Final Project

Active Brownian Particles in an External Field

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1 Introduction

Active Brownian Particles (ABPs) are a minimal model for self-propelled agents in a thermal environment. The three key ideas are:

- **Brownian particles:** random motion due to collisions with a surrounding fluid (thermal noise).
- **Active:** particles also possess an intrinsic self-propulsion at a characteristic speed.
- **External field:** particles respond to spatial variations in an environmental potential.

This work implements a simple discrete-time ABP model and examines how an external potential modifies the single-particle dynamics and the collective emergent behavior.

2 Background: Brownian and Active Motion

2.1 Regular Brownian Motion

Brownian motion refers to the random translational and rotational perturbations that particles experience due to collisions with many much lighter fluid molecules. In the overdamped limit we assume that collisions rapidly randomize the orientation of a particle's velocity vector while leaving its speed (magnitude of self-propulsion) approximately constant.

2.2 Active Brownian Motion

Active Brownian Particles add a self-propelled velocity of fixed magnitude v whose direction evolves under rotational diffusion. In 2D we denote by $\theta_i(t)$ the orientation angle of particle i at time t. The orientation is driven by stochastic noise and the position is updated by deterministic self-propulsion plus any external forces.

3 Model and Discrete-Time Updates

The discrete-time update rules used in the simulations are:

$$\theta_i(t+\Delta t) = \theta_i(t) + \sqrt{2D\,\Delta t}\,\xi_i(t), \tag{1}$$

$$x_i(t+\Delta t) = x_i(t) + \Big[v\cos\theta_i(t) \;-\; \mu\,\nabla_x V\big(x_i(t), y_i(t)\big)\Big]\Delta t, \tag{2}$$

$$y_i(t+\Delta t) = y_i(t) + \Big[v\sin\theta_i(t) \;-\; \mu\,\nabla_y V\big(x_i(t), y_i(t)\big)\Big]\Delta t. \tag{3}$$

Notes on the parameters and terms:

- $\xi_i(t)$ is a sample from a standard normal distribution $\mathcal{N}(0,1)$; samples across different t and i are independent.
- D is the rotational diffusion coefficient. D can be related to temperature T, viscosity of the fluid η, and particle radius R through $D \sim \dfrac{k_B T}{6\pi\eta R}$. $2D\Delta t$ is the variance of the probability distribution defining θ updates.
- v is the self-propulsion speed (kept constant).
- μ is the motility, controlling how strongly the particle responds to the potential gradient.
- $-\nabla V$ gives the force field associated with potential $V(x,y)$.

4 External Potential

To introduce nontrivial spatial dependence I used the following quartic potential, with four minima and one maximum:

$$V(x, y) = A\left[x^4 - x^2 + y^4 - y^2\right],$$

where A sets the potential strength/scale. This choice produces multiple attractor regions and rich gradients that interact with the self-propulsion to produce nonlinear phase-like behaviour.

The corresponding force components are

$$-\nabla_x V(x, y) = -A(4x^3 - 2x), \qquad -\nabla_y V(x, y) = -A(4y^3 - 2y).$$

5 Numerical Implementation

Simulations were performed with the updates described in Eqs. 1 – 3 with a fixed timestep Δt. Typical choices used in experiments:

- $v \sim \mathcal{O}(1)$ (units set by simulation).
- $\Delta t \ll 1$, chosen small enough to keep updates stable.
- A varied to explore weak-to-strong external fields.
- μ scanned to vary sensitivity to the field (small μ yields weak response; large μ yields strong response).
- D scanned to see effects of rotational diffusion (large D rapidly randomizes orientation; small D preserves orientation longer).

We also implemented basic collision detection, both between particles and with the walls of the container. In short, if the distance between the center of two particles becomes less than twice the radius they intersect with one another, which we correct for by pushing them apart and reversing direction. A similar rule is applied to collisions with the boundary, where a reflection along x or y is added when the center of a particle gets within one radius of a vertical or horizontal boundary, respectively.

6 Results and Observations

With no external field ($A = 0$) the particle performs persistent random walks whose persistence length depends on v/D. Adding the potential biases motion toward wells: when μA is moderate, particles tend to be attracted to stable wells; with strong motility the interplay between self-propulsion and the potential leads to trapping and escape events that depend sensitively on D and v.

When many-particle or long-time behavior is considered, the system can exhibit emergent patterns not present in the single deterministic parts: multiple steady-state populations, metastable trapping regions, and transitions between regimes as parameters vary. This qualitatively matches the expectation that ABPs interacting with nontrivial fields produce phase-like transitions and rich stochastic dynamics.

We generated a number of short videos showing the behavior of the system in different regimes, notably by varying D and μ. These animations and full code live on GitHub (see Appendix). As a dynamic phemenon, an image embedded in a PDF does not provide a particularly strong impression of the work, but one will be provided nonetheless.

7 Discussion and Conclusion

The combination of self-propulsion, rotational diffusion, and spatially varying forces produces nontrivial steady states and dynamics. The quartic potential used here is simple but already generates a landscape with multiple stable/unstable regions; scanning (D, μ, A, v) surfaces reveals parameter ranges where trapping or persistent circulation dominate.

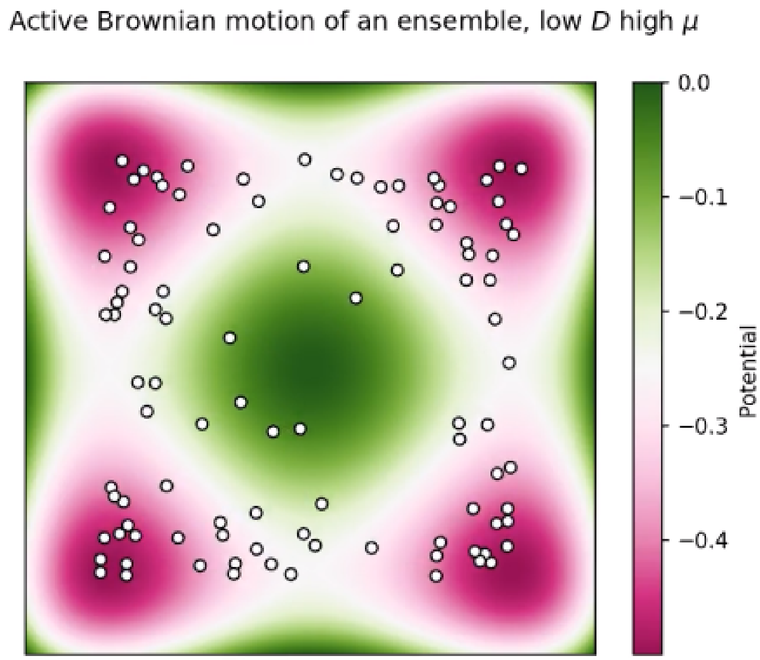

Figure 1: Example of ABP visualization.

Active Brownian Particles in an external field demonstrate how simple stochastic microscopic rules combine to produce complex, nonlinear macroscopic behaviour. The minimal ABP model with a quartic separable potential reveals trapping, drift, and parameter-dependent transitions that are qualitatively representative of chemotactic or field-responsive active matter.

8 Code Availability

The full code and animations mp4s are available on GitHub:

- `https://github.com/josephtemple/JT_PHYS4023` (project root)
- Particularly relevant folders: `Final Project` and `Final Project/videos` (see the repository for exported frames and movies).