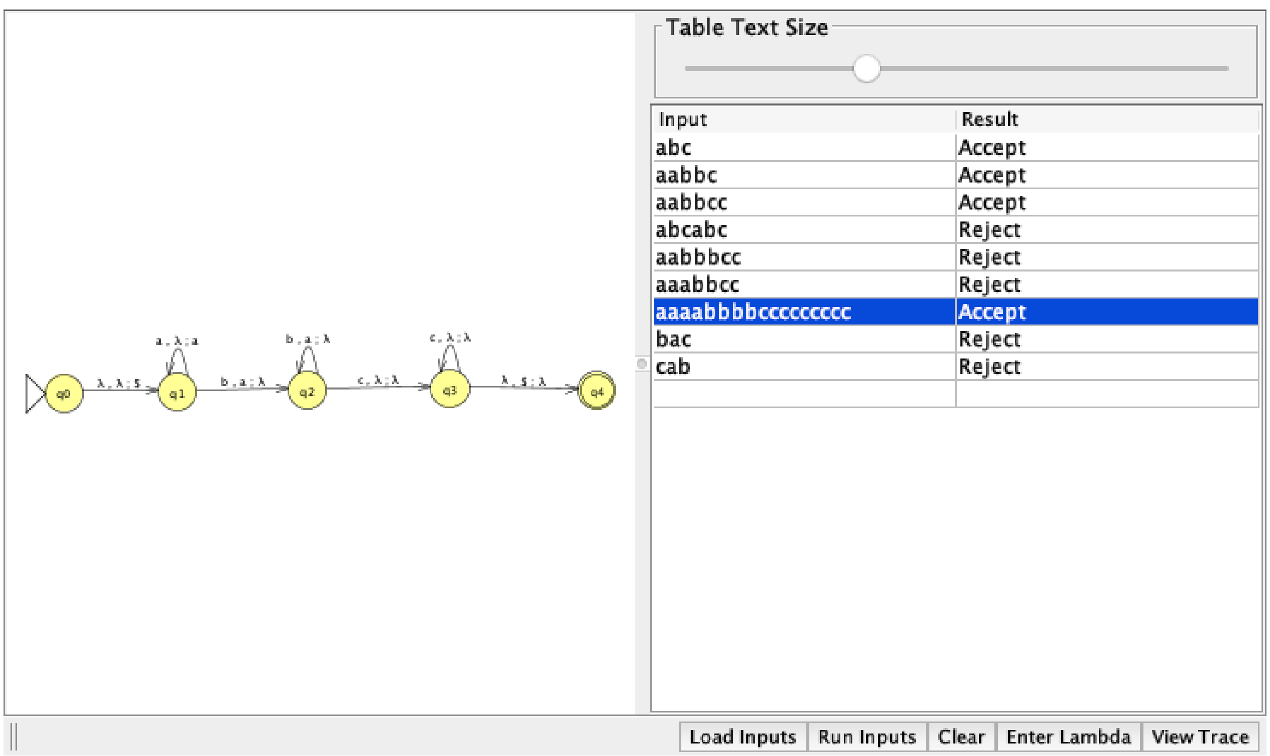
CSE 355: Intro. to Theoretical Computer Science

HW #4: CFL Part 2 and TM Part 1

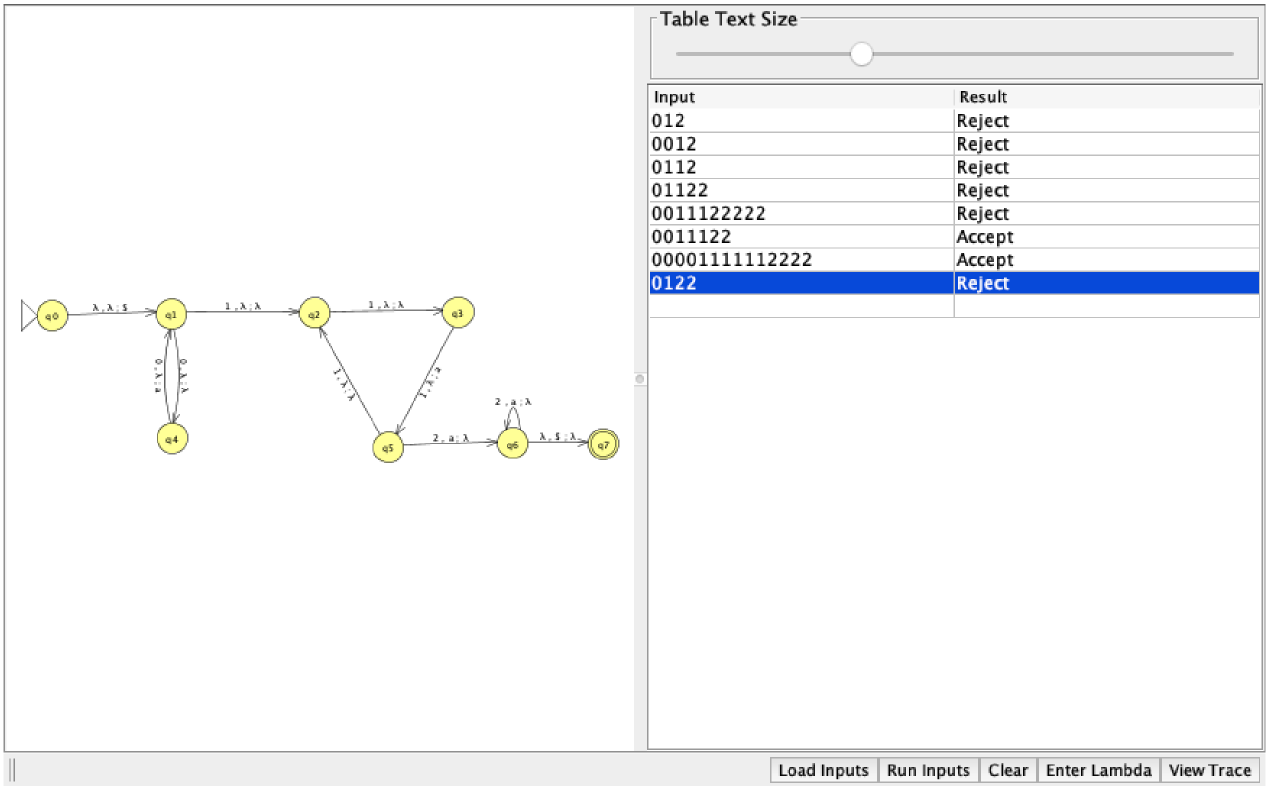
- This assignment is worth 100 points and 5% of your final grade.
- It is due on **Sunday, March 28, 2021 at 11:59pm** Arizona time.
- Copying answers from any sources (online, book, last-semester's notes, ...) without proper citation is considered a violation of academic integrity and will be dealt with accordingly. • Answers can be provided in handwritten or typed form.
- Unreadable and unclear answers will be graded with 0 point.
- Scan and submit your homework on Canvas **as a single PDF file** (in case you use Apps. taking pictures of your answers, please make sure they are neat and readable)
- **No late submissions will be accepted!** Submission through emails will **NOT** be accepted!

1. Draw the state machine of a deterministic PDA for the following languages (assume all missing transitions go to a dead state; you do not have to draw these transitions). 10 pts each.

   a. $L5 = \{a^i b^j c^k \mid i = j \text{ or } i, j, k \geq 0\}$



   b. $L7 = \{0^{2x} 1^{3y} 2^{x+y} \mid x, y \geq 0\}$



Input	Result
012	Reject
0012	Reject
0112	Reject
01122	Reject
0011122222	Reject
0011122	Accept
00001111112222	Accept
0122	Reject

2. Give the implementation-level description (English prose description of how the tape head moves and what is written to the tape) of the Turing machine that decides each of the following languages. 10 pts each.

   a. L_1 described by the regular expression ba* (bb)* a

   To accept the language

      1. If we read a ‘b’, mark it as blank and move right
      2. If we read an ‘a’, we mark it as blank and move right, Repeat this step until it reads a ‘b’ or blank.
         a. If we reads b, mark two b’s as blank and move right. Repeat this step until it reads an a. If we reads an a we go to step 3.
         b. If we reads an blank, then accept
      3. If we read an a mark it as blank and move right, if we read blank then accept.

   b. L2 = {w w = ∈ 0,1 | is a palindrome of even length }

      1. If we read a 0, mark it as blank and skip every symbol and move to the rightmost. Then, we move left and we should see a 0. Replace it as a blank. Move left and skip through all the symbols until we see a blank on the leftmost.
      2. If we read a 1, mark it as blank and skip every symbol and move to the rightmost.

Then, we move left and we should see a 1. Replace it as a blank. Move left and skip through all the symbols until we see a blank on the leftmost.

3. Repeat steps 1 and 2. Accept if the initial state reads blank.

c. $L3 = \{a^m b^n c^m d^n \mid m, n \geq 0\}$

1. If we read an a, mark it as blank and move to the right until we reads a ‘c’. Then, we mark c as blank and move left and skip through all the symbols until we see a blank on the leftmost.
2. If we read a ‘b’, mark it as blank and move to the right until we reads a ‘d’. Then, we mark d as blank and move left until we see a blank on the leftmost.
3. Accept if the initial state reads blank.

d. $L_4 = \{ww \mid w = \in \mid \{0,1\}^* \}$

1. If we read a 0 mark it as X and skip every symbol and move to the right until we reads a X, Y, or blank. Then move left. If we read a 0 we mark it as X and if we read a 1 we mark it as Y. Then, move all the way to the left until it reads a X or Y.
2. If we read a 1 mark it as Y and skip every symbol and move to the right until we reads a X, Y, or blank. Then move left. If we read a 0 we mark it as X and if we read a 1 we mark it as Y. Then, move all the way to the left until it reads a X or Y
3. Repeat step 1 and 2 until every symbol is convert to X and Y. We should now be in the middle point of the string (ex: XYXY S XYXY). Then do the following steps.
4. If we read a X mark it as blank and move to the left until we read a X and mark it as blank. Then, move all the way to the right until we read an X or Y.
5. If we read a Y mark it as blank and move to the left until we read a Y and mark it as blank. Then, move all the way to the right until we read an X or Y.
6. Accept if the tape consists all blanks.

3. Give the sequence of configurations that the specified machine below enters when started on the indicated input string. Total 10 pts.

a. # on M_1 (see Turing machine M_1 from our textbook pp. 166 Figure 3.10).

$q_1\# \rightarrow \#q8 \rightarrow q_{accept}$

b. 10110#10100 on M_1 (see Turing machine M_1 from our textbook pp. 166 Figure 3.10).

$q_1 10110\#10100 \rightarrow xq_3 0110\#10100 \rightarrow x0110q_3\#10100 \rightarrow x0110\#q_5 10100 \rightarrow$

$x0110\#xq_6 0100 \rightarrow x0110\#q_6 x0100 \rightarrow x0110q_7\#x0100 \rightarrow xq_7 0110\#x0100 \rightarrow$

$xq_1 0110\#x0100 \rightarrow xxq_2 110\#x0100 \rightarrow xx110q_2\#x0100 \rightarrow xx110\#q_4 x0100 \rightarrow$

$xx110\#xq_40100 \to xx110\#xxq_6100 \to xx110\#q_6xx100 \to xx110q_7\#xx100 \to$

$xxq_7110\#xx100 \to xxq_1110\#xx100 \to xxxq_310\#xx100 \to xxx10q_3\#xx100 \to$

$xxx10\#q_5xx100 \to xxx10\#xxq_5100 \to xxx10\#xxxq_600 \to xxx10\#q_6xxx00 \to$

$xxx10q_7\#xxx00 \to xxxq_710\#xxx00 \to xxxxq_110\#xxx00 \to xxxxq_30\#xxx00 \to$

$xxxx0q_3\#xxx00 \to xxxx0\#q_5xxx00 \to xxxx0\#xxxq_500$

c. 00 on M_2 (see Turing machine M_2 from our textbook pp. 165 Figure 3.8).
//Empty = u

$q_100 \to uq_20 \to uxq_3u \to uq_5x \to q_5ux \to uq_2x \to uxq_2u \to uxuq_{accept}$

d. 00000000 on M_2 (see Turing machine M_2 from our textbook pp. 165 Figure 3.8).

$q_100000000 \to uq_20000000 \to uxq_3000000 \to ux0q_400000 \to ux0xq_30000$
$\to ux0x0q_4000 \to ux0x0xq_300 \to ux0x0x0q_40 \to ux0x0x0xq_3u \to$
$ux0x0x0q_5x \to q_5ux0x0x0x \to uq_2x0x0x0x \to uxq_20x0x0x \to$
$uxxq_3x0x0x \to uxxxq_30x0x \to uxxxxq_4x0x \to uxxxxxq_40x \to uxxxxx0q_3x$
$\to uxxxxx0xq_3u \to uxxxxx0q_5x \to q_5uxxxxx0x \to uq_2xxxxx0x \to$
$uxxxxxq_20x \to uxxxxxxq_3x \to uxxxxxxxq_3u \to uxxxxxxq_5xu \to$
$q_5uxxxxxxxu \to uq_2xxxxxxxu \to uxxxxxxxq_2u \to uxxxxxxxuq_{accept}$

4. Convert the following Context Free Grammar into Chomsky Normal From (CNF). 10 pts.

S ⟶ ASA | aB
A ⟶ B | S
B ⟶ b | ε

Step 1: make new start variable

S’ → S
S ⟶ ASA | aB
A ⟶ B | S
B ⟶ b | ε

Step 2: remove epsilon, B→ ε

S’ → S
S ⟶ ASA | aB | a
A ⟶ B | S | ε
B ⟶ b

Step 3: remove epsilon, A → ε

S’ → S
S ⟶ ASA | aB | a | SA | AS | S
A ⟶ B | S
B ⟶ b

Step 4: remove unit rule, S’ → S

S’ → ASA | aB | a | SA | AS
S ⟶ ASA | aB | a | SA | AS
A ⟶ B | S
B ⟶ b

Step 5: remove unit rule, A → B, A→ S

S’ → ASA | aB | a | SA | AS
S ⟶ ASA | aB | a | SA | AS

A ⟶ b | ASA | aB | a | SA | AS
B ⟶ b

Step 6: change it to proper form

S’ → AC | DB | a | SA | AS
S ⟶ AC | DB | a | SA | AS
A ⟶ b | AC | DB | a | SA | AS
B ⟶ b
C → SA
D → a

5. For each of the following languages, determine whether it is context free or not, if it is, give a context free grammar for it; if not, prove it. [10 pts each]

a. $L = \{\omega \in \{0,\#\}^{\star} | \omega \text{ has } 0^n\#0^{2n}\#0^{3n} \text{ pattern, where } n \geq 0\}$

Assume L is context free. There exist a pumping constant p for L

$|vx| \geq 1$

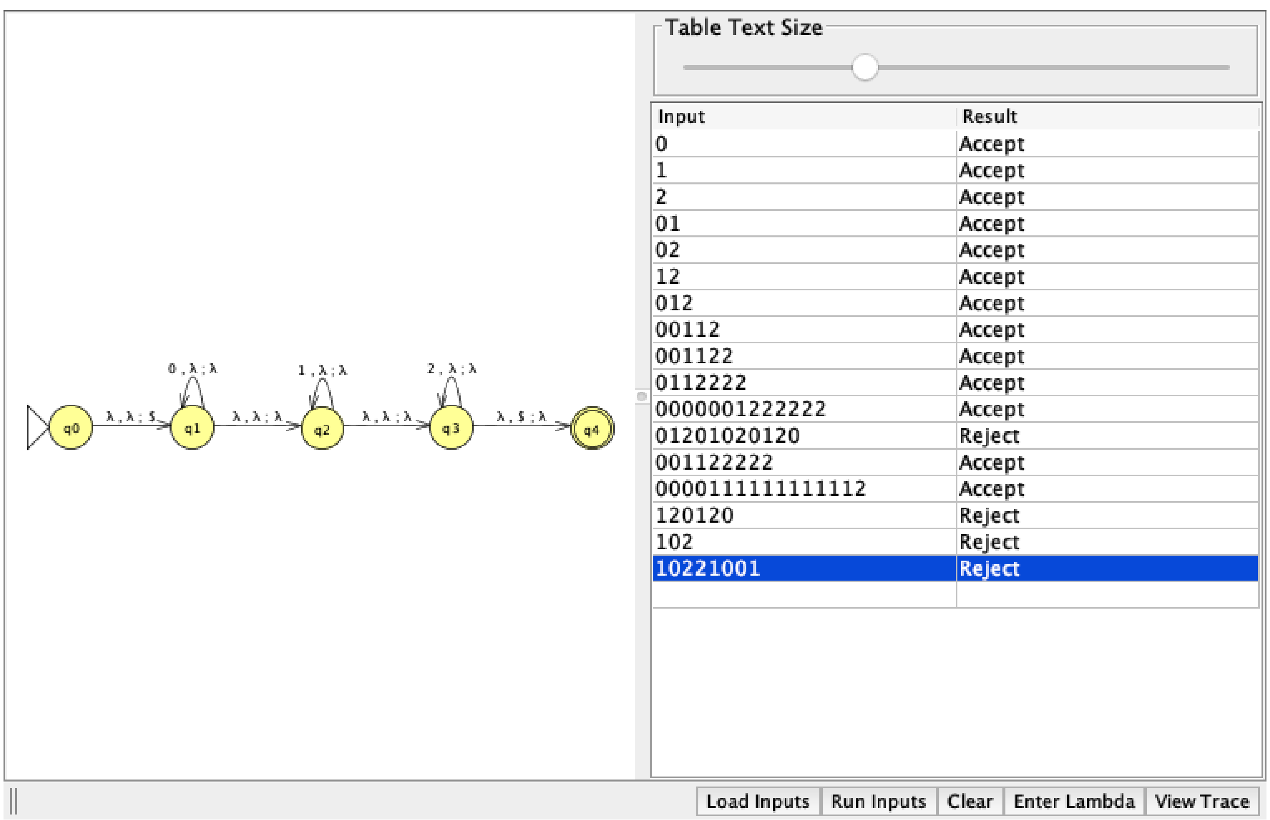
$|vwx| \leq p$

for all $i \geq 0$: $uv^ixy^iz \in L$

Lets pick $w = 0^p\#0^{2p}\#0^{3p}$

Let vxy contains only the part of 0^{2p}. When we pump v and y up we only increase number of 0s in 0^{2p}. The resulting string will be in the form of $0^p\#0^{2p+2b}\#0^{3p}$. This cannot be in L since 2p+2b cannot be 2n. We get a contradiction.

b. $L = \{\omega \in \{0, 1, 2\}^{\star} | \omega \text{ has } 0^i1^j2^k \text{ pattern, where } i,j, k \geq 0\}$



Input	Result
0	Accept
1	Accept
2	Accept
01	Accept
02	Accept
12	Accept
012	Accept
00112	Accept
001122	Accept
0112222	Accept
0000001222222	Accept
01201020120	Reject
001122222	Accept
0000111111111112	Accept
120120	Reject
102	Reject
10221001	Reject