Parameters and Equilibrium Conditions

```
P1 = [2 1 1 1 1 1];
P2 = [2 1 1 1 0.99 1];
P3 = [2 1 0.5 1 1 1];
P4 = [2 1 1 1 0.5 1];

E1 = [0 0 0];
E2 = [0 pi pi];

case1 = num2cell([P1,E1]);
case2 = num2cell([P1,E2]);
case3 = num2cell([P2,E1]);
case4 = num2cell([P2,E2]);
case5 = num2cell([P3,E1]);
case6 = num2cell([P3,E2]);
case7 = num2cell([P4,E1]);
case8 = num2cell([P4,E2]);

u = 0;
C = [1 0 0 0 0 0];
D = 0;
```

Case 1: *P1, E1*

```
% pull parameters and initial conditions for case 1
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case1{:});

mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -0.5000   -1.5000         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
         0
         0
    0.5000
    0.5000
    0.5000
```

```
sys = ss(A,B,C,D);
[v,d] = eig(sym(A))
```

v =

$$\begin{pmatrix} 1 & \frac{\sqrt{2}\,\mathrm{i}}{4} & -\frac{\sqrt{2}\,\mathrm{i}}{4} & 0 & 0 \\ 0 & \frac{\sqrt{2}\,\mathrm{i}}{2} & -\frac{\sqrt{2}\,\mathrm{i}}{2} & -\mathrm{i} & \mathrm{i} \\ 0 & \frac{\sqrt{2}\,\mathrm{i}}{2} & -\frac{\sqrt{2}\,\mathrm{i}}{2} & \mathrm{i} & -\mathrm{i} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2}\,\mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2}\,\mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mathrm{i} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathrm{i} \end{pmatrix}$$

Case 2: P1, E2

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case2{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

A = 6×6

```
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0    1.5000    0.5000         0         0         0
         0    0.5000    1.5000         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

B = 6×1

```
         0
         0
         0
    0.5000
   -0.5000
   -0.5000
```

```
sys = ss(A,B,C,D);
[v,d] = eig(sym(A))
```

v =

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \\ 0 & -1 & 1 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 1 & -1 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{2} \end{pmatrix}$$

Case 3: P2, E1

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case3{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -0.5051   -1.5152         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
         0
         0
    0.5000
    0.5000
    0.5051
```

```
sys = ss(A,B,C,D);
eig(A)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
```

```
   0.0000 + 0.0000i
  -0.0000 + 1.4178i
  -0.0000 - 1.4178i
   0.0000 + 1.0025i
   0.0000 - 1.0025i
```

Case 4: P2, E2

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case4{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0    1.5000    0.5000         0         0         0
         0    0.5051    1.5152         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
         0
         0
    0.5000
   -0.5000
   -0.5051
```

```
sys = ss(A,B,C,D);
eig(A)
```

```
ans = 6×1
         0
         0
    1.4178
    1.0025
   -1.4178
   -1.0025
```

Case 5: P3, E1

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case5{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.2500         0         0         0
         0   -1.5000   -0.2500         0         0         0
         0   -0.5000   -1.2500         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
         0
         0
    0.5000
    0.5000
    0.5000
```

```
sys = ss(A,B,C,D);
eig(A)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -0.0000 + 1.3229i
  -0.0000 - 1.3229i
   0.0000 + 1.0000i
   0.0000 - 1.0000i
```

Case 6: P3, E2

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case6{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
     -m1*l1*cos(theta1e), m1*l1^2, 0;
     -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.2500         0         0         0
         0    1.5000    0.2500         0         0         0
         0    0.5000    1.2500         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

```
B = 6×1
         0
         0
         0
    0.5000
   -0.5000
   -0.5000
```

```
sys = ss(A,B,C,D);
```

```
eig(A)
```

```
ans = 6×1
         0
         0
   -1.3229
   -1.0000
    1.0000
    1.3229
```

Case 7: P4, E1

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case7{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -1.0000   -3.0000         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

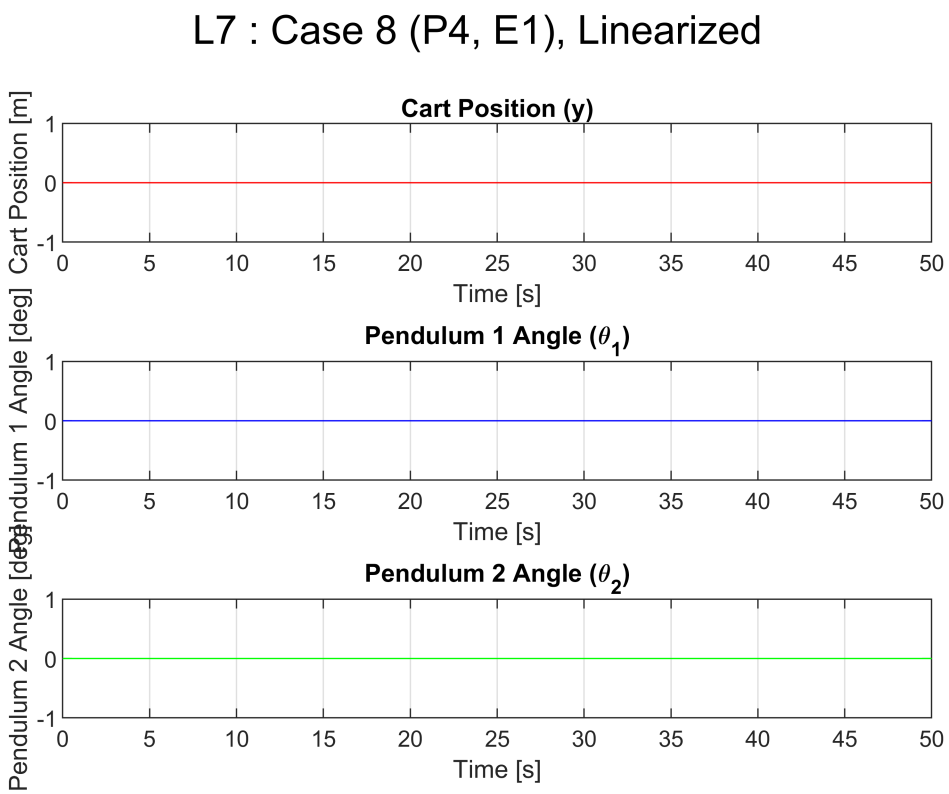
```
B = 6×1
         0
         0
         0
    0.5000
    0.5000
    1.0000
```

```
sys7 = ss(A,B,C,D);
eig(A)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -0.0000 + 1.8113i
  -0.0000 - 1.8113i
   0.0000 + 1.1042i
   0.0000 - 1.1042i
```

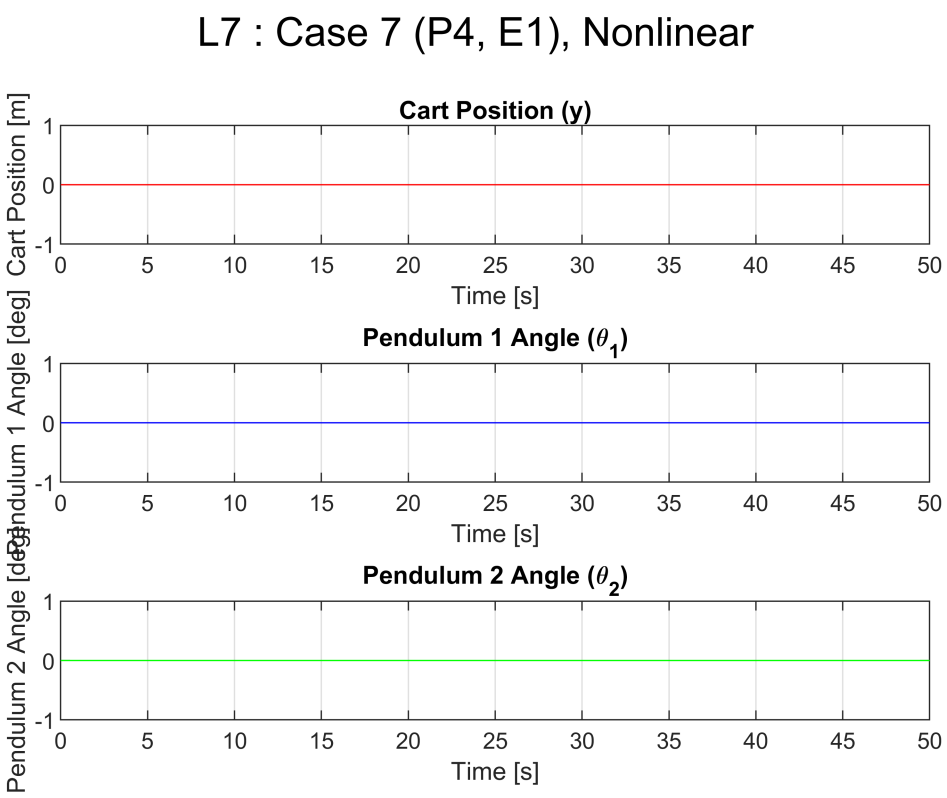
```
IC = [E1, [0 0 0]];
t = linspace(0,50,2^12);
[t,x] = ode45(@(t,x) xfun(x,A,B),t,IC);
figure(1)
subplot(3,1,1)
plot(t,x(:,1),'r')
title('Cart Position (y)')
xlabel('Time [s]')
ylabel('Cart Position [m]')
```

```
grid on
subplot(3,1,2)
plot(t,rad2deg(x(:,2)),'b')
title('Pendulum 1 Angle (\theta_{1})')
xlabel('Time [s]')
ylabel('Pendulum 1 Angle [deg]')
grid on
subplot(3,1,3)
plot(t,rad2deg(x(:,3)),'g')
title('Pendulum 2 Angle (\theta_{2})')
xlabel('Time [s]')
ylabel('Pendulum 2 Angle [deg]')
grid on
sgtitle('L7 : Case 8 (P4, E1), Linearized')
```




```
figure(2)
[t,x] = ode45(@(t,x) xfun2(x, m0, m1, m2, l1, l2, g, mt),t,IC);
subplot(3,1,1)
plot(t,x(:,1),'r')
title('Cart Position (y)')
xlabel('Time [s]')
ylabel('Cart Position [m]')
grid on
subplot(3,1,2)
plot(t,rad2deg(x(:,2)),'b')
title('Pendulum 1 Angle (\theta_{1})')
xlabel('Time [s]')
ylabel('Pendulum 1 Angle [deg]')
grid on
```

```
subplot(3,1,3)
plot(t,rad2deg(x(:,3)),'g')
title('Pendulum 2 Angle (\theta_{2})')
xlabel('Time [s]')
ylabel('Pendulum 2 Angle [deg]')
grid on
sgtitle('L7 : Case 7 (P4, E1), Nonlinear')
```



Case 8: P4, E2

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case8{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0    1.5000    0.5000         0         0         0
         0    1.0000    3.0000         0         0         0
```

```
B = [0;0;0;M^-1*W]
```

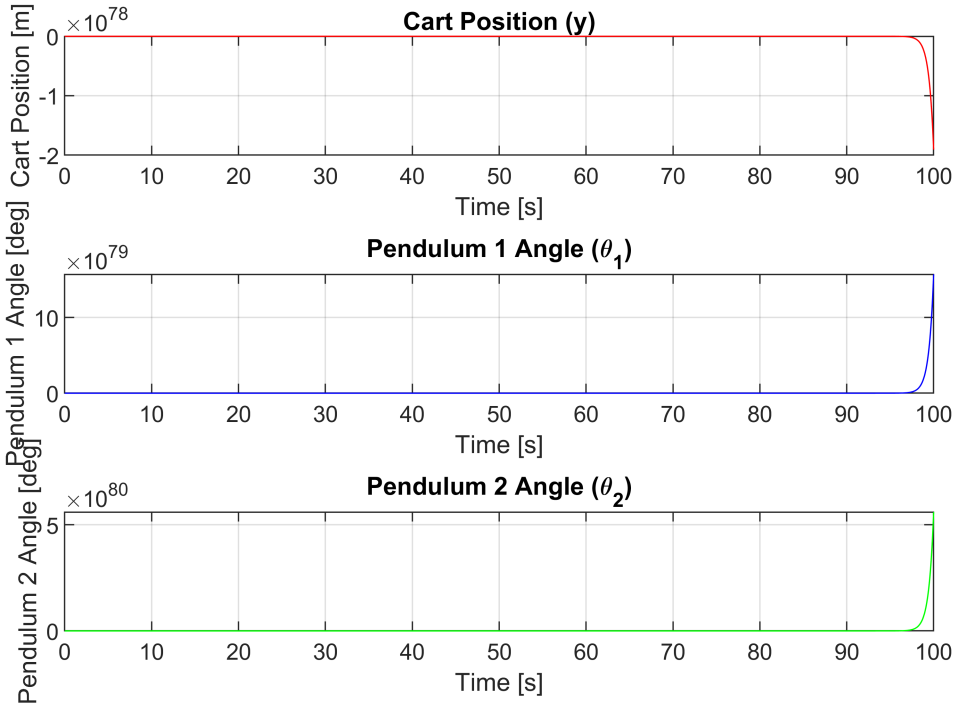
```
B = 6×1
         0
         0
         0
    0.5000
   -0.5000
   -1.0000
```

```
sys8 = ss(A,B,C,D);
eig(A)
```

```
ans = 6×1
         0
         0
    1.8113
    1.1042
   -1.8113
   -1.1042
```

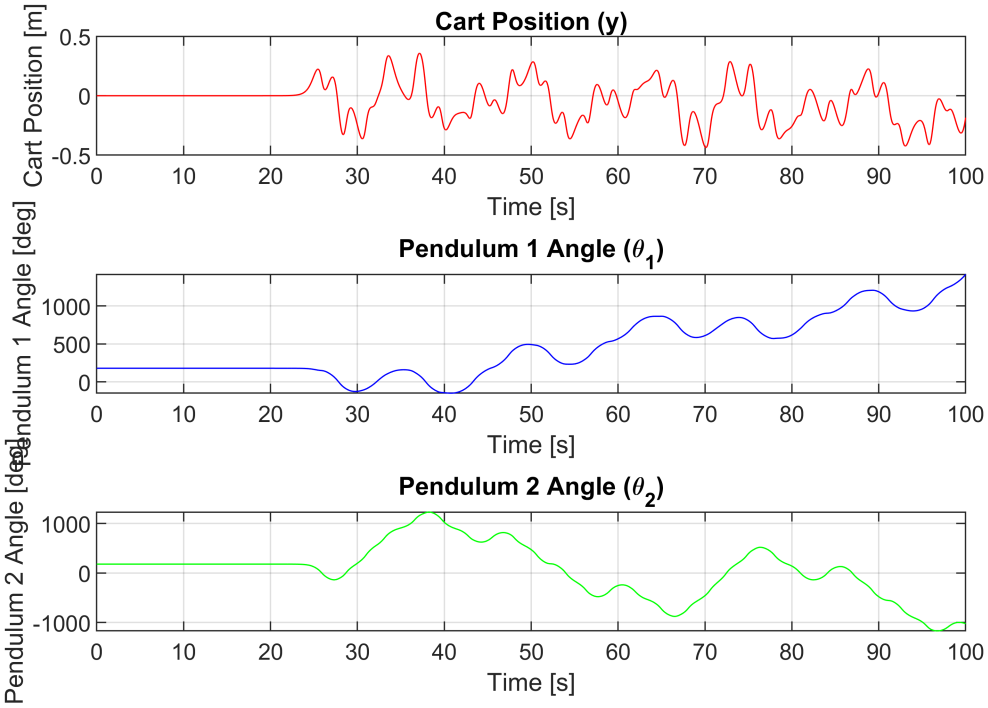
```
IC = [E2, [0 0 0]];
t = linspace(0,100,2^12);
[t,x] = ode45(@(t,x) xfun(x,A,B),t,IC);
figure(3)
subplot(3,1,1)
plot(t,x(:,1),'r')
title('Cart Position (y)')
xlabel('Time [s]')
ylabel('Cart Position [m]')
grid on
subplot(3,1,2)
plot(t,rad2deg(x(:,2)),'b')
title('Pendulum 1 Angle (\theta_{1})')
xlabel('Time [s]')
ylabel('Pendulum 1 Angle [deg]')
grid on
subplot(3,1,3)
plot(t,rad2deg(x(:,3)),'g')
title('Pendulum 2 Angle (\theta_{2})')
xlabel('Time [s]')
ylabel('Pendulum 2 Angle [deg]')
grid on
sgtitle('L8 : Case 8 (P4, E2), Linearized')
```

L8 : Case 8 (P4, E2), Linearized




```
figure(4)
[t,x] = ode45(@(t,x) xfun2(x, m0, m1, m2, l1, l2, g, mt),t,IC);
subplot(3,1,1)
plot(t,x(:,1),'r')
title('Cart Position (y)')
xlabel('Time [s]')
ylabel('Cart Position [m]')
grid on
subplot(3,1,2)
plot(t,rad2deg(x(:,2)),'b')
title('Pendulum 1 Angle (\theta_{1})')
xlabel('Time [s]')
ylabel('Pendulum 1 Angle [deg]')
grid on
subplot(3,1,3)
plot(t,rad2deg(x(:,3)),'g')
title('Pendulum 2 Angle (\theta_{2})')
xlabel('Time [s]')
ylabel('Pendulum 2 Angle [deg]')
grid on
sgtitle('L8 : Case 8 (P4, E2), Nonlinear')
```

L8 : Case 8 (P4, E2), Nonlinear



Linearized

```
function dxdt = xfun(x,A,B)
u = 0;
dxdt = A*x + B*u;
end
```

Nonlinear

```
function dxdt = xfun2(x, m0, m1, m2, l1, l2, g, mt)
u = 0;
M = [mt, -m1*l1*cos(x(2)), -m2*l2*cos(x(3));
 -m1*l1*cos(x(2)), m1*l1^2, 0;
 -m2*l2*cos(x(3)), 0, m2*l2^2];
G = [m1*l1*sin(x(2))*x(5)^2 + m2*l2*sin(x(3))*x(6)^2;
 m1*l1*g*sin(x(2));
 m2*l2*g*sin(x(3))];
W = [1;0;0];
q = x(1:3);
qdot = x(4:6);
qddot = inv(M)*(W*u-G);
dxdt = [qdot; qddot];
end
```