## Problem 3

```
Rv = 6051.9;
 mu = 324858.59882646;
 a = 8*Rv
 a = 4.8415e+04
 e = 0.7
 e = 0.7000
 inc deg = 30, inc = deg2rad(inc deg)
 inc_deg = 30
 inc = 0.5236
 om_deg = 60, om_deg = deg2rad(om_deg)
 om_deg = 60
 om = 1.0472
 w deg = 90, w = deg2rad(w deg)
 w_deg = 90
 w = 1.5708
Part a)
 % Original Orbit Characteristics
 ths deg = 235, ths = deg2rad(ths deg), th = w + ths, th deg = rad2deg(th)
 ths_deg = 235
 ths = 4.1015
 th = 5.6723
 th deg = 325
 p = a * (1-e^2)
 p = 2.4692e + 04
 iCr = findDCM(om,inc,th)
 iCr = 3 \times 3
     0.8398 -0.3276 0.4330
     0.4610
            0.8514 -0.2500
    -0.2868
            0.4096
                    0.8660
 rmag = p/(1+e*cos(ths))
 rmag = 4.1256e + 04
 r r = rmag*[1 0 0]'
 r_r = 3 \times 1
 10<sup>4</sup> ×
```

```
4.1256
        0
r = iCr * r_r
r = 3 \times 1
10<sup>4</sup> ×
   3.4645
   1.9021
  -1.1832
vmag = sqrt(mu*(2/rmag - 1/a))
vmag = 3.0064
hmag = sqrt(mu*p)
hmag = 8.9562e + 04
gamma = -acos(hmag/(rmag*vmag)), gamma_deg = rad2deg(gamma)
gamma = -0.7640
gamma\_deg = -43.7735
v_r = vmag*[sin(gamma), cos(gamma), 0]
v_r = 3 \times 1
  -2.0799
   2.1709
v = iCr * v r
v = 3 \times 1
  -2.4577
   0.8895
   1.4856
h = cross(r, v)
h = 3 \times 1
10<sup>4</sup> ×
   3.8781
  -2.2390
  7.7563
% New Orbit Characteristics
e1 = 0.5, w1 deg = w deg - 30, w1 = deg2rad(w1 deg), dw deg = -30, dw = deg2rad(dw deg)
e1 = 0.5000
w1_deg = 60
w1 = 1.0472
dw_deg = -30
dw = -0.5236
inc1 = inc, inc1_deg = inc_deg, om1 = om, om1_deg = om_deg
inc1 = 0.5236
```

```
inc1_deg = 30
om1 = 1.0472
om1_deg = 60
ths1 = ths - dw, ths1_deg = rad2deg(ths1)
ths1 = 4.6251
ths1_deg = 265
th1 = w1 + ths1, th1_deg = rad2deg(th1)
th1 = 5.6723
th1_deg = 325
r1mag = rmag, r1 = r, r1 r = r
r1mag = 4.1256e+04
r1 = 3 \times 1
10<sup>4</sup> ×
   3.4645
   1.9021
  -1.1832
r1 r = 3 \times 1
10^4 \times
   4.1256
        0
        0
p1 = r1mag * (1+e1*cos(ths1))
p1 = 3.9458e + 04
a1 = p1/(1-e1^2)
a1 = 5.2611e + 04
v1mag = sqrt(mu*(2/r1mag - 1/a1))
v1mag = 3.0941
h1mag = sqrt(mu*p1)
h1mag = 1.1322e+05
gamma1 = -acos(h1mag/(r1mag*v1mag)), gamma1 deg = rad2deg(gamma1)
gamma1 = -0.4801
gamma1\_deg = -27.5102
iCr1 = findDCM(om1,inc1,th1)
iCr1 = 3 \times 3
           -0.3276
                    0.4330
   0.8398
   0.4610
            0.8514
                    -0.2500
   -0.2868
            0.4096
                    0.8660
v1 r = v1mag*[sin(gamma1), cos(gamma1), 0]
```

```
v1_r = 3 \times 1
  -1.4292
   2.7443
        0
v1 = iCr1 * v1_r
v1 = 3 \times 1
   -2.0991
   1.6777
   1.5339
% Delta V Characteristics
dv1 = v1 - v
dv1 = 3 \times 1
   0.3586
   0.7882
   0.0483
dv1mag = norm(dv1)
dv1mag = 0.8673
dv1 r = v1 r - v r
dv1_r = 3 \times 1
   0.6507
   0.5734
        0
vhat = v/vmag
vhat = 3 \times 1
  -0.8175
   0.2959
   0.4941
nhat = h/hmag
nhat = 3 \times 1
   0.4330
   -0.2500
   0.8660
bhat = cross(vhat, nhat)
bhat = 3 \times 1
   0.3798
   0.9219
   0.0763
iCv = [vhat nhat bhat]
iCv = 3 \times 3
  -0.8175
           0.4330 0.3798
   0.2959 -0.2500 0.9219
   0.4941 0.8660 0.0763
```

```
dv1 vnb = iCv.' * dv1
dv1_vnb = 3 \times 1
  -0.0361
   0.0000
   0.8665
beta1 = asin(dv1 vnb(2)), beta1 deg = rad2deg(beta1)
beta1 = 2.2204e-16
beta1_deg = 1.2722e-14
alpha1 = acos(dv1 vnb(1)/dv1mag), alpha1 deg = rad2deg(alpha1)
alpha1 = 1.6124
alpha1_deg = 92.3850
alpha1_check = asin(dv1_vnb(3)/dv1mag); alpha1_check = [alpha1_check pi-alpha1_check]
alpha1 check = 1\times2
   1.5292
          1.6124
phi1 = alpha1 + gamma, phi1 deg = rad2deg(phi1)
phi1 = 0.8484
phi1_deg = 48.6115
dgamma1 = gamma1 - gamma, dgamma1_deg = rad2deg(dgamma1)
dgamma1 = 0.2838
dgamma1_deg = 16.2633
% check using dv1 r
beta1 = asin(dv1 r(3)), beta1 deg = rad2deg(beta1)
beta1 = 0
beta1_deg = 0
phi1 = asin(dv1 r(1)/dv1mag); phi1 = [phi1 pi-phi1]
phi1 = 1 \times 2
   0.8484
            2.2932
phil check = acos(dv1 r(2)/dv1mag), phil = phil(1), phil deg = rad2deg(phil)
phi1\_check = 0.8484
phi1 = 0.8484
phi1_deg = 48.6115
alpha1 = phi1 - gamma, alpha1 deg = rad2deg(alpha1)
alpha1 = 1.6124
alpha1_deg = 92.3850
r1a = p1/(1+e1*cos(pi))
r1a = 7.8917e + 04
```

```
rlp = p1/(1+e1*cos(0))
 r1p = 2.6306e + 04
 period1 = 2*pi*sqrt(a1^3/mu), period1 hr = period1/3600, period1 days = period1 hr/24
 period1 = 1.3303e+05
 period1_hr = 36.9529
 period1_days = 1.5397
 E1 = eccenAnom(ths1,e1), E1 deg = rad2deg(E1)
 E1 = -1.1245
 E1_{deg} = -64.4274
Part b)
 dv2mag = 1, phi2_deg = 40, phi2 = deg2rad(phi2_deg)
 dv2mag = 1
 phi2_deg = 40
 phi2 = 0.6981
 beta2 deg = -30, beta2 = deg2rad(beta2 deg)
 beta2 deg = -30
 beta2 = -0.5236
 % OG Orbit Characteristics
 ths_deg = 150, ths = deg2rad(ths_deg)
 ths_deg = 150
 ths = 2.6180
 th = w + ths, th deg = rad2deg(th)
 th = 4.1888
 th_deg = 240.0000
 iCr = findDCM(om, inc, th)
 iCr = 3 \times 3
     0.3995
             0.8080
                     0.4330
    -0.8080
             0.5335
                      -0.2500
            -0.2500
    -0.4330
                      0.8660
 rmag = p/(1+e*cos(ths))
 rmag = 6.2704e+04
 r r = rmag * [1;0;0]
 r_r = 3 \times 1
 10^4 \times
     6.2704
         0
         0
```

r = iCr \* r r

```
r = 3 \times 1
10^4 \times
   2.5051
  -5.0666
  -2.7152
vmag = sqrt(mu*(2/rmag - 1/a))
vmag = 1.9110
hmag = sqrt(mu*p)
hmag = 8.9562e + 04
gamma = acos(hmag/(rmag*vmag)), gamma_deg = rad2deg(gamma)
gamma = 0.7266
gamma_deg = 41.6312
v r = vmag*[sin(gamma), cos(gamma), 0]'
v_r = 3 \times 1
   1.2695
   1.4283
        0
v = iCr * v_r
v = 3 \times 1
   1.6613
  -0.2638
  -0.9068
h = cross(r, v)
h = 3 \times 1
10<sup>4</sup> ×
   3.8781
   -2.2390
   7.7563
dv2 r = dv2mag*[cos(beta2)*sin(phi2), cos(beta2)*cos(phi2), sin(beta2)]'
dv2 r = 3 \times 1
   0.5567
   0.6634
  -0.5000
dv2 = iCr * dv2 r
dv2 = 3 \times 1
   0.5419
   0.0291
   -0.8399
alpha2 = phi2-gamma
alpha2 = -0.0285
dv2\_vnb = [cos(beta2)*cos(alpha2);sin(beta2);cos(beta2)*sin(alpha2)]
```

```
dv2\_vnb = 3 \times 1
   0.8657
   -0.5000
   -0.0247
% Post Maneuver
v2 = dv2 + v
v2 = 3 \times 1
   2.2032
   -0.2347
  -1.7467
v2_r = dv2_r + v_r, v2mag = norm(v2)
v2_r = 3 \times 1
   1.8262
    2.0917
   -0.5000
v2mag = 2.8214
r2 = r, r2mag = rmag
r2 = 3 \times 1
10<sup>4</sup> ×
   2.5051
  -5.0666
  -2.7152
r2mag = 6.2704e+04
a2 = mu/2/(mu/r2mag - v2mag^2/2)
a2 = 1.3528e + 05
h2 = cross(r2, v2)
h2 = 3 \times 1
10<sup>5</sup> ×
   0.8213
   -0.1606
   1.0575
h2mag = norm(h2)
h2mag = 1.3486e+05
p2 = h2mag^2/mu
p2 = 5.5981e+04
e2 = sqrt(1-p2/a2)
e2 = 0.7656
ths2 = acos(1/e2 * (p2/r2mag - 1)), ths2_deg = rad2deg(ths2)
ths2 = 1.7113
ths2_deg = 98.0496
r2hat = r2/r2mag
```

```
r2hat = 3 \times 1
   0.3995
  -0.8080
  -0.4330
h2hat = h2/h2mag
h2hat = 3 \times 1
   0.6090
  -0.1191
   0.7842
th2hat = cross(h2hat, r2hat)
th2hat = 3 \times 1
   0.6852
   0.5770
  -0.4445
iCr2 = [r2hat, th2hat, h2hat]
iCr2 = 3 \times 3
   0.3995
          0.6852
                   0.6090
  -0.8080
          0.5770
                   -0.1191
          -0.4445
                   0.7842
  -0.4330
i2 = acos(h2hat(3)), i2 deg = rad2deg(i2)
i2 = 0.6694
i2_{deg} = 38.3556
om2 = asin(h2hat(1)/sin(i2)); om2 = [om2 pi-om2]
om2 = 1 \times 2
   1.3776
          1.7640
om2check = acos(-h2hat(2)/sin(i2)), om2 = om2(1), om2 deg = rad2deg(om2)
om2check = 1.3776
om2 = 1.3776
om2 deg = 78.9327
th2 = asin(r2hat(3)/sin(i2)); th2 = [th2 pi-th2]
th2 = 1 \times 2
          3.9139
  -0.7723
th2check = 2*pi-acos(th2hat(3)/sin(i2))
th2check = 3.9139
th2 = -(2*pi - th2(2)), th2 deg = rad2deg(th2)
th2 = -2.3693
th2_deg = -135.7493
w2 = th2 - ths2, w2_deg = rad2deg(w2)
w2 = -4.0806
w2_{deg} = -233.7989
```

```
r2a = p2/(1+e2*cos(pi))

r2a = 2.3886e+05

r2p = p2/(1+e2*cos(0))

r2p = 3.1706e+04

period2 = 2*pi*sqrt(a2^3/mu), period2_hr = period2/3600, period2_days = period2_hr/24

period2 = 5.4854e+05
period2_hr = 152.3712
period2_days = 6.3488

E2 = eccenAnom(ths2,e2), E2_deg = rad2deg(E2)

E2 = 0.7944
E2 deg = 45.5144
```

## **Function 1: Find Direction Cosine Matrix**

```
function iCr = findDCM(om,inc,th)
    col1 = [cos(om)*cos(th) - sin(om)*cos(inc)*sin(th);
        sin(om)*cos(th)+cos(om)*cos(inc)*sin(th);
        sin(inc)*sin(th)];

col2 = [-cos(om)*sin(th)-sin(om)*cos(inc)*cos(th);
        -sin(om)*sin(th)+cos(om)*cos(inc)*cos(th);

sin(inc)*cos(th)];

col3 = [sin(om)*sin(inc);
        -cos(om)*sin(inc);
        cos(inc)];

col2 = cross(col3,col1);
    iCr = [col1 col2 col3];
end
```

## **Function 4: Eccentric Anomaly**

```
function E = eccenAnom(ths,e)
   E = 2*atan(sqrt((1-e)/(1+e))*tan(ths/2));
end
```