Homework 2

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Problem 1:

In attempt to apply the cross-correlation techniques that were explored in the last homework with an actual GNSS signal, the PRN code generation method will be used. The first thing that needs to be done is to interpolate the PRN code that is generated using the designated PRN taps and the code from the first homework. Once the desired PRN code is generated and is confirmed to be correct, it needs to be interpolated to match the size (same number of discrete indices) as the incoming signal since the incoming signal is sent using a carrier signal. Once the PRN code is generated, it is then used to generate the local in-phase and quadrature signals. To do so, the following parameters will be used: f_s = 10 MHz as the sampling rate (each time step will be T_s = 1e-7 seconds or 0.1 microseconds), f_I = 2.716 MHz as the intermediate carrier frequency, and data length as 1 millisecond. Once these parameters are taken into consideration, the in-phase and quadrature signals can be generated using the discretized versions of equations (1) and (2), for problem 1, a doppler shift frequency of 500 Hz will be used. The time variable that will be used will go from 0 to 1ms with a total of 10000 steps which was calculated using the 1ms code duration and the sampling frequency. Once the in-phase and quadrature signals are generated, they will be multiplied (elementwise) with the interpolated PRN code. This will give the actual in-phase and quadrature signals. A second set of signals will be created using a much smaller time step for the purposes of plotting a continuous signal against the actual signal. This is seen in the first figure below where the red dots are the sampled signal, and the blue line is the continuous signal. These plots match up perfectly with the plots that were given in the assignment.

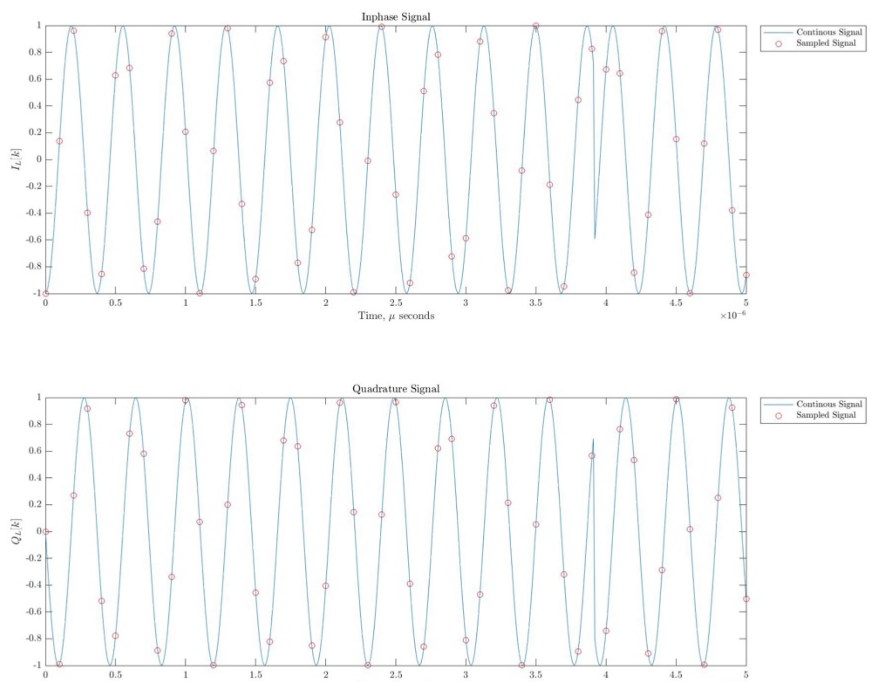

Figure 1 – In-phase and Quadrature Signals (Sampled and Continous)

Problem 2:

To calculate the maximum doppler frequency the equation $f_D = v_{relative} * \frac{f_{carrier}}{c}$, where c is the speed of light and $v_{relative}$ is the relative motion between a satellite and a stationary receiver (https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5467708/). Using this calculation with a carrier frequency of 1575.2 MHz (L1 frequency) and a relative velocity of 929 m/s, the maximum doppler shift frequency is 4881.9 Hz. This calculation is also the same as dividing the velocity by the wavelength of the carrier frequency. The maximum doppler shift frequency can now be used to create a range of which to test for doppler frequencies with the received signal, the maximum doppler shift frequency will be rounded up to 5000 Hz and the range that will be tested will go from -5000 Hz to 5000 Hz.

Problem 3:

Using the range of test doppler frequencies, a new set of in-phase and quadrature signals will be generated. Similarly, they will be multiplied by an interpolated PRN code (PRN 20 will be used as a first test, then the rest of the PRN codes will be tested for as well). After this is done, the resulting in-phase and quadrature signals will be cross-correlated with the received data signal to check if PRN 20 (and the resulting satellite) is contained within the received data. Once the two cross-correlations are generated for both the in-phase and quadrature, the elements of each matrix will be used to calculate the magnitude of the plot, this comes from Euler's law where a complex number can be represented as cosine and sine multiplied by j (imaginary number). The magnitude of this number will be calculated by adding the squares of the elements of the in-phase and quadrature signals. The test delays are also generated when using the cross-correlation process, since there are 10000 steps along the delay axis of the magnitude matrix and the code duration is 1ms long, the test delays will go from -.5 milliseconds to .5 milliseconds with 10000 steps in between since the delay can go in either direction. Plotting the cross-correlation magnitudes for PRN-20 results in the plot seen in figure 2. A peak can be seen in the plot that is much larger than the noise around it. It can be concluded that PRN-20 is present in the received signal, the peak of the magnitude comes to be around 35.6548 at test delay of -0.442 ms and the doppler frequency is at around -100 Hz (this value may be slightly incorrect due to a large step size for test dopplers to since smaller step sizes require more computational resources). Applying this same method for another PRN code (PRN-15), it can be seen that there is also a detected PRN code within the received data as seen in figure 3. This peak occurs around 0.337 ms for the delay, 1800 Hz for doppler shift frequency, and a peak of 72.5174.

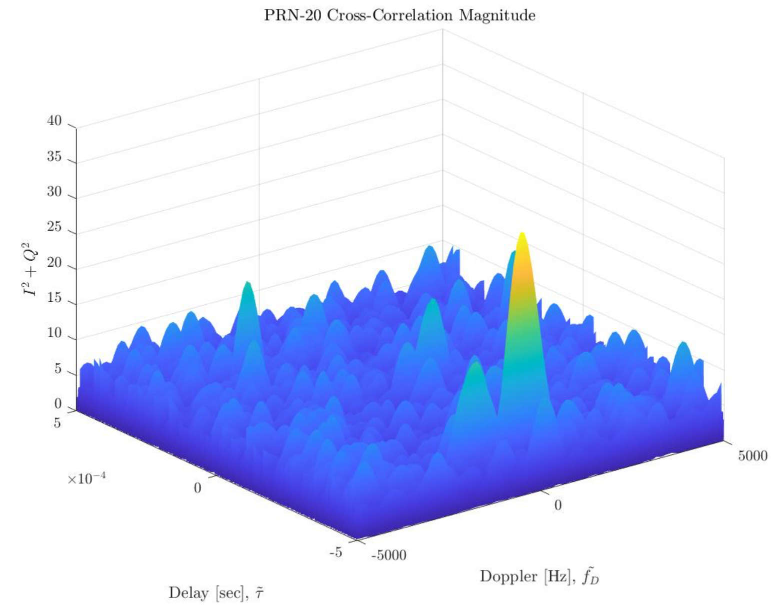

Figure 2 – Cross-Correlation Magnitude of PRN-20 and Received Data

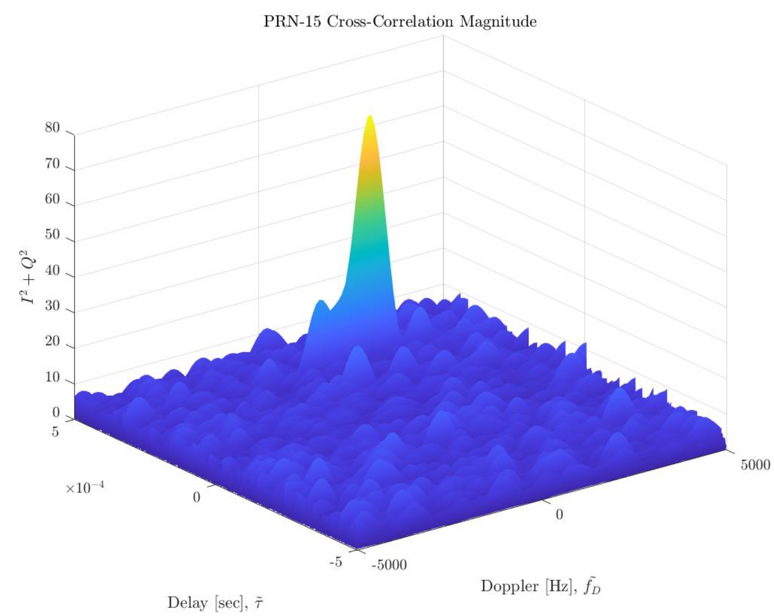

Figure 3 – Cross-Correlation Magnitude of PRN-15 and Received Data

<u>Problem 4</u>:

Using the peak data that was generated problem 4, the signal to noise ratio ($\frac{C}{N_0}$) can be found using this equation: $\frac{C}{N_0} = 10 * log_{10}\left(\frac{C_{peak}}{N_0}\right)$, this results in a power ratio between the peaks of each graph to the noise floor in units of dB. Since no defined noise floor is given, the magnitude plot of a PRN code that is not visible within the received date will be used as the noise floor. This is done by plotting every single non-repeated PRN correlation magnitude and finding the PRN numbers that are not used, the one with the cleanest noise will be used as the noise floor (N_0). Looking at the figures provided in the PDF containing the code, it can be seen that PRN 5, 13, 15, 20, 21 and 29 are detected within the received date while the rest from 1 through 36 are not detected. PRN 36 was used as the noise floor since it is (subjectively) very flat compared to most of the plots with no PRN signal detected as seen in figure 4. Using PRN-36 as the noise floor, the signal to noise ratio equation can be used to calculate signal to noise ratio of PRN-20 which results in 14.6387 dB. As mentioned before in problem 3, the doppler frequency that was calculated for PRN-20 was around -100 Hz, and the delay was around -0.422 ms.

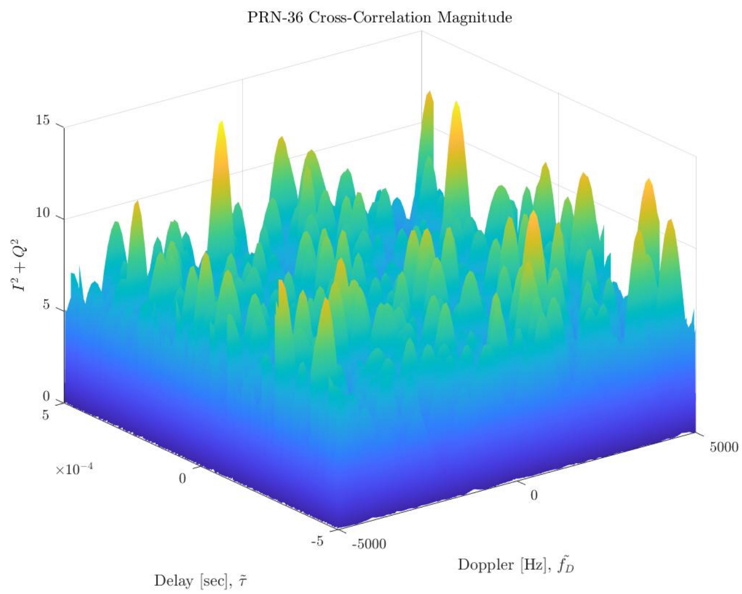

Figure 4 – Cross-Correlation Magnitude of PRN-36 and Received Data (Noise Floor)

<u>Problem 5</u>:

Using a combinations of the codes from problem 3 and 4, the sections of the methods are rationed in to separate functions so a loop can be used to iterated through each and every PRN code with their distinct taps. The signal to noise ratio code will be used, in this case, to determine if a PRN code is detected within the received signal. The limit between a detection and no detection was set at a signal to noise ratio of 13 dB (based off of the lowest power of the detected PRN-codes) and the functions were set to run for each possible PRN code. This resulted in a set of 36 plots, 36 sets of signal to noise ratios, 36 sets of delays (only 6 of which were relevant), 36 sets of doppler frequencies (only 6 of which were relevant) as seen in the code PDF. The detected PRN codes were 5, 13, 15, 20, 21, and 29 for a total of 6 detected satellites. Their respective parameters can be found below in table 1:

PRN Number	Delay (ms)	Doppler (Hz)	$\frac{C}{N_0}$ (dB)
5	-0.1956	-2700	16.0574
13	0.3003	-300	15.5133
15	0.3371	1800	17.7219

20	-0.4420	-100	14.6387
21	0.2808	1700	13.7227
29	-0.4464	-2200	18.5708

The plots for these PRN Code can also be seen in figures 5 through 10.

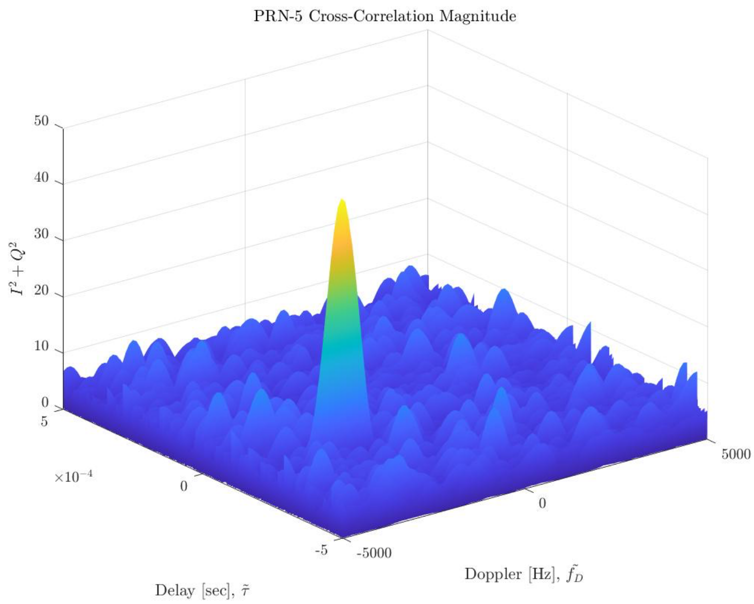

Figure 5 – Cross-Correlation Magnitude of PRN-5 and Received Data

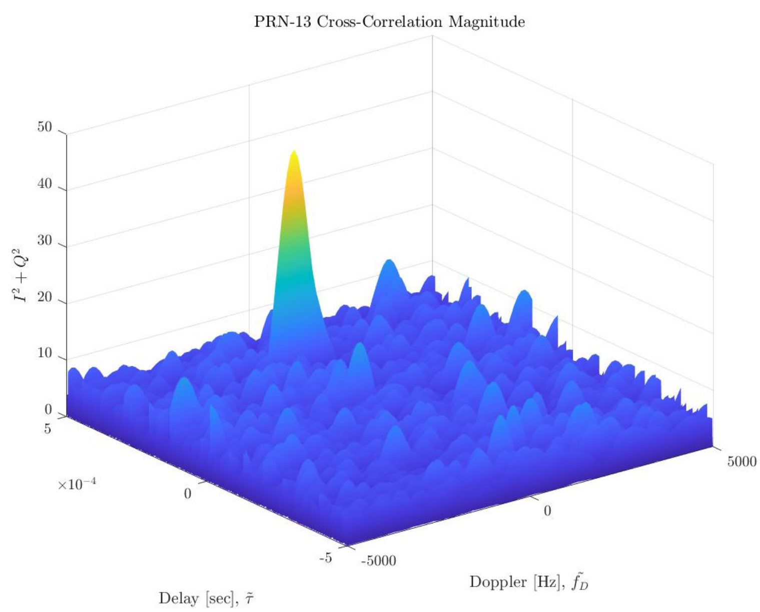

Figure 6 – Cross-Correlation Magnitude of PRN-13 and Received Data

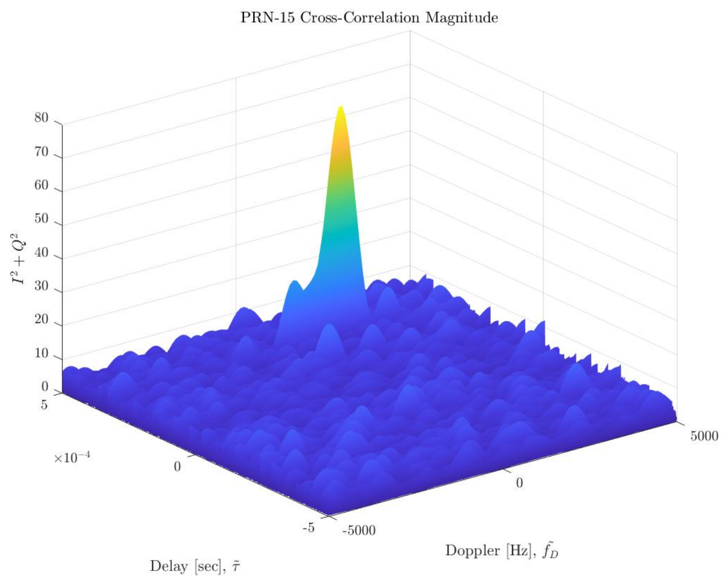

Figure 7 – Cross-Correlation Magnitude of PRN-15 and Received Data

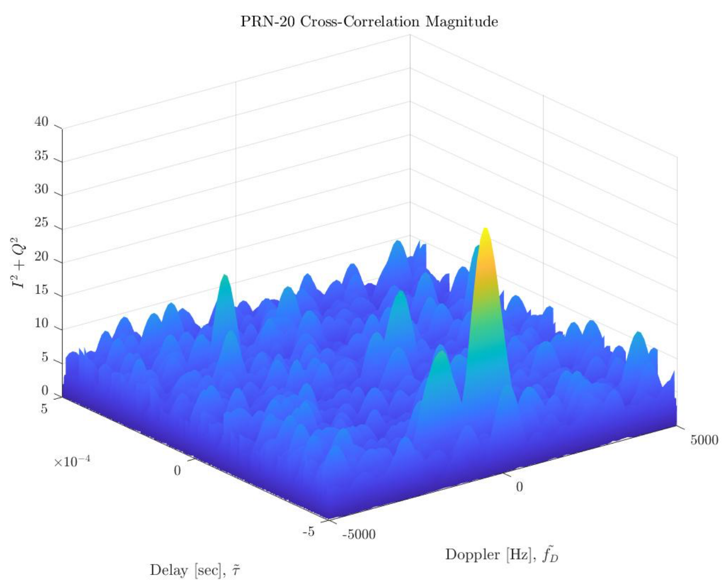

Figure 8 – Cross-Correlation Magnitude of PRN-20 and Received Data

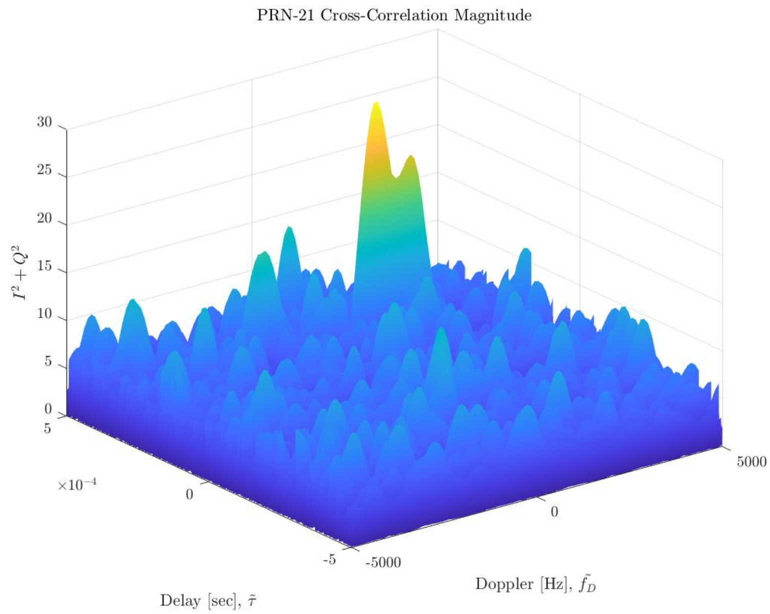

Figure 9 – Cross-Correlation Magnitude of PRN-21 and Received Data

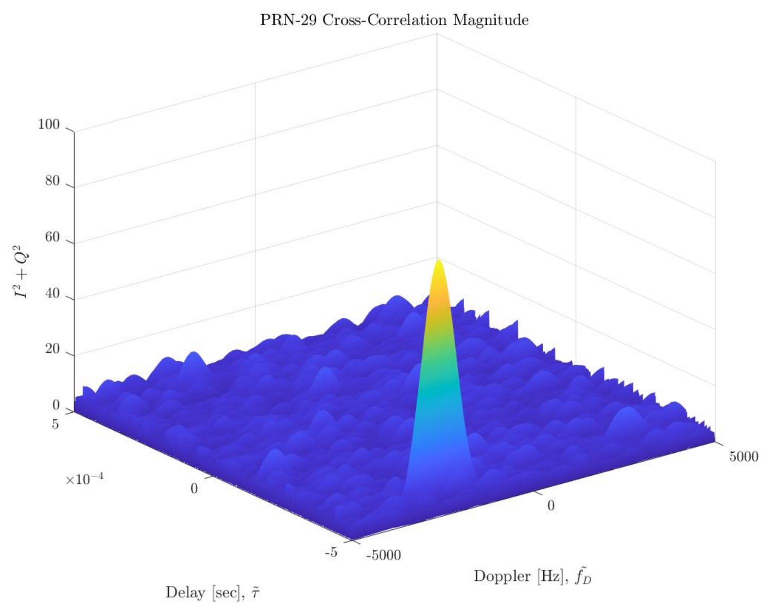

Figure 10 – Cross-Correlation Magnitude of PRN-29 and Received Data