If light waves are stretched by gravitational waves, how can we use light as a ruler to detect gravitational waves?

Peter R. Saulson
Department of Physics, Syracuse University, Syracuse, New York 13244-1130

(Received 19 August 1996; accepted 10 December 1996)

I give an answer to the frequently asked question of the article's title, based on an analogy between the description of gravitational waves in the transverse-traceless gauge and the description of an expanding universe in comoving coordinates. Both use freely falling masses to define the coordinate system. Taking advantage of the insight that has been achieved in cosmology, I show how to understand the operation of an interferometric gravitational wave detector in a way that resolves the apparent paradox.

I. INTRODUCTION

The search for gravitational waves has been an active field in experimental physics for over three decades. The first detectors, starting with those of the field's founder Joseph Weber and continuing to some of the best detectors operating today, respond to the stretching and compressing force that a gravitational wave would apply to a massive cylindrical bar. The resulting vibrations of the bar, persisting long after a pulse of gravitational waves has passed, are registered in a highly sensitive accelerometer mounted on the end of the bar. While progress in sensitivity has been dramatic over the years, there has been as yet no unequivocal detection of gravitational waves.[1]

Free-mass interferometers are about to make a substantial improvement in our ability to detect weak gravitational waves. These instruments differ from the resonant-mass detectors described above in several ways. Two of the most important are:

(1) Instead of using a single mass whose ends are pushed together or pulled apart, separate test masses hung as pendulums (up to kilometers apart) are used; since the motion of masses in response to a gravitational wave is proportional to their separation, this can provide a substantial advantage over the bars whose ends are separated by only a few meters.
(2) Since an accelerometer won't be sensitive to the relative motion of the separate masses, a sophisticated version of a Michelson interferometer is used to register the small changes in separation between the widely separated test masses.

The promise of interferometric gravitational wave detectors is just now about to be realized, with the construction of several kilometer-scale interferometers. The LIGO (Laser Interferometer Gravitational-Wave Observatory) Project is now building two 4-km interferometer facilities in the United States. The VIRGO Project, a French–Italian collaboration, is building a 3-km interferometer in Italy. Scientists in a number of other countries are also readying plans for the next generation of sensitive gravitational wave detectors.[2]

Understanding how an interferometer responds to a gravitational wave makes a wonderful thought experiment in relativity. The usual explanation says that the test masses move apart or together as the space expands or contracts between them. The response of an interferometer to such a distortion of space has been studied by a number of authors, and is by now well understood.[3] The basic idea is to consider the travel time of light pulses that take a round trip between two freely falling masses. As the masses are moved together or apart by the gravitational wave, that round-trip light travel time varies in an intuitively sensible way. If we replace the light pulses with steady beams of light, the travel time variations can be observed as phase shifts in the returning light. As we will show in Sec. II, a Michelson interferometer is a good way to make sensitive measurements of these variations in the phase of the light.

But there is another point of view, from which it is anything but obvious that interferometers should work as gravitational wave detectors. We usually understand the cosmological redshift with a simple picture, that in an expanding universe light waves themselves expand with the space around them. The following argument can then be made: If light waves expand in an expanding universe, isn't it also the case for the light waves traveling through an interferometer when a gravitational wave stretches the space in the arms? And if that is true, then how can the phase of the light be modulated by a gravitational wave? Won't the coexpansion of the arms and of the light wave ''rulers'' used to measure them render the effect of a gravitational wave invisible? From this point of view, how can an interferometer work?

In fact, much of this alternative point of view is correct. However, one crucial point is missing in the argument given above. Once this oversight is recognized and corrected, we can see that interferometers should indeed function as advertised.

The issue I am addressing here is one of conceptual understanding, not new physics. For this reason, I have kept the discussion at the heuristic level, with the absolute minimum of mathematics.

II. GRAVITATIONAL WAVES, LIGHT TRAVEL TIME, AND INTERFEROMETERS

To understand what a gravitational wave does, it is useful to consider a liberal sprinkling of freely falling test masses through space. These test masses mark out the coordinates we use when we compute in the coordinate system called the *transverse-traceless gauge*. A gravitational wave can be thought of as changing the distances between these test masses. The mathematical expression for this effect can be seen by writing down the space-time metric, which gives the interval ds between neighboring events. If the gravitational wave is propagating along the z axis, and has its polarization aligned with the x and y axes, then the metric has the simple form

$$ds^2 = -c^2 dt^2 + [1 + h(t)]dx^2 + [1 - h(t)]dy^2. \tag{2.1}$$

A physical interpretation of this expression is straightforward. A wave of amplitude $h(t)$ causes distances between freely falling test masses separated only in the x direction to change by a factor of

$$\sqrt{1+h(t)} \approx (1 + \tfrac{1}{2}h(t)). \tag{2.2}$$

At the same time, an equal and opposite change occurs in distances between masses separated only in the y direction.

If this physical interpretation makes sense, then there ought to be, at least in principle, a way to measure the presence of a gravitational wave. One way to do so is to send light signals between various masses, and look for variations in the round-trip travel time associated with these length changes. Such thought experiments have a long history in relativity. The key idea is to remember that two nearby events on the path of a light ray are separated by an interval $ds = 0$. Plugging that into Eq. (2.1), we can find the relationship between the space and time coordinates along the light path,

$$c^2 dt^2 = (1 + h(t))dx^2 + (1 - h(t))dy^2. \tag{2.3}$$

If we specialize to paths only along one of the coordinate axes, the right-hand side will have only one nonzero term. Then we can take the square root of each side, finding (for example, for a light ray traveling along the x axis) that the time it takes is given by

$$\tau \equiv \int dt = \frac{1}{c} \int \sqrt{1+h(t)}dx. \tag{2.4}$$

As long as h is small (as it will be in all realistic circumstances), we can use the approximation of Eq. (2.2), and the integral is simple to evaluate. It shows that the travel time grows or shrinks in just such a way as the intuitive interpretation suggested.

A Michelson interferometer is a simple arrangement of test masses that enables such measurements to be made. The basic layout is sketched in Fig. 1. This version has only three such masses: a mass serving as a beam splitter, whose location can be taken as the origin of coordinates, and two masses carrying highly reflective mirrors, one placed out along the x axis at a distance L, and the other an equal distance from the beam splitter along the y axis. Ideally, the masses would be truly free. In the laboratory we can approximate that freedom by hanging the masses as pendulums, which are free for influences that are rapid compared with the pendulum period.

To analyze what goes on in an interferometer, it is simplest if we imagine that the gravitational wave has the form of a step function $h(t) = h_0 H(t-\tau)$, where h_0 is the (distressingly small, perhaps 10^{-21}) amplitude of the wave, and $H(t-\tau)$ is the unit step function at time τ. Although gravitational waveforms should come in many varieties, there is a class of them that will in fact have a net dc shift in h. [And we lose no generality by considering only a pure step function, since we can always approximate an arbitrary wave form $h(t)$ by a suitable succession of positive and negative step functions.]

Imagine that we shine a strobe light, emitting a series of brief powerful pulses of light, along the x axis from the left

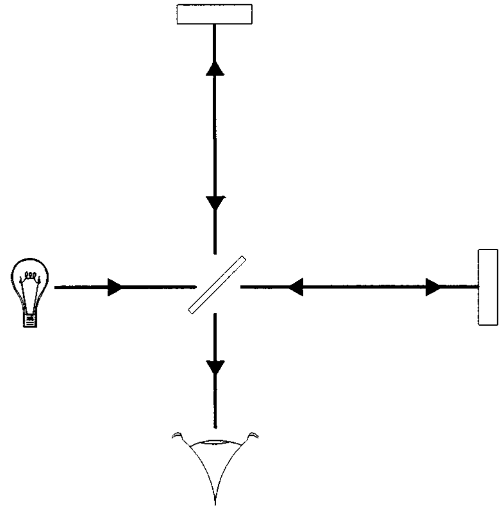

Fig. 1. A schematic diagram of an interferometric gravitational wave detector.

of the beam splitter toward the x test mass. At the beam splitter, part of the light is redirected toward the y test mass. Before the arrival of the gravitational wave, a pulse of light traveling along the x axis takes a time $\tau_s = 2L/c$ to make the round trip from the beam splitter and back. The pulse traveling along the y axis takes the same amount of time. So the two pulses derived from a single flash of the strobe return to the beam splitter at precisely the same time.

Once the step-function gravitational wave has arrived, the two arms will no longer have the same length. This will become apparent when the arrival times of the x and y pulses are compared. If h_0 is positive, then the x arm is lengthened, so the pulse in that arm will have to travel farther, causing it to arrive late. Meanwhile, the y arm is shortened, so its pulse returns to the beam splitter early. The presence of the gravitational wave is thus signaled by the fact that there is now a time separation between the arrival times of the two pulses, of value $\Delta\tau = (2L/c)h_0$. Measuring this arrival time difference is a way to measure the strength of the gravitational wave.

In practice, we would use optical components like the ones described, but would replace the strobe light with a laser, a steady source of coherent light. What was true for the arrival times of distinct pulses also ought to be true for the arrival times of individual wave ''crests'' in the laser light. So if the x arm is lengthened, the light in that arm would suffer a phase lag, while the light traveling in the y arm acquires a phase lead. Measuring the phase difference between two beams of light is the standard function of an interferometer. And here, that phase shift is related to the strength of the gravitational wave by the relation $\Delta\phi = (4\pi L/\lambda)h_0$. So a Michelson interferometer should be a good way to measure gravitational waves. This is what LIGO and VIRGO are designed to do.

The reader interested in more information about the operation of interferometric gravitational wave detectors may find it in a variety of more technical references.[4]

III. THE COSMOLOGICAL REDSHIFT

The argument given in Sec. II is a version of the standard explanation of how interferometers are used to detect gravitational waves. Next, we would like to motivate the frequently asked question used as the title of this paper. To do so, it seems useful to review the basics of relativistic cosmology, as a justification for the picture of light waves being stretched by the expansion of space.

Just as we did to understand gravitational waves, much of cosmology is best understood by assuming that the universe is liberally sprinkled with freely falling test masses, objects which feel no (non-gravitational) forces. In cosmology, these masses are the markers of what is referred to as the *comoving coordinate system*. In a homogeneous and isotropic universe, each of these masses is equivalent to any other and can be taken as a suitable origin of coordinates. If we restrict our attention to the case of a flat universe, then the metric can be written as

$$ds^2 = -c^2 dt^2 + R^2(t)[dx^2 + dy^2 + dz^2]. \tag{3.1}$$

This differs from the ordinary Minkowski metric describing the flat space of special relativity only by the presence of the dimensionless *cosmic scale factor* $R(t)$ multiplying all of the spatial parts. In an expanding universe, $R(t)$ is an increasing function of time. An observer on any one of the freely falling masses sees all of the rest of the masses receding in the pattern called Hubble's law. The recession of the other masses is observable as a redshift (lengthening of wavelength) of light signals sent from those other masses to the observer at the origin. For the nearby parts of the universe, this redshift behaves just as if it were the ordinary Doppler shift in the wavelength caused by the motion of those masses away from the observer at the origin.

It is possible to make a careful calculation of how the wavelength of a light signal is redshifted in an expanding universe, by considering the difference in the amount of time it takes two successive crests of a light wave to travel from its source to the observer.[5] This is similar to the calculation for the light travel time in an interferometer with a gravitational wave (although in cosmology we are only interested in one-way travel times from distant light sources to us). Light emitted with wavelength λ_0 when the cosmic scale factor was $R(t_0)$ is transformed by the cosmic expansion into light we receive [when the cosmic scale factor has grown to $R(t_1)$] with wavelength λ_1, where the relation between the two wavelengths has the simple form

$$\frac{\lambda_1}{\lambda_0} = \frac{R(t_1)}{R(t_0)}. \tag{3.2}$$

(Note that this convention of ordering t_0 and t_1 is the opposite of that typically used by cosmologists, who prefer to refer to the present with t_0.)

This result is so simple that it cries out for a heuristic explanation. I should say explanations, for there are several.[6] The most pictorially suggestive shows that the correct result follows from the statement that the wave itself expands with the expanding space in which it travels, so that its wavelength grows with the cosmic scale factor $R(t)$.

IV. A GRAVITATIONAL WAVE DOES STRETCH LIGHT

It should be clear from Secs. II and III that there is a deep analogy between the stretching of space in an expanding universe and the distortion of space caused by a gravitational wave. The pattern of distance changes is rather different; the universe's expansion causes all distances to grow by the same fraction, while a gravitational wave causes equal and opposite fractional changes for x and y distances, with no change for distances along the wave propagation direction z. Still, if we consider a single interferometer arm in a direction for which the effect of the gravitational wave is to stretch distances, then the effect of a gravitational wave is indeed very analogous to a cosmic expansion.

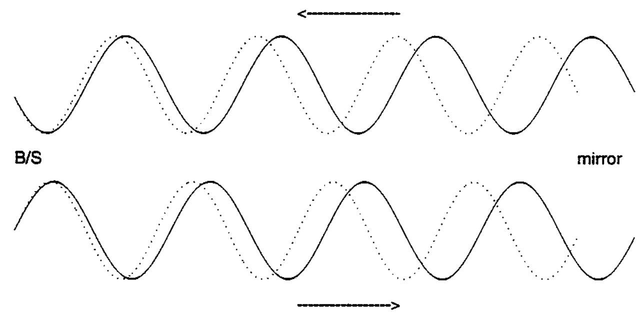

Fig. 2. Light before (dotted) and after (solid) the arrival of a gravitational wave. The beamsplitter is at left, end mirror at right. Outbound light is shown at the bottom, returning light at the top.

Now we are ready to ask whether the light in an interferometer also suffers a redshift when a gravitational wave arrives. When we see that it does, we will be ready to give a critical examination to the standard analysis of an interferometer's sensitivity to gravitational waves.

Consider first the times $t < \tau$, before the gravitational wave has arrived. The interferometer arm is being steadily supplied with light from a laser, which generates a new wave crest every ν^{-1} seconds. The distance between successive wave crests (i.e., the wavelength) is $\lambda = c\nu^1$ throughout the arm. Imagine that we have strewn small freely falling test masses so thickly that we can find one of them immediately adjacent to each wave crest.

Next, consider what happens at $t = \tau$, when the gravitational wave arrives. Suddenly, all distances between freely falling masses along the x axis are increased by a factor of $(1 + \frac{1}{2}h_0)$. What happens to the light? Remember that the key idea of relativity is that there is no absolute standard of rest. This means that the light doesn't have a choice to stay fixed with respect to any absolute coordinate system. The only thing that can happen is for each wave crest to remain next to whichever test mass it was next to just before the gravitational wave arrived. Thus, as the distances between the test masses suddenly grow, so does the distance between wave crests. In other words, the wavelength of the light is increased by the same factor of $(1 + \frac{1}{2}h_0)$. Light waves do indeed stretch as the gravitational wave stretches the interferometer arm, as we have illustrated in Fig. 2. (To make the effect visible, we have chosen extreme values for the ratio of light wavelength to arm length and for h_0.)

V. LENGTHS IN COSMOLOGY AND IN LABORATORY PHYSICS

Note that the language we have been using in this paper only makes sense if we imagine that we have standards of length other than either the separations of freely falling test masses or the wavelengths of light waves. We do. A good paradigm of a length standard is a perfectly rigid rod. Such a rod does not change its length in the presence of a gravitational wave, because the arbitrarily strong elastic forces between its parts resist the gravitational force carried by the gravitational wave. As we will see below, we can also use the travel time of light as a reliable ruler under most conditions, in spite of the stretching of light waves that goes on when space expands.

In fact, cosmology is susceptible to the same temptations to confusion in its language, yet there is no ambiguity about physically meaningful statements. There is much learned dispute over such things as

(1) whether there is any motion in an expanding universe, or whether instead all of the galaxies are at rest,[7] or
(2) whether the cosmological redshift is a form of Doppler shift, or whether instead it arises through another mechanism entirely.[8]

Still, the distance from us to another galaxy does grow as the universe expands. We could, in principle (and with sufficient patience), verify this by measuring the distance by the inverse-square law of brightness. And the rate of increase of distance between us and a galaxy is numerically equal to the velocity we infer from interpreting the redshift as arising from the Doppler effect, at least for nearby galaxies.

Similarly, we have sufficient physical understanding not to let relativistic language confuse us into thinking that there are no distance changes caused by a gravitational wave, even though it is convenient to define a coordinate system out of freely falling masses. There are changes in distance between two points whose coordinate separation remains fixed. This is the physical meaning behind saying that the metric has the form given in Eq. (2.1). The present case is quite parallel to the situation in cosmology, since both there and in the gravitational wave case the most convenient coordinate system is defined by freely falling masses.

VI. WITH PATIENCE, GRAVITATIONAL WAVES ARE OBSERVABLE

We are left with the question, ''Are gravitational waves observable by examining the light in an interferometer?'' It might seem that the recognition that the wave stretches with the space has in fact shown that light is unsuitable to reveal the length changes. We often say, after all, that we are using light as a ruler to measure the distortions of space. What good is a ruler that stretches to the same extent that space stretches?

To see why light still works perfectly well for our purpose, recall first that there is no direct sense in which we observe the wavelength of the light in the arms. Our observations are instead of the phase of the light, that is, of the arrival times of wave crests.

What happens when we observe the phase of the light wave in the x arm of our interferometer? Imagine that just before $t=\tau$, one light wave crest had returned precisely to the beam splitter. By the same argument as we used above, that wave crest has no choice other than to remain at the beam splitter when the gravitational wave arrives. So the gravitational wave causes no phase shift at the beam splitter immediately after its arrival.

The key is the word ''immediately.'' All of the other wave crests suddenly at $t=\tau$ become farther from the beam splitter than they were before. Gravitational wave or no, light travels through the arm at the speed of light, c. The physically observable meaning of the stretching of the space is that the light in it has to cover extra distance, and so will arrive late.

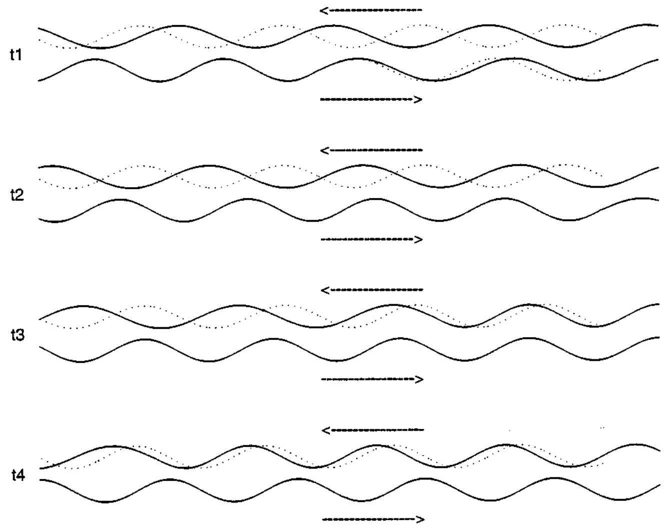

Fig. 3. Like Fig. 2, but at a succession of later moments. Note the buildup of phase shift between the light in the stretched arm (solid) compared to how it would have traveled through an unstretched arm.

And, since each successive wave crest has to cover a larger extra distance to make it back to the beam splitter, the total time delay (or phase shift) builds up steadily until all of the light that was in the interferometer arm at $t=\tau$ finally makes it back to the beam splitter. This phase shift *is* observable, and it builds up over the storage time $\tau_s=2L/c$ of the interferometer arm. This process is illustrated in Fig. 3.

We've concentrated our attention on the light that was present in the arm at $t=\tau$, but it is important also to understand the light that entered afterwards. The laser has been steadily pumping out wave crests every ν^{-1} seconds. Those waves that entered the stretched space after $t=\tau$ are not stretched; they travel at the speed c through the space they find themselves traveling in, and so have the ordinary wavelength $\lambda=c\nu^{-1}$.

In case this seems surprising, recall that precisely the same effect would occur in cosmology if we could imagine a universe that, instead of steadily expanding, expanded in a single step function. This follows directly from Eq. (3.2) above: all that matters is the ratio of the values of the scale factor at the emission and reception times. In a step-function cosmic expansion, all light waves emitted before the expansion are stretched by the ratio $R(t_1)/R(t_0)$, but all light waves emitted afterwards are unstretched. In fact, it is precisely from this example that Harrison argues that the cosmological redshift is a completely different phenomenon from the Doppler shift.[9] In our gravitational wave interferometer, light emitted after $t=\tau$ experiences the same ''scale factor'' $(1+\frac{1}{2}h_0)$ both at emission and at its eventual reception, hence it exhibits no change in wavelength.

How does this affect the observed phase shift at the beam splitter? Once light is arriving that has not had its wavelength stretched, then the phase shift no longer grows with each cycle. These latter wave crests still arrive late, since they have had to travel their whole path through the lengthened interferometer arm, so the overall phase shift is just a constant, the dc response of the interferometer to the step-function gravitational wave.

After $t=\tau+\tau_s$, the interferometer arm contains only light of the original wavelength. However, since by any real physical standard the arm has been lengthened by the gravitational wave, all of the successive wave crests arrive late in a way that is measurable at the beam splitter. If we care to, we can say that we are measuring the arm length by seeing how many wavelengths of light fit in it, but we only do so by timing when the wave crests finally arrive back at the beam splitter. It is precisely for this reason that people have found it simplest to use travel-time arguments to calculate the detailed response of an interferometer to gravitational waves. Perhaps it is better to say that we use the laser as a clock than to say we use its light as a ruler.

VII. CONCLUSION

There is no conflict between our standard description of how an interferometer works and a picture of light waves being stretched by a gravitational wave. The exercise of reconciling the two pictures is valuable, though, both for sharpening one's physical understanding and for critically examining the language that we use to describe the sometimes subtle phenomena of general relativity.

ACKNOWLEDGMENTS

For stimulating conversations, I would like to thank Jennie Traschen, Rafael Sorkin, Bill Startin, and Michael Telegin. This work has been supported by Syracuse University and by National Science Foundation Grants Nos. PHY-9113902 and PHY-9602157.

[1]Excellent reviews can be found in the following: K. S. Thorne, ''Gravitational radiation,'' in *300 Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge U.P., Cambridge, 1987), pp. 330–458, and *The Detection of Gravitational Waves*, edited by D. G. Blair (Cambridge U.P., Cambridge, 1991).

[2]A. Abramovici *et al.*, ''LIGO: The Laser Interferometer Gravitational-wave Observatory,'' Science **256**, 325–333 (1992); C. Bradaschia *et al.*, ''The VIRGO Project: A wide band antenna for gravitational wave detection,'' Nucl. Instrum. Methods Phys. Res. A **289**, 518–525 (1990); A. Abramovici *et al.*, ''Gravitational wave astrophysics,'' in *Proceedings of the 1994 Snowmass Summer Study: Particle and Nuclear Astrophysics and Cosmology in the Next Millenium*, edited by E. W. Kolb and R. D. Peccei (World Scientific, Singapore, 1995), pp. 389–425.

[3]The basic idea, in the form of a thought experiment, goes back at least as far as F. A. E. Pirani, ''On the physical significance of the Riemann tensor,'' Acta Phys. Pol. **15**, 389–405 (1956). A first estimate of experimental sensitivity can be found in M. E. Gertsenshtein and V. I. Pustovoit, ''On the detection of low frequency gravitational waves,'' J. Exp. Theoret. Phys. (U.S.S.R.) **43**, 605–607 (1962), in English translation in Sov. Phys. JETP **16**, 433–435 (1963). Another treatment of the theory and much more detailed discussion of experimental issues was carried out by R. Weiss, ''Electromagnetically coupled broadband gravitational antenna,'' Q. Prog. Rep., MIT Res. Lab. Electron. **105**, 54–75 (1972). The most detailed discussion to date of the interferometer response as a function of signal frequency and angle of incidence was carried out by R. Weiss, P. S. Linsay, P. R. Saulson, and S. E. Whitcomb, in *A Study of a Long Baseline Gravitational Wave Antenna System* (MIT, Cambridge, MA, 1983). For a summary of that calculation, see P. R. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific, Singapore, 1994), pp. 18–25.

[4]See, for example, D. G. Blair, in Ref. 1; or P. R. Saulson, in Ref. 3.

[5]One clear version of this calculation can be found in S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), pp. 415–418.

[6]These are especially well described in P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton U.P., Princeton, NJ, 1993), pp. 93–98.

[7]F. H. Shu, *The Physical Universe* (University Science Books, Mill Valley, CA, 1982), p. 374.

[8]E. R. Harrison, *Cosmology, the Science of the Universe* (Cambridge U.P., Cambridge, 1981); J. V. Narlikar, ''Spectral shifts in general relativity,'' Am. J. Phys. **62**, 903–907 (1994).

[9]E. R. Harrison, in Ref. 8, pp. 245, 246.