CSE 355: Intro. to Theoretical Computer Science Recitation #5 (20 pts)

Use pumping Lemma to prove that the following languages are not regular [5 pts each].

Conditions :

(1) $xy^iz \in$ A for every i >= 0

(2) |y| > 0

(3) |xy| <= p

1. $L_1 = \{0^n1^n2^n | \ n \geq 0, \Sigma = \{0,1,2\} \}$

Assume L is regular. There exist a pumping constant p for L

x = 0^α

y = 0^β

z = $0^{p-\alpha-\beta}1^p2^p$

Choose one i so that xy^iz is not in L = $\{0^n1^n2^n \mid n \geq 0\}$

i = 2:

$xy^iz = 0^\alpha\ 0^\beta\ 0^\beta\ 0^{p-\alpha-\beta}1^p2^p$

= $xy^iz = 0^{p+\beta}\ 1^p2^p$

In L if and only if $P + \beta = P$. Where $\beta = 0$. Which L is not regular because |y| > 0.

2. $L_2 = \{\omega\omega\omega \mid \omega \in \{a, b\}^*\}$

= $a^pb^pa^pb^pa^pb^p$

Assume L is regular. There exist a pumping constant p for L

x = a^{α}

y = a^{β}

z = $a^{p-\alpha-\beta} b^p a^p b^p a^p b^p$

Choose one i so that xy^iz is not in L $= \{\omega\omega\omega \mid \omega \in \{a, b\}^*\}$

i = 0:

$$xy^iz = a^{\alpha}\ a^{\beta}\ a^{p-\alpha-\beta} b^p a^p b^p a^p b^p$$

= $$xy^iz = a^{p-\beta}\ b^p a^p b^p a^p b^p$$

$In\ L\ if\ and\ only\ if\ P - \beta = P.$ Where $\beta = 0$. Which L is not regular because |y| > 0.

3. $L_3 = \{\omega\omega^R\beta \mid \omega, \beta \in \{0,1\}^+\}$

$= 0^p 1^p 1^p 0^p 1^p$

Assume L is regular. There exist a pumping constant p for L

x = 0^d

y = 0^e

z = $0^{p-d-e} 1^p 1^p 0^p 1^p$

Choose one i so that xy^iz is not in L $= \{\omega\omega^R\beta \mid \omega, \beta \in \{0,1\}^+\}$

i = 2:

$$xy^iz = 0^d\ 0^e\ 0^e\ 0^{p-d-e} 1^p 1^p 0^p 1^p$$

= $$xy^iz = 0^{p+e}\ 1^p 1^p 0^p 1^p$$

$In\ L\ if\ and\ only\ if\ P + e\ =\ P.$ Where $e\ =\ 0$. Which L is not regular because $|y| > 0$.

4. $L_4 = \{1^i 0^j 1^k \mid i > j \ \text{ and } \ i < k \ \text{ and } \ i, j, k > 0\}$

$= 1^{p+1}0^{p}1^{p+2}$

Assume L is regular. There exist a pumping constant p for L

$x = 1^{\alpha}$

$y = 1^{\beta}$

$z = 1^{p-\alpha-\beta+1}0^{p}1^{p+2}$

Choose one i so that xy^iz is not in $L = \{1^i 0^j 1^k \mid i > j \ \text{ and } \ i < k \ \text{ and } \ i, j, k > 0\}$

i = 2:

$$xy^iz = 1^{\alpha}\ 1^{\beta}\ 1^{\beta}\ 1^{p-\alpha-\beta+1}0^{p}1^{p+2}$$

$$= \quad xy^iz = 0^{p+\beta+1}\ 1^{p}1^{p}0^{p}1^{p}$$

$In\ L\ if\ and\ only\ if\ P + \beta\ =\ P.$ Where $\beta\ =\ 0$. Which L is not regular because $|y| > 0$.