

Metronome timing array:

(a metronome-microphone demo of a pulsar timing array)

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Les Houches Summer School
25 July 2018

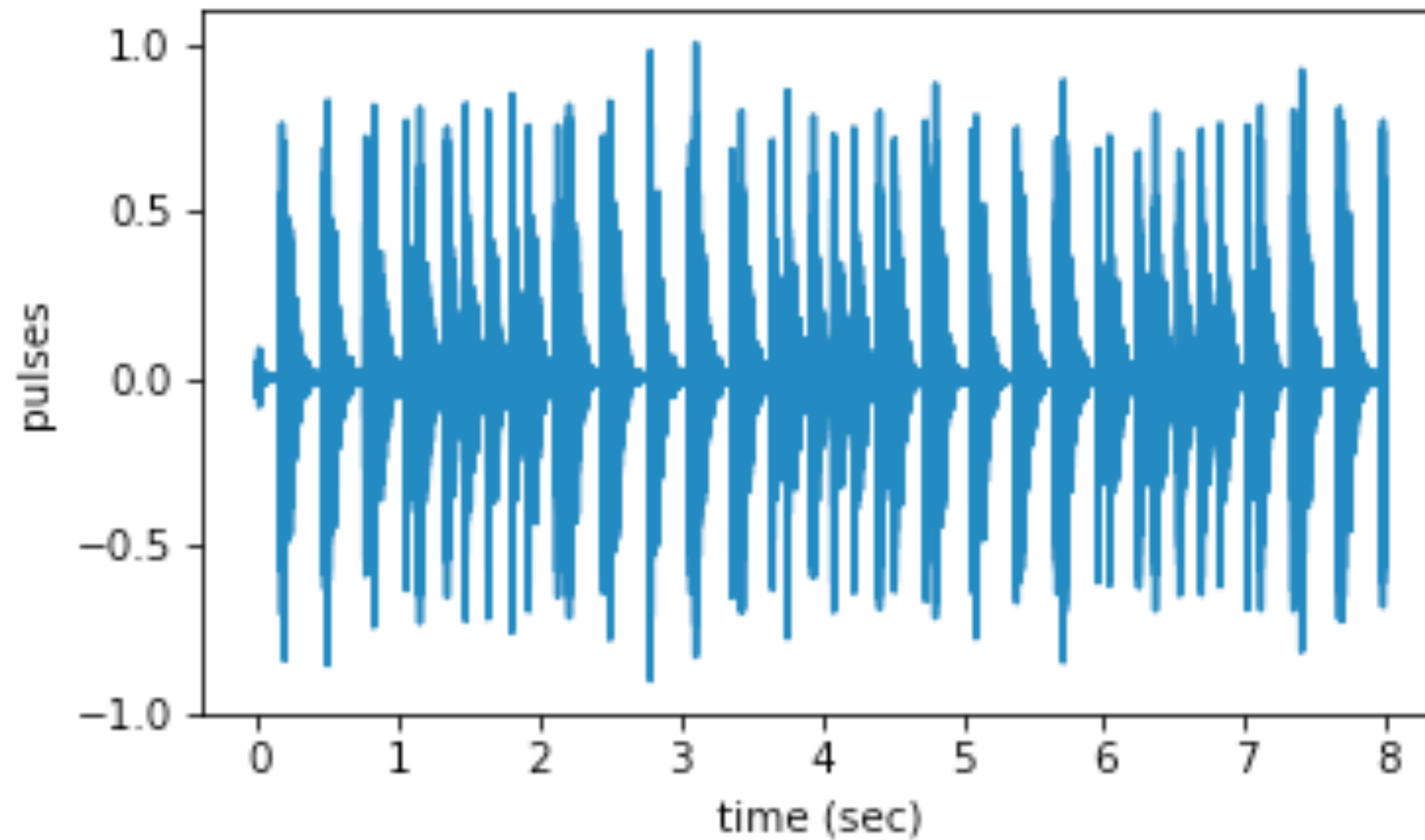
(work in collaboration with M. Lam, M. Normandin, J. Key, and J. Hazboun)

code and sample data: <https://github.com/josephromano/leshouches/tree/master/pta-demo>
paper (draft): <https://github.com/josephromano/leshouches/blob/master/pta-demo/manuscript/pta-demo.pdf>

Purpose of demo

- Illustrate how a PTA is used to search for GWs in the context of a simple acoustical model
- Introduce concepts / techniques used by real pulsar astronomers:
 - **folding** (for calculating pulse periods and pulse profiles)
 - **detrending** (for improved estimates of pulse periods)
 - **matched filtering** (for calculating measured times-of-arrivals)
 - **timing model** (for calculating expected times-of-arrivals)
 - **correlation analyses** (for extracting common GW component)

Q: Is there evidence of a “GW” in the data?



Q: Is there a common disturbance to the pulse arrival times (TOAs), and if so, is this disturbance correlated across metronomes as expected for a “GW” (i.e., microphone motion)?

- disturbance = measured TOAs - expected TOAs = timing residuals
- common = correlation between the timing residuals

$$\rho_{12} \equiv \langle x_1 x_2 \rangle / \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \quad \langle x_1 x_2 \rangle \equiv \frac{1}{T_{\text{obs}}} \int_0^{T_{\text{obs}}} dt x_1(t) x_2(t)$$

- measured TOAs: obtained using matched filtering with pulse profile

$$C(\Delta t) = \mathcal{N} \int dt y(t) p(t - \Delta t)$$

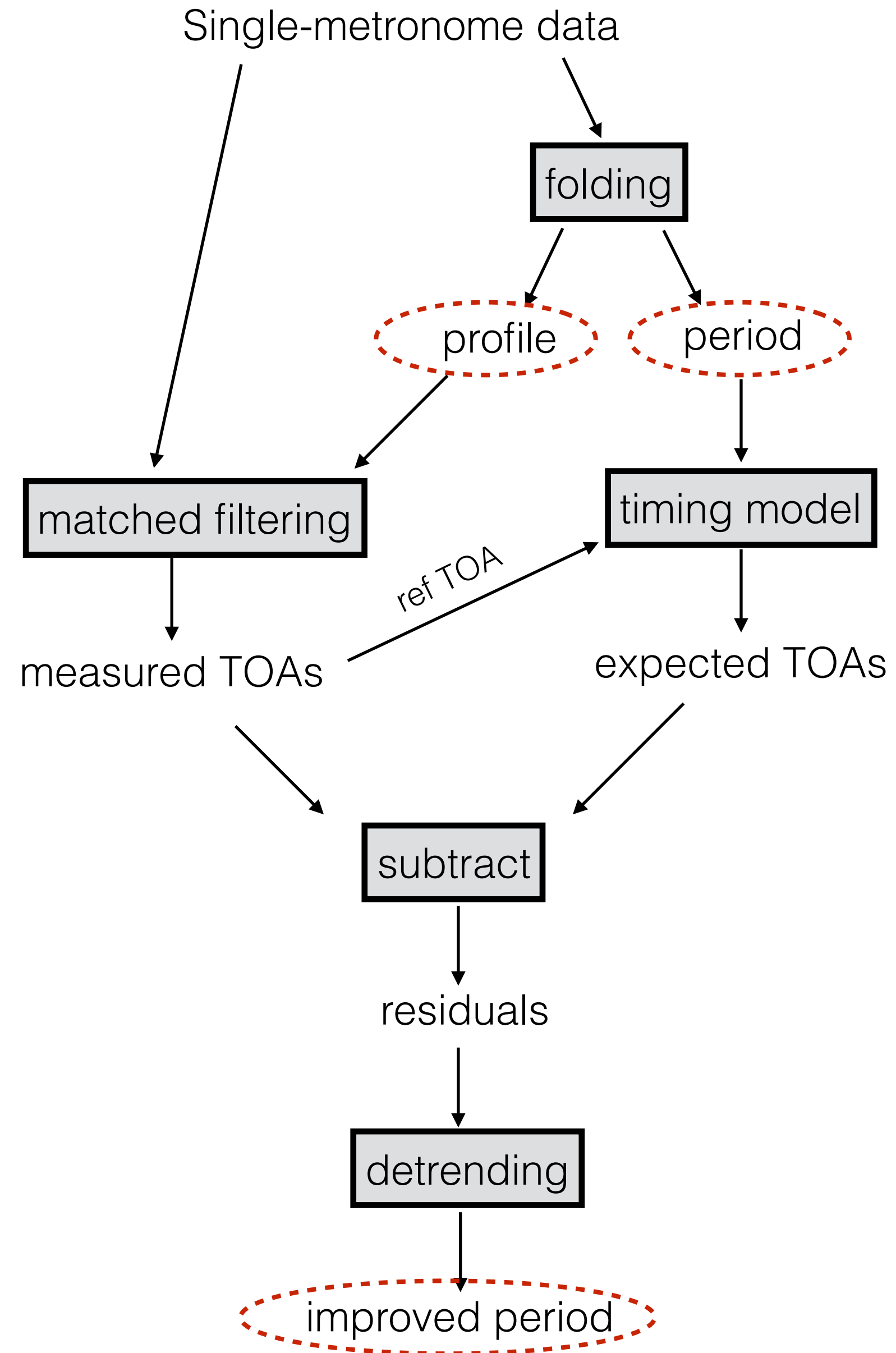
- expected TOAs: timing model using pulse period and reference TOA

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0) T_p$$

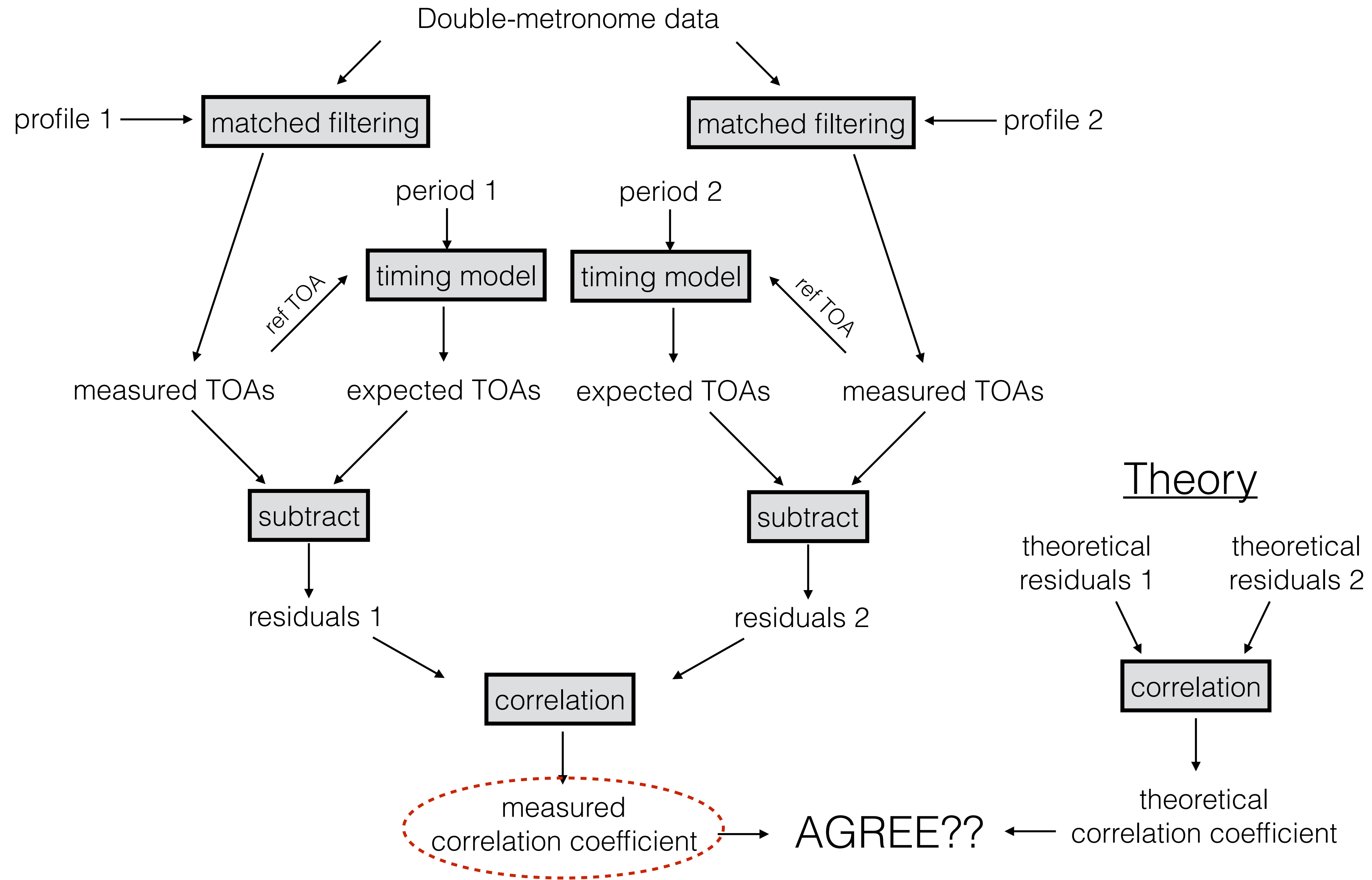
- pulse profile, period: folding and detrending single-metronome data
- expected “GW” correlation (for uniform circular motion):

$$\delta\tau_I(t) = \frac{\Delta L_I(t)}{c_s} \simeq -\frac{1}{c_s} \hat{u}_I \cdot \vec{r}(t) \quad \Rightarrow \quad \boxed{\rho_{12} \simeq \cos \zeta}$$

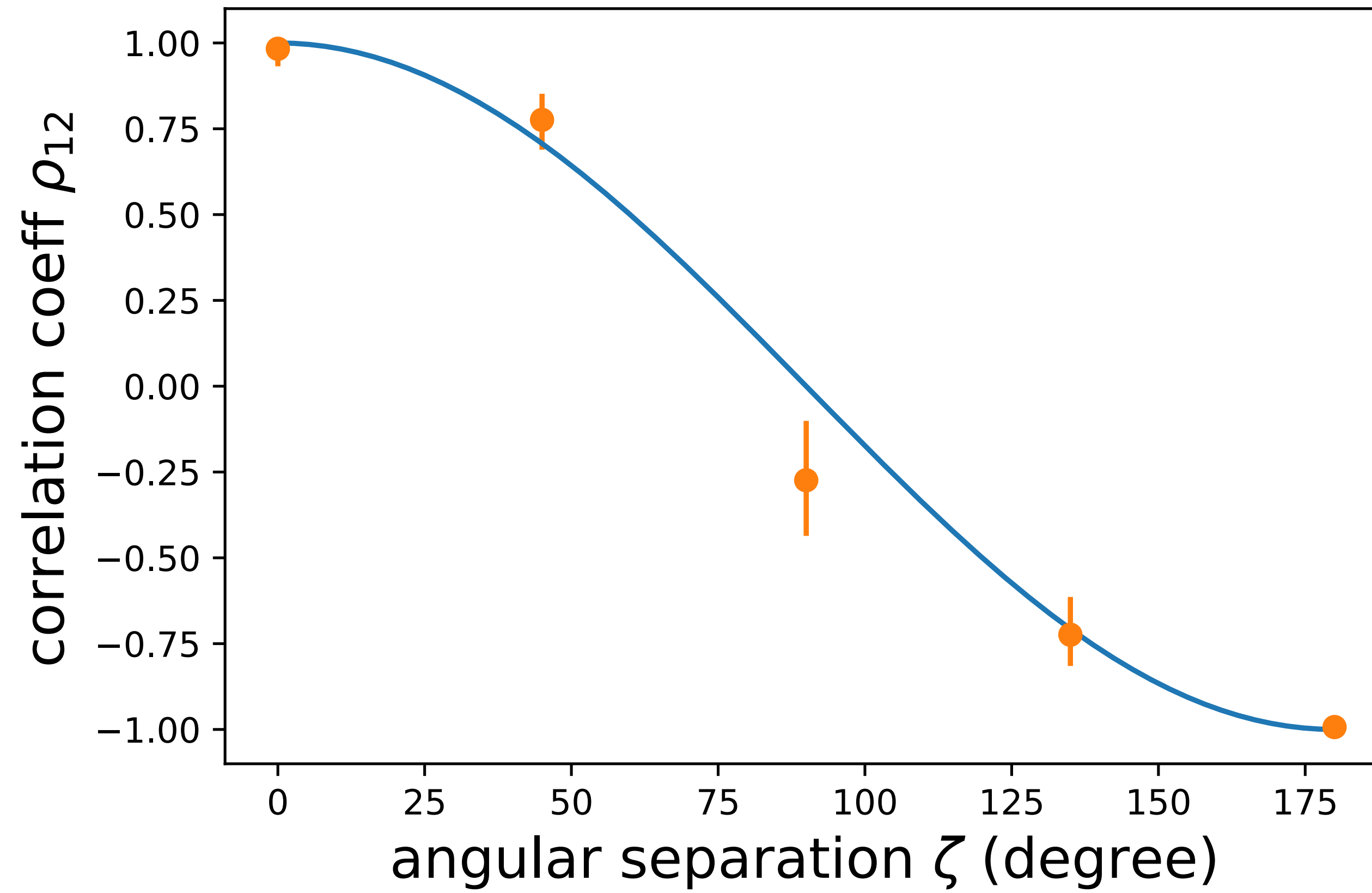
Single-metronome analysis



Double-metronome analysis

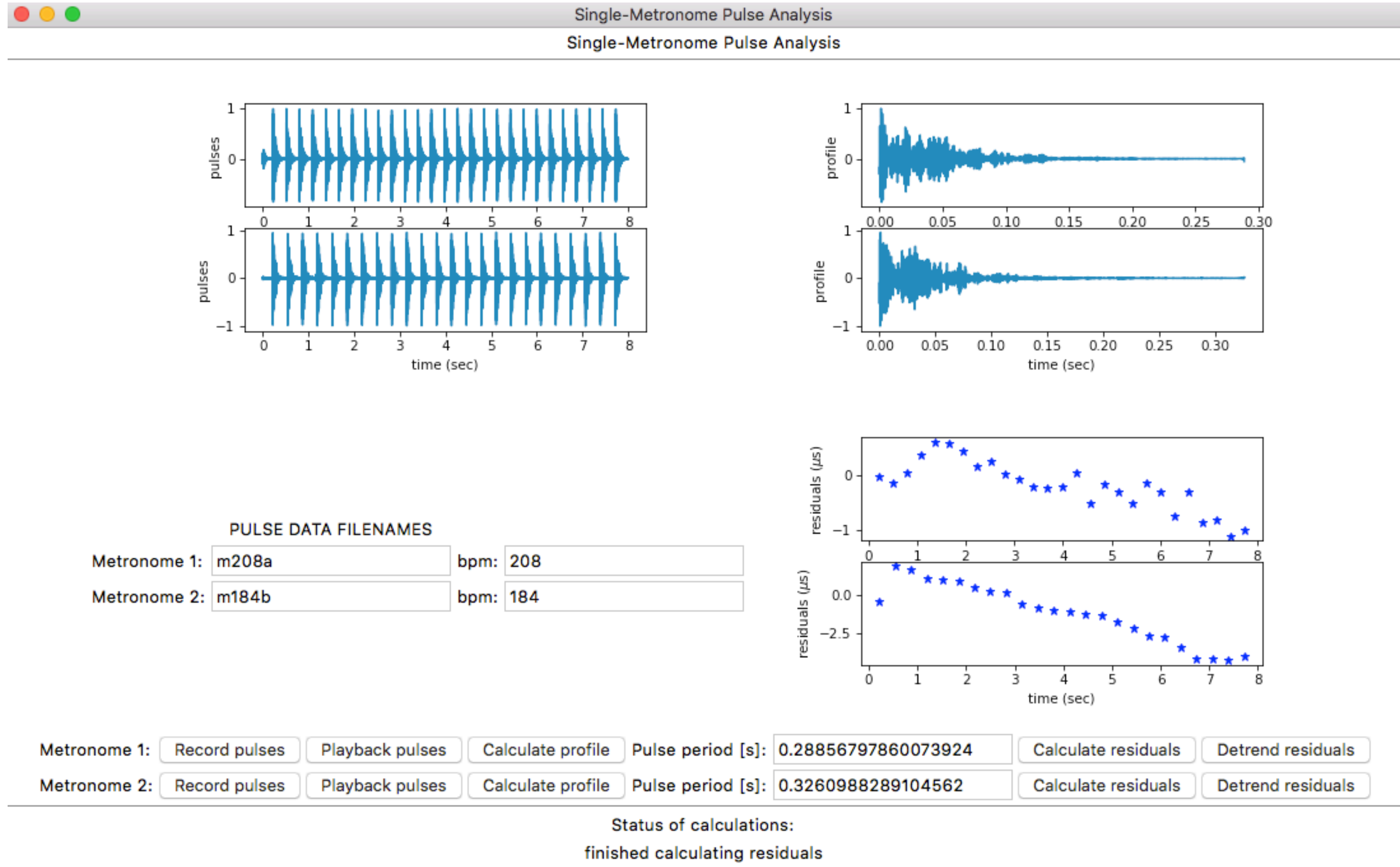


Metronome correlation

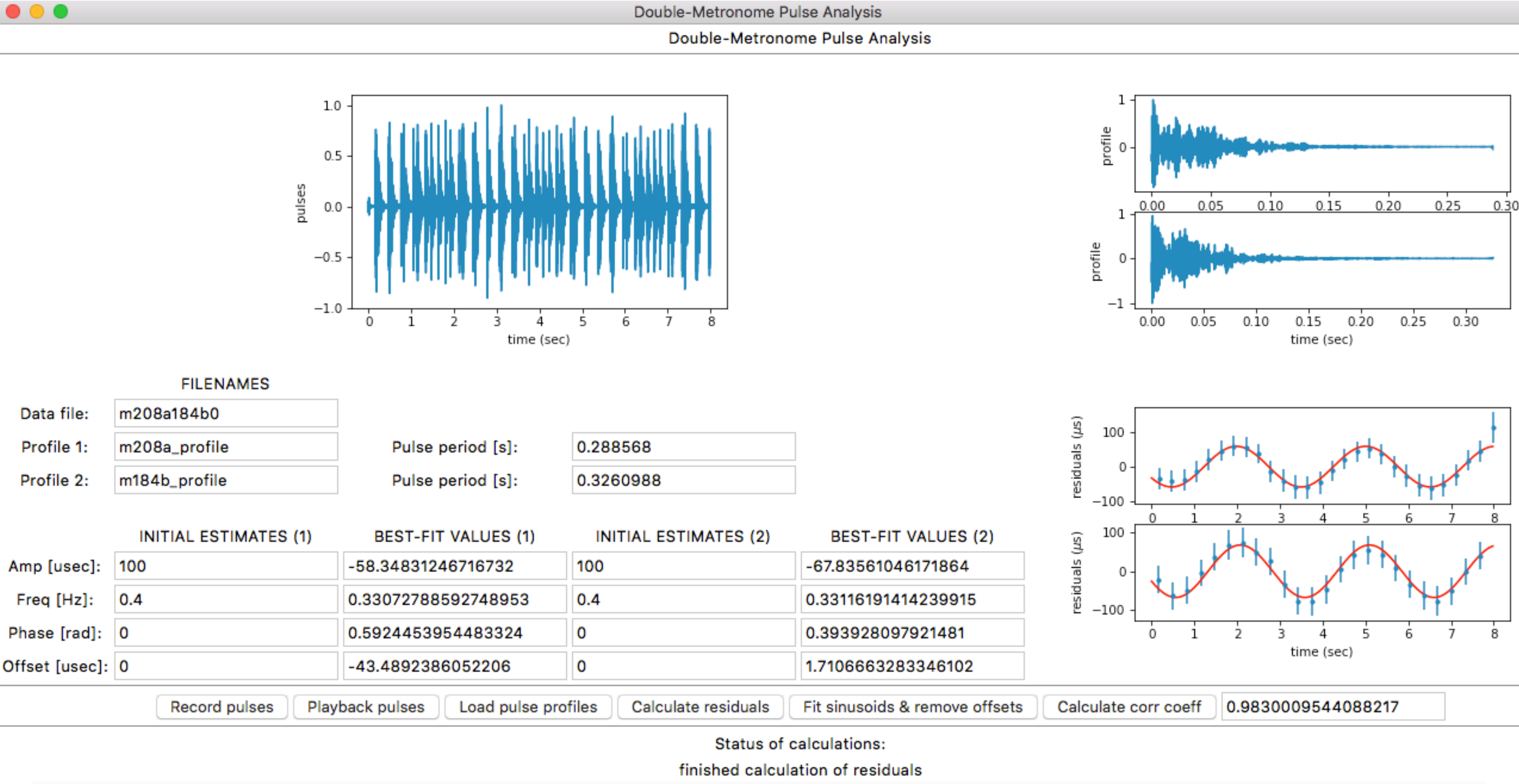


Output of the GUIs

Single-metronome analysis



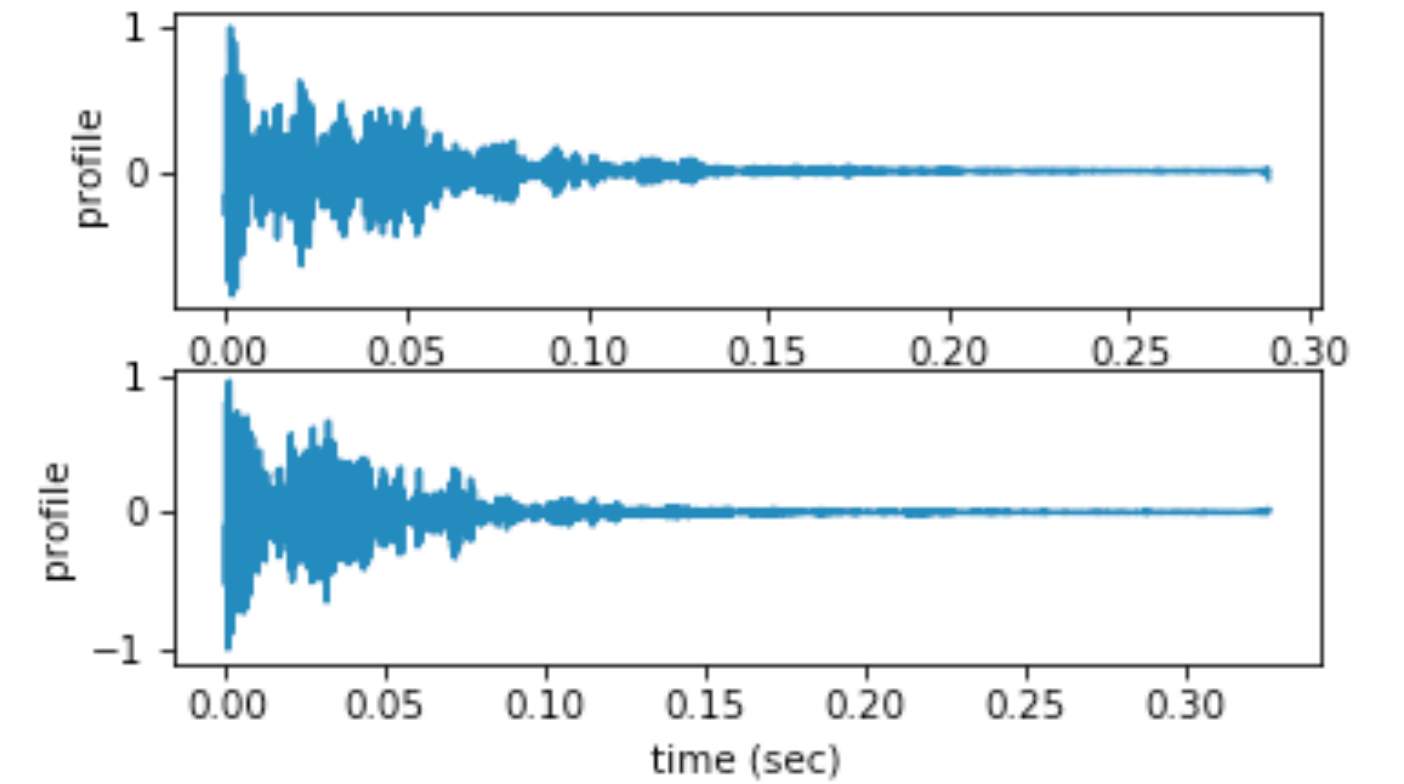
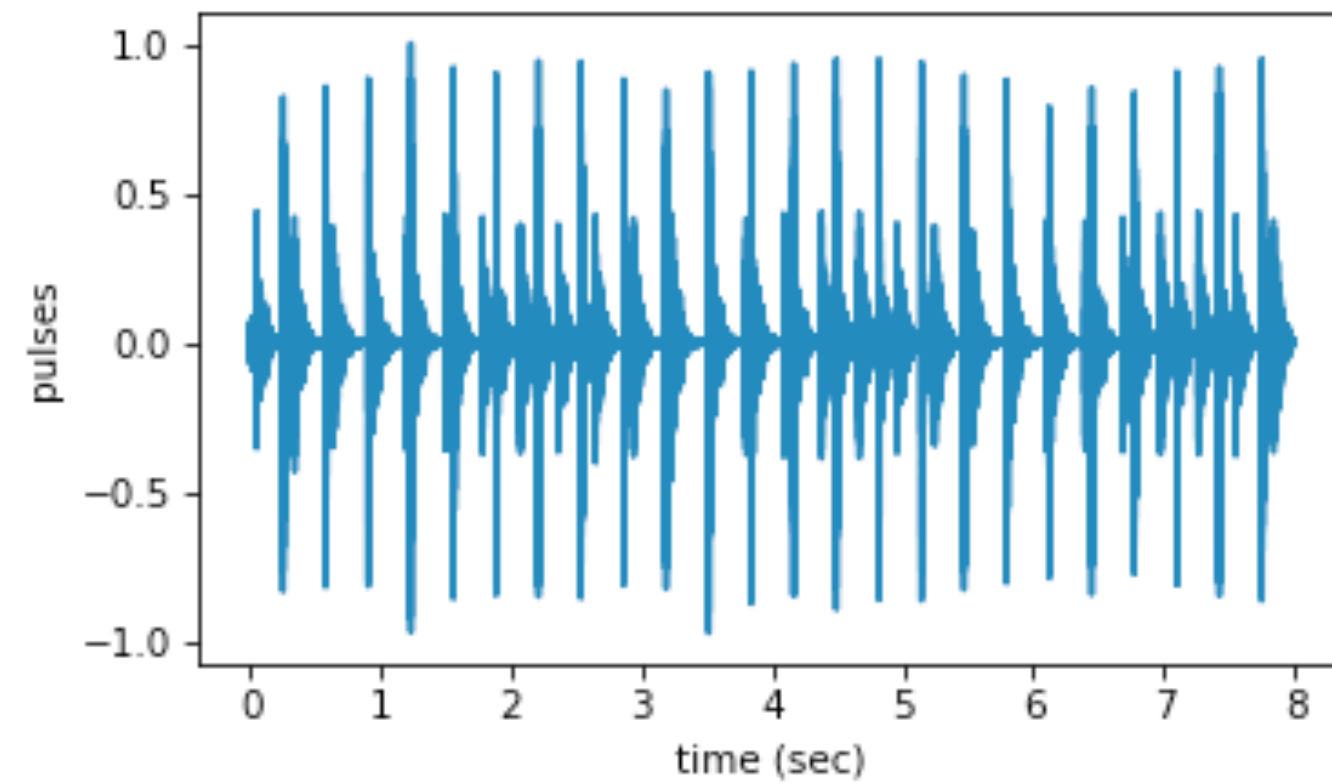
Double-metronome analysis ($\zeta=0$ degrees)



Double-metronome analysis ($\zeta=45$ degrees)

Double-Metronome Pulse Analysis

Double-Metronome Pulse Analysis



FILENAMES

Data file: m208a184b45

Profile 1: m208a_profile

Profile 2: m184b_profile

Pulse period [s]: 0.288568

Pulse period [s]: 0.3260988

INITIAL ESTIMATES (1)

BEST-FIT VALUES (1)

INITIAL ESTIMATES (2)

BEST-FIT VALUES (2)

Amp [usec]:

100

-83.54267144872672

100

103.51040350638623

Freq [Hz]:

0.4

0.33844784664075234

0.4

0.3394604420911975

Phase [rad]:

0

0.082340180957162

0

2.544467981756352

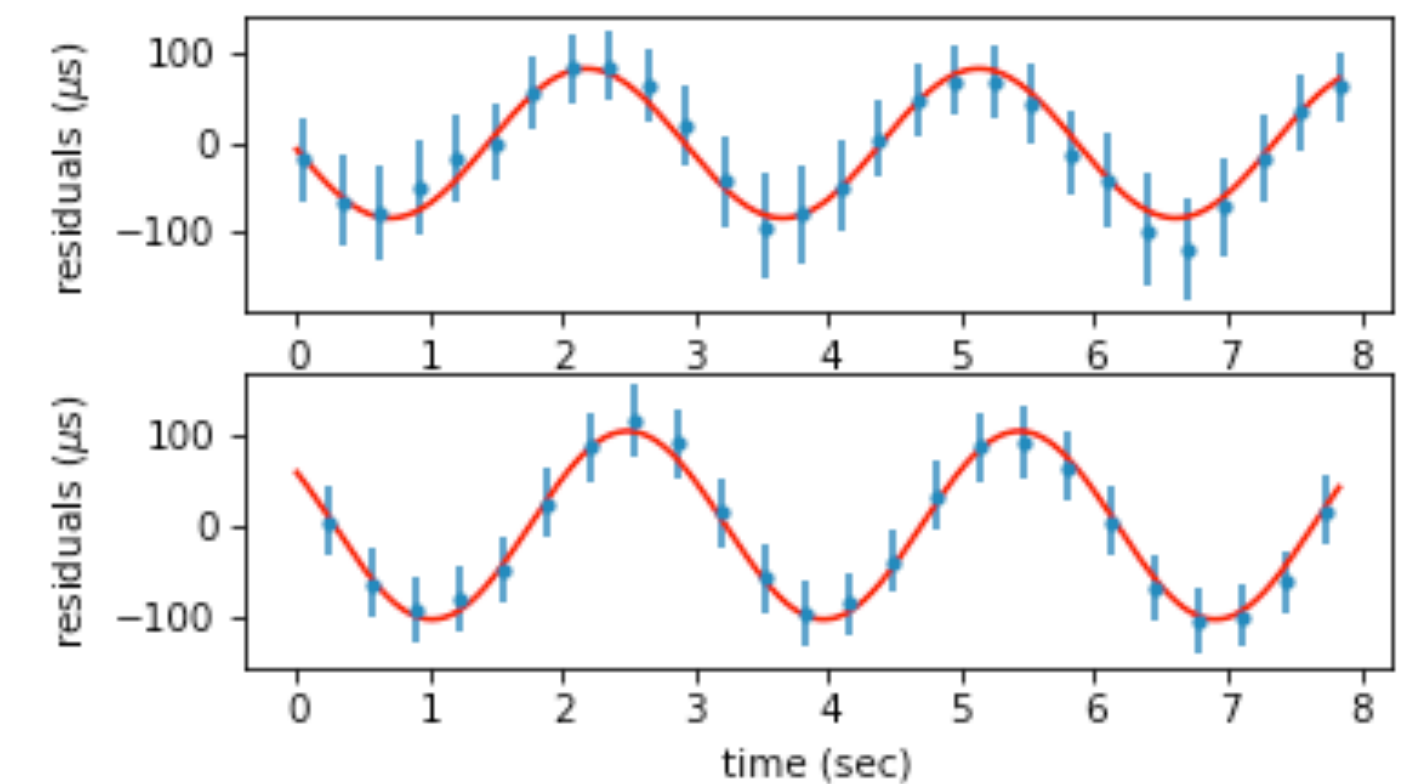
Offset [usec]:

0

-83.59186463058177

0

62.255379154251095



Record pulses

Playback pulses

Load pulse profiles

Calculate residuals

Fit sinusoids & remove offsets

Calculate corr coeff

0.7762191706588184

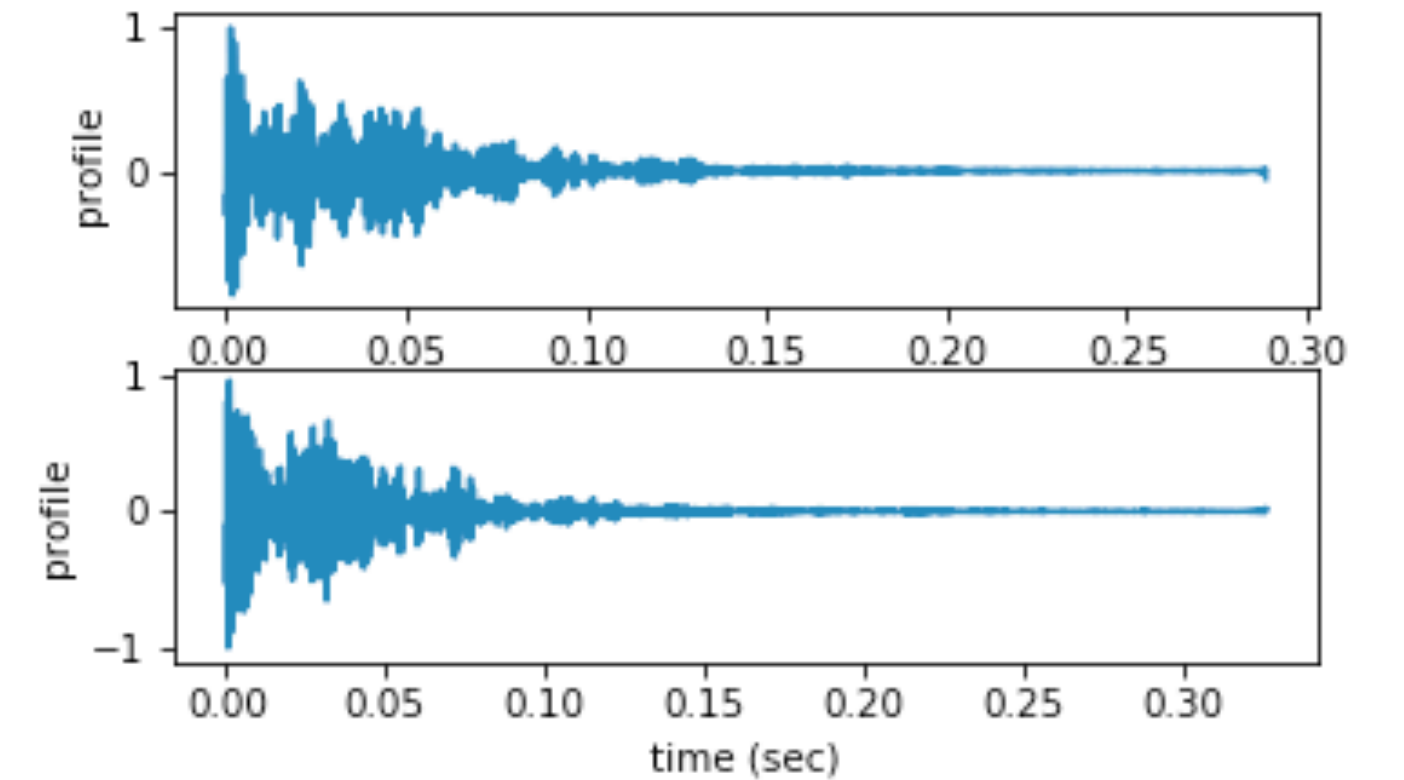
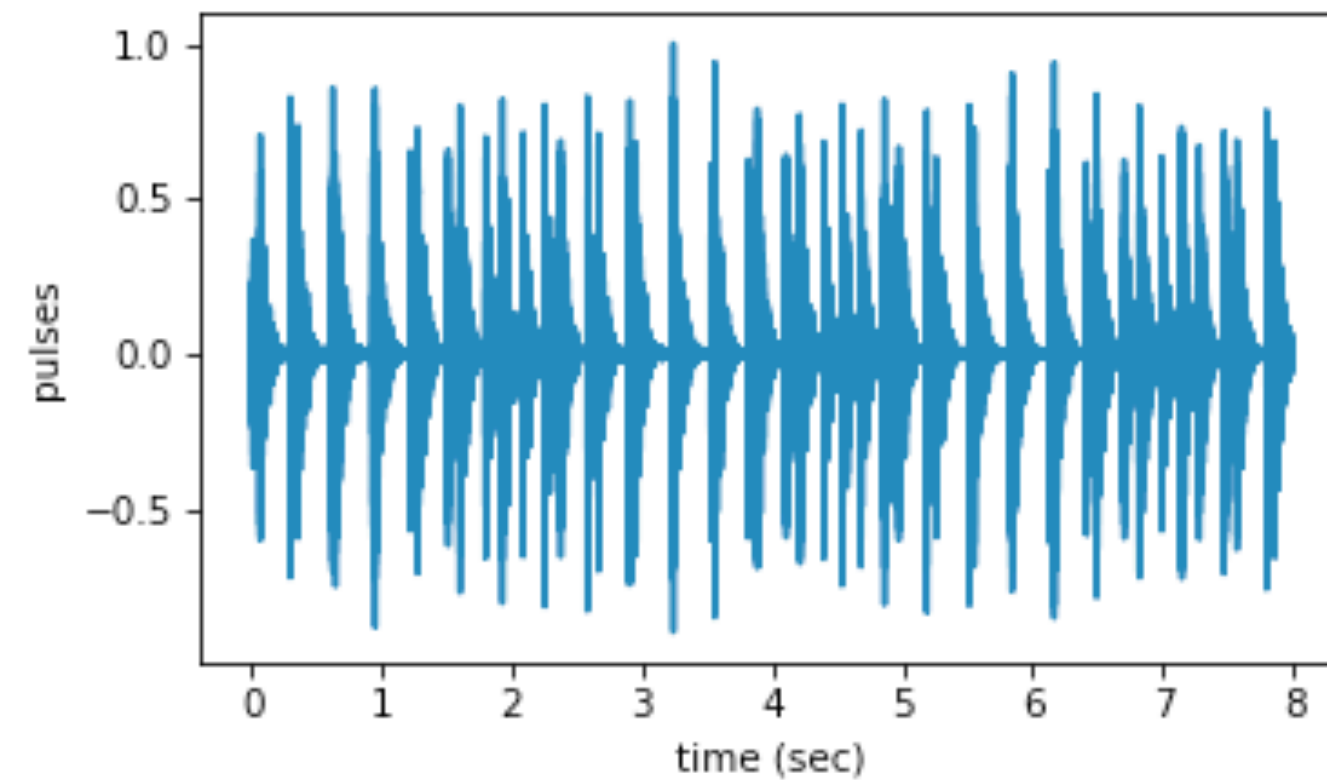
Status of calculations:

finished calculation of residuals

Double-metronome analysis ($\zeta=90$ degrees)

Double-Metronome Pulse Analysis

Double-Metronome Pulse Analysis



FILENAMES

Data file:

Profile 1:

Profile 2:

Pulse period [s]:

Pulse period [s]:

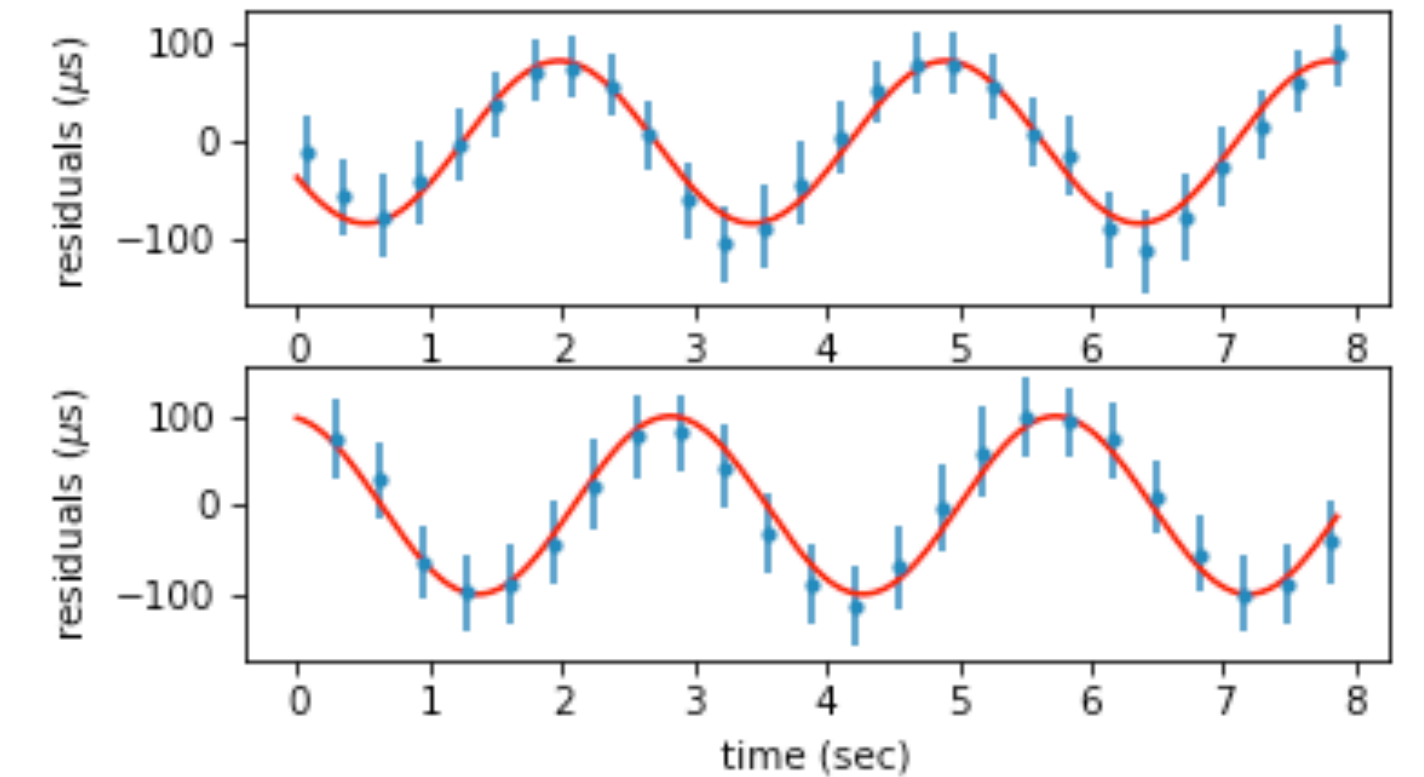
INITIAL ESTIMATES (1)

BEST-FIT VALUES (1)

INITIAL ESTIMATES (2)

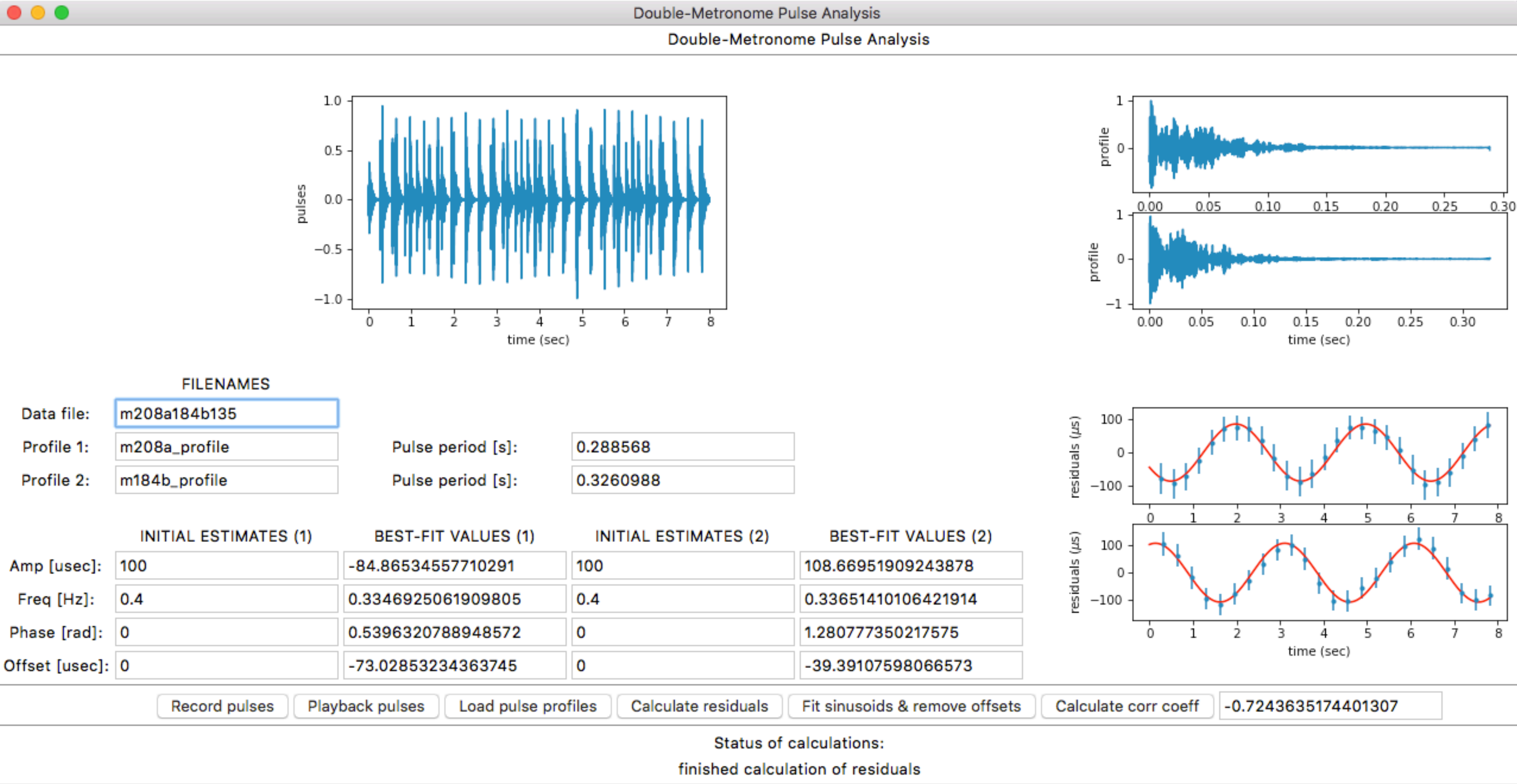
BEST-FIT VALUES (2)

| | | | | |
|----------------|----------------------------------|--|----------------------------------|---|
| Amp [usec]: | <input type="text" value="100"/> | <input type="text" value="-83.2204679782364"/> | <input type="text" value="100"/> | <input type="text" value="100.56846337609925"/> |
| Freq [Hz]: | <input type="text" value="0.4"/> | <input type="text" value="0.34235401761876616"/> | <input type="text" value="0.4"/> | <input type="text" value="0.3435231659876081"/> |
| Phase [rad]: | <input type="text" value="0"/> | <input type="text" value="0.45096522793443056"/> | <input type="text" value="0"/> | <input type="text" value="1.7673131445900625"/> |
| Offset [usec]: | <input type="text" value="0"/> | <input type="text" value="-88.73206094670937"/> | <input type="text" value="0"/> | <input type="text" value="-95.5978511142905"/> |

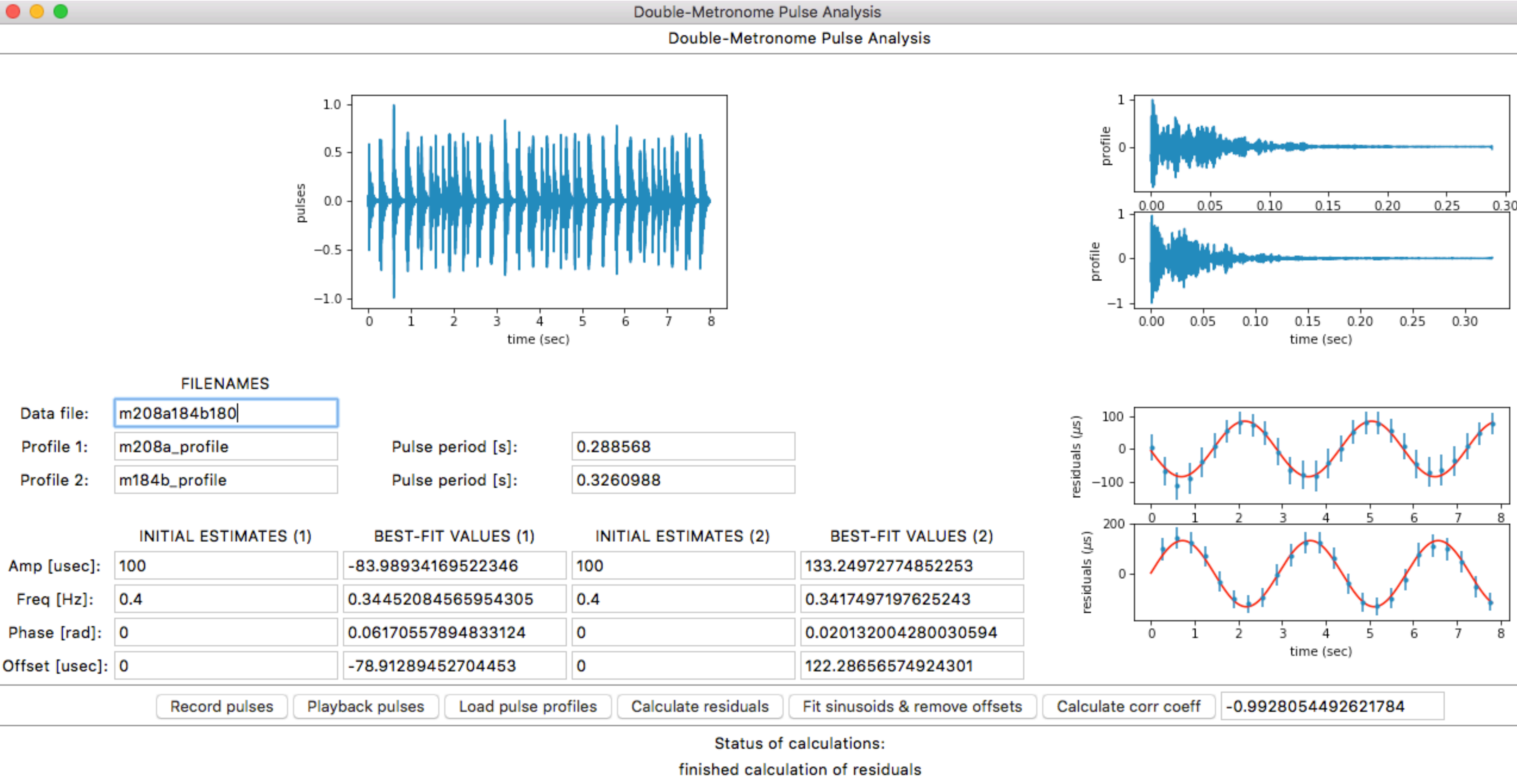


Status of calculations:
finished calculation of residuals

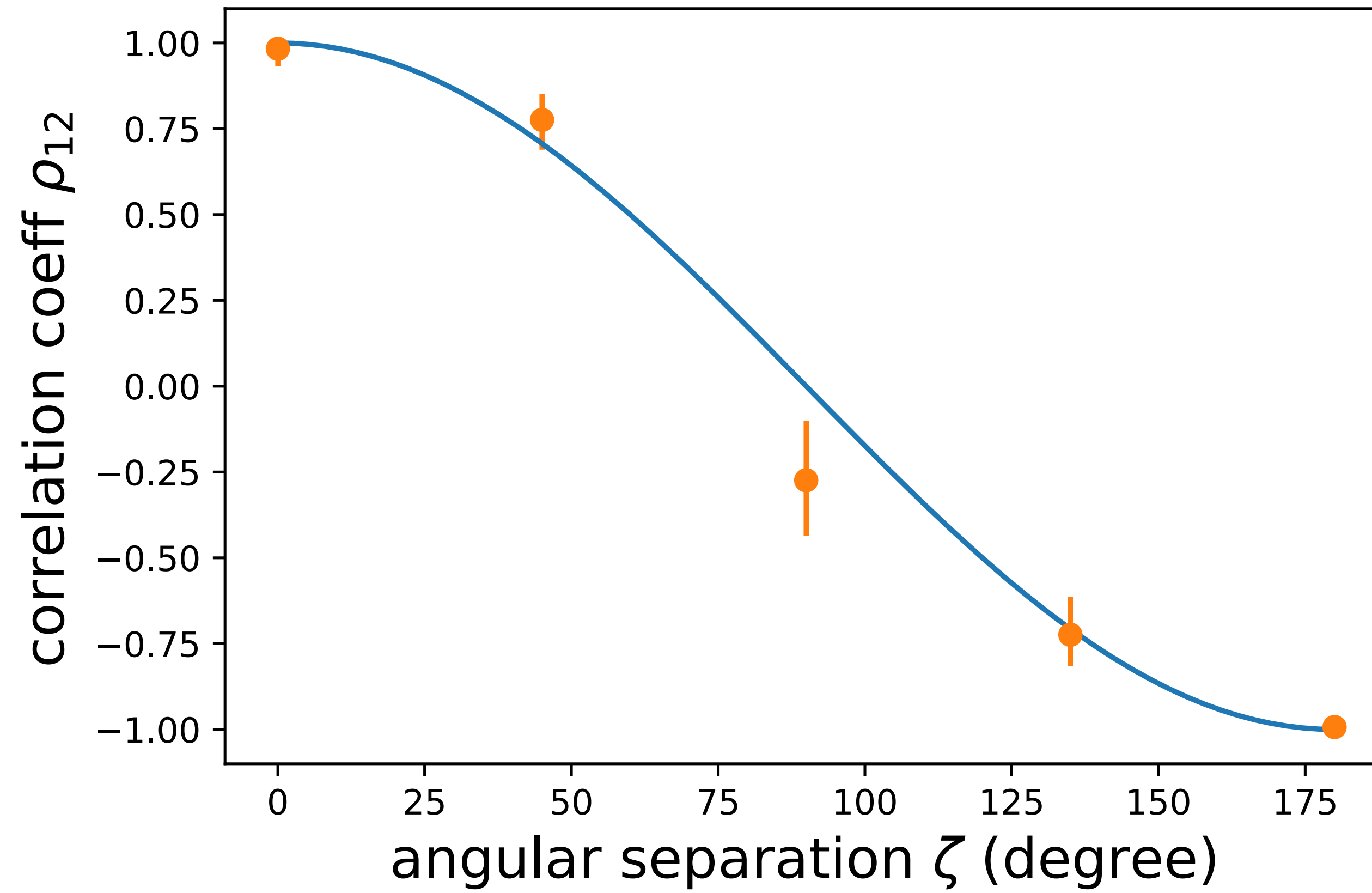
Double-metronome analysis ($\zeta=135$ degrees)



Double-metronome analysis ($\zeta=180$ degrees)

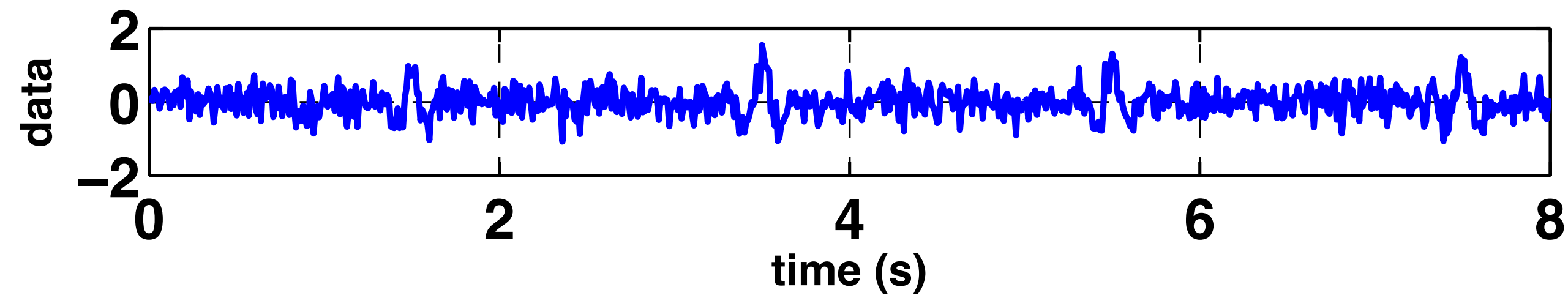


Metronome correlation

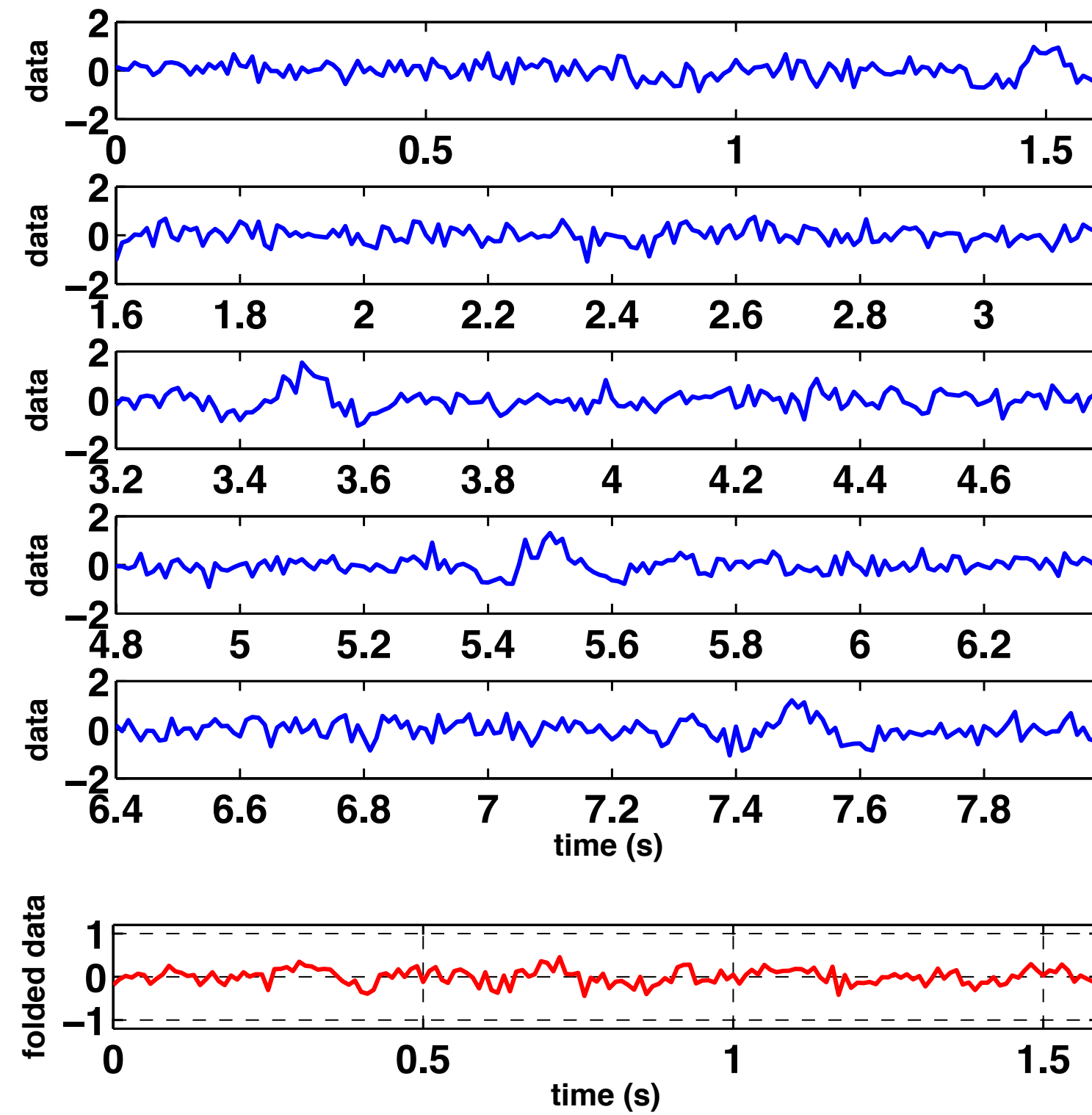


More details

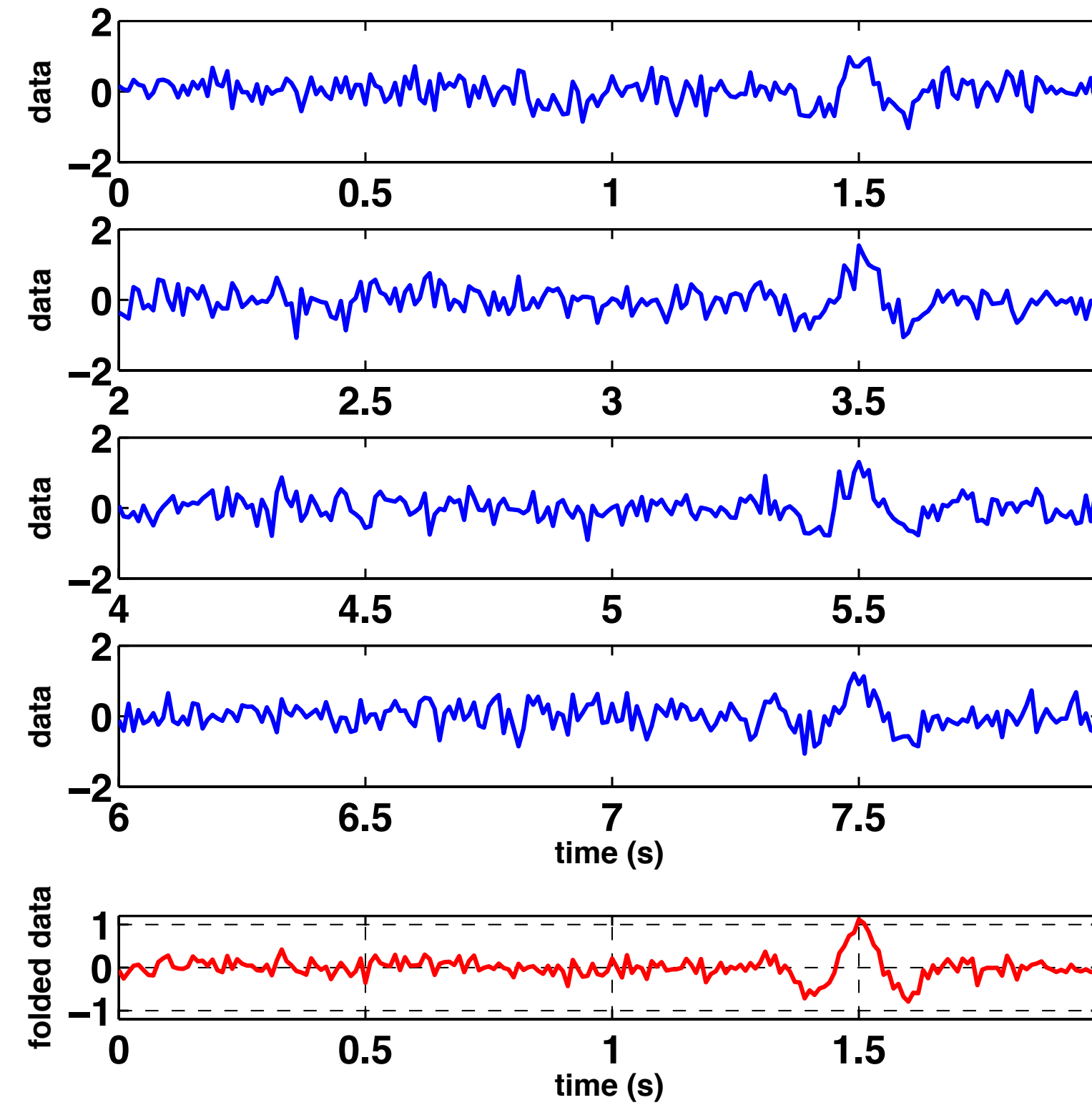
“Fold” data to determine pulse period and pulse profile



fold with incorrect T_p



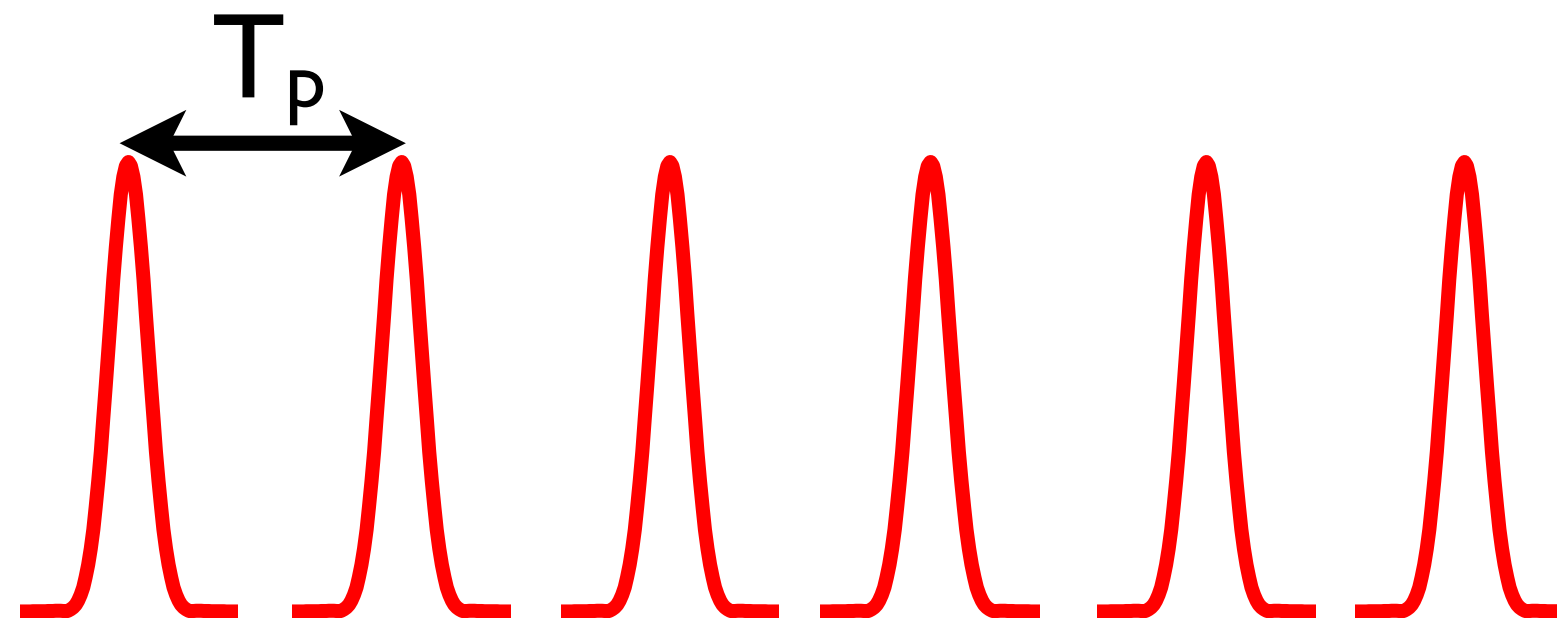
fold with correct T_p



Timing model

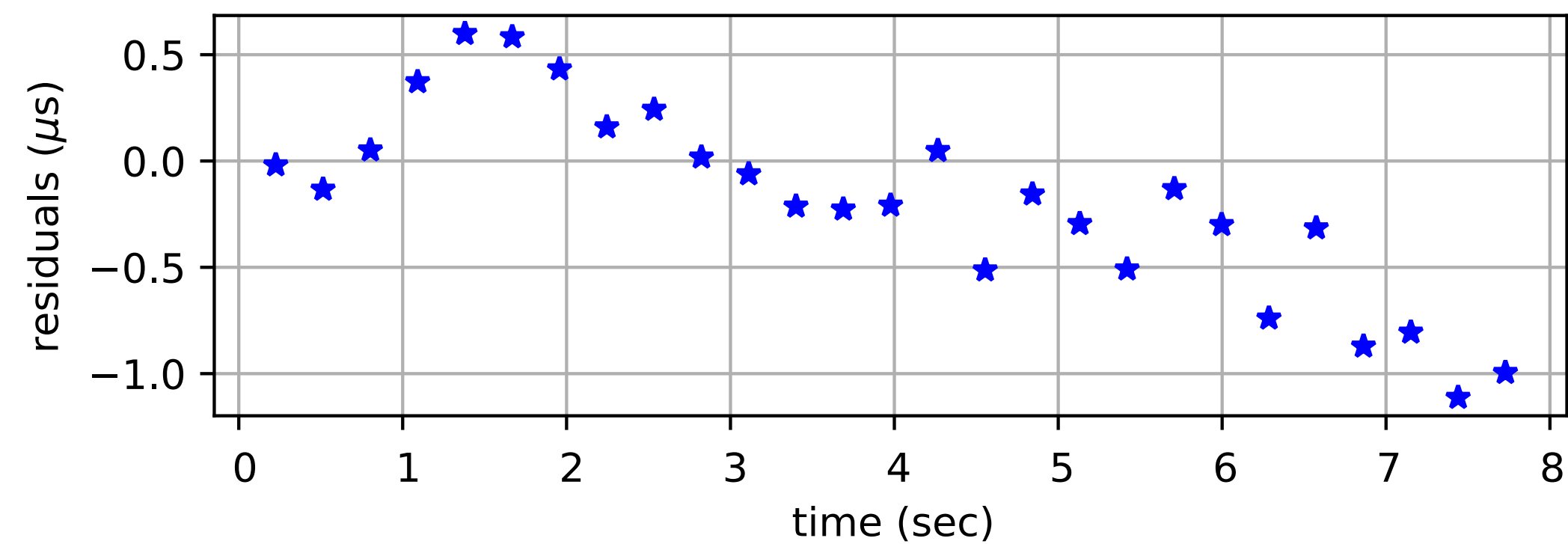
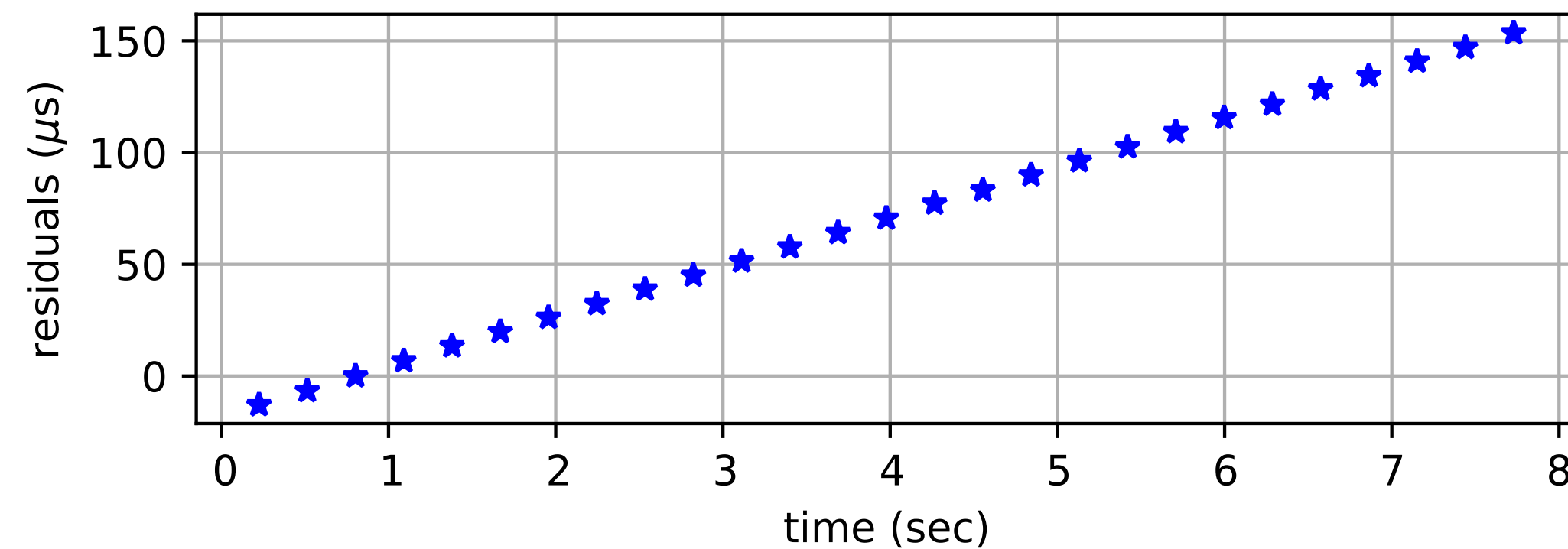
Pulses should arrive regularly with period T_p relative to some reference pulse

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_p$$



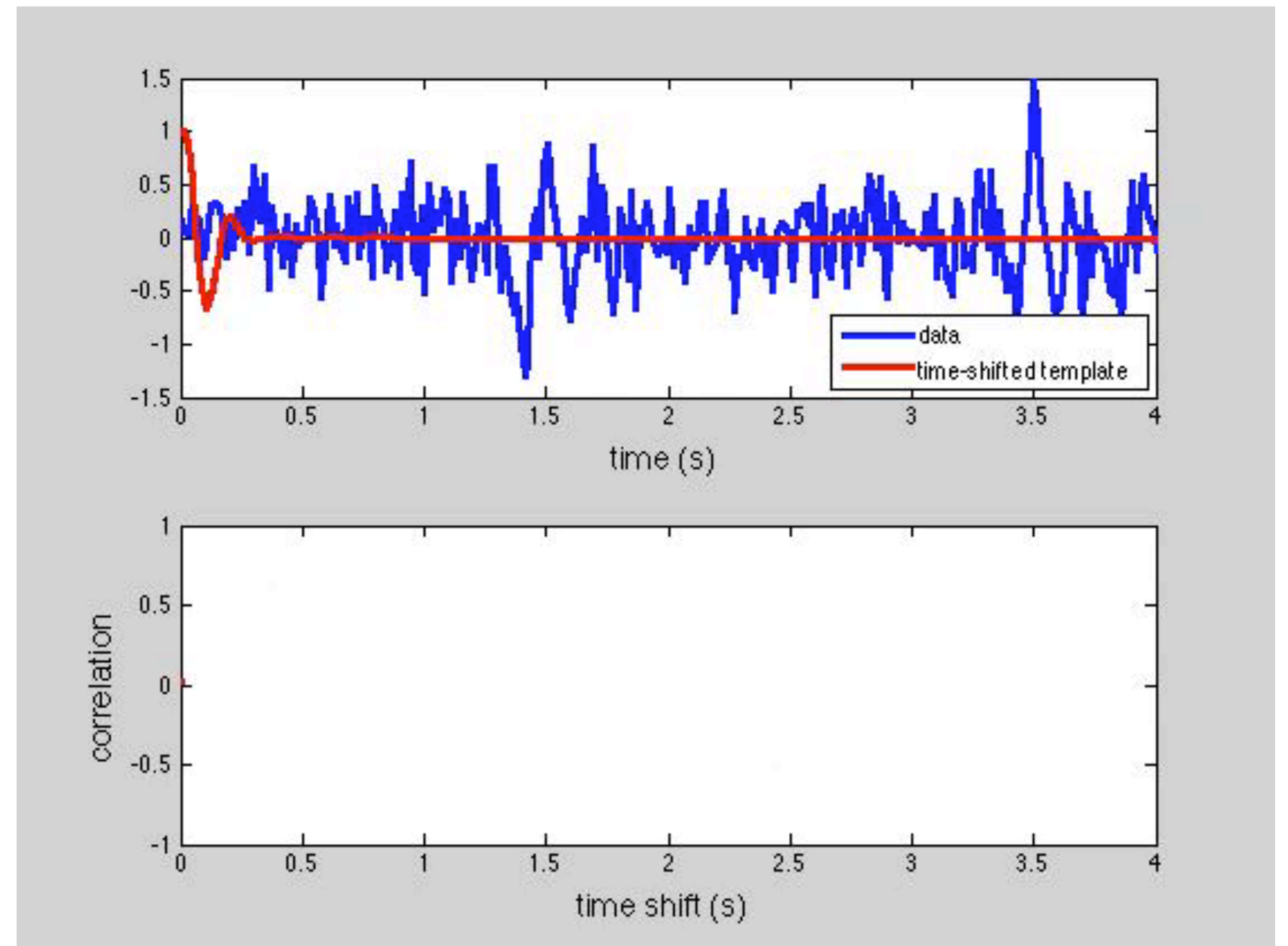
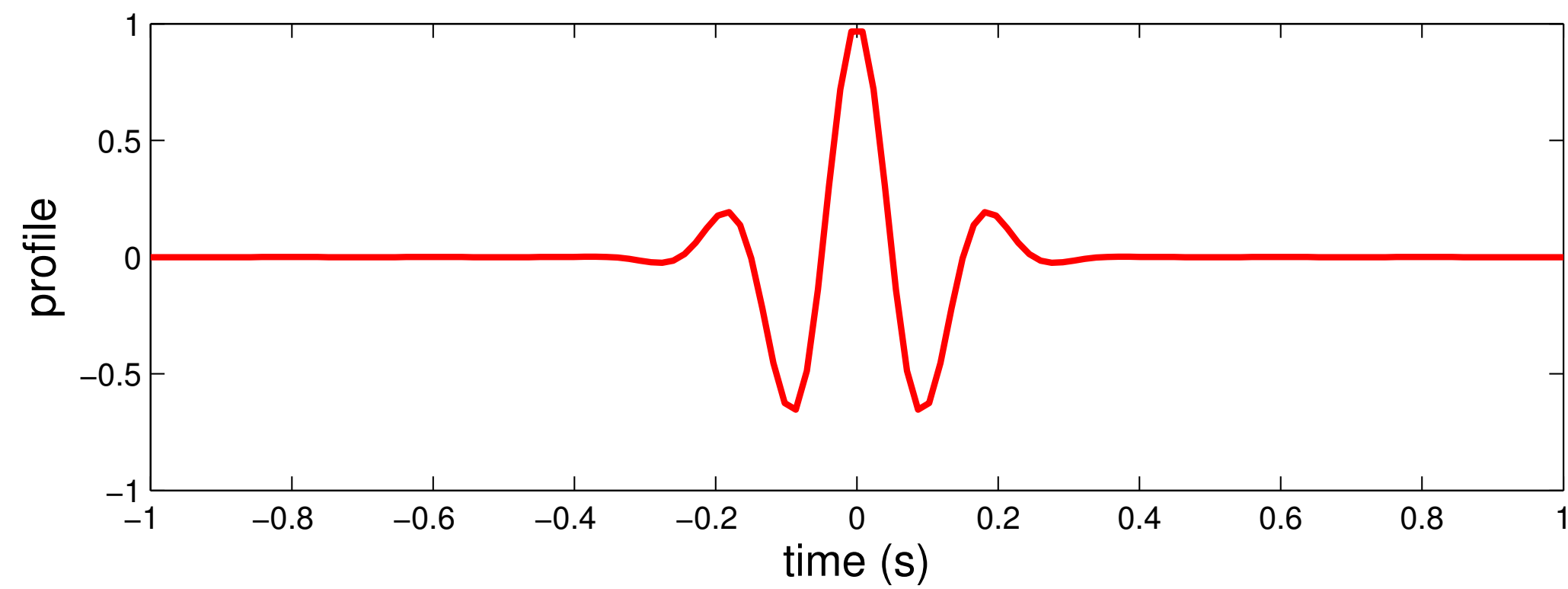
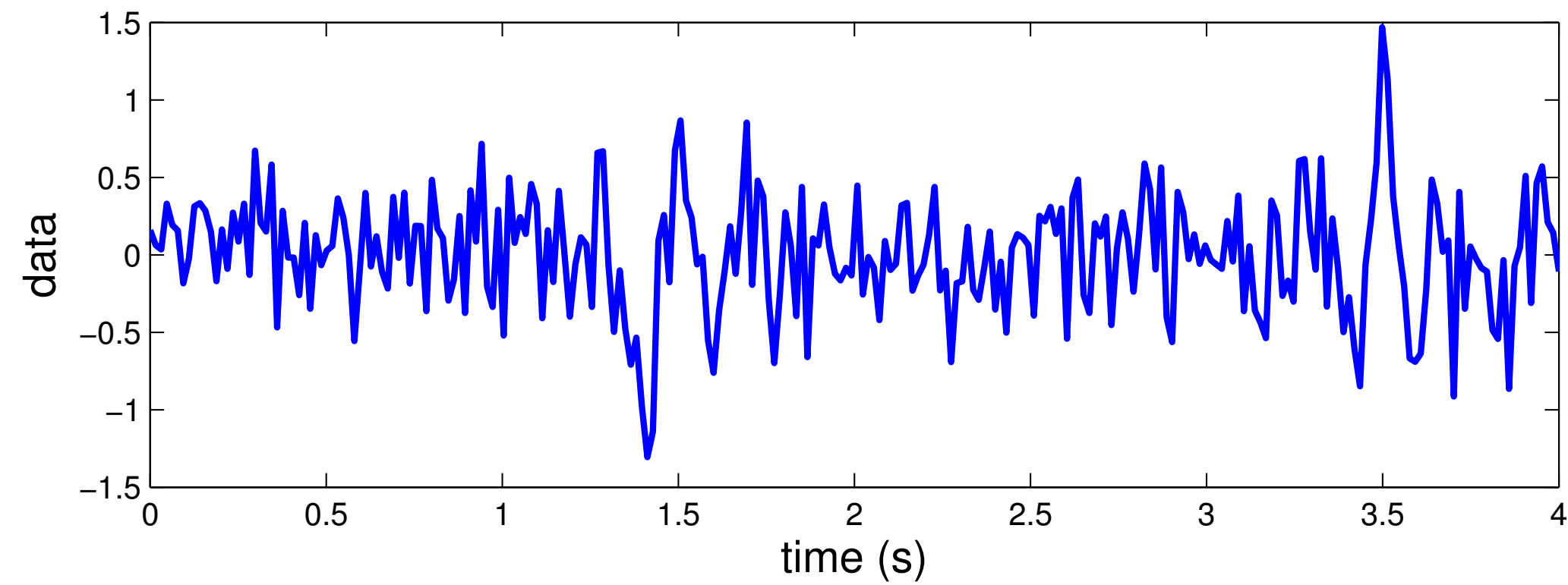
Remove linear trend to more accurately determine pulse period

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_p$$

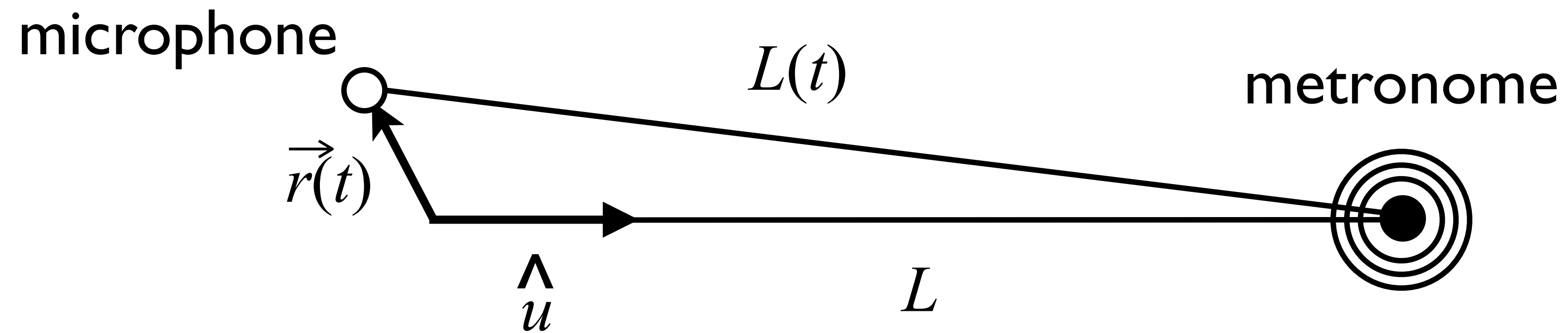


Matched-filtering determination of measured TOAs

$$C(\Delta t) = \mathcal{N} \int dt y(t)p(t - \Delta t)$$

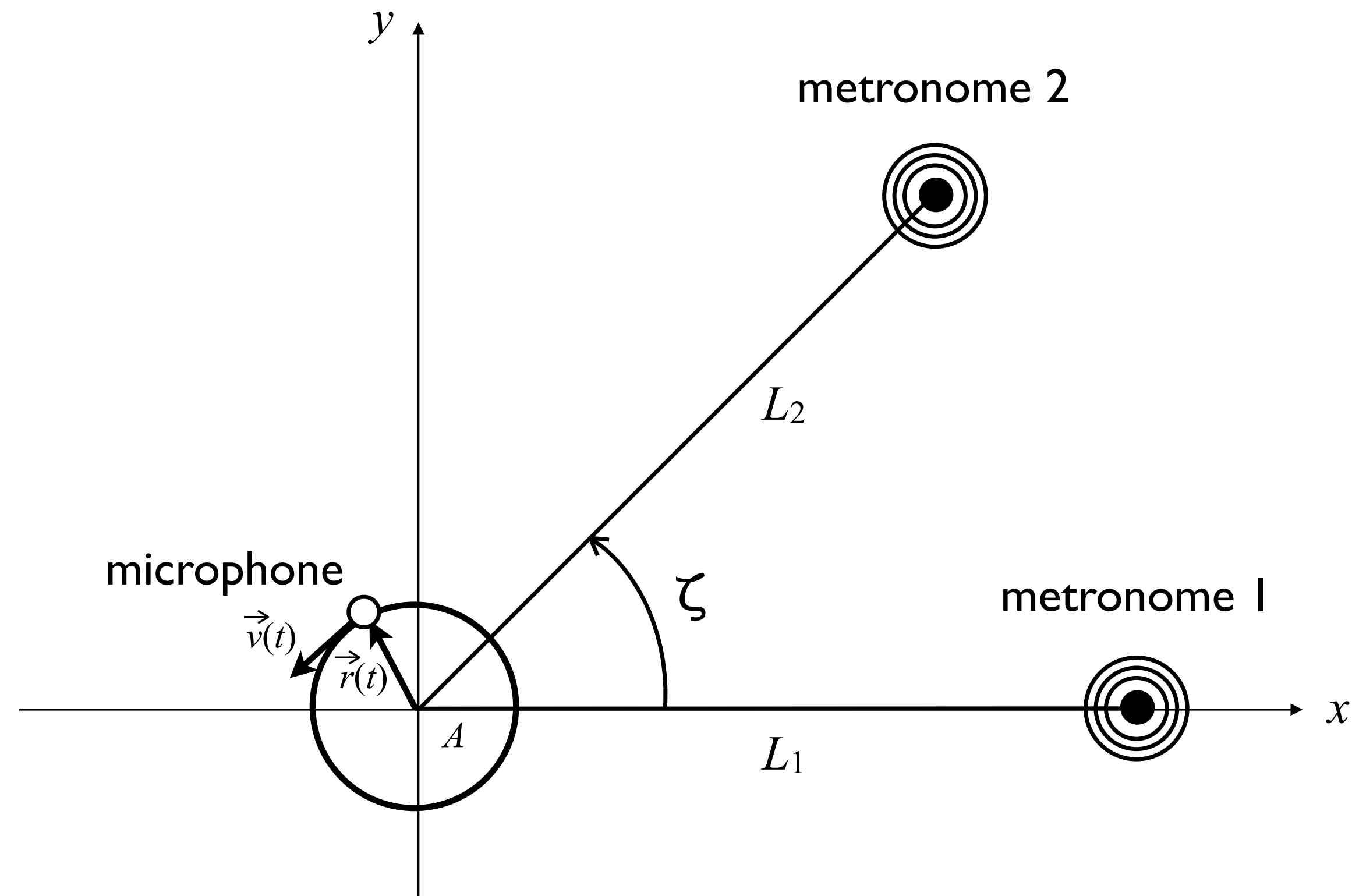


Timing-residual response to microphone motion



$$\delta\tau(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{1}{c_s} \hat{u} \cdot \vec{r}(t)$$

Two metronomes - uniform circular motion

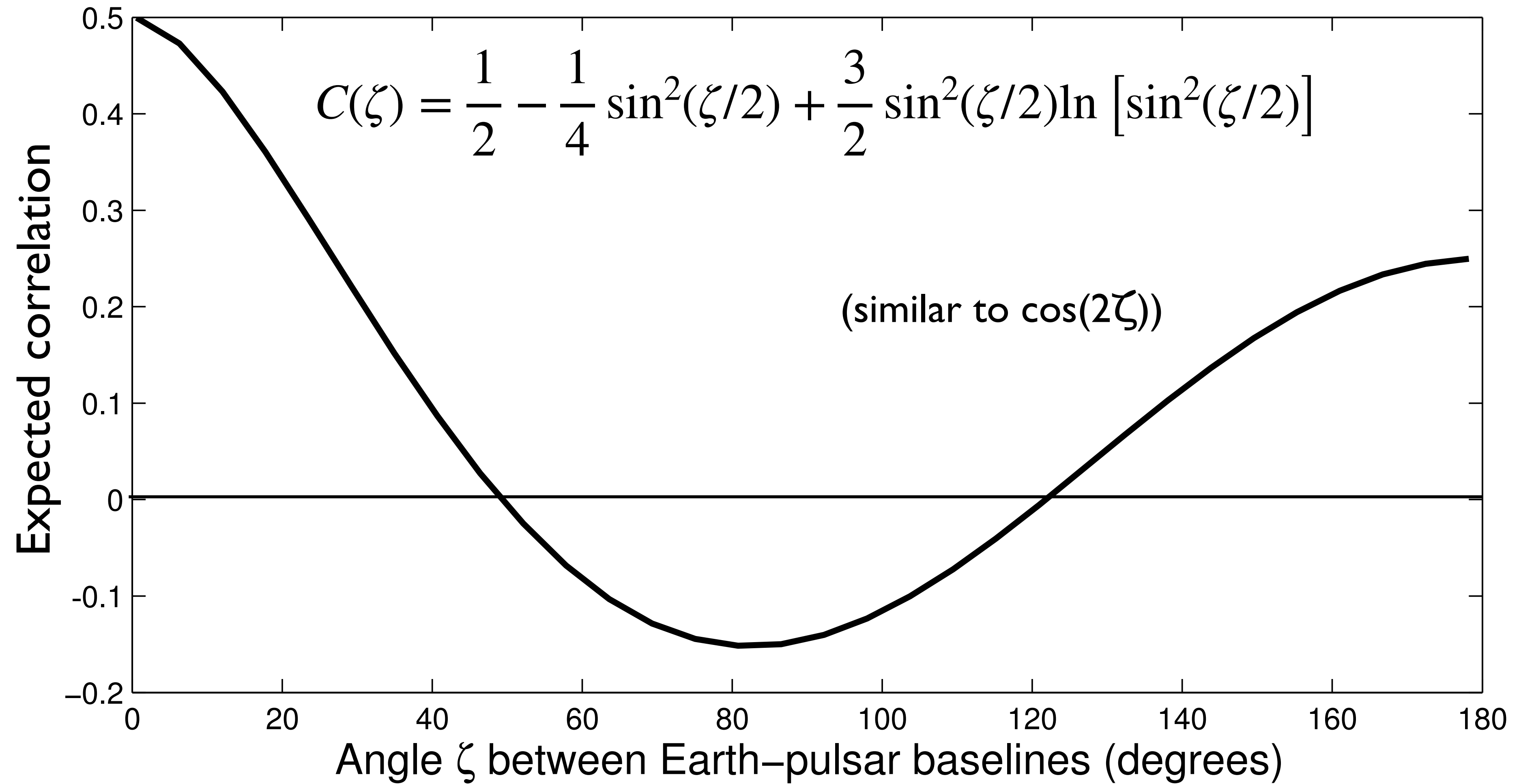


$$\vec{r}(t) = A [\cos(2\pi f_0 t + \phi_0) \hat{x} + \sin(2\pi f_0 t + \phi_0) \hat{y}]$$

$$\delta\tau_I(t) \simeq -\frac{A}{c_s} \cos(2\pi f_0 t + \phi_0 - \theta_I), \quad I = 1, 2$$

$$\rho_{12} \simeq \cos \zeta, \quad \zeta \equiv \theta_1 - \theta_2$$

Expected PTA correlation - Hellings & Downs curve (isotropic, unpolarized GW background)



Metronome demo numbers

$$c_s = 340 \text{ m/s (in air)}$$

$$\text{amplitude} \approx 5 \text{ cm}$$

$$\text{amplitude} / c_s = 1 \times 10^{-4} \text{ sec}$$

$$184 \text{ bpm: } T_p = 0.3261 \text{ sec}$$

$$208 \text{ bpm: } T_p = 0.2885 \text{ sec}$$

Pulsar timing numbers

$f \sim 1/\text{few weeks to } 1/10 \text{ years } (10^{-7} \text{ Hz to } 10^{-9} \text{ Hz})$

$\lambda \sim 0.1 \text{ to } 10 \text{ lyr } (\text{GW wavelength})$

$L \sim \text{few} \times 1000 \text{ lyr } (\text{distance to pulsars})$

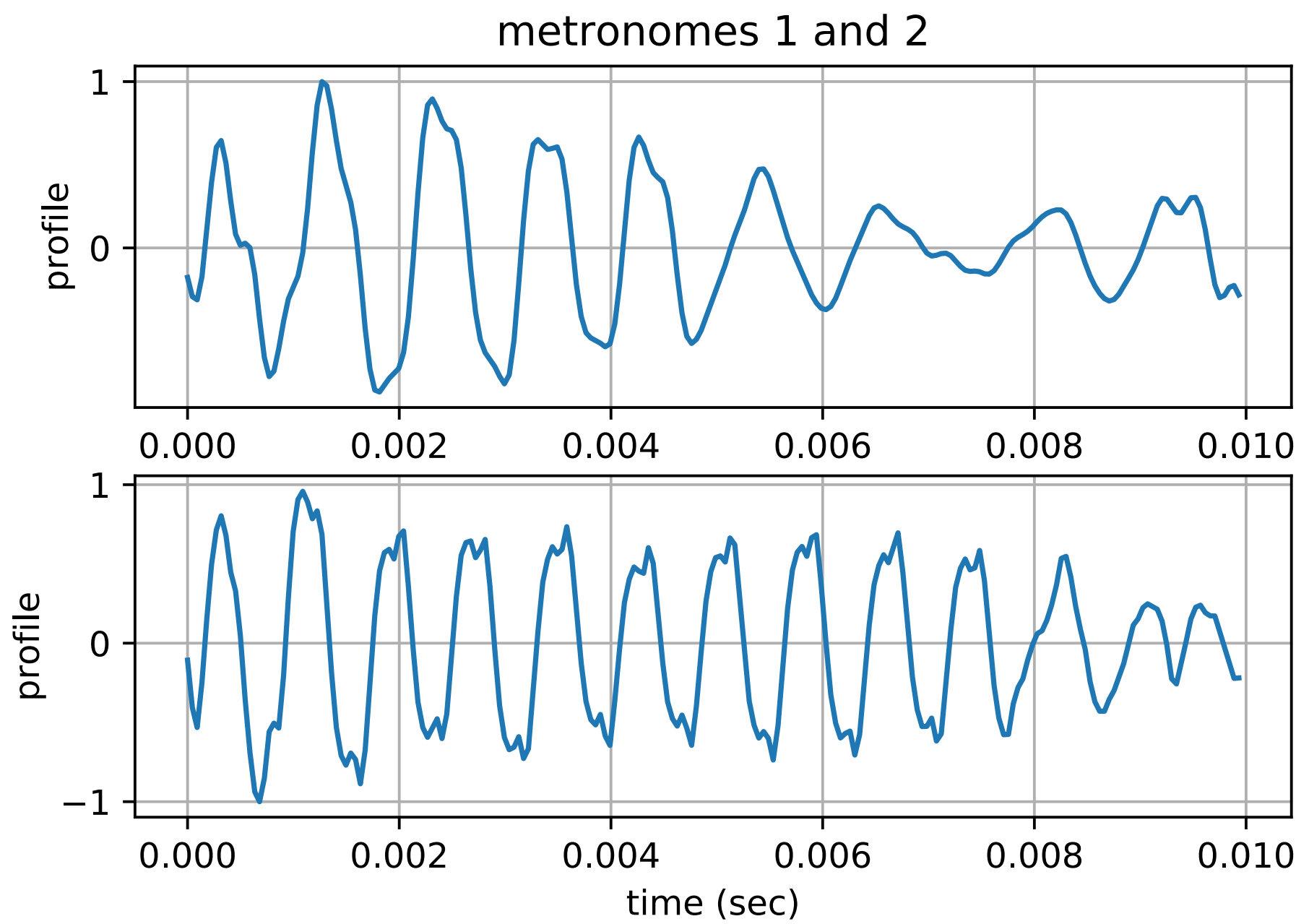
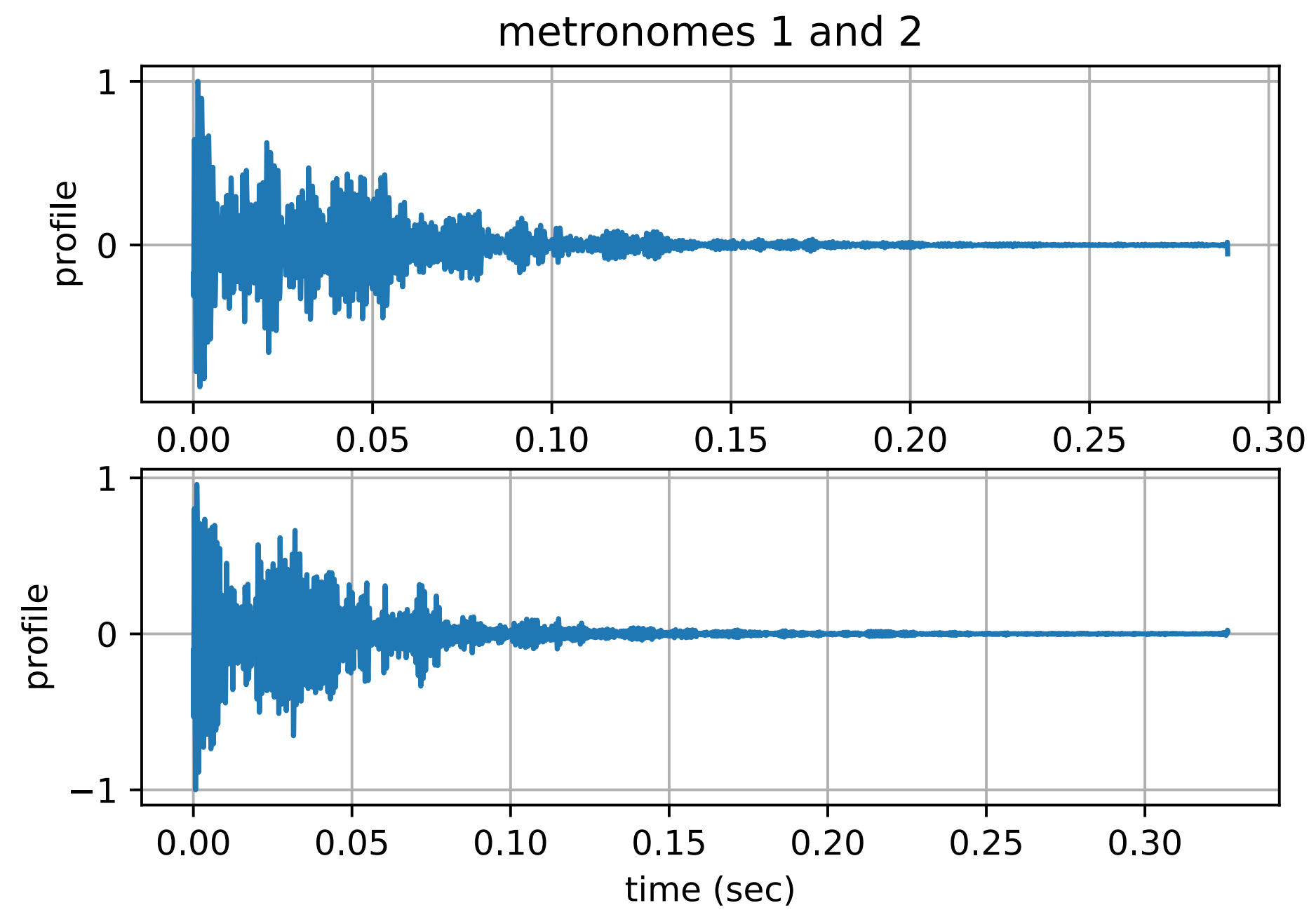
$\lambda \ll L$ (short-wavelength limit)

sensitivity $\sim \sigma_{\text{rms}}/T_{\text{obs}} \sim 100 \text{ ns}/10 \text{ yr} \sim 10^{-15}$

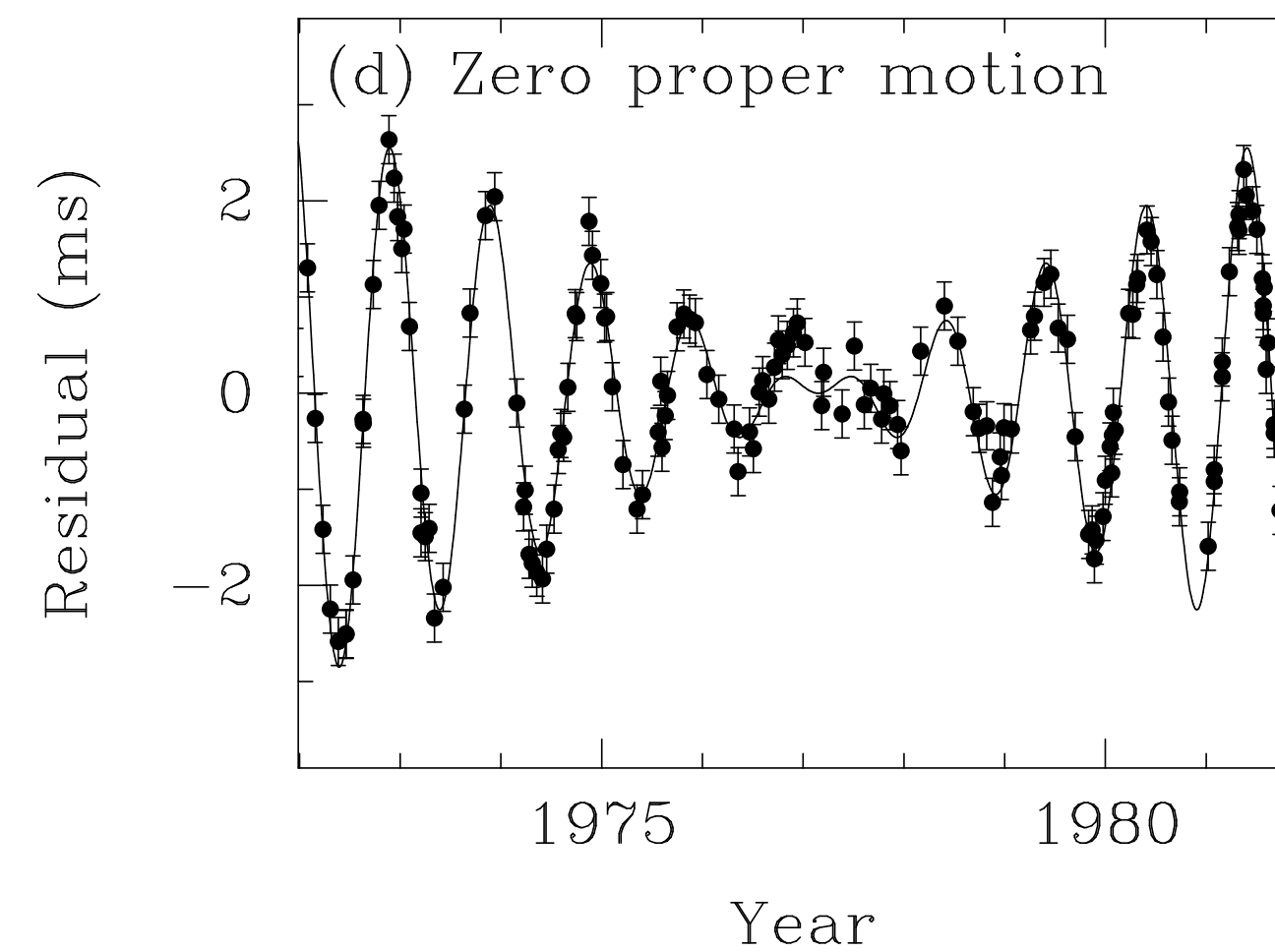
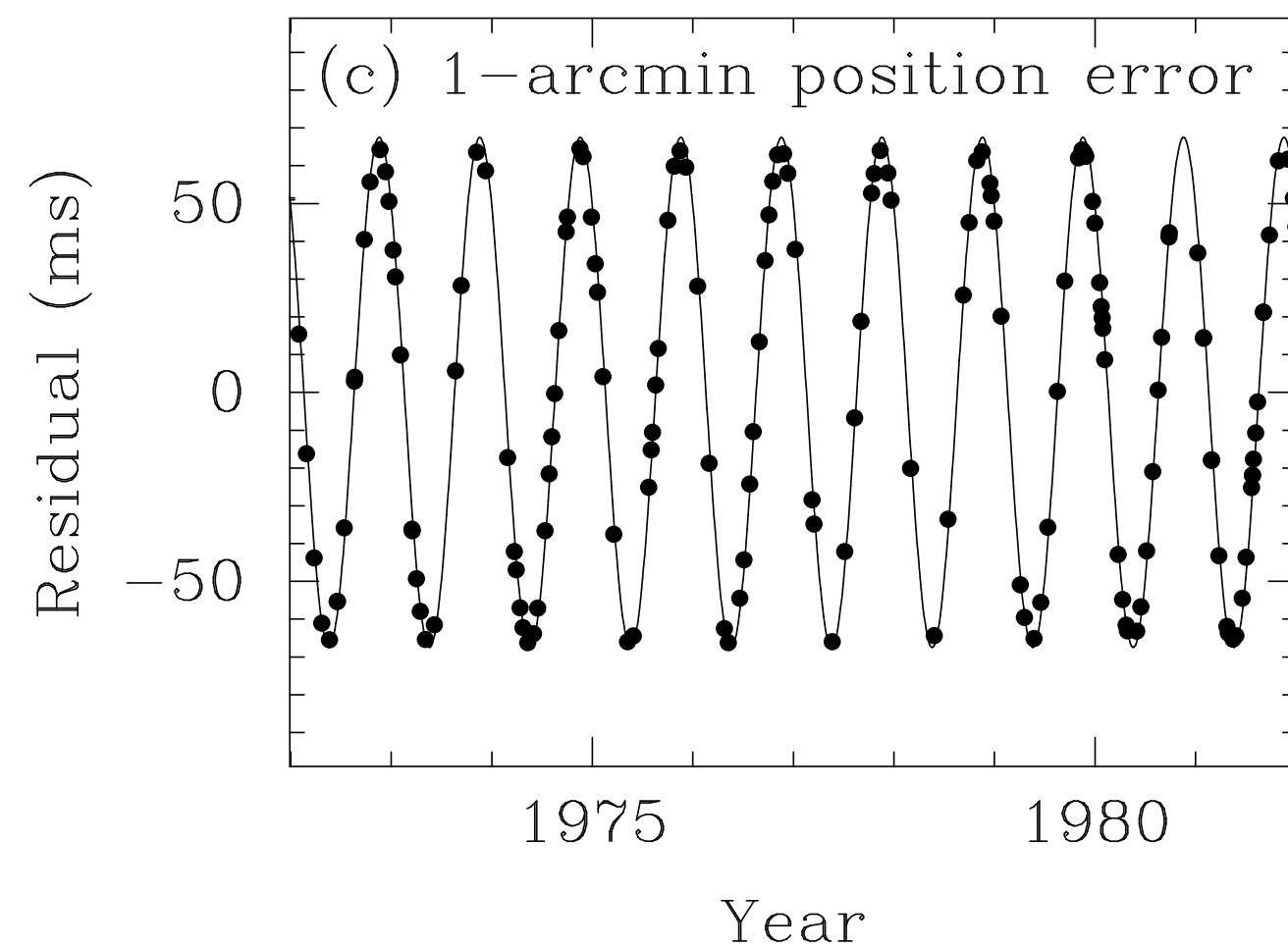
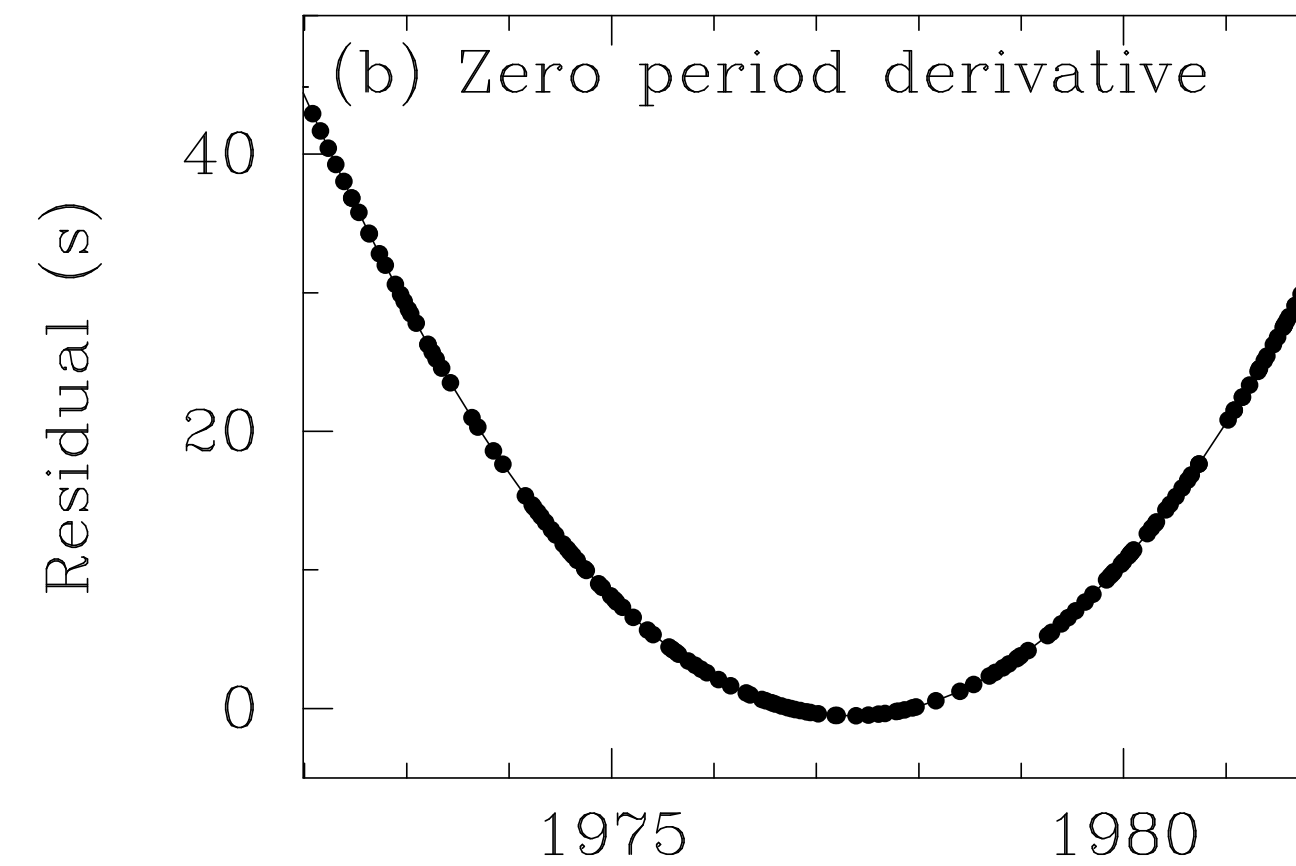
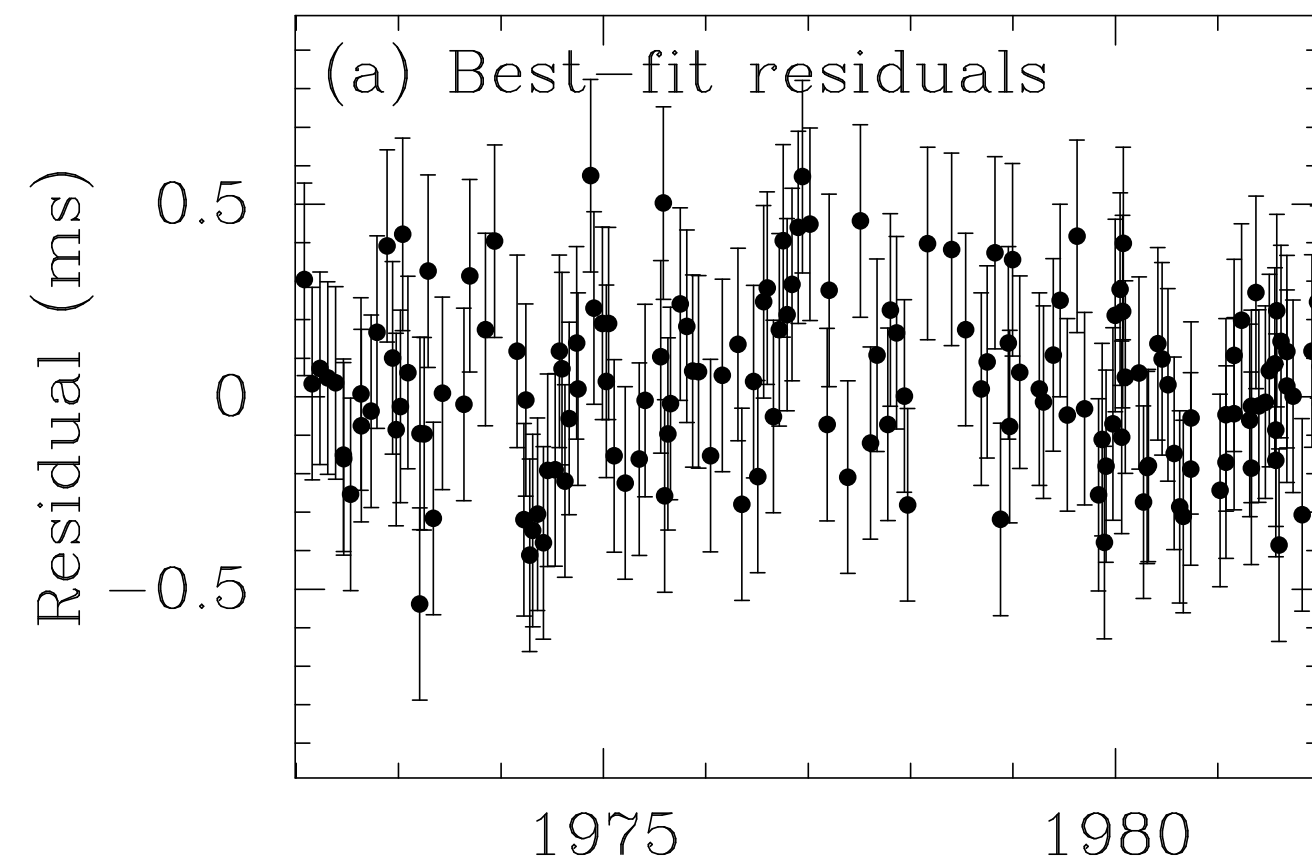


can detect changes $\sim 10 \text{ km}$ in the position
of a pulsar at a distance of $\sim 1000 \text{ lyr}$

Metronome pulse profiles



Errors in the timing model show up as deterministic features in the timing residuals

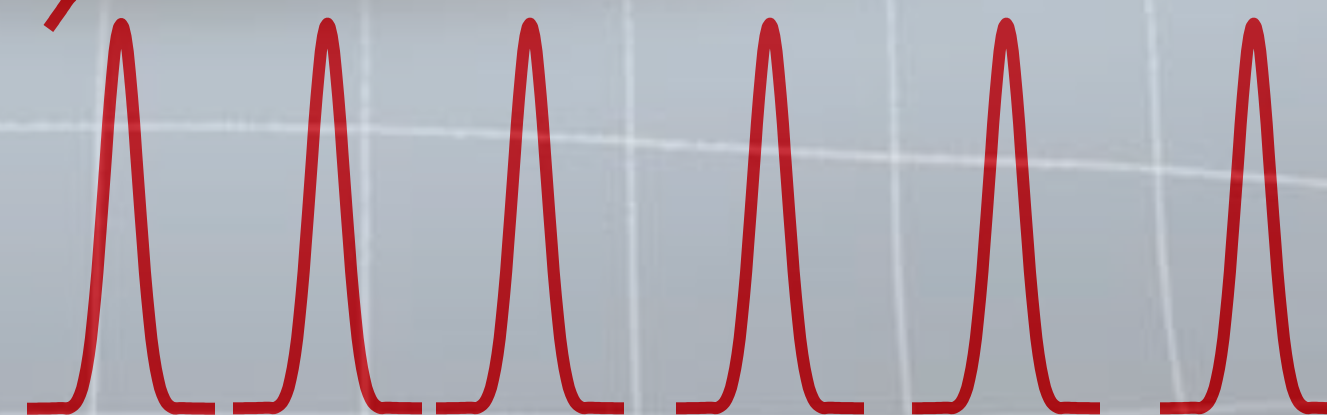


(from Lorimer LRR-2008-8)

Pulsar timing array

GWs cause pulses to arrive
ahead or behind schedule,
correlated across pulsars

radio
telescope



$$\delta\tau(t) = \frac{1}{c} u^a u^b \int ds h_{ab}(t(s), \vec{x}(s))$$

pulsar

pulsar

Metronome timing array

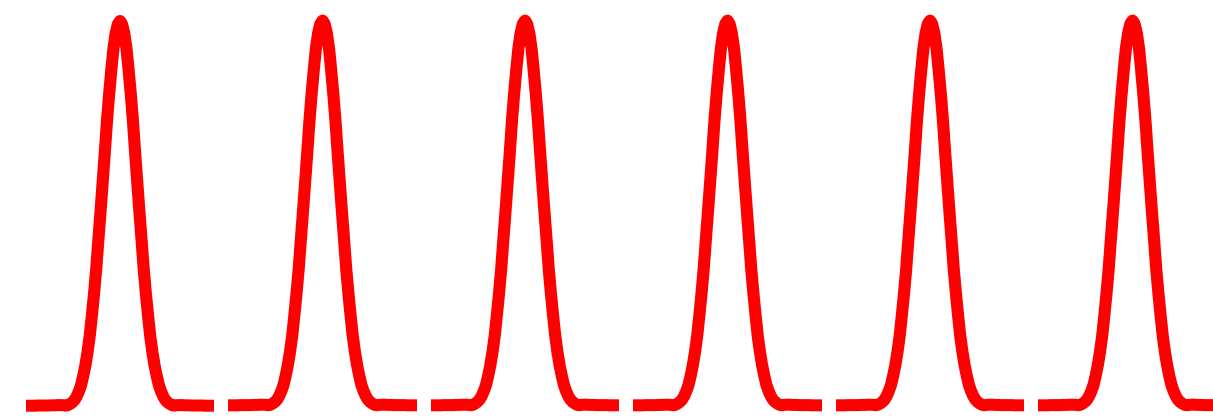
microphone motion
causes pulses to arrive
ahead or behind schedule,
correlated across metronomes



metronome

$$\delta\tau(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{1}{c_s} \hat{u} \cdot \vec{r}(t)$$

microphone



metronome