L27 - 10-30

Wednesday, October 30, 2024 11:31 AM

Theorem(1) $\{X_i\}_{i\ge1}$ Ind, $E(X_i)=0$, $i\ge1$ (theorem 2.5.3)

if $\sum_{i=1}^{\infty} E(X_i^2)<\infty$ then $\sum X_i$ Converges a.s.

(2) $\{X_i\}_{i\ge1}$ if $\sum_{i=1}^{\infty} Var(X_i)<\infty$ then $\sum_{i=1}^{\infty}(X_i-E(X_i))$ con as

$E[X_i-E(X_i)]^2$

$X_1, X_2, \ldots$ ind.

$EX_n=0$.

if $\sum_{n=1}^{\infty} Var(X_n)<\infty$

then $\sum_{n=1}^{\infty} X_n(\omega)$ Converges in P.

Remark no Assumptions.

if $\sum_{i=1}^{\infty} E(X_i^2)<\infty$ then $\sum_{i=1}^{\infty} X_i^2$ Conv as.

$E\left(\sum_{i=1}^{\infty} X_i^2\right)<\infty$

Focus on version (1).

Proof $P(\max_{M\le m\le N}|S_m-S_n|>\varepsilon)\le \frac{Var(S_N-S_M)}{\varepsilon^2}=\frac{\sum_{i\ge M}^{N} Var(X_i)}{\varepsilon^2}$

$P(\sup_{M\le m}\{|S_m-S_M|\}>\varepsilon)\le \frac{\sum_{i=M}^{\infty} Var(X_i)}{\varepsilon^2}\xrightarrow[M\to\infty]{}0$

Sum of Var Converges
$\therefore$ tail goes to 0

a_n Converges As $n\to\infty$ iff $\sup_{n,m\ge M}|a_n-a_m|\xrightarrow[M\to\infty]{}0$ Cauchy Criteria

$\sup_{n,m\ge 0}|a_n-a_m|\downarrow 0$

$P(\sup_{n,m\ge M}|S_n-S_m|>2\varepsilon)\le$

By Δ ineq.

$2\varepsilon\le|S_n-S_m|\le|S_n-S_M|+|S_m-S_M|$

Let $W_M=\sup_{m\ge M}|S_n-S_m|$

By monotone Argument.

we get $W_M\xrightarrow[M\to\infty]{P}0$, $W_M\overset{a.s.}{\downarrow}W\ge0$

$\Rightarrow W=0$ a.s.

$\Rightarrow \sum_{i=1}^{\infty}X_i$ Conv a.s.

$\lim_{n\to\infty}S_n$ exist a.s.

Kolgomorov 3 series theorem.

Kol 3-Series theorem

Let $\{X_k\}_{k\ge1}$ be ind. Let $A>0$ Notation. $Y_k=X_k\mathbb{1}_{\{|X_k|\le A\}}$ $k=1,2.$

$\mu_k=E(Y_k)$

necessary conditions that X_k goes to Zero.

$\sum_{k=1}^{\infty}X_k$ conv. iff (a) $\sum_{k=1}^{\infty}P(|X_k|>A)<\infty$ And.

(b) $\sum_{k=1}^{\infty}\mu_k$ Converges,

Says $X_k>A$ Finitely often, not I.O.

$\lim_{n\to\infty}S_n=\lim_{n\to\infty}S_{n+1}=S$

$S_{n+1}-S_n=X_{n+1}$

(c) $\sum_{k=1}^{\infty} V_k(Y_k)<\infty$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} S_{n+1} = S$

$S_{n+1} - S_n = X_{n+1}$

(b) $\sum_{k=1} M_k$ Converges.

(c) $\sum_{k=1}^{\infty} Var(Y_k) < \infty$

Says $X_k > A$ finitely often, not I.O.

Proof $\Leftarrow$

By Thm 2.1

(c) implies. $\sum_{k=1}^{\infty} Y_k - M_k$ Conv as.

Book Proves. By C.L.T.

$\overset{(b)}{\Rightarrow} \sum_{k=1}^{\infty} Y_k$ Conv. as.

By BCII $\overset{(a)}{\Rightarrow} \sum_{k=1}^{\infty} X_k$ Conv. as.