Homework 1

January 24, 2020

1 Introduction

For this homework, you will implement some Haskell functions using pattern matching.

1. Your solution should be a working Haskell program in a text file that I can run with the provided `main`.
2. You may add comments to your program such as `-- Question 1`.
3. Use 'let' or 'where' as necessary for local variables and local functions.

2 Questions

1. Use recursion to define a function `range` so that `range from to count` returns `count` number of equally-spaced numbers between `from` and `to` (but not including `to`).

   ```
   range 0 1 10;
   ```

   returns

   ```
   [0.0,0.1,0.2,0.30000000000000004,0.4,0.5,0.6,0.7,0.7999999999999999,0.8999999999999999]
   ```

   The imprecision is acceptable.

2. Define a function `rd` so that `rd n x` rounds a list x to the nth digits of precision.

   ```
   rd 1 (range 0 1 10)
   ```

   returns [0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9].

   For the next two questions, we use a 2-tuple to represent a complex number. For example, $1 + i \cdot 2$ is written as `(1, 2)`

3. Define a function `absolute` to compute the absolute value of a list of complex numbers. The absolute value of $a + i \cdot b$ is $\sqrt{a^2 + b^2}$.

   ```
   absolute [(3.0, 4.0), (4.0, 3.0)]
   ```

   returns [5.0,5.0].

4. Define a function `dft` to compute the Discrete Fourier Transform (DFT) of a list of reals $\{x_0, x_1, \ldots, x_N\}$. You should use the following equation to compute each element of the DFT:

   $$X_k = \sum_{n=0}^{N-1} x_n \cdot [\cos(2\pi kn/N) - i \cdot \sin(2\pi kn/N)]$$

   The result is a list of 2-tuples, where each tuple X_k is

   $$(\sum_{n=0}^{N-1} x_n \cdot \cos(2\pi kn/N), \sum_{n=0}^{N-1} x_n \cdot \sin(2\pi kn/N))$$

   In Haskell, `pi`, `sin`, and `cos` are predefined.

3 Testing

You can test the implementation using the following `main`.

```
main = do
         let n = 64
         let s = map (\t -> sin(10*2*pi*t) + sin(20*2*pi*t)/2) $ range 0 1 n
         let result = map (\x -> x/n) $ absolute $ dft s
         print(rd 3 s)
         print(rd 2 result)
```

Running main should result the following output.

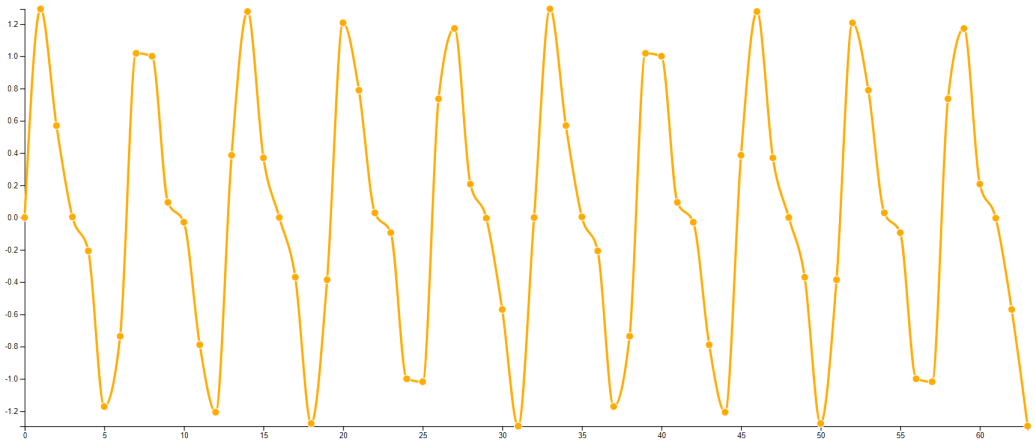
```
[0.0,1.293,0.57,4.0e-3,-0.207,-1.172,-0.736,1.018,1.0,9.4e-2,-2.9e-2,-0.789,-1.207,
 0.386,1.277,0.37,0.0,-0.37,-1.277,-0.386,1.207,0.789,2.9e-2,-9.4e-2,-1.0,-1.018,
 0.736,1.172,0.207,-4.0e-3,-0.57,-1.293,0.0,1.293,0.57,4.0e-3,-0.207,-1.172,-0.736,
 1.018,1.0,9.4e-2,-2.9e-2,-0.789,-1.207,0.386,1.277,0.37,0.0,-0.37,-1.277,-0.386,
 1.207,0.789,2.9e-2,-9.4e-2,-1.0,-1.018,0.736,1.172,0.207,-4.0e-3,-0.57,-1.293]

[0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.5,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.25,
 0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.
 0.25,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.5,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0]
```

The variable s is the addition of two sine waves: one at 10 Hz and one at 20 Hz (with half the magnitude). Mathematically, s is written as

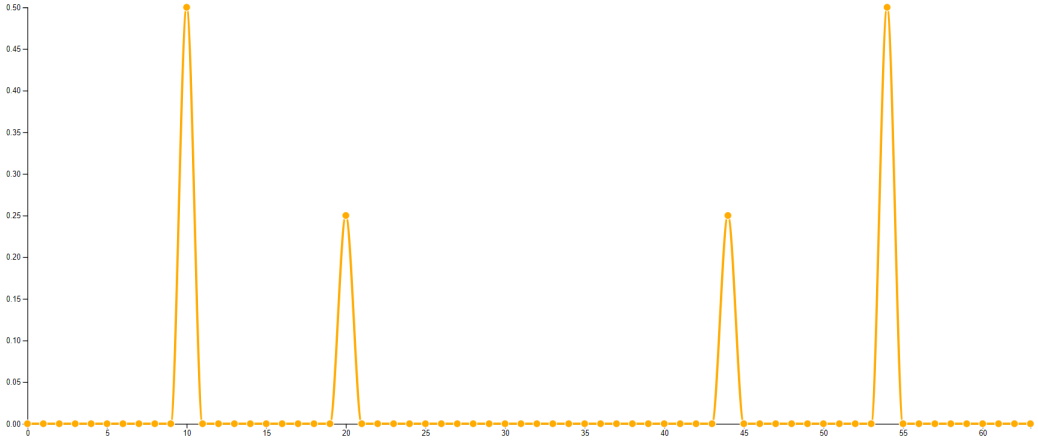
$$\sin(10 \times 2\pi t) + \sin(20 \times 2\pi t)/2$$

The wave is sampled at 64 Hz for 1 second. That is, s is a list of 64 points sampled between 0 and 1 second.

The list s is shown in the following diagram.



The absolute value of the DFT of s divided by 64 is shown below as well, where you can find a peak value of 0.5 at 10th and a peak value of 0.25 at 20th position (and there are also symmetric peaks from the right end).



4 Submission

Please write your solution in a text file by the name of `hwk1.hs` and submit it to the dropbox.