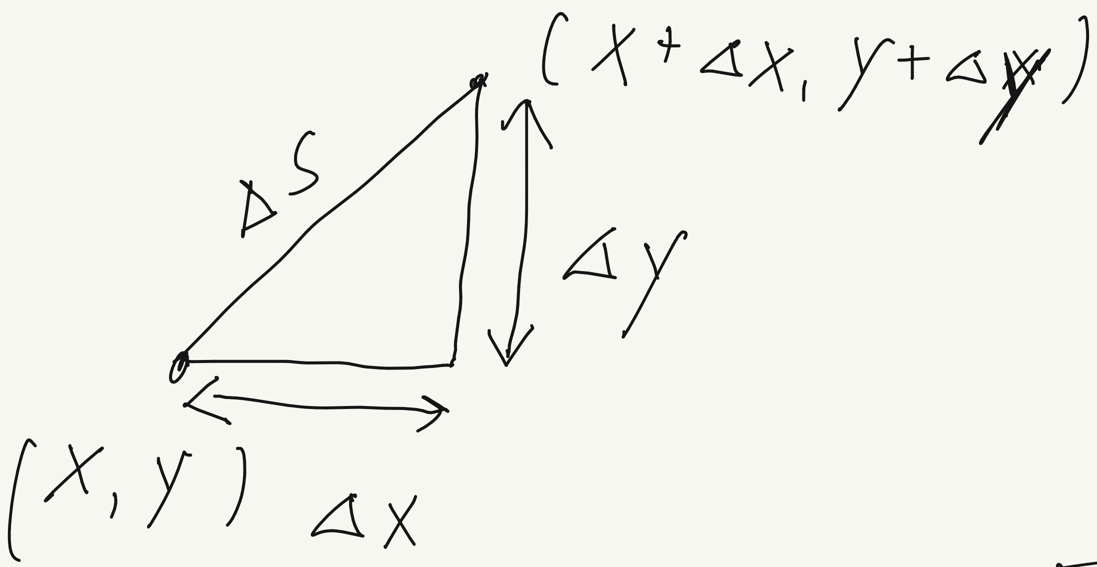
$$x^2 + y^2 = a^2$$

$$\theta = \frac{s}{a}$$

$$= \frac{1}{a}\int^{P_2} \sqrt{dx^2 + dy^2}$$

$$= \int^{P_1} d\theta$$

$$ds^2 = dx^2 + dy^2$$



$$\left.\begin{aligned} \sin\theta &\equiv y/a \\ \cos\theta &\equiv x/a \end{aligned}\right\} \text{definitions}$$

$$x = a\cos\theta$$
$$y = a\sin\theta$$

$$\boxed{x^2 + y^2 = a^2} \rightarrow \cancel{a^2}\cos^2\theta + \cancel{a^2}\sin^2\theta = \cancel{a^2}$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

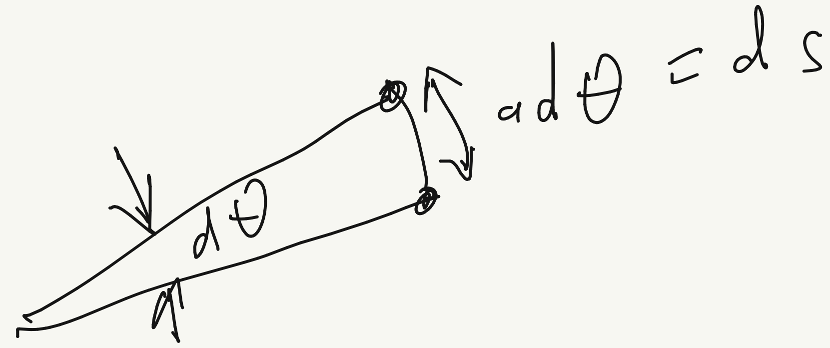
$$\boxed{\frac{d}{d\theta}\sin\theta = \cos\theta}$$

Proof: $\displaystyle \frac{d}{d\theta}\sin\theta = \frac{d}{d\theta}\left(\frac{y}{a}\right)$

$$= \frac{1}{a}\frac{dy}{d\theta}$$

$$= \frac{dy}{\sqrt{dx^2+dy^2}}$$

$$= \frac{1}{\frac{\sqrt{dx^2+dy^2}}{dy}} = \frac{1}{\sqrt{\left(\frac{dx}{dy}\right)^2+1}}$$



$$a\,d\theta = \sqrt{dx^2+dy^2}$$

$$x^2+y^2=a^2 \rightarrow 2x\,dx+2y\,dy=0$$

$$\frac{dx}{dy}=-\frac{y}{x}$$

$$\frac{d}{d\theta}\sin\theta=\frac{1}{\sqrt{\left(\frac{-y}{x}\right)^2+1}}=\frac{1}{\sqrt{\frac{y^2}{x^2}+1}}=\frac{x}{\sqrt{x^2+y^2}}=\frac{x}{a}=\cos\theta$$

$$\boxed{\frac{d}{d\theta}\sin\theta=\cos\theta}$$

Similarly $\boxed{\frac{d\cos\theta}{d\theta}=-\sin\theta}$

$$\int\frac{d(\sin\theta)}{\cos\theta}=\int d\theta=\theta+\text{const}$$

$t=\sin\theta$

$$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-t^2}$$

$$\boxed{\int\frac{dt}{\sqrt{1-t^2}}=\theta+\text{const}=\sin^{-1}t+\text{const}}$$