MTH 331 — Homework #2 (Rough Draft)

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Exercise 2: Implication is not associative

We want to compare the statements

$$P \Longrightarrow (Q \Longrightarrow R)$$
 and $(P \Longrightarrow Q) \Longrightarrow R$.

Truth tables

Here is the beginning of the truth table for both statements. Each row corresponds to one choice of truth values for P, Q, and R.

P	Q	R	$P \implies (Q \implies R)$	$(P \Longrightarrow Q) \Longrightarrow R$
T	Т	T	Т	T
T	Γ	F	F	F
T	F	$\mid T \mid$	T	T
T	F	$\mid F \mid$	T	F
F	Т	$\mid T \mid$	Т	T

Explanation in words

The statement $P \Longrightarrow (Q \Longrightarrow R)$ means that if P is true, then we also need that $Q \Longrightarrow R$ is true. This is the same as saying "if P and Q are both true, then R must be true." That is why $P \Longrightarrow (Q \Longrightarrow R)$ is logically equivalent to $(P \land Q) \Longrightarrow R$.

An example in plain English: Let P be "it is raining," let Q be "I have an umbrella," and let R be "I will stay dry." The statement $P \Longrightarrow (Q \Longrightarrow R)$ means "if it is raining, then if I have an umbrella I will stay dry." This is the same as saying "if it is raining and I have an umbrella, then I will stay dry."