## Problem 1

```
addpath(genpath(fileparts(which('pathfile.m'))))
interr = 'latex';
% interr = 'none';
set(groot, 'defaulttextinterpreter',interr);
set(groot, 'defaultAxesTickLabelInterpreter',interr);
set(groot, 'defaultLegendInterpreter',interr);
R_E = 6378.1363;
R_M = 1738.2;
mu_E = 398600.4415;
mu_M = 4902.8005821478;
a_M = 384400.00; % moon SMA around Earth
```

## Part a)

Hohmann Transfer Characterisites

```
rp_T mag = 250 + R_E, ra_T mag = a_M
rp_T_mag = 6.6281e+03
ra_T_mag = 384400
a T = (rp T mag + ra T mag)/2
a_T = 1.9551e + 05
e T = 1 - rp T mag/a T
e_T = 0.9661
v1_n_mag = sqrt(mu_E*(2/rp_T_mag - 1/a_T))
v1 n mag = 10.8737
v1 n = [v1 n mag 0]'
v1 n = 2 \times 1
  10.8737
v2 \ 0 \ mag = sqrt(mu \ E*(2/ra \ T \ mag - 1/a \ T))
v2_0_mag = 0.1875
v2 \ 0 \ R = [0 \ v2 \ 0 \ mag]'
v2 \ 0 \ R = 2 \times 1
   0.1875
n T = sqrt(mu E/a T^3)
n_T = 7.3030e-06
```

Moon Orbit about Earth

```
v M mag = sqrt((mu E)/a M)
 v_M = 1.0183
 v_M R = [0, v_M mag]'
 v_M_R = 2 \times 1
     1.0183
Departure Parking Orbit
 r1_mag = 250 + R_E
 r1_mag = 6.6281e + 03
 v1_0_mag = sqrt(mu_E/r1_mag)
 v1_0_mag = 7.7548
 v1 0 = v1 0 mag * [1 0]'
 v1_0 = 2 \times 1
     7.7548
 dv1 = v1 n-v1 0, dv1 mag = norm(dv1)
 dv1 = 2 \times 1
     3.1188
 dv1_mag = 3.1188
Arrival Parking orbit
 rp M = 195 + R M
 rp_M = 1.9332e+03
Arrival Hyperbolic Orbit
 v inf M = v2 0 R - v M R
 v_{inf_M} = 2 \times 1
    -0.8308
 v_inf_M mag = norm(v_inf_M)
 v_{inf_M_mag} = 0.8308
 v2 n mag = sqrt(mu M/rp M)
 v2_n_mag = 1.5925
 dv2 mag = sqrt(v inf M mag^2 + 2*mu M/rp M) - v2 n mag
```

2

 $dv2\_mag = 0.8080$ 

## Total

```
dvtotal mag = dv1 mag + dv2 mag
 dvtotal_mag = 3.9268
Post Encounter Orbit
 a_h = -mu_M/v_inf_M_mag^2
 a h = -7.1030e + 03
 e h = 1 - rp M/a h
 e_h = 1.2722
 delta = 2*(asin(1/e h)), delta deg = rad2deg(delta)
 delta = 1.8088
 delta_deg = 103.6376
 ths inf = acos(-1/e_h), ths_inf_deg = rad2deg(ths_inf)
 ths_inf = 2.4752
 ths_inf_deg = 141.8188
 v2 n mag = sqrt(v M mag^2 + v inf M mag^2 - 2*v M mag*v inf M mag*cos(delta))
 v2_n_m = 1.4581
 gamma 2 n = 0.5868
 gamma 2 n deg = 33.6223
 v2 n R = v2 n mag*[sin(gamma 2 n);cos(gamma 2 n)]
 v2_n_R = 2 \times 1
    0.8074
    1.2142
 v_2_0_R = [0 v_2_0_mag]'
 v_2_0_R = 2 \times 1
    0.1875
 dveq R = v2 n R - v 2 0 R
 dveq R = 2 \times 1
    0.8074
    1.0267
 dveq_mag = norm(dveq_R)
 dveq_mag = 1.3061
```

```
ths 2 n = atan((ra T mag * v2 n mag^2 / mu E)*sin(gamma 2 n)*cos(gamma 2 n)...
     /((ra_T_mag * v2_n_mag^2 / mu_E)*cos(gamma 2 n)^2-1))
ths_2_n = 1.1512
ths 2 \text{ n deg} = \text{rad2deg(ths } 2 \text{ n)}
ths_2_n_deg = 65.9585
iCr2 = [cos(pi), -sin(pi); sin(pi) cos(pi)]
iCr2 = 2 \times 2
          -0.0000
  -1.0000
   0.0000 -1.0000
v2 \ 0 = iCr2 * v2 \ 0 R
v2 0 = 2 \times 1
  -0.0000
  -0.1875
v2 n = iCr2 * v2 n R
v2_n = 2 \times 1
  -0.8074
  -1.2142
dveq = v2_n - v2_0
dveq = 2 \times 1
  -0.8074
  -1.0267
a post = -mu_E*(v2_n mag^2 - 2*mu_E/ra_T_mag)^-1
a_post = -7.6283e + 06
e post = sqrt((ra T mag * v2 n mag^2/mu E -1)^2*cos(gamma 2 n)^2+sin(gamma 2 n)^2)
e post = 1.0352
rp post = a post*(1-e post)
rp_post = 2.6853e + 05
dv_d = -dveq, dv_d = norm(dv d)
dv d = 2 \times 1
   0.8074
   1.0267
dv_d_mag = 1.3061
v2 d = v2 0
v2_d = 2 \times 1
  -0.0000
  -0.1875
```

```
v2 \circ = v2 n
 v2 o = 2 \times 1
    -0.8074
    -1.2142
 dgamma2 = 0 - gamma_2_n
 dgamma2 = -0.5868
  [alpha, beta] = alphabeta(norm(v2 o), norm(v2 d), norm(-dveq), dgamma2)
 alpha = 3.0620
 beta = 0.0796
 alpha deg = rad2deg(alpha), beta deg = rad2deg(beta)
 alpha deg = 175.4411
 beta_deg = 4.5589
Part b)
Transfer Orbit Characteritics
 trans angle = 173 * pi / 180;
 r1 \text{ mag} = R E + 150
 r1_mag = 6.5281e+03
 r2_mag = a_M
 r2_{mag} = 384400
 ta1 = 0
 ta1 = 0
 ta2 = trans angle
 ta2 = 3.0194
 e T = (r2 mag/r1 mag - 1)*(1-r2 mag/r1 mag * cos(ta2))^-1
 e_T = 0.9737
 a_T = r1_mag/(1-e_T)
 a_T = 2.4858e + 05
 rp = r1_mag
 rp = 6.5281e + 03
 ra = a T*(1+e T)
```

ra = 4.9064e + 05

```
n = sqrt(mu E/a T^3)
 n = 5.0940e-06
 period = 2*pi/n, period hr = period/3600, period day = period/3600/24
 period = 1.2335e + 06
 period_hr = 342.6253
 period_day = 14.2761
 [TOF T,dt1,dt2] = timeofflight(ta1,ta2,a T,e T,r1 mag,mu E)
 TOF T = 2.6708e + 05
 dt1 = 0
 dt2 = 2.6708e + 05
 TOF T day = TOF T/3600/24
 TOF_T_day = 3.0912
 energy_T = -mu_E/(2*a_T)
 energy_T = -0.8017
 n_M = sqrt(mu_E/a_M^3)
 n M = 2.6491e-06
 phase = pi - n M*TOF T, phase deg = rad2deg(phase)
 phase = 2.4341
 phase_deg = 139.4630
 v1 \ 0 \ mag = sqrt(mu \ E/r1 \ mag)
 v1 0 mag = 7.8140
 v1 n mag = sqrt(mu E*(2/r1 mag - 1/a T))
 v1_n_mag = 10.9779
 dv1 mag = v1 n mag - v1 0 mag
 dv1_mag = 3.1639
Part c)
 r2_mag
 r2_{mag} = 384400
 v2 \ 0 \ mag = sqrt(mu \ E*(2/r2 \ mag - 1/a \ T))
 v2_0_mag = 0.6859
 h T = sqrt(mu E*a T*(1-e T^2))
```

 $h_T = 7.1665e + 04$ 

```
gamma2 0 = asin(mu E*e T/v2 0 mag/h T*sin(ta2)), gamma2 0 deg = rad2deg(gamma2 0)
gamma2 0 = 1.2955
gamma2_0_deg = 74.2271
pi-gamma2 0
ans = 1.8461
gamma2_0 check = acos(sqrt(mu_E*a_T*(1-e_T^2))/r2_mag/v2_0_mag)
gamma2_0_check = 1.2955
v2_n mag = v2_0 mag
v2_n_mag = 0.6859
gamma2 n = -gamma2 0
gamma2_n = -1.2955
c = r2 \text{ mag}*v2 \text{ n mag}^2/\text{mu E};
e_T2 = sqrt((c - 1)^2 * cos(gamma2_n)^2 + sin(gamma2_n)^2)
e_T2 = 0.9737
ta2 n = -ta2
ta2 n = -3.0194
ta2_n_deg = rad2deg(ta2_n)
ta2_n_deg = -173
dw = 2*pi + (ta2 n - ta2)
dw = 0.2443
v2_0 = v2_0 mag*[sin(gamma2_0); cos(gamma2_0)]
v2 0 = 2 \times 1
   0.6600
   0.1864
v2 n = v2 n mag*[sin(gamma2 n); cos(gamma2 n)]
v2_n = 2 \times 1
  -0.6600
   0.1864
dv2_eq = v2_n-v2_0
dv2_eq = 2 \times 1
  -1.3201
dv2 eq mag = norm(dv2 eq)
```

```
dv2\_eq\_mag = 1.3201
dgamma = gamma2_n - gamma2_0, dgamma_deg = rad2deg(dgamma)
dgamma = -2.5910
dgamma \ deg = -148.4542
[alpha,beta] = alphabeta(v2 0 mag,v2 n mag,dv2 eq,dgamma), alpha = - alpha
alpha = 2.8663
beta = 0.2753
alpha = -2.8663
alpha_deg = rad2deg(alpha)
alpha_deg = -164.2271
beta deg = rad2deg(beta)
beta deg = 15.7729
v_M mag = sqrt(mu_E/a_M)
v_M = 1.0183
v M = v M mag*[0;1]
v M = 2 \times 1
   1.0183
v2_{inf_0} = v2_0 - v_M
v2_inf_0 = 2 \times 1
   0.6600
  -0.8319
v2_inf_mag = norm(v2_inf_0)
v2_{inf_mag} = 1.0619
v2 inf n = v2 n - v M
v2_inf_n = 2 \times 1
  -0.6600
  -0.8319
a h = -mu M/v2 inf mag^2
a_h = -4.3478e + 03
L = a\cos(dot(v2_inf_n, v2_inf_0)/v2_inf_mag^2)
```

```
L = 1.3414
% delta = pi-L
delta = 2*asin(v2_0_mag/v2_inf_mag * sin(gamma2_0))
                                          8
```

```
delta_deg = rad2deg(delta)

delta_deg = 76.8594

e_h = 1/sin(delta/2)

e_h = 1.6089

rp_h = a_h*(1-e_h)

rp_h = 2.6472e+03

alt_p = rp_h - R_M

alt_p = 909.0458
```

## Part d)

```
% plotorbit(ai,ei,ths1,ths2,rotate) % plot orbit inputs
plotorbit(rp,0,0,2*pi,0) % parking orbit
hold on
% plotorbit(a M, 0, 0, 2*pi, 0) % Moon orbit
plotorbit(a T,e T,0,ta2,0)
plotorbit(a T,e T,-ta2,0,dw)
plotunit([0,0],0,'g--',10)
plotpos (a T, e T, ta2, 'b--', 0, 10)
plotvel(a_T,e_T,ta2,v2_0,'r',0,0,3e5)
plotvel(a T,e T,ta2,v2 n,'b',0,0,3e5)
plotvel(a T,e T,ta2,dv2 eq,'k',[1;v2 0],0,3e5)
plotvel(a T,e T,ta2,v2 inf 0,'r--',[1;v M],0,3e5)
plotvel(a T,e T,ta2,v2 inf n,'b--',[1;v M],0,3e5)
xlim([-6 \ 1]*1e5), ylim([-3.5 \ 1.5]*1e5)
set(gcf, 'position', [0,0,1000,1000])
title('Lunar Free Return')
xlabel('$\hat{e}$ direction')
ylabel('$\hat{p}$ direction')
```

