

R is also for Filipino Researchers

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What is R? (R Foundation 2017)

"R is a language and environment for statistical computing and graphics. It is a GNU project which is similar to the S language and environment which was developed at Bell Laboratories (formerly AT&T, now Lucent Technologies) by John Chambers and colleagues. R can be considered as a different implementation of S. There are some important differences, but much code written for S runs unaltered under R.

R provides a wide variety of statistical (linear and nonlinear modelling, classical statistical tests, time-series analysis, classification, clustering, ...) and graphical techniques, and is highly extensible. The S language is often the vehicle of choice for research in statistical methodology, and R provides an Open Source route to participation in that activity."

(Some) Advantages of using R (from an R fanboy)

- R is the most comprehensive statistical analysis software available.
- R is a programming language developed by research and practicing statisticians for statisticians.
- The graphical capabilities and options in R far surpasses the available graphical capabilities in other statistical packages.
- R is free and open source licensed to The R Foundation for Statistical Computing under the GNU General Public License
- R has a large community of users, developers, and bug-fixers. You can contribute to the development of R, too, by becoming an active member of the community.
- R has over 10,000 packages available in CRAN and more available in bioconductor and Github repositories.
- There are a lot of free books, websites, and coursewares available for learning R.

(Some) Disadvantages of Using R

- R has a steep learning curve (?)
- Documentation is sometimes lacking
- The quality of some packages is sometimes questionable
- There is in general no one to complain to when something goes wrong
- R's memory management sucks (?)

Who Uses R? (Bhalla 2017)

- Facebook - For behavior analysis related to status updates and profile pictures.
- Google - For advertising effectiveness and economic forecasting.
- Twitter - For data visualization and semantic clustering
- Microsoft - Acquired Revolution R company and use it for a variety of purposes.
- Uber - For statistical analysis
- Airbnb - Scale data science.
- IBM - Joined R Consortium Group
- ANZ - For credit risk modeling

Why use R?

- It is free (and open source)!
- R is the most popular tool for analytics/data science (Piatetsky 2016).
- Ranked 5th in most popular software based on number of job offerings: SQL, Python, Java, Hadoop, R, C/C++/C#, SAS, Apache Spark, Tableau, Apache Hive (Muenchen 2017)
- R has surpassed SAS in scholarly use—but still way behind SPSS (Muenchen 2016)

Is "R also for Filipino Researchers"?

Yes.

I have no experience with coding so R frightens me.

- You use Microsoft Excel, right?

Why use Rstudio with R?

The screenshot displays the RStudio IDE interface. The top menu bar includes File, Edit, Code, View, Plots, Session, Build, Debug, Profile, Tools, and Help. The main editor window shows a script file named 'R4Filipino.Rmd' with R code and comments. The code includes a comparison of three variables, a comment about R for Filipino researchers, and a plot command. The Environment pane on the right shows the Global Environment with variables like 'nod4', 'nod5', 'nod6', 'nod7', 'nod8', 'mtcars_mpg', 'mtcars2', and 'mtcars3'. The Console pane at the bottom shows the output of the R code, including coefficients, residual standard error, multiple R-squared, adjusted R-squared, F-statistic, and a plot command.

```
## comparison of three
54
55
56
57 ## Is "R also for Filipino Researchers"?
58
59 Yes.
60
61 ## I have no experience with coding so R frightens me.
62
63
64
65 ## Why use Rstudio with R?
66
67
68 ## Script file and Rmarkdown
69
70 ## Why use Rstudio with R?
71
```

Environment

Variable	Value
nod4	List of 13
nod5	List of 12
nod6	List of 12
nod7	List of 12
nod8	List of 12
mtcars_mpg	2 obs. of 4 variables
mtcars2	32 obs. of 13 variables
mtcars3	32 obs. of 3 variables

Console

```
~R_workshop_urdaneta/
R> plot(mpg~log(displacement), data=mtcars)
R>
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	70.07	3.82	18.4	< 2e-16
log(displacement)	-9.55	0.72	-13.3	2.4e-13

Residual standard error: 2.14 on 27 degrees of freedom
Multiple R-squared: 0.867, Adjusted R-squared: 0.862
F-statistic: 176 on 1 and 27 DF, p-value: 2.41e-13

R> plot(mpg~log(displacement), data=mtcars)

R>

Modify components of a theme

Description

Use theme() to modify individual components of a theme, allowing you to control the appearance of all non-data components of the plot. theme() only affects a single plot: see theme_update() if you want modify the active theme, to affect all subsequent plots.

Usage

```
theme(line, rect, text, title, aspect.ratio, axis.title, axis.title.x,
axis.title.x.top, axis.title.x.bottom, axis.title.y, axis.title.y.left,
axis.title.y.right, axis.text, axis.text.x, axis.text.x.top,
axis.text.x.bottom, axis.text.y, axis.text.y.left, axis.text.y.right,
axis.ticks, axis.ticks.x, axis.ticks.x.top, axis.ticks.x.bottom, axis.ticks
```

Let's load the required packages first

```
library(tidyverse)  
library(agricolae)
```

Introducing the mtcars data set

```
?mtcars  
write.csv(mtcars, "mtcars.csv")
```

Using different ways to load data set into R

- Using the R console

```
mt1 <- read.table("mtcars.csv", sep = ",", header=TRUE)
mtcars2 <- read.csv("mtcars.csv")
mt3 <- read_csv("mtcars.csv")
mt4 <- data.table::fread("mtcars.csv")
```

- From Rstudio, click **File > Import Dataset**.

Linear Model for comparing the means of two groups

Means Model

$$y_{ij} = \mu_i + \varepsilon_{ij}, i = 1, 2$$

- $H_0 : \mu_1 = \mu_2$
- $H_a : \mu_1 \neq \mu_2$

Linear Model for comparing the means of two groups

Effects Model

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij}, i = 1, 2$$

where:

- y_{ij} = is the j th value of **mpg** in the i th group of **am**
- μ = mean of Y
- α_i = effect of the i th group of **am** on **mpg**
- ε = the random error due to the j th value of **mpg** in the i th value of **am**

Hypotheses

- $H_0 : \alpha_1 = \alpha_2$
- $H_a : \alpha_1 \neq \alpha_2$

A research question: Is there a difference in mileage for automatic and manual cars?

- $H_a : \alpha_1 = \alpha_2 = 0$ (or $\mu_1 = \mu_2$) The mileage per gallon differ based on transmission type of the car.
- $H_0 : \alpha_1 \neq \alpha_2$ (or $\mu_1 \neq \mu_2$) The mileage per gallon do not differ based on transmission type of the car.

A research question: Is there a difference in mileage for automatic and manual cars?

```
table(mtcars$am)
```

```
0 1  
19 13
```

```
aggregate(mtcars$mpg, by = list(mtcars$am), FUN="mean")
```

```
  Group.1      x  
1      0 17.15  
2      1 24.39
```

```
aggregate(mtcars$mpg, by = list(mtcars$am), FUN="var")
```

```
  Group.1      x  
1      0 14.70  
2      1 38.03
```

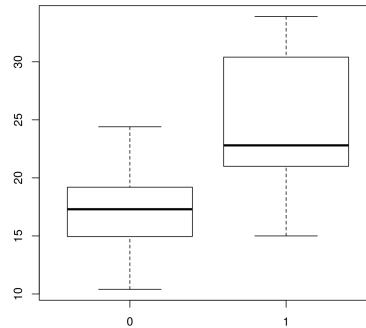

A research question: Is there a difference in mileage for automatic and manual cars?

```
mtcars_mpg <- mtcars %>%  
  group_by(am) %>%  
  summarise(  
    mean_mpg = mean(mpg),  
    var_mpg = var(mpg),  
    n = n()  
  )  
mtcars_mpg
```

```
# A tibble: 2 x 4  
  am mean_mpg var_mpg    n  
<dbl>   <dbl>   <dbl> <int>  
1     0    17.15    14.70    19  
2     1    24.39    38.03    13
```

Continuation of Exploratory Data Analysis

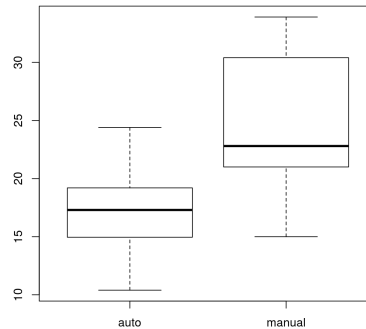
```
boxplot(mpg~am, data = mtcars)
```



Changing the labels of a plot; creating a new variable in a data set (**data.frame**)

Let us put some labels for the levels of **am**.

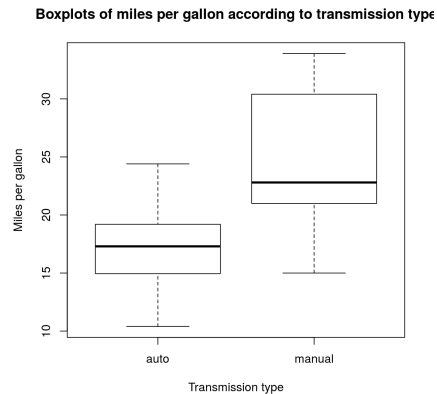
```
mtcars2$amf <- factor(mtcars2$am, levels = c(0,1), labels = c("auto", "manual"))  
boxplot(mpg~amf, data=mtcars2)
```



Changing the x and y labels and putting a title

Let us put some labels for the levels of `am`.

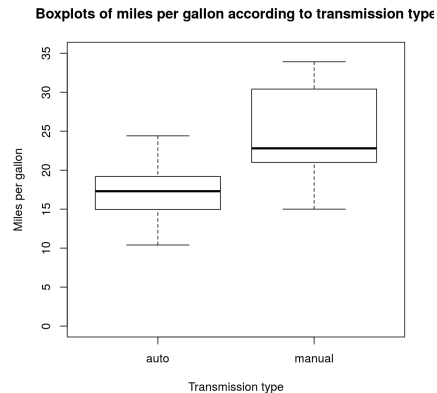
```
boxplot(mpg~am, data=mtcars2,  
        main = "Boxplots of miles per gallon according to transmission type",  
        xlab = "Transmission type",  
        ylab = "Miles per gallon")
```



Changing the range of values in the y -axis

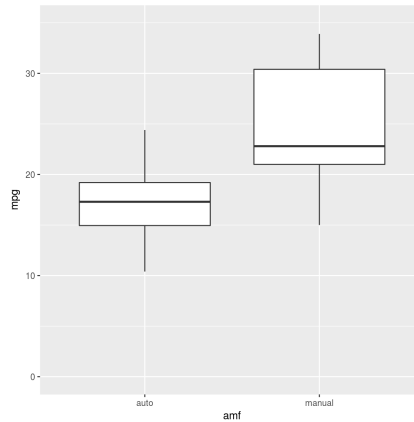
Let us start the boxplot at 0.

```
boxplot(mpg~amf, data=mtcars2,  
        main = "Boxplots of miles per gallon according to transmission type",  
        xlab = "Transmission type",  
        ylab = "Miles per gallon",  
        ylim = c(0, 35))
```



Plotting with ggplot2

```
ggplot(mtcars2, aes(x = amf, y = mpg)) + geom_boxplot() + ylim(0,35)
```

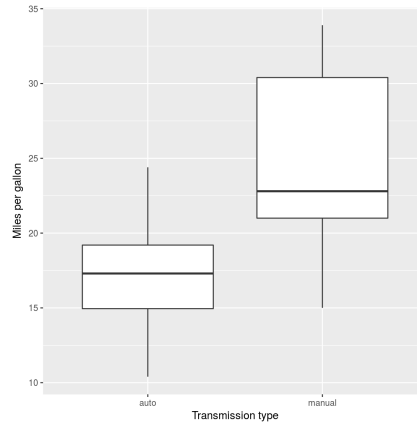


What is the difference?

```
ggplot(mtcars2, aes(x = amf, y = mpg)) + geom_boxplot() + coord_cartesian(ylim = c(0,35))
```

ggplot2 uses the language of graphics

```
p <- ggplot(mtcars2, aes(x = amf, y = mpg)) +  
  geom_boxplot() +  
  xlab("Transmission type") +  
  ylab("Miles per gallon")  
p
```



A review of t test

What are the assumptions of the independent samples t test?

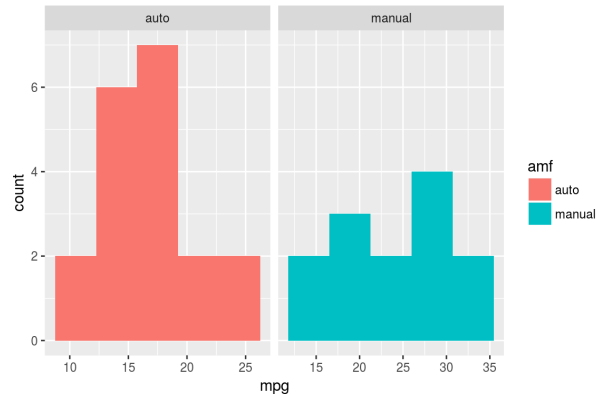
1. Dependent variable should be measured on a continuous scale (interval or ratio level)
2. Independent variable consist of two categorical, independet groups
3. Observations are independent of each other
4. No significant outliers
5. Dependent variable should be (approximately) normally distributed for each group of the independent variable
6. Variances should be homogenous

Applying the assumptions

1. What is the independent variable? What is the independent variable?
2. Is the dependent variable measured on a continuous scale?
3. Does the independent variable consist of two categorical, independent groups?
4. Are observations independent of each other?
5. Are there no significant outliers?
6. Is the dependent variable normally distributed for each group of the independent variable?
7. Are the variances homogenous?

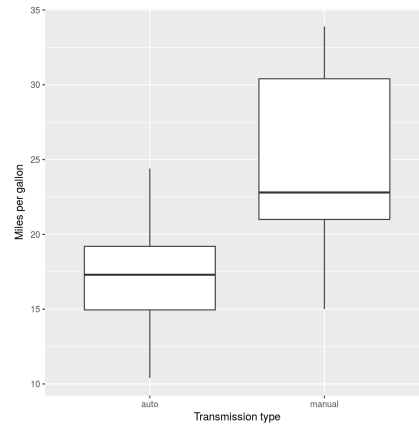
Normality for each group of the independent variable

```
ggplot(mtcars2, aes(x = mpg, fill = amf)) + geom_histogram(bins = 5) + facet_wrap(~amf, scales = "free_x")
```



Homogeneity of variance

p



Homogeneity of variance (cont...)

```
var.test(mpg~amf, data=mtcars2)
```

F test to compare two variances

data: mpg by amf

F = 0.39, num df = 18, denom df = 12, p-value = 0.07

alternative hypothesis: true ratio of variances is not equal to 1

95 percent confidence interval:

0.1244 1.0703

sample estimates:

ratio of variances

0.3866

t test results

```
(t1 <- t.test(mpg ~ amf, data = mtcars2))
```

Welch Two Sample t-test

data: mpg by amf

t = -3.8, df = 18, p-value = 0.001

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

-11.28 -3.21

sample estimates:

mean in group auto	mean in group manual
17.15	24.39

How to use `t.test`?

?`t.test`

How about independent samples t test?

```
(t2 <- t.test(mpg~amf, data=mtcars2, var.equal=TRUE))
```

Two Sample t-test

data: mpg by amf

t = -4.1, df = 30, p-value = 3e-04

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

-10.848 -3.642

sample estimates:

mean in group auto mean in group manual

17.15

24.39

Non-parametric alternative

```
(w <- wilcox.test(mpg ~ amf, data = mtcars2, conf.int = TRUE))
```

```
Warning in wilcox.test.default(x = c(21.4, 18.7, 18.1, 14.3, 24.4, 22.8, :  
cannot compute exact p-value with ties
```

```
Warning in wilcox.test.default(x = c(21.4, 18.7, 18.1, 14.3, 24.4, 22.8, :  
cannot compute exact confidence intervals with ties
```

Wilcoxon rank sum test with continuity correction

```
data: mpg by amf  
W = 42, p-value = 0.002  
alternative hypothesis: true location shift is not equal to 0  
95 percent confidence interval:  
-11.7 -2.9  
sample estimates:  
difference in location  
-6.8
```


Confidence intervals

```
t1$conf.int
```

```
[1] -11.28 -3.21  
attr(,"conf.level")  
[1] 0.95
```

```
t2$conf.int
```

```
[1] -10.848 -3.642  
attr(,"conf.level")  
[1] 0.95
```

```
w$conf.int
```

```
[1] -11.7 -2.9  
attr(,"conf.level")  
[1] 0.95
```

Determining other values from the tests

```
names(t1)
```

```
[1] "statistic"  "parameter"  "p.value"    "conf.int"   "estimate"  
[6] "null.value" "alternative" "method"     "data.name"
```

Conclusion of comparison of milleage according to transmission type

The milleage per gallon differ between manual and automatic tramission-type vehicles by about 7.24 miles per gallon at 0.05 significance level (or 95% confidence level).

When to use one-tailed t test

```
?t.test
```

```
(t3 <- t.test(mpg ~ amf, data = mtcars2, alternative = "less"))
```

Welch Two Sample t-test

```
data: mpg by amf
t = -3.8, df = 18, p-value = 7e-04
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
 -Inf -3.913
sample estimates:
 mean in group auto mean in group manual
           17.15           24.39
```

```
t3 %>% broom::tidy()
```

	estimate	estimate1	estimate2	statistic	p.value	parameter	conf.low
1	-7.245	17.15	24.39	-3.767	0.0006868	18.33	-Inf
	conf.high			method	alternative		
1	-3.913			Welch Two Sample t-test	less		

Comparison of the means of three groups

```
table(mtcars2$cyl)
```

```
 4  6  8  
11  7 14
```

```
mtcars3 <- mtcars2 %>% select(mpg, cyl)  
head(mtcars3)
```

```
   mpg  cyl  
1 21.0    6  
2 21.0    6  
3 22.8    4  
4 21.4    6  
5 18.7    8  
6 18.1    6
```

Linear Model for the problem of comparing three means

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij}, i = 1, 2, 3$$

where:

- y_{ij} = is the j th value of **mpg** in the i th group of **cy1**
- μ = mean of Y
- α_i = effect of the i th group of **cy1** on **mpg**
- ε = the random error due to the j th value of **mpg** in the i th value of **cy1**

Hypotheses

- $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = 0$ (equivalently: $\mu_1 = \mu_2 = \mu_3$)
- $H_a : \alpha_i \neq 0$ for at least 1 i (equivalently: $\mu_a \neq \mu_b$ for at least one pair a and b)

Exploratory data analysis of mpg in terms of cyl

```
mtcars3 %>% group_by(cyl) %>% summarise(mean = mean(mpg), var = var(mpg), sd = sd(mpg), n = n())
```

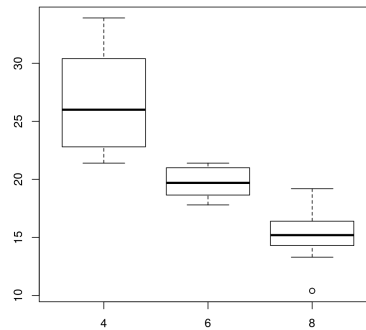
```
# A tibble: 3 x 5  
  cyl mean   var    sd    n  
  <int> <dbl> <dbl> <dbl> <int>  
1     4 26.66 20.339 4.510   11  
2     6 19.74  2.113 1.454    7  
3     8 15.10  6.554 2.560   14
```

Hypotheses

- H_a : There are differences in mean millege per gallon depending on number of the car's cylinders.
- H_o : There are no differences in mean millege per gallong according to the number of the car's cylinders.

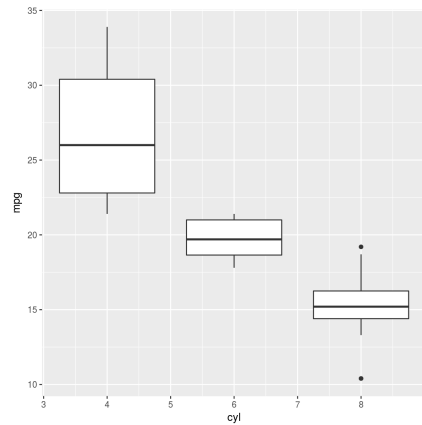
Investigation of Normality and equality of variances

```
boxplot(mpg ~ cyl, data = mtcars3)
```



Investigation of Normality and equality of variances

```
ggplot(mtcars3, aes(x=cyl, y=mpg, group=cyl)) + geom_boxplot()
```



Analysis of variance

```
mtcars3$cylf <- as.factor(mtcars3$cyl)
mod <- aov(mpg~cylf, data=mtcars3)
summary(mod)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
cylf	2	825	412	39.7	5e-09
Residuals	29	301	10		

```
TukeyHSD(mod, "cylf")
```

Tukey multiple comparisons of means
95% family-wise confidence level

```
Fit: aov(formula = mpg ~ cylf, data = mtcars3)
```

```
$cylf
      diff      lwr      upr p adj
6-4  -6.921 -10.769 -3.0722 0.0003
8-4 -11.564 -14.771 -8.3565 0.0000
8-6  -4.643  -8.328 -0.9581 0.0112
```

More on post-hoc analysis

```
with(mtcars3,  
  pairwise.t.test(mpg, cylf, p.adjust.method = "bonferroni")  
)
```

Pairwise comparisons using t tests with pooled SD

data: mpg and cylf

```
      4      6  
6 4e-04 -  
8 3e-09 0.01
```

P value adjustment method: bonferroni

More on post-hoc analysis (cont...)

```
scheffe.test(mod, "cylf", console=TRUE)
```

```
Study: mod ~ "cylf"
```

```
Scheffe Test for mpg
```

```
Mean Square Error : 10.39
```

```
cylf, means
```

	mpg	std	r	Min	Max
4	26.66	4.510	11	21.4	33.9
6	19.74	1.454	7	17.8	21.4
8	15.10	2.560	14	10.4	19.2

```
Alpha: 0.05 ; DF Error: 29
```

```
Critical Value of F: 3.328
```

```
Groups according to probability of means differences and alpha level( 0.05 )
```

```
Means with the same letter are not significantly different.
```

	mpg	groups
4	26.66	a
6	19.74	b
8	15.10	c

Non-parametric Kruskal-Wallis Test

```
kruskal.test(mpg~cyl, data=mtcars)
```

```
Kruskal-Wallis rank sum test
```

```
data: mpg by cyl
```

```
Kruskal-Wallis chi-squared = 26, df = 2, p-value = 3e-06
```

Non-parametric Kruskal-Wallis Test

```
with(mtcars, agricolae::kruskal(mpg, cyl, p.adj="BH", console = TRUE))
```

```
Study: mpg ~ cyl  
Kruskal-Wallis test's  
Ties or no Ties
```

```
Critical Value: 25.75  
Degrees of freedom: 2  
Pvalue Chisq : 2.566e-06
```

```
cyl, means of the ranks
```

```
      mpg  r  
4 26.955 11  
6 17.429  7  
8  7.821 14
```

```
Post Hoc Analysis
```

```
P value adjustment method: BH  
t-Student: 2.045  
Alpha : 0.05
```

```
Groups according to probability of treatment differences and alpha level.
```

```
Treatments with the same letter are not significantly different.
```

```
      mpg groups  
4 26.955      a  
6 17.429      b  
8  7.821      c
```

Randomized Complete Block Design

Suppose we want to know the effect of the number of cylinders to mpg when we group the observations by type of transmission, which we know has an effect on mpg. That is, we want to isolate the effect of cyl on mpg when we group the observations by am.

The linear model is

$$y_{ij} = \mu + \alpha_i + \rho_j + \varepsilon_{ij}$$

where

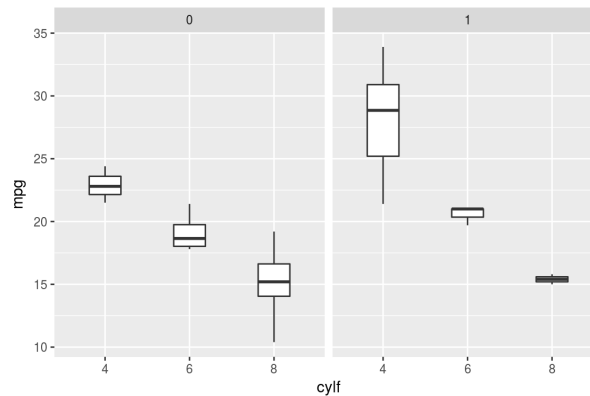
- μ = hypothesized mean
- ρ_j = the effect of the j th blocking factor (am) to mpg
- α_i = effect of the the i th cyl to mpg
- ε_{ij} = the random effect on the ij -th observation

Then the hypotheses are

- $H_0 : \alpha_i = 0$. The number of cylinders has no effect on millege per gallon.
- $H_a : \alpha_i \neq 0$. The number of cylinders affect millege per gallon.

Exploratory Data Analysis for RCBD

```
mtcars4 <- mtcars2 %>% select(mpg, am, cyl)
mtcars4$cylf <- as.factor(mtcars4$cyl)
mtcars4$amf <- as.factor(mtcars4$am)
ggplot(mtcars4, aes(x = cylf, y = mpg)) +
  geom_boxplot() +
  facet_wrap(~amf)
```



RCBD

```
mod2 <- aov(mpg ~ amf + cylf, data = mtcars4)
summary(mod2)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
amf	1	405	405	42.9	4.2e-07
cylf	2	456	228	24.2	8.0e-07
Residuals	28	264	9		

```
TukeyHSD(mod2, "cylf")
```

Tukey multiple comparisons of means
95% family-wise confidence level

```
Fit: aov(formula = mpg ~ amf + cylf, data = mtcars4)
```

```
$cylf
      diff      lwr      upr p adj
6-4 -4.757  -8.434 -1.0798 0.0092
8-4 -7.330 -10.394 -4.2655 0.0000
8-6 -2.573  -6.093  0.9475 0.1853
```

Two-way ANOVA (Two-factor CBD)

What if prior to the experiment, we don't know the effect of any of **am** and **cy1** on **mpg**? We want to see how these factors affect **mpg** and whether they affect **mpg** independently or not.

- Model: $y_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ij}$

where

- μ = hypothesized mean
- α_i = the effect of the i th type of transmission (**am**) to **mpg**
- β_j = effect of the j th number of cylinders (**cy1**) to **mpg**
- γ_{ij} = the interaction effect of the ij -th type of transmission and number of cylinders
- ε_{ij} = the random effect on the ij -th observation

Three pairs of hypotheses for Two-way ANOVA

There are three pairs of hypotheses to be tested:

1. Interaction effects

- $H_0 : \gamma_{ij} = 0$. There is no interaction between **am** and **cyl**.
- $H_a : \gamma_{ij} \neq 0$. There is an interaction between **am** and **cyl**.

1. Effect of **am**

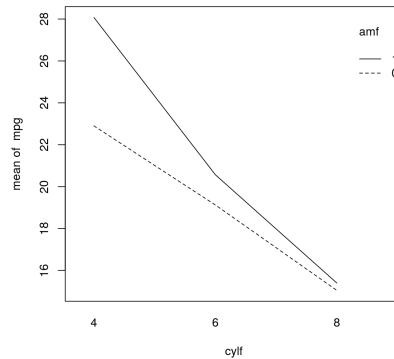
- $H_0 : \alpha_i = 0$. Controlling for other variables, **am** has no effect on **mpg**.
- $H_a : \alpha_i \neq 0$. Controlling for other variables, **am** has an effect on **mpg**.

1. Effect of **cyl**

- $H_0 : \beta_j = 0$. Controlling for other variables, **cyl** has no effect on **mpg**.
- $H_a : \beta_j \neq 0$. Controlling for other variables, **cyl** has an effect on **mpg**.

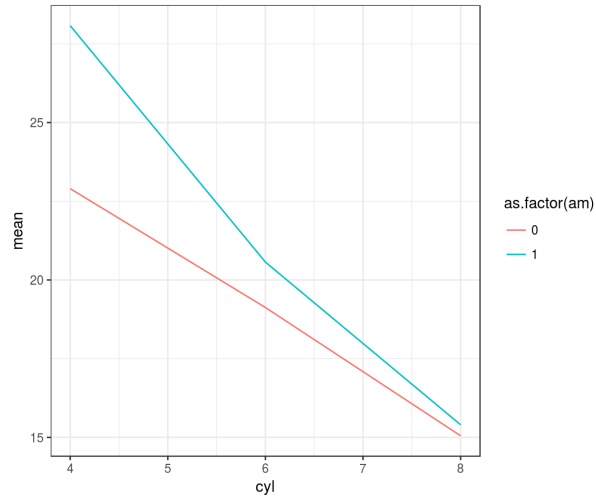
Interaction plot

```
with(mtcars4, interaction.plot(cylf, amf, mpg, fun = mean))
```



Interaction plot with **ggplot2**

Challenge: Create an interaction plot using **ggplot2**.



How to do two-way ANOVA in R

```
mod4 <- aov(mpg ~ amf * cylf, data = mtcars4)
summary(mod4)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
amf	1	405	405	44.06	4.8e-07
cylf	2	456	228	24.82	9.4e-07
amf:cylf	2	25	13	1.38	0.27
Residuals	26	239	9		

Conclusions from two-way ANOVA

1. There is no interaction between `am` and `cyl`.
2. `cyl` affects `mpg`.
3. `am` affects `mpg`.

Post hoc analyses for two-way ANOVA

```
TukeyHSD(mod4, "cylf")
```

```
Tukey multiple comparisons of means  
95% family-wise confidence level
```

```
Fit: aov(formula = mpg ~ amf * cylf, data = mtcars4)
```

```
$cylf
```

	diff	lwr	upr	p adj
6-4	-4.757	-8.400	-1.1137	0.0088
8-4	-7.330	-10.365	-4.2937	0.0000
8-6	-2.573	-6.061	0.9151	0.1788

```
TukeyHSD(mod4, "amf")
```

```
Tukey multiple comparisons of means  
95% family-wise confidence level
```

```
Fit: aov(formula = mpg ~ amf * cylf, data = mtcars4)
```

```
$amf
```

	diff	lwr	upr	p adj
1-0	7.245	5.001	9.488	0

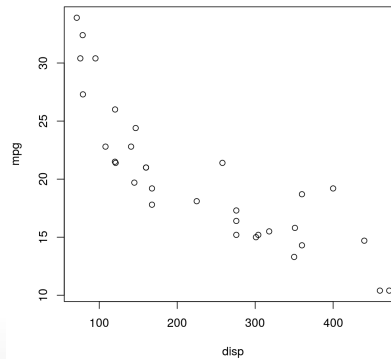
Finding relationships

Try any of the following codes to plot **mpg** against **disp** in the **mtcars** package.

```
plot(mpg~disp, data = mtcars)
```

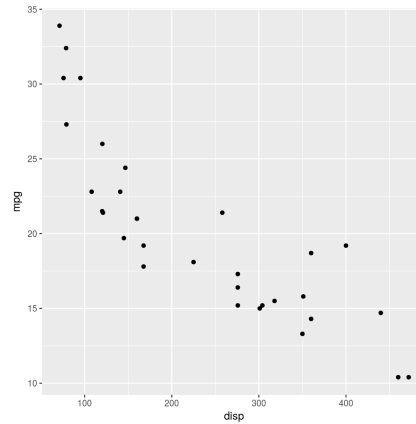
```
plot(mtcars$disp, mtcars$mpg)
```

```
with(mtcars, plot(disp, mpg))
```



Scatterplot with **ggplot2**

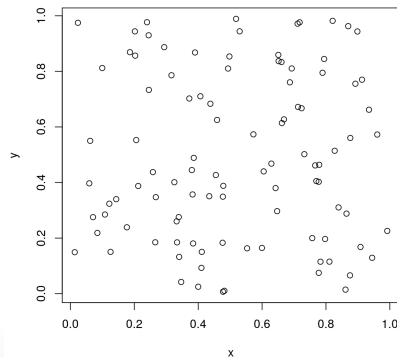
```
ggplot(mtcars, aes(x = disp, y = mpg)) + geom_point()
```



Correlation between **disp** and **mpg**

- Research problem: What is the relationship between **disp** and **mpg**?
- More specific research problem: Is there a linear relationship between **disp** and **mpg**?
- How does a scatterplot of no relationship between two variables look like?

```
set.seed(1); x = runif(100)  
set.seed(2); y = runif(100)  
plot(x,y)
```



```
with(mtcars, cor(disp, mpg))
```

Remember, correlation does not imply causation

But in controlled experiments where you test the variation in the dependent variable by manipulating the values of the independent variable, you can investigate causation.

Suppose we want to investigate whether **disp** has an effect on **mpg**.

The model is a linear regression of **mpg** on **disp**:

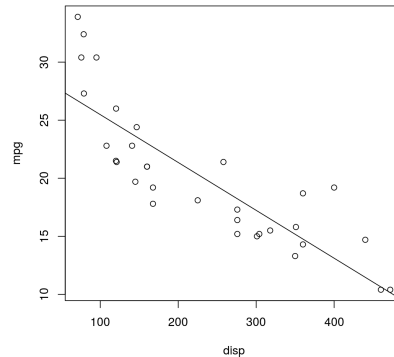
$$y = \beta_0 + \beta x + \varepsilon$$

where

- $y = \text{mpg}$
- $x = \text{disp}$
- $\beta_0 = \text{intercept}$
- $\beta = \text{the increase in mpg for every 1 unit increase in disp}$
- $\varepsilon = \text{random error}$

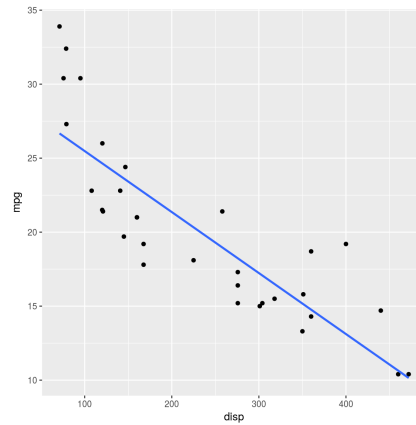
Plotting the line of best fit

```
mod5 <- lm(mpg~disp, data = mtcars)
with(mtcars, plot(disp, mpg))
abline(mod5)
```



Plotting the line of best fit with ggplot2

```
ggplot(mtcars, aes(displacement, mpg)) + geom_point() + geom_smooth(method="lm", se=FALSE)
```



Testing the linear fit

```
summary(mod5)
```

```
Call:
```

```
lm(formula = mpg ~ disp, data = mtcars)
```

```
Residuals:
```

Min	1Q	Median	3Q	Max
-4.892	-2.202	-0.963	1.627	7.231

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	29.59985	1.22972	24.07	< 2e-16
disp	-0.04122	0.00471	-8.75	9.4e-10

```
Residual standard error: 3.25 on 30 degrees of freedom
```

```
Multiple R-squared: 0.718, Adjusted R-squared: 0.709
```

```
F-statistic: 76.5 on 1 and 30 DF, p-value: 9.38e-10
```


Results

We have the following results from this output:

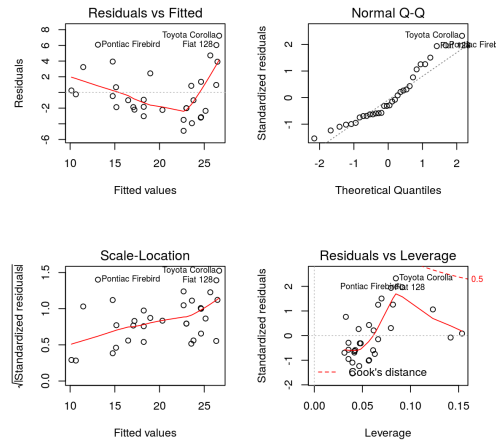
- The line of best fit has an equation: $y = 29.60 - 0.0412x$.
- **disp** has an affect on **mpg** at the .05 significance level ($p = 9.38 \times 10^{-10}$)
- **disp** explains about 72% of the variation in **mpg**

Four Principal Assumptions of linear regression

- **Linearity and additivity** of the relationship between dependent and independent variables / **Linearity of residuals**
- **Statistical independence** of the errors/residuals
- **Homoscedasticity** (equal variance) of the errors/residuals
- **Normality** of errors/residuals

Testing the linear fit

```
par(mfrow=c(2,2))  
plot(mod5)
```

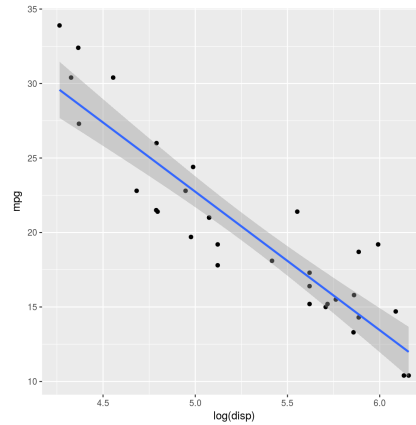


Interpreting the diagnostic plots

- The **residuals vs fitted** plot shows if residuals have non-linear patterns. This plot should show equally spread residuals around a horizontal line without distinct pattern.
- The **Normal Q-Q Plot** shows if the residuals are normally distributed. The residuals should follow a straight line well.
- The **Scale-Location Plot** shows if residuals are spread equally along the ranges of predictors. This plot can be used to check the assumption of equal variance (homoscedasticity). It should show a horizontal line with equally (randomly) spread points.
- The **Residuals vs Leverage Plot** helps us find influential cases (or subjects/observations) if any. There should be no points outside the dashed lines (or Cook's distance)

Transformations

```
ggplot(mtcars, aes(log(displacement), mpg)) + geom_point() + geom_smooth(method="lm")
```



Regression with log transformation

```
mod6 <- lm(mpg~log(displacement), data=mtcars)
summary(mod6)
```

Call:

```
lm(formula = mpg ~ log(displacement), data = mtcars)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-3.808	-1.634	-0.675	1.443	5.676

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	69.205	4.185	16.5	< 2e-16
log(displacement)	-9.293	0.787	-11.8	8.4e-13

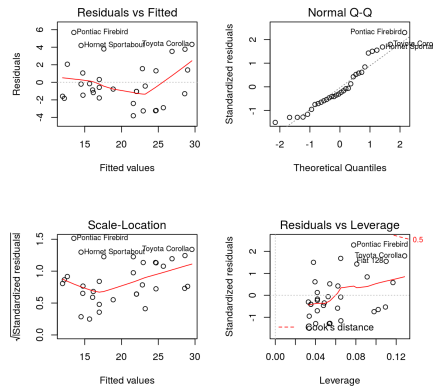
Residual standard error: 2.58 on 30 degrees of freedom

Multiple R-squared: 0.823, Adjusted R-squared: 0.817

F-statistic: 139 on 1 and 30 DF, p-value: 8.4e-13

Diagnostic plots of mod6

```
par(mfrow=c(2,2))  
plot(mod6)
```

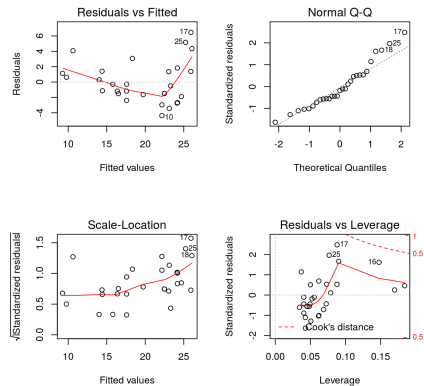


Interpretation of `mod6`

- The linear fit improved as the log of `disp` now explains about 82% of the variation in `mpg`.
- However, how do we now interpret the results?

Removing the influential observations

```
mtcars7 <- mtcars %>% filter(!rownames(.) %in% c("Pontiac Firebird",
"Toyota Corolla",
"Hornet Sportabout"))
mod7 <- lm(mpg~disp, data = mtcars7)
par(mfrow=c(2,2))
plot(mod7)
```



Effect of removing influential observations

```
summary(mod7)
```

```
Call:
```

```
lm(formula = mpg ~ disp, data = mtcars7)
```

```
Residuals:
```

Min	1Q	Median	3Q	Max
-4.37	-1.63	-0.50	1.38	6.47

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	29.26606	1.09492	26.73	< 2e-16
disp	-0.04237	0.00429	-9.87	1.9e-10

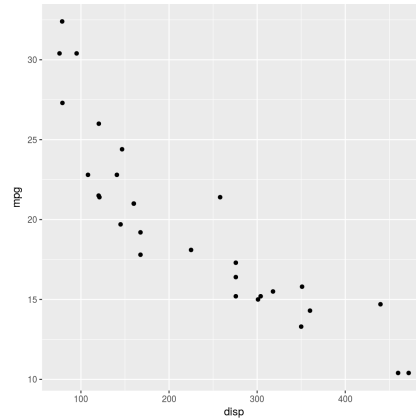
```
Residual standard error: 2.73 on 27 degrees of freedom
```

```
Multiple R-squared: 0.783, Adjusted R-squared: 0.775
```

```
F-statistic: 97.3 on 1 and 27 DF, p-value: 1.9e-10
```

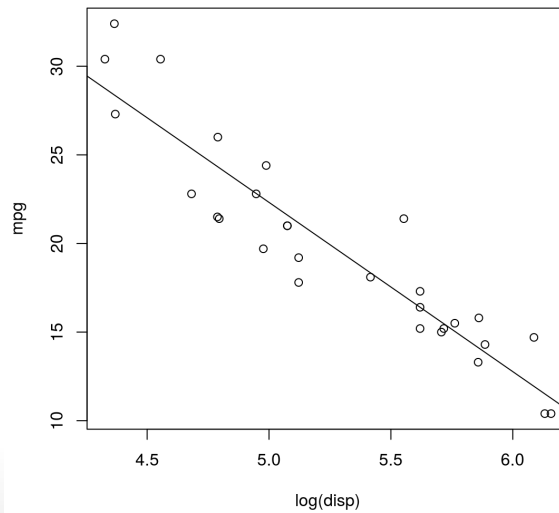
Plot without influential observations

```
ggplot(mtcars7, aes(displacement, mpg)) + geom_point()
```



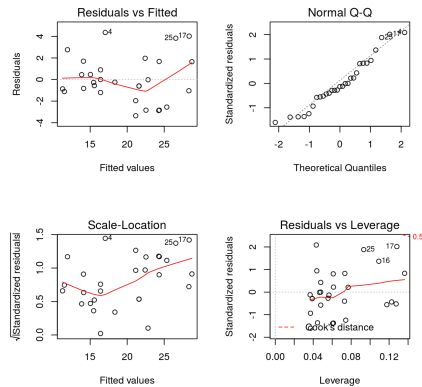
Trying transformations

```
mod8 <- lm(mpg~log(displacement), data=mtcars7)
plot(mpg~log(displacement), data=mtcars7)
abline(mod8)
```



Diagnostic plots of mod8

```
par(mfrow=c(2,2))  
plot(mod8)
```



Fit of mod8

```
summary(mod8)
```

Call:

```
lm(formula = mpg ~ log(displacement), data = mtcars)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-3.358	-1.116	-0.271	1.652	4.362

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	70.07	3.82	18.4	< 2e-16
log(displacement)	-9.55	0.72	-13.3	2.4e-13

Residual standard error: 2.14 on 27 degrees of freedom

Multiple R-squared: 0.867, Adjusted R-squared: 0.862

F-statistic: 176 on 1 and 27 DF, p-value: 2.41e-13

Multiple regression

```
head(mtcars)
```

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.460	20.22	1	0	3	1

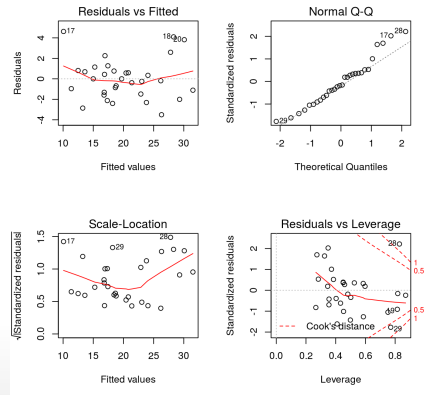
```
mtcars5 <- mtcars %>%  
  mutate(  
    cyl = as.factor(cyl),  
    vs = as.factor(vs),  
    am = as.factor(am),  
    gear = as.factor(gear),  
    carb = as.factor(carb)  
  )
```

Continuation of Multiple Regression

```
mod7 <- lm(mpg~., data=mtcars5)
par(mfrow=c(2,2))
plot(mod7)
```

Warning: not plotting observations with leverage one:
30, 31

Warning: not plotting observations with leverage one:
30, 31



Step-wise regression

```
mod8 <- step(mod7, direction="both", trace=FALSE)
summary(mod8)
```

Call:

```
lm(formula = mpg ~ cyl + hp + wt + am, data = mtcars5)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-3.939	-1.256	-0.401	1.125	5.051

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	33.7083	2.6049	12.94	7.7e-13
cyl6	-3.0313	1.4073	-2.15	0.0407
cyl8	-2.1637	2.2843	-0.95	0.3523
hp	-0.0321	0.0137	-2.35	0.0269
wt	-2.4968	0.8856	-2.82	0.0091
am1	1.8092	1.3963	1.30	0.2065

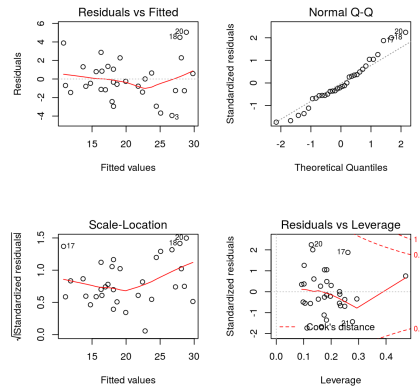
Residual standard error: 2.41 on 26 degrees of freedom

Multiple R-squared: 0.866, Adjusted R-squared: 0.84

F-statistic: 33.6 on 5 and 26 DF, p-value: 1.51e-10

Diagnostic plots of result of step-wise regression

```
par(mfrow=c(2,2))  
plot(mod8)
```



Prediction (for Demonstration Only)

<https://stats.stackexchange.com/questions/244017/prediction-vs-inference>

- Inference: Given a set of data you want to infer how the output is generated as a function of the data.
- Prediction: Given a new measurement, you want to use an existing data set to build a model that reliably chooses the correct identifier from a set of outcomes.
- Inference: You want to find out what the effect of Age, Passenger Class and, Gender has on surviving the Titanic Disaster. You can put up a logistic regression and infer the effect each passenger characteristic has on survival rates.
- Prediction: Given some information on a Titanic passenger, you want to choose from the set {lives,dies} and be correct as often as possible. (See bias-variance tradeoff for prediction in case you wonder how to be correct as often as possible.)

Let us use **mod5** for prediction

```
newdata <- data.frame(displacement = sample(mtcars$displacement, 5) + rnorm(5))
predict(mod5, newdata, interval="prediction")
```

	fit	lwr	upr
1	14.75	7.895	21.61
2	18.24	11.480	24.99
3	23.67	16.875	30.47
4	13.09	6.149	20.03
5	23.79	16.991	30.59

For More on Prediction and Machine Learning

- <https://www.datacamp.com/community/tutorials/machine-learning-in-r>
- <https://machinelearningmastery.com/machine-learning-in-r-step-by-step/>
- <https://www.coursera.org/learn/practical-machine-learning>
- <https://www.kaggle.com>
- <https://www.kdnuggets.com/2017/04/10-free-must-read-books-machine-learning-data-science.html>

Challenge: multiple linear regression

Using the **diamonds** data set, create a model for pricing diamonds based on the other variables.

```
?diamonds  
head(diamonds)
```

Jump start your self-learning of the R statistical package

```
install.packages("swirl")  
library(swirl)  
swirl()
```

Thank you!

```
library(ggplot2)
dat <- data.frame(x=seq(0, 2*pi, length.out=100))
shape <- function(x) 2-2*sin(x) + sin(x)*(sqrt(abs(cos(x))))/(sin(x)+1.4)
ggplot(dat, aes(x=x)) + stat_function(fun=shape) + coord_polar(start=-pi/2)
```


References

Bhalla, Deepanshu. 2017. "List of Companies Using R." Data Science Central. <https://www.datasciencecentral.com/profiles/blogs/list-of-companies-using-r>.

Muenchen, Robert A. 2016. "R Passes SAS in Scholarly Use (finally)." <http://r4stats.com/2016/06/08/r-passes-sas-in-scholarly-use-finally/>.

———. 2017. "The Popularity of Data Science Software." Accessed January 1. <http://r4stats.com/articles/popularity/>.

Piatetsky, Gregory. 2016. "R, Python Duel As Top Analytics, Data Science software—KDnuggets 2016 Software Poll Results." <https://www.kdnuggets.com/2016/06/r-python-top-analytics-data-mining-data-science-software.html>.

R Foundation. 2017. "What Is R?" Accessed October 31. <https://www.r-project.org/about.html>.