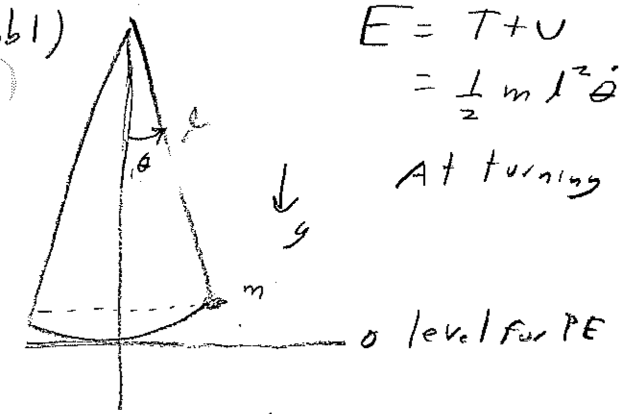
Section II:

(Prob 1)



$$L = T - U \qquad U = mg(l - l\cos\theta)$$
$$= mgl(1-\cos\theta)$$

$$E = T + U$$
$$= \frac{1}{2} m l^2 \dot\theta^2 + mgl(1-\cos\theta)$$

At turning points $\dot\theta = 0 \rightarrow E = mgl(1-\cos\theta_0)$

$$\frac{1}{2} m l^2 \dot\theta^2 + U(\theta) = E$$

$$\frac{1}{2} m l^2 \dot\theta^2 = E - U(\theta)$$

$$\frac{d\theta}{dt} = \dot\theta = \sqrt{\frac{2}{ml^2}(E - U(\theta))}$$

$$dt = \frac{d\theta}{\sqrt{\frac{2}{ml^2}(E-U(\theta))}}$$

$$\rightarrow \qquad T(\theta_0) = 4\int_0^{\theta_0} \frac{d\theta}{\sqrt{\frac{2}{ml^2}(E-U(\theta))}}$$

$$= 2\sqrt{2}\sqrt{m}\, l \int_0^{\theta_0} \frac{d\theta}{\sqrt{E - U(\theta)}}$$

$$= 2\sqrt{2}\sqrt{m}\, l \int_0^{\theta_0} \frac{d\theta}{\sqrt{mgl(1-\cos\theta_0) - mgl(1-\cos\theta)}}$$

$$= \frac{2\sqrt{2}\cancel{\sqrt{m}}\, l}{\sqrt{\cancel{m}gl}}$$

$$= 2\sqrt{2}\sqrt{\frac{l}{g}} \int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

Now write in terms of a complete elliptic integral

NOTE: $\cos\theta = \cos\left(2\left(\frac{\theta}{2}\right)\right) = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$

and $\cos\theta_0 = 1 - 2\sin^2\left(\frac{\theta_0}{2}\right)$

$$\int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} = \int_0^{\theta_0} \frac{d\theta}{\sqrt{1 - 2\sin^2(\frac{\theta}{2}) - (1 - 2\sin^2(\frac{\theta_0}{2}))}}$$

$$= \int_0^{\theta_0} \frac{d\theta}{\sqrt{2}\sqrt{\sin^2(\frac{\theta_0}{2}) - \sin^2(\frac{\theta}{2})}}$$

$$= \int_0^{\theta_0} \frac{d\theta}{\sqrt{2}\,\sin(\frac{\theta_0}{2})\sqrt{1 - \frac{\sin^2(\frac{\theta}{2})}{\sin^2(\frac{\theta_0}{2})}}}$$

<u>Let:</u> $t = \dfrac{\sin(\frac{\theta}{2})}{\sin(\frac{\theta_0}{2})} \rightarrow dt = \dfrac{\cos(\frac{\theta}{2})\,\frac{1}{2}\,d\theta}{\sin(\frac{\theta_0}{2})}$

$$\rightarrow d\theta = \frac{dt\ \sin(\frac{\theta_0}{2})\cdot 2}{\cos(\frac{\theta}{2})}$$

$$= \frac{2\,dt\ \sin(\frac{\theta_0}{2})}{\sqrt{1 - \sin^2(\frac{\theta}{2})}}$$

$$= \frac{2\,dt\ \sin(\frac{\theta_0}{2})}{\sqrt{1 - \sin^2(\frac{\theta_0}{2})t^2}}$$

$\theta = 0 \leftrightarrow t = 0$

$\theta = \theta_0 \leftrightarrow t = 1$

Thus,

$$\int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} = \int_0^1 \frac{2\,dt\ \cancel{\sin(\frac{\theta_0}{2})}}{\sqrt{2}\ \cancel{\sin(\frac{\theta_0}{2})}\sqrt{1-t^2}\sqrt{1-k^2t^2}}$$

$$= \sqrt{2}\int_0^1 \frac{dt}{\sqrt{1-t^2}\sqrt{1-k^2t^2}} \quad \text{where } k = \sin(\tfrac{\theta_0}{2})$$

$$= \sqrt{2}\ F(\phi = \tfrac{\pi}{2}, k) \equiv \sqrt{2}\,K(\sin(\tfrac{\theta_0}{2}))$$

thus,

$$T(\theta_0) = 2\sqrt{2}\sqrt{\frac{\ell}{g}} \int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

$$= 2\sqrt{2}\sqrt{\frac{\ell}{g}}\,\sqrt{2}\;K\left(\sin\left(\frac{\theta_0}{2}\right)\right)$$

$$= 4\sqrt{\frac{\ell}{g}}\;K\left(\sin\left(\frac{\theta_0}{2}\right)\right)$$

$$= 4\sqrt{\frac{\ell}{g}} \int_0^1 \frac{dt}{\sqrt{1-t^2}\,\sqrt{1-\sin^2\left(\frac{\theta_0}{2}\right)t^2}}$$

0th order:

$\theta_0 \to 0$

$$\to \quad T = 4\sqrt{\frac{\ell}{g}} \int_0^1 \frac{dt}{\sqrt{1-t^2}}$$

$$= 4\sqrt{\frac{\ell}{g}} \int_0^{\pi/2} dx$$

$$= 2\pi\sqrt{\frac{\ell}{g}}$$

$t = \sin x$
$dt = \cos x\, dx$
$\sqrt{1-t^2} = \cos x$
$t = 0, 1 \leftrightarrow x = 0, \pi/2$

1st correction:

$\sin^2\left(\frac{\theta_0}{2}\right) \approx \left(\frac{\theta_0}{2}\right)^2$

$$\to \quad T(\theta_0) \approx 4\sqrt{\frac{\ell}{g}} \int_0^1 \frac{dt}{\sqrt{1-t^2}\,\sqrt{1-\frac{\theta_0^2}{4}t^2}}$$

$$T(\theta_0) \approx 4\sqrt{\frac{\ell}{g}} \int_0^1 \frac{dt}{\sqrt{1-t^2}}\left[1 + \frac{\theta_0^2}{8}t^2\right]$$

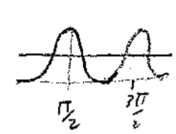
$$= 4\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2} + \frac{\theta_0^2}{8}\int_0^1 \frac{t^2\,dt}{\sqrt{1-t^2}}\right]$$

$$= 4\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2} + \frac{\theta_0^2}{8}\int_0^{\pi/2} \sin^2 x\,dx\right]$$

$$= 4\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2} + \frac{\theta_0^2}{8}\,\frac{\pi}{4}\right]$$

$$= 2\pi\sqrt{\frac{\ell}{g}}\left[1 + \frac{\theta_0^2}{16}\right]$$

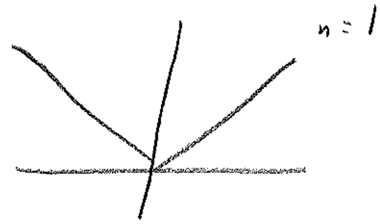
$t = \sin x$
$dt = \cos x\, dx$
etc.

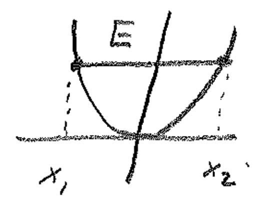


$$\int_0^{2\pi} \sin^2 x\,dx = \pi$$

Sec 11, Prob 2:

a) $U = A|x|^n$





etc

$$T(E) = \sqrt{2m} \int_{x_1(E)}^{x_2(E)} \frac{dx}{\sqrt{E-U(x)}}$$

$$= 2\sqrt{2m} \int_0^{x_0(E)} \frac{dx}{\sqrt{E-U(x)}}$$ since $U = A|x|^n$ is an even function

Turning pt: $E = U(x_0)$

$$= A|x_0|^n$$

$$x_0 = \left(\frac{E}{A}\right)^{\frac{1}{n}} = x_0(E)$$

$$T(E) = 2\sqrt{2m} \int_0^{x_0(E)} \frac{dx}{\sqrt{A|x_0|^n - A|x|^n}}$$

$$= \frac{2\sqrt{2m}}{\sqrt{A}} \int_0^{x_0(E)} \frac{dx}{\sqrt{x_0^n - x^n}}$$ since $x_0 > 0$, $0 < x < x_0(E)$

Let $y = \frac{x}{x_0(E)}$ $\qquad x = 0 \rightarrow y = 0$

$x = x_0(E) \rightarrow y = 1$

$dy = \frac{dx}{x_0}$

$\rightarrow x^n = y^n x_0^n$

$$\longrightarrow T(E) = \frac{2\sqrt{2m}}{\sqrt{A}} \int_0^1 \frac{x_0\,dy}{x_0^{n/2}\sqrt{1-y^n}}$$

$$T(E) = \frac{2\sqrt{2m}}{\sqrt{A}} \frac{1}{x_0^{(n/2-1)}} \int_0^1 \frac{dy}{\sqrt{1-y^n}}$$

Beta function:

$$B(x,y) = \int_0^1 dt \quad t^{x-1} (1-t)^{y-1} dt$$

$$= \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \longleftarrow \text{Gamma functions}$$

Now: $\int_0^1 \frac{dy}{\sqrt{1-y^n}}$ let $t = y^n \rightarrow$ $y=0 \rightarrow t=0$, $y=1 \rightarrow t=1$

$$dt = n y^{n-1} dy$$
$$= n(t^{\frac{1}{n}})^{n-1} dy$$
$$= n\, t^{(1-\frac{1}{n})} dy$$

Thus, $$\int_0^1 \frac{dy}{\sqrt{1-y^n}} = \int_0^1 \frac{dt}{n\, t^{(1-1/n)} \sqrt{1-t}}$$

$$= \frac{1}{n} \int_0^1 t^{\frac{1}{n}-1} (1-t)^{-\frac{1}{2}} dt$$

$$\rightarrow T(E) = \frac{2\sqrt{2m}}{\sqrt{A}} \frac{1}{x_0^{(n/2-1)}} \frac{1}{n} \int_0^1 dt \quad t^{\frac{1}{n}-1} (1-t)^{\frac{1}{2}-1} dt$$

$$= \frac{2\sqrt{2m}}{\sqrt{A}} \frac{1}{x_0^{(n/2-1)}} \frac{1}{n} B\left(\frac{1}{n}, \frac{1}{2}\right)$$

$$= \frac{2\sqrt{2m}}{\sqrt{A}} \frac{1}{\left(\frac{E}{A}\right)^{\frac{1}{2}-\frac{1}{n}}} \frac{1}{n} \frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$$

③

<u>Now:</u> $\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$ $\qquad \left(\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\,dx \right)$

$\Gamma(1) = 1$

$$\Gamma(z) = \frac{\Gamma(z+1)}{z} \quad \rightarrow \quad \Gamma(z-1) = \frac{\Gamma(z)}{z-1}$$

$$\boxed{T(E) = \frac{2\sqrt{2m}}{A^{\frac{1}{n}}} \frac{1}{E^{\frac{1}{2}-\frac{1}{n}}} \frac{\sqrt{\pi}}{n} \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n}+\frac{1}{2})}}$$

<u>NOTE:</u> $n = 2 \rightarrow U = Ax^2$, $A = \frac{1}{2}k$ (SHO)

$E = \frac{1}{2}k x_0^2$

$$T(E) = \frac{\cancel{2}\sqrt{2m}}{\sqrt{A}} \cancel{\left(\frac{1}{E^0}\right)} \frac{\sqrt{\pi}}{\cancel{2}} \frac{\Gamma(\frac{1}{2})}{\Gamma(1)}$$

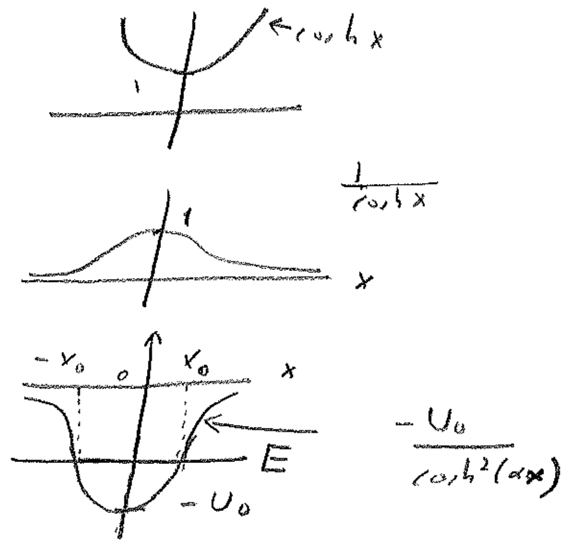
$$= \frac{\sqrt{2m}}{\sqrt{\frac{1}{2}k}} \frac{\pi}{1}$$

$$= 2\pi\sqrt{\frac{m}{k}} \qquad \text{(for SHO)}$$

<u>Better to combine terms:</u>

$$\boxed{T(E) = \frac{2}{n}\sqrt{\frac{2m\pi}{E}} \left(\frac{E}{A}\right)^{\frac{1}{n}} \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n}+\frac{1}{2})}}$$

(b) $U = \dfrac{-U_0}{\cosh^2(\alpha x)}$, $-U_0 < E < 0$



Since even potential

$$T(E) = 2\sqrt{2m}\int_0^{x_0(E)} \frac{dx}{\sqrt{E - U(x)}}$$

at turning points $\pm x_0$:

$$E = U(x_0)$$
$$= \frac{-U_0}{\cosh^2(\alpha x_0)} \quad \rightarrow \quad \cosh(\alpha x_0) = \sqrt{\frac{-U_0}{E}} = \sqrt{\frac{U_0}{|E|}}$$

$$\boxed{x_0 = \frac{1}{\alpha}\cosh^{-1}\left(\sqrt{\frac{U_0}{|E|}}\right)}$$

Thus,

$$T(E) = 2\sqrt{2m}\int_0^{x_0(E)} \frac{dx}{\sqrt{\dfrac{-U_0}{\cosh^2(\alpha x_0)} + \dfrac{U_0}{\cosh^2(\alpha x)}}}$$

$$= 2\sqrt{2m}\int_0^{x_0(E)} \frac{dx}{\sqrt{U_0}\sqrt{\dfrac{\cosh^2(\alpha x_0) - \cosh^2(\alpha x)}{\cosh^2(\alpha x_0)\cosh^2(\alpha x)}}}$$

$$= 2\sqrt{\frac{2m}{U_0}}\cosh(\alpha x_0)\int_0^{x_0} \frac{\cosh(\alpha x)\, dx}{\sqrt{\cosh^2(\alpha x_0) - \cosh^2(\alpha x)}}$$

$$= 2\sqrt{\frac{2m}{U_0}}\int_0^{x_0} \frac{\cosh(\alpha x)\,dx}{\sqrt{1 - \left(\dfrac{\cosh(\alpha x)}{\cosh(\alpha x_0)}\right)^2}}$$

Let $u = 1 - \left(\frac{\cosh(\alpha x)}{\cosh(\alpha x_0)}\right)^2$ $ch^2 - sh^2 = 1$

$$du = -\frac{2\cosh(\alpha x)}{\cosh^2(\alpha x_0)}\,\alpha\sinh(\alpha x)\,dx$$

Thus, $\cosh(\alpha x)\,dx = -\frac{1}{2}\frac{\cosh^2(\alpha x_0)\,du}{\alpha\sinh(\alpha x)}$

$$= -\frac{1}{2\alpha}\frac{\cosh^2(\alpha x_0)}{\sqrt{\cosh^2(\alpha x)-1}}\,du$$

$$= -\frac{1}{2\alpha}\frac{\cosh^2(\alpha x_0)\,du}{\sqrt{\cosh^2(\alpha x_0)(1-u)-1}}$$

$$= -\frac{1}{2\alpha}\frac{\cosh^2(\alpha x_0)\,du}{\sqrt{\sinh^2(\alpha x_0)-\cosh^2(\alpha x_0)\,u}}$$

$\rightarrow$ $$\int_0^{x_0}\frac{\cosh(\alpha x)\,dx}{\sqrt{1-\left(\frac{\cosh(\alpha x)}{\cosh(\alpha x_0)}\right)^2}} = \int_{1-\left(\frac{1}{\cosh^2(\alpha x_0)}\right)}^{0} -\frac{1}{2\alpha}\frac{\cosh^2(\alpha x_0)\,du}{\sqrt{u}\sqrt{\sinh^2(\alpha x_0)-\cosh^2(\alpha x_0)\,u}}$$

$$= -\frac{1}{2\alpha}\cosh^{\cancel{2}}(\alpha x_0)\int_{\tanh^2(\alpha x_0)}^{0}\frac{du}{\sqrt{u}\,\cancel{\cosh(\alpha x_0)}\sqrt{\tanh^2(\alpha x_0)-u}}$$

$$= -\frac{1}{2\alpha}\cosh(\alpha x_0)\int_{\tanh^2(\alpha x_0)}^{0}\frac{du}{\sqrt{u}\sqrt{\tanh^2(\alpha x_0)-u}}$$

$$\int_0^{x_0} \frac{\cosh(\alpha x)\,dx}{\sqrt{1-\left(\frac{\cosh(\alpha x)}{\cosh(\alpha x_0)}\right)^2}} = \frac{1}{2\alpha}\cosh(\alpha x_0)\int_0^a \frac{du}{\sqrt{(a-u)u}} \quad , \quad a = \tanh^2(\alpha x_0)$$

Now:

$$\begin{aligned}(a-u)u &= au - u^2 \\ &= -(u^2 - au) \\ &= -\left(u^2 - au + \frac{a^2}{4} - \frac{a^2}{4}\right) \\ &= -\left[\left(u-\frac{a}{2}\right)^2 - \frac{a^2}{4}\right]\end{aligned}$$

Let: $\left(u - \frac{a}{2}\right) = x \cdot \frac{a}{2}$, $du = \frac{a}{2}\,dx$

$$\begin{aligned}\sqrt{(a-u)u} &= \sqrt{-\left[x^2\frac{a^2}{4} - \frac{a^2}{4}\right]} \\ &= \frac{a}{2}\sqrt{1-x^2}\end{aligned}$$

$u = 0 \rightarrow x = -1$

~~u~~
$u = a \rightarrow x = 1$

$$\text{Integral} = \frac{1}{2\alpha}\cosh(\alpha x_0)\cancel{\frac{a}{2}}\int_{-1}^{1}\frac{dx}{\cancel{\frac{a}{2}}\sqrt{1-x^2}}$$

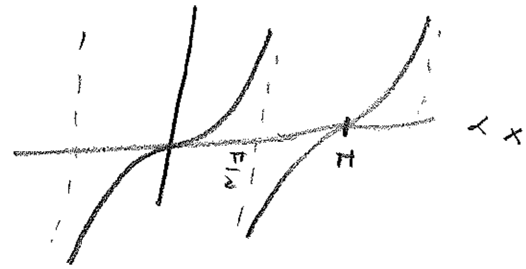
$$= \frac{1}{2\alpha}\cosh(\alpha x_0)\int_{-\pi/2}^{\pi/2}\frac{\cancel{\cos\theta}\,d\theta}{\cancel{\sqrt{1-\sin^2\theta}}}$$

$x = \sin\theta$
$dx = \cos\theta\,d\theta$
$x = \mp 1 \leftrightarrow \theta = \mp \frac{\pi}{2}$

$$= \frac{1}{2\alpha}\cosh(\alpha x_0)\,\pi$$

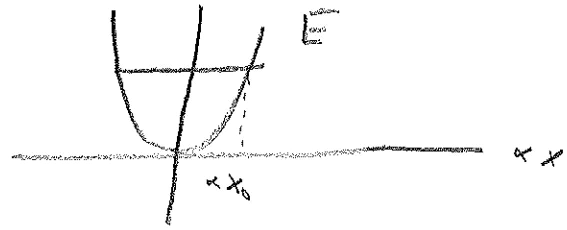
Thus, $T(E) = \cancel{2}\sqrt{\frac{2m}{U_0}}\,\frac{1}{\cancel{2}\alpha}\cosh(\alpha x_0)\,\pi = \boxed{\frac{\pi}{\alpha}\sqrt{\frac{2m}{|E|}} = T(E)}$

c) $U = U_0 \tan^2(\alpha x)$



Turning pt:

$$E = U(x_0)$$
$$= U_0 \tan^2(\alpha x_0)$$



$$\rightarrow \sqrt{\frac{E}{U_0}} = \tan(\alpha x_0)$$

$$\boxed{x_0 = \frac{1}{\alpha}\tan^{-1}\sqrt{\frac{E}{U_0}}} = x_0(E)$$

Even potential:

$$T(E) = 2\sqrt{2m}\int_0^{x_0(E)} \frac{dx}{\sqrt{E - U(x)}}$$

$$= 2\sqrt{2m}\int_0^{x_0} \frac{dx}{\sqrt{U_0}\sqrt{\tan^2(\alpha x_0) - \tan^2(\alpha x)}}$$

$$= 2\sqrt{\frac{2m}{U_0}}\,\frac{1}{\tan(\alpha x_0)}\int_0^{x_0} \frac{dx}{\sqrt{1 - \frac{\tan^2(\alpha x)}{\tan^2(\alpha x_0)}}}$$

Let $u = 1 - \left(\frac{\tan(\alpha x)}{\tan(\alpha x_0)}\right)^2$ $\quad x = 0 \rightarrow u = 1$, $\quad x = x_0 \rightarrow u = 0$

$$du = -2\left(\frac{\tan(\alpha x)}{\tan^2(\alpha x_0)}\right)\alpha \sec^2(\alpha x)\,dx$$

$$= -2\left(\frac{\tan(\alpha x)}{\tan(\alpha x_0)}\right)^2 \frac{\alpha(1 + \tan^2\alpha x)}{\tan(\alpha x)}dx = \frac{-2\alpha(1-u)\,dx}{\sqrt{1-u}\;\tan(\alpha x_0)}$$

$$\left(\frac{\tan(\alpha x)}{\tan(\alpha x_0)}\right)^2 = 1 - u$$

$$\tan \alpha x = \sqrt{1-u}\;\tan(\alpha x_0)$$

$$1 + \tan^2\alpha x = 1 + (1-u)\tan^2(\alpha x_0)$$

$$\int_0^{x_0} \frac{dx}{\sqrt{1-\left(\frac{\tan(\alpha x)}{\tan(\alpha x_0)}\right)^2}} = ?$$

Let:

$$\sin\theta = \frac{\tan(\alpha x)}{\tan(\alpha x_0)} \qquad \begin{array}{l} x=0 \rightarrow \theta = 0 \\ x = x_0 \rightarrow \theta = \pi/2 \end{array}$$

$$\cos\theta\, d\theta = \frac{1}{\tan(\alpha x_0)} \sec^2(\alpha x)\, \alpha\, dx$$

$$= \frac{\alpha}{\tan(\alpha x_0)} \left[1 + \tan^2(\alpha x)\right] dx$$

$$= \frac{\alpha}{\tan(\alpha x_0)} \left(1 + \tan^2(\alpha x_0)\sin^2\theta\right) dx$$

Thus, $$dx = \frac{\tan(\alpha x_0)}{\alpha} \frac{\cos\theta\, d\theta}{\left[1 + \tan^2(\alpha x_0)\sin^2\theta\right]}$$

$$\rightarrow \int_0^{x_0} \frac{dx}{\sqrt{1-\left(\frac{\tan(\alpha x)}{\tan(\alpha x_0)}\right)^2}} = \int_0^{\pi/2} \frac{\tan(\alpha x_0)}{\alpha} \frac{\cancel{\cos\theta}\, d\theta}{\cancel{\sqrt{1-\sin^2\theta}}\left[1 + \tan^2(\alpha x_0)\sin^2\theta\right]}$$

$$= \frac{\tan(\alpha x_0)}{\alpha} \int_0^{\pi/2} \frac{d\theta}{\left(1 + \tan^2(\alpha x_0)\sin^2\theta\right)}$$

$$= \frac{a}{\alpha} \int_0^{\pi/2} \frac{d\theta}{\left(1 + a^2\sin^2\theta\right)} \qquad \left(a = \tan(\alpha x_0)\right)$$

☆ $\boxed{\text{Trick}}$ divide numerator and denominator by $\frac{1}{\cos^2\theta}$

$$\frac{d\theta}{\cos^2\theta} = \sec^2\theta\, d\theta = d\left[\tan\theta\right]$$

$s^2 + c^2 = 1$
$t^2 + 1 = \sec^2$

$$\frac{1 + a^2\sin^2\theta}{\cos^2\theta} = \sec^2\theta + a^2\tan^2\theta = 1 + \tan^2\theta(1+a^2)$$

Thus,

$$\frac{a}{\alpha}\int_0^{\pi/2} \frac{d\theta}{1+a^2\sin^2\theta} = \frac{a}{\alpha}\int_0^{\pi/2} \frac{\frac{d\theta}{\cos^2\theta}}{\left(\frac{1+a^2\sin^2\theta}{\cos^2\theta}\right)}$$

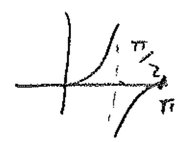
$$= \frac{a}{\alpha}\int_0^{\pi/2} \frac{\sec^2\theta\, d\theta}{1+(1+a^2)\tan^2\theta}$$

<u>Let</u>: $\tan x = \sqrt{1+a^2}\tan\theta$

$$1+(1+a^2)\tan^2\theta = 1+\tan^2 x = \sec^2 x$$

$$\sec^2 x\, dx = \sqrt{1+a^2}\,\sec^2\theta\, d\theta$$

$$\rightarrow \sec^2\theta\, d\theta = \frac{1}{\sqrt{1+a^2}}\sec^2 x\, dy$$

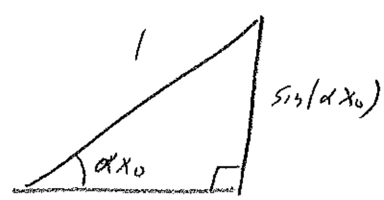


Thus,

$$\frac{a}{\alpha}\int_0^{\pi/2} \frac{d\theta}{1+a^2\sin^2\theta} = \frac{a}{\alpha}\frac{1}{\sqrt{1+a^2}}\int_0^{\pi/2}\frac{\cancel{\sec^2 x}\,dx}{\cancel{\sec^2 x}}$$

$$= \frac{a}{\alpha}\frac{1}{\sqrt{1+a^2}}\frac{\pi}{2}$$

$$= \frac{\tan(\alpha x_0)}{\alpha}\frac{1}{\sec(\alpha x_0)}\frac{\pi}{2}$$



~~sec(αx₀)~~

$$1+\tan^2(\alpha x_0) = \sec^2(\alpha x_0)$$

$$\sec(\alpha x_0) = \sqrt{1+\tan^2(\alpha x_0)} = \sqrt{1+\frac{E}{V_0}} = \sqrt{\frac{V_0+E}{V_0}}$$

$$\rightarrow T(E) = \cancel{2}\sqrt{\frac{2m}{V_0}}\frac{1}{\cancel{\tan(\alpha x_0)}}\frac{\cancel{\tan(\alpha x_0)}}{\alpha}\frac{1}{\sec(\alpha x_0)}\frac{\pi}{\cancel{2}}$$

$$= \frac{\pi}{\alpha}\sqrt{\frac{2m}{V_0}}\frac{1}{\sec(\alpha x_0)} = \boxed{\frac{\pi}{\alpha}\sqrt{\frac{2m}{E+V_0}}}$$