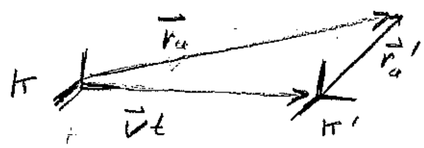
Section 8:

K, K': two inertial frames, K' moving with velocity $\vec{V}$ wrt K.



$$\vec{r}_a = \vec{r}_a{}' + \vec{V}t$$

$$\vec{v}_a = \vec{v}_a{}' + \vec{V}$$

Then
$$\begin{aligned}
L &= \sum_a \tfrac{1}{2} m_a v_a^2 - U \\
&= \sum_a \tfrac{1}{2} m_a \left(\vec{v}_a{}' + \vec{V}\right)\cdot\left(\vec{v}_a{}' + \vec{V}\right) - U \\
&= \sum_a \tfrac{1}{2} m_a \left(v_a'^2 + V^2 + 2\vec{V}\cdot\vec{v}_a{}'\right) - U \\
&= \sum_a \tfrac{1}{2} m_a v_a'^2 - U + \tfrac{1}{2}\left(\sum_a m_a\right)V^2 + \vec{V}\cdot\sum_a m_a \vec{v}_a{}' \\
&= L' + \tfrac{1}{2}\mu V^2 + \vec{V}\cdot\sum_a m_a \frac{d\vec{r}_a{}'}{dt} \\
&= L' + \tfrac{1}{2}\mu V^2 + \vec{V}\cdot\frac{d}{dt}\underbrace{\left(\sum_a m_a \vec{r}_a{}'\right)}_{\mu \underbrace{\vec{R}'}_{\text{com}}} \qquad \text{where } \mu = \sum_a m_a \\
&= L' + \tfrac{1}{2}\mu V^2 + \mu\vec{V}\cdot\frac{d\vec{R}'}{dt}
\end{aligned}$$

Action:
$$\begin{aligned}
S &= \int_{t_1}^{t_2} L\,dt \\
&= \int_{t_1}^{t_2}\left(L' + \tfrac{1}{2}\mu V^2 + \mu\vec{V}\cdot\frac{d\vec{R}'}{dt}\right)dt \\
&= S' + \tfrac{1}{2}\mu V^2(t_2 - t_1) + \mu\vec{V}\cdot\left(\vec{R}'(t_2) - \vec{R}'(t_1)\right)
\end{aligned}$$

setting $t_1 = 0$, $t_2 = t$ $\longrightarrow$ $S = S' + \tfrac{1}{2}\mu V^2 t + \mu\vec{V}\cdot\vec{R}'(t)$