MTH 331 — Homework #1 (Rough Draft)

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Exercise 2: Finite and Countably Infinite Sets

We want to decide whether the sets A, B, C, and Q^+ are finite or countably infinite.

Set A

The set is

$$A = \{0, 0.01, 0.02, 0.03, \dots, 1\}.$$

This has only finitely many elements (from 0 up to 1 in increments of 0.01). In fact, there are 101 of them, so A is finite.

Set B

The set is

$$B = \{0, 0.9, 0.99, 0.999, \dots, 1\}.$$

Although it looks different, we can list its elements in a sequence:

$$0, 0.9, 0.99, 0.999, \dots$$

This is an infinite list, and every element is accounted for by some natural number n. So B is countably infinite.

Set C

The set includes the same type of decimals as in B, but repeated for $1, 2, \ldots$ and so on. It can still be arranged into a single infinite list, which shows that C is countably infinite.

Set Q^+

The positive rationals can also be arranged into a sequence (for example, by writing them in a grid and reading diagonally). Therefore Q^+ is countably infinite.