L01 - 08-26 Lecture

Friday, August 23, 2024 11:04 AM

Syllabus: D2L. submit homework there.

"This material is Basic, should Be part of The Language."

Cardinality of set

\# of point in The set. $\bar{\bar{A}}$ ← current cardinality Notation.

if $A = \{a_1, a_2, a_3\} \rightarrow \bar{\bar{A}} = 3$

$A = \{1, 2, 3, \ldots\}$ natural numbers.

$\bar{\bar{A}}$ is called "countable"

Example of countable sets.

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

if you can map 1-1 Betten $\mathbb{N}$ Then its countable.

$\mathbb{Q} = \{\frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0\}$ countable.

$\tilde{\mathbb{Q}} = ((m,n);\ m, n \in \mathbb{Z}\}$ set of All Pairs of $\mathbb{Z}$

$(1,1) \rightarrow (1,2), (1,3) \rightarrow (1,4)$

$(2,1), (2,2), (2,3) \ldots$

$(3,1), (3,2), (3,3)$

Rational $\mathbb{Q} \subseteq \mathbb{R}$ - Real number.

Every cauchy sequence has a limit in $\mathbb{R}$.

$\text{CarD}(\mathbb{R}) > \text{CarD}(\mathbb{Z})$

uncountable — countable.

sumtime called Continium.

Canto. Proff.

set A. , PowerSet (A) denoted 2^A ← the set of all subset of A including $\emptyset$

Ex $A = \{1, 2, 3\} \rightarrow 2 \cdot 2 \cdot 2 = 2^3 = 8$.

Every has possiblity of Being included or Not.

$n < 2^n$

Assume that $\bar{\bar{A}} = \bar{\bar{2^A}}$ bijective.

$\exists f : A \rightarrow 2^A$, f is 1-1 & onto

select a subset of A

$B = \{a \in A : a \notin f(a)\}$. ← Define B as..

Define a is not in $f(a)$.

Assume $f(b) = B$, $b \in A$

if $b \notin B$ then $b \in B$

if $b \in B$ then. $b \notin B$.

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thus implies $\bar{\bar{2^{\mathbb{Q}}}} > \bar{\bar{\mathbb{Q}}}$ which is countable

$A = \{x \in \mathbb{R} : 0 < x < 1\}$

$x = \sum_{i=1}^{\infty} \frac{\epsilon_i}{2^i} \quad \epsilon_i \in \{0, 1\}$

$x \rightarrow (0, 1, 0, 1, 0, 0, 0, 1, \ldots \}$

Every subset is of The form.

Convergence of series.

$a_1, a_2, \ldots = \{a_k : a_k \in \mathbb{R}\}$. sequence of Real # Then Boi D Series

$S_n = \sum_{i=1}^{n} a_i$, $n = 1, 2, 3 \ldots$

$\sum_{i=1}^{\infty} a_i$ - Converge.

Partial sum $S_n \underset{n \to \infty}{\rightarrow} S$. $S \in \mathbb{R}$ or $\lim_{n \to \infty}$

Diverge.

$\lim_{n \to \infty} a_i$ Does not Exist or is not finite Eg. $-\infty, \infty$

sometimes.

$\sum_{n=1}^{\infty} a_n < \infty$ if $a \geq 0$ then $S_n \uparrow (S_{n+1} \geq S_n)$

and $\lim_{n \to \infty} S_n < \infty$ then $\sum_{n=1}^{\infty} a_n < \infty$

Says "Limit is finite."

countable - Finite or Countably infinite.

Cardinality Denoted As $\bar{\bar{\ }}$.

thus $\bar{\bar{A}}$ is The cardinality of A

decribes uncountable

$\text{CarD}(\mathbb{R}) > \text{CarD}(\mathbb{Q})$

Power set Denoted 2^A for set A

Cantor's Proff $\bar{\bar{A}} < \bar{\bar{2^A}}$

By Contradiction

Assume $f : A \rightarrow 2^A$ is Bijection. $\rightarrow \forall a \ \exists f(a) \subseteq A$

Define: $B \subset A, B = \{a \in A \mid a \notin f(a)\}$

Since $B \subset A$, $\exists b \in A : f(b) = B$

Also, $b \notin B$ or $b \in B$.

Assume $b \in B$, $\rightarrow b \notin f(b)$, By Def. B.

$\rightarrow b \notin B$, By $f(b) = B$

$\rightarrow \leftarrow$

Assume $b \notin B$ $\rightarrow b \in f(B)$, By Def B.

$\rightarrow b \in B$, By $f(b) = B$

$\rightarrow \leftarrow$.

Since $b \notin$ or $b \notin B$. $\rightarrow \nexists b \in A : f(b) = B$.

Notation

Sequence $a : \mathbb{N} \rightarrow X$: a_n is nth term in X (Function, set)

An ordered list of Element following Role.

Series $S_n = a_1 + a_2 + \ldots + a_n$ (finite) or $S_n = \sum_{i=1}^{\infty} a_i = a_1 + a_2 + \ldots$ (infinite)

Sum of terms in sequence.

Partial sum $S_N = \sum_{i=1}^{N} a_i$

Note $f : A \rightarrow 2^A$ is a map Between Elements of A and a set, subset of A

$1 \rightarrow \{1, 2\}$.

$B \subset A \rightarrow B \in 2^A$

f: $1 \rightarrow \{2\}$

$2 \rightarrow \{3\}$

$3 \rightarrow \{2, 3\}$

$\therefore B = \{1, 2\}$

$b \in B$ $\quad f(1) = \{2\}$

$f(2) = \{3\}$

and $\lim_{n\to\infty} S_n < \infty$ then $\sum_{n=1}^{\infty} a_n < \infty$

Says "Limit is finite."

$\sum_{i=1}^{\infty} a_i = \sum_{i=1}^{\infty} \frac{1}{i} = \infty$; $\sum_{i=1}^{\infty} \frac{1}{(i)^2} < \infty$

infinite. finite

Ex $a_i = (-1)^i$ $\sum_{i=1}^{\infty} a_i$ DNE because $-1, 1$

Sum of terms in sequence.

Partial sum $S_N = \sum_{i=1}^{N} a_i$

Convergences if $S_N \to S \Rightarrow S_n \to S$

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measure theorey.

Algebra

Some set Ω, $\mathcal{A} = 2^{\Omega}$ is "Algebra" if

① $A, B \in \mathcal{A} \Rightarrow A \cap B \in \mathcal{A}$ closed under intersection As well

② $A \in \mathcal{A} \Rightarrow A^c \in \mathcal{A}$

③ $\phi \in \mathcal{A} \Rightarrow \Omega \in \mathcal{A}$

Also $A, B \in \mathcal{A} \Rightarrow A \cup B \in \mathcal{A}$

proff $A \cap B \Rightarrow (A^c \cap B^c)^c = A \cup B$.

Example $\Omega = \mathbb{R}$

$\mathcal{A} = \{ A \subseteq \mathbb{R} : A = \bigcup_{i=1}^{k} (a_i, b_i] \}$. $a < b_i < a_{i+1} <$

a_1 b_1 a_2 b_2 ...

Assume Disjoint.

Measure $(\Omega, \mathcal{A})$

$\mu : A \to \mathbb{R}^+$, $A \in \mathcal{A}$ maps Every Algebra To Real positive.

① $\mu(A) \geq 0$, $A \in \mathcal{A}$

② $\mu(\phi) = 0$

③ $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$ if $A_i \in \mathcal{A} \wedge A_i \cap A_j = \phi \ \forall i,j$ and $\bigcup_{i=1}^{\infty} A_i \in \mathcal{A}$