Problem 2

```
Re = 6378.1;
mu = 398600.4415;
r1 = Re * [-5.025, -0.087, -2.06]'
```

```
r1 = 3×1
10^4 ×
   -3.2050
   -0.0555
   -1.3139
```

```
v1 = [-0.2186, -2.904, -1.483]'
```

```
v1 = 3×1
   -0.2186
   -2.9040
   -1.4830
```

Part a)

```
r1mag = norm(r1)
```

```
r1mag = 3.4643e+04
```

```
v1mag = norm(v1)
```

```
v1mag = 3.2681
```

```
a1 = -mu/(2*(v1mag^2/2 - mu/r1mag))
```

```
a1 = 3.2324e+04
```

```
h1 = cross(r1,v1)
```

```
h1 = 3×1
10^4 ×
   -3.7332
   -4.4658
    9.2952
```

```
h1mag = norm(h1)
```

```
h1mag = 1.0967e+05
```

```
p1 = h1mag^2/mu
```

```
p1 = 3.0176e+04
```

```
e1 = sqrt(1-p1/a1)
```

```
e1 = 0.2578
```

```
ths1 = acos(1/e1*(p1/r1mag - 1))
```

```
ths1 = 2.0947
```

```
ths1_deg = rad2deg(ths1)
```

```
ths1_deg = 120.0169
```

```
h1hat = h1/h1mag
```

```
h1hat = 3×1
   -0.3404
   -0.4072
    0.8475
```

```
r1hat = r1/r1mag
```

```
r1hat = 3×1
   -0.9251
   -0.0160
   -0.3793
```

```
ths1hat = cross(h1hat,r1hat)
```

```
ths1hat = 3×1
    0.1680
   -0.9132
   -0.3713
```

```
% 3D Angles
iCr1 = [r1hat, ths1hat, h1hat]
```

```
iCr1 = 3×3
   -0.9251    0.1680   -0.3404
   -0.0160   -0.9132   -0.4072
   -0.3793   -0.3713    0.8475
```

```
i1 = acos(h1hat(3)), i1_deg = rad2deg(i1)
```

```
i1 = 0.5595
i1_deg = 32.0550
```

```
om1 = asin(h1hat(1)/sin(i1)); om1 = [om1 pi-om1], om1_deg = rad2deg(om1)
```

```
om1 = 1×2
   -0.6963    3.8379
om1_deg = 1×2
  -39.8944  219.8944
```

```
om1check = acos(-h1hat(2)/sin(i1)); om1check = om1check*[1 -1], om1check_deg = rad2deg
```

```
om1check = 1×2
    0.6963   -0.6963
om1check_deg = 1×2
   39.8944  -39.8944
```

```
om1 = om1(1), om1_deg = rad2deg(om1)
```

```
om1 = -0.6963
om1_deg = -39.8944
```

```
th1 = asin(r1hat(3)/sin(i1)); th1 = [th1 pi-th1], th1_deg = rad2deg(th1)
```

```
th1 = 1×2
   -0.7961    3.9377
th1_deg = 1×2
  -45.6109  225.6109
```

```
th1check = acos(ths1hat(3)/sin(i1)); th1check = [th1check, 2*pi-th1check], th1check_deg
```

```
th1check = 1×2
    2.3455    3.9377
th1check_deg = 1×2
  134.3891  225.6109
```

```
th1 = th1(2), th1_deg = rad2deg(th1)
```

```
th1 = 3.9377
th1_deg = 225.6109
```

```
v1r = iCr1.' * v1
```

```
v1r = 3×1
    0.8112
    3.1658
    0.0000
```

```
r1r = iCr1.' * r1
```

```
r1r = 3×1
10^4 ×
    3.4643
         0
    0.0000
```

```
% Orbit Characteristics
E1 = eccenAnom(ths1,e1), E1_deg = rad2deg(E1)
```

```
E1 = 1.8529
E1_deg = 106.1637
```

```
M1 = E1-e1*sin(E1), M1_deg = rad2deg(M1)
```

```
M1 = 1.6053
M1_deg = 91.9784
```

```
w1 = th1 - ths1, w1_deg = rad2deg(w1)
```

```
w1 = 1.8430
w1_deg = 105.5940
```

Part b)

```
ths2 = pi-w1, ths2_deg = rad2deg(ths2)
```

```
ths2 = 1.2986
ths2_deg = 74.4060
```

```
n1 = sqrt(mu/a1^3)
```

```
n1 = 1.0864e-04
```

```
E2 = eccenAnom(ths2,e1), E2_deg = rad2deg(E2)
```

```
E2 = 1.0559
E2_deg = 60.4973
```

```
dt1 = orbittime(e1,E1-2*pi,n1)
```

```
dt1 = -4.3058e+04
```

```
dt2 = orbittime(e1,E2,n1)
```

```
dt2 = 7.6540e+03
```

```
period1 = 2*pi/n1
```

```
period1 = 5.7835e+04
```

```
dt = dt2-dt1, dt_hr = dt/3600
```

```
dt = 5.0712e+04
dt_hr = 14.0867
```

```
p1 = a1*(1-e1^2)
```

```
p1 = 3.0176e+04
```

```
r2mag = p1/(1+e1*cos(ths2))
```

```
r2mag = 2.8220e+04
```

```
th2 = w1+ths2, th2_deg = rad2deg(th2)
```

```
th2 = 3.1416
th2_deg = 180
```

```
iCr2 = findDCM(om1,i1,th2)
```

```
iCr2 = 3×3
   -0.7672   -0.5436   -0.3404
    0.6414   -0.6503   -0.4072
    0.0000   -0.5307    0.8475
```

```
r2r = r2mag * [1 0 0]'
```

```
r2r = 3×1
10^4 ×
    2.8220
         0
         0
```

```
r2 = iCr2 * r2r
```

```
r2 = 3×1
10^4 ×
   -2.1651
    1.8100
    0.0000
```

```
v2mag = sqrt(mu*(2/r2mag - 1/a1))
```

```
v2mag = 3.9897
```

```
gamma2 = acos(h1mag/r2mag/v2mag), gamma2_deg = rad2deg(gamma2)
```

```
gamma2 = 0.2281
gamma2_deg = 13.0720
```

```
v2r = v2mag*[sin(gamma2), cos(gamma2), 0]'
```

```
v2r = 3×1
    0.9024
    3.8863
         0
```

```
v2 = iCr2 * v2r
```

```
v2 = 3×1
   -2.8049
   -1.9483
   -2.0626
```

```
dv_vnb = [0.2, 0.7, 0.3]' % delta_v in vnb frame using v2hat as vhat
```

```
dv_vnb = 3×1
    0.2000
    0.7000
    0.3000
```

```
dvmag = norm(dv_vnb)
```

```
dvmag = 0.7874
```

```
vhat = v2/v2mag
```

```
vhat = 3×1
   -0.7030
   -0.4883
   -0.5170
```

```
nhat = h1hat
```

```
nhat = 3×1
   -0.3404
   -0.4072
    0.8475
```

```
bhat = cross(vhat,nhat)
```

```
bhat = 3×1
   -0.6244
    0.7718
    0.1200
```

```
iCv = [vhat,nhat,bhat]
```

```
iCv = 3×3
   -0.7030   -0.3404   -0.6244
   -0.4883   -0.4072    0.7718
   -0.5170    0.8475    0.1200
```

```
dv = iCv * dv_vnb % in xyz
```

```
dv = 3×1
   -0.5662
   -0.1512
    0.5259
```

```
beta = asin(dv_vnb(2)/dvmag); beta = [beta pi-beta], beta_deg = rad2deg(beta)
```

```
beta = 1×2
    1.0952    2.0464
beta_deg = 1×2
   62.7480  117.2520
```

```
dv_vb = [dv_vnb(1), 0, dv_vnb(3)]' % in vnb
```

```
dv_vb = 3×1
    0.2000
         0
    0.3000
```

```
dv_vb_inertial = iCv * dv_vb % in xyz
```

```
dv_vb_inertial = 3×1
   -0.3279
    0.1339
   -0.0674
```

```
phi = acos(dot(dv_vb_inertial,iCr2(:,2))/norm(dv_vb))
```

```
phi = 1.2109
```

```
alpha = phi - gamma2, alpha_deg = rad2deg(alpha)
```

```
alpha = 0.9828
alpha_deg = 56.3099
```

```
beta_check = acos(0.2/(dvmag*cos(alpha)))
```

```
beta_check = 1.0952
```

```
v3 = v2 + dv
```

```
v3 = 3×1
   -3.3711
   -2.0995
   -1.5367
```

```
r3 = r2
```

```
r3 = 3×1
10^4 ×
   -2.1651
    1.8100
    0.0000
```

Part c)

```
rN = r3, vN = v3
```

```
rN = 3×1
10^4 ×
   -2.1651
    1.8100
    0.0000
vN = 3×1
   -3.3711
   -2.0995
   -1.5367
```

```
rNmag = norm(rN)
```

```
rNmag = 2.8220e+04
```

```
vNmag = norm(vN)
```

```
vNmag = 4.2583
```

```
aN = -mu/(2*(vNmag^2/2 - mu/rNmag))
```

```
aN = 3.9404e+04
```

```
hN = cross(rN,vN)
```

```
hN = 3×1
10^5 ×
   -0.2781
   -0.3327
    1.0647
```

```
hNmag = norm(hN)
```

```
hNmag = 1.1497e+05
```

```
pN = hNmag^2/mu
```

```
pN = 3.3159e+04
```

```
eN = sqrt(1-pN/aN)
```

```
eN = 0.3981
```

```
thsN = acos(1/eN*(pN/rNmag -1)), thsN_deg = rad2deg(thsN)
```

```
thsN = 1.1157
thsN_deg = 63.9247
```

```
rNhat = rN/rNmag
```

```
rNhat = 3×1
   -0.7672
    0.6414
    0.0000
```

```
hNhat = hN/hNmag
```

```
hNhat = 3×1
```

```
   -0.2419
   -0.2894
    0.9261
```

```
thNhat = cross(hNhat,rNhat)
```

```
thNhat = 3×1
   -0.5940
   -0.7106
   -0.3772
```

```
iCrN = [rNhat,thNhat,hNhat]
```

```
iCrN = 3×3
   -0.7672   -0.5940   -0.2419
    0.6414   -0.7106   -0.2894
    0.0000   -0.3772    0.9261
```

```
rNr = iCrN.' * rN
```

```
rNr = 3×1
10^4 ×
    2.8220
   -0.0000
         0
```

```
vNr = iCrN.' * vN
```

```
vNr = 3×1
    1.2398
    4.0739
    0.0000
```

```
iN = acos(hNhat(3)), iN_deg = rad2deg(iN)
```

```
iN = 0.3868
iN_deg = 22.1609
```

```
omN = asin(hNhat(1)/sin(iN)); omN = [omN pi-omN]
```

```
omN = 1×2
   -0.6963    3.8379
```

```
omNcheck = acos(-hNhat(2)/sin(iN)), omN = omN(1), omN_deg = rad2deg(omN)
```

```
omNcheck = 0.6963
omN = -0.6963
omN_deg = -39.8944
```

```
thN = asin(rNhat(3)/sin(iN)); thN = [thN pi-thN]
```

```
thN = 1×2
    0.0000    3.1416
```

```
thNcheck = acos(thNhat(3)/sin(iN))
```

```
thNcheck = 3.1416 - 0.0000i
```

```
thN = thN(2), thN_deg = rad2deg(thN)
```

```
thN = 3.1416
thN_deg = 180
```

```
wN = thN - thsN, wN_deg = rad2deg(wN)
```

```
wN = 2.0259
wN_deg = 116.0753
```

```
iCrNcheck = findDCM(omN,iN,thN)*iCrN.'
```

```
iCrNcheck = 3×3
    1.0000    0.0000    0.0000
   -0.0000    1.0000    0.0000
   -0.0000   -0.0000    1.0000
```

```
% % Orbit Characteristics
% E1 = eccenAnom(ths1,e1), E1_deg = rad2deg(E1)
% M1 = E1-e1*sin(E1), M1_deg = rad2deg(M1)
```

Function 1: Eccentric Anomaly

```
function E = eccenAnom(th,e)
    E = 2*atan(sqrt((1-e)/(1+e))*tan(th/2));
end
```

Function 2: Time Since Periapsis

```
function dt = orbittime(e,E,n)
    dt = (E - e*sin(E))/n;
end
```

Function 3: Find Direction Cosine Matrix

```
function iCr = findDCM(om,inc,th)
    col1 = [cos(om)*cos(th) - sin(om)*cos(inc)*sin(th);
        sin(om)*cos(th)+cos(om)*cos(inc)*sin(th);
        sin(inc)*sin(th)];
%     col2 = [-cos(om)*sin(th)-sin(om)*cos(inc)*cos(th);
%         -sin(om)*sin(th)+cos(om)*cos(inc)*cos(th);
%         sin(inc)*cos(th)];
    col3 = [sin(om)*sin(inc);
        -cos(om)*sin(inc);
        cos(inc)];

    col2 = cross(col3,col1);
    iCr = [col1 col2 col3];
end
```