

Haskell Learning

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Contents

1	Introduction	2
1.1	Resources	2
2	Types	2
2.1	Hask	2
2.1.1	Bottom Type	2
2.1.2	Hask is not a category	2
2.2	Constructive vs Intuitionistic logic	3
2.3	Curry Howard Isomorphism	3
3	Common Structures	3
3.1	Applicatives	3

1 Introduction

This document is for me to document my learning particularly with respect to Haskell.

1.1 Resources

Listed here are any resources I used or have found useful in my learning process

- Category theory for programmers by Bartosz Milewski
- Hask is not a category by Andrej Bauer
- Haskell Wiki
 - The category Hask
- Fast and loose reasoning is morally correct by Jeremy Gibbons
- The Algebra (and calculus!) of Algebraic Data types by John Burget
- Type Theory and Functional Programming by Simon Thompson

2 Types

2.1 Hask

”Objects of Hask are Haskell Types”.

Haskell types can be **thought of** like sets ¹. e.g. `Int` could be thought of as the set of values from $\{-2^{29}, \dots, 2^{29} - 1\}$ and `Boolean` as the set containing two elements $\{True, False\}$

2.1.1 Bottom Type

These types actually contain an additional value of \perp which is a value included in every type allowing for computations which don’t terminate.

\perp is necessary because of the existence of general recursion and is witnessed by the equation $x = x$.

2.1.2 Hask is not a category

Because of the existence of both \perp and the fact that Haskell is a non-strict language with the function `seq` defined, Hask actually violates the category laws of category theory.

Working in a limited subset

¹There is a distinction between set and object of category `hask` (or `set`)

2.2 Constructive vs Intuitionistic logic

Philosophically the difference is in

In classical (intuitionistic) logic every proposition is assigned a value T or F by law of excluded middle (think truth tables). **this is not the case for constructive logic** In constructive logic we lose out on law of excluded middle and by extension proofs by contradiction. Instead in order to prove things we must show that a proposition is *witnessed* (in this context inhabited).

2.3 Curry Howard Isomorphism

Isomorphism between types and Proofs. Haskell types have a mapping to **Propositional** Logic Note that more powerful type systems can map to more powerful systems of logic (Dependent types can correspond to predicate logic for example)

- Think of a Haskell type as a set and a program as a proof that such a set is inhabited
- From my understanding: Your compiler also functions as a proof checker
- From a practical standpoint
- Note: Haskell

Note that the propositional logic system that Haskell types map to is **unsound** due to the bottom type (\perp) which inhabits every type including void. Consider the following example:

```
absurd :: () -> Void
absurd a = undefined
```

Because the void type corresponds to *False* and the unit type to *True* we have $True \rightarrow False$ which is clearly unsound. However we note that the bottom type terminates the program so I'm not entirely sure whether you can consider it a value.

3 Common Structures

3.1 Applicatives