Randomized Bidding Auction Analysis

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7/17/2018

Abstract

This project involves an analysis of a simple online auction system. A certain number of agents each present a unique bid in a random order. The system maintains a variable equal to the highest bid so far. Based on the number of agents, this report will look at the theoretical analysis of the expected number of updates to the highest bid variable, and test that analysis with an implementation of such an auction.

Problem Statement

The given task is to consider a simple online auction system. There are n bidding agents, each with a unique bid, presented one at a time in a random order. The system keeps track of the highest bid seen so far. We will calculate the expectation for the number of times the highest bid will update while the system traverses the bids in order, and test our theoretical analysis against an implementation of the auction.

Hypothesis

The expected number of times the highest bid will be updated is equal to *ln n + O(1)*, where n is the number of bidding agents. This is the same as in the hiring problem, and can be shown as follows:

For all the bidding agents b_1 to b_i, the chances that an agent b_j's bid is accepted is equal to the chances b_j is the highest among bids b_1 to b_j, which is $1/j$, given the bids are in random order.

So the total expected number of updates is equal to

$$\sum_{i=1}^{n} P(b_i) = \sum_{i=1}^{n} \frac{1}{i}$$

$$= ln(n) + O(1)$$

Therefore, we hypothesize that, given enough random permutations of n bids, the average number of updates needed for each permutation will be $ln(n) + O(1)$.

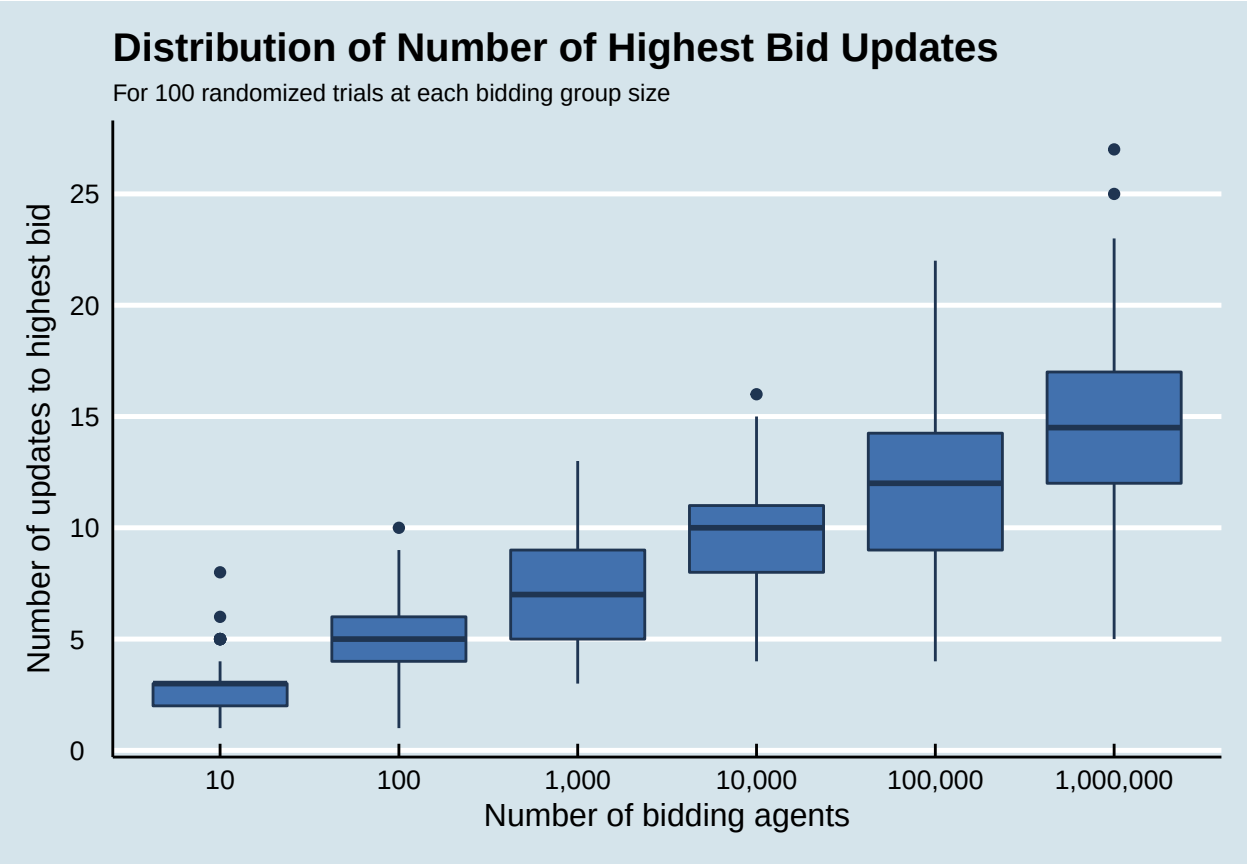
Experimental Design

To test this hypothesis, we will implement this auction process, and record the number of updates needed for several different sizes of bids.

There will be six different groups of bids: 10 bids, 100 bids, 1,000 bids, 10,000 bids, 100,000 bids, and 1,000,000 bids. Each bid group size will have 100 different trials of random permutations.

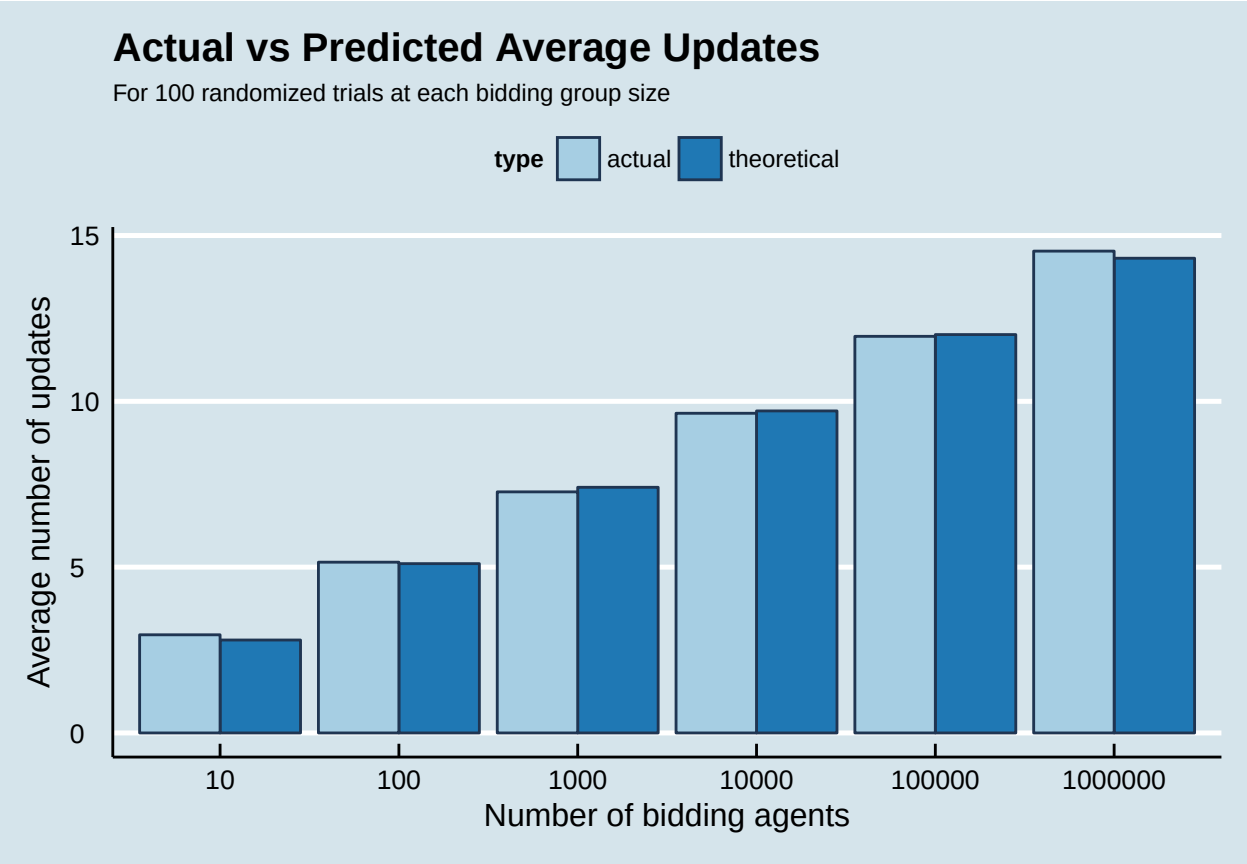
Results

The auction process and experiment was implemented in R, and led to the following results, as presented by a boxplot of number of updates needed in all 100 trials at each group size:



Analysis

The following plots the average number of updates needed from the trials per group size and compares it to the theoretical prediction. Here, the constant $O(1)$ added to $ln(n)$ is equal to 0.5.



Conclusions

The results strongly support our hypothesis. The trials resulted in a fairly normal distribution of average updates at each group level, and the average results very strongly correlate with our theoretical analysis. It is likely, then, that our theoretical analysis is correct.

Citation

The problem considered here is very similar to the hiring problem presented in the following text.

Cormen, T. et al. (2009). *Introduction to algorithms*, 3rd ed. Cambridge, MA: The MIT Press. pp. 120-121.