ECO 420Y — Homework 4

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```r
library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```r
miu1 = 5;  s1 = 4
miu2 = 8;  s2 = 5
miu3 = 7;  s3 = 4.5
miuv = c(miu1, miu2, miu3)

targetmiu = seq(5, 8, 0.10)

# ---- OLD frontier ----
rho12 = -0.5; rho13 = -0.3; rho23 = 0.4
s12 = rho12 * s1 * s2
s13 = rho13 * s1 * s3
s23 = rho23 * s2 * s3
sigma_old = matrix(c(s1^2, s12,  s13,
                     s12,  s2^2, s23,
                     s13,  s23,  s3^2),
                   nrow = 3, ncol = 3, byrow = TRUE)

sigp_old = rep(0, length(targetmiu))
w_old = vector("list", length(targetmiu))
for (i in 1:length(targetmiu)) {
  op = portfolio.optim(t(miuv), pm = targetmiu[i], covmat = sigma_old)
  sigp_old[i] = sqrt(t(op$pw) %*% sigma_old %*% op$pw)
  w_old[[i]] = op$pw
}

idx_old = which.min(sigp_old)
gmvp_mu_old  = targetmiu[idx_old]
gmvp_sig_old = sigp_old[idx_old]
gmvp_w_old   = w_old[[idx_old]]

cat("=== OLD correlations (rho12=-0.5, rho13=-0.3, rho23=0.4) ===\n")
```

=== OLD correlations (rho12=-0.5, rho13=-0.3, rho23=0.4) ===

```
cat(sprintf("GMVP: mu = %.3f, sigma = %.3f\n", gmvp_mu_old, gmvp_sig_old))
```

GMVP: mu = 6.300, sigma = 2.015

```
cat(sprintf("weights: w1 = %.3f, w2 = %.3f, w3 = %.3f\n\n",
            gmvp_w_old[1], gmvp_w_old[2], gmvp_w_old[3]))
```

weights: w1 = 0.498, w2 = 0.296, w3 = 0.206

```
# ---- NEW frontier ----
rho12 = 0.5; rho13 = 0.3; rho23 = 0.4
s12 = rho12 * s1 * s2
s13 = rho13 * s1 * s3
s23 = rho23 * s2 * s3
sigma_new = matrix(c(s1^2, s12,  s13,
                     s12,  s2^2, s23,
                     s13,  s23,  s3^2),
                   nrow = 3, ncol = 3, byrow = TRUE)

sigp_new = rep(0, length(targetmiu))
w_new = vector("list", length(targetmiu))
for (i in 1:length(targetmiu)) {
  op = portfolio.optim(t(miuv), pm = targetmiu[i], covmat = sigma_new)
  sigp_new[i] = sqrt(t(op$pw) %*% sigma_new %*% op$pw)
  w_new[[i]] = op$pw
}

idx_new = which.min(sigp_new)
gmvp_mu_new  = targetmiu[idx_new]
gmvp_sig_new = sigp_new[idx_new]
gmvp_w_new   = w_new[[idx_new]]

cat("=== NEW correlations (rho12=0.5, rho13=0.3, rho23=0.4) ===\n")
```

=== NEW correlations (rho12=0.5, rho13=0.3, rho23=0.4) ===

```
cat(sprintf("GMVP: mu = %.3f, sigma = %.3f\n", gmvp_mu_new, gmvp_sig_new))
```

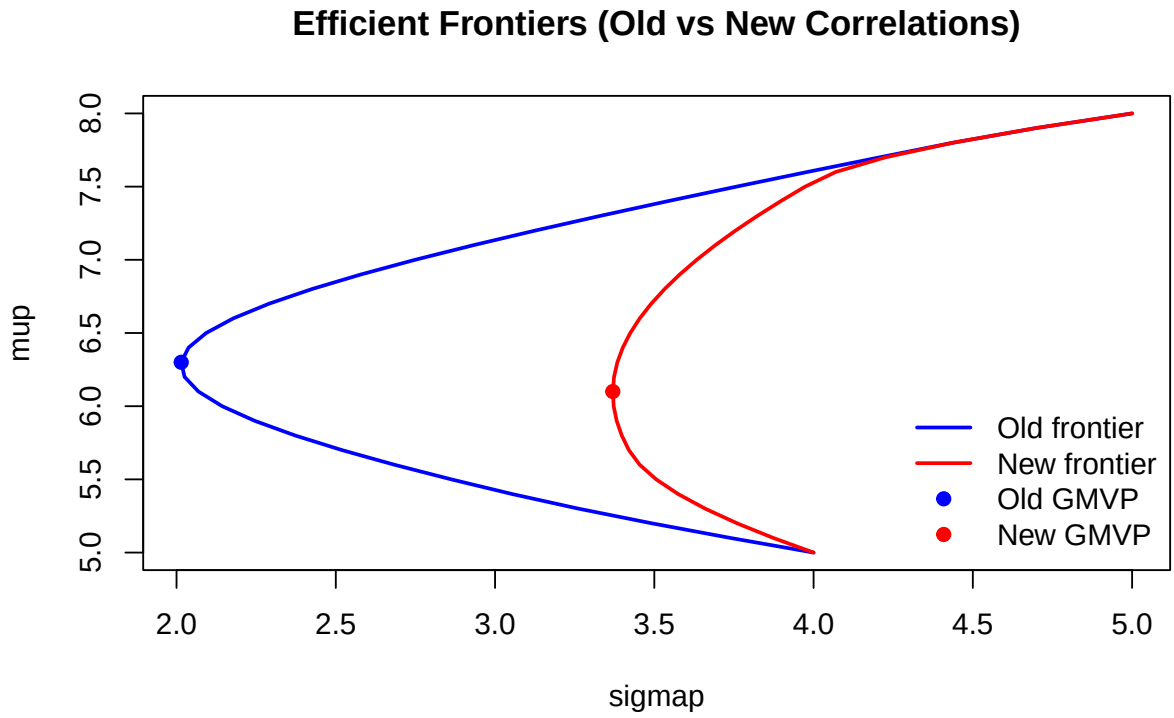
GMVP: mu = 6.100, sigma = 3.370

```
cat(sprintf("weights: w1 = %.3f, w2 = %.3f, w3 = %.3f\n\n",
            gmvp_w_new[1], gmvp_w_new[2], gmvp_w_new[3]))
```

weights: w1 = 0.509, w2 = 0.118, w3 = 0.374

```
# ---- Plot ----
plot(sigp_old, targetmiu, type = "l", lwd = 2,
     xlab = "sigmap", ylab = "mup",
     main = "Efficient Frontiers (Old vs New Correlations)",
     col = "blue")
```

```r
lines(sigp_new, targetmiu, lwd = 2, col = "red")
points(gmvp_sig_old, gmvp_mu_old, pch = 19, col = "blue")
points(gmvp_sig_new, gmvp_mu_new, pch = 19, col = "red")
legend("bottomright",
       legend = c("Old frontier", "New frontier", "Old GMVP", "New GMVP"),
       lty = c(1,1,NA,NA), pch = c(NA,NA,19,19),
       col = c("blue","red","blue","red"),
       bty = "n", lwd = c(2,2,NA,NA))
```



Comments and Intuition

- With the old correlations (negative between 1–2 and 1–3), the frontier is farther left and the GMVP risk is lower.
- With new correlations (all positive), the frontier shifts rightward and the GMVP has higher .

For weights w,

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \rho_{ij} \sigma_i \sigma_j.$$

Negative ρ_{ij} make cross terms reduce variance (offsetting moves → strong diversification). Positive ρ_{ij} make them add variance (assets move together → weaker diversification). Hence higher/more-positive correlations mean less diversification, a rightward shifted frontier, and a riskier GMVP.