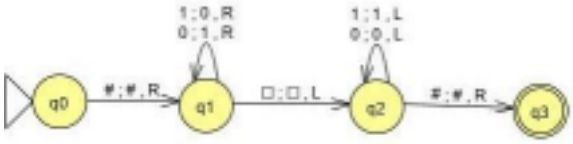
CSE 355: Intro to Theoretical Computer Science
Recitation #11 (20 pts)

1. [5 pts] Assume there is a special symbol # placed at the left end of the Turing machine's tape (to mark the end of the type). Given the following TM's state diagram, explain in English what this TM does?



The machine moves from left to right on the input and flips every bit, 1 to 0 and 0 to 1, until it reads a blank. Then, it will move back to the start of the tape.

The machine flips all bits and halts at the start of the tape.

2. [5 pts] Given the following TM state diagram, give the sequence of configurations for the following two strings:

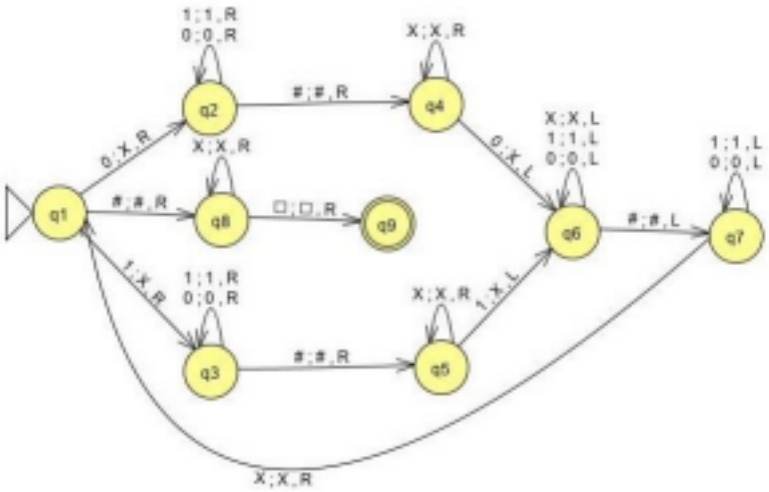
2.1) `11`

$q_1 11 \rightarrow x q_3 1 \rightarrow x 1 q_3$

2.2) `01#01`

`U = blank`

$q_1 01\#01 \rightarrow x q_2 1\#01 \rightarrow x1 q_2 \#01 \rightarrow x1\#q_4 01 \rightarrow x1\#q_6 x1 \rightarrow x1 q_6 \#x1 \rightarrow$
$x q_7 1\#x1 \rightarrow q_7 x1\#x1 \rightarrow x q_1 1\#x1 \rightarrow xx q_3 \#x1 \rightarrow xx\#q_5 x1 \rightarrow xx\#x q_5 1 \rightarrow xx\#x q_6 x \rightarrow$
$xx\#q_6 xx \rightarrow xx q_6 \#xx \rightarrow x q_7 x\#xx \rightarrow xx q_1 \#xx \rightarrow xx\#q_8 xx \rightarrow xx\#xx q_8 U \rightarrow xx\#xx U q_9$



2.3) Use set notation to describe the language accepted by this Turing machine.

$L = \{w\#w \mid w \in \{1,0\}^*\}$

3. [5 pts] Exam the formal definition of the Turing-Machine to answer the following question:

3.1) Can a TM ever write a blank symbol ␣ on its type?

Yes, A Turing machine can write any characters in Γ on its tape and $␣ \in \Gamma$. Thus the Turing machine can write the blank symbol on its tape.

3.2) Can the type alphabet Γ be the same as input alphabet Σ?

No, since Σ cannot contain the blank symbol and Γ can contain the blank symbol. So they cannot be the same.

3.3) Can the Turing machine's head ever be in the same location in two successive steps?

Yes, when you were at the leftmost end of the tape(start), then, if you try to go left again the head will stay at the same place.

3.4) Can a Turing machine just contain a single state?

No, since a Turing machine must have an accept state and a reject state so the minimum requires state is two. In the formal definition, it is a 7-tuple, which contains accepting states and rejecting states.

4. [5 pts] Give the informal description of the TM that decides the following language, assume $\Sigma = \{0, 1\}$. (Note: check textbook pp. 160, TM M_1 for an example of such description)

$L = \{\omega \mid \omega$ contains equal number of 0s and 1s$\}$

1. Scan the tape and if it reads a 0 mark it as blank. If there is no 0 on the tape then go to step 4. Otherwise, move back to the start of the tape.
2. Scan the tape and if it reads a 1 mark it as blank. If there is no unmarked 1 on the tape. Reject.
3. Move the head to the start of the tape and Repeat step 1 and step 2.
4. Move the head to the start of the tape and scan the tape to see if there is 1 on the tape. If there is no 1 on the tape, Accept. Otherwise, if there is 1 on the tape, Reject.