MTH 331 — Homework #1 (Rough Draft)

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Exercise 2: Finite and Countably Infinite Sets

Are the sets A, B, C, and $\mathbb{Q}^+$ finite or countably infinite?

Set A

The set is

$$A = \{0, 0.01, 0.02, 0.03, \ldots, 1\}.$$

This has only finitely many elements (from 0 up to 1 in increments of 0.01). If you counted them all up, there are 101 of them, so A is finite.

Set B

The set is

$$B = \{0, 0.9, 0.99, 0.999, \ldots, 1\}.$$

This is different from Set A, but you can list its elements in a sequence:

$$0,\ 0.9,\ 0.99,\ 0.999,\ \ldots$$

This list will go on forever and every element is some natural number n. So B is countably infinite.

Set C

The set includes the same type of decimals as in B, but repeated for $1, 2, \ldots$ and so on. It can still be arranged into a single infinite list, therefore C is countably infinite.

Set $\mathbb{Q}^+$.

The positive rationals can also be arranged into a sequence, such as writing them in a grid. Therefore $\mathbb{Q}^+$. is countably infinite.