PHYS 4023
Fall 2025

Homework #2: Numerics & Derivatives
Due Sept. 26th

1. (Problem 2.6) Take the standard quadratic equation $ax^2 + bx + c = 0$ whose solutions are given by the following formula:

$$x_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Take $b > 0$ for correctness. It is easy to see that when $b^2 >> ac$ we don't get a catastrophic cancellation inside the square root term (but it may lead to a 'benign' cancellation). Furthermore, $\sqrt{b^2 - 4ac} \approx b$. However, this means that x_+ will have a catastrophic cancellation in the numerator. We will employ a numerical trick to help us conserve significant figures. Observe that the product of the two roots obey the relation $x_+x_- = c/a$. The answer now presents itself. Use the quadratic formula to calculate x_-, for which no catastrophic cancellation takes place. Then use the relation $x_+x_- = c/a$ to find x_+. Notice that you ended up calculating x_+ via division only.

Write a python code that evaluates and prints out (A) x_- (B) x_+ using the 'bad' formula (C) x_+ using the 'good' formula. Take $a = c = 1$ and $b = 10^8$. Discuss your answers.

2. (Problem 2.8) Study the following function:

$$f(x) = \frac{1 - \cos(x)}{x^2}$$

(A) start by plotting the function using a grid of points $x = 0.1 \times i$ for $i = 1, 2, \cdots, 100$. This should give some idea of values to expect for $f(x)$ at small x. (B) Verify your hunch taking the limit as $x \to 0$ and using L'Hopitals rule. (C) Now see what you find for the function $f(x)$ when $x = 1.2 \times 10^{-8}$. Does this make sense, even qualitatively? (D) Use a trig identity to avoid the cancellation in the original function. Evaluate your new function and compare with your analytical answer for $x \to 0$.

3. (Not from textbook) Consider a set of velocity data that was collected from a sounding rocket as it was launched from the ground level. The first 30 seconds of data are provided as part of the table below

t (s)	v(t) (m/s)
0	0.0
5	111.8
10	225.1
15	351.9
20	519.2
25	702.6
30	897.7

(A) Create a plot of the rocket's velocity vs time (B) Use the forward difference approximation to determine the acceleration of the rocket at the listed times int he table (note that you doing the forward difference will leave you with only $N - 1$ after doing the forward difference and that's ok!). Create a plot of the acceleration vs. time. (C) Use the forward difference formula a second time on the accelerations that you determined in part B. The derivative of acceleration is known as the 'jerk'. Plot the jerk vs time for the rocket. Additionally, print the value of the maximum jerk.

4. (Problem 3.8) This problem studies the 2nd derivative of the function

$$f(x) = \frac{1 - \cos(x)}{x^2}$$

which you first encountered in Problem 2.8. Take $x = 0.004$

(A) Start by analytically find the second derivative. Evaluate using the provided x value. (B) Give h the values $10^{-1}, 10^{-2}, \cdots, 10^{-6}$ and produce a log-log plot of the absolute error using the central difference approximation to the 2nd derivative (see course notes). (C) Introduce a new set of points on the previous plot, this time evaluating the first central difference approximation to the second derivative not of $f(x)$ but of the analytically rewritten version that you created in problem 2.8 (recall the trig. substitution used in part C).

5. (Problem 3.25) Examine the derivative of a *Noisy Function* (e.g., from imperfect measurements of an object's velocity v vs time t). We will model the effect of noise by superimposing a highly oscillatory behavior on a slowly varying function. Consider the following equation:

$$f(x) = 2 + 5\sin(x) + 0.1\sin(30x)$$

The last term has small amplitude, but contains significant oscillations that will impact the function's derivative. Take 128 equally place points from 0 to 2π and produce a table (or set of arrays) containing x and $f(x)$. (A) Plot $f(x)$ with $g(x)$, where $g(x)$ is a function only containing the first two terms of $f(x)$. Notice the two curves basically lie on top of each other. (B) Create a plot that contains (i) the analytically computed derivative $f'(x)$ (ii) the analytically computed derivative $g'(x)$ (iii) the forward difference approximation to $f'(x)$ using the array of points found earlier. Observe both analytic and forward difference versions of $f'(x)$ are very different from the 'underlying' behavior of the system represented by $g'(x)$. (C) Introduce a new set of points into your latest plot: (iv) the forward difference approximation to $f'(x)$ using grid points that are twice removed (e.g., to estimate the derivative of the data point x_{10} use the function values at x_{12} and x_{10}). Observe this set of points exhibits smaller amplitude 'noise' due to the fact that the effective step size h is larger thereby 'smoothing' the data. (D- not in the textbook but added by Dr. Hankins) Investigate use of the smoothing function that was introduced in Lab #3. Determine a value of smoothing for the data that produces a forward difference approximation for the derivative that agrees well with the analytic result for $g'(x)$.