MTH 331 — Homework #3 Draft

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Exercise 1: Slogans, logical form, and inference rules

Orville Redenbacher slogan. Let P mean "you will like it better." Let Q mean "my name is Orville Redenbacher." The slogan

"You will like it better or my name is not Orville Redenbacher."

has logical form

$$P \vee \neg Q.$$

If the slogan is true and the speaker's name really is Orville Redenbacher, then Q is true. From Q and $P \vee \neg Q$ we can conclude P.

Inference rule.

$$\frac{P \vee \neg Q \qquad Q}{P}$$

This works because $P \vee \neg Q$ is equivalent to $Q \Rightarrow P$. With Q true, modus ponens lets us conclude P.

Truth table check. In the rows where both premises are true, P is also true.

P	Q	$\neg Q$	$P \vee \neg Q$	Both premises true? $\Rightarrow P$
T	T	F	T	Yes, P is T
F	T	F	F	No
T	F	T	T	No
F	F	T	T	No

As a conditional. Since $\neg Q \vee P$ is equivalent to $Q \Rightarrow P$, the slogan can also be written as

$$Q \Rightarrow P.$$

With Q true, modus ponens again gives P.

Bumper sticker. "If you're not outraged, you're not paying attention." Let P mean "you are paying attention." Let Q mean "you are outraged." Logical form:

$$\neg Q \Rightarrow \neg P.$$

The contrapositive is $P \Rightarrow Q$, meaning if you are paying attention then you are outraged.

Why word it this way. The original versions sound sharper. The Orville slogan sounds confident, and the bumper sticker is provocative. The simpler conditionals are logically the same but the given wording is more attention-grabbing.

Exercise 3: Parity statements with three integers

Assume $m, n, r \in \mathbb{Z}$. An integer is even if it is $2k$ for some k, and odd if it is $2k + 1$.

(1) If m, n, r are all odd then $m + n + r$ is odd. ***True.***

Write $m = 2a + 1$, $n = 2b + 1$, $r = 2c + 1$. Then

$$m + n + r = (2a + 1) + (2b + 1) + (2c + 1) = 2(a + b + c + 1) + 1,$$

which is odd.

(2) If $m + n + r$ is odd then m, n, r are all odd. ***False.***

Example: $m = 2$, $n = 2$, $r = 1$. Then $m + n + r = 5$ is odd, but m and n are even.

(3) If at least one of m, n, r is odd then $m + n + r$ is odd. ***False.***

Example: $m = 1$, $n = 1$, $r = 2$. At least one number is odd, but $m + n + r = 4$ is even. Another example: $m = 1$, $n = 2$, $r = 3$ gives $1 + 2 + 3 = 6$, which is even.

(4) If $m + n + r$ is odd then at least one of m, n, r is odd. ***True.***

Proof by contrapositive. Suppose none of m, n, r is odd. Then all are even: $m = 2a$, $n = 2b$, $r = 2c$. Then

$$m + n + r = 2(a + b + c),$$

which is even. So if the sum is odd, not all three can be even. That means at least one must be odd.

Why I used these methods. Parts (1) and (4) are easy to prove directly or directly with a contrapositive. Parts (2) and (3) are false, and counterexamples are the clearest way to show that.