HMMA 307 : Modèles linéaires avancés

Lasso vs FoBa

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https://github.com/opheliecoiffier/LASSOvsFoBa

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Introduction

To measure the quality of prediction, we use linear prediction model :

$$f(x) = \beta^{\mathsf{T}} x$$

With

$$\hat{\beta} = \underset{\beta \in \mathbb{R}^d}{\arg\min} \sum_{i=1}^{n} \Phi(\beta^{\mathsf{T}} x_i, y_i) + \lambda g(\beta)$$

The Lasso method

Method uses L_1 regularization.
We have

$$\hat{\beta} = \arg\min_{\beta \in \mathbb{R}^d} \sum_{i=1}^{n} \Phi(\beta^\intercal x_i, y_i) + \lambda ||\beta||_1$$

2 issues :

- strong assumptions
- large regularization parameter (λ)

The OMP method

"Orthogonal Match Pursuit" : forward greedy algorithm

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Principle : Add feature at every step to reduce the squared error
& Calculate its error

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Main issue : Never correct mistakes made earlier steps
Solution : Backward greedy algorithm ?

The FoBa algorithm (Lars method)

"Forward-Backward" greedy algorithm

Principle :

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1) Use Forward greedy step

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3) Use Backward greedy step $\&$ adaptively Backward step

The FoBa algorithm (Lars method)

"Forward-Backward" greedy algorithm

Principle :
1) Use Forward greedy step
2) Until squared error increase is no more than half of the squared error decrease in the earlier forward step
3) Use Backward greedy step & adaptively Backward step
*) **Backward functions :**

- Remove any errors caused by earlier forward steps
- Keep reasonable number of basis functions
- (adaptively Backward) Make sure that we progress

Comparison between Lasso and FoBa

Similarity	Difference
Path-following algorithm	Conditions to find threshold
Large regularization parameter	Bias
Mistakes in earlier step	

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Introduction

Loss function used :

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2$$

Principle :

- Randomly partition into 50 training points and *nb* test points
- Predict variables
- Compare MSE for each methods and for each groups

Boston Housing data

Data : 506 census tracts from 1970 census
14 features

Y : housing price

Principle : 50 training points / 456 test points
We repeat 50 times MSE calculations and compare MSE obtained
With a sparsity between 1 and 10.

Boston Housing data

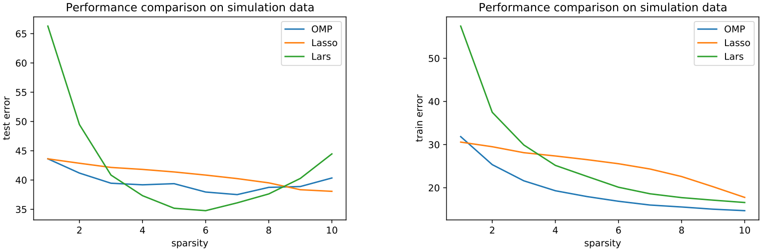


Figure: MSE depending on sparsity for the 3 methods and for the 2 groups.

Ionosphere data

Data : 351 data points
35 features

Y : binary value (Y$\in \{0,1\}$)

Principle : 50 training points / 301 test points
We repeat 50 times MSE calculations and compare MSE obtained
With a sparsity between 1 and 10.

Ionosphere data

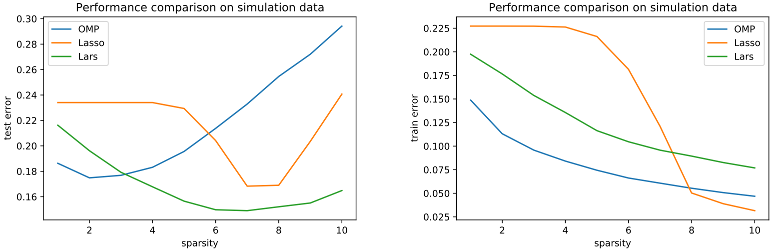
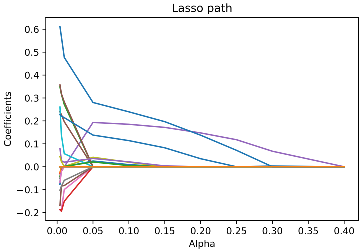


Figure: MSE depending on sparsity for the 3 methods and for the 2 groups.

Issues

- Representation of Lasso



alpha	Nb non-zero coefs
0.400	1
0.300	2
0.250	2
0.200	4
0.150	4
0.100	4
0.050	8
0.010	13
0.006	15
0.004	18

Figure: Lasso path and non-zero coefficients for Ionosphere data

- Differences between our algorithms and these in the article

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Conclusion

We find these results :

- **Test points :** FoBa for *Ionosphere* data & FoBa/OMP for *Boston Housing* data
- **Training points :** OMP (depending on sparcity for *Ionosphere* data

The article finds :

- **Test points :** FoBa for small sparsity and for *Boston housing* data & Lasso for *Ionosphere* data
- **Training points :** the mixed algorithm (FoBa)

Finally, we must choose the best algorithm and the best method depending on data, sparsity and mathematics conditions that we can check.

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