Metronome-microphone demo of a pulsar timing array

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Purpose

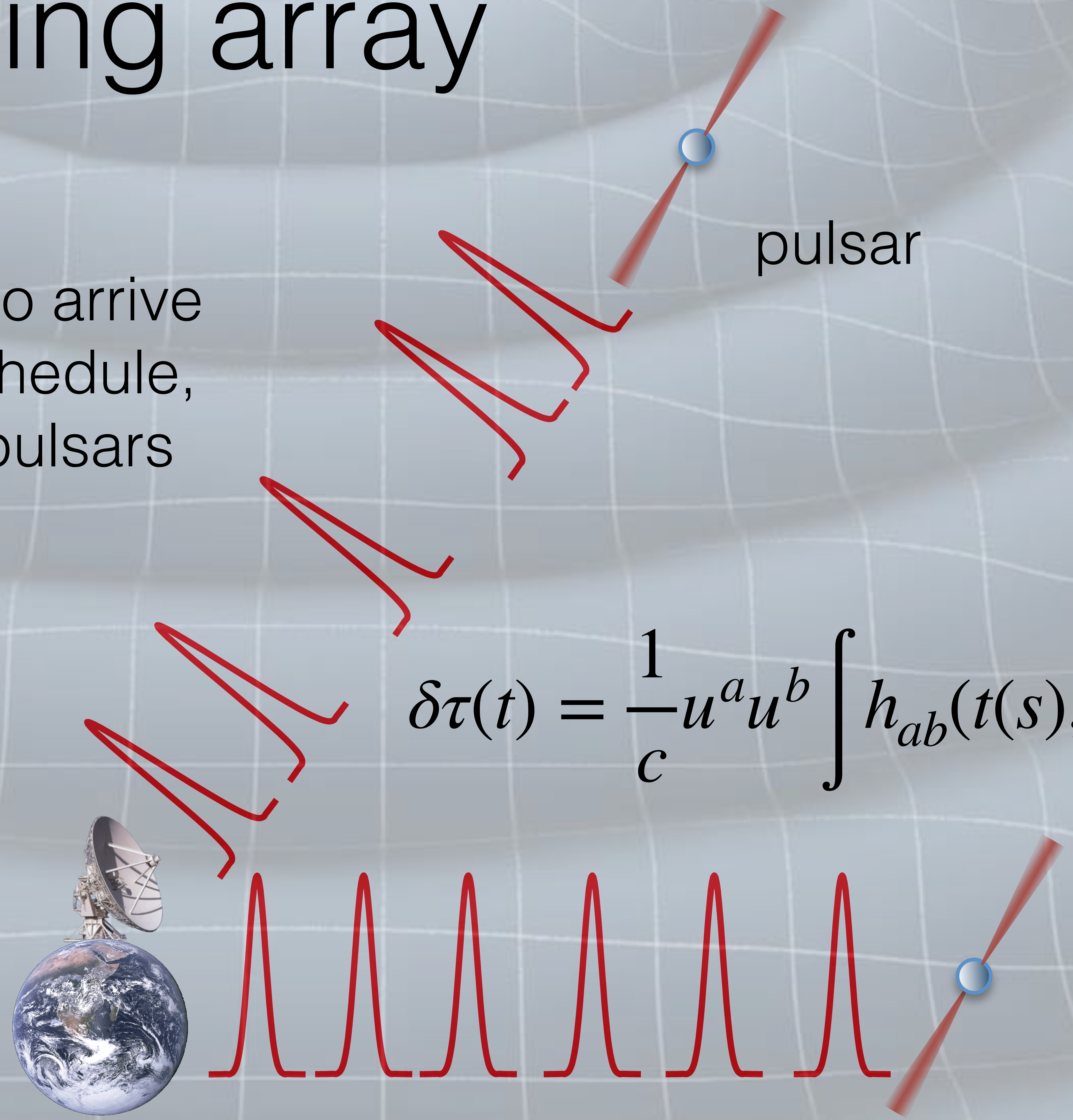
- Illustrate how a PTA is used to search for GWs in the context of a simple acoustical model
- Become familiar with techniques used by pulsar astronomers:
  - folding (for calculating pulse periods and pulse profiles)
  - detrending (for better estimating pulse period)
  - matched filtering (for calculating measured times-of-arrivals)
  - timing models (for calculating expected times of arrivals)
  - correlation analyses (for extracting common GW component)

https://github.com/josephromano/pta-demo

Pulsar timing array

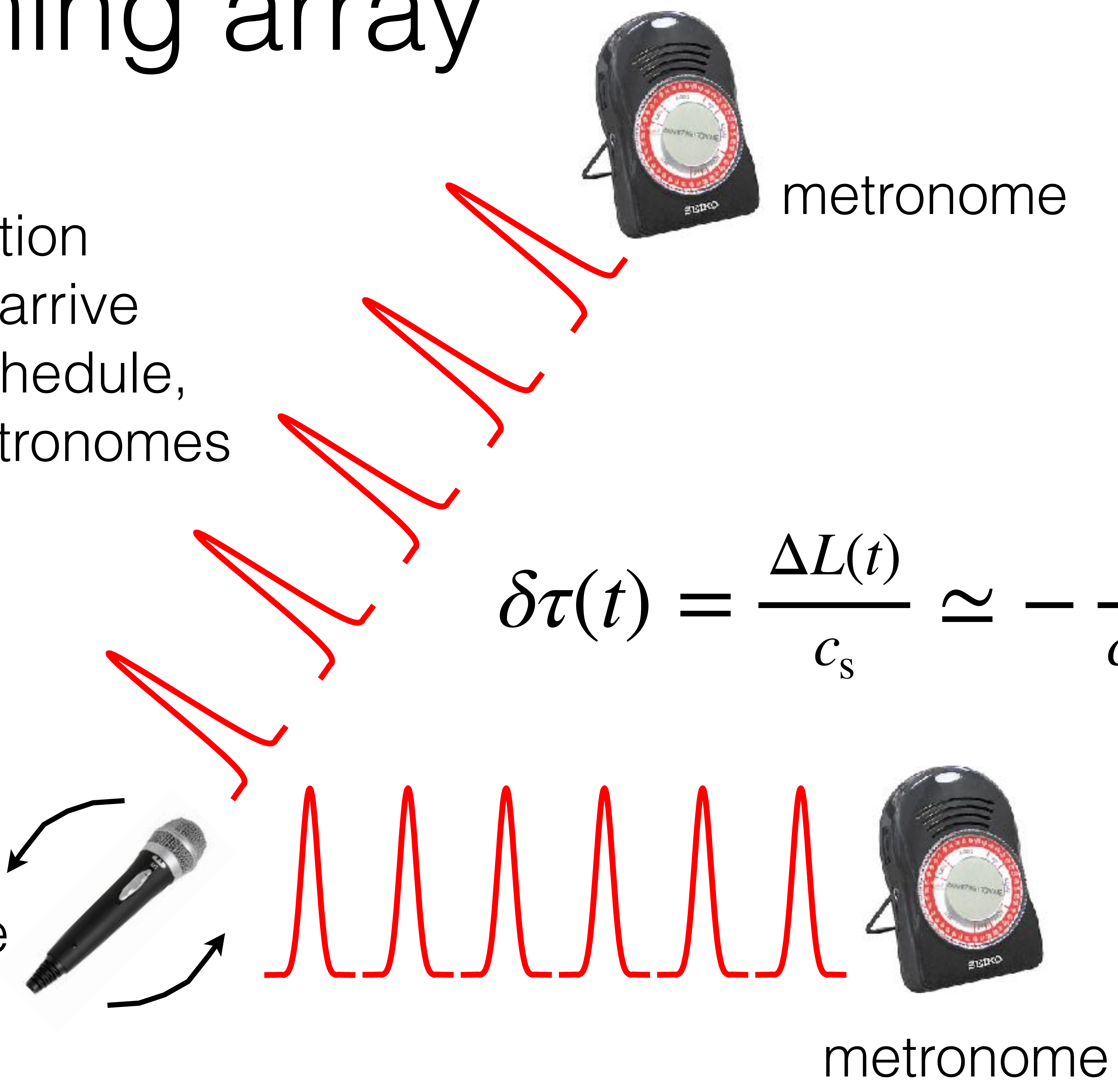
GWs cause pulses to arrive ahead or behind schedule, correlated across pulsars

$$\delta\tau(t) = \frac{1}{c} u^a u^b \int h_{ab}(t(s), \vec{x}(s))\, \mathrm{d}s$$



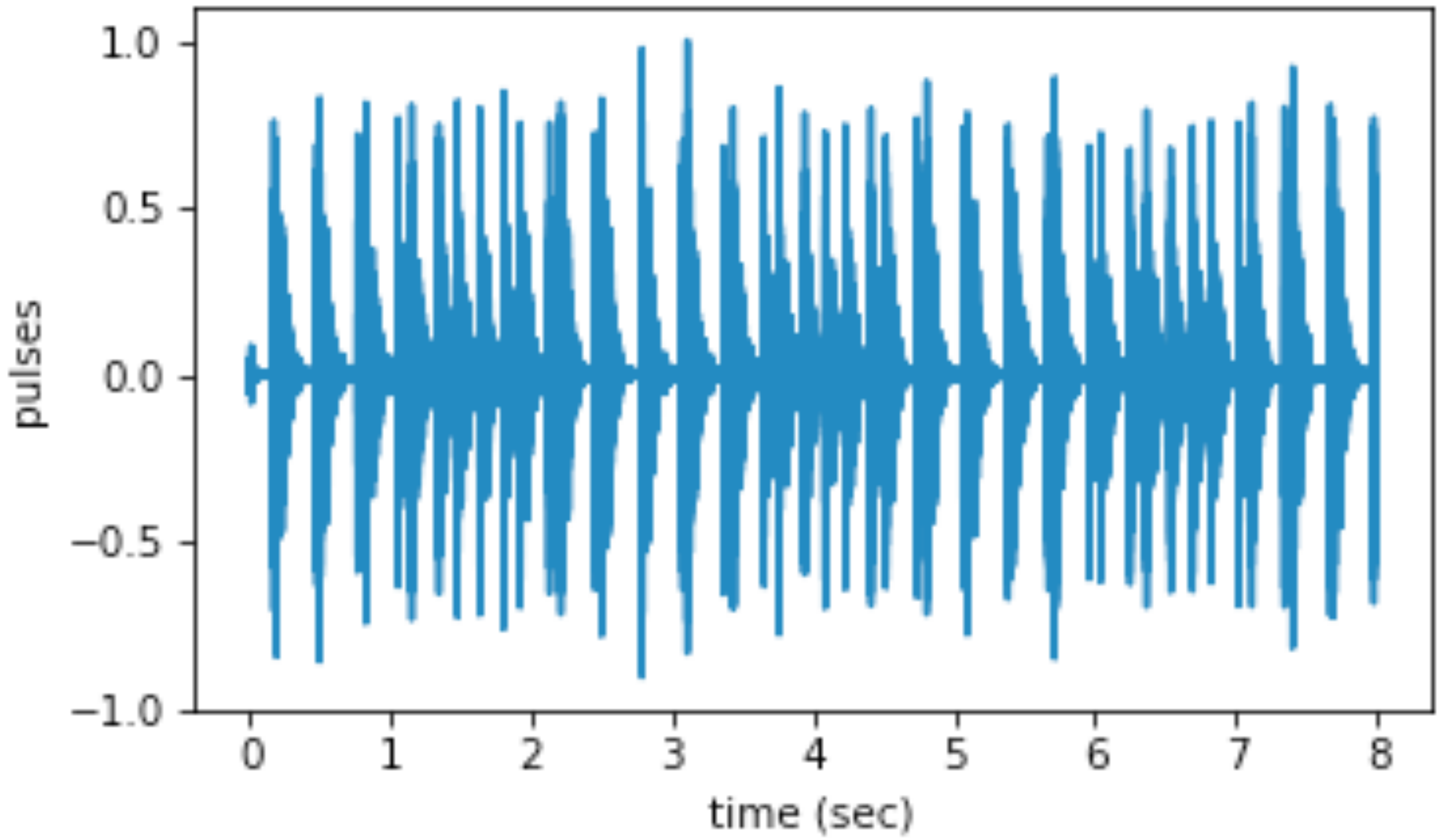
Metronome timing array

microphone motion
causes pulses to arrive
ahead or behind schedule,
correlated across metronomes



$$\delta\tau(t) = \frac{\Delta L(t)}{c_\mathrm{s}} \simeq -\frac{1}{c_\mathrm{s}}\hat{u}\cdot\vec{r}(t)$$

Q: Is there evidence of a “GW” in the data?



Q: Is there evidence of a “GW” in the data?

Is there a common disturbance to the pulse arrival times (TOAs), and if so, is this disturbance correlated across metronomes as expected for a “gravitational wave” (i.e., microphone motion)?

- disturbance = timing residuals = measured TOAs - expected TOAs
- common = correlated

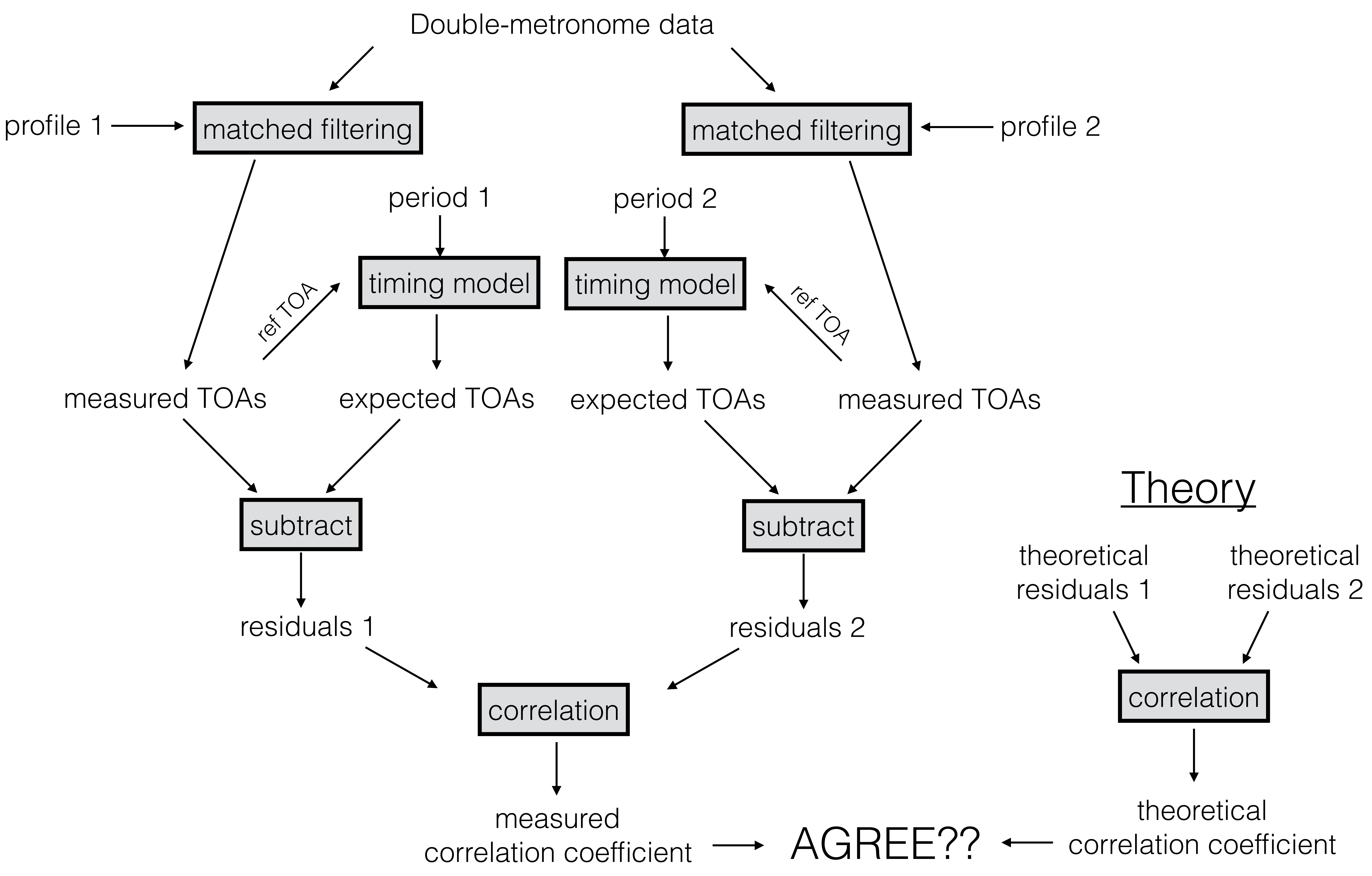
$$\rho_{12} \equiv \langle x_1 x_2 \rangle / \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \qquad \langle x_1 x_2 \rangle \equiv \frac{1}{T_{\rm obs}} \int_0^{T_{\rm obs}} \mathrm{d}t \, x_1(t) x_2(t)$$

- measured TOAs: calculated using matched filtering with pulse profile

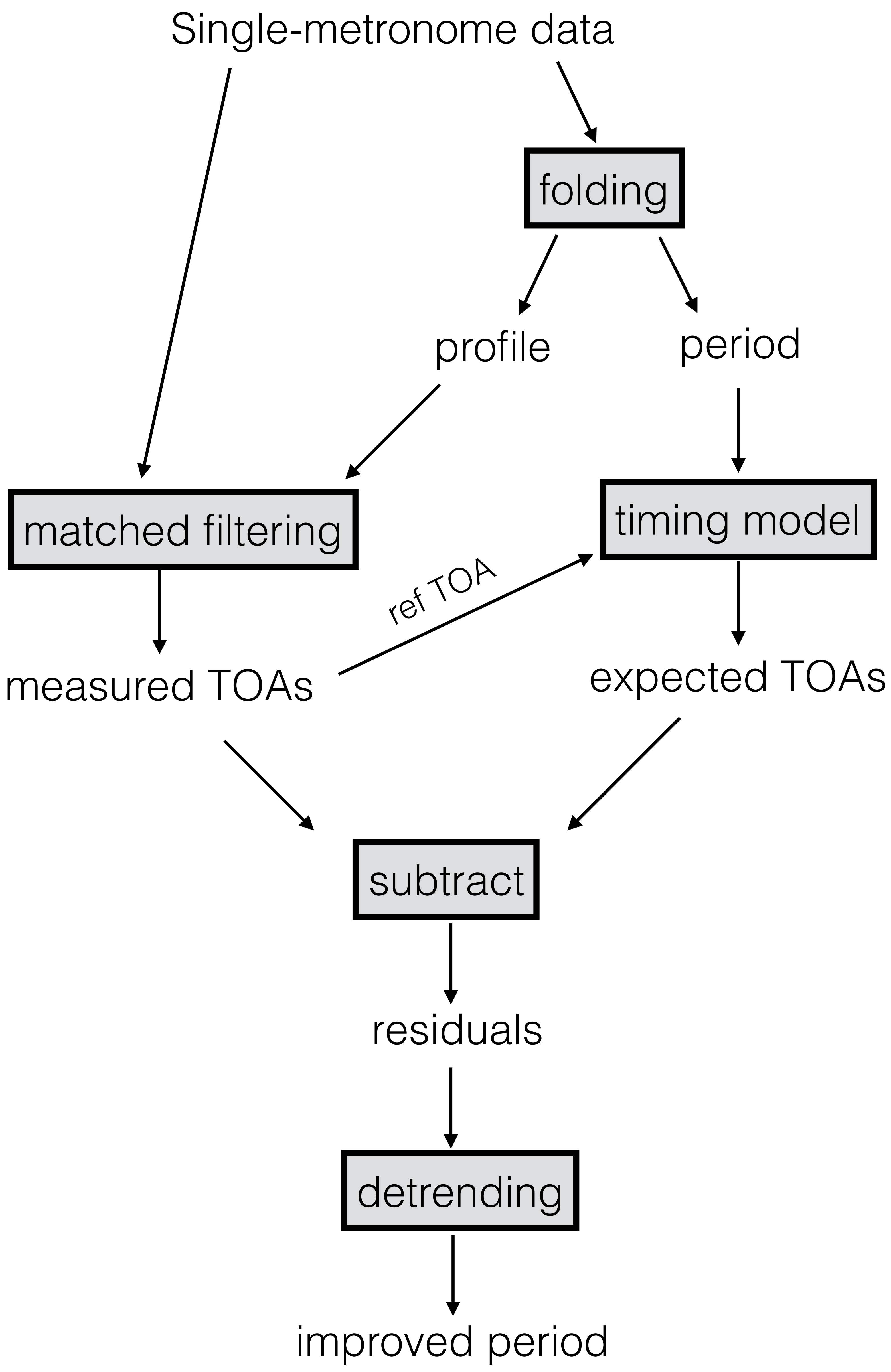
$$C(\Delta t) = \mathcal{N} \int \mathrm{d}t \, y(t) p(t - \Delta t)$$

- expected TOAs: timing model using pulse period and reference TOA
- pulse profile, period: folding and detrending single-metronome data
- expected “GW” correlation (for uniform circular motion): $\boxed{\rho_{12} \simeq \cos\zeta}$

Double-metronome analysis

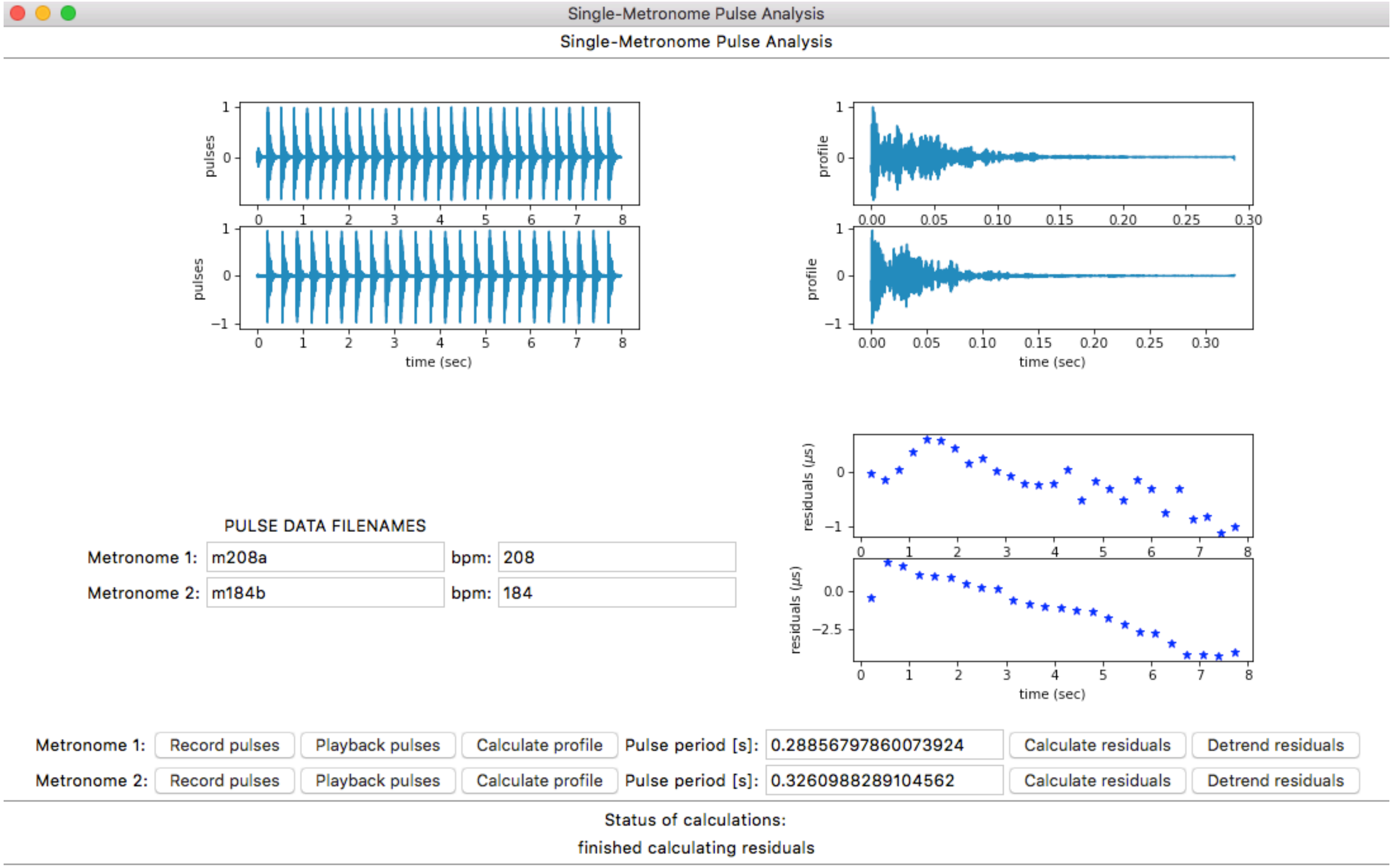


Single-metronome analysis

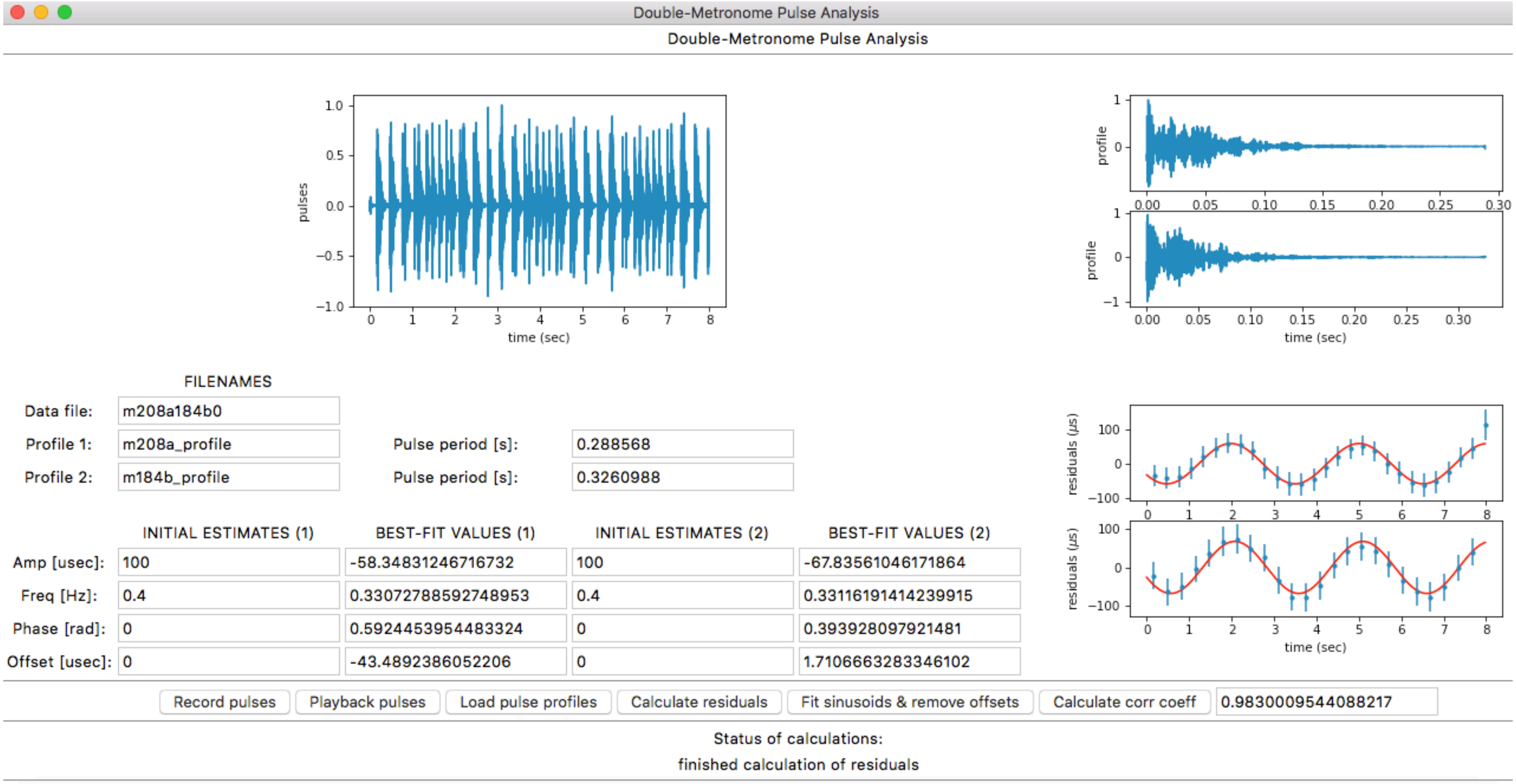


Output of the GUIs

Single-metronome analysis



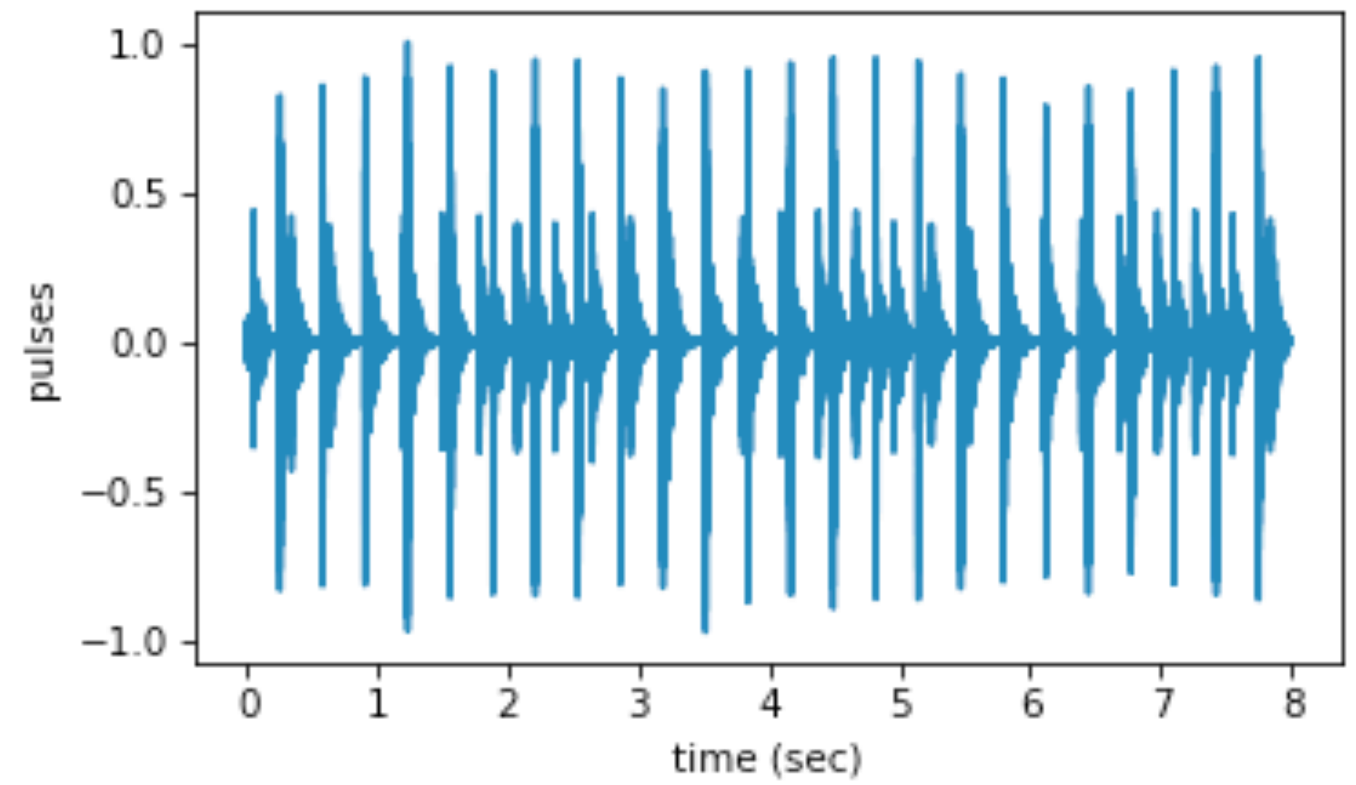
Double-metronome analysis (ζ=0 degrees)

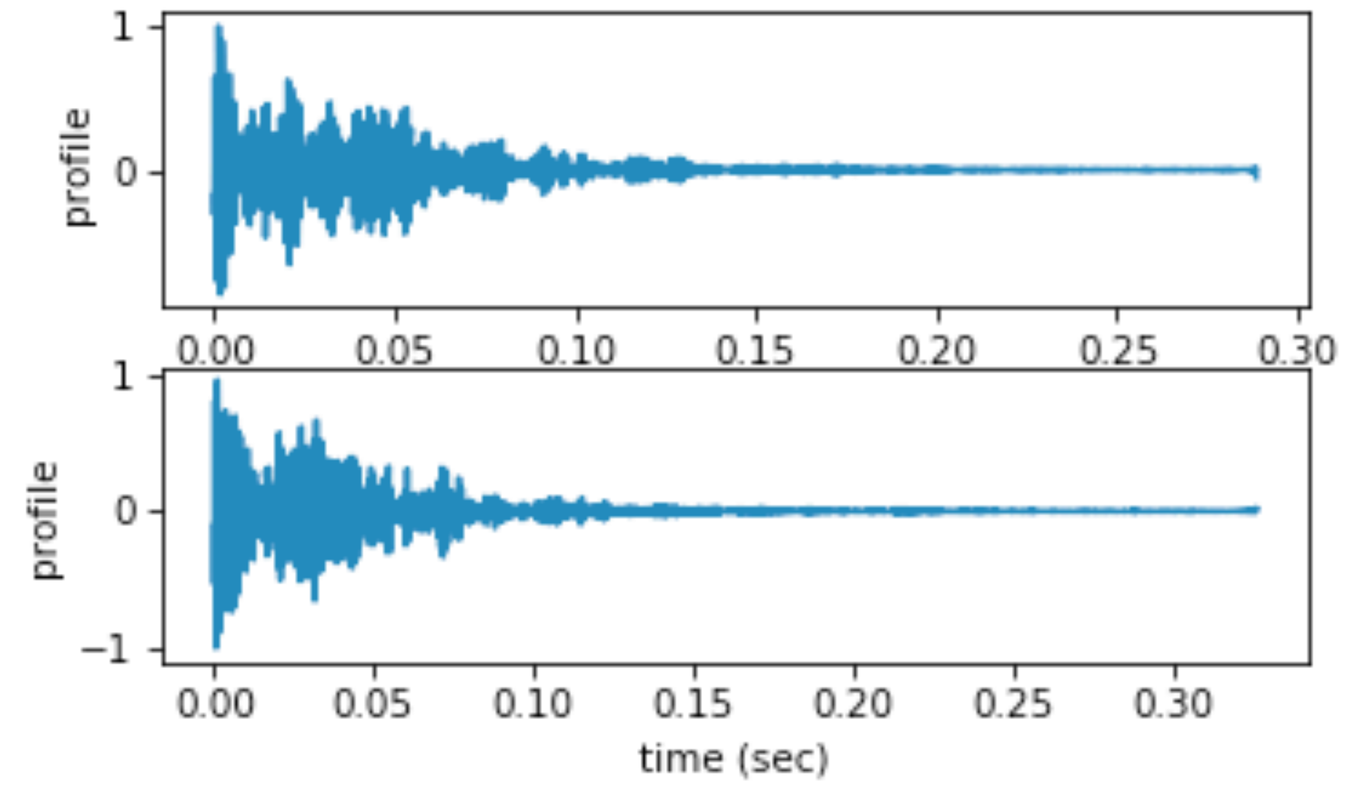


Double-metronome analysis (ζ=45 degrees)

Double-Metronome Pulse Analysis

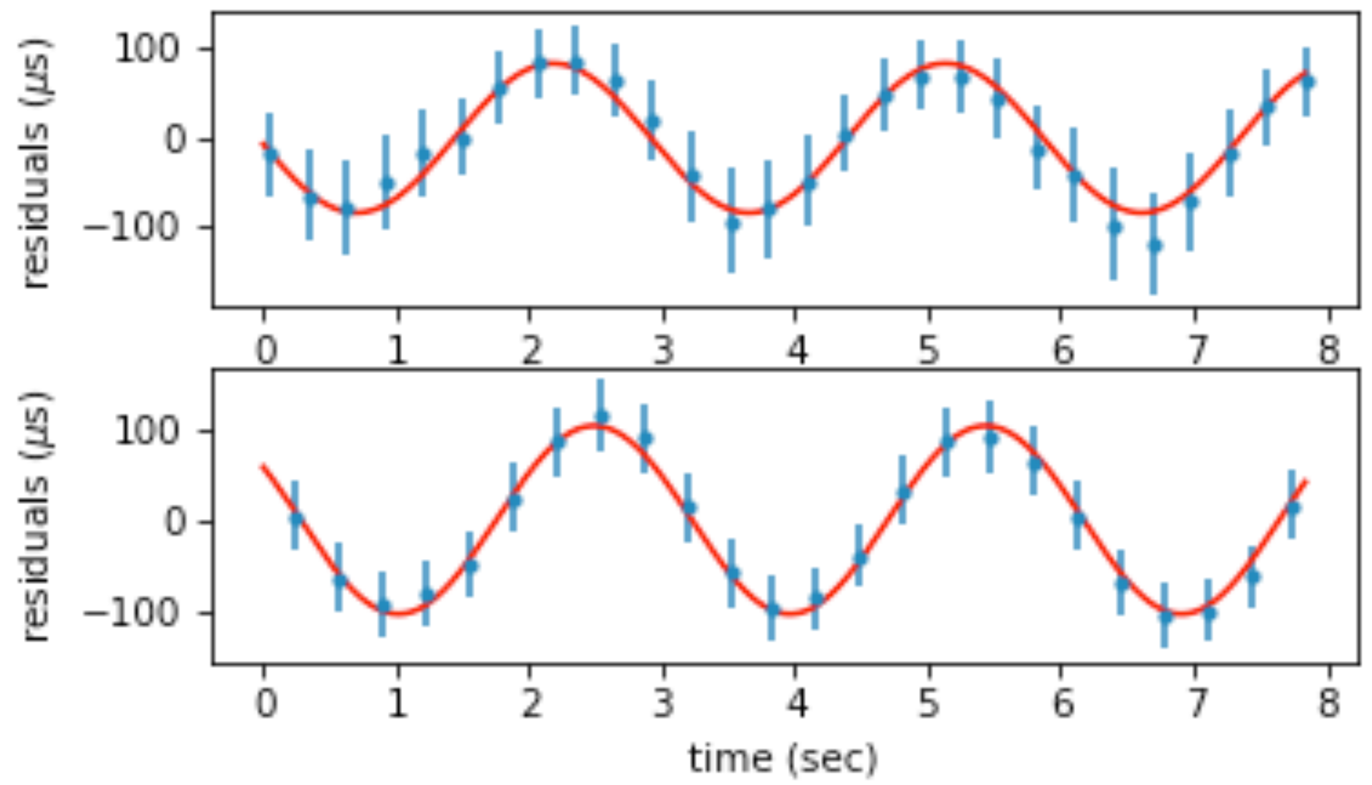
Double-Metronome Pulse Analysis





FILENAMES

Data file:	m208a184b45		
Profile 1:	m208a_profile	Pulse period [s]:	0.288568
Profile 2:	m184b_profile	Pulse period [s]:	0.3260988



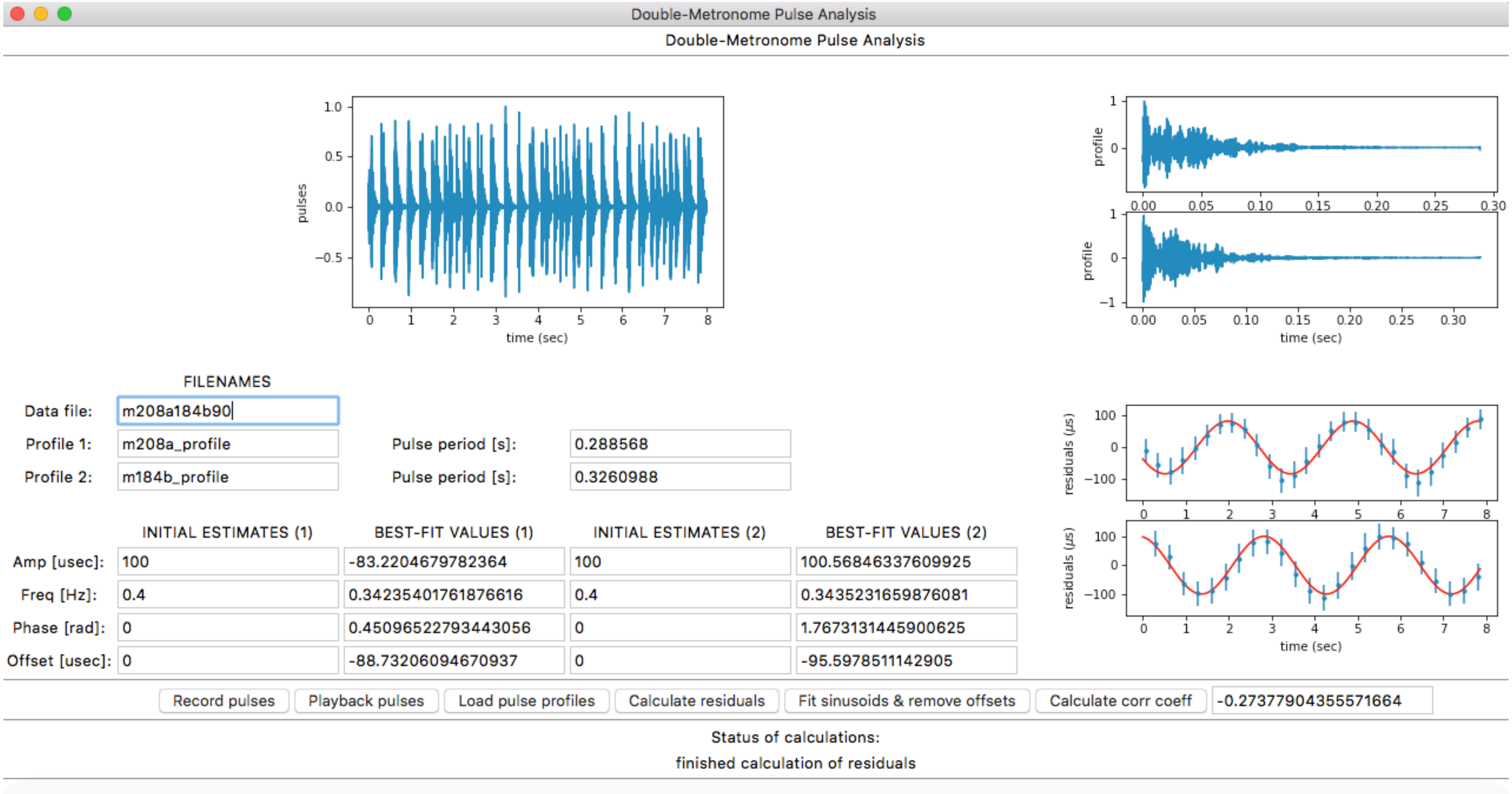
	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-83.54267144872672	100	103.51040350638623
Freq [Hz]:	0.4	0.33844784664075234	0.4	0.339460442091975
Phase [rad]:	0	0.082340180957162	0	2.544467981756352
Offset [usec]:	0	-83.59186463058177	0	62.255379154251095

Record pulses | Playback pulses | Load pulse profiles | Calculate residuals | Fit sinusoids & remove offsets | Calculate corr coeff | 0.7762191706588184

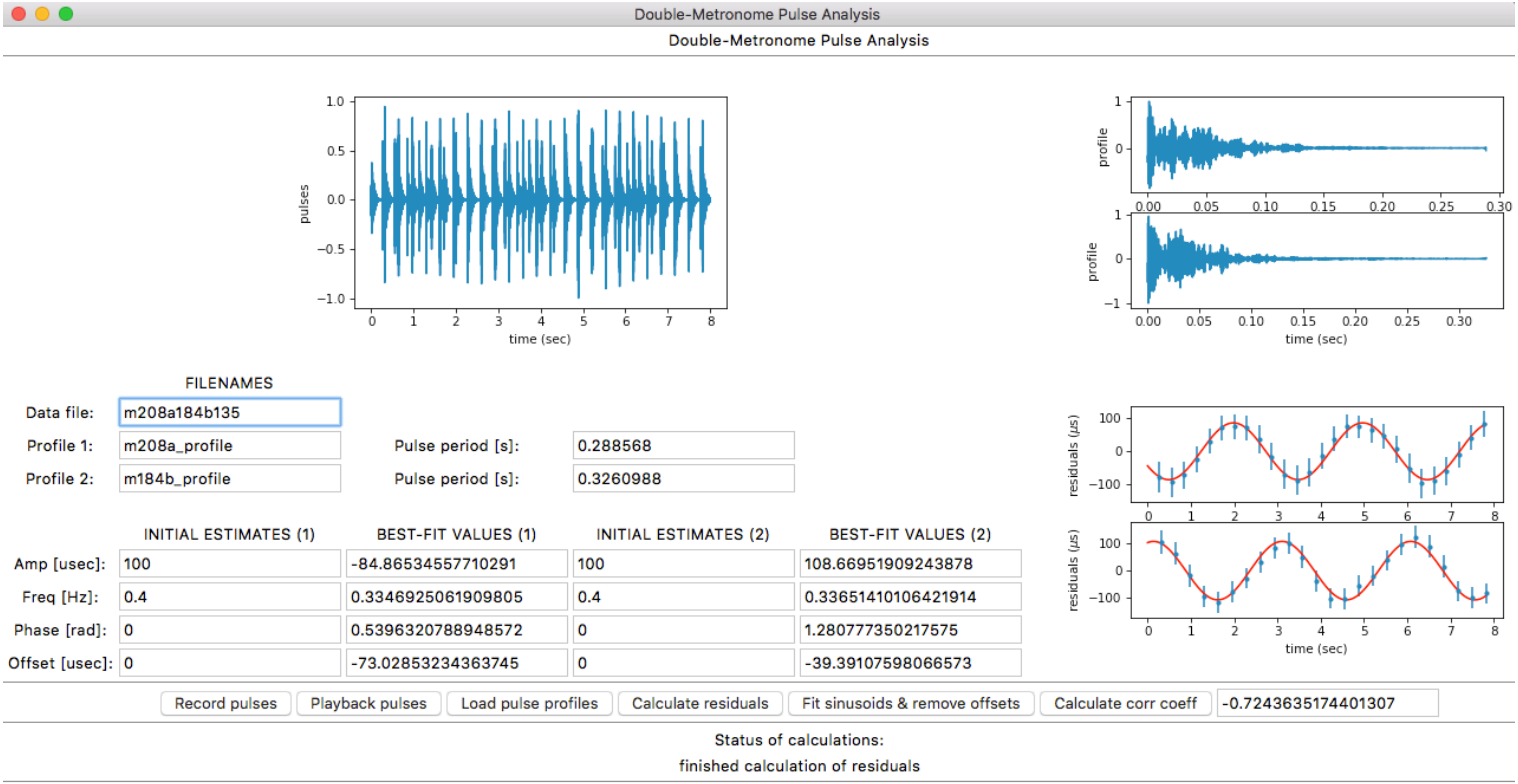
Status of calculations:

finished calculation of residuals

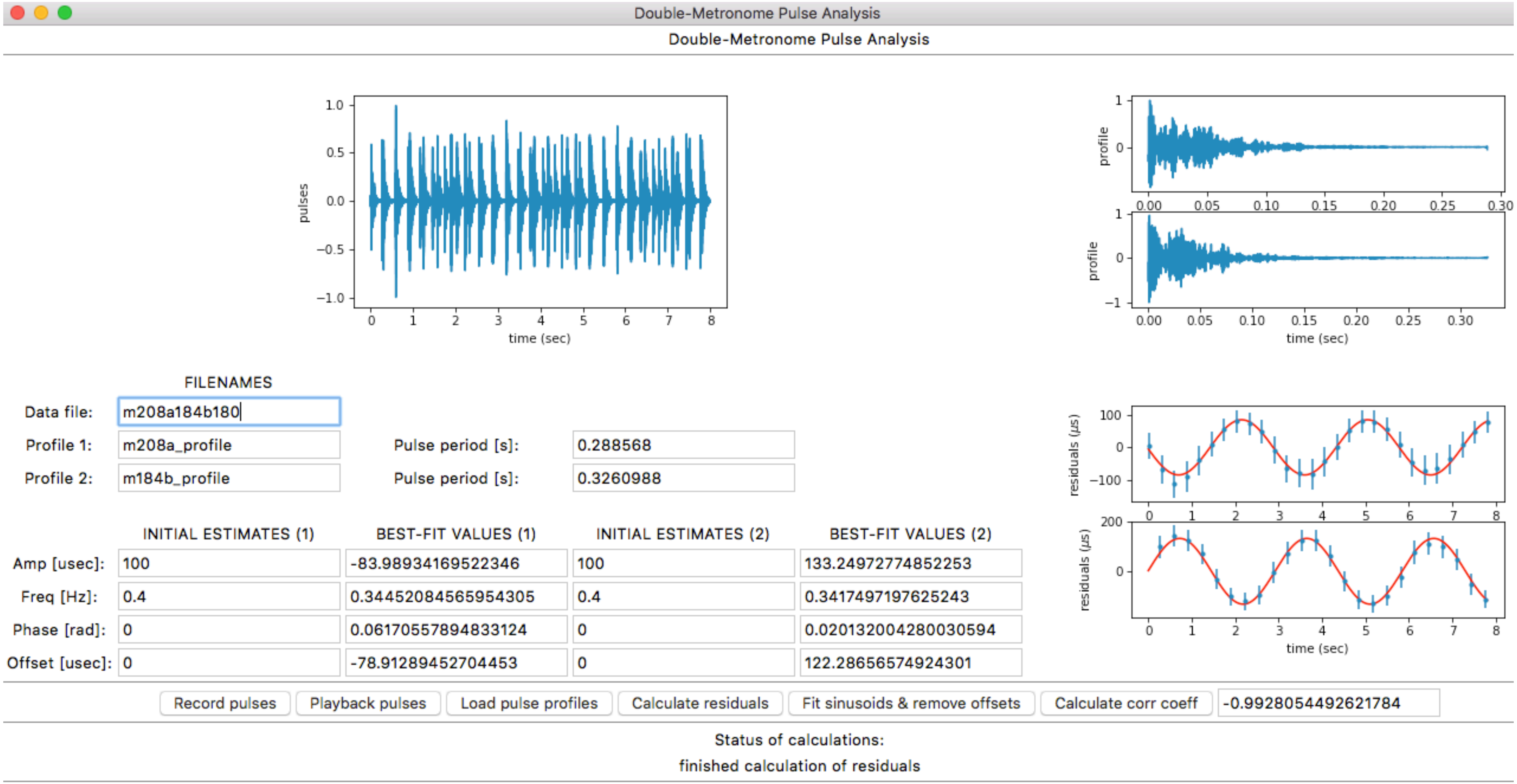
Double-metronome analysis (ζ=90 degrees)



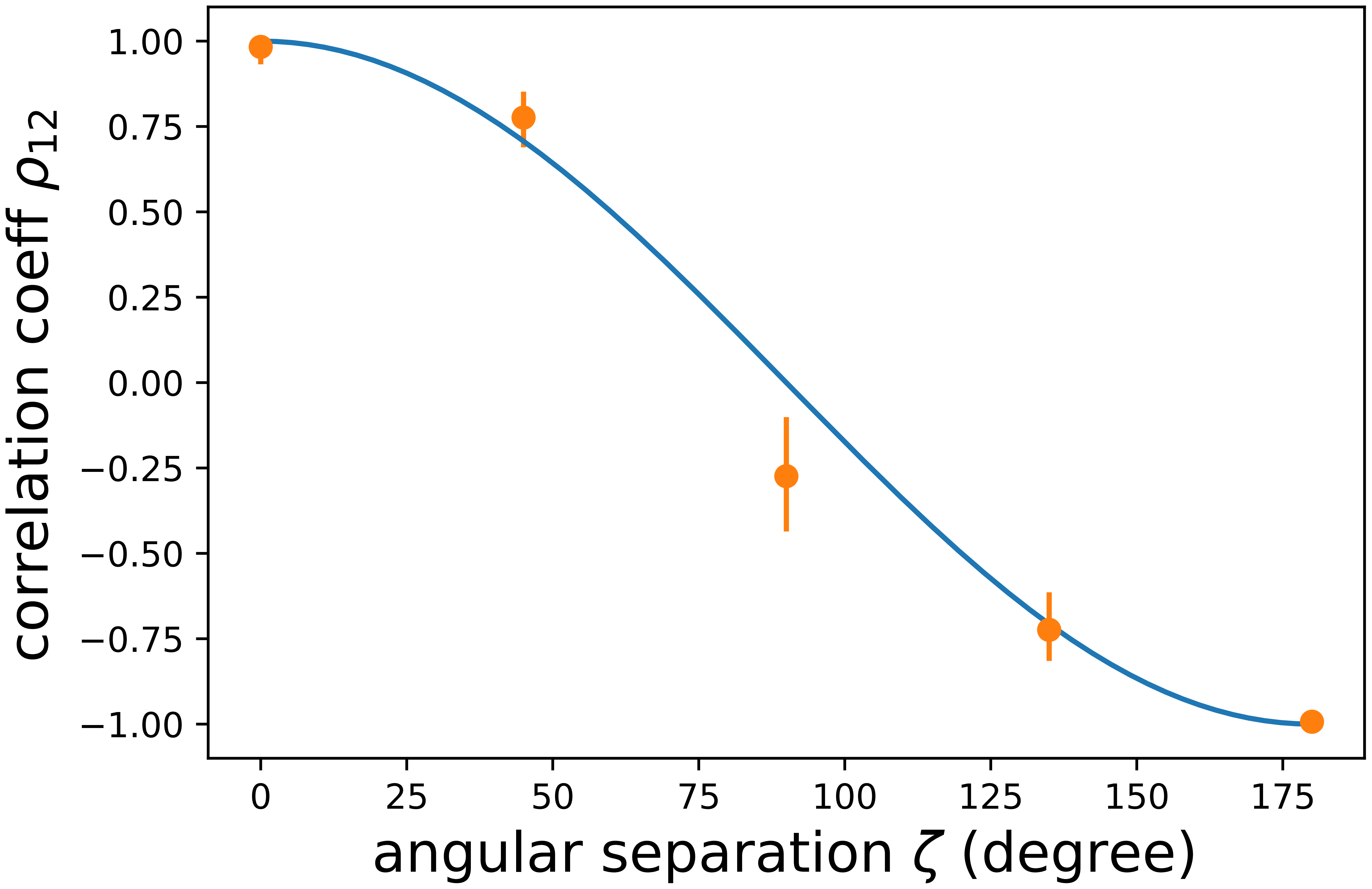
Double-metronome analysis (ζ=135 degrees)



Double-metronome analysis (ζ=180 degrees)

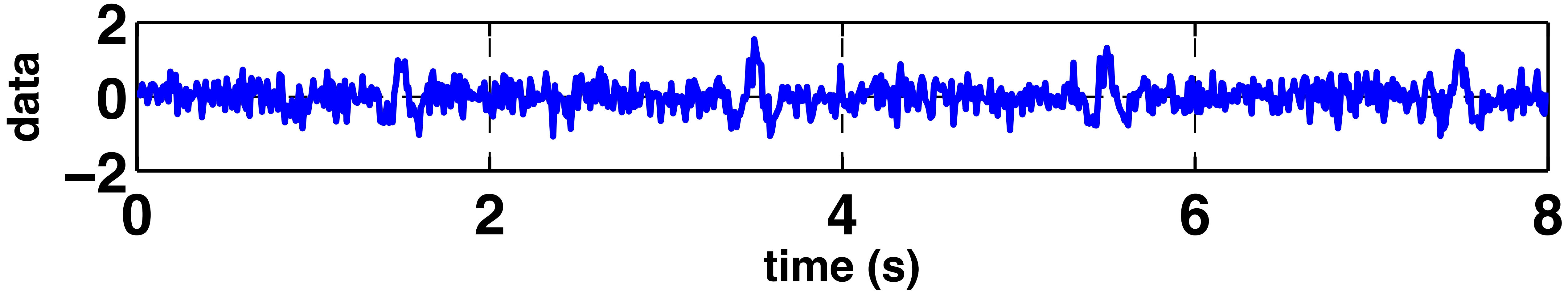


Metronome correlation

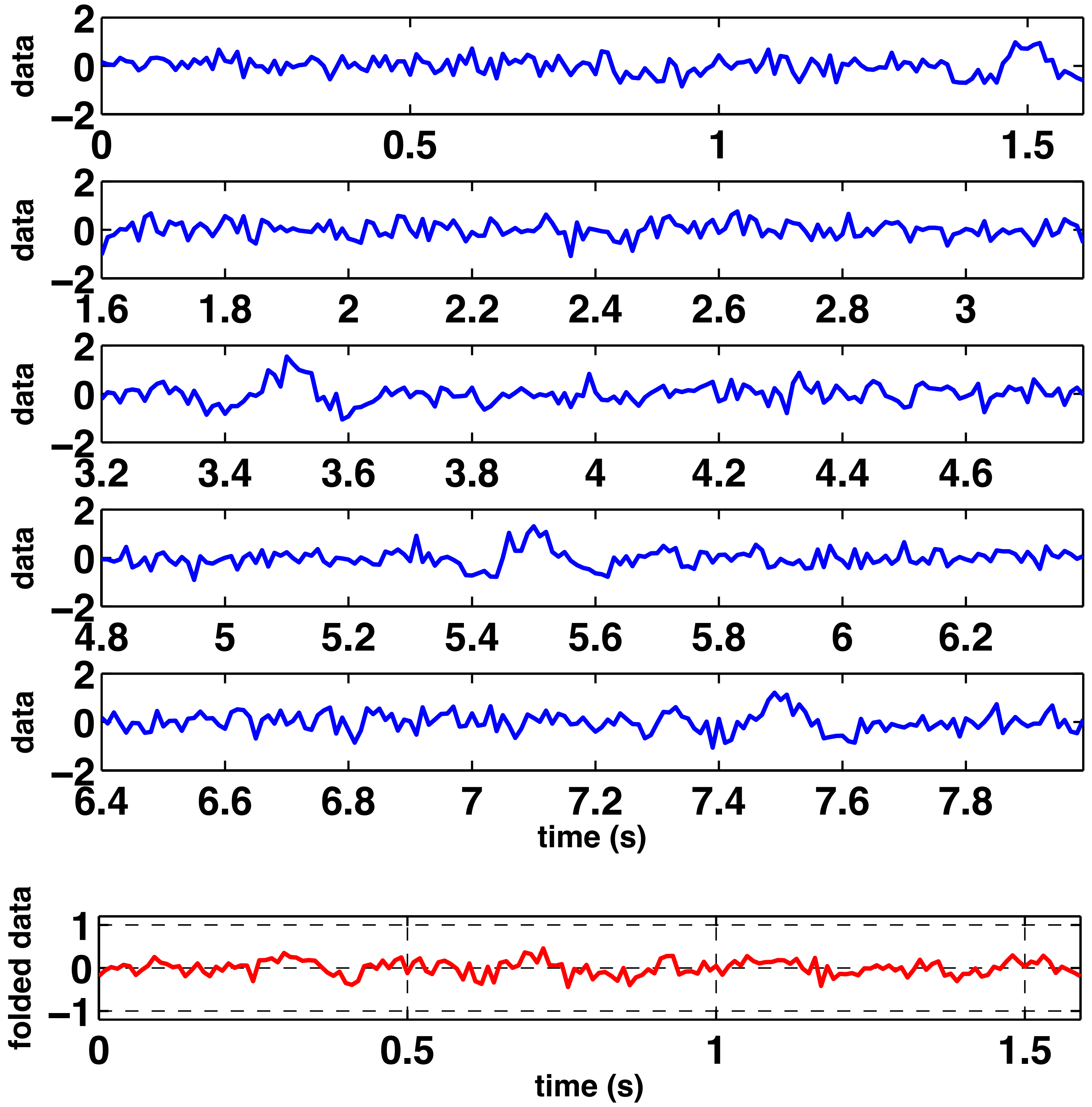


More details

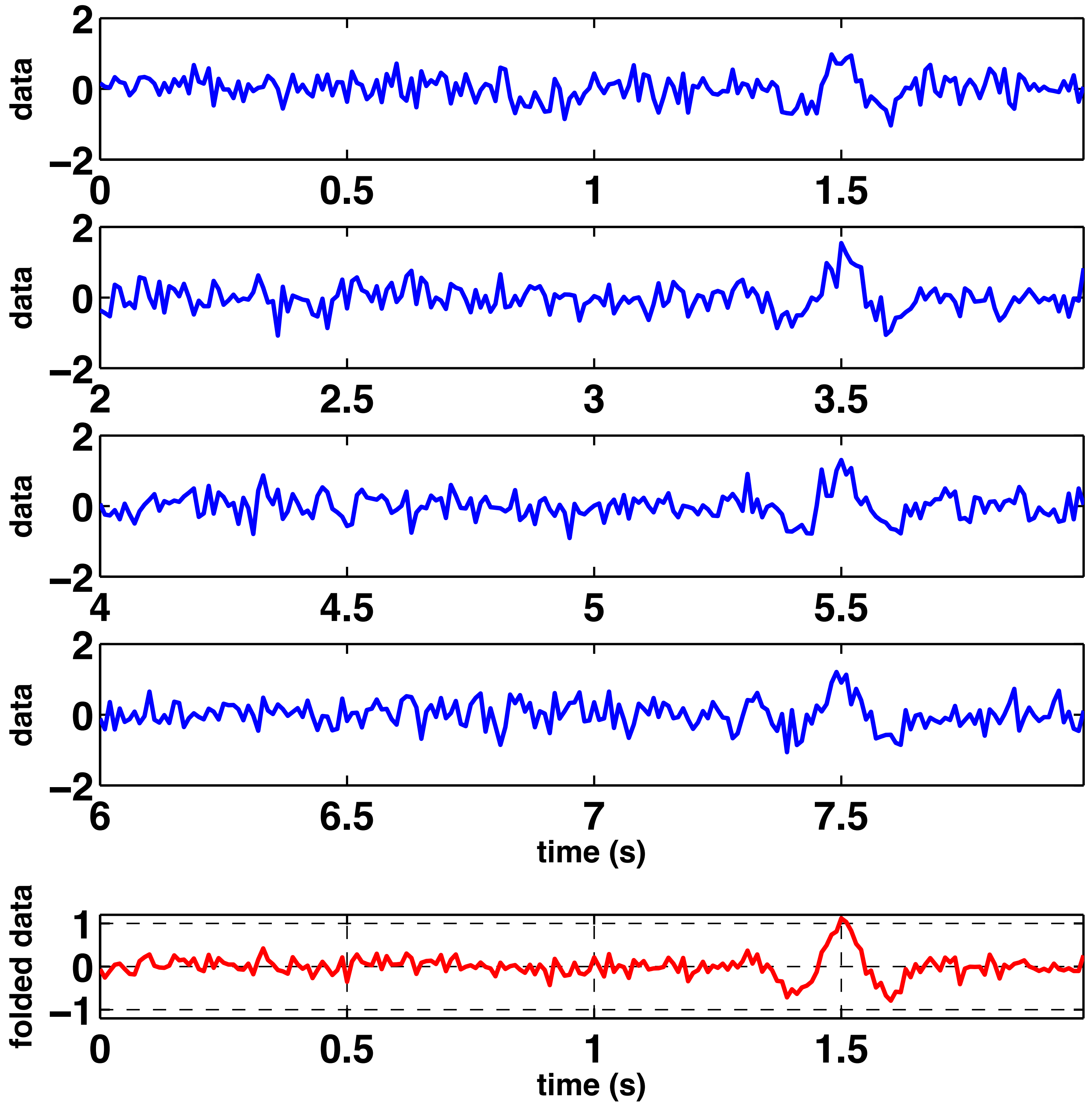
“Fold” data to determine pulse period and pulse profile



fold with incorrect T_P



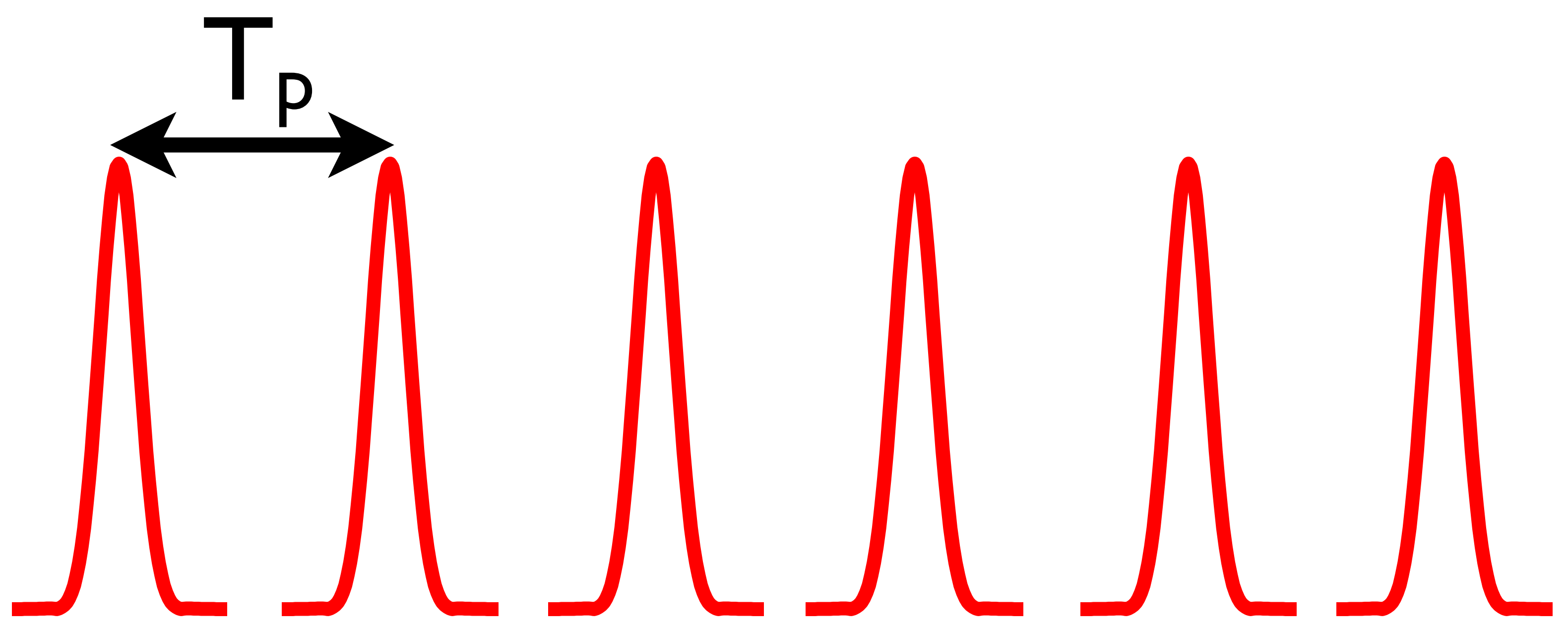
fold with correct T_P



Timing model

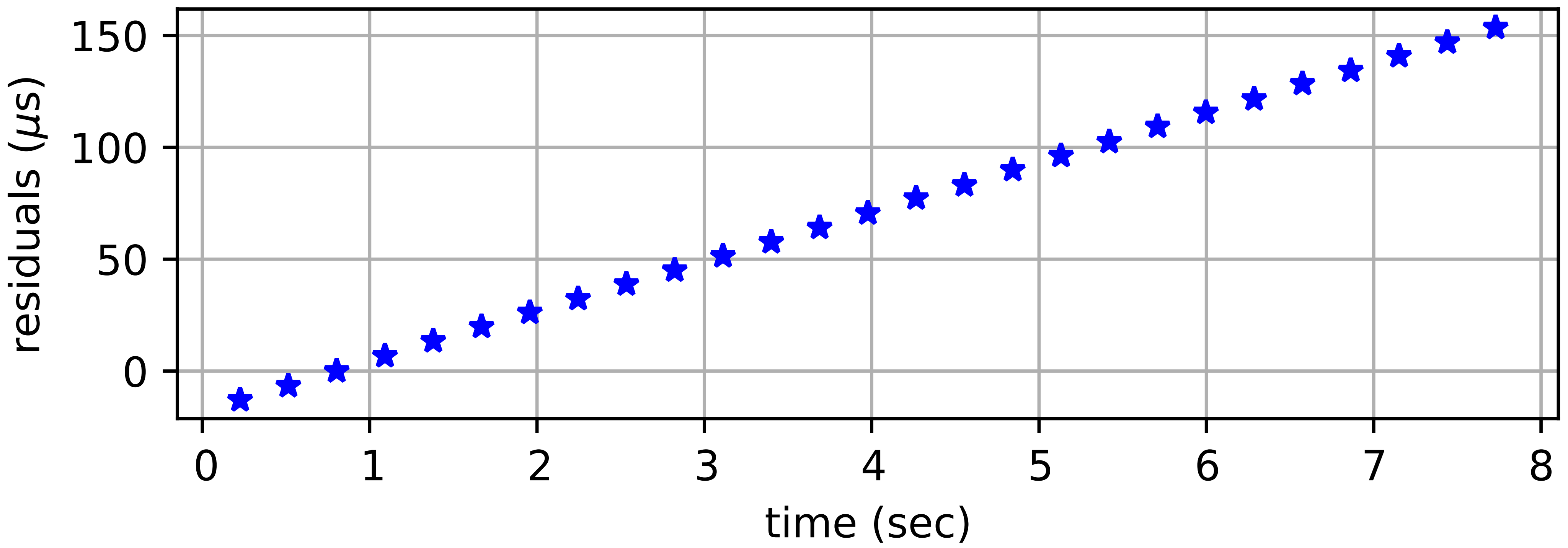
Pulses should arrive regularly with period T_p relative to some reference pulse

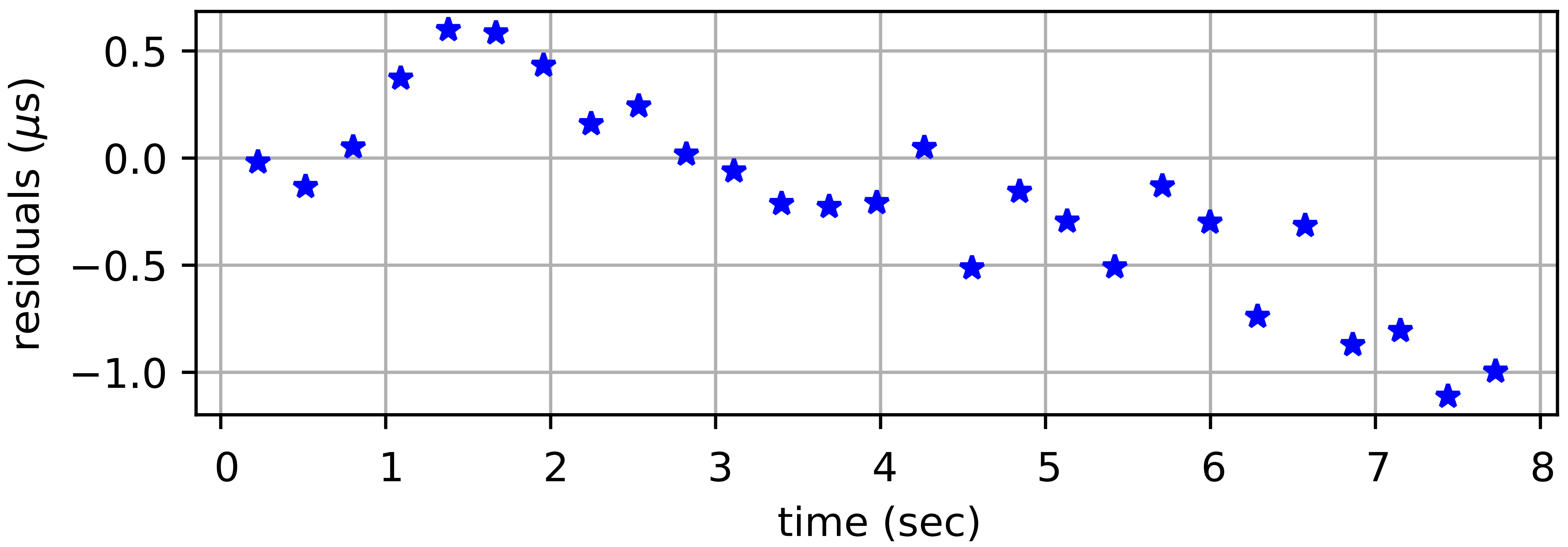
$$\tau^{\mathrm{expected}}[i] = \tau^{\mathrm{measured}}[i_0] + (i - i_0)T_\mathrm{p}$$



Remove linear trend to more accurately determine pulse period

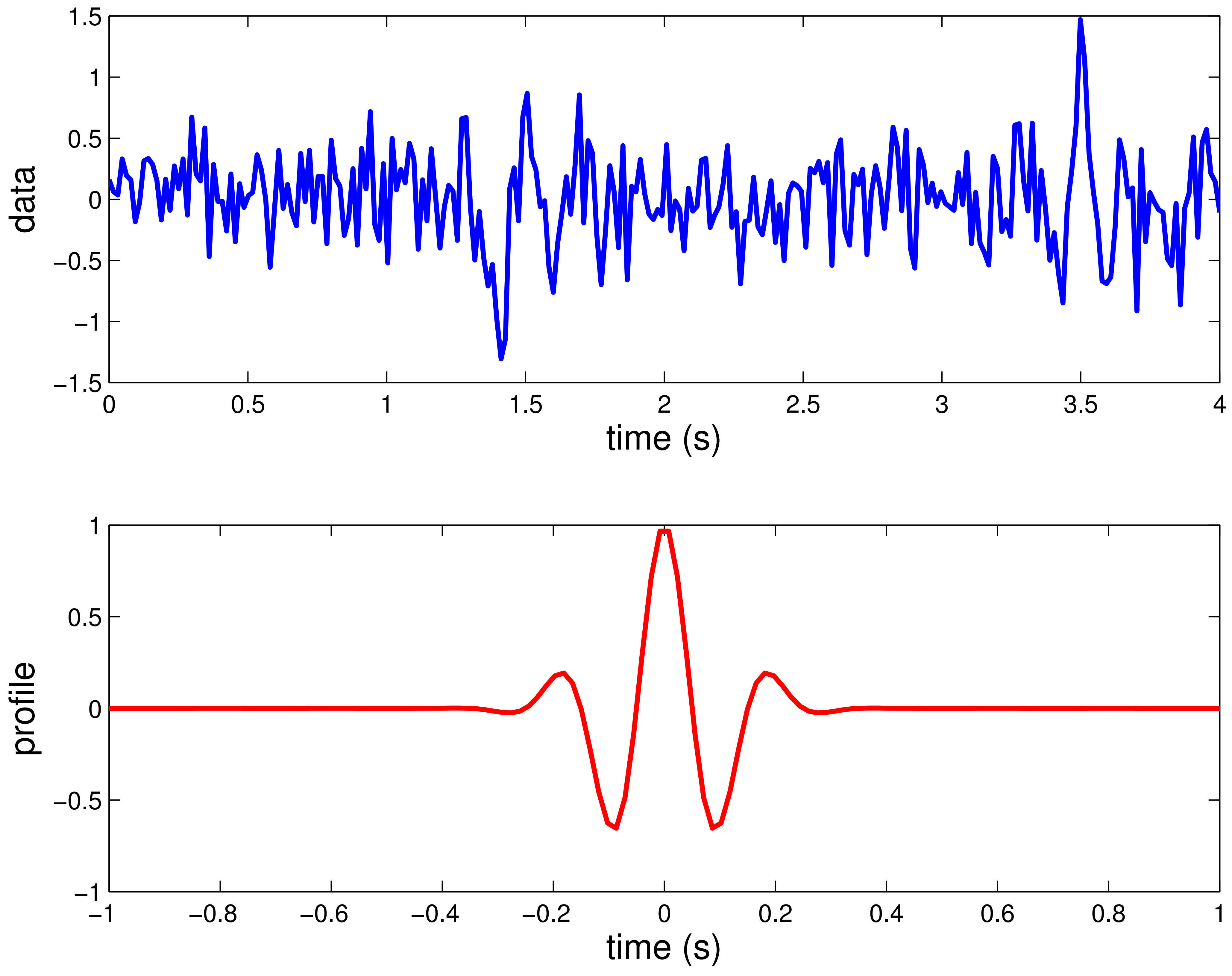
$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$

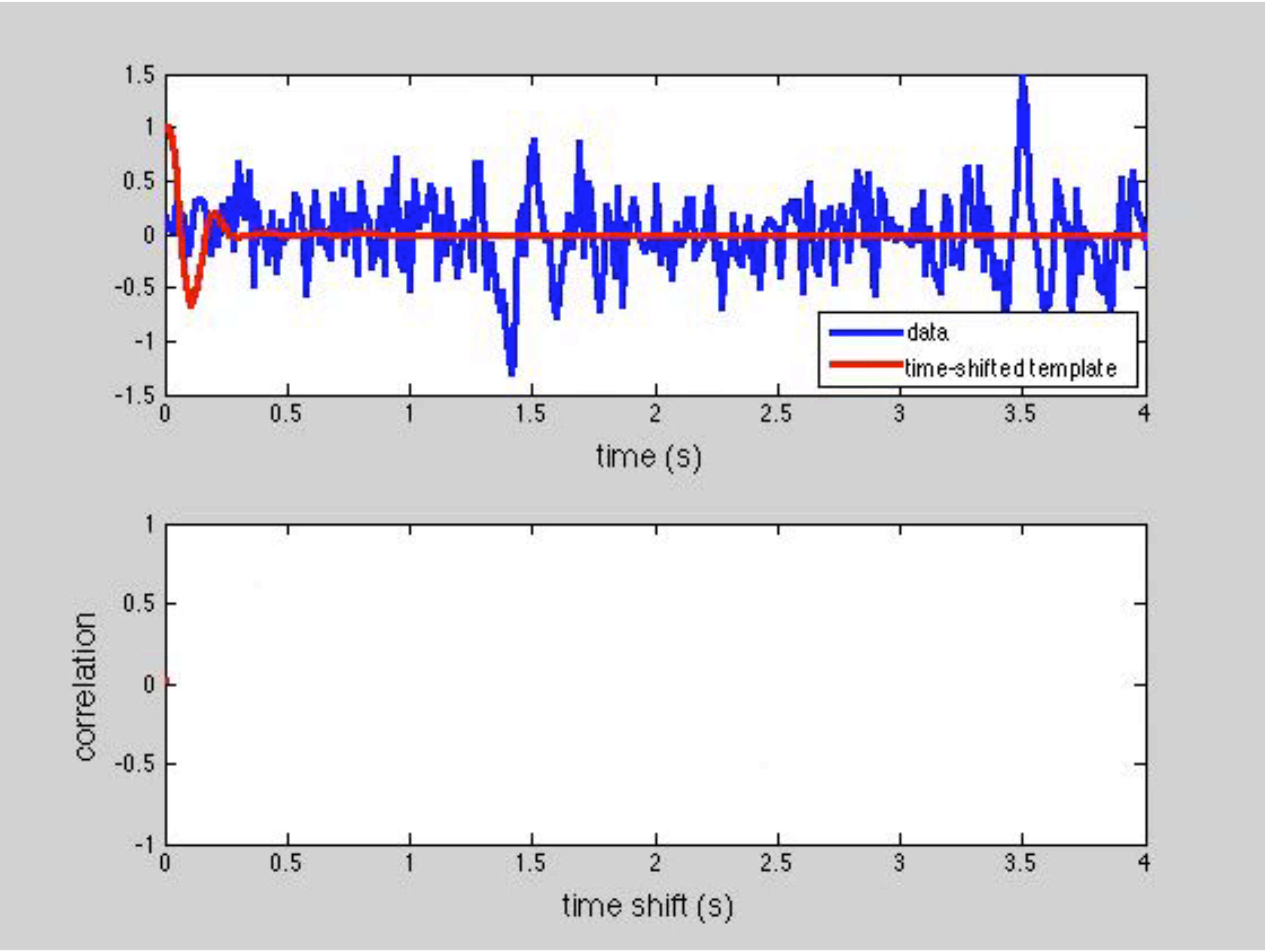




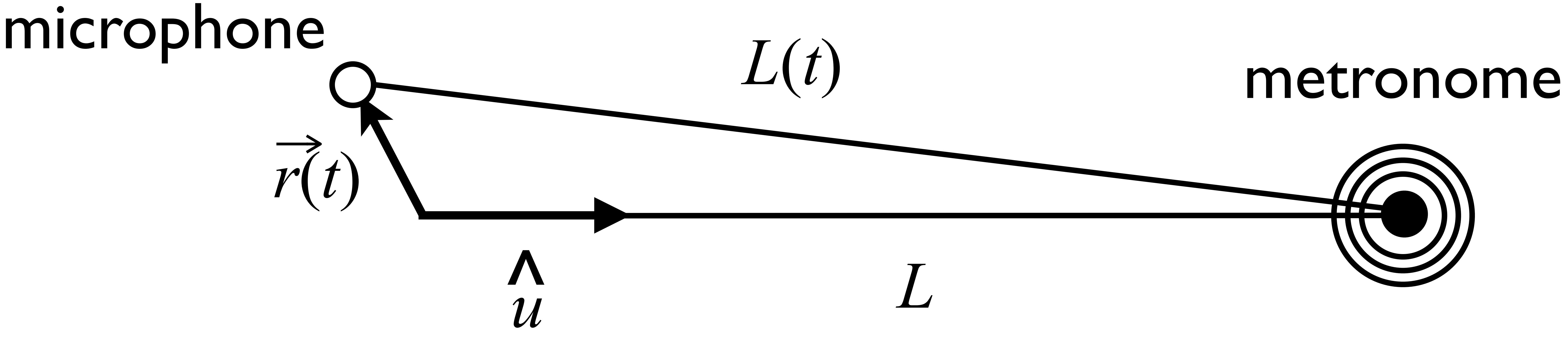
Matched-filtering determination of measured TOAs

$$C(\Delta t) = \mathcal{N} \int \mathrm{d}t \; y(t) p(t - \Delta t)$$



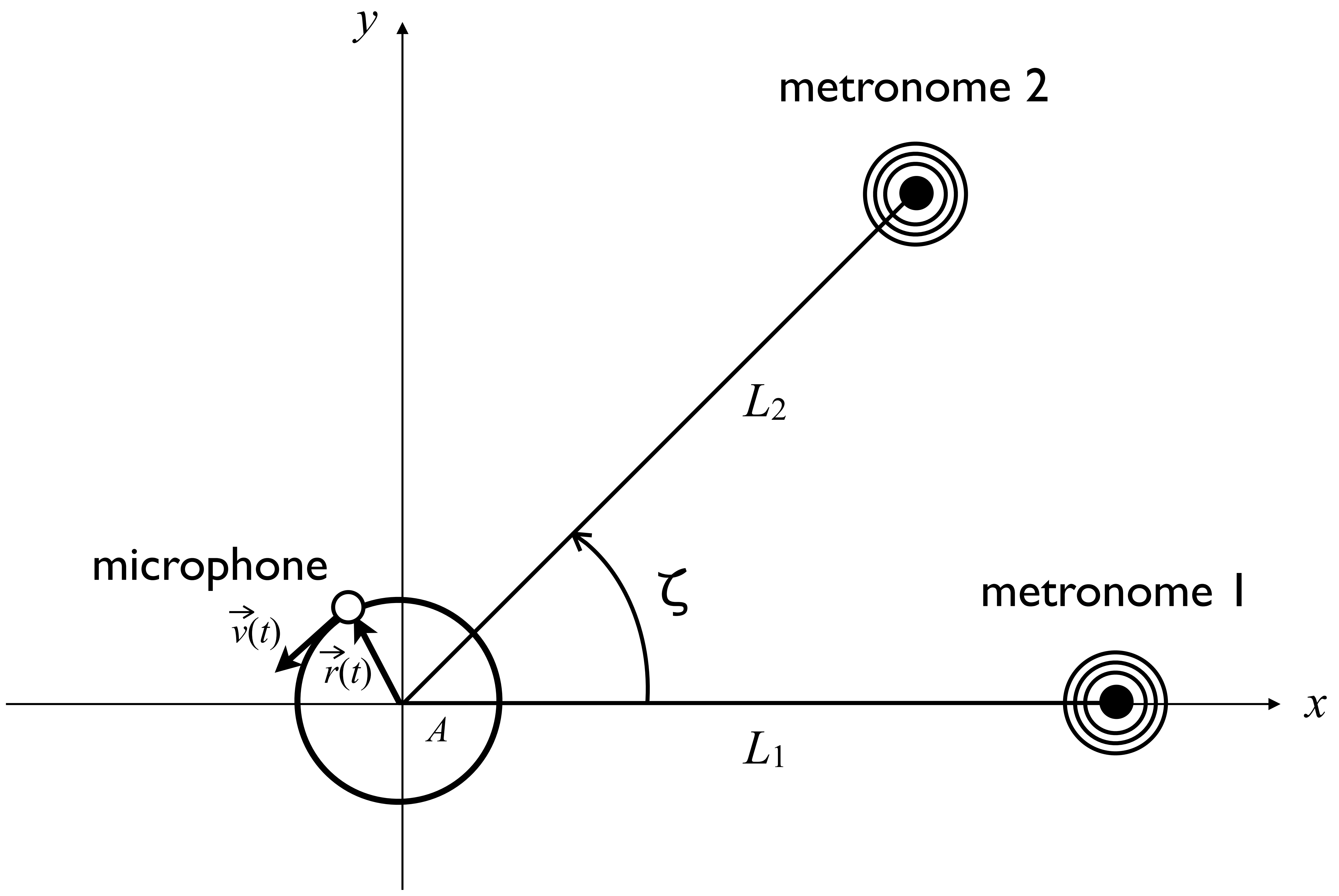


Timing-residual response to microphone motion



$$\delta\tau(t) = \frac{\Delta L(t)}{c_{\rm s}} \simeq -\frac{1}{c_{\rm s}}\hat{u}\cdot\vec{r}(t)$$

Two metronomes - uniform circular motion

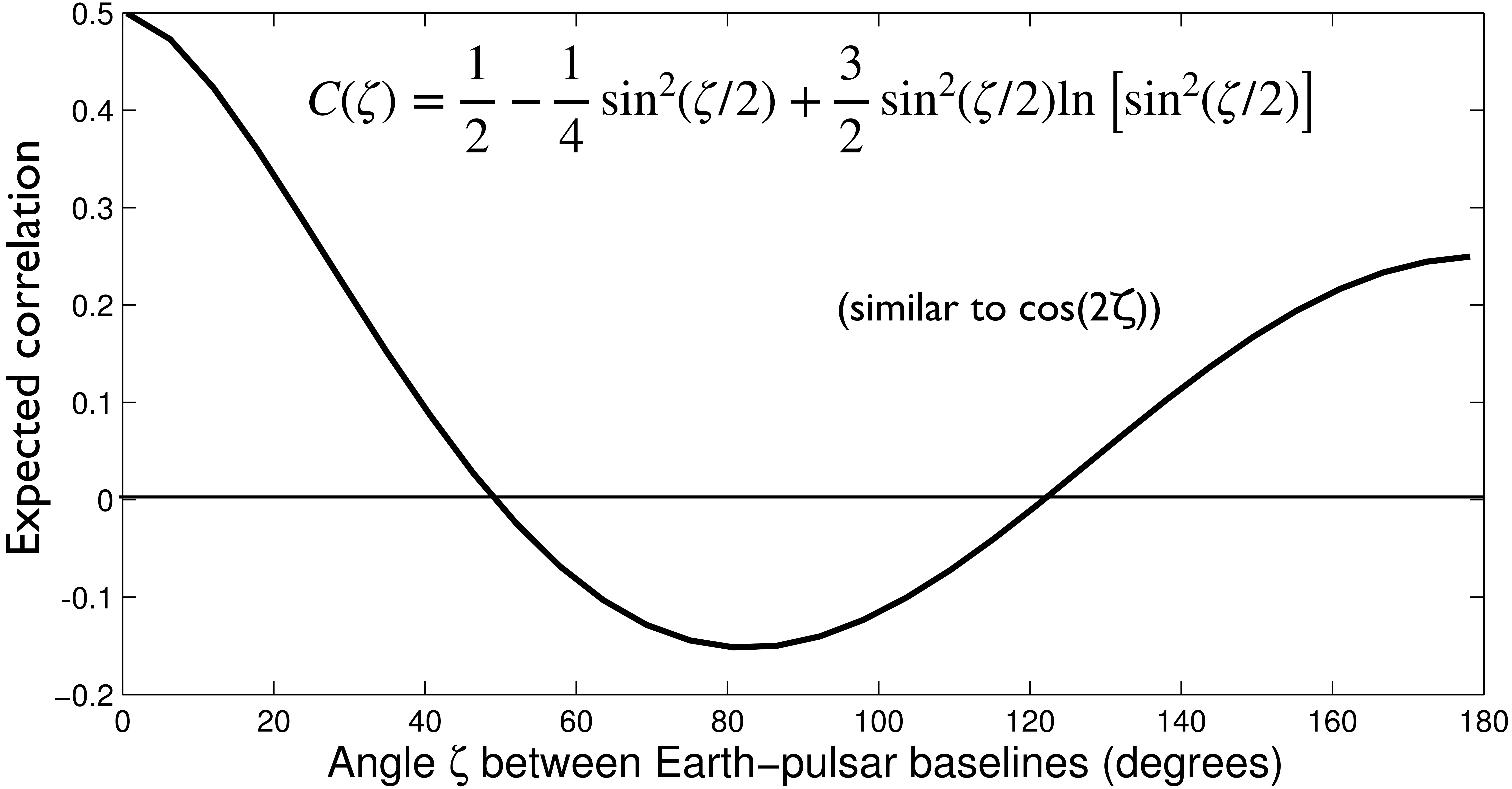


$$\vec{r}(t) = A\left[\cos(2\pi f_0 t + \phi_0)\hat{x} + \sin(2\pi f_0 t + \phi_0)\hat{y}\right]$$

$$\delta\tau_I(t) \simeq -\frac{A}{c_s}\cos(2\pi f_0 t + \phi_0 - \theta_I)\,, \quad I = 1,2$$

$$\rho_{12} \simeq \cos\zeta\,, \quad \zeta \equiv \theta_1 - \theta_2$$

Expected PTA correlation - Hellings & Downs curve
(isotropic, unpolarized GW background)



Metronome demo numbers

c_s = 340 m/s (in air)

amplitude $\approx$ 5 cm

amplitude / c_s = 1×10^{-4} sec

184 bpm: T_p = 0.3261 sec

208 bpm: T_p = 0.2885 sec

Pulsar timing numbers

$f \sim$ 1/few weeks to 1/10 years (10^{-7} Hz to 10^{-9} Hz)

$\lambda \sim$ 0.1 to 10 lyr (GW wavelength)

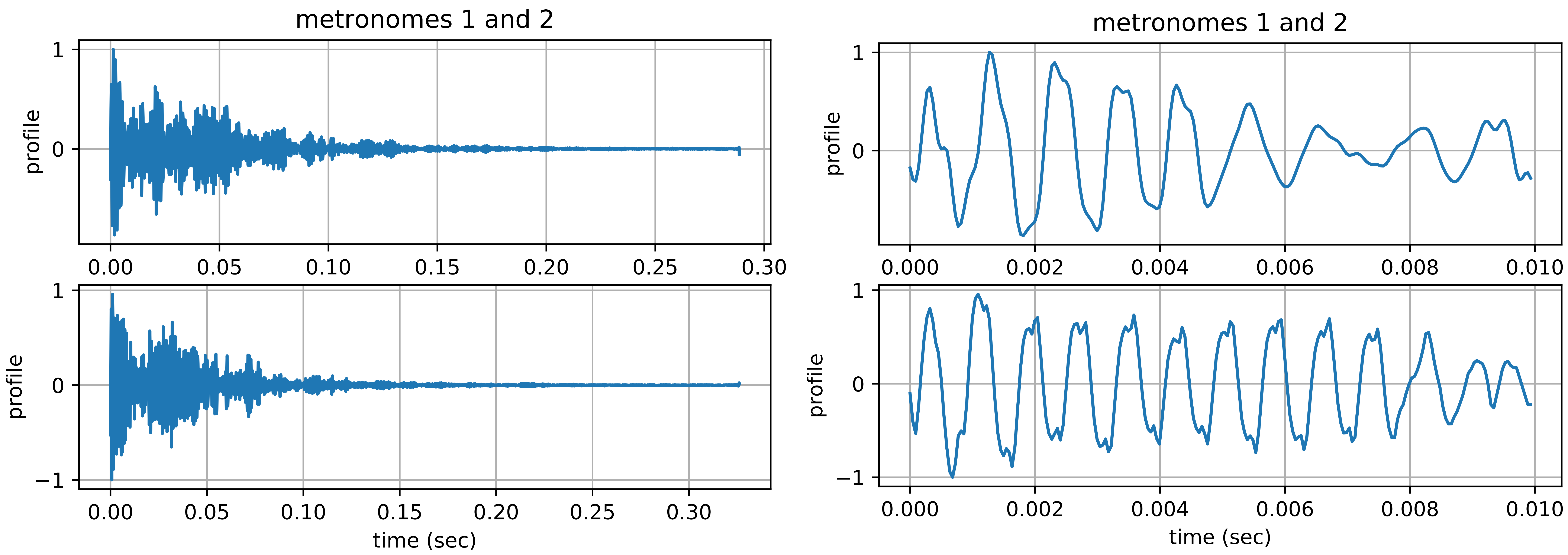
$L \sim$ few x 1000 lyr (distance to pulsars)

$\lambda << L$ (short-wavelength limit)

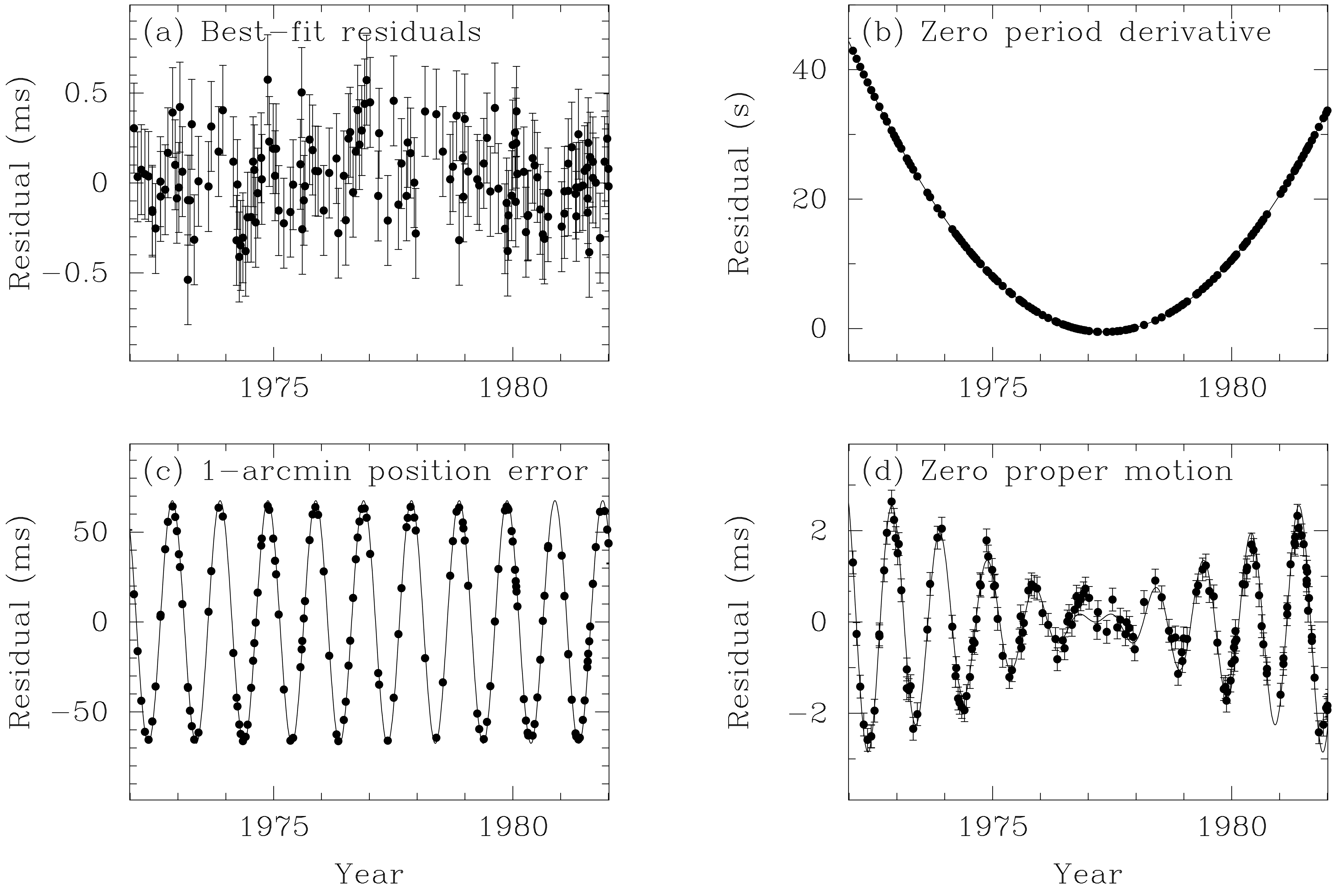
sensitivity $\sim \sigma_{rms}/T_{obs} \sim$ 100 ns/10 yr $\sim 10^{-15}$

can detect changes ~10 km in the position of a pulsar at a distance of ~1000 lyr

Metronome pulse profiles



Errors in the timing model show up as deterministic features in the timing residuals



(from Lorimer LRR-2008-8)