Metronome timing array:

(a metronome-microphone demo of a pulsar timing array)

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Les Houches Summer School
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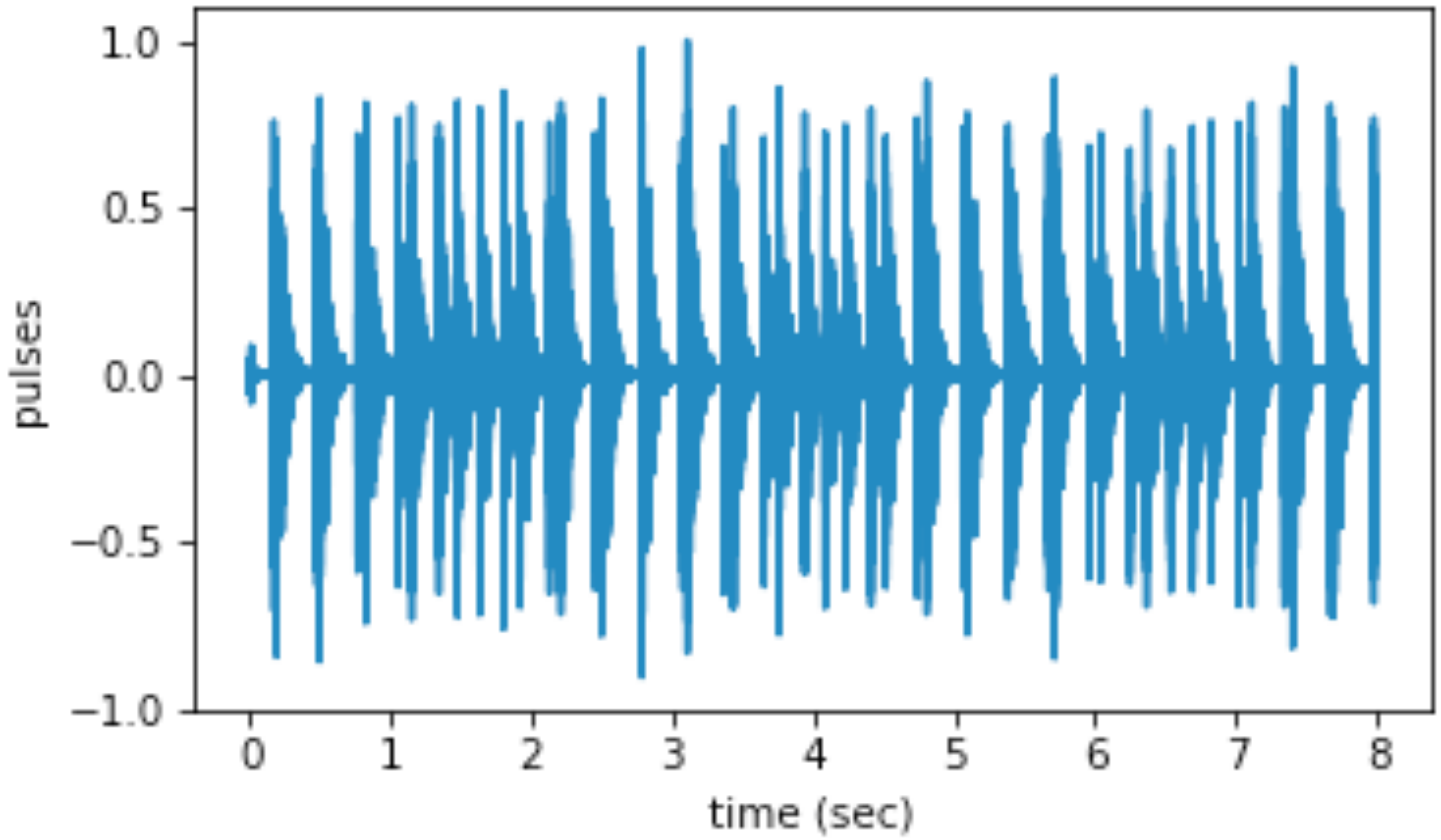
(work in collaboration with M. Lam, M. Normandin, J. Key, and J. Hazboun)

code and sample data: https://github.com/josephromano/leshouches/tree/master/pta-demo
paper (draft): https://github.com/josephromano/leshouches/blob/master/pta-demo/manuscript/pta-demo.pdf

Purpose of demo

- Illustrate how a PTA is used to search for GWs in the context of a simple acoustical model
- Introduce concepts / techniques used by real pulsar astronomers:
    - folding (for calculating pulse periods and pulse profiles)
    - detrending (for better estimating pulse period)
    - matched filtering (for calculating measured times-of-arrivals)
    - timing model (for calculating expected times-of-arrivals)
    - correlation analyses (for extracting common GW component)

Q: Is there evidence of a “GW” in the data?



Q: Is there a common disturbance to the pulse arrival times (TOAs), and if so, is this disturbance correlated across metronomes as expected for a "GW" (i.e., microphone motion)?

- disturbance = measured TOAs - expected TOAs = timing residuals
- common = correlation between the timing residuals

$$\rho_{12} \equiv \langle x_1 x_2 \rangle / \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \qquad \langle x_1 x_2 \rangle \equiv \frac{1}{T_{\rm obs}} \int_0^{T_{\rm obs}} {\rm d}t\, x_1(t) x_2(t)$$

- measured TOAs: obtained using matched filtering with pulse profile

$$C(\Delta t) = \mathcal{N} \int {\rm d}t\, y(t) p(t - \Delta t)$$

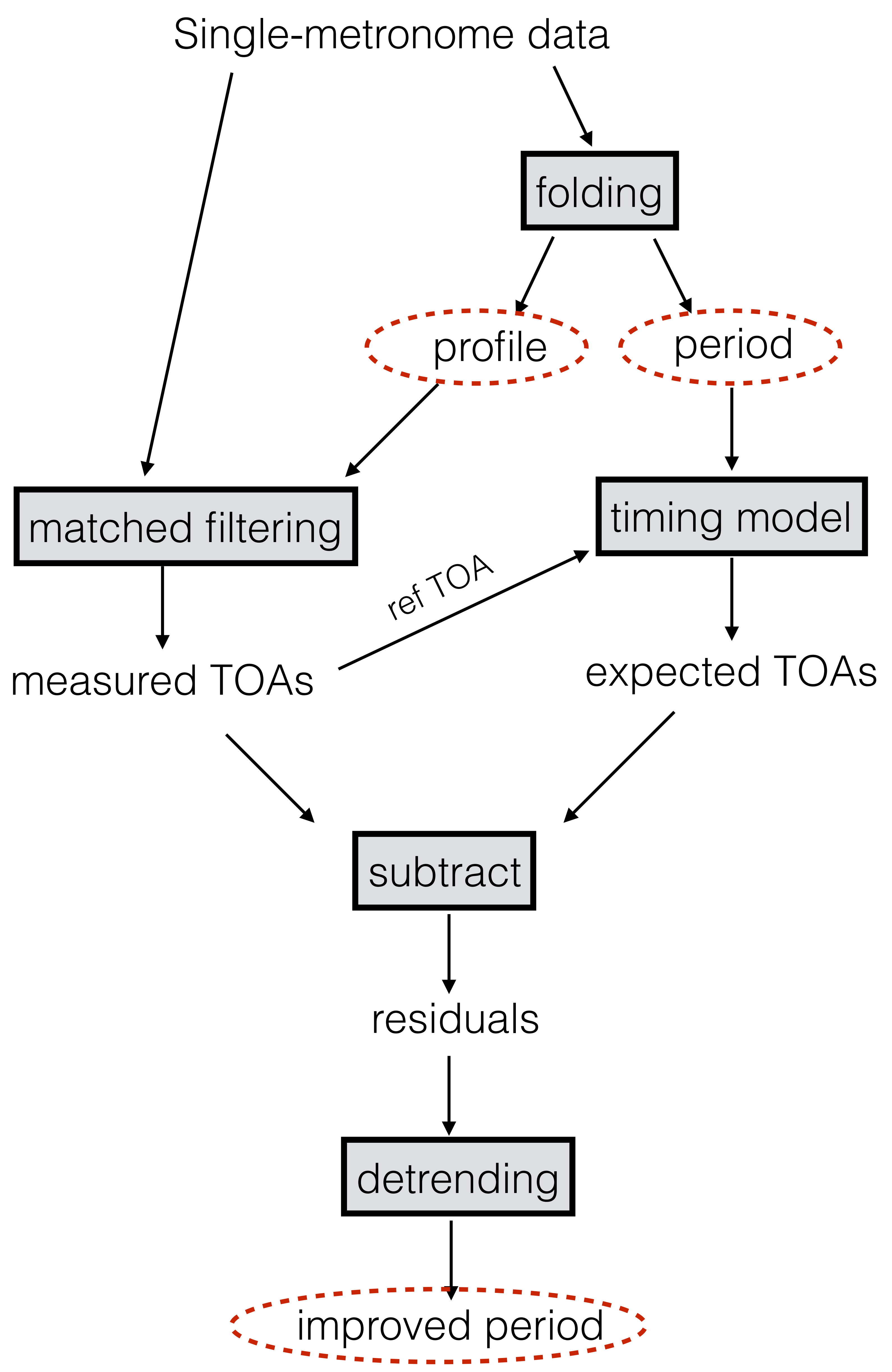
- expected TOAs: timing model using pulse period and reference TOA

$$\tau^{\rm expected}[i] = \tau^{\rm measured}[i_0] + (i - i_0) T_{\rm p}$$

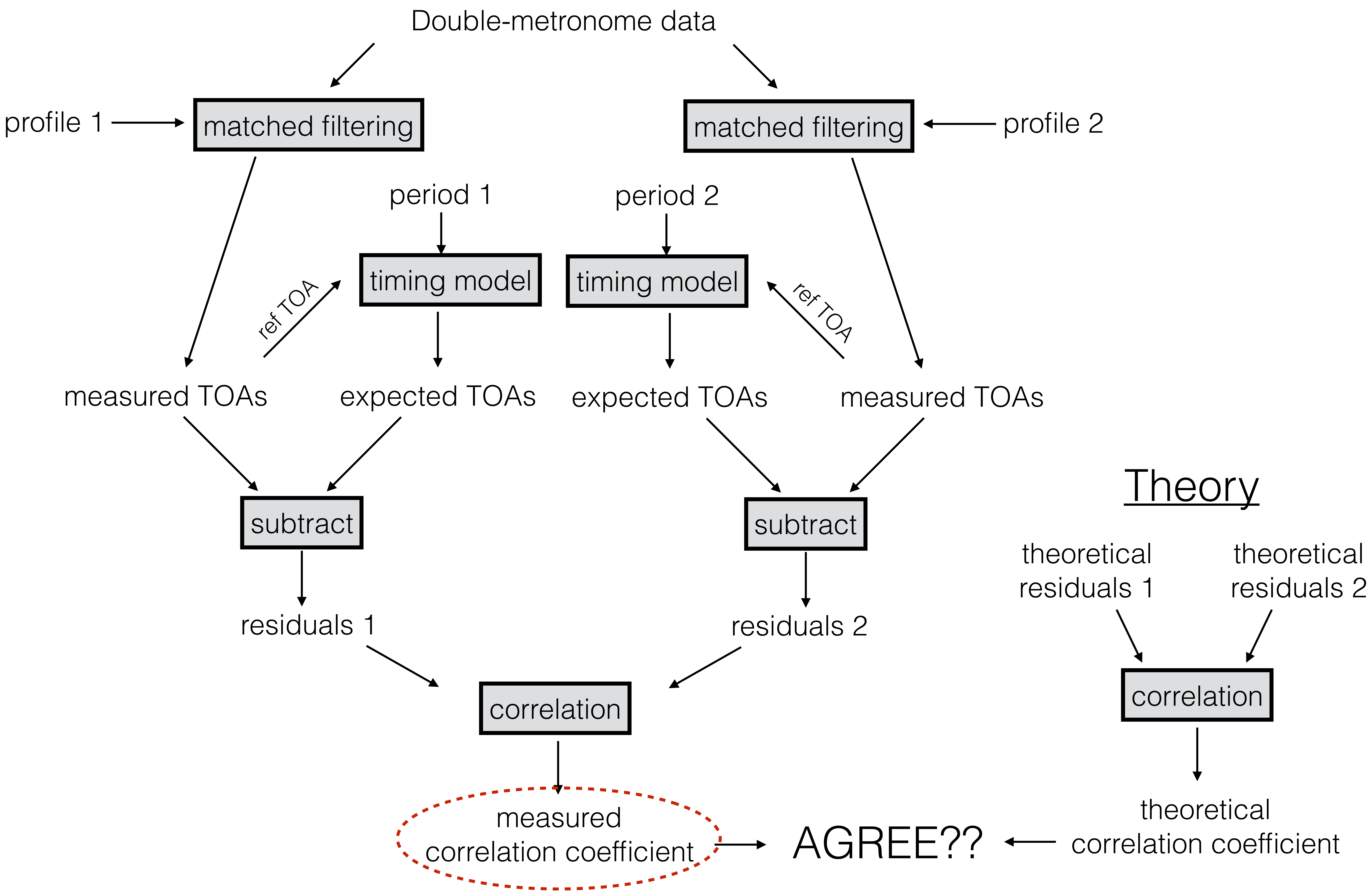
- pulse profile, period: folding and detrending single-metronome data
- expected "GW" correlation (for uniform circular motion):

$$\delta\tau_I(t) = \frac{\Delta L_I(t)}{c_{\rm s}} \simeq -\frac{1}{c_{\rm s}} \hat{u}_I \cdot \vec{r}(t) \quad \Longrightarrow \quad \boxed{\rho_{12} \simeq \cos\zeta}$$

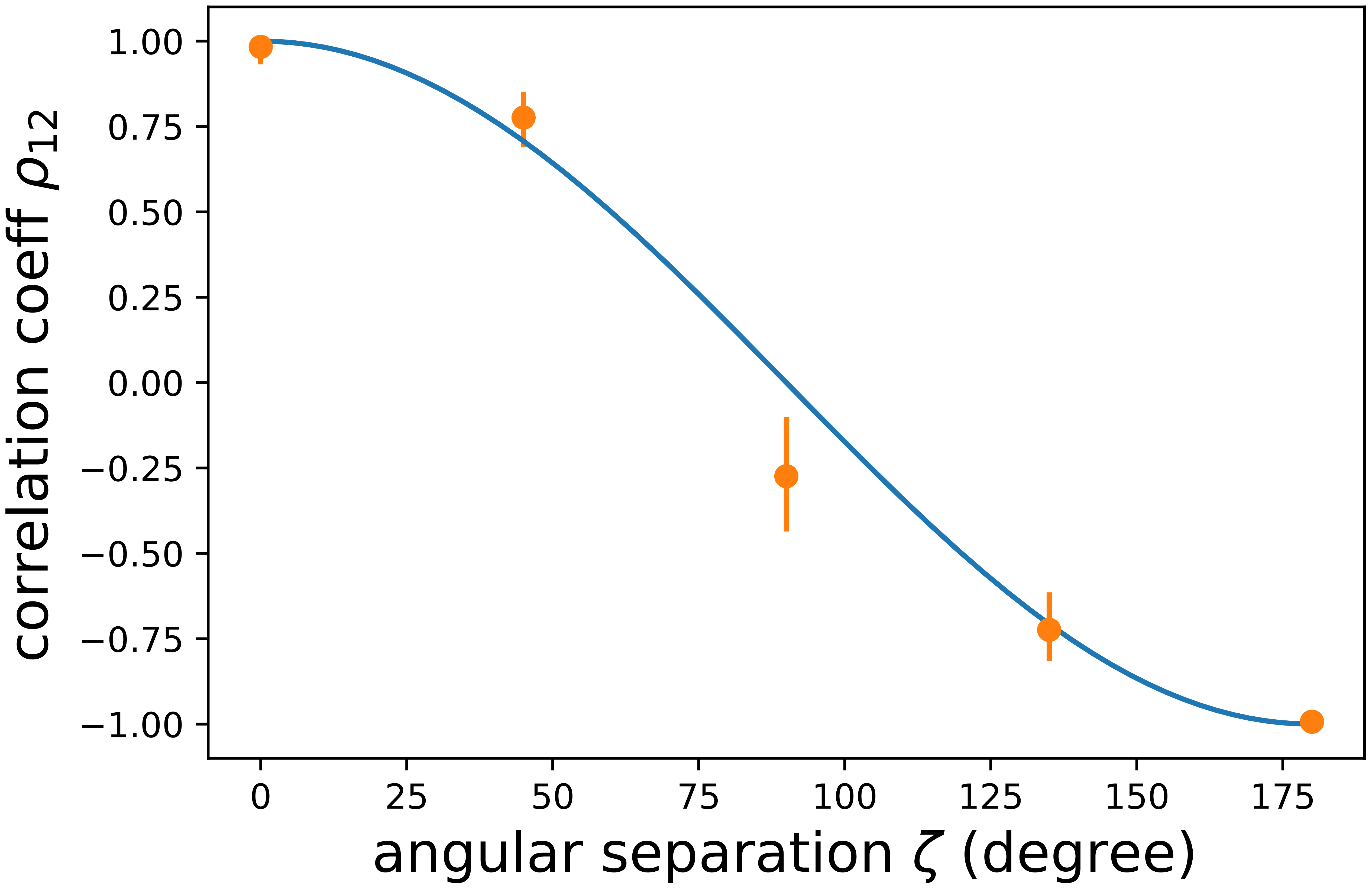
Single-metronome analysis



Double-metronome analysis

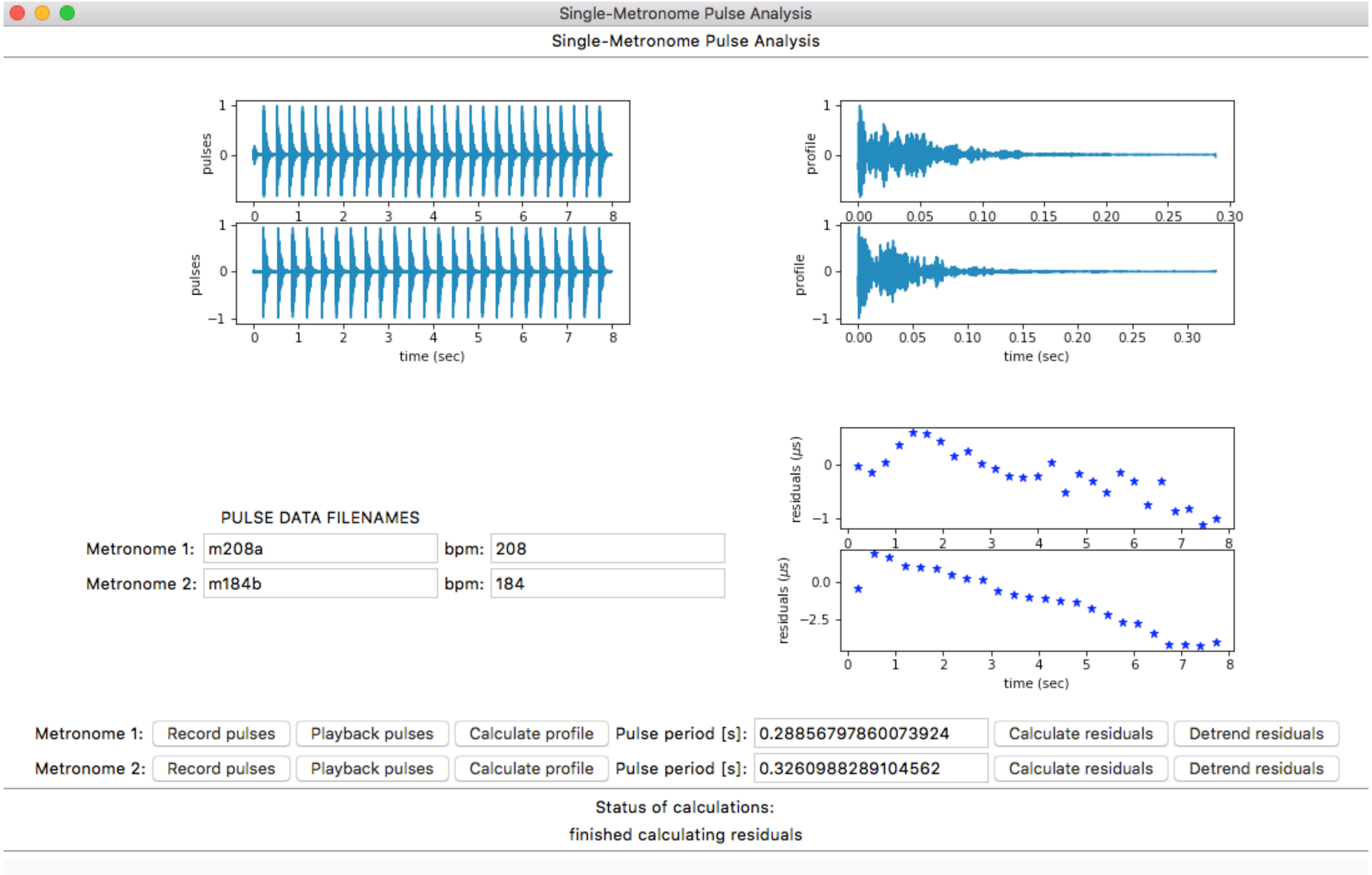


Metronome correlation

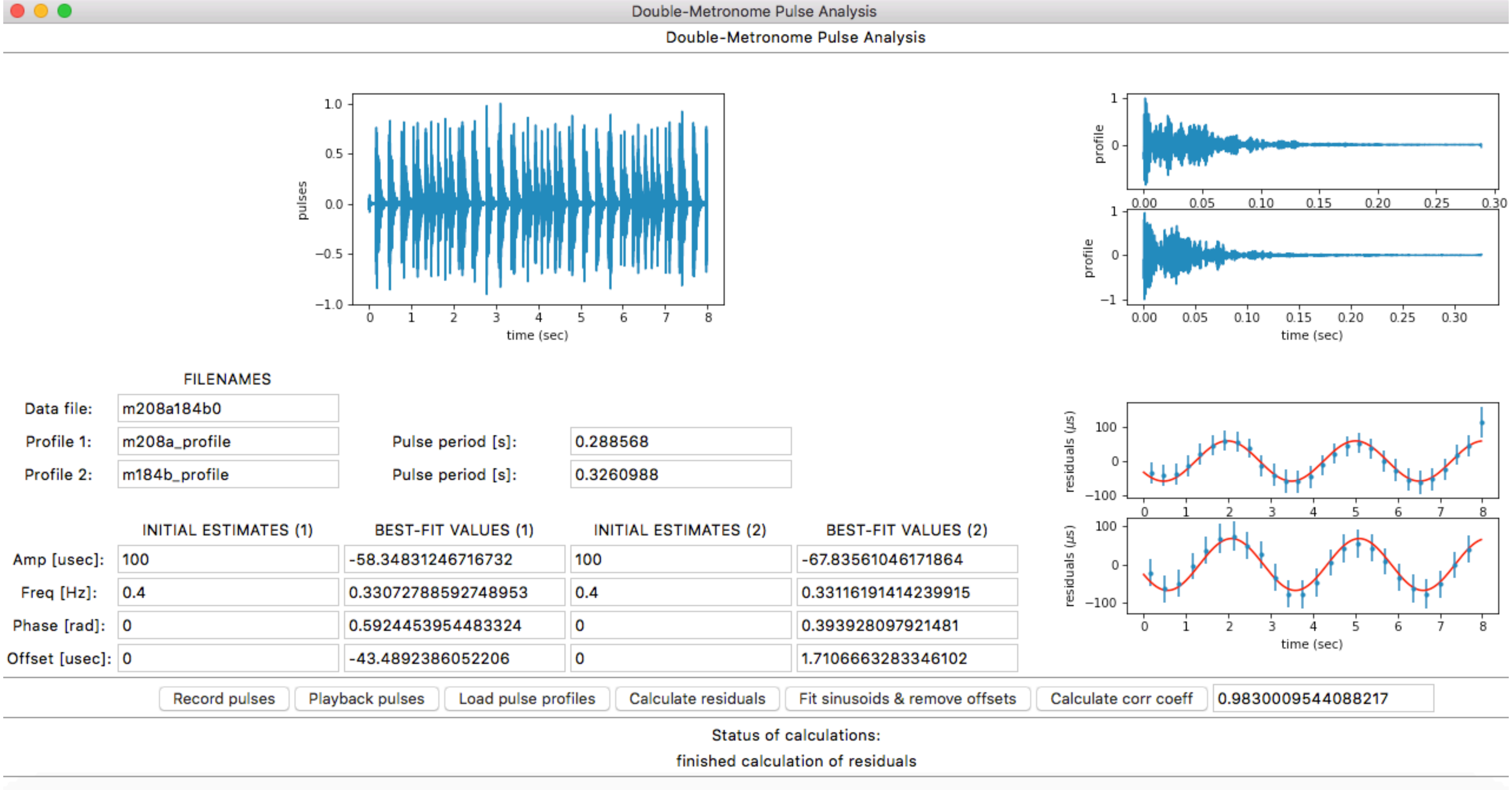


Output of the GUIs

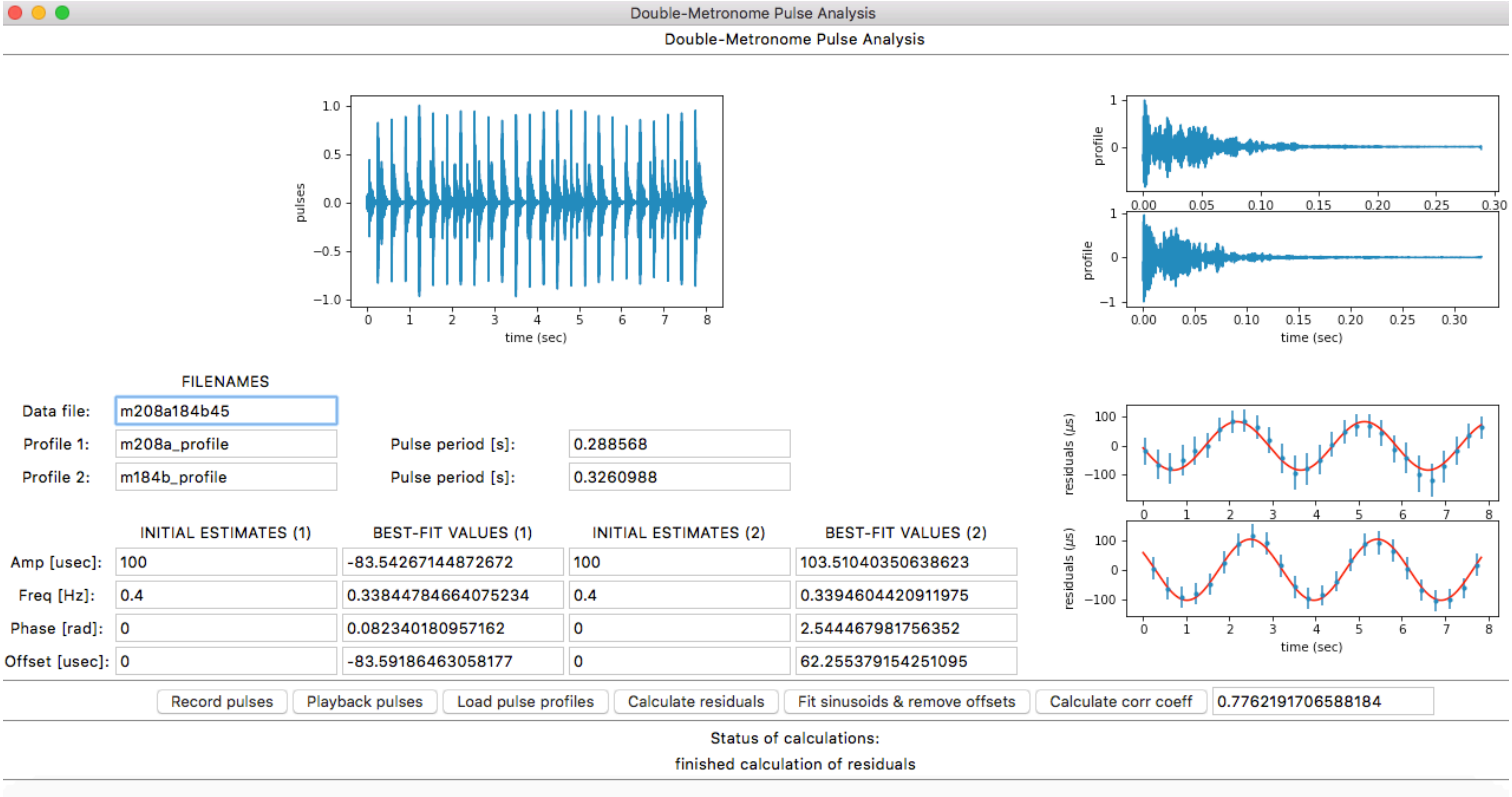
Single-metronome analysis



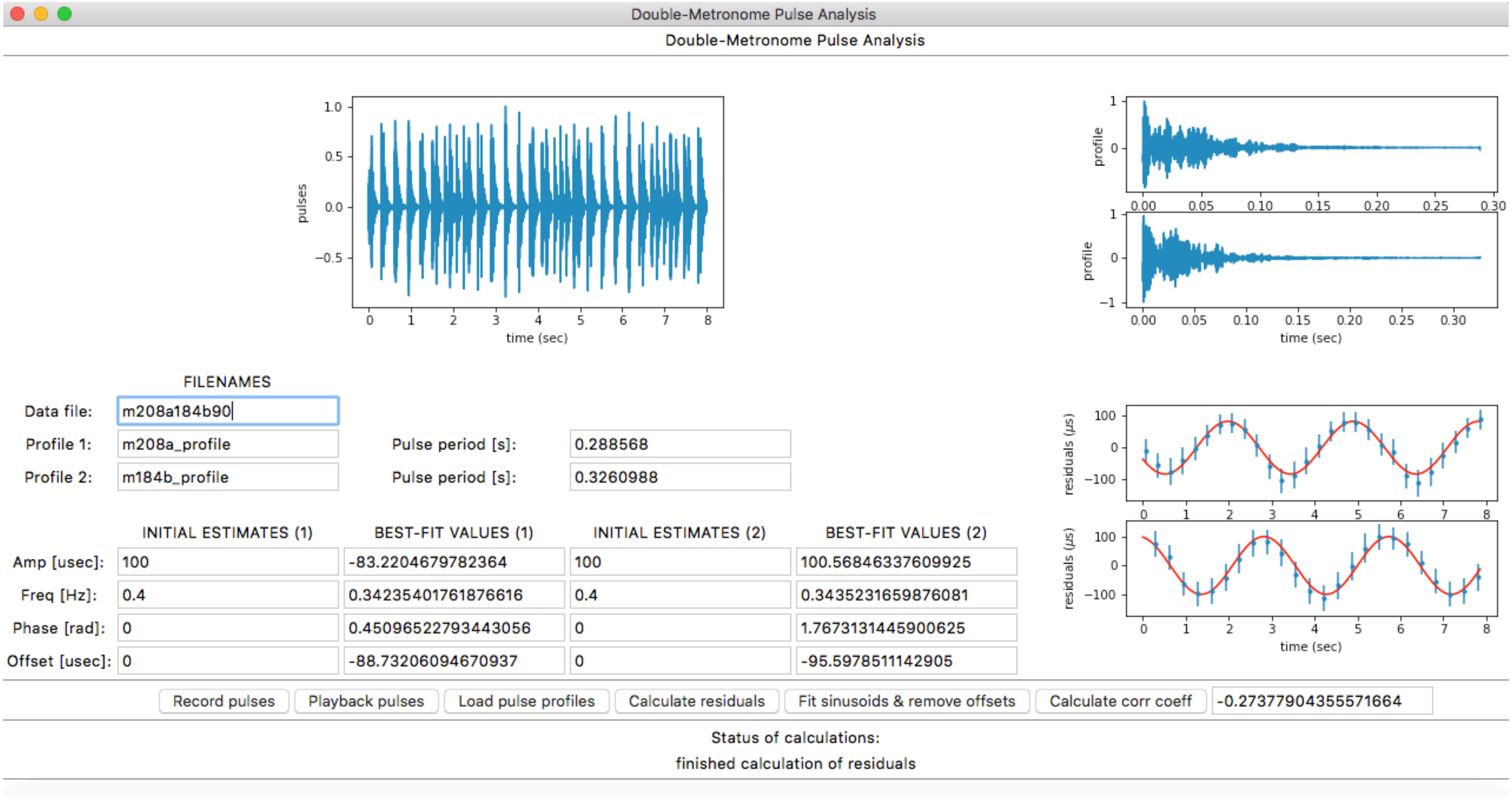
Double-metronome analysis (ζ=0 degrees)



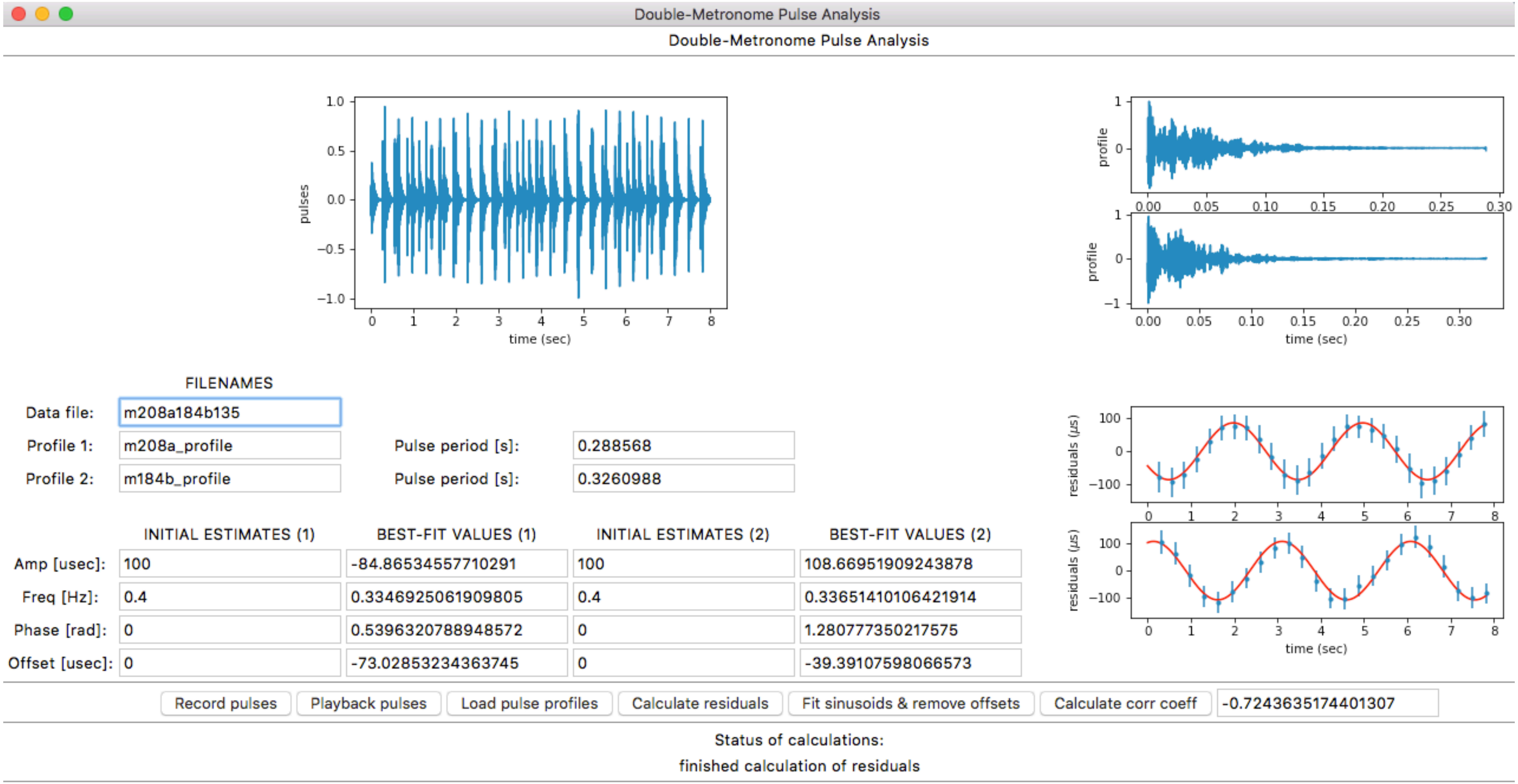
Double-metronome analysis (ζ=45 degrees)



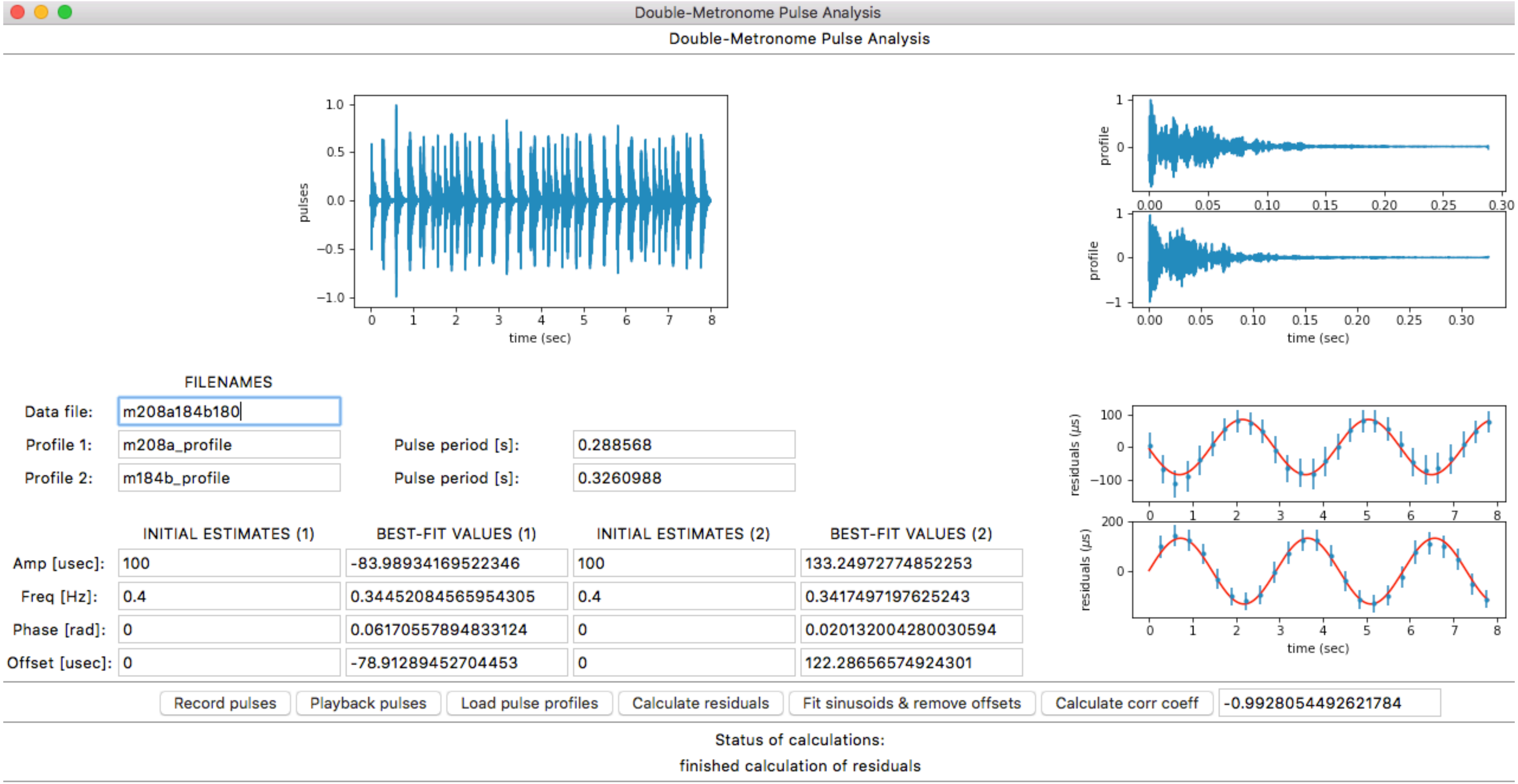
Double-metronome analysis (ζ=90 degrees)



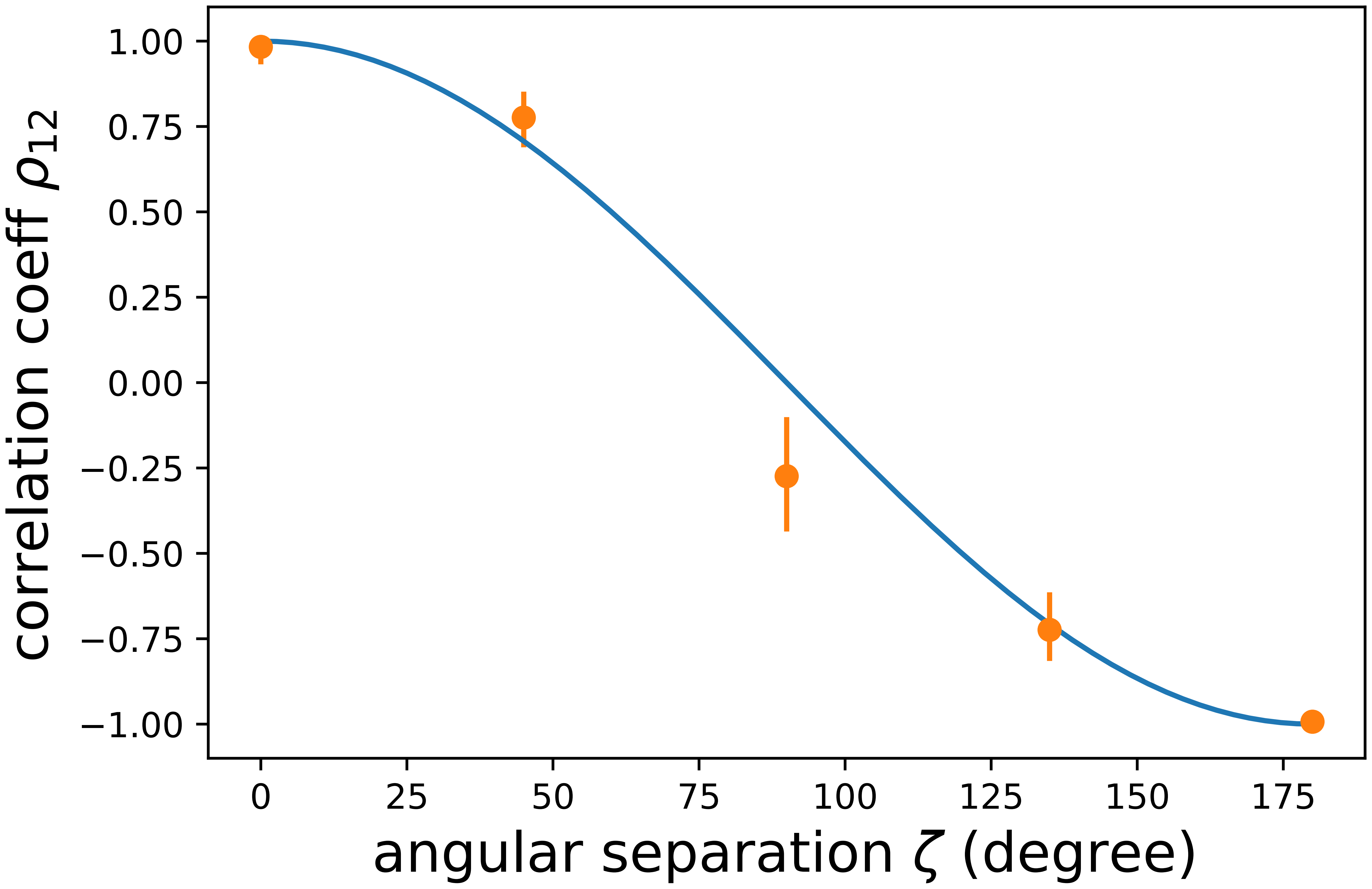
Double-metronome analysis (ζ=135 degrees)



Double-metronome analysis (ζ=180 degrees)

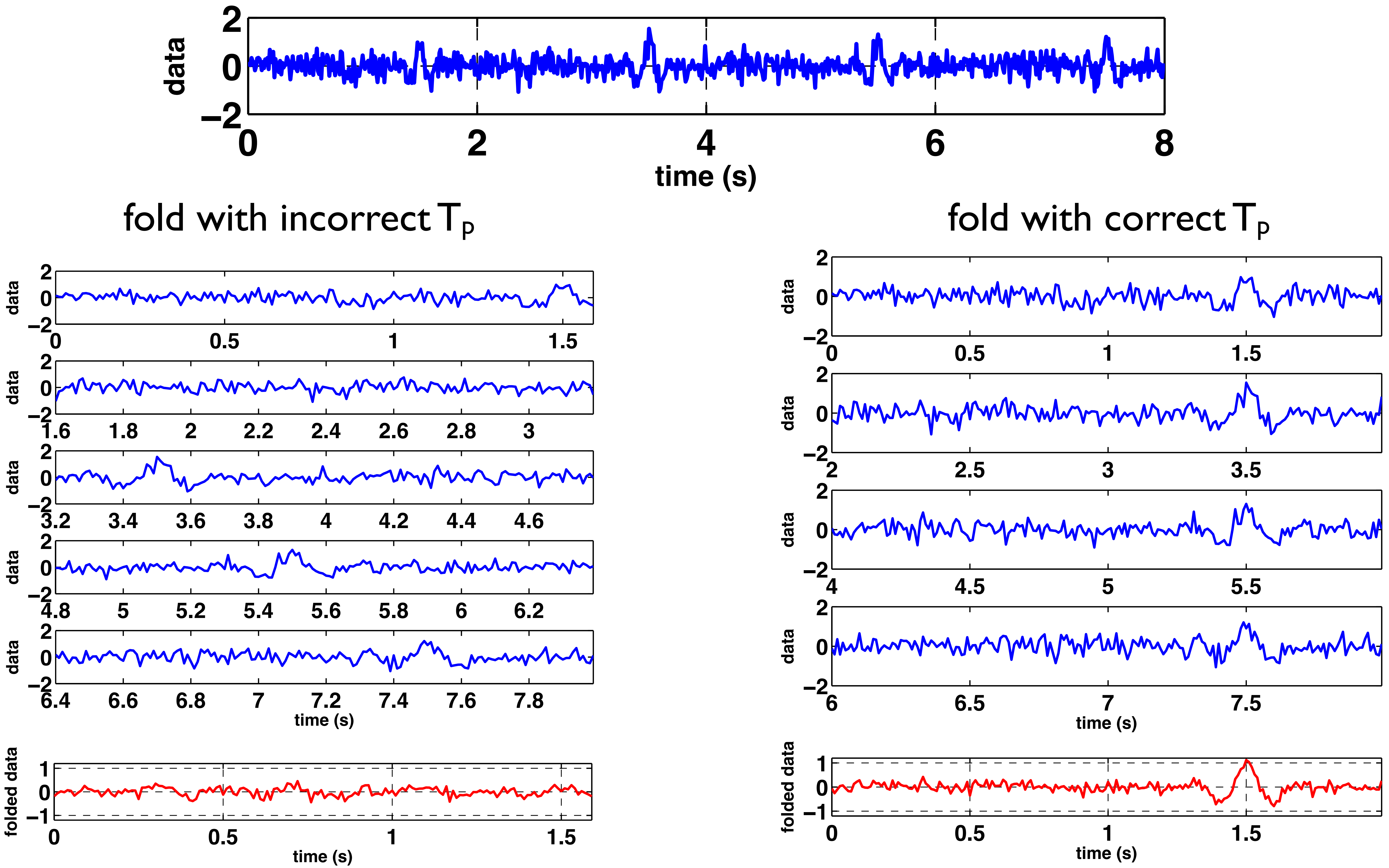


Metronome correlation



More details

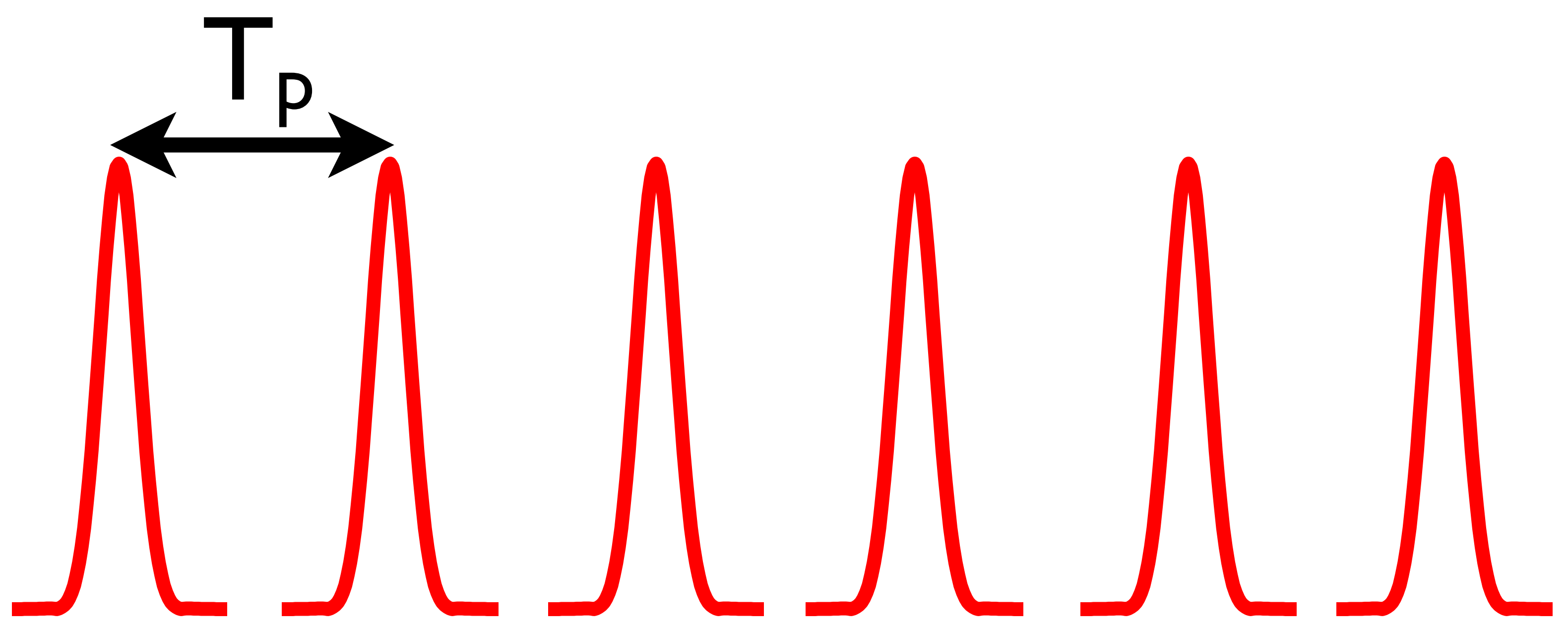
"Fold" data to determine pulse period and pulse profile



Timing model

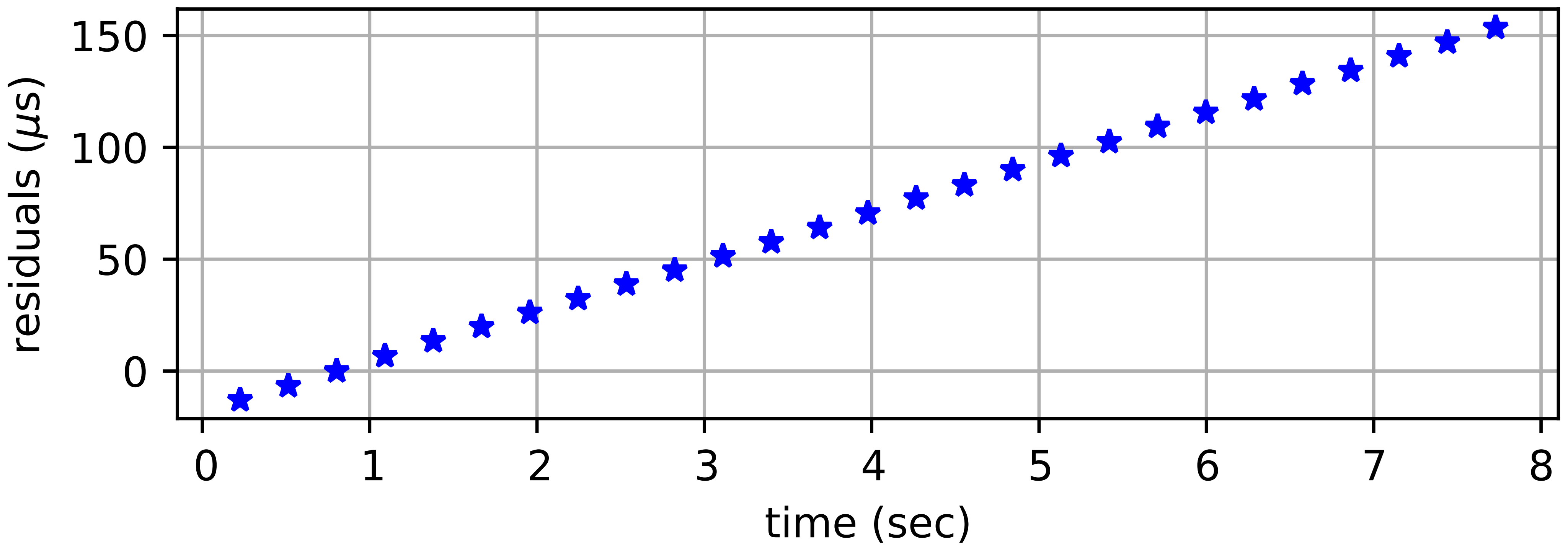
Pulses should arrive regularly with period T_p relative to some reference pulse

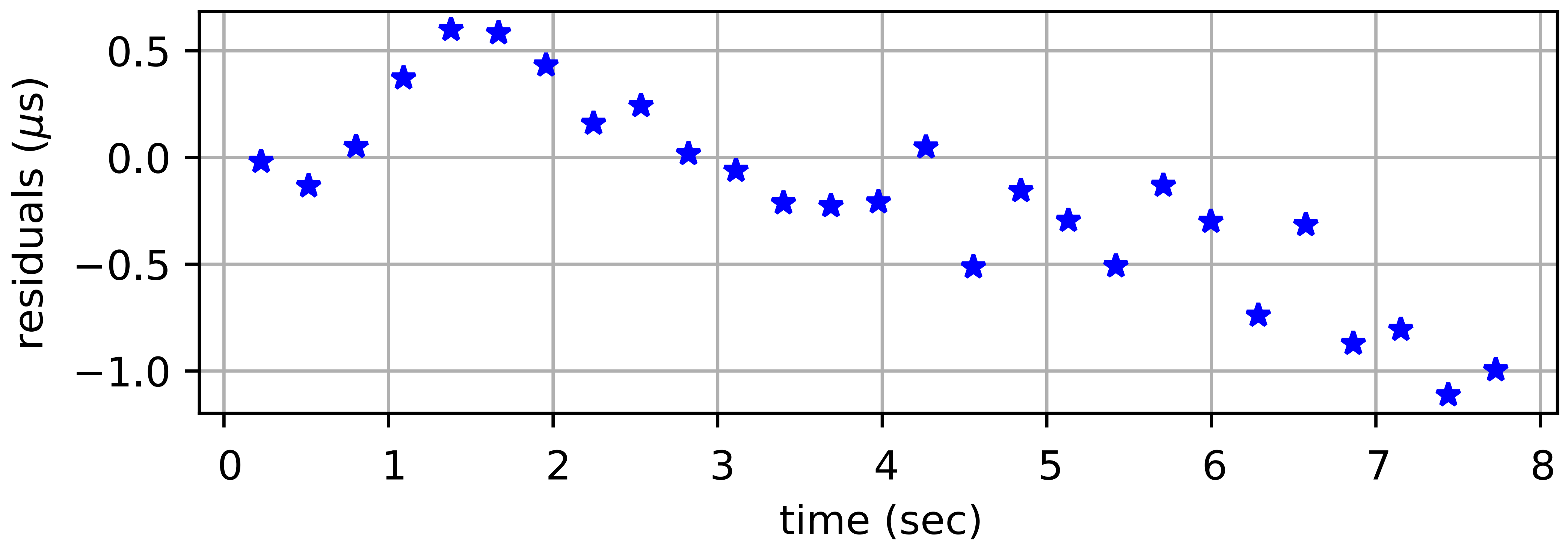
$$\tau^{\mathrm{expected}}[i] = \tau^{\mathrm{measured}}[i_0] + (i - i_0)T_\mathrm{p}$$



Remove linear trend to more accurately determine pulse period

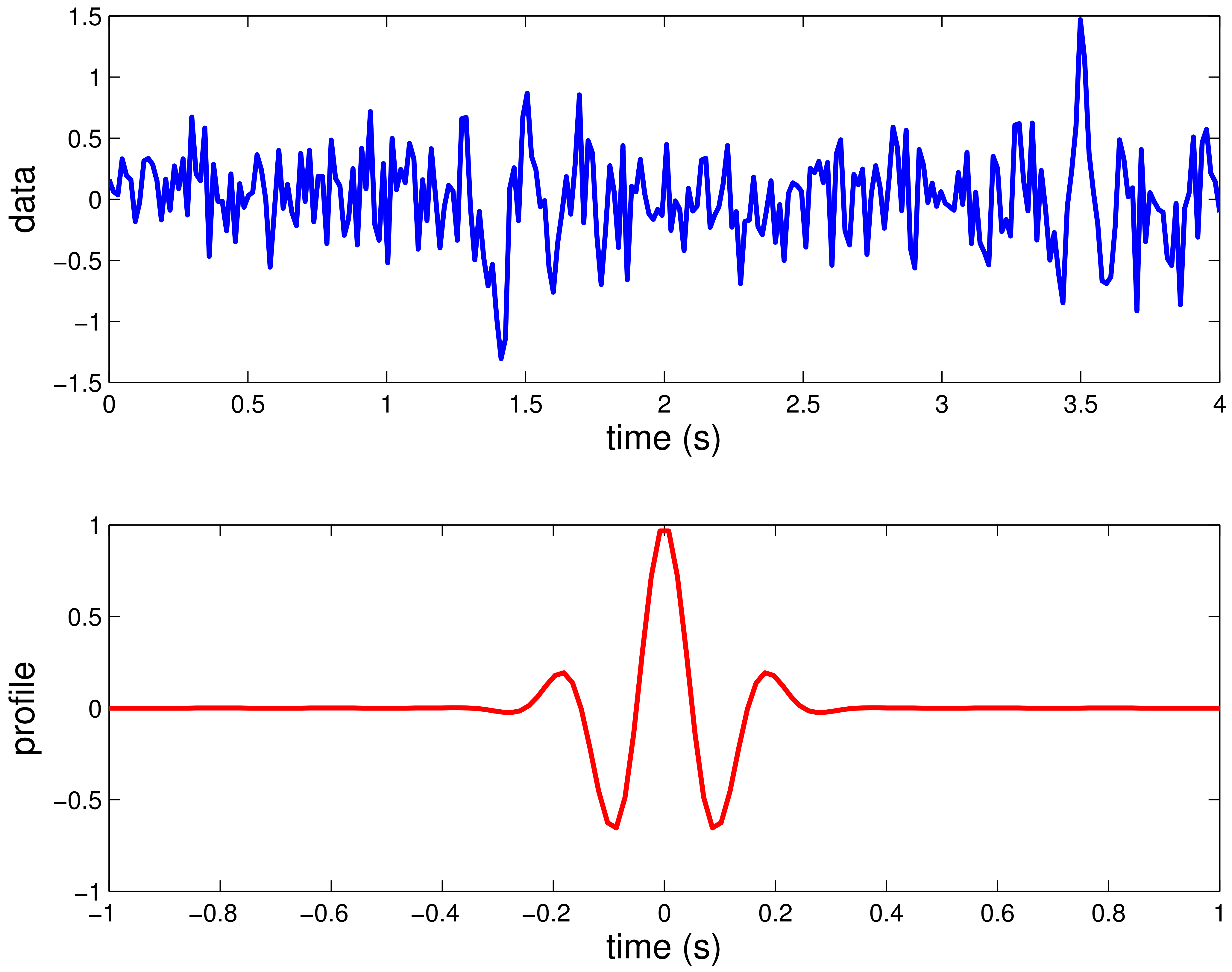
$$\tau^{\mathrm{expected}}[i] = \tau^{\mathrm{measured}}[i_0] + (i - i_0)T_{\mathrm{p}}$$

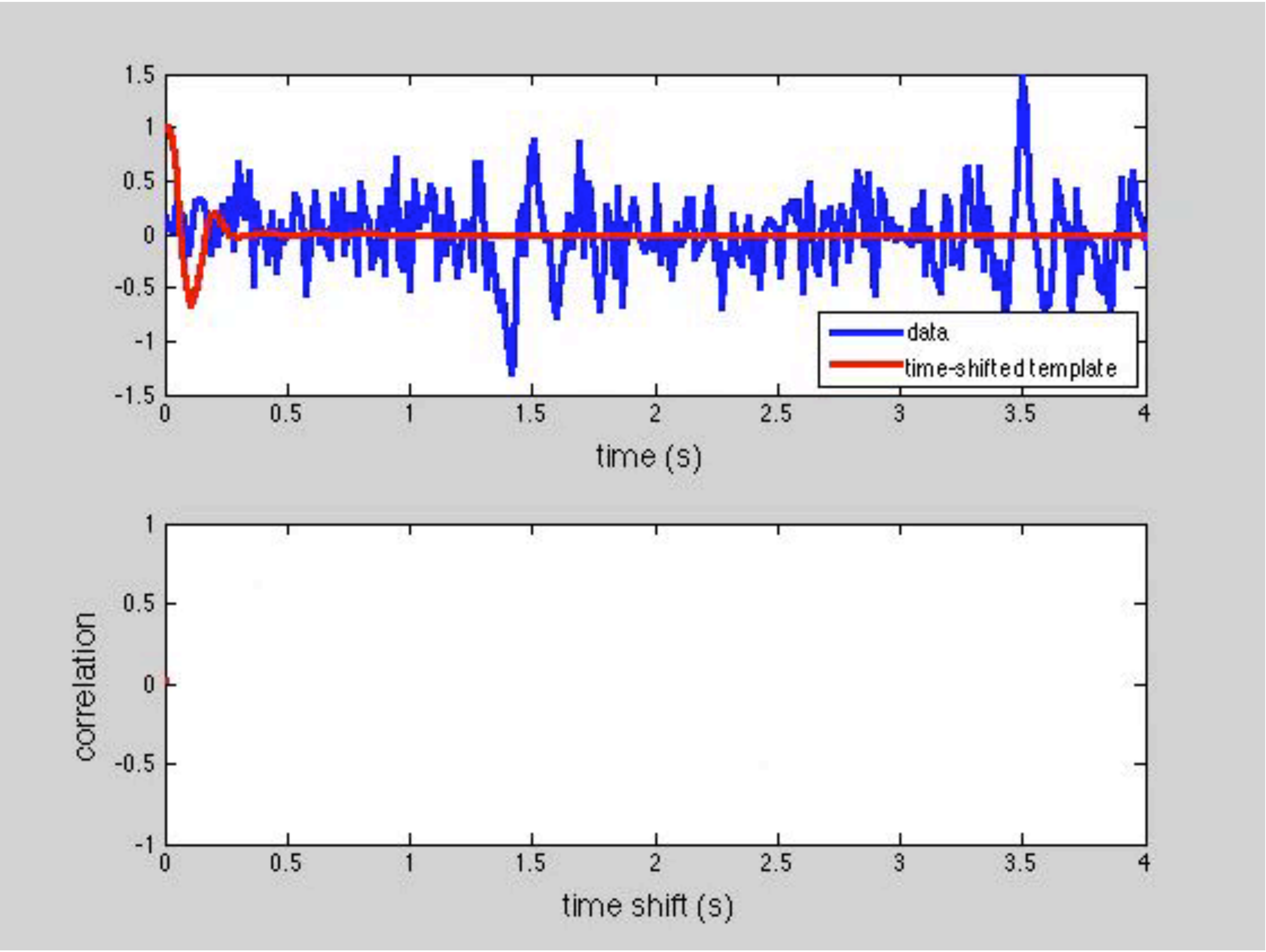




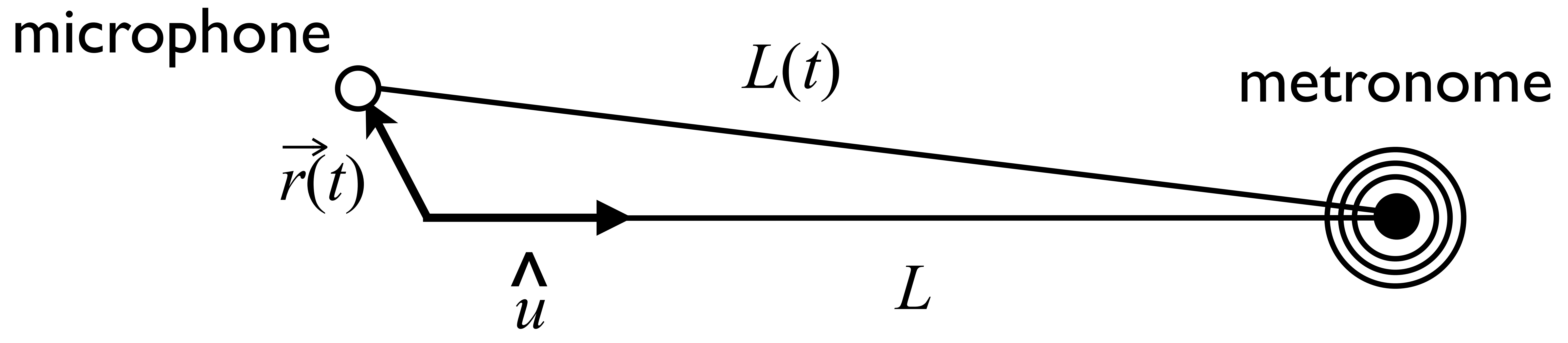
Matched-filtering determination of measured TOAs

$$C(\Delta t) = \mathcal{N} \int \mathrm{d}t \; y(t) p(t - \Delta t)$$



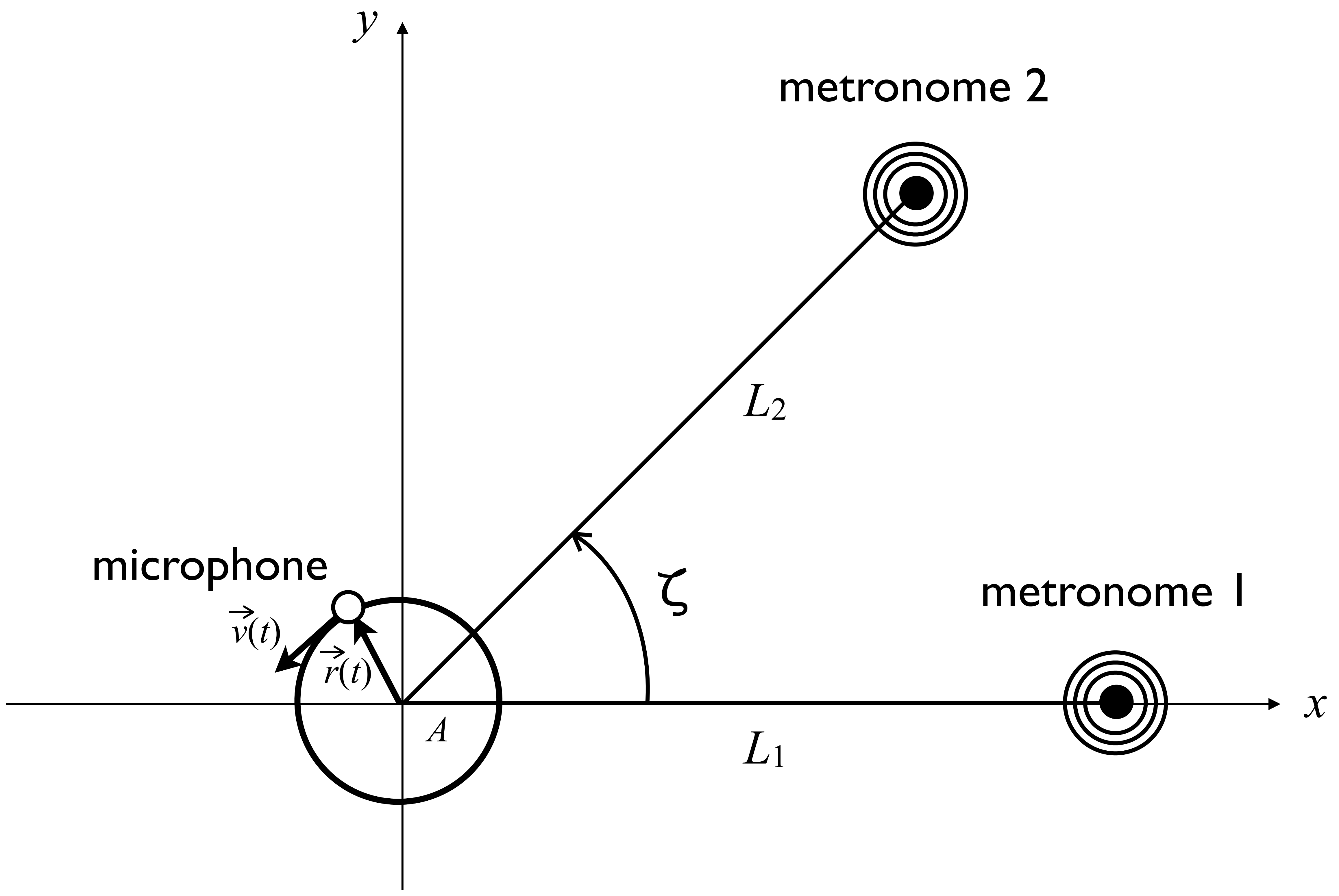


Timing-residual response to microphone motion



$$\delta\tau(t) = \frac{\Delta L(t)}{c_\mathrm{s}} \simeq -\frac{1}{c_\mathrm{s}}\hat{u}\cdot\vec{r}(t)$$

Two metronomes - uniform circular motion

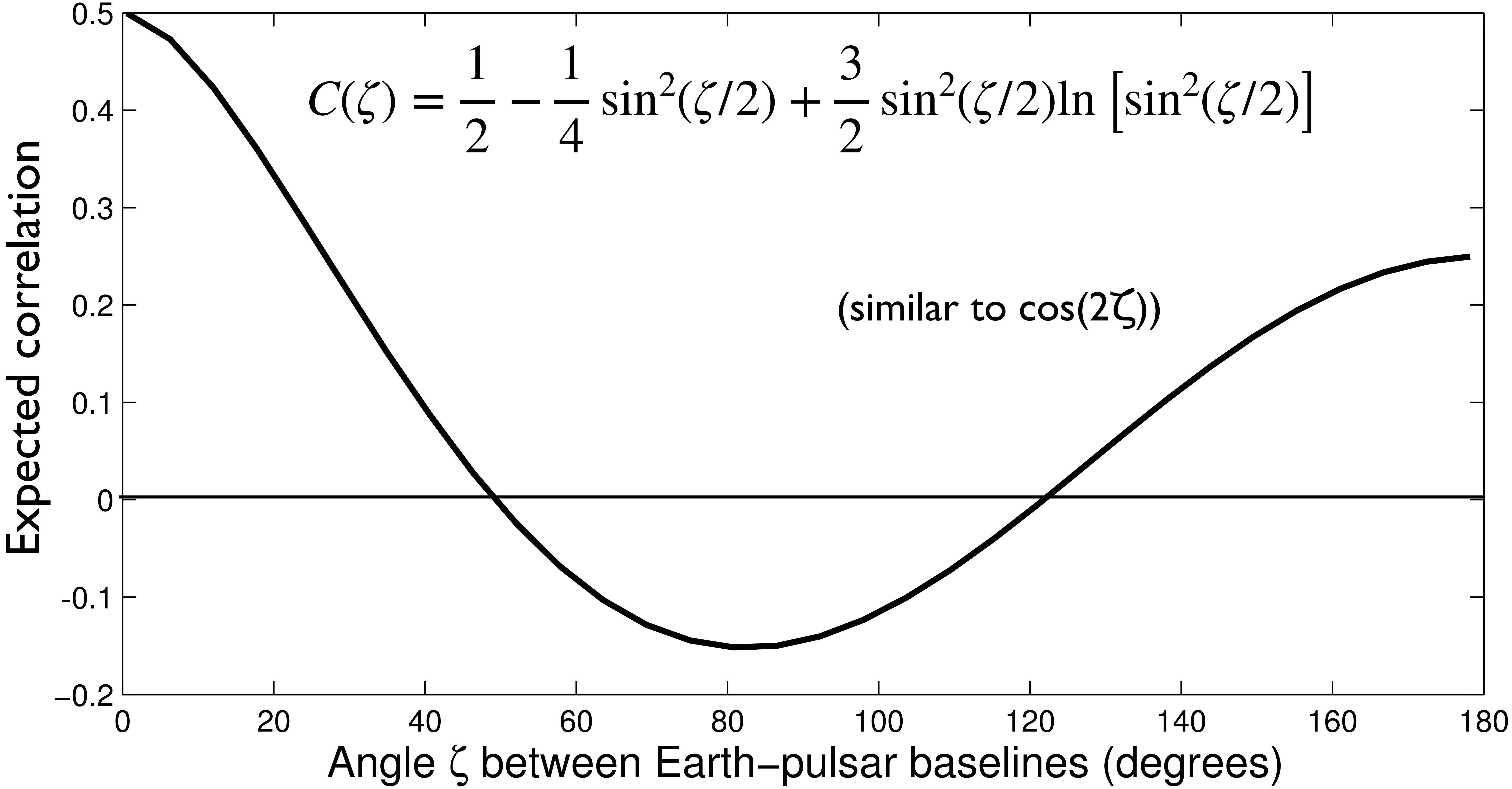


$$\vec{r}(t) = A\left[\cos(2\pi f_0 t + \phi_0)\hat{x} + \sin(2\pi f_0 t + \phi_0)\hat{y}\right]$$

$$\delta\tau_I(t) \simeq -\frac{A}{c_s}\cos(2\pi f_0 t + \phi_0 - \theta_I)\,, \quad I = 1,2$$

$$\rho_{12} \simeq \cos\zeta\,, \quad \zeta \equiv \theta_1 - \theta_2$$

Expected PTA correlation - Hellings & Downs curve (isotropic, unpolarized GW background)



Metronome demo numbers

c_s = 340 m/s (in air)

amplitude $\approx$ 5 cm

amplitude / c_s = 1×10^{-4} sec

184 bpm: T_p = 0.3261 sec

208 bpm: T_p = 0.2885 sec

Pulsar timing numbers

$f \sim$ 1/few weeks to 1/10 years (10^{-7} Hz to 10^{-9} Hz)

$\lambda \sim$ 0.1 to 10 lyr (GW wavelength)

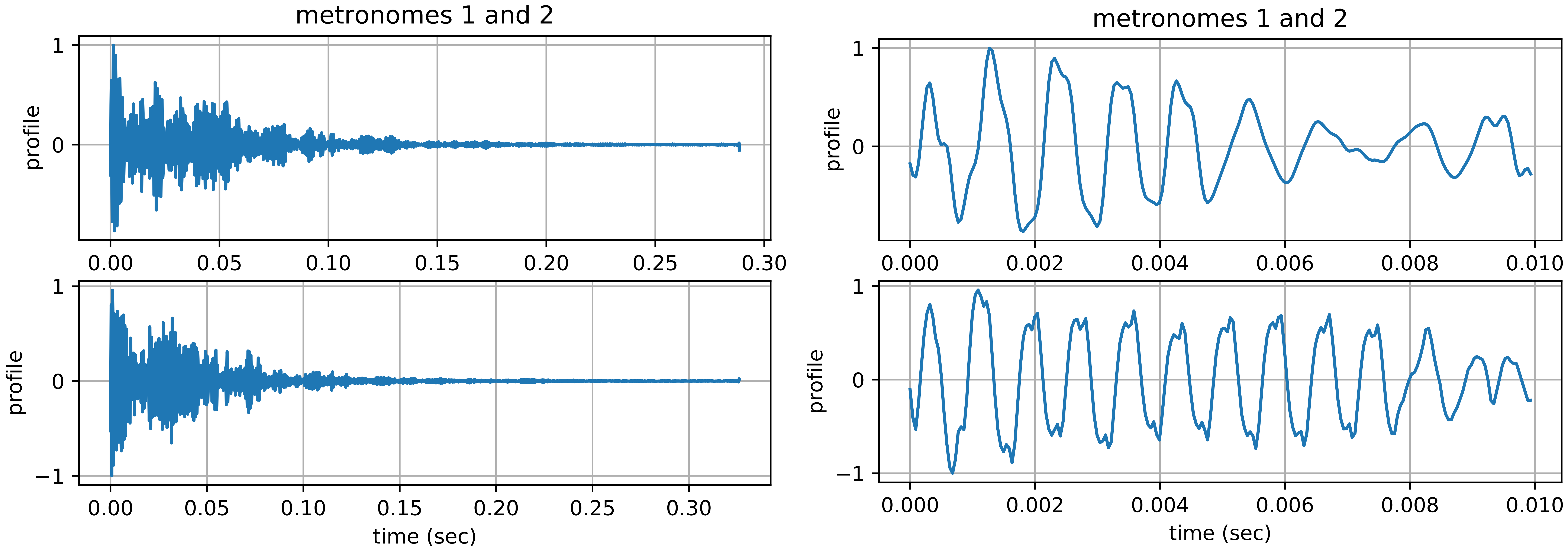
$L \sim$ few x 1000 lyr (distance to pulsars)

$\lambda << L$ (short-wavelength limit)

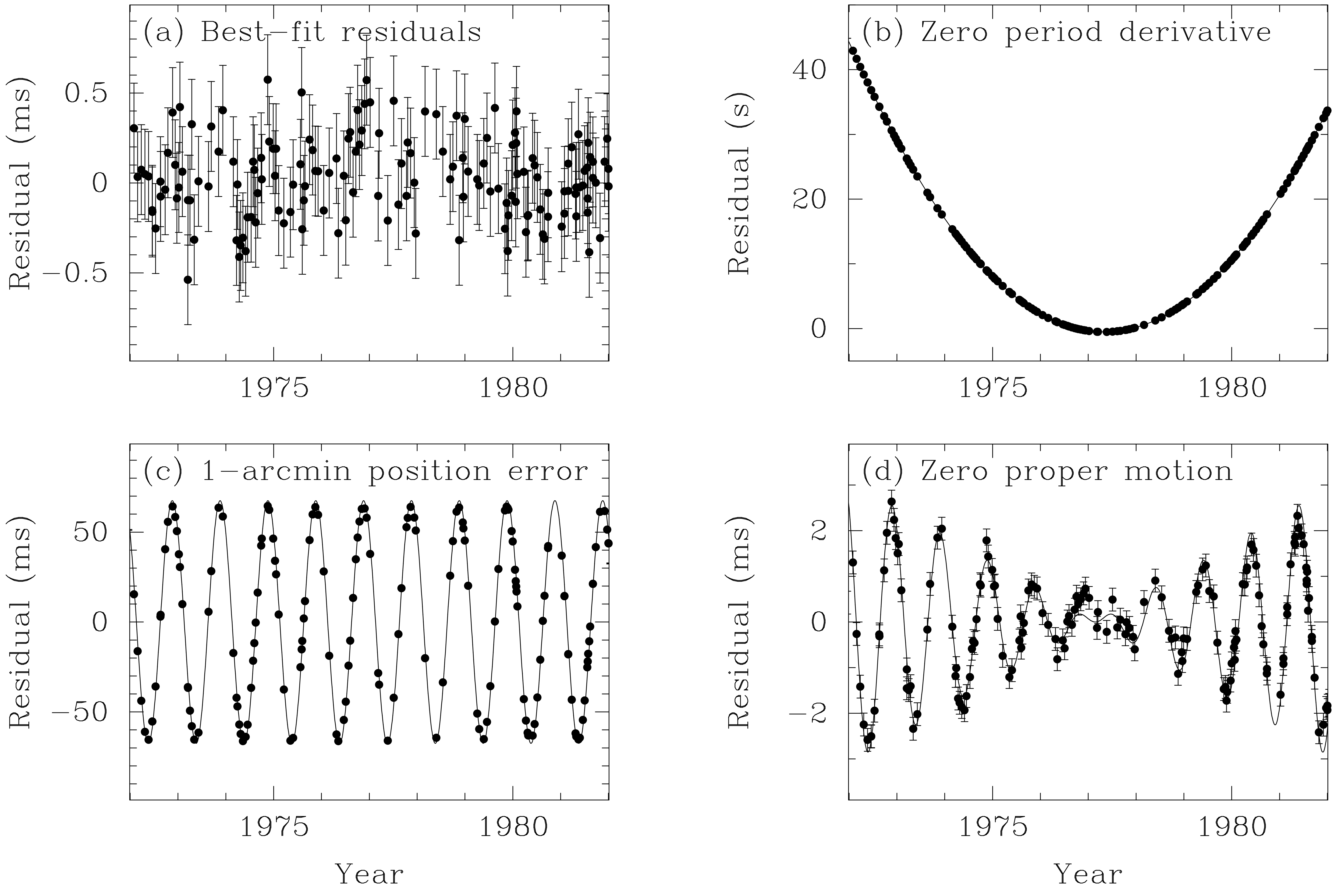
sensitivity $\sim \sigma_{rms}/T_{obs} \sim$ 100 ns/10 yr $\sim 10^{-15}$

can detect changes ~10 km in the position of a pulsar at a distance of ~1000 lyr

Metronome pulse profiles



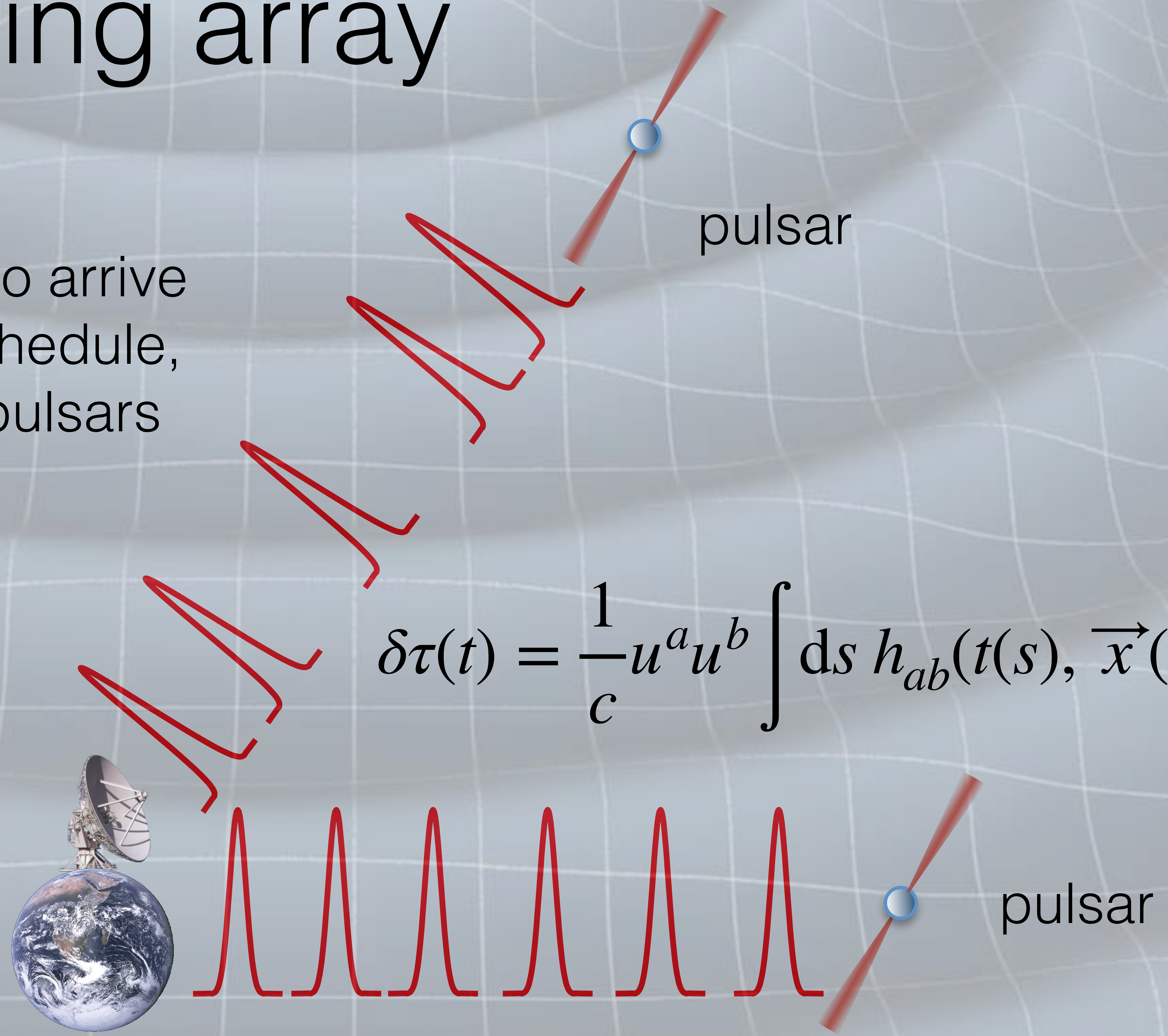
Errors in the timing model show up as deterministic features in the timing residuals



(from Lorimer LRR-2008-8)

Pulsar timing array

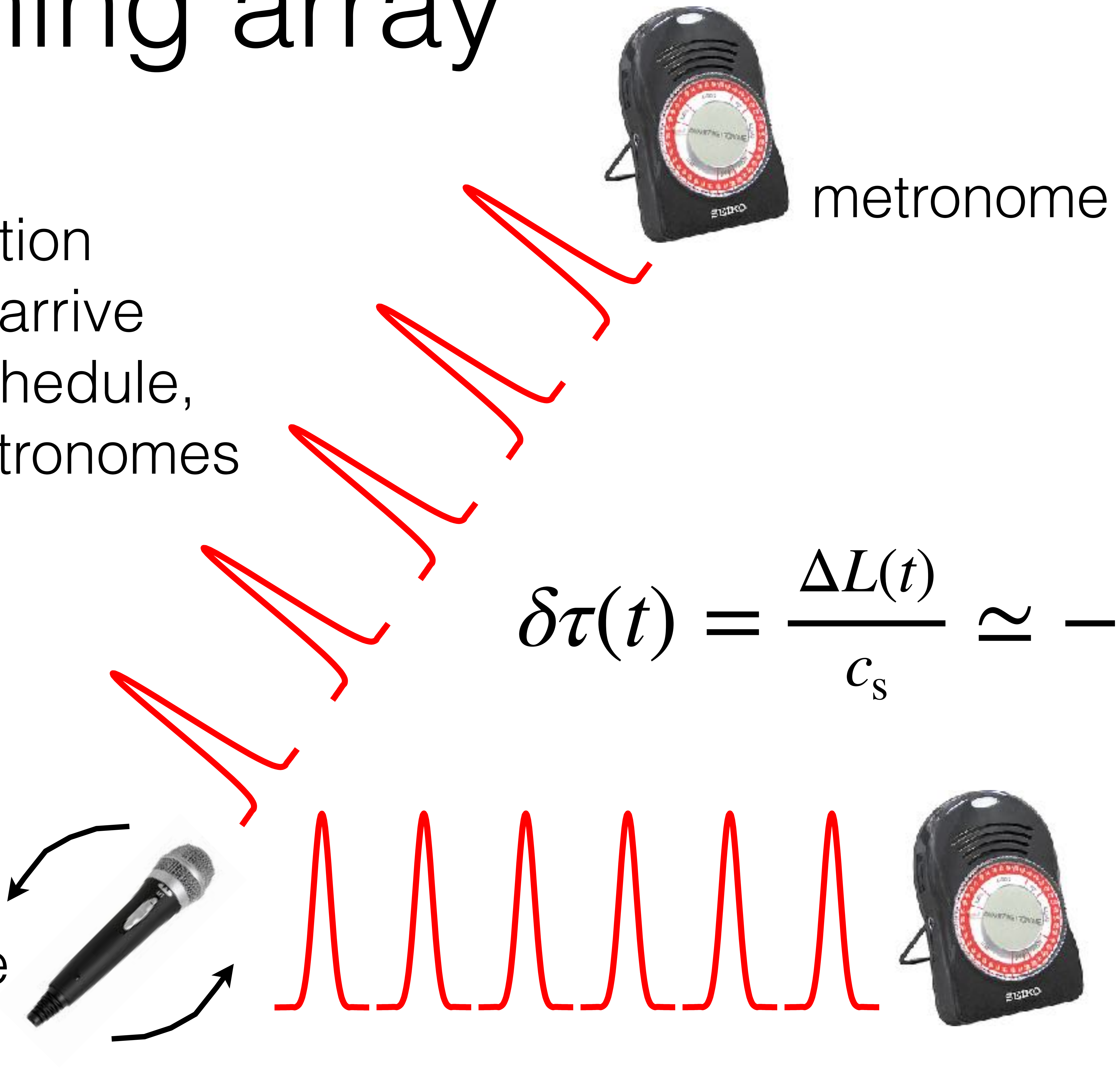
GWs cause pulses to arrive ahead or behind schedule, correlated across pulsars



$$\delta\tau(t) = \frac{1}{c} u^a u^b \int \mathrm{d}s \, h_{ab}(t(s), \vec{x}(s))$$

Metronome timing array

microphone motion
causes pulses to arrive
ahead or behind schedule,
correlated across metronomes



$$\delta\tau(t) = \frac{\Delta L(t)}{c_\mathrm{s}} \simeq -\frac{1}{c_\mathrm{s}}\hat{u}\cdot\vec{r}(t)$$