HMMA 308 : Machine learning

Lasso vs FoBa

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https://github.com/opheliecoiffier/LASSOvsFoBa

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Introduction

To measure the quality of prediction, we use linear prediction model :

$$f(x) = \beta^\mathsf{T} x$$

With

$$\hat{\beta} = \arg\min_{\beta \in \mathbb{R}^d} \sum_{i=1}^{n} \Phi(\beta^\mathsf{T} x_i, y_i) + \lambda g(\beta)$$

The Lasso method

Method uses L_1 regularization.
We have

$$\hat{\beta} = \arg\min_{\beta \in \mathbb{R}^d} \sum_{i=1}^{n} \Phi(\beta^{\intercal} x_i, y_i) + \lambda ||\beta||_1$$

2 issues :

- strong assumptions
- large regularization parameter (λ)

The OMP method

"Orthogonal Match Pursuit" : forward greedy algorithm

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& Calculate its error

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Main issue : Never correct mistakes made earlier steps
Solution : Backward greedy algorithm ?

The FoBa algorithm (Lars method)

"Forward-Backward" greedy algorithm

Principle :

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1) Use Forward greedy step

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3) Use Backward greedy step & adaptive Backward step

The FoBa algorithm (Lars method)

"Forward-Backward" greedy algorithm

Principle :
1) Use Forward greedy step
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3) Use Backward greedy step & adaptive Backward step
*) **Adaptive backward functions :**

- Make sure that we progress

Comparison between Lasso and FoBa

Similarity	**Difference**
Path-following algorithm	Conditions to find threshold
Large regularization parameter	Bias
Mistakes in earlier step	

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Introduction

Loss function used :

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2$$

Principle :

- Randomly partition into 50 training points and *nb* test points
- Predict variable
- Compare MSE for each methods and for each groups

Boston Housing data

Data : 506 census tracts from 1970 census
14 features

Y : housing price

Principle : 50 training points / 456 test points
We repeat 50 times MSE calculations and compare MSE obtained
With a sparsity between 1 and 10.

***Boston Housing* data**

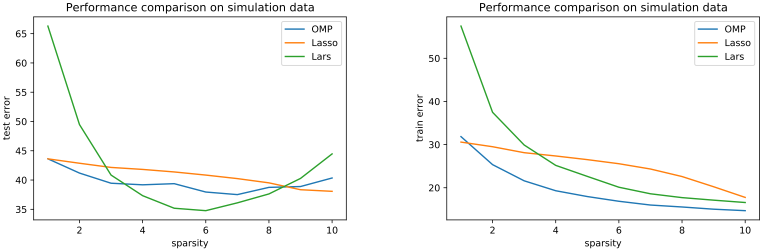


Figure: MSE depending on sparsity for the 3 methods and for the 2 groups.

Ionosphere data

Data : 351 data points
35 features

Y : binary variable (Y$\in \{0, 1\}$)

Principle : 50 training points / 301 test points
We repeat 50 times MSE calculations and compare MSE obtained
With a sparsity between 1 and 10.

Ionosphere data

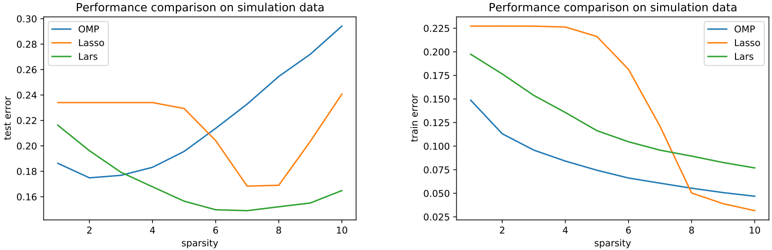
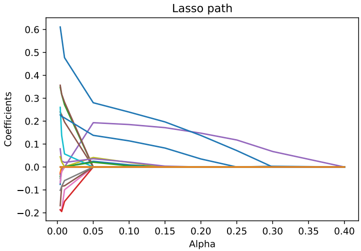


Figure: MSE depending on sparsity for the 3 methods and for the 2 groups.

Issues

- Representation of Lasso



alpha	Nb non-zero coefs
0.400	1
0.300	2
0.250	2
0.200	4
0.150	4
0.100	4
0.050	8
0.010	13
0.006	15
0.004	18

Figure: Lasso path and non-zero coefficients for Ionosphere data

- Differences between our algorithms and these in the article

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Conclusion

The main question is about the learning sparse representations using greedy algorithms.

FoBa is useful with training points.

We need to use a novel combination of the forward-greedy and backward-greedy algorithms to obtain a smaller error.

But, it depends on the data : with Ionosphere data (test points), with the L1 regularization, the "bias" is an advantageous.

Finally, we must choose the best algorithm and the best method depending on data, sparsity and mathematics conditions that we can check.

Bibliography

- (BSD License), scikit-learn developers. *scikit-learn*.
  https://scikit-learn.org/stable/modules/classes.html/module-sklearn.linear_model.
  2020.
- Coiffier, Ophélie. *Github LASSOvsFoBa*.
  https://github.com/opheliecoiffier/LASSOvsFoBa. 2020.
- Salmon, Joseph. *HMMA 308 - Apprentissage statistique*. 2019.
- Sigillito, Vince. *Ionosphere Data Set*.
  https://archive.ics.uci.edu/ml/datasets/ionosphere.
- University, Johns Hopkins. *Index of /ml/machine-learning-databases/housing*.
  https://archive.ics.uci.edu/ml/machine-learning-databases/housing/.
- Zhang, Tong. *Adaptive Forward-Backward Greedy Algorithm for Learning Sparse Representations*.
  http://tongzhang-ml.org/papers/it11-foba.pdf.