

Metronome timing array:

(a metronome-microphone demo of a pulsar timing array)

Joe Romano Les Houches Summer School 25 July 2018

(work in collaboration with M. Lam, M. Normandin, J. Key, and J. Hazboun)

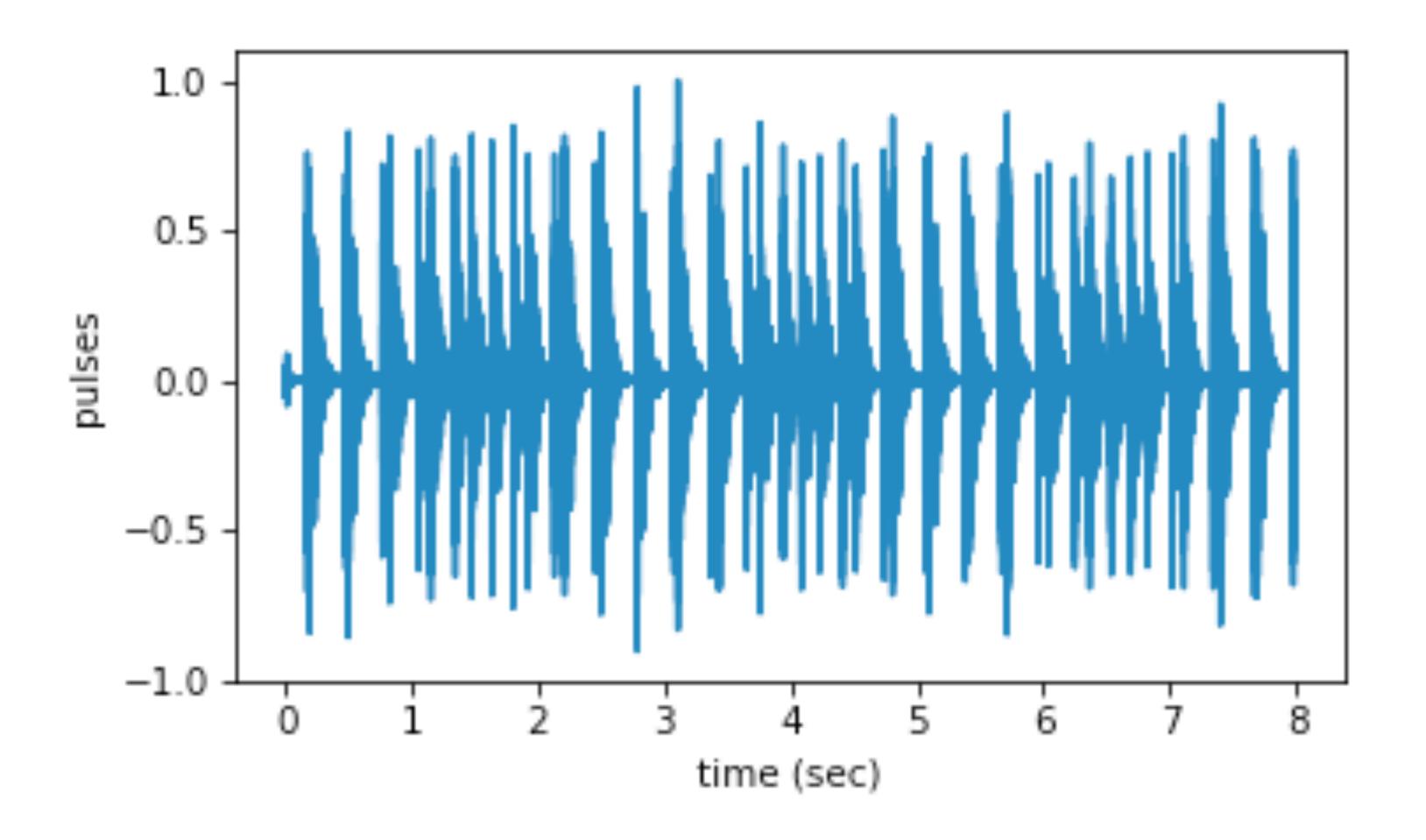
code and sample data: https://github.com/josephromano/leshouches/tree/master/pta-demo

paper (draft): https://github.com/josephromano/leshouches/blob/master/pta-demo/manuscript/pta-demo.pdf

Purpose of demo

- · Illustrate how a PTA is used to search for GWs in the context of a simple acoustical model
- Introduce concepts / techniques used by real pulsar astronomers:
 - folding (for calculating pulse periods and pulse profiles)
 - detrending (for better estimating pulse period)
 - matched filtering (for calculating measured times-of-arrivals)
 - timing model (for calculating expected times-of-arrivals)
 - correlation analyses (for extracting common GW component)

Q: Is there evidence of a "GW" in the data?



Q: Is there a common disturbance to the pulse arrival times (TOAs), and if so, is this disturbance correlated across metronomes as expected for a "GW" (i.e., microphone motion)?

- disturbance = measured TOAs expected TOAs = timing residuals
- common = correlation between the timing residuals

$$\rho_{12} \equiv \langle x_1 x_2 \rangle / \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \qquad \langle x_1 x_2 \rangle \equiv \frac{1}{T_{\text{obs}}} \int_0^{T_{\text{obs}}} dt \, x_1(t) x_2(t)$$

measured TOAs: obtained using matched filtering with pulse profile

$$C(\Delta t) = \mathcal{N} \int dt \ y(t)p(t - \Delta t)$$

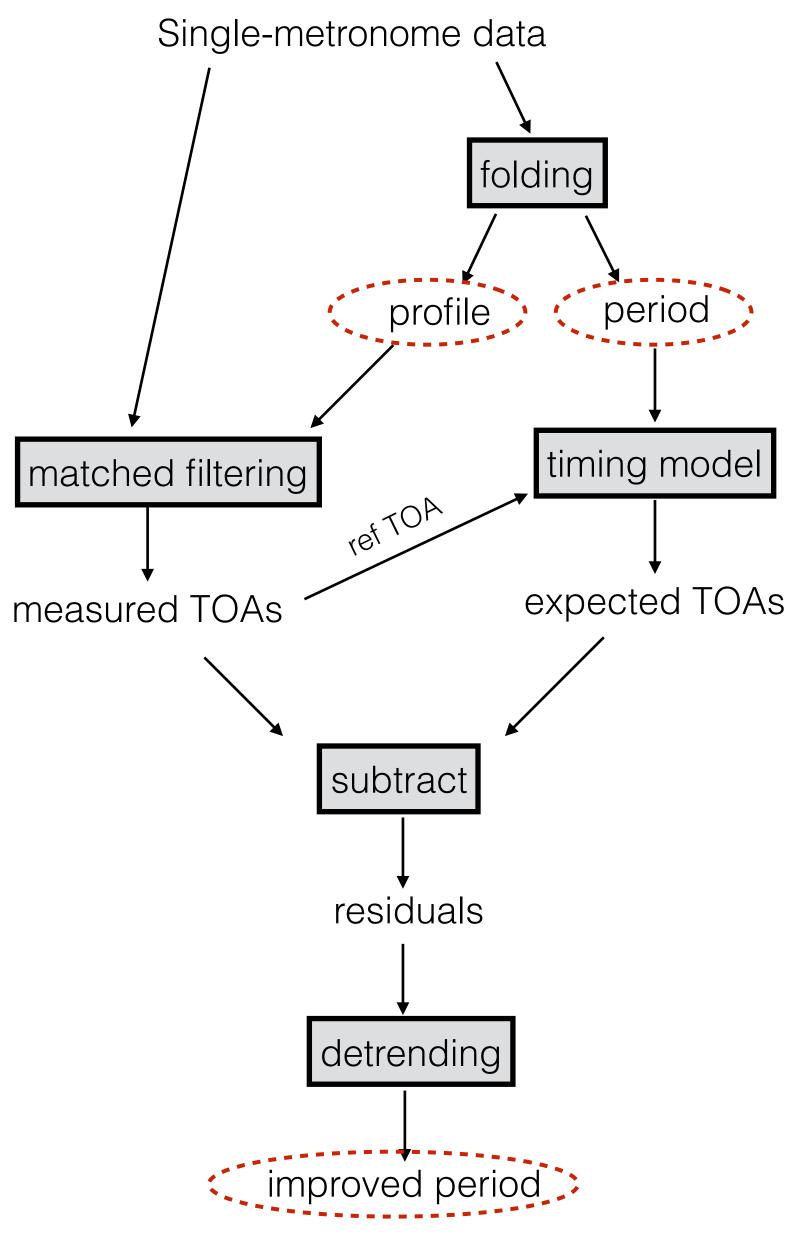
expected TOAs: timing model using pulse period and reference TOA

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$

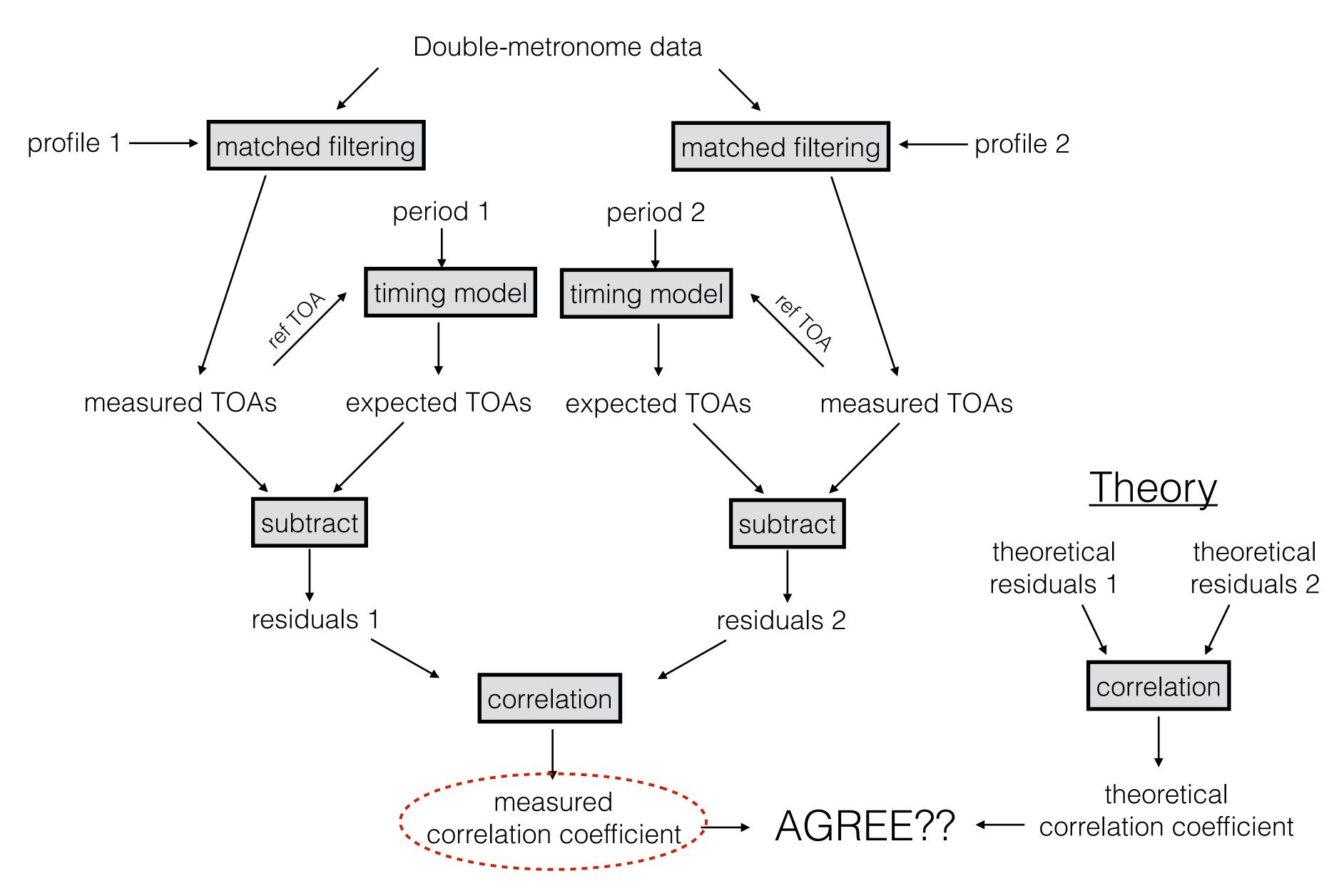
- pulse profile, period: folding and detrending single-metronome data
- expected "GW" correlation (for uniform circular motion):

$$\delta \tau_I(t) = \frac{\Delta L_I(t)}{c_{\rm S}} \simeq -\frac{1}{c_{\rm S}} \hat{u}_I \cdot \vec{r}(t) \implies \rho_{12} \simeq \cos \zeta$$

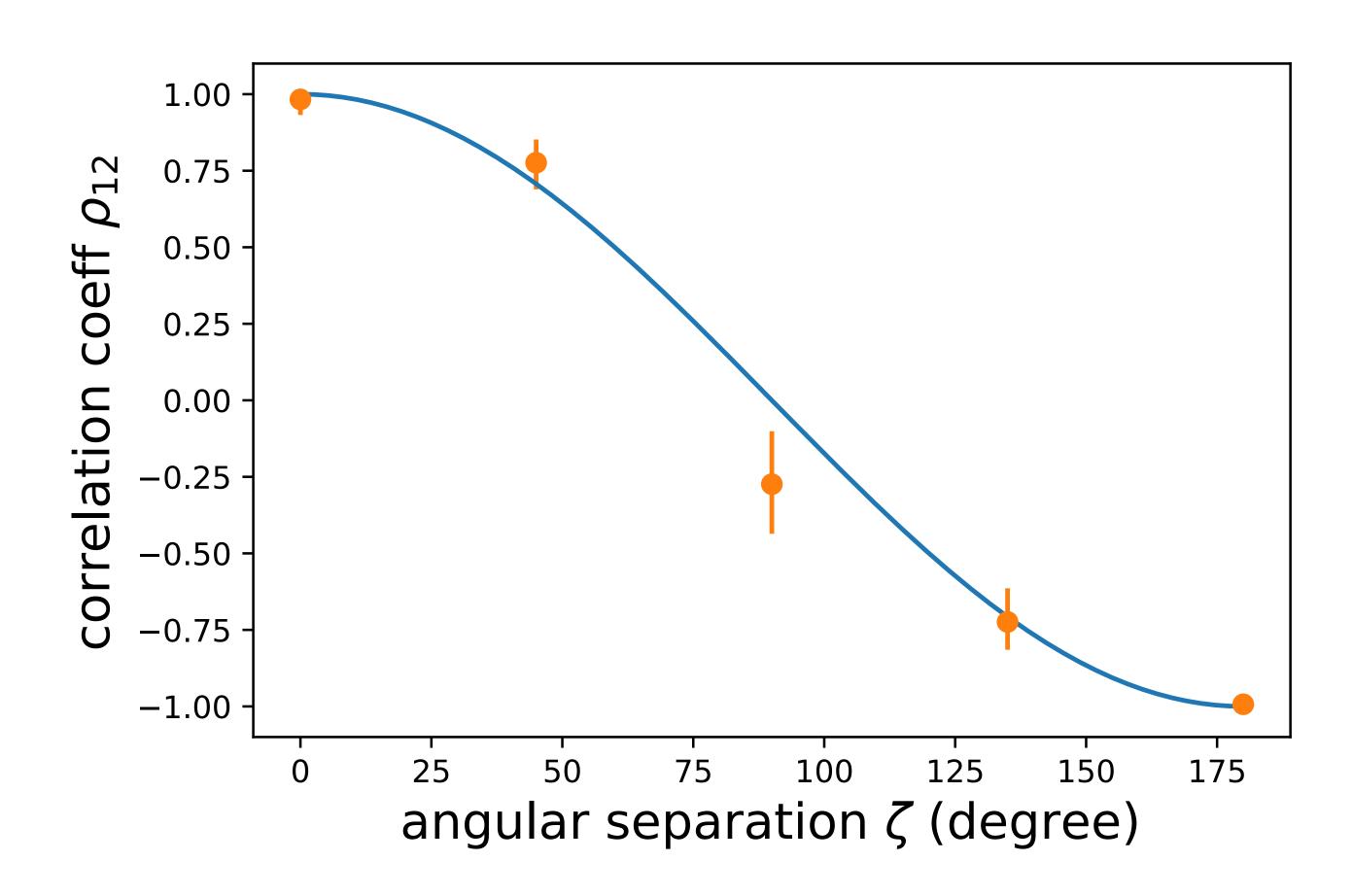
Single-metronome analysis



Double-metronome analysis



Metronome correlation

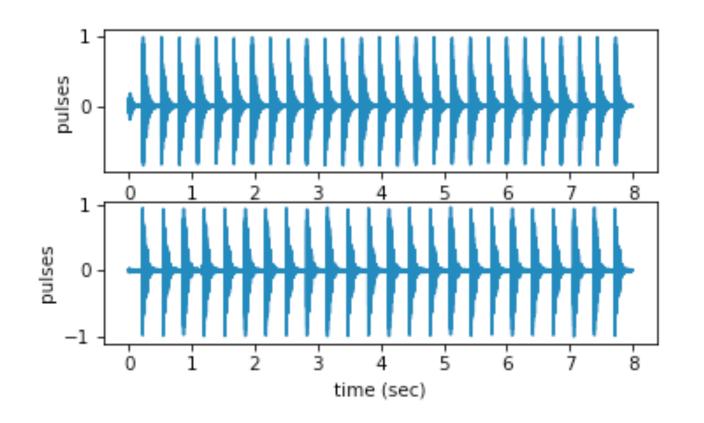


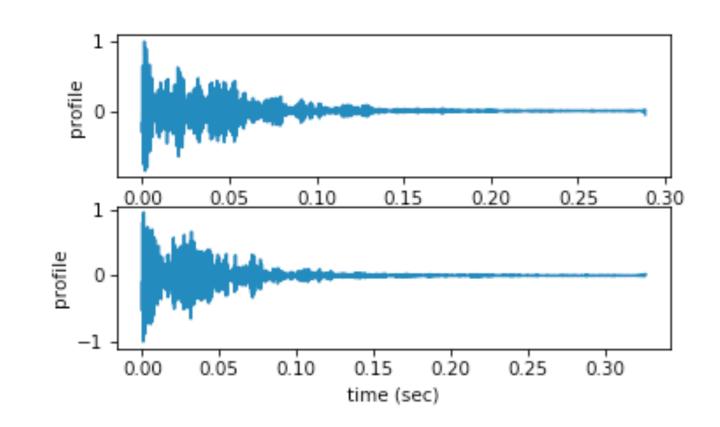
Output of the GUIs

Single-metronome analysis



Single-Metronome Pulse Analysis

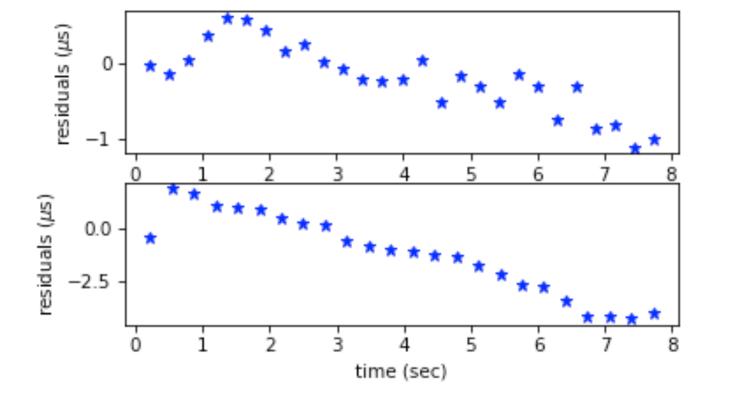




PULSE DATA FILENAMES

 Metronome 1:
 m208a
 bpm:
 208

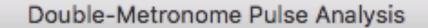
 Metronome 2:
 m184b
 bpm:
 184



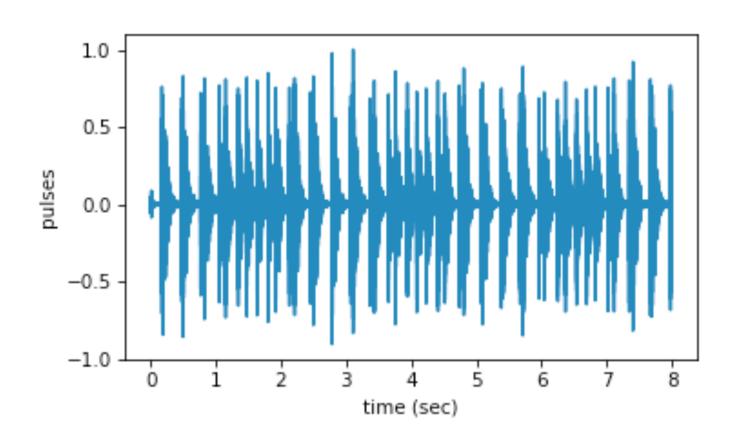
Metronome 1: Record pulses Playback pulses Calculate profile Pulse period [s]: 0.28856797860073924 Calculate residuals Detrend residuals

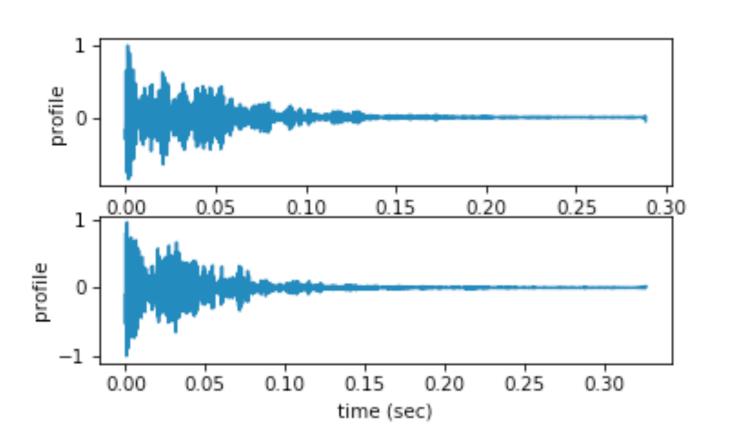
Metronome 2: Record pulses Playback pulses Calculate profile Pulse period [s]: 0.3260988289104562 Calculate residuals Detrend residuals

Double-metronome analysis (ζ =0 degrees)



Double-Metronome Pulse Analysis





FILENAMES

INITIAL ESTIMATES (1)

Data file: m208a184b0

Profile 1: m208a_profile

Profile 2:

m184b_profile

Pulse period [s]:

BEST-FIT VALUES (1)

0.288568

Pulse period [s]: 0.3260988

residuals (µs)	444	
- 100 - 001 - 001 - 001 - 001	0 1 2 3 4 5 6 7 8	8
	0 1 2 3 4 5 6 7 8 time (sec)	8

Amp [usec]:	100	-58.34831246716732	100	-67.83561046171864
Freq [Hz]:	0.4	0.33072788592748953	0.4	0.33116191414239915
Phase [rad]:	0	0.5924453954483324	0	0.393928097921481
Offset [usec]:	0	-43.4892386052206	0	1.7106663283346102

Record pulses

Playback pulses

Load pulse profiles

Calculate residuals

INITIAL ESTIMATES (2)

Fit sinusoids & remove offsets

BEST-FIT VALUES (2)

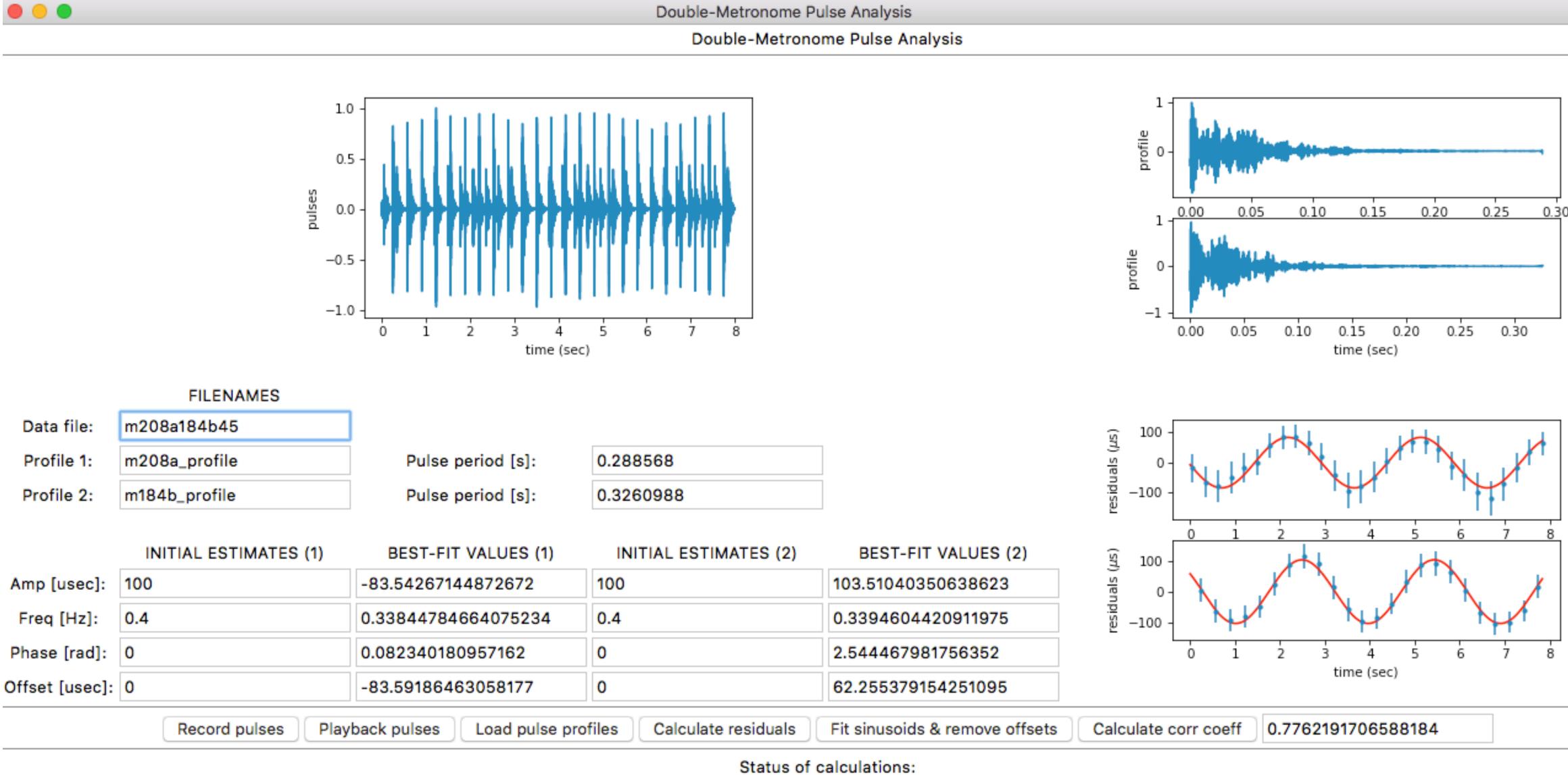
Calculate corr coeff

0.9830009544088217

Status of calculations:

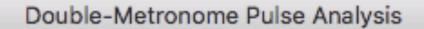
finished calculation of residuals

Double-metronome analysis (ζ =45 degrees)

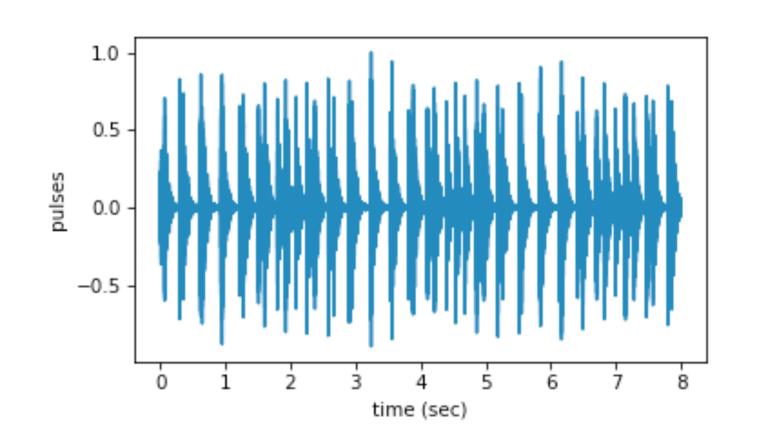


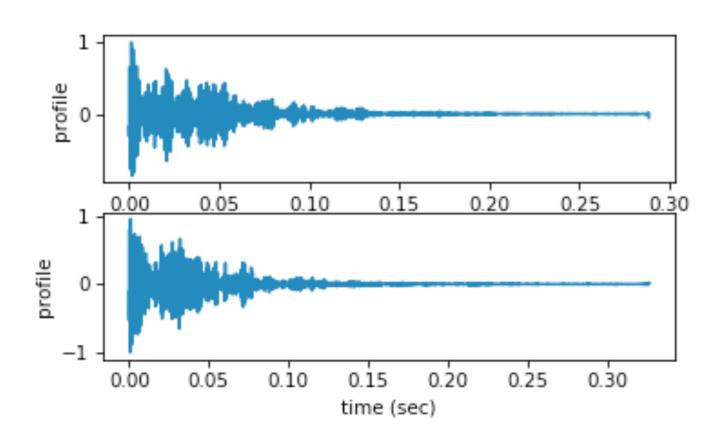
finished calculation of residuals

Double-metronome analysis (ζ =90 degrees)



Double-Metronome Pulse Analysis





FILENAMES

Data file:

m208a184b90

Profile 1:

Profile 2:

m208a_profile

m184b_profile

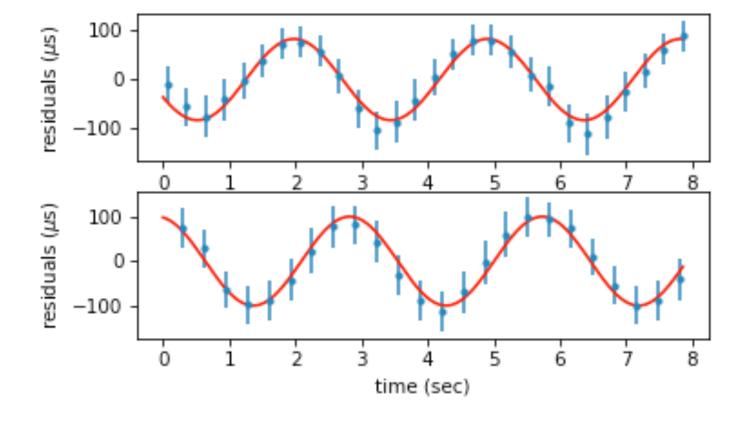
Pulse period [s]:

0.288568

Pulse period [s]:

0.3260988

	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-83.2204679782364	100	100.56846337609925
Freq [Hz]:	0.4	0.34235401761876616	0.4	0.3435231659876081
Phase [rad]:	0	0.45096522793443056	0	1.7673131445900625
Offset [usec]:	0	-88.73206094670937	0	-95.5978511142905



Record pulses

Playback pulses

Load pulse profiles

Calculate residuals

Fit sinusoids & remove offsets

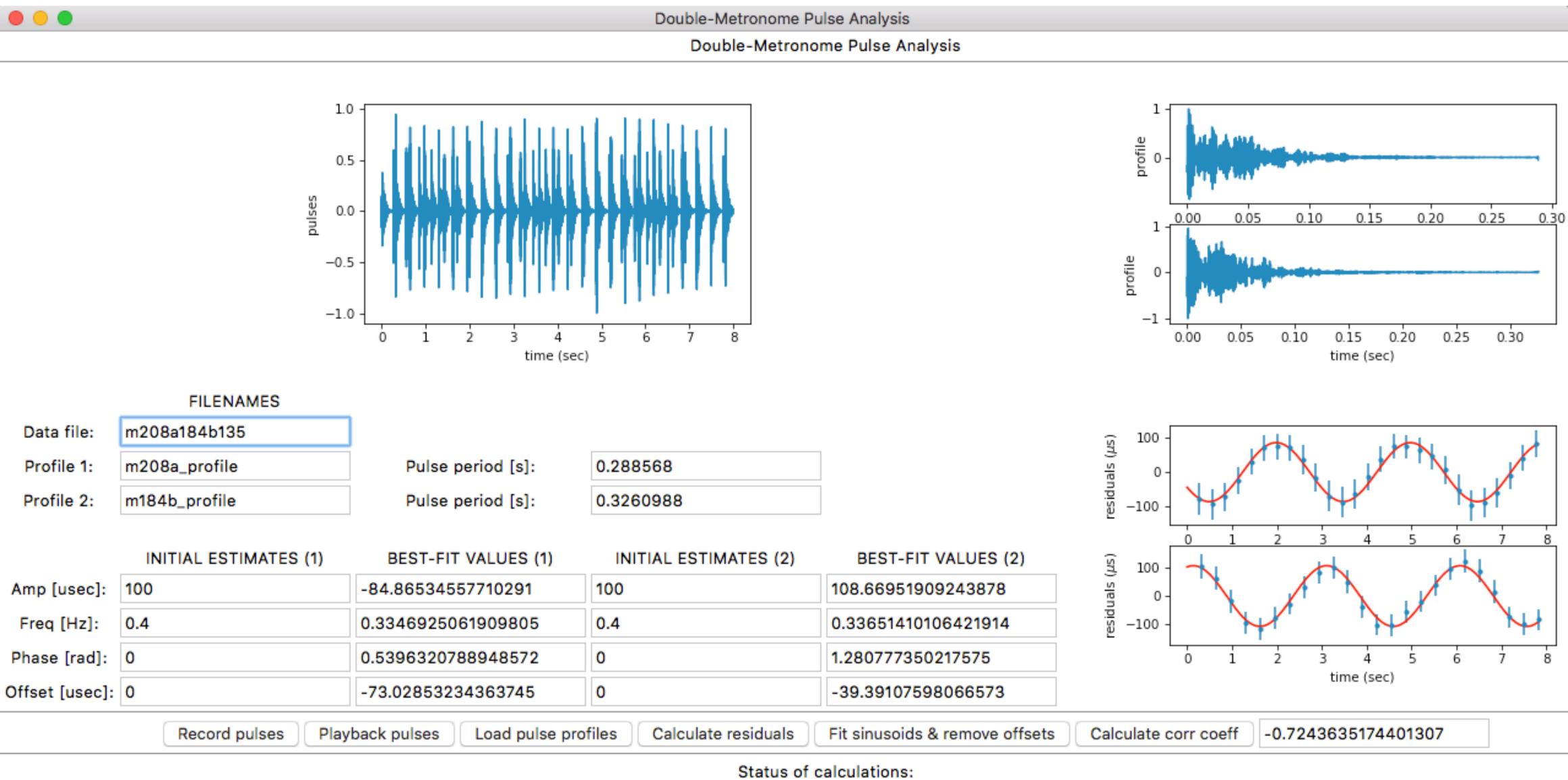
Calculate corr coeff

-0.27377904355571664

Status of calculations:

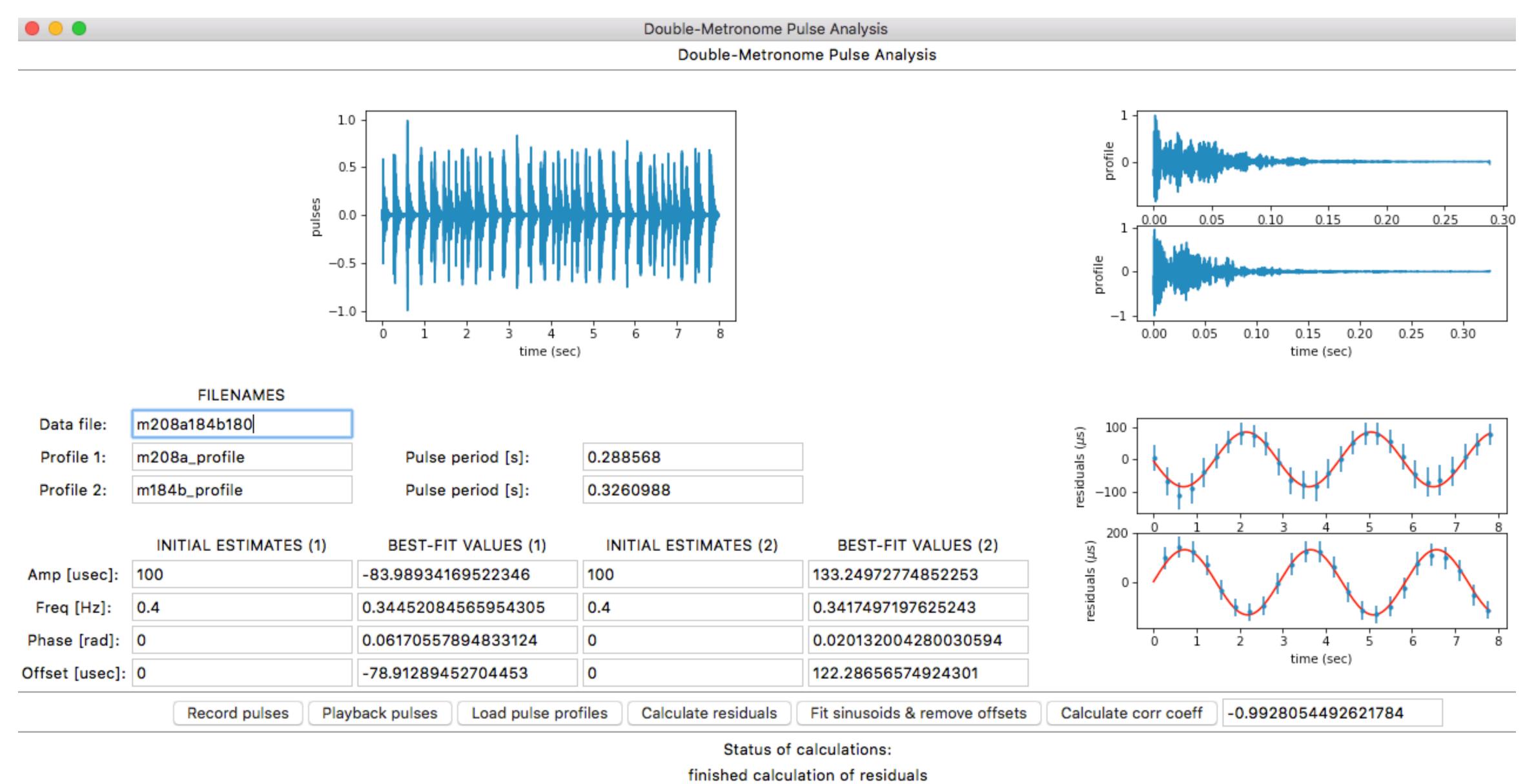
finished calculation of residuals

Double-metronome analysis (ζ =135 degrees)

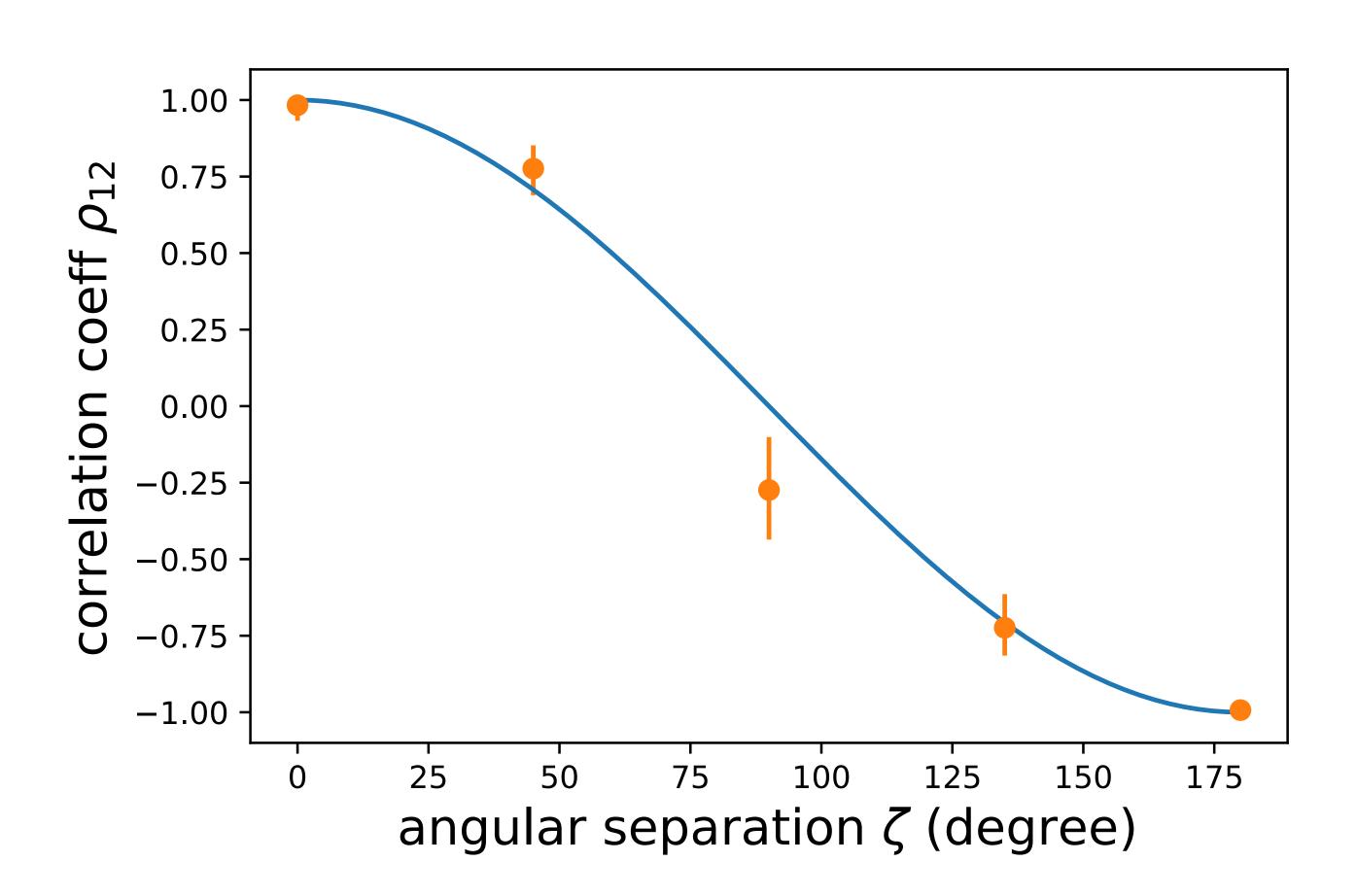


Status of calculations: finished calculation of residuals

Double-metronome analysis (ζ =180 degrees)

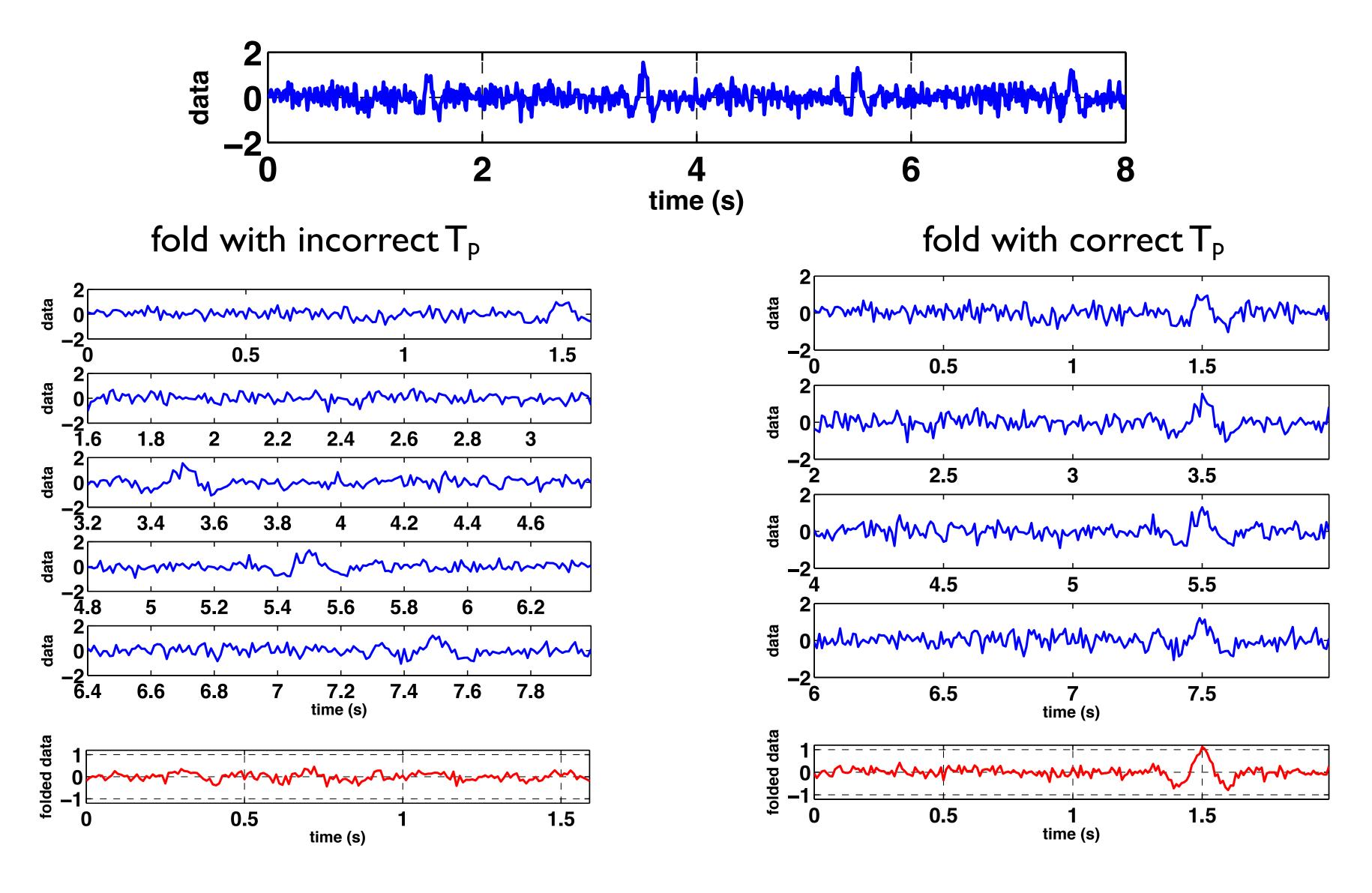


Metronome correlation



More details

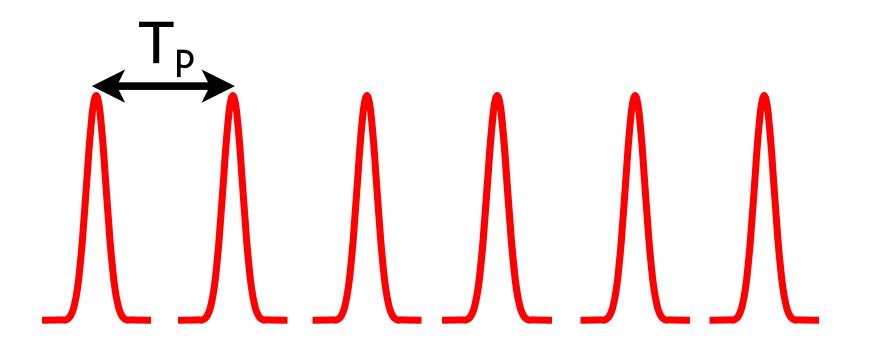
"Fold" data to determine pulse period and pulse profile



Timing model

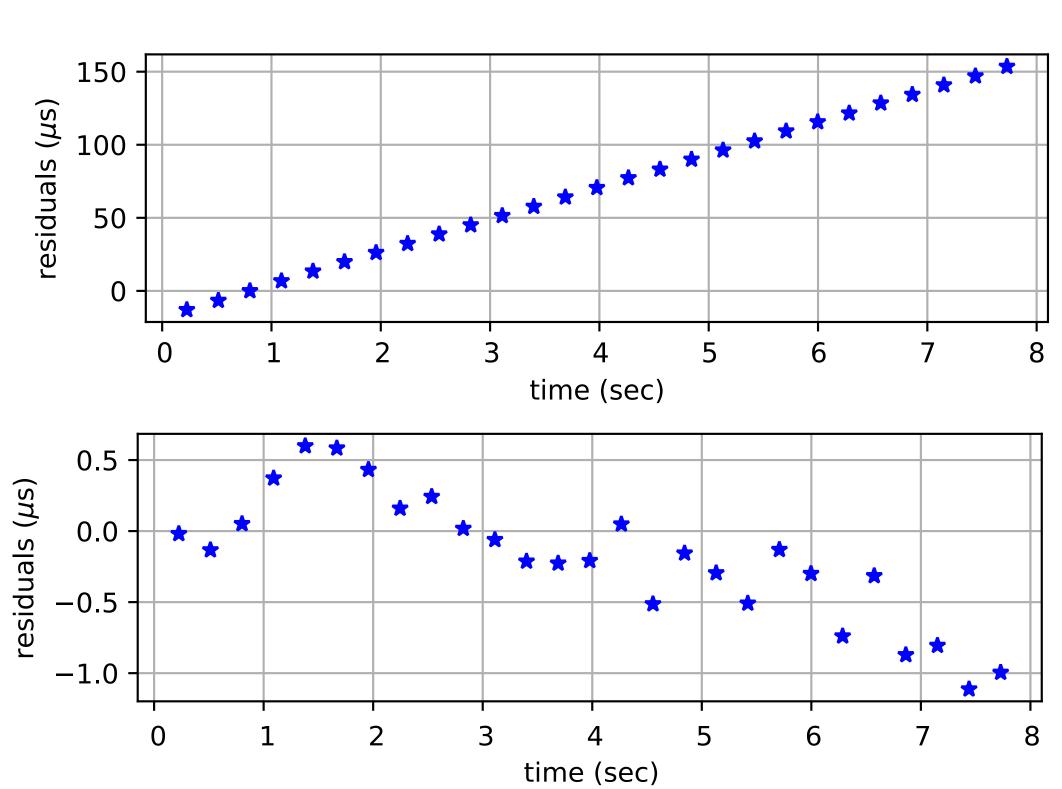
Pulses should arrive regularly with period T_p relative to some reference pulse

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$



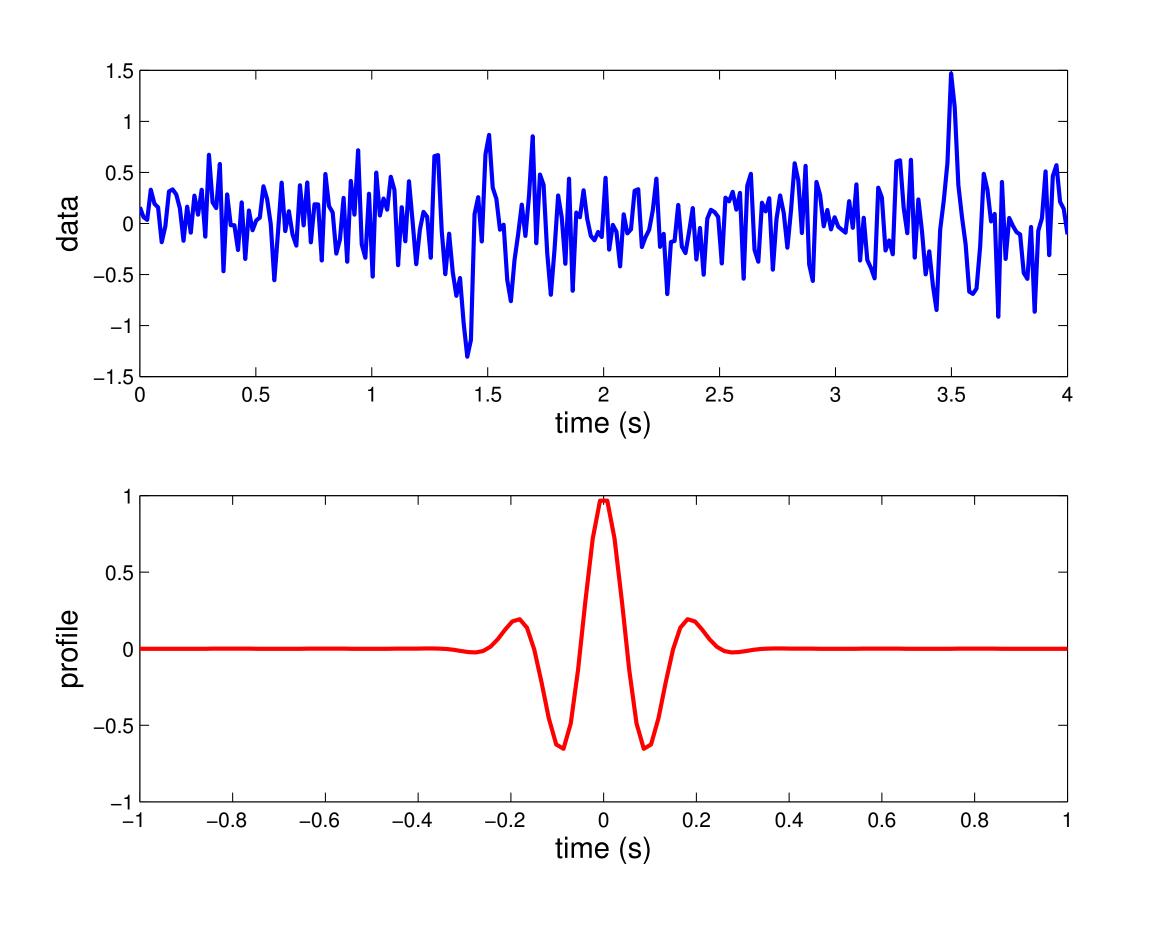
Remove linear trend to more accurately determine pulse period

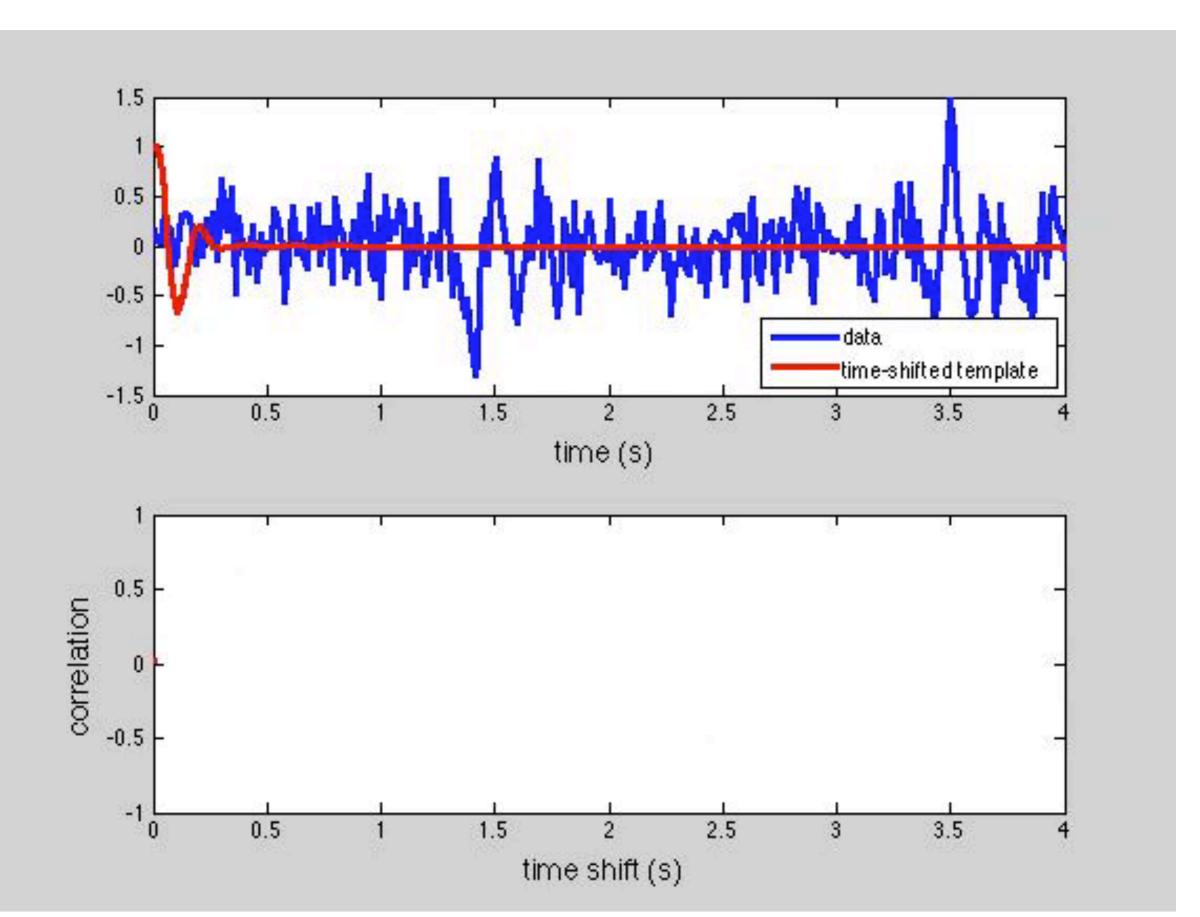
$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$



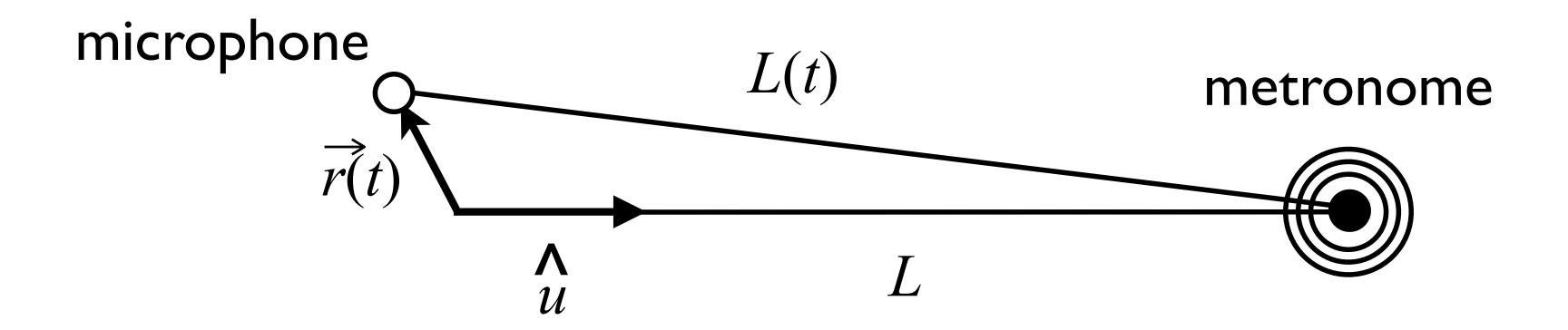
Matched-filtering determination of measured TOAs

$$C(\Delta t) = \mathcal{N} \int dt \ y(t) p(t - \Delta t)$$



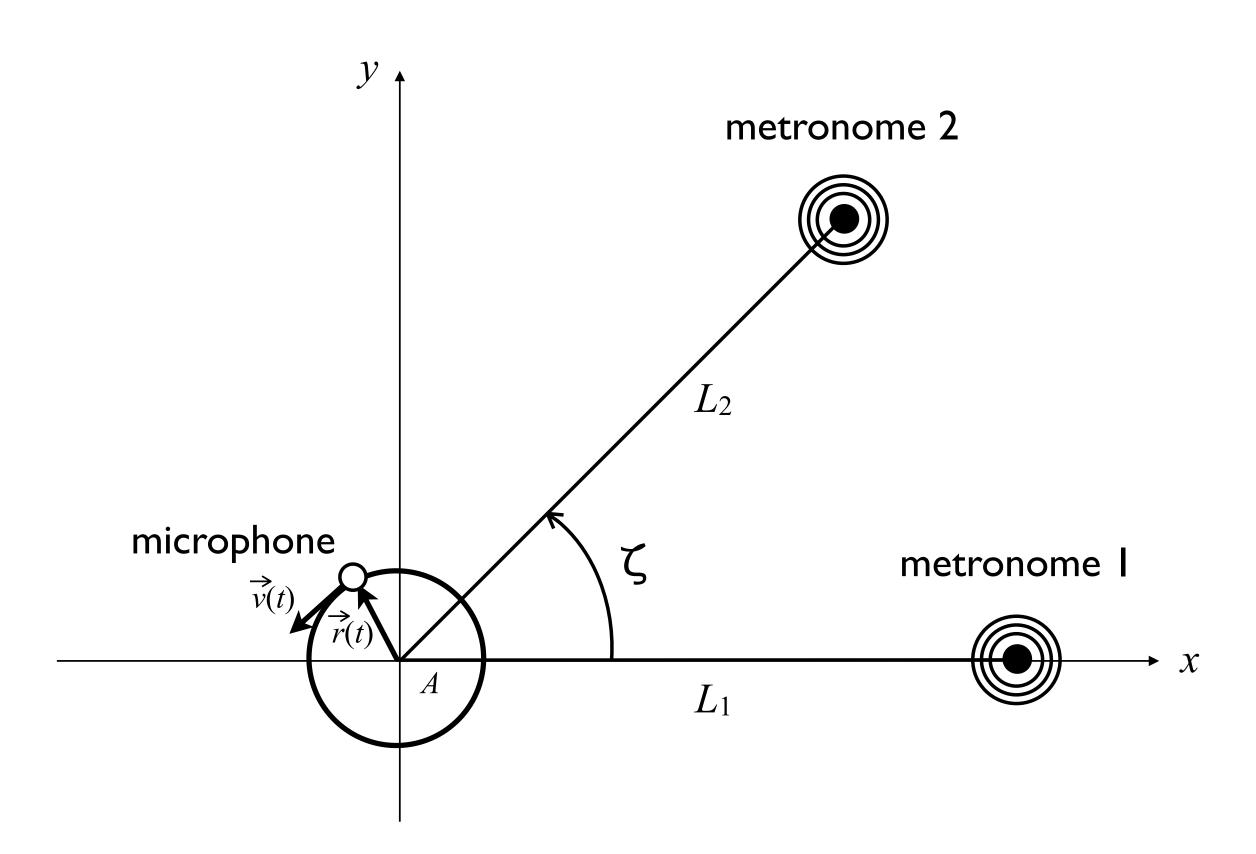


Timing-residual response to microphone motion



$$\delta \tau(t) = \frac{\Delta L(t)}{c_{\rm s}} \simeq -\frac{1}{c_{\rm s}} \hat{u} \cdot \vec{r}(t)$$

Two metronomes - uniform circular motion

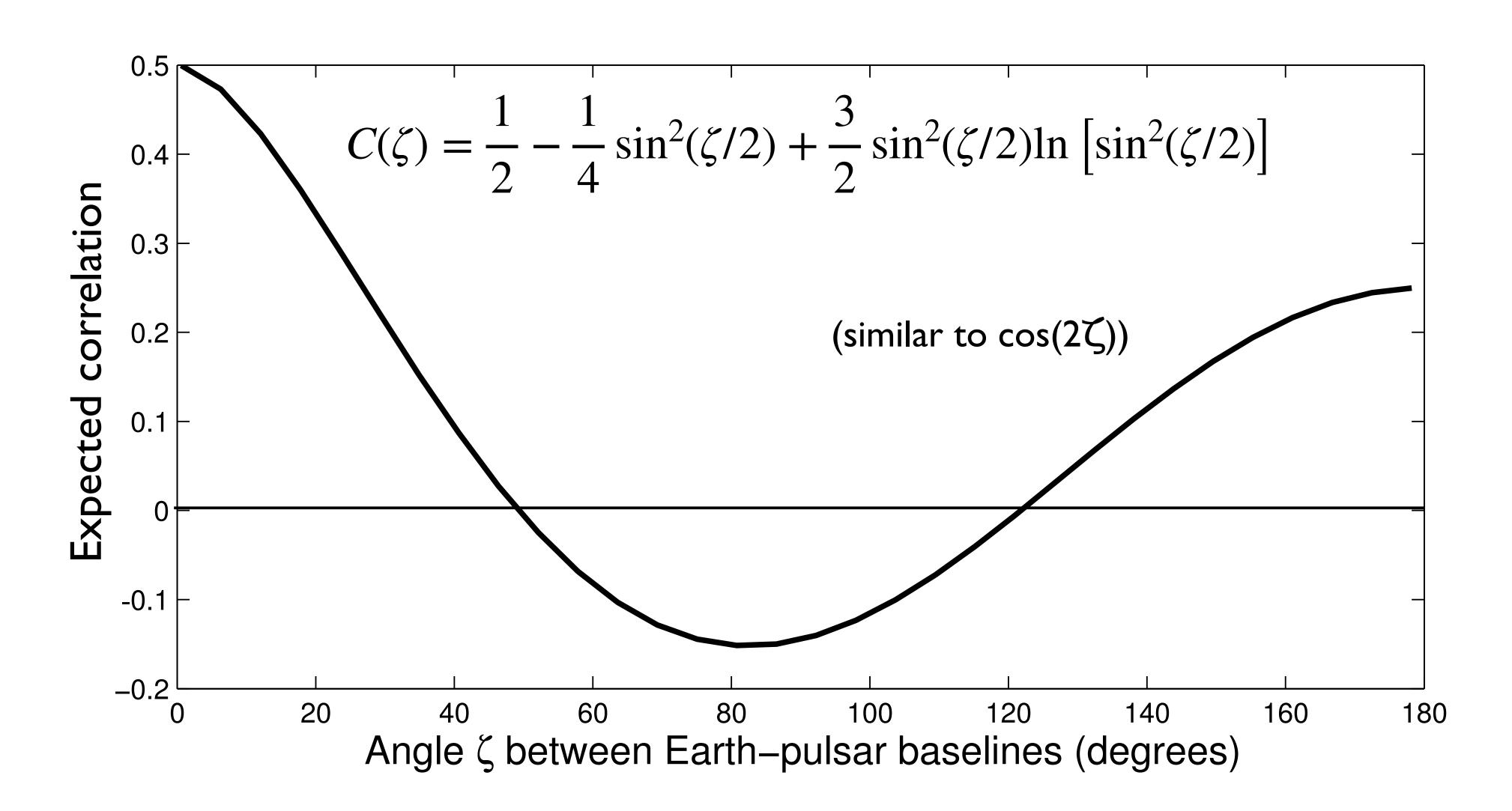


$$\vec{r}(t) = A \left[\cos(2\pi f_0 t + \phi_0)\hat{x} + \sin(2\pi f_0 t + \phi_0)\hat{y} \right]$$

$$\delta \tau_I(t) \simeq -\frac{A}{c_s} \cos(2\pi f_0 t + \phi_0 - \theta_I), \quad I = 1,2$$

$$\rho_{12} \simeq \cos \zeta, \quad \zeta \equiv \theta_1 - \theta_2$$

Expected PTA correlation - Hellings & Downs curve (isotropic, unpolarized GW background)



Metronome demo numbers

```
c_s = 340 \text{ m/s (in air)}
```

amplitude ≈ 5 cm

amplitude / $c_s = 1 \times 10^{-4} \text{ sec}$

184 bpm: $T_p = 0.3261 \text{ sec}$

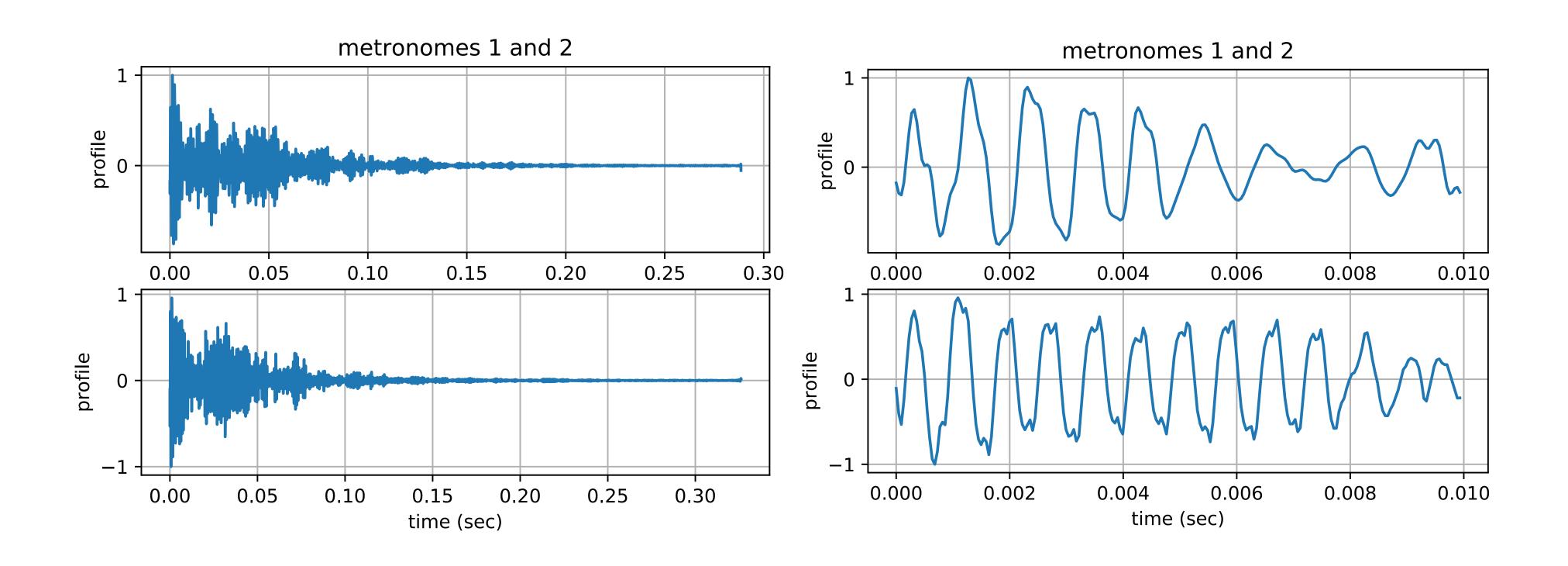
208 bpm: $T_p = 0.2885$ sec

Pulsar timing numbers

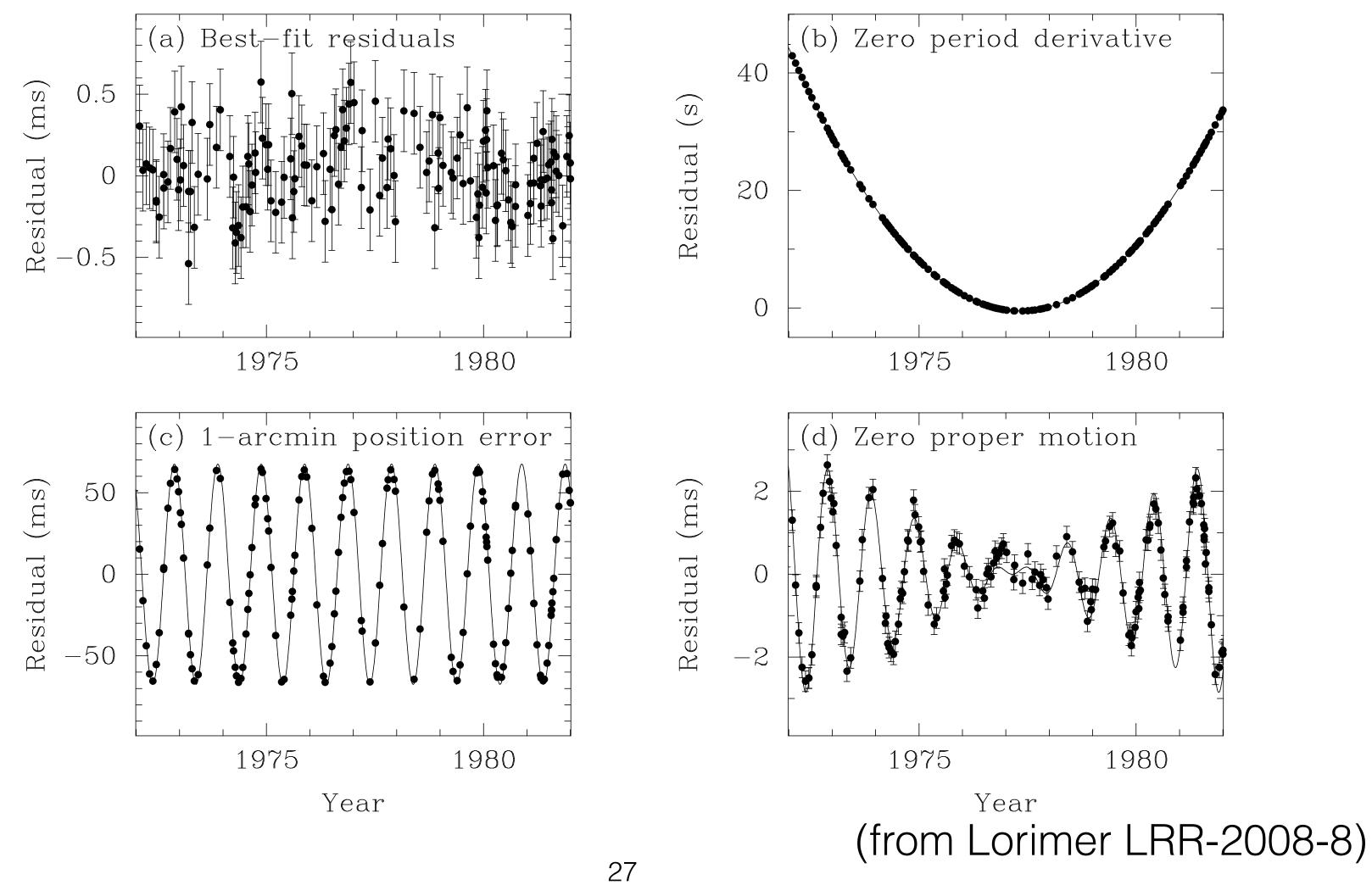
```
f ~ 1/few weeks to 1/10 years (10-7 Hz to 10-9 Hz) \lambda ~ 0.1 to 10 lyr (GW wavelength) L ~ few x 1000 lyr (distance to pulsars) \lambda << L (short-wavelength limit) sensitivity ~ \sigma_{rms}/T_{obs} ~ 100 ns/10 yr ~ 10-15
```

can detect changes ~10 km in the position of a pulsar at a distance of ~1000 lyr

Metronome pulse profiles



Errors in the timing model show up as deterministic features in the timing residuals



Pulsar timing array

GWs cause pulses to arrive ahead or behind schedule, correlated across pulsars

pulsar

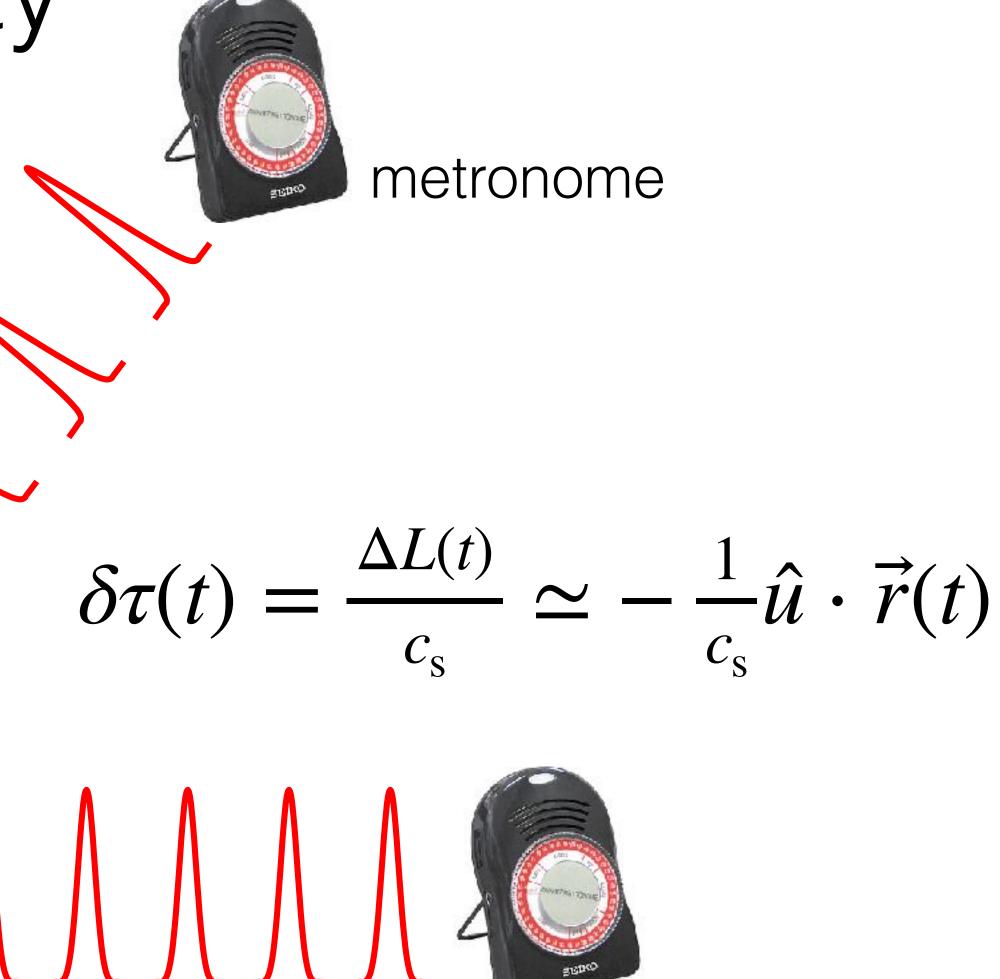
$$\delta \tau(t) = \frac{1}{c} u^a u^b \int ds \ h_{ab}(t(s), \overrightarrow{x}(s))$$

radio telescope

pulsar

Metronome timing array

microphone motion causes pulses to arrive ahead or behind schedule, correlated across metronomes



metronome