04-18

Friday, April 18, 2025 11:34 AM

Last Time $T_a = \inf\{t>0 : B_t = a\}$ $a>0$

$P_0(T_a < t) = 2P_0(B_t > a) = 2P_0(\sqrt{t}\,Z > a) = 2P(Z > \frac{a}{\sqrt{t}})$

$f_{T_a}(t) = \frac{\partial}{\partial t}\left[2\cdot P(Z > \frac{a}{\sqrt{t}})\right] = (2\pi t^3)^{-1/2} a e^{-\frac{a^2}{2t}}$ $t>0$

if $t\to\infty$ $e^{-\frac{a^2}{t}} \to 0$. but $(2\pi)^{-1/2} t^{3/2}$

$E(T_a) = \int^\infty$

Also said $T_a \overset{D}{=} a^2 T_1$

Both there is a nicer way.

$X_t = a\cdot B_t$ $\quad cov(X_{t_1}, X_{t_2}) = a^2(t_1\wedge t_2)$

$S = \inf\{t : X_t = a\} \overset{D}{=} T_1$

X_t & Y_t have same Distributn

$Y_t = B(a^2 t)$. $t\ge 0$

covariance

$E[Y_{t_1}Y_{t_2}] = a^2 t_2 \wedge a^2 t_1 = a^2(t_2\wedge t_1)$

$\tilde{S} = \inf\{t : Y_t = a\}$. $\Rightarrow$ $S \overset{D}{=} \tilde{S}$

$a^2\tilde{S} = T_a \Rightarrow a^2$

$a^2 T_1 \overset{D}{=} T_a$

$T_2 \overset{D}{=} 2^2 T_1 = 4T_1$

type of Brownian Motion.

$\{T_a\}_{a\ge 0}$ has Indepent & stationary Increments.

① $T_b - T_a \perp\!\!\!\perp \mathcal{F}_{T_a}$ $\quad 0<a<b$

② $T_b - T_a \overset{D}{=} T_{b-a}$ "

Look At

same Distribution.

$T_2 = T_1 + (T_2 - T_1) = T_1 + \tilde{T}_1$ $\quad T_1 \overset{D}{=} \tilde{T}_1$

this Happens we call stable when $n^2 T_1 = T_n = \sum_{i=1}^n X_i$

$n^2 T_1 \overset{D}{=} T_n = X_1 + X_2 + \dots + X_n$

$\begin{array}{ccc} \| & \| & \| \\ t_1 & t_2 - t_1 & T_n - T_{n-1} \end{array}$

$\{X_k\}_{1\le k\le n}$ iid $X_k \overset{D}{=} T_1$

$T_n \overset{D}{=} n^2 T_1$

standard normal.

$1 \le k \le n$

$T_n \overset{\mathcal{D}}{=} n^2 T_1$

standard normal.

$\sum_{n=1}^{n} z^k \overset{\mathcal{D}}{=} \sqrt{n}\, z$,

we only have stable Distrib. when $0 < \alpha \le 2$

T_1 is stable Dis with Parameter $\alpha = 1/2$.

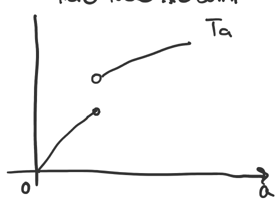
$\sum_{k=1}^{n} Z_k = \sqrt{n} Z = n^{1/2} Z$

$T_a \ge 0$, $a \ge 0$

$T_0 \equiv 0$ For SBM

$T_b > T_a$, $b = a \ge 0$

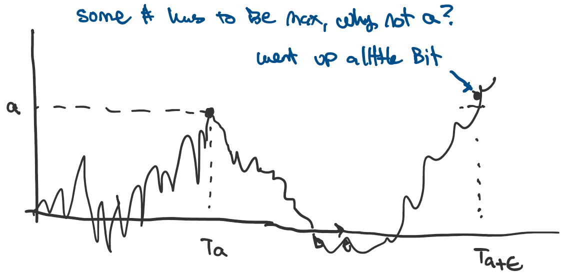
Here there Are Jumps.



Left continuous with Left Limits

some # has to be max, why not a?

went up a little Bit



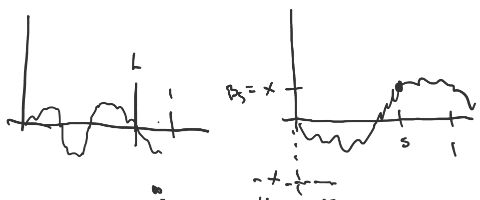
①

$P_0(L \le s)$ $0 \le s \le 1$

$\sup(T \le 1, B_t = 0)$

$P(T_0 > 1-s)$

$P_0(T_x > 1-s)$



$P_0(L \le s) = 2\int_{x=0}^{\infty} (2\pi s)^{-1/2} e^{-\frac{x^2}{2}} \; P_0(T_x > 1-s)\, dx$

$N(0,s)$ $\quad 1_{\{L \le s\}} = 1_{\{T_0 > 1-s\}} \circ \theta_s$

$P_0(T_x > 1-s)\,dx$

$\int_{n=1-s}^{\infty} (2\pi n^3)^{-3/2} x e^{-\frac{x^2}{2}} \, d\Omega$?

$\frac{1}{\pi}\int_{t=0}^{\infty} (t(1+t))^{-1/2} dt = \frac{2}{\pi} \arcsin(\sqrt{s})$

$t = \frac{s}{}$

$= \frac{2}{\pi} \arcsin(\sqrt{s})$

about zero's about of SBM.

$A(\omega) = \{t \ge 0 ; B_t(\omega) = \omega(t) = 0\}$

claim with Prob 1 there is not one point.

$t\ A(\Omega)$ s.t. t is isolated

other at zero.