Parameters and Equilibrium Conditions

```
P1 = [2 1 1 1 1 1];
P2 = [2 1 1 1 0.99 1];
P3 = [2 1 0.5 1 1 1];
P4 = [2 1 1 1 0.5 1];

E1 = [0 0 0];
E2 = [0 pi pi];

case1 = num2cell([P1,E1]);
case3 = num2cell([P2,E1]);
case7 = num2cell([P4,E1]);

u = 0;
C = [1 0 0 0 0 0];
D = 0;
```

Case 1: P1, E1

```
% pull parameters and initial conditions for case 1
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case1{:});

mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -0.5000   -1.5000         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
         0
         0
    0.5000
    0.5000
    0.5000
```

```
eig(A)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
   0.0000 + 1.4142i
   0.0000 - 1.4142i
```

```
  -0.0000 + 1.0000i
  -0.0000 - 1.0000i
```

```
Qc = ctrb(A,B)
```

```
Qc = 6×6
         0    0.5000         0   -0.5000         0    1.0000
         0    0.5000         0   -1.0000         0    2.0000
         0    0.5000         0   -1.0000         0    2.0000
    0.5000         0   -0.5000         0    1.0000         0
    0.5000         0   -1.0000         0    2.0000         0
    0.5000         0   -1.0000         0    2.0000         0
```

```
nc = rank(Qc)
```

```
nc = 4
```

```
ans == 6
```

```
ans = 6×1 logical array
   0
   0
   0
   0
   0
   0
```

```
[~,unctrbeig] = PBHtest(A,B,C)
```

```
unctrbeig = 1×3 cell
```

	1	2	3
1	6×2 complex...	[-0 + 1i;-0...	6×2 complex...

```
[Tc,S,~] = svd(Qc)
```

```
Tc = 6×6
   -0.3489   -0.0000    0.9372   -0.0000    0.0000   -0.0000
   -0.6627         0   -0.2467         0    0.7071   -0.0000
   -0.6627         0   -0.2467         0   -0.7071    0.0000
   -0.0000   -0.3489    0.0000    0.9372   -0.0000   -0.0000
    0.0000   -0.6627   -0.0000   -0.2467    0.0000   -0.7071
    0.0000   -0.6627   -0.0000   -0.2467    0.0000    0.7071
S = 6×6
    3.4565         0         0         0         0         0
         0    3.4565         0         0         0         0
         0         0    0.2287         0         0         0
         0         0         0    0.2287         0         0
         0         0         0         0    0.0000         0
         0         0         0         0         0    0.0000
```

```
Anew = Tc^-1*A*Tc
```

```
Anew = 6×6
    0.0000    1.0000    0.0000   -0.0000   -0.0000   -0.0000
   -1.9878   -0.0000   -0.7399   -0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0000    1.0000    0.0000    0.0000
   -0.0328    0.0000   -0.0122   -0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0000    0.0000    0.0000   -1.0000
   -0.0000    0.0000   -0.0000    0.0000    1.0000   -0.0000
```

```
Auu = Anew(nc+1:end,nc+1:end)
```

```
Auu = 2×2
    0.0000   -1.0000
    1.0000   -0.0000
```

```
[ob,co] = abilT(A,B,C)
```

ob = 1×3 cell

	1	2	3
1	6×6 double	4×4 double	[0,-1;1,0]

co = 1×3 cell

	1	2	3
1	6×6 double	4×4 double	[0,-1;1,...

Case 3: P2, E1

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case3{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)]
```

```
G = 3×3
         0         0         0
         0    1.0000         0
         0         0    0.9900
```

```
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -0.5051   -1.5152         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
         0
         0
    0.5000
    0.5000
    0.5051
```

```
eig(A)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -0.0000 + 1.4178i
  -0.0000 - 1.4178i
   0.0000 + 1.0025i
   0.0000 - 1.0025i
```

```
Qc = ctrb(A,B)
```

```
Qc = 6×6
         0    0.5000         0   -0.5025         0    1.0101
         0    0.5000         0   -1.0025         0    2.0127
         0    0.5051         0   -1.0178         0    2.0484
    0.5000         0   -0.5025         0    1.0101         0
    0.5000         0   -1.0025         0    2.0127         0
    0.5051         0   -1.0178         0    2.0484         0
```

```
rank(Qc)
```

```
ans = 6
```

```
ans == 6
```

```
ans = logical
   1
```

```
[~,unctrbeig] = PBHtest(A,B,C)
```

```
unctrbeig = 1×3 cell
```

	1	2	3
1	[]	[]	[]

```
[~,S,~] = svd(Qc)
```

```
S = 6×6
    3.5019         0         0         0         0         0
         0    3.5019         0         0         0         0
         0         0    0.2290         0         0         0
         0         0         0    0.2290         0         0
         0         0         0         0    0.0016         0
         0         0         0         0         0    0.0016
```

Case 7: P4, E1

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case7{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -1.0000   -3.0000         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
```

```
         0
         0
    0.5000
    0.5000
    1.0000
```

```
eig(A)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -0.0000 + 1.8113i
  -0.0000 - 1.8113i
   0.0000 + 1.1042i
   0.0000 - 1.1042i
```

```
Qc = ctrb(A,B)
```

```
Qc = 6×6
         0    0.5000         0   -0.7500         0    2.3750
         0    0.5000         0   -1.2500         0    3.6250
         0    1.0000         0   -3.5000         0   11.7500
    0.5000         0   -0.7500         0    2.3750         0
    0.5000         0   -1.2500         0    3.6250         0
    1.0000         0   -3.5000         0   11.7500         0
```

```
rank(Qc)
```

```
ans = 6
```

```
ans == 6
```

```
ans = logical
   1
```

```
[~,unctrbeig] = PBHtest(A,B,C)
```

```
unctrbeig = 1×3 cell
```

	1	2	3
1	[]	[]	[]

```
[~,S,~] = svd(Qc)
```

```
S = 6×6
   13.1371         0         0         0         0         0
         0   13.1371         0         0         0         0
         0         0    0.3513         0         0         0
         0         0         0    0.3513         0         0
         0         0         0         0    0.1083         0
         0         0         0         0         0    0.1083
```