Metronome-microphone demo of a pulsar timing array

Joe Romano
Les Houches Summer School
25 July 2018

(work in collaboration with M. Lam, M. Normandin, J. Key, and J. Hazboun)

code and sample data: https://github.com/josephromano/leshouches/tree/master/pta-demo
paper (draft): https://github.com/josephromano/leshouches/blob/master/pta-demo/manuscript/pta-demo.pdf

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    - timing models (for calculating expected times-of-arrivals)

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    - matched filtering (for calculating measured times-of-arrivals)
    - timing models (for calculating expected times-of-arrivals)
    - correlation analyses (for extracting common GW component)

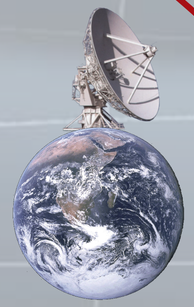
Pulsar timing array

GWs cause pulses to arrive ahead or behind schedule, correlated across pulsars

pulsar

$$\delta\tau(t) = \frac{1}{c} u^a u^b \int h_{ab}(t(s), \vec{x}(s))\, \mathrm{d}s$$

radio telescope



pulsar

Metronome timing array

metronome

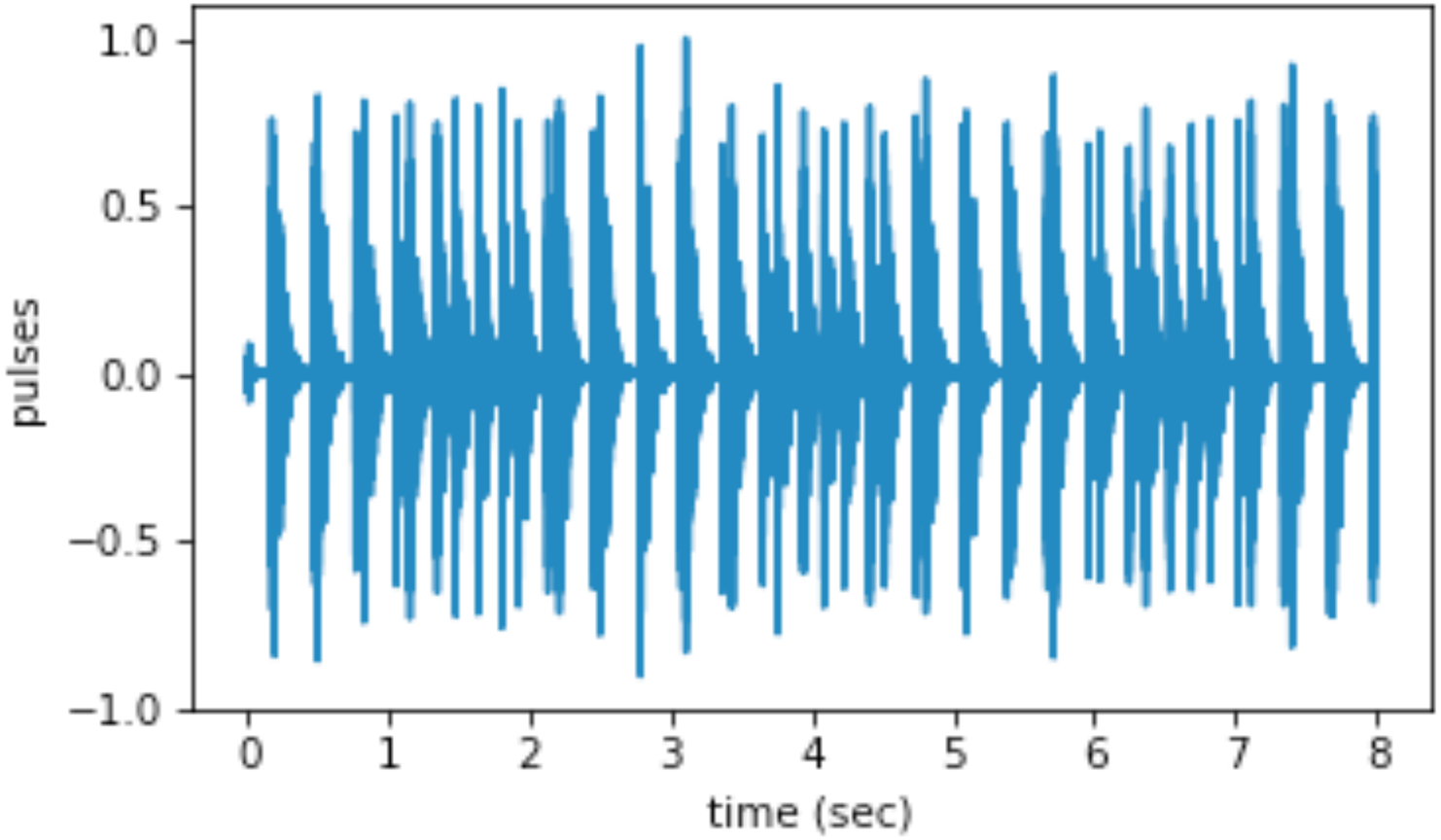
microphone motion
causes pulses to arrive
ahead or behind schedule,
correlated across metronomes

$$\delta\tau(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{1}{c_s}\hat{u}\cdot\vec{r}(t)$$

microphone

metronome

Q: Is there evidence of a “GW” in the data?



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$$\rho_{12} \equiv \langle x_1 x_2 \rangle / \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \qquad \langle x_1 x_2 \rangle \equiv \frac{1}{T_{\rm obs}} \int_0^{T_{\rm obs}} {\rm d}t \; x_1(t) x_2(t)$$

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$$\tau^{\rm expected}[i] = \tau^{\rm measured}[i_0] + (i - i_0) T_{\rm p}$$

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$$C(\Delta t) = \mathcal{N} \int {\rm d}t \; y(t) p(t - \Delta t)$$

- expected TOAs: timing model using pulse period and reference TOA

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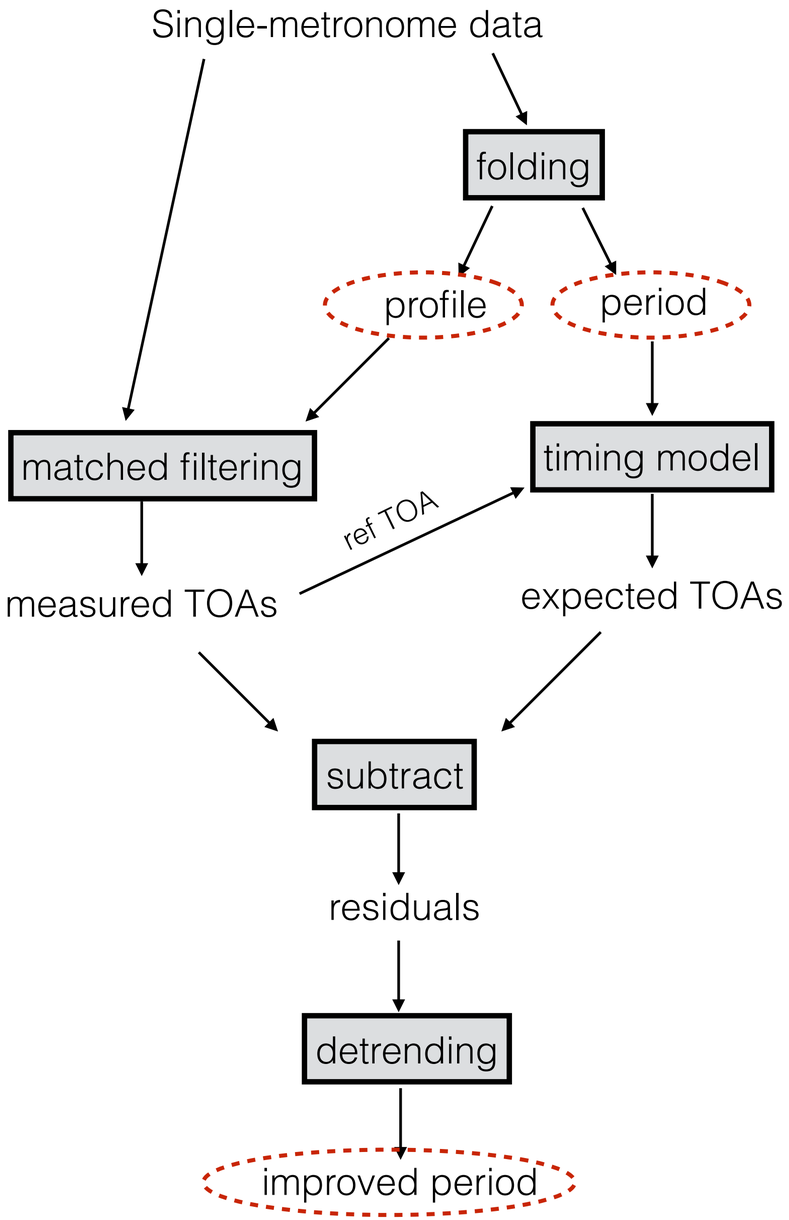
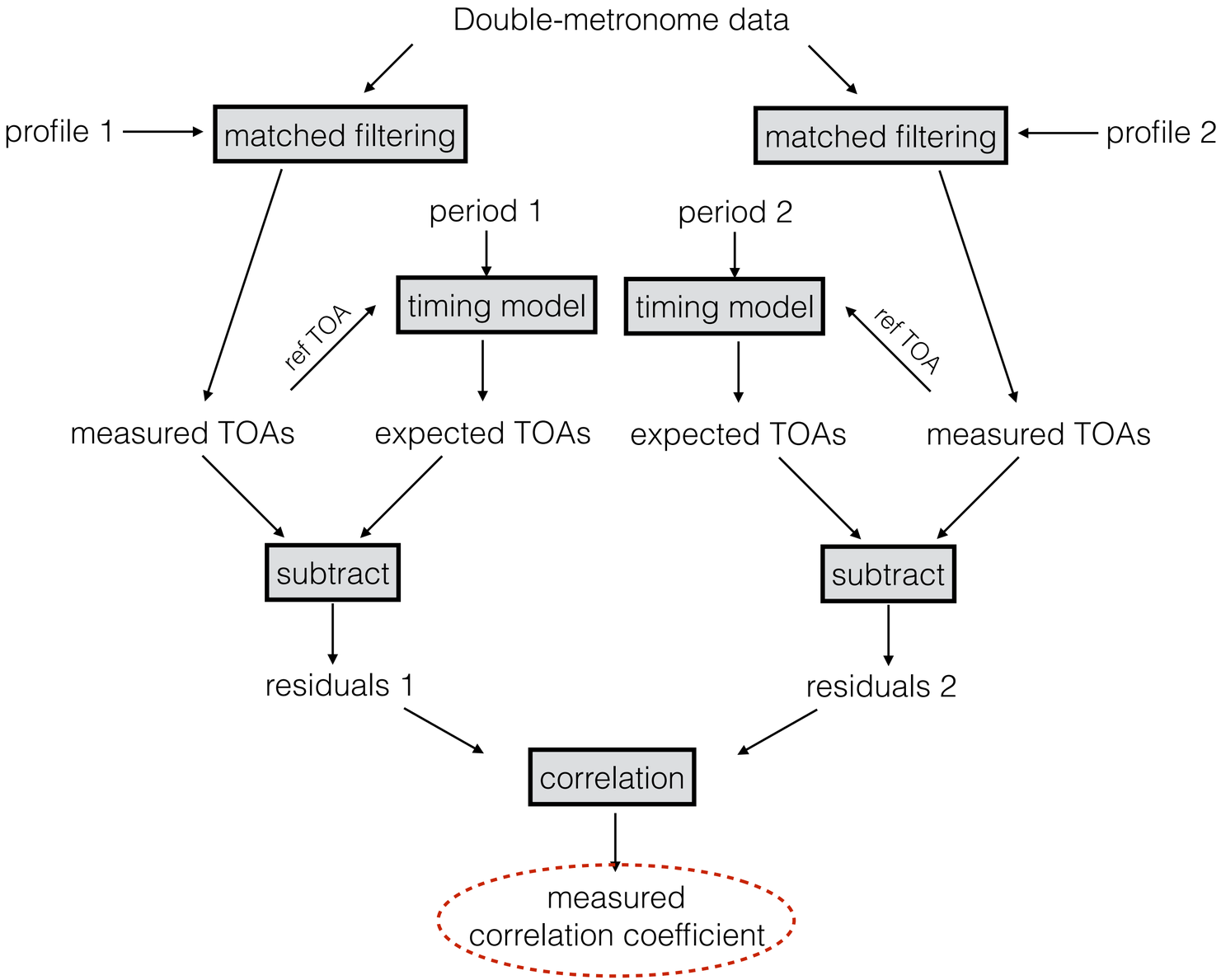
- pulse profile, period: folding and detrending single-metronome data
- expected “GW” correlation (for uniform circular motion): $\rho_{12} \simeq \cos\zeta$

Single-metronome analysis



Double-metronome analysis



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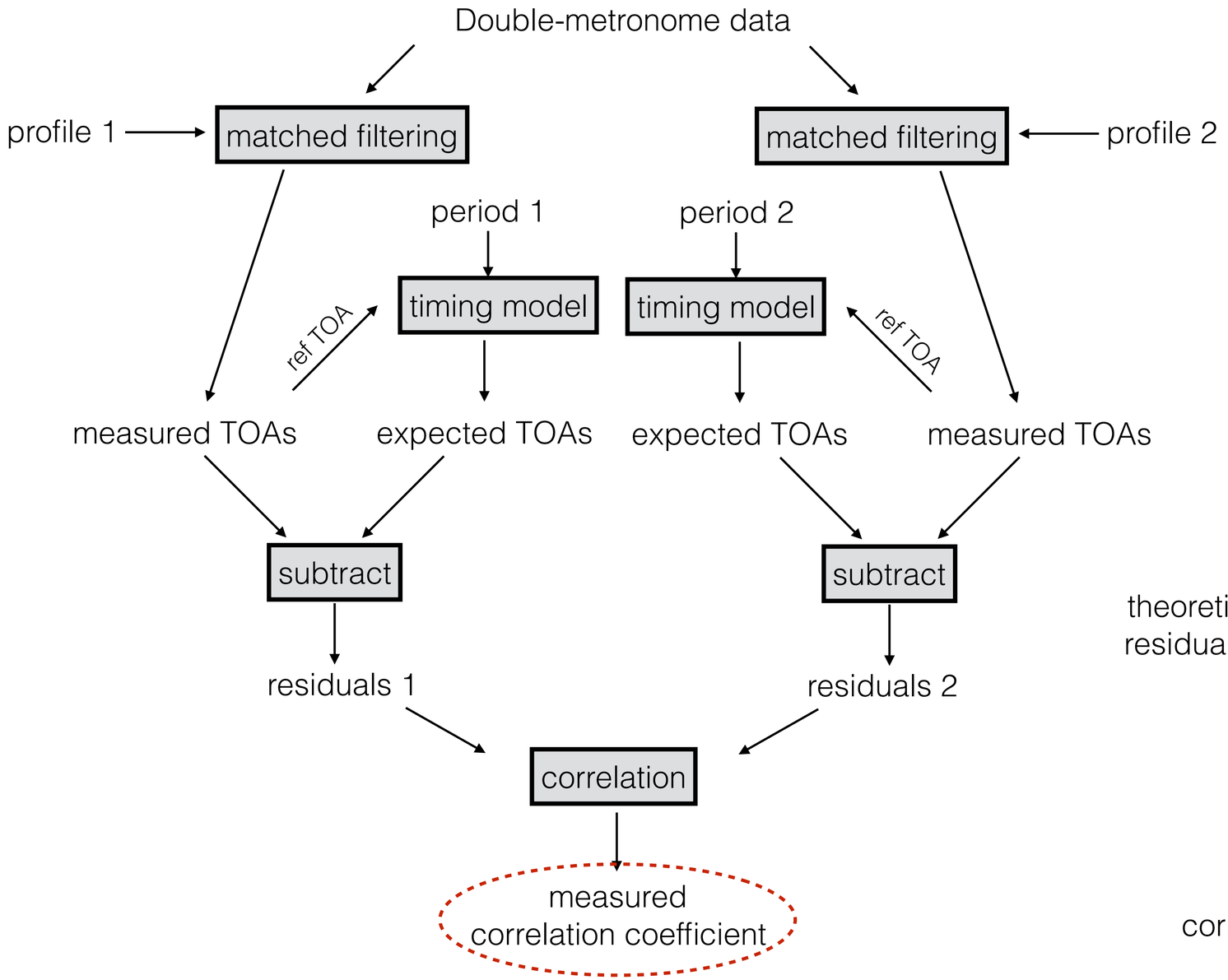
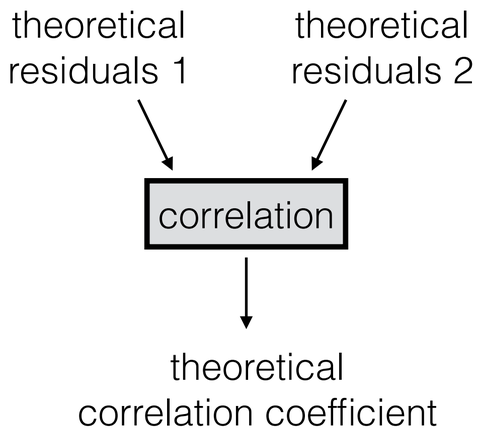
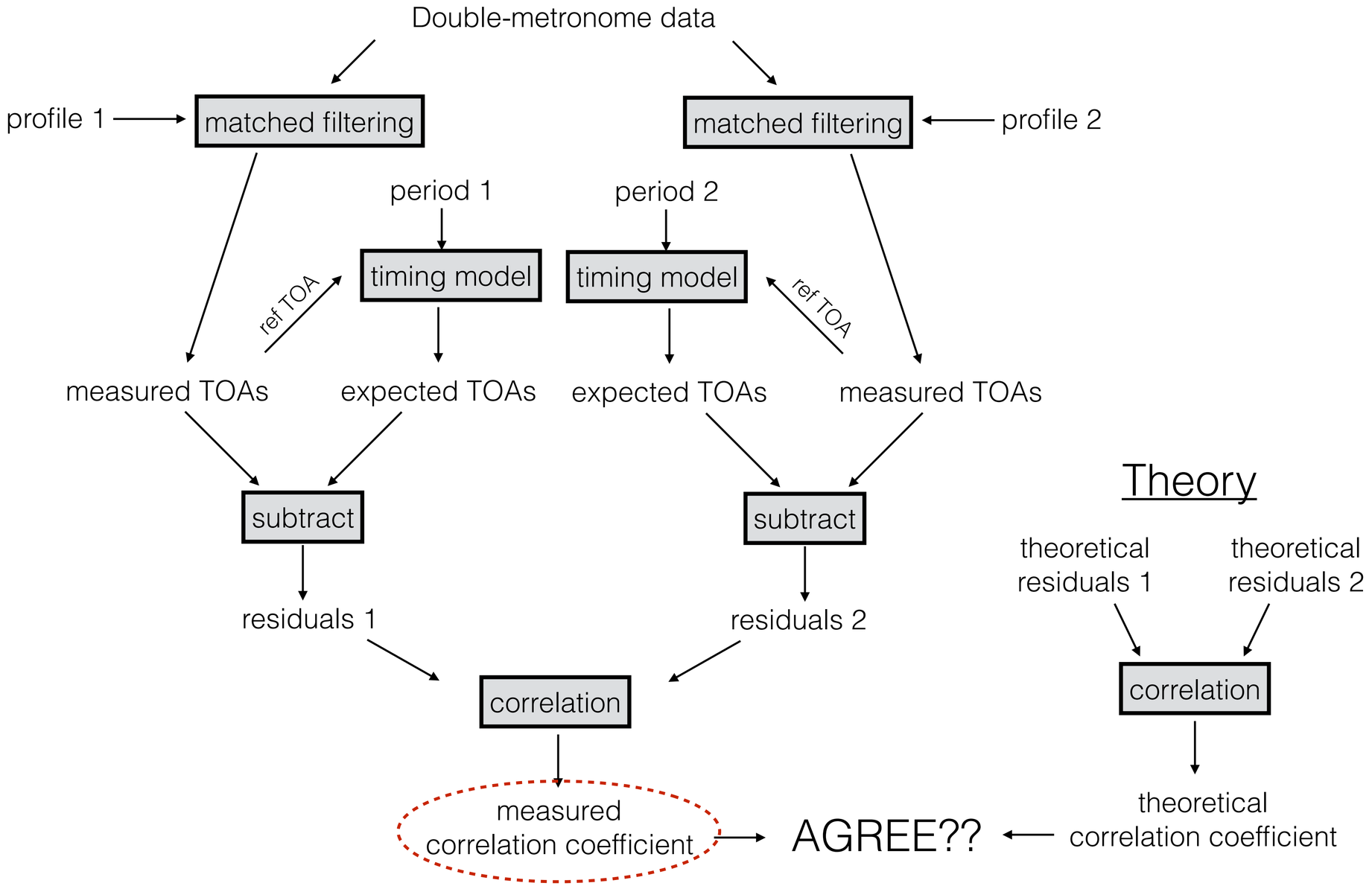


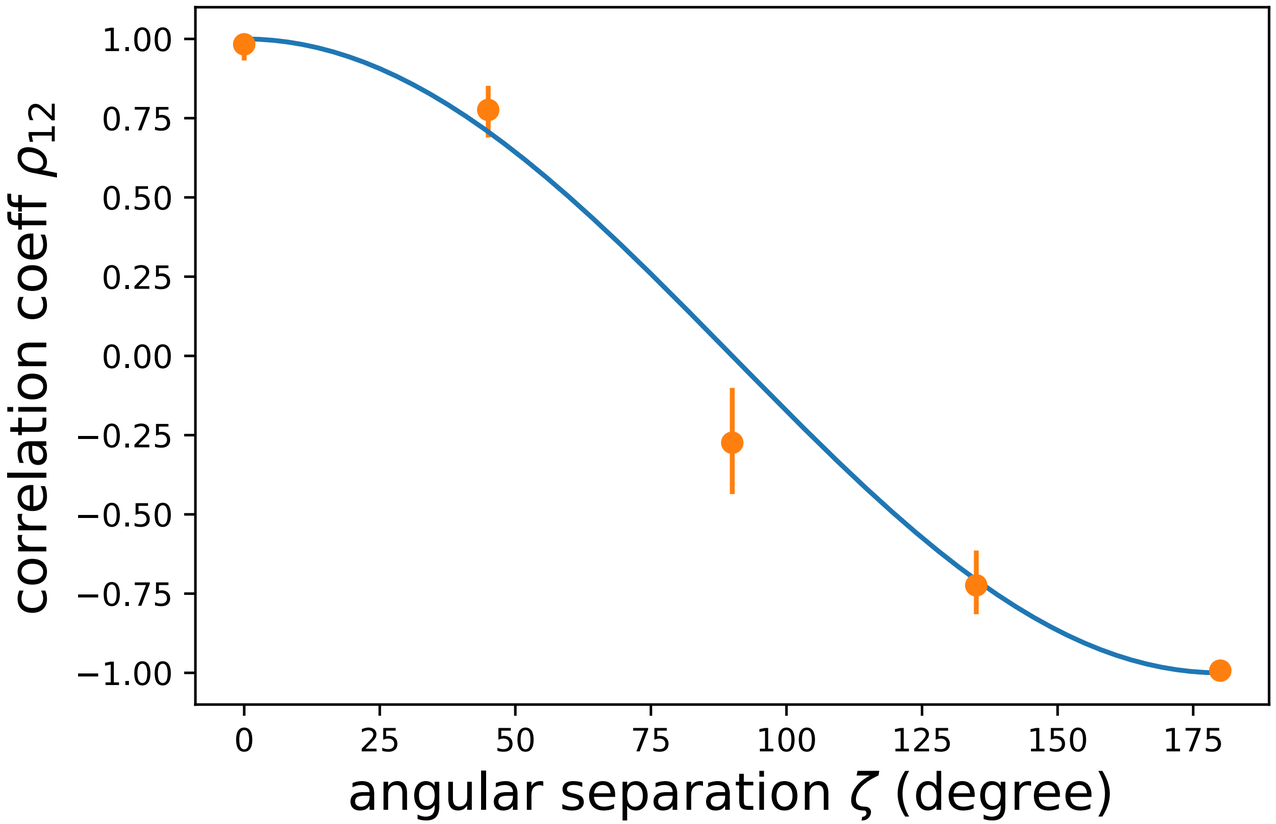
Theory



Double-metronome analysis



Metronome correlation

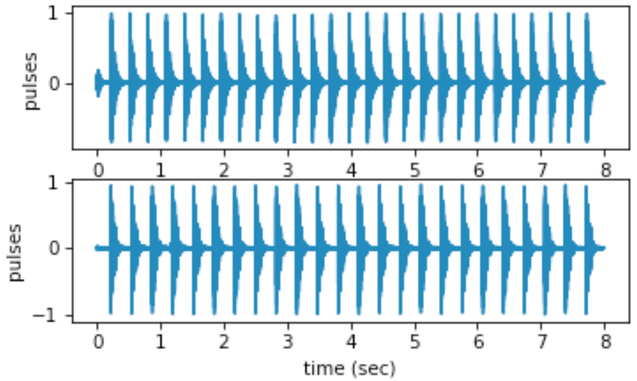


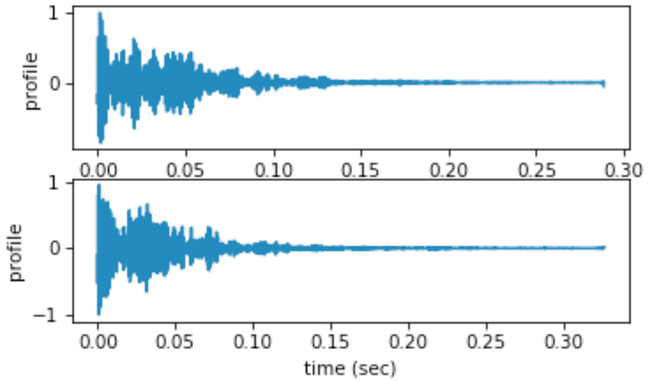
Output of the GUIs

Single-metronome analysis

Single-Metronome Pulse Analysis

Single-Metronome Pulse Analysis

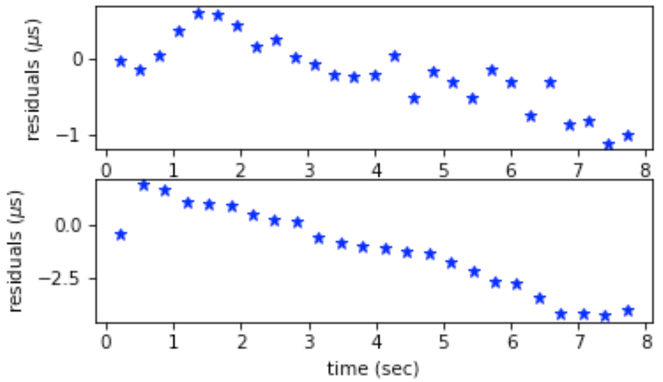




PULSE DATA FILENAMES

Metronome 1: m208a bpm: 208

Metronome 2: m184b bpm: 184



Metronome 1: Record pulses | Playback pulses | Calculate profile | Pulse period [s]: 0.28856797860073924 | Calculate residuals | Detrend residuals

Metronome 2: Record pulses | Playback pulses | Calculate profile | Pulse period [s]: 0.3260988289104562 | Calculate residuals | Detrend residuals

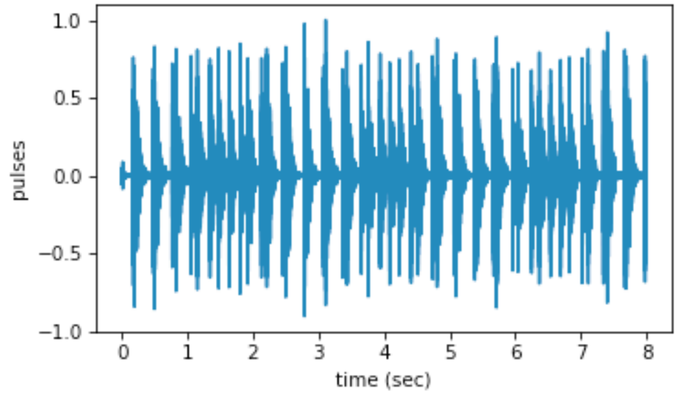
Status of calculations:

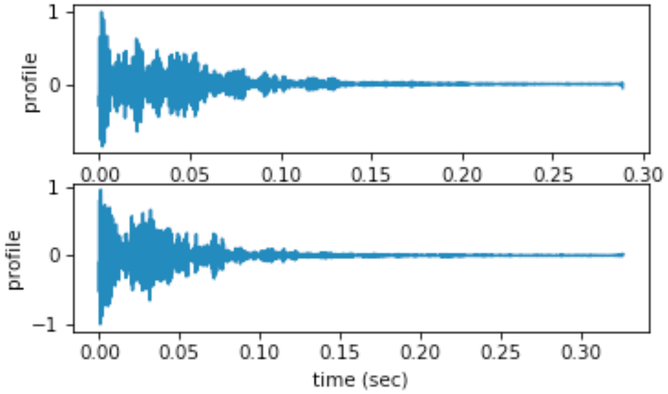
finished calculating residuals

Double-metronome analysis (ζ=0 degrees)

Double-Metronome Pulse Analysis

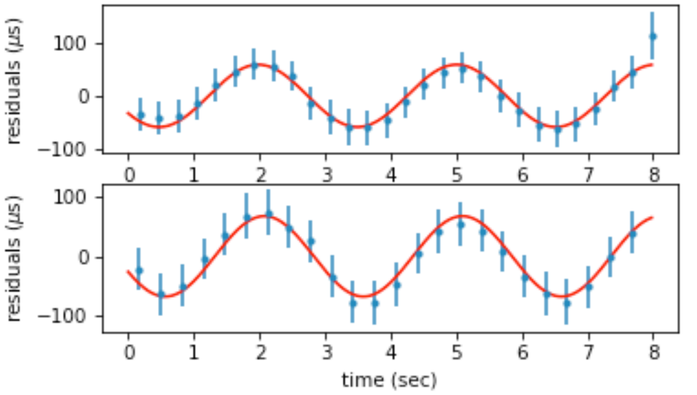
Double-Metronome Pulse Analysis





FILENAMES

Data file:	m208a184b0		
Profile 1:	m208a_profile	Pulse period [s]:	0.288568
Profile 2:	m184b_profile	Pulse period [s]:	0.3260988



	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-58.34831246716732	100	-67.83561046171864
Freq [Hz]:	0.4	0.33072788592748953	0.4	0.33116191414239915
Phase [rad]:	0	0.5924453954483324	0	0.393928097921481
Offset [usec]:	0	-43.4892386052206	0	1.7106663283346102

Record pulses | Playback pulses | Load pulse profiles | Calculate residuals | Fit sinusoids & remove offsets | Calculate corr coeff | 0.9830009544088217

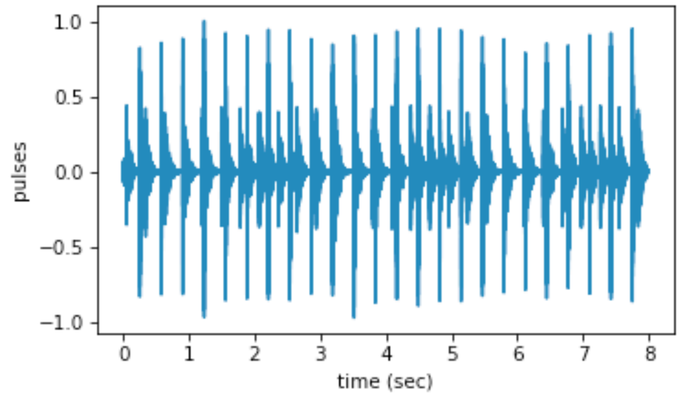
Status of calculations:

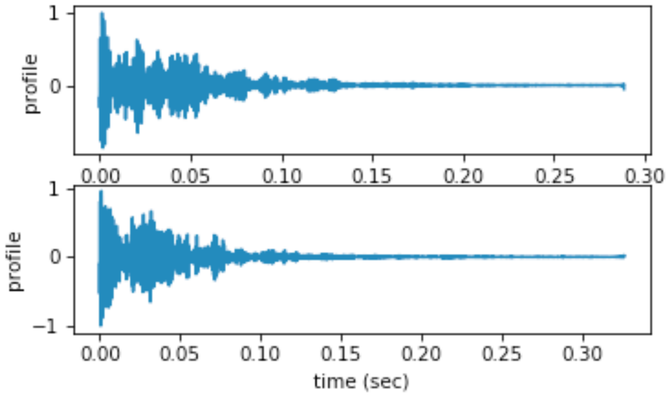
finished calculation of residuals

Double-metronome analysis (ζ=45 degrees)

Double-Metronome Pulse Analysis

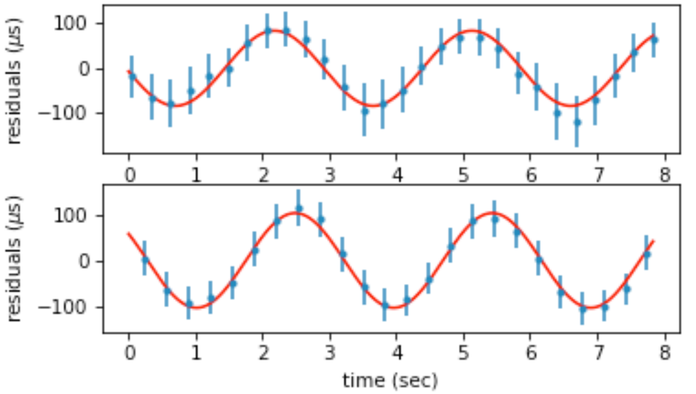
Double-Metronome Pulse Analysis





FILENAMES

Data file:	m208a184b45		
Profile 1:	m208a_profile	Pulse period [s]:	0.288568
Profile 2:	m184b_profile	Pulse period [s]:	0.3260988



	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-83.54267144872672	100	103.51040350638623
Freq [Hz]:	0.4	0.33844784664075234	0.4	0.3394604420911975
Phase [rad]:	0	0.082340180957162	0	2.544467981756352
Offset [usec]:	0	-83.59186463058177	0	62.255379154251095

Record pulses | Playback pulses | Load pulse profiles | Calculate residuals | Fit sinusoids & remove offsets | Calculate corr coeff | 0.7762191706588184

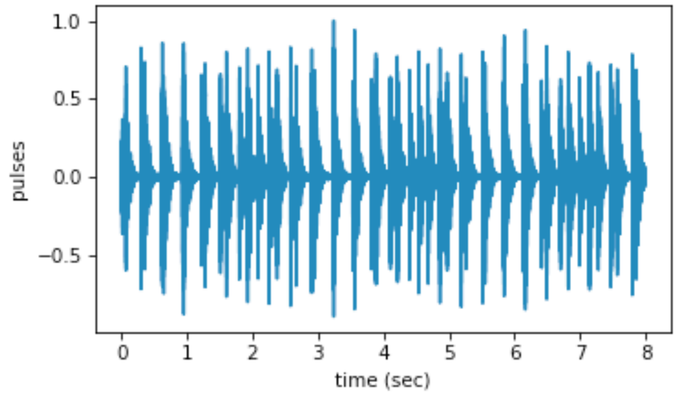
Status of calculations:

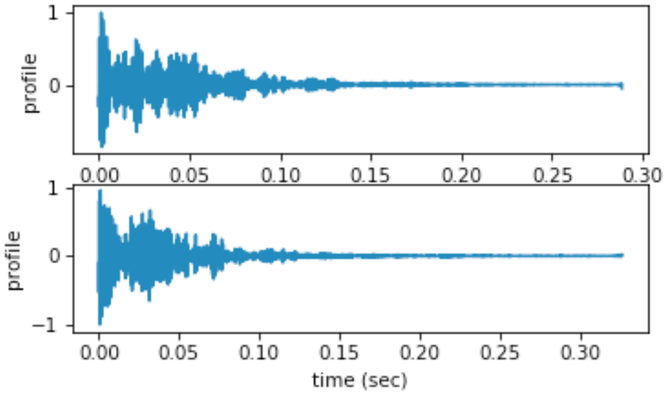
finished calculation of residuals

Double-metronome analysis (ζ=90 degrees)

Double-Metronome Pulse Analysis

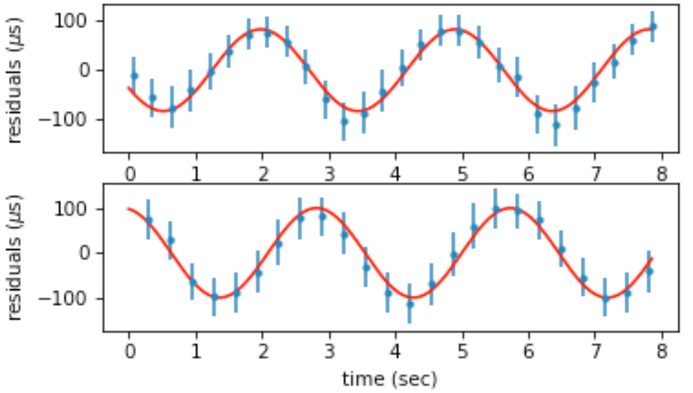
Double-Metronome Pulse Analysis





FILENAMES

Data file:	m208a184b90		
Profile 1:	m208a_profile	Pulse period [s]:	0.288568
Profile 2:	m184b_profile	Pulse period [s]:	0.3260988



	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-83.2204679782364	100	100.56846337609925
Freq [Hz]:	0.4	0.34235401761876616	0.4	0.3435231659876081
Phase [rad]:	0	0.45096522793443056	0	1.7673131445900625
Offset [usec]:	0	-88.73206094670937	0	-95.5978511142905

Record pulses | Playback pulses | Load pulse profiles | Calculate residuals | Fit sinusoids & remove offsets | Calculate corr coeff | -0.27377904355571664

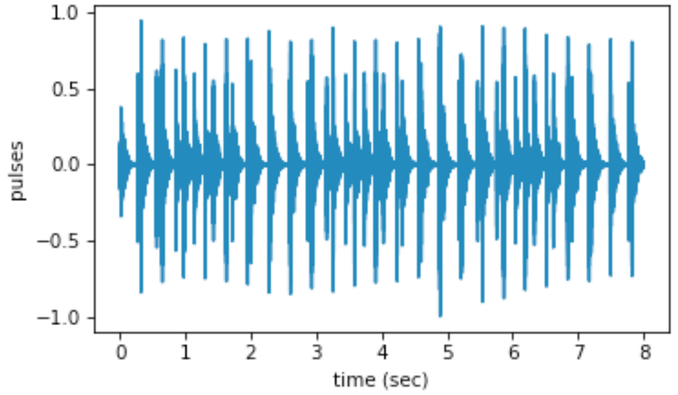
Status of calculations:

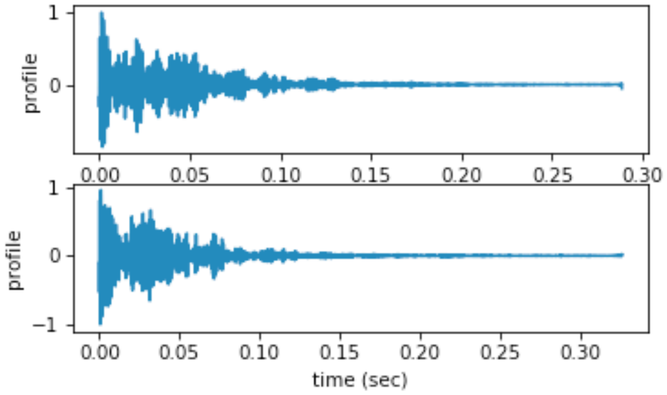
finished calculation of residuals

Double-metronome analysis (ζ=135 degrees)

Double-Metronome Pulse Analysis

Double-Metronome Pulse Analysis

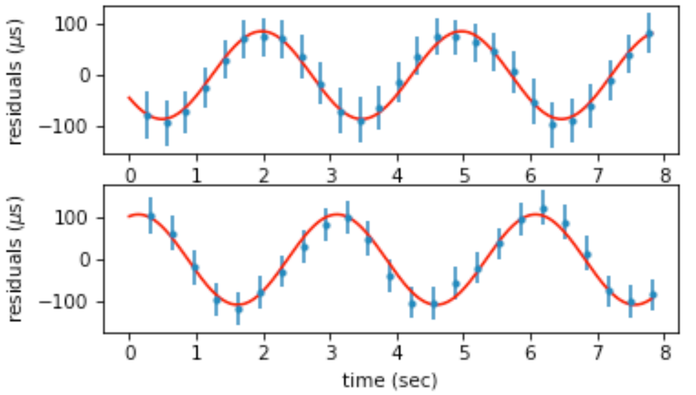




FILENAMES

Data file:	m208a184b135		
Profile 1:	m208a_profile	Pulse period [s]:	0.288568
Profile 2:	m184b_profile	Pulse period [s]:	0.3260988

	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-84.86534557710291	100	108.66951909243878
Freq [Hz]:	0.4	0.3346925061909805	0.4	0.33651410106421914
Phase [rad]:	0	0.5396320788948572	0	1.280777350217575
Offset [usec]:	0	-73.02853234363745	0	-39.39107598066573



Record pulses | Playback pulses | Load pulse profiles | Calculate residuals | Fit sinusoids & remove offsets | Calculate corr coeff | -0.7243635174401307

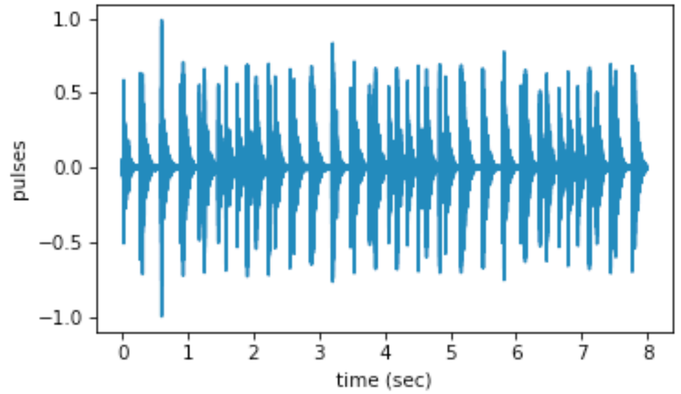
Status of calculations:

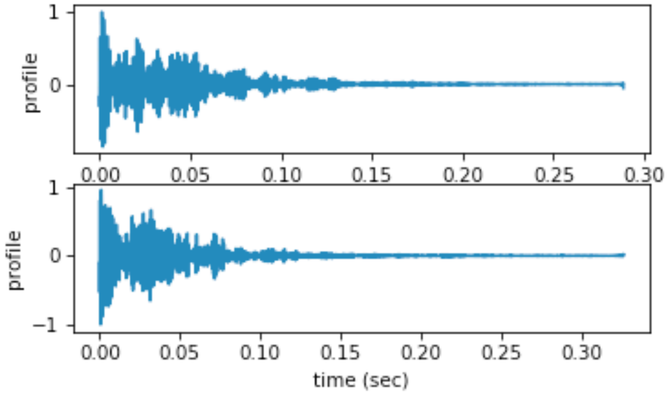
finished calculation of residuals

Double-metronome analysis (ζ=180 degrees)

Double-Metronome Pulse Analysis

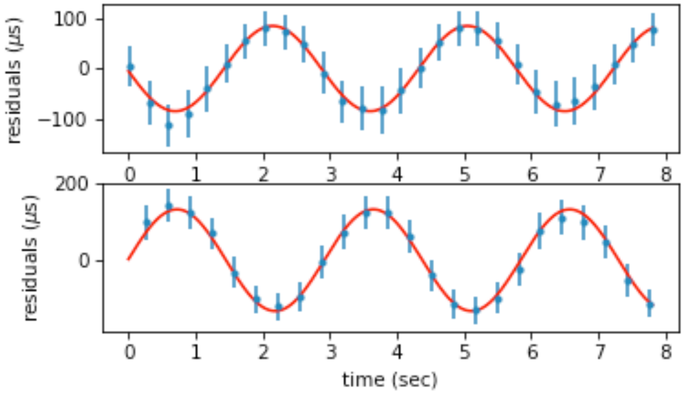
Double-Metronome Pulse Analysis





FILENAMES

Data file:	m208a184b180		
Profile 1:	m208a_profile	Pulse period [s]:	0.288568
Profile 2:	m184b_profile	Pulse period [s]:	0.3260988



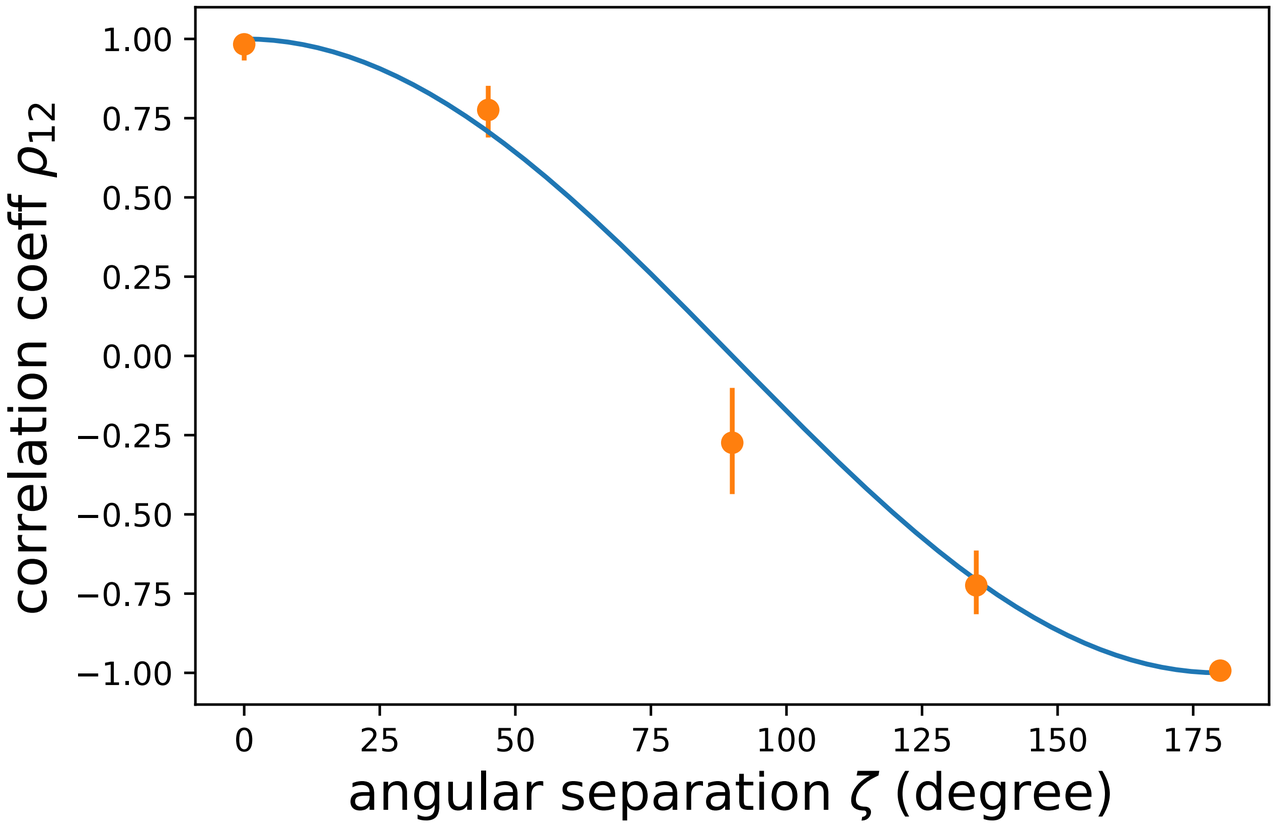
	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-83.98934169522346	100	133.24972774852253
Freq [Hz]:	0.4	0.34452084565954305	0.4	0.3417497197625243
Phase [rad]:	0	0.06170557894833124	0	0.020132004280030594
Offset [usec]:	0	-78.91289452704453	0	122.28656574924301

Record pulses | Playback pulses | Load pulse profiles | Calculate residuals | Fit sinusoids & remove offsets | Calculate corr coeff | -0.9928054492621784

Status of calculations:

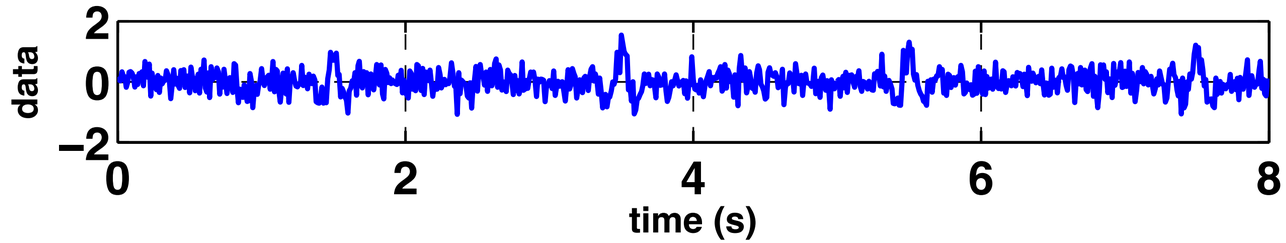
finished calculation of residuals

Metronome correlation

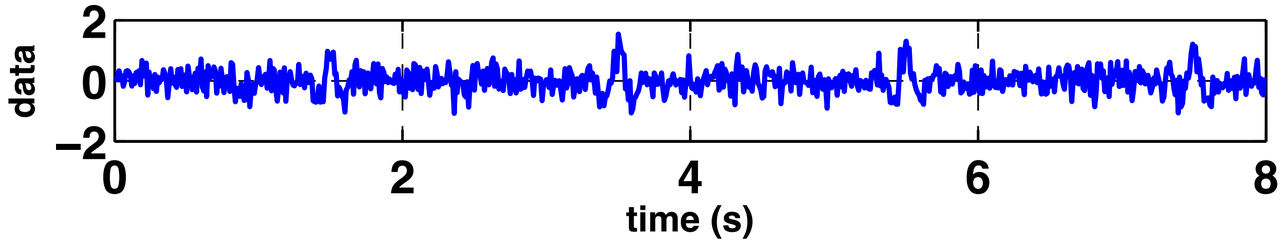


More details

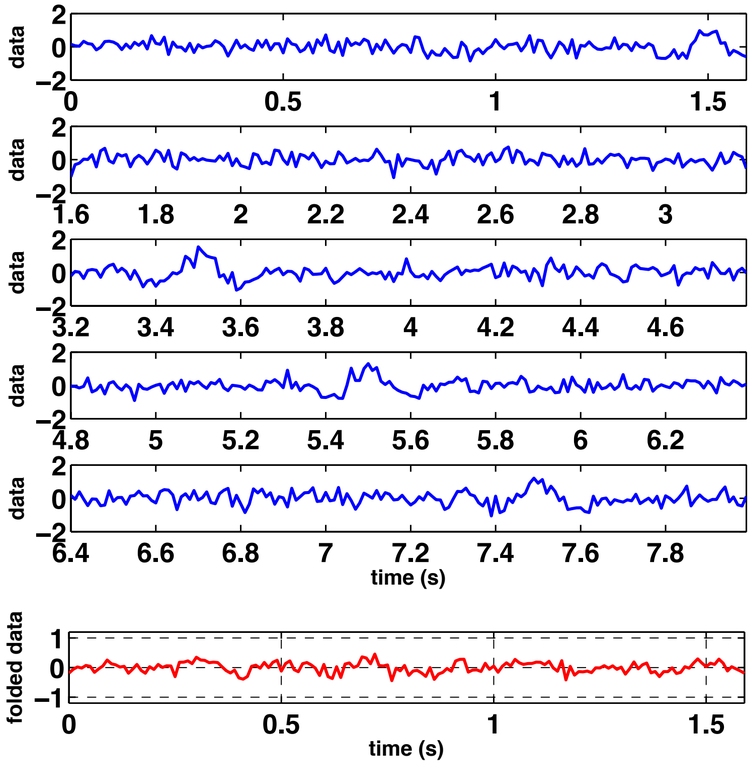
“Fold” data to determine pulse period and pulse profile



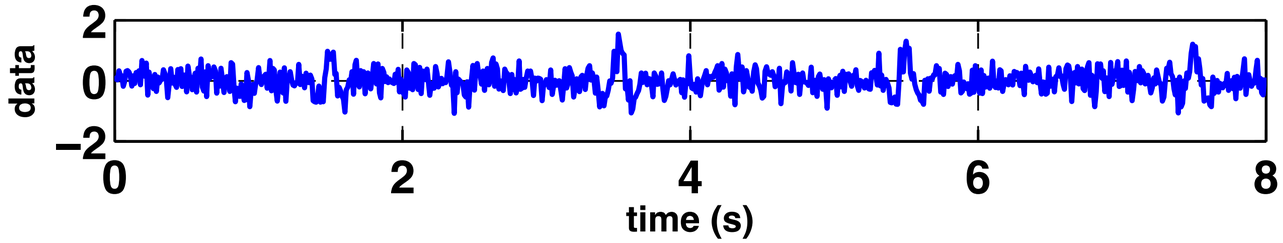
“Fold” data to determine pulse period and pulse profile



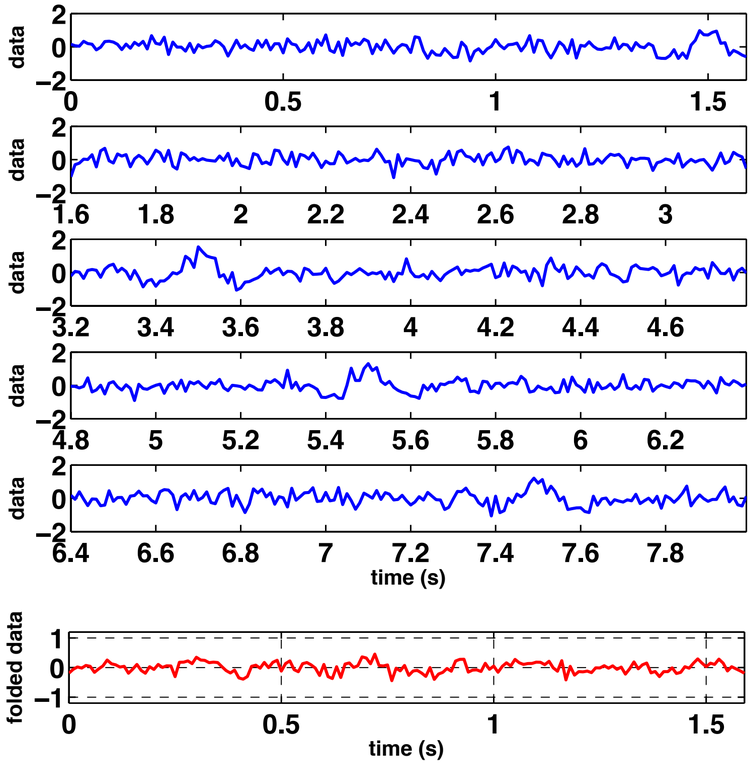
fold with incorrect T_p



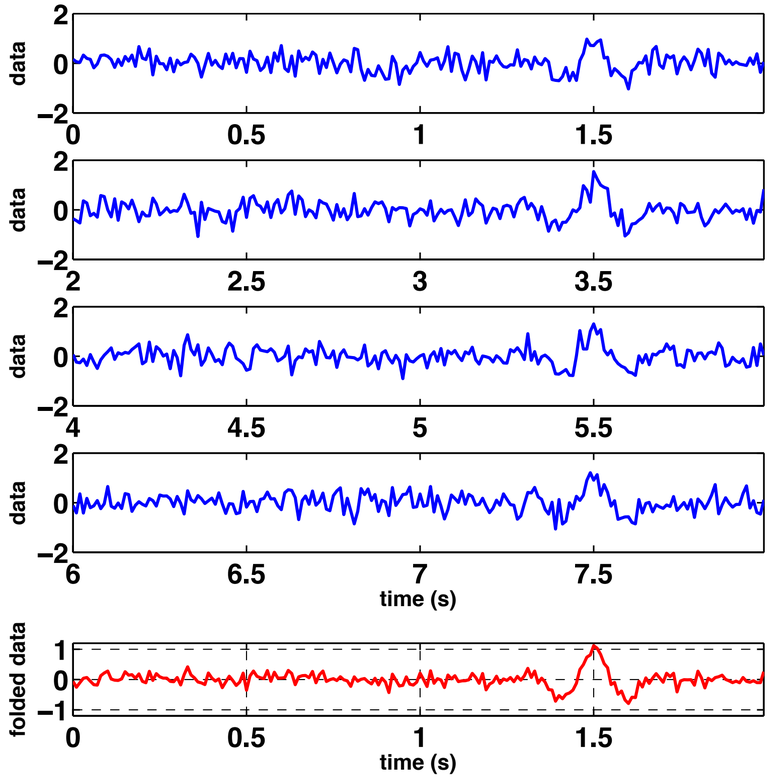
“Fold” data to determine pulse period and pulse profile



fold with incorrect T_p



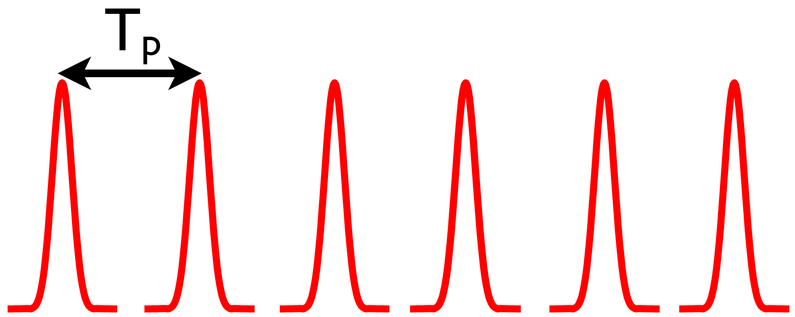
fold with correct T_p



Timing model

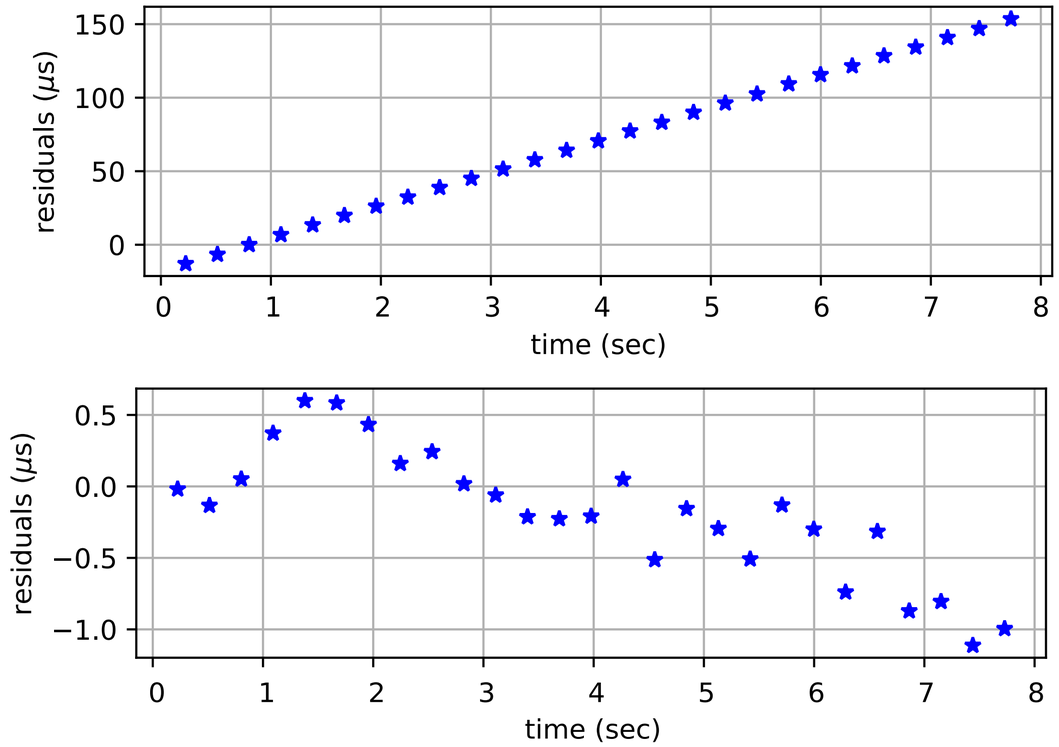
Pulses should arrive regularly with period T_p relative to some reference pulse

$$\tau^{\mathrm{expected}}[i] = \tau^{\mathrm{measured}}[i_0] + (i - i_0)T_\mathrm{p}$$



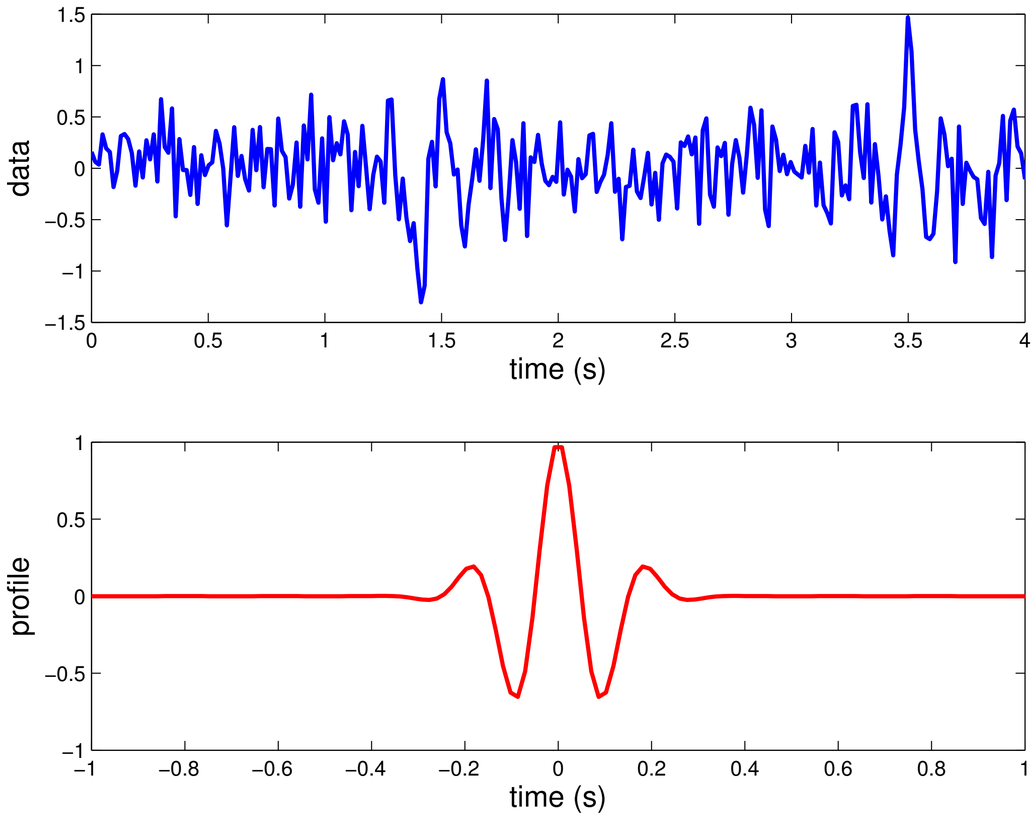
Remove linear trend to more accurately determine pulse period

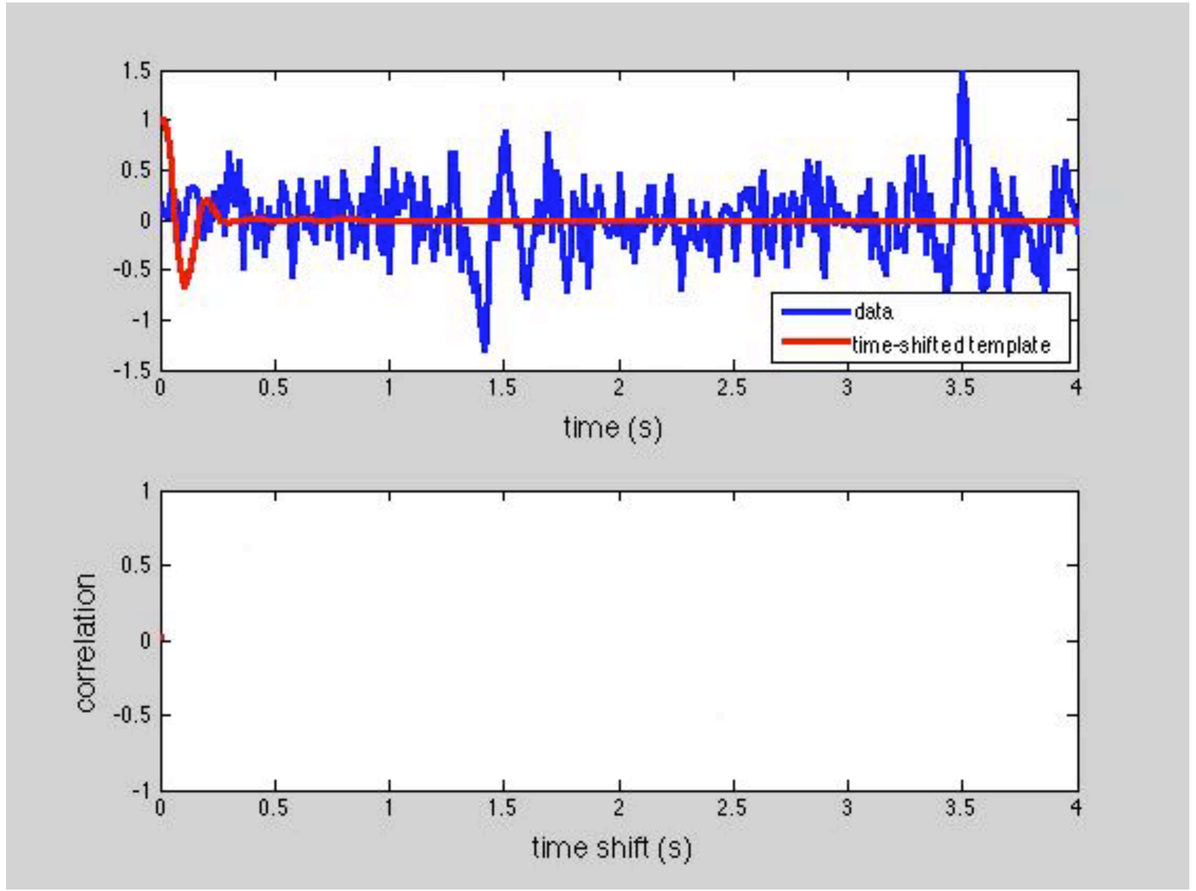
$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$



Matched-filtering determination of measured TOAs

$$C(\Delta t) = \mathcal{N} \int \mathrm{d}t \; y(t) p(t - \Delta t)$$





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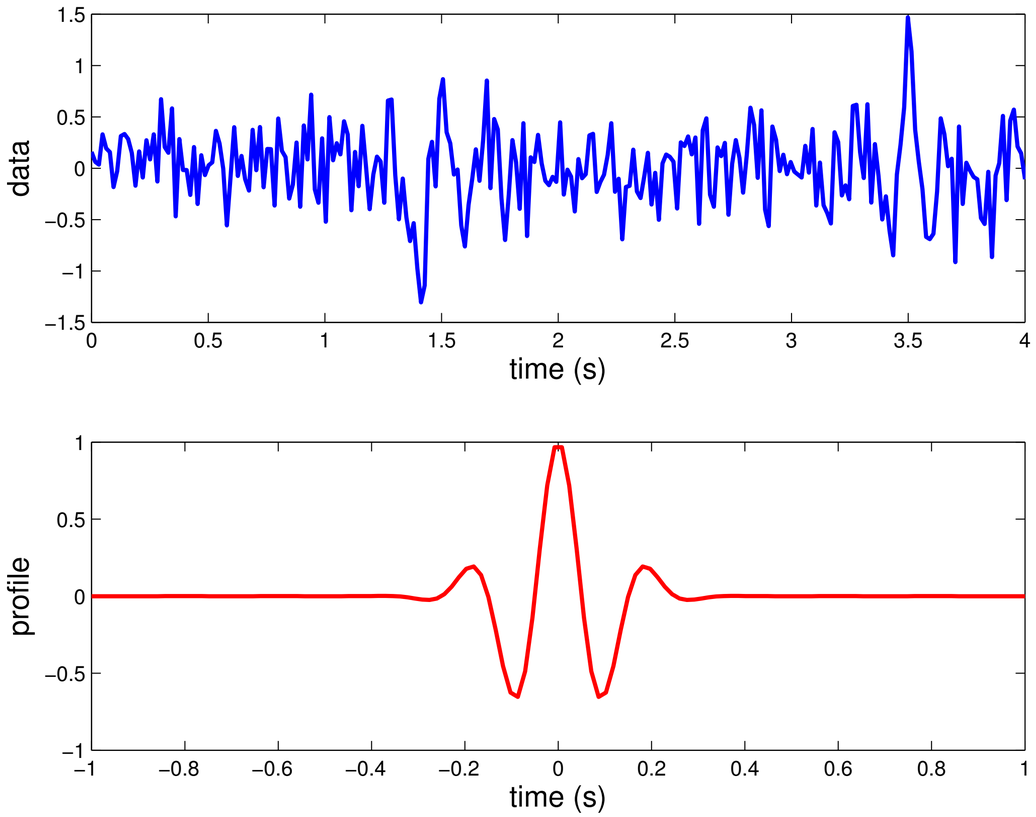
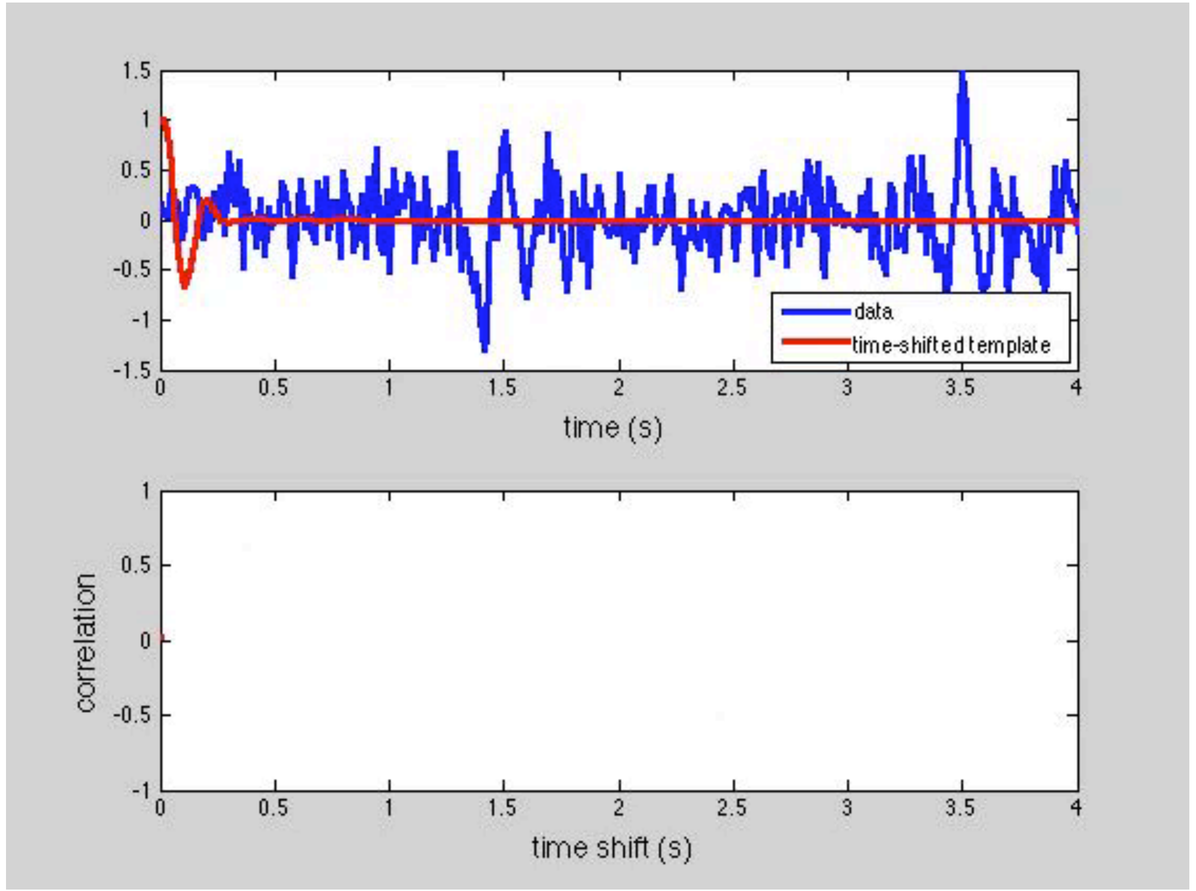
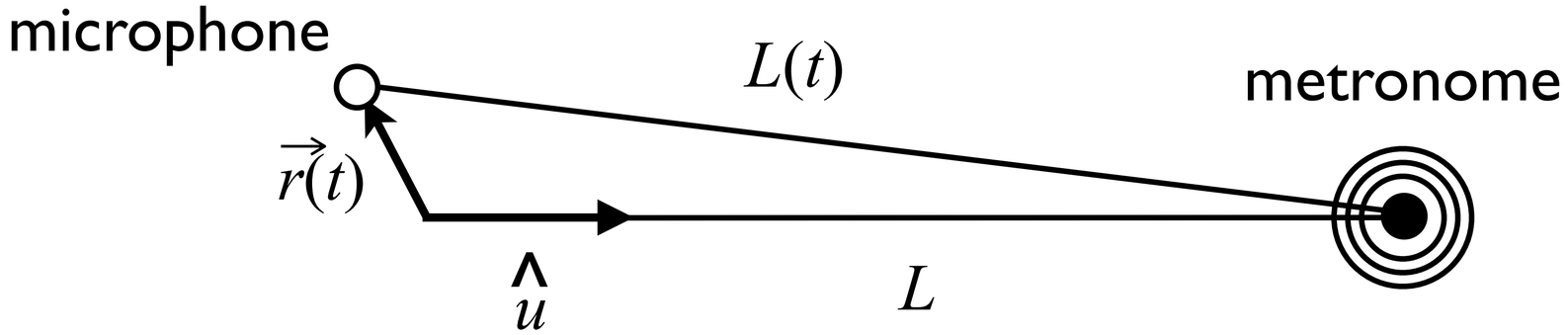
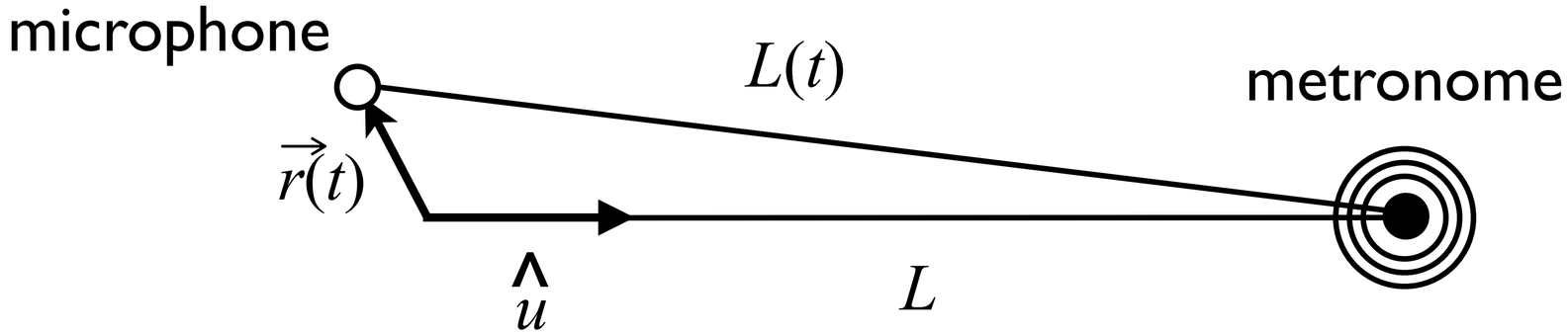




Timing-residual response to microphone motion

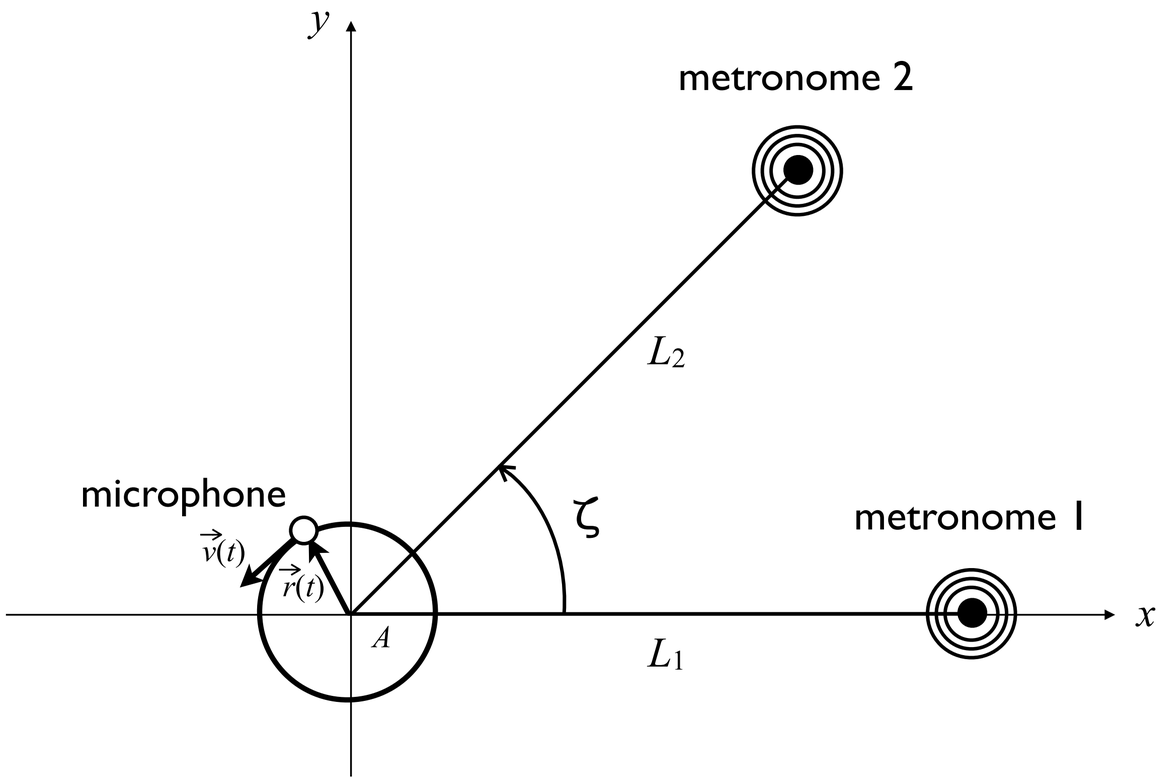


Timing-residual response to microphone motion



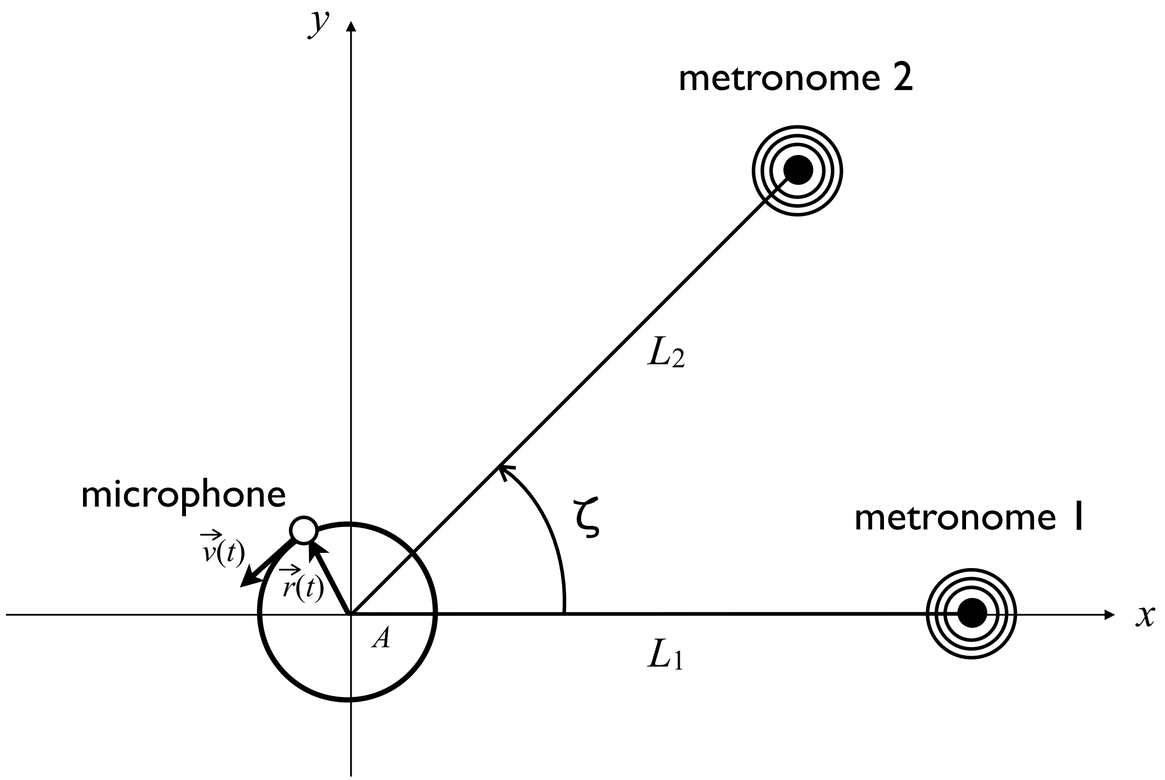
$$\delta\tau(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{1}{c_s}\hat{u}\cdot\vec{r}(t)$$

Two metronomes - uniform circular motion



$$\vec{r}(t) = A\left[\cos(2\pi f_0 t + \phi_0)\hat{x} + \sin(2\pi f_0 t + \phi_0)\hat{y}\right]$$

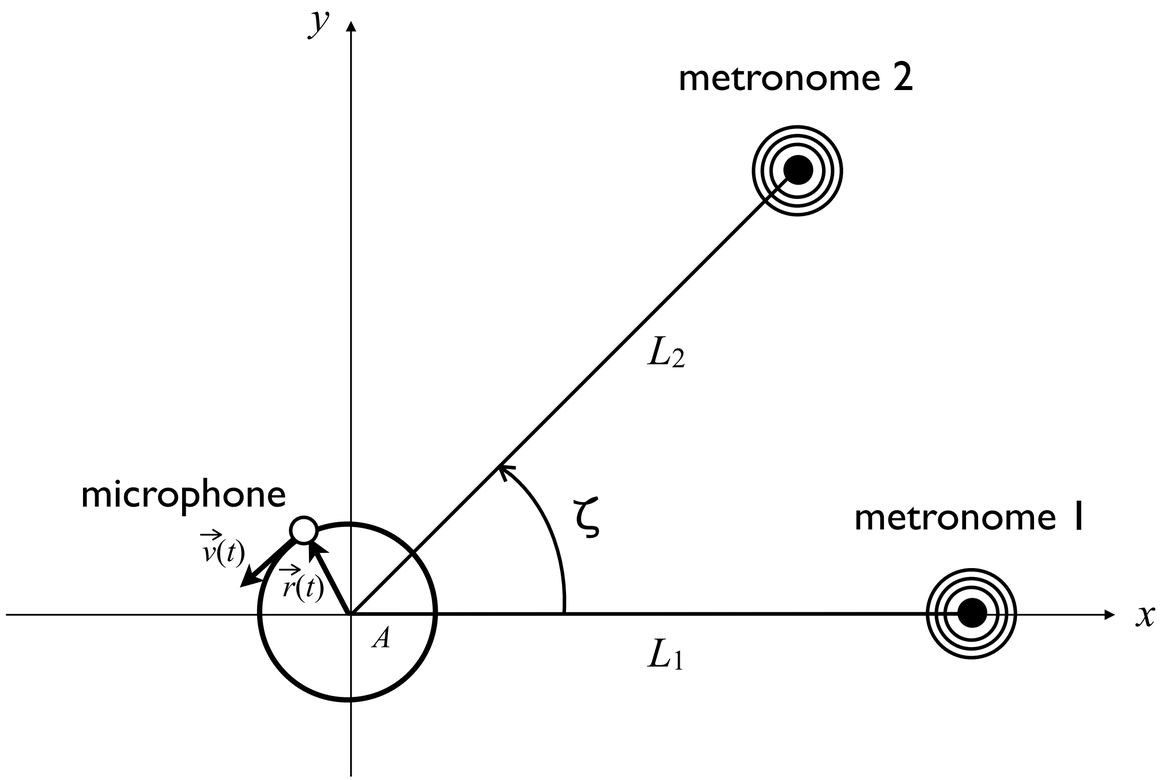
Two metronomes - uniform circular motion



$$\vec{r}(t) = A\left[\cos(2\pi f_0 t + \phi_0)\hat{x} + \sin(2\pi f_0 t + \phi_0)\hat{y}\right]$$

$$\delta\tau_I(t) \simeq -\frac{A}{c_s}\cos(2\pi f_0 t + \phi_0 - \theta_I)\,, \quad I = 1,2$$

Two metronomes - uniform circular motion



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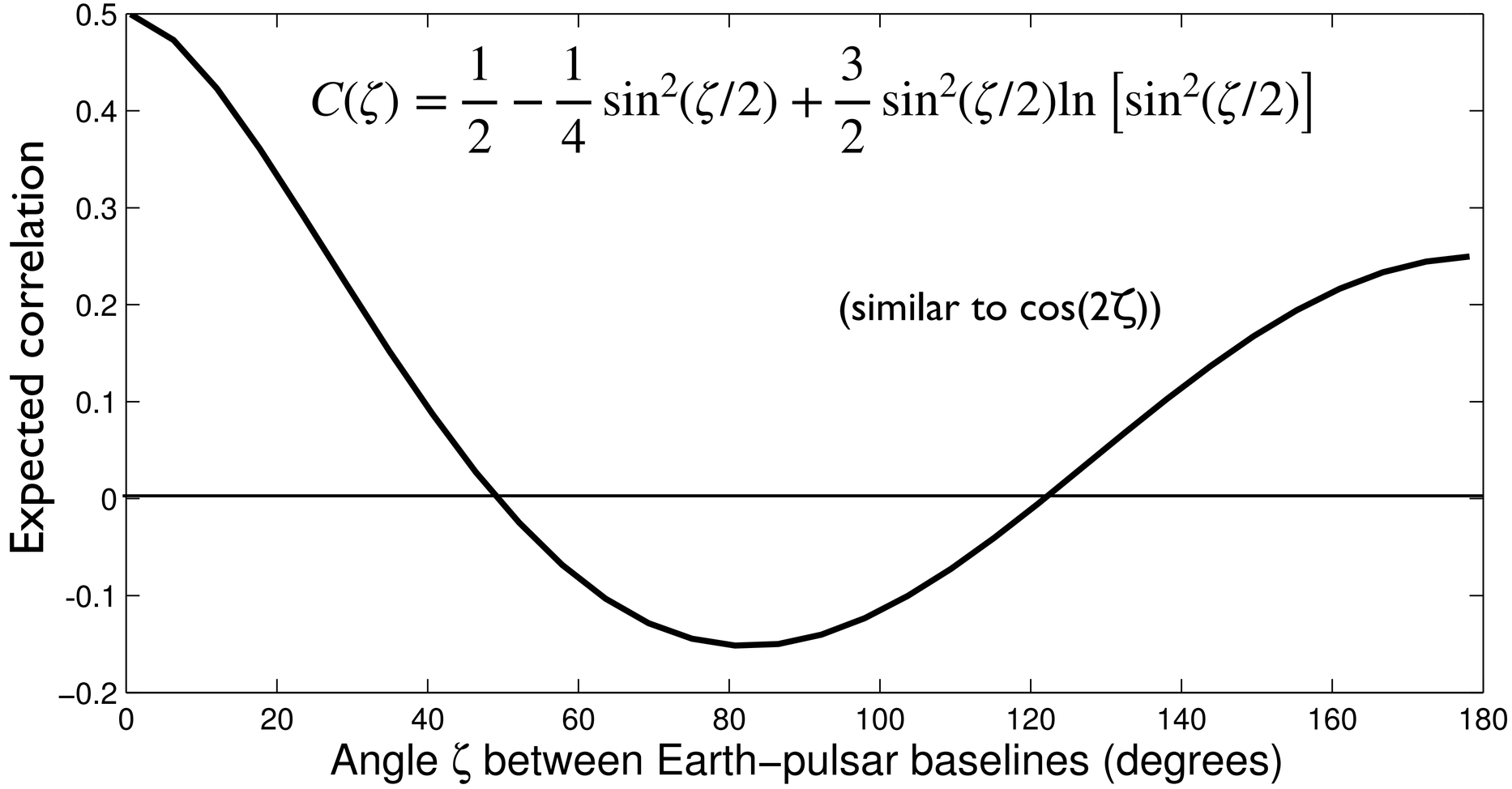
$$\delta\tau_I(t) \simeq -\frac{A}{c_s}\cos(2\pi f_0 t + \phi_0 - \theta_I)\,, \quad I = 1,2$$

$$\rho_{12} \simeq \cos\zeta\,, \quad \zeta \equiv \theta_1 - \theta_2$$

Expected PTA correlation - Hellings & Downs curve (isotropic, unpolarized GW background)



Metronome demo numbers

c_s = 340 m/s (in air)

amplitude ≈ 5 cm

amplitude / c_s = 1 x 10^{-4} sec

184 bpm: T_p = 0.3261 sec

208 bpm: T_p = 0.2885 sec

Pulsar timing numbers

f ~ 1/few weeks to 1/10 years (10^{-7} Hz to 10^{-9} Hz)

λ ~ 0.1 to 10 lyr (GW wavelength)

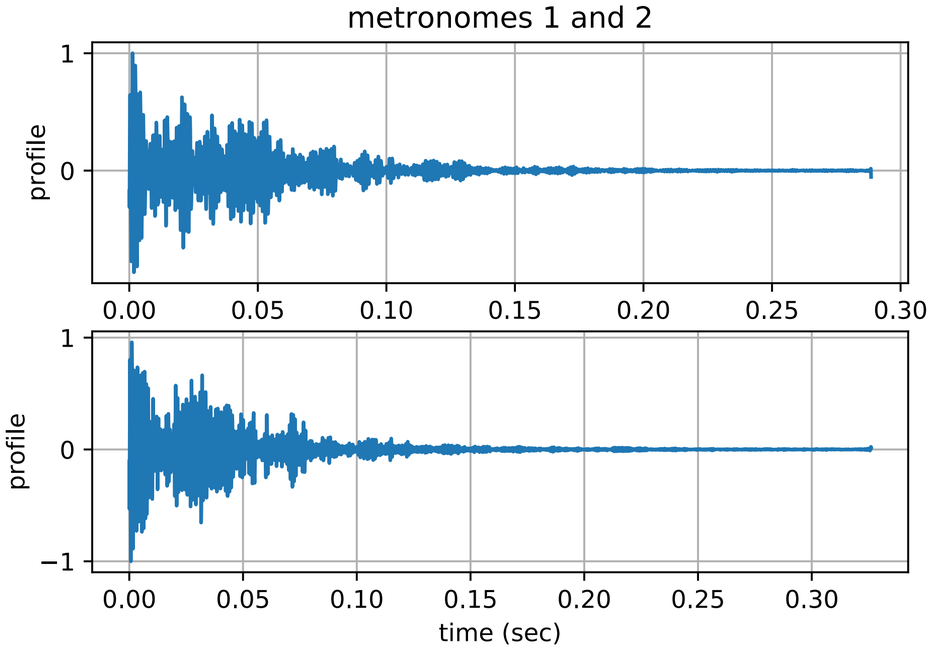
L ~ few x 1000 lyr (distance to pulsars)

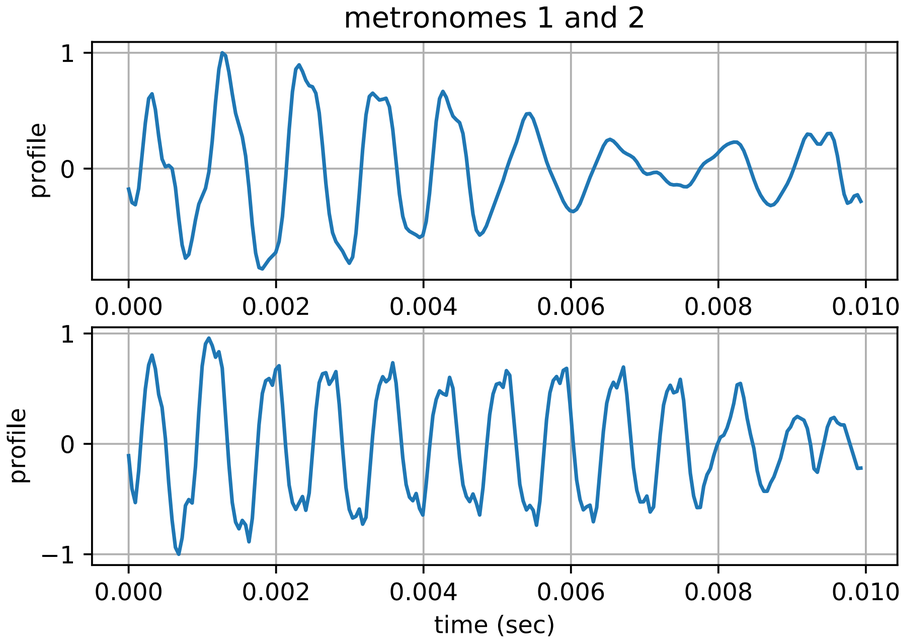
λ << L (short-wavelength limit)

sensitivity ~ σ_{rms}/T_{obs} ~ 100 ns/10 yr ~ 10^{-15}

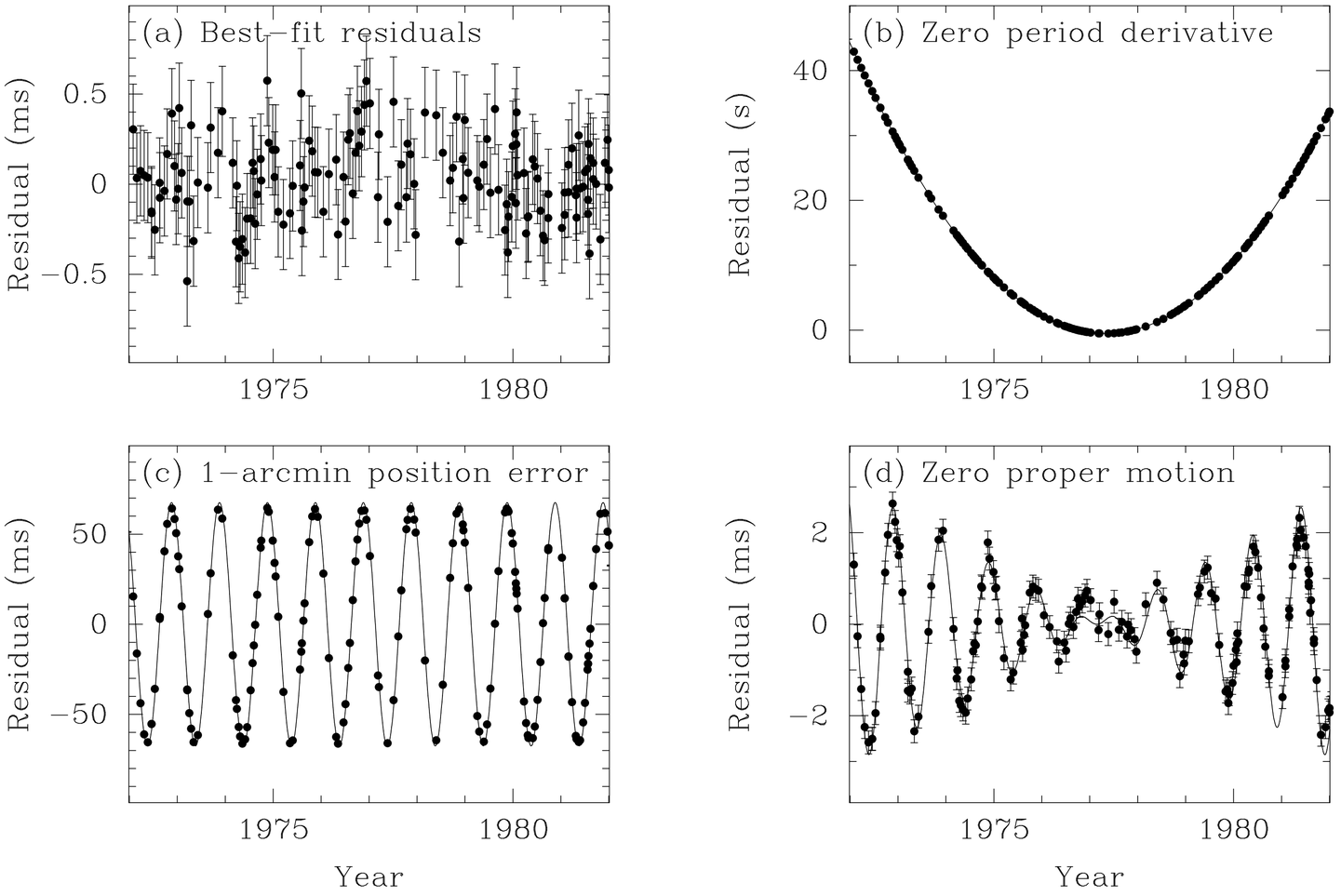
can detect changes ~10 km in the position of a pulsar at a distance of ~1000 lyr

Metronome pulse profiles





Errors in the timing model show up as deterministic features in the timing residuals



(from Lorimer LRR-2008-8)