Parameter Identification of the 2DOF Helicopter Model

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1 – Introduction

1.1 – Objectives, Goals, and Purposes

The main objective of this lab is to use experimental data to determine the parameters of the model helicopter including the moment of inertia, damping, and input gains. This is done in order to model a controller to be able to control that is implemented to control the rotors on the helicopter model that respond to give inputs.

1.2 – Intended Methods

To get the parameters, the motors will be turned off and the centers of mass will be measured, and the resting angle will be measured as well. The motors will be turned on, only one at a time, to measure the response of each rotor to obtain parameters such as damping and how voltage converts to the movement of the rotors.

2 – Procedure

2.1 – Definition of Variables

- θ is the pitch angle
- ψ is the yaw angle (corresponding to the fixed vertical axis)
- R_c is the horizontal distance of the center of mass from the pivot point ($R_c > 0$ by design)
- h is the vertical distance of the center of mass from the pivot point ($h > 0$ by design)
- l is the distance from the front (back) propeller axis to the pivot point ($l = 0.184$ m)
- L is the total length of the helicopter ($L = 0.483$ m).
- x is the distance from the small mass to the pivot point ($x = 0.120$ m)
- m_h is the of the helicopter without the small mass ($m_h = 1.17$ kg)
- m_s is small mass added to the helicopter ($m_s = 0.156$ kg)
- $m = m_h + m_s$ is the total mass of the helicopter ($m = 1.326$ kg)
- $m_{motors} = 0.754$ kg is the mass of pitch and yaw propellers, propeller shields and motors
- $m_b = 0.416$ kg is the mass moving about the pitch axis.
- J_p is the moment of inertia of the helicopter relative to the pitch axis
- J_y is the moment of inertia of the helicopter relative to the yaw axis
- $J_{shaft} = 0.0039\ kg \cdot m^2$ is the moment of inertia of metal shaft about yaw axis at end point.
- $c_p = B_p$ is the coefficient of viscous friction corresponding to the pitch axis
- $c_y = B_y$ is the coefficient of viscous friction corresponding to the yaw axis
- v_p is input the voltage to the pitch or front motor
- v_y is input the voltage to the yaw or back motor
- k_{pp} is the gain from the pitch motor to the pitch angle

- k_{py} is the gain from the yaw motor to the pitch angle
- k_{yy} is the gain from the yaw motor to the yaw angle
- k_{yp} is the gain from the pitch motor to the yaw angle.

2.2 – Schematic and Description of Apparatus

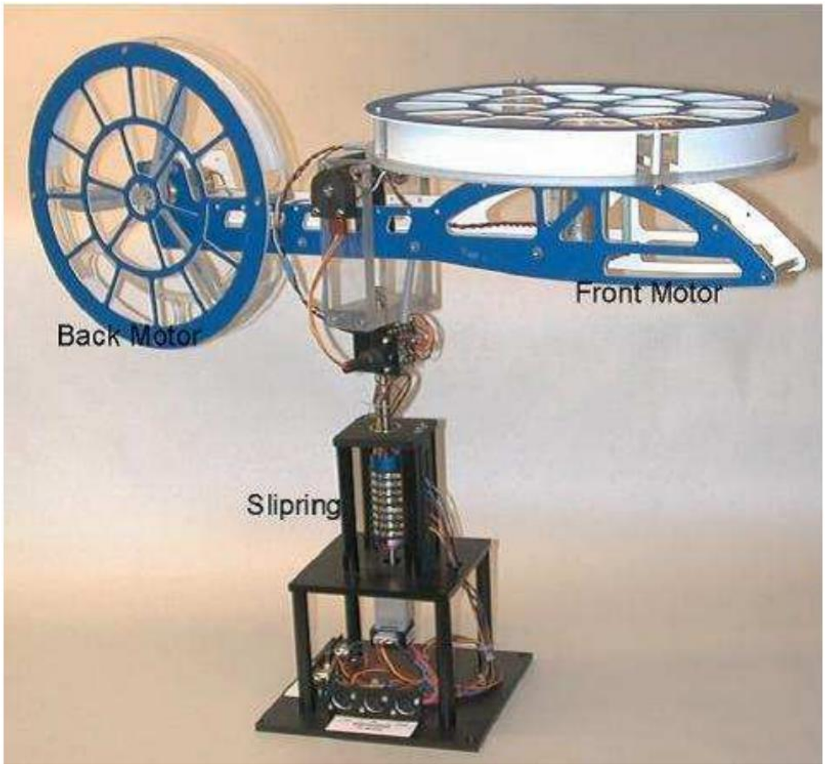


Figure 1 – Helicopter Model Apparatus

This lab makes use of a model helicopter that is set on a 2 degree of freedom mount which restricts movement of the helicopter to only pitch and yaw movements. The inputs into the apparatus is the voltage coming from the power supply and the output is the rotation of the rotors which pitch and yaw the model helicopter.

2.3 – Procedure of Experiments

Part i – Identification of Rc and h

a) Measure the distance between mass and pivot point, this is position x0
b) Set theta_0 = 0 in MATLAB
c) Run Simulink model with all specified gains set to 0
d) Move mass away from pivot point to level to the model and at equilibrium, measure distance from pivot, this is x1
e) Record value in pitch scope, this is p0

Part ii – Identification of Jp and cp

a) Put mass in position x1, set theta_0 = 0 in MATLAB
b) Set slider gains to 0 in Simulink model and run
c) Run the Simulink model and manually displace the model between +-15 or +-20 degrees then let go of the model and let it settle
d) Save pitch data

Part iii – Identification of Jy and cy

a) Set mass to x0, set theta_0 = 0 in MATLAB
b) Secure helicopter to set pitch to stay at 0
c) Set slider gains to 0 in Simulink model and run
d) Tap the helicopter model to spin it clockwise
e) Record the yaw rate data
f) Repeat step d) but spin it counterclockwise
g) Record the yaw rate data

Part iv – Identification of kpp and kpy

a) Secure model to prevent yaw movement and mass to x0
b) Use the recorded p0, set theta_0 = -p0 in radians, this is zero angle for level flight
c) Set slider gain for the back motor to 0
d) Use front motor slider gain to find the voltage required to overcome Coulomb friction, v+
e) Make 6 measurement between v+ and 11 volts and record pitch angle for each measurement
f) Repeat step c-e for the back motor and measurements between v+ and 12 volts

3 – Results

Parameter	Value	Units
p0	-0.5185	rad
m	1.326	kg
h	0.0138	m
Rc	0.0079	m

Jp	0.0425	kg*m^2
Jy	0.0425	kg*m^2
Jshaft	0.0039	kg*m^2
cp	0.0101	kg*m^2/s
cy	0.0051	kg*m^2/s
kpp	0.0493	N/V
kpy	0.0036	N*m/V

4 – Analysis and Discussion

The nonlinear equations of motion of the helicopter model can be seen below in figure 2 and the linearized equations of motion in figure 3. The assumptions that were made are that the angles and rates do not go too high such that the system is still near the linearization points. It is also assumed that the function between the voltage and the force output is linear and multiplied by the gains. It is also assumed that the pitch and yaw moments of inertia are equal.

$$
\begin{aligned}
&J_p\ddot{\theta} + J_y \sin(\theta)\cos(\theta)\dot{\psi}^2 + mg(h\sin(\theta) + R_c\cos(\theta)) + c_p\dot{\theta} \\
&= lF_p(v_p) + T_p(v_y) \\
&\left(J_y\cos(\theta)^2 + J_{shaft}\right)\ddot{\psi} - 2J_y\cos(\theta)\sin(\theta)\dot{\theta}\dot{\psi} + c_y\dot{\psi} \\
&= lF_y(v_y)\cos(\theta) + T_y(v_p)\cos(\theta).
\end{aligned}
$$

Figure 2 – Nonlinear Equations of Motion of the Helicopter Model

$$
\begin{aligned}
J_p\delta\ddot{\theta} + c_p\delta\dot{\theta} + mg\left(h\cos(\theta_e) - R_c\sin(\theta_e)\right)\delta\theta &= lk_{pp}\delta v_p + k_{py}\delta v_y \\
\left(J_y\cos(\theta_e)^2 + J_{shaft}\right)\delta\ddot{\psi} + c_y\delta\dot{\psi} &= lk_{yy}\left(\cos(\theta_e)\delta v_y - v_{ye}\sin(\theta_e)\delta\theta\right) \\
&\quad + k_{yp}\left(\cos(\theta_e)\delta v_y - v_{pe}\sin(\theta_e)\delta\theta\right) \\
\delta v_p &= v_p - v_{pe} \\
\delta v_y &= v_y - v_{ye}.
\end{aligned}
$$

Figure 3 – Linearized Equations of Motion

The equilibrium points for the yaw angle are at any point because the yaw angle will not cause instability in the system. On the other hand, the equilibrium point for the pitch is at -0.5185 radians.

To derive the linear equations of motion, define the linearized variables as:

$$\delta\theta = \theta - \theta_e \quad \text{and} \quad \delta\psi = \psi - \psi_e,$$

where the variables with the e subscript are the equilibrium points. Then take the derivative of the nonlinear EOM with respect to each order of variable (theta, thetadot, thetaddot, psi, psidot, etc.) and substitute in the equilibrium points, multiply them by their respective variables (theta → delta theta, etc) and add them all together.

4.i – Identification of Rc and h

P_0 is the resting angle of the helicopter when no forces are acting upon it and the mass is closest to the pivot point. After allowing the helicopter to oscillate to rest, the zero pitch is at -0.5185 radians. To find Rc and h, use the equations:

$$R_c = \frac{m_s(x_1 - x)}{m}. \quad \text{and} \quad h = \frac{R_c}{\tan(|p_0|)}.$$

Where Rc is going to be the horizontal distance for the center of mass of the system and h is the horizontal component of the distance for the center of mass. Using our data, the calculated values for Rc is 0.0079m and h is 0.0138m.

4.ii – Identification of Jp and cp

From the linear approximation, Jp and cp can be calculated using the following equations:

$$\begin{aligned} J_p &= J_p' - m_s(x_1^2 - x^2) \\ c_p &= 2\sigma J_p'. \end{aligned}$$

$$\sigma = \frac{\omega}{2\pi(n-1)} \ln\left(\frac{\theta(t_0)}{\theta(t_0 + 2\pi(n-1)/\omega)}\right). \quad \text{and} \quad \omega = 2\pi\frac{\text{number of cycles}}{\text{time}}.$$

$$J_p' = \frac{mgh}{\omega^2 + \sigma^2}.$$

Doing these calculations results in a Jp of 0.0425 kgm^2 and a cp of 0.0101kgm^2/s. The sigma value can also be used to calculate the envelope equation for the natural pitch angle data as seen in figure 4:

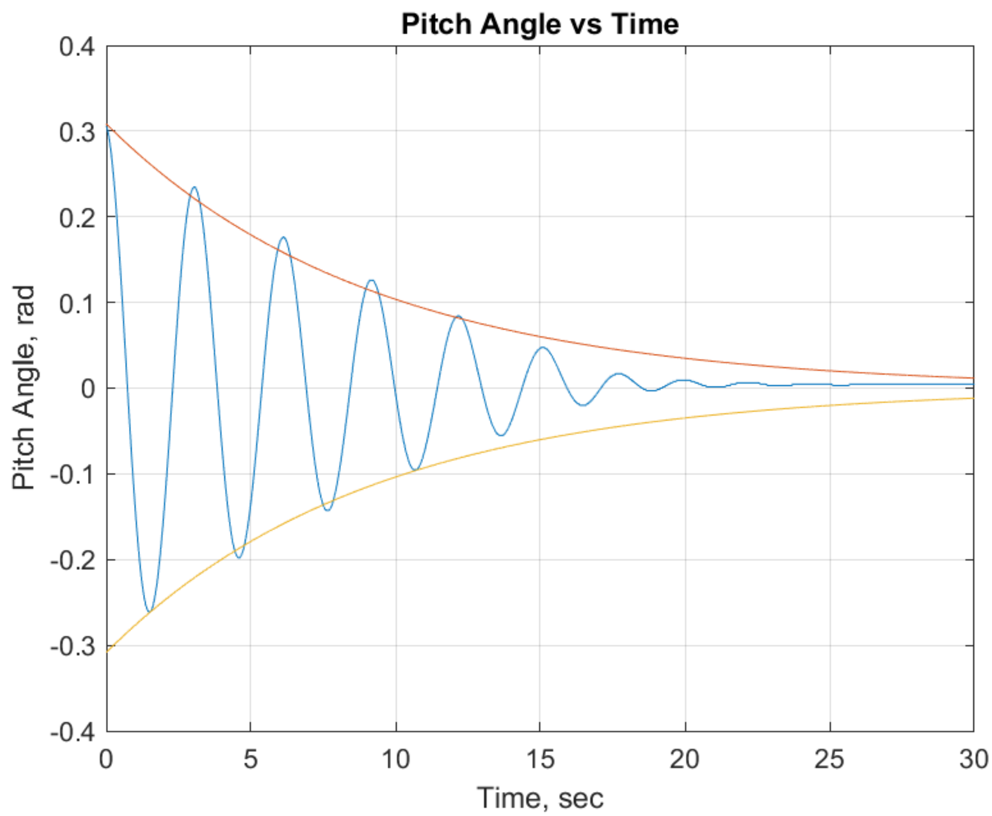


Figure 4 – Natural Pitch Angle Data with Envelopes

Where sigma was calculated to be 0.1090 1/(kgm^2), omega = 2.0847 rad/s, and Jp' was 0.0462kgm^2. Below, in figure 5, a Simulink model was used with the improved cp and Jp' values compared to the experimental data, the model can be found in the appendix. It can be seen that the simulated data matches closely in the beginning and matches the frequency quite well, but the experimental data seems to dampen out much quicker than the simulated data. This could be due to the linearizations that were used that could have affected the parameters for the simulated data.

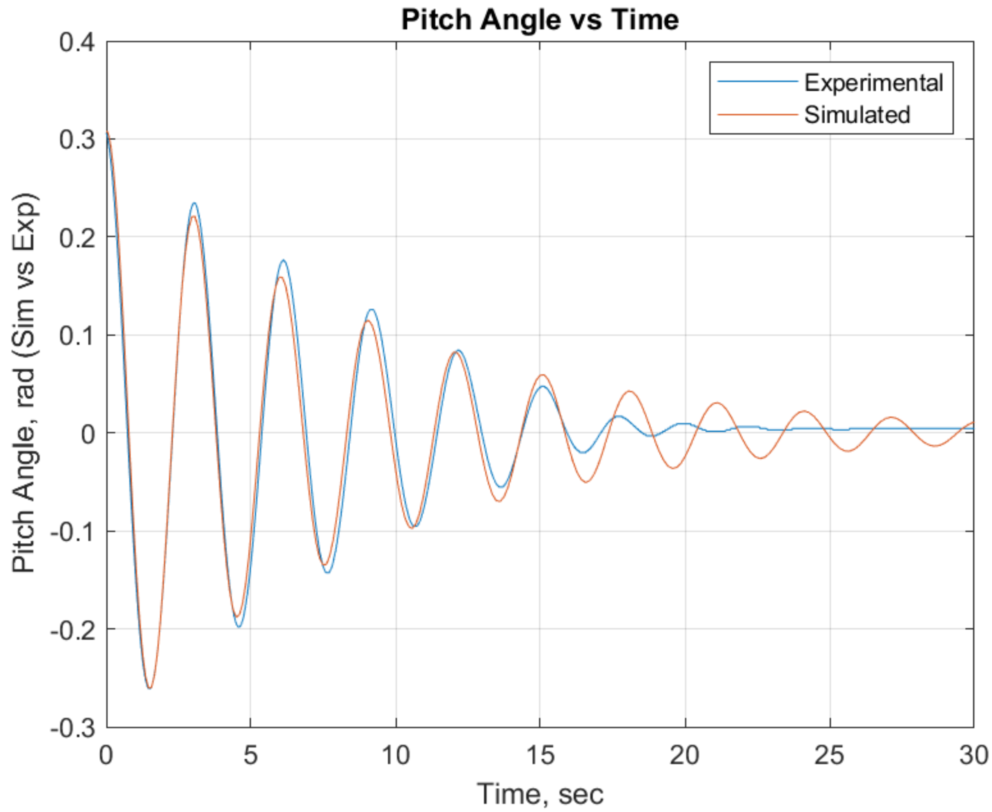


Figure 5 – Experimental vs Simulated Pitch Data

4.iii – Identification of Jy and cy

To find the moment of inertia (Jy) and damping/friction (cy), the equations below are used:

$$\dot{\psi}(t) = \dot{\psi}(0)e^{-\sigma t} \quad \text{where} \quad \sigma = \frac{c_y}{J_y + J_{shaft}}.$$

$$\begin{aligned} J_y &\approx J_p \\ c_y &\approx \sigma(J_p + J_{shaft}). \end{aligned}$$

Where the sigma, Jp, and Jshaft are known from the lab manual or calculated in the previous sections and whose values can be found in appendix. Applying these equations will yield a moment of inertia that is the same as the pitching moment of inertia and a yaw damping constant of 0.0051, which is roughly half of that of the pitching friction. These equations come from the assumptions that were previously mentioned. Below are the plots for clockwise spin which plots the yaw rate and the simulated yaw rate on the same graph. After that, the counterclockwise data is presented with its respective simulated data. Both data sets follow fairly closely to the simulated data sets with a slight offset that may be due to the approximation when calculating sigma from the previous sections and the choice of initial average yaw rate.

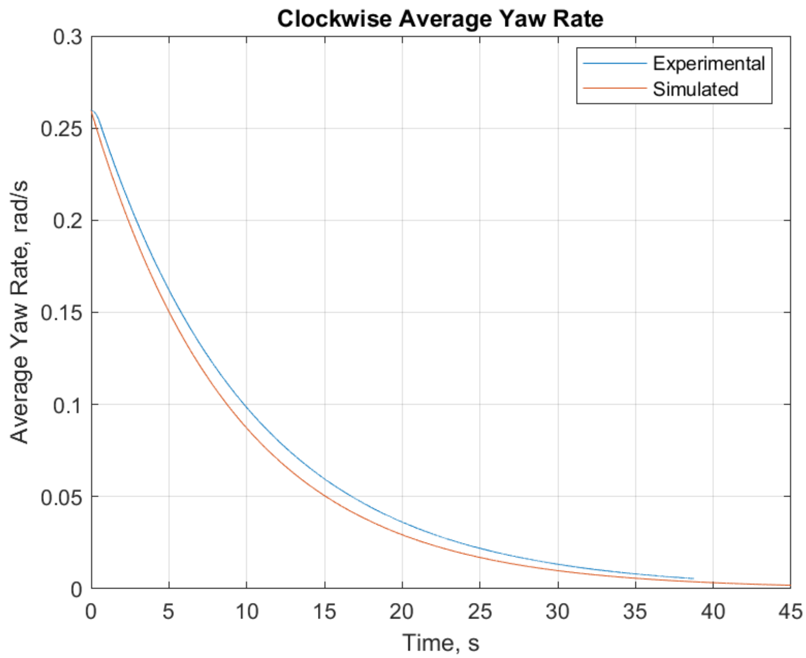


Figure 6 – Clockwise Average Yaw Rate vs Time

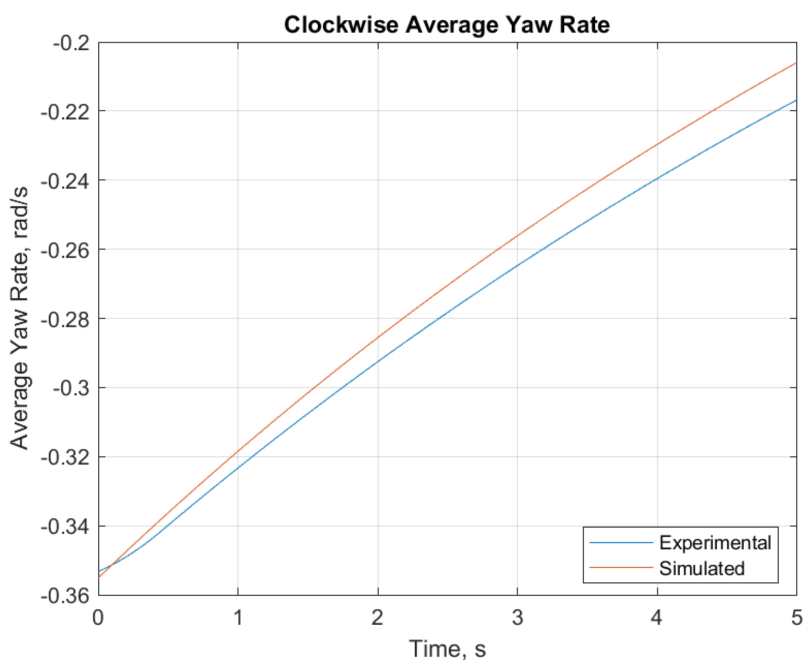


Figure 7 – Counterclockwise Average Yaw Rate vs Time

4.iv – Identification of Kpp and Kpy

To calculate Kpp and Kpy, the equations below will be used:

$$mg(h\sin(\theta(\infty)) + R_c\cos(\theta(\infty))) = lk_{pp}v_p - f_{cp}(v_p, 0).$$

$$mg(h\sin(\theta(\infty)) + R_c\cos(\theta(\infty))) = k_{py}v_y - f_{cp}(0, v_y).$$

These equations come from the Coulomb friction that is present in the system. The first equation models that system when only the front motor is turned on at steady state and the second is the same for the back motor. From these equations the information can be extracted below to calculate Kpp and Kpy:

$$\begin{aligned} k_{pp} &= \text{Slope of } (5.1)/l \\ k_{py} &= \text{Slope of } (5.2). \end{aligned}$$

To get data, we ran the two rotors (separately) and slowly increasing the voltage until an pitch angle change is detected. The voltage and angles were recorded to use the previous equations. Below are plots of the voltages and angles used in the equations above that help calculate Kpp and Kpy along with a line of best fit that shows the trend in the discrete points:

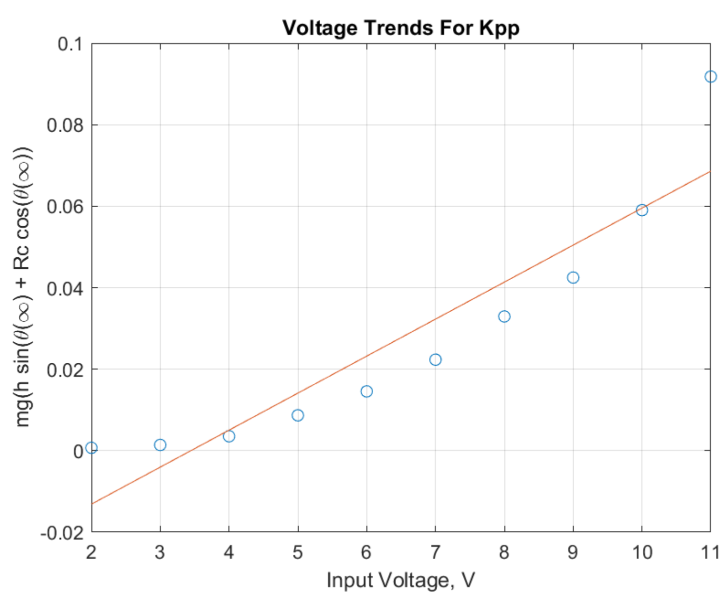


Figure 8 – Voltage Trends for Calculating Kpp

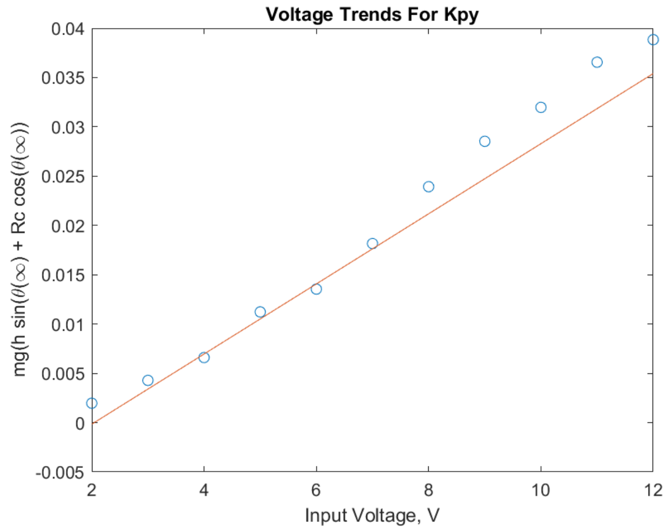


Figure 9 – Voltage Trends for Calculating Kpy

From these graphs, the slope of each fit line can be used to calculate Kpp and Kpy using the second set of equations. This yields a Kpp value of 0.0493 N/V and a value for Kpy of 0.0036 Nm/V. Looking at the plots, it seems that the relationships are fairly linear once the volage is able to overcome the Coulomb friction.

5 – Conclusions and Discussions

5.1 – Main Points

In this experiment, the parameters if the model helicopter were found in order to properly design a controller for the system in the next experiment. These parameters were found through experimental means and calculations that used these experimental data sets.

5.2 – Limitations

A large assumption that was made for this experiment was that the equations of motion were linearized about a point. This means that the further the motion of the system deviates from the equilibrium point, the more inaccurate it will be. Some limitations during the experiment was the high amount of noise in the data, this cause small orders of error in the final calculations, alone with the fact that for certain parts of the lab where we had to physically interact with the apparatus, we would not do so perfectly to get the cleanest results.

5.3 – Personal Lessons and Suggestions

Some personal lessons I learned was to be sure, when physical manipulation was required during an experiment, do so such that the resulting data will be as clean as possible. I was able to learn how to derive the parameters of a model from experimental means and be able to use the equations of motion of a system to be able to do so. As for suggestion, I have none that do not pertain to getting better or fixing some of the equipment.

6 – Appendix

```
addpath('lab4files')

addpath('simfiles')
sec3 = load('ThetaData3')
sec4_i = load('Psidotdata4cw')
sec4_ii = load('Psidotdata4ccw')
ms =  0.156; mh = 1.326; m = ms+mh; g = 9.81;
x = 0.12; x1 = 19.5/100, x0 = 13.75/100
```

Part i

```
p0 = -0.5185
Rc = ms * (x1 - x)/m
h = Rc / tan(abs(p0))
```

Part ii

```
th3 = sec3.Theta.signals.values;
time_ii = sec3.Theta.time-14.582;
Jshaft = 0.0039;
n = 5
t_ii = 15.07 % s
om = 2*pi * n / t_ii % rad/s
th0 = 0.30833 % rad
o = om/(2*pi*(n-1)) * log(th0/0.082835)
Jp_prime = m*g*h/ (om^2 + o^2)
Jp = Jp_prime - ms * (x1^2 - x^2)
cp = 2*o*Jp_prime


eeee = th0 * exp(-time_ii * o);
plot(time_ii,th3)
```

```
hold on
plot(time_ii,eeee)
plot(time_ii,-eeee)
xlim([0 30])
title('Pitch Angle vs Time')
xlabel('Time, sec')
ylabel('Pitch Angle, rad')
grid on
hold off



out = sim('part_iisim.slx');
thiisim = out.th.data;
timesim = out.th.time;
plot(time_ii,th3)
hold on
plot(timesim,thiisim)
xlim([0 30])
title('Pitch Angle vs Time')
xlabel('Time, sec')
ylabel('Pitch Angle, rad (Sim vs Exp)')
grid on
legend('Experimental','Simulated')
hold off
```

Part iii)

```
Jy = Jp
yawratecw = sec4_i.Average_psi_dot.signals.values;
exptime = sec4_i.psi_dot.time-5.96;
t = 0:1/1000:45;
y0 = .259466;
cy = o*(Jp + Jshaft)
simyawrate = y0 * exp(-o*t)
plot(exptime,yawratecw)
xlim([0 45])
hold on
grid on
title('Clockwise Average Yaw Rate')
ylabel('Average Yaw Rate, rad/s')
xlabel('Time, s')
plot(t,simyawrate)
legend('Experimental','Simulated','Location',"best")
hold off
```

```
yawrateccw = sec4_ii.Average_psi_dot.signals.values;
exptime = sec4_ii.psi_dot.time-9.3480;
plot(exptime,yawrateccw)
xlim([0 5])
hold on
y0 = -.355121;
simyawrate = y0 * exp(-o*t);
grid on
title('Counter-Clockwise Average Yaw Rate')
ylabel('Average Yaw Rate, rad/s')
xlabel('Time, s')
plot(t,simyawrate)
legend('Experimental','Simulated','Location',"best")
hold off
```

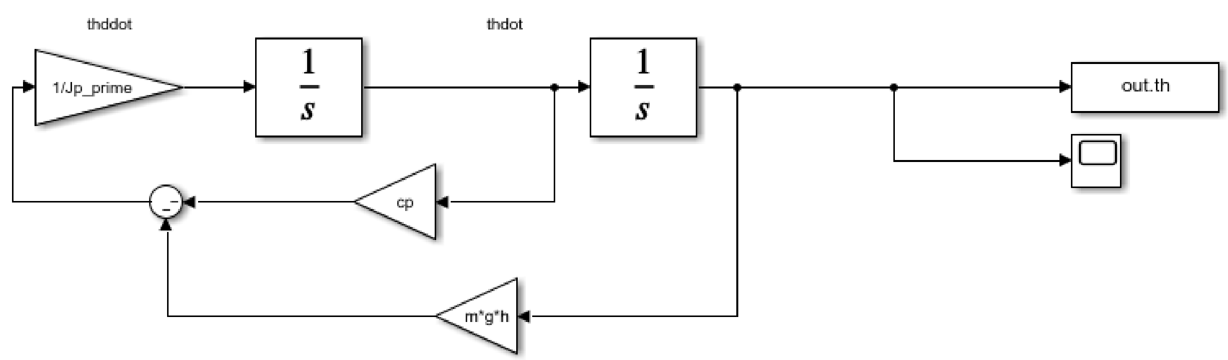
Part iv:

```
pitchvolt = [2 3 4 5 6 7 8 9 10 11]; pitchangle = [-0.5154 -.5124 -.5032 -
.481 -.4556 -.4219 -.3758 -.334 -.2608 -.111];
yawvolt = [2 3 4 5 6 7 8 9 10 11 12]; yawangle = [-.51 -.5 -.49 -.47 -.46
-.44 -.415 -.395 -.38 -.36 -.35];
l = 0.184;
pitcheqn = m*g*(h*sin(pitchangle) + Rc*cos(pitchangle));
coef = polyfit(pitchvolt,pitcheqn,1);
xp = linspace(pitchvolt(1), pitchvolt(end),2^12); yp = polyval(coef,xp);
kpp = (yp(2)-yp(1))/(xp(2)-xp(1))/l


plot(pitchvolt,pitcheqn,'o')
hold on
plot(xp,yp)
grid on
xlabel('Input Voltage, V')
ylabel('mg(h sin(\theta(\infty) + Rc cos(\theta(\infty))')
title('Voltage Trends For Kpp')
hold off


yaweqn = m*g*(h*sin(yawangle) + Rc*cos(yawangle));
coef = polyfit(yawvolt,yaweqn,1);
xy = linspace(yawvolt(1),yawvolt(end),2^12); yy = polyval(coef,xp);
kpy = (yy(2)-yy(1))/(xy(2)-xy(1))


plot(yawvolt,yaweqn,'o')
hold on
plot(xy,yy)
```

```
xlabel('Input Voltage, V')
ylabel('mg(h sin(\theta(\infty) + Rc cos(\theta(\infty))')
title('Voltage Trends For Kpy')
hold off
```

Parameters

ms	0.1560	kg
mh	1.3260	kg
x	0.1200	m
x1	0.1950	m
Rc0	0.0079	m
N	5	cycles
σ	0.1090	1/(kg*m^2)
ω	2.0847	rad/s
Jp’	0.0462	kg*m^2
cp	0.0101	kg*m^2/s
σ (cw)	0.1090	1/(kg*m^2)
σ (ccw)	0.1090	1/(kg*m^2)



Voltages and Pitch Angles

Voltage (V)	Pitch Angle (Radians)
2	-.5154
3	-.5124
4	-.5032
5	-.481
6	-.4556
7	-.4219
8	-.3758
9	-.334
10	-.2608
11	-.111

Voltage (V)	Pitch Angle (Radians)
2	-.5154
3	-.5001
4	-.4981
5	-.4714
6	-.4632
7	-.4421
8	-.4155
9	-.3956
10	-.3819
11	-.3673
12	-.3529