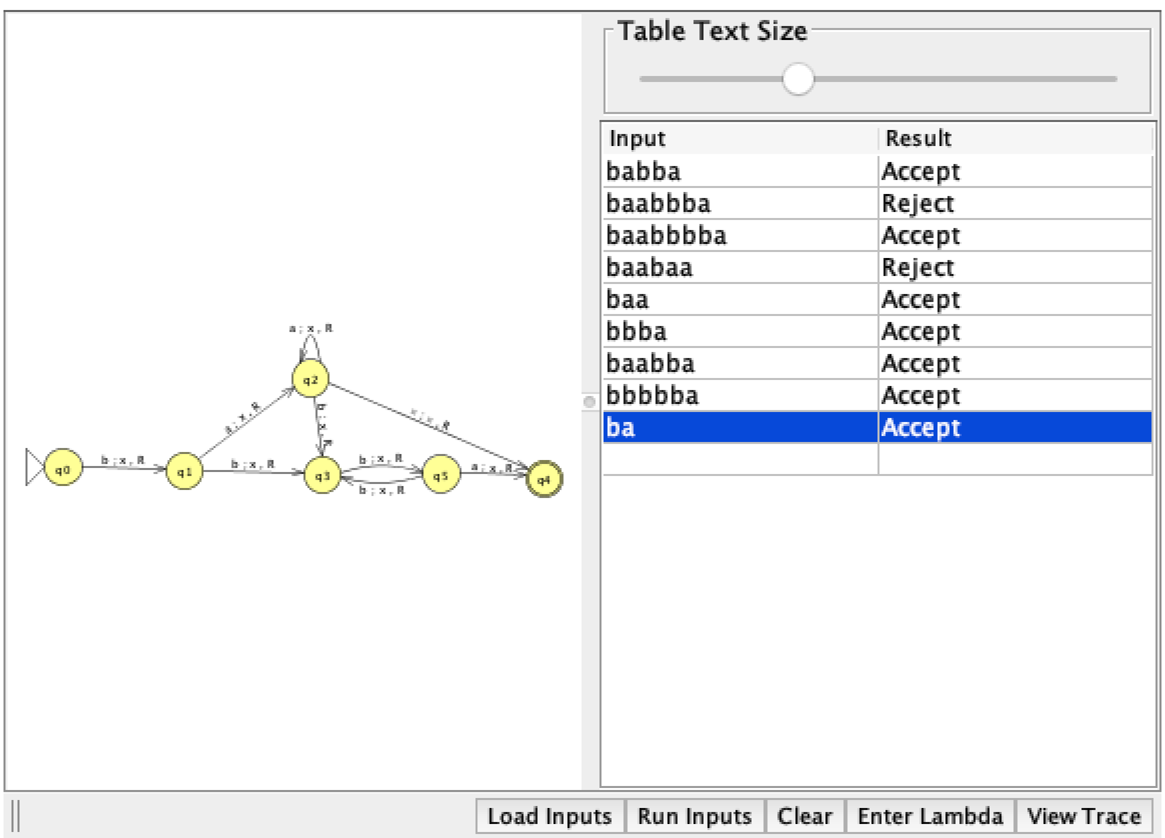
CSE 355: Intro. to Theoretical Computer Science

HW #5: TM Part 2 and Decidability

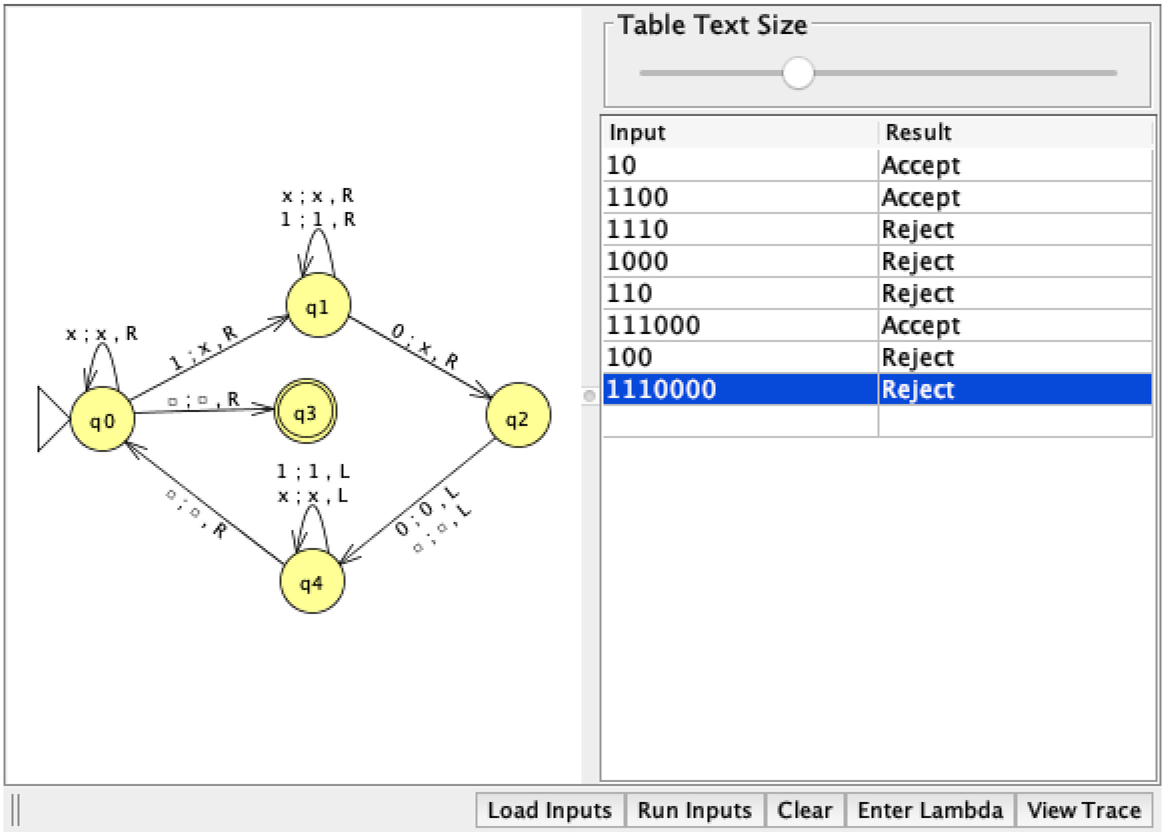
- This assignment is worth 100 points and 5% of your final grade.
- It is due on **Friday, Apr. 9, 2021 at 11:59pm** Arizona time.
- Copying answers from any sources (online, book, last-semester's notes, ...) without proper citation is considered a violation of academic integrity and will be dealt with accordingly. • Answers can be provided in handwritten or typed form.
- Unreadable and unclear answers will be graded with 0 point.
- Scan and submit your homework on Canvas **as a single PDF file** (in case you use Apps. taking pictures of your answers, please make sure they are neat and readable)
- **No late submissions will be accepted!** Submission through emails will **NOT** be accepted!

1. For each part below, use JFLAP to draw the state diagram of an ordinary Turing machine (deterministic, and not any of the variants discussed in class) that recognizes the given language (assume all missing transitions go to an implicit rejecting state; you do not have to draw these transitions, unless specified otherwise). *30 points*

a. L1 described by the regular expression $ba^*(bb)^*a$

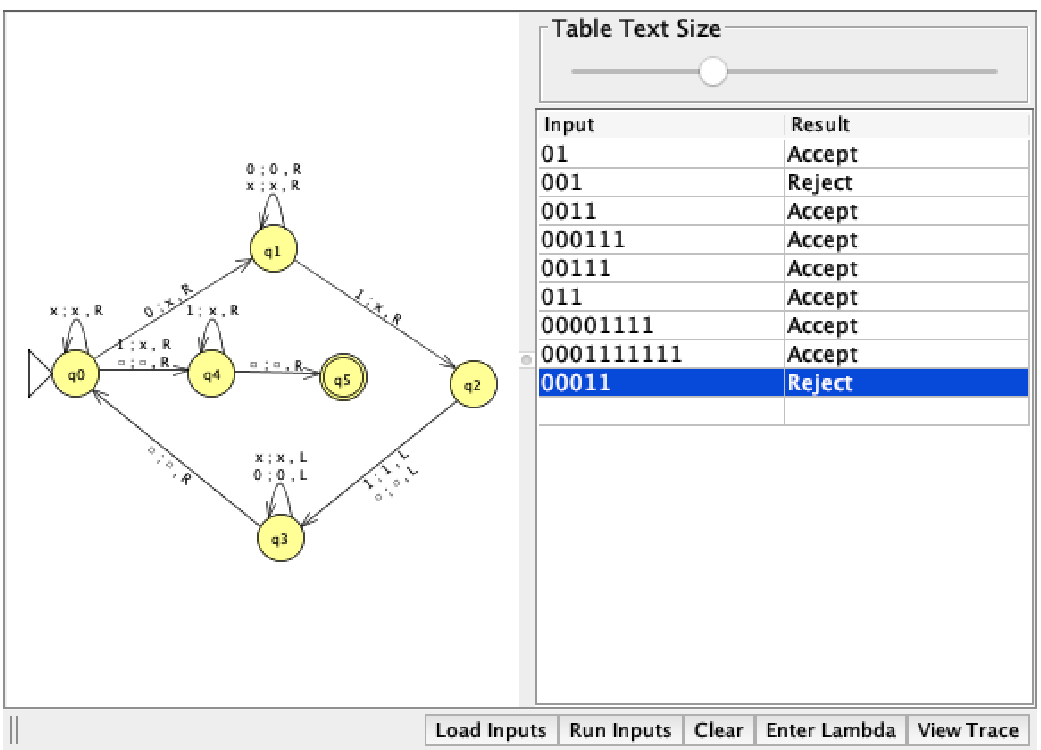


b. L2 = $\{0^n1^n \mid n \geq 0\}$



Input	Result
10	Accept
1100	Accept
1110	Reject
1000	Reject
110	Reject
111000	Accept
100	Reject
1110000	Reject

c. L3 = $\{0^i1^j \mid i \geq 0 \text{ and } j \geq i\}$



Input	Result
01	Accept
001	Reject
0011	Accept
000111	Accept
00111	Accept
011	Accept
00001111	Accept
0001111111	Accept
00011	Reject

2. Prove that ***Turing-decidable*** languages are closed under the following operations (*Hint: prove by constructing a new TM). *20 points*

a. Complementation

Let us consider M to be a Turing machine define on language L. Now, we construct a TM M' that decides L.

On input w:

1. Run M on w
2. Accept it if M rejects, reject if M accepts.

b. Intersection

Let L1 and L2 be decidable languages and M1 and M2 be the TM that decide them. We construct a TM M'.

On input w:

1. Run M1 on w, if reject then reject
2. Run M2 on w, if reject then reject
3. Accepts

Because M1 and M2 are both decidable languages. They will be guaranteed to halt. If one of the machines does not accept, then it will reject. Otherwise, if both of the machines accept, then the intersection of the two languages is decidable.

3. Prove that ***Turing-recognizable*** languages are closed under the following operations ((*Hint: prove by constructing a new TM). *20 points*

a. Union

Let L1 and L2 be recognizable language and M1 and M2 be the TM that recognize them. We construct a TM M'.

On input w:

1. Run M1 and M2 on w in parallel. Thus, M1 executes one step and M2 executes one step.
2. If either M1 or M2 accepts, accepts. If both reject, reject.

b. Concatenation

Let L1 and L2 be recognizable language and M1 and M2 be the TM that recognize them. We construct a TM M'.

On input w:

1. Divide w into two parts(non-deterministically) w1 and w2. W1 will be from the empty to w. The rest of it will be w2
2. Run M1 on w1. If accepts, then proceed to step 3, otherwise, reject.
3. Run M2 on w2, if accepts, accepts, otherwise, reject.

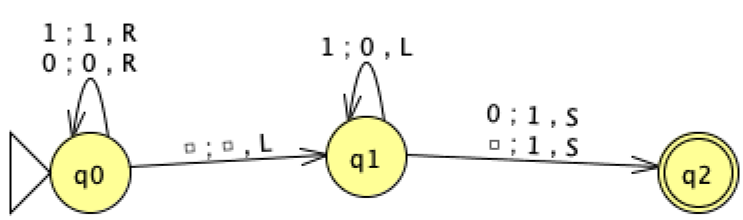
4. Consider the problem of determining whether a DFA accepts strings containing an equal number of 0s and 1s. Convert this problem into a language, and then prove that it is decidable. *10 points*

$$S \rightarrow 0S1 \mid 1S0 \mid \varepsilon$$

The above language is Context-free grammar. Which a DFA can not accept language that is generated by a CFG. So DFA does not accept the language that has an equal number of 0s and 1s. So it is decidable.

5. Use [JFLAP] to draw the state diagram of a Turing Machine that increments a binary number by 1. Also explain in English how your machine works. *20 points*

A) [15 pts] **Hints**: *for example, if the input string is 11 (binary number 3), then the output should be 100 (binary number 4). While we manually add two numbers, we start adding from the unit digit. For this task, consider an infinite tape in both directions for this TM.*



B) [5 pts] Show the computation (sequence of configurations) of above Turing Machine for input string 111.

U = blank

$$q_0111 \rightarrow 1q_011 \rightarrow 11q_01 \rightarrow 111q_0U \rightarrow 11q_11 \rightarrow 1q_110 \rightarrow Uq_1100 \rightarrow q_1U000 \rightarrow q_21000$$