AAE 575 Homework 6

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Problem 1:

The goal of the first problem is to be able to calculate the orbits of the set of satellites from the ephemeris files provided. To do so, the functions that were created in homework 5 were used. These functions took the ephemeris data file and desired PRN number to parse the ephemeris data and calculate the ephemeris data using the algorithms (including perturbations) to convert orbital geometry and time parameters and convert them to the x, y, and z coordinates in the Earth Centered Earth Fixed (ECEF) frame. The time steps that were used in these calculations was the time of reception at the receiver subtracted by the time of application and the estimated delay. Implementing this, the following orbits were calculated and plotting in figure 1 below. The orbits that were plotting are calculated at the time of transmission for each time step.

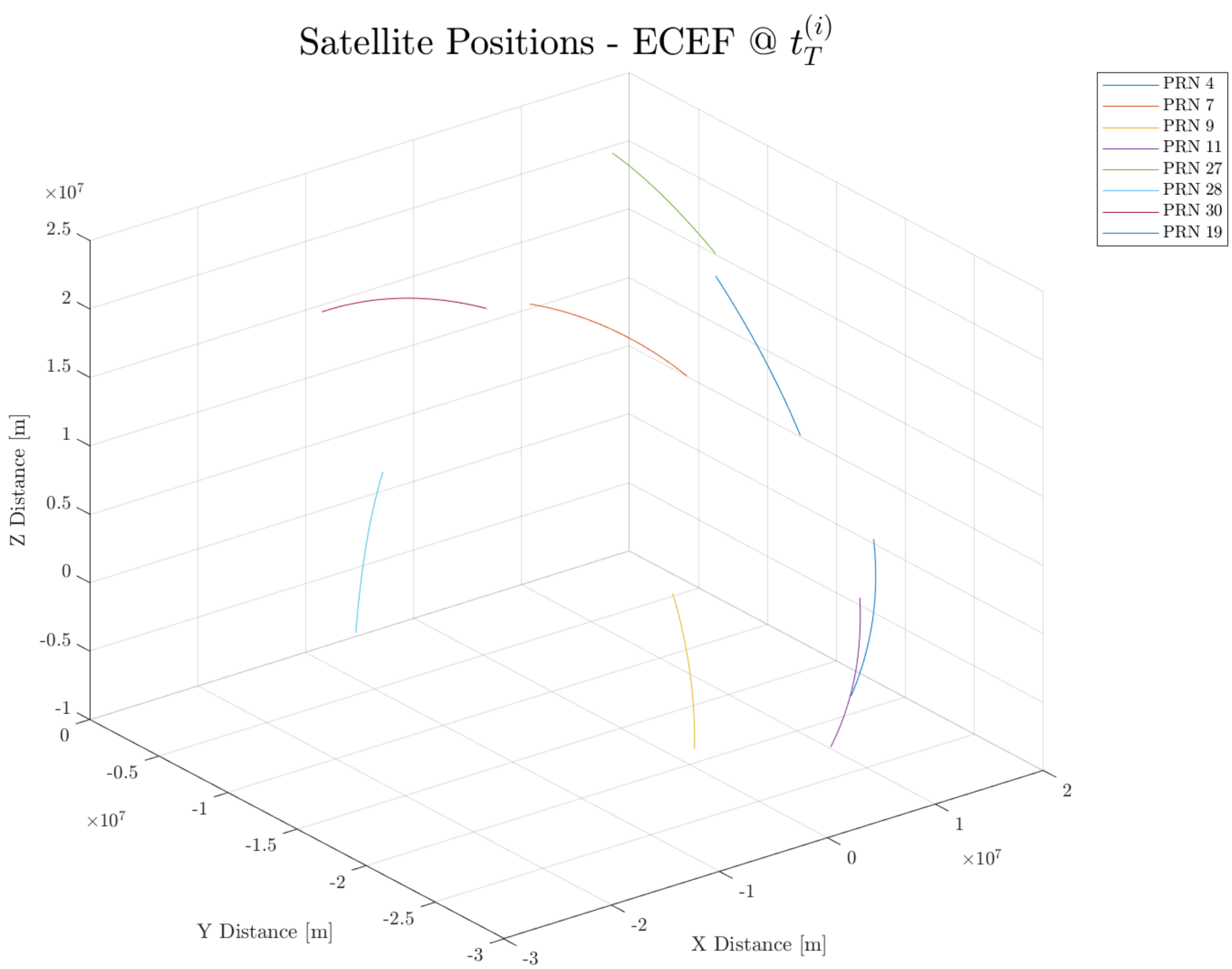

Figure 1 – Satellite Positions in ECEF at time of Transmission

Problem 2:

Since problem 1 calculates the positions of each satellite at the time of transmission, a rotation matrix, taking input of the Earth rotational rate along with an estimate of the signal delay using the pseudoranges, will be used to calculate the position of the satellites at the time of arrival to the receiver. This is done since the transmission delay is about 70 to 80 milliseconds and at the speed of which the satellites are moving, the position should have changed with values on the order of ~100 kilometers. The calculation of the rotation matrix for this operation is seen in figure 2.

$$\begin{bmatrix} x^i_{t_R} \\ y^i_{t_R} \\ z^i_{t_R} \end{bmatrix} = \begin{bmatrix} \cos(\omega_E \tau) & \sin(\omega_E \tau) & 0 \\ -\sin(\omega_E \tau) & \cos(\omega_E \tau) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^i_{t_T} \\ y^i_{t_T} \\ z^i_{t_T} \end{bmatrix}$$

$$\tau = t_R - t^i_T$$

Figure 2 – Transmission Delay Rotation Matrix

Once this is done, a rotation matrix will be calculated for each time step using each time delay and applying to each of the positions at each time step for each satellite. The magnitude of the difference between the position of the satellites at the time of transmission and time of arrival are fairly small compared to the magnitudes of the position of the satellites. The comparison between the positions at the time of arrival and time of transmission can be seen for PRN 4 in figure 3 below, but since the differences are so small, it cannot be easily seen.

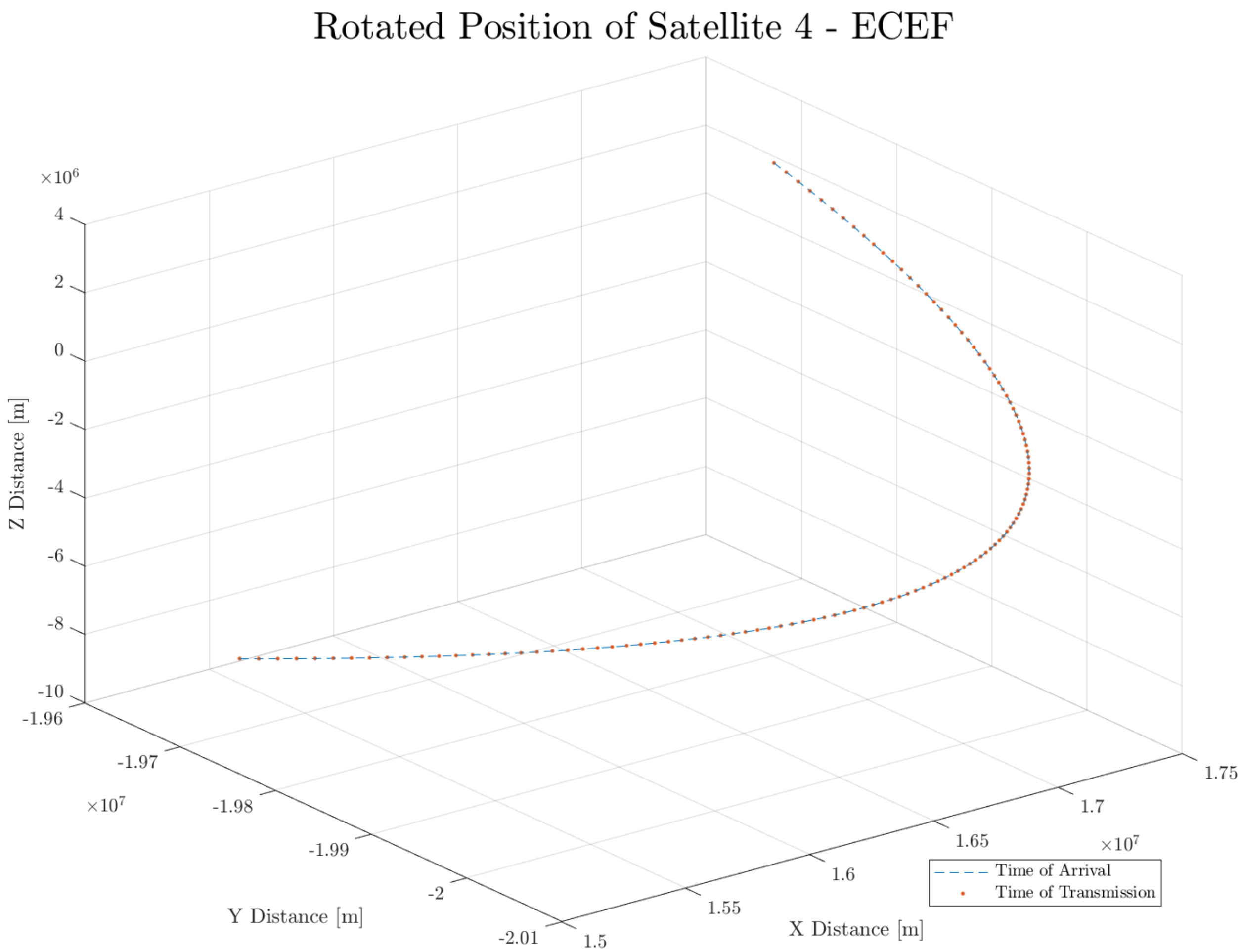

Figure 3 – Satellite Positions in ECEF at Time of Arrival vs Transmission

<u>Problem 3</u>:

The Klobuchar model will be used to calculate the ionospheric delay for each satellite. The model constants that will be used are as seen in figure 4. The satellite positions calculated in problems 1 and corrected in 2 will be used to calculate the geodetic coordinates of the satellites. The built in MATLAB function that will be used is ecef2aer() which inputs the ECEF coordinates of the satellite positions and the geodetic coordinates of the guess location, and the WGS84 ellipsoid parameter. Once this is done, the function created in homework 3 will be used to calculate the ionospheric delay using the Klobuchar model. This function takes the location of the guess point of the receiver, the geodetic coordinates of the satellites, and the alpha and beta constants to calculate the delay. The ionospheric delays calculated using this model can be seen in figure 5 below.

α_0	α_1	α_2	α_3
0.01397 E-6	0.022352 E-6	-0.11921 E-6	-0.11921 E-6
β_0	β_1	β_2	β_3
110590	163840	-65536	-524290

Figure 4 – Klobuchar Model Constants

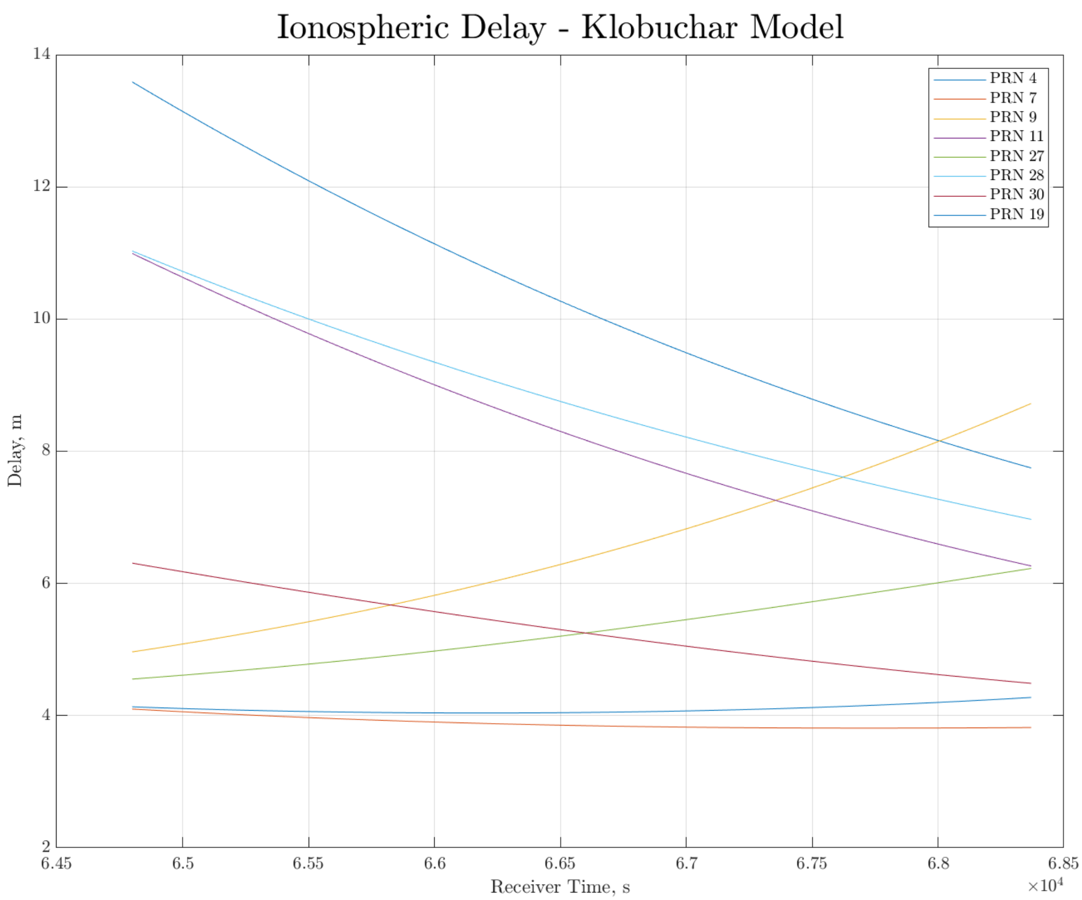

Figure 5 – Ionospheric Delay – Klobuchar Model

Problem 4:

The next delay that needs to be calculated is the delay due to the troposphere. To do this, the Saastamoinen model will be used. The given parameters for the model are as follows – Relative Humidity: 40%, Surface Temperature: 292.85K, Surface Pressure: 983.1 mbar. Along with the Saastamoinen model, the Chao mapping functions will be applied to the wet and dry delays to get the final tropospheric delay. The Saastamoinen was also created in homework 3 and the function will take in the model parameters, the geodetic location of the guess point, and the geodetic positions of the satellites and return the tropospheric delay for each satellite. The resulting calculated delays can be seen in figure 6.

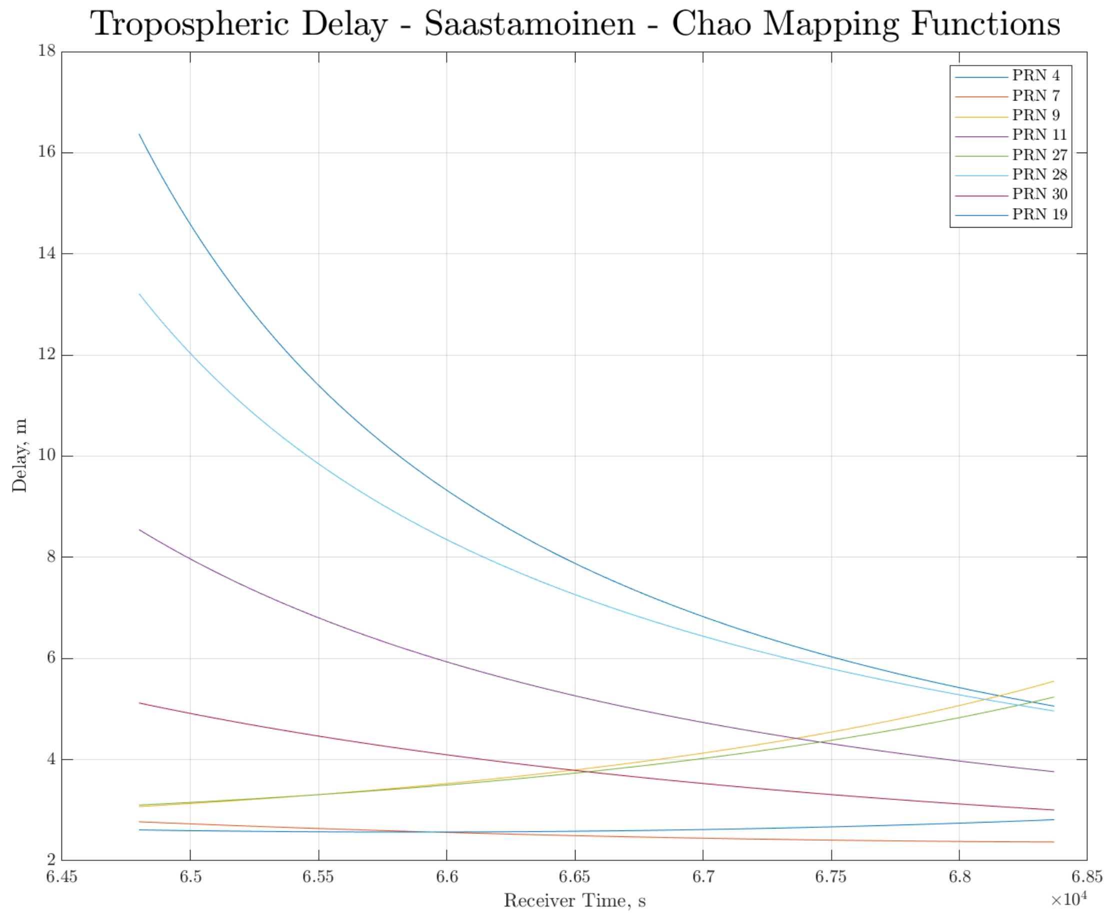

Figure 6 – Tropospheric Delay using Saastamoinen Model and Chao Mapping

<u>Problem 5</u>:

Finally, the corrected pseudorange can be found using the previously calculated ionospheric and tropospheric delays. Three additional corrections will be used to calculate the corrected pseudorange and are as follows: 1) satellite clock bias, 2) relativity correction, and 3) differential group delay. The satellite clock bias delays are given in the sv_clocks.mat file which are multiplied by the speed of light, c, to get the delay in meters. The relativity correction can be found using the equations found in figure 7 which will also be multiplied by the speed of light to get the delay in meters. The differential group delay can be found in the ephemeris of each satellite and will also be multiplied by the speed of light to get the delay in meters. Once all of these corrections are found, the corrected pseudorange equation can be seen in figure 8, which is plotted on figure 9. Finally, using the given corrected pseudorange values, the error between that and the calculated corrected pseudoranges can be found which is plotted in figure 10.

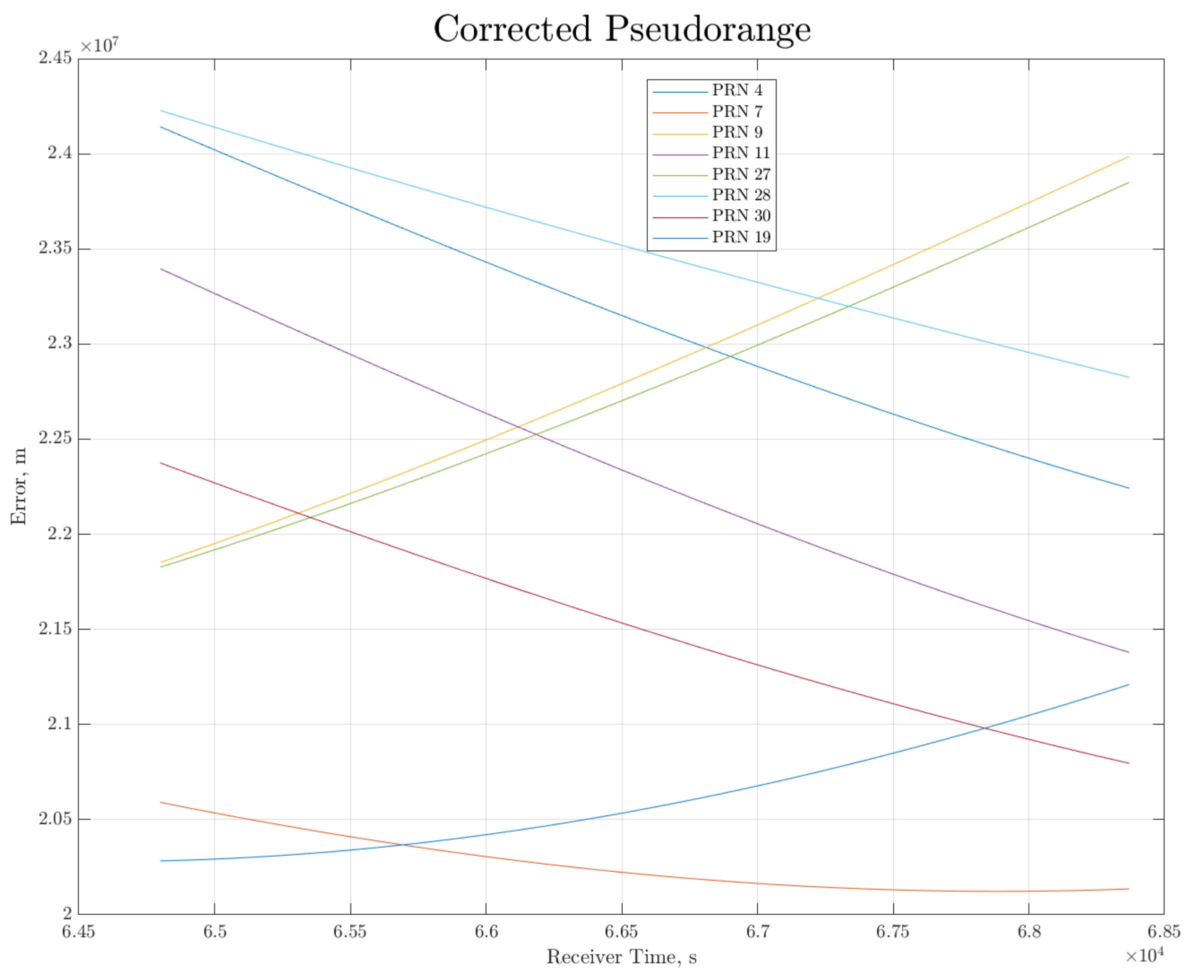

Figure 9 – Corrected Pseudorange

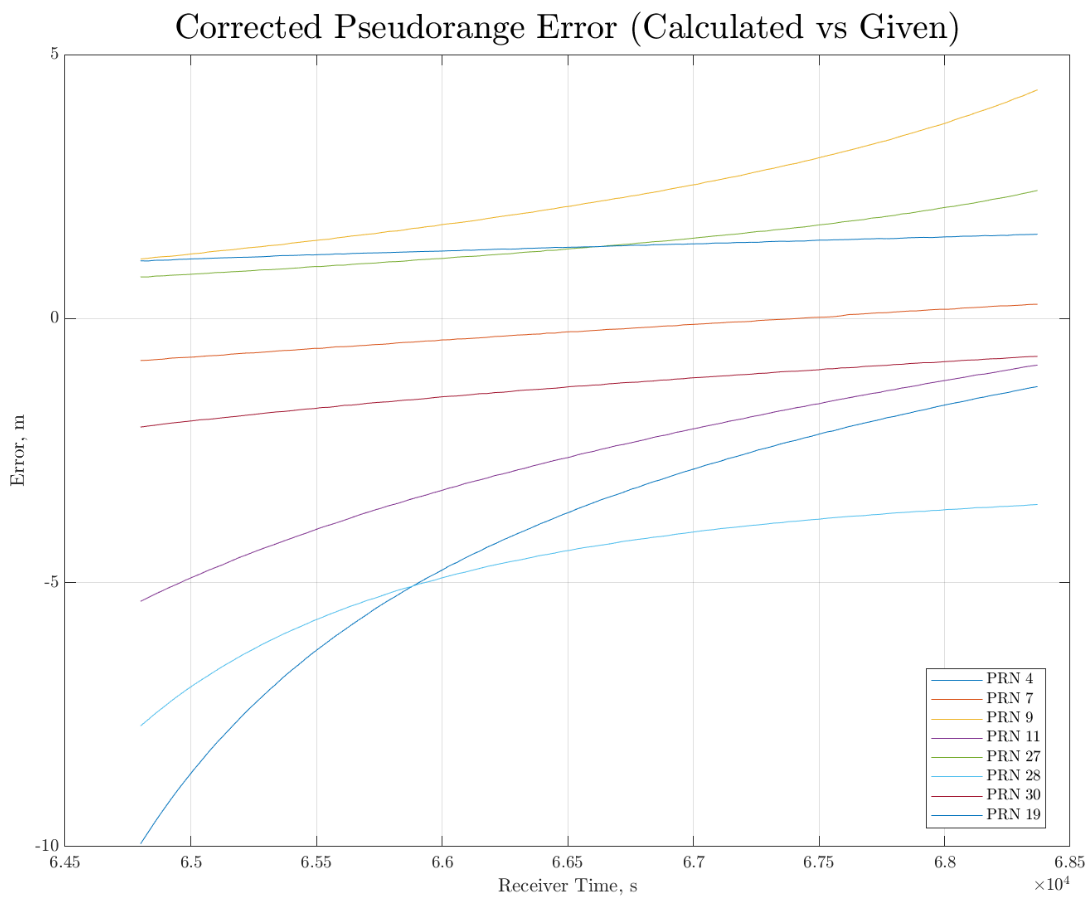

Figure 10 – Corrected Pseudorange Error

<u>Problem 6</u>:

Continuing, from problem 6-10, the pseudoranges and satellite positions that will be used the further calculations will be from the provided values instead of the values calculated in problems 1-5.

To solve for problem 6, an iterative process will be used to calculate the state of the receiver, the x, y, and z positions in ECEF and the clock bias. An iterative process will be used since the system is nonlinear, thus a linearization must be made about a guess point that is given and iterated until the change in the position is smaller than the specified 5 centimeters. To start, two vectors are defined, the observation vector which is **Y**, and **X** which is the state of the position; these two can be seen in figure 11. The first step of the iteration process starts with the linearization of the system which is defined by:

$$\mathbf{Y} = \begin{bmatrix} \rho_c^{(1)} \\ \rho_c^{(2)} \\ \vdots \\ \rho_c^{(N)} \end{bmatrix} \quad \text{and} \quad \mathbf{X} = \begin{bmatrix} x_R \\ y_R \\ z_R \\ ct_R \end{bmatrix}$$

Figure 11 – Observation and State Vectors

$$\mathbf{x} = \mathbf{X} - \mathbf{X^*}$$

$$\mathbf{y} = \mathbf{Y} - \mathbf{Y^*} = \mathbf{Y} - \mathrm{h}(\mathbf{X^*},t)$$

Now, the iteration process can be started. The zeroth step is to start with a guess of the state vector which contains the given x, y, and z positions and the receiver clock bias and assign it to **X***.

$$\mathbf{X_0^*} = [x,y,z,ct_b]^T$$

The first step is to find **y** from the equations from above where Y is the observed pseudoranges and h(**X***,t) is the estimate, where $r^{(i)}_{comp}$ is the geometric distance from **X*** and each satellite.

$$\mathbf{y} = [\rho^{(1)}_{obs}, \ldots, \rho^{(N)}_{obs}] - [r^{(1)}_{comp} - ct_b^{(1)}, \ldots, r^{(N)}_{comp} - ct_b^{(N)}]$$

The second step is to compute the linear observation model matrix, **H**:

$$\mathbf{H} = \begin{bmatrix} \frac{\mathbf{x}^{*(1)} - \mathrm{x}^{(1)}}{\mathrm{r}^{(1)}_{\mathrm{comp}}} & \frac{\mathbf{y}^{*(1)} - \mathrm{y}^{(1)}}{\mathrm{r}^{(1)}_{\mathrm{comp}}} & \frac{\mathbf{z}^{*(1)} - \mathrm{z}^{(1)}}{\mathrm{r}^{(1)}_{\mathrm{comp}}} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\mathbf{x}^{*(i)} - \mathrm{x}^{(N)}}{\mathrm{r}^{(N)}_{\mathrm{comp}}} & \frac{\mathbf{y}^{*(i)} - \mathrm{y}^{(N)}}{\mathrm{r}^{(N)}_{\mathrm{comp}}} & \frac{\mathbf{z}^{*(i)} - \mathrm{z}^{(N)}}{\mathrm{r}^{(N)}_{\mathrm{comp}}} & 1 \end{bmatrix}$$

Then, the third step is to solve for $\hat{\mathbf{x}}$ in the following equation:

$$(\mathbf{H}^T\mathbf{H})\hat{\mathbf{x}} = \mathbf{H}^T\mathbf{y}$$

The $\hat{\mathbf{x}}$ vector is the least squares estimate for the change in the state which is calculated in step four:

$$\hat{\mathbf{X}} = \mathbf{X}^* + \hat{\mathbf{x}}$$

The final step is to update $\mathbf{X}^*$ using the $\hat{\mathbf{X}}$ that was calculated in the previous step. Once this is done, the process is started back at step one until the magnitude of $\hat{\mathbf{x}}$ is less than 5 centimeters.

Once the state converges, the final $\hat{\mathbf{X}}$ is the final estimated state, which is the position of the receiver in ECEF and the receiver clock bias in meters, of the system and can be used for further processing. The following plot (figure 12) is the error between the actual position of the P775 receiver and the position that was calculated using the iterative process described above. As seen in figure 13, the errors are fairly small across the time period that was recorded and calculated on being within ±3 meters for all time steps with the error reducing towards the middle of the data. Later, in problem 9 and 10, statistics will be used to characterize the variance and other parameters of the navigation solution.

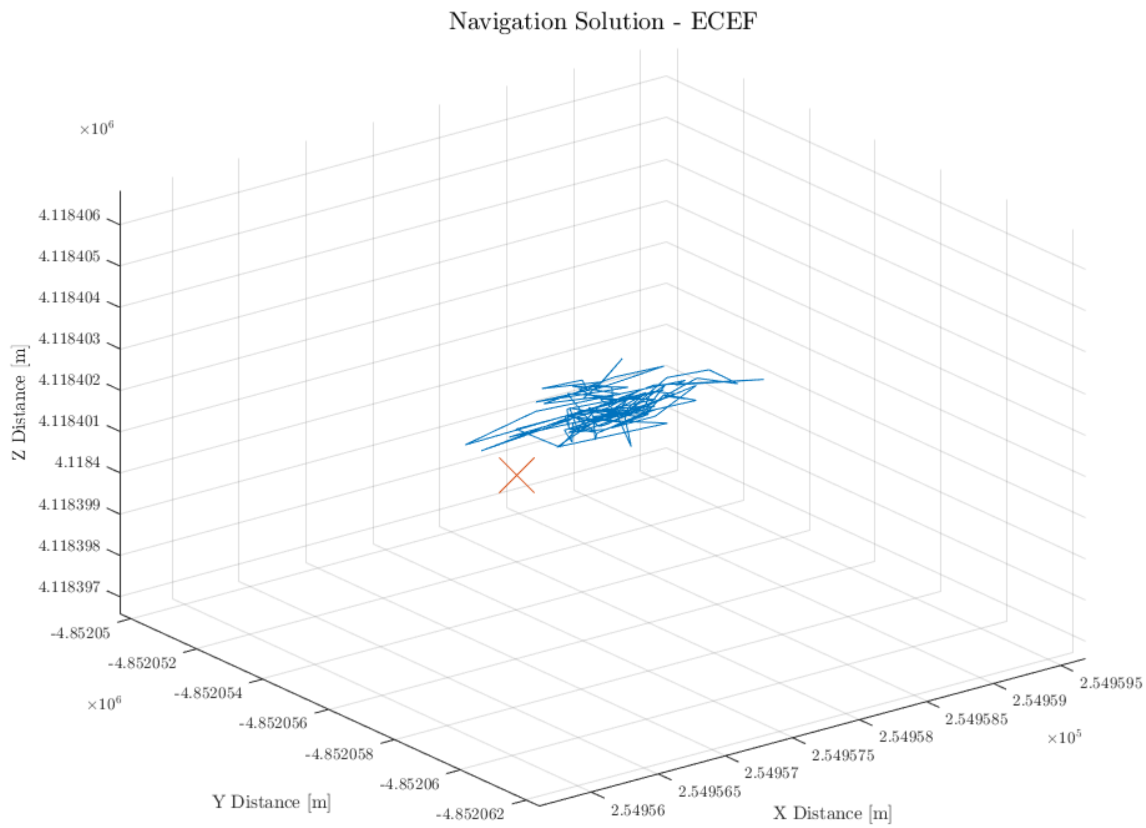

Figure 12 – Navigation Solution

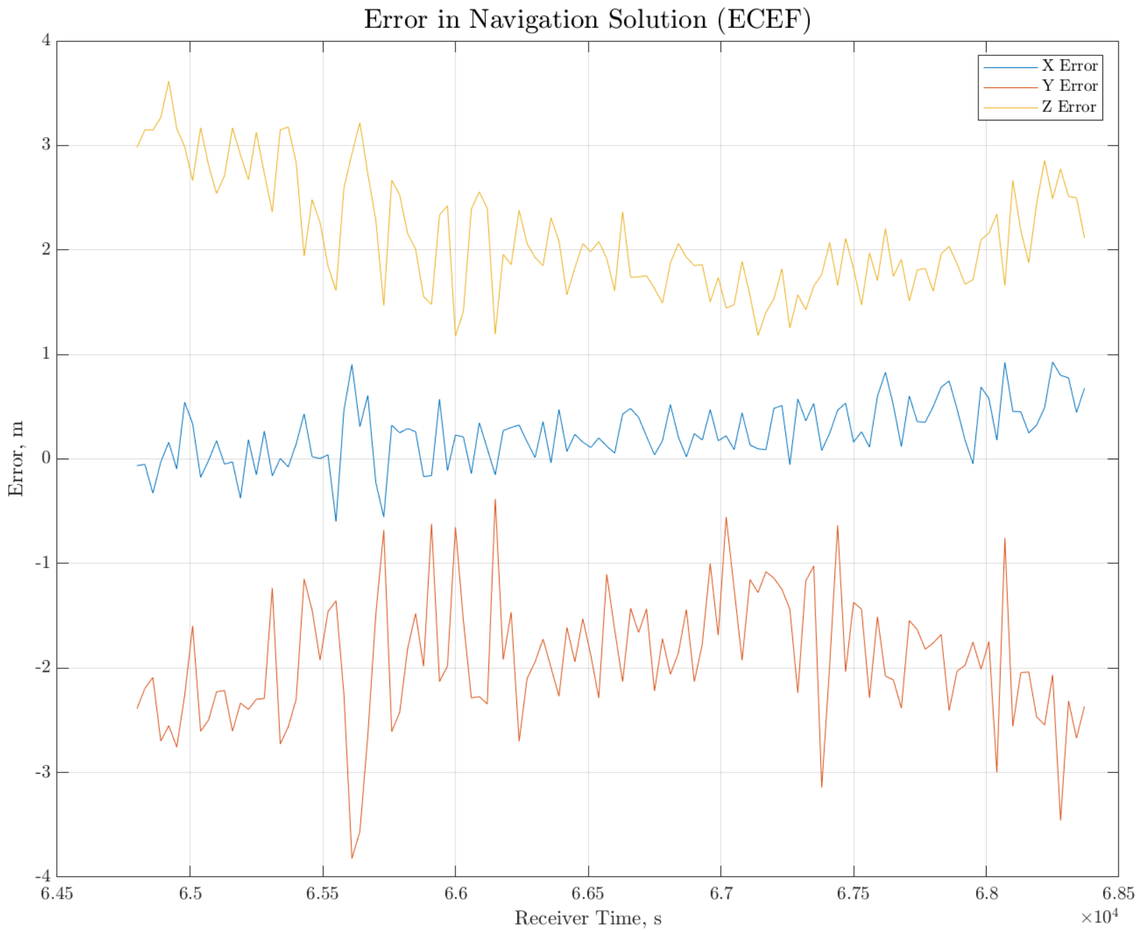

Figure 13 – Navigation Solution Errors

Problem 8:

After the position of the receiver is estimated and calculated, the coordinates can be transformed from ECEF to its respective geodetic coordinates using an iterative process. The first calculations that need to be done are to find the parallel:

$$p = \sqrt{x^2 + y^2}$$

Once the length of the parallel can be found, the iterative process can be found as follows:

1) $N(\phi) = \dfrac{a^2}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$

2) $h = \dfrac{p}{\cos\phi} - N(\phi)$

3) $\phi = \dfrac{z}{\frac{b^2}{a^2}N(\phi) + h}$

Where, for the first iteration, the starting geodetic latitude is set equal to the geocentric latitude. After many iterations, the latitude will converge into the geodetic latitude and the ellipsoidal height.

For the longitude, this value will be computed the same way as the geocentric longitude which is as follows (making sure to use the correct quadrant checks, ie. atan2()):

$$\lambda = arctan\left(\frac{y}{x}\right)$$

Once these coordinates are found, the error between the estimated value and the give n P775 coordinates can be found and plotted which are shown in figure 15 and the actual value for the geodetic coordinates are plotted in figure 14.

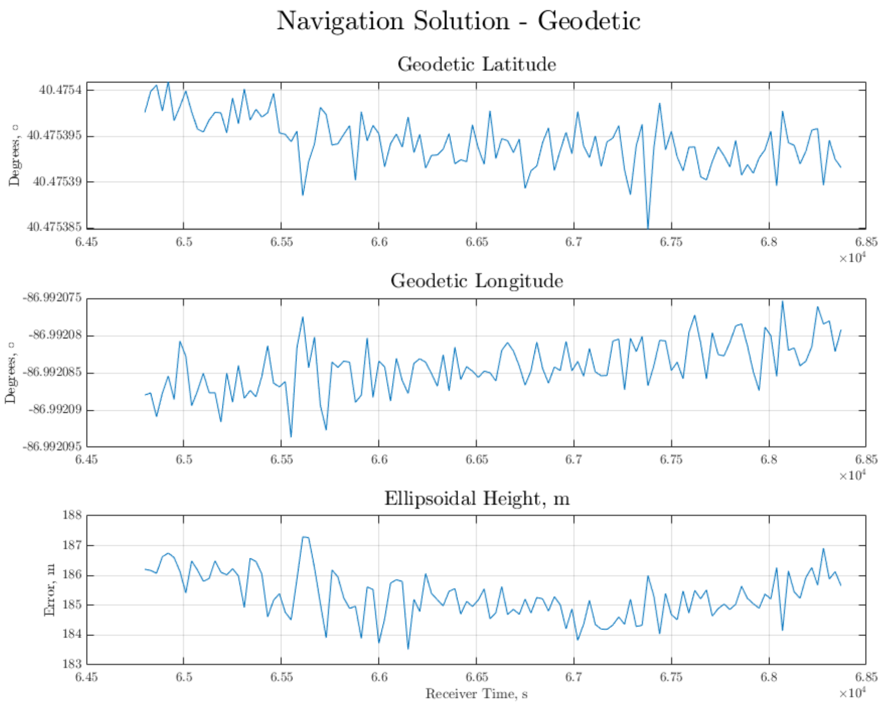

Figure 14 – Geodetic Coordinates of Navigation Solution

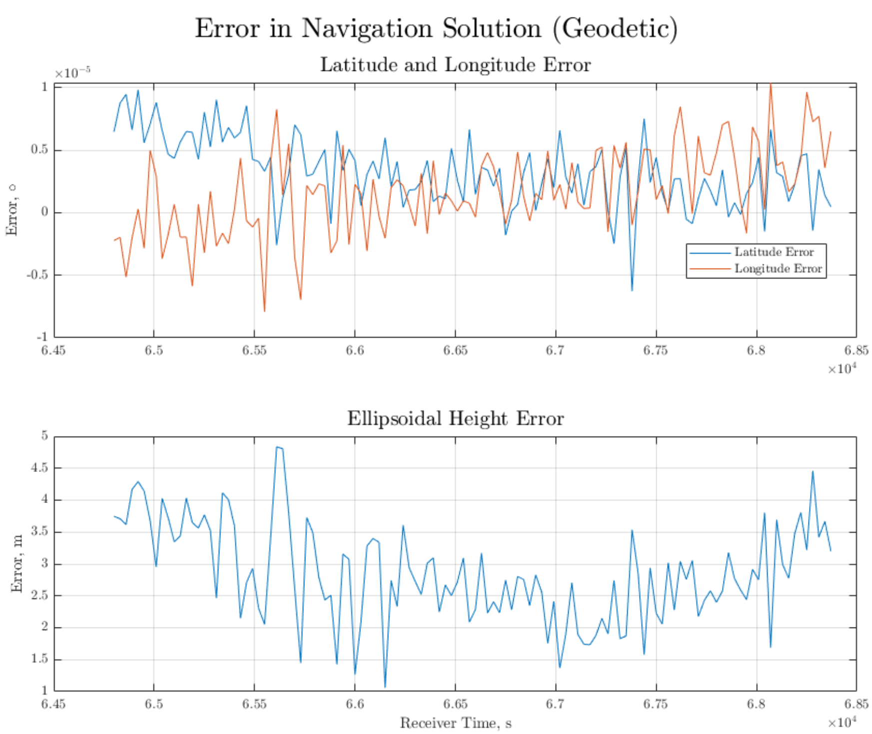

Figure 15 – Error in Geodetic Navigation Solution

Problem 9:

Dilution of precision is the cause of error due to the geometry of the navigation system. There are five main types of dilution of precision: GDOP, TDOP, PDOP, HDOP, and VDOP which are the geometric, time, 3D positions, horizontal, and vertical dilutions of precision respectively. To calculate the first three dilutions of precision, the **H** matrix from problem 6 must be used to get the covariance matrix, the following will result in the covariance matrix in the ECEF frame:

$$G = (\mathbf{H}^T\mathbf{H})^{-1}$$

From this covariance matrix, the GDOP can be found by taking the square root of the trace of the covariance matrix, while the TDOP can be found by finding the square root of the fourth diagonal of the matrix, and the PDOP can be found taking the square root of the first three diagonals of the covariance matrix. Looking at the components of the covariance matrix:

$$\mathbf{G} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{xt} & \sigma_{yt} & \sigma_{zt} & \sigma_t^2 \end{bmatrix}$$

It can be seen that GDOP is the magnitude of the standard deviations of the entire solution state, TDOP is the standard deviation of the receiver clock bias values, and PDOP is the magnitude standard deviation of the x, y, and z values for the receiver. Calculating this matrix and performing the calculations for the dilutions of precision resulted in the following for the first time step:

GDOP	TDOP	PDOP	HDOP	VDOP
2.362653813259	2.058518841046	1.159583210626	1.092277013117	1.744829718212

Table 1 – Dilution of Precision (1st Time Step)

To calculate the dilution of precision for the horizontal and vertical states, a transformation matrix must be used on the G matrix to transform the x, y, and z components to the ENU frame. The transformation matrix is as follows:

$$R = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\phi & -\sin\lambda\sin\phi & \cos\phi \\ \cos\lambda\cos\phi & \sin\lambda\cos\phi & \sin\phi \end{bmatrix}$$

$$T = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix}$$

The transformation from ECEF to the ENU is a 3x3 matrix so it must be placed into a 4x4 matrix in order to allow for multiplication with the G matrix, this is because the G matrix includes the x, y, and z variances and covariances but also the time variances and covariances which does not need to be transformed. Once the covariance matrix is transformed, the diagonal contains the necessary values needed to calculate the dilutions of precision. HDOP can be found by taking the square root of the first two diagonals of the transformed covariance matrix added together and VDOP can be found by taking the square root of the third diagonal in the transformed covariance matrix. These values can be seen in table 1.

When plotting the respective dilution of precision for each parameter, it can be seen that there is change in the dilutions of precision over time with a peak towards the middle which can also be seen in the errors of the ECEF positions in problem 6 with figure 12 and in problem 8 with figure 14. The higher the dilution of precision the higher the error tends to be.

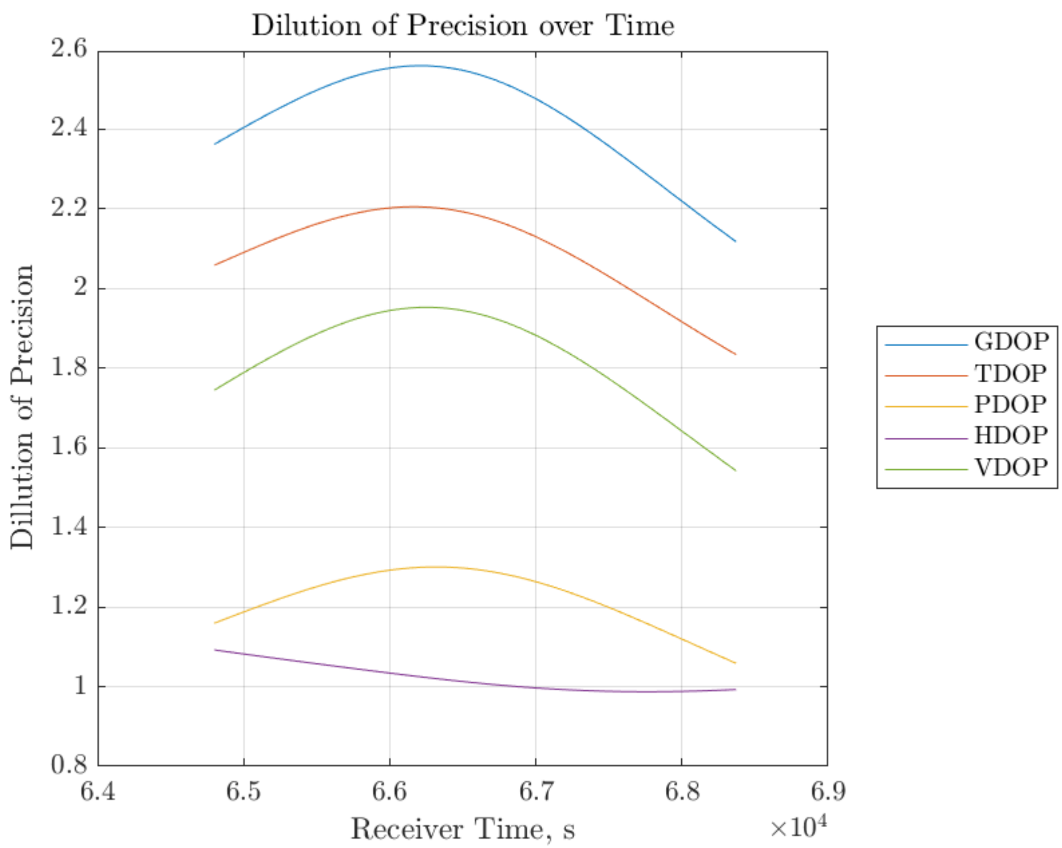

Figure 15 – Dilution of Precision Over Time

<u>Problem 10</u>:

Similar statistics can be found from the data across all the time periods. To do so, the navigation solution must be done to each time step in the given data sets. Once this is done the statistics can be found. The first statistic that can be found is the root sum square of the position which is done by taking the variance of the x, y, and z positions of the receiver and taking the square root of the norm of the three as seen below:

$$\text{RSS} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

Along with the root sum square, the root mean square can also be found accordingly:

$$\text{RMS} = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{N}}$$

Where N is the number of time steps in the solution data and each σ represents the standard deviation in the error for each measurement. Once the RSS is calculated, it can be used to compare to the PDOP values that were calculated in problem 9. Since the definition of PDOP is the same as the definition of the root mean square in position. Comparing the two values, they are fairly close as the RMS method results in a value of 2.2600 while the PDOP method results in a value of around 2.05 throughout the time steps.

HDOP and VDOP can also be compared to statistics that can be calculated from the position data. First the navigation solution data must be converted from the ECEF frame to the ENU coordinates using the transformation matrix from problem 8 for each time step. Once this is done, the standard deviation can be taken for each of the xEast, yNorth, and zUp directions. Taking the ECEF coordinates and transforming them to the ENU coordinates effectively takes an error from the local point (the actual position of the P775 receiver) and the calculated positions. Once this is done, the statistical HDOP can be found taking the norm of the xEast and yNorth standard deviations and the statistical VDOP can be found by taking the square root of the square (absolute value) of the zUp standard deviation as seen below:

$$HDOP = \sqrt{\sigma_{xE}^2 + \sigma_{yN}^2}$$

$$VDOP = \sqrt{\sigma_{\mathrm{zU}}^2}\mathrm{D}$$

The values the of the statistically calculated dilutions of precision are calculated to be 0.4270 for HDOP and 0.7737 for VDOP. This shows a slightly lower values than calculated from the G matrices, but the ratio between the HDOP and VDOP values are proportional to the statistically calculated ones. The values for all of the statistically calculated DOPs can be found in table 2.

RMS	RSS (PDOP)	HDOP (stat)	VDOP (stat)
3.015974059643285	2.259968408304989	0.426967256777942	0.773741066780687

Table 2 – Statistically Calculated Dilutions of Precision