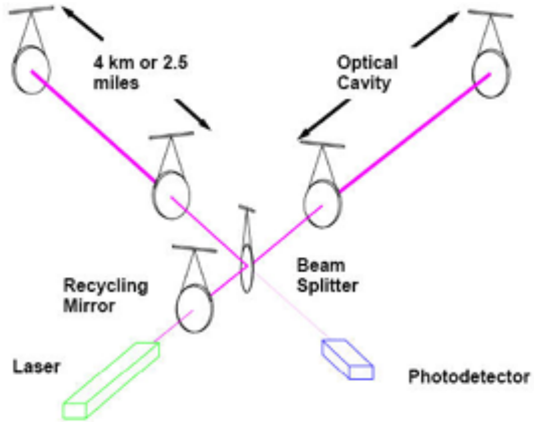

Figure 2 shows a basic schematic of the LIGO project, with laser light travelling down the interferometer arms and back.

Initially, it may seem as though light would not be able to be used to detect gravitational waves, as it is stretched or compressed by the gravitational wave, disallowing one to measure the stretch or compression of space-time. However, as explained in a paper by Peter Saulson (If light waves are stretched by gravitational waves, how can we use light as a ruler to detect gravitational waves?), the laser light that continually enters an interferometer arm has a fixed wavelength (relative to the light that is currently propagating in the arm), allowing one to detect changes in the arrival times of the returning wave fronts. The effect of the wave propagating along the z-axis can be expressed mathematically using the space-time interval ds, where $h_+(t)$ is the amplitude of the wave:

$$ds^2 = -c^2dt^2 + \big(1 + h_+(t+z)\big)dx^2 + \big(1 - h_+(t+z)\big)dy^2 + dz^2. \qquad (1)$$

Two nearby events on the path of an electromagnetic wave front are separated by an interval of *ds*=0. Using this and setting *c*=1, one can solve for *dt*, which will ultimately be used for calculations involving the detection of the gravitational wave:

$$dt^2 = \big(1 + h_+(t+z)\big)dx^2 + \big(1 - h_+(t+z)\big)dy^2 + dz^2. \qquad (2)$$

Also, for weak gravitational waves, we can use the approximation given by the following equation, which represents the magnitude of the stretching or compressing factor of the gravitational wave along the x-axis,

$$\sqrt{1 + h_+(t)} \approx (1 + \tfrac{1}{2}h_+(t)). \qquad (3)$$

Using (2) and (3), the response of an interferometer to a passing gravitational wave can be derived. The propagation time of the wave front outward along the x-axis is given by

$$T_{out,x}(t) = L + \tfrac{1}{2}\int_0^L h_+(t - L + x)dx, \qquad (4)$$

where L is the length of the interferometer arm in the absence of a gravitational wave. Similarly, the returning time of the wave front is given by

$$T_{back,x}(t) = L + \frac{1}{2}\int_0^L h_+(t-x)dx. \qquad (5)$$

The total round trip time along the x arm, which is used in the program, can be found by adding $T_{out,x}(t\text{-}L)$ and $T_{back,x}(t)$ which gives

$$T_{tot,x}(t) = 2L + \frac{1}{2}\int_0^L [h_+(t-x) + h_+(t-L+x)]dx. \qquad (6)$$

In order to find the difference in wave front propagation times, it is necessary to take the difference in total round trip times between the x and y arms:

$$\Delta T(t) = T_{tot,x}(t) - T_{tot,y}(t). \qquad (7)$$

Since a similar result holds for wave front propagation down the y arm compared to the x arm (with h_+ simply replaced by $-h_+$), (6) and (7) lead to

$$\Delta T(t) = \int_0^L [h_+(t-x) + h_+(t-2L+x)]dx. \qquad (8)$$

This calculation can be done in either the time domain or the frequency domain. The frequency domain results are found by taking the Fourier transform of both sides of (8) by using the definition

$$\tilde{h}(f) = \int_{-\infty}^{\infty} h(t)e^{-i2\pi ft}dt. \qquad (9)$$

After transforming both sides, the final expression in the frequency domain can be found:

$$\widetilde{\Delta T}(f) = R(f)\tilde{h}(f), \qquad (10)$$

where

$$R(f) = 2L\,\mathrm{sinc}(2\pi fL)e^{-i2\pi fL}. \qquad (11)$$

Depending on the situation and desired calculation, expressions such as (6), (8), and (10) can be used in the detection of a gravitational wave. While the actual detection of a gravitational wave has, at this point, still eluded all efforts of programs such as LIGO and VIRGO (the European version of an interferometer based detection system), much theoretical and experimental progress has been made since the days of Newton. Though the detection of gravitational waves may still be years away, it is certain that their detection will be a profound discovery.