CSE 355: Intro to Theoretical Computer Science

HW #2: Regular Expressions and Nonregular Languages

- This assignment is worth 100 points and 5% of your final grade.
- It is due on **Friday, Feb. 26, 2021 at 11:59pm** Arizona time.
- Copying answers from any sources (online, book, last-semester's notes, ...) without proper citation is considered a violation of academic integrity and will be dealt with accordingly. • Answers can be provided in handwritten or typed form (typed form preferred). • For grading purpose, do NOT just submit/write the answers, instead copy each question and put your answer under it. Write legibly, unreadable and unclear answers will be graded with 0 point. • Scan and submit your homework on Canvas **as a single PDF file** (in case you use Apps. taking pictures of your answers, please make sure they are neat and readable)
- **No late submissions will be accepted!** Submission through emails will **NOT** be accepted!

1. Let $\Sigma = \{a,b\}$. As what we did in recitation #4, for each part below, write the formal description of the set described by the regular expression. *20 points*

a) $a^+(bb)^* \cup b^*a(aa)^*$

The above string {w | w is a string that contains 'a' follows by even numbers of b's or b's follows by odd numbers of a's}

b)$(\Sigma^*aba\Sigma^* \cup \Sigma^*bab\Sigma^*) \circ \Sigma^*abb\Sigma^*$

The above string {w | w is a string that contains a substring 'aba' and follows by a substring 'abb', or a substring 'bab' and follows by a substring 'abb'.}

c) $a^* \cup (a^*ba^*ba^*ba^*)^*$

The above string {w | w is a string that contains numbers of a's, or contains at least 3 b's

d) $ba\Sigma^*a\Sigma^*\Sigma^*$

The above string {w | w is a string that contains 'ba' and follows by a substring that contains at least one 'a'.

2. Let $\Sigma = \{0,1\}$. For each part below, write the **regular expression** that describes the set.

15 points

a) $L_1 =$ { w | w is of even length and |w| > 0}

$(0 \mid 1)(0 \mid 1)^+$

b) L_2 = {w|w has an even number of 0s and an odd number of 1s}

$(00 \mid 11 \mid (01 \mid 10)\ (00 \mid 11)^*\ (01 \mid 10)\ )^*(0 \mid (01 \mid 10)\ (11 \mid 00)^*0)$

c) L_3 ={w | w is any string except 0, 01, 11, 1100, 1101, 1110, 1111}

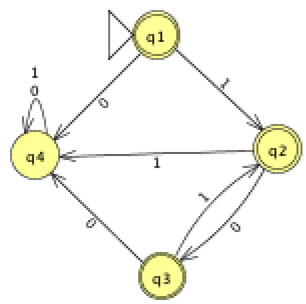
$\varepsilon \mid 1 \mid 00 \mid 10 \mid (0 \mid 1)(0 \mid 1)(0 \mid 1) \mid 10(0 \mid 1)(0 \mid 1) \mid (0 \mid 1)(0 \mid 1)(0 \mid 1)(0 \mid 1)(0 \mid 1)^+$

3. Use the GNFA method discussed in class to convert the following DFA into an equivalent regular expression. Note: 1) eliminate states in order of q_1, q_2, q_3,…, 2) you must show all steps to receive full credit. *15 points*

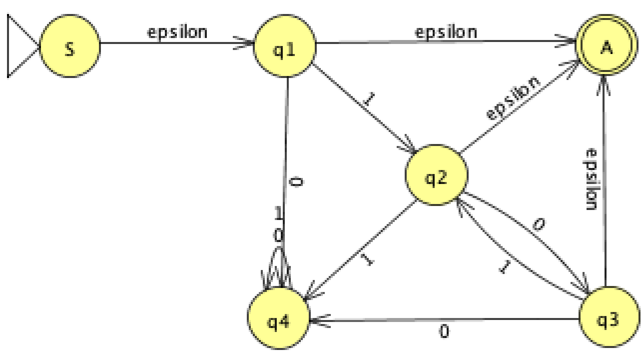
Let DFA $M = (\{q_1, q_2, q_3, q_4\}, \{0,1\}, \delta, q_1\ \{q_1, q_2, q_3\})$ where

δ	0	1
q_1	q_4	q_2
q_2	q_3	q_4
q_3	q_4	q_2
q_4	q_4	q_4

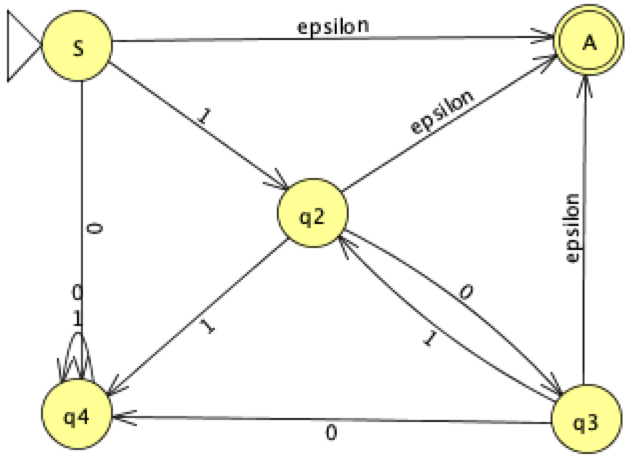
Original DFA:



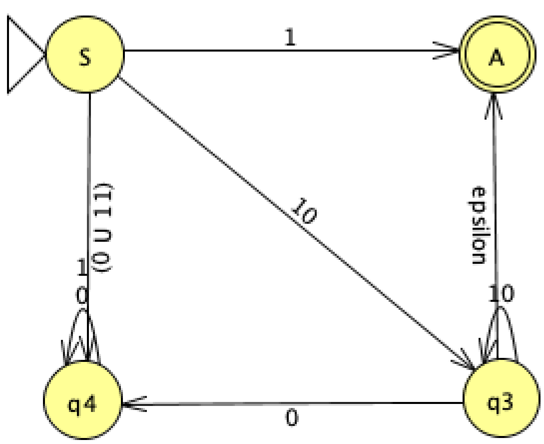
Add start and final state:



RIP q1:

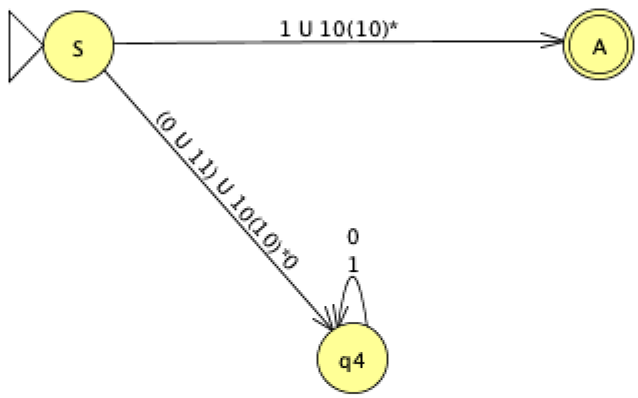

S
A
q2
q4
q3
epsilon
1
epsilon
0
0
1
1
0
1
epsilon
0

Rip q2:

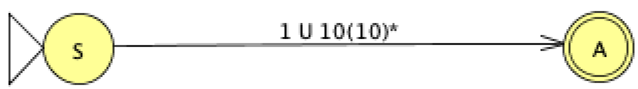

S
A
q4
q3
1
10
(0 U 11)
1
0
epsilon
10
0

Rip q3:

0:



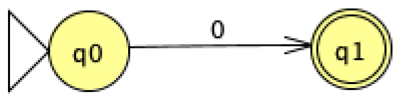
Rip q4:



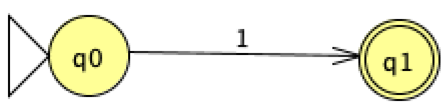
4. Let $\Sigma = \{0,1\}$. Use the procedure given in Lemma 1.55 to convert the following regular expression into an NFA. You must show all steps to receive full credit. *10 points*

$$(01)^{+}10(\varepsilon \cup \Sigma)$$

0:



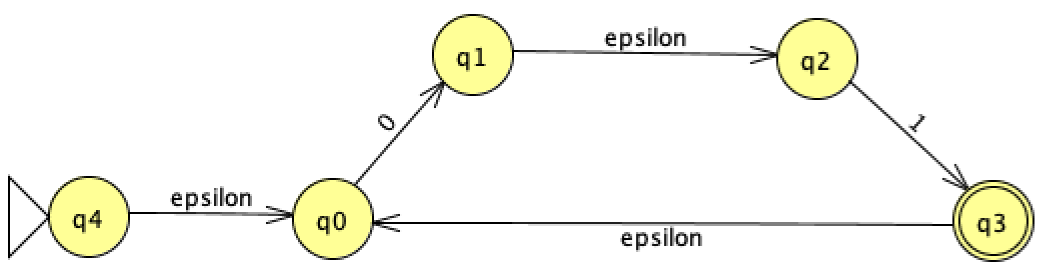
1:



01:

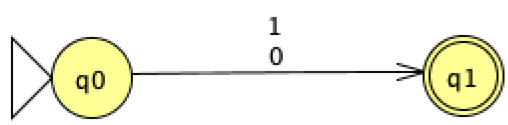

q0
0
q1
epsilon
q2
1
q3

$(01)^+$:

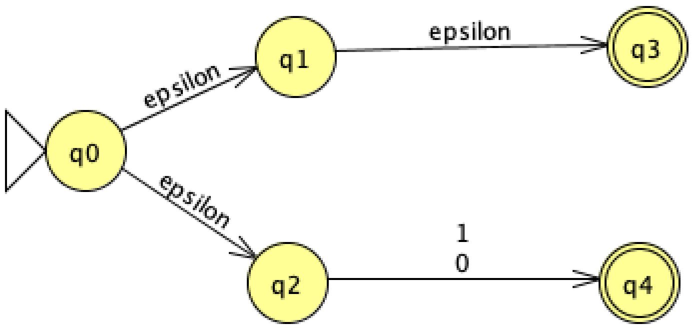

q1
epsilon
q2
0
1
q4
epsilon
q0
q3
epsilon

ε:

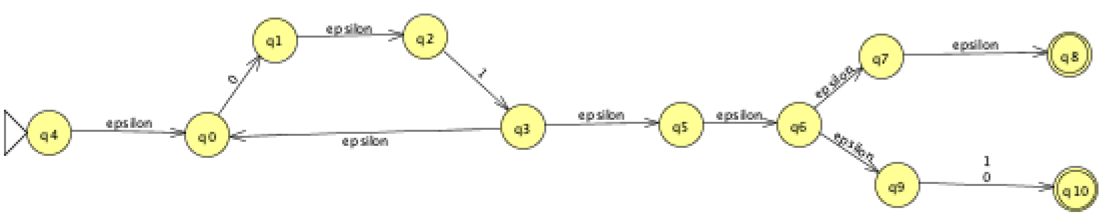
Σ:



(ε ∪ Σ):



$(01)^{+}10(\varepsilon \cup \Sigma)$:



5. For each part, show that the language is regular, or use the pumping lemma to show that the language is nonregular. You must justify your answers to receive full credit. *40 points, 8 pts each*

a) $L_4 = \{0^i 1^j \mid j \leq i \text{ and } i, j > 0\}$

b) $L_5 = \{a^i b^j c^k \mid i < j + k \text{ and } i, j, k > 0\}$

c) $L_6 = \{xy \mid |x| = |y| \text{ and } x, y \in \{0,1\}^*\}$

d) $L_7 = \{xy \mid |x| \geq 2|y| \text{ and } x, y \in \{0,1\}^*\}$

e) $L_9 = \{0^i 1^j \mid 0 \leq i, j \leq 3\}$

Conditions :

(1) $xy^iz \in$ A for every i >= 0

(2) |y| > 0

(3) |xy| <= p

a)

Assume L is regular. There exist a pumping constant p for L4

x = 0^{α}

y = 0^{β}

z = $0^{p-\alpha-\beta}1^{p}$

Choose one i so that xy^iz is not in L = $\{ 0^i 1^j \mid j \leq i \text{ and } i,\ j > 0 \}$

i = 0:

$xy^iz = 0^{\alpha}\ 0^{p-\alpha-\beta}1^{p}$

= $xy^iz = 0^{p-\beta}\ 1^{p}$

In L if and only if $P - \beta = P$. Where $\beta = 0$. Which L is not regular because |y| > 0.

b)

Assume L is regular. There exist a pumping constant p for L5

x = a^{α}

y = a^{β}

z = $a^{p-\alpha-\beta} b^{j} c^{k}$

Choose one i so that $xy^{i}z$ is not in L = $\{ a^{i} b^{j} c^{k} \mid i < j + k \text{ and } i,\ j,\ k > 0 \}$

i = 3 , j = 1, k = 1:

$$xy^{i}z = a^{\alpha}\ a^{\beta}\ a^{\beta} a^{\beta}\ a^{p-\alpha-\beta} b^{j} c^{k}$$

= $$xy^{i}z = a^{p+2\beta}\ b^{j} c^{k}$$

In L if and only if $P + 2\beta = P$. Where $\beta \neq 0$. If $\beta = 1$ then

$2 < 1 + 1 = 2 < 2$ which contradict i < j + k

c)

Assume L is regular. There exist a pumping constant p for L6

x = 0^{α}

y = 0^{β}

z = $0^{p-\alpha-\beta} 1^{p}$

Choose one i so that $xy^{i}z$ is not in L = $\{ xy \mid |x| = |y| \text{ and } x,\ y\ \varepsilon\ \{1,\ 0\}* \}$

i = 2:

$$xy^{i}z = 0^{\alpha} 0^{\beta} 0^{\beta}\ 0^{p-\alpha-\beta} 1^{p}$$

= $$xy^{i}z = 0^{p+\beta}\ 1^{p}$$

In L if and only if $P + \beta = P$. *where* $\beta = 0$. *which it contradict* $|y| > 0$

d)

$0^p 1^p 1^p$

Assume L is regular. There exist a pumping constant p for L7

x = 0^α

y = 0^β

z = $0^{p-\alpha-\beta} 1^p\ 1^p$

Choose one i so that $xy^i z$ is not in L = $\{\ xy\ |\ |x|\ \geq 2|y|\ \ and\ x,\ y\ \varepsilon\ \{1,\ 0\}*\ \ \}$

i = 1:

$xy^i z\ =\ 0^\alpha\ 0^\beta\ 0^{p-\alpha-\beta} 1^p 1^p$

= $xy^i z\ =\ 0^p\ 1^{2p}$

$In\ L\ if\ and\ only\ if\ P\ \ =>\ \ 2P.$ Where it is not possible so it contradicts itself.

e)

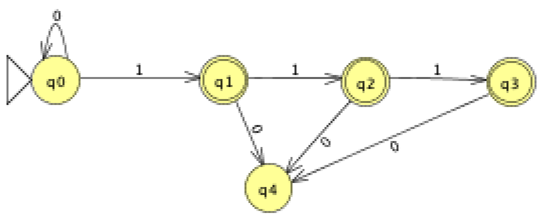


Table Text Size

Input	Result
0	Reject
00	Reject
000	Reject
0001	Accept
00011	Accept
000111	Accept
111	Accept
1	Accept
11	Accept
1111	Reject
11111	Reject
1110	Reject
1100	Reject
01010	Reject

Load Inputs | Run Inputs | Clear | Enter Lambda | View Trace