Each of my programs takes in the name of a graph file in Graph File Format at the command line. The graph is an Arxiv High Energy Physics Theory (HEP-TH) collaboration network from the Stanford Large Network Dataset Collection.

Regression performs two linear regressions on the degree distribution of the HEP-TH graph: one fitted to an exponential function, and one fitted to a power function. It then gives me a table of the Degree d and Probability pr(d). Regression then outputs the respective equations for $log \ pr(d)$ and pr(d) and correlation coefficients for both linear regressions.

connectedComponents finds all the connected components of the HEP-TH graph. It outputs the total number of connected components, then a table of each component's number and the number of vertices in that component. Next, it prints the number of each component, and the number of each vertex in that component. Finally, it tells me the size of the smallest and largest components, and the percentage of the graph's vertices that are contained in the largest component.

TopRank finds the top-40-ranked vertices of the HEP-TH graph, with respect to both degree centrality and closeness centrality, and outputs the results.

2. Regression.java uses classes from the Parallel Java 2 Library, so the classpath must be set accordingly before this code can be compiled.

Once the files have been compiled, the command lines to run my programs are as follows:

```
java Regression <fileName>
where <filename> = the name of a file in Graph File Format

java ConnectedComponents <fileName>
where <filename> = the name of a file in Graph File Format

java TopRank <fileName>
where <filename> = the name of a file in Graph File Format
```

 import java.math.BigDecimal; import java.math.MathContext; import java.text.DecimalFormat; import java.util.TreeMap;

```
import edu.rit.numeric.ListXYSeries;
import edu.rit.numeric.XYSeries;
* Perform regressions on a graph, and analyze the results
* @author Joseph Ville
* Usage: java Regression <fileName>
               <fileName> = the name of the graph file to be analyzed
public class Regression
       private static Collaboration collab;
       private static String fileName;
       private static Vertex[] vertices;
       private static XYSeries.Regression expReg;
       private static XYSeries.Regression powReg;
        * Main method for this program
        * @param args - the command line arguments
       public static void main(String[] args)
              if(args.length != 1)
              {
                     usage();
              fileName = args[0];
              collab = new Collaboration();
              vertices = collab.readFile(fileName);
              TreeMap<Integer, Integer> occurrences = degreeOccurrences(vertices);
              executeRegressions(occurrences);
              MathContext mathContext = new MathContext(5);
              // power
              double cP = Math.exp(powReg.a);
              System.out.print("log pr(d) = " + new BigDecimal(Math.log(cP),
mathContext) + " + " + new BigDecimal(powReg.b, mathContext) + " log d");
```

```
System.out.println(", corr = " + new BigDecimal(powReg.corr,
mathContext));
              System.out.println("pr(d) = " + new BigDecimal(cP, mathContext) + " d^" +
new BigDecimal(powReg.b, mathContext));
              // exponential
              double cE = Math.exp(expReg.a);
              double d = Math.exp(expReg.b);
              System.out.print("log pr(d) = " + new BigDecimal(Math.log(cE),
mathContext) + " + " + new BigDecimal(Math.log(d), mathContext) + " d");
              System.out.println(", corr = " + new BigDecimal(expReg.corr,
mathContext));
              System.out.println("pr(d) = " + new BigDecimal(cE, mathContext) + " * " +
new BigDecimal(d, mathContext) + "^d");
      }
       * Performs linear regressions on an exponential function and a power function
       * @param degreeCounts - map of degrees to the number of occurrences
       public static void executeRegressions(TreeMap<Integer, Integer> degreeCounts)
              ListXYSeries expXYSeries = new ListXYSeries();
              ListXYSeries powXYSeries = new ListXYSeries();
              DecimalFormat d = new DecimalFormat("#");
              DecimalFormat d1 = new DecimalFormat("0.00000E00");
              System.out.println("d\tcount\tpr");
              for(Integer key : degreeCounts.keySet())
              {
                     double probD = (double)degreeCounts.get(key) / collab.V();
                     System.out.println(key + "\t" + d.format(degreeCounts.get(key)) +
"\t" + sFormat(d1.format(probD).toString()));
                     expXYSeries.add(key, Math.log(probD));
                     powXYSeries.add(Math.log(key), Math.log(probD));
              }
              expReg = expXYSeries.linearRegression();
              powReg = powXYSeries.linearRegression();
      }
       * Find the number of occurrences of each degree in an of an
       * array of vertices
       * @param vertices
       */
```

```
public static TreeMap<Integer, Integer> degreeOccurrences(Vertex[] vertices)
               // Using a TreeMap so it will be ordered automatically by vertex degree
               TreeMap<Integer, Integer> occurrences = new TreeMap<Integer,
Integer>();
               for(Vertex v : vertices)
                      if(occurrences.containsKey(v.degree()))
                      {
                              int value = occurrences.get(v.degree());
                              occurrences.put(v.degree(), ++value);
                      }
                      else
                      {
                              occurrences.put(v.degree(), 1);
               }
               return occurrences;
       }
        * Format the given string so that "E"s are "e"s
        * @param input - the given string
        * @return the formatted string
       public static String sFormat(String input)
               return input.replaceAll("E", "e");
       }
        * Print a usage message and exit.
       public static void usage()
       {
               System.err.println("Usage: java Regression <fileName>\n" +
                              "<fileName> = the name of the graph file to be analyzed");
               System.exit(0);
       }
}
```

```
import java.util.ArrayList;
import java.util.Collections;
import java.util.LinkedList;
import java.util.TreeMap;
* Find the connected components of a graph
* @author Joseph Ville
* Usage: java ConnectedComponents <fileName>
               <fileName> the name of the graph file to be analyzed
public class ConnectedComponents
       private static String fileName;
       private static int smallestCCSize;
       private static int largestCCSize;
       private static int largestCC;
       private boolean printData;
       private static TreeMap<Integer, ArrayList<Integer>> ccIndexes;
        * Default constructor
        */
       public ConnectedComponents()
              smallestCCSize = Integer.MAX_VALUE;
              largestCCSize = Integer.MIN_VALUE;
       }
        * Construct an object of this class
        * @param printResults
        */
       public ConnectedComponents(boolean printData)
              this.printData = printData;
              smallestCCSize = Integer.MAX_VALUE;
              largestCCSize = Integer.MIN_VALUE;
       }
        * Main method for this class
```

```
* @param args - the command line arguments
        */
       public static void main(String[] args)
              if(args.length != 1)
              {
                      usage();
              fileName = args[0];
              Collaboration collab = new Collaboration();
              Vertex[] vertices = collab.readFile(fileName);
              ConnectedComponents cc = new ConnectedComponents(true);
              cc.findComponents(vertices);
              DecimalFormat df = new DecimalFormat("0.####");
              System.out.println("Size of smallest CC = " + smallestCCSize);
              System.out.println("Size of largest CC = " + largestCCSize);
              System.out.println(largestCCSize + "/" + collab.V() + " = " +
df.format((double)largestCCSize / collab.V()));
       }
        * Find all connected components of an array of vertices
        * @param vertices - the array of vertices
        * @return the total number of connected components in the graph
       public int findComponents(Vertex[] vertices)
              LinkedList<Integer> unvisited = new LinkedList<Integer>();
              LinkedList<Integer> queue = new LinkedList<Integer>();
              ccIndexes = new TreeMap<Integer, ArrayList<Integer>>();
              ArrayList<Integer> components;
              for(int a = 0; a < vertices.length; <math>a++)
              {
                      unvisited.add(a);
              }
              String outString = "connected components\nComp\tSize\n";
              int i = -1, m = -1, n = 0;
              while(unvisited.size() > 0)
                      i = unvisited.poll();
```

```
queue.add(i);
               components = new ArrayList<Integer>();
               components.add(i);
              while(queue.size() > 0)
               {
                      m = queue.poll();
                      for(Integer neighbor : vertices[m].getNeighbors())
                             if(unvisited.contains(neighbor))
                             {
                                     unvisited.remove(neighbor);
                                     queue.add(neighbor);
                                     components.add(neighbor);
                             }
                      }
              }
               if(components.size() > largestCCSize)
                      largestCC = n;
                      largestCCSize = components.size();
              if(components.size() < smallestCCSize)</pre>
                      smallestCCSize = components.size();
               outString = outString.concat(n + "\t" + components.size() + "\n");
               Collections.sort(components);
               ccIndexes.put(n++, components);
       }
       if(printData)
               System.out.print(n + " " + outString);
               printFormat(ccIndexes);
       }
       return n;
}
* Print the CCs and their associated indexes in a readable format
* @param ccIndexes - a map of indexes to their CCs
```

```
*/
       public static void printFormat(TreeMap<Integer, ArrayList<Integer>> ccIndexes)
              for(int i = 0; i < ccIndexes.size(); i++)
                      System.out.printf("Comp %d = ", i);
                      for(int j = 0; j < cclndexes.get(i).size(); j++)</pre>
                              System.out.printf("%d ", ccIndexes.get(i).get(j));
                      System.out.println();
               }
       }
        * @return the largest connected component
       public ArrayList<Integer> getLargestComponent()
               return ccIndexes.get(largestCC);
       }
        * Print a usage message and exit
       public static void usage()
               System.err.println("Usage: java ConnectedComponents <fileName>\n" +
                              "<fileName> = the name of the graph file to be analyzed");
               System.exit(0);
}// end class ConnectedComponents
import java.util.AbstractMap.SimpleEntry;
import java.util.ArrayList;
import java.util.Collections;
import java.util.Comparator;
import java.util.LinkedList;
import java.util.List;
* Analyze a graph and make conclusions about the top-40-ranked vertices
* @author Joseph Ville
```

```
* Usage: java TopRank <fileName>
                <fileName> = the name of the graph file to be analyzed
public class TopRank
       private static String fileName;
       private static Vertex[] vertices;
        * Main method for this program
        * @param args - the command line arguments
       public static void main(String[] args)
              if(args.length != 1)
                      usage();
              fileName = args[0];
              Collaboration collab = new Collaboration();
              ConnectedComponents cc = new ConnectedComponents(false);
              vertices = collab.readFile(fileName);
              int n = cc.findComponents(vertices);
              ArrayList<Integer> largestCC = cc.getLargestComponent();
              int vertex;
              List<SimpleEntry<Integer, Integer>> cloCent = degreeCent(largestCC);
              System.out.println("Rank\tVertex\tDegCen");
              for(int i = 0; i < 40; i++)
              {
                     vertex = largestCC.get(i);
                      System.out.println((i+1) + "\t" + cloCent.get(vertex).getKey() + "\t"
+ cloCent.get(vertex).getValue());
              }
              List<SimpleEntry<Integer, Float>> avgDistances =
closenessCent(largestCC);
              System.out.println("Rank\tVertex\tCloCen");
              for(int i = 0; i < 40; i++)
              {
                      vertex = largestCC.get(i);
```

```
System.out.println((i+1) + "\t" + avgDistances.get(vertex).getKey()
+ "\t" + avgDistances.get(vertex).getValue());
              }
       }
        * Compute the closeness centrality of the given connected component
        * @param cc - the connected component
        * @return - an ArrayList<SimpleEntry<Integer, Float>> which stores the vertex
number and its
                     associated closeness centrality
        */
       public static List<SimpleEntry<Integer, Float>>
closenessCent(ArrayList<Integer> cc)
       {
              List<SimpleEntry<Integer, Float>> avgDistances = new
ArrayList<SimpleEntry<Integer, Float>>(cc.size());
              int current = 0;
              for(int i = 0; i < cc.size(); i++)
                     current = cc.get(i);
                     avgDistances.add(new SimpleEntry<Integer, Float>(current,
avgDistance(current)));
              }
              // sort the distances
              Collections.sort(avgDistances, new Comparator<SimpleEntry<Integer,
Float>>(){
                             @Override
                             public int compare(SimpleEntry<Integer, Float> arg0,
SimpleEntry<Integer, Float> arg1)
                             {
                                    return
arg0.getValue().compareTo(arg1.getValue());
              return avgDistances;
       }
        * Compute the degree centrality of the given connected component
        * @param cc - the connected component
        * @return an ArrayList<SimpleEntry<Integer, Integer>> which stores the vertex
```

number and its

```
associated degree centrality
        */
       public static List<SimpleEntry<Integer, Integer>> degreeCent(ArrayList<Integer>
cc)
       {
              List<SimpleEntry<Integer, Integer>> degrees = new
ArrayList<SimpleEntry<Integer, Integer>>(cc.size());
              int current = 0;
              for(int v = 0; v < cc.size(); v++)
                      current = cc.get(v);
                      degrees.add(new SimpleEntry<Integer, Integer>(current,
vertices[current].degree()));
              Collections.sort(degrees, new Comparator<SimpleEntry<Integer,
Integer>>(){
                      @Override
                      public int compare(SimpleEntry<Integer, Integer> arg0,
SimpleEntry<Integer, Integer> arg1)
                      {
                             return arg1.getValue().compareTo(arg0.getValue());
                      }
              });
              return degrees;
       }
        * Compute the average of the distances between the given vertex and every
other vertex
        * @param vertex - the vertex to find distances from
        * @return the average distance from this vertex to every other vertex
        */
       public static float avgDistance(int vertex)
              LinkedList<Integer> queue = new LinkedList<Integer>(); // queue of
vertex indices
              boolean[] seen = new boolean[
                                                    vertices.length]; // whether this
vertex has been seen
              int[] distances = new int[vertices.length]; // distances from this to all other
vertices
              for(int i = 0; i < vertices.length; i++) // initialize the arrays
                      distances[i] = Integer.MAX_VALUE;
```

```
seen[i] = false;
               }
               seen[vertex] = true;
               queue.add(vertex);
               int a = 0;
               int count = 0;
               distances[vertex] = 0;
               while(queue.size() != 0)
               {
                       a = queue.poll(); // remove the head of the queue
                      for(Integer b : vertices[a].getNeighbors()) // loop through all
neighbors of current vertex
                      {
                              if(!seen[b])
                                      seen[b] = true;
                                      queue.add(b); // add b to end of queue
                                      count++;
                                      distances[b] = distances[a] + 1;
                              } // end if
                      } // end for
               } // end while
               int sum = 0;
               for(int i = 0; i < vertices.length; i++)
               {
                      if(distances[i] != Integer.MAX_VALUE)
                      {
                              sum += distances[i];
                       }
               }
               return (float) sum / count;
       }
        * Print a usage message and exit
       public static void usage()
               System.err.println("Usage: java TopRank <fileName>\n" +
                              "<fileName> = the name of the graph file to be analyzed");
```

```
System.exit(0);
       }
}
import java.io.FileReader;
import java.io.BufferedReader;
* Analyzes a Collaboration Graph
* @author Joseph Ville
public class Collaboration
       private int V;
       private int E;
       private Vertex[] vertices;
        * Construct an object of this class
       public Collaboration()
       }
        * Reads and processes a file in the Graph File Format
        * @param fileName - the file to process
        * @return vertices - an array of vertices for the graph
        * @throws Exception
        */
       public Vertex[] readFile(String fileName)
               String line = "";
               try
               {
                      BufferedReader buff = new BufferedReader(new
FileReader(fileName));
                      boolean gLine = false;
                      boolean eLine = false;
```

```
while((line = buff.readLine()) != null)
                               String[] lineArr;
                               // ignore any blank line
                               if(!line.equals(null) && !line.isEmpty())
                                       lineArr = line.split(" ");
                                       /* Required. Edges of the graph.
                                       First field - source vertex #, 0 <= an int <= V-1
                                       Second field - destination vertex #, 0 <= an int <=
V-1
                                       Third field - edge weight, a floating pt #. If 3rd field
omitted.
                                               assume to be 1 by default.
                                       */
                                       if(lineArr[0].equals("e"))
                                       {
                                               eLine = true;
                                               int index = Integer.parseInt(lineArr[1]);
                                               int neighbor = Integer.parseInt(lineArr[2]);
                                               initializelfNull(vertices, index);
                                               initializelfNull(vertices, neighbor);
                                               vertices[index].addNeighbor(neighbor);
                                               vertices[neighbor].addNeighbor(index);
                                       /* Required. Occurs once, at beginning of file
                                       The parameters of the graph.
                                       First field is # of vertices V, an int \geq 0
                                       Second field is # of edges E, an int >= 0
                                       else if(lineArr[0].equals("g"))
                                       {
                                               gLine = true;
                                               int numVertices =
Integer.parseInt(lineArr[1]); // V and E are guaranteed to be >= 0
                                               int numEdges = Integer.parseInt(lineArr[2]);
                                               /* ok to do this here, because this will be
executed before
                                               any of the other if statements on this or any
other
```

```
pass of the while loop */
                                       this.V = numVertices;
                                       this.E = numEdges;
                                       vertices = new Vertex[V];
                               }
                               // ignore any lines beginning with v, c, or d
                               else if(lineArr[0].equals("v"))
                                       continue;
                               else if(lineArr[0].equals("d"))
                                       continue;
                               else if(lineArr[0].equals("c"))
                               {
                                       continue;
                       }// end if
               }// end while
               buff.close();
               // make sure the file contained the required lines
               if(gLine == false || eLine == false)
                       System.err.println("The file is in an invalid format.");
                       System.exit(0);
               }
       catch(NumberFormatException nfe)
       {
               System.err.println("A number in the file had invalid format");
               System.exit(0);
       }
       catch(Exception ex)
               System.err.println("There was an error reading the file");
               System.exit(0);
       return vertices;
}// end readFile()
```

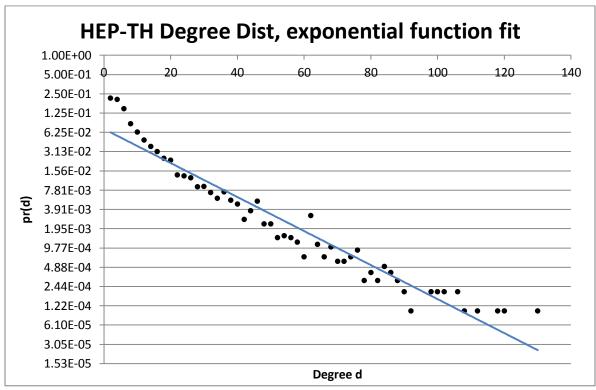
```
* @return the number of vertices
        */
        public int V()
                return V;
        }
         * @return the number of edges
        public int E()
               return E;
        }
        * Check if a vertex is null, and if it is, initialize it.
        * @param v - the vertex to check
        */
        private void initializeIfNull(Vertex[] vertices, int v)
                if(vertices[v] == null)
                       vertices[v] = new Vertex();
}// end class Collaboration
import java.util.ArrayList;
* A class to store all the attributes of a vertex object
* @author Joseph Ville
*/
public class Vertex
        private ArrayList<Integer> neighbors; // neighbors of the current vertex
        * Construct an object of this class, by setting
        * the class variables to their default values
        public Vertex()
```

```
{
               neighbors = new ArrayList<Integer>();
       }
        * Find the degree of this vertex. Will only work after
        * neighbors list has finished populating.
        * @return the degree of this vertex
        */
       public int degree()
               return neighbors.size();
        * Get the list of vertices that are adjacent to the current vertex
        * @return the list of neighbors
        */
       public ArrayList<Integer> getNeighbors()
               return neighbors;
       }
        * Appends a vertex number to the end of this vertex's neighbor list
        * @param n - the neighbor to be added
       public void addNeighbor(int n)
               this.neighbors.add(n);
}// end class Vertex
```

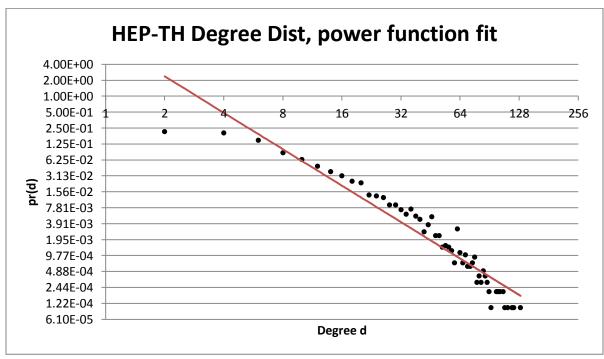
4. Command line: java Regression CA-HepTh-graph.txt

d	count	pr
2	2110	2.13628E-01
4	2015	2.04009E-01
6	1446	1.46401E-01
8	847	8.57548E-02
10	628	6.35821E-02
12	468	4.73828E-02
14	371	3.75620E-02
16	311	3.14873E-02
18	244	2.47039E-02

20 227 2.29827E-02 24 129 1.30606E-02 26 120 1.21494E-02 28 87 8.80834E-03 30 88 8.90959E-03 32 71 7.18842E-03 34 58 5.87223E-03 36 73 7.39091E-03 38 54 5.46725E-03 40 47 4.75853E-03 42 27 2.73362E-03 44 37 3.74608E-03 46 52 5.26476E-03 48 23 2.32864E-03 50 23 2.32864E-03 50 23 2.32864E-03 50 23 2.32864E-03 50 23 2.32864E-03 51 1.51868E-03 54 15 1.51868E-03 56 14 1.41743E-03 58 12 1.21494E-03 60 7 7.08717E-04 62 <			
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30 88 8.90959E-03 32 71 7.18842E-03 34 58 5.87223E-03 36 73 7.39091E-03 38 54 5.46725E-03 40 47 4.75853E-03 42 27 2.73362E-03 44 37 3.74608E-03 46 52 5.26476E-03 48 23 2.32864E-03 50 23 2.32864E-03 54 15 1.51868E-03 56 14 1.41743E-03 58 12 1.21494E-03 60 7 7.08717E-04 62 31 3.13860E-03 64 11 1.11370E-03 70 6 6.07472E-04 7	26	120	1.21494E-02
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34 58 5.87223E-03 36 73 7.39091E-03 38 54 5.46725E-03 40 47 4.75853E-03 42 27 2.73362E-03 44 37 3.74608E-03 46 52 5.26476E-03 48 23 2.32864E-03 50 23 2.32864E-03 50 23 2.32864E-03 52 14 1.41743E-03 54 15 1.51868E-03 56 14 1.41743E-03 58 12 1.21494E-03 60 7 7.08717E-04 62 31 3.13860E-03 64 11 1.11370E-03 66 7 7.08717E-04 68 10 1.01245E-03 70 6 6.07472E-04 72 6 6.07472E-04 74 7 7.08717E-04 78 3 3.03736E-04 80 <td>30</td> <td>88</td> <td>8.90959E-03</td>	30	88	8.90959E-03
36 73 7.39091E-03 38 54 5.46725E-03 40 47 4.75853E-03 42 27 2.73362E-03 44 37 3.74608E-03 46 52 5.26476E-03 48 23 2.32864E-03 50 23 2.32864E-03 52 14 1.41743E-03 54 15 1.51868E-03 56 14 1.41743E-03 58 12 1.21494E-03 60 7 7.08717E-04 62 31 3.13860E-03 64 11 1.11370E-03 66 7 7.08717E-04 68 10 1.01245E-03 70 6 6.07472E-04 72 6 6.07472E-04 74 7 7.08717E-04 76 9 9.11208E-04 80 4 4.04981E-04 82 3 3.03736E-04 84	32	71	7.18842E-03
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40 47 4.75853E-03 42 27 2.73362E-03 44 37 3.74608E-03 46 52 5.26476E-03 48 23 2.32864E-03 50 23 2.32864E-03 52 14 1.41743E-03 54 15 1.51868E-03 56 14 1.41743E-03 58 12 1.21494E-03 60 7 7.08717E-04 62 31 3.13860E-03 64 11 1.11370E-03 66 7 7.08717E-04 68 10 1.01245E-03 70 6 6.07472E-04 72 6 6.07472E-04 74 7 7.08717E-04 76 9 9.11208E-04 78 3 3.03736E-04 80 4 4.04981E-04 82 3 3.03736E-04 84 5 5.06227E-04 86 4 4.04981E-04 92 1 1.01245E-04 </td <td>36</td> <td>73</td> <td>7.39091E-03</td>	36	73	7.39091E-03
42 27 2.73362E-03 44 37 3.74608E-03 46 52 5.26476E-03 48 23 2.32864E-03 50 23 2.32864E-03 52 14 1.41743E-03 54 15 1.51868E-03 56 14 1.41743E-03 58 12 1.21494E-03 60 7 7.08717E-04 62 31 3.13860E-03 64 11 1.11370E-03 66 7 7.08717E-04 68 10 1.01245E-03 70 6 6.07472E-04 72 6 6.07472E-04 74 7 7.08717E-04 76 9 9.11208E-04 80 4 4.04981E-04 82 3 3.03736E-04 84 5 5.06227E-04 86 4 4.04981E-04 88 3 3.03736E-04 90 2 2.02491E-04 100 2 2.02491E-04 </td <td>38</td> <td>54</td> <td>5.46725E-03</td>	38	54	5.46725E-03
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	130	1	1.01245E-04



 $pr(d) = 0.070479 * 0.94064^d$ Correlation = -0.96800



 $pr(d) = 11.529 \ d^{2.2857}$

Correlation = -0.95723

The exponential function fit has a stronger correlation coefficient, therefore the HEP-TH graph more closely resemble a small-world graph.

The HepTh graph more closely resembles a small-world graph, because the exponential function has the stronger correlation coefficient.

5. Command line: java ConnectedComponents CA-HepTh-graph.txt 429 connected components

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42 2

43 2

44 7

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Comp 69 = 7190 7191

Comp 70 = 72157216

Comp 71 = 7217 7218 7219 7220 7221

Comp 72 = 7234 7235 7236

Comp 73 = 72607261

Comp 74 = 7275 7276 7277 7278

Comp 75 = 72807281

Comp 76 = 72997300

Comp 77 = 7303 7304 7305 7306 7307 7308 7309 7310

Comp 78 = 73247325

Comp 79 = 7338 7339 7340 7341

Comp 80 = 7356 7357 7358 7359 7360

Comp 81 = 73617362

Comp 82 = 73687369

Comp 83 = 73747375

Comp 84 = 7405 7406

Comp 85 = 7454 7455

Comp 86 = 7463 7464

Comp 87 = 7491 7492 7493

Comp 88 = 75047505

Comp 89 = 7521 7522 7523

Comp 90 = 7535 7536 7537 7538 7539 7540 7541

Comp 91 = 75457546

Comp 92 = 75757576

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Comp 93 = 75777578
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Comp 94 = 7579 7580 7581 7582 7583 8010 8011 8012

Comp 95 = 75847585

Comp 96 = 7586 7587 7588 7589

Comp 97 = 7621 7622 7623 7624 7625 8953 8954 8955

Comp 98 = 76547655

Comp 99 = 76677668

Comp 100 = 7682 7683 7684 7685 7686 7687

Comp 101 = 77007701

Comp 102 = 77147715

Comp 103 = 7728 7729 7730

Comp 104 = 77507751

Comp 105 = 77527753

Comp 106 = 7756 7757 7758 7759 7760 7761 7762

Comp 107 = 77637764

Comp 108 = 7769 7770 7771

Comp 109 = 78237824

Comp 110 = 78347835

Comp 111 = 7845 7846 7847

Comp 112 = 7898 7899 7900 7901 7902

Comp 113 = 7918 7919 7920 7921 7922 7923

Comp 114 = 79337934

Comp 115 = 7936 7937 7938

Comp 116 = 7973 7974 7975

Comp 117 = 79777978

Comp 118 = 8025 8026 8027

Comp 119 = 8035 8036 8037

Comp 120 = 8054 8055

Comp 121 = 8057 8058 8059 8060

Comp 122 = 8061 8062

Comp 123 = 8091 8092 8093 8094

Comp 124 = 8100 8101 8102

Comp 125 = 8105 8106

Comp 126 = 81228123

Comp 127 = 8124 8125 8126 8127 8128

Comp 128 = 81298130

Comp 129 = 8134 8135 8136

Comp 130 = 8139 8140

Comp 131 = 8148 8149 8150 8151

Comp 132 = 8158 8159

Comp 133 = 8160 8161 8162

Comp 134 = 81728173

Comp $135 = 8181 \ 8182$

Comp 136 = 8183 8184

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Comp 137 = 8194 8195 8196 8197 8198
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Comp 138 = 8199 8200 8201

Comp 139 = 8202 8203 8204 8205

Comp 140 = 8211 8212

Comp 141 = 8222 8223 8224 8225 8226 8227 8228 8229 8230

Comp 142 = 8234 8235 8236 8237

Comp 143 = 8245 8246 8247 8977 8978

Comp 144 = 8248 8249 8250 8251 8252 8253 8254

Comp 145 = 8255 8256

Comp 146 = 8257 8258

Comp 147 = 8260 8261 8262 8263 8264

Comp 148 = 8267 8268 8269 8270

Comp 149 = 8295 8296

Comp 150 = 8305 8306 8307 8308 8309 8310 8417

Comp 151 = 83188319

Comp 152 = 8344 8345

Comp 153 = 8359 8360

Comp 154 = 8367 8368 8369

Comp 155 = 8375 8376

Comp 156 = 8378 8379 8380 8381

Comp 157 = 83898390

Comp 158 = 8401 8402 8403

Comp 159 = 8405 8406

Comp 160 = 8409 8410 8411 8412 8413 8414

Comp 161 = 8418 8419

Comp 162 = 8420 8421

Comp 163 = 8422 8423 8424

Comp 164 = 8433 8434

Comp 165 = 8435 8436

Comp 166 = 8437 8438

Comp 167 = 8439 8440

Comp 168 = 8471 8472

Comp 169 = 8473 8474

Comp 170 = 8503 8504

Comp 171 = 8507 8508 8509 8510

Comp 172 = 8513 8514

Comp 173 = 8524 8525 8526

Comp 174 = 8541 8542 8543

Comp 175 = 8604 8605

Comp 176 = 8606 8607

Comp 177 = 8608 8609

Comp 178 = 8613 8614

Comp 179 = 8616 8617

Comp 180 = 8618 8619

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Comp 181 = 8620 8621
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Comp 182 = 8633 8634 8635

Comp 183 = 8661 8662

Comp 184 = 8678 8679 8680

Comp 185 = 86928693

Comp 186 = 8717 8718 8719 8720 8721

Comp 187 = 8730 8731

Comp 188 = 8733 8734 8735 8736 8737 9669 9670

Comp 189 = 8738 8739

Comp 190 = 8748 8749 8750 8751

Comp 191 = 8753 8754

Comp 192 = 8755 8756 8757 8758

Comp 193 = 8768 8769 8770

Comp 194 = 8771 8772

Comp 195 = 8774 8775

Comp 196 = 8778 8779 8780 8781 8782 8783 8784 8785 8786 8787 8788 8789 8790 8878

8879 8880 8881 8908 8938 8939 8940

Comp 197 = 8792 8793

Comp 198 = 8832 8833

Comp 199 = 8835 8836

Comp 200 = 88428843

Comp 201 = 88628863

Comp 202 = 8864 8865

Comp 203 = 8872 8873

Comp 204 = 8884 8885

Comp 205 = 8890 8891

Comp 206 = 8905 8906

Comp 207 = 8909 8910 8911

Comp 208 = 8912 8913

Comp 209 = 8922 8923

Comp 210 = 8924 8925

Comp 211 = 8931 8932

Comp 212 = 8933 8934 8935 8936 8937

Comp 213 = 8943 8944 8945 8946 8947 8948 8949

Comp 214 = 8951 8952

Comp 215 = 8957 8958 8959 8960

Comp 216 = 8974 8975

Comp 217 = 8988 8989 8990

Comp 218 = 8991 8992

Comp 219 = 9001 9002 9003 9004

Comp 220 = 9008 9009 9010

Comp 221 = 9015 9016

Comp 222 = 9024 9025 9026 9027

Comp 223 = 9031 9032 9033 9034

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Comp 224 = 9036 9037
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Comp 225 = 9039 9040

Comp 226 = 90439044

Comp 227 = 9053 9054

Comp 228 = 9055 9056

Comp 229 = 9059 9060

Comp 230 = 9074 9075 9076

Comp 231 = 9085 9086

Comp 232 = 9089 9090 9091

Comp 233 = 9093 9094

Comp 234 = 9095 9096 9097

Comp 235 = 9098 9099 9100 9101

Comp 236 = 9104 9105 9106

Comp 237 = 9107 9108

Comp 238 = 9109 9110 9111

Comp 239 = 9117 9118

Comp 240 = 9119 9120 9121 9122 9123

Comp 241 = 9124 9125 9126 9127 9128

Comp $242 = 9131 \ 9132$

Comp $243 = 9140 \ 9141$

Comp 244 = 9143 9144

Comp $245 = 9145 \ 9146$

Comp $246 = 9147 \ 9148$

Comp 247 = 9156 9157 9158 9159

Comp 248 = 9176 9177

Comp 249 = 9182 9183 9184 9185

Comp 250 = 9186 9187 9188 9189

Comp 251 = 91949195

Comp 252 = 9196 9197 9198 9199

Comp 253 = 9202 9203

Comp 254 = 9208 9209

Comp 255 = 9213 9214

Comp 256 = 9218 9219

Comp 257 = 9221 9222

Comp 258 = 9223 9224

Comp 259 = 9226 9227 9228

Comp 260 = 92329233

Comp 261 = 9236 9237 9238

Comp 262 = 9239 9240

Comp 263 = 9246 9247 9248

Comp 264 = 9276 9277

Comp 265 = 9278 9279 9280 9281 9282 9283 9284 9285

Comp 266 = 9286 9287 9288 9289

Comp 267 = 9293 9294

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Comp 268 = 9297 9298
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Comp 269 = 9300 9301 9302

Comp 270 = 9304 9305

Comp 271 = 9306 9307 9308

Comp 272 = 9310 9311 9312 9313 9314 9315 9316

Comp 273 = 9317 9318 9319 9320 9321 9322

Comp 274 = 9327 9328

Comp 275 = 9330 9331 9332

Comp 276 = 93339334

Comp 277 = 9338 9339 9340 9341

Comp 278 = 93429343

Comp $279 = 9344 \ 9345$

Comp 280 = 9352 9353 9354 9355 9356

Comp $281 = 9358 \ 9359$

Comp 282 = 9366 9367 9368

Comp 283 = 9370 9371

Comp 284 = 9378 9379 9380

Comp 285 = 9383 9384 9385

Comp 286 = 9389 9390 9391 9392

Comp 287 = 93949395

Comp 288 = 9396 9397

Comp 289 = 9401 9402 9403 9404 9701 9702

Comp 290 = 9405 9406

Comp 291 = 9408 9409

Comp 292 = 9413 9414 9415 9416 9417 9418

Comp 293 = 9419 9420

Comp 294 = 9428 9429

Comp 295 = 9435 9436 9437 9438

Comp 296 = 9440 9441 9442

Comp 297 = 9443 9444

Comp 298 = 9446 9447

Comp 299 = 9448 9449 9450 9451

Comp 300 = 9452 9453 9454 9455 9456

Comp 301 = 9457 9458

Comp 302 = 9460 9461

Comp 303 = 9462 9463 9464

Comp 304 = 9465 9466 9467 9468 9469 9470 9471 9472 9473

Comp 305 = 9474 9475

Comp 306 = 9477 9478

Comp 307 = 9479 9480 9481 9482 9483

Comp 308 = 9487 9488

Comp 309 = 9494 9495 9496 9497

Comp 310 = 9498 9499

Comp 311 = 9500 9501 9502 9503

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Comp 312 = 9506 9507
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Comp 313 = 9515 9516 9517 9518

Comp 314 = 9519 9520

Comp 315 = 9521 9522

Comp 316 = 9527 9528

Comp 317 = 9529 9530 9531 9532

Comp 318 = 9534 9535 9536 9537

Comp 319 = 9539 9540

Comp 320 = 9542 9543

Comp 321 = 9545 9546

Comp 322 = 9549

Comp 323 = 9554 9555

Comp 324 = 9570 9571 9572

Comp 325 = 9573 9574

Comp 326 = 9581 9582

Comp 327 = 9583 9584

Comp 328 = 9585 9586 9587

Comp 329 = 9590 9591

Comp 330 = 9600

Comp 331 = 9602 9603

Comp 332 = 9604 9605 9606

Comp 333 = 9607 9608

Comp 334 = 9609 9610

Comp 335 = 9611 9612

Comp 336 = 9613 9614

Comp 337 = 9615 9616

Comp 338 = 9617 9618

Comp 339 = 9620 9621 9622

Comp 340 = 9625 9626

Comp 341 = 9627 9628

Comp 342 = 9633 9634

Comp 343 = 9636 9637 9638

Comp 344 = 9639 9640 9641

Comp 345 = 96429643

Comp 346 = 9646 9647

Comp 347 = 9652 9653

Comp 348 = 96549655

Comp 349 = 9658 9659

Comp 350 = 9660 9661

Comp 351 = 96639664

Comp 352 = 9667 9668

Comp 353 = 9671 9672

Comp 354 = 9673 9674 9675

Comp 355 = 9680 9681 9682 9683

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Comp 356 = 9685 9686
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Comp 357 = 9687 9688

Comp 358 = 9689 9690

Comp 359 = 9691 9692 9693

Comp 360 = 9696 9697 9698

Comp 361 = 9704 9705 9706

Comp 362 = 9707 9708

Comp 363 = 9709 9710

Comp 364 = 9711 9712

Comp 365 = 9715 9716

Comp 366 = 9718 9719

Comp 367 = 9720 9721

Comp 368 = 9722 9723

Comp $369 = 9724 \ 9725$

Comp 370 = 9727 9728

Comp 371 = 9729 9730

Comp 372 = 9731 9732 9733

Comp 373 = 9735 9736

Comp 374 = 9737 9738 9739 9740

Comp 375 = 9741 9742

Comp 376 = 9743 9744

Comp 377 = 9747 9748

Comp 378 = 9749 9750 9751

Comp 379 = 9752 9753

Comp 380 = 9754 9755

Comp 381 = 9756 9757 9758 9759

Comp 382 = 9760 9761

Comp 383 = 9762 9763

Comp 384 = 9764 9765

Comp 385 = 9766 9767

Comp $386 = 9768 \ 9769$

Comp 387 = 9770 9771

Comp 388 = 9772 9773

Comp 389 = 9774 9775

Comp 390 = 9777 9778 9779

Comp 391 = 9783 9784

Comp 392 = 9785 9786

Comp 393 = 9787 9788

Comp 394 = 9789 9790

Comp 395 = 9791 9792

Comp 396 = 9793 9794 9795

Comp 397 = 9796 9797

Comp 398 = 9798 9799

Comp 399 = 9802 9803 9804

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Comp 400 = 98059806
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Comp 401 = 9807 9808 9809

Comp 402 = 9810 9811

Comp 403 = 9812 9813

Comp 404 = 9814 9815

Comp 405 = 9816 9817 9818 9819

Comp 406 = 9820 9821

Comp 407 = 9822 9823 9824

Comp 408 = 9825 9826

Comp 409 = 98279828

Comp 410 = 98319832

Comp 411 = 9834 9835

Comp 412 = 9836 9837

Comp 413 = 9838 9839

Comp 414 = 9840 9841 9842 9843

Comp 415 = 9844 9845 9846 9847

Comp 416 = 9848 9849

Comp 417 = 9850 9851 9852

Comp 418 = 9853 9854

Comp 419 = 9855 9856

Comp 420 = 98579858

Comp 421 = 9859 9860

Comp 422 = 9861 9862 9863

Comp 423 = 9864 9865

Comp 424 = 9866 9867

Comp 425 = 9868 9869

Comp 426 = 9870 9871

Comp 427 = 9872 9873

Comp 428 = 9874 9875 9876

Size of smallest CC = 1

Size of largest CC = 8638

8638/9877 = 0.87456

The HepTh graph contains 429 connected components. The smallest connected component contains one vertex, and largest component contains 8638 vertices.

The largest component contains 8638 of the 9877 total vertices in the graph. 8338/9877 = 0.87456, so the largest component contains 87.456% of the graph's vertices. Thus, the graph does contain a giant component.

6. Command line: java TopRank CA-HepTh-grap.txt

Rank	Vertex	DegCen
1	86	130
2	15	120
3	54	118
4	920	112
5	163	108
6	35	106
7	1546	106
8	1420	102
9	1873	102
10	8	100
11	38	100
12	2	98
13	277	98
14	67	92
15	40	90
16	1907	90
17	220	88
18	252	88
19	1305	88
20	32	86
21	386	86
22		86
23		86
24		84
25		84
26		84
27		84
28		84
29		82
30		82
31	1904	82
32		80
33		80
34		80
35		80
36		78
37		78
38		78
39		76
40	173	76

7. Command line: java TopRank CA-HepTh-graph.txt

Rank	Vertex	CloCen
1	15	4.045849
2	920	4.074215
3	40	4.095751
4	38	4.121107
5	1	4.147042
6	1461	4.154799
7	1094	4.163946
8	54	4.168924
9	1332	4.169851
10	442	4.183281
11	13	4.190228
12	8	4.19486
13	163	4.195323
14	30	4.211879
15	17	4.213268
16	62	4.221605
17	35	4.222184
18	277	4.222415
19	1420	4.228551
20	429	4.229015
21	14	4.240245
22	67	4.242446
23		4.247771
24		4.248582
25	386	4.249508
26	31	4.250434
27	26	4.253676
28	252	4.259697
29	87	4.264675
30		4.264675
31		4.269191
32	851	4.272548
33	171	4.274517
34	618	4.278685
35	76	4.2832
36	173	4.291884
37		4.296515
38		4.298483
39		4.300104
40	24	4.310293

8. As stated in my answer to question 4, the HEP-TH graph is more closely related to a small-world graph than a scale-free graph. This is because the correlation coefficient of the exponential function fit of the degree distribution was stronger than that of the power function fit. However, the graph appears to exhibit properties of a scale-free graph as well a small-world graph.

My results from question 5 show that most of the vertices are indeed connected to just a few nearby vertices, but that a few are connected to distant random vertices. This is to be expected in a small-world graph.

My answer to question 4 shows that there are many vertices of low degree and a few vertices of high degree, known as hubs. This is a characteristic of a scale-free graph. This characteristic is not as strong as the properties of the small-world graph, however. This accounts for the fact that the correlation coefficient is stronger towards a small-world graph.

Based on my results, it appears that many high energy physics theory researchers collaborated on a few papers with other nearby researchers, and a few of them collaborated on a smaller number of papers with researchers to whom they were distantly connected.

9. In this project I learned a lot about programmatic analysis of graphs. I found it beneficial to learn how it is useful to fit a dataset to logarithmic functions. I thought it was fascinating to see how so much information can be drawn from the same set of data by looking at the data in a few different ways. I also learned better how program design so greatly affects the ability of the program to process large amounts of data. Originally, I was concerned with running out of time and memory, but then I redesigned some parts of the programs to more efficiently handle the volume of data.