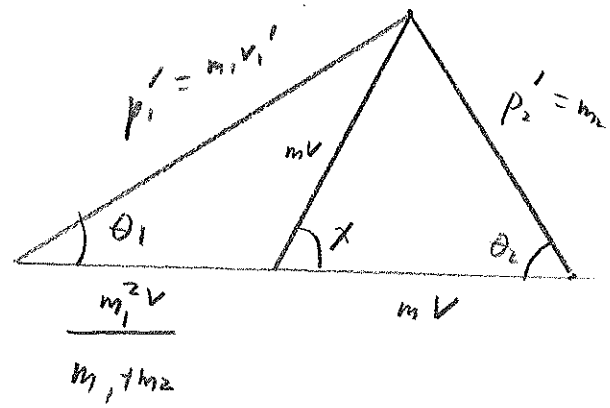
Sec 17, Problem

Express v_1', v_2' in terms of θ_1, θ_2 (m_2 at rest)



From (17.5), (1):

$$V_2' = \frac{2m_1 V}{m_1+m_2}\sin\left(\tfrac{1}{2}\chi\right)$$

$$= \frac{2m_1 V}{m_1+m_2}\sin\left(\frac{\pi}{2}-\theta_2\right)$$

$$\theta_2 = \tfrac{1}{2}(\pi-\chi) = \frac{\pi}{2}-\frac{\chi}{2}$$

$$\rightarrow \frac{\chi}{2} = \frac{\pi}{2}-\theta_2$$

$$= \frac{2m_1 V}{m_1+m_2}\left(\underbrace{\sin\frac{\pi}{2}}_{=1}\cos\theta_2 - \underbrace{\cos\frac{\pi}{2}}_{0}\sin\theta_2\right)$$

$$= \frac{2m_1 V}{m_1+m_2}\cos\theta_2$$

$$= 2\left(\frac{m}{m_2}\right)V\cos\theta_2 \quad \text{where} \quad m \equiv \frac{m_1 m_2}{m_1+m_2}$$

From left triangle:

$$(mV)^2 = \frac{m_1^4 V^2}{(m_1+m_2)^2} + m_1^2 v_1'^2 - 2\frac{m_1^2 V}{m_1+m_2}m_1 V_1'\cos\theta_1$$

$$\frac{m_1^2 m_2^2}{(m_1+m_2)^2}V^2 = \frac{m_1^4 V^2}{(m_1+m_2)^2} + m_1^2 V_1'^2 - \frac{2m_1^3 V V_1'\cos\theta_1}{(m_1+m_2)}$$

Quadratic equation for v_1':

$$v_1'^2 - 2\left(\frac{m_1 v}{m_1+m_2}\right)\cos\theta_1 \, v_1' + \frac{v^2(m_1^2-m_2^2)}{(m_1+m_2)^2} = 0$$

Thus,

$$v_1' = \frac{2\left(\frac{m_1 v}{m_1+m_2}\right)\cos\theta_1 \pm \sqrt{4\left(\frac{m_1 v}{m_1+m_2}\right)^2\cos^2\theta_1 - \frac{4v^2(m_1^2-m_2^2)}{(m_1+m_2)^2}}}{2}$$

$$= \left(\frac{m_1 v}{m_1+m_2}\right)\cos\theta_1 \pm \left(\frac{m_1 v}{m_1+m_2}\right)\sqrt{\cos^2\theta_1 - \left(1-\left(\frac{m_2}{m_1}\right)^2\right)}$$

$$= \left(\frac{m_1 v}{m_1+m_2}\right)\left[\cos\theta_1 \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \sin^2\theta_1}\right]$$

NOTE: $\theta_{1,max}$ determined by $\left(\frac{m_2}{m_1}\right)^2 - \sin^2\theta_{1,max} = 0$

$$\rightarrow \boxed{\sin\theta_{1,max} = \frac{m_2}{m_1}}$$

Rewrite:

$$\boxed{\left(\frac{v_1'}{v}\right) = \left(\frac{m_1}{m_1+m_2}\right)\cos\theta_1 \pm \frac{1}{m_1+m_2}\sqrt{m_2^2 - m_1^2\sin^2\theta_1}}$$

For $m_1 < m_2$, only one solution. When $\theta_1 = 0$, should get $v_1' = v \rightarrow +\sqrt{\ }$

Alternative calculation for v_2':

Right triangle: 

$$\cancel{(mv)}^2 = \cancel{(mv)}^2 + (m_2 v_2')^2 - 2\, mv\, m_2 v_2' \cos\theta_2$$

$$2\, mv\, m_2 v_2' \cos\theta_2 = m_2^2 v_2'^2$$

$$\rightarrow \boxed{v_2' = 2\left(\frac{m}{m_2}\right) v \cos\theta_2}$$

where $m = \dfrac{m_1 m_2}{m_1 + m_2}$