Optimal Control
with Aerospace Applications - Longuski
Notes

Joseph Le

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Chapter 1

Parameter Optimization

1.1 Introduction

Two main branches of optimization: parameter optimization and optimal control theory. Parameter optimization is finite dimensional, and a function is minimized via a set of optimal parameters. Optimal control is infinite dimensional find a trajectory $\boldsymbol{x}(t)$, generally an n-vector, that minimizes a **functional**.

Parameter optimization seeks to find maxima and minima using calculus. The problem is stated as:
Find:

$$\boldsymbol{x}$$

to minimize:

$$J = f(\boldsymbol{x}) \tag{1.1}$$

where J is a scalar cost function or index of performance and $\boldsymbol{x}$ is a constant n-vector. The solution, $\boldsymbol{x}^*$, can be found by (if f has continuous partial derivatives and x_i are independent)

$$\frac{\partial f}{\partial x_i} = 0, \quad i = 1, 2, 3, \ldots, n \tag{1.2}$$

Equation 1.2 is a necessary condition for an extreumum (maximum or minimum, note that f can be maximized by minimizing $-f$). The stationary point $\boldsymbol{x}^*$ is the local minimum if the matrix $\frac{\partial^2 f}{\partial x_i \partial x_j}$ evaluated at $\boldsymbol{x}^*$ is a **positive-definite** matrix, which is a sufficient condition for a local minimum. The second partial derivatives of f must also be continuous for the matrix to be well defined.

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