Upgrading Program: Learning Institute for Teachers (UPLIFT)

"Life is not about being comfortable, it's about making a difference."

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Mathematics

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Preface

Objectives

1. To be able to apply different perspectives in solving mathematics problems.
2. To be able to integrate technology for mathematics inquiry and problem solving.
3. To be able to use appropriate pedagogy and strategies in teaching mathematics.
4. To be able to enhance English communication skills to improve delivery of content and in constructing quality assessment tools.
5. To be able to clarify common misconceptions in mathematics.
6. To be able to appreciate mathematics.

Contents

Module 1

SETS

INTRODUCTION

This module aims to provide the material necessary to introduce the mathematical concept of sets and set operations to DepEd Grade 7 students.

OBJECTIVES

This module is made with the goal of helping the learners:

1. Be able to explore set concepts and set operations.
2. Describe and illustrate well-defined sets, subsets, the universal set, and the null set.
3. Define and describe the union and intersection of sets, and the complement of a set.
4. Use Venn diagrams to represent sets, subsets, and set operations.
5. Solve math problems using sets.

TIME ALLOTMENT

Morning Activities

Discussion on Set Concepts and Exercises	1 hour
Discussion on Venn Diagrams and Problem Solving	1.5 hours
Q and A	0.5 hours

Afternoon Activities

Activity 1: Populating a Venn Diagram	45 minutes
Activity 2: Constructing a Venn Diagram problem	1 hour

MATERIALS

- Manila paper
- Masking tape
- Markers

DISCUSSION

A set is a **well-defined** collection of objects. Well-defined means that it must be clear from the definition if an object is a member of a set or not. This concept can be reinforced to students by providing examples similar to the one below:

Given the following collections of objects, identify if the collection can be classified as a set, and if not, modify the definition to make it well-defined.

1. The collection of all participants in this seminar.
2. The collection of all rich people in Manila (or in your town/city).
3. The collection of young teachers in your school.
4. The collection of firstborn (panganay) students in the class.
5. The collection of difficult to pronounce words in a dictionary.
6. The collection of very large numbers.
7. The collection of integers that are factors of five.

The objects in a set are called the **elements** or **members** of the set. These objects can be anything: people, fruits, numbers, other sets, etc. For example, the number 7 is an element of the set of all odd integers. It is also possible for a set element to be a member of another set. For example, the number 7 is also a member of the set of all prime numbers.

If an object x is an element of a set A, it is said that x belongs to A. This relationship is symbolized as $x \in A$. The symbol $\in$ is derived from the greek epsilon, ϵ. The expression $x \notin A$ is used to mean that "x is not in A".

Equality of sets

Two sets A and B are said to be equal if they contain exactly the same elements, that is, every element of A is also an element of B and vice versa. The equality remains even if each set is defined differently. For example, sets A and B can be defined as follows:

A = the set of all positive integers less than 10 that are even.
B = the set of all values y such that $y = 2n$ where n is a natural number from 1 to 4.

Both sets will contain the elements 2, 4, 6, and 8. We can say that the two sets are equal ($A = B$) even if they have different definitions.

The empty set

A set with no elements is referred to as the empty set or the null set. The null set can be denoted by the symbols $\varnothing$ or $\{\}$. Take note that $\varnothing \neq \{\varnothing\}$ and using the symbol $\{\varnothing\}$ to denote the empty set is a common mistake.

Defining sets

A set can be defined by listing its elements inside curly braces, separated by commas. This definition is sometimes referred to as the list method. See the examples below:

1. $A = \{1, 2, 3, 4, 5\}$
2. $B = \{\text{Petri}, \text{Mardan}, \text{Natnat}, \text{Sherwin}, \text{Mark}\}$ can be used for non-numeric elements
3. $C = \{1, 2, 3, \ldots, 98, 99, 100\}$ ellipses can be used to imply elements The order of elements in a list is immaterial. The set $\{1, 2, 3\}$ is equal to the set $\{3, 1, 2\}$. Repetition or multiplicity of elements is also immaterial, and the sets $\{1, 2, 3\}$, $\{1, 1, 1, 2, 3\}$, and $\{3, 3, 1, 2, 2, 3\}$ are all equal.

A set can be defined using a rule that specifies what elements are members of this set. This rule must be well-defined in accordance to our definition of what a set is. Examples of sets defined using the rule method are:

1. The set of all students in a your class whose surnames start with a vowel.
2. The set of teachers in your school who have been teaching for three years or more.
3. The set of all positive irrational numbers.
4. The set of all negative integers divisible by five.
5. The set of all ordered pairs x, y that satisfy the equation $y = 2x + 1$ where x and y are integers.
6. The set of real numbers between 1 and 2. (*This example can be used to illustrate the limitations of the list method as its elements cannot be listed.*)

Sets can also be defined using the set-builder notation. This is a mathematical notation for describing a set by stating the properties that its members must satisfy. The set-builder notation has the form $A = \{x|\Phi x\}$ where a vertical bar separates x and Φx, and Φx indicates the properties that x must satisfy. Examples of sets defined using set builder notation are:

1. $A = \{x|x \in \mathbb{R}, x > -3\}$ *The set of all real numbers greater than* -3.
2. $B = \{x|x \in \mathbb{R}, x = x^2\}$ The set of all real numbers satisfying the equation $x = x^2$, specifically $\{0, 1\}$

Subsets and Supersets

Given two sets A and B, A is a subset of B, indicated by $A \subseteq B$ if every element of A is also an element of B. By this definition, it follows that if two sets are equal then they are subsets of each other. It also follows that a set is a subset of itself. If $A \subseteq B$, and $A \neq B$, then we say that A is a proper subset of B, or $A \subset B$.

If A is a subset of B, then it follows that B is a superset of A, indicated by $B \supseteq A$.

The null set is a subset of every set, and every set is a subset of itself.

Example: Given the following sets:

$A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$ $C = \{1, 2, 3, 4, 5\}$ $D = \{0, 1, 2, 3, 4, 5, 6\}$

We can conclude the following relationships:

1. $A \subseteq B$ A is a subset of B
2. $A \not\subseteq B$ A is not a proper subset of B
3. $B \subset D$ B is a proper subset of D
4. $D \subset C$ D is a proper superset of C

Universal set and Complementation

The universal set is the set of all elements of all sets under consideration for a given problem. It can be said that all sets in a given problem are subsets of some universal set. The universal set is usually given the symbol U.

In the previous example, we can take set $D = \{0, 1, 2, 3, 4, 5, 6\}$ as the universal set as it contains all the elements of all the sets under consideration. When studying the properties of the set of integers, or rational numbers, or irrational numbers, the set of real numbers can sometimes be taken as the universal set.

The complement of a set A, denoted by A' or $\bar{A}$ (read as "A-prime" or "A-complement") is the set of all elements in the universal set that are not in A. Using the previous example, if $A = \{1, 2, 3\}$ and $U = \{0, 1, 2, 3, 4, 5, 6\}$, then $\bar{A} = \{0, 4, 5, 6\}$.

Set operations

The union of two sets A and B, denoted by $A \cup B$, is the set of all objects that are elements of either A or B. The intersection of two sets A and B, denoted by $A \cap B$, is the set of all objects that are elements of both A and B.

If two sets A and B have no common elements, then the two sets are said to be disjoint. The intersection of disjoint sets is the null set.

The difference of two sets A and B, denoted by $A - B$ or $A\backslash B$, is the set of all elements that belong to A but not to B. This is also referred to as the relative complement of B with respect to A. Note that for the operation $A - B$, B does not have to be a subset of A, or B can contain elements that are not in A.

Sample exercise on Set Operations:

Given the following sets, determine the result of the indicated set operations:

$$A = \{1,3,5,7,9\} \qquad D = \{1,2,3,5,8\}$$
$$B = \{2,3,4,5\} \qquad E = \{7,8,9\}$$
$$C = \{5,6,8,9\} \qquad F = \{5,8,10\}$$
$$U = \{\text{set of counting the numbers from 1 to 10}\}$$

1. $B \cap C$ $\{5\}$
2. $A \cup F$ $\{1,3,5,7,8,9,10\}$
3. $B \cap E$ $\varnothing$ B and E are disjoint
4. $C - F$ $\{6,9\}$
5. $A \cap B \cap F$ $\{5\}$
6. $A - B \cup C - D$ $\{1,6,7,9\}$
7. $\bar{D} \cup \bar{F}$ $\{1,2,3,4,6,7,9,10\}$
8. $\bar{D} \cap \bar{A}$ $\{1,2,3,5,7,8,9\}$
9. $B \cup C \cap D$ $\{2,3,5,8\}$
10. $E \bar{\cup} F \cap \bar{C}$ $\{1,2,3,4\}$

Power Sets

The power set of a given set A is the set of all subsets of A, including the null set and A itself. For example, the power set of A, denoted by PA, given that $A = \{1,2,3\}$ is:

$$PA = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}.$$

The number of subsets of a set A is given by 2^n, where n is the number of elements of A.

Cardinality

The cardinality of a set A, referred to as $|A|$, is the number of elements of set A.

Venn Diagrams

Venn diagrams are diagrams that show all possible logical relations between given sets. Venn diagrams normally consists of overlapping circles, where the interior of each circle represents the elements of a given set.

A Venn diagram must have 2^n regions, where n is the number of sets under consideration.

Examples of Venn Diagrams are shown in Figures (1.1) and (1.2).

Venn diagrams can be used to illustrate set operations by shading the appropriate region in a Venn diagram. For the diagrams in Figure (1.3), set A is represented by the left circle and set B is represented by the right circle.

Examples of set operations illustrated using Venn diagrams:

1. $A \cap B \cap C$
2. $A \cup B \cap C$
3. $B - C - A$
4. $A - C \cup B$
5. $A \cap B' \cap C'$
6. $C \cap B - A'$
7. $[A \cup B \cap C] - B' \cup A'$
8. $[B - C \cup A - B]' \cap C$

The results are in in Figures (1.4(a)) to (1.4(h)).

It is possible to construct Venn diagrams for more than three sets, but there must be 2^n regions on the resulting diagram. See Figure (1.5) which shows Venn diagrams for four and five sets.

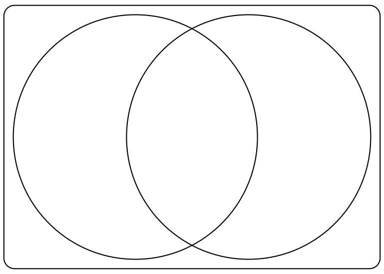


Figure 1.1

Venn diagram for two sets (4 regions)

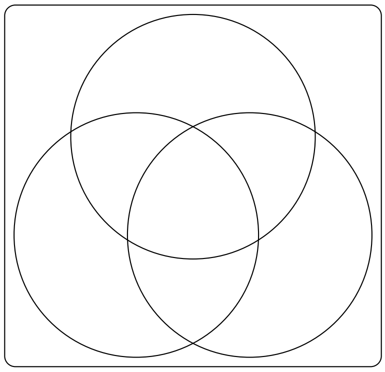
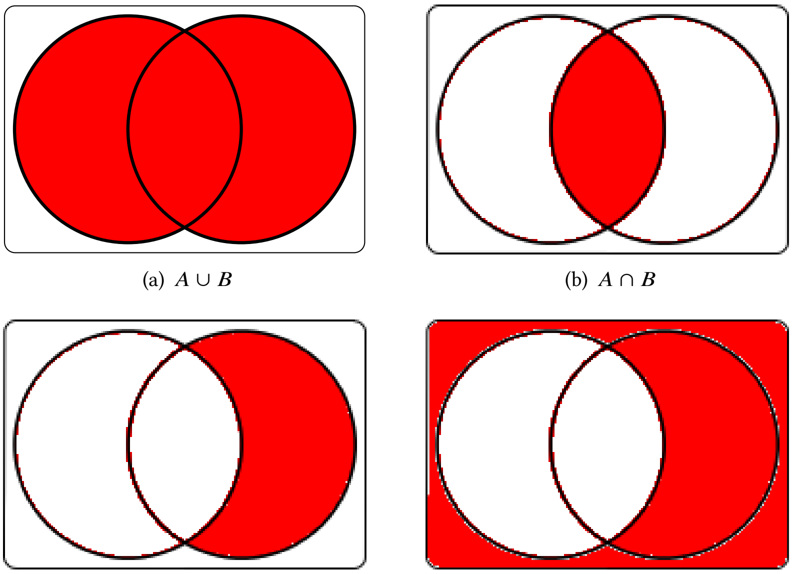


Figure 1.2

Venn diagram for two sets (8 regions)



(a) $A \cup B$ (b) $A \cap B$

(c) $A - B$ (d) $A \cap B$

Figure 1.3

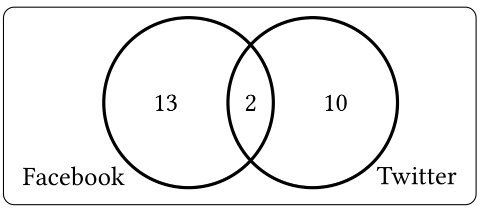
Illustrating set operations by shading the appropriate region in a Venn diagram.

Venn Diagram problems

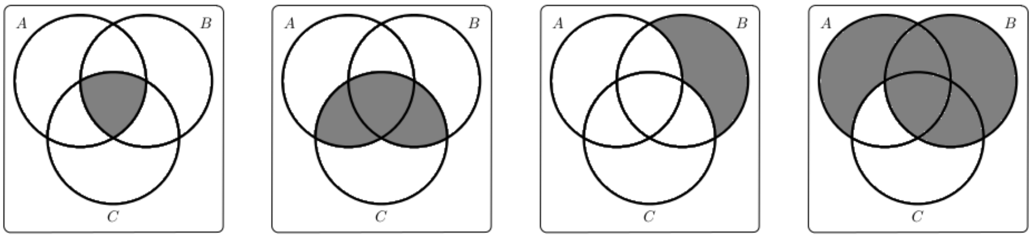
A common exercise involving Venn diagrams is populating a Venn diagram with values corresponding to the information given in a problem. A sample problem is given below along with step by step instructions on populating the Venn diagram. It is important to note that the words “or” and “and”, when referring to set operations, denote union and intersection respectively.

Example 1

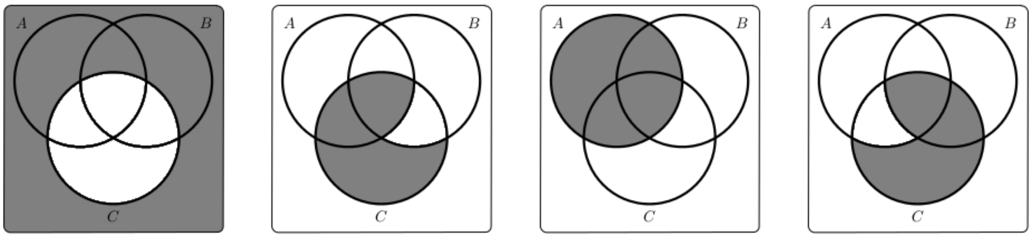
1. Create a Venn diagram representing the number of students using each online service.



2. How many students use both Facebook and Twitter? Ans: 2



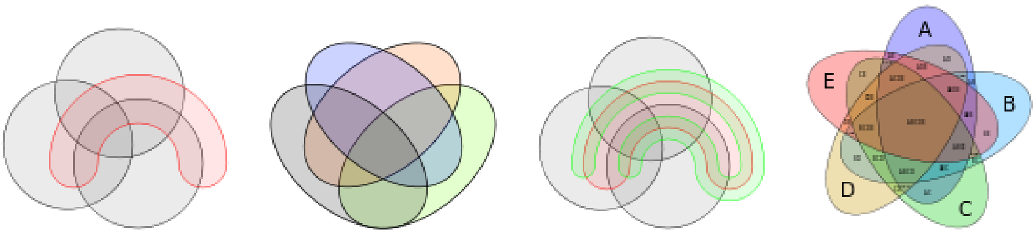
(a) $A \cap B \cap C$ (b) $A \cup B \cap C$ (c) $B - C - A$ (d) $A - C \cup B$



(e) $A \cap B' \cap C'$ (f) $C \cap B - A'$ (g) $[A \cup B \cap C] - B' \cup A'$ (h) $[B - C \cup A - B]' \cap C$

Figure 1.4

Operations on sets using Venn diagrams.



(a) 4-set Venn Diagram (b) 4-set Venn Diagram (c) 5-set Venn Diagram (d) 6-set Venn Diagram

Figure 1.5

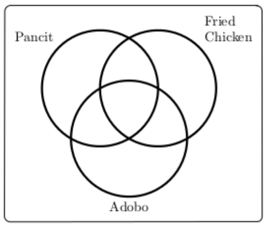
Some Venn diagrams for more than three sets.

Example 2

100 people seated at different tables in a party were asked if their group had ordered any of the following items: Pancit, Fried Chicken, or Pork Adobo

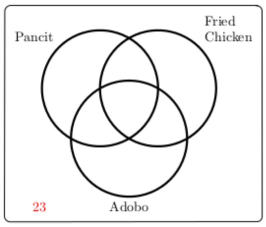
1. 23 people ordered none of these items.
2. 11 people ordered all three of these items.
3. 29 had ordered Fried Chicken or Pork Adobo but did not order Pancit.
4. 41 people ordered Pork Adobo.
5. 46 people ordered at least two of these items.
6. 13 had ordered Pancit and Pork Adobo but not ordered Fried Chicken.
7. 26 people ordered Pancit and Fried Chicken.

Create a Venn diagram representing the number of people who ordered each dish.

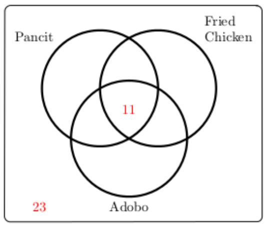


Step 1 Draw a Venn diagram for the three sets: People who ordered Pancit, Fried Chicken or Pork Adobo.

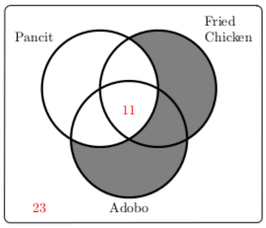
Step 2 Use the given information to populate regions on your Venn Diagram.



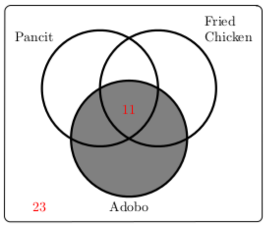
"23 people ordered none of these items."



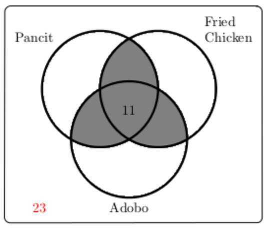
"11 people ordered all three of these items."
This statement corresponds to the intersection of all three sets.



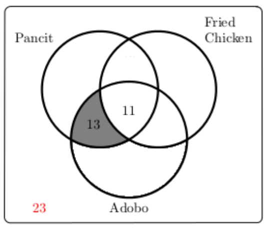
"29 had ordered Fried Chicken or Pork Adobo but did not order Pancit"
This statement corresponds to the expression $FC \cup Adobo - Pancit$ and the shaded region on the right. We don't have enough data on the diagram yet to use this piece of information.



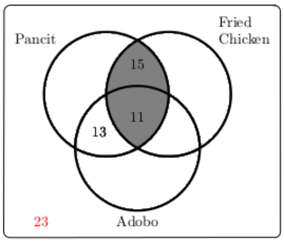
"41 people ordered Pork Adobo"
See the diagram for the region represented by this statement. We don't have enough data yet to use this information.



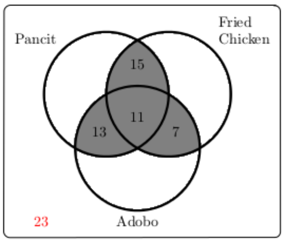
"46 people ordered at least two of these items"
See the diagram for the region represented by this statement. We don't have enough data yet to use this information.



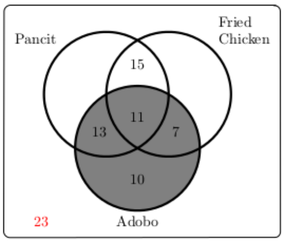
"13 had ordered Pancit and Pork Adobo but not ordered Fried Chicken"



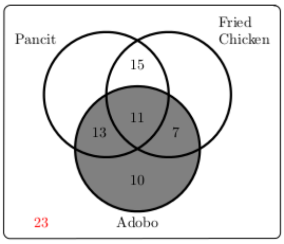
"26 people ordered Pancit and Fried Chicken"
Since the statement corresponds to the shaded area on the right, we can conclude that 15 people ordered Pancit and Fried Chicken but NOT Adobo.



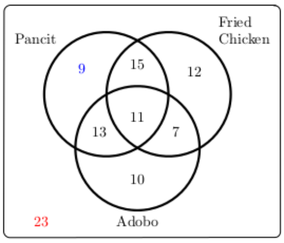
We now have enough data to process Statement 5: "46 people ordered at least two of these items"
Since the statement corresponds to the shaded area on the right, we can conclude that 7 people ordered both Fried Chicken and Adobo but NOT Pancit.



We can refer again to Statement 4: "41 people ordered Pork Adobo"
From the data on the diagram, we can conclude that 10 people ordered Pork Adobo only.



Going back to Statement 3: "29 had ordered Fried Chicken or Pork Adobo but did not order Pancit"
We now have enough data on the diagram to conclude that 12 people ordered Fried Chicken only.



Using the initial information that there are 100 patrons, we can complete the diagram by determining how many customers ordered Pancit only.

The completely filled-up Venn Diagram is the final answer.

Here are additional problems on filling up Venn Diagrams:

1. A study was made of 1000 bodies of water in the country to determine what pollutants were in them.

   (a) 177 areas were clean

   (b) 101 areas were polluted only with crude oil

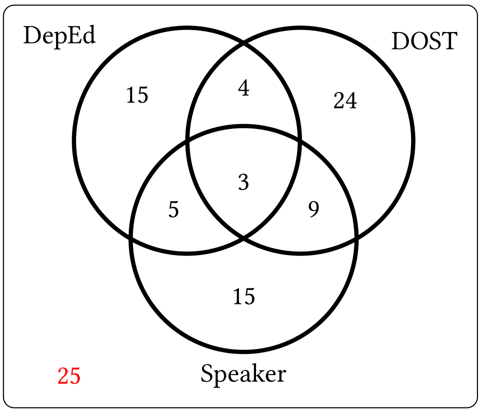
   (c) 439 areas were polluted with phosphates.

   (d) 72 areas were polluted with sulfur compound and crude oil, but not with phosphates.

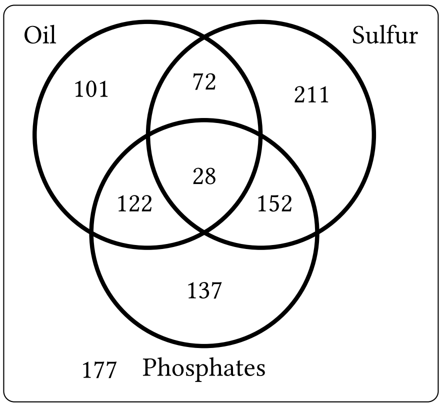
   (e) 289 areas were polluted with phosphates, but not with crude oil.

   (f) 463 areas were polluted with sulfur compounds.

   (g) 137 areas were polluted with only phosphates.



2. 100 students were asked if they knew who any of the following are: The DepEd secretary, The Speaker of the House, and The DOST secretary.
   (a) 25 people did not know any of these.
   (b) 3 people knew all three.
   (c) 48 people knew who the Speaker of the House or the DOST secretary were but didn't know who the DepEd secretary was.
   (d) 40 people knew who the DOST secretary was.
   (e) 21 people knew who at least two of these were.
   (f) 7 people knew who the DepEd secretary and the DOST secretary were but didn't know who the Speaker of the House was.
   (g) 8 people knew who the DepEd secretary and the Speaker of the House were.



SUGGESTED ACTIVITIES

Resource Speakers may choose which class activity will be done by the participants depending on your level of comfortableness with the activity, available materials, and remaining time.

Activity 1: Constructing a Venn Diagram from a Survey Data

Objectives:

To construct a Venn Diagram problem based on survey data

Preparation:

Group the participants into 10 groups. Each group is given several minutes to construct a simple preference survey. (e.g. TV channels they like to watch: kapamilya, kapuso, kapatid; sports they play: basketball, volleyball, bowling).

SURVEY FORM			
Name of Participant	Choice 1	Choice 2	Choice 3
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Figure 1.6

Activity 1: Constructing a Venn Diagram from a Survey Data

Activity:

1. Each group will survey all the participants based on the survey questions that they constructed. There should be 3 choices and the participants can tick all choices if applicable.
2. After the members of the group surveys ALL the participants, the group tallies the data on a tally sheet (each group should be provided with a list of all participants).
3. After the tallying all the data, the group constructs the Venn diagram.
4. From the Venn diagram constructed, the group must design a Venn diagram problem.
5. Each group will write down their problem on a sheet of Manila paper and post their problems on the wall.
6. After the break, groups pair up and solve each others problem.
7. After the alloted time, each group present their solution, and the problems and solutions are critiqued.

Note: You may use a sample sheet like that found in Figure (1.6) For an easier activity instead of 3 choices, 2 choices could be used for the survey.

Activity 2: Populating a Venn Diagram

Objective:

To integrate the Venn Diagram concepts with other subjects taken up by Grade 7 students. For this activity we can integrate some historical facts of SEA nations.

Preparation:

1. Construct a large Venn Diagram on the classroom floor with tape. Label each circle with a category. For this activity we can use the SEA countries Vietnam, Thailand, and the Philippines.

2. Prepare notecards containing facts about the three countries. Here are some examples, which can be expanded as needed:
   (a) Monarchy *Thailand only*
   (b) Colonized by Europeans *Vietnam and Philippines*
   (c) Occupied by Japanese *All three countries*
   (d) Predominantly Christian *Philippines only*
3. Optional: Pass around a handout containing historical information about these three countries. This can also be coordinated with the History teacher if these topics are already covered.

Activity

1. Have each student draw from a pile of notecards containing facts about the countries.
2. Randomly select a student to read their fact to the class and then stand in the appropriate area of the Venn Diagram.
3. Encourage discussion and provide feedback as to the correct placement of the characteristics.
4. Repeat steps 4 and 5 until all students are standing in the correct place in the Venn Diagram (use a model Venn Diagram if necessary).
5. Summarize the results of the activity.

This activity can be revised to integrate Venn diagram concepts with other academic topics like science or literary characters, or things that children enjoy like superhero superpowers.

Activity 3: Constructing a Venn Diagram Problem

Objectives:

1. To construct a Venn Diagram problem based on personal information of the participants/students.
2. To determine the least amount of information that can be given about a problem that can still be used to populate a Venn Diagram.

Preparation:

Prepare two surveys on personal preferences of students or participants. There should be three possible options for the participants to choose from (actors/actresses, food, subjects, etc.). The participants should be allowed to choose more than one option.

Activity:

1. Distribute the participants into two groups. Have each group fill up a different survey to generate the data they will need for their Venn Diagram activity.
2. Each group should populate their own Venn Diagrams based on the results of the survey they filled up. A copy of the populated Venn Diagrams are given to the facilitator.
3. On a piece of manila paper, each group should write down phrases that give information about their Venn Diagram. The goal is to give as little information as possible while ensuring that there is enough information to populate a Venn Diagram.
4. The two teams swap information, and they will race to populate the Venn Diagram based on information provided by the other group. The group that completes first is declared the winner.
5. If the losing team can show that it is impossible to populate a Venn Diagram based on the given information, then they are declared the winner instead.
6. The two teams will share their techniques and difficulties encountered in constructing a Venn Diagram problem.

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Module 2

SET OF REAL NUMBERS

INTRODUCTION

This module is designed to describe, represent and compare the different subsets of real numbers in hierarchical form. Mainly, this module talks about applications of various procedures and manipulations on the different subsets of the set of real numbers. The topics included in this module are: definitions of different kinds of numbers; the location of numbers on the number line; properties of operations (addition and multiplication) that characterizes each set of numbers; operations involved in each set of numbers (particularly integers and rational numbers); description and representation of real-life situations involving integers, rational numbers, square roots of rational numbers and irrational numbers; and solving problems involving real numbers. An assessment exam and alternative assessment in terms of group activity are also given at the end of the module.

OBJECTIVES

This module is made towards the goal of helping the learners:

1. Identify the different sets of numbers using properties of operations (addition and multiplication).
2. Describe and illustrate the absolute value of a number on a number line as the distance of the number from 0.
3. Perform fundamental operations on integers: addition, subtraction, multiplication, and division.
4. Define and illustrate rational numbers and arrange them on a number line.
5. Express rational numbers from fraction form to decimal form (both terminating and repeating) and vice versa.
6. Perform operations on rational numbers and illustrate their properties.
7. Describe principal roots and tell whether they are rational or irrational.
8. Write in scientific notation.

DISCUSSION

Numbers and Numerals

Numbers are ideas that we associate with quantities of things around us. **Numerals** are the symbols we use to represent these ideas. Different people use different symbols to represent numbers. The **number line** is a line with points and is often used to illustrate order of numbers because each point which is associated with, or represents a number has a definite location in relation to other points that also represent other numbers.

The System of Counting Numbers ($\mathbb{N}$)

$\mathbb{N}$ and the Number Line

Naturally, the first number we use when we count is the number one (1), followed by two (2), three (3), and so on. We call these numbers **counting** or **natural numbers**. The set of counting numbers is represented by $N = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$. The number line can be used to show the correct order of these numbers.

Subsets of $\mathbb{N}$

Some of the subsets of the set of natural numbers are the following:

1. The set of **prime numbers**. A **prime number** is a natural number greater than 1 whose only positive divisors are 1 and itself. Let us represent the set of prime numbers as set P.

$$P = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$$

2. The set of **composite numbers**. A **composite number** is a natural number that has a positive other than 1 and itself. In other words, a composite number is a natural number greater than 1 that is *not* a prime number. Let us represent the set of composite numbers as set C.

$$C = \{4, 6, 8, 9, 10, 12, 14, 15, 16, \ldots\}$$

3. The set of **odd numbers**. An **odd number** is a number that is not divisible by 2. Let us represent the set of odd numbers as set O.

$$O = \{1, 3, 5, 7, 9, 11, 13, 15, 17, \ldots\}$$

4. The set of **even numbers**. An **even number** is a number that can be divided by 2. Let us represent the set of even numbers as set E.

$$E = \{2, 4, 6, 8, 10, 12, 14, 16, \ldots\}$$

Number Theory

The basic number theory concepts or terms included in discussing counting numbers are tDivisibility Tests. The method of determining which counting number is divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and other numbers. he following.

1. **Prime Factors**. These are factors that are prime numbers.
2. **Prime Factorization**. This is the method of expressing a composite number as a product of primes.
3. **GCF and LCM**. The methods of finding the **greatest common factor (GCF)** and **least common multiple (LCM)** of certain sets of numbers.
4. **Divisibility Tests**. The method of determining which counting number is divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and other numbers.

Properties of Operations (+ and $\times$) in $\mathbb{N}$

The following properties are always true in $\mathbb{N}$.

Property	Examples
1. Closure Property for Addition (CLPA)	$1 + 3 = 4, 4 \in N$
2. Closure Property for Multiplication (CLPM)	$4x5 = 20, 20 \in N$
3. Commutative Property for Addition (CPA)	$3 + 5 = 5 + 3$
4. Commutative Property for Multiplication (CPM)	$4 \times 8 = 8 \times 4$
5. Associative Property for Addition (APA)	$2 + 5 + 3 = 2 + 5 + 3$
6. Associative Property for Multiplication (APM)	$4 \times 3 \times 2 = 4 \times 3 \times 2$
7. Distributive Property for Multiplication over Addition (DPMA)	$4 \times 5 + 3 = 4 \times 5 + 4 \times 3$
8. Identity Property of Multiplication (IPM)	$8 \times 1 = 1 \times 8 = 8$

The System of Whole Numbers ($\mathbb{W}$)

$\mathbb{W}$ and the Number Line

Recall from Module 1 when a set has no element, it is called an **empty set** or a **null set**. The cardinal number of the empty set is represented by a number which is called **zero (0)**. The union of the unit set $\{0\}$ and the set $\mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$ is the set of **whole numbers** represented by $\mathbb{W} = \{0, 1, 2, 3, 4, \ldots\}$. The set $\mathbb{N}$ is a subset of the set $\mathbb{W}$, therefore *all subsets of* $\mathbb{N}$ (i.e., set of prime numbers, set of composite numbers, set of even numbers, and set of odd numbers) *are all subsets of* $\mathbb{W}$.

Properties of Operations ($+$ and $\times$) in $\mathbb{W}$

Property	Examples
1. Closure Property for Addition (CLPA)	$1 + 3 = 4, 4 \in N$
2. Closure Property for Multiplication (CLPM)	$4x5 = 20, 20 \in N$
3. Commutative Property for Addition (CPA)	$3 + 5 = 5 + 3$
4. Commutative Property for Multiplication (CPM)	$4 \times 8 = 8 \times 4$
5. Associative Property for Addition (APA)	$2 + 5 + 3 = 2 + 5 + 3$
6. Associative Property for Multiplication (APM)	$4 \times 3 \times 2 = 4 \times 3 \times 2$
7. Distributive Property for Multiplication over Addition (DPMA)	$4 \times 5 + 3 = 4 \times 5 + 4 \times 3$
8. Identity Property of Multiplication (IPM)	$8 \times 1 = 1 \times 8 = 8$
9. Identity Property of Addition (IPA)	$5 + 0 = 0 + 5 = 5$

The System of Integers ($\mathbb{Z}$)

$\mathbb{Z}$ and the Number Line

In the sets $\mathbb{N}$ and $\mathbb{W}$, subtraction can only be performed by deducting a smaller number from a larger number. When a larger number is subtracted from a smaller number, the result is a number that does not belong to these two sets. Another set of numbers called the **negative numbers**, whose values are less than zero, is formed. The union of these negative numbers and $\mathbb{W}$ is called the set of integers or signed numbers represented by $\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$. Since $\mathbb{W}$ is a subset of $\mathbb{Z}$, all subsets of W are also subsets of $\mathbb{Z}$. The number line shows the position of the points that represent the integers. The order of the numbers depends on their position relative to the point which represents zero, also called the **origin**. The **negative numbers**, also called the **additive inverses** of the natural numbers (**positive numbers**), are located at the left side of zero, while all positive numbers are located at the right side of zero. Negative numbers are less than zero, while positive numbers are greater than zero. Zero is neither positive nor negative.

Properties of Operations ($+$ and $\times$) in $\mathbb{Z}$

The following properties are true in $\mathbb{Z}$.

Property	Examples
1. Closure Property for Addition (CLPA)	$-2 + -4 = -6, 1 + 3 = 4, -6, 4 \in \mathbb{Z}$
2. Closure Property for Multiplication (CLPM)	$-3 \times 5 = -15, 4x5 = 20, -15, 20 \in \mathbb{Z}$
3. Commutative Property for Addition (CPA)	$-3 + 5 = 5 + -3$
4. Commutative Property for Multiplication (CPM)	$4 \times -8 = -8 \times 4$
5. Associative Property for Addition (APA)	$2 + -5 + 3 = 2 + -5 + 3$
6. Associative Property for Multiplication (APM)	$-4 \times 3 \times 2 = -4 \times 3 \times 2$
7. Distributive Property for Multiplication over Addition (DPMA)	$-4 \times 5 + 3 = -4 \times 5 + -4 \times 3$, $5 + 3 \times -4 = 5 \times -4 + 3 \times -4$
8. Identity Property of Multiplication (IPM)	$-8 \times 1 = 1 \times 8 = -8$
9. Identity Property of Addition (IPA)	$-5 + 0 = 0 + -5 = -5$
10. Additive Inverse Property (AIP)	$5 + -5 = 0$

Absolute Value of an Integer

The absolute value of an integer is the distance of the point representing the integer from the point representing zero. The absolute value of $+5$, represented by $|+5|$, and the absolute value of -5, represented by $|-5|$, are both equal to 5, since the points representing them are both 5 units away from the point representing 0.

Operations on $\mathbb{Z}$

Addition of Integers The sum of two integers depends on the signs of both numbers. If the numbers have the same sign, the sum is preceded by the common sign. If the numbers have different signs, the difference of the two numbers is taken and the sign of the answer is the sign of the integer with the greater absolute value.

Example 2.1

1. (a) $+3 + +4 = +7$ (b) $-3 + -4 = -7$
2. (a) $+3 + -4 = -1$ (b) $-3 + +4 = +1$

Subtraction of Integers Subtraction of integers is the addition of the additive inverse of the subtrahend to the minuend. In symbols, $a - b = a + -b$.

Example 2.2

1. $9 - 3 = 9 + -3 = 6$
2. $-9 - 3 = -9 + -3 = -12$
3. $-9 - -3 = -9 + --3 = -9 + 3 = -6$
4. $9 - -3 = 9 + --3 = 9 + 3 = 12$

Multiplication of Integers The product of two integers with the same sign is positive while the product of two integers with different signs is negative.

Division of Integers The quotient of two integers with the same sign is positive, while the quotient of two integers with different signs is negative. Zero can never be a divisor.

The System of Rational Numbers $\mathbb{Q}$

$\mathbb{Q}$ and the Number Line

When an integer is divided by another nonzero integer, and the quotient is an integer having no remainder, the first integer is said to be divisible by the nonzero integer. But what happens when an integer is divided by a nonzero integer which is not a divisor or a factor of the first integer? The quotient is definitely not an integer. Another set of numbers is formed which will satisfy the given condition, and we call this set as the set of **fractions**. The fractions can either be **proper** or **improper** fractions. The union of the set of integers and the set of fractions is called the set of **rational numbers** which is represented by $\mathbb{Q} = \{\ldots, -4, -7/2, -3, -5/2, -2, -3/2, -1, -1/2, 0, 1/2, 1, 3/2, 2, 5/2, 3, 7/2, 4, \ldots\}$. In other words, a rational number can be expressed as a ratio or quotient of two integers. All rational numbers can be represented by fractions. Since the set of integers is a subset of the set of rational numbers, all subsets of the set of integers are also subsets of set $\mathbb{Q}$. The number line can be used to show the location of points represented by the rational numbers.

Properties of Operations (+ and ×) in $\mathbb{Q}$

The following properties are true in $\mathbb{Q}$.

Property	Examples
1. Closure Property for Addition (CLPA)	$-\frac{1}{2}+-\frac{3}{4}=-\frac{5}{4}, 1+3=4, -\frac{5}{4}, 4 \in \mathbb{Q}$
2. Closure Property for Multiplication (CLPM)	$-\frac{3}{4}\times\frac{2}{3}=-\frac{2}{5}, 4x5=20, -\frac{2}{5}, 20 \in \mathbb{Q}$
3. Commutative Property for Addition (CPA)	$-\frac{3}{2}+\frac{5}{4}=\frac{5}{4}+-\frac{3}{2}$
4. Commutative Property for Multiplication (CPM)	$\frac{4}{7}\times 8=8\times\frac{4}{7}$
5. Associative Property for Addition (APA)	$\frac{2}{3}+-\frac{5}{2}+3=\frac{2}{3}+-\frac{5}{2}+3$
6. Associative Property for Multiplication (APM)	$-\frac{4}{3}\times 3\times\frac{2}{5}=-\frac{4}{3}\times 3\times\frac{2}{5}$
7. Distributive Property for Multiplication over Addition (DPMA)	$-4\times\frac{5}{2}+\frac{3}{4}=-4\times\frac{5}{2}+-4\times\frac{3}{4}$, $\frac{5}{2}+\frac{3}{4}\times-4=\frac{5}{2}\times-4+\frac{3}{4}\times-4$
8. Identity Property of Multiplication (IPM)	$-\frac{8}{9}\times 1=1\times-\frac{8}{9}=-\frac{8}{9}$
9. Identity Property of Addition (IPA)	$-\frac{5}{7}+0=0+-\frac{5}{7}=-\frac{5}{7}$
10. Additive Inverse Property (AIP)	$\frac{5}{9}+-\frac{5}{9}=0$
11. Multiplicative Inverse Property (MIP)	$2\frac{1}{2}=1$

Fractions and Decimals

A subset of $\mathbb{Q}$ is the set of fractions. A fraction can be defined as the quotient of a whole number divided by a natural number. A proper fraction has a numerator less than the denominator, while an improper fraction has a greater numerator than denominator. Any fraction can be converted to decimal form, either to a terminating or repeating decimal as the case may be.

To convert a fraction to decimal, the numerator is divided by the denominator. A proper fraction can be converted to either a terminating or repeating decimal. An improper fraction can be converted to a decimal form with a whole number part, or to a mixed number form. The mixed number form is a combination of a whole number and a proper fraction.

To convert a terminating decimal to fraction, the decimal is multiplied by a fraction equivalent to 1 (whose numerator and denominator is a power of 10) and then simplified by dividing both numerator and denominator by the greatest common factor. For example, 0.45 will be multiplied by $\frac{100}{100}$, to get $\frac{45}{100}$, which is further simplified to $\frac{9}{20}$.

To convert a repeating decimal to fraction, the following examples can be followed.

Example 1. Convert 0.333 . . . to fraction.

Solution.

$$\begin{array}{r} \text{Let } n = 0.333\ldots \\ 10n = 3.333\ldots \\ \hline 9n = 3 \\ \therefore n = \frac{1}{3} \end{array}$$

Example 2. Convert 0.4545 . . . to a fraction.

Solution.

$$\begin{array}{r} \text{Let } n = 0.4545\ldots \\ 100n = 45.4545\ldots \\ -n = 0.4545\ldots \\ \hline 99n = 45 \\ \therefore n = \frac{5}{11} \end{array}$$

Example 3. Convert 4.3636 . . . to a fraction.

Solution.

$$\begin{array}{r} \text{Let } n = 4.3636\ldots \\ 100n = 436.3636\ldots \\ -n = 4.3636\ldots \\ \hline 99n = 432 \\ \therefore n = \frac{48}{11} \end{array}$$

Operations on Natural Numbers

Addition and Subtraction Rational numbers in fraction form can only be added or subtracted if they have the same denominators. These fractions are called similar fractions. Always convert dissimilar fractions to fractions with the same denominators first before adding or subtracting.

Multiplication When multiplying fractions, cancellation can be used to reduce the product to its simplest form. Cancellation is done by dividing the numerator and denominator by the greatest common factor.

Division Division of a fraction by another fraction is done by multiplying the dividend by the reciprocal or multiplicative inverse of the divisor. Then the product is reduced to its simplest form.

The System of Irrational Numbers $\mathbb{Q}'$

The Set of $\mathbb{Q}'$ and the Number Line

The numbers that cannot be represented as a ratio of two integers in simplest form are called irrational numbers. The union of the set of rational numbers and the set of irrational numbers comprise the set of real numbers which is represented by $\Re$. The set of irrational numbers is the complement of the set of rational numbers in the universal set of real numbers, and is represented by

$$\mathbb{Q}' = \{\ldots \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{11}, \pi, \ldots\}.$$

Square Root of a Number as a Rational Number or Irrational Number

Square roots of numbers sometimes result in rational or irrational numbers.

The square roots of perfect squares, or of fractions having perfect squares in both the numerator and denominator, are rational numbers.

Example 2.3

1. $\sqrt{49} = \sqrt{7^2} = 7$
2. $\sqrt{121} = \sqrt{11^2} = 11$

Otherwise, the square roots are irrational numbers.

Scientific Notation

When writing very large or very small numbers, like the mass of the earth, (6 000 000 000 000 000 000 000 000 kilograms) or the mass of an electron (0.000 000 000 000 000 000 000 000 000 000 911 kilogram), the most convenient form to use is the scientific notation. In this notation, numbers are represented by the product of a multiplying factor and a power of ten. A power of ten is the number 10, raised to an integral exponent. The decimal point is moved until only one nonzero digit is on the left side of it. Then the number of places the decimal point is moved is used as the exponent of 10. The sign of the exponent depends on the movement of the decimal point: to the right (−) or to the left (+).

Example 2.4

1. $6\,000\,000\,000\,000\,000\,000\,000\,000 = 6.0 \times 10^{24}$
2. $0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,911 = 9.11x10^{-31}$

EXERCISES

Tests for Divisibility.

Draw ☺ on the third column if the number in the first column is divisible by the number on the second column. Otherwise, draw ☹.

1. 67348	4	________
2. 73488	8	________
3. 83671	3	________
4. 5334	6	________
5. 5991	6	________
6. 4425575	9	________
7. 47830	5	________
8. 53867	11	________
9. 1183	7	________
10. 13855	17	________

Properties of Real Numbers

1. How many prime numbers are there from 1 to 50? From 51 to 100? From 101 to 150? From 151 to 200? (Use Sieve of Eratosthenes)

2. How many factors does 72 have? 120? 144? 84? 360?

3. Express each of the following numbers in scientific notation:

   (a) 45 000 000 000 000 000 000 000 000 000 000

   (b) 0.000 000 000 000 000 173

   (c) 930 000 000 000 000 000 000

   (d) 0.000 000 000 000 000 000 000 000 62

   (e) 508 000 000 000 000 000

SUGGESTED ACTIVITIES

Activity 1: Illustrating the Set of Numbers using Venn Diagrams

1. Illustrate the relationship of each of the following sets of numbers using the Venn Diagrams:

   (a) Prime Numbers and Composite Numbers

   (b) Prime Numbers and Even Numbers

   (c) Composite Numbers and Odd Numbers

   (d) Composite Numbers and Even Numbers

   (e) Natural Numbers and Whole Numbers

2. Show the Hierarchy of the System of Real Numbers using the following:

   (a) Venn Diagram

   (b) Flow Chart

   (c) Concept Map

Activity 2: Categorizing Numbers into the Different Sets

Check the appropriate boxes that categorize the given numbers.

Number	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$	$\Re$
45.9						
$-1\,444$						
$3\sqrt{6}$						
$\frac{5}{9}$						
125%						
27.5						
3 678 125						
$5\sqrt{2693}$						
8.246						
$7\sqrt{9}$						

Activity 3: The Boat is Sinking (Divisibility Version)

Objective:

To practice divisibility rules in a fun way.

Preparation:

Each participant is given a sheet of paper (can be a scratch paper) and they should write their favourite one digit on the sheet of paper (it should be large enough to occupy the whole sheet).

Activity:

1. The resource person calls on the participants (just like in the party game, "The Boat is Sinking") to group themselves (using their own one-digit numbers as digit) to form numbers like:
   (a) two-digit numbers divisible by 2
   (b) three-digit numbers divisible by 3
   (c) four-digit numbers divisible by 4
   (d) five-digit numbers divisible by 5
2. Participants that cannot form groups will be disqualified from the activity. After some time, remaining participants are declared winners.

Activity 4: Words of Wisdom

Objectives:

To be able to conduct a drill on arithmetic (operations on integers).

Preparation:

Each participant is given a copy of the worksheet.

Activity:

1. After each participant has a copy of the worksheet, the resource person announces the operation for each column: Col A (×), Col B (−), Col C (+) and Col D (÷).
2. The worksheet is quite self-explanatory. If they have found "the phrase" and they are satisfied, they may ask the resource person if they are correct.
3. After the allotted time, the worksheet's answers are revealed.

WORDS OF WISDOM (*an arithmetic drill*)

PART 1: BASIC ARITHMETIC

Using the indicated operation, compute as fast as you can but do not be careless.

Row				Col A				Col B				Col C				Col D
1	1	6	=		2	1	=		3	6	=		21	7	=	
2	2	7	=		3	6	=		2	1	=		36	6	=	
3	3	8	=		2	7	=		4	6	=		27	3	=	
4	2	2	=		4	8	=		7	7	=		48	8	=	
5	4	4	=		7	2	=		8	8	=		72	9	=	
6	7	3	=		8	4	=		6	2	=		84	4	=	
7	8	5	=		6	3	=		5	4	=		63	7	=	
8	6	9	=		5	5	=		3	3	=		55	5	=	
9	4	6	=		3	9	=		1	5	=		39	3	=	
10	3	1	=		1	6	=		2	9	=		16	2	=	

PART 2: FROM NUMBERS TO LETTERS TO WORDS

From Part 1, compute from the corresponding column and row.
Convert the value to the corresponding letter in the standard English alphabet. (e.g. 1 → A, 2 → B)
Then, rearrange the letters to form a word.

WORD 1			
A4+D6	=		
A2+C8	=		
B1⋆C1	=		
A1+D4	=		
B4+D9	=		
D5/B6	=		
B7/D4	=		
A9+B10	=		
D7-B3	=		
B5⋆C2	=		
C3+D2	=		
A3+B10	=		
C4-D3	=		
C2⋆C9	=		
WORD			

WORD 2			
C7/C1	=		
D6/C2	=		
B2⋆B9	=		
A6-A5	=		
B6/C5	=		
WORD			

WORD 3			
D10-B8	=		
A8/C9	=		
A4⋆B5	=		
A2+C7	=		
WORD			

WORD 4			
A1⋆D1	=		
C3-B3	=		
A9-C6	=		
A7⋆D5	=		
C4+D3	=		
WORD			

WORD 5			
C6+C10	=		
A3-D8	=		
A6-D2	=		
B1⋆D1	=		
C6+C10	=		
WORD			

WORD 6			
D7+D8	=		
B7/C2	=		
A10-B4	=		
C10-C8	=		
A10⋆C9	=		
WORD			

PART 3: FROM WORDS TO A PHRASE

Rearrange the words in part 2 to form a phrase.

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Module 3

MEASUREMENT

INTRODUCTION

This module presents a discussion on measurement that includes key concepts like definition of measurement, development of measurement from primitive to present times, uses and importance of measurement, systems of measurement, types of measurements such as linear measures, liquid measures (capacity/volume), and mass or weight. Conversion factors from one unit to another either within a system or from one system to another are also discussed. Measurement of time, temperature, and utilities usage (meter reading) are also included. Solving problems using the formulas on measurement of geometric figures such as perimeter, area, and volume are also explored.

OBJECTIVES

After completing this module, the learner should be able to extend concepts of measurements to include different types of measures and all the subsets of the set of real numbers to solve measurement problem. In particular, the learner should be able to:

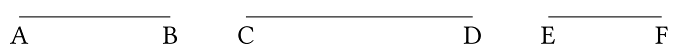
1. describe what it means to measure;
2. describe the development of measurement from the primitive to the present international system of units;
3. convert measurements from one unit to another for each type of measurement including the English system;
4. estimate or approximate the measures of quantities particularly length, weight/mass, volume/capacity, time, angle and temperature (use of instruments);
5. use appropriate instruments to measure quantities such as length, weight/mass, volume, time, angle, and temperature; and
6. solve problems involving formulas in finding perimeter, area, and volume.

DISCUSSION

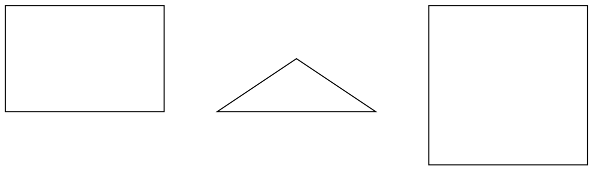
Motivation

Answer the following questions.

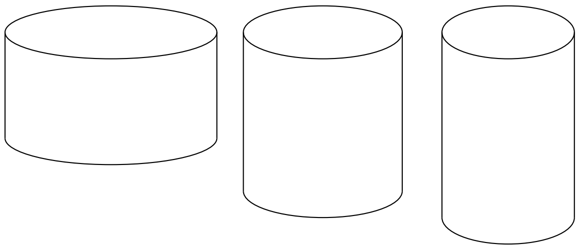
1. Which of the following line segments is the longest? Which is the shortest?



2. Which of the given figures occupy more space than the others?



3. Which of the given containers have more liquid than the others?



How do you answer these questions? To be able to answer these questions you need to compare one figure with the other figures. Then you compare also the quantities or the properties of these figures. The process of comparing one quantity with another quantity is known as **measurement**.

Definition 3.1: Measurement

Measurement is a quantitative description of a fundamental property or physical phenomenon. When measuring, comparison of an unknown quantity with a certain standard called **unit of measurement** is being done.

The science of measurement is called **metrology**. The English word *measurement* originates from the Latin *mensura* and the verb *metiri* through the Middle French *mesure*.

Historical Background

Standard units of measurement were already used in ancient times. It evolved over the course of human history to help prevent problems on fraud in commerce. Laws regulating measurement were developed so that communities would have certain common benchmarks. Generally, units of measurement are essentially arbitrary; groups of people make them up as the need arises, and then agree on certain standards when they use them.

Ancient people used human body parts (arms, hands, feet) to measure length. The width of a finger was called a **finger** or **digit**. Another unit of measure used was the **palm** which was the width of a person's four fingers excluding the thumb. A **span** was the width of three palms. The **cubit** was described as the distance from a man's elbow to the tip of his middle finger. Genesis 6: 15 of the Bible stated that Noah's ark was 300 cubits long, 50 cubits wide and 30 cubits high. One cubit was equivalent to seven palms. The width of a person's thumb was called **uncia**. One foot was equivalent to twelve uncias. The **yard**, based on the distance from the tip of a person's nose to the tip of his middle finger, was equivalent to three feet. Ancient people also used the **king's foot** as the standard unit of measure for buying, selling, and trading as long as the king ruled. A **fathom** was the distance from one outstretched hand to the other, and was equivalent to four cubits.

Eventually, size variations of the different body parts created measurement problems. Different measurements would be given for the same length using the same unit. To eliminate such confusion, international conventions were conducted to set standard units of measure.

International Treaties (Optional)

Units of measurement are generally defined on a scientific basis as overseen by governmental or supra-governmental agencies, and established in international treaties. **The General Conference on Weights and Measures (CGPM)** which was established in 1875 by the Treaty of the meter oversees the **International System of Units (SI)** and has custody of the **International Prototype Kilogram**. The **meter** was redefined in 1983 by the CGPM as the *distance traveled by light in free space in 1/299,792,458* *of a second* while in 1960 the **international yard** was defined by the governments of the United States, United Kingdom, Australia and South Africa as being *exactly* 0.9144 meters.

In the United States, the **National Institute of Standards and Technology (NIST)**, a division of the United States Department of Commerce, regulates commercial measurements. In the United Kingdom, the one doing this job is the National Physical Laboratory (NPL). In Australia it is done by the Commonwealth Scientific and Industrial Research Organization. In South Africa the one doing this is the Council for Scientific and Industrial Research, while in India the National Physical Laboratory of India does this job.

Units and Systems of Measurement

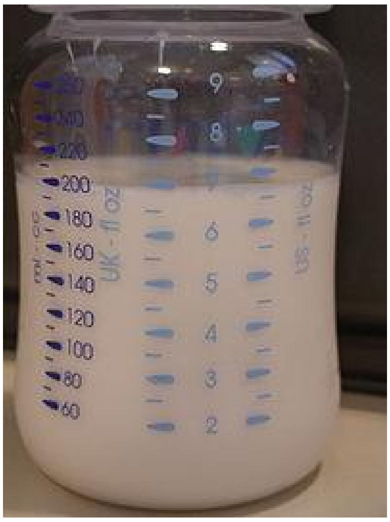


Figure 3.1

A baby bottle that measures in all three measurement systems, Imperial (U.K.), U.S. customary, and metric.

Figure 3.2

Four measuring devices having metric calibrations

Linear Measures

To measure lengths or distances, linear measures are used. An inch is the smallest and commonly used linear unit in the English system. In this system, other units such as foot, yard, and mile are used for longer distances. In the metric system the basic unit of length is the meter. The meter is the basis of other units of measurements such as the decimeter, centimeter and millimeter. To measure longer distances, the decameter, hectometer and kilometer are used.

Liquid Measures

To measure volume (the space occupied by a liquid substance), liquid measures are used. The basic liquid measures in the English system are the fluid ounce and the gallon. In the metric system, liter is the basic unit.

The Dry Measure

The dry measure is used for measuring grains, fruits, vegetables, and the like. The English system uses pints and quarts as the units for dry measures and for liquid measures. The metric system also uses the same unit for measuring both liquid and dry substances.

Mass and Weight

Mass was used by Newton to mean the quantity of matter, and it mass manifests itself gravitationally and inertially. The earth's gravitational pull on an object depends on the object's mass. The greater the mass of the object, the stronger the earth's gravitational pull on it. Mass depends only on the number and kinds of atoms that compose an object. It does not depend on location. The amount of material in an object remains the same whether it is located on Earth, on the moon or in outer space. Mass is also a measure of the inertia of an object. The greater the mass of an object, the greater is its inertia and the more force is needed to change its state of motion. Mass is measured in ounces (oz), pounds (lb) and tons in the English system, while it is measured in grams (g) and kilograma (kg) in the metric system.

Weight is described as a force. Weight is the earth's gravitational force acting on an object. It is measured in Newtons (N) like any other force. It depends on the object's location and how strongly the object is attracted by the earth's gravity. The weight of an object in a place without gravity would be zero.

Mass and weight are different from each other. Mass is the amount of matter in an object while weight is how that matter is strongly attracted by the force of gravity. Moreover, mass does not change and does not depend on the location, while weight varies with location. However, they are proportional to each other in a certain place. Greater masses have greater weights while smaller masses have smaller weights. This is the reason why in some cases, mass and weight are used interchangeably. In free fall, (no net gravitational forces) objects lack weight but retain their mass.

One device for measuring weight or mass is called a weighing *scale* or, often, simply a scale. A spring scale measures force but not mass, and a balance compares weight. Both require a gravitational field to operate. Some of the most accurate instruments for measuring weight or mass are based on load cells with a digital read-out, but require a gravitational field to function and would not work in free fall.

English Customary Weights and Measures

Linear Measure or Distance

Short distance units in all traditional measuring systems are based on the dimensions of the human body. The inch represents the width of a thumb; in many languages, the word for "inch" is also the word for "thumb." The foot (12 inches) was originally the length of a human foot, but has evolved to be longer than most people's feet. The yard (3 feet) was used in England as the name of a 3-foot measuring stick, but it is also understood to be the distance from the tip of the nose to the end of the middle finger of the outstretched hand. If you stretch your arms out to the sides as far as possible, your total "arm span," from one fingertip to the other, is a fathom (6 feet).

Based on history, many other "natural units" of the same kind were used such as the following: the digit (the width of a finger, 0.75 inch), the nail (length of the last two joints of the middle finger, 3 digits or 2.25 inches), the palm (width of the palm, 3 inches), the hand (4 inches), the shaftment (width of the hand and outstretched thumb, 2 palms or 6 inches), the span (width of the outstretched hand, from the tip of the thumb to the tip of the little finger, 3 palms or 9 inches), and the cubit (length of the forearm, 18 inches).

Mass/Weight

The basic traditional unit of weight, the pound, originated as a Roman unit and was used throughout the Roman Empire. The Roman pound was divided into 12 ounces, but many European merchants preferred to use a larger pound of 16 ounces, perhaps because a 16-ounce pound is conveniently divided into halves, quarters, or eighths. During the Middle Ages there were many different pound standards in use, some of 12 ounces and some of 16. The use of these weight units naturally followed trade routes, since merchants trading along a certain route had to be familiar with the units used at both ends of the trip.

The oldest English weight system has been used since the time of the Saxon kings. It is based on the 12-ounce troy pound, which provided the basis on which coins were minted and gold and silver were weighed. Since Roman coins were still in circulation in Saxon times, the troy system was designed to model the Roman system directly. The troy pound weighs 5760 grains, and the ounces weigh 480 grains. Twenty pennies weighed an ounce, and therefore a pennyweight is $480/20 = 24$ grains. The troy system continued to be used by jewelers and also by druggists until the nineteenth century. Even today gold and silver prices are quoted by the troy ounce in financial markets everywhere.

Since the troy pound was smaller than the commercial pound units used in most of Europe, medieval English merchants often used a larger pound called the "mercantile" pound (*libra mercatoria*). This unit contained 15 troy ounces, so it weighed 7200 grains. This unit seemed about the right size to merchants, but its division into 15 parts, rather than 12 or 16, was very inconvenient. Around 1300 the mercantile pound was replaced in English commerce by the 16-ounce avoirdupois pound. This is the pound unit still in common use in the U.S. and Britain. Modeled on a common Italian pound unit of the late thirteenth century, the avoirdupois pound weighs exactly 7000 grains. The avoirdupois ounce, 1/16 pound, is divided further into 16 drams.

Unfortunately, the two English ounce units don't agree: the avoirdupois ounce is $7000/16 = 437.5$ grains while the troy ounce is $5760/12 = 480$ grains. Conversion between troy and avoirdupois units is so awkward, no one wanted to do it. The troy system quickly became highly specialized, used only for precious metals and for pharmaceuticals, while the avoirdupois pound was used for everything else.

Liquid Measure or Capacity or Volume

The names of the traditional volume units are the names of standard containers. Until the eighteenth century, it was very difficult to measure the capacity of a container accurately in cubic units, so the standard containers were defined by specifying the weight of a particular substance, such as wheat or beer, that they could carry. Thus the gallon, the basic English unit of volume, was originally the volume of eight pounds of wheat. This custom led to a multiplicity of units, as different commodities were carried in containers of slightly different sizes.

Gallons are always divided into 4 quarts, which are further divided into 2 pints each. For larger volumes of dry commodities, there are 2 gallons in a peck and 4 pecks in a bushel. Larger volumes of liquids were carried in barrels, hogsheads, or other containers whose size in gallons tended to vary with the commodity, with wine units being different from beer and ale units or units for other liquids.

For liquids Americans preferred to use the traditional British wine gallon, which Parliament defined to equal exactly 231 cubic inches in 1707. As a result, the U.S. volume system includes both "dry" and "liquid" units, with the dry units being about 1/6 larger than the corresponding liquid units.

On both sides of the Atlantic, smaller volumes of liquid are traditionally measured in fluid ounces, which are at least roughly equal to the volume of one ounce of water. To accomplish this in the different systems, the smaller U.S. pint is divided into 16 fluid ounces, and the larger British pint is divided into 20 fluid ounces.

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Because of their many eccentricities, English customary units clearly are more cumbersome to use than metric units in trade and in science. As metrication proceeds, they are less and less in use. On the other hand, these traditional units are rich in cultural significance. We can trace their long histories in their names and relationships.

The units of measure of length, volume and mass /weight in English system are given in the Table (3.1) below. These units are used as conversion factors from one unit of measure to another.

Table 3.1

English System of Measurement

Linear Measures	Equivalent Unit Measure
12 inches (in)	1 foot (ft)
3 feet	1 yard (yd)
$5\frac{1}{2}$ yards	1 rod
40 rods	1 furlong
8 furlongs	1 statute mile
3 miles	1 league
3 inches	1 palm
4 inches	1 hand
6 inches	1 span
18 inches	1 cubit
21.8 inches	1 Bible cubit
$2\frac{1}{2}$ feet	1 military pace
5, 280 feet or 1,760 yards	1 mile
Liquid Measures	**Equivalent Unit Measures**
2 cups (c)	1 pint
2 pints (pt)	1 quart
4 quarts (qt)	1 gallon (gal)
3 teaspoons (tsp)	1 tablespoon (tbs)
8 fluid ounces (fl. oz)	1 cup
16 fluid ounces	1 pint
Dry Measures	**Equivalent Unit Measures**
(Used for measuring grains, fruits, vegetables, etc.)	
2 pints	1 quart
8 quarts	1 peck(pk)
4 pecks	1 bushel (bu)
4 quarts	1 gallon
Measures of Mass /Weight	**Equivalent Unit Measures**
Avoirdupois Weight	
27 11/32 grams	1 dram
16 drams	1 ounce (oz)
28.3495 grams	1 ounce
16 ounces	1 pound (lb)
454 grams	1 pound
28 pounds	1 quarter (qtr)
4 quarters	1 cwt
2,000 pounds	1 short ton
2,240 pounds	1 long ton
Troy Weight	**Equivalent Unit Measure**
24 grains	1 pennyweight (pwt)
20 pennyweight	1 ounce
12 ounces	1 pound
(Used for weighing gold, silver and jewels)	

The Metric System

The metric system is a decimal system of measurement based on its units for length, the meter and for mass, the kilogram. It exists in several variations, with different choices of base units. Since the 1960s, the International System of Units (SI) is the internationally recognized metric system. Metric units of mass, length, and electricity are widely used around the world for both everyday and scientific purposes.

The metric system features a single base unit for many physical quantities. Other quantities are derived from the standard SI units. Multiples and fractions of the units are expressed as powers of 10 of each unit. Unit conversions are always simple because they are in the ratio of ten, one hundred, one thousand, etc, so that convenient magnitudes for measurements are achieved by simply moving the decimal place: 1.234 meters is 1234 millimeters or 0.001234 kilometers. The use of fractions, such as 2/5 of a meter, is not prohibited, but uncommon. All lengths and distances, for example, are measured in meters, or thousandths of a meter (millimeters), or thousands of meters (kilometers). There is no profusion of different units with different conversion factors as in the Imperial system which uses, for example, inches, feet, yards, fathoms, rods.

The metric system replaces all the traditional units, except the units of time and of angle measure, with units satisfying three conditions:

1. One fundamental unit is defined for each quantity. These units are now defined precisely in the International System of Units.
2. Multiples and fractions of these fundamental units are created by adding prefixes to the names of the defined units. These prefixes denote powers of ten, so that metric units are always divided into tens, hundreds, thousands, etc. The original prefixes included milli- for 1/1,000, centi- for 1/100, deci- for 1/10, deka- for 10, hecto- for 100, and kilo- for 1,000.
3. The fundamental units are defined rationally and are related to each other in a rational fashion.

The metric units were defined in a different way from any traditional units of measure. The Earth was selected as the "measuring stick". The meter was defined to be one ten-millionth of the distance from the Equator to the North Pole. The liter was to be the volume of one cubic decimeter, and the kilogram was to be the weight of a liter of pure water.

The metric system was first proposed in 1791. It was adopted by the French revolutionary assembly in 1795, and the first metric standards (a standard meter bar and kilogram bar) were adopted in 1799. There was considerable resistance to the system at first, and its use was not made compulsory in France until 1837. The first countries to actually require use of the metric system were Belgium, the Netherlands, and Luxembourg, in 1820.

The units of measure of length, volume and mass/weight in metric system are given in the Table (3.2) below. These units are used as conversion factors from one unit of measure to another. Dekameter decameter

International System of Units

The International System of Units (abbreviated as SI from the French language name *Système International d'Unités*) is the modern revision of the metric system. It is the world's most widely used system of units, both in everyday commerce and in science. The SI was developed in 1960 from the meter-kilogram-second (MKS) system, rather than the centimeter-gram-second (CGS) system, which, in turn, had many variants. During its development, the SI also introduced several newly named units that were previously not a part of the metric system. The original SI units for the six basic physical quantities were:

meter (m)	SI unit of length
second (s)	SI unit of time
kilogram (kg)	SI unit of mass
ampere (A)	SI unit of electric current
degree Kelvin (K)	SI unit of thermodynamic temperature
candela (cd)	SI unit of luminous intensity

The mole was subsequently added to this list and the degree Kelvin renamed the kelvin.

There are two types of SI units, base units and derived units. Base units are the simple measurements for time, length, mass, temperature, amount of substance, electric current and light intensity. Derived

Table 3.2

Metric System

Linear Measures	Equivalent Unit Measure
10 millimeters (mm)	1 centimeter (cm)
10 centimeters	1 decimeter (dm)
10 decimeters	1 meter (m)
100 centimeters	1 meter
1,000 millimeters	1 meter
10 meters	1 decameter (dkm)
10 decameters	1 hectometer (hm)
100 meters	1 hectometer
10 hectometers	1 kilometer (km)
1,000 meters	1 kilometer

Liquid Measures	Equivalent Unit Measures
10 milliliters (ml)	1 centiliter (cl)
10 centiliters	1 deciliter (dl)
10 deciliters	1 liter (l)
100 centiliters	1 liter
1,000 milliliters	1 liter
10 liters	1 decaliter (dkl)
10 decaliters	1 hectoliter (hl)
10 hectoliters	1 kiloliter (kl)
1 cubic centimeter (cc)	1 milliliter (ml)
1,000 cubic centimeters	1,000 milliliters
1,000 milliliters	1 liter
1,000 liters	1 cubic meter (m^3)

Measures of Mass /Weight	Equivalent Unit Measures
10 milligrams	1 centigram (cg)
10 centigrams	1 decigram (dg)
10 decigrams	1 gram (g)
100 centigrams	1 gram
1,000 milligrams	1 gram
10 grams	1 decagram (dkg)
10 decagrams	1 hectogram (hg)
100 grams	1 hectogram
10 hectograms	1 kilogram (kg)
100 kilograms	1 quintal (ql)
10 quintals	1 ton (t)

units are constructed from the base units, for example, the watt, i.e. the unit for power, is defined from the base units as $m^2 \cdot kg \cdot s^{-3}$. Other physical properties may be measured in compound units, such as material density, measured in kg/m^3.

Converting Prefixes

The SI allows easy multiplication when switching among units having the same base but different prefixes. To convert from meters to centimeters it is only necessary to multiply the number of meters by 100, since there are 100 centimeters in a meter. Inversely, to switch from centimeters to meters one multiplies the number of centimeters by 0.01 or divide centimeters by 100.

Length

Figure 3.3

A 2-metre carpenter's ruler

A ruler or rule is a tool used in, for example, geometry, technical drawing, engineering, and carpentry, to measure lengths or distances or to draw straight lines. Strictly speaking, the *ruler* is the instrument used to **rule** straight lines and the calibrated instrument used for determining length is called a *measure*, however common usage calls both instruments *rulers* and the special name *straightedge* is used for an unmarked rule. The use of the word *measure*, in the sense of a measuring instrument, only survives in the phrase *tape measure*, an instrument that can be used to measure but cannot be used to draw straight lines. As can be seen in the photographs on this page, a two-meter carpenter's rule can be folded down to a length of only 20 centimeters, to easily fit in a pocket, and a five-meter-long tape measure easily retracts to fit within a small housing.

Some Special Names

We also use some special names for some multiples of some units as in Table (3.3)

Table 3.3

SI Special Names

100 kilograms	1 quintal
1000 kilogram	1 metric tonne
10 years	1 decade
100 years	1 century
1000 years	1 millennium

Difficulties

Since accurate measurement is essential in many fields, and since all measurements are necessarily approximations, a great deal of effort must be taken to make measurements as accurate as possible. For example, consider the problem of measuring the time it takes an object to fall a distance of one meter (about 39 in). Using physics, it can be shown that, in the gravitational field of the Earth, it should take any object about 0.45 second to fall one meter. However, the following are just some of the sources of error that arise:

1. This computation used for the acceleration of gravity 9.8 meters per second squared (32 ft/s^2). But this measurement is not exact, but only precise to two significant digits.
2. The Earth's gravitational field varies slightly depending on height above sea level and other factors.
3. The computation of 0.45 second involved extracting a square root, a mathematical operation that required rounding off to some number of significant digits, in this case two significant digits.

So far, we have only considered scientific sources of error. In actual practice, dropping an object from a height of a meter stick and using a stopwatch to time its fall, we have other sources of error:

1. Most common, is simple carelessness.
2. Determining the exact time at which the object is released and the exact time it hits the ground. There is also the problem that the measurement of the height and the measurement of the time both involve some error.
3. Air resistance

Scientific experiments must be carried out with great care to eliminate as much error as possible, and to keep error estimates realistic.

Metric-English Relationship

Table (3.4) shows the comparison between the Metric and English Systems of Measurements.

Conversion from One Unit to Another

The following illustrative examples show how to convert one measure from one unit to another.

English Units of Measure

Example 3.1

1. Convert 7 yards to feet. Conversion factor: 1 yd = 3 ft

   Solution: $7\,\text{yd} \cdot \dfrac{3\,\text{ft}}{1\,\text{yd}} = \boxed{21\,\text{ft}}$

2. Convert 72 inches to feet. Conversion factor: 1 ft = 12 in

   Solution: $72\text{in} \cdot \dfrac{1\text{ft}}{12\,\text{in}} = \boxed{6\,\text{ft}}$

3. Convert 14.25 gallons to quarts. Conversion factor: 1 gal = 4 qt

   Solution: $14.2\,\text{gal} \cdot \dfrac{4\,\text{qt}}{1\,\text{gal}} = \boxed{57\,\text{qt}}$

Metric Units of Measure

Example 3.2

1. Convert 8 meters to centimeters. Conversion factor: 1 m = 100 cm

   Solution: $8\,\text{m} \cdot \dfrac{100\,\text{cm}}{1\,\text{m}} = \boxed{800\,\text{cm}}$

2. Convert 3 liters to millimeters. Conversion factor: 1 L = 1000 mL

   Solution: $3\,\text{L} \cdot \dfrac{1000\,\text{mL}}{1\,\text{L}} = \boxed{3,000\,\text{mL}}$

Table 3.4

Metric-English Conversions

Linear Measures	Equivalent Measure
Metric	English
1 millimeter	0.04 inches
1 centimeter	0.39 inches
1 decimeter	3.94 inches
1 meter	39.37 inches
1 meter	3.28 feet
1 meter	1.09 yards
1 decameter	32.81 feet
1 hectometer	328.08 feet
1 kilometer	3,280.80 feet
1 kilometer	0.62 mile
25.40 millimeter	1 inch
2.54 centimeters	1 inch
0.03 meter	1 inch
0.30 meter	1 foot
0.91 meter	1 yard
1.61 kilometers	1 mile

Liquid Measures	Equivalent Measures
Metric	English
1 milliliter	0.03 fluid ounce
1 centiliter	0.34 fluid ounce
1 deciliter	3.38 fluid ounce
1 liter	1.06 liquid quarts
1 liter	0.91 dry quart
1 decaliter	2.64 gallons
1 hectoliter	26.42 gallons
1 kiloliter	264.18 gallons
0.95 liter	1 liquid quart
1.10 liters	1 dry quart
3.80 liter	1 gallon

Measures of Mass	Equivalent Measures
Metric	English
1 milligram	0.02 grain
1 centigram	0.15 grain
1 decigram	1.54 grains
1 gram	0.04 ounce
1 decagram	0.35 ounce
1 hectogram	3.53 ounces
1 kilogram	2.20 pounds
1 metric ton	2,204.62 pounds
1 metric ton	1.10 tons
28.35 grams	1 ounce
0.45 kilogram	1 pound
0.91 metric ton	1 ton

3. Convert 250 millimeters to decimeters. Conversion factors: $1\,\text{cm} = 10\,\text{mm}$, $1\,\text{dm} = 10\,\text{cm}$, hence, $1\,\text{dm} = 100\,\text{m}$

   $$\text{Solution: } 250\,\text{mm} \cdot \frac{1\,\text{cm}}{10\,\text{mm}} \cdot \frac{1\,\text{dm}}{10\,\text{cm}} = \boxed{2.5\,\text{dm}}$$

4. Convert 4,625 centimeters to meters. Conversion factor: $1\,\text{m} = 100\,\text{cm}$

Solution: $4,625\,\text{cm} \cdot \dfrac{1\,\text{m}}{100\,\text{cm}} = \boxed{46.25\,\text{m}}$

Metric to English and Vice Versa

Example 3.3

1. Convert 12 meters to yards. Conversion factor: $1\,\text{m} = 1.09\,\text{yd}$

   Solution: $12\,\text{m} \cdot \dfrac{1.09\,\text{yd}}{1\,\text{m}} = \boxed{13.08\,\text{yd}}$

2. Convert 10 miles to meters. Conversion factor: $1\,\text{mi} = 1.61\,\text{km}$, $1\,\text{km} = 1000\,\text{m}$

   Solution: $10\,\text{mi} \cdot \dfrac{1.61\,\text{km}}{1\,\text{mi}} \cdot \dfrac{1000\,\text{m}}{1\,\text{km}} = \boxed{16,100\,\text{m}}$

3. Convert 400 oz to grams. Conversion factor: $1\,\text{oz} = 28.35\,\text{g}$

   Solution: $400\,\text{oz} \cdot \dfrac{28.35\,\text{g}}{10\,\text{oz}} = \boxed{11,340\,\text{g}}$

Exercises

A. Write the abbreviations of the following units of measure.

1. foot
2. meter
3. inch
4. hectometer
5. millimeter
6. kilometer
7. yard
8. mile
9. gallon
10. pint
11. liter
12. pound
13. ounces
14. grain
15. millimeter

B. Write either weight, linear, time, or capacity in each blank to denote the type of measure.

1. foot ________
2. gallon ________
3. pint ________
4. yard ________
5. gram ________
6. cup ________
7. liter ________
8. peck ________
9. day ________
10. century ________

C. Fill in the blank with the correct number.

1. ________ ft = 1 yd
2. ________ t = 1 yd
3. ________ dm = 1 m
4. ________ in = 1 yd
5. ________ ft = 1 mi
6. ________ m = 1 dkm
7. ________ cm = 1 dm
8. ________ dm = 1 km

D. Convert. Use mixed numbers or decimals if necessary.

1. 24 yd = ________ ft
2. 6 mi = ________ yd
3. 72 in = ft
4. 17 ft = ________in
5. 7 mi = ________ft
6. 1/4 mi = ________yd
7. 90 in = ________yd
8. 7 ft = ________yd
9. 6 mi = ________in
10. 14 m = ________dm
11. 85 km = ________dkm
12. 12 mm = ________m
13. 8 m = ________cm
14. 1hm = ________m
15. 45 m = ________hm
16. 83.5 km = ________hm
17. 47 dm = ________m
18. 145 km = ________m
19. 15 ft = ________in
20. 3/4 ft = ________in
21. 1.3 mi = ________ft
22. 380,160 = ________mi

Perimeter

Perimeter is the measure of the total distance around a plane figure. To find the perimeter of any polygon, the lengths of the sides are added. There are formulas that are used to make the computation easier and faster.

Table (3.5) shows most common shapes and the formula to find the perimeter of each figure.

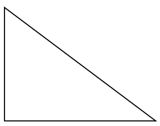
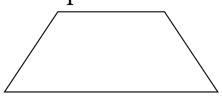
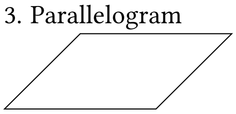
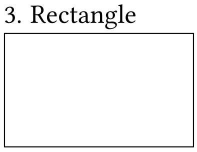
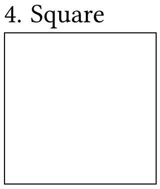
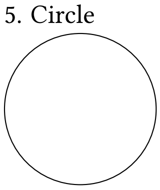
Figure	Formula	Description
1. Triangle 	$P = a + b + c$	a, b, and c are the lengths of the sides
2. Trapezoid 	$P = b_1 + b_2 + s_1 + s_2$	b_1 and b_2 are the bases or parallel sides while s_1 and s_2 are the nonparallel sides
3. Parallelogram 	$P = 2l + 2w$	l is the length and w is the width of the parallelogram
3. Rectangle 	$P = 2L + 2W$	L is the length and W is the width of the rectangle
4. Square 	$P = 4s$	s is the length of side of the square
5. Circle 	$C = 2\pi r = \pi d$	r is the radius of the circle and d is the diameter; $\pi \approx 3.14$

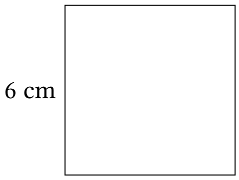
Table 3.5

Note: A circle is a plane figure whose perimeter is known as the circumference of the circle.

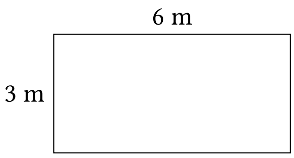
Exercises

Find the perimeter of the following.

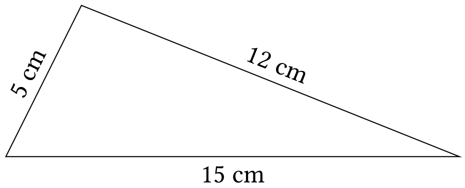
1. Square



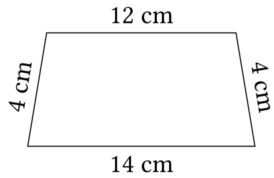
2. Rectangle



3. Triangle



4. Isosceles Trapezoid



Area

The area of a plane figure is the total number of unit regions occupied or contained in the given plane figure. The unit region is expressed in square units.

Example 3.4

1. The rectangle below has width = 3 units, length = 5 units. There are 15 square units in the figure. Therefore, the area of the rectangle is 15 square units.

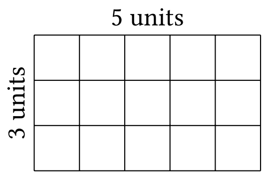


Table (3.6) shows most common shapes and the formula to find the area of each figure.

Table 3.6

Formulas for Areas of Some Common Shapes

1. Triangle	$A = \frac{bh}{2}$	A is the area, b is the base and h is the height or altitude
2. Square	$A = s^2$	s is the length of the side
3. Rectangle	$A = lw$	l is the length and w is the width
4. Parallelogram	$A = bh$	b is the base and h is the height or altitude
5. Trapezoid	$A = \frac{hb_1 + b + 2}{2}$	h is the altitude and b_1 and b_2 are the bases
6. Circle	$A = \pi r^2$	r is the radius of the circle; $\pi \approx$ 3.14

Exercises

Find the area of the following figures:

1. A square whose side is 9 cm.
2. A rectangle whose width is 2 m and whose length is 5 m.
3. A triangle whose height is 2 cm and whose base is 8 cm.
4. A circle whose diameter is 20 ft.
5. A trapezoid with a height of 8 cm and bases 6 cm and 9 cm.

Volume

If we want to find how much a box or any container will hold, we need a measurement of a space. Space involves three dimensions: length, width and height. Any figure that represents a space is three-dimensional and the measurement of this space is called volume. The volume is measured in cubic units.

The standard English cubic measures are the cubic inch, cubic foot and the cubic yard.

1 cubic foot (cu. ft) = 1,728 cubic inches (cu. in)
1 cubic yard (cu. yd) = 27 cubic feet (cu. ft)

The standard metric cubic measures are the cubic centimeters and the cubic meters.

Table (3.7) shows the formula that are used to solve for the volume of some regular solids.

Time Measure

Table (3.8) shows the units used to measure time.

Time reading is done through a clock with 12 numbers on it. The two hands are called the minute hand and the hour hand. The hour hand indicates the hour reading and the minute hand represents the minute reading. Some clock has a second hand that reads the second.

Temperature

There are two thermometer scales that are commonly used to measure temperature. These are the Fahrenheit (F) and the Centigrade or Celsius (C). The two scales give temperature readings in degrees (°).

Example 3.5

1. 45° F is read as "forty-five degrees Fahrenheit"
2. 20° C is read as "twenty degrees Centigrade"

The freezing point of water on the Fahrenheit scale is 32° F while the boiling point of water is 212° F. On the Centigrade scale, the freezing point of water is 0° C while the boiling point is 100°. The thermometer is the instrument used to measure temperature.

To change one unit of measure to another unit, we use formulas. The formula used to change Fahrenheit to Centigrade or Celsius is

$$C = \frac{5}{9}F - 32 \tag{3.1}$$

Example 3.6

Table 3.7

Formulas for Volumes of Regular Solids

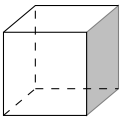
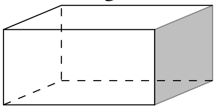
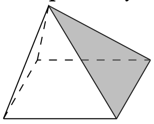
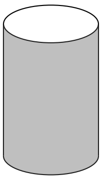
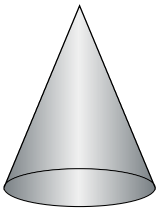
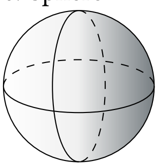
Figure	Formula	Meaning of Symbols
1. Cube 	$V = s^3$	V = volume; s = length of edge
2. Rectangular solid 	$V = lwh$	V = volume; l = length; w = width; h = height
3. Square Pyramid 	$V = \frac{1}{3}lwh$	V = volume; l = length; w = width; h = height
4. Circular Cylinder 	$V = \pi r^2 h$	V = volume; r = radius; h = height
5. Cone 	$V = \frac{1}{3}\pi r^2 h$	V = volume; r = radius; h = height
6. Sphere 	$V = \frac{4}{3}\pi r^3$	V = volume; r = radius; $\pi \approx 3.14$

Table 3.8

Units of Time

Time Measures	Equivalent Unit Measure
60 seconds	1 minute
60 minutes	1 hour
24 hours	1 day
7 days	1 week
28,29,30 or 31 days	1 calendar month
30 days	1 month (computing interest)
365 days	1 year
366 days	1 leap year

1. Change 59° F to ° C.

Solution:

$$
\begin{aligned}
C &= \frac{5}{9}F - 32 \\
&= \frac{5}{9}27 \\
&= \boxed{15}
\end{aligned}
$$

Therefore, 59° F = 15° C.

To change Centigrade to Fahrenheit, we use the formula

$$F = \frac{9}{5}C + 32 \tag{3.2}$$

Example 3.7

1. Change 20° C to ° F

   Solution:

$$
\begin{aligned}
F &= \frac{9}{5}C + 32 \\
&= \frac{9}{5}20 + 32 \\
&= 36 + 32 \\
&= \boxed{68}
\end{aligned}
$$

Therefore, 20° C = 68° F.

Exercises

A. Change each of the following to ° C.

1. 72° F.
2. 120° F.
3. 160° F.
4. 84° F.
5. 98° F.
6. 130° F.

B. Change each of the following to ° F.

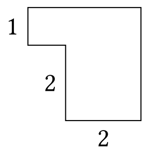
1. 12° C.
2. 30° C.
3. 35° C.
4. 94° C.
5. 98° C.
6. 75° C.

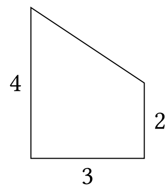
WORKSHOP ACTIVITIES

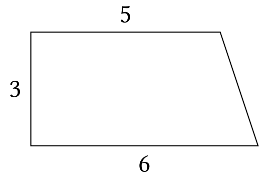
Activity 1: Exercises on Problem Solving Involving Measurements

1. If one side of a regular pentagon is 7 cm long, what is its perimeter? (Hint: A regular pentagon has 5 equal sides.)
2. The diameter of a bicycle tire is 50 cm. What is the circumference of the tire?
3. The perimeter of a square is 48 cm. How long is each side?
4. A large pizza has a circumference of 215.2 cm. What is its diameter?
5. The diameter of a wheel is 72 cm. Find its circumference.

   Find the area of each of the following by subdividing it into common shapes.

6. 

7. 

8. 

9. Which is warmer, 48° Cor 112° F?
10. Which is colder, 18° Cor 53 ° F?
11. What is the melting point of gold in ° Fif its melting point is 1,115 ° C?
12. What is the melting point of silver in ° Cif its melting point is 1,814 ° F?

Activity 2: Math Trail

Objective:

To apply measurement skills, conversion, and problem solving in real-life situations.

Preparation:

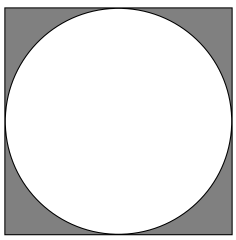
Group the participants into groups. Each group is oriented on the rules of the “Math Trail”/Amazing Race”.

Instructions:

Using the same materials (e.g. ruler, meterstick, protractor, calculator), each group will race in a specified area (an auditorium or field) to do tasks (e.g. measure, compute) in a given time limit.

Sample Task 1:

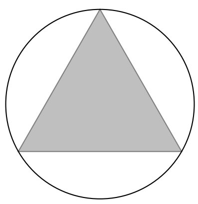
Assume that the figures drawn are a square and a circle. Find the area of the shaded region rounded to the nearest mm^2.



Sample Task 2:

Assume that the figures drawn are a triangle and a circle.

Find the volume of the cylinder whose base radius is the same as the circumference of the circle and whose height is the same as the perimeter of the triangle (rounded to the nearest mL).



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Module 4

ALGEBRAIC MANIPULATIONS AND COMMON MISCONCEPTIONS

INTRODUCTION

This module presents common misconceptions of students in using GEMDAS in their algebra classes, and some mental math and shortcuts in addition, subtraction, multiplication and division. Also, we review the properties of real numbers as a field, rules on exponents, and hierarchy of operations. Examples, exercises, and oral assessment are also given at the end of the module.

OBJECTIVES

After completing this module, you should be able to:

1. Recognize properties of real numbers and use them correctly;
2. Recognize properties of exponents and use them correctly;
3. Remember GEMDAS and apply it in simplifying numeric and algebraic expressions;
4. Learn and be aware of the common misconceptions and errors in using GEMDAS and simplifying numeric and algebraic expressions; and
5. Acquire new tricks and shortcuts in mental math.

DISCUSSION

Before we discuss some of the common misconceptions in using GEMDAS, it is just right to go over with some theoretical background and discussions that covers these misconceptions.

Review on Properties of Real Numbers

The properties of real numbers can be used to rewrite algebraic expressions. The properties are true for variables and algebraic expressions as well as for real numbers.

At this point, you are advised to review the properties of real numbers by revisiting Chapter 2.

Properties of Exponents

Just as multiplication by a positive integer can be described as repeated addition, *repeated multiplication* can be written in what is called **exponential form**. Let n be a positive integer and let a be a real number. Then the product of n factors of a is given by

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n\ a\text{'s}} \qquad n \text{ factors, } a \text{ is the base and } n \text{ is the } \textbf{exponent}. \tag{4.1}$$

Let a and b represent real numbers, and let m and n be integers. Then the properties in Table (4.1) are always true.

Table 4.1

Properties of Exponents

Property	Example
$a^m a^n = a^{m+n}$	$x^5 x^4 = x^{5+4} = x^9$
$ab^m = a^m b^m$	$2x^3 = 2^3 x^3 = 8x^3$
$a^{m n} = a^{mn}$	$x^{23} = x^{2\cdot 3} = x^6$
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{x^6}{x^2} = x^{6-2} = x^4, x \neq 0$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$

Historical Note

Originally, Arabian mathematicians used their words for colors to represent quantities (*cosa*, *censa*, *cubo*). These words were eventually abbreviated to *co*, *ce*, *cu*. **Rene Descatrtes** (1596-1650) simplified this even further by introducing the symbols x, x^2 and x^3.

Order of Operations

The order of operations is a set of rules that tells what sequence to use when simplifying expressions that contain more than one operation. Grouping symbols include parentheses , brackets [], braces {}, fraction bars, radical symbols, and absolute value symbols.

Table 4.2

Order of Operations

First:	Perform operations inside grouping symbols.
Second:	Simplify powers.
Third:	Perform multiplication and division from left to right.
Fourth:	Perform addition and subtraction from left to right.

If an expression contains more than one grouping symbol, simplify the innermost set first. Within each set, follow the order of operations.

Remember that you have to use the order of operations: **G**rouping, **E**xponents, **M**ultiplication/**D**ivision, and **A**ddition/**S**ubtraction (**GEMDAS**).

EXERCISES

Simplify the following expressions.

1. $14 + 125 \times 4 - 125/5$
2. $14 + 3 \times 14 - 5$
3. $14 + 18 \times 9 - 30/5$
4. $2 + 18 \times 32 - 40/2$
5. $10 + 8^2 - 6$
6. $2 + 84 \times 9 - 112/7$
7. $13 + 12 \times 5 - 25/5$
8. $12 + 2 \times 216 - 108/9$
9. $16 + 75 \times 8 - 195/3$
10. $7 + 12 \times 4 - 56/8$

Common Misconceptions in Calculation

Two of the most basic problems in elementary algebra are simplifying numerical expressions and evaluating algebraic expressions. Simplifying a numerical expression means performing the indicated operations in proper sequence to obtain a single number. Evaluating an algebraic expression consists of replacing each of the variables in the expression with given numbers and simplifying the resulting numerical expression.

The following examples show common exercises that students encounter early in many algebra textbooks.

Example 4.1

1. Simplify $-2 + 54 - 6^2$

$$\boxed{\begin{aligned} -2 + 54 - 6^2 \\ -2 + 5{-}2^2 \\ -2 + 54 \\ -2 + 20 \\ 18 \end{aligned}}$$

2. Evaluate $5x^2 - 2xy$ using $x = -2$ and $y = -3$.

$$\boxed{\begin{aligned} 5{-}2^2 - 2{-}2{-}3 \\ 54 - 12 \\ 20 - 12 \\ 8 \end{aligned}}$$

Marquis (1988) collected some of the common misconceptions and errors in using GEMDAS and described the universality of a certain set of errors made by students who are attempting to transform algebraic expressions. She provided a list of twenty-two such errors in Figure (4.1). Now, let us demonstrate misconceptions in using GEMDAS and incorrect use of algebra in the following cases, taken from Larson and Hostetler (2010), *Common Algebra and Trigonometry Mistakes made in Calculus* (2012), Dawkins (2012) and Merlin (2008). **Dividing by Zero** Everyone knows that $\dfrac{0}{2} = 0$, the problem is that far too many students say that $\frac{2}{0} = 0$ or $\frac{2}{0} = 2$. Remember that division by zero is undefined. **Bad/Lost/Assumed Parenthesis** There are a lot of errors that students commonly make here. The first error is that students get lazy and decide that parenthesis are not needed at certain steps or tend to forget about them in the very next step. The other error is that students sometimes don't understand just how important parentheses really are as often seen in errors made in exponentiation.

Example 4.2

1. Square $4x$.

Correct	Incorrect
$4x^2 = 4^2x^2 = 16x^2$	$4x^2$

When dealing with exponents remember that only the quantity immediately to the left of the exponent gets the exponent. So in the incorrect case, x is the quantity immediately to the left of the exponent, so we are squaring only x, and 4 is not squared. In the correct case, the parenthesis is immediately to the left of the exponent so this signifies that everything inside the parenthesis should be squared.

2. Square -3.

$$\begin{aligned} -3^2 &\neq -3^2 \\ -3{-}3 &\neq -33 \\ 9 &\neq -9 \end{aligned}$$

Figure 4.1

Common misconceptions and errors in using GEMDAS

1. $|-3| = -3$
2. $3^2 \cdot 3^3 = 9^5$
3. $a^2 \cdot a^5 = ab^7$
4. $x + y - 3z + w = x + y - 3z + w$
5. $\frac{r}{4} - \frac{6-s}{2} = \frac{r-12-2s}{4}$
6. $3a + 4b = 7ab$
7. $3x^{-1} = \frac{1}{3x}$
8. $\sqrt{x^2+y^2} = x + y$
9. $\frac{x+y}{x+z} = \frac{y}{z}$
10. $\frac{1}{x-y} = \frac{-1}{x+y}$
11. $\frac{x}{y} + \frac{r}{s} = \frac{x+r}{y+s}$
12. $x\left(\frac{a}{b}\right) = \frac{ax}{bx}$
13. $\frac{xa+xb}{x+xd} = \frac{a+b}{d}$
14. $\sqrt{-x}\sqrt{-y} = \sqrt{xy}$
15. If $22 - z < 12$ then $z < -4$
16. $\frac{1}{1-\frac{x}{y}} = \frac{y}{1-x}$
17. $a^2 \cdot a^5 = a^{10}$
18. $3a^4 = 3a^4$
19. $\frac{a}{b} - \frac{b}{a} = \frac{a-b}{ab}$
20. $x + 4^2 = x^2 + 16$
21. $\frac{r}{4} - \frac{6-s}{4} = \frac{r-6-s}{4}$
22. $a^{25} = a^7$

Remember that exponent comes before the negative symbol. So, on the right side, only 3 gets squared before negated.

3. Convert $\sqrt{7x}$ to fractional exponents.

$$\sqrt{7x} = 7x^{\frac{1}{2}} \qquad \sqrt{7x} = 7x^{\frac{1}{2}}.$$

Improper Distribution Be careful when using distribution property, two errors that are common to students are as follows.

Example 4.3

1. Multiply $42x^2 - 10$.

Correct	Incorrect
$42x^2 - 10 = 8x^2 - 40$	$42x^2 - 10 = 8x^2 - 10$

Make sure you distribute the 4 all the way through the parenthesis. Too often students just multiply the first term by the 4 and ignore the second term, especially when the second term is just a number.

2. Multiply $32x - 5^2$.

Correct

$$\begin{aligned} 32x - 5^2 &= 34x^2 - 20x + 25 \\ &= 12x^2 - 60x + 75 \end{aligned}$$

Incorrect

$$\begin{aligned} 32x - 5^2 &= 6x - 15^2 \\ &= 36x^2 - 180x + 225 \end{aligned}$$

Remember that exponentiation must be performed before you distribute any coefficients through the parenthesis.

The Mad Slasher The Mad Slasher is when, in the haste of the moment, start crossing out similar looking expression on the numerator and denominator. This mistake is understood by realizing that the operation you are doing when cancelling is division; and when you cancel the denominator with a single term you are essentially only dividing that single term by the denominator.

Some bad examples of mad slashing are:

Example 4.4

1. $\dfrac{4+8}{4} = \dfrac{12}{4} = 3$ definitely does not equal $\dfrac{\cancel{4}+8}{\cancel{4}} = 1 + 8 = 9$.
2. $\dfrac{x+4}{x} = \dfrac{\cancel{x}+4}{\cancel{x}} = 4$.
3. $\dfrac{x-3x-x^2-x^2x+1}{x-3} = \dfrac{\cancel{x-3}x-x^2x+1}{\cancel{x-3}} = x - x^2x + 1$

Linear Dysfunction Disorder (LDD) Much like in the Mad Slasher case, an instinctual desire to simplify as much as possible can lead the students to a case of LDD, which is an incorrect use of linearity properties. The following are some examples of LDD.

Incorrect	Correct
$x + 9^2 = x^2 + 9^2 = x^2 + 81$	$\begin{aligned} x + 9^2 &= x + 9x + 9 \\ &= x^2 + 9x + 9x + 81 \\ &= x^2 + 18x + 81 \end{aligned}$
$\sqrt{x^2+64} = \sqrt{x^2} + \sqrt{64} = x + 8$	$\sqrt{x^2+64} =$ (cannot be simplified further)
$\dfrac{x^2}{x^2-4} = \dfrac{x^2}{x^2} - \dfrac{x^2}{4} = 1 - \dfrac{x^2}{4}$	$\dfrac{x^2}{x^2-4} = \dfrac{x^2}{x+2x-2} =$ (cannot be simplified further)
$2^{x+4} = 2^x + 2^4$	$2^{x+4} = 2^x + 2^4$ (by the law of exponents)

Mental Math

Figuring out answers in our heads is an important skill. Give it a starting role in your math teaching. - Marilyn Burns

Every day in the advanced world, we face situations that call for adding, subtracting, multiplying, or dividing. We figure tips in restaurant, decide when to leave home to get to the movies on time, estimate the price of sale item, keep track of what we are spending while shopping in the supermarket, double and halve recipes, and so on. Figuring in our own head is such an important life skill that is why mental math is, in one way or another, a very important skill that we and our students should learn.

We present some mental math tricks and shortcuts found in Benjamin and Sherner (2006), *Mental math with tricks and shortcuts* (2012), and Stephens (2012).

Mental Math Tricks

In this part, we list down some mental math tricks and shortcuts.

Addition

Adding left to right

Example 4.5

1. $326 + 678 + 245 + 567 =$

   Add the hundreds digit first from left to right, then add the tens digits, then the units digits.

   900, 1100, 1600, 1620, 1690, 1730, 1790, 1796, 1804, 1809, then 1816

2. $1757 + 5783 =$

   6000, 6700, 7400, 7450, 7530, 7537, then 7540

   Note: Look for opportunities to combine numbers to form 10, 100, 1000 and etc. between numbers that are not necessarily next to each other. Practice!

Distributive property

Example 4.6

1. $5 \times 17 + 5 \times 3 =$ Note that using Distributive Property of Multiplication over Addition:

   $5 \times 17 + 5 \times 3 = 5 \times 17 + 3 = 5 \times 20 = 100$

2. 6×78

   Note that using Distributive Property of Multiplication over Addition:

   $6 \times 78 = 6 \times 70 + 8 = 6 \times 70 + 6 \times 8 = 420 + 48 = 468$

3. $7 \times 99 =$

   Note that using Distributive Property of Multiplication over Addition:

$$7 \times 99 = 7 \times 100 - 1 = 7 \times 100 - 7 \times 1 = 700 - 7 = 693$$

Subtraction

Round the Subtrahend

Example 4.7

1. $496 - 279 =$

   Add a number to the subtrahend to round up to the nearest multiple of 10, then add same number to the minuend and subtract.

$$496 - 279 = 496 + 4 - 279 + 4 = 500 - 283 = 217$$

Round the Minuend

Example 4.8

1. $496 - 279 =$

   Add a number to the minuend to round up to the nearest multiple of 10, then add same number to the subtrahend and subtract.

$$496 - 279 = 496 + 21 - 279 + 21 = 517 - 300 = 217$$

Multiplication and Squaring

Multiply by 50, 25 or 75

Example 4.9

1. 24×50

   Multiply by 100 and divide by 2 (or vice-versa).

   $$24 \times 50 = 24 \times 100 \div 2 = 2400 \div 2 = 1200$$

2. $96 \times 25 =$

   Multiply by 100 and divide by 4 (or vice-versa).

   $$96 \times 25 = 96 \times 100 \div 4 = 9600 \div 4 = 2400$$

3. 56×75

   Multiply by 100 and divide by 4, then multiply by 3.

   $$56 \times 75 = 56 \times 100 \div 4 \times 3 = 5600 \div 4 \times 3 = 1400 \times 3 = 4200$$

Squaring

Example 4.10

1. $61^2 =$

   Using the special product: $a + b^2 = a^2 + 2ab + b^2$

   $$61^2 = 60 + 1^2 = 60^2 + 2601 + 1^2 = 3600 + 120 + 1 = 3721$$

2. $78^2 =$

   Using the special product: $a - b^2 = a^2 - 2ab + b^2$

   $$78^2 = 80 - 2^2 = 802 - 2802 + 2^2 = 6400 - 320 + 4 = 6080 + 4 = 6084$$

Multiplying two numbers using difference of two squares

Example 4.11

1. $21 \times 19 =$

   Using the special product: $a^2 - b^2 = a + ba - b$

   $$21 \times 19 = 20 + 1 \times 20 - 1 = 202 - 12 = 400 - 1 = 399$$

2. $48 \times 52 =$

   Using the special product: $a - b^2 = a^2 - 2ab + b^2$

   $$48 \times 52 = 50 - 2 \times 50 + 2 = 502 - 22 = 2500 - 4 = 2496$$

Division

Divide by 5, 25 and 50

Example 4.12

1. $365 \div 5 =$

   Multiply by 2 and divide by 10.

$$365 \div 5 = 365 \times 2 \div 10 = 730 \div 10 = 73$$

2. $234 \div 50 =$

   Multiply by 2 and divide by 100.

$$234 \div 50 = 234 \times 2 \div 100 = 468 \div 100 = 4.68$$

3. $212 \div 25 =$

   Multiply by 4 and divide by 100.

$$212 \div 25 = 212 \times 4 \div 100 = 848 \div 100 = 8.48$$

Divide by the factors of the divisors one at a time

Example 4.13

1. $728 \div 14 =$

   The factors of 14 are 2 and 7.

$$728 \div 14 = 728 \div 7 \div 2 = 104 \div 2 = 52$$

2. $1344 \div 24 =$

   The factors of 24 are 6 and 4.

$$1344 \div 24 = 1344 \div 6 \div 4 = 224 \div 4 = 56$$

Suggested Activities

Activity 1: Common Misconceptions

Correct the mathematical statement and describe the error.

Potential Error	Correct Form	Comment
1. $a - x - b = a - x - b$		
2. $a + b^2 = a^2 + b^2$		
3. $\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{2}ab$		
4. $3x + 6^2 = 3x + 2^2$		
5. $\frac{a}{x+b} = \frac{a}{x} + \frac{a}{b}$		
6. $\left(\frac{1}{3}\right)x = \frac{1}{3x}$		
7. $\left(\frac{1}{x}\right) + 2 = \frac{1}{x+2}$		

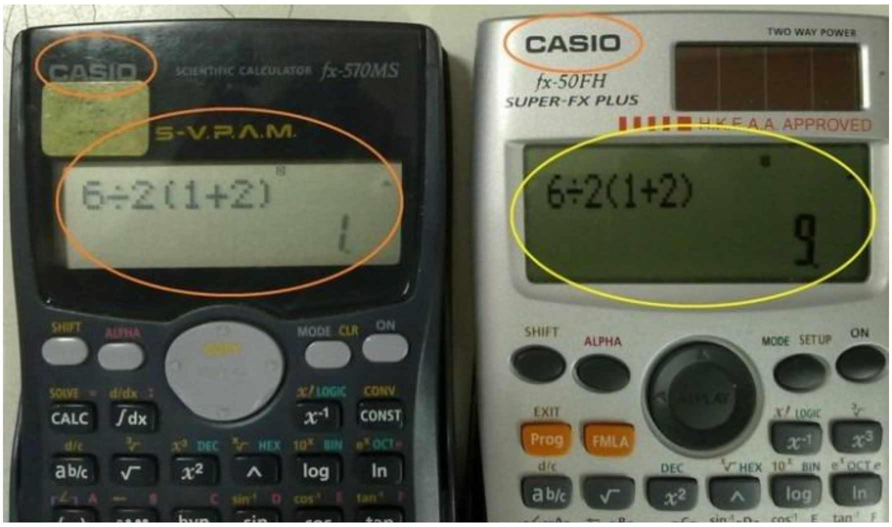
8. $x^{23} = x^5$
9. $2x^3 = 2x^3$
10. $\dfrac{1}{x^2 - x^3} = x^{-2} - x^{-3}$
11. $\sqrt{x^2 + a^2} = x + a$
12. $\dfrac{a + bx}{a} = 1 + bx$
13. $\dfrac{a}{a} = 1$
14. $\dfrac{a + ax}{a} = a + x$
15. $\sqrt{-x + a} = -\sqrt{x - a}$
16. $\dfrac{1}{a^{-1} + b^{-1}} = \left(\dfrac{1}{a + b}\right)^{-1}$
17. $x2x - 1^2 = 2x^2 - x^2$
18. $\dfrac{2x^2 + 1}{5x} = \dfrac{2x + 1}{5}$
19. $\dfrac{3}{x} + \dfrac{4}{y} = \dfrac{7}{x + y}$
20. $a\left(\dfrac{x}{y}\right) = \dfrac{ax}{ay}$

Activity 2: Calculator Debate

Divide the participants into 6 groups. Give a copy of the picture in Figure (4.2) and make each group discuss why the 2 calculators (same brand at that!) have different answers. Determine if one of the calculators is correct. Facilitate a debate and resolve the issue.

Figure 4.2

Which is correct?



Activity 3: Oral Quiz Bee

Materials

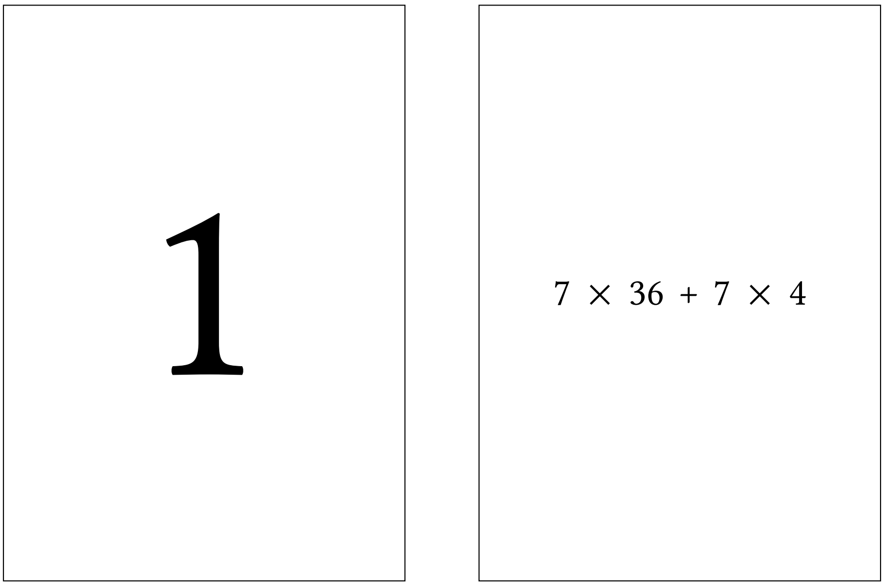
Flashcard, Marker, Masking tape, Improvised buzzer

Preparation for the Quiz Bee

The teacher-in-charge will prepare flashcards where the questions or mathematical statements will be written. Number of the corresponding question or mathematical statement will be put on in front part of the flashcard. And then the flashcards will be pasted on the board as in Table (4.3) First row contains

Figure 4.3

Sample Flash Flash Card



(a) Front of flash card (b) Back of flash card

Table 4.3

Placement of flashcards

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

easy question, second and third rows contain average questions, while fourth and fifth rows contain difficult questions.

Mechanics

1. Students will be divided into 5 groups. Each group will have a team captain who will serve as the leader of the group.

2. Each group will be provided with an improvised buzzer.

3. The questions to be asked are shown with corresponding points and time limits.

Easy	2 point	15 seconds
Average	3 points	20 seconds
Difficult	5 points	30 seconds

4. No writing of computations, just solve mentally.

5. The teacher-in-charge will choose the first question to be asked. Then the first group who buzzed up will be given the opportunity to answer the question. If the first group did not answer the question correctly, other groups will be given the chance to answer.

6. The group who answered the question correctly will have the right to choose the number of the next question to be asked.

7. Scores will be tallied accordingly.

Set of Questions

#	Question/Math Expression	WHAT TO DO?		
		Simplify/Solve	Correct and Describe the Error	TRUE or FALSE. Check for divisibility
1	$7 \times 36 + 7 \times 4$	*		
2	$14 + 3 \times 14 - 5$	*		
3	$a + b^2 = a^2 + b^2$		*	
4	$174 + 268 + 275 + 547$	*		
5	$67348 \div 4$			*
6	$7 + 12 \times 4 - 56/8$	*		
7	$2 + 18 \times 32 - 40/2$	*		
8	$\frac{a + bx}{a} = 1 + bx$		*	
9	$\frac{a}{a} = 1$, for all a		*	
10	$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2$	*		
11	$28 \div 11 + 82 \div 11$	*		
12	$4425575 \div 9$			*
13	7.5×16	*		
14	$\frac{1}{x^2 - x^3} = x^{-2} - x^{-3}$		*	
15	$a\left(\frac{x}{y}\right) = \frac{ax}{ay}$		*	
16	$\sqrt{7921}$	*		
17	$53867 \div 11$			*
18	$\frac{2x^2 + 1}{5x} = \frac{2x + 1}{5}$		*	
19	$73488 \div 8$			*
20	$\sqrt{21316}$	*		
21	$\frac{1}{a^{-1} + b^{-1}} = \left(\frac{1}{a + b}\right)^{-1}$		*	
22	$12 + 2 \times 216 - 108/9$	*		
23	998^2	*		
24	If $a = 3$ and $b = 5$, then $a - ba^2 + ab + b^2$	*		
25	$13855 \div 17$			*

Activity 4: Proving Misconception

The following is the proof that $1 = 2$. What is wrong with the proof?

1.	$a = b$	We'll start assuming this to be true.
2.	$ab = b^2$	Multiply both sides by a.
3.	$ab - b^2 = a^2 - b^2$	Subtract b^2 from both sides.

4.	$ba - b = a + ba - b$	Factor both sides.
5.	$b = a + b$	Divide both sides by $a - b$
6.	$b = 2b$	Recall we started off by assuming $a = b$.
7.	$1 = 2$	Divide both sides by b.

Activity 5: Mental Challenge

Answer the following mentally.

1. $174 + 268 + 275 + 547$
2. $7 \times 36 + 7 \times 4$
3. 39^2
4. 46^2
5. 37×43
6. 77^2
7. 7.5×16
8. $876 - 289$
9. $921 - 388$
10. $3912 \div 12$

Answers. 1. 1264 2. 280 3. 1521 4. 2116 5. 15916. 5929 7. 120 8. 587 9. 533 10. 326

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Module 5

TRANSLATING VERBAL PHRASES TO MATHEMATICAL EXPRESSION & TRANSLATING OPEN SENTENCES TO EQUATIONS

INTRODUCTION

This module presents a discussion on translating verbal phrases to mathematical expression and translating open sentences to equation which includes assigning variable to one of the unknown quantities, representing other unknowns in terms of same variable and forming mathematical expression or equation from the given verbal phrase or open sentence. You will find in here several examples and exercise to enhance your skill in translating verbal phrases to mathematical expressions and vice versa. The skills and knowledge in translating verbal phrases to mathematical phrase is very useful in solving application type problems.

OBJECTIVES

After completing this module, you should be able to:

1. Explain the importance of translating verbal phrase to mathematical phrase in solving application type problems;
2. Assign variable to one of the unknown quantities and represent the other unknown in terms of same variable;
3. Translate verbal phrase to mathematical phrase;
4. Write word phrase for mathematical expression or equation; and
5. Apply it in solving application type problem.

DISCUSSION

In the course of your discussion with your family, you happen to ask your sibling about this riddle: *"Five years ago, I was half the age I will be in eight years. How old am I now?"*

It seems confusing on how to get it but in this lesson, we will apply some processes to solve such kind of word problems. Let us first determine words and phrases that are commonly used to represent an algebraic expression.

Definition 5.1: Algebraic expressions

Algebraic expressions are made up of constants and variables connected by arithmetic operations. These operations include addition, subtraction, multiplication and division.

Table (5.1) show examples of algebraic expressions. In each of these algebraic expressions, we see that

Figure 5.1

Examples of algebraic expressions

Algebraic expression	Constant(s)	Variable(s)
$5x$	5	x
$3z - 2$	3 and 2	z
$\frac{2p+1}{5}$	2, 1, and 5	p
$53c + d$	5, 4, and 3	c and d
$-5w - y$	-5 and -1	w and y

the constants and the variables are all attached by arithmetic operations. So, we need to find out which phrases are used to stand for different operations. Then, we can represent a verbal phrase as an algebraic expression.

It is important to remember that a variable is used to represent an unknown value. In some cases, a phrase or sentence will tell us which variable we should use to represent the unknown value. However, it is more common for the reader to create the variable using a let statement.

Definition 5.2: Let statement

A **let statement** is used to help solve a word problem by creating a variable to represent the unknown value in the problem, e.g., "Let x = the unknown number."

Now that we know how to write a let statement, Let us apply this in the problem posed at beginning of the lesson.

Let a = Cousin Jesus' age now

The following expressions all imply addition.

Let n represent the unknown number.

Algebraic expression	Constant(s)	Variable(s)
plus	6 plus *a number*	$6 + n$
added to	*a number* added to 6	$6 + n$
increased by	*a number* increased by 6	$n + 6$
more than	6 more than *a number*	$n + 6$
sum	the sum of 6 and *a number*	$6 + n$
total	the total of 6 and *a number*	$6 + n$

The following expressions all imply subtraction.

Let n represent the unknown number.

Key Words	Word Expression	Algebraic Expression
minus	5 minus *a number*	$5 - n$
	a number minus 5	$n - 5$
diminished by	*a number* diminished by 5	$n - 5$
	5 diminished by *a number*	$5 - n$
decreased by	*a number* decreased by 5	$n - 5$
	5 decreased by *a number*	$5 - n$
subtracted from	5 subtracted from *a number*	$n - 5$
	a number subtracted from 5	$5 - n$
less than	5 less than *a number*	$n - 5$
	a number less than 5	$5 - n$

Think Back 5.1

"Five minus a number" and *"a number minus five"* are not equivalent. There is no commutative property for subtraction since $5 - 7$ and $7 - 5$ are not equivalent. Remember that $5 - 7 = -2$ and $7 - 5 = +2$. Similarly, $5 - n$ and $n - 5$ are not equivalent. Expressions must be translated from English to algebraic form exactly.

Try translating the following phrases on your own before looking at the answers.

Example 5.1

1. Underline the key words in each expression, and then write the algebraic expression implied by each phrase below. Let n = the number.

   Word Expression — Algebraic Expression

   (a) A number subtracted from 13
   (b) 16 more than a number
   (c) A number increased by 10
   (d) A number decreased by 10
   (e) The sum of a number and 5
   (f) 12 less than a number

(a) $13 - n$ (b) $16 + n$ (c) $n + 10$ (d) $n - 10$ (e) $n + 5$ (f) $n - 12$

Let's look at some expressions that imply multiplication.
This time, let x represent the unknown number.

Key Words	Word Expression	Algebraic Expression
times	3 times a number	$3x$
multiplied by	a number multiplied by 5	$5x$
product	the product of 3 and a number	$3x$
twice	twice a number	$2x$
double	double a number	$2x$
triple	triple a number	$3x$
of	1/4 of a number	$\frac{1}{4}x$

Fact! 5.1

Multiplication is cummutative. Remember $3 \cdot 4 = 4 \cdot 3$. However, when writing algebraic expressions, the constant is always written first. We write $4x$ rather than $x4$. Therefore a number multiplied by 4 is written $4x$ instead of $x4$.

The last operation we need to look at is division.

The following expressions all imply division.

Let a represent the unknown number this time.

Key Words	Word Expression	Algebraic Expression
Quotient	The quotient of 9 and a number	$\frac{9}{n}$
	The quotient of a number and 9	$\frac{n}{9}$
Divided by	A number divided by 9	$\frac{n}{9}$
	9 divided by a number	$\frac{9}{n}$

Think Back 5.2

Division is not commutative. $\frac{2}{3} \neq \frac{3}{2}$. Likewise $\frac{7}{a} \neq \frac{a}{7}$.

Notice that we no longer use the division symbol "÷".

Try these practice problems on your own.

Word Expression	Algebraic Expression
Three less than a number r	
Four more than the number x	
The quotient ten and a number g	
Two-thirds of a number a	
A number c times 16	
Nine less than a number k	

$r - 3$, $x + 4$, $\frac{10}{g}$, $\frac{2a}{3}$, $16c$, $k - 4$

When we translate a word expression into an algebraic expression, it is very important to preserve the order of operations.

The algebraic expression, $3x + 2$, is not equivalent to the algebraic expression, $3x + 2$.

The algebraic expressions on the right represent the word expressions on the left. The variable n will be used to hold the place of the unknown number.

Word Expression	Algebraic Expression
four times a number	$4x$
five times the sum of a number and eight	$5x + 8$
Ana's age in 12 years if her present age is a	$a + 12$
5.2 times the square of the length of the radius	$5.2r^2$
The sum of two consecutive integers if r is the first integer	$r + r + 1$
Thrice the difference of a number and one	$3x - 1$
Fe's age 9 years ago	$f - 9$
A given height subtracted from nine times that height	$9h - h$

Try these practice problems on your own.

Word Expression	Algebraic Expression
1. The difference of 7 and twice a number y	
2. Twice the difference of 7 and a number y	
3. Twice the difference of a number y and 7	
4. Rey's age in h years if he is 11 now	

We are already done with translating phrases into algebraic expressions, next is to complete the mathematical sentence in order to create an algebraic equation.

Definition 5.3: Algebraic equation

An **algebraic equation** has an algebraic expression set equal to a number or another expression, e.g., $3x + 2 = 7$, $3n - 7 = 4n$, and $\frac{a}{5} + 5 = 10a - 8$.

The key word for translating complete sentences into an equation is the word "is". All forms of the word "is" represent the equal sign in an equation.

In this example, let us express the mathematical sentence into an algebraic equation. "Five years ago, I was half the age I will be in eight years."

Five years ago	$a - 5$
I was half the age	$\frac{1}{2}$
I will be in eight years	$a + 8$

So, now we will write this sentence as an algebraic equation.

$$a - 5 = \frac{1}{2}a + 8$$

Let's try one more together.

Example 5.2

1. Five more than twice a number is three times the difference of that number and two.

   Solution:

   Let n = the number.

   Five more than twice a number is three times the difference of that number and two.

   The mathematical equation is $2n + 5 = 3n - 2$.

Try these practice problems on your own.

Sentence	Equation
Thrice the sum of eight and a number is twelve less than that same number	
Nine years from now, Ana will be four times her age from 5 years ago.	

Example 5.3

1. Three consecutive integers sum to 99.

   Solution

   Consecutive integers increase by one each time.

   Let x = the first integer
   $x + 1$ = the second integer
   $x + 2$ = the third integer

   We add our three integers and set that equal to 99.

$$x + x + 1 + x + 2 = 99$$

Let's try one that is a little harder.

Example 5.4

1. Jessica has Php 3.15 in 25-centavo, 10-centavo, and 5-centavo coins. She has twice as many 5-centavo coins as 10-centavo coins as and two less 25-centavo than 5-centavo.

   Solution

   Let $d =$ number of 10-centavo
   $2d =$ number of 5-centavo
   $2d - 2 =$ number of 25-centavo

Try these practice problems on your own.Translate the following sentences to equations.

Sentence	Equation
Three consecutive integers sum to 96.	
Ana has Php 2.00 in the denomination of 25-centavo, 10-centavo, and 5- centavo coins. The number 10-centavo is twice the number of 5-centavo and the number of 25-centavo is thrice the number of 5-centavo coins. Form an e2quatiuon representing the total amount of her coins.	

Here is a summary table which lists some key words and phrases that are used to describe common mathematical operations.

Table 5.1

Common key words and phrases for mathematical operations

Operation	Key Word/Phrase	Example	*Translation*
Addition (+)	plus	a number plus two	$n + 2$
	more than	Six more than a number	$n + 6$
	the sum of	The sum of a number and three	$n + 3$
	the total of	the total of five and a number	$n + 5$
	increased by	A number increased buy one	$n + 1$
	added to	Thirteen added to a number	$n + 13$
Subtraction (−)	minus	A number minus eight	$n - 8$
	less than	Two less than a number	$n - 2$
	the difference of	The difference of a number and five	$n - 5$
	less	Eleven less number	$11 - n$
	decreased by	A number decreased by six	$n - 6$
	subtracted by	Three subtracted by a number	$3 - n$
Multiplication (×)	times	Three times a number	$3n$
	the product of	The product of five and a number	$5n$
	twice, double	Twice a number, double a number	$2n$
	multiplied by	A number multiplied by four	$4n$
	of	Two-fifths of a number	$\frac{2n}{5}$
Division (÷)	The quotient of	The quotient of 2 and a number	$\frac{2}{n}$
	Divided by	Thirty divided by a number	$\frac{30}{n}$
	The ratio of	The ratio of a number and six	$\frac{n}{6}$
Powers (x^n)	the square of	The square of a number or number squared	n^2
	the cube of	The cube of a number or number cubed	n^3
Equals (=)	equals	Three less than a number equals nine	$n - 3 = 9$
	is	Two times a number is 14	$2n = 14$

Continued on next page

Table 5.1

(continued)

Operation	Key Word/Phrase	Example	*Translation*
	is the same as	Fifteen is the same as thrice a number	$15 = 3n$
	yields	A number added to one yields four	$n + 1 = 4$
	amounts to	Six less than a number amounts to twenty-four	$n - 6 = 24$

SUGGESTED ACTIVITIES

Activity 1: Translate each word phrase to algebraic expression

1. The sum of n and 5.
2. The product of x and y
3. Nine more than a number
4. Twice of a number
5. Three-fourths of a number
6. A number decreased by 8
7. 10 more than the unknown
8. The quotient of a number and 3
9. Six more than thrice a number
10. Five times the sum of a number and 4
11. The ratio of x and $2y$
12. One less than 9 times a number
13. Five times a number increased by 4
14. One more than a number
15. The product of twice a number and 8 increased by six

Answers

1. $n + 5$
2. xy
3. $x + 9$
4. $2x$
5. $\frac{3x}{4}$
6. $x - 8$
7. $x + 10$
8. $\frac{x}{3}$
9. $3x + 6$
10. $5x + 4$

11. $\frac{x}{2y}$
12. $9x - 1$
13. $5x + 4$
14. $x + 1$
15. $2x8 + 6$

Activity 2: Translating Exercise

Write a word phrase for each algebraic expression: (Note: There are many ways in writing a word phrase for certain algebraic expression)

1. $\frac{2}{y}$
2. $x + 4$
3. $x - 6$
4. $7x + 4$
5. $2x - 6$
6. $3n + 2$
7. $52 - 4a$
8. $x^2 + 2$
9. $\frac{2x+3}{7}$
10. $5 - \frac{3}{x}$

Answers

1. The quotient of 2 and y.
2. A number increased by 4.
3. Six less than a number.
4. Four more than seven times a number.
5. Twice a number less than six.
6. Thrice the sum of n and 2.
7. Fifty-two decreased by four times a number
8. The square of x added to 2.
9. The quotient of twice of x added to 3 and 7.
10. Five decreased by the quotient of 3 and x.

Activity 3: Translating Activity 3

1. $7x + 3 = 17$
2. $4x + 5 < 20$
3. $3x8 - 7 = 31$
4. $x + \frac{x}{3} \geq 16$
5. $8 - 4x = 7$
6. $x + x + 2 = 14$
7. $\frac{x}{5} - 100$
8. $3x + 9 = 36$
9. $3x + 9 = 36$
10. $2x + 3x = 75$

Answers

1. Seven times a number increased by 3 is 17.
2. Four times the sum of a number and five is less than twenty.
3. The product of thrice a number and 8 decreased by 7 is 31.
4. A number increased one-third of itself is greater than or equal to 16.
5. The difference of eight and four time a number is 7.
6. The sum of two consecutive even integers is 14.
7. The quotient of a number and 5 decreased by 15 is 100.
8. Thrice the sum of a number and 9 is 36
9. The sum of thrice a number and 9 is 36.
10. Twice a number added by thrice of same number is 75.

Activity 4: Translating Activity 4

Give the equation/inequality for the given sentence/situation.

A. Number Problems

(a) Twice a number increased by 12 is 48.
(b) The difference of twice a number and 5 is 7.
(c) The product of a number and 11 is less than or equal to 165.
(d) Decreasing a number by 25 gives 42.
(e) The larger number is four more than the smaller number. If their sum is 72, form an equation stating their sum.
(f) The sum of two numbers is 92. One number is twelve less than the other. Find numbers.
(g) The sum of two consecutive integers is sixty-three.
(h) A number added to half of itself is 9.

B. Age Problems

(a) Rona's age five years ago was 25.
(b) Ten less thrice of Fe's age is 58.

(c) Cora is three years older than Din. If the sum of their ages is 35 years, form an equation representing the sum of their ages.

(d) Ana is five years older than Jen. If the sum of their ages is 11, form an equation stating the sum of their ages.

C. Money Problems

(a) Fe saves 5-centavo and 10-centavo coins. If she has 28 coins worth Php 2.60, form an equation representing the total amount of her coins.

(b) Jen has 26 coins in the denomination of 5 centavos and 25 centavo. If the total amount of her coins is Php 3.10, form an equation representing the total amount of her coins.

D. Investment Problem

(a) An amount of Php 10,000 is invested at 3.5% simple interest for one year. Form an equation representing the interest.

(b) Len invested some money at 3% and Php 4000 less than that amount at 5%. The two investments produced a total of Php 200 interest in one year. Form an equation representing the total interest of the two investments.

E. Mixture Problem

(a) A mechanic added 25 mLof water to 125 mLof a 20% solution of antifreeze in water. Form an equation representing the concentration of antifreeze in the new solution?

(b) A certain amount of water was added to a 50 mLsolution of 25% acid in water. If the resulting mixture has concentration of 10% acid, form an equation representing the concentration of the new mixture.

F. Uniform Motion Problem

(a) Two trains leave from a station at the same time. They travel in opposite directions, one at 62 km/h and the other at 48 km/h. If they are 550 km after certain time, form an equation representing the total distance covered by the two trains.

(b) Two hikers leave on same the place at the same time and travels in opposite direction, one travels 2 kph faster than the other. If they 168 km apart after 4 hours, form an equation representing the total distance covered by the hikers after 4 hours.

Activity 5: Math BINGO

Level 1

Step 1. The participants will have to translate the verbal phrases to mathematical expressions and randomly place them on their bingo cards.

Step 2. The facilitators will check if everybody has finished accomplishing their bingo cards.

Step 3. The resource person will announce the pattern to be accomplished. There could be more than 1 winning pattern for higher chances of winning.

Step 4. The resource person will randomly read verbal phrase one at a time and the participants will mark the corresponding mathematical expression on their bingo card.

Step 5. If a pattern is completed, the participant will just have to "announce". The facilitator and resource person will verify the correctness of the bingo card. If there is a wrong entry, the participant is disqualified and the activity continues. Otherwise, the winner is proclaimed.

Step 6. The game may continue for the other patterns or the game may be restarted for a different pattern.

Figure 5.2

Bingo Cards for Activity (5)

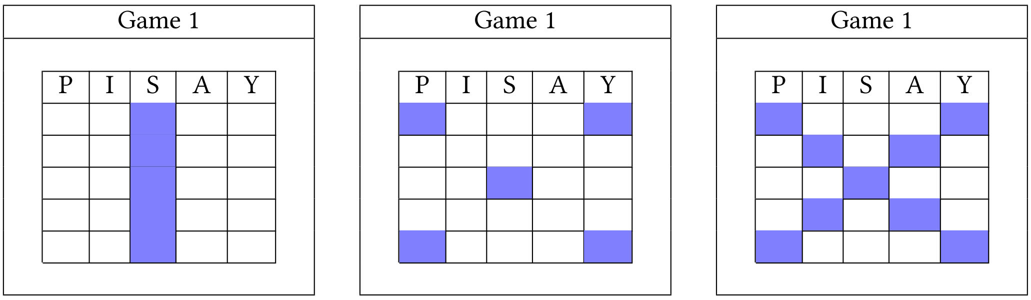


Table 5.2

Questions for (5.2)

Verbal Phrases (In the Bingo Box)	Mathematical Expressions (Card Entries)
a number decreased by seventy-eight	$n - 78$
the difference between a number and twenty-seven	$n - 27$
the quotient of a number and fifty-three	$\frac{n}{53}$
the sum of four and a number	$4 + n$
the quotient of sixty-five and a number	$\frac{65}{n}$
a number increased by fifty-eight	$n + 58$
the difference between thirty-six and a number	$36 - n$
a number added to fifty-six	$56 + n$
seventy-three less than a number	$n - 73$
the product of a number and forty-four	$44n$
a number decreased by thirty-three	$n - 33$
twenty-eight times a number	$28n$
the sum of twenty-three and a number	$23 + n$
the quotient of a number and seventy-nine	$\frac{n}{79}$
the difference between a number and fifty-nine	$n - 59$
the sum of a number and thirty-seven	$n + 37$
the quotient of sixty-one and a number	$\frac{61}{x}$
a number increased by ninety-three	$x + 93$
the product of thirty-two and a number	$32x$
the product of twenty-five and a number	$25x$
the difference between a number and eight	$x - 8$
nine more than a number	$x + 9$
fifty-nine times a number	$59x$
the quotient of a number and ninety-seven	$\frac{x}{97}$
seventeen times a number	$17x$
the sum of sixty-one and a number	$61 + x$
a number decreased by eighty-six	$x - 86$
the difference between ninety and a number	$90 - x$
eighty-one less than a number	$x - 81$
The sum of a number and 3	$x + 3$
The product of a number and 3	$3x$
The sum of two numbers	$x + y$
Three times the sum of two numbers	$3x + y$
Three times a number	$3x$
Three less than a number	$x - 3$
A number, less 3	$x - 3$
Three more than a number	$x + 3$
A number, plus 3	$x + 3$

Continued on next page

Table 5.2

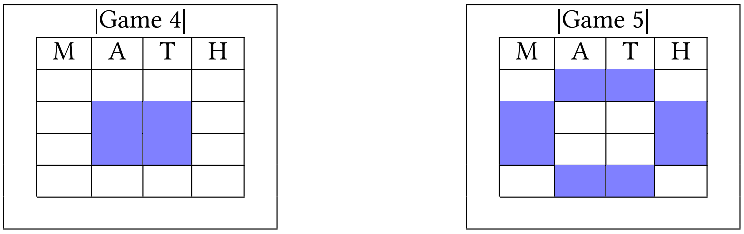
(continued)

Verbal Phrases (In the Bingo Box)	Mathematical Expressions (Card Entries)
The square of a number	x^2
The square of three times a number	$3x^2$
Three times the square of a number	$3x^2$
One-third of a number	$\frac{x}{3}$
One less than 3 times a number	$3x - 1$
Two more than 5 times a number	$5x + 2$
The square of the sum of two numbers	$x + y^2$
The sum of the squares of two numbers	$x^2 + y^2$

Activity 6: Bingo Activity 2

Figure 5.3

Bingo Cards for Activity 2



Follow the same procedures in Activity (5).

Table 5.3

Questions for (5.3)

Verbal Phrases (In the Bingo Box)	Mathematical Expressions (Card Entries)
A number increased by nine is fifteen. Twice a number is eighteen.	
Four less than a number is twenty.	
A number divided by six is eight.	
Twice a number, decreased by twenty-nine, is seven.	
Thirty-two is twice a number increased by eight.	
The quotient of fifty and five more than a number is ten.	
Twelve is sixteen less than four times a number.	
Eleni is x years old. In thirteen years she will be twenty-four years old.	
Each piece of candy costs 25 cents. The price of h pieces of candy is	
Suzanne made a withdrawal of d dollars from her savings account. Her old balance was $350, and her new balance is	
A large pizza pie with 15 slices is shared among p students so that each student's share is 3 slices.	

Continued on next page

Table 5.3

(continued)

Verbal Phrases (In the Bingo Box)	Mathematical Expressions (Card Entries)
Lorene has some nickels, twice as many dimes as nickels, and half as many quarters as nickels. All total she has 77 coins.	
Jack is 25 years younger than his mother. Together, their ages add up to 89.	
Cucumbers cost 15¢ each and tomatoes costs 9¢ each. Mom buys six more tomatoes than cucumbers. Her total bill is $1.26.	
Marie bought 12 fruits, of which there were x apples, five fewer oranges than apples, and one pineapple.	
Tara is twice as old as Gwen. Their sister, Amy, is 5 years older than Gwen. If the sum of their ages is 29 years, find each of their ages.	
Carol is 25 years older than her cousin Amanda. Cousin Bill is 3 times as old as Amanda. The sum of their ages is 90. Find each of their ages.	
Derrick is 5 less than twice as old as Brandon. The sum of their ages is 43. How old are Derrick and Brandon?	
Beth's mom is 6 times older than Beth. Beth's dad is 7 years older than Beth's mom. The sum of their ages is 72. How old are each of them?	
Ruby is 2 years more than three times as old as her son, Raul. If the difference between their ages is 26, how old are Ruby and Raul?	
Sam is ten less than 3 times as old as Alex. If the difference between their ages is 28, how old is Sam and Alex.	
Eileen is 6 years older than Karen. John is three times as old as Karen. The sum of their ages is 56. How old are Eileen, Karen and John?	
Taylor is 18 years younger than Jim. Andrew is twice as old as Taylor. The sum of their ages is 26. How old are Taylor, Jim and Andrew?	
Jick is 3 years less than four times as old as Jall. If the difference between their ages is 72, how old are Jick and Jall?	
Rufio is 15 years younger than Rafe. Rafe is four times older than Raquel. If the sum of their ages is 48 then how old are they?	
Robert and Roberta are twins. The product of their ages is 196.	

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Module 6

SOLVING LINEAR EQUATIONS AND LINEAR INEQUALITIES

INTRODUCTION

This section covers equalities and inequalities in one variable (also known as linear equations and linear inequalities). Equations and inequalities are relations between two quantities. Many real-life situations can be modeled by equations and inequalities. Learning how to solve these mathematical models enables one to use mathematical solutions to answer real-world problems.

OBJECTIVES

1. Differentiate equations from inequalities.
2. Find the solution of an equation and inequality involving one variable
   (a) from a given replacement set;
   (b) intuitively by guess and check;
   (c) by algebraic procedures (applying the properties of equalities and inequalities); and/or
   (d) by graphing.
3. Check a solution of an equation and inequality.
4. Solve problems involving equations (separate module) and inequalities.

DISCUSSION

The mathematical sentence "$4 + 3 = 7$" is true while "$3 > 4$" is false. On the other hand, the sentences "$x - 3 = 5$" and "$x + 4 > 1$" cannot be determined as true or false until we have a replacement for the variable x. The sentences "$x - 3 = 5$" and "$x + 4 > 1$" are examples of **open sentences**. An open sentence of the first degree in one variable can either be an **equation** or an **inequality**. First degree equations and inequalities in one variable are known as **linear equations** or **linear inequalities**.

An **equation** is a statement that expresses the equality of two mathematical expressions. A lunear equation is of the form $\mathbf{ax} + \mathbf{b} = \mathbf{c}$ where a, b and c are any real numbers. To solve an equation is to find a value of the variable that makes the equation true. This value which can replace the variable in the equation is called a **root** or a **solution** of the equation. For example, 2 is a solution of the equation $2x - 3 = 1$ because when 2 is substituted to x, $22 - 3 = 1$ which may be simplified to the true statement $1 = 1$.

The **replacement set** or **domain** is the set of possible values that can be used in place of the variable in an open sentence. A collection of all solutions or all elements of the replacement set of an equation is called the **solution set** of the equation. Equations having the same solution are said to be **equivalent equations**.

Solving equations and inequalities can be done in four ways: a) from a given **replacement set**; b) **guess and check**–one guesses and substitutes values into the equation then check if a true statement will result; c) **algebraic procedures**; and d) **graphing**. Since there is only one or unique solution of an equation, the graphical method (graphical method) is more emphasized in solving inequalities.

Property 6.1: frametitle=Properties of Equality

- **Reflexivity**: For any real number a, $a = a$; that is, anything is equal to itself.
- **Symmetry**: For any real numbers a and b, if $a = b$, then $b = a$.
- **Transitivity**: For any real numbers a, b and c, if $a = b$ and $b = c$, then $a = c$.
- **Adition Property of Equality (APE)**: Any real number added to both sides of an equation does not change the nature of the equation. That is, if a, b and c are real numbers, and $a = b$, then $a + c = b + c$.
- **Multiplication Property of Equality MPE**: ny real number multiplied to both sides of an equation does not change the nature of the equation. That is, if a, b and c are real numbers, and $a = b$, then $ac = bc$.

Example 6.1

1. Find the solution set of $2 - 5z = 7$. The replacement set is $\{-1, 0, 1\}$. **(From a given replacement set)**

   Solution

Replace z with -1.	Replace z with 0.	Replace z with 1.
$2 - 5z = 7$	$2 - 5z = 7$	$2 - 5z = 7$
$2 - 5{-1} = 7?$	$2 - 50 = 7?$	$2 - 51 = 7?$
$2 + 5 = 7?$	$2 - 0 = 7?$	$2 - 5 = 7?$
$7 = 7$ **True**	$2 = 7$ **False**	$-3 = 7$ **False**

   The value of -1 for z makes $2 - 5z = 7$ true. The solution set is $\{-1\}$.

   When no number from the replacement set makes the open sentence true, then the solution set is null.

2. Check whether 0, 1, 4 and 6 are solutions of $3x - 15 = 3$. **(By guess and check.)**

   Solution

$x = 0:$	$30 - 15 = 3?$	No, 0 is not a solution of the given equation because $-15 \neq 3$.
$x = 1:$	$31 - 15 = 3?$	No, 1 is not a solution because $-12 \neq 3$
$x = 4:$	$34 - 15 = 3?$	No, 4 is not a solution because $-3 \neq 3$
$x = 6:$	$36 - 15 = 3?$	Yes, 6 is a solution of the equation because $3 = 3$.

3. Solve the following equations: **(By algebraic procedure)**

   1. $2x + 1 = 2x + 3$

      Since the terms $2x$ of the equation cancels out after adding $-2x$ on both sides, the equation simplifies to $1 = 3$, which is a false statement. This equation is an example of a **contradiction**, an equation that cannot be true for any value of the variable. Its solution set is the null set.

   2. $2x + 1 = x + 3$

      Upon solving, the value $x = 2$ makes the equation true. This linear equation is an example of a **conditional equation**, an equation which is true only for a particular value of the variable. For linear conditional equations, exactly one real number will satisfy it. Therefore, its solution set is a unit set or a singleton (a one-element set).

3. $2x + 1 = 2x + 1$
   Since the terms $2x$ of the equation cancels out after adding $-2x$ on both sides, the equation simplifies to $1 = 1$, which is a true statement. This equation is an example of an identity, an equation that is true for any value of the variable. Its solution set is the set of real numbers.

Exercises

I. Find the solution set of each equation if the replacement set is $\{-2, 0, 2\}$.

1. $x + 3 = 1$
2. $5 - 2y = 1$
3. $x = -x$
4. $\frac{3y}{2} + \frac{1}{2} = 2$
5. $4t - 4t - 3 = 12$
6. $-2z - 5 = 8 - 2z$

1. $\{-2\}$ 2. $\{2\}$ 3. $\{0\}$ 4. $\{\}$ 5. $\{-2, 0, 2\}$ 6. $\{\}$

II. Find the solution set of the following equations and classify if it is a contradiction, conditional or identity.

1. $x + 3 = 1$
2. $5 - 2y = 1$
3. $x = -x$
4. $\frac{3y}{2} + \frac{1}{2} = 2$
5. $4t - 4t - 3 = 12$
6. $-2z - 5 = 8 - 2z$

1. $\{-2\}$ conditional 2. $\{2\}$ conditional 3. $\{0\}$ conditional 4. $\{1\}$ conditional 5. $\mathbb{R}$ identity 6. $\{\}$ contradiction

III. Give the solution sets of the following equations.

1. $3x + 4 = 2x - 6 + x + 16$
2. $2.3x - 2x + 1 = 0.7$
3. $5x + 7 = 7x + 5 - 2x + 2$
4. $83p - 1 - 53p + 2 = 32p - 1$
5. $\frac{x+1}{3} - \frac{x+2}{3} = \frac{x}{2}$
6. $2x + 1 + 7x - 1 = 3x + 1 + 6x - 1$

1. $\mathbb{R}$ 2. $\{6\}$ 3. $\{\}$ 4. $\{5\}$ 5. $\{-\frac{2}{3}\}$ 6. $\{\}$

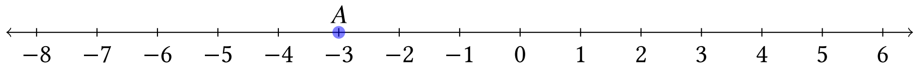
GRAPHING EQUATIONS AND INEQUALITIES

A **number line** represents the order of real numbers. Also, real numbers can be represented as points in the number line. On the number line shown in Figure (6.1), point A is the graph of -3.

If a number is found on the left of another number on the number line, it is always less than the number on its right. For example, -5 is less than -4 because -5 is on the left of -4.

Figure 6.1

A point on the number line.



Exercises

Graph these numbers on the number line.

1. 2.5 2. $-1\frac{1}{2}$ 3. $\frac{5}{4}$ 4. $-3\frac{2}{3}$

Draw the graph of the following.

1. $x < 4$ 2. $x \geq \frac{1}{2}$ 3. $x \neq 1$ 4. $x \leq \frac{1}{3}$

INEQUALITIES IN ONE VARIABLE

Another type of an open sentence is an inequality. A linear inequality is of the form $ax + b < c$ or $ax + b > c$ or $ax + b \leq c$ or $ax + b \geq c$ where a, b and c are real numbers. Take note of the inequality symbols and their meanings.

Table 6.1

Inequality symbols and their meaning

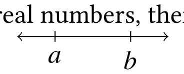
Inequality Symbol	Meaning
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to
$\neq$	not equal to

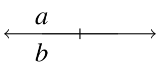
(6.1)

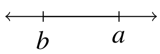
A solution of an inequality is a number that satisfies the inequality statement when it is substituted. For example, 1 is a solution of $x + 1 < 6$ because $2 < 6$ is a true statement. But 1 is not the only solution to $x + 1 < 6$. We can have 2, 3, 4, 5 or 2.2 because all these make the inequality statement true. We can write the *solution* as $x < 5$.

Property 6.2: frametitle=Trichotomy Property

If a and b are real numbers, then one and only one of the following is true:

- $a > b$ 

- $a = b$ 

- $a > b$ 

Solving inequalities follows the same procedure as solving equations. The same properties hold except for the multiplication property. A negative real number multiplied to both sides of an inequality *reverses* the *direction* of the inequality. Why is this so?

Let's take a look. Try multiplying or dividing the statement below by a negative number, say -2.

$2 < 3$	true statement
$-22 \overset{?}{<} -23$	multiply by -2
$-4 \overset{?}{} -6$	Is this statement true?
	No, −4 is greater than −6
$-4 \overset{\checkmark}{>} -6$	To make the statement true, reverse the sign.

Let's summarize the properties of inequality.

Property 6.3: frametitle=Properties of Inequality

The following **properties of inequality** hold for any real numbers a, b and c.

- **Transitivity**: If $a < b$ and $b < c$, then $a < c$.
- **Addition Property of Inequality** (**API**): If $a < b$, then $a + c < b + c$.
- **Multiplication Property of Inequality** (**MPI**): If $a < b$ and $c > 0$ $(c < 0)$, then $ac < bc$ $(ac > bc)$.

The solution of inequalities is more concretized when it is simultaneously presented with a graph.

The following are graphing symbols that will be used in solving inequalities.

Property 6.4: frametitle=Graphing Symbols

 greater than (the open circle indicates that this is *not* equal to the number graphed)

 greater than or equal to (the closed circle indicates that this is equal to the number graphed)

 less than

 less than or equal to

Example 6.21.

1. Solve the following inequalities then graph the solution set on a number line:

   1. $3x + 2 < 2$

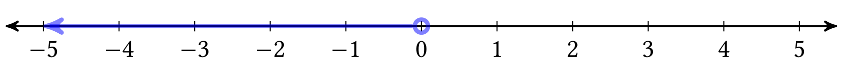
      Solution

$$
\begin{aligned}
3x + 2 + -2 &< 2 + -2 && \text{API (add } -2 \text{ to both sides)} \\
3x + 0 &< 0 && \text{Additive inverse property} \\
3x &< 0 && \text{Additive identity property} \\
\left(\frac{1}{3}\right)3x &< \left(\frac{1}{3}\right)0 && \text{MPI} \\
x &< 0 && \text{Zero property}
\end{aligned}
$$

We can give the solution set as $x < 3$ or as $\{x|x < 3\}$ and we can graph this as in (6.2). The

Figure 6.2

Solution of $x < 3$



open circle on 0 indicates that 0 is not part of the solution set.

Check: Take a test value on the interval, say $x = -1$.

$$
\begin{aligned}
3x + 2 &< 2 \\
3{-1} + 2 &\overset{?}{<} 2 \\
-3 + 2 &\overset{?}{<} 2 \\
-1 &\overset{\checkmark}{<} 2
\end{aligned}
$$

2. $12 - 4x - 5$

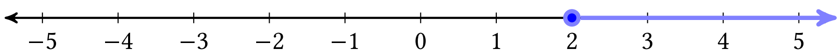
Solution

$12 \geq -4x + 20$	Distributive Property
$12 - 20 \geq -4x + 20 - 20$	API
$-8 \geq -4x$	Simplify
$(\frac{1}{-4}) \leq (\frac{1}{-4})-4x$	MPI
$2 \leq x$	Multiplicative inverse property
$x \geq 2$	It's easier when the variable comes first.

Solution set: $\{x|x \geq 2\}$

Figure 6.3

Notice the closed interval on $x = 2$. This indicates that 2 is part of the solution set.



Check Test value: $x = 3$

$$12 \overset{?}{\geq} -4x - 5$$
$$12 \overset{?}{\geq} -43 - 5$$
$$12 \overset{?}{\geq} -4 {-} 2$$
$$12 \overset{\checkmark}{\geq} 8$$

APPLICATION OF LINEAR INEQUALITIES

The skill of being able to translate sentences into mathematical symbols is a must in successful problem solving. Some of the **inequality key words** are the following.

Property 6.5: frametitle=Inequality key words

at least $\longrightarrow$ greater than or equal to
no more than $\longrightarrow$ less than or equal to
more than $\longrightarrow$ greater than
at most $\longrightarrow$ less than or equal to

Example 6.3

1. Taxi operators in Metro Manila charges ₱40 as flag down rate, in addition to ₱3.50 per 300 meters or 2 minutes of waiting time. Macy has no more than ₱250 to spend on a ride.

   - Write an inequality that represents Macy's situation, assuming that there was no instance that the taxi stopped moving (no waiting time).
   - How many kilometers can Macy travel without exceeding her limit?
     Solution: Let k = number of km Macy can travel without exceeding her limit

$$\frac{₱3.50}{300\text{ m}} \times \frac{100\text{ m}}{1\text{ km}} = ₱\frac{35}{3}/\text{ km}$$

$$\underbrace{\frac{P35}{3}}_{\text{P 35/3 per km}} + \underbrace{40}_{\text{flag down rate}} \underbrace{\leq}_{\text{no more than}} \underbrace{250}_{\text{amount to spend}}$$

$$\frac{35}{3}k \leq 250 - 40 \iff \frac{35}{3}k \leq 210 \iff 35k \leq 630 \iff k \leq 18\,\text{km}$$

Thus, Macy cannot go beyond 18 km.

2. Salyn has ₱20,000 in a savings account at the beginning of the year. She wants to have at least ₱14,000 in her account by the end of the summer. She withdraws ₱500 every week for miscellaneous expenses.
   - Write an inequality that represents Salyn's situation.
   - How many weeks can Salyn withdraw money from her account?

   Solution

   Let w = number of weeks Salyn can withdraw money from her savings account

$$20{,}000 \underbrace{-}_{\text{withdraw}} 500w \underbrace{\geq}_{\text{at least}} \underbrace{14{,}000}_{\text{desired amount left at the end of summer}}$$

$$\begin{aligned} -500w &\geq 14,000 - 20,000 \\ -500w &\geq -6,000 \\ \frac{-500w}{-500} &\geq \frac{-6,000}{-500} && \text{division by a negative number} \\ w &\leq 12 \text{ weeks} \end{aligned}$$

Thus, Salyn has a maximum of 12 weeks to withdraw ₱500 weekly.

3. Owen wants to order USBs on the Internet. Each USB costs ₱250.00 and shipping for the entire order is ₱200.00. Owen has at most ₱5,300 to spend.
   - Write an inequality that represents Owen's situation.
   - How many USB's can Owen order with her ₱5,300?

   Solution: Let u = number of USBs that Owen can buy

$$250u + 200 \underbrace{\leq}_{\text{at most}} 5{,}300$$

$$\begin{aligned} 250u &\leq 5,100 \\ \frac{250u}{250} &\leq \frac{5,100}{250} \\ \therefore u &\leq 20.4 \end{aligned}$$

So, Owen can order a maximum of 20 USB's.

Enrichment Lesson: Step-by-step Solving Using the Properties of Real Numbers

Solve $4x - 3 = 25$

Solution:

$4x - 3 + 3 = 25 + 3$	Addition Property of Equality
$4x + -3 + 3 = 25 + 3$	Definition of Subtraction
$4x + -3 + 3 = 25 + 3$	Commutative Property of Addition
$4x + 0 = 25 + 3$	Inverse Property of Addition
$4x = 25 + 3$	Identity Property of Addition
$4x = 28$	Addition Fact
$4x\frac{1}{4} = 29\frac{1}{4}$	Multiplication Property of Equality
$4x\frac{1}{4} = 7$	Multiplication Fact
$\frac{1}{4}4x = 7$	Commutative Property of Multiplication
$(\frac{1}{4} \cdot 4)x = 7$	Associative Property of Multiplication
$1 \cdot x = 7$	Inverse Property of Multiplication
$x = 7$	Identity Property of Multiplication

Check

$$47 - 3 \stackrel{?}{=} 25$$
$$28 - 3 \stackrel{?}{=} 25$$
$$25 \stackrel{\checkmark}{=} 25$$

Therefore, $x = 7$ is a root or solution of the given equation.

1. Solve $62x - \frac{1}{2} + 15 = -3x + 1$ using the procedure above.

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Module 7

SOLVING WORD PROBLEMS USING BLOCK MODEL

INTRODUCTION

This module is designed to present an alternative method in solving word problems using an approach that has been used by Singaporean students. For the most part, this module will discuss a few examples on how this method is applied. After the method has been discussed, the participants will design their own problems using the block model and present their output to the other participants.

OBJECTIVES

After completing this module, the participant should be able to:

1. Analyze and solve a given word problem using the block model;
2. Design word problems using the block model.

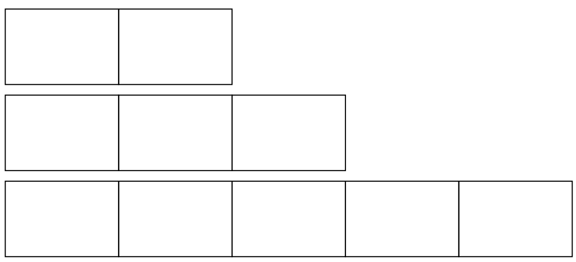
DISCUSSION

During the early 1980s, primary pupils in Singapore were taught basic skills and processes in problem solving as well as one heuristic which uses “drawing a diagram” to be able to answer the problem. This method, which was introduced by Dr. Kho Tek Hong and his team, has been used to solve many challenging arithmetic word problems as well as questions that were designed for secondary students. In using this method, students draw bars or rectangles in representing the given problem, and from these representations they analyze what is/are given then solve the problem. Let us consider the following examples:

Example 7.1

1. A piece of wire is cut into 3 pieces in the ratio 2:3:5. If the longest piece is longer than the middle piece by 8 cm, find the length of the wire.

   From this problem, we can represent the three pieces of wire using bars or rectangles that are of the same dimensions using the ratio 2:3:5, such as

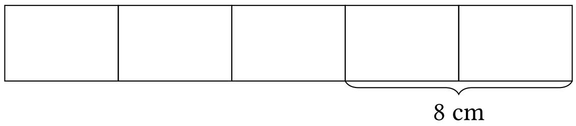


(shortest piece)

(middle piece)

(longest piece)

Notice that from the given problem, it was stated that the longest piece is longer than the middle piece by 8 cm.



(longest piece)

(longest piece)

This implies that the two bars, which are 8 cm long, have a length of 4 cm each. This means that the shortest piece has a length of 8 cm, the middle piece is 12 cm (3 bars $\times$ 4 cm), and the longest piece is 20 cm (5 bars $\times$ 4 cm). Therefore, the length of the wire is 8 cm + 12 cm + 20 cm = 40 cm.

2. The difference of two numbers is 38. One number is only one-third of the other. What are the two numbers?

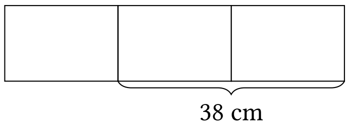
Solution

(1^{st} number)

(2^{nd} number)

It is given that their difference is 38.

(1^{st} number)



(2^{nd} number)

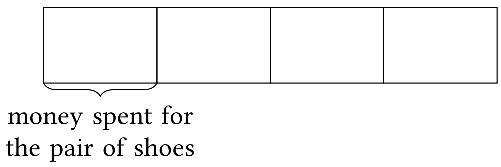
This means that the two bars of the 2^{nd} number are worth 38, which implies that each bar is worth 19. Therefore the first number is 19 and the second number is 57 (i.e., 3×19).

3. Mila spent 1/5 of her money on a pair of shoes and 2/5 of the remainder on a dress. She had Php 768 left. How much money did she have at first?

From this problem, we can represent Mila's money at first and part of this money that she spent for the pair of shoes.

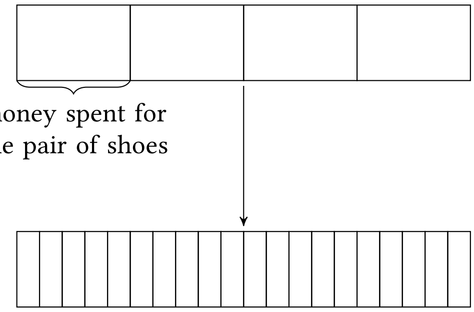
(money at first)

After buying the pair of shoes, she's left with this amount of money.

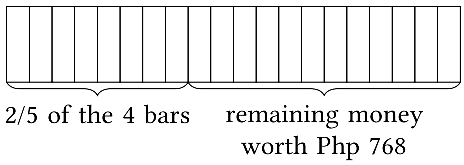


(money at first)

Now, she spent 2/5 of this amount on a dress. How do we represent 2/5 of these 4 bars? To be able to do this, we must recall the concept of LCD. What is the LCM of 4 and 5? We must divide the 4 bars into 20 smaller bars.



Two-fifths of these smaller bars are 8 smaller bars. The remaining 12 smaller bars are worth Php 768.

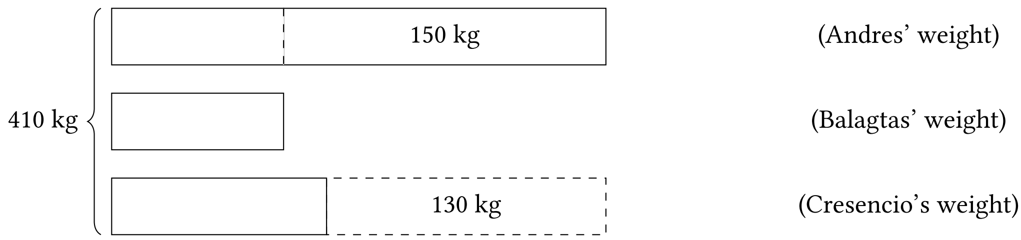


This implies that each of the smaller bars is worth Php 64. Now solving backwards, she had Php 1280 before she bought the shoes, and she had Php 1600 at first.

4. Andres weigh 150 kg more than Balagtas. Cresencio weigh 130 kg less than Andres. Altogether the three weigh 410 kg. What is the weight of Andres?

   Solution

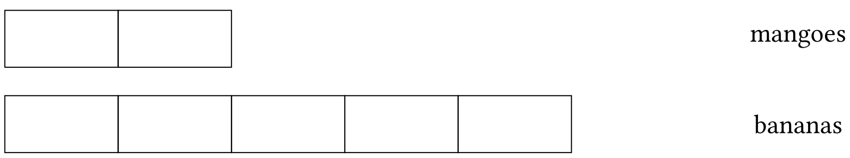
   We can represent the given problem with following diagram.



   We can see from the diagram that the difference between Andres' and Balagtas' weights is 150 kg, and the difference between Andres' and Cresencio's weights is 20 kg. Notice that this 20 kg difference is the additional weight of Cresencio compared to Balagtas' weight. Thus, if we subtract 150 kg and 20 kg from their total weight, this would give us the weight of the three(assuming their weights are the same) $410 - 150 - 20 = 240$. Now if their weights are the same, this implies that each of them weigh 80 kg. But since Andres weighs 150 kg more than Balagtas, therefore he weighs $150 + 80 = 230$ kg.

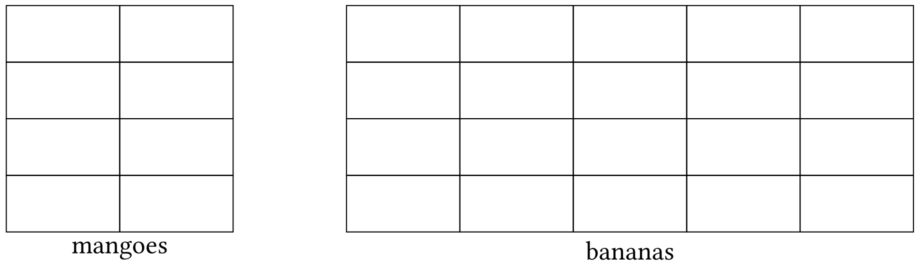
5. Mrs. Evangelista bought some mangoes and bananas. The ratio of the number of mangoes to the number of bananas that she bought was 2 : 5. She gave 3/4 of the mangoes to her niece Amparo and 34 bananas to her nephew Jose. The ratio of the mangoes to bananas is now 2 : 3. How many mangoes and bananas did Mrs. Evangelista buy?

   Easily, we can represent the number of mangoes to the number of bananas as follows:

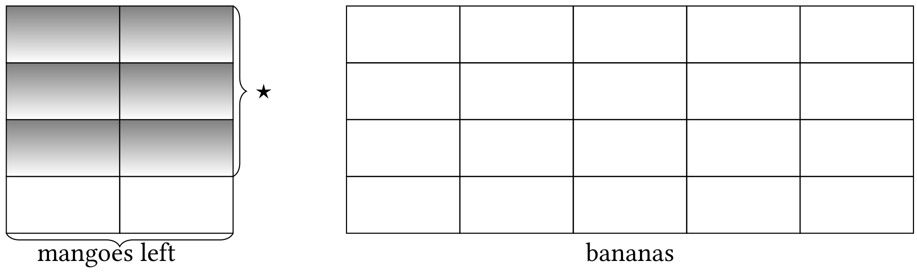


   Now, she gave 3/4 of the mangoes to her niece and 34 bananas to her nephew. From the diagram, it is difficult to represent 3/4 of the mangoes. So we will devise a way on how to represent this scenario in such a way that we are not changing the given in the problem.

6. Let us make the previous diagram such that we can represent 3/4 of the mangoes more clearly. Let us draw 3 more sets of the mangoes and 3 more sets of the bananas so as not to change the ratio 2 : 5.

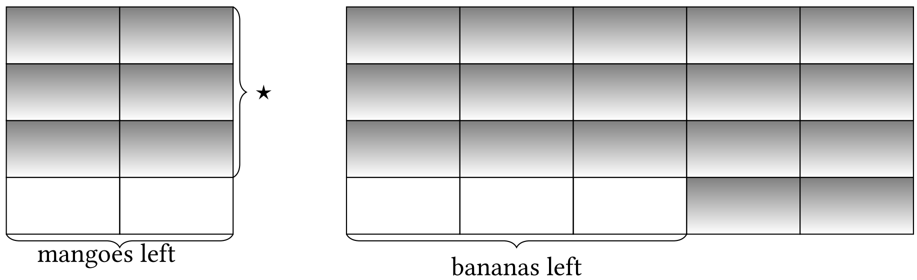


Now from this diagram, we can take away 3/4 of the mangoes.



★ mangoes given to her niece

Now how do we represent the 34 bananas given to her nephew? Take note that each of the individual rectangle does not correspond to 1 banana. But we know that after giving the 34 bananas, Mrs. Evangelista was left with mangoes and bananas in the ratio 2:3. Therefore, from the 20 rectangles of the bananas we can take away 17 of them so she's left with the ratio 2:3.

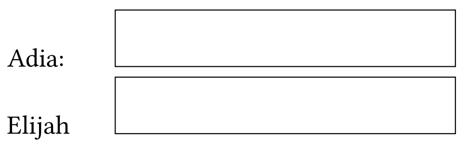


The 17 rectangles are equal to the 34 bananas given to his nephew, which means that each rectangle represents 2 bananas. And each of the rectangles on the left represents 2 mangoes. From this, we can conclude that originally, Mrs. Evangelista bought 168×2 mangoes and 4020×2 bananas.

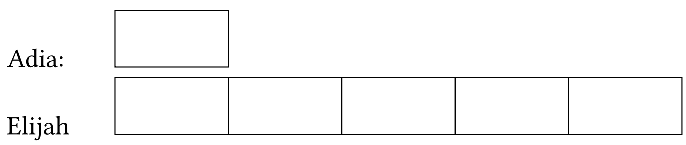
7. Adia and Elijah had the same amount of money at first. When Adia gave Php 18 to Elijah, Elijah then had five times as much money as his sister. How much money do Adia and Elijah have together?

   Solution

   Before Adia gave Php 18 to Elijah, They both have the same amount of money, which we can represent as follows:

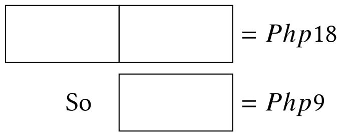


But after receiving Php 18 from his sister, Elijah had five times as much money as Adia.



Notice that if we move the two blocks from Elijah's to Adia's, they will have the same number of blocks. This implies that the two blocks are equivalent to the Php 18 that Adia gave to Elijah.

Therefore,



Since they have a total of 6 blocks, $then69 = 54$. Hence, Adia and Elijah have Php 54 altogether.

Andres weigh 150 kg more than Balagtas. Cresencio weigh 130 kg less than Andres. Altogether the three weigh 410 kg. What is the weight of Andres?

EXERCISES

Answer the following problems using the block model.

1. One number is greater than half of another number by 15. What are the two numbers if their sum is 48?
2. Mrs. Cruz is using the following recipe in making a juice cocktail: 1/2 cup of pineapple juice + 3/4 cup of mango juice + 1 1/4 cup of apple juice. She will be hosting a party this coming Saturday. How much of each type of juice will she need if she wants to make 5 liters of this juice cocktail?
3. A sum of money was divided among Elena, Felicisima, and Gracia in the ratio 2:4:5. Had the sum of money been equally divided among them, Elena's share would have been larger by Php 3000. What was the total sum of money?
4. Ben and Caloy are coin collectors. Initially 1/5 of Caloy's coins were equal to 1/3 of Ben's coins. If Ben gave 24 coins to Caloy, Caloy would have thrice as many coins as Ben. How many coins did each of them have originally?

SUGGESTED ACTIVITY

The participants are tasked to create/design your own word problems (3 word problems each) using the block model. The level of difficulty of the problems should be easy, average and difficult. Make sure to have your solution for each of the problem. Write your word problems and the corresponding solutions to the papers provided for you. You are given 1 1/2 hours to accomplish the task. After which, some of you will be asked to present his/her output to the rest of the participants.

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Module 8

GEOMETRY: INQUIRY-BASED APPROACH

INTRODUCTION

This module presents Inquiry-Based Learning (IBL) as an alternative teaching strategy in introducing geometry. The first part of the model will let the participants experience an inquiry- based activity. After which, the activity will be examined on how it was implemented. From the discussion, definition as well as some models of inquiry-based learning will be discussed. In the afternoon, participants will have to develop their own lesson plan, activity or worksheet using inquiry-based learning. Outputs will then be presented for comments and critiquing. Aside from the models, sample activities, lesson plans, and assessment will also be included.

OBJECTIVES

After completing this module, you should be able to:

1. Experience inquiry-based lesson.
2. Define Inquiry-Based Learning including its factors and essential elements.
3. Identify different models for Inquiry-Based Learning.
4. Develop inquiry-based activity sheets for geometry.

MATERIALS

LCD Projectors	Manila Papers	White Board Markers
Permanent Markers	Colored and White Chalks	Bond Markers

For Group Activity 1 (1 set per group of 5 or 6 participants)

Manipulatives	Pencils	Ruler/Meter Stick	Permanent Marker
Manila Paper	Crayons	Masking tape	

For Group Activity 2 (1 set per group of 5 or 6 participants)

Manila Paper	Permanent Marker	Masking Tape

GROUP ACTIVITY 1

Participants are to be divided into groups of 5 or 6. This can be done through ice breakers such as "The Boat is Sinking..." or other games (You may also refer to the site http://www.girlscoutsnorcal.org/documents/LE-Fun_Splitters.pdf).

After the groups have settled down and formed a circle, each group will be given an envelope containing the set of manipulatives with the instructions and guide questions. One set of manipulatives contains the following figures: square, rectangle, rhombus, parallelogram, trapezoid, isosceles trapezoid, and kite. Figures may be of different sizes. Answers and outputs will be written/drawn on a manila paper.

PROCEDURE

1. Open the envelope and place the contents on the table/floor. Guide Questions:
   (a) Are you familiar with these shapes/figures?
   (b) Do you know the names of these figures?
2. Classify the given figures. Guide Questions:
   (a) How would you classify these figures?
   (b) How many classifications can you make?
3. Write a general description for each type of classification that you made, as well as specific descriptions for the figures.
4. Match the following words with the descriptions that you wrote:

Quadrilateral	Parallelogram	Kite	Trapezoid
Rectangle	Rhombus	Square	Isosceles Trapezoid

5. Draw a concept diagram showing the classification of the figures and how each group of figures is related to one another.

   Guide questions:
   (a) How are these figures related to one another?
   (b) Does your concept map clearly show the hierarchy of the different figures?
6. Based on your concept diagram, determine if the following statement is **Always**, **Sometimes**, or **Never True**.
   (a) A kite is a parallelogram.
   (b) A rhombus is a square.
   (c) A square is a rhombus.
   (d) Squares are rectangles.
   (e) Rectangles are squares.

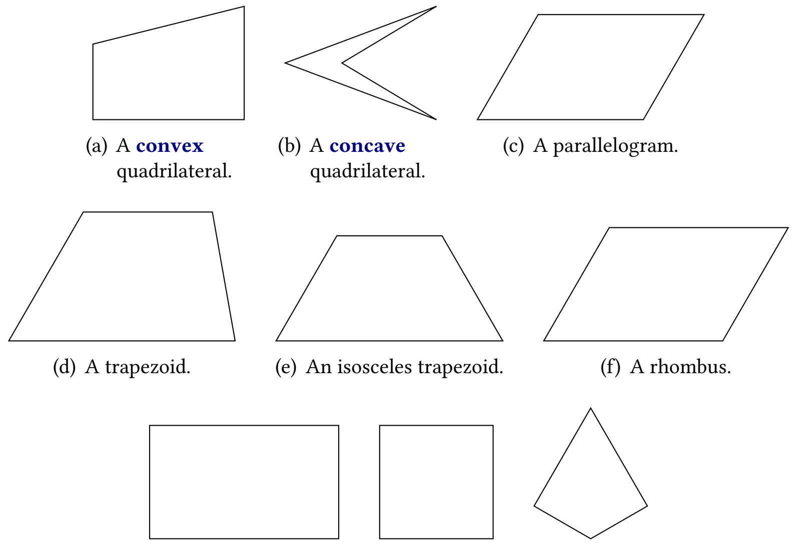
Definition 8.1: Quadrilaterals

Let A, B, C, and D be four points in the same plane. If no three of these points are collinear, and the segments AB, BC, CD, and DA intersect only at their endpoints, then the union of these four segments is called a **quadrilateral**. A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel. A **trapezoid** is a quadrilateral in which one and only one pair of opposite sides is parallel. An **isosceles trapezoid** is a trapezoid whose non-parallel sides are congruent. A **rhombus** is a parallelogram whose sides are all congruent. A **rectangle** is a parallelogram whose angles are all right angles. A **square** is a rectangle whose sides are all congruent. A **kite** is a quadrilateral with two pairs of congruent adjacent sides.

Examples of quadrilaterals are show in Figure (8.1).

Additional Questions:

1. When can a rectangle be a square?
2. Can a rectangle be a rhombus? How?
3. Can a kite be a rhombus? How?



(a) A **convex** quadrilateral. (b) A **concave** quadrilateral. (c) A parallelogram.

(d) A trapezoid. (e) An isosceles trapezoid. (f) A rhombus.

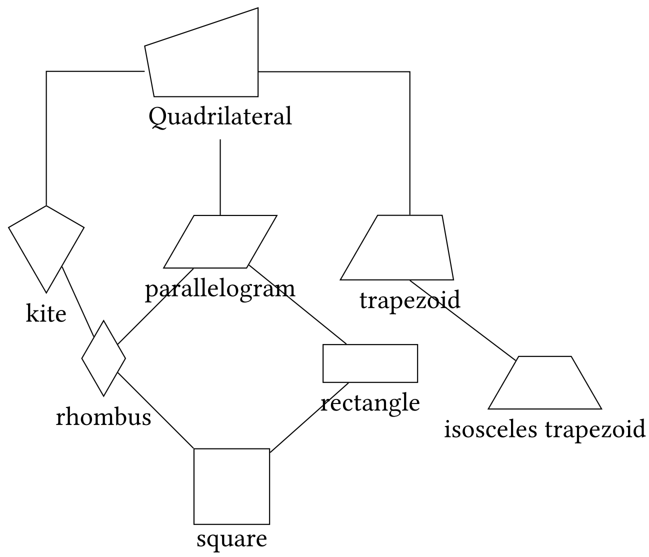
(g) A rectangle. (h) A square. (i) A kite.

Figure 8.1

Examples of parallelograms.

Figure 8.2

Concept Diagram for Quadrilaterals



Answers:

1. A rectangle can be a square if all the sides of the rectangle are congruent.
2. If a rectangle is a square, then it automatically becomes a rhombus.
3. A kite can become a rhombus if the two pairs of congruent sides are congruent to one another.

DISCUSSION

After Activity 1 has been implemented and discussed, the participants will now examine what happened during the activity itself. Each step of the activity will be examined for their reaction or behavior.

Questions for Discussion

1. After the envelopes were distributed, what did the members do? How did they behave? Were they excited, happy, anxious, or indifferent?

2. When you were asked to classify the figures, what was the first thing that came to your mind? What were the initial questions raised during the discussion?

3. Did everyone participate or contribute in doing the activity? What were the difficulties encountered by the group? How did you overcome these difficulties?

4. What was the role of the teacher during the entire activity?

In Activity 1, the teacher did not directly lecture on the topic to be discussed but instead, the participants were given an activity to discover the relationship between the figures given. Participants were doing more of the learning through the manipulatives and the guide questions given by the teacher.

This type of activity is an example of an inquiry-based approach of learning.

What is Inquiry-Based Learning?

As defined by the Brigham Young University's Center for Teaching and Learning, Inquiry- Based Learning is

> "an inductive teaching methodology that centers around students focusing on questions and/or research (Spronken-Smith et al., 2008). Teachers engage students by allowing them to facilitate their own learning with support and may allow students to create their own learning situations (Spronken-Smith et al., 2008; Feletti, 1993). "Inquiry learning" was founded around the scientific method while "inquiry-based learning" was developed as a flexible alternative to problem-based learning (Feletti, 1993)."

Inquiry-based learning is a teaching strategy which involves the learner more than the traditional way of teaching. In this strategy, learners are guided to understand the concepts. Dictionary defines inquiry as seeking knowledge, information, or truth through questioning. Inquiry-based learning is not just asking questions, but it is a way of converting data and information into useful knowledge. A useful application of inquiry-based learning involves many different factors, which are, a different level of questions, a focus for questions, a framework for questions, and a context for questions.

What are the types of questions?

Definition 8.2: Types of questions in IBL

1. **Inference questions**
   - Mainly asked by students who want to gain extra knowledge about a particular topic
   - Students can be encouraged by finding clues, examining them, and discussing them to justify the inferences of the topic
2. **Interpretation questions**
   - Mainly force the students to understand the consequences of the ideas or information
3. **Transfer questions**
   - Make students take their information to a new place, level or stage
4. **Hypothesis questions**
   - Mainly based on what can be predicted or tested through thinking

What are the key principles for Inquiry-Based Learning?

Definition 8.3: Key Principles for IBL

1. **Principle 1:** All learning activities should focus on using information-processing skills (from observations to synthesis) and applying the discipline "ground rules" as a means to learn content set in a broad conceptual context.
2. **Principle 2:** Inquiry learning puts the learner at the center of an active learning process, and the systemic elements (the teacher, instructional resources, technology, and so forth) are prepared or aligned to support the learner.
3. **Principle 3:** The role of the teacher becomes one of facilitating the learning process. The teacher also becomes a learner by finding out more about the learner and the process of inquiry learning.
4. **Principle 4:** What is assessed is what is valued. Therefore, more emphasis needs to be placed on assessing the development of information-processing skills, nurtured habits of mind, or "ground rules" of the discipline, and conceptual understandings rather than just the content of the field.

Inquiry-Based Learning generally follows the basic inquiry process as shown in Figure (8.3). There are five key elements in inquiry learning–Ask, Investigate, Create, Discuss and Reflect.

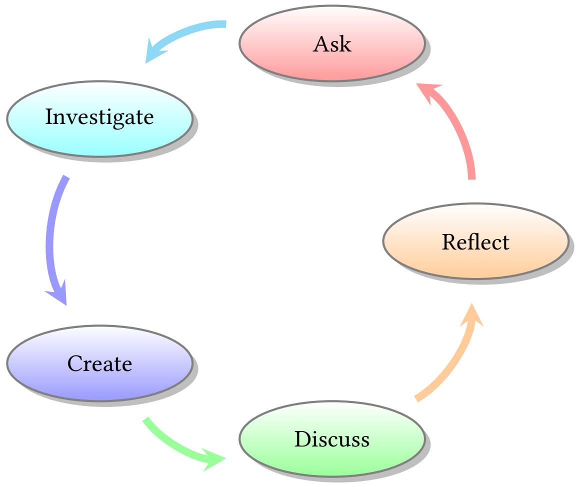


Figure 8.3

The **Basic Inquiry Process**. Based on: `http://www.cii.illinois.edu/InquiryPage/inquiry/process.html`

Ask It begins with the desire to discover. Meaningful questions are inspired by genuine curiosity about real-world experiences. A question or a problem comes into focus at this stage, and the learner begins to define or describe what it is.

Some real examples of questions in this stage of the process are:

- "What makes a poem poetry?"
- "Where do chickens come from and how does an egg 'work'?"
- "Why does the moon change shape?"

Of course, questions are redefined throughout the learning process. We never fully leave one stage and go neatly to the next. As one teacher at a recent Inquiry Workshop pointed out, "It's messy, but it works!" Questions naturally lead to the next stage in the process: Investigation.

Investigate Taking the curious impulse and putting it into action is what we call the investigation process. At this stage the learner begins to gather information: researching resources, studying, crafting an experiment, observing, or interviewing, to name a few. The learner may recast the question, refine a line of query, or plunge down a new path that the original question did not or could not anticipate. The information- gathering stage becomes a self-motivated process that is wholly owned by the engaged learner.

Create As the information gathered in the investigation stage begins to coalesce, the learner begins to make connections. The ability at this stage to synthesize meaning is the creative spark that forms all new knowledge. The learner now undertakes the creative task of shaping significant new thoughts, ideas, and theories outside of his/her prior experience.

Discuss At this point in the circle of inquiry, learners share their new ideas with others. The learner begins to ask others about their own experiences and investigations. Shared knowledge is a community-building process, and the meaning of their investigation begins to take on greater relevance in the context of the learner's society. Comparing notes, discussing conclusions, and sharing experiences are all examples of this process in action.

Reflect Reflection is just that: taking the time to look back at the question, the research path, and the conclusions made. The learner steps back, takes inventory, makes observations, and possibly makes new decisions. Has a solution been found? Do new questions come into light? What might those questions be?

What follows are other models which are accepted and used in inquiry learning.

Predict–Observe–Explain (POE)

The POE strategy was developed by White and Gunstone (1992) to uncover individual students' predictions, and their reasons for making these, about a specific event.

When to Use

POE is a strategy often used in science. It works best with demonstrations that allow immediate observations, and suits Physical and Material World contexts. A similar strategy also works well in mathematics, particularly in statistics.

- It can be used for finding out students' initial ideas;
- providing teachers with information about students' thinking;
- generating discussion;
- motivating students to want to explore the concept; and
- generating investigations.

The Theory

Constructivist theories of learning consider that students' existing understandings should be considered when developing teaching and learning programmes. Events that surprise create conditions where students may be ready to start re-examining their personal theories.

How the Strategy Works

- Unless students are asked to predict first what will happen, they may not observe carefully.
- Writing down their prediction motivates them to want to know the answer.
- Asking students to explain the reasons for their predictions gives the teacher indications of their theories. This can be useful for uncovering misconceptions or developing understandings they have. It can provide information for making decisions about the subsequent learning.
- Explaining and evaluating their predictions and listening to others' predictions helps students to begin evaluating their own learning and constructing new meanings.

What to Do

- Set up a demonstration of an event, related to the focus topic, that may surprise students, and which can be observed.
- Tell the students what you are going to be doing.

Step 1: Predict

- Ask the students to independently write their prediction of what will happen.
- Ask them what they think they will see and why they think this.

Step 2: Observe

- Carry out the demonstration.
- Allow time to focus on observation.
- Ask students to write down what they observe.

Step 3: Explain

- Ask students to amend or add to their explanation to take account of the observation.
- After students have committed their explanations to paper, discuss their ideas together

Problem-Based Learning

At the University of Delaware, they use Problem-based learning in their undergraduate courses. According to them, in a problem-based learning (PBL) model, students engage complex, challenging problems and collaboratively work toward their resolution. PBL is about students connecting disciplinary knowledge to real-world problems–the motivation to solve a problem becomes the motivation to learn.

With PBL, the teacher gives the students problems to work on and not the lectures or exercises or assignments. Content is not readily given. Instead, students should work on the given problem to draw out the necessary concepts to be learned. The teacher acts as a facilitator who will guide the students in solving the problems instead of being the source of solutions.

Problem-based learning will provide the students opportunities for the following:

- examine and try out what they know;
- discover what they need to learn;
- develop their people skills for achieving higher performance in teams;
- improve their communications skills;
- state and defend positions with evidence and sound argument;
- become more flexible in processing information and meeting obligations; and
- practice skills that they will need after their education.

An example of how the students can learn through PBL is shown below using a simplified model for PBL.

1. *Explore the issues.*

   Your teacher introduces an "ill-structured" problem to you. Discuss the problem statement and list its significant parts. You may feel that you don't know enough to solve the problem but that is the challenge!

   You will have to gather information and learn new concepts, principles, or skills as you engage in the problem-solving process.

2. *List "What do we know?"* What do you know to solve the problem? This includes both what you actually know and what strengths and capabilities each team member has. Consider or note everyone's input, no matter how strange it may appear: it could hold a possibility!

3. *Develop, and write out, the problem statement in your own words.*

   A problem statement should come from your/the group's analysis of what you know, and what you will need to know to solve it. You will need:

   - a written statement
   - the agreement of your group on the statement; and
   - feedback on this statement from your instructor. (This may be optional, but is a good idea.)
     Note: The problem statement is often revisited and edited as new information is discovered, or "old" information is discarded.

4. *List out possible solutions.* List them all, then order them from strongest to weakest. Choose the best one, or most likely to succeed.

5. *List actions to be taken with a timeline.*

   - What do we have to know and do to solve the problem?
   - How do we rank these possibilities?
   - How do these relate to our list of solutions? Do we agree?

6. *List "What do we need to know?"*

   Research the knowledge and data that will support your solution. You will need to information to fill in missing gaps.

   - Discuss possible resources, such as experts, books, web sites, etc.
   - Assign and schedule research tasks, and their corresponding deadlines.

   If your research supports your solution, and if there is general agreement, go to item (7). If not, go back to item (4).

7. *Write up your solution with its supporting documentation, and submit it.* You may need to present your findings and/or recommendations to a group or your classmates. This should include the problem statement, questions, data gathered, analysis of data, and support for solutions or recommendations based on the data analysis, in short, the process and outcome.

8. *Present and defend your conclusions.* The goal is to present not only your conclusions, but the foundation upon which they rest. Prepare to:

   - state clearly both the problem and your conclusion;
   - summarize the process you used, options considered, and difficulties encountered;
   - convince, not overpower; (Bring others to your side, or to consider without prejudice your supporting documentation and reason.)
   - help others learn, as you have learned; and
   - if challenged and you have an answer, present it clearly. (If you don't have an answer, acknowledge it and refer it for more consideration.)

5E Instructional Model

The 5E's instructional model provides a format for lessons that builds on what students already know. The 5E's sequence the learning experience so that learners construct their understanding of a concept across time. Each phase of the learning sequence can be described using five words that begin with "E": Engage, Explore, Explain, Extend, and Evaluate.

Figure 8.4

Source: *The 5E Instructional Model.* URL: http://www.nasa.gov/audience/foreducators/nasaeclips/5eteachingmodels/index (visited on 04/14/2012)

Table 8.1

Sequences of the 5E Instructional Model

Learning Phase	Student Rule	Teacher Rule
Engage/Excite	Students are introduced to the concept. Students make connections to prior knowledge and what is to be studied. Student thinking is clarified. Students become mentally engaged in the new learning experience.	Teachers ask questions of students and engage them in the guided inquiry lessons. They use strategies that make connections between the past and present learning experience. Teachers set a level of anticipation.
Explore	Students explore or experiment at this point. They engage in observations, use science tools and materials (manipulatives), collect data, and record data.	Teachers set up the investigation and guide students in inquiry, asking probing questions to clarify understanding.
Explain	Students verbalize their understandings from the "explore" phase, look for patterns in their data, and describe what they observed. This can be done in small and/or whole groups.	Teachers ask probing questions that encourage students to look for patterns or irregularities in their data.
Elaborate	Students expand their learning, practice skills and behavior, and make connections or applications to related concepts and in the world around them.	Teachers provide learning opportunities for students to apply their knowledge and to gain a deeper understanding. Activities can include reading articles and books, writing, designing other experiments, and exploring related topics on the Internet.
Evaluate	Students answer questions, pose questions, and illustrate their knowledge (understandings) and skills (abilities).	Teachers diagnose student understanding through an ongoing process. Assessment can be both formative (ongoing and dynamic) and summative (end-of- lesson final test or product).

7E Instructional Model

The 7E Instructional model is based on the 5E model but expands the engagement phase to elicit and engage. The elaborate and evaluate phases are expanded to elaborate, evaluate and extend, as well. The purpose of the 7E model is to emphasize the importance of recognizing prior knowledge and expanding knowledge gained through conceptual change (Eisenkraft, 2003).

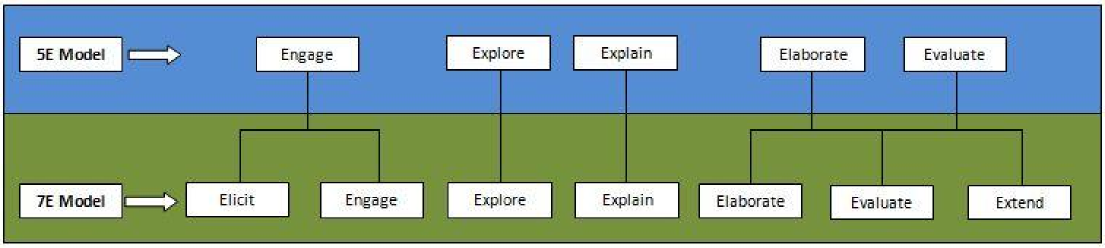


Figure 8.5

Source: *The 7E Instructional Model Used in Active Chemistry.* URL: http://www.its-about-time.com/htmls/ac/7e.pdf (visited on 04/15/2012)

These instructional elements will help to structure lessons to be Inquiry based.

Table 8.2

Instructional Elements of the 7E Instructional Model

Elicit	Identify what students know.
Engage	Creates interest students.
Explore	Provides activities and experiments for students to collect and analyse data
Explain	Encourage students to explain concepts followed by formal explanation by teacher.
Elaborate	Provide opportunity for students to apply knowledge and skills learnt in new but similar situations.
Evaluate	Assess understanding and abilities of the students.
Extend	Students try to apply new knowledge and skills in completely unfamiliar situations.

4E × 2 Instructional Model

The 4E × 2 Instructional Model was designed to unite three major learning constructs (inquiry instruction, formative assessment, and reflective practice) that have all been shown to improve learning. When well integrated these learning constructs help facilitate deeper teaching and more powerful learning experiences. All the lessons designed and available under the lesson plan tab use the 4E × 2 Model.

GROUP ACTIVITY 2

The participants will again be grouped into 5 or 6. The groupings in the morning may be retained for this activity.

Each group must come up with their own inquiry-based activity or lesson plan. Each group can choose one topic among the following to work on. Outputs shall be written on the Manila Paper.

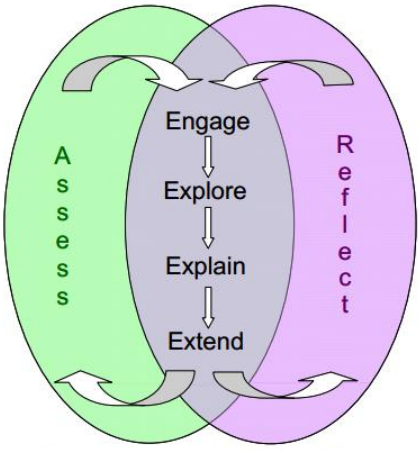


Figure 8.6

4E ×2 Instructional Model. Source: J. C. Marshall, B. Horton, and D. Llewellyn (2008). *EQUIP: Electronic Quality of Inquiry Protocol.* URL: www.clemson.edu/iim (visited on 04/20/2012)

Points, Lines and Planes	Subsets of a Line
Different Kinds of Angles	Angle Pairs
Vertical Angles and Angles formed by Transversals	Classification of Triangles
Relationship of Sides and Angles in Triangles	Convex Polygons
Quadrilaterals	Exterior and Interior Angles of Convex Polygons
Circles	

SAMPLE ACTIVITIES, LESSON PLAN and LESSON PLAN TEMPLATES

- Sum of Interior Angles: *Sum of Interior Angles.* URL: http://learningwithalisha.com/products/sum_of_interior_angles_activity.pdf (visited on 04/18/2012)
- Geometry and the Real World: Shamsu Abdul-Aziz. *Geometry in the Real World.* URL: http://teachers.yale.edu/curriculum/search/viewer.php?id=initiative_10.04.01_u#d (visited on 04/19/2012)
- 5E Lesson Plan: The Geometry for Pentominoes: *5E Lesson Plan. The Geometry of Pentominoes.* University of Houston. URL: http://www.teachhouston.uh.edu/TeachHouston_document/lesson_plans1/The_Geometry_of_Pentominoes.html (visited on 04/20/2012)

After the workshop, each group will present their output to the body for critiquing and suggestions.

How to Evaluate Inquiry-based Instruction

The Electronic Quality of Inquiry Protocol (EQUIP) instrument is designed to measure the quantity and quality of inquiry instruction being facilitated in K-12 math and science classrooms. The instrument does not seek to measure all forms of quality instruction–only those that are inquiry-based in nature. Marshall, Horton, and Llewellyn (2012) provides a form which can be used for peer evaluation and classroom observations.

Included in this file are four rubrics to evaluate instructional, discourse, assessment, and curriculum factors. For each construct measured, the evaluator will determine if the implementation of the lesson plan is at pre-inquiry (level 1), developing inquiry (level 2), proficient inquiry (level 3) or exemplary inquiry (level 4). Pre-inquiry level suggests that it is more teacher-centered rather than student centered while a rating of 4 would indicate more student participation in acquiring learning.

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Marshall, J. C., B. Horton, and D. Llewellyn. *4E + 2 Instructional Model.* URL: http://itcenter.clemson.edu/iim/wp-content/uploads/2008/05/4e-x-2-overview-graphic.pdf (visited on 04/17/2012).

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Module 9

SOLVING WORD PROBLEMS IN GEOMETRY

INTRODUCTION

This module is intended to give an overview on solving word problems in geometry. It presents the different steps in problem solving as designed by Polya. It also contains sample word problems for the participants to solve. Assessment questions are also given.

OBJECTIVES

After completion of the module, the participants are expected to:

1. Apply the different steps in problems solving
2. Determine the appropriate strategy to use in solving a word problem.
3. Use variety of problem solving strategies in solving word problems.
4. Solve the word problems accurately.
5. Appreciate the importance of problem solving in a mathematics curriculum.
6. Gain confidence in doing problem solving
7. Create more interesting word problems.
8. Integrate or connect geometry to origami, the art of paper folding.

DISCUSSION

The National Council of Teachers of Mathematics (NCTM) gives the following definitions on the nature of mathematics: mathematics is a study of patterns and relationships, it is a way of thinking, it is an art, it is a language and it is a tool. NCTM identified five broad goals required to meet the students' mathematical needs for the 21st century. They are as follows: students should (1) value mathematics, (2) reason mathematically, (3) communicate mathematics, (4) solve problems and (5) develop confidence.

Problem solving is the heart of mathematics. It is a process. It is the means by which an individual uses previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation. Students must be exposed to a variety of problems – problems that vary in context, in level of difficulty and in methods in solving the problem. But what is really a problem or a word problem specifically? A word problem is a mathematical proposition that a student has never experienced solving. What are the purposes of word problems? The following are the different purposes of word problems: (1) to prepare the students for real life, (2) to develop children's logical and abstract thinking and mental discipline, and (3) to motivate students. Hence a word problem should simulate real life situations, events, and places that students can experience firsthand. In this way the students will be able to see the importance of mathematics in their life.

George Polya, a mathematician, devised the following **four-step method** in solving word problems:

1. **Read and understand the problem**

   Read and reread the problem. Ask the following questions: What is the situation all about? What information is given? What are the different assumptions? What information is missing? What are you being asked to find or to do? You can make a list in this format

   Information given (GIVEN): ____________________

   Goals of the problem (REQUIRED): ____________________

2. **Plan how to solve the problem. Try a strategy**

   In developing a plan to solve the problem ask the following questions: Have you ever worked a similar problem before? Will you estimate or calculate? What strategy (ies) can you use? Consider the different strategies in solving problem that you know. The following could be your shopping list on the different problem solving strategies: draw a diagram, make a table, write an equation, guess and check (trial and error), look for a pattern, solve a simpler problem, work backward. Word problems can be solved using different strategies.

3. **Solve the problem/ Carry out the plan.**

   As you implement the strategy that you choose, ask the following questions: Did you calculate correctly? What is the solution? Did you interpret correctly? Did you answer the question?

4. **Look back**

   Most students forget this last part especially if they are solving a problem set or many problems. The most important question that should be asked here is: "Is the calculated answer reasonable?"

However there is also a danger in over-using Polya's four steps in problem solving. Students may think that problem solving is one directional that follows the given steps above as if it's the prescribed recipe. Emphasize to students that with challenging problems, the actual problem solving becomes a process whereby the solver keeps a mental "check" of the progress, and corrects himself if progress is not made. He may go one route, notice it won't work, go backwards a bit, and then, take another route. In other words, devising plans and carrying them out can occur somewhat simultaneously, and we can go back and forth. Lastly, not all problems can be solved using the steps above.

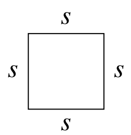
Problems Involving Perimeters

Example 9.1

1. The perimeter of a square is 28 cm. Find the length of each side.

   Solution

   Geometry Concept: A square is a quadrilateral (4-sided polygon) with four congruent sides. Its four interior angles are all right angles. The formula for the perimeter (P) of a square is $P = 4s$ and its area (A) is $A = s^2$ where s is the side of the square.



$$P = 4s$$
$$28 = 4s$$
$$\therefore s = 7$$

   Therefore, the square has side of length 7 cm.

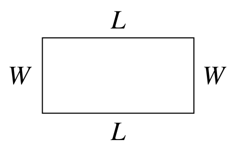
Example 9.2

1. The length of a rectangular garden is 3 meters more than its width. If the perimeter of the garden is 50 meters, what is the area of the rectangle?

   Solution

   Geometry Concept

   A rectangle is a quadrilateral (4-sided polygon) with two pairs of congruent and parallel sides. Its four interior angles are all right angles. The formula for the perimeter (P) of a rectangle is $P = 2L + 2W$ where L is the length and W is the width of the rectangle.

   Representation

   $$W = \text{width of the rectangle}$$
   $$L = W + 3$$

   Equation:

$50 = 2W + 3 + 2W$	Using the formula for perimeter.
$50 = 2W + 6 + 2W$	
$44 = 4W$	
$W = 11$	
$L = 11 + 3 = 14$	
$A = 1411 = 154$	Using the formula for area.

   Therefore, the rectangle has an area of 154 cm^2.

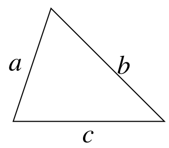
Example 9.3

1. The longest side of a triangle is twice as long as the shortest side and is also 2 cm longer than the third side. If the perimeter of the triangle is 33 cm, what is the length of each side?

   Solution

   Geometry Concept:

   A triangle is a 3-sided polygon. The formula for the perimeter (P) of a triangle is $P = a + b + c$ where a, b and c are the sides of the triangle.

   Representation

$a = x$	shortest side of the triangle
$b = 2x$	longest side
$c = 2x - 2$	third side

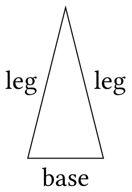
Equation

$$
\begin{aligned}
33 &= x + 2x + 2x - 2 && \text{(Using the formula for the perimeter.)} \\
35 &= 5x \\
x &= 7 \\
a &= x = 7 \\
b &= 2x = 14 \\
c &= 2x - 2 = 12
\end{aligned}
$$

Therefore, the sides of the triangle are 7 cm, 12 cm and 14 cm.

Example 9.4

1. What is the measure of the base of an isosceles triangle if its perimeter is 50 cm and the length of one of its congruent sides exceeds the length of the base by 10 cm?



Solution

Geometry Concept

An isosceles triangle is a triangle with two congruent sides. The congruent sides are called legs while the non-congruent side is called the base.

Representation

$$
\begin{aligned}
x &= \text{length of the base of the triangle} \\
x + 10 &= \text{length of the leg of the triangle}
\end{aligned}
$$

Equation

$$
\begin{aligned}
50 &= x + 2x + 10 && \text{(Using the formula for perimeter of triangles.)} \\
50 &= x + 2x + 20 \\
30 &= 3x \\
x &= 10
\end{aligned}
$$

Therefore, the base of the isosceles triangle is 10 cm.

PROBLEMS INVOLVING ANGLES OF POLYGONS

Example 9.5

1. The first angle of a triangle is triple that of the smallest angle. The second angle is 5 degrees more than the first angle. What is the measure of the smallest angle?

Solution

Geometry Concept

A triangle has three interior angles and their measures sum up to 180 degrees.

Representation

$$\begin{aligned} x &= \text{smallest angle of the triangle} \\ 3x &= \text{first angle of the triangle} \\ 3x + 5 &= \text{second angle of the triangle} \end{aligned}$$

Equation

$$\begin{aligned} x + 3x + 3x + 5 &= 180 \qquad \text{(Sum of the angles of a triangle is } 180^\circ\text{.)} \\ 7x &= 175 \\ x &= 25 \end{aligned}$$

Therefore, the measure of the smallest angle of the triangle is 25 degrees.

Example 9.6

1. One of the base angles of an isosceles triangle measures 15 degrees less than its vertex angle. Find the measure of its vertex angle.

   Geometry Concept

   In an isosceles triangle, it has two congruent angles opposite the congruent sides. The angle opposite the base is called the vertex angle.

   Solution

   Representation

$$\begin{aligned} x &= \text{vertex angle} \\ x - 15 &= \text{base angle} \end{aligned}$$

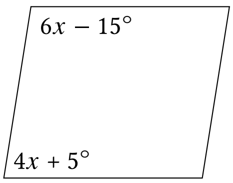
   Equation

$$\begin{aligned} 2x - 15 + x &= 180 \qquad \text{(Sum of the angles of a triangle is } 180^\circ\text{.)} \\ 2x - 30 + x &= 180 \\ 3x &= 210 \\ x &= 70 \end{aligned}$$

   Therefore, the measure of the vertex angle of the triangle is 70 degrees.

Example 9.7

1. Given the parallelogram on the right, find the value of x.



   Geometry Concept

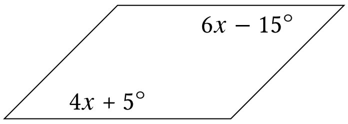
A parallelogram is a quadrilateral with two pairs of congruent and parallel sides. Its two pairs of opposite angles are congruent and the sum of the measures of any two consecutive angles is 180 degrees.

Solution

$$\begin{aligned} 4x + 5 + 6x - 15 &= 180 && \text{(The given angles are consecutive.)} \\ 10x - 10 &= 180 \\ 10x &= 190 \\ x &= 19 \end{aligned}$$

Example 9.8

1. Given the rhombus, find the measure of the angles of the rhombus.



Solution

Geometry Concept

A rhombus is a parallelogram with four congruent sides. Just like a parallelogram, its two pairs of opposite angles are congruent and the sum of the measures of any two consecutive angles is 180 degrees.

$$\begin{aligned} 6x - 15 &= 4x + 5 && \text{(The given angles are opposite.)} \\ 2x &= 20 \\ x &= 10 \\ 4x + 5 &= 45 \end{aligned}$$

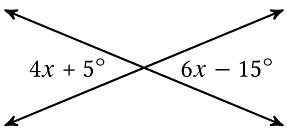
The given opposite angles have a measure of 45 degrees each while its other two angles measure $180 - 45 = 135$ degrees each.

Therefore, the measures of the angles of the rhombus are 45°, 45°, 135°, and 135°.

PROBLEMS INVOLVING COMPLEMENTARY, SUPPLEMENTARY AND CONGRUENT ANGLES

Example 9.9

1. Given two intersecting lines, find the value of x.



Solution

Geometry Concept

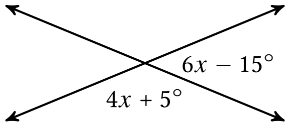
The two intersecting lines formed angles and the two given angles are "opposite" each other. They are called **vertical angles**. Vertical angles are congruent.

Since the given angles are vertical angle, they are congruent. Refer to Example 8.

Therefore, the value of x is 10.

Example 9.10

1. Given two intersecting lines, find the value of x.



Solution

Geometry Concept

The two intersecting lines formed angles and note that when the measures of the two given angles are added, we get the measure of a straight angle. These two angles are called linear pairs. The degree measures of a **linear pair** sum up to 180 degrees.

Since the given angles are linear pairs, they sum up to 180°. Refer to Example 7.

Therefore, the value of x is 19.

Example 9.11

1. The supplement of an angle is 20 degrees more than thrice the measure of the angle. What is the measure of the angle?

Solution

Geometry Concept: When the measures of two angles sum up to 180 degrees, they are called **supplementary angles**.

$$\begin{aligned} x &= \text{the given angle} \\ 180 - x &= \text{the measure of the supplement of the angle} \\ 180 - x &= 3x + 20 \\ 160 &= 4x \\ x &= 40 \end{aligned}$$

Therefore, the measure of the angle is 40 degrees.

Example 9.12

1. The supplement of a certain angle is four times its complement. What is the measure of the angle?

Solution

Geometry Concept: When the measures of two angles sum up to 90 degrees, they are called **complementary angles**.

$$\begin{aligned} x &= \text{the given angle} \\ 180 - x &= \text{the measure of the supplement of the angle} \\ 90 - x &= \text{the measure of the complement of the angle} \\ 180 - x &= 490 - x \\ 180 - x &= 360 - 4x \\ 3x &= 180 \\ x &= 60 \end{aligned}$$

Therefore, the measure of the angle is 60 degrees.

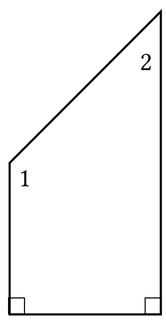
EXERCISES

1. If the width of a rectangle is 2 cm more than one-half its length and its perimeter is 40 cm, what are the dimensions of the rectangle?
2. Each side of a triangle with sides 3 cm, 4 cm and 5 cm is extended the same amount. If the new perimeter of the triangle is twice its original perimeter, what is the length of each side of the new triangle?
3. In a triangle, the ratio of the measures of its sides is 2:3:4 and the perimeter is 72 cm. Find the measures of the sides of the triangle.
4. In a triangle, the ratio of the measures of its angles is 2:3:4. Find the measures of the angles of the triangle.
5. If the numerical value of the area of a rectangle is twice the numerical value of its perimeter, and the length of one of its sides is 4.5, what are the lengths of its the other three sides?
6. The measure of an angle is thrice the measure of its supplement. Find the measure of the supplement of the angle.
7. What is the measure of an angle if the measure of its supplement is 10 degrees more than twice its complement?
8. The sum of the measures of an acute angle and an obtuse angle is 1400. The sum of twice the supplement of the obtuse angle and three times the complement of the acute angle is 3400. Find the measures of the angles.
9. When a beam of light is reflected from a smooth surface, the angle formed by the incoming beam with the surface is congruent to the angle formed by the reflected beam and the surface. Given the following information, the measure of $\angle ABC$ is 90, the measure of $\angle BCD$ is 75, and the beam of light makes an angle of 35° with line segment AR. At what angle does the beam reflect from line segment AB the second time. You can indicate computed measures on the figure.

   $m\angle EAF =$ ____________
10. The Leaning Tower of Pisa in Italy makes an angle with the ground of about 84° on one side the figure and is denoted by °. Find the measure of the other angle that the tower makes with the ground.

ADDITIONAL PROBLEMS

1. The design of the patio of Mr. Archangel is shown at the right. If the measure of $\angle 1$ is three times as large as the measure of $\angle 2$, calculate the measure of $\angle 1$.

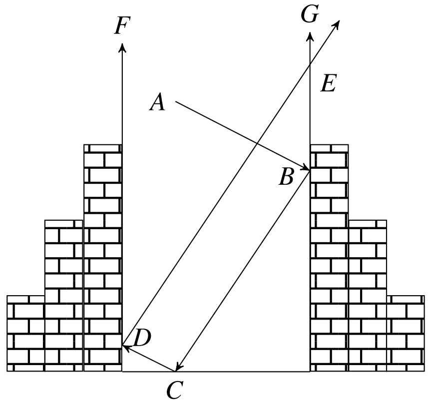


Solution

We have here a convex quadrilateral. The some of the measures of the interior angles of a quadrilateral is 360. Two angles are already right angles, hence, the sum of their measures is 180. The sum of the measures of $\angle 1$ and $\angle 2$ is 180.

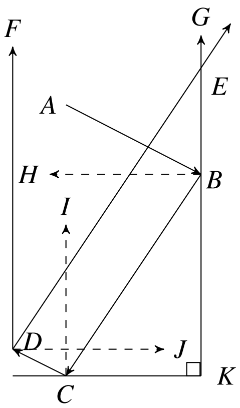
Setting up an equation, we have the following solution: Let x be the measure of $\angle 2$. The measure of $\angle 1$ is $3x$ and $x + 3x = 180$ will be our working equation. Simplifying we have $4x = 180$ and simplifying further $x = 45$. Therefore the measure of $\angle 1$ is 135.

2. At night time, it is usually warmer during summer at the urban regions that at the rural regions. Part of the reasons to this is the amount of heat absorbed by the establishments during the day which they will emit at night time. The narrow city streets and tall building trap and absorbs the sun's heat during the day. When sunlight shines on flat land, some of the light scatters back into the sky. In the city, light scattered by the ground or building often hits another building. Instead of escaping into the atmosphere the heat is absorbed. The more heat is absorbed the greater is the heat released at night time. The reflection of light is governed by the law of reflection which states that the angle of incidence is equal to the angle of reflection. The angle of incidence is the angle formed by the incident ray and the line perpendicular to the surface called the normal line. On the other hand, the angle of reflection is the angle formed by the reflected ray and the normal line. The figure below shows two parallel buildings on the opposite sides of the road. $\overrightarrow{AB}$ is the incident line and is reflected along the vertical side of the building. It is reflected as $\overrightarrow{BC}$, then reflected again as $\overrightarrow{CD}$ and finally as $\overrightarrow{DE}$. If the measure of $\angle ABG$ is 35, what is the measure of $\angle FDE$?



Solution

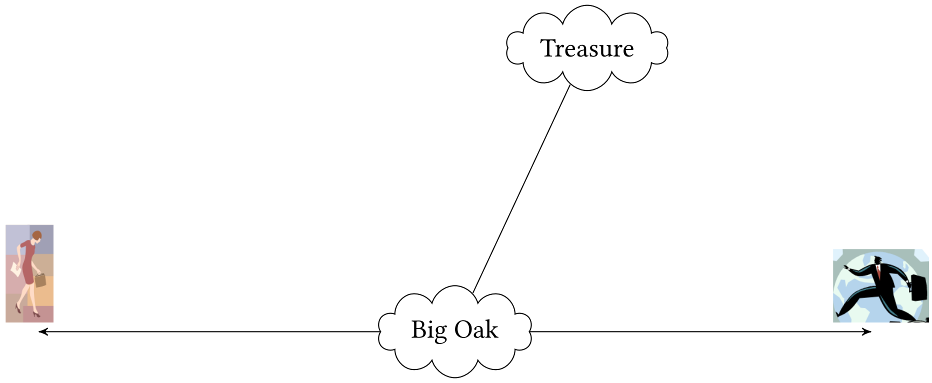
We know that the measure of $\angle ABG$ is 35. We are tasked to find the measure of $\angle FDE$. We also know that the angle of incidence is equal to the angle of reflection. To solve the problem, we need to redraw the figure and in each reflection process we draw the normal line. We will also use the theorem complements of congruent angles are congruent. (The normal line is perpendicular to the surface. The angle of incidence and the angle formed by the incident ray and the surface are complementary). For the purpose of clearer solution, we will enlarge the figure and remove the buildings. The broken arrows are the normal line.



The measure of $\angle ABG$ is 35. Therefore its complement ($\angle ABH$) has a measure of 55. Complementary angles are pair of angles whose some of their measures is equal to 90. $\angle ABH$ is the angle of incidence while $\angle HBC$ is the angle of reflection. By the law of reflection these angles ($\angle ABH$ and $\angle HBC$) have equal measure. The normal ray ($\overrightarrow{BH}$) is parallel to the surface of the road (since the road and the side of the building are perpendicular and the normal line is perpendicular to the side of the building. Lines perpendicular to the same line are parallel). $\overline{CB}$ is the transversal and $\angle HBC$ and $\angle BCK$ are alternate interior angles. (Alternate interior angles of parallel lines cut by a transversal are congruent). Therefore the measure of $\angle BCK$ is 55. The measure of $\angle BCI$ (angle of incidence) is 35 since $\angle BCK$ and $\angle BCI$ are complementary.

Using the same processes, the measure of $\angle BCI$ is 35, the measure of $\angle ICD$ is 35, $\angle CDJ$ is 55, $\angle JDE$ is also 55 and finally the measure of $\angle FDE$ is 35.

3. Kaitlin and Henry are participating in a treasure hunt. They are on the same straight path, walking toward each other. When Kaitlin reaches the Big Oak, she will turn 115° onto another path that leads to the treasure. At what angle will Henry turn when he reaches the Big Oak to continue on to the treasure?



Solution

To solve the problem, we need to draw a diagram that illustrates the problem. Below is the diagram that illustrates the problem. We need to assume that Big Oak, Kaitlin and Henry are on the same line. The angle formed by the rays joining Kaitlin, Big Oak and treasure has a measure equal to 115°. Therefore we need to get the measure of the supplement of that angle since it's the angle where Henry will turn if he will reach Big Oak. Hence, Henry should turn at 65°-angle to continue to the treasure.

SUGGESTED ACTIVITY

This is adopted from Sourcebook on Practical Work for Teacher Trainers: High School Mathematics I and II Volume 2.

Exploring the Geometry in a Paper Cup

Objectives

1. To apply geometric concepts and principles
2. To justify or prove geometric relationships

Materials

1. 5 pieces of coupon bond
2. 2 square papers

Instructional Procedures

Introductory Activity

1. *Constructing perpendicular lines by paper folding*

   Get a piece of paper. Fold the paper to show two (2) creases perpendicular to each other. Call the fist crease $\overline{AB}$ and the second crease $\overline{CD}$. Explain why you think the creases you made are perpendicular.

2. *Constructing perpendicular bisector by paper folding*

   Get another piece of paper. Make a crease and call it $\overline{AB}$. Make another crease $\overline{CD}$ that bisects $\overline{AB}$. How do you know that with your procedure the second crease bisects the first? Does the first crease bisect the second?

3. *Constructing bisector of angles by paper folding*

   Get another piece of paper. Make a crease to make an angle with one side on the lower edge of the paper. Call this angle $\angle BAC$. Did you make an acute, right or obtuse angle? Show how you will bisect $\angle BAC$ by folding. Call this bisector $\overline{AD}$.

Lesson Proper

1. *Making a cup*

   Let us do an activity on paper folding. Use the square paper and follow the procedure in the activity sheet that will be distributed.

2. *Post-Activity Discussion*

   Let us examine the mathematics in the activity you did. Let us discuss your answers to the questions in the activity sheet.

   (a) Which paper folding procedure in our previous activity was used in steps 1, 2 and 3?
   (b) Let us fill the table with the names of the polygons found in the figure.
   (c) Identify some figures which are symmetrical and give the line of symmetry for each (you can omit this since it's not included in the topics to be discussed)
   (d) Now we consider the congruent angles and congruent segments that resulted from the folding. Use the upper triangular half of the figure only.
   (e) List down other possible problems that can be formulated based on the resulting figure in the activity

Assessment

Give a piece of square paper. Call the diagonal fold $\overline{AB}$.

1. Fold the paper to show an angle of measure 67.5° with vertex at A.

2. Using the same paper used in number 1, fold the paper to show 2 creases perpendicular to $\overline{AB}$.

3. Trace the following figures. Name the points as needed and indicate the measure of the angles.

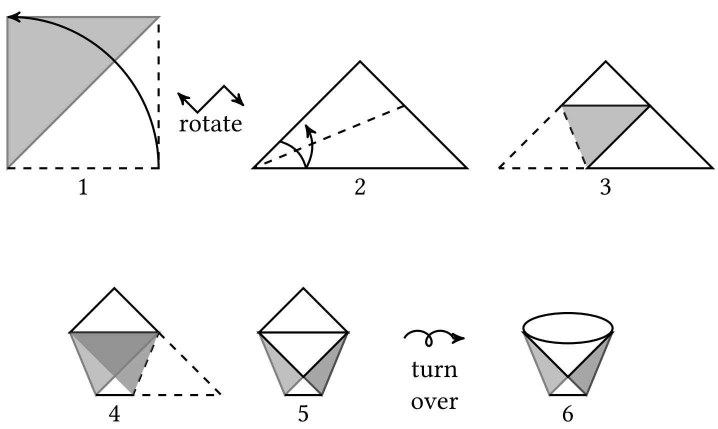
   (a) Two triangles of different sizes but of the same set of angle measure.
   (b) Two triangles that are congruent. Explain why they are congruent.
   (c) A trapezoid. Find the measure of its angles and explain why you think it is a trapezoid.

   If the above questions (assessment of this paper folding activity) are still too high for your students, then you can use the following questions instead.

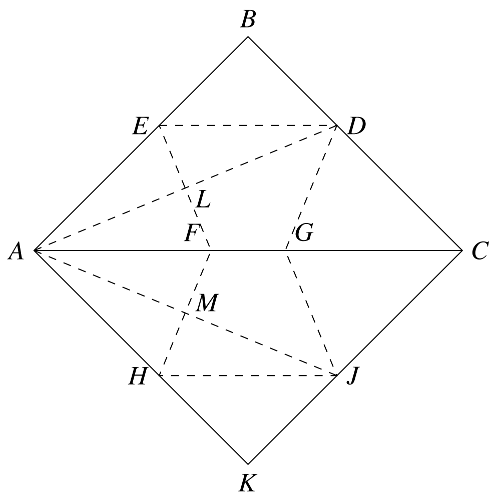
   (a) What do the creases represent?
   (b) Name congruent segments.
   (c) Name congruent angles.
   (d) Name acute, right and obtuse angles.
   (e) Name complementary, supplementary and vertical angles.
   (f) Name an angle bisector and the angle it bisects
   (g) Name segments which are perpendicular to each other.
   (h) Name segments which are parallel.

Making a Paper Cup

A. To make a cup, fold the square paper by following the procedure below.



B. We study some mathematics related to the figures. carefully unfold the final figure. Label the points as shown.



Answer the following questions. Discuss your answers with your group.

(a) Study steps 1 to 3. Which folding in our introductory paper folding procedures correspond to each step?

(b) What polygons do you recognize? Name some of them and classify them as indicated in the following table.

Polygons	Some Convex Polygons	Some Concave Polygons
Triangle		
Quadrilateral		
Pentagon		
Hexagon		

(c) Study the symmetrical figures. Can you identify symmetrical figures? Name some and identify the line of symmetry for each.

(d) Name all the line segments that are congruent by superimposing. Name also the congruent angles.

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Module 10

INTRODUCTION TO STATISTICS

INTRODUCTION

This module present a discussion on statistics to include key concepts, uses and importance of statistics and probability, data collection and the different forms of data representation. It will also discuss the measures of central tendency for ungrouped and grouped data. Every lesson will present examples, application and sample assessment tools. You will appreciate more the importance of statistics which you will also share to your students.

OBJECTIVES

After completing this module, you should be able to:

1. Explain the basic concepts, uses, and the branches of statistics;
2. Collect or gather statistical data and organize in a frequency table according to some systematic considerations;
3. Use appropriate graph to represent organized data: pie, bar graph, line graph, and histogram;
4. Analyze, interpret accurately and draw conclusions from graphic and tabular presentations of statistical data;
5. Find the mean, median and mode of statistical data; and
6. Describe the data using information from the mean, median and mode.

DISCUSSION

Definition 10.1: Statistics

Statistics is the science of conducting studies to collect, organize, summarize, analyze and draw conclusions from data (Bluman, 2004). It is a science of collection and classification of facts in the basis of relative number or occurrence.

The **collection** or **gathering of data** may be undertaken using interview, questionnaires, tests, observation, registration, and experiments. **Organization of data** is the manner of presenting the gathered data into tabular, textual or graphical form. The **summarized data** will be subjected to **analysis** in order to extract relevant data and information using statistical tools or techniques. **Interpretation of data** refers to the drawing of conclusions or inferences from the analyzed data (Basilio, Faith B., Edna A. Chua, Maria T. Juwanan, 2003).

Uses of Statistics

Statistics has many uses. Among the contributions of statistics are the following:

- Statistics provides us with the ways and means of expressing our thoughts in the most definite and exact way feasible. Example: There are more females in the first year than males; it will rain on June.
- Statistics provides us with numbers and figures with which to describe completely and accurately the characteristics of certain phenomena. Example: There are 25 females in the first year and 20 are males; it will rain on June 10.
- Statistics enables us to express the results of research activities meaningfully and present them in a form easily understood and read. Example: in tables or graphs.
- Statistics allows us to draw inferences or conclusions from characteristics of given data.

In *Training Workshop on Teaching Basic Statistics* (2007), it was mentioned that the eminent statistician Bradley Efron summarized how diverse the uses of statistics:

> During the 20th Century statistical thinking and methodology has become the scientific framework for literally dozens of field including education, agriculture, economics, biology, and medicine, and with increasing influence recently on the hard sciences such as astronomy, geology, and physics. In other words, we have grown from a small obscure field into a big obscure field.

BRANCHES OF STATISTICS

There are two areas of statistics namely: descriptive statistics and inferential statistics.

Definition 10.2: Descriptive Statistics

Descriptive Statistics tend to bring out into light only the significant aspects of data. A large mass of data come into man's hands for his use. Oftentimes these data are complex that yield so much information.

Example 10.1

1. the rating obtained by the senior students of public high schools in the National College Entrance Examination will yield an almost unlimited amount of information such as: the highest ratings; the lowest rating; the most frequent rating; the range of the ratings; the number of students who belong to the upper 75%, 50%, or 25% of the group; and many more.

Definition 10.3: Inferential Statistics

Inferential Statistics is concerned in drawing conclusions or inferences or from the given characteristics of the samples to specific properties of the population. In inferential statistics, testing the significant difference and independence between two or more variables are given emphasis. An assertion or hypothesis about the population is made and is intended to be rejected or accepted depending on the result of a test based from available samples.

Before we proceed with the discussion of statistics, there are some terms commonly used in statistics that we have to define.

Definition 10.4: Universe, Variable, Parameters

The **universe** or **population** is the set of all entities under study. Meanwhile, a **variable** is the attribute of interest observable of each entity in the universe. Parameters are numerical measures that describe the population of interest. Example, you can consider all the students in 7-Quartz as the population, a possible variable is their math grade, while a possible parameter is the average math grade of the class (*Training Workshop on Teaching Basic Statistics* 2007).

The *population* refers to groups or aggregates of people, animals, objects, materials, happenings or things of any form (Punzalan and Uriarte 2000). While **sample** is a subset or a representative group of a population to represent the characteristics or traits (Basilio et al. 2003). For example, in a baranggay with 5,000 families, a sample could be a group of 100 families.

A **parameter** is any measure of a given characteristic of the *entire* population, for example the average number of children in each family in a certain baranggay. On the other hand, a **statistic** is any measure of a given characteristics based on a *part (sample)* of a population under study. Example, the corresponding statistic to the previous example is the average number of children in a sample of 100 from a population of 5,000 families.

VARIABLES AND TYPES OF DATA

The building blocks of statistical science are data (*Training Workshop on Teaching Basic Statistics* 2007).

Definition 10.5: Data

Data are the values (measurement or observations) that the variables can assume (Bulman 2004). **Variables** refer to the fundamental quantity that changes/vary in value from one observation to another within a given domain and under given set of conditions (Del Rosario 2005). Quantities that do not take different values from one observation to another are called constants. Variables can be classified as **quantitative** and **qualitative**. Quantitative are numerical data while qualitative are categorical data.

Some examples of quantitative variables are age, height, weight, scores in the exam, and temperature. Examples of qualitative data include food preference, student's section, city address, favorite color, and favorite subject.

PRESENTATION OF DATA

Data gathered can be presented in many ways: tabular, graphical and even textual. In this section, it will present the mechanics on how to present data using frequency distribution, histogram, frequency polygon, bar graph, line graph, and pie chart.

Definition 10.6: Frequency distribution

A **frequency distribution** is the organization of raw data in table form, using classes and frequencies (Bulman 2004).

Frequency Distributions for qualitative data look like tallies of the number of data that corresponds to the different values of the variable.

Example 10.2

1. A social worker wanted to determine the religious affiliation of twenty families in a certain barangay. The data set is

Aglipayan	Protestant	Roman Catholic	Muslim
Born Again Christian	Aglipayan	Protestant	Roman Catholic
Aglipayan	Muslim	Protestant	Roman Catholic
Roman Catholic	Protestant	Roman Catholic	Roman Catholic
Roman Catholic	Protestant	Roman Catholic	Roman Catholic

To make a table which summarizes the data, follow these steps:

(a) Make a table as shown.

A	B	C	D
Class	Tally	Frequency	Percent
Aglipayan			
Protestant			
Roman Catholic			
Born Again			
Christian			
Muslim			
	Total	(n)	

2. Tally the data and place the results in column B.

3. Count the tallies and place the corresponding frequency in column C.

4. Find the percentage in each class by using the formula:

$$\text{Percentage} = \frac{f}{n} \cdot 100\%$$

5. Find the totals for column C and D to complete the table.

Table 10.1

A Distribution of the Religious Affiliation of Twenty Families

A	B	C	D
Class	Tally	Frequency	Percent
Aglipayan	///	3	15
Protestant	~~////~~	5	25
Roman Catholic	~~////~~ ////	9	45
Born Again Christian	/	1	5
Muslim	//	2	10
	Total (n)	29	100

So from the given example, more families are affiliated with Roman Catholic than any other religion.

Frequency Distributions for Quantitative Data are also called Grouped Distributions. This is because data which are close enough are grouped into one and these groups are called **classes**. Each class has a **lower class limit** and an **upper class limit** which serve as guides as to which data will be counted to each class. The **class boundaries** are the midpoints of the upper class limits of one class and the lower class limit of the next class. The **class width** is the difference between two successive class boundaries. It also corresponds to the difference between two successive lower class limits or two successive upper class limits. The **class mark** is the midpoint of each class. The **cumulative frequency** is a continuous tally of the frequencies. Here's an example of a Quantitative Frequency Distribution:

Table 10.2

Distribution of Math Long Test Scores of a sample of 24 students

Lower Class Limit	Upper Class Limit	Lower Class Boundary	Upper Class Boundary	Class Mark	Frequency	Cumulative Frequency
10	14	9.5	14.5	12	4	4
15	19	14.5	19.5	17	9	13
20	24	19.5	24.5	22	6	19
25	29	24.5	29.5	27	5	24

Example 10.3

1. Given the scores of sophomore students in Mathematics 1 periodical test, construct the frequency distribution.

50	55	60	41	34	58	37	35	56	59
48	20	65	63	61	28	36	55	53	60
33	45	56	60	42	56	41	45	57	59

Solution

(a) Find the **range** by using the formula:

$$\begin{aligned}\text{Range} &= \text{Highest Score} - \text{Lowest Score}\\ &= 65 - 20\\ &= \boxed{45}\end{aligned}$$

(b) Determine the **number of classes** (k). The ideal number of class intervals is between 5 and 20 depending on the nature of data. You can also use the formula to determine the number of intervals

$$\begin{aligned}k &= \sqrt{n}\\ k &= \sqrt{30}\\ &= 5.4\\ &\approx 5\end{aligned}$$

Another way of finding the number of intervals is using the **Sturges' Rule**: $K = 1 + 3.3 \log n$; where n is the number of samples.

For the purposes of this module as Introduction to Statistics, the teacher should set the number of intervals (k). Usually, limit the data to $20 - 50$, using $4 - 6$ classes.

(c) Determine the **class width/interval** (i) using this formula:

$$\begin{aligned}\text{Class Width } i &= \frac{R}{k}\\ &= \frac{45}{5}\\ &= \boxed{9}\end{aligned}$$

Usually, the computed value for the class width is not a whole number. If this is the case, *round up to the nearest whole number.*

If the computed value for the class width is a whole number, consider adding 1. Not doing so might result to at least one of the data points being excluded in the table.

(d) Determine the **class limits**. The lower limit of the lowest class should be a multiple of the class width.

Since $i = 9$ and the lowest score is 20 which is not a multiple of the class width thus, 18 will be the lower limit. Therefore, the lowest class interval will be $18 - 26$.

(e) Determine the **class boundaries** by subtracting 0.5 from each lower class limit and adding 0.5 to each upper limit: $17.5 - 26.5$, $26.5 - 35.5$.

(f) Determine the **class mark/midpoint** of every class. It is obtained by getting the sum of the lower and upper limits of a class divided by two.

$$\text{Class Mark} = \frac{18 + 26}{2} = 22$$

(g) Tally the data.

(h) Find the numerical frequencies from the tallies.

(i) Find the cumulative frequencies.

Class Limits	Class Boundaries	Class Marks	Tally	Frequency	Cumulative Frequency
18 - 26	17.5 - 26.5	22	𝍸𝍸𝍸𝍸||	1	1
27 - 35	26.5 - 35.5	31	𝍸𝍸𝍸𝍸𝍸𝍸|	4	5
36 - 44	35.5 - 44.5	40	𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸	5	10
45 - 53	44.5 - 54.5	49	𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸||||	5	15
54 - 62	54.5 - 62.5	58	𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸|||	13	28
63 - 71	62.5 - 71.5	67	𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸||	2	30
			Total n	30	

Table 10.3

A Distribution of the Mathematics 1 Periodical Test Scores of Thirty Sophomore Students

There are also ways to summarize data in the form of graphs or charts. The main advantage of using charts and graphs is that they give a visual representation of the data. This makes interpretation of the data easier compared to just seeing data in tables.

Here are some examples of usual graphs and charts you can use in order to summarize data.

Definition 10.7: Histogram

A **histogram** displays data by using adjacent vertical bars of various heights to represent the frequencies of the classes. The histogram is a graphical representation of a frequency distribution table.

Example 10.4

1. Construct a histogram to represent the frequency distribution below for the scores of sophomore students in Mathematics 1 Periodical Test.

Class limits	Class Boundaries	Frequency
18 - 26	17.5 - 26.5	1
27 - 35	26.5 - 35.5	4
36 - 44	35.5 - 44.5	5
45 - 53	44.5 - 54.5	5
54 - 62	54.5 - 62.5	13
63 - 71	62.5 - 71.5	2
		$n = 30$

Solution

(a) Draw and label the x and y axes.

(b) Represent the frequency on the y-axis and the class boundaries on the x-axis.

(c) Using the frequencies as the heights, draw vertical bars for each class.

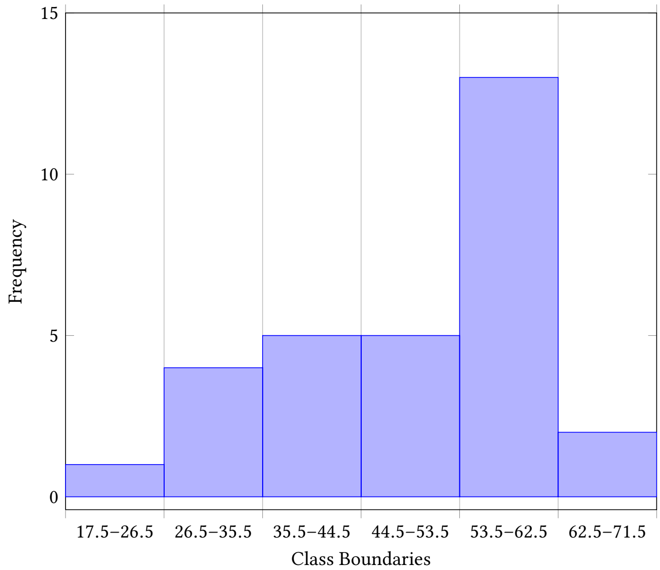


Figure 10.1

Scores of Sophomore Students in Mathematics 1 Periodical Test (Histogram)

The histogram presents that the class boundary with the greatest frequency is 54.5 - 62.5 while the lowest is 17.5 - 26.5.

Definition 10.8: Frequency Polygon

A **frequency polygon** connects the points representing relevant data in the coordinate plane.

Example 10.5

1. Construct a frequency polygon from the histogram for the scores of sophomore students in Mathematics 1 Periodical Test.

   Solution

   (a) Draw and label the x and y axes.

   (b) Represent the frequency on the y-axis and the class mark on the x-axis.

   (c) Connect the points.

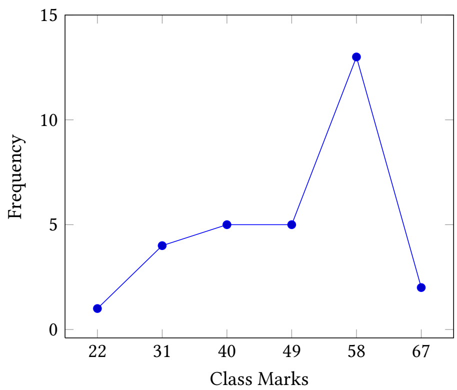


Figure 10.2

Scores of Sophomore Students in Mathematics 1 Periodical Test (Line Graph)

Definition 10.9: Bar Graph

A **bar graph** is an illustration of the data using bars in the coordinate plane. This can be used for quantitative data.

Example 10.6

1. The table shows the educational attainment distribution of parent-respondents.

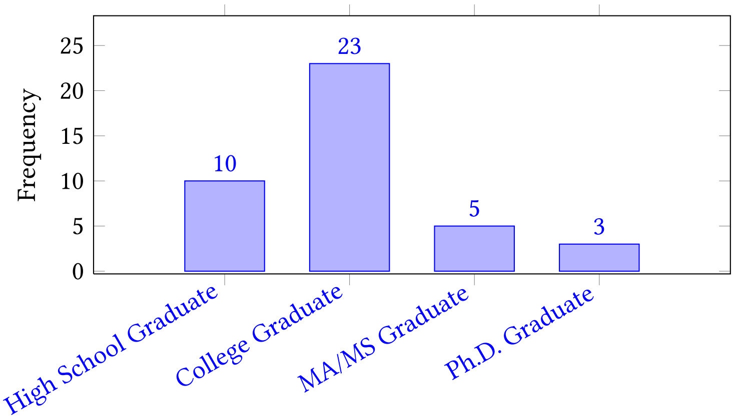
Educational Attainment	Frequency
High School Graduate	10
College Graduate	23
MA/MS Graduate	5
PhD Graduate	3

Solution

(a) Draw and label the x and y axes.

(b) Represent the frequency on the y-axis and the category on the x-axis.

(c) Draw the bars for each class. Place corresponding category on top of the bars.



Educational Attainment of Parent-Respondents

The bar graph shows that *College Graduate* had the highest frequency and Ph D graduate the lowest.

Definition 10.10: Pie Chart

A **pie chart** is a circle that is divided into sections according to the percentage of frequencies in each category of the distribution. This is usually used to show the different components of the population.

Example 10.7

1. The table below shows the enrolment data of PSHS-Ilocos Region Campus for School Year 2011 – 2012. Construct a pie graph for the data.

Year Level	Frequency
1st Year	88
2nd Year	78
3rd Year	65
4th Year	48

Solution

(a) A circle has 360°, the frequency for each class should be converted into a proportional part of the circle. The formula is as follows:

$$\text{Degrees} = \frac{f}{n} \cdot 360$$

where f = frequency of each class and n = sum of the frequencies.

Year Level	Frequency	Proportional Part
1st Year	88	$\frac{88}{279} \cdot 360° = 113.55° \approx 113°$
2nd Year	78	$\frac{78}{279} \cdot 360° = 100.65° \approx 101°$
3rd Year	65	$\frac{65}{279} \cdot 360° = 83.87° \approx 84°$
4th Year	48	$\frac{48}{279} \cdot 360° = 61.94° \approx 62°$

(b) Convert the frequency of each class to percentage using the formula:

$$\% = \frac{f}{n} \cdot 100\%$$

(c) Draw the graph using a protractor and a compass. Label each section with the name and percentages.

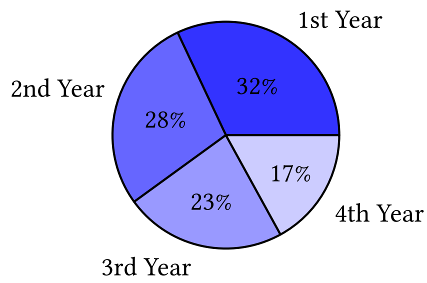


Figure 10.3

Enrolment Data of PSHS-Ilocos Region Campus for School Year 2011–2012

From Figure (10.3), it could be observed that the enrolment of freshmen has the greatest, with 32% compared to the other year levels.

2. Below is a pie chart showing the population of the four provinces of Region 1 on 2010. What can you infer? Make necessary conclusion for this chart.

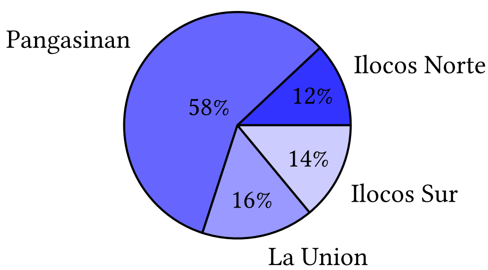


Figure 10.4

Population for Region 1 Based on 2010 Censuses

Solution

Possible Answers: From the four provinces, Pangasinan has the most number of living individuals. More than fifty percent of the region's population hales in Pangasinan.

MEASURES OF CENTRAL TENDENCY

A data can be summarized by using a single number, which is the concentration point of scores. The three (3) measures of central tendency are the **mean**, **median**, and the **mode**.

The table below presents the properties of the different measures of central tendency which was adopted from Bluman Bulman (2004).

Measure	Symbol	Definition	Properties and Uses
Mean	$\mu, \bar{X}$	Sum of values, divided by total number of values	1. One computes the mean by using all values of the data. 2. The mean varies less than the median or mode when samples are taken from the same population and all three measures are computed for these samples. 3. The mean is used in computing other statistics, such as the variance. 4. The mean for the data set is unique and not necessarily one of the data values. 5. The mean cannot be computed for an open-ended frequency distribution. 6. The mean is affected by extremely high or low values, called outliers, and may not be the appropriate average to use in these situations.
Median	MD	Middle point in data set that has been ordered	1. The median is used when one must find the center or middle value of a data set. 2. The median is used when one must determine whether the data values fall into the upper half or lower half of the distribution. 3. The median is affected less than the mean by extremely high or extremely low values.

Continued to next page...

Table 10.4

(continued)

Measure	Symbol	Definition	Properties and Uses
Mode	Mo	Most frequent data value	1. The mode is used when the most typical case is desired. 2. The mode is the easiest average to compute. 3. The mode can be used when the data are nominal, such as religious preference, gender, or political affiliation. 4. The mode is not always unique. A data set can have more than one mode, or the mode may not exist for a data set.

Example 10.8

1. The data represent the lengths of service (in years) for a sample of nine (9) PSHS- IRC teachers. Find the mean, median and mode.

9,9,8,2,3,5,4,2,7

Solution

(a) **Finding the mean of a list of data**

$$\begin{aligned}\bar{X} &= \frac{\sum X}{n}\\ &= \frac{9+9+8+2+3+5+4+2+7}{9}\\ &= \frac{49}{9}\\ &= \boxed{5.44}\end{aligned}$$

Hence, the mean of the length of service (in years) of PSHS-IRC teachers is 5.44 years.

(b) **Finding the median of a list of data**

Step 1. Arrange the data in an array (increasing/decreasing order).

9, 9, 8, 7, 5, 4, 3, 2, 2

Step 2. Select the middle value. That is the median of the data set.

$$9, 9, 8, 7\underbrace{\boxed{5}}_{\text{Median}}, 4, 3, 2, 2$$

Hence, the median length of service (in years) of PSHS-IRC teachers is 5.

This method works only if there is an odd number of data in the set. However, if the number of data is even, there will be two middle values. To get the median of that data set, get the average of the two middle values.

Suppose the data set is 9, 9, 8, 7, 6, 5, 4, 3, 2, 2. The two middle values are 6 and 5. So the median is the average of these two numbers, which is 5.5.

(c) **Finding the mode of a list of data**

Step 1. Arrange the data in an array (increasing/decreasing order).

9, 9, 8, 7, 5, 4, 3, 2, 2

Step 2. Select the value that occurs most often in a data set.
Since, 9 and 2 occur 2 times, the modes are 9 and 2. This data set is said to be **bimodal**. If the data has no repeated value, then there is no mode. Also, if for example all the values occur twice, thrice or whatever number of times, there is also no mode.

ENRICHMENT LESSON

Example 10.9

1. Below is a distribution of Mathematics 1 periodical test scores of thirty sophomore students. Compute the mean, median and mode.

Class Limits	Class Boundaries	Frequency f
18 - 26	17.5 - 26.5	1
27 - 35	26.5 - 35.5	4
36 - 44	35.5 - 44.5	5
45 - 53	44.5 - 54.5	5
54 - 62	54.5 - 62.5	13
63 - 71	62.5 - 71.5	2
		$n = 30$

Solution

(a) **Finding the mean of grouped data**

Step 1. Make a table as shown.

A Class Limits	B Class Boundaries	C Class f	D Frequency Marks X_m	E $f \cdot X_m$
18 - 26	17.5 - 26.5	1		
27 - 35	26.5 - 35.5	4		
36 - 44	35.5 - 44.5	5		
45 - 53	44.5 - 54.5	5		
54 - 62	54.5 - 62.5	13		
63 - 71	62.5 - 71.5	2		
		$n = 30$		

Step 2. Compute the class marks/ midpoints of each class and complete column D.

A Class Limits	B Class Boundaries	C Class f	D Frequency Marks X_m	E $f \cdot X_m$
18 - 26	17.5 - 26.5	1	22	
27 - 35	26.5 - 35.5	4	31	
36 - 44	35.5 - 44.5	5	40	
45 - 53	44.5 - 54.5	5	49	
54 - 62	54.5 - 62.5	13	58	
63 - 71	62.5 - 71.5	2	67	
		$n = 30$		

Step 3. For each class, multiply the frequency by the class mark, and fill-in column E.

A Class Limits	B Class Boundaries	C Class f	D Frequency Marks X_m	E $f \cdot X_m$
18 - 26	17.5 - 26.5	1	22	124
27 - 35	26.5 - 35.5	4	31	200
36 - 44	35.5 - 44.5	5	40	200
45 - 53	44.5 - 54.5	5	49	245
54 - 62	54.5 - 62.5	13	58	754
63 - 71	62.5 - 71.5	2	67	134
		$n = 30$		

Step 4. Compute $\sum f \cdot X_m$.

A Class Limits	B Class Boundaries	C Class f	D Frequency Marks X_m	E $f \cdot X_m$
18 - 26	17.5 - 26.5	1	22	124
27 - 35	26.5 - 35.5	4	31	200
36 - 44	35.5 - 44.5	5	40	200
45 - 53	44.5 - 54.5	5	49	245
54 - 62	54.5 - 62.5	13	58	754
63 - 71	62.5 - 71.5	2	67	134
		$n = 30$		$\sum f \cdot X_m = 1479$

Step 5. Identify the **cumulative frequency below the median class** f_c.
The cumulative frequency below the median class f_c is 10.

Step 6. Identify the class width i.
The class width i is 9.

Step 7. Use the formula below to compute for the median.

$$\text{Md} = L + \left[\frac{0.5n - f_c}{f_m}\right] i$$
$$\text{Md} = 44.5 + \left[\frac{0.530 - 10}{5}\right] 9$$
$$\text{Md} = 44.5 + \left[\frac{15 - 10}{5}\right] 9$$
$$\text{Md} = 44.5 + 19$$
$$\text{Md} = \boxed{53.5}$$

The median score of the Mathematics 1 periodical test of thirty sophomore students is 53.5.

(b) **Finding the mode of grouped data (True Mode)**

Step 1. Compute the median.
From the previous computation, the median is 53.5.

Step 2. Compute the mean.
The computed mean was 49.3.

Step 3. Use the formula below to compute:
Design or create an activity of your own in which you can apply the concepts of the mean, median, and mode.

$$\text{Mo} = 3\text{Md} - 2\bar{X}$$
$$\text{Mo} = 353.5 - 249.3$$
$$\text{Mo} = 160.5 - 98.6$$
$$\text{Mo} = \boxed{61.9}$$

The true mode is 61.9.

SUGGESTED ACTIVITIES

The activities below will further enhance your skills in statistics. Through these activities, you will appreciate statistics and its applications.

1. The two situations that follow involve the use of descriptive statistics and inferential statistics.

   Situation 1. A statistics teacher ranks his students according to their average grades.

   Situation 1 involves the use of descriptive statistics since it is only a matter of computing the average grade of the students and then you just rank them from highest to lowest or vice versa.

   Situation 2. A statistics teacher employs one teaching technique in one class and another teaching technique in another class and then gives the same examination. Using the results, he determines which technique is more effective.

   Situation 2 involves the use of inferential statistics since it utilizes statistical tools to determine which of the two techniques used by the teacher is more effective, i.e. using pre-test/post-test and comparing means using appropriate statistical tool.

   Write two examples of situation which involves the use of descriptive and inferential statistics.

2. Construct a bar graph and pie chart for the following table on the number of participants in the UPLIFT 2012.

Province	Number of Participants

3. Below is a table showing the population of the four provinces of Ilocos Region.

Region	Total Population			Population Growth Rate		
	May 1, 1990	May 1, 2000	May 1, 2010	1990-2000	2000-2010	1990-2010
Region I-Ilocos Region	**3,550,642**	**4,200,478**	**4,748,372**	**1.69**	**1.23**	**1.46**
Ilocos Norte	461,661	514,241	568,017	1.08	1.00	1.04
Ilocos Sur	519,966	594,206	658,587	1.34	1.03	1.19
La Union	548,742	657,945	741,906	1.83	1.21	1.52
Pangasinan	2,020,273	2,434,086	2,779,862	1.88	1.34	1.61

Table 10.5

2010 Census and Housing Population. Population and Annual Growth Rates for Region 1 Based on 1990, 2000, and 2010 Censuses. Source: `http://www.census.gov.ph/data/census2010/index.html`(2012)

   What can you infer in the table? Make necessary conclusion for this table.

4. Figures (10.5) and (10.6) present the Basic Literacy Rate of Population 10 to 64 years by age in the Philippines for the year 2003.

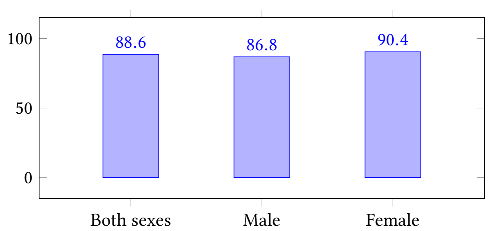


Figure 10.5

Basic literacy rate of population 10 to 64 years by sex, Philippines 2003. Source: http://62.0.5.133/www.census.gov.ph/data/sectordata/f103_lsff2.gif

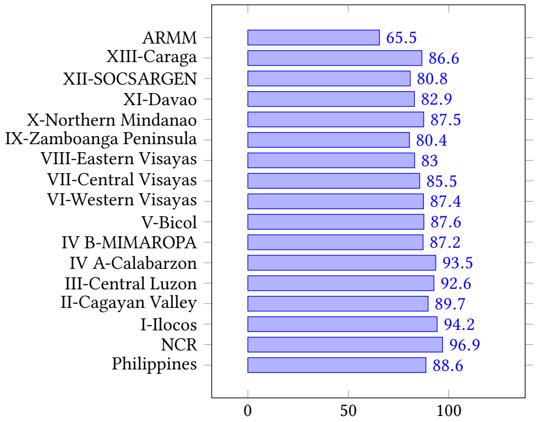


Figure 10.6

Basic literacy rate of population 10 to 64 years by region, Philippines 2003. Source: http://62.0.5.133/www.census.gov.ph/data/sectordata/f103_lsff2.gif

5. Below is a pie chart which shows the percentage of OFWs by place of work from April to September 2000.

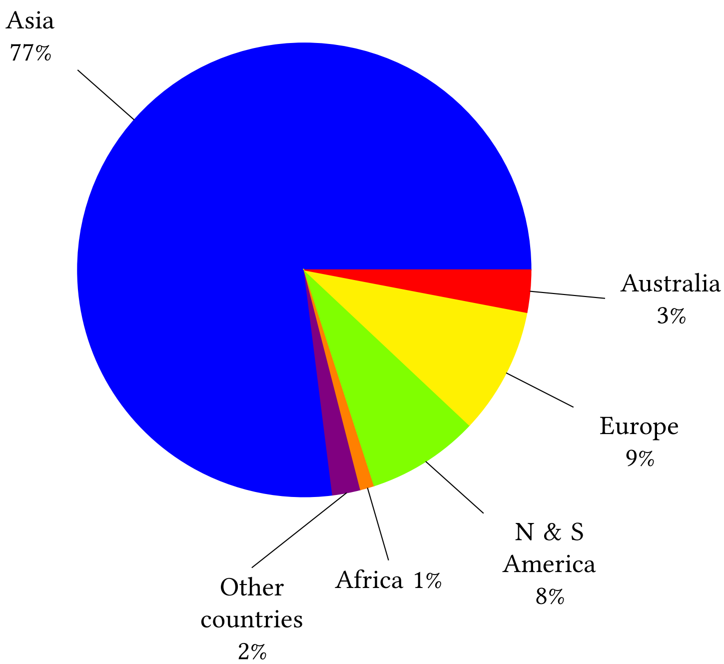


Figure 10.7

Percentage of OFWs by Place of Work: April to September 2000. Source: http://www.census.gov.ph/

What conclusion can you draw from the chart?

6. The data represent the monthly salary for a sample of government employees from San Ildefonso, Ilocos Sur. Find the mean, median and mode.

₱31,200	₱23,500	₱25,300	₱31,200	₱9,000
₱31,200	₱8,000	₱15,700	₱16,300	₱10,000
₱21,000	₱9,500	₱12,000	₱16,000	₱14,000

7. The ages of 40 teachers in a certain school are as follows:

43	58	21	24	31	49	40	51	55	28
50	33	62	30	25	39	59	29	36	42
38	46	42	52	26	50	41	37	35	40
47	35	57	55	36	45	32	45	42	36

(a) Set up a frequency distribution using 6 classes with the smallest starting at 21.

(b) Derive a cumulative frequency distribution for (7a).

(c) Draw a histogram (7a).

(d) Calculate the mean, median, and the mode of the distribution.

MISLEADING STATISTICS

Work with a group to explore how statistics can be misleading. Think about how the data could be represented in other non-misleading way.

1. Boy 1: Have you seen the poster outside this building saying

Electronic Load Earn Money The Easiest Way Average Php 2,000 per week For information, call 888-02-02

I think it will be suited for us.

Boy 2: Yes, but I doubt because it could be misleading.

(a) Discuss reasons why the poster Boy 1 and Boy 2 saw could be misleading.

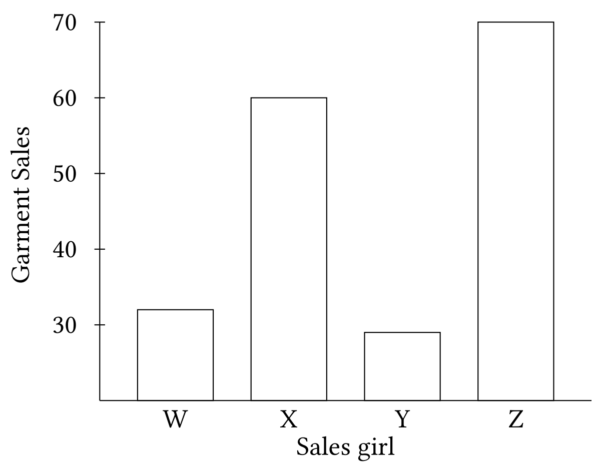
(b) Create a hypothetical frequency table that indicates why the "average" earning of Php 2,000 per week is a misleading statistic.

2. A school head evaluates his colleagues in terms of some criteria. He gives O or outstanding rating if his colleague averages 96-100, VS or very satisfactory if 90-95, S or satisfactory if 85-89, and P or poor if 80-84. Teacher X got these scores: 90, 95, 80, 84, 98, and 83. The school head gave him an S. How might Teacher X convince the head that he deserve a higher grade?

3. The bars on this graph appear to indicate that Salesgirl X has sold 3 times as many garments as Salesgirl W. Salesgirl W appears to have sold twice as many as Salesgirl Y. And the difference between the sales of Salesgirl X and Salesgirl Z is not very large.

(a) Discuss why the graph is misleading.

(b) Create a graph that accurately reflects the data.



CARRYING OUT A SURVEY

This is to formalize the learnings you gained from this module. The objectives of the activity are the following:

1. To conduct a simple survey from your co-participants.
2. To prepare a graphical or tabular presentation of your survey.

Procedure

1. Group yourselves into 5. Work together to write a plan for a survey using questionnaire method.
2. Think of any topic that is of interest to you and construct a questionnaire.
3. The plan should include: title of the survey, purpose, importance of what you want to do, population and sample, and the questionnaire.
4. Conduct the survey.
5. Prepare the graphical or tabular presentation of your survey.
6. Make inference and necessary conclusions on your survey.

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Module 11

ORGANIZING A MATH FIELD TRIP

INTRODUCTION

This module presents a guide on how to organize a field trip for Mathematics class. This encourages the teacher to have an excursion as an alternative activity to promote learning in Mathematics. This also emphasizes that experience is the best education thus, when students actually apply their knowledge in a setting outside of school, these experiences strengthen their learning experience.

OBJECTIVES

After completing this module, you should be able to:

1. Learn different guidelines in organizing a field trip for Mathematics class; and

2. Appreciate outdoor activities like field trips to promote learning in Mathematics.

DISCUSSION

Importance of Field Trips

Field trips can be a wonderful avenue for students to have a firsthand experience on the areas of the subject matter. It is an opportunity for the students to explore the rich resources of the local community. Through such, students' knowledge and understanding of the subject will increase while exposing them to reality. Moreover, certain values such as sense of responsibility, independence, team work and cooperation among students will be developed.

Field Trip for Mathematics Class

Math field trips can cover a whole range of venues and activities. Here are a few sample math field trip venues to think about:

- garden to see the growth patterns in plants (i.e. Fibonacci, etc.);
- farm to calculate the farm's yield;
- forest area to record the ages of trees;
- factory to explain how math affects each level of the process and how math is used in the design and manufacture of the products (This could also be a venue to appreciate linear programming.);
- airport where students can plot plane takeoff/landing trajectories;
- tall landmark such as a nearby building for the students to calculate the height of the building using trigonometry;

- train station to help students understand the passage of time, estimating time, calculating the difference between arriving and departing trains, and managing time to arrive on time for a departing train;
- sport event to record statistics at the event (For example, the number of assists and three-point shots in a basketball game. Averages can be calculated as part of classroom activities on the next school day.);
- circus or theme park to look into the essence of symmetry and rotations;
- local park to apply the concept of measurements (perimeter and area);
- bank to give students exposure to money management as they learn about the importance of savings and checking accounts; and
- historical building to look into Golden ratio, tessellations and isometries.

Tips in Organizing a Field Trip

The success of the field trip lies on a good plan. Careful attention should be given to trip selection, pre-visit preparation, the trip itself, and evaluation. When considering a field trip, teachers are advised to first consult with their administrator regarding existing school board policies and follow those recommended procedures.

Herein is a guide adopted from (2012).

Trip Selection

1. Identify the rationale, objectives and plan of evaluation for the field trip.
2. Select the site to be visited. Contact the educational coordinator for the site and arrange the date and time. Obtain the pre-trip information package if one is available. Record addresses, directions, names of contact persons and their phone numbers, e-mail addresses, etc.
3. Conduct a pre-visit to familiarize yourself with the major features of the venue. Take photographs to share with students prior to the visit. Explore the place to get ideas for activities.

Logistics Planning

1. File the activity permit.
2. Make transportation arrangements. File request for use of school bus, if possible and applicable.
3. Make arrangements for meal or packed lunch, if needed.
4. Outline the schedule for the day.
5. Make a seat plan for the bus.
6. Arrange for special equipment and supplies, such as video camera and digital camera.
7. Prepare name tags for students and chaperones.
8. Collect money for admission fees.
9. Compose parent permission letter including
    - Date and location of the field trip and transportation arrangements
    - Educational purpose of the field trip
    - Provision for students with special needs
    - Cost of the trip
    - List of things that the students need to bring
    - Lunch arrangements
    - Money needed

- Attire for the trip
- Trip schedule
- Whether a child will need prescribed medication administered
- Reply slip with parent's name and signature

10. Send a letter to parents or include in the class newsletter a request for volunteers as chaperones.
11. Communicate assigned duties/responsibilities, review field trip objectives, and list activities and schedule.
12. Provide alternative activities for students who will not be going on the trip.
13. Submit a list of students who will be attending the field trip to other teachers if their classes will be affected.
14. Collect the money for the trip and deposit it to the school's account. If required, send the advanced fee to the field trip site.
15. Create a list of all student names and home phone numbers for use in an emergency.

Preparing Students Before the Trip

1. Discuss the purpose of the field trip and how it relates to the current unit of study.
2. Show photographs of the field trip site.
3. Explore the website of the location you will be visiting.
4. As a class, brainstorm a set of standards of conduct for the trip and discuss suggested spending money, lunch plans, appropriate clothing to wear for the trip including gear for rainy weather.
5. Assign students "specialists" roles in one aspect of the topic that they will be studying during the field trip. Students could be grouped in different subject areas related to the field trip topic to research.
6. Discuss with students how to ask good questions and brainstorm a list of open-ended observation questions to gather information during the visit. Have the students record the questions on their field trip journals.
7. Overview the field trip schedule.

Final Planning

Check all permission slips the day before the field trip.

Conducting the Trip

On the day of the trip:

- Pass out name tags.
- Divide class into small groups and assign chaperones to groups.
- Assign each student a partner/buddy.
- Secure a class list and student emergency forms in a folder.
- Bring an emergency kit.
- Take inventory of food, specific equipment, and other supplies pertinent to the trip.

Activities During the Trip

Plan activities that allow students to work alone, in pairs or in small groups. Activities might include:

- Adventure game "Math Trail..."
- Mystery with clues provided
- Sketch pages with partial drawings of objects found in the venue for students to complete the drawings based on their observations
- Hand drawn postcards to write near the end of the tour that will summarize the field trip visit

Provide time for students to observe, ask questions, and record key words, ideas and phrases as journal entries in their field book after exploring the place.

Ask follow-up questions as students make observations.

- How did they apply the concept of mathematics?
- Can you observe any pattern?
- What particular concept in mathematics is being illustrated?
- How are these two objects different from one another?
- In what ways do these two objects relate to one another?

Schedule a particular segment of the field trip for a scavenger hunt where students look for particular objects and record them in their field book or on an observation sheet.

Provide time for students to work in their field book writing questions, describing favorite displays or making sketches of artifacts, structures, scenery, etc. If they cannot complete their sketches, encourage them to label them for future completion as to color, detail, etc.

Provide time for students to use (tape recorder, camcorder, digital camera) for recording important resources viewed/heard.

Post-Field Trip Activities

Just as quality pre-planning is essential to the success of a field trip, planning for appropriate follow-up activities will facilitate student learning and multiply the value of hands-on experiences outside the classroom. The following activities provide a general guide when planning for post-field trip classroom experiences.

- Provide time for students to share general observations and reactions to field trip experiences.
- Share specific assignments students completed while on the field trip.
- Create a classroom bulletin board displaying materials developed or collected while on the field trip.
- Develop a classroom exhibit that replicates and extends displays students observed on the field trip.
- Link field trip activities to multiple curricular areas. For example, students can develop vocabulary lists based on field trip observations; record field trip observations in a classroom journal; construct math problems related to actual field trip budget planning; etc.
- Share and evaluate student assignments/activities from their field books.
- Have the class compose and send thank-you letters to the field trip site host, chaperones, school administrators and other persons that supported the field trip. Include favorite objects or special information learned during the field trip.
- Create a short news report about what happened on the field trip. Publicize the trip via an article in your local newspaper, school bulletin board, trip presentation for parent's night, or class web page.

Evaluating the Trip

Complete a "Teacher Journal" regarding the field trip. This will provide a good reference for future field trips.

- What was of unique educational value in this field trip?
- Did the students meet the objectives/expectations?
- Was there adequate time?
- Was there adequate staff and adult supervision?
- What might be done differently to make this an even better experience in the future?
- What special points should be emphasized next time?
- What special problems should be addressed in the future?
- What would improve a visit to this site in the future?

Share the evaluation with the students, volunteers, hosts from the field trip site, and school administrators.

SUGGESTED ACTIVITY

OBJECTIVES

1. Apply the concepts of the mean, median, and mode in real life situations.
2. Create/design an activity that requires the use of the concept of the mean, median, and mode.

SUGGESTED TIME ALLOTMENT

1 day

MATERIALS

- meter stick
- tape measure
- writing materials

PROCEDURES

The class will be divided into 3 groups. Each group will have a designated area or location where they will collect their data.

1. From the designated area (preferably a park or an area where there are variety of plants), classify each tree by their names. Count the number of each class of tree and record the result as the table below:

Name of Tree	Number of Trees

Based on the data, what is the most abundant tree in that location? We call this the mode. What can you infer based on the data?

2. From the same designated area, look for the ornamental plants (preferably 30 - 50 ornamental plants). Measure the height of the ornamental plants in centimeters using the meter stick. Arrange the heights from smallest to tallest or from tallest to highest. Record the result as the table below.

Ornamental Plant	Height in cm
1	
2	
3	
4	
5	

   Based on the data, what is the middle most height? We call this the median. What can you infer based on the data?

3. On the same designated area, measure the girth (stem circumference) of the trees in cm using the tape measure. The number of trees to be measured is preferably 30 - 50 trees. Record the result as the table below:

Tree	Girth in cm
A	
B	
C	
D	
E	

   Based on the data, sum up the all the girths of trees and then divide it by the number of trees measured. What is the result? We call this the mean.

   What can you infer based on the data?

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