7. Sound Intensity, Hearing, Just Noticeable Difference (JND)

PURPOSE AND BACKGROUND

We can hear a wide range of sound intensities and frequencies. The intensity between the threshold of hearing and the threshold of pain varies by a factor of 10^{12}, i.e., by 12 orders of magnitude or 120 decibel. The corresponding range in the amplitude of *air pressure fluctuations* is a factor of 10^6. In view of this extreme range in sound intensity level, numbers are most conveniently expressed in power-of-ten notation and with a decibel or dB-scale.

Here we study sound intensity levels (SIL) and the frequency response of the human ear. We also discuss "just noticeable differences" (JND) in intensity and frequency that the ear can discern.

The ear is sensitive to a range in frequencies from about 20 Hz to 20 kHz. This audible range thus covers a factor of 1000 or 10^3 in frequency, which is not nearly as large as the intensity range of 10^{12}. In order to cover these large ranges, the ear response is compressed or logarithmic with respect to both frequency and sound intensity.

I Theory and Experiment

The amplitude of a sound wave corresponds to air pressure fluctuations (compressions and rarefactions of the air) in a longitudinal wave.

The *threshold of hearing* is a sound intensity at the ear of

$$I_0 = 1 \times 10^{-12}\ \mathrm{W/m^2} \quad \text{at} \quad f = 1000\ \mathrm{Hz} \tag{1}$$

This is the reference intensity for sound intensity measurements.

The *sound intensity level* (SIL) is defined by comparing any intensity I to the threshold of hearing I_0 according to

$$\mathrm{SIL} = 10\ \log(\mathrm{I/I_0})\ \mathrm{dB} \tag{2}$$

where the logarithm is taken to the base 10. The inverse equation is

$$I = I_0\, 10^{(\mathrm{SIL}/10\ \mathrm{dB})} \tag{3}$$

SIL is measured in *decibels* or dB.

For example, let the sound intensity in a room be $I = 1 \times 10^{-6}\ \mathrm{W/m^2}$. The SIL is then

$$\mathrm{SIL} = 10\ \log\left(\frac{1 \times 10^{-6}}{1 \times 10^{-12}}\right)\ \mathrm{dB} = 10\ \log 10^6\ \mathrm{dB} = 10 \times 6\ \mathrm{dB} = 60\ \mathrm{dB} \tag{4}$$

The SIL also can be used to express a change in intensity from one value to another, without referring to the threshold of hearing I_0. We are then dealing with a *change* in SIL, denoted by $\Delta\,\mathrm{SIL}$, and not the SIL itself.

For instance, if the intensity I doubles to $2I$, we have

$$\Delta\,\mathrm{SIL} = 10\ \log(2I/I)\ \mathrm{dB} = 10\ \log 2\ \mathrm{dB} = 10 \times 0.3\ \mathrm{dB} = 3\ \mathrm{dB} \tag{5}$$

Therefore, a doubling in intensity corresponds to an increase of 3 dB in the SIL.

1. The sound intensity level in a typical environment is generally much higher than the threshold of hearing. For example, a typical sound intensity level that one might measure for the background noise in the laboratory is $\mathrm{SIL} = 70$ dB. What is the sound intensity I of this background noise, expressed in units of $\mathrm{W/m^2}$? Hint: Use equation (3) to find I.

2. Assume that the sound intensity level for one student clapping in the laboratory is $\mathrm{SIL}_1 = 75$ dB. Calculate the theoretical increase in the sound intensity level, $\Delta\,\mathrm{SIL}$, if the intensity I_{10} for ten students clapping is ten times the intensity I_1 for one student clapping.

3. Using your answer to the previous question, what would you expect the sound intensity level SIL_{10} to be for 10 students clapping? Hint: $\Delta\,\mathrm{SIL} = \mathrm{SIL}_{10} - \mathrm{SIL}_1$

4. At a frequency $f = 1000$ Hz, an intensity of $I = 1\ \mathrm{W/m^2}$ becomes quite painful to the ear. What is the sound intensity level in dB of a 1000 Hz sinusoidal tone at the threshold of pain?

II Frequency Response of the Ear

The ear can hear sound over a wide range of frequencies from about 20 Hz to 20 kHz. However, the perceived loudness varies quite dramatically with frequency. The so-called *Fletcher-Munson curves* in Figure 1 show lines of equal perceived loudness. The curve at the bottom marked "0 phons"

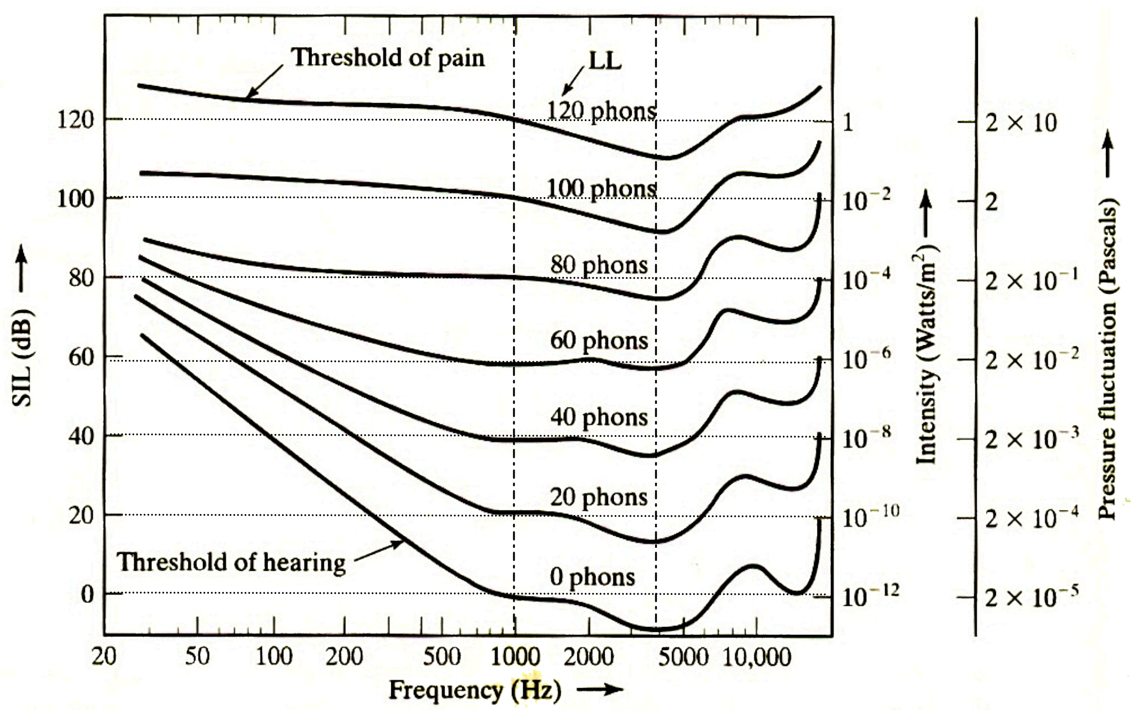

Figure 1: Fletcher-Munson curves of equal loudness. (From "Physics of Sound" by R.A. Berg and D.G. Stork.)

represents the threshold of hearing, and the line marked "120 phons" represents the threshold of pain. Each curve has a "phon" designation and indicates equal perceived loudness as a function of frequency. The "decibel" and "phon" scales agree by convention at a frequency of 1000 Hz (see Figure 1). For example, if a loudspeaker produces a 1000 Hz tone with $\mathrm{SIL} = 60$ dB at your location, you perceive this sound intensity as a loudness of 60 phon. If on the other hand the speaker produces a tone at 100 Hz with the same $\text{SIL} = 60$ dB, you hear this as less loud than the 1000 Hz tone. In order for the two frequencies to sound equally loud, the speaker must produce the 100 Hz tone at about $\text{SIL} = 70$ dB instead. Verify this on the curve labeled "60 phons".

You can also see from Figure 1 that the human ear is most sensitive to sound around 4000 Hz, where the Fletcher-Munson curves dip lowest. Therefore, if you follow a Fletcher-Munson curve from 4000 Hz to lower frequencies, the sound intensity must be raised to be perceived as equally loud. The same applies to higher frequencies above 4000 Hz.

1. A loudspeaker produces a 1000 Hz tone at an $\text{SIL} = 40$ dB. From the Fletcher-Munson curve labeled "40 phon" in Figure 1, what SIL would be needed at 200 Hz for the tone to be perceived as equally loud?

2. What SIL would be needed at 4000 Hz?

III Just Noticeable Difference in Intensity

The just noticeable difference (JND) in intensity is the smallest change in SIL that the ear can discern. Usually a 25% or 1 dB change in intensity is detected. This depends somewhat on sound intensity and frequency as can be seen in Figure 2. As the intensity or frequency decreases, the ear becomes less sensitive to changes in intensity.

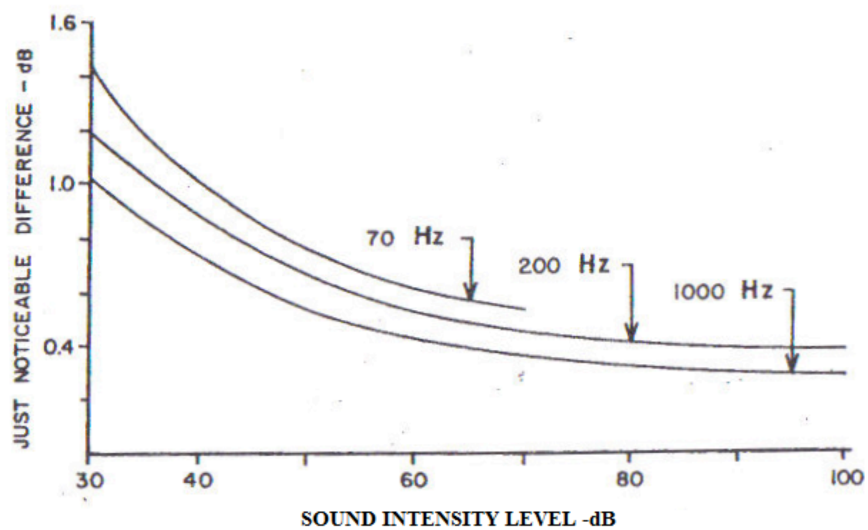

Figure 2: Just noticeable difference curves in intensity for 70 Hz, 200 Hz, and 1000 Hz sinusoidal tones. (From "Physics of Sound" by R.A. Berg and D.G. Stork.)

1. Express a 25% change in intensity I as a change in dB. Hint: Calculate $\Delta\,\text{SIL}$ for $I_2 = 1.25 I_1$.

2. From Figure 2, what is the value of the JND in intensity for the 1000 Hz curve at 80 dB?

3. Suppose you compare the intensity of a square wave at $f = 1000$ Hz to that of a sine wave at $f = 1000$ Hz. For which do you get a smaller JND in intensity—i.e., for which can you hear smaller differences in SIL? Can you give a reason for this? (Hint: Consider the harmonics in the square wave.)

IV Just Noticeable Difference in Frequency

In addition to being able to discern changes in sound intensity, we have an even better ability to notice changes in frequency. Figure 3 shows the JND in frequency, comparing it to the size of the critical bands on the cochlea.

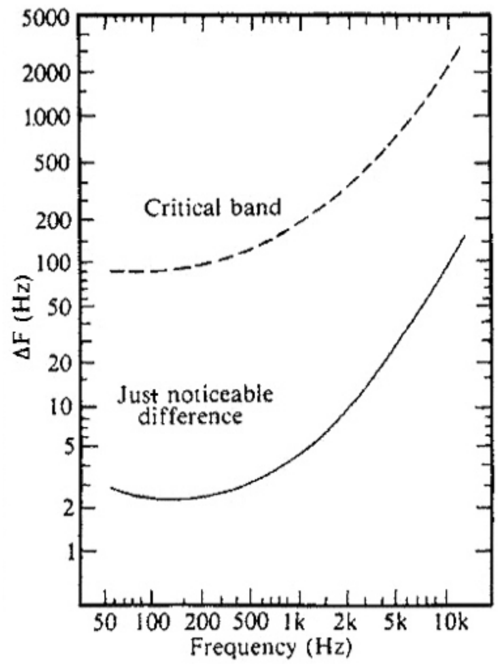

Figure 3: Just noticeable difference in frequency, comparing it to the size of the critical bands on the cochlea. (From "Science of Sound," by Rossing, Moore, and Wheeler.)

To experimentally determine the just noticeable difference in frequency, we play two pure tones one right after the other, starting with the same frequency. We then increase one frequency slightly and keep playing both tones in succession. The JND in frequency is when you can first discern a difference in the frequency (i.e., pitch) of the two tones. One can express the JND in frequency as the either the difference between the two frequencies or as a percentage relating the frequency difference to the starting frequency.

1. According to Figure 3 what is the JND in frequency at a frequency of 200 Hz? (Express the JND both as a difference of frequencies and as a percentage.)
2. What is the JND in frequency at a frequency of 2000 Hz?

V Loudness in Sones

The decibel values that we have discussed above are based on *objective* measurements of the sound intensity. There also exists a *subjective* "sone" scale that tells what sounds "twice as loud" to many persons. Such a "twice as loud curve" is shown as a straight line in Figure 4. On the sone scale, 1 sone corresponds to a loudness level of 40 phon for a pure sine wave with $f = 1000$ Hz. (Recall that for the special case of a pure tone at a frequency of 1000 Hz, the number of phon is the same as the number of dB.)

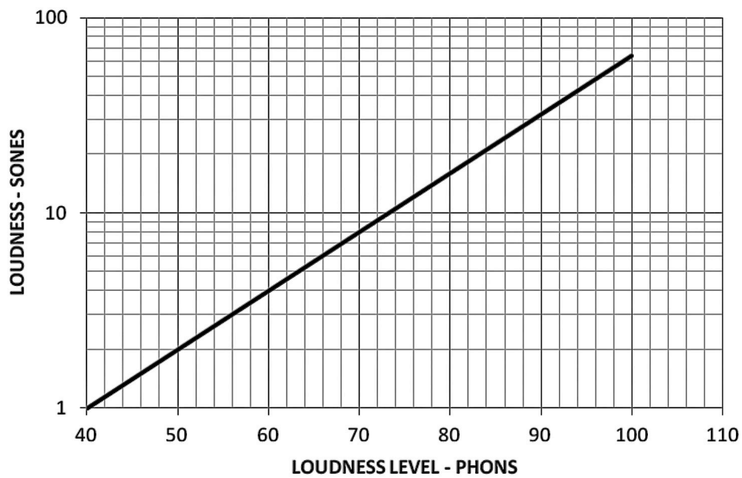

Figure 4: Sone scale, with "twice as loud" meaning a doubling in the sone number. The reference is 1 sone at a loudness level of 40 phon. The phon scale is the same as the dB scale for a pure tone at 1000 Hz.

Figure 4 shows that, in order for sound to be perceived as twice as loud, the sound intensity level must be higher by 10 phon (or 10 dB at 1000 Hz). For example, for an increase in loudness from 1 sone to 2 sone, the sound intensity increases by 10 phon from 40 phon to 50 phon. Generally, for every increase in sound intensity by 10 phon, the sone number doubles. Example: For a doubling in loudness from 4 to 8 sone, the sound intensity increases from 60 to 70 phon.

1. According to Figure 4, what is the increase in phon for a doubling in loudness from 10 to 20 sone?
2. How many times louder does a 90 phon tone sound than a 60 phon tone?

Application: The sone scale is used for specifying the loudness of fans and appliances. For instance, quiet bathroom fans have a rating of 1 to 2 sones; louder ones have a rating of 3 to 4 sones or more.