Parameters and Equilibrium Conditions

```
P1 = [2 1 1 1 1 1];
P2 = [2 1 1 1 0.99 1];
P3 = [2 1 0.5 1 1 1];
P4 = [2 1 1 1 0.5 1];

E1 = [0 0 0];
E2 = [0 pi pi];

case1 = num2cell([P1,E1]);
case3 = num2cell([P2,E1]);
case7 = num2cell([P4,E1]);

u = 0;
C = [1 0 0 0 0 0];
D = 0;
```

Case 1: P1, E1

```
% pull parameters and initial conditions for case 1
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case1{:});

mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -0.5000   -1.5000         0         0         0
```

```
Qo = obsv(A,C)
```

```
Qo = 6×6
    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0
         0   -0.5000   -0.5000         0         0         0
         0         0         0         0   -0.5000   -0.5000
         0    1.0000    1.0000         0         0         0
         0         0         0         0    1.0000    1.0000
```

```
rank(Qo)
```

```
ans = 4
```

```
eig(A)
```

```
ans = 6×1 complex
```

```
   0.0000 + 0.0000i
   0.0000 + 0.0000i
   0.0000 + 1.4142i
   0.0000 - 1.4142i
  -0.0000 + 1.0000i
  -0.0000 - 1.0000i
```

```
PBHtest(A,C)
```

```
ans = 2×1 complex
  -0.0000 + 1.0000i
  -0.0000 - 1.0000i
```

Case 3: P2, E1

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case3{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)]
```

```
G = 3×3
         0         0         0
         0    1.0000         0
         0         0    0.9900
```

```
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -0.5051   -1.5152         0         0         0
```

```
Qo = obsv(A,C)
```

```
Qo = 6×6
    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0
         0   -0.5000   -0.5000         0         0         0
         0         0         0         0   -0.5000   -0.5000
         0    1.0025    1.0076         0         0         0
         0         0         0         0    1.0025    1.0076
```

```
rank(Qo)
```

```
ans = 6
```

```
eig(A)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -0.0000 + 1.4178i
```

```
  -0.0000 - 1.4178i
   0.0000 + 1.0025i
   0.0000 - 1.0025i
```

```
PBHtest(A,C)
```

```
ans =

  0×1 empty double column vector
```

Case 7: P4, E1

```
[m0, m1, m2, l1, l2, g, ye, theta1e, theta2e] = deal(case7{:});
mt = m0 + m1 + m2;
M = [mt, -m1*l1*cos(theta1e), -m2*l2*cos(theta2e);
    -m1*l1*cos(theta1e), m1*l1^2, 0;
    -m2*l2*cos(theta2e), 0, m2*l2^2];
G = [0,0,0;0,m1*l1*g*cos(theta1e),0;0,0,m2*l2*g*cos(theta2e)];
W = [1 0 0]';
A = [zeros(3), eye(3); M^-1*(-G), zeros(3)]
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -0.5000   -0.5000         0         0         0
         0   -1.5000   -0.5000         0         0         0
         0   -1.0000   -3.0000         0         0         0
```

```
ans =
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -1 & -3 & 0 & 0 & 0 \end{pmatrix}$$

```
Qo = obsv(A,C)
```

```
Qo = 6×6
    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0
         0   -0.5000   -0.5000         0         0         0
         0         0         0         0   -0.5000   -0.5000
         0    1.2500    1.7500         0         0         0
         0         0         0         0    1.2500    1.7500
```

```
rank(Qo)
```

```
ans = 6
```

```
eig(A)
```

```
ans = 6×1 complex
```

```
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -0.0000 + 1.8113i
  -0.0000 - 1.8113i
   0.0000 + 1.1042i
   0.0000 - 1.1042i
```

```
PBHtest(A,C)
```

```
ans =

  0×1 empty double column vector
```

```
function unobsveig = PBHtest(A,C)
s = eig(A);
n = size(A,2);
for i = 1:length(s)
    T = [A-s(i)*eye(size(A));C];
    r = rank(T);
    test(i) = (r==n);
end
unobsveig = s(~test);
end
```