Metronome-microphone demo of a pulsar timing array

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(work in collaboration with M. Lam, M. Normandin, J. Key, and J. Hazboun)



Purpose

- · Illustrate how a PTA is used to search for GWs in the context of a simple acoustical model
- Become familiar with techniques used by pulsar astronomers:
 - folding (for calculating pulse periods and pulse profiles)
 - detrending (for better estimating pulse period)
 - matched filtering (for calculating measured times-of-arrivals)
 - timing models (for calculating expected times-of-arrivals)
 - correlation analyses (for extracting common GW component)

https://github.com/josephromano/pta-demo



GWs cause pulses to arrive ahead or behind schedule, correlated across pulsars

pulsar

$$\delta \tau(t) = \frac{1}{c} u^a u^b \int h_{ab}(t(s), \overrightarrow{x}(s)) ds$$

radio telescope

pulsar

Metronome timing array

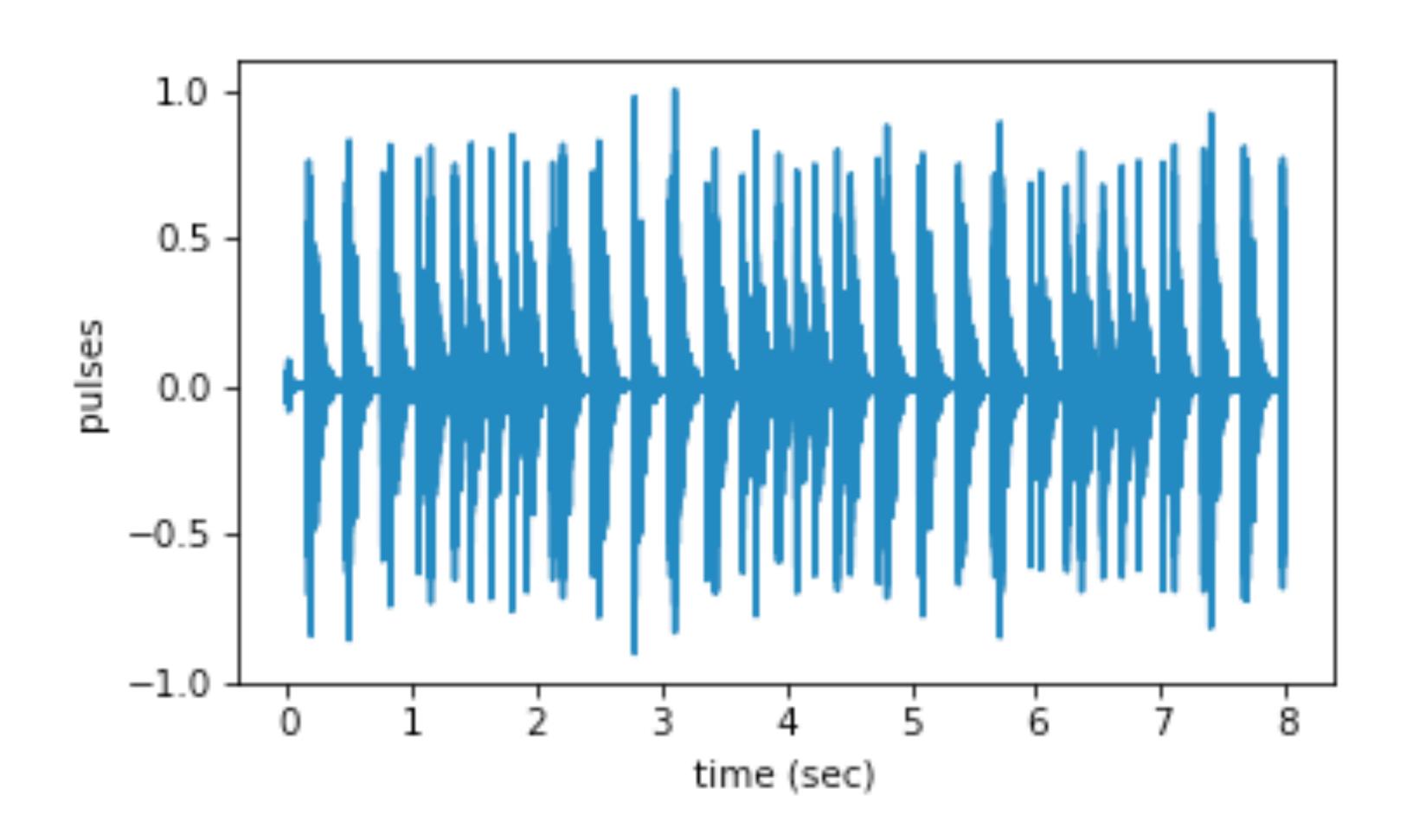
microphone motion causes pulses to arrive ahead or behind schedule, correlated across metronomes



metronome

$$\delta \tau(t) = \frac{\Delta L(t)}{c_{\rm s}} \simeq -$$

Q: Is there evidence of a "GW" in the data?



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Is there a common disturbance to the pulse arrival times (TOAs), and if so, is this disturbance correlated across metronomes as expected for a "gravitational wave" (i.e., microphone motion)?

- disturbance = timing residuals = measured TOAs expected TOAs
- common = correlated

$$\rho_{12} \equiv \langle x_1 x_2 \rangle / \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \qquad \langle x_1 x_2 \rangle \equiv \frac{1}{T_{\text{obs}}} \int_0^{T_{\text{obs}}} dt \, x_1(t) x_2(t)$$

measured TOAs: calculated using matched filtering with pulse profile

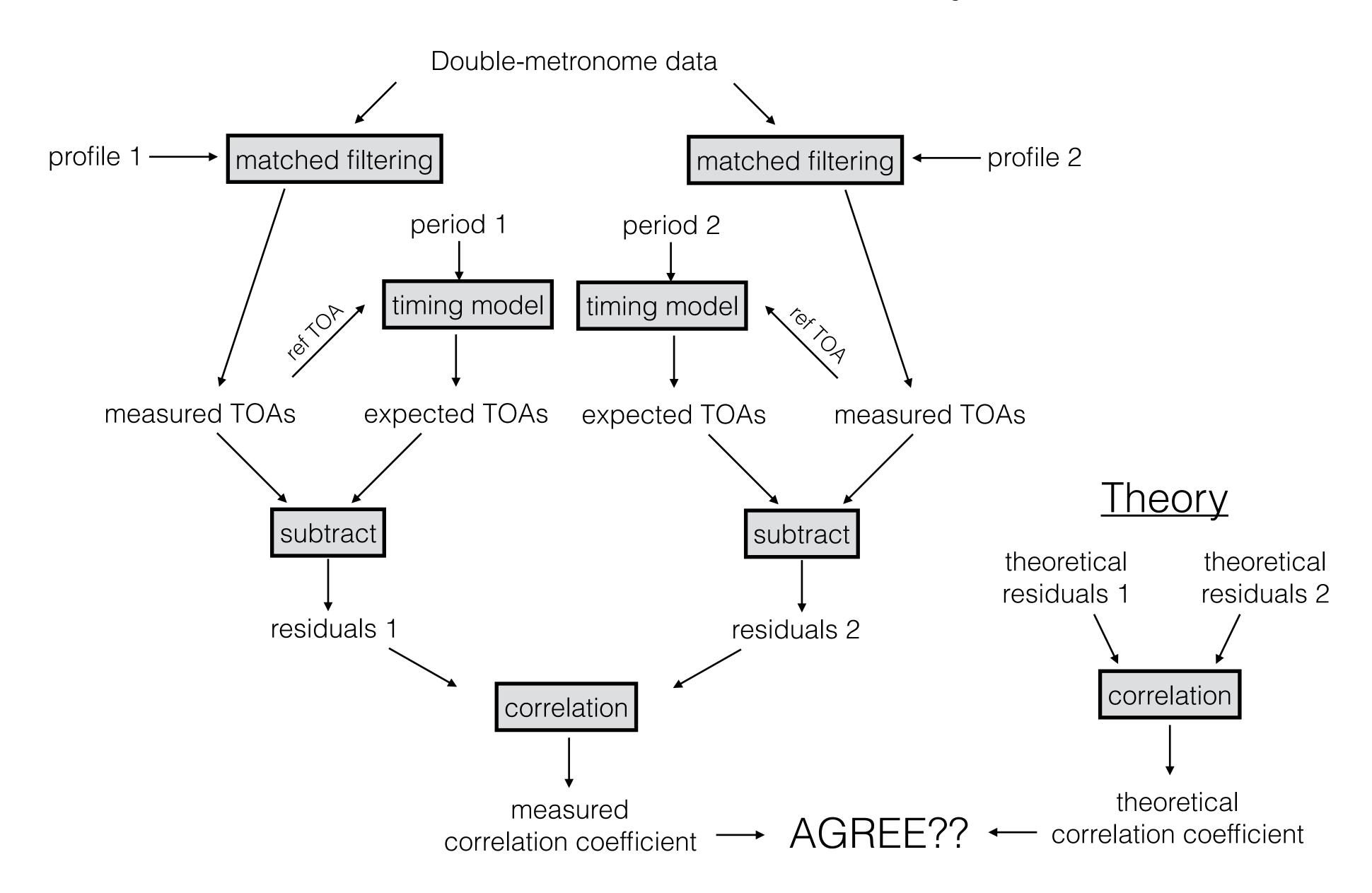
$$C(\Delta t) = \mathcal{N} \int dt \ y(t)p(t - \Delta t)$$

expected TOAs: timing model using pulse period and reference TOA

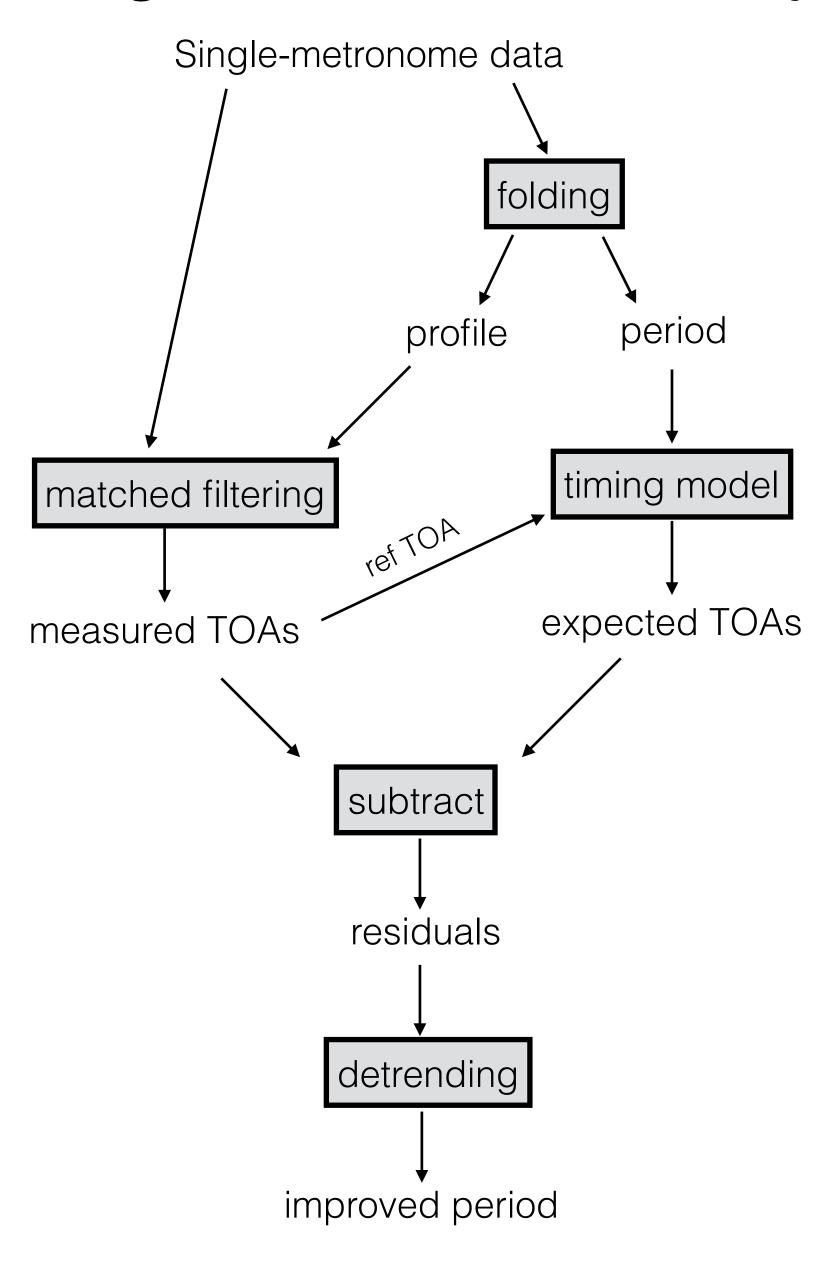
$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$

- pulse profile, period: folding and detrending single-metronome data
- expected "GW" correlation (for uniform circular motion): $\rho_{12} \simeq \cos \zeta$

Double-metronome analysis



Single-metronome analysis

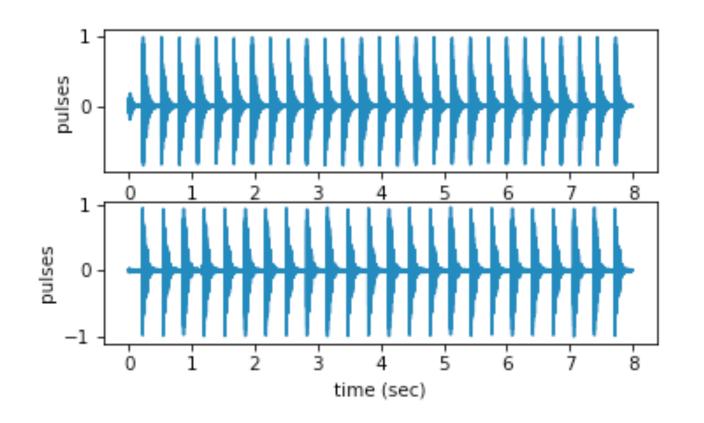


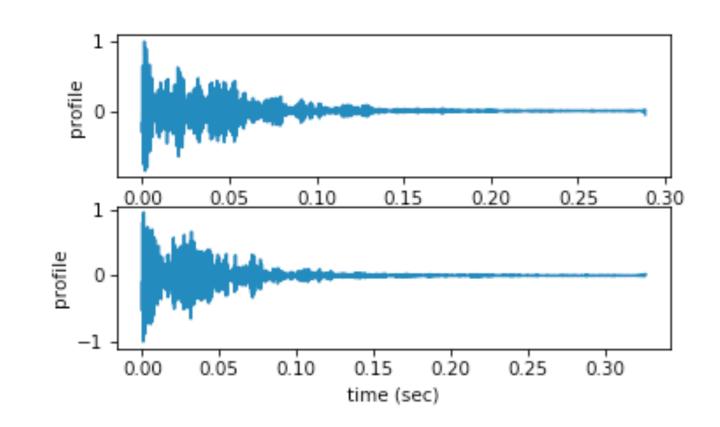
Output of the GUIs

Single-metronome analysis



Single-Metronome Pulse Analysis

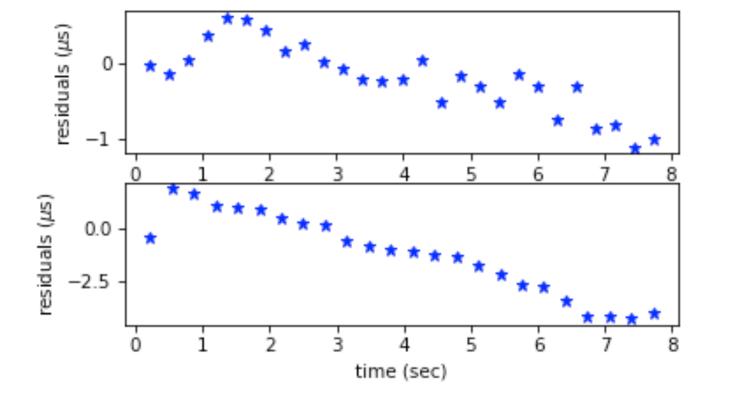




PULSE DATA FILENAMES

 Metronome 1:
 m208a
 bpm:
 208

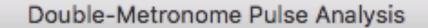
 Metronome 2:
 m184b
 bpm:
 184



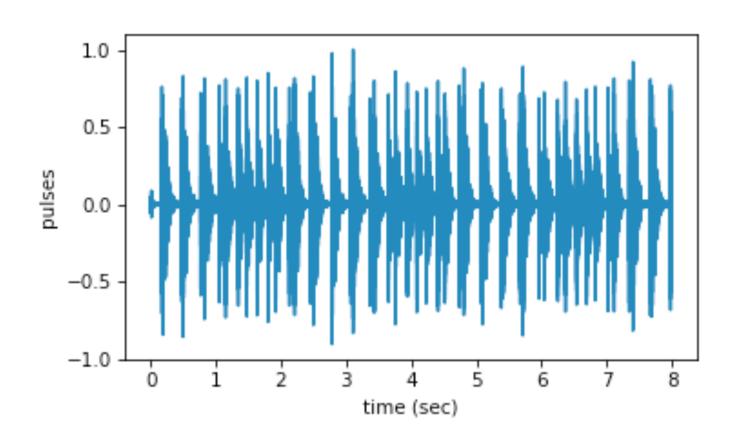
Metronome 1: Record pulses Playback pulses Calculate profile Pulse period [s]: 0.28856797860073924 Calculate residuals Detrend residuals

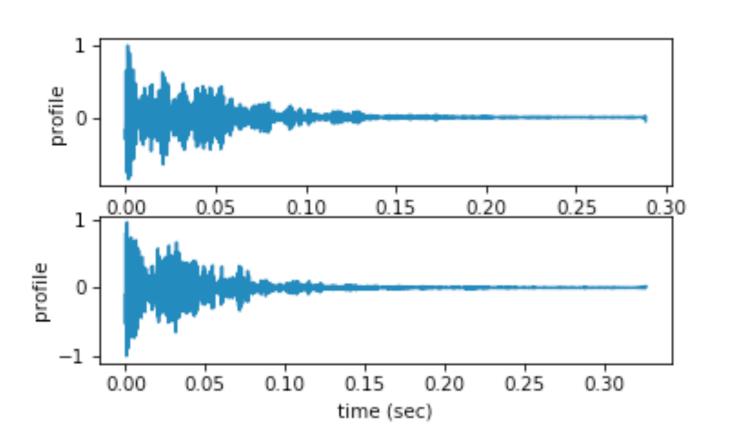
Metronome 2: Record pulses Playback pulses Calculate profile Pulse period [s]: 0.3260988289104562 Calculate residuals Detrend residuals

Double-metronome analysis (ζ =0 degrees)



Double-Metronome Pulse Analysis





FILENAMES

INITIAL ESTIMATES (1)

Data file: m208a184b0

Profile 1: m208a_profile

Profile 2:

m184b_profile

Pulse period [s]:

BEST-FIT VALUES (1)

0.288568

Pulse period [s]: 0.3260988

residuals (µs)	444	
- 100 - 001 - 001 - 001 - 001	0 1 2 3 4 5 6 7 8	8
	0 1 2 3 4 5 6 7 8 time (sec)	8

Amp [usec]:	100	-58.34831246716732	100	-67.83561046171864
Freq [Hz]:	0.4	0.33072788592748953	0.4	0.33116191414239915
Phase [rad]:	0	0.5924453954483324	0	0.393928097921481
Offset [usec]:	0	-43.4892386052206	0	1.7106663283346102

Record pulses

Playback pulses

Load pulse profiles

Calculate residuals

INITIAL ESTIMATES (2)

Fit sinusoids & remove offsets

BEST-FIT VALUES (2)

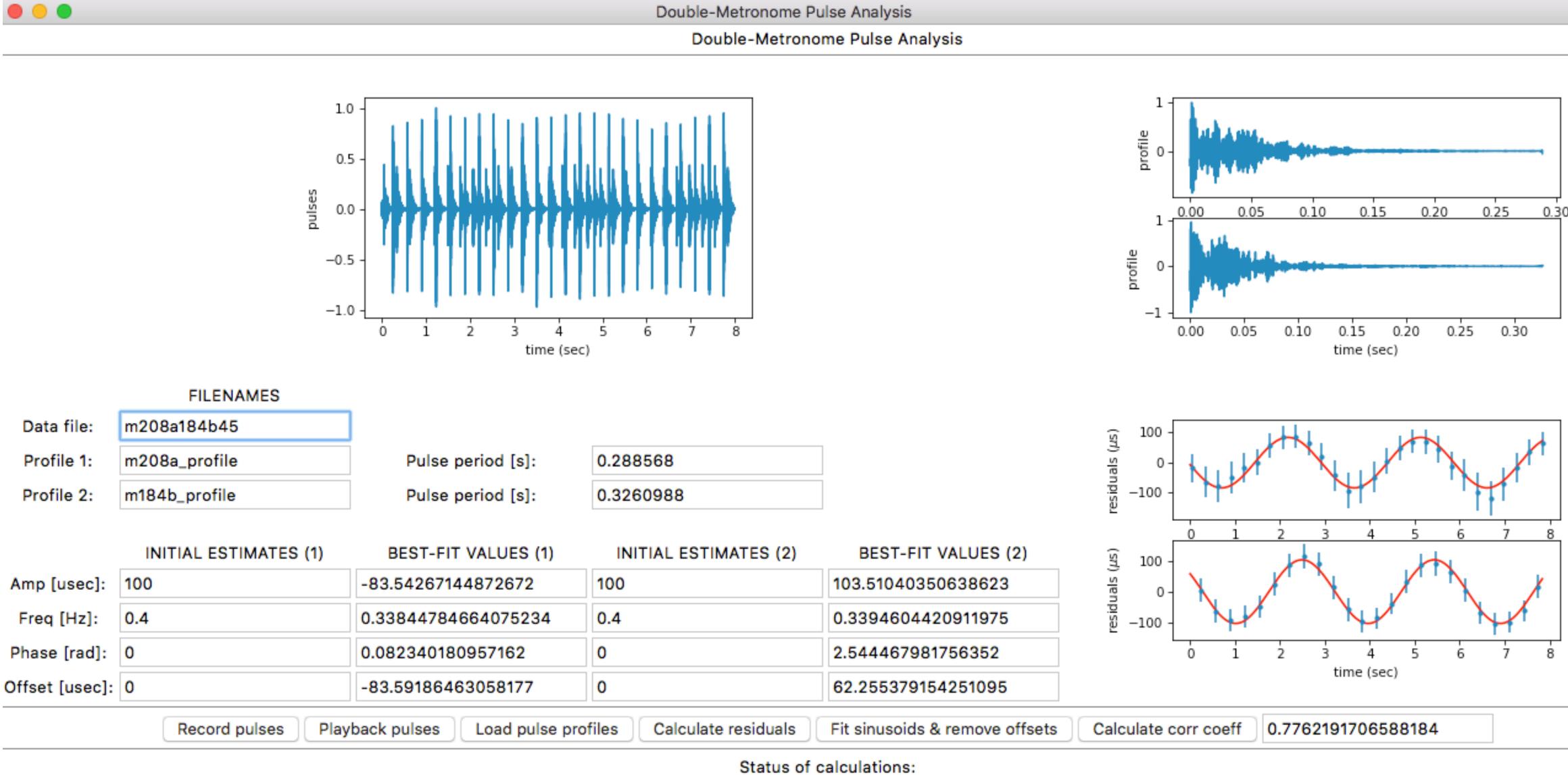
Calculate corr coeff

0.9830009544088217

Status of calculations:

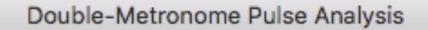
finished calculation of residuals

Double-metronome analysis (ζ =45 degrees)

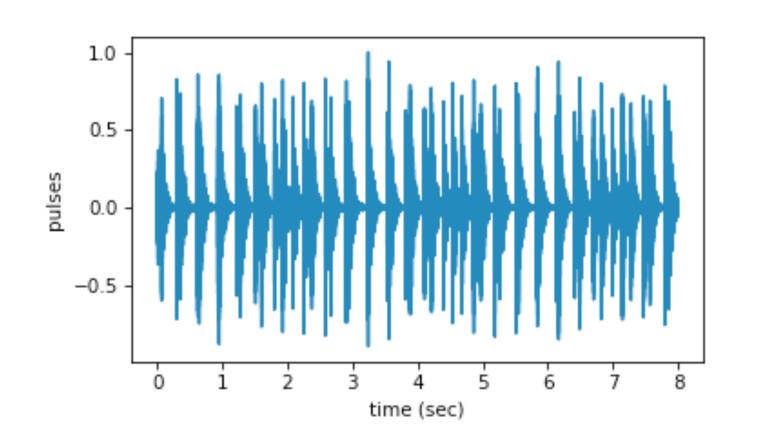


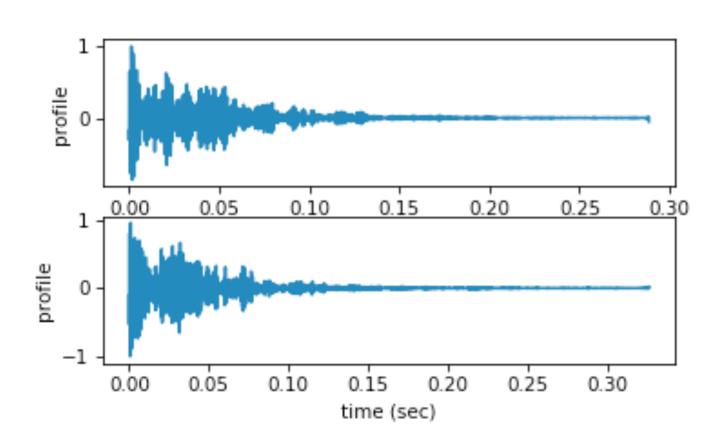
finished calculation of residuals

Double-metronome analysis (ζ =90 degrees)



Double-Metronome Pulse Analysis





FILENAMES

Data file:

m208a184b90

Profile 1:

Profile 2:

m208a_profile

m184b_profile

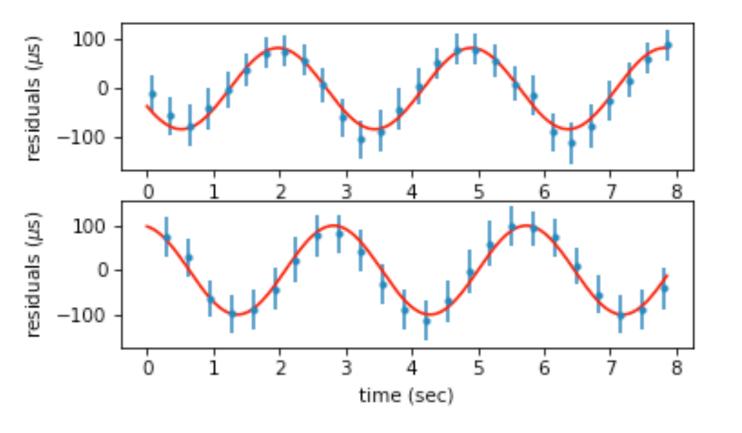
Pulse period [s]:

0.288568

Pulse period [s]:

0.3260988

	INITIAL ESTIMATES (1)	BEST-FIT VALUES (1)	INITIAL ESTIMATES (2)	BEST-FIT VALUES (2)
Amp [usec]:	100	-83.2204679782364	100	100.56846337609925
Freq [Hz]:	0.4	0.34235401761876616	0.4	0.3435231659876081
Phase [rad]:	0	0.45096522793443056	0	1.7673131445900625
Offset [usec]:	0	-88.73206094670937	0	-95.5978511142905



Record pulses

Playback pulses

Load pulse profiles

Calculate residuals

Fit sinusoids & remove offsets

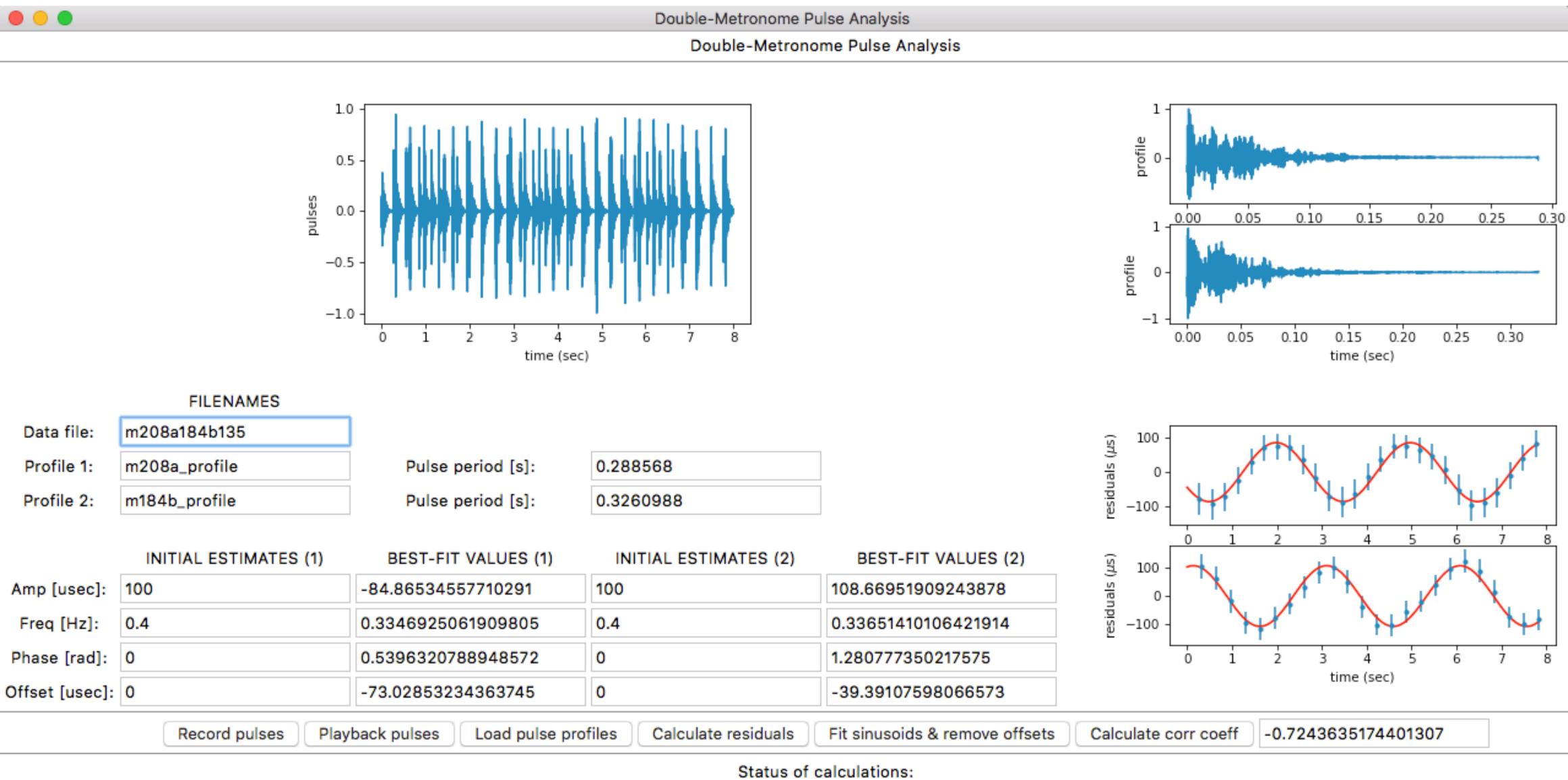
Calculate corr coeff

-0.27377904355571664

Status of calculations:

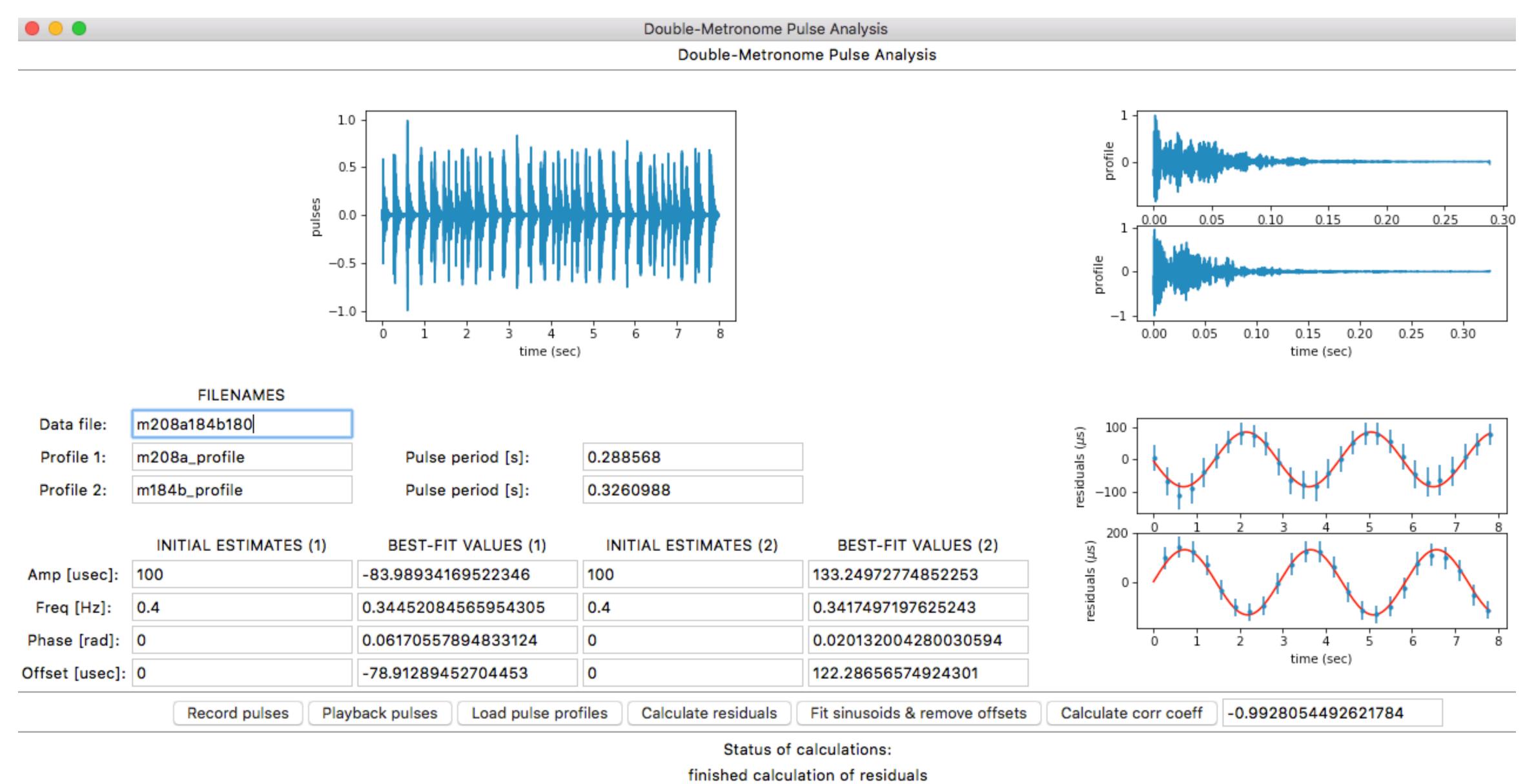
finished calculation of residuals

Double-metronome analysis (ζ =135 degrees)

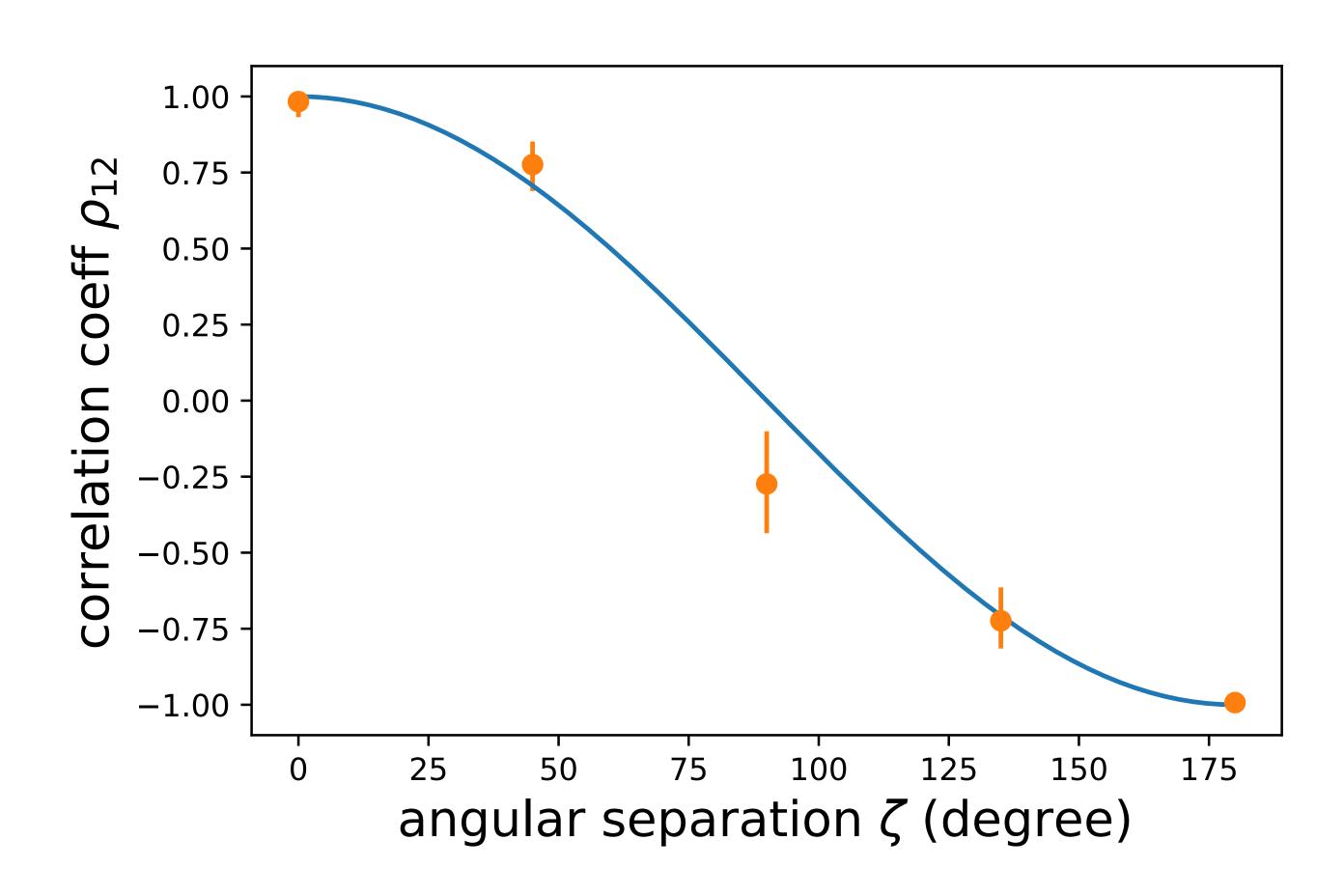


Status of calculations: finished calculation of residuals

Double-metronome analysis (ζ =180 degrees)

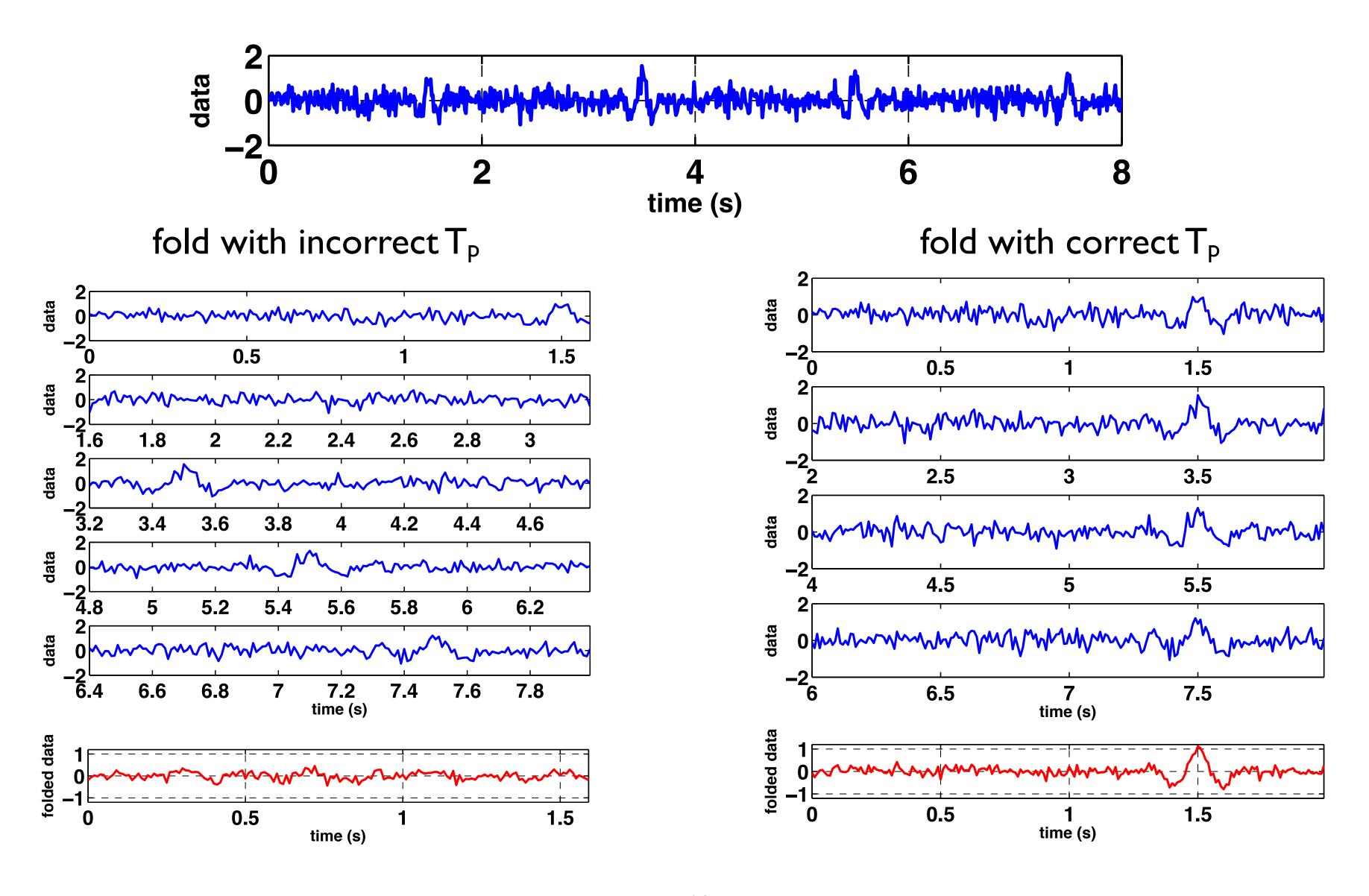


Metronome correlation



More details

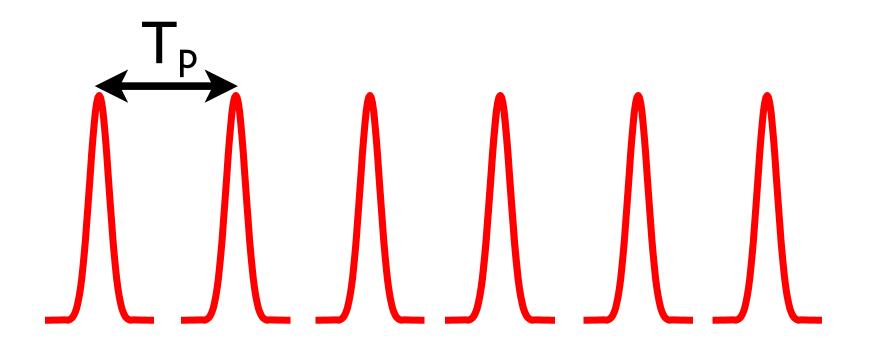
"Fold" data to determine pulse period and pulse profile



Timing model

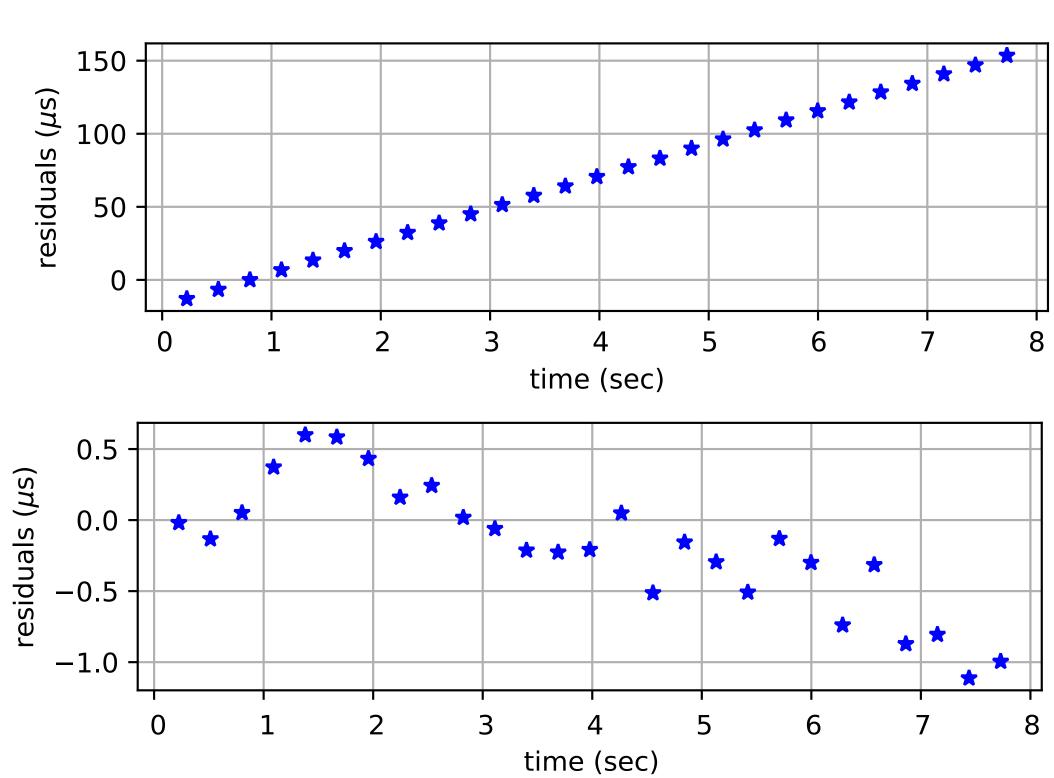
Pulses should arrive regularly with period T_p relative to some reference pulse

$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$



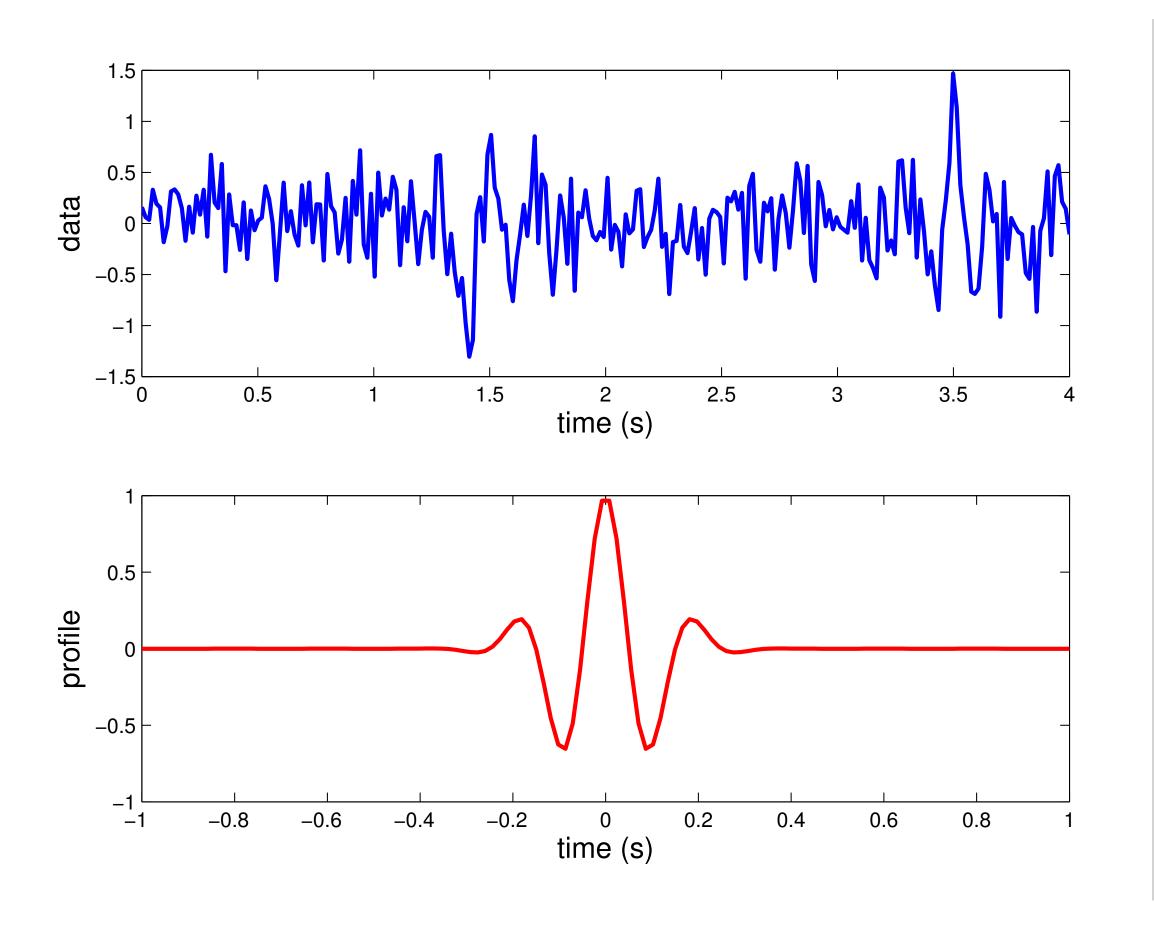
Remove linear trend to more accurately determine pulse period

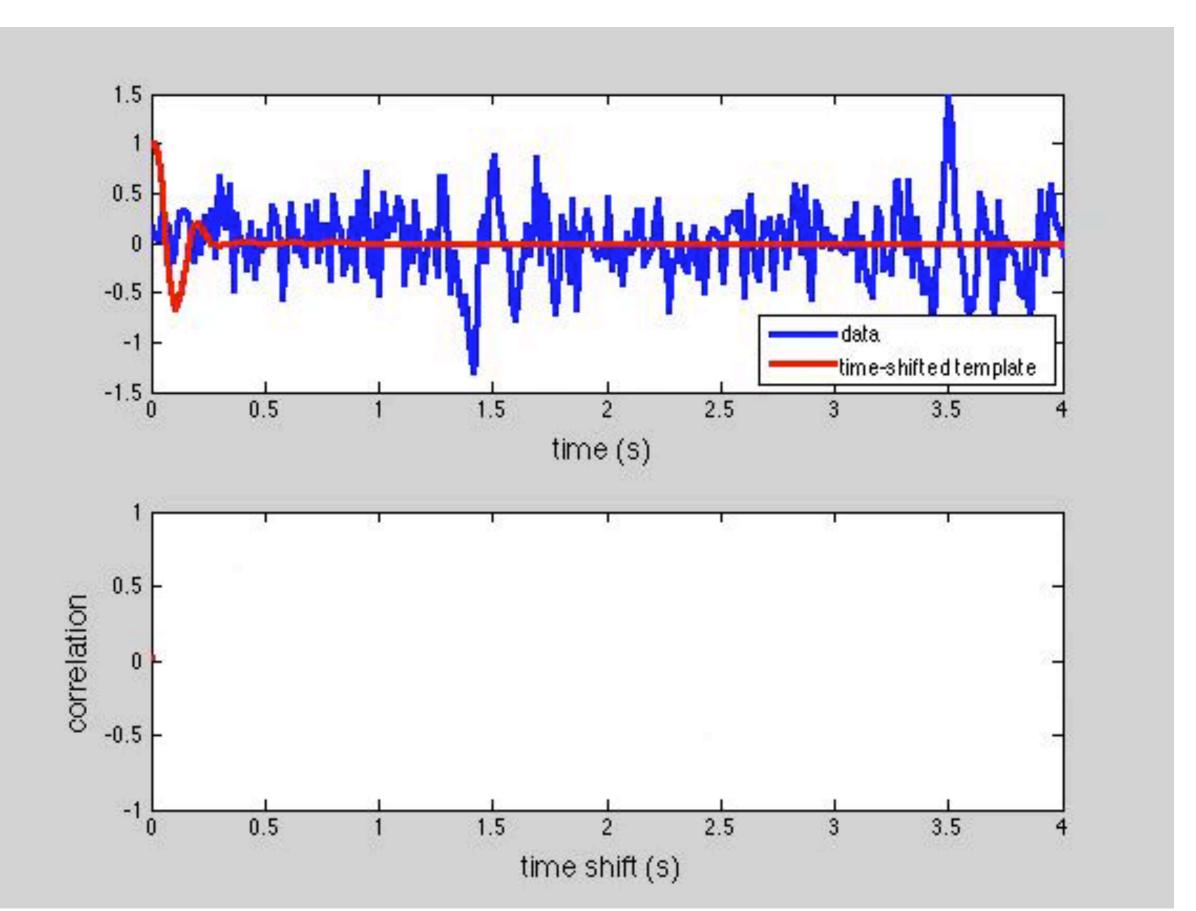
$$\tau^{\text{expected}}[i] = \tau^{\text{measured}}[i_0] + (i - i_0)T_{\text{p}}$$



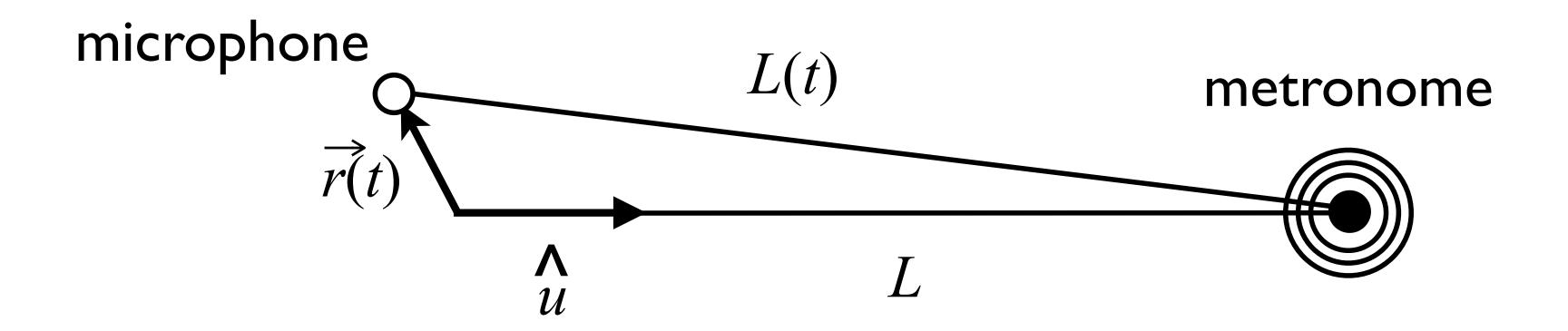
Matched-filtering determination of measured TOAs

$$C(\Delta t) = \mathcal{N} \int dt \ y(t) p(t - \Delta t)$$



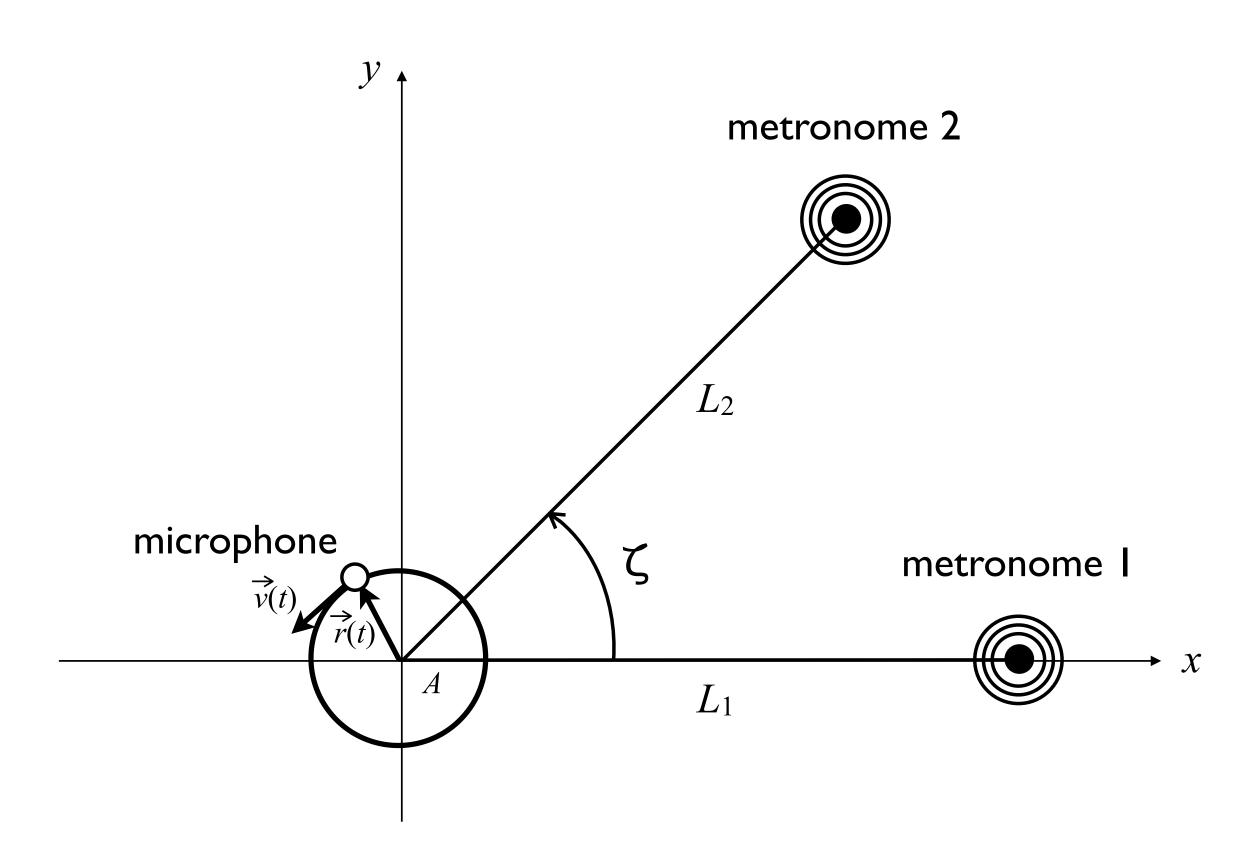


Timing-residual response to microphone motion



$$\delta \tau(t) = \frac{\Delta L(t)}{c_{\rm s}} \simeq -\frac{1}{c_{\rm s}} \hat{u} \cdot \vec{r}(t)$$

Two metronomes - uniform circular motion

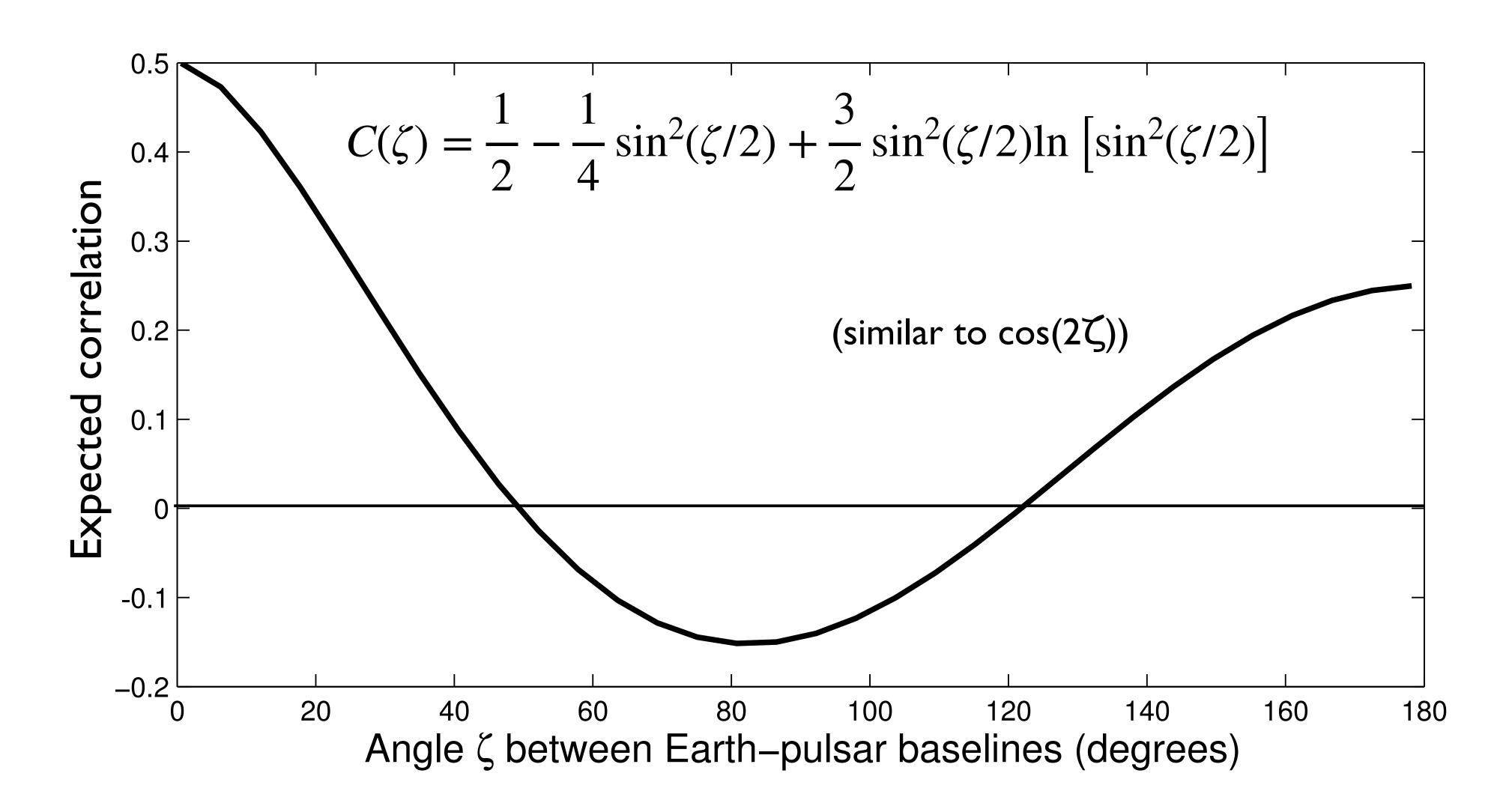


$$\vec{r}(t) = A \left[\cos(2\pi f_0 t + \phi_0)\hat{x} + \sin(2\pi f_0 t + \phi_0)\hat{y} \right]$$

$$\delta \tau_I(t) \simeq -\frac{A}{c_s} \cos(2\pi f_0 t + \phi_0 - \theta_I), \quad I = 1,2$$

$$\rho_{12} \simeq \cos \zeta, \quad \zeta \equiv \theta_1 - \theta_2$$

Expected PTA correlation - Hellings & Downs curve (isotropic, unpolarized GW background)



Metronome demo numbers

```
c_s = 340 \text{ m/s (in air)}
```

amplitude ≈ 5 cm

amplitude / $c_s = 1 \times 10^{-4} \text{ sec}$

184 bpm: $T_p = 0.3261 \text{ sec}$

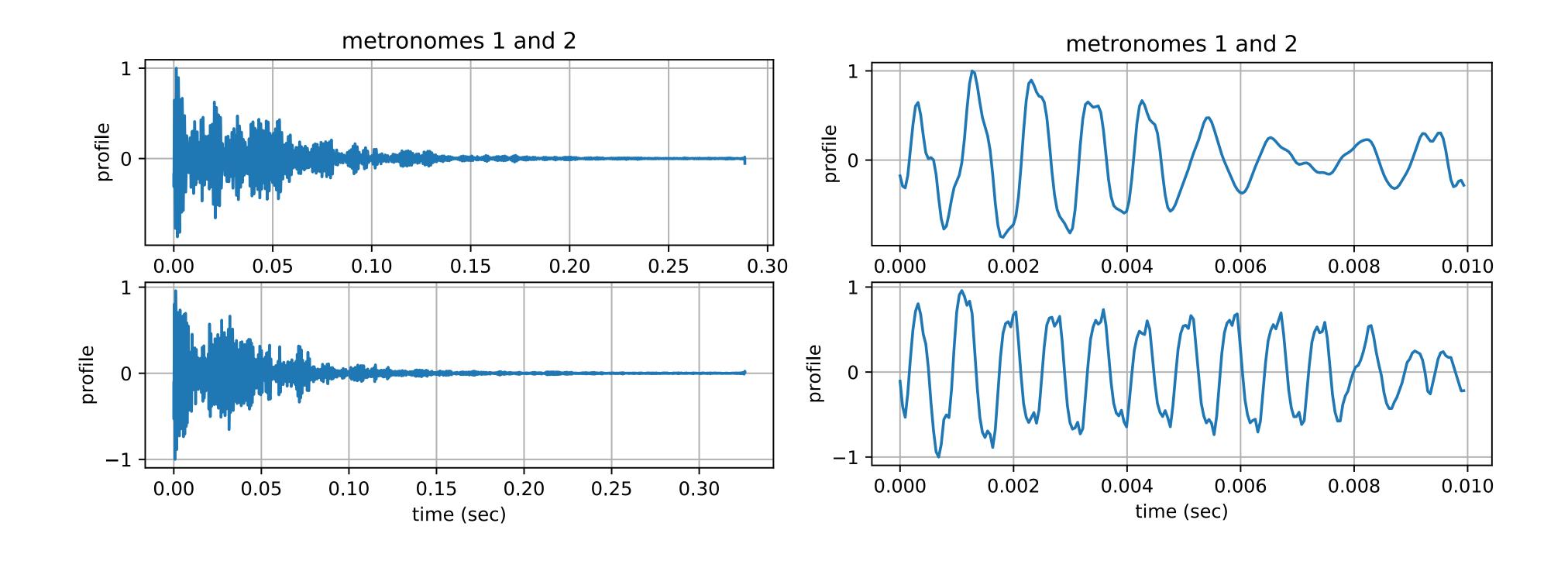
208 bpm: $T_p = 0.2885$ sec

Pulsar timing numbers

```
f ~ 1/few weeks to 1/10 years (10-7 Hz to 10-9 Hz) \lambda ~ 0.1 to 10 lyr (GW wavelength) L ~ few x 1000 lyr (distance to pulsars) \lambda << L (short-wavelength limit) sensitivity ~ \sigma_{rms}/T_{obs} ~ 100 ns/10 yr ~ 10-15
```

can detect changes ~10 km in the position of a pulsar at a distance of ~1000 lyr

Metronome pulse profiles



Errors in the timing model show up as deterministic features in the timing residuals

