**PART-C (15 MARK)**

**UNIT I INTRODUCTION**

1. **Explain how analysis of linear search is done with a suitable illustration. (10)**

**(ii) Define recurrence equation and explain how solving recurrence equations are done.(6) (NOV/DEC 2011) (R)**

**(i) Analysis of Linear Search**

**Linear Search** is a simple searching algorithm that checks each element of the list or array one by one until the desired element is found or the entire list is traversed. The key advantage of Linear Search is its simplicity, but it is not the most efficient when dealing with large datasets.

**Linear Search Algorithm:**

python

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def linear\_search(arr, key):

for i in range(len(arr)):

if arr[i] == key:

return i # Return index if found

return -1 # Return -1 if key is not found

**Time Complexity Analysis:**

* **Best Case:** The best case occurs when the element is found in the first position. In this case, the algorithm makes only **1 comparison**.
  + **Best case time complexity:** O(1)O(1)O(1) (constant time)
* **Worst Case:** The worst case occurs when the element is not found in the array, or the element is at the very last position. The algorithm will have to examine every element of the list to conclude that the element is not present.
  + **Worst case time complexity:** O(n)O(n)O(n), where nnn is the size of the list, because the algorithm makes nnn comparisons in the worst case.
* **Average Case:** In the average case, the algorithm will have to search through approximately half of the list before finding the target (if it exists).
  + **Average case time complexity:** O(n)O(n)O(n)

**Illustration of Linear Search:**

Let's consider an array: **arr = [4, 2, 7, 1, 9]**, and we want to search for the element **7**.

1. Start at the first element: arr[0] = 4. This is not equal to 7.
2. Move to the next element: arr[1] = 2. This is also not equal to 7.
3. Move to the next element: arr[2] = 7. We find that this is equal to 7, so we return the index 2.

Thus, the Linear Search would find the element 7 at index **2** after checking 3 elements.

**Time complexity for this search is O(3)O(3)O(3),** which is effectively **O(n)**.

**Conclusion:**

* **Best Case:** O(1)O(1)O(1)
* **Worst Case:** O(n)O(n)O(n)
* **Average Case:** O(n)O(n)O(n)

Linear Search is simple but inefficient for large datasets, especially when the desired element is near the end or not present at all.

**(ii) Recurrence Equation and Solving Recurrence Equations**

A **recurrence equation** is an equation that defines a function in terms of itself. In computer science, recurrence equations are commonly used to describe the running time of recursive algorithms. These equations capture the problem's behavior as it is broken down into smaller subproblems.

**Definition of Recurrence Equation:**

A recurrence equation defines a sequence where each term is expressed as a function of its preceding terms. In the context of algorithms, recurrence equations are used to describe the running time of recursive algorithms.

For example, the recurrence relation for the Merge Sort algorithm is:

T(n)=2T(n2)+O(n)T(n) = 2T\left(\frac{n}{2}\right) + O(n)T(n)=2T(2n​)+O(n)

* T(n)T(n)T(n) represents the time taken for a problem of size nnn.
* 2T(n2)2T\left(\frac{n}{2}\right)2T(2n​) represents the time to recursively solve two subproblems, each of size n/2n/2n/2.
* O(n)O(n)O(n) represents the linear time to merge the two halves together.

**Solving Recurrence Equations:**

There are several methods to solve recurrence equations. Common methods include:

1. **Substitution Method (Induction)**:
   * This method involves making a guess about the solution and proving it by induction.
   * You assume a form for the solution, then prove it works through mathematical induction.

**Example:**

Consider the recurrence relation for a simple recursive algorithm:

T(n)=T(n−1)+O(1)T(n) = T(n - 1) + O(1)T(n)=T(n−1)+O(1)

* + **Base Case:** T(1)=O(1)T(1) = O(1)T(1)=O(1).
  + Assume T(k)=O(k)T(k) = O(k)T(k)=O(k) for some kkk.
  + Prove that T(k+1)=O(k+1)T(k + 1) = O(k + 1)T(k+1)=O(k+1).
  + By solving, we see that T(n)=O(n)T(n) = O(n)T(n)=O(n).

1. **Recursion Tree Method**:
   * This method visualizes the recurrence as a tree, where each level of the tree represents a recursive call, and you sum the work done at each level.
   * This method is particularly useful for recurrences that split into multiple subproblems.

**Example:**

Consider the recurrence for Merge Sort:

T(n)=2T(n2)+O(n)T(n) = 2T\left(\frac{n}{2}\right) + O(n)T(n)=2T(2n​)+O(n)

The recursion tree for this would look like:

* + At level 0 (root), the work is O(n)O(n)O(n).
  + At level 1, we have two subproblems, each of size n/2n/2n/2, and the total work at this level is 2×O(n/2)=O(n)2 \times O(n/2) = O(n)2×O(n/2)=O(n).
  + At level 2, we have four subproblems, each of size n/4n/4n/4, and the total work at this level is 4×O(n/4)=O(n)4 \times O(n/4) = O(n)4×O(n/4)=O(n).
  + This continues until the problem size reaches 1.

By summing up the work at all levels, we get:

T(n)=O(n)+O(n)+O(n)+⋯=O(nlog⁡n)T(n) = O(n) + O(n) + O(n) + \dots = O(n \log n)T(n)=O(n)+O(n)+O(n)+⋯=O(nlogn)

1. **Master Theorem**:
   * The Master Theorem provides a straightforward way to solve recurrences of the form:

T(n)=aT(nb)+O(nd)T(n) = aT\left(\frac{n}{b}\right) + O(n^d)T(n)=aT(bn​)+O(nd)

where a≥1a \geq 1a≥1, b>1b > 1b>1, and d≥0d \geq 0d≥0.

* + The Master Theorem has three cases:
    1. If a>bda > b^da>bd, then T(n)=O(nlog⁡ba)T(n) = O(n^{\log\_b a})T(n)=O(nlogb​a).
    2. If a=bda = b^da=bd, then T(n)=O(ndlog⁡n)T(n) = O(n^d \log n)T(n)=O(ndlogn).
    3. If a<bda < b^da<bd, then T(n)=O(nd)T(n) = O(n^d)T(n)=O(nd).

**Example:**

For the recurrence relation:

T(n)=2T(n2)+O(n)T(n) = 2T\left(\frac{n}{2}\right) + O(n)T(n)=2T(2n​)+O(n)

We can apply the Master Theorem:

* + a=2a = 2a=2, b=2b = 2b=2, and d=1d = 1d=1.
  + Since a=bda = b^da=bd, the solution is T(n)=O(nlog⁡n)T(n) = O(n \log n)T(n)=O(nlogn).

1. **Give the recursive algorithm which finds the number of binary digits in the binary representation of a positive decimal integer. Find the recurrence relation and complexity.**

To find the number of binary digits (or bits) in the binary representation of a positive decimal integer, we can repeatedly divide the number by 2 (using integer division) and count how many times we need to divide until the number becomes 0.

A recursive approach to solve this problem would look like this:

**Recursive Algorithm:**

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def count\_binary\_digits(n):

# Base case: if n is 0, the number of binary digits is 0

if n == 0:

return 0

# Recursive case: reduce n by half and add 1 to the count

return 1 + count\_binary\_digits(n // 2)

**Explanation of the Algorithm:**

1. **Base Case:** If n == 0, the function returns 0 because the number 0 has no binary digits in this context (or equivalently, we can consider 0 to have 1 digit, but we'll assume the former here).
2. **Recursive Case:** For any other number n > 0, the function divides the number by 2 (n // 2), effectively removing the least significant binary digit (rightmost bit), and recursively counts the remaining binary digits, adding 1 for the current binary digit.

For example:

* To find the binary digits of 13:
  + The binary representation of 13 is 1101 (which has 4 binary digits).
  + The algorithm works as follows:
    - count\_binary\_digits(13) → 1 + count\_binary\_digits(6)
    - count\_binary\_digits(6) → 1 + count\_binary\_digits(3)
    - count\_binary\_digits(3) → 1 + count\_binary\_digits(1)
    - count\_binary\_digits(1) → 1 + count\_binary\_digits(0)
    - count\_binary\_digits(0) → 0 (base case)
  + The total count of binary digits is 1 + 1 + 1 + 1 = 4.

**Recurrence Relation:**

Let T(n)T(n)T(n) represent the number of binary digits in the binary representation of a positive integer nnn.

The recurrence relation can be written as:

T(n)=1+T(n2)forn>0T(n) = 1 + T\left(\frac{n}{2}\right) \quad \text{for} \quad n > 0T(n)=1+T(2n​)forn>0

Where:

* 111 accounts for the current binary digit (bit),
* T(n2)T\left(\frac{n}{2}\right)T(2n​) represents the recursive call on the integer obtained by dividing nnn by 2.

**Base Case:**

T(0)=0T(0) = 0T(0)=0

**Time Complexity:**

The number of recursive calls is determined by how many times we can divide nnn by 2 until nnn becomes 0. This is equivalent to finding the number of bits required to represent nnn in binary, which is the logarithm (base 2) of nnn.

Thus, the number of recursive calls is proportional to log⁡2n\log\_2 nlog2​n.

* **Time Complexity:** The time complexity is O(log⁡n)O(\log n)O(logn), since the algorithm reduces the problem size by half in each recursive step.

**Conclusion:**

* **Recursive Algorithm**: A recursive approach to find the number of binary digits in a decimal integer involves dividing the number by 2 and counting the number of divisions (recursive calls).
* **Recurrence Relation**:T(n)=1+T(n2),T(0)=0T(n) = 1 + T\left(\frac{n}{2}\right), \quad T(0) = 0T(n)=1+T(2n​),T(0)=0
* **Time Complexity**: The time complexity of this algorithm is O(log⁡n)O(\log n)O(logn), where nnn is the input number.

**UNIT II**

# GRAPH ALGORITHMS

1. **Discuss about the algorithm and pseudocode to find the Minimum Spanning Tree using Prim’s Algorithm. Find the Minimum Spanning Tree for the graph. Discuss about the efficiency of the algorithm.**

**Prim's Algorithm for Minimum Spanning Tree (MST)**

Prim's algorithm is a greedy algorithm used to find the Minimum Spanning Tree (MST) of a connected, undirected graph. A Minimum Spanning Tree is a subset of the edges of the graph that connects all the vertices without any cycles and with the minimum possible total edge weight.

Prim's algorithm works by building the MST one vertex at a time, starting from an arbitrary vertex and gradually adding the smallest edge that connects a vertex in the MST to a vertex outside the MST.

**Steps of Prim's Algorithm:**

1. **Start from an arbitrary vertex:** Choose any vertex as the starting point.
2. **Edge selection:** Look at all edges that connect a vertex in the MST to a vertex outside the MST. Select the edge with the smallest weight.
3. **Add the edge to the MST:** Add the selected edge and its corresponding vertex to the MST.
4. **Repeat the process:** Repeat steps 2 and 3 until all vertices are included in the MST.

**Pseudocode for Prim's Algorithm:**

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Prim(Graph G, start\_vertex):

Initialize min\_heap (priority queue) to keep track of edges with minimum weights

Initialize the `key` array to store the minimum edge weight for each vertex

Initialize the `parent` array to store the parent vertex of each vertex in the MST

Initialize a `visited` array to keep track of the vertices included in the MST

For each vertex v in G:

key[v] = ∞ (except for the start\_vertex, which is 0)

parent[v] = NIL

key[start\_vertex] = 0

Push the start\_vertex into the priority queue with priority 0

while the priority queue is not empty:

u = ExtractMin(min\_heap) // Get the vertex with the smallest key

visited[u] = true

For each neighbor v of u:

if v is not in MST and the weight of edge (u, v) is smaller than key[v]:

key[v] = weight(u, v)

parent[v] = u

Push v into the priority queue with updated key[v]

// The `parent` array contains the MST structure

// Return the MST formed from the `parent` and `key` arrays

**Explanation of the Pseudocode:**

1. **Initialization:**
   * min\_heap: A priority queue (min-heap) is used to always fetch the vertex with the minimum edge weight.
   * key: This array keeps track of the minimum weight edge that connects a vertex to the MST.
   * parent: This array stores the parent of each vertex, which will form the MST structure.
   * visited: This array keeps track of whether a vertex is included in the MST.
2. **Prim's Main Loop:**
   * The algorithm repeatedly extracts the vertex with the minimum edge weight from the priority queue.
   * For each unvisited neighbor of this vertex, the algorithm updates the key and parent arrays if the weight of the edge to the neighbor is smaller than the current key for that neighbor.
   * The neighbor is then pushed back into the priority queue with the updated key value.
3. **Termination:**
   * The algorithm terminates when all vertices have been added to the MST.

**Example of Prim’s Algorithm:**

Let's consider the following graph:

lua

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(1)---(3)---(5)

| / |

(2) (4) (6)

| / |

(7)---(8)---(9)

Edges and their weights:

* (1, 2) = 4, (1, 3) = 1, (2, 4) = 3, (2, 7) = 5, (3, 4) = 6, (3, 5) = 2, (4, 7) = 7, (5, 6) = 8, (7, 8) = 9, (8, 9) = 10.

We start from vertex 1. Prim's algorithm will proceed as follows:

1. Start with vertex 1, select the minimum edge (1, 3) with weight 1.
2. Add vertex 3, now the edges available are (3, 5) with weight 2, (3, 4) with weight 6. Select (3, 5) with weight 2.
3. Add vertex 5, now the edges available are (5, 6) with weight 8. Select (5, 6) with weight 8.
4. Add vertex 6, now the edges available are (6, 9) with weight 10. Select (6, 9) with weight 10.
5. Add vertex 9, now the edges available are (9, 8) with weight 9. Select (9, 8) with weight 9.
6. Add vertex 8, now the edges available are (8, 7) with weight 5. Select (8, 7) with weight 5.
7. Add vertex 7, now the edges available are (7, 4) with weight 7. Select (7, 4) with weight 7.
8. Add vertex 4, the MST is complete.

**MST Result:**

The MST contains the edges:  
(1, 3), (3, 5), (5, 6), (6, 9), (9, 8), (8, 7), (7, 4).

**Efficiency of Prim’s Algorithm:**

The time complexity of Prim’s algorithm depends on the data structures used to implement it:

1. **Using a simple array or list for the priority queue:**
   * **Time Complexity:** O(V2)O(V^2)O(V2), where VVV is the number of vertices in the graph. This is because for each vertex, we might have to scan through all other vertices to find the minimum weight edge.
2. **Using a binary heap (priority queue):**
   * **Time Complexity:** O((V+E)log⁡V)O((V + E) \log V)O((V+E)logV), where EEE is the number of edges. This is more efficient than the simple array approach because it allows us to extract the minimum key and update the key values in logarithmic time.
3. **Using a Fibonacci heap:**
   * **Time Complexity:** O(E+Vlog⁡V)O(E + V \log V)O(E+VlogV). This is the most efficient version in theory, but the constant factors can make it slower in practice compared to the binary heap implementation.
4. **Distinguish between breadth first search and depth first search with example**

**Breadth-First Search (BFS) vs Depth-First Search (DFS)**

Breadth-First Search (BFS) and Depth-First Search (DFS) are two fundamental graph traversal algorithms used to explore or search through graphs. They differ in the approach they take to visit the vertices of the graph.

Here is a detailed comparison between BFS and DFS:

**1. BFS (Breadth-First Search):**

**Definition:**

* BFS is an algorithm for traversing or searching a graph or tree in a breadthward motion.
* It explores all the vertices at the present depth level before moving on to the vertices at the next depth level.

**Working Principle:**

* BFS starts at the root (or an arbitrary node in the case of a graph) and explores all the neighbors at the present depth level before moving on to nodes at the next depth level.
* It uses a **queue** to keep track of the vertices to be explored.

**Steps:**

1. Start with a chosen node, mark it as visited.
2. Add the starting node to a queue.
3. While the queue is not empty, dequeue a node and explore all its unvisited neighbors.
4. For each unvisited neighbor, mark it as visited and enqueue it.
5. Repeat until all nodes are visited.

**Data Structure Used:**

* **Queue** (FIFO: First In, First Out)

**Time Complexity:**

* **O(V + E)** where VVV is the number of vertices and EEE is the number of edges in the graph.

**Space Complexity:**

* **O(V)**, as we store the visited nodes and the queue.

**Example:** Consider the following graph:

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A

/ \

B C

/ \

D E

BFS starting from node A:

1. Start with A, mark A as visited, and enqueue A.
2. Dequeue A, visit its neighbors B and C, mark them as visited, and enqueue them.
3. Dequeue B, visit its neighbor D, mark it as visited, and enqueue it.
4. Dequeue C, visit its neighbor E, mark it as visited, and enqueue it.
5. Dequeue D and E, but they have no unvisited neighbors.

**BFS traversal result:**

* A → B → C → D → E

**2. DFS (Depth-First Search):**

**Definition:**

* DFS is an algorithm for traversing or searching a graph or tree in a depthward motion.
* It explores as far as possible along each branch before backtracking.

**Working Principle:**

* DFS starts at the root (or an arbitrary node in the case of a graph) and explores each branch completely before moving to the next branch.
* It uses a **stack** (or recursion) to keep track of the vertices to be explored.

**Steps:**

1. Start with a chosen node, mark it as visited.
2. Visit an unvisited neighbor of the current node and push it to the stack.
3. Repeat the process for the current node's neighbor until no unvisited neighbors are left.
4. Backtrack (pop the stack) to the previous node and explore the next unvisited neighbor.
5. Continue until all nodes are visited.

**Data Structure Used:**

* **Stack** (LIFO: Last In, First Out) or **Recursion**

**Time Complexity:**

* **O(V + E)** where VVV is the number of vertices and EEE is the number of edges in the graph.

**Space Complexity:**

* **O(V)**, as we store the visited nodes and the stack.

**Example:** Consider the same graph as before:

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Copy

A

/ \

B C

/ \

D E

DFS starting from node A:

1. Start with A, mark A as visited, and push it onto the stack.
2. Visit neighbor B, mark B as visited, and push it onto the stack.
3. Visit neighbor D, mark D as visited, and push it onto the stack.
4. Since D has no unvisited neighbors, backtrack to B.
5. Since B has no unvisited neighbors, backtrack to A.
6. Visit neighbor C, mark C as visited, and push it onto the stack.
7. Visit neighbor E, mark E as visited, and push it onto the stack.
8. Since E has no unvisited neighbors, backtrack to C.

**DFS traversal result:**

* A → B → D → C → E

**Key Differences Between BFS and DFS:**

| **Criteria** | **Breadth-First Search (BFS)** | **Depth-First Search (DFS)** |
| --- | --- | --- |
| **Traversal Strategy** | Explores level by level, visiting all neighbors at the current level before moving deeper. | Explores as deep as possible along one branch before backtracking. |
| **Data Structure** | Queue (FIFO) | Stack (LIFO) or Recursion |
| **Time Complexity** | O(V + E) | O(V + E) |
| **Space Complexity** | O(V) (due to the queue) | O(V) (due to the stack/recursion) |
| **Complete Search** | Guarantees finding the shortest path in an unweighted graph. | Does not guarantee the shortest path. |
| **Use Case** | Suitable for finding the shortest path, level order traversal, and search in unweighted graphs. | Suitable for tasks like topological sorting, solving puzzles, and exploring deep paths in the graph. |
| **Implementation** | Can be implemented using a queue and iterative method. | Can be implemented using recursion or stack (either explicit or via function call stack). |

**Example with Comparison:**

**Example with Comparison:**

Given the graph:

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Copy

A

/ \

B C

/ \

D E

* **BFS traversal:** A → B → C → D → E
* **DFS traversal:** A → B → D → C → E

**UNIT III**

**ALGORITHM DESIGN TECHNIQUES**