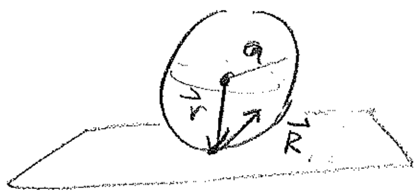
§ 38, Prob 1:

Homog sphere rolling without slipping on a horizontal plane subject to external force $\vec{F}$, torque $\vec{K}$.

$$I = \frac{2}{5}\mu a^2$$



$\uparrow \hat{n} = \hat{z}$

$\vec{R}$: reaction force (points in arbitrary direction)

Force: $$\frac{d\vec{P}}{dt} = \vec{F} + \vec{R}$$

Torque: $$\frac{d\vec{M}}{dt} = \vec{K} + \vec{r}\times\vec{R} = \vec{K} - a\,\hat{n}\times\vec{R}$$

Roll w/out slipping: $$\vec{V} = \vec{\Omega}\times\vec{R}_{com} = \vec{\Omega}\times a\hat{n}$$

Using $\vec{P} = \mu\vec{V}$, $\vec{M} = I\vec{\Omega}$ (homog. sphere)

$$\mu\frac{d\vec{V}}{dt} = \vec{F} + \vec{R} \qquad (1)$$

$$I\frac{d\vec{\Omega}}{dt} = \vec{K} - a\,\hat{n}\times\vec{R} \qquad (2)$$

$$\frac{d\vec{V}}{dt} = a\frac{d\vec{\Omega}}{dt}\times\hat{n} \qquad (3)$$

— solve for $\vec{R}$: $$\vec{R} = \mu\frac{d\vec{V}}{dt} - \vec{F}$$

— solve for $\frac{d\vec{\Omega}}{dt}$: $$\frac{d\vec{\Omega}}{dt} = \frac{\vec{K}}{I} - \frac{a}{I}\,\hat{n}\times\vec{R}$$

$$= \frac{\vec{K}}{I} - \frac{a}{I}\,\hat{n}\times\left(\mu\frac{d\vec{V}}{dt} - \vec{F}\right)$$

— substitute last expression into (3):

$$\frac{d\vec{V}}{dt} = a\left[\frac{\vec{K}}{I}\times\hat{n} - \frac{a}{I}\left(\hat{n}\times\left(\mu\frac{d\vec{V}}{dt} - \vec{F}\right)\right)\times\hat{n}\right]$$

Thus,

$$\frac{d\vec{V}}{dt} = a\left[\frac{\vec{\Gamma}}{I}\times\hat{n} + \frac{a}{I}\hat{n}\times\left(\mu\hat{n}\times\frac{d\vec{V}}{dt} - \hat{n}\times\vec{F}\right)\right]$$

<u>U/c</u>: $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

$$\hat{n}\times\left(\hat{n}\times\frac{d\vec{V}}{dt}\right) = \hat{n}\left(\hat{n}\cdot\frac{d\vec{V}}{dt}\right) - \frac{d\vec{V}}{dt}$$

$$= \hat{z}\,\frac{dV_z}{dt} - \frac{d\vec{V}}{dt}$$

$$= -\left(\frac{dV_x}{dt}\right)\hat{x} - \left(\frac{dV_y}{dt}\right)\hat{y}$$

$$\hat{n}\times(\hat{n}\times\vec{F}) = \hat{n}(\hat{n}\cdot\vec{F}) - \vec{F}$$

$$= \hat{z}\,F_z - \vec{F}$$

$$= -F_x\hat{x} - F_y\hat{y}$$

<u>Also</u>: $\vec{\Gamma}\times\hat{n} = (\Gamma_x\hat{x} + \Gamma_y\hat{y} + \Gamma_z\hat{z})\times\hat{z}$

$$= -\Gamma_x\hat{y} + \Gamma_y\hat{x}$$

$$\rightarrow \frac{d\vec{V}}{dt} = \frac{a}{I}\left(-\Gamma_x\hat{y} + \Gamma_y\hat{x}\right) + \frac{a^2\mu}{I}\left(-\left(\frac{dV_x}{dt}\right)\hat{x} - \left(\frac{dV_y}{dt}\right)\hat{y}\right)$$

$$- \frac{a^2}{I}\left(-F_x\hat{x} - F_y\hat{y}\right)$$

$$= \frac{5}{2m}\left(-\frac{\Gamma_x}{a}\hat{y} + \frac{\Gamma_y}{a}\hat{x}\right) - \frac{5}{2}\left(\left(\frac{dV_x}{dt}\right)\hat{x} + \left(\frac{dV_y}{dt}\right)\hat{y}\right)$$

$$+ \frac{5}{2m}\left(F_x\hat{x} + F_y\hat{y}\right)$$

Take $\hat{x}, \hat{y}, \hat{z}$ components:

$$\boxed{\frac{dV_z}{dt} = 0}$$ [com doesn't accelerate in z-direction]

$$\frac{dV_x}{dt} = \frac{5}{2M}\left(\frac{\Gamma_y}{a} + F_x\right) - \frac{5}{2}\frac{dV_x}{dt}$$

$$\frac{7}{2}\frac{dV_x}{dt} = \frac{5}{2\mu}\left(\frac{\Gamma_y}{a} + F_x\right)$$

$$\boxed{\frac{dV_x}{dt} = \frac{5}{7\mu}\left(\frac{\Gamma_y}{a} + F_x\right)}$$

$$\frac{dV_y}{dt} = \frac{5}{2\mu}\left(-\frac{\Gamma_x}{a} + F_y\right) - \frac{5}{2}\frac{dV_y}{dt}$$

$$\frac{7}{2}\frac{dV_y}{dt} = \frac{5}{2\mu}\left(-\frac{\Gamma_x}{a} + F_y\right)$$

$$\boxed{\frac{dV_y}{dt} = \frac{5}{7\mu}\left(-\frac{\Gamma_x}{a} + F_y\right)}$$

solve above for V_x, V_y in terms of $F_x, F_y, \Gamma_x, \Gamma_y$

Rolling without slipping:

$$\vec{V} = \vec{\Omega} \times a\hat{n} = a\vec{\Omega} \times \hat{z}$$

$$= a\left(\Omega_x \hat{x} + \Omega_y \hat{y} + \Omega_z \hat{z}\right) \times \hat{z}$$

$$= -a\Omega_x \hat{y} + a\Omega_y \hat{x}$$

Thus, $V_z = 0$

$V_x = a\Omega_y \longrightarrow$ $\boxed{\Omega_y = \frac{V_x}{a}}$

$V_y = -a\Omega_x \longrightarrow$ $\boxed{\Omega_x = -\frac{V_y}{a}}$

To Find Ω_z:

$$\text{Take } \left(I \frac{d\vec{\Omega}}{dt} = \vec{K} - a\, \hat{n} \times \vec{R} \right) \cdot \hat{n}$$

$$I \frac{d\Omega_z}{dt} = \vec{K} \cdot \hat{n} - a \cancelto{0}{(\hat{n} \times \vec{R}) \cdot \hat{n}}$$

$$\boxed{\frac{d\Omega_z}{dt} = \frac{K_z}{I} = \frac{5 K_z}{2 \mu a^2}}$$

To Find $\vec{R}$:

$$\vec{R} = \mu \frac{d\vec{V}}{dt} - \vec{F}$$

$$\boxed{R_x} = \mu \frac{dV_x}{dt} - F_x$$

$$= \frac{5}{7}\left(\frac{K_y}{a} + F_x \right) - F_x$$

$$= \boxed{\frac{5}{7}\left(\frac{K_y}{a} - 2F_x \right)}$$

$$\boxed{R_y} = \mu \frac{dV_y}{dt} - F_y$$

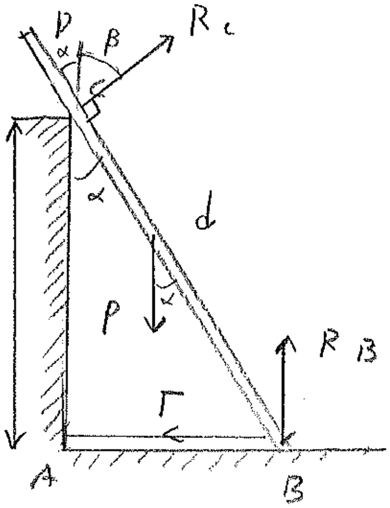
$$= \frac{5}{7}\left(-\frac{K_x}{a} + F_y \right) - F_y$$

$$= \boxed{-\frac{5}{7}\left(\frac{K_x}{a} + 2F_y \right)}$$

$$\boxed{R_z} = \mu \cancelto{0}{\frac{dV_z}{dt}} - F_z$$

$$= \boxed{-F_z}$$

§ 38, Prob 2 :



$$\left.\begin{aligned}\sum \vec{F} &= 0\\ \sum \vec{r}\times\vec{F} &= 0\end{aligned}\right\}\text{ on rod}$$

y: $\quad -P + R_c\cos\beta + R_B = 0 \qquad (1)$

x: $\quad -T + R_c\sin\beta = 0 \qquad (2)$

torque around B: $\quad P\frac{l}{2}\sin\alpha - R_c d = 0 \qquad (3)$

$\beta = \frac{\pi}{2} - \alpha$

$\sin\beta = \cos\alpha$

$\cos\beta = \sin\alpha$

$h = d\cos\alpha$

Rewrite:

$-P + R_c\sin\alpha + R_B = 0$

$-T + R_c\cos\alpha = 0$

$\frac{1}{2}Pl\sin\alpha - R_c\frac{h}{\cos\alpha} = 0$

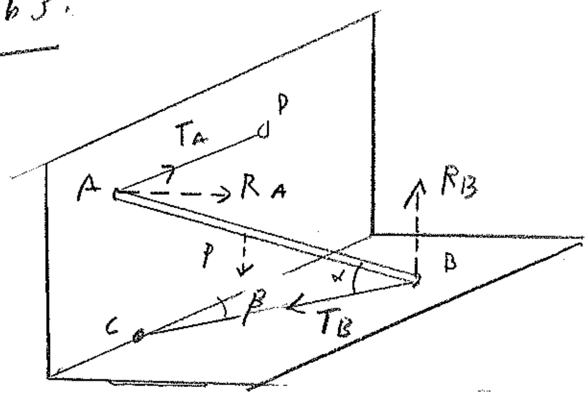
Solve: $\quad R_c\frac{h}{\cos\alpha} = \frac{1}{2}Pl\sin\alpha$

$$\boxed{R_c = \frac{1}{2}\frac{P}{h}l\sin\alpha\cos\alpha = \frac{1}{4}\frac{Pl}{h}\sin 2\alpha}$$

Also: $\quad \boxed{T = R_c\cos\alpha = \frac{1}{4}\frac{Pl}{h}\sin 2\alpha\cos\alpha = \frac{1}{2}\frac{Pl}{h}\sin\alpha\cos^2\alpha}$

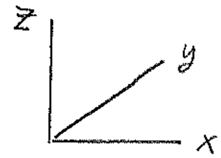
and $\quad \boxed{\begin{aligned}R_B &= P - R_c\sin\alpha\\ &= P - \frac{1}{4}\frac{Pl}{h}\sin 2\alpha\sin\alpha\\ &= P - \frac{1}{2}\frac{Pl}{h}\sin^2\alpha\cos\alpha\\ &= P\left(1 - \frac{1}{2}\frac{l}{h}\sin^2\alpha\cos\alpha\right)\end{aligned}}$

§38, Prob 3.



$$\sum \vec{F} = 0$$

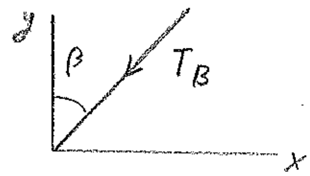
$$\sum \vec{r} \times \vec{F} = 0$$



Z: $-P + R_B = 0 \quad \rightarrow \quad \boxed{R_B = P}$

x: $R_A - T_B \sin\beta = 0$

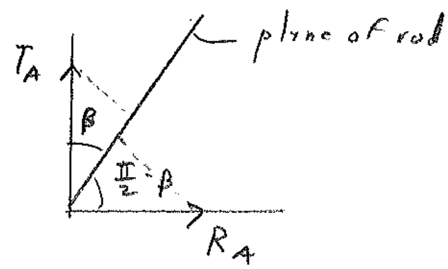
$\rightarrow \boxed{R_A = T_B \sin\beta}$



y: $T_A - T_B \cos\beta = 0$

$\rightarrow \boxed{T_A = T_B \cos\beta}$

Torque about B
(in plane of rod)



$$P \frac{l}{2} \cos\alpha - T_A \cos\beta \, l \sin\alpha$$

$$- R_A \cos\left(\frac{\pi}{2} - \beta\right) l \sin\alpha = 0$$

$$\rightarrow \frac{1}{2} P \cancel{l} \cos\alpha - T_A \cos\beta \sin\alpha \cancel{l} - R_A \sin\beta \sin\alpha \cancel{l} = 0$$

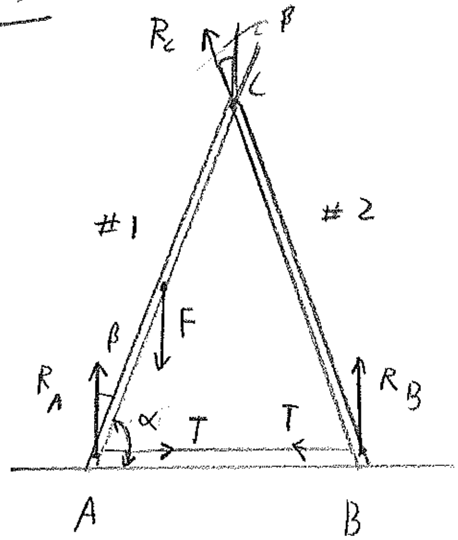
$$\frac{1}{2} P \cos\alpha - T_B \cos^2\beta \sin\alpha - T_B \sin^2\beta \sin\alpha = 0$$

$$\frac{1}{2} P \cos\alpha - T_B \sin\alpha = 0$$

So $\boxed{T_B = \frac{1}{2} P \cot\alpha}$

$\rightarrow R_A = \frac{1}{2} P \cot\alpha \sin\beta, \quad T_A = \frac{1}{2} P \cot\alpha \cos\beta$

§38, Prob 4.



$\sum \vec{F} = 0$, $\sum \vec{r} \times \vec{F} = 0$ for both rods

$\boxed{\#1}$: $\underline{y}$: $R_A - F + R_c \cos\beta = 0$

$\underline{x}$: $T - R_c \sin\beta = 0$

$\underline{\text{torque at } A}$: $-F \frac{l}{2} \cos\alpha + R_c \, l \sin 2\beta = 0$

$\beta = \frac{\pi}{2} - \alpha$

$\sin\beta = \cos\alpha$

$\cos\beta = \sin\alpha$

$\sin 2\beta = \sin(\pi - 2\alpha)$
$= \sin 2\alpha$
$= 2\sin\alpha\cos\alpha$

Rewrite:

$$R_A - F + R_c \sin\alpha = 0 \qquad (1)$$

$$T - R_c \cos\alpha = 0 \qquad (2)$$

$$-\frac{F}{2} + 2R_c \sin\alpha = 0 \qquad (3)$$

Thus,

$$\frac{F}{2} = 2R_c \sin\alpha$$

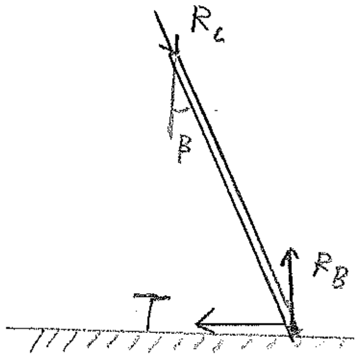
$$\boxed{R_c = \frac{F}{4\sin\alpha}}$$

and $\boxed{T} = R_c \cos\alpha$
$= \frac{F}{4\sin\alpha} \cos\alpha$
$= \boxed{\frac{F}{4} \cot\alpha}$

and $\boxed{R_A} = F - R_c \sin\alpha$
$= F - \frac{F}{4\cancel{\sin\alpha}} \cancel{\sin\alpha}$
$= \boxed{\frac{3}{4} F}$

To find R_B need to use static equilibrium equations for rod #2

$\boxed{\#2}$



$$y: \quad R_B - R_c \cos\beta = 0$$

$$R_B - R_c \sin\alpha = 0$$

$$\rightarrow \boxed{R_B = R_c \sin\alpha = \left(\frac{F}{4 \cancel{\sin\alpha}}\right) \cdot \cancel{\sin\alpha} = \frac{F}{4}}$$

check tension: $x: \quad -T + R_c \sin\beta = 0$

$$\boxed{T = R_c \sin\beta = R_c \cos\alpha} \quad \checkmark \text{ (as before)}$$