AAE 36401 - Pre - Lab 1

i) $m = M + M_J = M_c + M_w + M_J = 0.57 + 0.37 + 0.1331 = \boxed{1.0731 \text{ kg} = m}$

$M_J = \dfrac{\eta_g K_g^2 J_m}{r_{mp}^2} = 0.1331$

$\boxed{\gamma = \dfrac{\eta_g K_g \eta_m K_t}{R_m r_{mp}} = 1.7235}$

$\boxed{B_{emf} = \dfrac{\eta_g K_g^2 \eta_m K_t K_m}{R_m r_{mp}^2} = 7.7236 \ \frac{kg}{s}}$

ii) $k = 10$ V/m, $B_{eq} = 5.4$ kg/s

CE: $ms^3 + cs^2 + \gamma k s + \gamma k_i = 0 = \overbrace{s(ms^2 + cs + \gamma k)}^{D(s)} + \overbrace{\gamma k_i}^{k_i N(s)}$

$\rightarrow G(s) = \dfrac{N(s)}{D(s)} = \dfrac{\gamma}{s(ms^2 + cs + \gamma k)} \rightarrow G(s)K(s) = kL(s) = \dfrac{k_i \gamma}{s(ms^2 + cs + \gamma k)}$

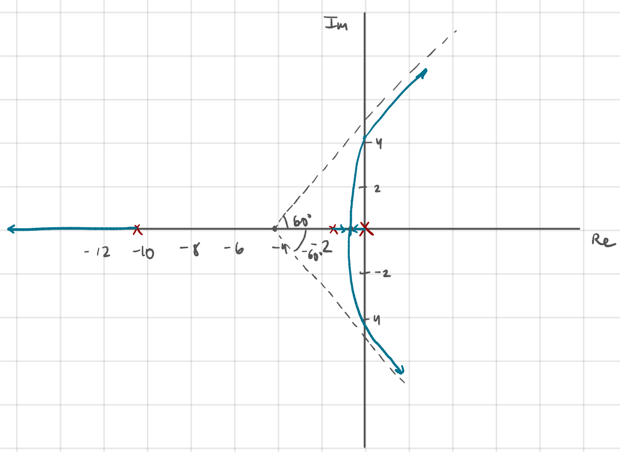
$CE = 1 + K(s)L(s) = 0 = 1 + \dfrac{N(s)}{D(s)}K(s) = D(s) + K(s)N(s)$

denom of $L(s)$ numerator of $L(s)$

$c = B_{emf} + B_{eq} = 13.1236$

Root Locus

$L(s) = \dfrac{\gamma}{s(ms^2 + cs + \gamma k)}$



① poles: MATLAB → root([m, c, γk, 0]) = 0, -10.7328, -1.4964

zeros: N/A

② symmetry

③ Real axis

④ $\theta_a = \dfrac{180 + 360 l}{n - m} = \dfrac{180 + 360(0,1,2)}{3} = 60°, 180°, 300°$

$n = 3$
$m = 0$

$\sigma_a = \dfrac{\sum poles - \sum zeros}{n - m} = -4.0764$

⑤ $\dfrac{d}{ds}\left(\dfrac{1}{L(s)}\right) = \dfrac{3ms^2 + 2cs + \gamma k}{\gamma} = 0 \rightarrow$ zeros: $s = -7.4326$, -0.7203

⑥ no complex poles

⑦ Intersection

$1 + k_i L(j\omega) = 0 = m(j\omega)^3 + c(j\omega)^2 + \gamma k j\omega + \gamma k_i$

$-m\omega^3 j - c\omega^2 + \gamma k \omega j + \gamma k_i = 0$

Re: $\gamma k_i = c\omega^2 \rightarrow \boxed{k_i = \dfrac{c\omega^2}{\gamma} = 131.2556}$ max k_i to be stable

Im: $-m\omega^3 + \gamma k \omega = 0 \rightarrow \omega(\gamma k - \omega^2) = 0$

$\omega = 0, \pm 4.1516$

iii) $M_p = 2\% = e^{-\left(\frac{\pi \zeta}{\sqrt{1-\zeta^2}}\right)} \rightarrow \zeta_D = \dfrac{-\ln(M_p/100)}{\sqrt{\pi^2 + \ln(M_p/100)^2}} = 0.7797$

$t_s = 0.5$ second $(2\%) = \dfrac{4}{\zeta \omega_n} \rightarrow \omega_{n0} = \dfrac{4}{\zeta} = 10.2603$

desired poles: $s_d = -\zeta_0 \omega_{n0} \pm j \omega_{n0}\sqrt{1 - \zeta_0^2} = -8 \pm 6.4245j$

$$\angle G(s) = \angle \frac{Y}{ms^2 + cs} = \cancel{\angle Y^{\circ}} - \angle m s_D^2 + c s_D = -\angle s_D(m s_D + c) = -\left(\angle s_D + \angle m s_D + c\right) = -\left(\tan^{-1}\left(\frac{Im}{Re}\right) + \tan^{-1}\left(\frac{Im}{s_D}\right)\right) = -197.6763^{\circ}$$

$$\angle G(s) + \phi = -180 \rightarrow \phi = 17.6763^{\circ}$$

poles : $s = 0, -12.2293$

$$K_d(s) = k_d(s + z_d)$$

$$\angle K_d(s) = \phi = \angle k_d(s + z_d) = \phi = \cancel{\angle k_d^{\circ}} + \angle s_D + z_d = \phi = \tan^{-1}\left(\frac{Im(s_D)}{Re(s_D - z_D)}\right)$$

$$Re(s_D - z_d) = \frac{Im(s_D)}{\tan\phi}$$

$$z_d = -\left(\frac{Im(s_D)}{\tan\phi} - Re(s_D)\right) = -27.9188$$

$$|G(s_D)K(s_D)| = |-1| = 1 = \left|\frac{Y}{m s_D^2 + c s_D}\right| \left|k_d(s_D + z_d)\right| = 1$$

$$k_d = \frac{1}{|s_D + z_d| \left|\frac{Y}{m s_D^2 + c s_D}\right|} = 1.3466$$

$$K_D(s) = k_d(s + z_d)$$

$$|z_d| = \left|\frac{k_p}{k_d}\right| \rightarrow k_p = |k_d \cdot z_d| = 37.5962$$

system type is type 1 so step input steady state error is 0 already, I controller not needed.

iv)