Problem 3

Preliminary Calculations

```
alt = 2000; % km
r12dot = [-1.2, 6.7, 0]; % km/s, [r_hat, th_hat]
m2 = 650; % kg, spacecraft mass
r_Earth = 6378.1363; % km, radius of Earth
r12 = [alt+r_Earth, 0, 0]; % km, [r_hat, th_hat, k_hat]
G = 6.67408e-20; % km^3/(kg*s^2)
mu_Earth = 398600.4415; % Earth gravitational parameter
m1 = mu_Earth/G; % kg, Earth mass
mu = G*(m1+m2); % km^2/s^2
```

b) Compute values: $\bar{C}_3, \bar{h}, T, C_4, \varepsilon, \dot{A}$

```
C3 = (m1*m2)/(m1+m2)*cross(r12,r12dot) % Total system momentum
```

```
C3 = 1×3
10^7 ×
        0         0    3.6487
```

```
% C3 = (m1*m2)/(m1+m2)*(norm(r12)*r12dot(2))
h12 = (norm(r12)*r12dot(2)) % specific angular momentum
```

```
h12 = 5.6134e+04
```

```
Adot = h12/2 % areal velocity
```

```
Adot = 2.8067e+04
```

```
T = 1/2 * (m1*m2)/(m1+m2)*(r12dot * r12dot') % kinetic energy
```

```
T = 1.5057e+04
```

```
U = G*m1*m2/norm(r12) % gravitational potential energy
```

```
U = 3.0925e+04
```

```
C4 = T-U % total energy
```

```
C4 = -1.5867e+04
```

```
massfrac = (m1+m2)/(m1*m2)
```

```
massfrac = 0.0015
```

```
E = C4 * massfrac % specific energy
```

```
E = -24.4113
```

c) Determine Orbital Characteristics: p, e, a, τ, θ^*

```
p = h12^2/mu % semi-latus rectum
```

```
p = 7.9051e+03
```

```
e = sqrt(1+2*E*h12^2/mu^2) % eccentricity
```

```
e = 0.1782
```

```
a = p/(1-e^2) % semi-major axis
```

```
a = 8.1643e+03
```

```
b = a*sqrt(1-e^2) % semi-minor axis
```

```
b = 8.0336e+03
```

```
tau = 2/h12 * (pi*a*b) % orbital period
```

```
tau = 7.3415e+03
```

```
flightpath = atan2(r12dot(1),r12dot(2)) % flight path angle, radians
```

```
flightpath = -0.1772
```

```
flightpath_deg = rad2deg(flightpath)
```

```
flightpath_deg = -10.1543
```

```
true_anom = sign(flightpath)*acos((p/norm(r12)-1)/e) % true anomaly, radians
```

```
true_anom = -1.8932
```

```
true_anom_deg = rad2deg(true_anom) % true anomaly, degrees
```

```
true_anom_deg = -108.4751
```

d) Find $\dot{\vec{r}}$ and $\vec{r}$ in terms of $\hat{e}$ and $\hat{p}$

```
C = [cos(true_anom) -sin(true_anom) 0;sin(true_anom) cos(true_anom) 0; 0 0 1]; % coordi
r12_i = C * r12' % Position vector in Earth frame
```

```
r12_i = 3×1
10^3 ×
   -2.6550
   -7.9463
         0
```

```
r12dot_i = C * r12dot' % Velocity vectory in Earth frame
```

```
r12dot_i = 3×1
    6.7350
   -0.9850
         0
```

e) Compare $\dot{\vec{r}}$ and v_c

```
vc = sqrt(mu/norm(r12)) % Circular velocity at point in orbit
```

vc = 6.8976

```
r12dot_mag = norm(r12dot) % Magnitude of velocity at point in orbit
```

r12dot_mag = 6.8066