

Metronome-microphone demo of a pulsar timing array

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Les Houches Summer School
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(work in collaboration with M. Lam, M. Normandin, J. Key, and J. Hazboun)



Purpose

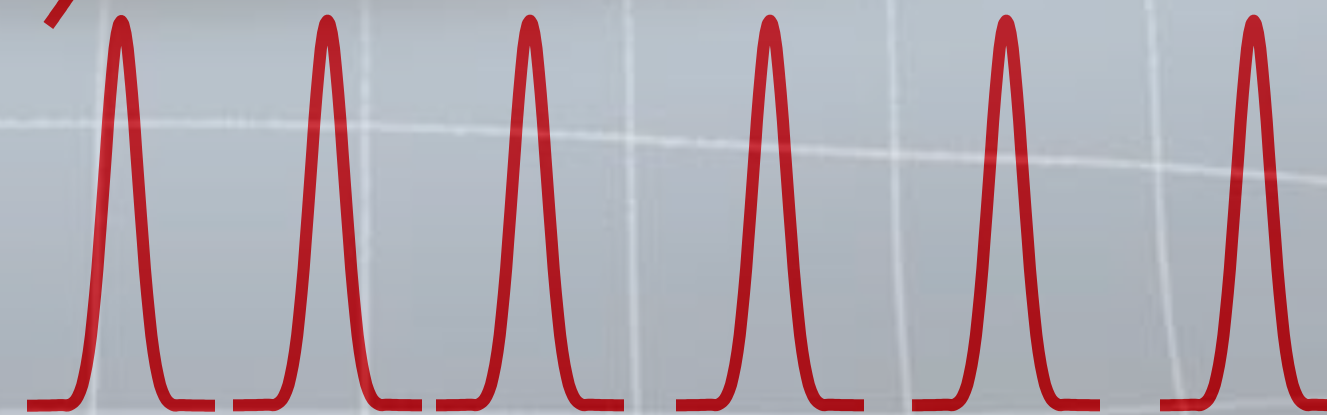
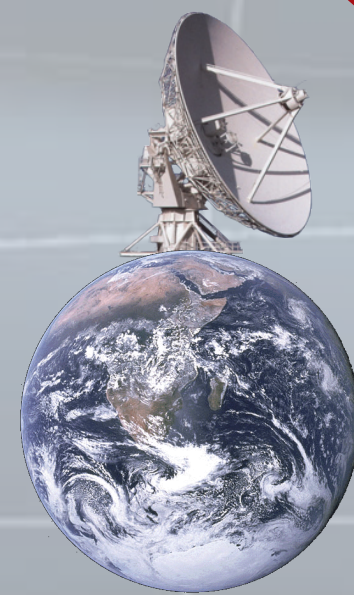
- Illustrate how a PTA is used to search for GWs in the context of a simple acoustical model
- Become familiar with techniques used by pulsar astronomers:
 - **folding** (for calculating pulse periods and pulse profiles)
 - **detrending** (for better estimating pulse period)
 - **matched filtering** (for calculating measured times-of-arrivals)
 - **timing models** (for calculating expected times-of-arrivals)
 - **correlation analyses** (for extracting common GW component)

<https://github.com/josephromano/pta-demo>

Pulsar timing array

GWs cause pulses to arrive
ahead or behind schedule,
correlated across pulsars

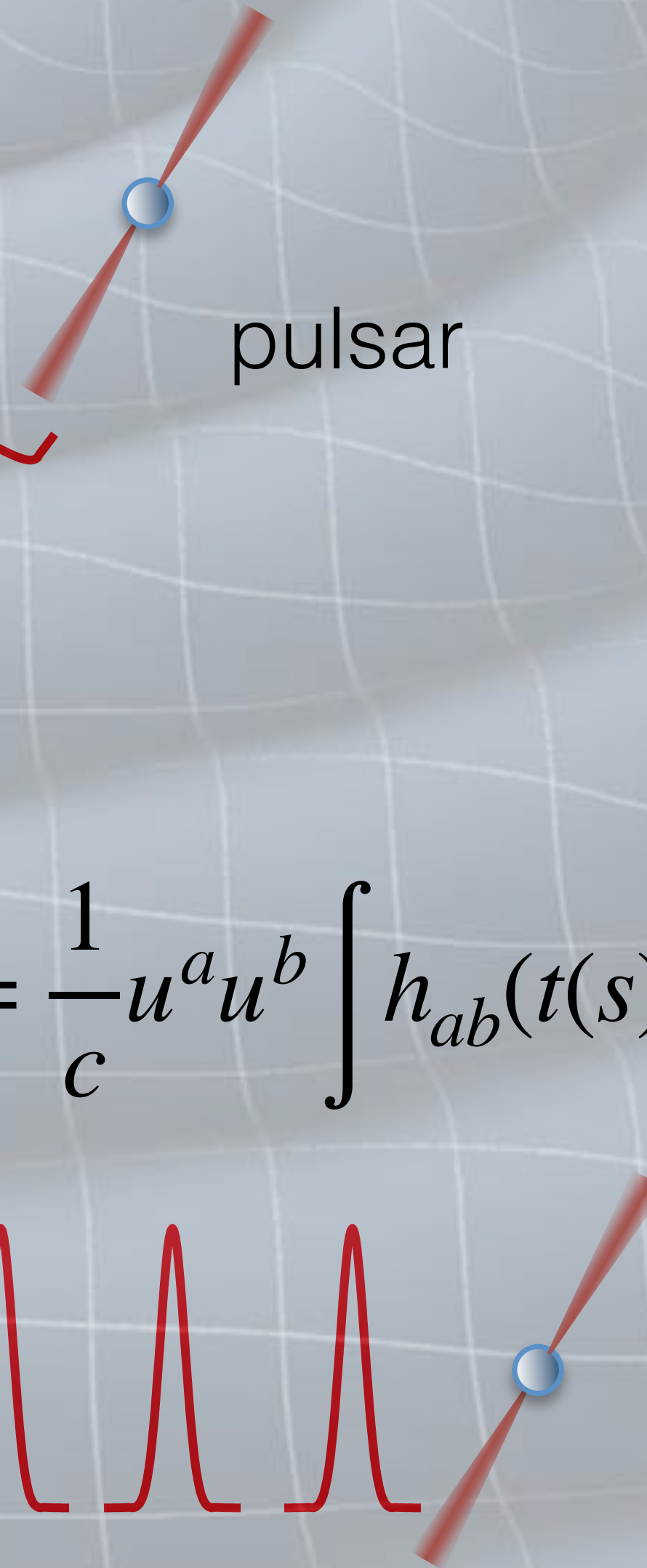
radio
telescope



$$\delta\tau(t) = \frac{1}{c} u^a u^b \int h_{ab}(t(s), \vec{x}(s)) ds$$

pulsar

pulsar



Metronome timing array

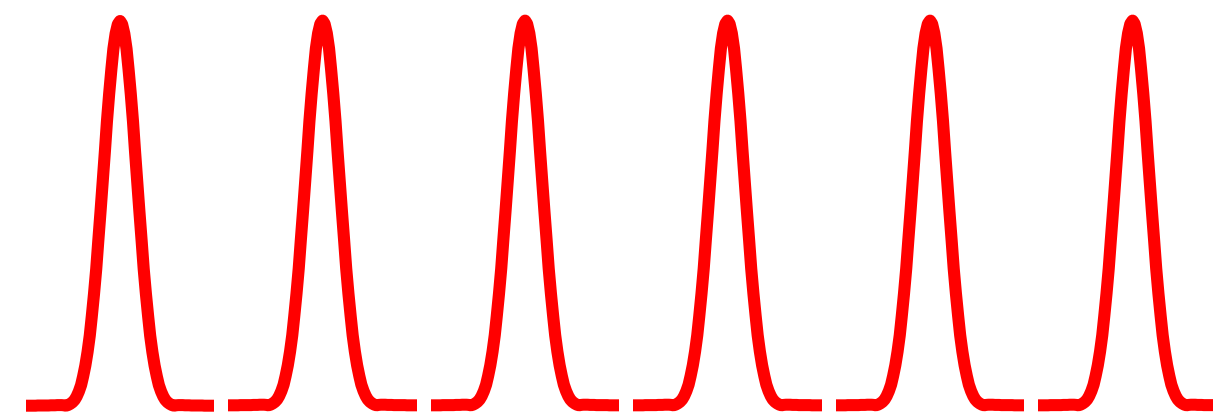
microphone motion
causes pulses to arrive
ahead or behind schedule,
correlated across metronomes



metronome

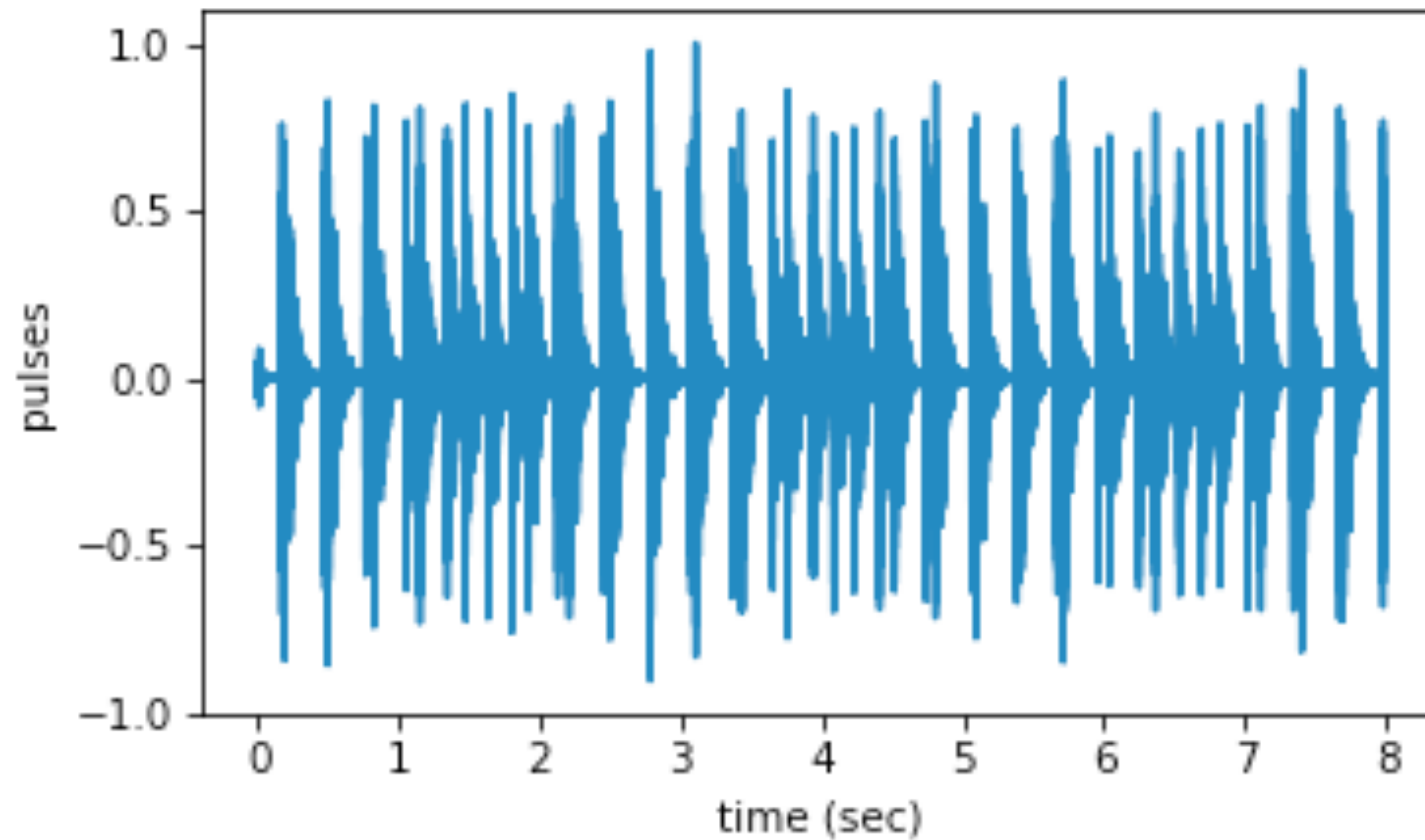
$$\delta\tau(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{1}{c_s} \hat{u} \cdot \vec{r}(t)$$

microphone



metronome

Q: Is there evidence of a “GW” in the data?



Q: Is there evidence of a “GW” in the data?

Is there a common disturbance to the pulse arrival times (TOAs), and if so, is this disturbance correlated across metronomes as expected for a “gravitational wave” (i.e., microphone motion)?

- disturbance = timing residuals = measured TOAs - expected TOAs
- common = correlated

$$\rho_{12} \equiv \langle x_1 x_2 \rangle / \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \qquad \langle x_1 x_2 \rangle \equiv \frac{1}{T_{\text{obs}}} \int_0^{T_{\text{obs}}} dt x_1(t) x_2(t)$$

- measured TOAs: calculated using matched filtering with pulse profile

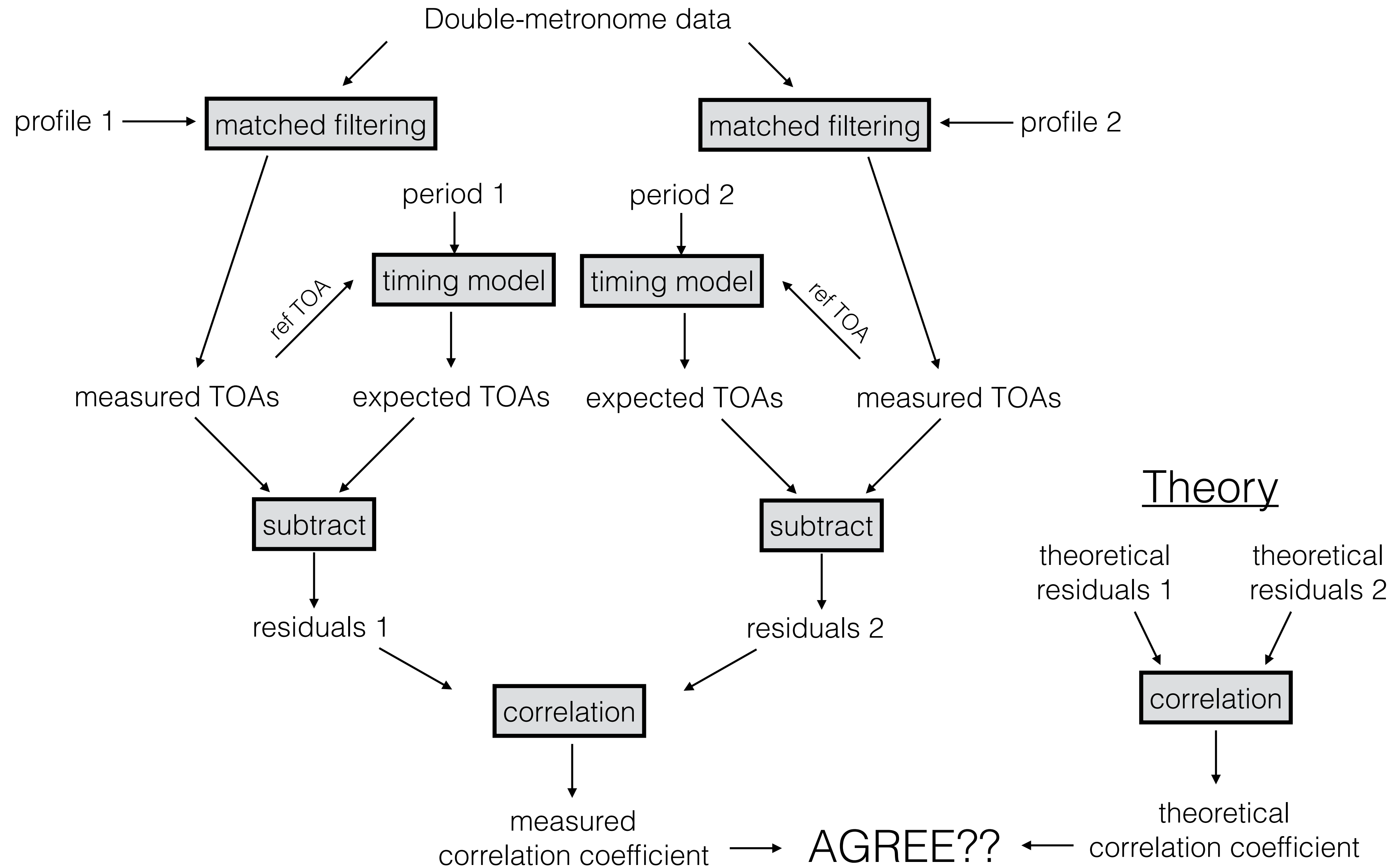
$$C(\Delta t) = \mathcal{N} \int dt y(t) p(t - \Delta t)$$

- expected TOAs: timing model using pulse period and reference TOA

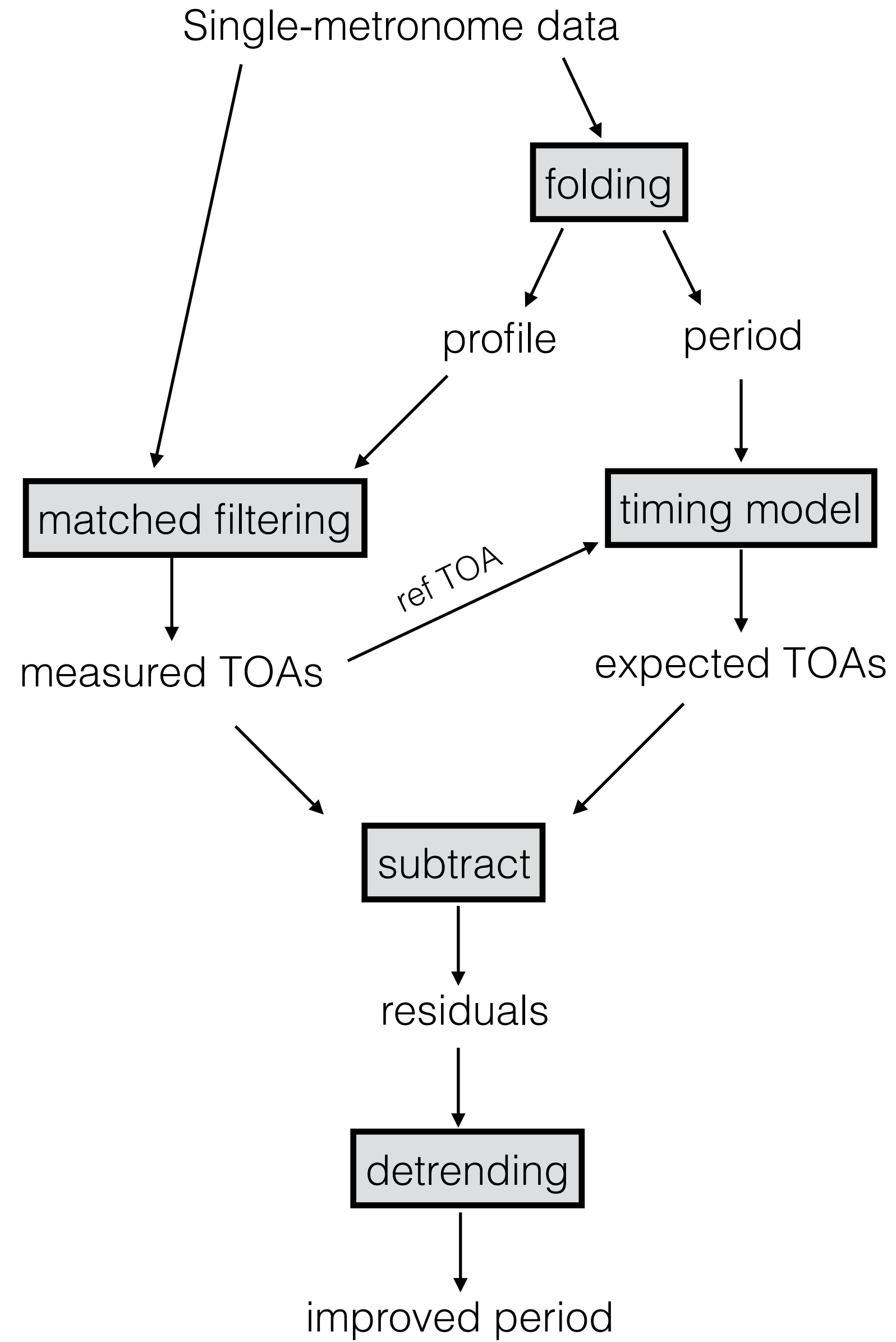
$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0) T_p$$

- pulse profile, period: folding and detrending single-metronome data
- expected “GW” correlation (for uniform circular motion): $\rho_{12} \simeq \cos \zeta$

Double-metronome analysis

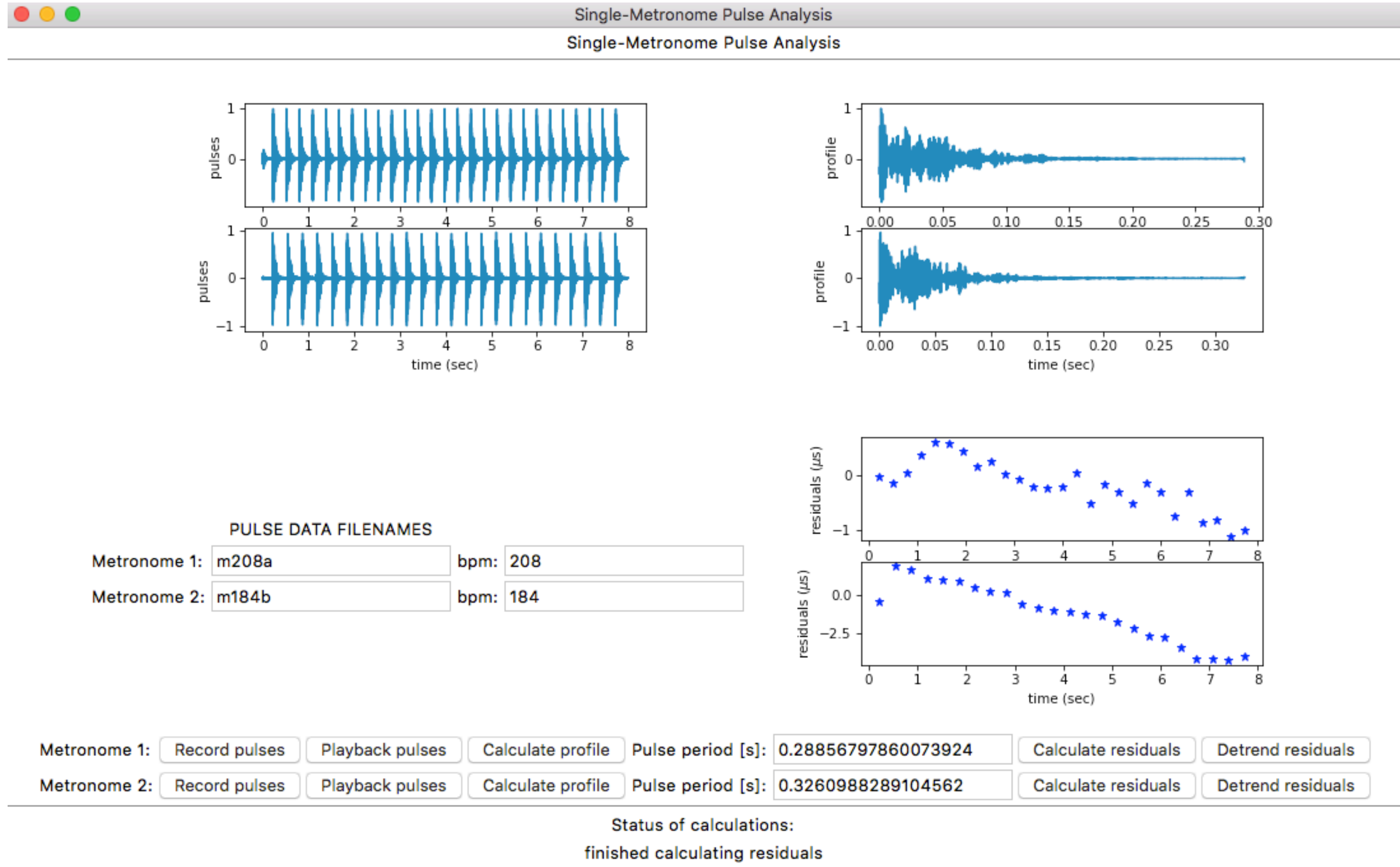


Single-metronome analysis



Output of the GUIs

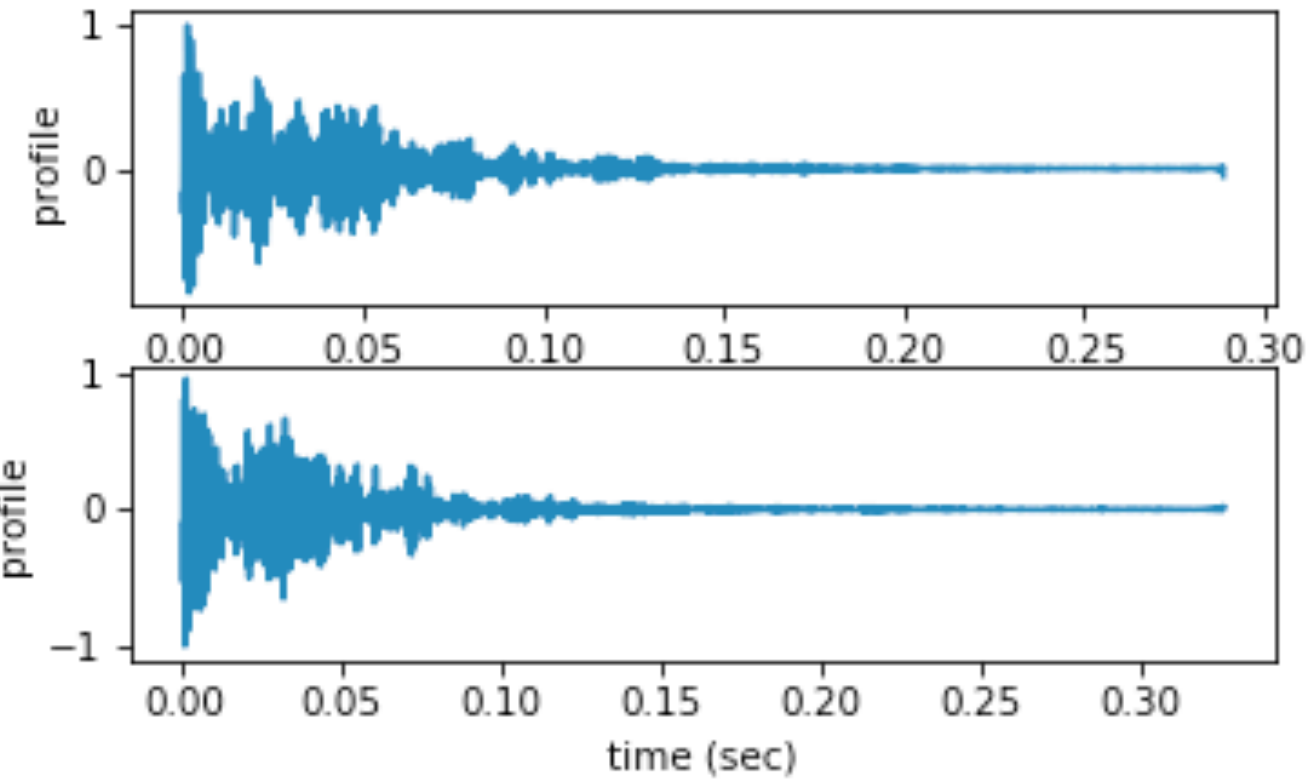
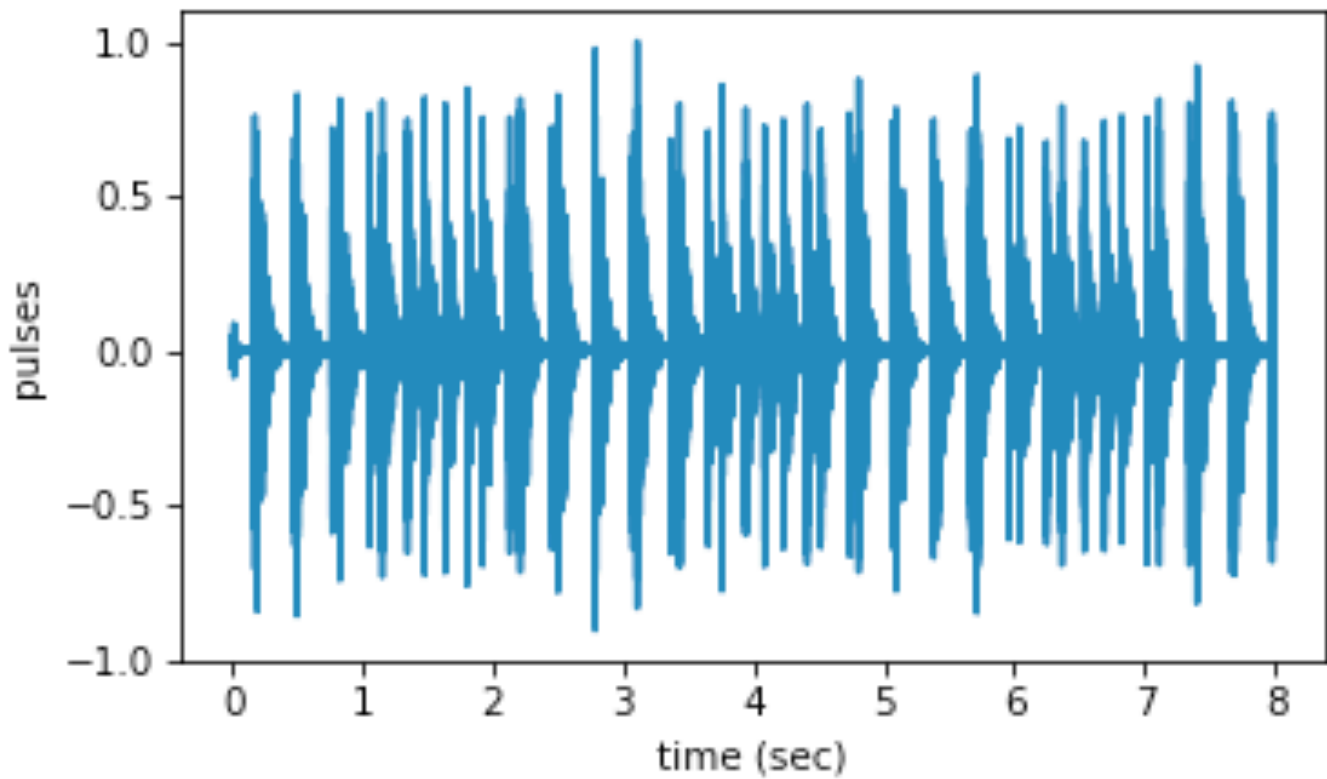
Single-metronome analysis



Double-metronome analysis ($\zeta=0$ degrees)

Double-Metronome Pulse Analysis

Double-Metronome Pulse Analysis

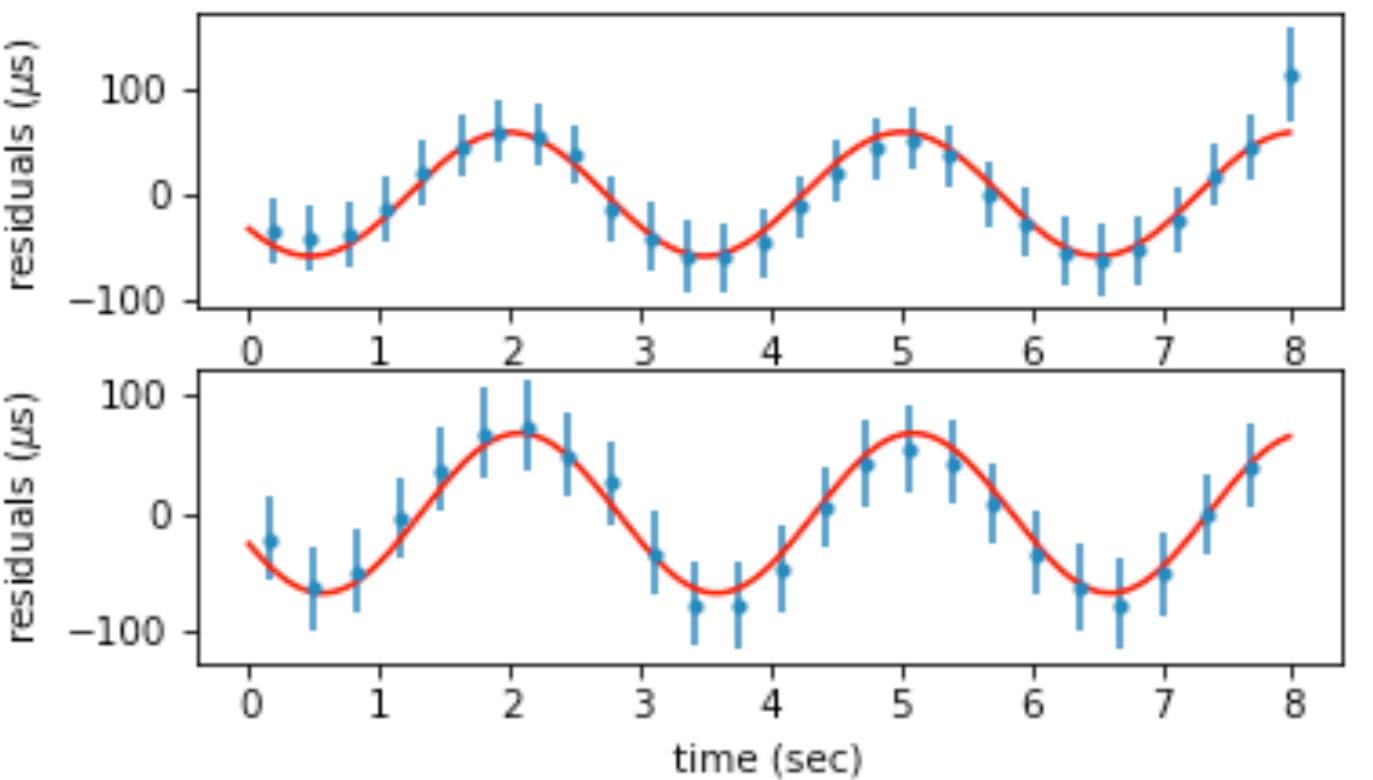


FILENAMES

Data file:
Profile 1:
Profile 2:

Pulse period [s]:
Pulse period [s]:

	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	<input type="text" value="100"/>	<input type="text" value="-58.34831246716732"/>	<input type="text" value="100"/>	<input type="text" value="-67.83561046171864"/>
Freq [Hz]:	<input type="text" value="0.4"/>	<input type="text" value="0.33072788592748953"/>	<input type="text" value="0.4"/>	<input type="text" value="0.33116191414239915"/>
Phase [rad]:	<input type="text" value="0"/>	<input type="text" value="0.5924453954483324"/>	<input type="text" value="0"/>	<input type="text" value="0.393928097921481"/>
Offset [usec]:	<input type="text" value="0"/>	<input type="text" value="-43.4892386052206"/>	<input type="text" value="0"/>	<input type="text" value="1.7106663283346102"/>

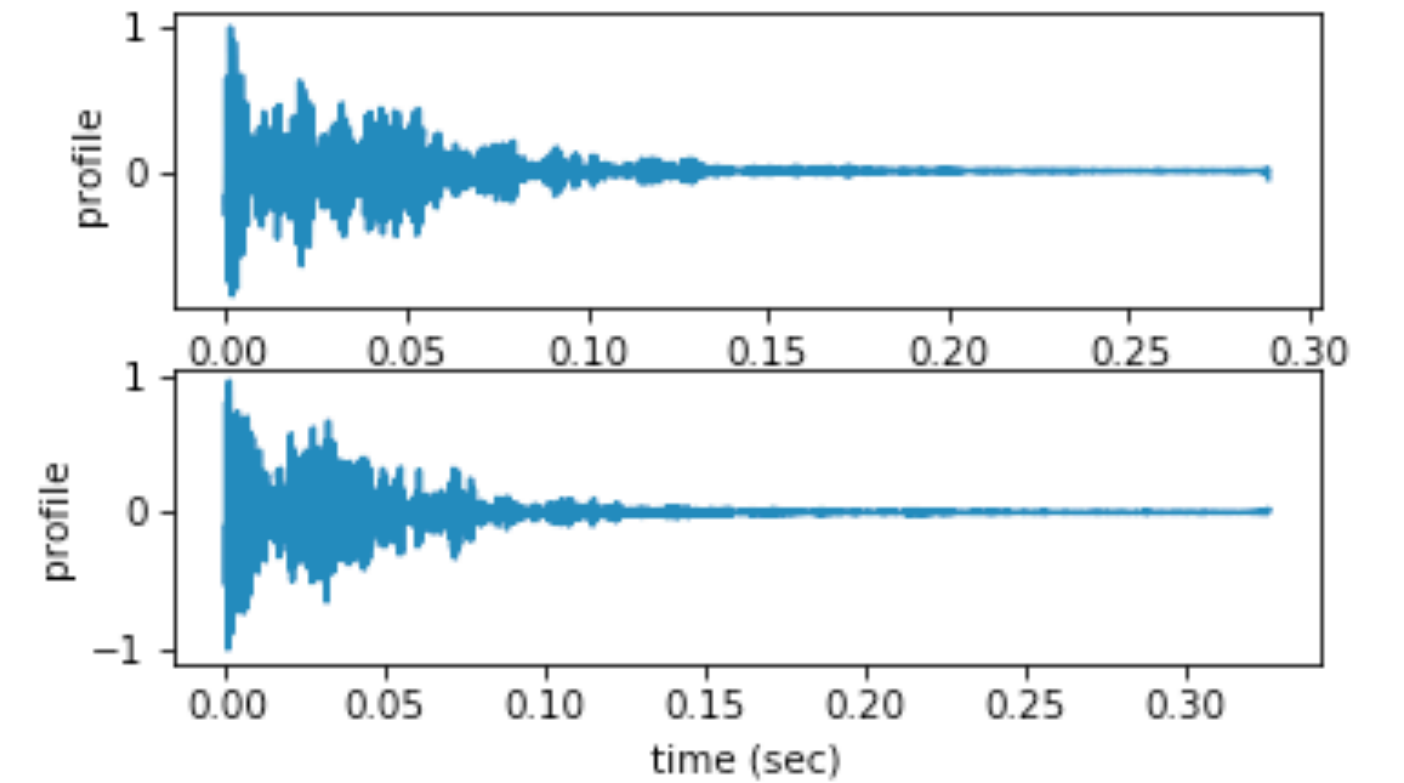
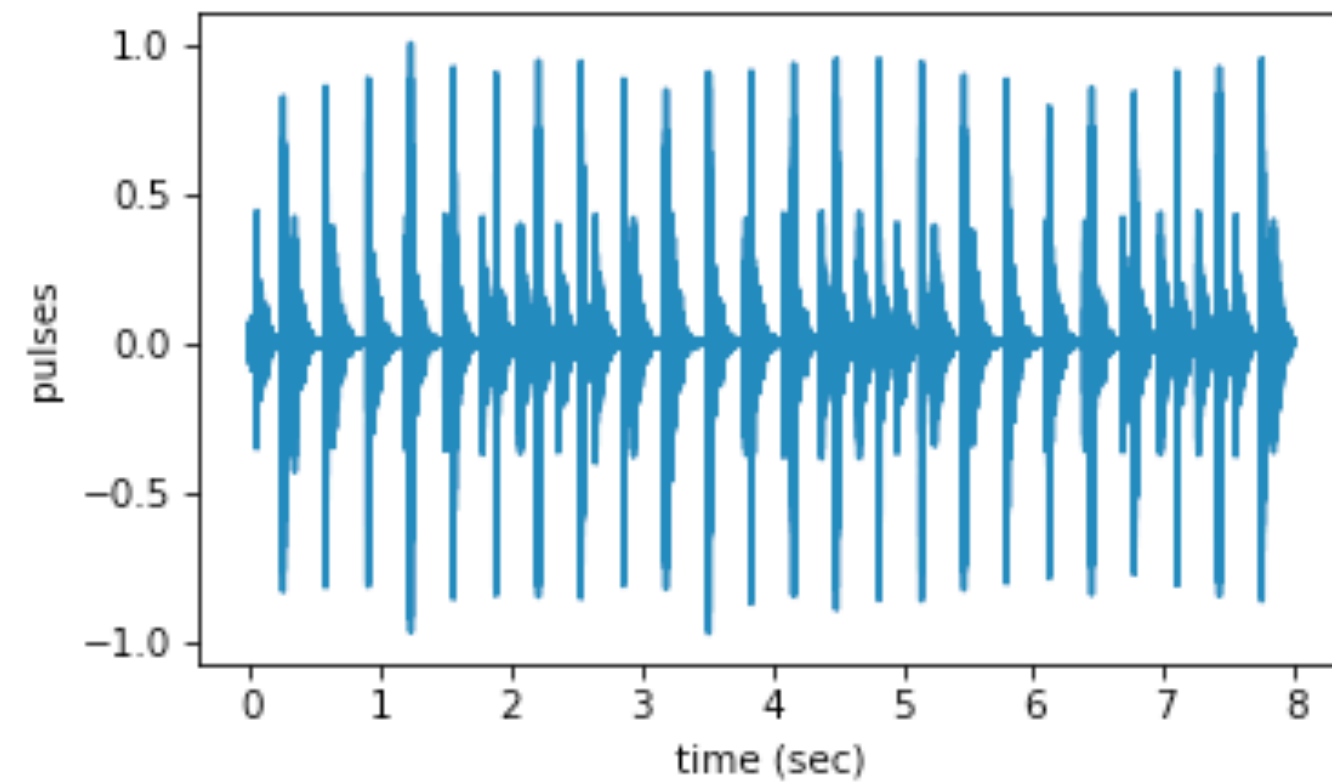


Status of calculations:
finished calculation of residuals

Double-metronome analysis ($\zeta=45$ degrees)

Double-Metronome Pulse Analysis

Double-Metronome Pulse Analysis

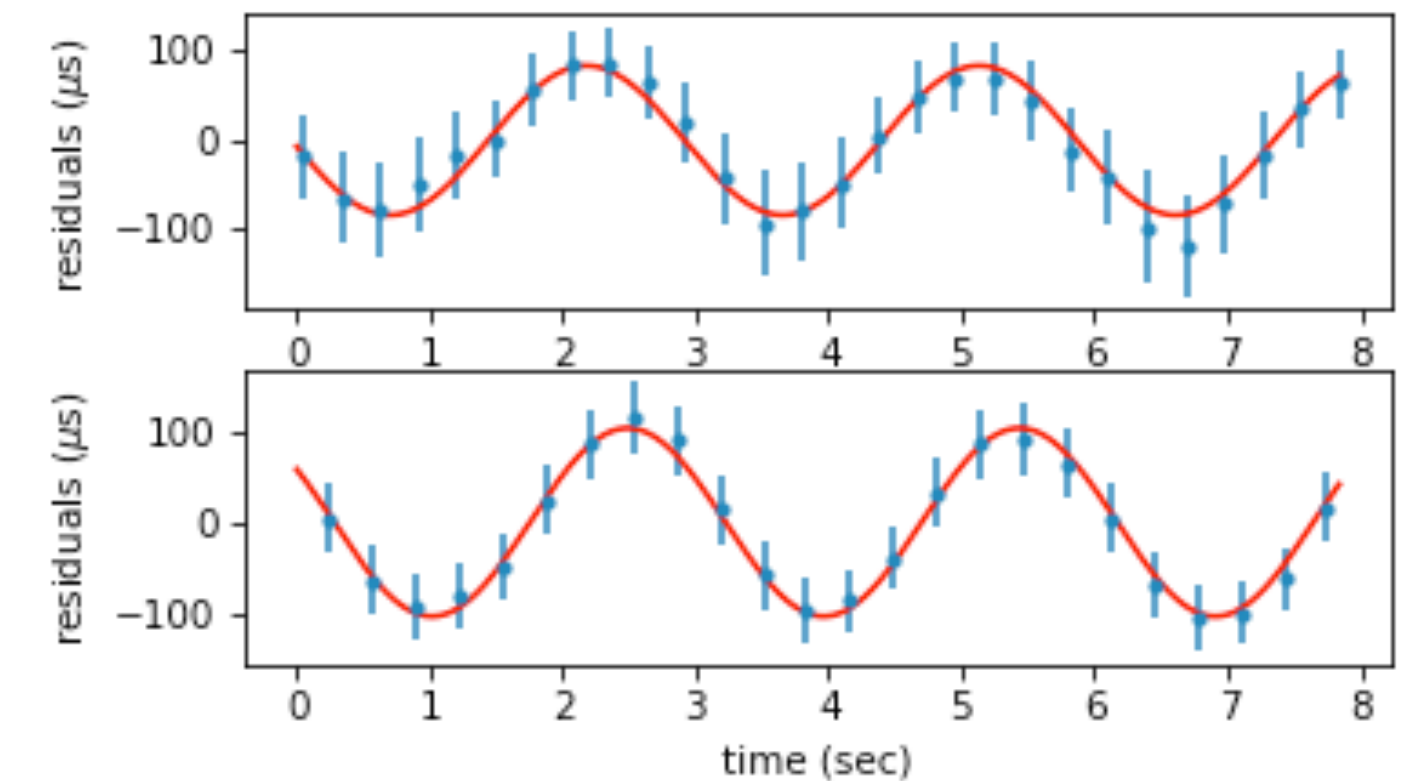


FILENAMES

Data file:
Profile 1:
Profile 2:

Pulse period [s]:
Pulse period [s]:

	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	<input type="text" value="100"/>	<input type="text" value="-83.54267144872672"/>	<input type="text" value="100"/>	<input type="text" value="103.51040350638623"/>
Freq [Hz]:	<input type="text" value="0.4"/>	<input type="text" value="0.33844784664075234"/>	<input type="text" value="0.4"/>	<input type="text" value="0.3394604420911975"/>
Phase [rad]:	<input type="text" value="0"/>	<input type="text" value="0.082340180957162"/>	<input type="text" value="0"/>	<input type="text" value="2.544467981756352"/>
Offset [usec]:	<input type="text" value="0"/>	<input type="text" value="-83.59186463058177"/>	<input type="text" value="0"/>	<input type="text" value="62.255379154251095"/>



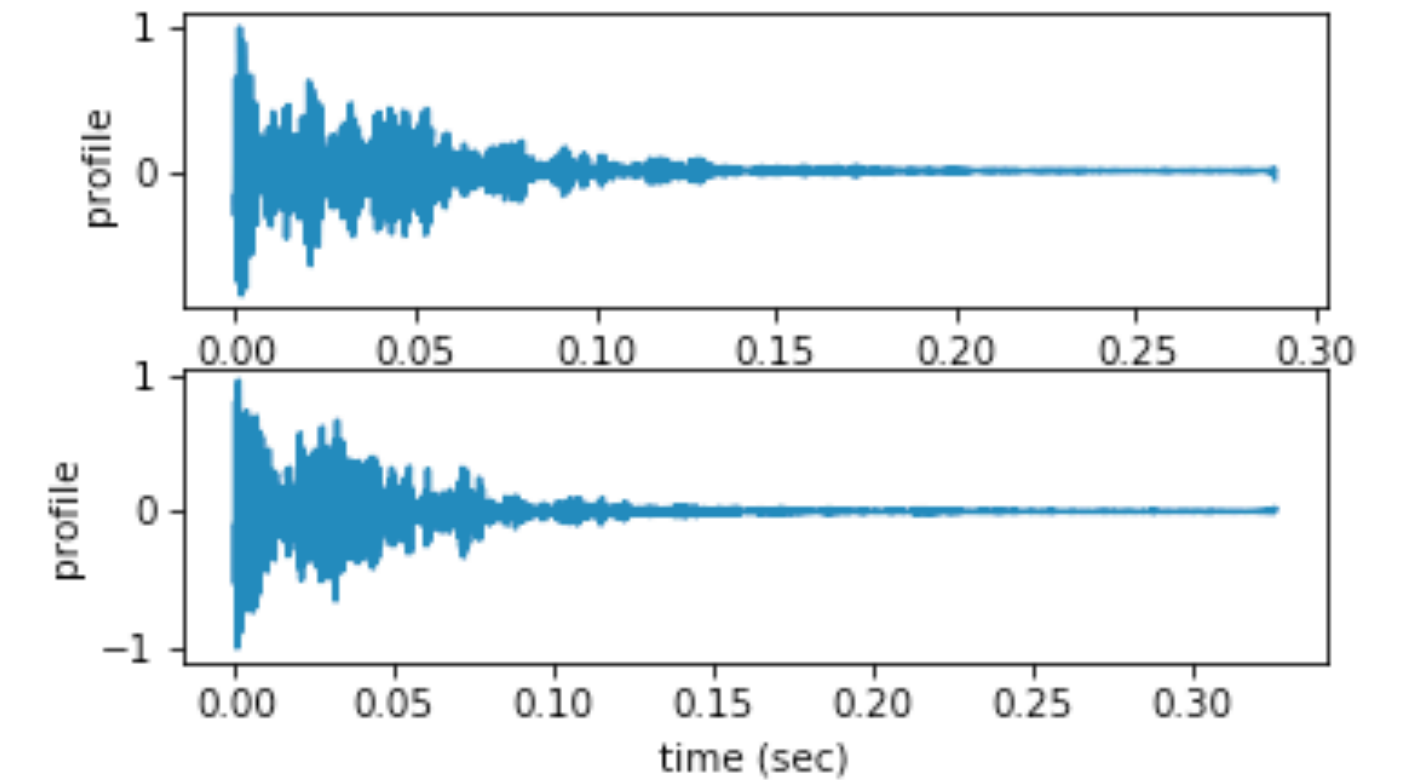
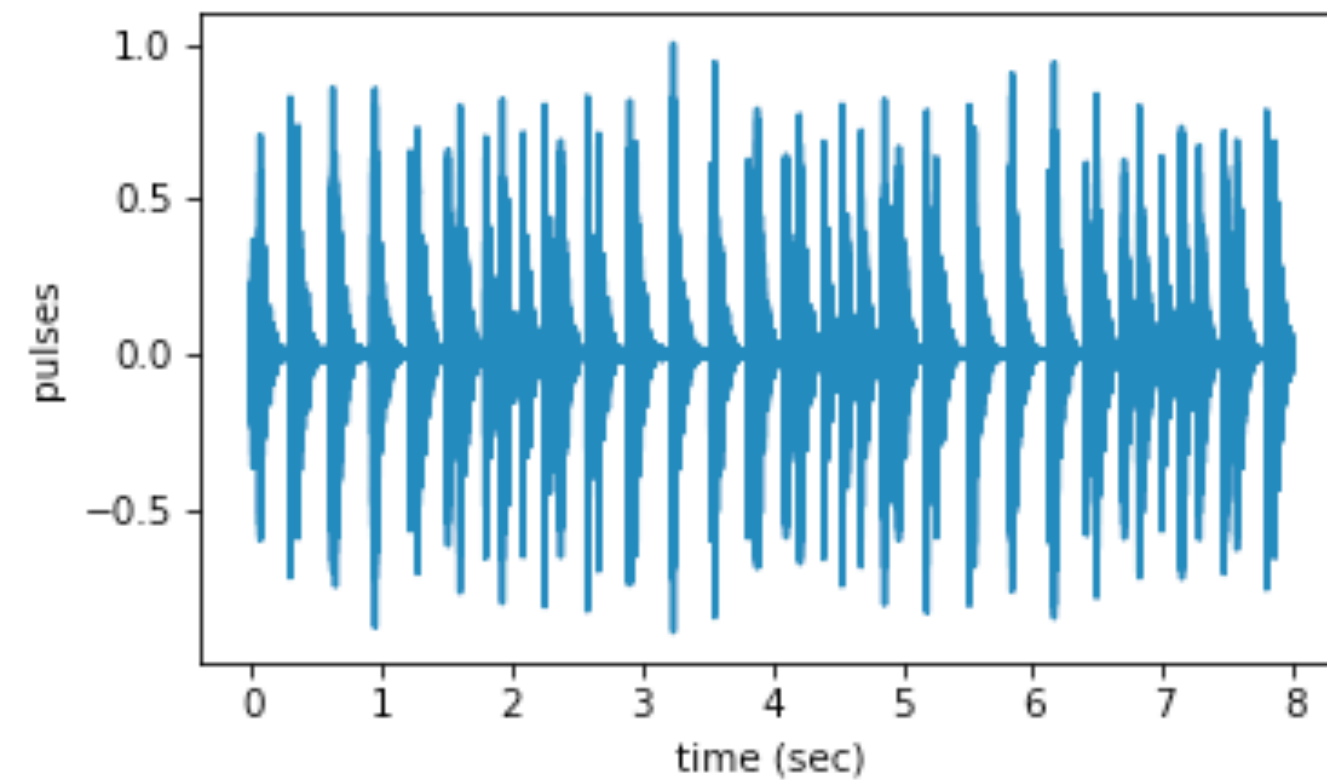
Status of calculations:

finished calculation of residuals

Double-metronome analysis ($\zeta=90$ degrees)

Double-Metronome Pulse Analysis

Double-Metronome Pulse Analysis



FILENAMES

Data file:

Profile 1:

Profile 2:

Pulse period [s]:

Pulse period [s]:

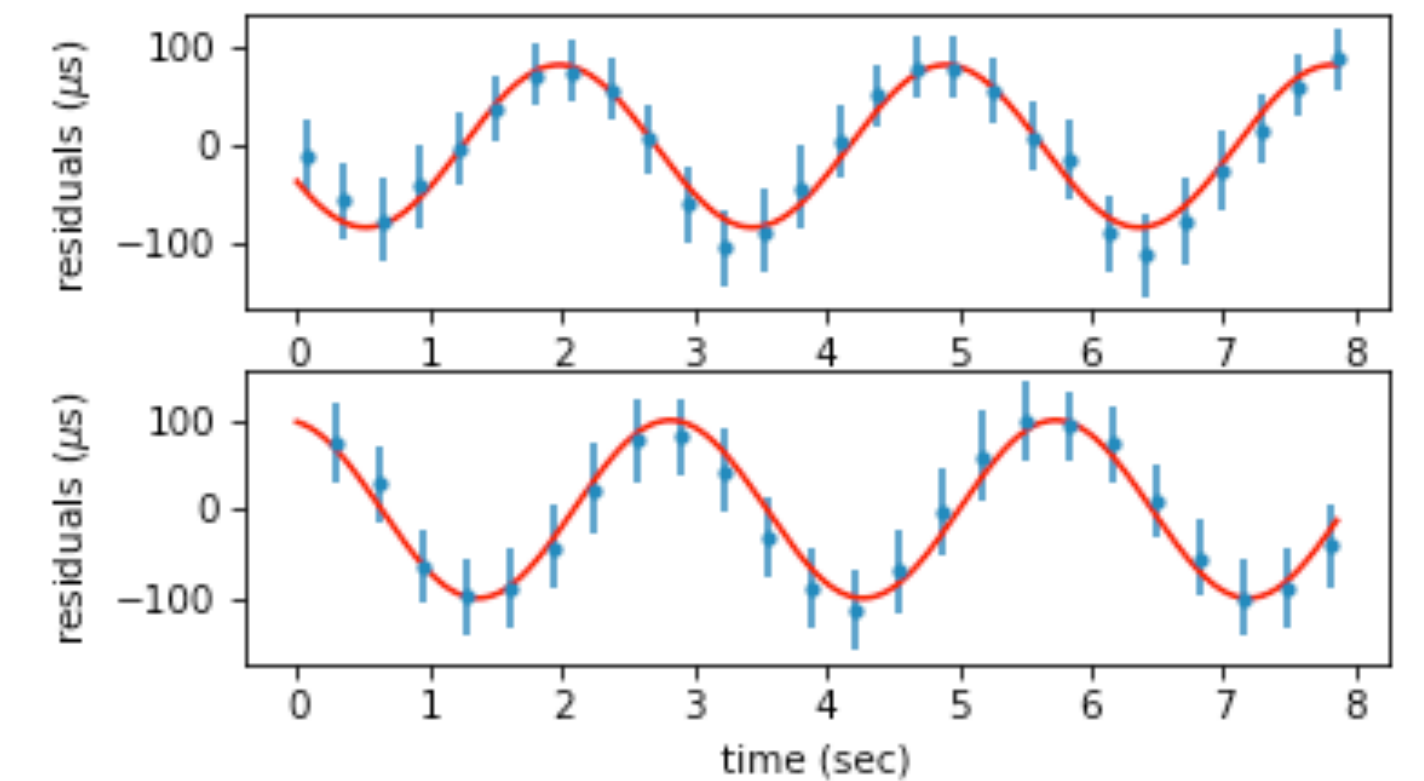
INITIAL ESTIMATES (1)

BEST-FIT VALUES (1)

INITIAL ESTIMATES (2)

BEST-FIT VALUES (2)

Amp [usec]:	<input type="text" value="100"/>	<input type="text" value="-83.2204679782364"/>	<input type="text" value="100"/>	<input type="text" value="100.56846337609925"/>
Freq [Hz]:	<input type="text" value="0.4"/>	<input type="text" value="0.34235401761876616"/>	<input type="text" value="0.4"/>	<input type="text" value="0.3435231659876081"/>
Phase [rad]:	<input type="text" value="0"/>	<input type="text" value="0.45096522793443056"/>	<input type="text" value="0"/>	<input type="text" value="1.7673131445900625"/>
Offset [usec]:	<input type="text" value="0"/>	<input type="text" value="-88.73206094670937"/>	<input type="text" value="0"/>	<input type="text" value="-95.5978511142905"/>



Record pulses

Playback pulses

Load pulse profiles

Calculate residuals

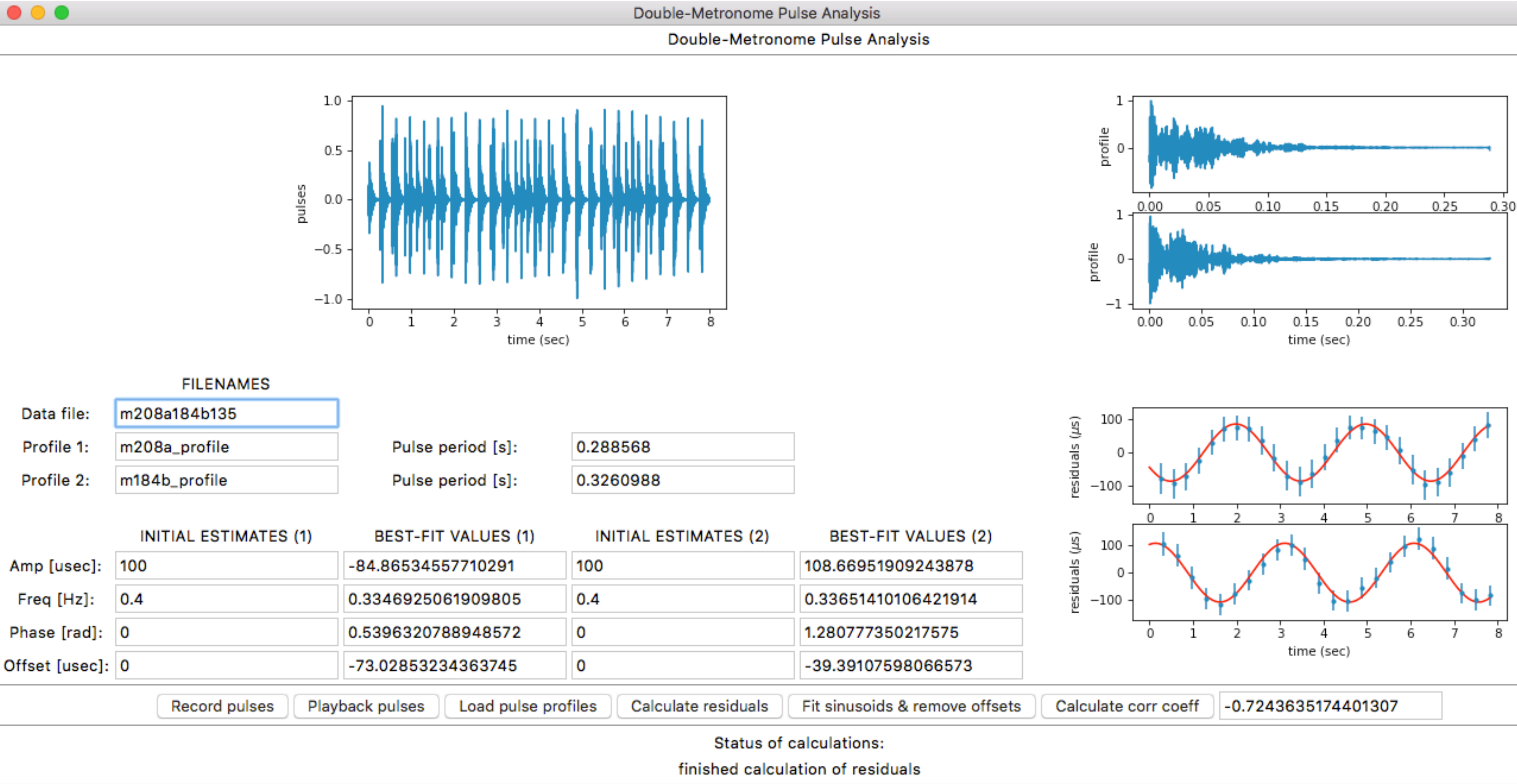
Fit sinusoids & remove offsets

Calculate corr coeff

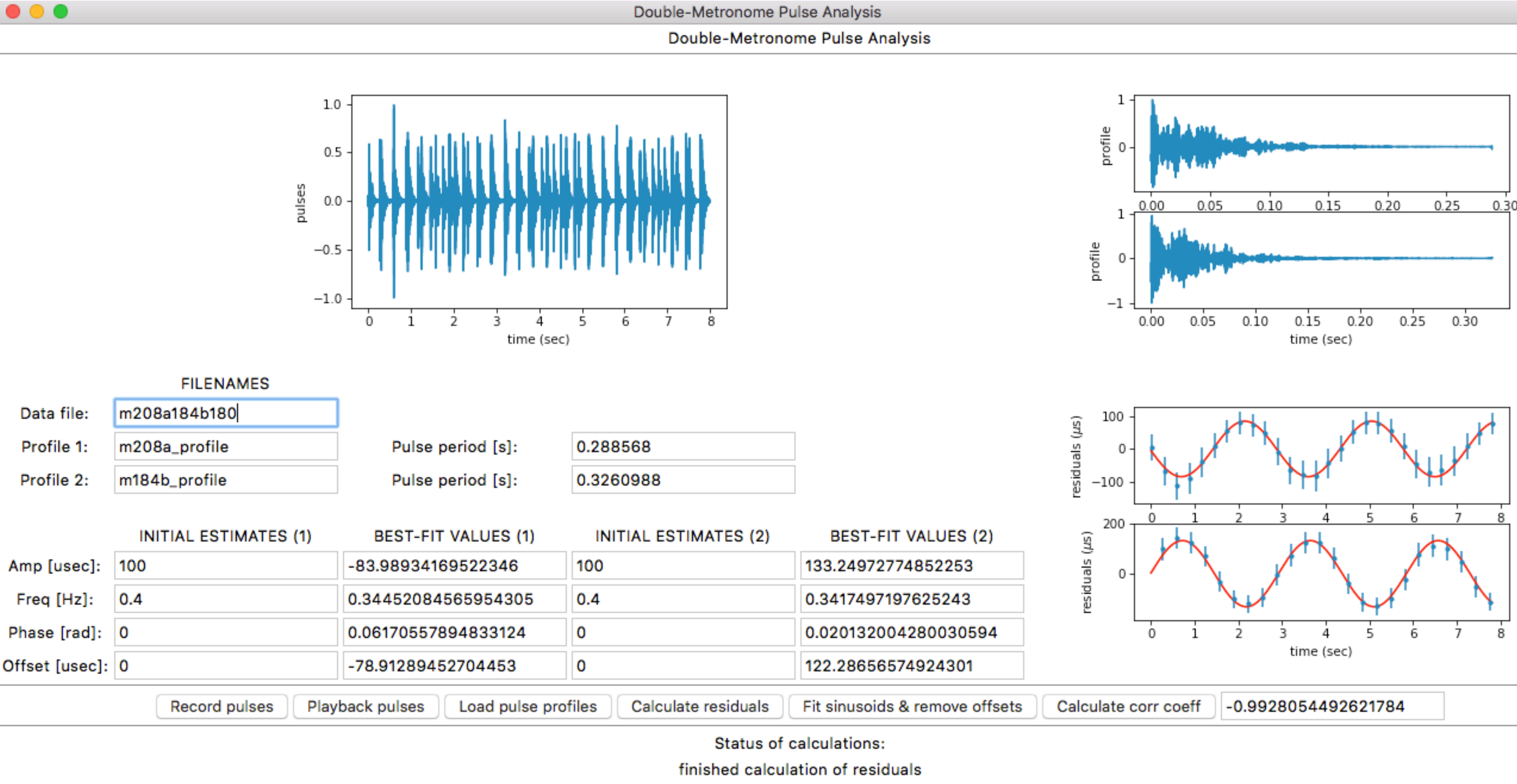
Status of calculations:

finished calculation of residuals

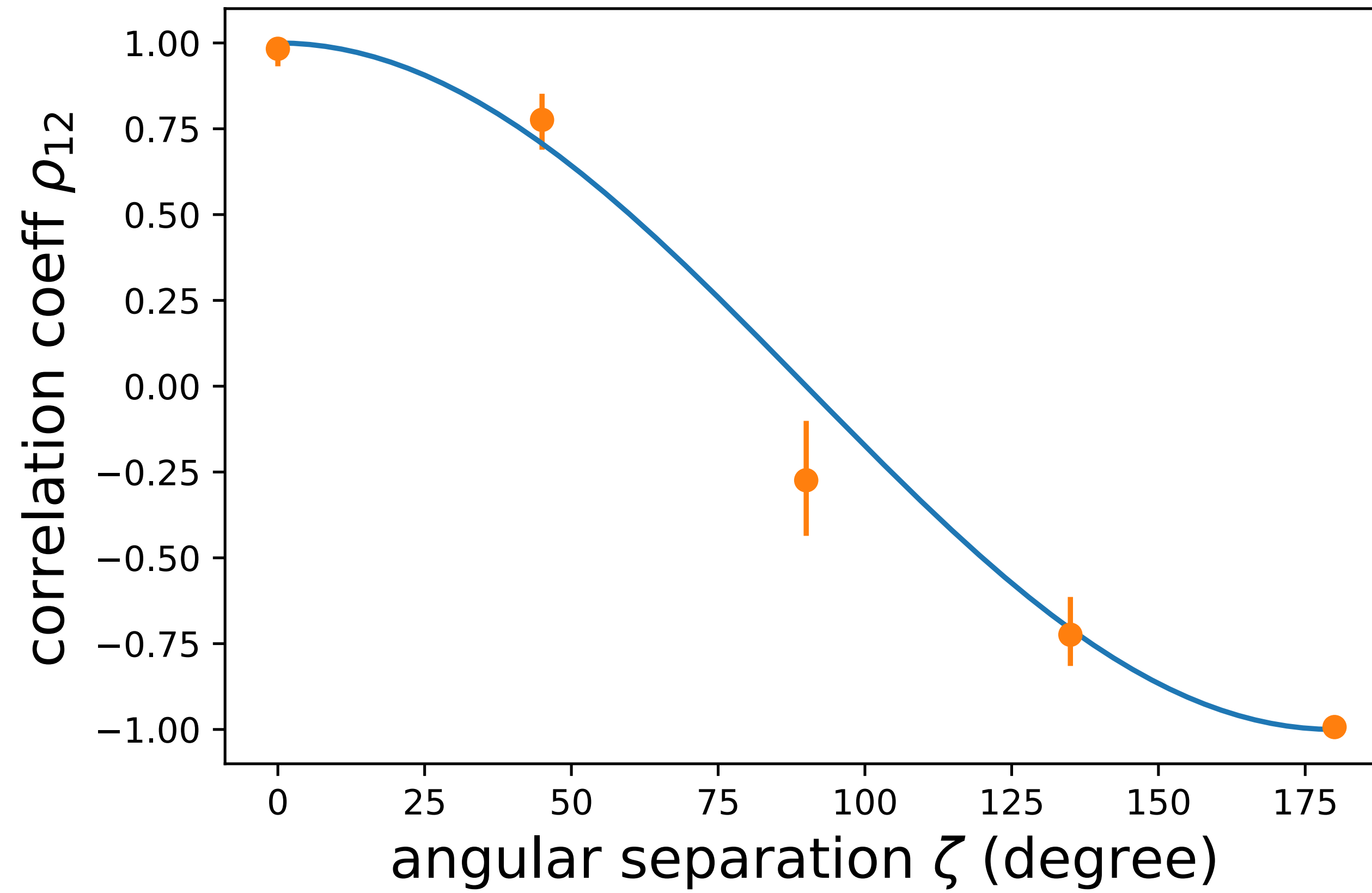
Double-metronome analysis ($\zeta=135$ degrees)



Double-metronome analysis ($\zeta=180$ degrees)

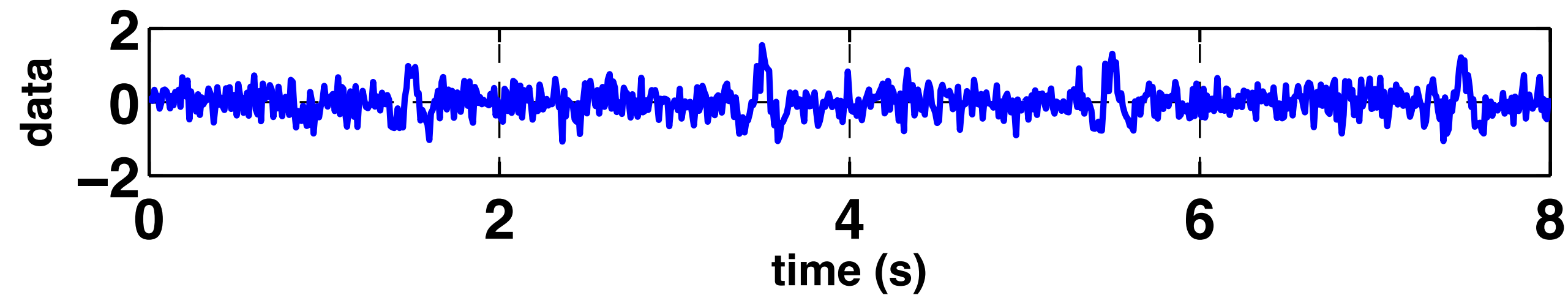


Metronome correlation

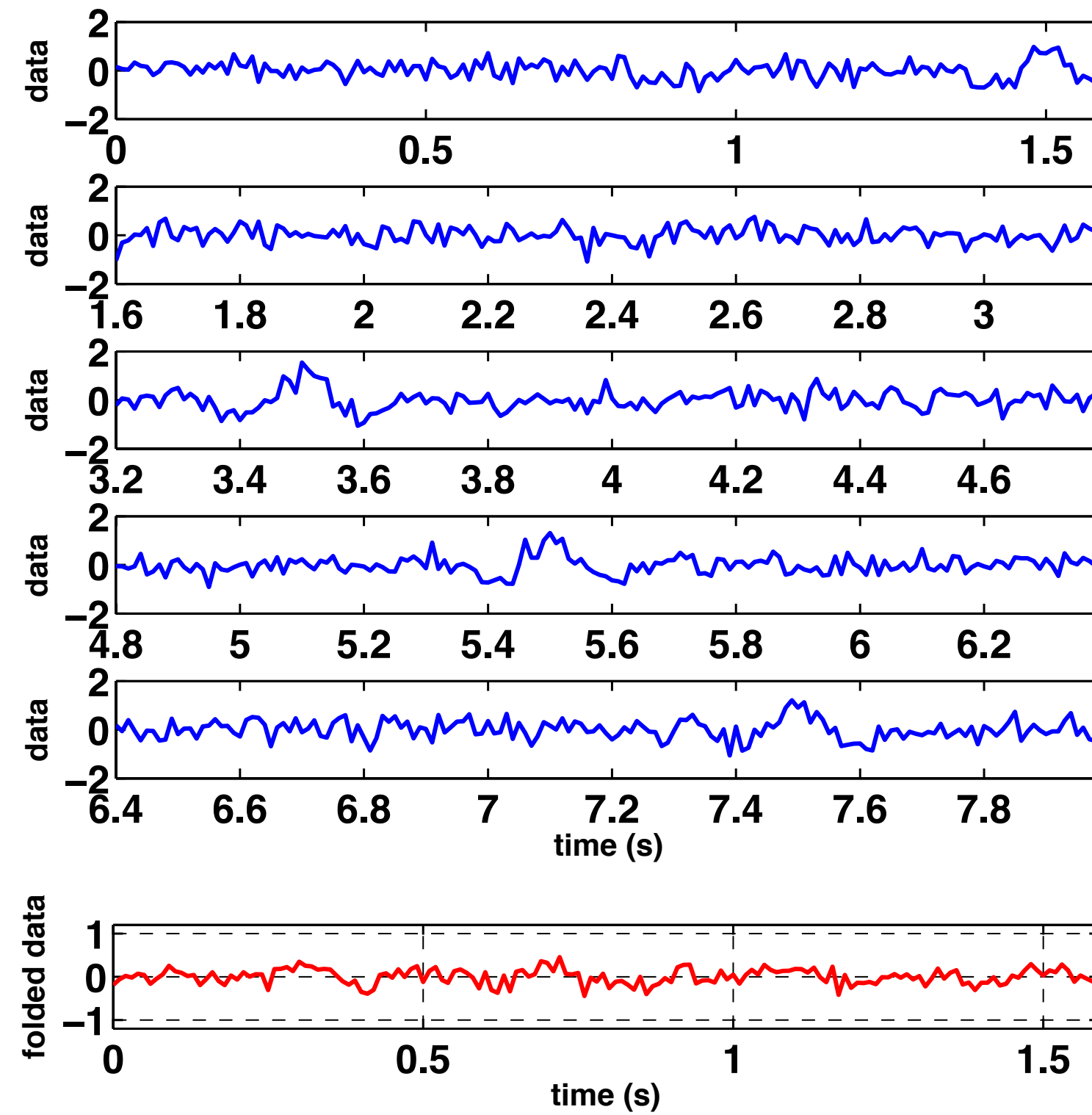


More details

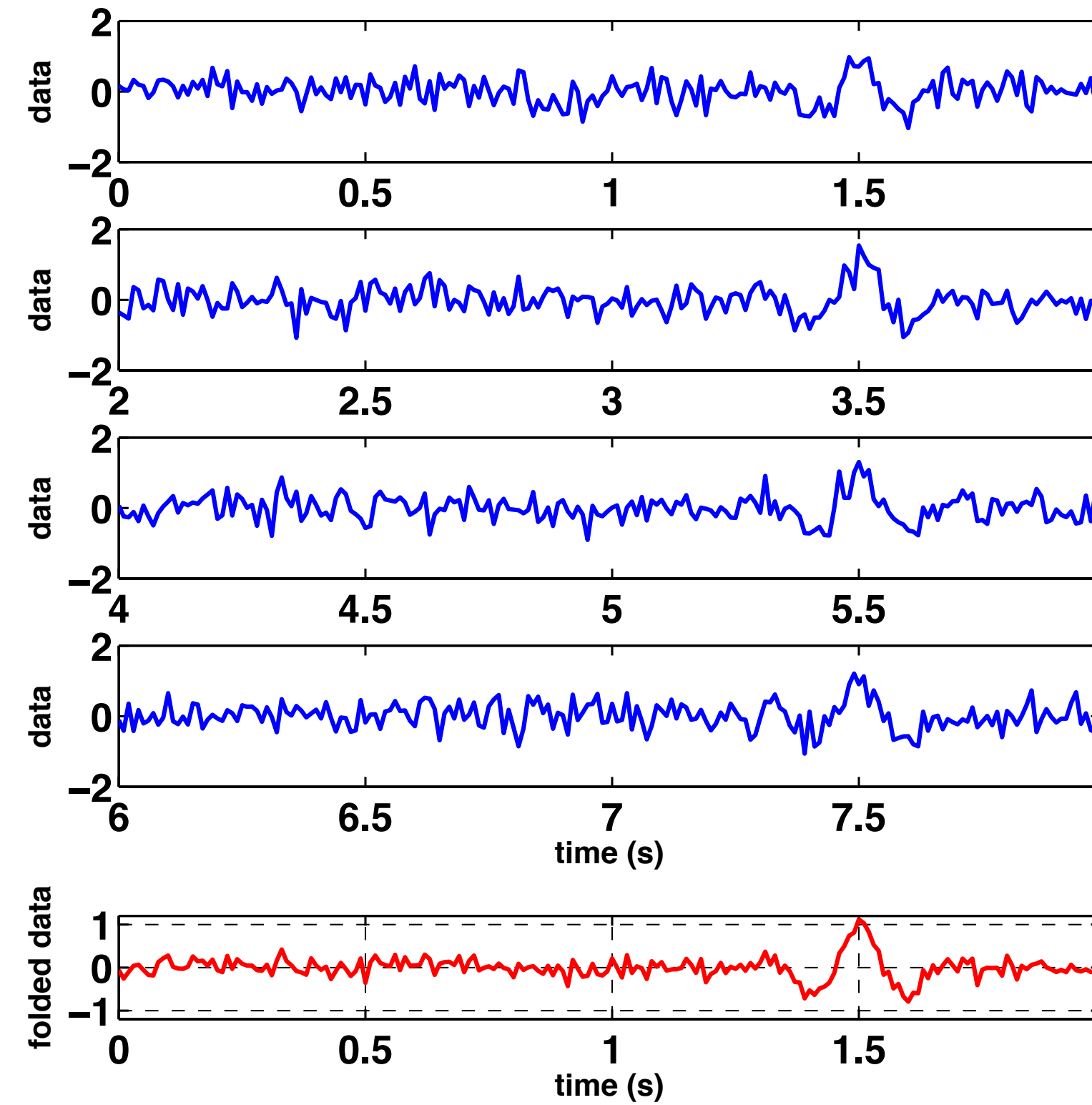
“Fold” data to determine pulse period and pulse profile



fold with incorrect T_p



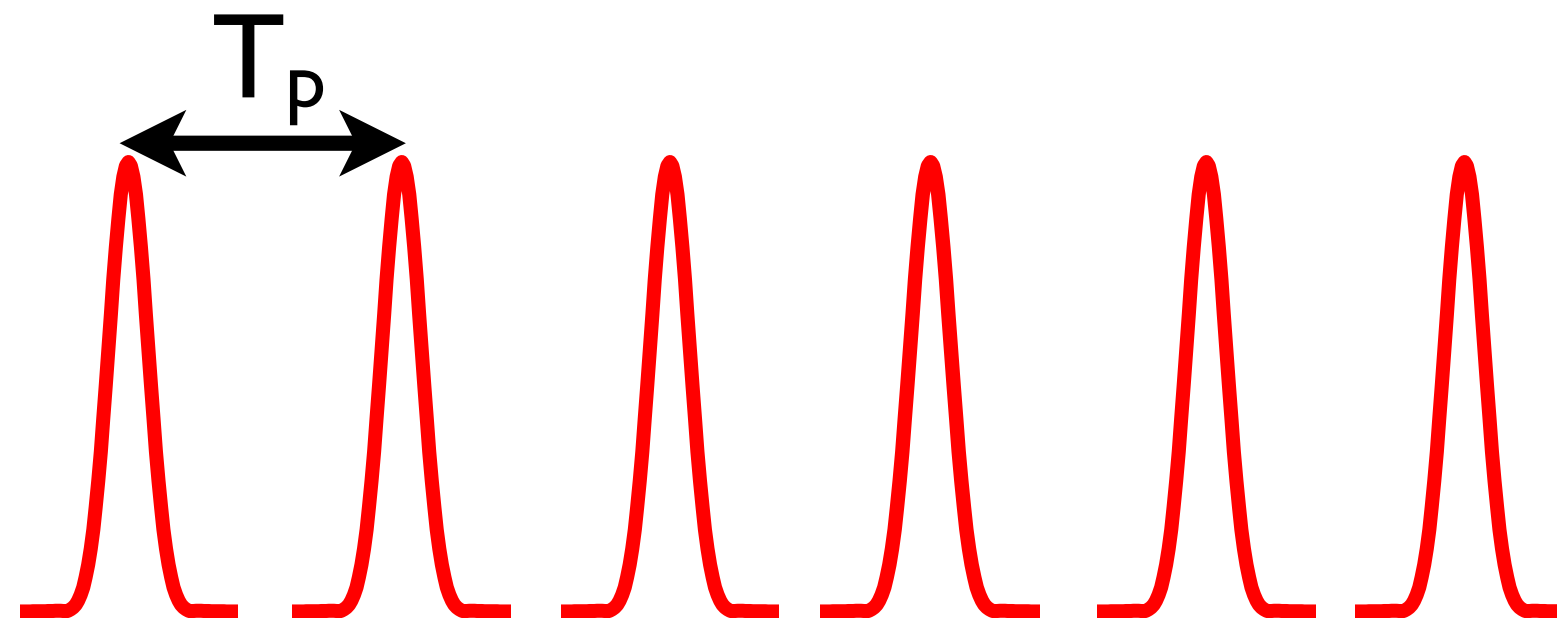
fold with correct T_p



Timing model

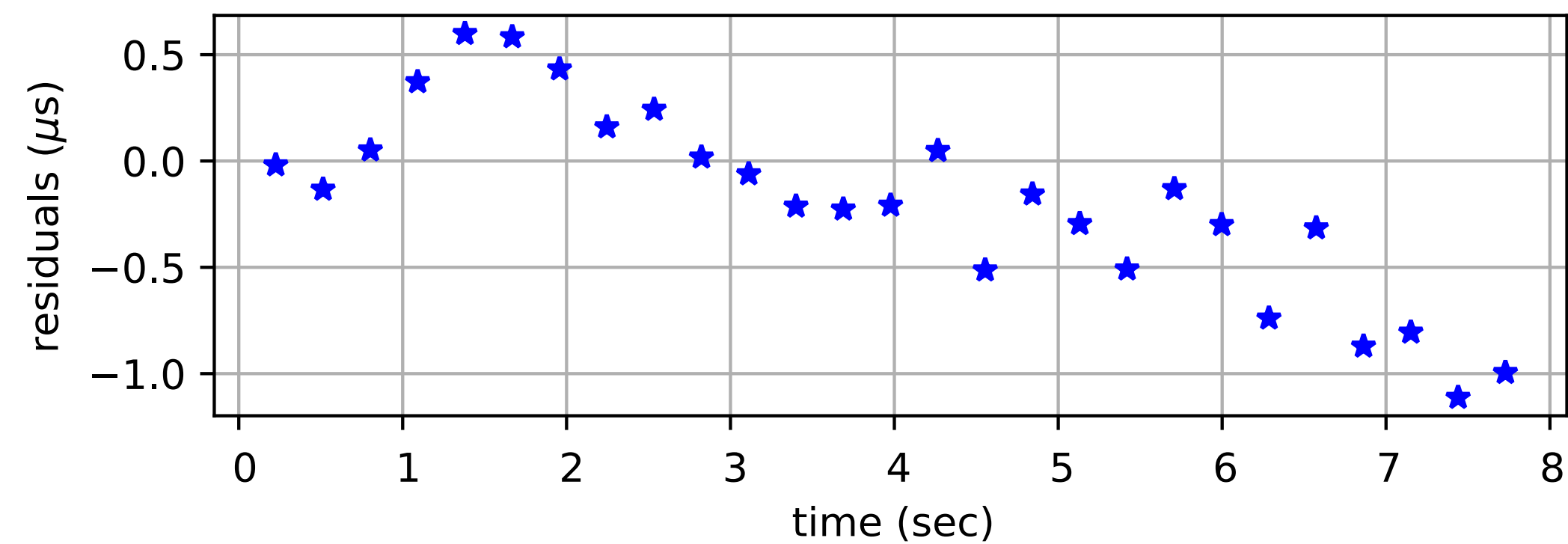
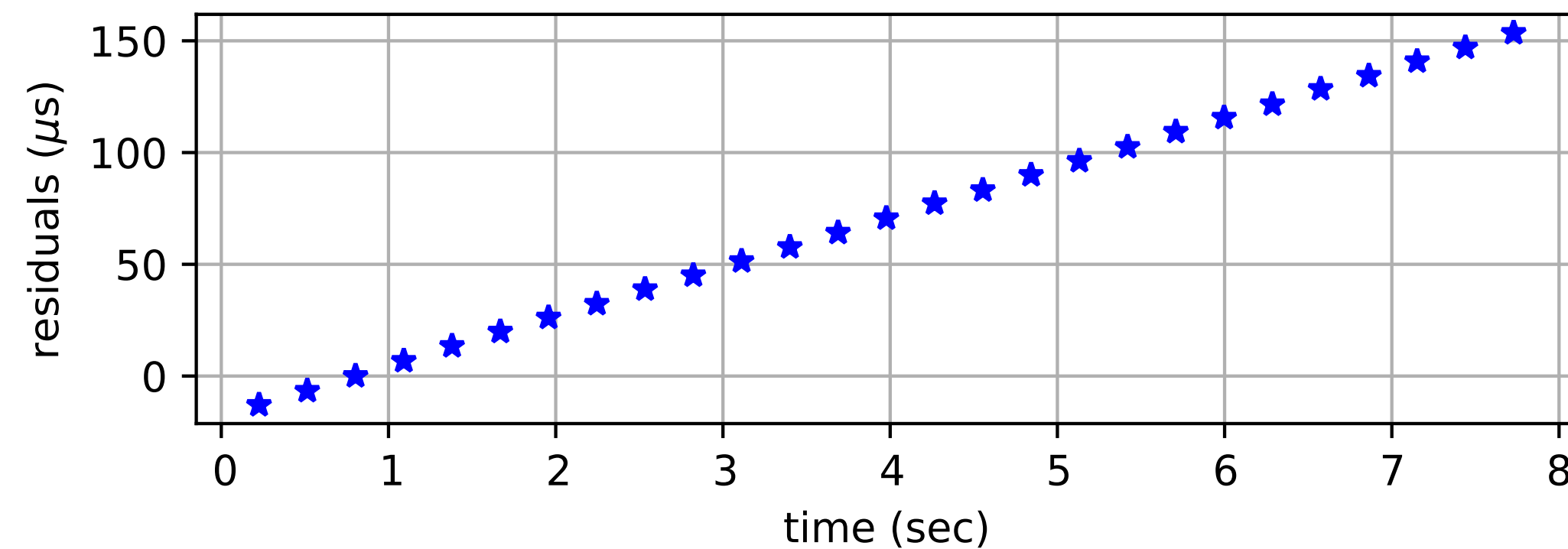
Pulses should arrive regularly with period T_p relative to some reference pulse

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_p$$



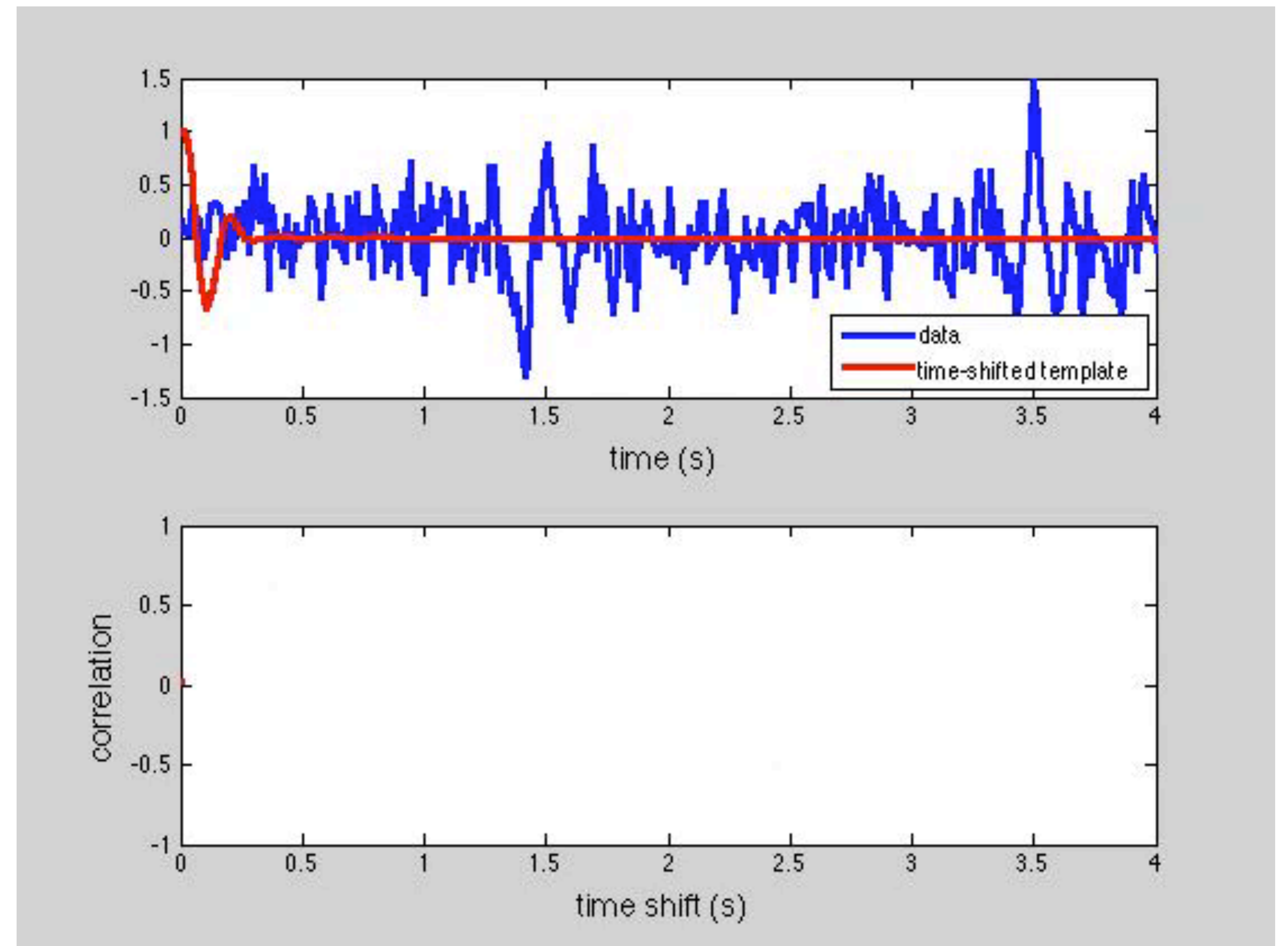
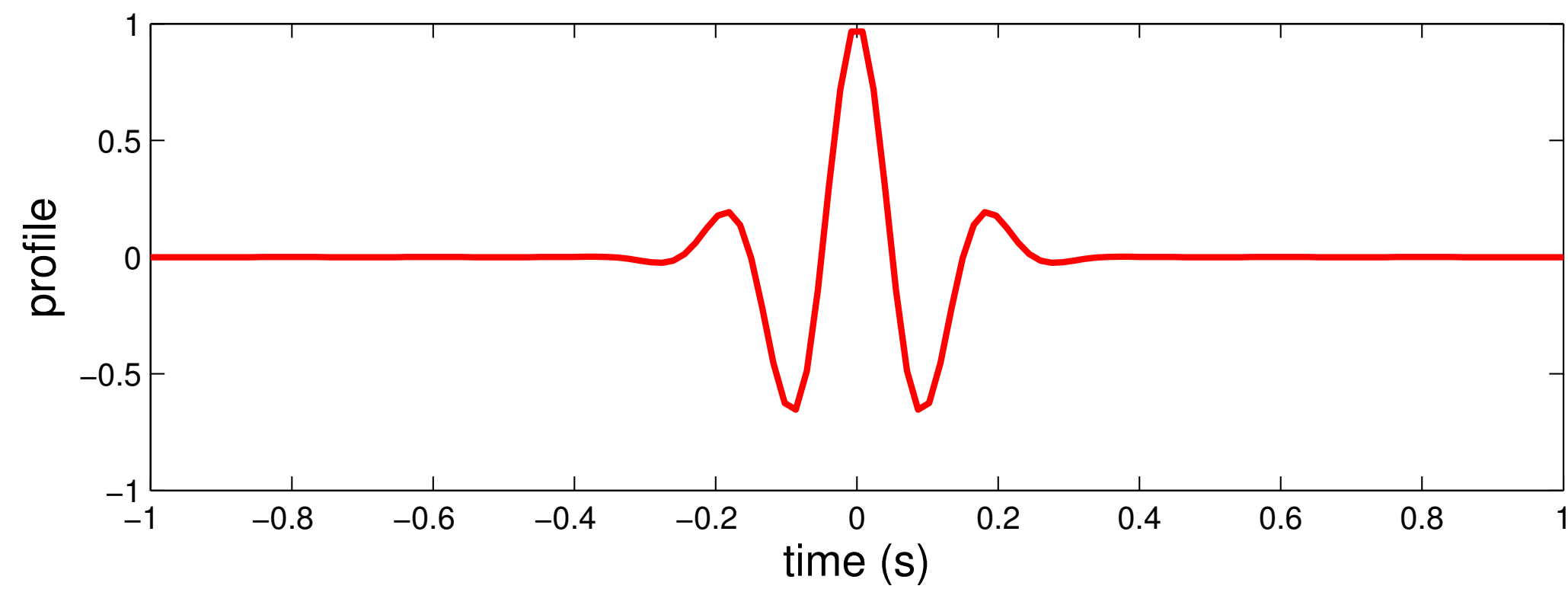
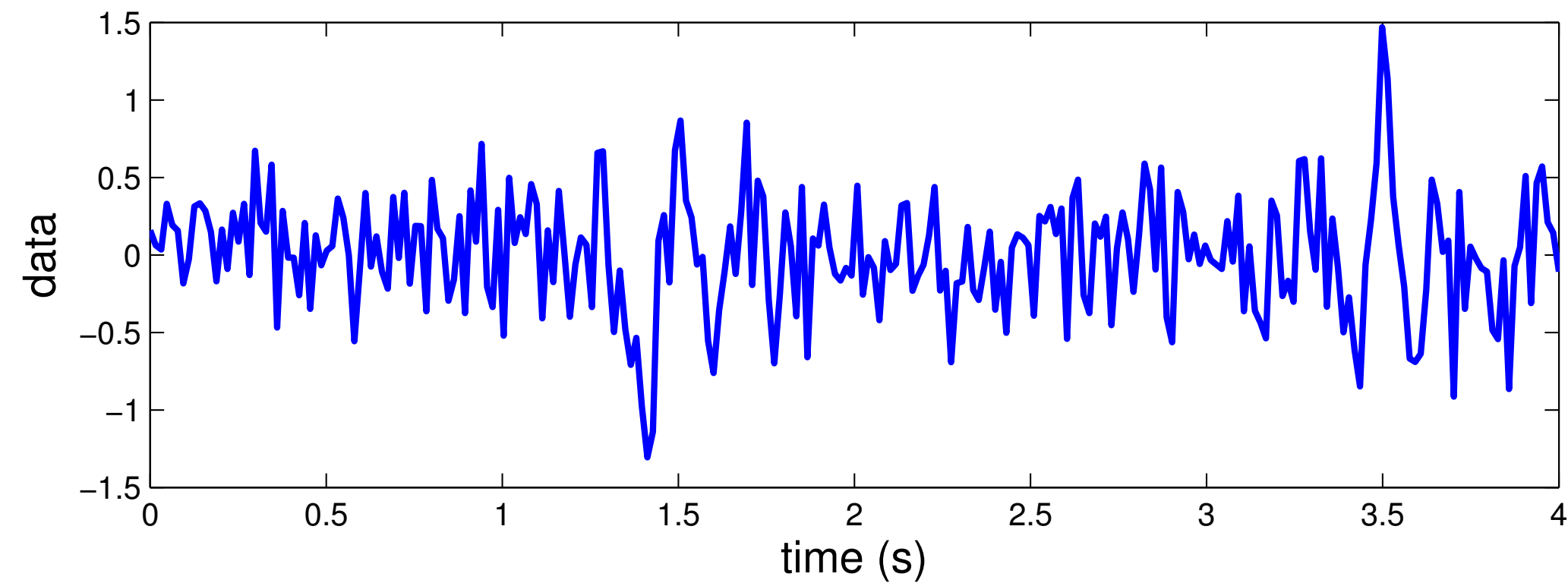
Remove linear trend to more accurately determine pulse period

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_p$$

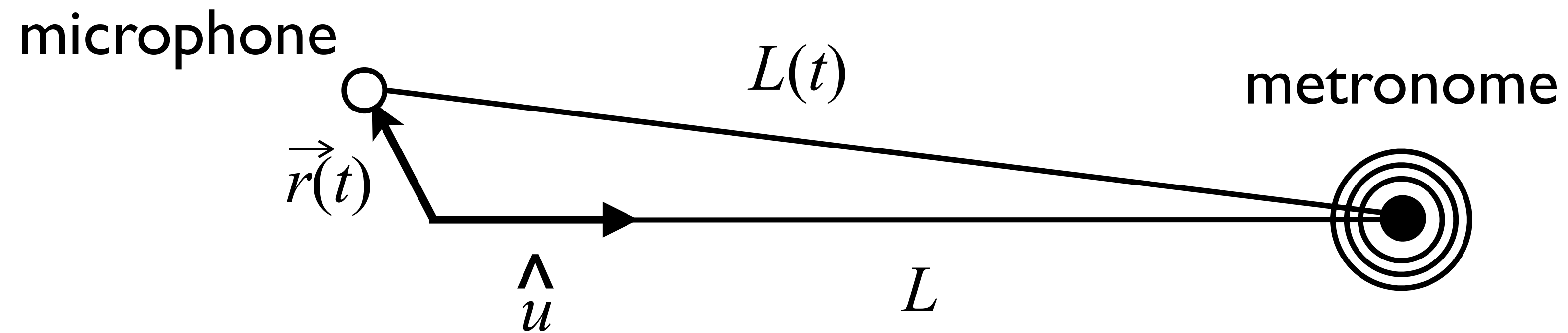


Matched-filtering determination of measured TOAs

$$C(\Delta t) = \mathcal{N} \int dt y(t)p(t - \Delta t)$$

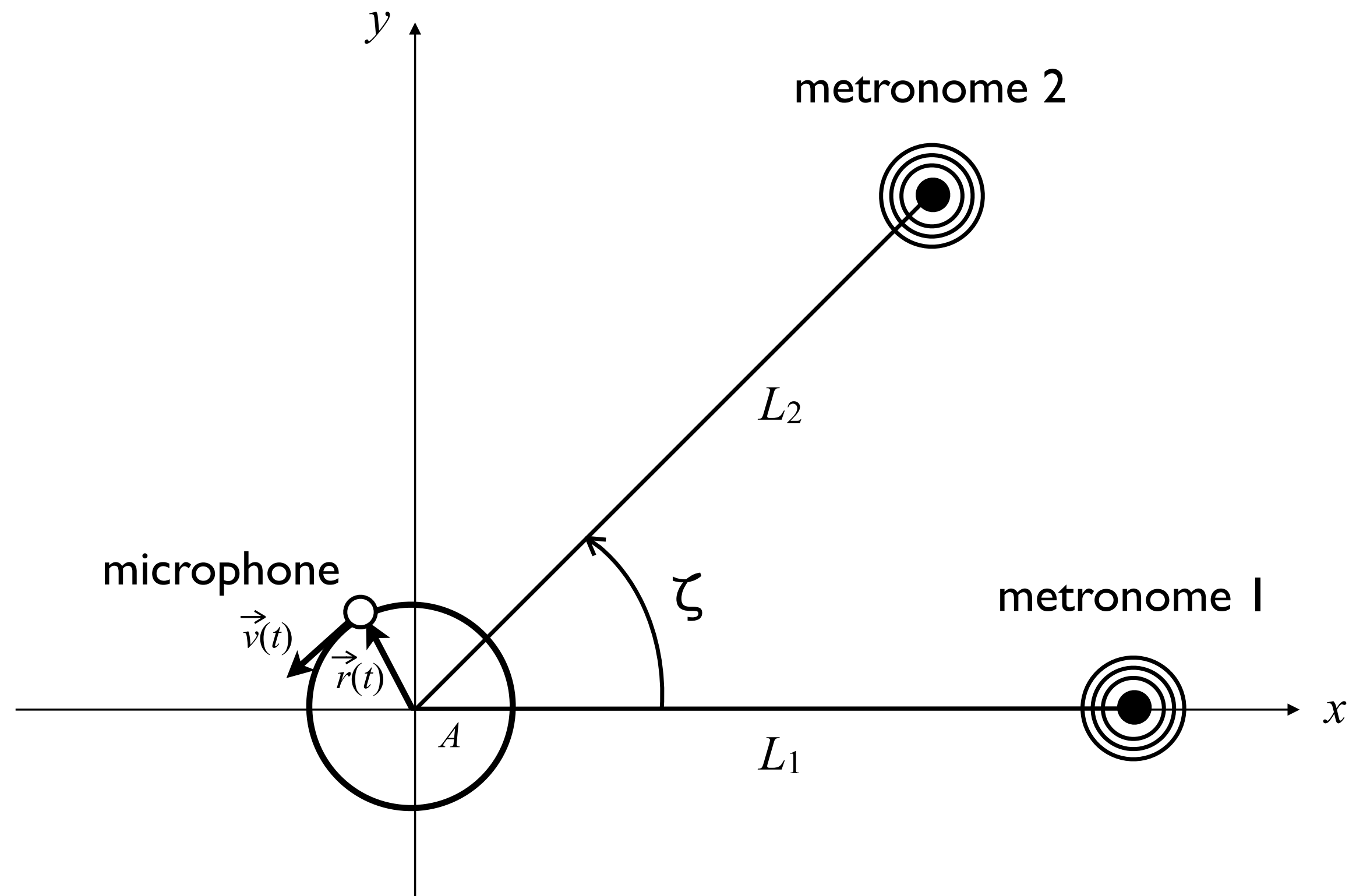


Timing-residual response to microphone motion



$$\delta\tau(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{1}{c_s} \hat{u} \cdot \vec{r}(t)$$

Two metronomes - uniform circular motion

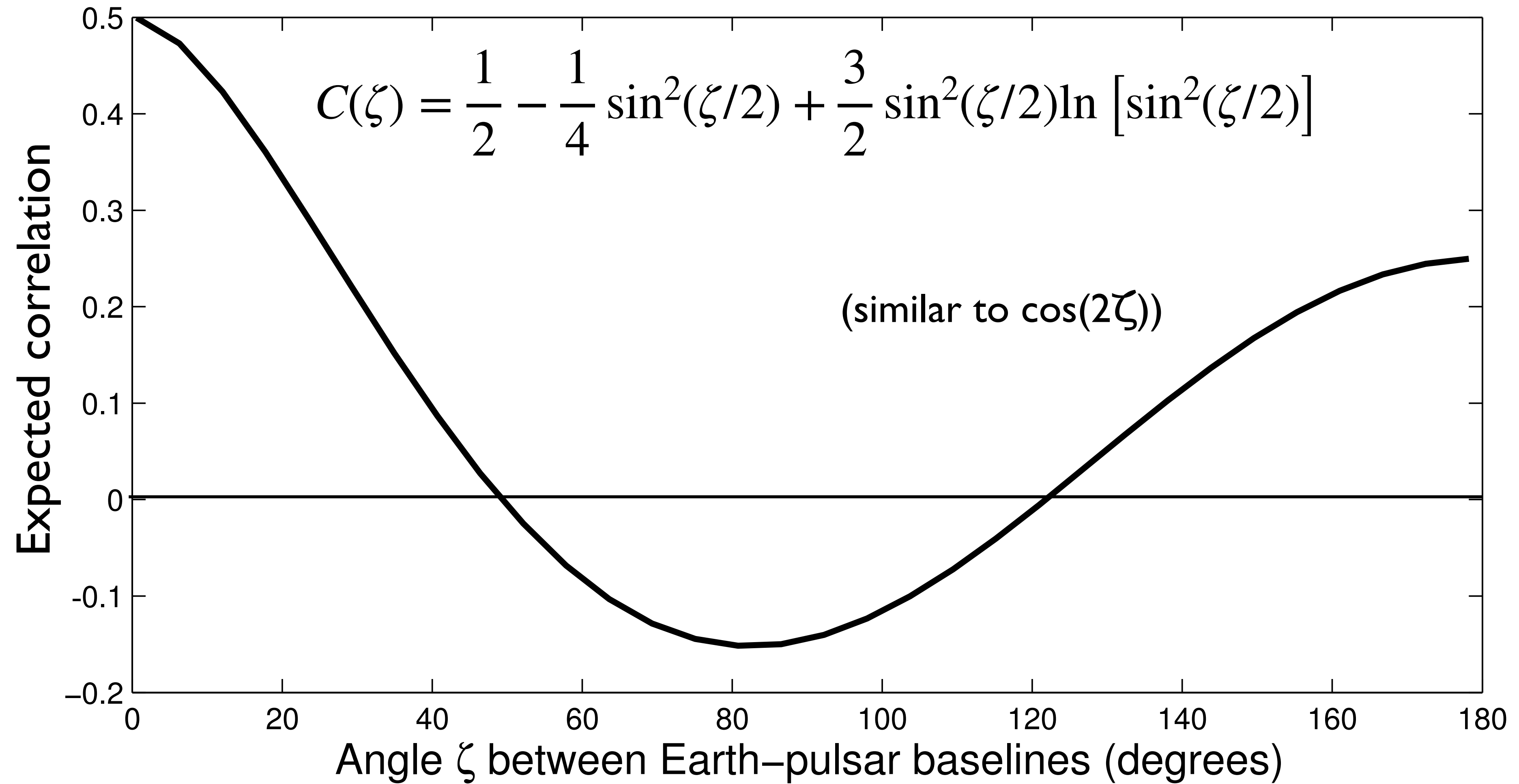


$$\vec{r}(t) = A [\cos(2\pi f_0 t + \phi_0) \hat{x} + \sin(2\pi f_0 t + \phi_0) \hat{y}]$$

$$\delta\tau_I(t) \simeq -\frac{A}{c_s} \cos(2\pi f_0 t + \phi_0 - \theta_I), \quad I = 1, 2$$

$$\rho_{12} \simeq \cos \zeta, \quad \zeta \equiv \theta_1 - \theta_2$$

Expected PTA correlation - Hellings & Downs curve (isotropic, unpolarized GW background)



Metronome demo numbers

$$c_s = 340 \text{ m/s (in air)}$$

$$\text{amplitude} \approx 5 \text{ cm}$$

$$\text{amplitude} / c_s = 1 \times 10^{-4} \text{ sec}$$

$$184 \text{ bpm: } T_p = 0.3261 \text{ sec}$$

$$208 \text{ bpm: } T_p = 0.2885 \text{ sec}$$

Pulsar timing numbers

$f \sim 1/\text{few weeks to } 1/10 \text{ years } (10^{-7} \text{ Hz to } 10^{-9} \text{ Hz})$

$\lambda \sim 0.1 \text{ to } 10 \text{ lyr (GW wavelength)}$

$L \sim \text{few} \times 1000 \text{ lyr (distance to pulsars)}$

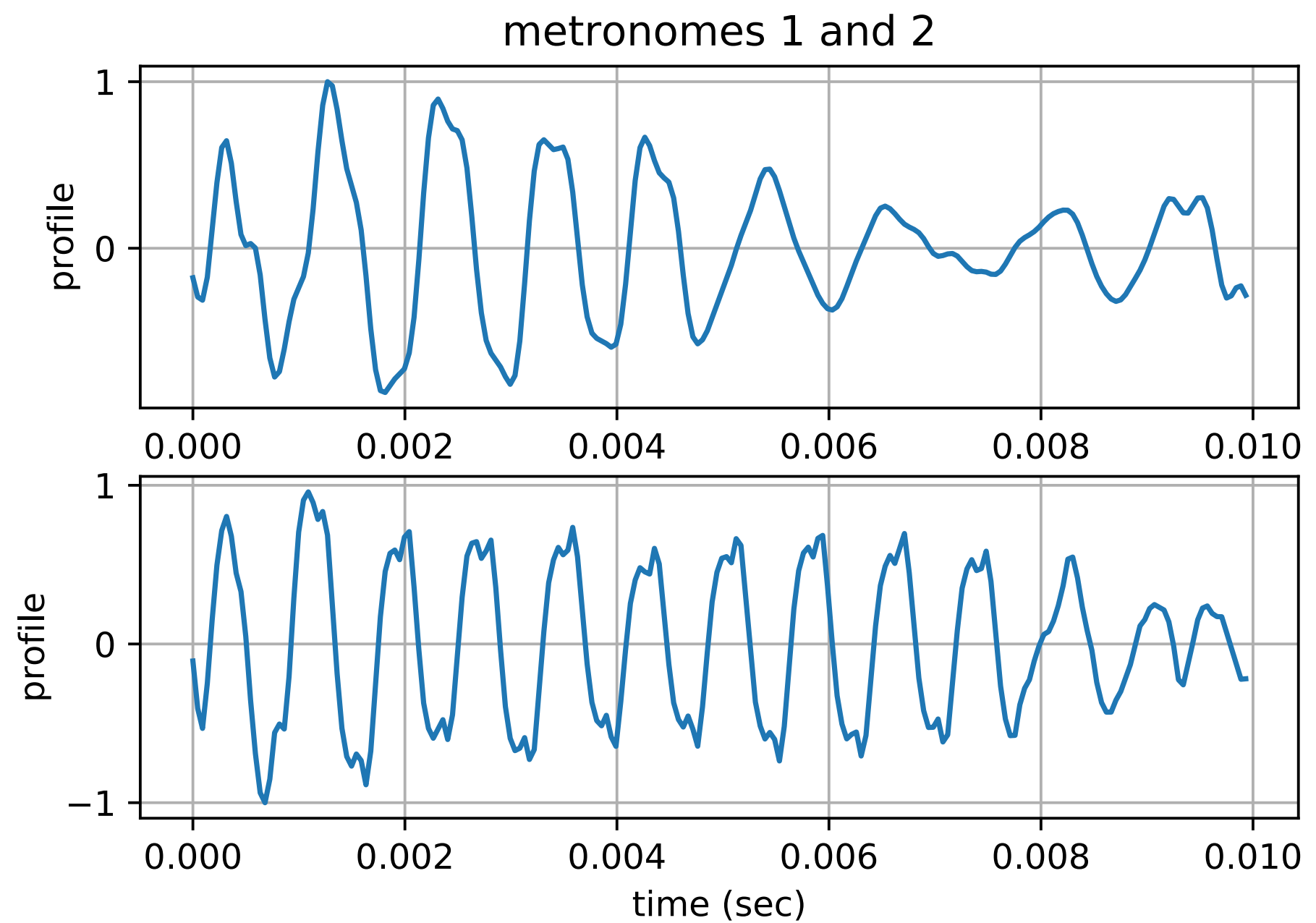
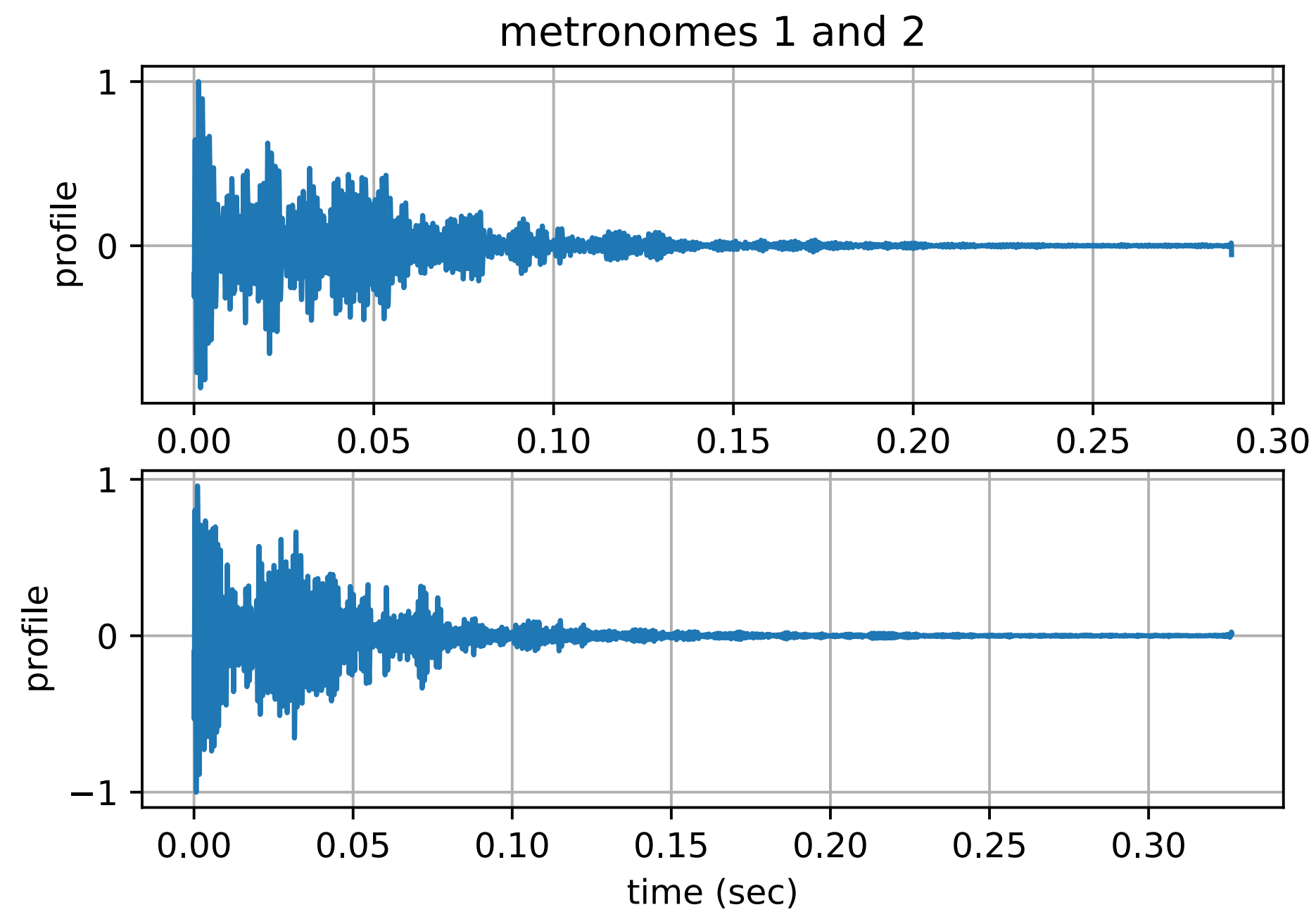
$\lambda \ll L \text{ (short-wavelength limit)}$

sensitivity $\sim \sigma_{\text{rms}}/T_{\text{obs}} \sim 100 \text{ ns}/10 \text{ yr} \sim 10^{-15}$

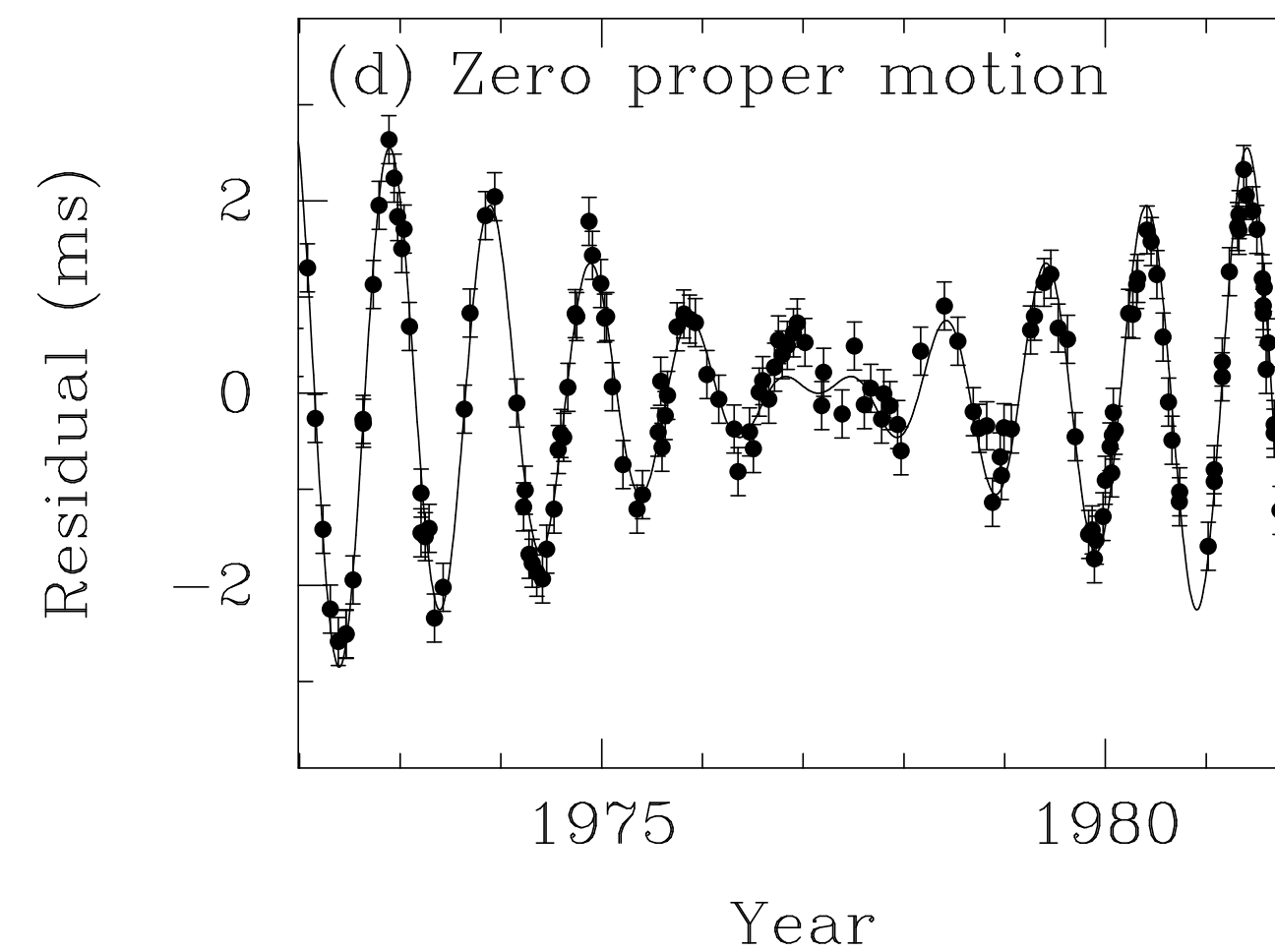
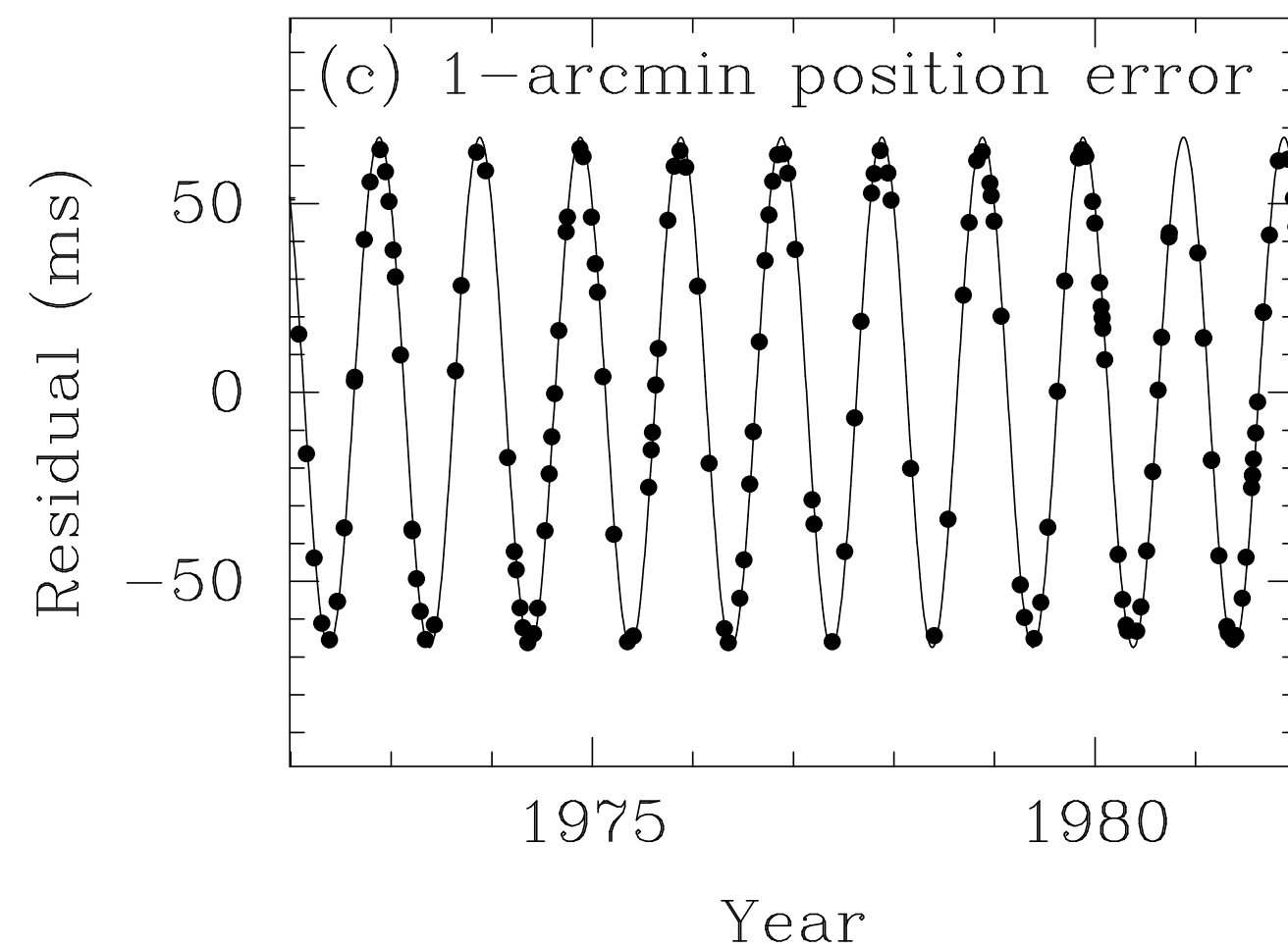
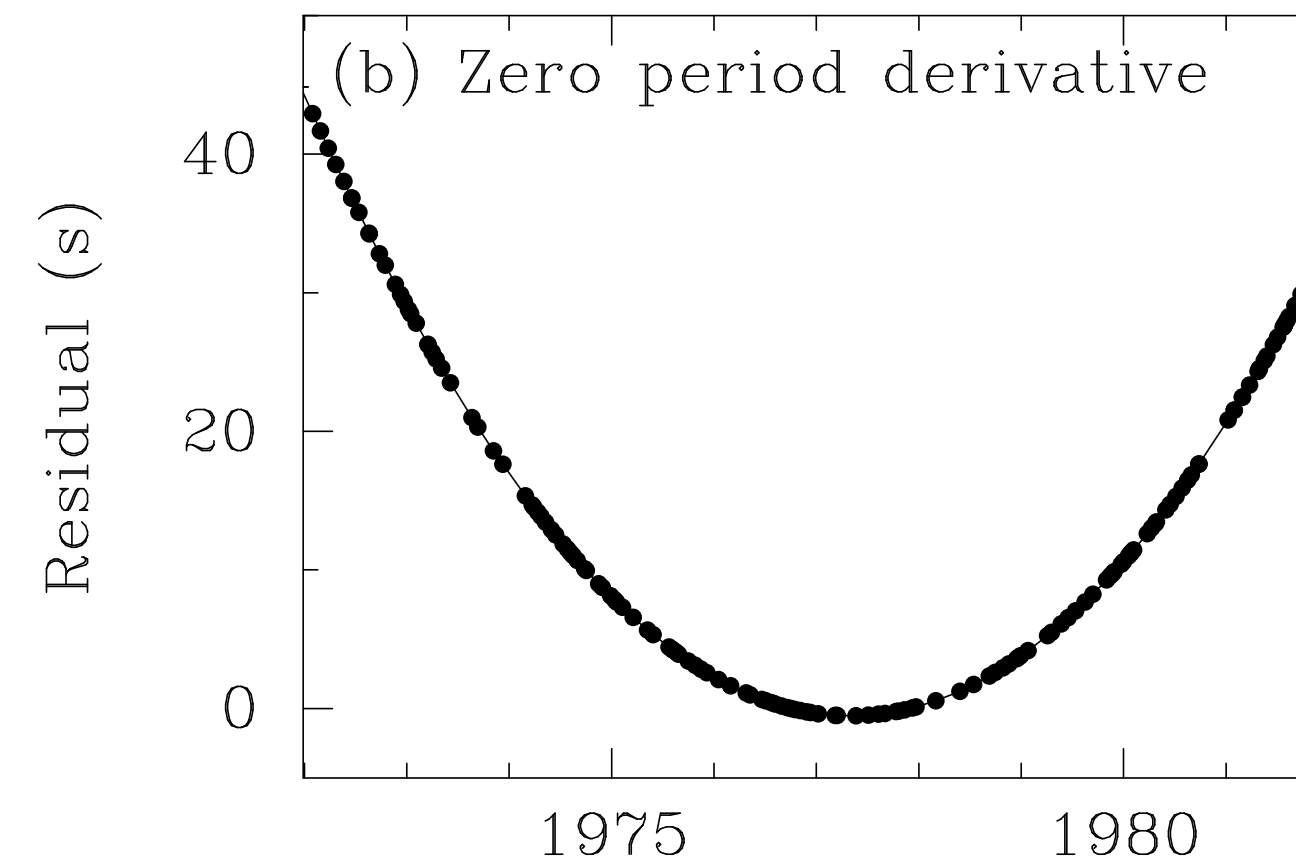
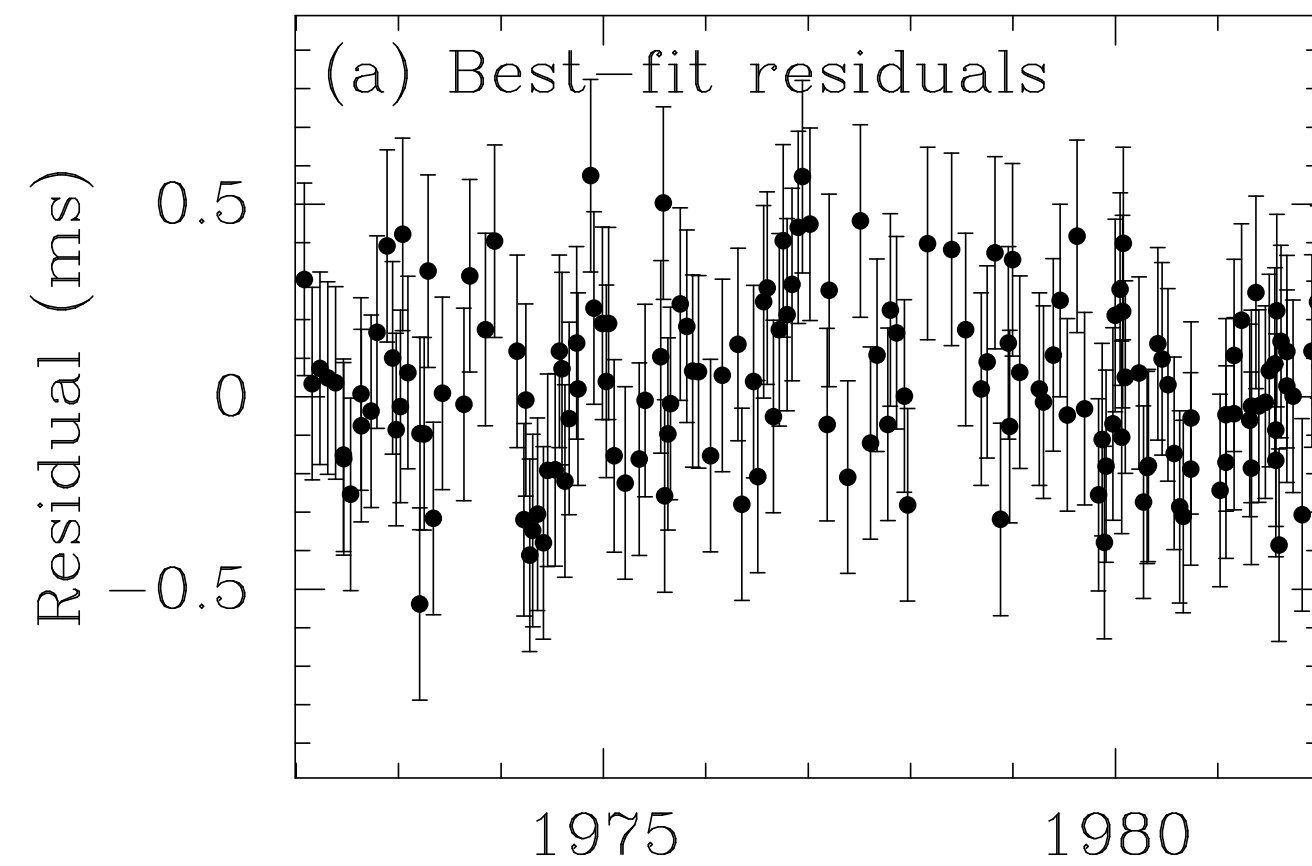


can detect changes $\sim 10 \text{ km}$ in the position
of a pulsar at a distance of $\sim 1000 \text{ lyr}$

Metronome pulse profiles



Errors in the timing model show up as deterministic features in the timing residuals



(from Lorimer LRR-2008-8)