

Problem 1

```
interr = 'latex';  
% interr = 'none';  
set(groot, 'defaulttextinterpreter', interr);  
set(groot, 'defaultAxesTickLabelInterpreter', interr);  
set(groot, 'defaultLegendInterpreter', interr);
```

Preliminary Calculations and Constants

```
Re = 6378.1; % Earth radius, km  
mu = 398600.4415; % Earth mu, km^3/s^2
```

Current Orbit Characteristics:

```
e1 = 0; a1 = 1.025*Re;  
v1_mag_0 = sqrt(mu/a1)
```

```
v1_mag_0 = 7.8084
```

Final Orbit Characteristics:

```
e2 = 0; a2 = 6.611*Re;  
v2_mag_0 = sqrt(mu/a2)
```

```
v2_mag_0 = 3.0746
```

i) Transfer Orbit Characteristics: Hohmann

```
[aT_1, eT_1, TOF_total_1, ...  
 v1_0_1, gamma_1_N_1, v1_N_1, dv1_1, alpha1_1, beta1_1, ...  
 v2_0_1, gamma_2_N_1, v2_N_1, dv2_1, alpha2_1, beta2_1, ...  
 dv_mag_total_1] = coplanartransfer(a1,a2,0,pi,mu)
```

```
aT_1 = 2.4352e+04  
eT_1 = 0.7315  
TOF_total_1 = 1.8909e+04  
v1_0_1 = 2x1  
    0  
    7.8084  
gamma_1_N_1 = 0  
v1_N_1 = 2x1  
    0  
    10.2749  
dv1_1 = 2x1  
    0  
    2.4665  
alpha1_1 = 0  
beta1_1 = 3.1416  
v2_0_1 = 2x1  
    0  
    1.5931  
gamma_2_N_1 = 2.9802e-08  
v2_N_1 = 2x1  
    0  
    3.0746  
dv2_1 = 2x1
```

```

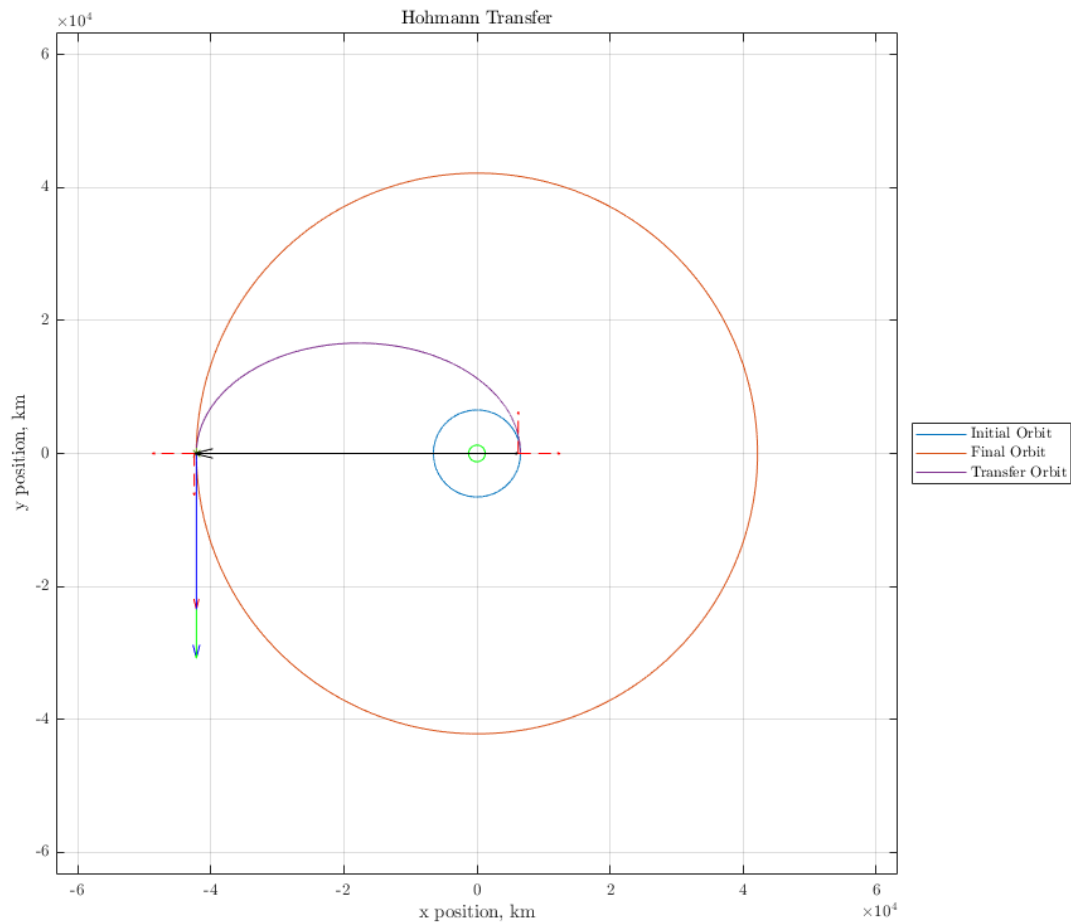
0
1.4815
alpha2_1 = 6.1848e-08
beta2_1 = 3.1416
dv_mag_total_1 = 3.9480

```

```

plotinitfin(a1,0,a2,0)
hold on
plottransfer(aT_1,eT_1,0,pi)
title('Hohmann Transfer')
xlabel('x position, km'), ylabel('y position, km')
plotvel(aT_1,eT_1,pi,v1_0_1,'r',0,0) % pre, pos1
plotvel(aT_1,eT_1,pi,v1_N_1,'b',0,0) % post, pos1
plotvel(aT_1,eT_1,pi,dv1_1,'g',[1; v1_0_1],0) % dv, pos1
plotpos(aT_1,eT_2,0,'k',0)
plotpos(aT_1,eT_2,pi,'k',0)
legend('Initial Orbit','Final Orbit', '', 'Transfer Orbit','','','','','location','ea
set(gcf,'position',[0,0,1000,1000])
hold off

```



```
phase1 = pi - TOF_total_1 * sqrt(mu/a2^3), phase1_deg = rad2deg(phase1)
```

```
phase1 = 1.7628
phase1_deg = 101.0003
```

ii) Transfer Orbit Characteristics: 150 Degree Transfer Angle

```
[aT_2, eT_2, TOF_total_2, ...
 v1_0_2, gamma_1_N_2, v1_N_2, dv1_2, alpha1_2, beta1_2, ...
 v2_0_2, gamma_2_N_2, v2_N_2, dv2_2, alpha2_2, beta2_2, ...
 dv_mag_total_2] = coplanartransfer(a1,a2,30*pi/180,pi,mu)
```

```
aT_2 = 2.4165e+04
eT_2 = 0.7449
TOF_total_2 = 1.8376e+04
v1_0_2 = 2x1
```

```

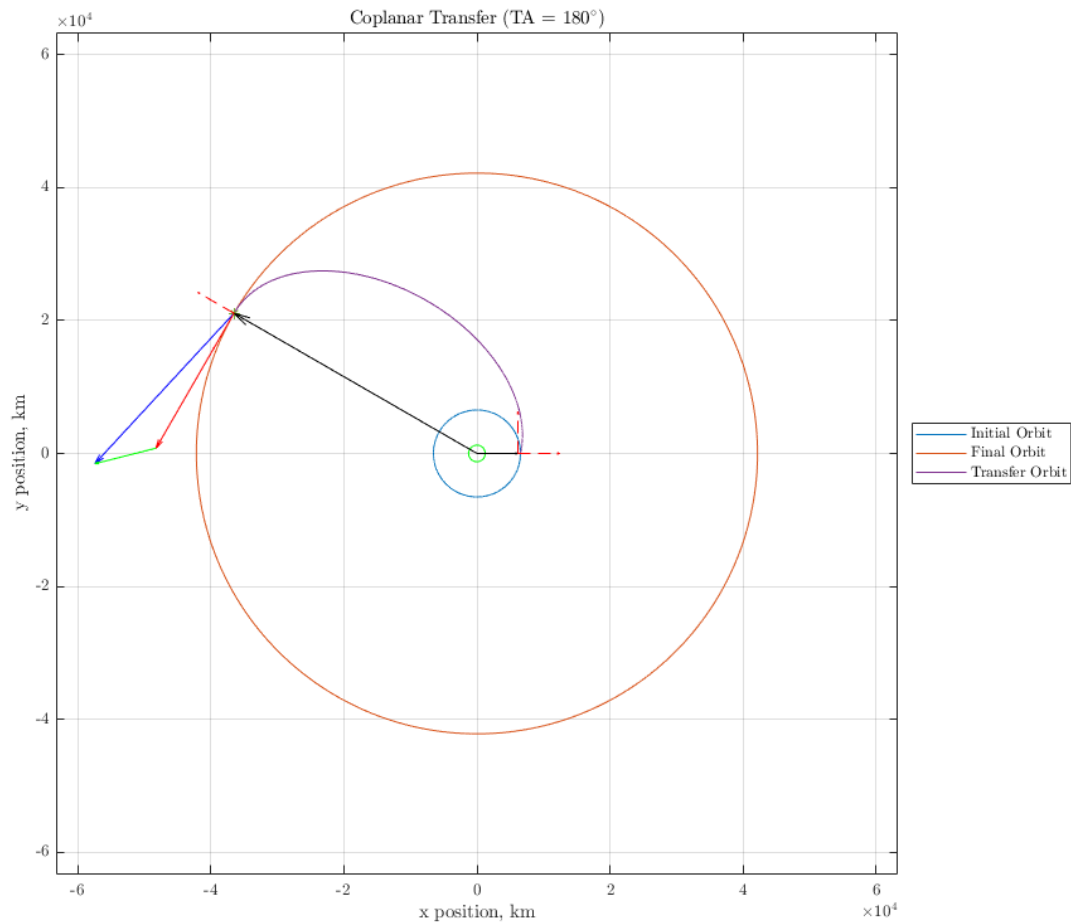
0
7.8084
gamma_1_N_2 = 0.2227
v1_N_2 = 2x1
    2.2675
    10.0152
dv1_2 = 2x1
    2.2675
    2.2069
alpha1_2 = 0.7990
beta1_2 = 2.3426
v2_0_2 = 2x1
    0
    1.5528
gamma_2_N_2 = 0
v2_N_2 = 2x1
    0
    3.0746
dv2_2 = 2x1
    0
    1.5218
alpha2_2 = 0
beta2_2 = 3.1416
dv_mag_total_2 = 4.6859

```

```

plotinitfin(a1,0,a2,0)
hold on
plottransfer(aT_2,eT_2,30*pi/180,pi)
title('Coplanar Transfer (TA = $180^{\circ}$)')
xlabel('x position, km'), ylabel('y position, km')
plotvel(aT_2,eT_2,pi,v1_0_2,'r',0,30*pi/180) % pre, pos1
plotvel(aT_2,eT_2,pi,v1_N_2,'b',0,30*pi/180) % post, pos1
plotvel(aT_2,eT_2,pi,dv1_2,'g',[1; v1_0_1],30*pi/180) % dv, pos1
plotpos(aT_2,eT_2,0,'k',0)
plotpos(aT_2,eT_2,pi,'k',30*pi/180)
legend('Initial Orbit','Final Orbit', '', 'Transfer Orbit','','','','','location','ea
set(gcf,'position',[0,0,1000,1000])
hold off

```



```
phase2 = pi - TOF_total_2 * sqrt(mu/a2^3), phase2_deg = rad2deg(phase2)
```

```
phase2 = 1.8017
phase2_deg = 103.2291
```

Function 1: Transfer Characteristics (Circle to Circle)

Can change to work with any starting and ending orbits, once that works, bi-elliptic/multiburn transfer can be done through recursion or multiple function calls

Transfer starting at periaapsis of transfer orbit to transfer angle on orbit

a1 and a2 in km

ths1 and ths2 are initial and final true anomalies on transfer orbit

```
function [aT, eT, TOF_total, ...
    v1_0, gamma_1_N, v1_N, dv1, alpha1, beta1, ...
    v2_0, gamma_2_N, v2_N, dv2, alpha2, beta2, ...
    dv_mag_total] = coplanartransfer(a1,a2,ths1,ths2,mu)
```

Transfer Orbit Characteristics:

```
eT = (a2 - a1)/(a2 + a1 * cos(th1));
aT = a2/(1+eT);
E1 = eccenAnom(th1,eT);
E2 = eccenAnom(th2,eT);
TOF1 = sqrt(aT^3/mu) * (E1 - eT*sin(E1));
TOF2 = sqrt(aT^3/mu) * (E2 - eT*sin(E2));
TOF_total = TOF2-TOF1;
```

1st Thrust Point Conditions (Pre-Maneuver: Initial Orbit): (change this to be able to accomodate other starting orbit)

```
r1_mag = a1; v1_mag_0 = sqrt(mu*(2/r1_mag-1/a1));
v1_0 = [0 v1_mag_0]';
```

1st Thrust Point Conditions (Post-Maneuver: Transfer Orbit):

```
v1_mag_N = sqrt(mu*(2/r1_mag - 1/aT));
gamma_1_N = findFPA(aT,eT,r1_mag,mu,ths1);
v1_N = v1_mag_N * [sin(gamma_1_N),cos(gamma_1_N)]';
dv1 = v1_N - v1_0; dv1_mag = norm(dv1);
[alpha1,beta1] = alphabeta(v1_mag_0,norm(v1_N),dv1_mag,gamma_1_N);
```

2nd Thrust Point Conditions (Pre-Maneuver: Transfer Orbit):

```
r2_mag = a2;
v2_mag_0 = sqrt(mu*(2/r2_mag - 1/aT)); v2_0 = [0 v2_mag_0]';
```

2nd Thrust Point Conditions (Post-Maneuver: Final Orbit): (change this to be able to accomodate other final orbit)

```
v2_mag_N = sqrt(mu*(2/r2_mag - 1/a2)); v2_N = [0 v2_mag_N]';
gamma_2_N = findFPA(aT,eT,r2_mag,mu,ths2);
dv2 = v2_N - v2_0; dv2_mag = norm(dv2);
dv_mag_total = dv1_mag + dv2_mag;
[alpha2,beta2] = alphabeta(v2_mag_0,norm(v2_N),dv2_mag,gamma_2_N);
end
```

Function 2: Eccentric Anomaly

```
function E = eccenAnom(th,e)
    E = 2*atan(sqrt((1-e)/(1+e))*tan(th/2));
end
```

Function 3: Find FPA

```
function FPA = findFPA(a,e,r,mu,ths)
    vmag = sqrt(mu*(2/r-1/a));
    p = a*(1-e^2);
```

```

h = sqrt(mu*p);
FPA = acos(h/r/vmag);
if wrapTo2Pi(th) <= 180
    FPA = abs(FPA);
else
    FPA = -abs(FPA);
end
end

```

Function 4: Alpha and Beta

```

function [alpha, beta] = alphabeta(v0,vN,dv,dgamma)
angles = asin(vN/dv * sin(dgamma));
angles = [angles pi-angles];

if v0 > vN
    alpha = max(angles);
    beta = min(angles);
elseif v0 < vN
    alpha = min(angles);
    beta = max(angles);
elseif dv == 0
    alpha = 0;
    beta = pi;
end

if dgamma < 0
    alpha = -abs(alpha);
else
    alpha = abs(alpha);
end
end

```

Function 5: Plotting Initial and Final Orbits

```

function plotinitfin(ai,ei,af,ef)
ths_plot = linspace(0,2*pi,2^12)';
ri = (ai*(1-ei^2))./(1+ei*cos(ths_plot));
rf = (af*(1-ef^2))./(1+ef*cos(ths_plot));
ri = ri .* [cos(ths_plot),sin(ths_plot)];
rf = rf .* [cos(ths_plot),sin(ths_plot)];
plot(ri(:,1),ri(:,2)), hold on, plot(rf(:,1),rf(:,2))
grid on
axis equal
maxlim = max([ai af]); maxlim = [-maxlim maxlim]*1.5;
xlim(maxlim), ylim(maxlim)
plot(0,0,'go','MarkerSize', 10)
end

```

Function 6: Plotting Transfer Orbit

```

function plottransfer(aT,eT,ths1,ths2)
ths_plot = linspace(ths1,ths2,2^12);
r_T_plot = aT * (1-eT^2)./(1+eT*cos(ths_plot));

```

```

rx_T = r_T_plot.*cos(th_s_plot-th_s1); ry_T = r_T_plot.*sin(th_s_plot-th_s1);
plot(rx_T,ry_T)
plot(r_T_plot(end).*cos(th_s2-th_s1),r_T_plot(end).*sin(th_s2-th_s1),'*')
end

```

Function 7: Velocity Vectors

```

function plotvel(a,e,ths,v,color,dvoption,angle)
rmag = a*(1-e^2)/(1+e*cos(ths));
size = 3e3;
iCr = [cos(ths-angle) -sin(ths-angle); sin(ths-angle) cos(ths-angle)];
v = iCr*v;
if dvoption(1) == 1
    v0 = dvoption(2:3);
    v0 = iCr * v0;
    quiver(rmag*cos(ths-angle)+v0(1)*size,rmag*sin(ths-angle)+v0(2)*size,v(1),v(2),size,color)
else
    quiver(rmag*cos(ths-angle),rmag*sin(ths-angle),v(1),v(2),size,color)
end
end

```

Function 8: Position Vectors

```

function plotpos(a,e,ths,color,angle)
rmag = a*(1-e^2)/(1+e*cos(ths)); r = [rmag 0]';
iCr = [cos(ths-angle) -sin(ths-angle); sin(ths-angle) cos(ths-angle)];
r = iCr * r;
quiver(r(1),r(2),color,'Autoscale','off')
plotunit(r,ths-angle,color)
end

```

Function 9: Unit Vectors

```

function plotunit(pos,th,color)
x = pos(1); y = pos(2); size = 7e3;
iCr = [cos(th) -sin(th); sin(th) cos(th)];
hat1 = iCr * [1 0]' * size;
hat2 = iCr * [0 1]' * size;
quiver(x,y,hat1(1),hat1(2),'r--')% x1
quiver(x,y,hat2(1),hat2(2),'r--')% x2
end

```