CSE 355: Intro. to Theoretical Computer Science

Homework #1: DFAs and NFAs

- This assignment is worth 100 points and 5% of your final grade.
- It is due on **Friday, Feb. 5th, 2021 at 11:59pm** Arizona time.
- Copying answers from any sources (online, book, last-semester's notes, ...) without proper citation is considered a violation of academic integrity and will be dealt with accordingly.
- Answers can be provided in handwritten or typed form (typed form preferred).
- For grading purpose, do NOT just submit/write the answers, instead copy each question and put your answer under it. Write legibly, unreadable and unclear answers will be graded with 0 point.
- Scan and submit your homework on Canvas **as a single PDF file** (in case you use Apps. taking pictures of your answers, please make sure they are neat and readable)
- **No late submissions will be accepted!** Submission through emails will **NOT** be accepted!

1. For each part below, give the formal definition *and* state diagram for a deterministic finite automaton (DFA) that accepts the language specified. *20 points*

a) $\{ L_1\ w = \in \{0,1\}^* \mid \text{w contains at most three 0s} \}$

Definition: Language L contains strings of 1s and 0s but would only accept less than or equal to three 0s.

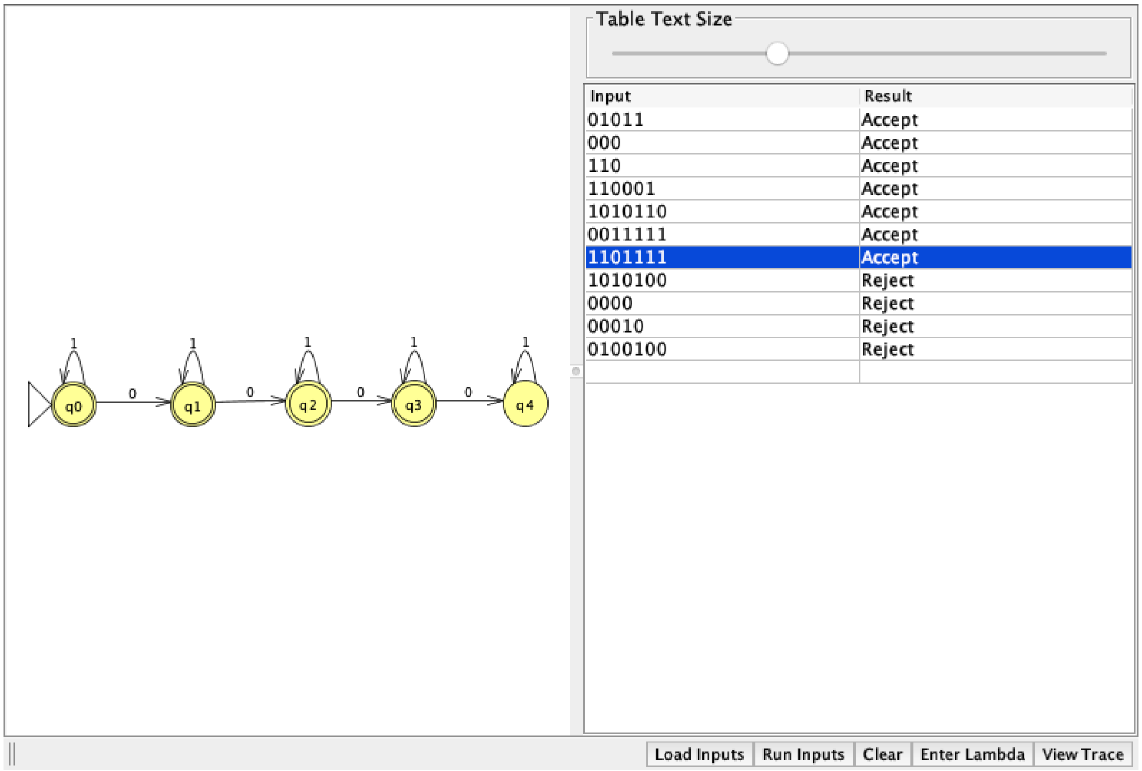
Q: {q0, q1, q2, q3, q4}

Σ : {0, 1}

q_{start}: q0 is the start state

F: {q0, q1, q2, q3}

δ :



b)$\{\ L_2\, w\ = \in \{\,0,1\}^*$ | w starts with a 1 and its number of 0s is a multiple of three$\}$

Definition: Language L contains strings of 1s and 0s but would only accept strings starting with 1, and the number of 0s have to be divisible by three.

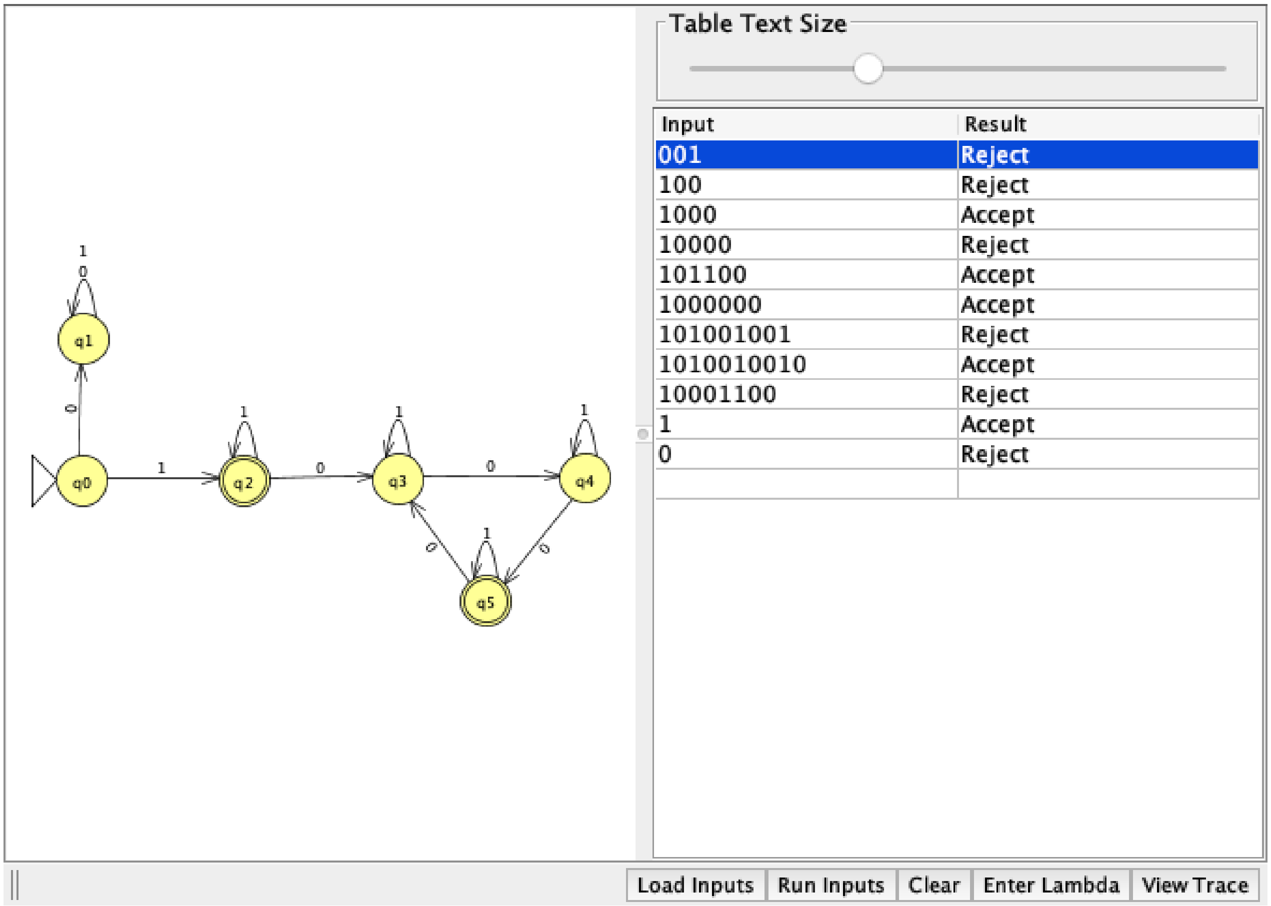
Q: {q0, q1, q2, q3, q4, q5}

Σ : {0, 1}

q_{start}: q0 is the start state

F: {q2, q5}

δ :



c) $L_3 = \{ \varepsilon ,a,baa,cab\}$

Definition: Language L only accepts strings ε, a, baa, cab.

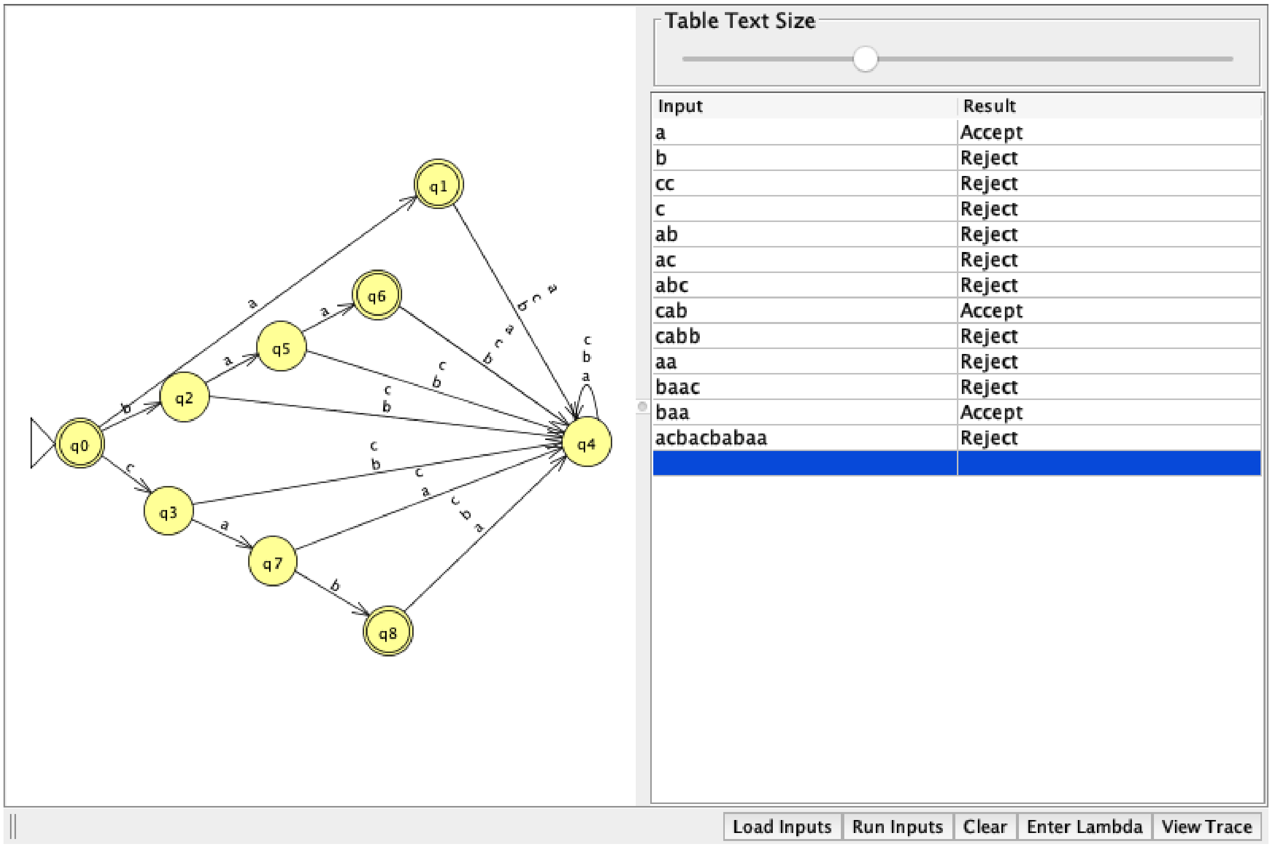
Q: {q0, q1, q2, q3, q4, q5, q6, q7, q8}

Σ : {a, b, c, ε }

q_{start}: q0 is the start state

F: {q1, q6, q8}

δ :



d)L_4 is the empty set. Let$\Sigma = \{a,b,c\}$.

Definition: Language L accepts empty set, other than empty set would be rejected.

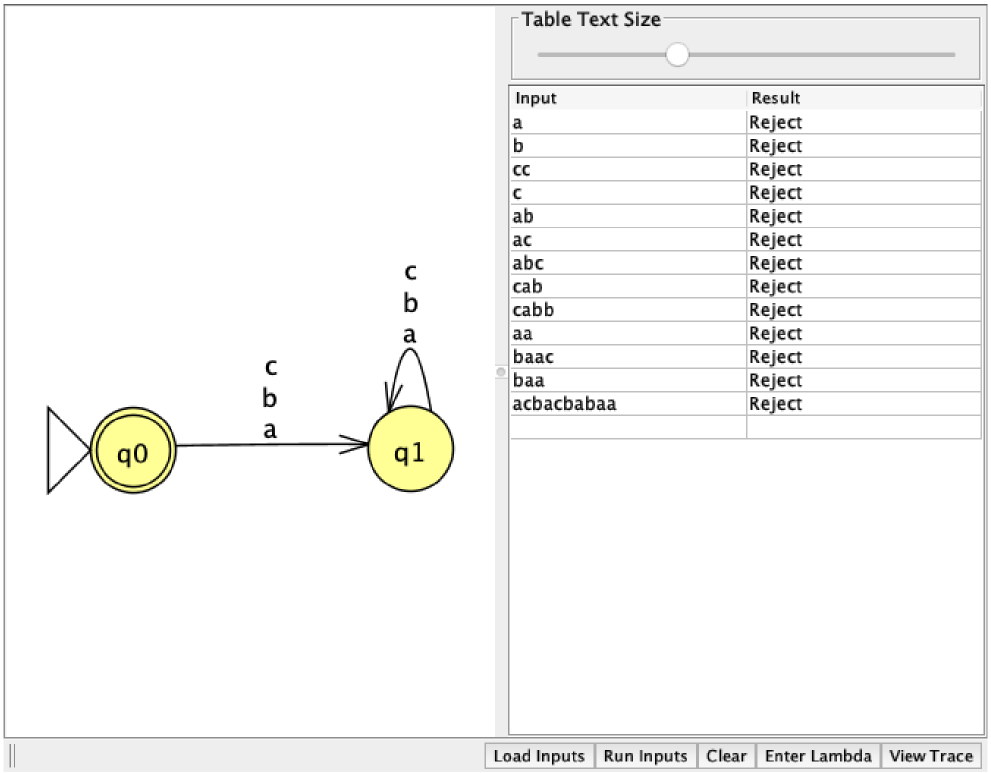
Q: {q0, q1}

Σ : {a, b, c}

q_{start}: q0 is the start state

F: {q0}

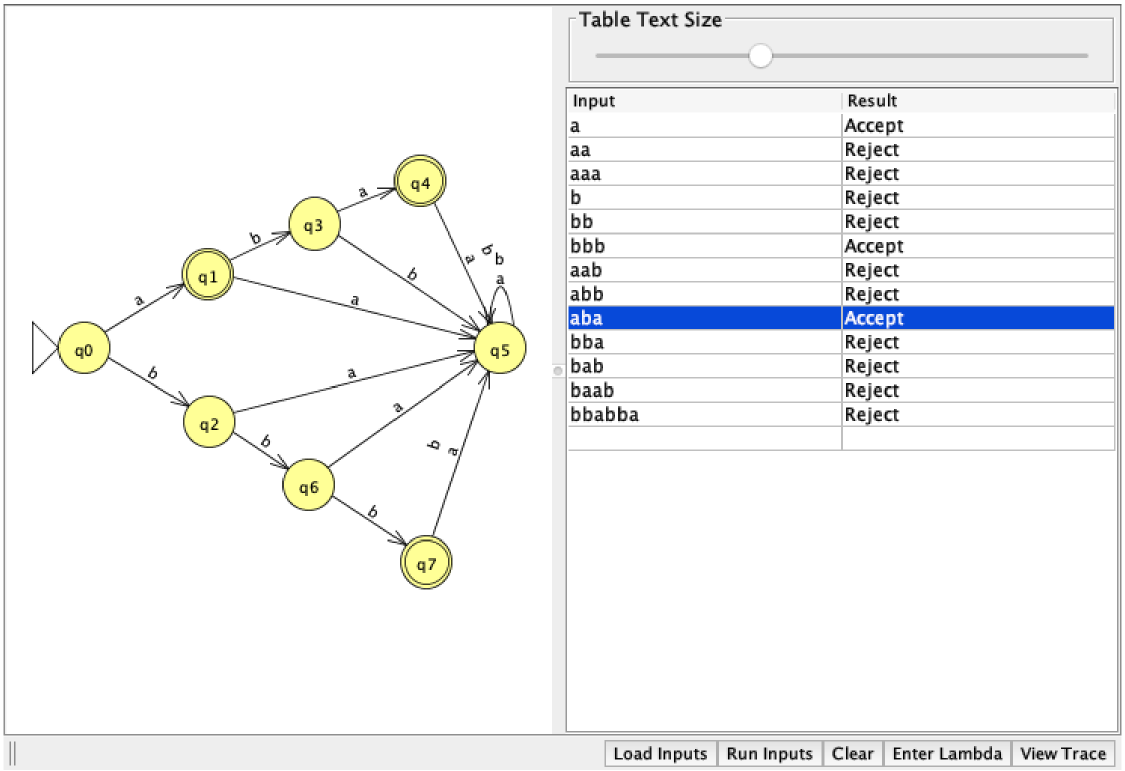
δ :



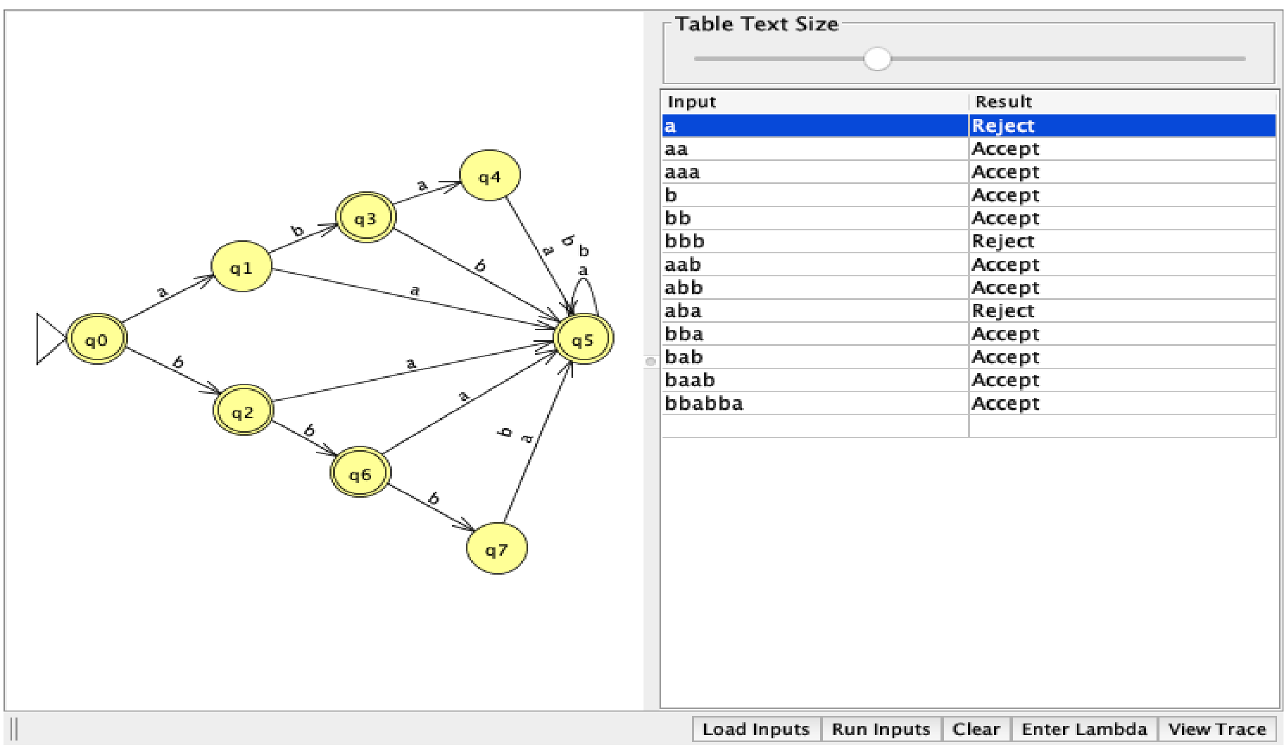
Input	Result
a	Reject
b	Reject
cc	Reject
c	Reject
ab	Reject
ac	Reject
abc	Reject
cab	Reject
cabb	Reject
aa	Reject
baac	Reject
baa	Reject
acbacbabaa	Reject

2. The language below is a complement of a simpler language. First, identify the simpler language and give the state diagram of the DFA that recognizes it. Then, use it to give the state diagram of the DFA that recognizes the language below. *10 points*

$\{L_5\ w = \in \{\ a,b\}^* \mid w$ is any string except a, aba, and bbb$\}$



Input	Result
a	Accept
aa	Reject
aaa	Reject
b	Reject
bb	Reject
bbb	Accept
aab	Reject
abb	Reject
aba	Accept
bba	Reject
bab	Reject
baab	Reject
bbabba	Reject

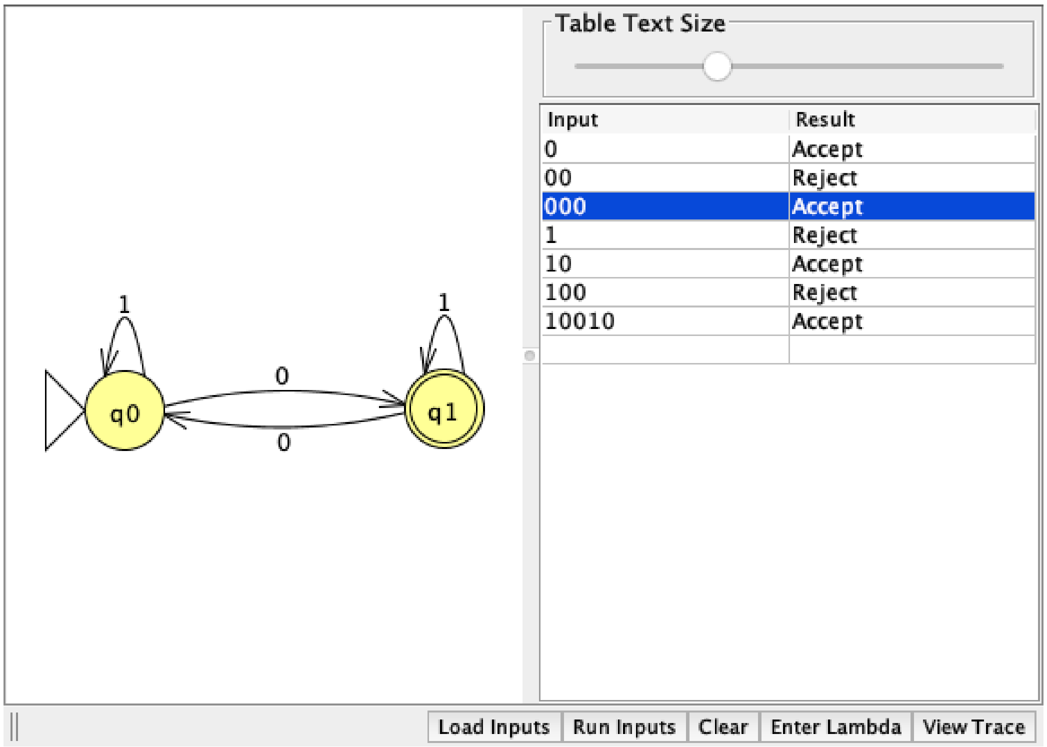


Input	Result
a	Reject
aa	Accept
aaa	Accept
b	Accept
bb	Accept
bbb	Reject
aab	Accept
abb	Accept
aba	Reject
bba	Accept
bab	Accept
baab	Accept
bbabba	Accept

3. The language below is the intersection of two simpler languages. First, identify the simpler languages and give the state diagrams of the DFAs that recognize them. Then, use the product construction from the

proof of Theorem 1.25 in the book to build a DFA that recognizes the language specified below; give its state diagram before and after simplification if there are any unneeded states or states that can be combined.
10 points

$L_6\ w = \in \{\ 0,1\}^* \mid w$ contains an odd number of 0s and the sum of its 0s and 1s is equal to 1}



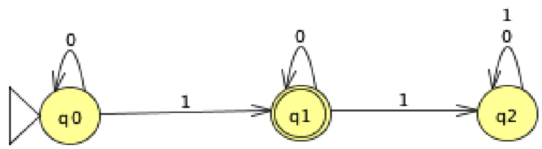


Table Text Size

Input	Result
0	Reject
00	Reject
000	Reject
1	Accept
10	Accept
100	Accept
10010	Reject
111	Reject
1000001	Reject
1010000	Reject

Load Inputs | Run Inputs | Clear | Enter Lambda | View Trace

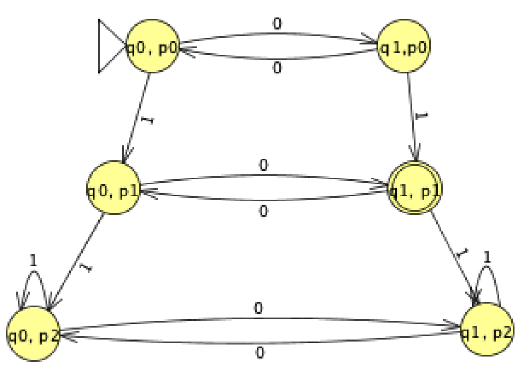


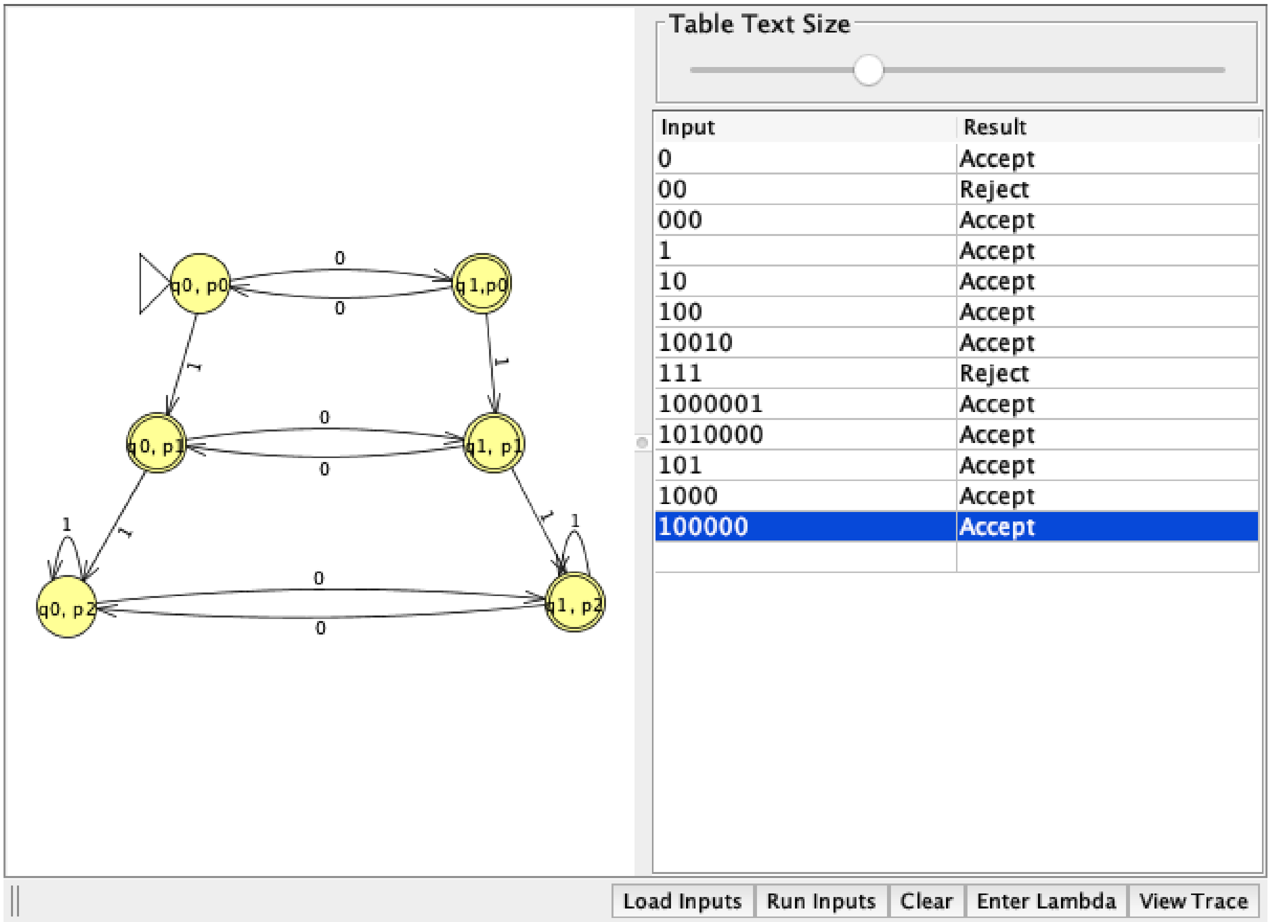
Table Text Size

Input	Result
0	Reject
00	Reject
000	Reject
1	Reject
10	Accept
100	Reject
10010	Reject
111	Reject
1000001	Reject
1010000	Reject
101	Reject
1000	Accept
100000	Accept

Load Inputs | Run Inputs | Clear | Enter Lambda | View Trace

4. Consider instead that the language from problem 3 was the union of its two simpler languages. Give the state diagram of your final DFA from problem 3, but with updated accept states for the language below. *5 points*

L_7 w = ∈ { 0,1} | w contains an odd number of 0s or the sum of its 0s and 1s is equal to 1}



Input	Result
0	Accept
00	Reject
000	Accept
1	Accept
10	Accept
100	Accept
10010	Accept
111	Reject
1000001	Accept
1010000	Accept
101	Accept
1000	Accept
100000	Accept

5. For each part, give the formal definition *and* state diagram for a nondeterministic finite automaton (NFA) that accepts the specified language. *10 points*

a) {L_8 = {w ∈ {a, b}* | ab is a prefix of w and bb is a suffix of w}

Definition: L contains a string of 'a's and 'b's. The string would be accepted if it starts with ab and ends with bb.

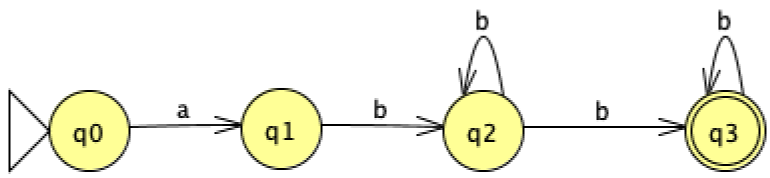
Q: {q0, q1, q2, q3}

Σ : {a, b}

q_{start}: q0 is the start state

F: {q3}

δ :



b) $\{L_9\, w = \in \{0,1\}^* \,|w$ represents a binary number of 4 to 6 digits in length and whose value is odd $\}$

Definition: L contains strings of 0s and 1s. It accepts length of 4 to 6 digits and the value must be odd. Which any number that ends with one.

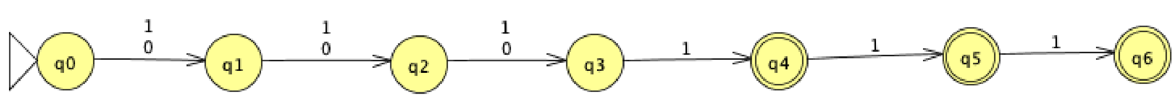
Q: {q0, q1, q2, q3, q4, q5, q6}

Σ : {1,0}

q_{start}: q0 is the start state

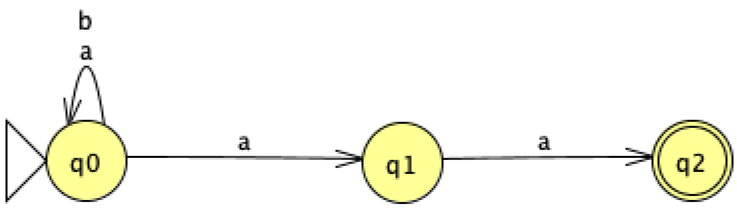
F: {q4, q5, q6}

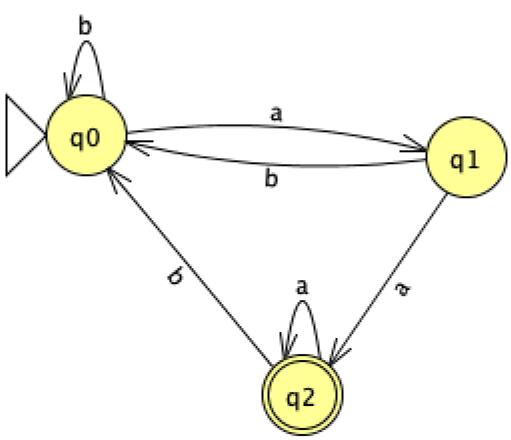
δ :



6. First, give the state diagram for the NFA that recognizes the language below using no more than 3 states. Next, use the powerset construction from the proof of Theorem 1.39 in the book to convert the NFA into a DFA. If there are any unneeded states or states that can be combined, you may simplify your DFA, but show your DFA's state diagram before and after simplification. *10 points*

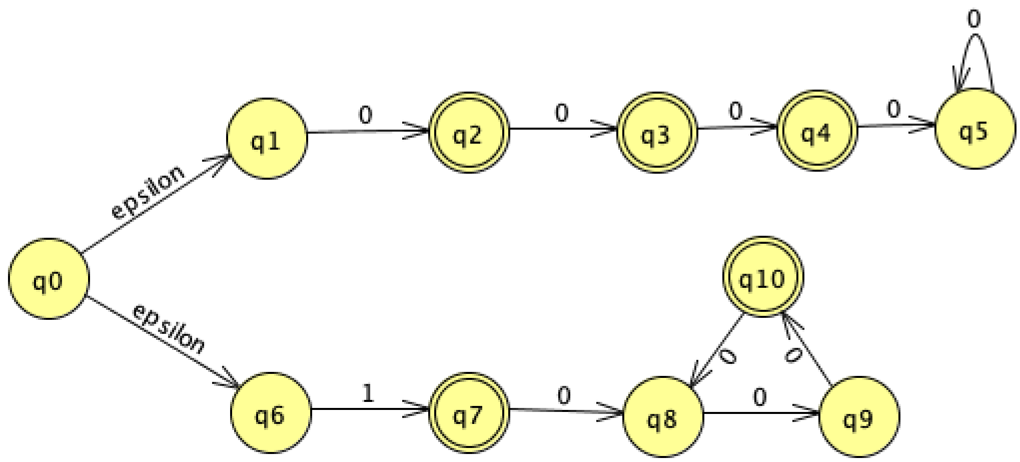
$\{L_{10}\ w = \in \{a,b\}^* \mid \text{aa is a suffix of w}\}$





7. For each part, use the appropriate construction (from the proof of Theorem 1.45, 1.47, and/or 1.49 in the book) to give the state diagram of an NFA recognizing the specified language. *15 points*

a)$L_1 \cup L_2$



b)$L_3 \circ L_4$

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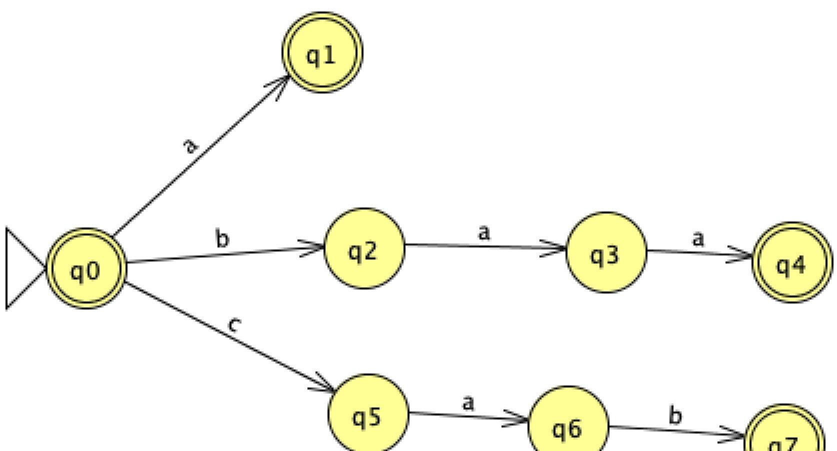

c) $L^*_4 \cup L^*_4 \circ L_4$

= Φ U Φ o Φ = L4

8. For each part below, show that the regular languages are closed under the specified operation. *20 points*

a) Complementation.

The complement of a language L is all strings that are not accepted in L. However, it is still created using the same alphabet. Which means interchange the states of accept and reject of L then L will be accepted.

b) String reversal.

To accept a reverse string. All it needs to do is change the direction of the transition and interchange the start and final states. Then we will have a DFA that accepts the reverse of the string.