

Problem 1

```
addpath(genpath(fileparts(which('pathfile.m'))))
interr = 'latex';
% interr = 'none';
set(groot,'defaulttextinterpreter',interr);
set(groot, 'defaultAxesTickLabelInterpreter',interr);
set(groot, 'defaultLegendInterpreter',interr);
Re = 6378.1;
mu = 398600.4415;
```

Given:

```
r1 = 2*Re, r2 = 4*Re
```

```
r1 = 1.2756e+04
r2 = 2.5512e+04
```

Part a)

```
aT = 1/2 * (r1 + r2)
```

```
aT = 1.9134e+04
```

```
v1 = sqrt(mu/r1)
```

```
v1 = 5.5900
```

```
v1_n = sqrt(mu*(2/r1-1/aT))
```

```
v1_n = 6.4547
```

```
v2_0 = sqrt(mu*(2/r2-1/aT))
```

```
v2_0 = 3.2274
```

```
v2 = sqrt(mu/r2)
```

```
v2 = 3.9527
```

```
dv1 = v1_n - v1
```

```
dv1 = 0.8648
```

```
dv2 = v2 - v2_0
```

```
dv2 = 0.7253
```

```
dvtot = dv1 + dv2
```

```
dvtot = 1.7295
```

```
n = sqrt(mu/aT^3)
```

```
n = 2.3853e-04
```

```
TOF = 2*pi/n, TOF_hr = TOF/3600
```

```
TOF = 2.6341e+04  
TOF_hr = 7.3169
```

```
n2 = sqrt(mu/r2^3);  
TA = pi;  
phase = TA-n2*TOF, phase_deg = rad2deg(phase)
```

```
phase = -0.9395  
phase_deg = -53.8269
```

```
ts = 2*pi/(n-n2), ts_hr = ts/3600
```

```
ts = 7.5156e+04  
ts_hr = 20.8768
```

Part b)

```
TA_deg = 270; TA = deg2rad(TA_deg)
```

```
TA = 4.7124
```

```
inangle = 2*pi - TA
```

```
inangle = 1.5708
```

```
c = sqrt(r1^2 + r2^2 - 2*r1*r2*cos(TA))
```

```
c = 2.8524e+04
```

```
s = (r1+r2+c)/2
```

```
s = 3.3396e+04
```

```
amin = s/2
```

```
amin = 1.6698e+04
```

```
P1Fmin = 2*amin - r1
```

```
P1Fmin = 2.0640e+04
```

```
P2Fmin = 2*amin - r2
```

```
P2Fmin = 7.8838e+03
```

```
energymin = -mu/2/amin
```

```
energymin = -11.9355
```

```
zeta = asin(r1/c*sin(inangle))
```

```
zeta = 0.4636
```

```
emin = sqrt(r2^2+P2Fmin^2-2*r2*P2Fmin*cos(zeta))/2/amin
```

```
emin = 0.5628
```

```
vD = sqrt(mu*(2/r1-1/amin))
```

vD = 6.2148

vA = sqrt(mu*(2/r2-1/amin))

vA = 2.7160

pmin = amin*(1-emin^2)

pmin = 1.1409e+04

tsD = -acos(1/emin * (pmin/r1 - 1)), tsd_deg = rad2deg(tsD)

tsD = -1.7595

tsd_deg = -100.8123

tsA = tsD+TA, tsa_deg = rad2deg(tsa)

tsA = 2.9529

tsa_deg = 169.1877

hmin = sqrt(mu*pmin)

hmin = 6.7438e+04

gammaDmin = -acos(hmin/r1/vD), gammaDmin_deg = rad2deg(gammaDmin)

gammaDmin = -0.5536

gammaDmin_deg = -31.7175

gammaAmin = acos(hmin/r2/vA), gammaAmin_deg = rad2deg(gammaAmin)

gammaAmin = 0.2318

gammaAmin_deg = 13.2825

dvD = sqrt(v1^2 + vD^2 - 2*v1*vD*cos(gammaDmin))

dvD = 3.2814

dvA = sqrt(v2^2 + vA^2 - 2*v2*vA*cos(gammaAmin))

dvA = 1.4505

betaD = asin(vD/dvD * -sin(gammaDmin)), betaD_deg = rad2deg(betaD)

betaD = 1.4782

betaD_deg = 84.6965

alphaD = pi-abs(betaD), alphaD_deg = rad2deg(alphaD)

alphaD = 1.6634

alphaD_deg = 95.3035

betaA = pi- asin(v2/dvA * sin(gammaAmin))

betaA = 2.4650

alphaA = pi-abs(betaA), alphaA_deg = rad2deg(alphaA)

alphaA = 0.6766

alphaA_deg = 38.7636

```
iCr1 = [cos(0),-sin(0);sin(0) cos(0)];
iCr2 = [cos(tsA-tsD), -sin(tsA-tsD); sin(tsA-tsD) cos(tsA-tsD)];
dvtot = dvD+dvA
```

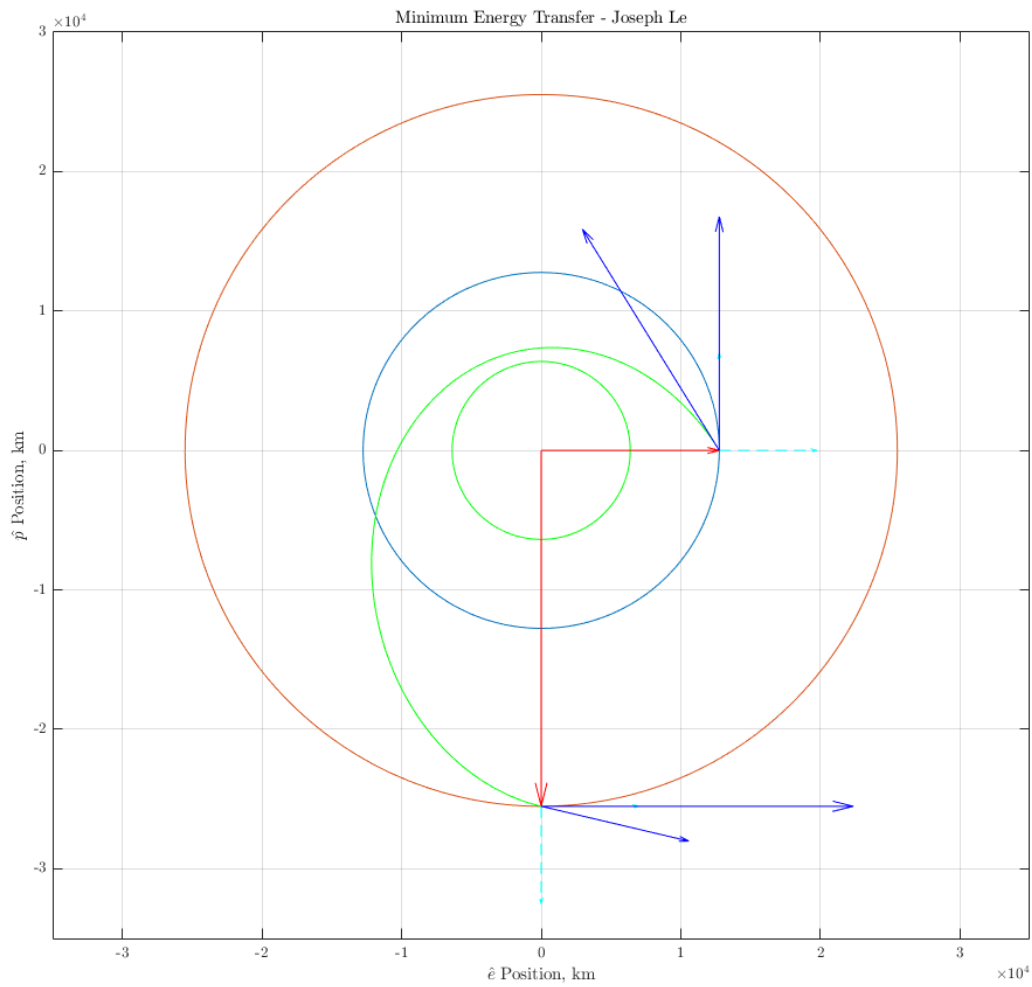
```
dvtot = 4.7318
```

```
plotorbit(r1,0,0,2*pi,0,'')
hold on
plotorbit(r2,0,0,2*pi,0,'')
plotorbit(Re,0,0,2*pi,0,'g')
plotpos(r1,0,0,'r',0,1)
plotpos(r2,0,7*pi/2,'r',0,1)
plotorbit(amin,emin,tsD,tsA,tsD,'g')
TOF = 1.0723e4;
n2 = sqrt(mu/r2^3);
phase = TA-n2*TOF, phase_deg = rad2deg(phase)
```

```
phase = 3.0511
phase_deg = 174.8123
```

```
plotvel(r1,0,0,v1*iCr1*[sin(0);cos(0)],'b',0,0,3e3)
plotvel(amin,emin,tsD,vD*iCr1*[sin(gammaDmin);cos(gammaDmin)],'b',0,tsD,3e3)
plotvel(r2,0,deg2rad(270),v1*iCr2*[sin(0);cos(0)],'b',0,0,4e3)
plotvel(amin,emin,tsA,vA*iCr2*[sin(gammaAmin);cos(gammaAmin)],'b',0,tsD,4e3)

xlim([-3.5 3.5]*1e4)
ylim([-3.5 3]*1e4)
xlabel('$\hat{e}$ Position, km')
ylabel('$\hat{p}$ Position, km')
title('Minimum Energy Transfer - Joseph Le')
set(gcf,'position',[0,0,1500,1500])
hold off
```



Part c)

```
TOF_hr = 6.25, TOF = TOF_hr * 3600
```

```
TOF_hr = 6.2500  
TOF = 22500
```

```
amin
```

```
amin = 1.6698e+04
```

```
TOFpar = 1/3 * sqrt(2/mu) * (s^(3/2) + (s-c)^(3/2))
```

```
TOFpar = 4.8109e+03
```

```
[a,alpha,beta] = bisection(TOF,amin,c,mu,'2B',.001)
```

```
a = 2.0449e+04  
alpha = 4.0268  
beta = -0.7048
```

```
alpha_deg = rad2deg(alpha), beta_deg = rad2deg(beta)
```

```
alpha_deg = 230.7187  
beta_deg = -40.3831
```

```
p = 4*a*(s-r1)*(s-r2)/c^2 * sin((alpha+beta)/2)^2
```

```
p = 1.6227e+04
```

```
e = sqrt(1-p/a)
```

```
e = 0.4544
```

```
rD = r1, rA = r2
```

```
rD = 1.2756e+04  
rA = 2.5512e+04
```

```
vD = sqrt(mu*(2/rD - 1/a))
```

```
vD = 6.5577
```

```
vA = sqrt(mu*(2/rA - 1/a))
```

```
vA = 3.4286
```

```
rp = a*(1-e)
```

```
rp = 1.1157e+04
```

```
ra = a*(1+e)
```

```
ra = 2.9742e+04
```

```
tsD = -acos(1/e * (p/r1 - 1)), tsD_deg = rad2deg(tsD)
```

```
tsD = -0.9289  
tsD_deg = -53.2229
```

```
tsA = tsD+TA, tsA_deg = rad2deg(tsA)
```

```
tsA = 3.7835  
tsA_deg = 216.7771
```

```
h = sqrt(mu*p)
```

```
h = 8.0424e+04
```

```
gammaD = -acos(h/r1/vD), gammaD_deg = rad2deg(gammaD)
```

```
gammaD = -0.2787  
gammaD_deg = -15.9673
```

```
gammaA = -acos(h/r2/vA), gammaA_deg = rad2deg(gammaA)
```

```
gammaA = -0.4042  
gammaA_deg = -23.1586
```

```
dvD = sqrt(v1^2 + vD^2 - 2*v1*vD*cos(gammaD))
```

```
dvD = 1.9404
```

```
dvA = sqrt(v2^2 + vA^2 - 2*v2*vA*cos(gammaA))
```

```
dvA = 1.5680
```

```
dvtot = dvD+dvA
```

```
dvtot = 3.5084
```

```
betaD = asin(vD/dvD * sin(gammaD)), betaD_deg = rad2deg(betaD)
```

```
betaD = -1.1936  
betaD_deg = -68.3869
```

```
alphaD = pi-abs(betaD), alphaD_deg = rad2deg(alphaD)
```

```
alphaD = 1.9480  
alphaD_deg = 111.6131
```

```
betaA = pi- asin(v2_n/dvA * sin(gammaAmin))
```

```
betaA = 2.5239
```

```
alphaA = pi- abs(betaA), alphaA_deg = rad2deg(alphaA)
```

```
alphaA = 0.6177  
alphaA_deg = 35.3913
```

```
iCr1 = [cos(0),-sin(0);sin(0) cos(0)];  
iCr2 = [cos(tsA-tsD), -sin(tsA-tsD); sin(tsA-tsD) cos(tsA-tsD)];
```

```
phase = TA-n2*TOF, phase_deg = rad2deg(phase)
```

```
phase = 1.2264  
phase_deg = 70.2683
```

```
plotorbit(r1,0,0,2*pi,0,'') % orbit 1  
hold on  
plotorbit(r2,0,0,2*pi,0,'') % orbit 2  
plotorbit(Re,0,0,2*pi,0,'g') % Earth  
plotpos(r1,0,0,'r',0,1) % position 1,D  
plotpos(r2,0,7*pi/2,'r',0,1) % position 2,A  
plotorbit(a,e,tsD,tsA,tsD,'g')
```

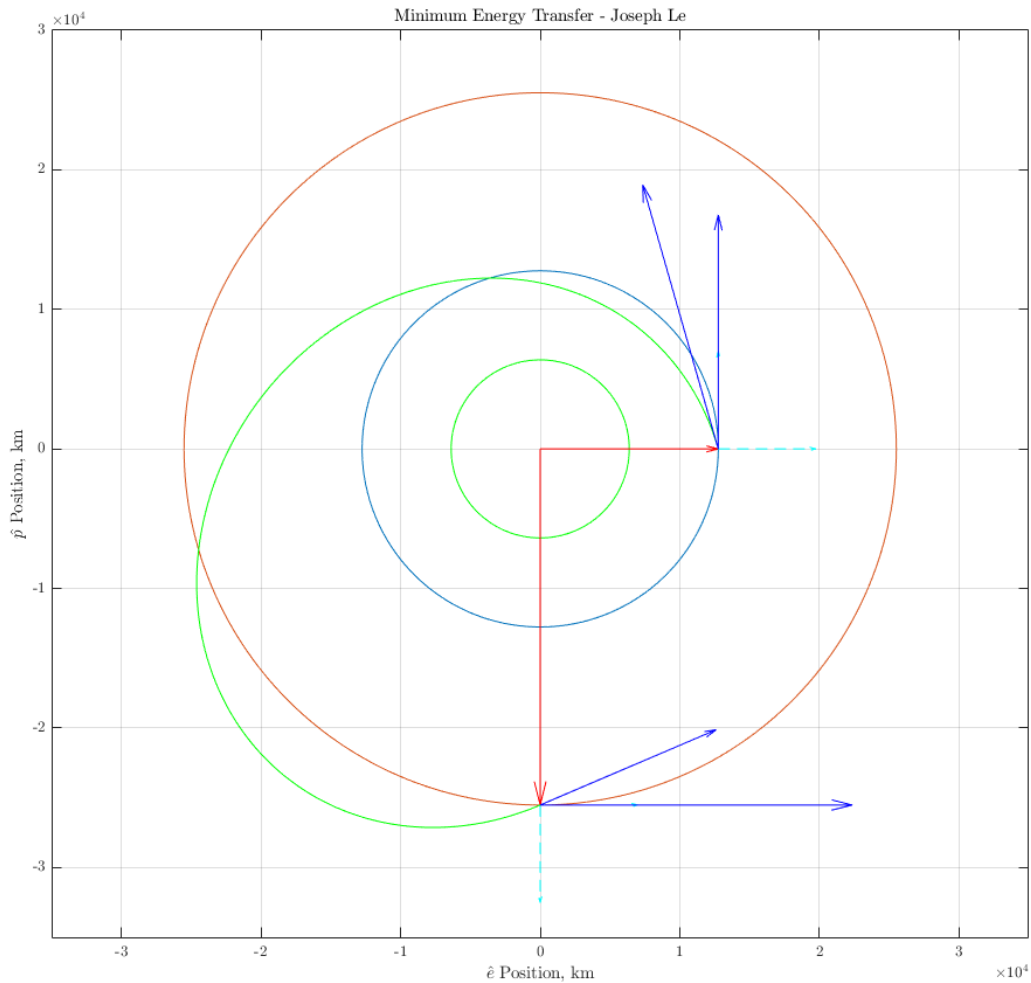
```
plotvel(r1,0,0,v1*iCr1*[sin(0);cos(0)],'b',0,0,3e3) % circular velocity at 1  
plotvel(a,e,tsD,vD*iCr1*[sin(gammaD);cos(gammaD)],'b',0,tsD,3e3) % departure velocity
```

```

plotvel(r2,0,deg2rad(270),v1*iCr2*[sin(0);cos(0)],'b',0,0,4e3) % circular velocity at 2
plotvel(a,e,tsA,vA*iCr2*[sin(gammaA);cos(gammaA)],'b',0,tsD,4e3) % arrival velocity

xlim([-3.5 3.5]*1e4)
ylim([-3.5 3]*1e4)
xlabel('$\hat{e}$ Position, km')
ylabel('$\hat{p}$ Position, km')
title('Minimum Energy Transfer - Joseph Le')

```



```

set(gcf,'position',[0,0,1500,1500])

```

```

function [a,alpha,beta] = bisectionSMA(TOF,amin,c,mu,type,tol)
s = amin*2;
if type(2) == 'A' | type(2) == 'B'
    amax = 100*amin;
elseif type(2) == 'H'

```



```

    amax = -100*amin;
    amin = 0;
end

TOFtest = TOF + 100000;
% for i = 1:10
while ~(abs(TOF - TOFtest) < tol)
    aguess = (amax+amin)/2;
    if isequal(type, '1A')
        alpha = 2*asin(sqrt(s/2/aguess));
        beta = 2*asin(sqrt((s-c)/2/aguess));
        TOFtest = sqrt(aguess^3/mu) * ((alpha - beta) - (sin(alpha) - sin(beta)));
    elseif isequal(type, '1B')
        alpha = 2*pi-2*asin(sqrt(s/2/aguess));
        beta = 2*asin(sqrt((s-c)/2/aguess));
        TOFtest = sqrt(aguess^3/mu) * ((alpha - beta) - (sin(alpha) - sin(beta)));
    elseif isequal(type, '2A')
        alpha = 2*asin(sqrt(s/2/aguess));
        beta = -2*asin(sqrt((s-c)/2/aguess));
        TOFtest = sqrt(aguess^3/mu) * ((alpha - beta) - (sin(alpha) - sin(beta)));
    elseif isequal(type, '2B')
        alpha = 2*pi-2*asin(sqrt(s/2/aguess));
        beta = -2*asin(sqrt((s-c)/2/aguess));
        TOFtest = sqrt(aguess^3/mu) * ((alpha - beta) - (sin(alpha) - sin(beta)));
    elseif isequal(type, '1H')
        alphap = 2*asinh(sqrt(s/2/abs(aguess)));
        betap = 2*asinh(sqrt((s-c)/2/abs(aguess)));
        TOFtest = sqrt(abs(aguess)^3/mu) * ((sinh(alphap)-alphap)-(sinh(betap)-betap));
    elseif isequal(type, '2H')
        alphap = 2*asinh(sqrt(s/2/abs(aguess)));
        betap = -2*asinh(sqrt((s-c)/2/abs(aguess)));
        TOFtest = sqrt(abs(aguess)^3/mu) * ((sinh(alphap)-alphap)-(sinh(betap)-betap));
    end

    if type(2) == 'A'
        if TOFtest > TOF
            amin = aguess;
        elseif TOFtest < TOF
            amax = aguess;
        else
            break
        end
    elseif type(2) == 'H' | type(2) == 'B'
        if TOFtest < TOF
            amin = aguess;
        elseif TOFtest > TOF
            amax = aguess;
        else
            break
        end
    end

end

end
end

```

```
a = aguess;  
end
```