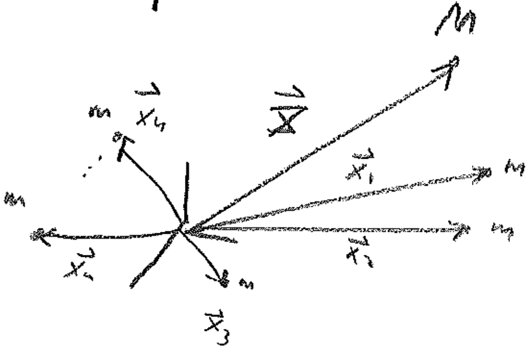
Sec 13:

n particles of mass m, 1 particle of mass M

Work in com frame and write as a reduced n-particle problem



com Frame:

$$m(\vec{x}_1 + \vec{x}_2 + \cdots + \vec{x}_n) + M\vec{X} = 0$$

Relative position vectors

$$\vec{x}_1 - \vec{X} \equiv \vec{r}_1$$
$$\vec{x}_2 - \vec{X} \equiv \vec{r}_2$$
$$\vdots$$
$$\vec{x}_n - \vec{X} \equiv \vec{r}_n$$

Multiply last n equations by m and subtract from 1st:

$$M\vec{X} + nm\vec{X} = -m(\vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_n)$$
$$(M+nm)\vec{X} = -m(\vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_n)$$
$$\vec{X} = -\frac{m}{M+nm}(\vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_n)$$

Then:

$$\left.\begin{aligned} \vec{x}_1 &= \vec{r}_1 + \vec{X} \\ \vec{x}_2 &= \vec{r}_2 + \vec{X} \\ &\vdots \\ \vec{x}_n &= \vec{r}_n + \vec{X} \end{aligned}\right\} \text{ where } \vec{X} \text{ given by above}$$

$$T = \frac{1}{2}m\left(|\dot{\vec{x}}_1|^2 + |\dot{\vec{x}}_2|^2 + \cdots + |\dot{\vec{x}}_n|^2\right) + \frac{1}{2}M|\dot{\vec{X}}|^2$$
$$= \frac{1}{2}m\Big(|\dot{\vec{r}}_1|^2 + |\dot{\vec{X}}|^2 + 2\dot{\vec{r}}_1\cdot\dot{\vec{X}}$$
$$+ |\dot{\vec{r}}_2|^2 + |\dot{\vec{X}}|^2 + 2\dot{\vec{r}}_2\cdot\dot{\vec{X}}$$
$$+ \cdots + |\dot{\vec{r}}_n|^2 + |\dot{\vec{X}}|^2 + 2\dot{\vec{r}}_n\cdot\dot{\vec{X}}\Big) + \frac{1}{2}M|\dot{\vec{X}}|^2$$

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$$T = \frac{1}{2} m\left(|\dot{\vec{r}}_1|^2 + |\dot{\vec{r}}_2|^2 + \cdots + |\dot{\vec{r}}_n|^2\right) \leftarrow \text{relative KE}$$

$$+ \frac{1}{2} m n |\dot{\vec{X}}|^2 + m\underbrace{\left(\dot{\vec{r}}_1 + \dot{\vec{r}}_2 + \cdots + \dot{\vec{r}}_n\right)}_{-\frac{(M+nm)}{m}\dot{\vec{X}}} \cdot \dot{\vec{X}} + \frac{1}{2} M |\dot{\vec{X}}|^2$$

$$\dot{\vec{X}} = -\left(\frac{m}{M+nm}\right) \sum_a \dot{\vec{r}}_a$$

$$= \frac{1}{2} m\left(|\dot{\vec{r}}_1|^2 + |\dot{\vec{r}}_2|^2 + \cdots + |\dot{\vec{r}}_n|^2\right)$$

$$+ \frac{1}{2} |\dot{\vec{X}}|^2 \left(mn - 2(M+nm) + M \right)$$

$$= \frac{1}{2} m\left(|\dot{\vec{r}}_1|^2 + |\dot{\vec{r}}_2|^2 + \cdots + |\dot{\vec{r}}_n|^2\right) - \frac{1}{2}(M+nm)|\dot{\vec{X}}|^2$$

$$= \frac{1}{2} m \sum_a |\dot{\vec{r}}_a|^2 - \frac{1}{2}\left(\frac{m^2}{M+nm}\right)\left|\sum_a \dot{\vec{r}}_a\right|^2$$

$$U = U(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n, \vec{X})$$

$$= U\big(|\vec{x}_1 - \vec{x}_2|, \ldots, |\vec{x}_1 - \vec{x}_n|, \quad |\vec{x}_1 - \vec{X}|,$$
$$|\vec{x}_2 - \vec{x}_3|, \ldots, |\vec{x}_2 - \vec{x}_n|, \quad |\vec{x}_2 - \vec{X}|,$$
$$\ldots, |\vec{x}_n - \vec{x}_1|, \ldots, |\vec{x}_n - \vec{x}_{n-1}|, \quad |\vec{x}_n - \vec{X}| \big)$$

$$= U\big(|\vec{r}_1 - \vec{r}_2|, \ldots, |\vec{r}_1 - \vec{r}_n|, |\vec{r}_1|,$$
$$|\vec{r}_2 - \vec{r}_3|, \ldots, |\vec{r}_2 - \vec{r}_n|, |\vec{r}_2|,$$
$$\ldots, |\vec{r}_n - \vec{r}_1|, \ldots, |\vec{r}_n - \vec{r}_{n-1}|, |\vec{r}_n| \big)$$

which depends only on magnitude of relative position vectors