ECO 420Y - Homework 2

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Question 1 — Solve $110x^3 + 10x^2 + 10x - 95 = 0$ by Bisection

```
d = function(x) { return(110*x^3 + 10*x^2 + 10*x - 95) }
i = 0; smallx = 0; bigx = 1
while (i < 30) {
  root = (smallx + bigx)/2
  if (d(root) * d(bigx) >= 0) { bigx = root } else { smallx = root }
  cat("--------------------------", "\n")
  cat("Root is ", root, "\n")
  cat("d(root) is ", d(root), "\n")
  i = i + 1
}
```

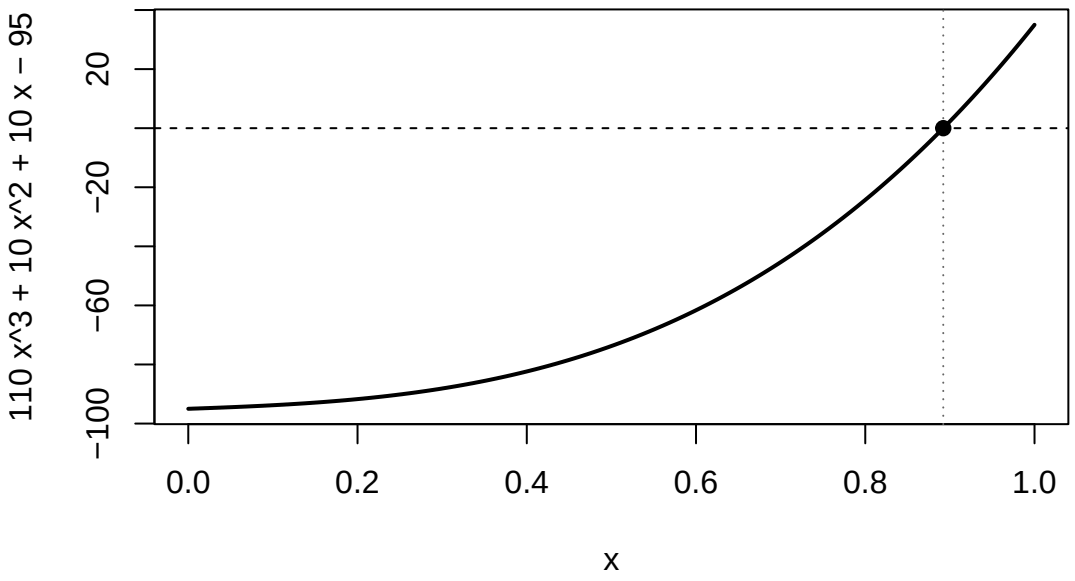
```
## --------------------------
## Root is  0.5
## d(root) is  -73.75
## --------------------------
## Root is  0.75
## d(root) is  -35.46875
## --------------------------
## Root is  0.875
## d(root) is  -4.902344
## --------------------------
## Root is  0.9375
## d(root) is  13.80127
## --------------------------
## Root is  0.90625
## d(root) is  4.147644
## --------------------------
## Root is  0.890625
## d(root) is  -0.4515457
## --------------------------
## Root is  0.8984375
## d(root) is  1.829343
## --------------------------
## Root is  0.8945312
```

```
## d(root) is  0.6842417
## --------------------------
## Root is  0.8925781
## d(root) is  0.1151862
## --------------------------
## Root is  0.8916016
## d(root) is  -0.1684699
## --------------------------
## Root is  0.8920898
## d(root) is  -0.02671442
## --------------------------
## Root is  0.892334
## d(root) is  0.04421774
## --------------------------
## Root is  0.8922119
## d(root) is  0.008747126
## --------------------------
## Root is  0.8921509
## d(root) is  -0.008984781
## --------------------------
## Root is  0.8921814
## d(root) is  -0.0001191111
## --------------------------
## Root is  0.8921967
## d(root) is  0.004313936
## --------------------------
## Root is  0.892189
## d(root) is  0.002097395
## --------------------------
## Root is  0.8921852
## d(root) is  0.0009891375
## --------------------------
## Root is  0.8921833
## d(root) is  0.0004350121
## --------------------------
## Root is  0.8921824
## d(root) is  0.0001579502
## --------------------------
## Root is  0.8921819
## d(root) is  1.941951e-05
## --------------------------
## Root is  0.8921816
## d(root) is  -4.98458e-05
## --------------------------
## Root is  0.8921818
## d(root) is  -1.521315e-05
## --------------------------
## Root is  0.8921818
```

```
## d(root) is  2.10318e-06
## -------------------------
## Root is  0.8921818
## d(root) is  -6.554985e-06
## -------------------------
## Root is  0.8921818
## d(root) is  -2.225902e-06
## -------------------------
## Root is  0.8921818
## d(root) is  -6.136115e-08
## -------------------------
## Root is  0.8921818
## d(root) is  1.020909e-06
## -------------------------
## Root is  0.8921818
## d(root) is  4.797741e-07
## -------------------------
## Root is  0.8921818
## d(root) is  2.092065e-07
```

Plot

```r
curve(110*x^3 + 10*x^2 + 10*x - 95, from = 0, to = 1,
      n = 1000, lwd = 2, col = "black",
      xlab = "x", ylab = "110 x^3 + 10 x^2 + 10 x - 95",
      )
abline(h = 0, lty = 2)
abline(v = root, col = "gray40", lty = 3)
points(root, 0, pch = 19)
```



Question 2 — Maximize $-x^2 + \log x$ using Newtons Method

```
f = function(x) { return(-x^2 + log(x)) }
fprime = function(x) { return(-2*x + 1/x) }
fprime2 = function(x) { return(-2 - 1/x^2) }
tol = 0.001
x0 = 1
while (abs(fprime(x0)) > tol) {
  step = - fprime(x0) / fprime2(x0)
  x1 = x0 + step
  while ((x1 <= 0) || (f(x1) <= f(x0))) {
    step = step/10
    x1 = x0 + step
  }
  cat("----------------------------", "\n")
  cat("New value is ", x1, "\n")
  cat("Gradient is ", fprime(x1), "\n")
  cat("f(x) is ", f(x1), "\n")
  x0 = x1
}
```

```
## ----------------------------
## New value is  0.6666667
## Gradient is  0.1666667
## f(x) is  -0.8499096
```

```
## --------------------------
## New value is  0.7058824
## Gradient is  0.004901961
## f(x) is  -0.8465766
## --------------------------
## New value is  0.7071057
## Gradient is  4.2478e-06
## f(x) is  -0.8465736
```

Plot

```
x_star = x0
curve(-x^2 + log(x), from = 0.1, to = 2,
      n = 1000, lwd = 2, col = "black",
      xlab = "x", ylab = "f(x) = -x^2 + log(x)"
      )
abline(v = x_star, col = "gray40", lty = 3)
points(x_star, f(x_star), pch = 19)
```