ECE 410 Lab #3 Report

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Numerical Linear Algebra and Controllability

Output 1

```
A =

[ 0, 1,                 0, 0]
[ 0, 0,           (g*m)/M, 0]
[ 0, 0,                 0, 1]
[ 0, 0, (g*(M + m))/(M*l), 0]


B =

       0
     1/M
       0
 1/(M*l)
```

Output 2

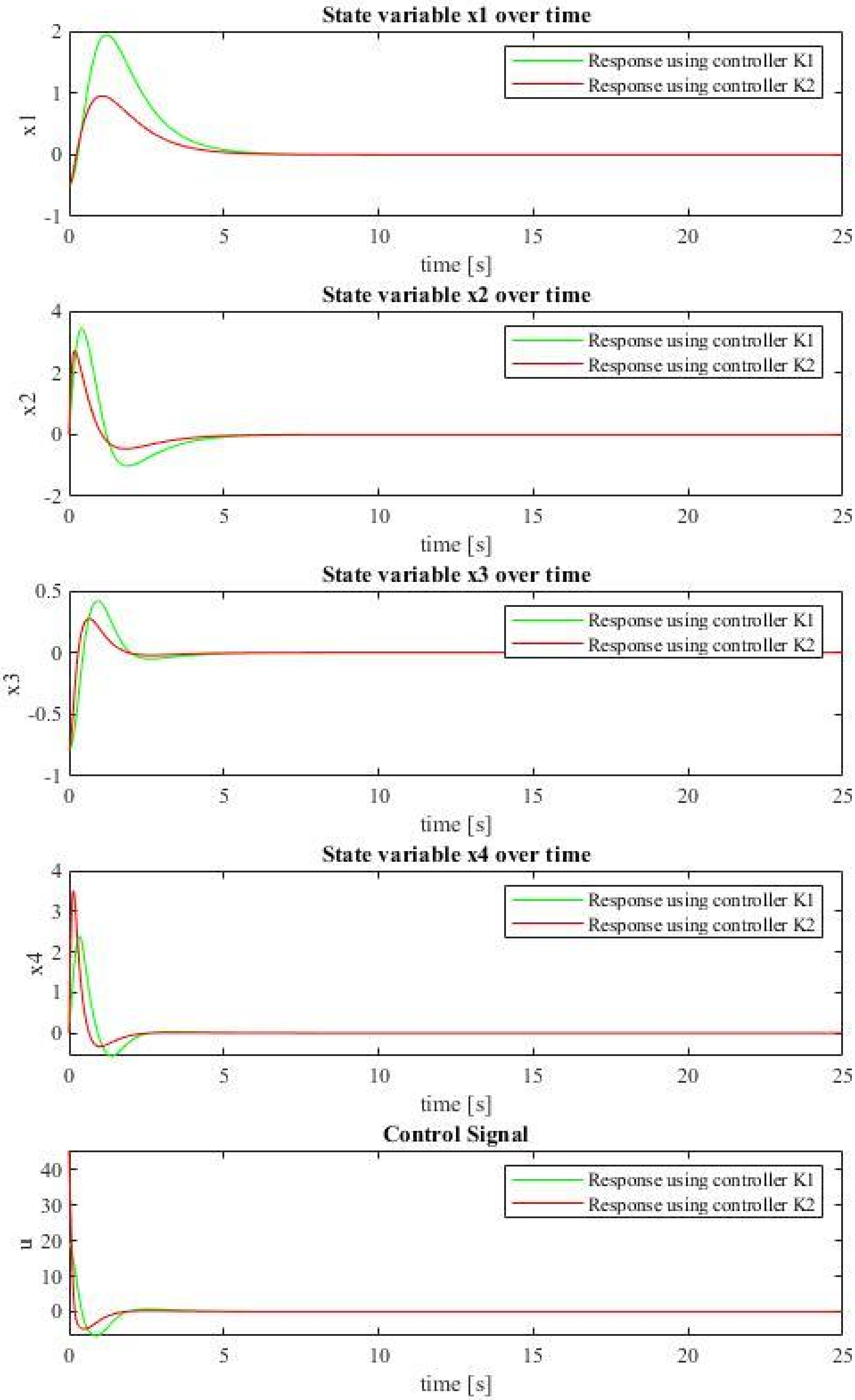
```
K1 =

   -0.8678   -1.8078   25.4587    4.1403


K2 =

   -4.3388   -8.1715   60.6213   11.9110
```

Output 2 - Control responses using Pole Placement



Describe the differences between the transient response for the two sets of gains. How is the change in one pole reflected across the different state variables? Is a single state affected, or are all states affected?

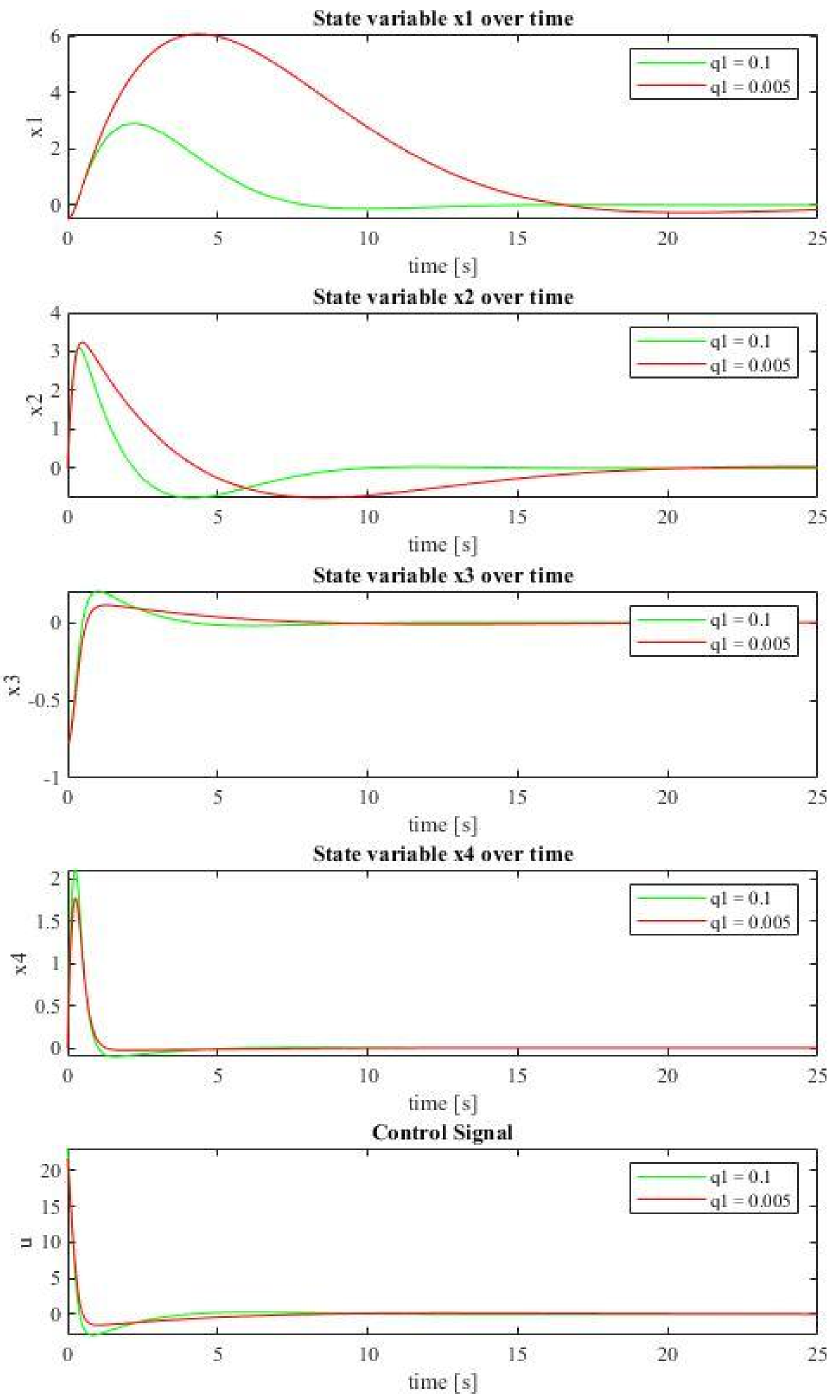
With a change in only one desired eigenvalue, all state variables are affected. They converge to equilibrium faster and they have less overshoot. This is true as well for the control input u.

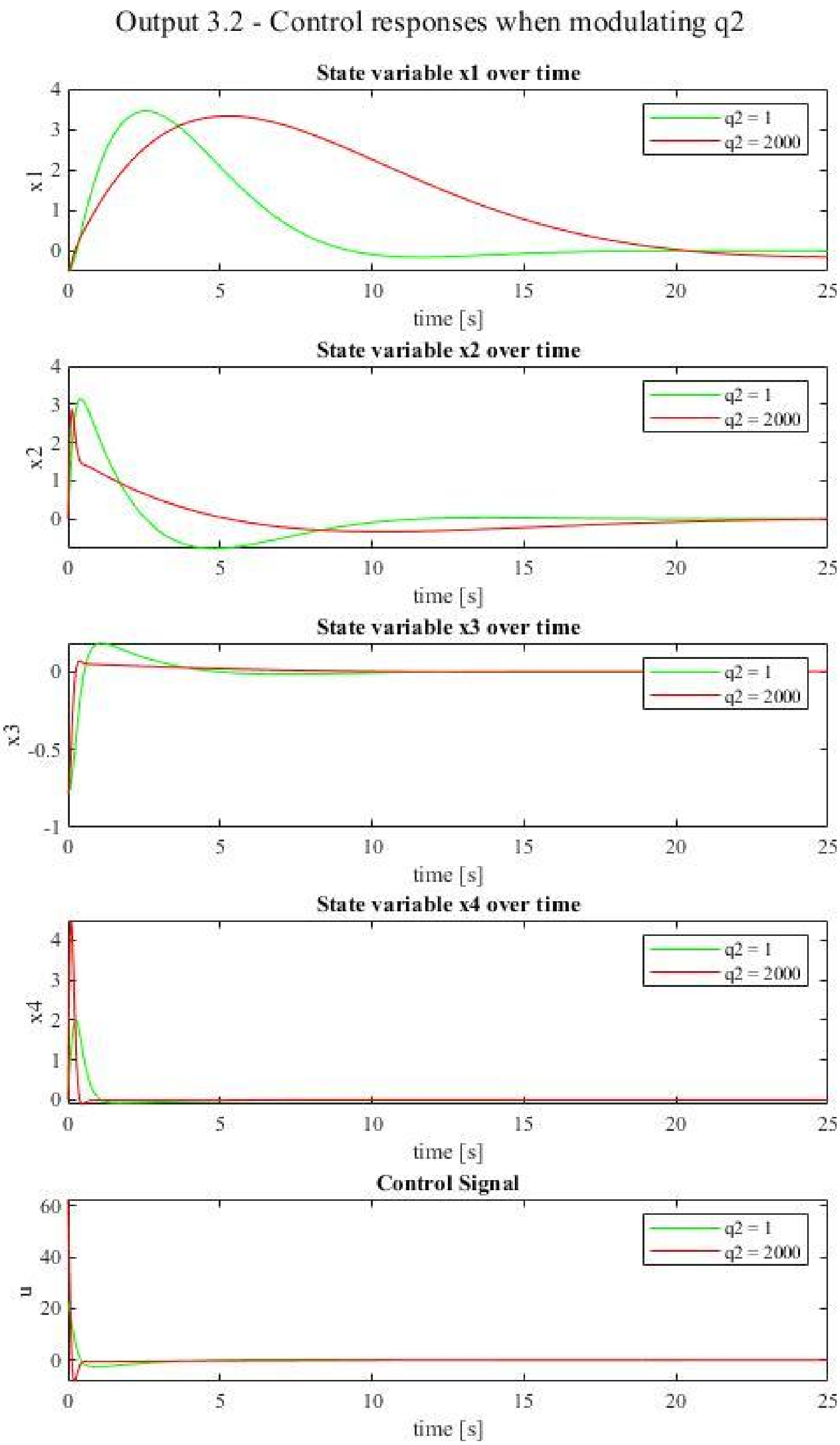
Comment on this: suppose you are satisfied with the convergence to zero of the pendulum angle, but the cart position converges to zero too slowly. Using pole assignment, do you think it would be possible to only speed up the cart convergence to zero? Explain why yes, or why not.

We've just seen with the above example that changing eigenvalues may allow the convergence of a system to occur at earlier times (i.e. faster). Observe how the controller K2 causes the states and the control output to stabilize at 0 faster than that of K1. Mathematically this checks out, since eigenvalues appear in the solution y(t) as exponential terms, which converge to zero at higher rates depending on the value of the eigenvalue. By setting poles with larger Real magnitudes (but negative Real parts), we can speed up the convergence of this system.

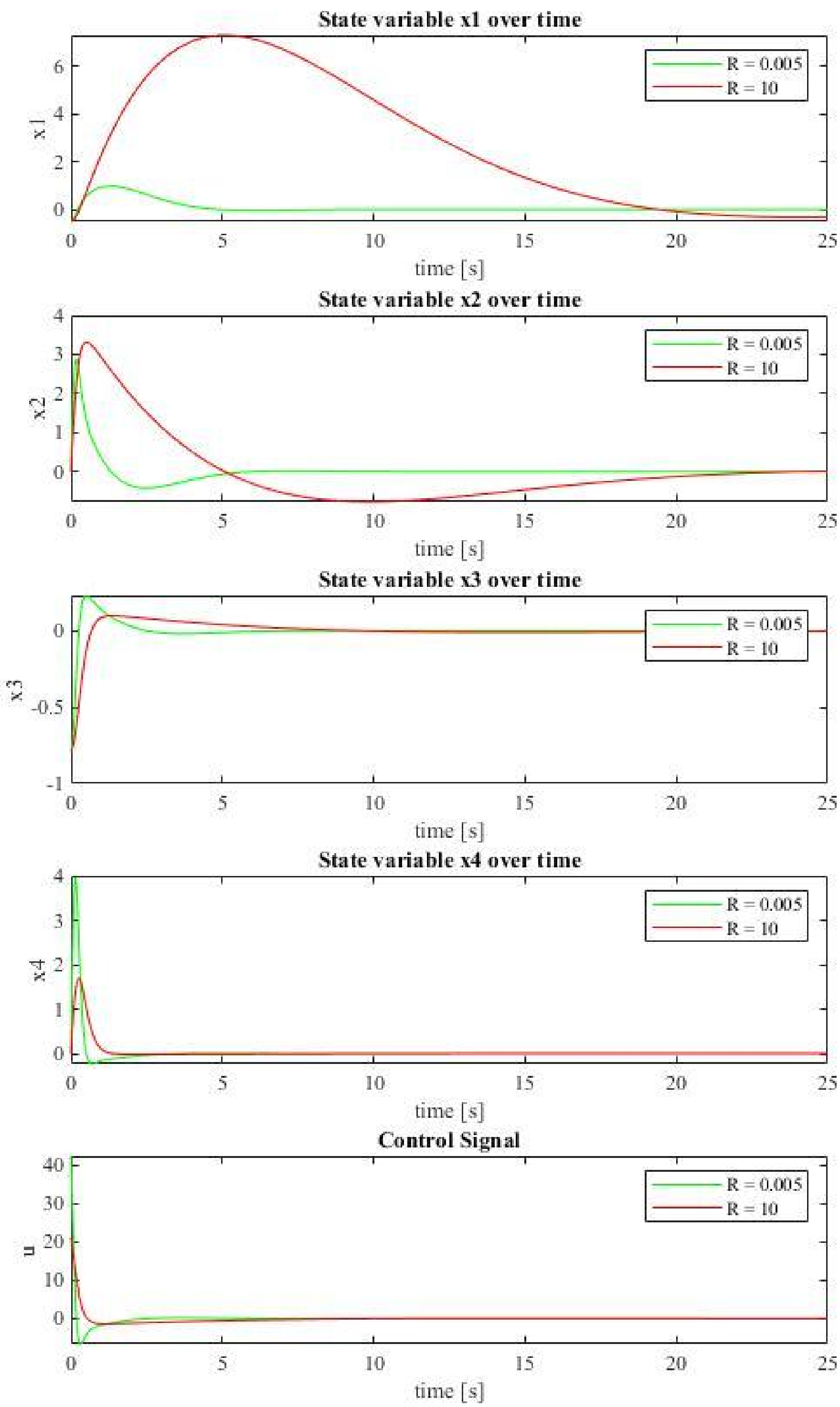
Output 3

Output 3.1 - Control responses when modulating q1




Output 3.2 - Control responses when modulating q2
State variable x1 over time
x1
time [s]
q2 = 1
q2 = 2000
State variable x2 over time
x2
time [s]
State variable x3 over time
x3
time [s]
State variable x4 over time
x4
time [s]
Control Signal
u
time [s]

Output 3.3 - Control responses when modulating R

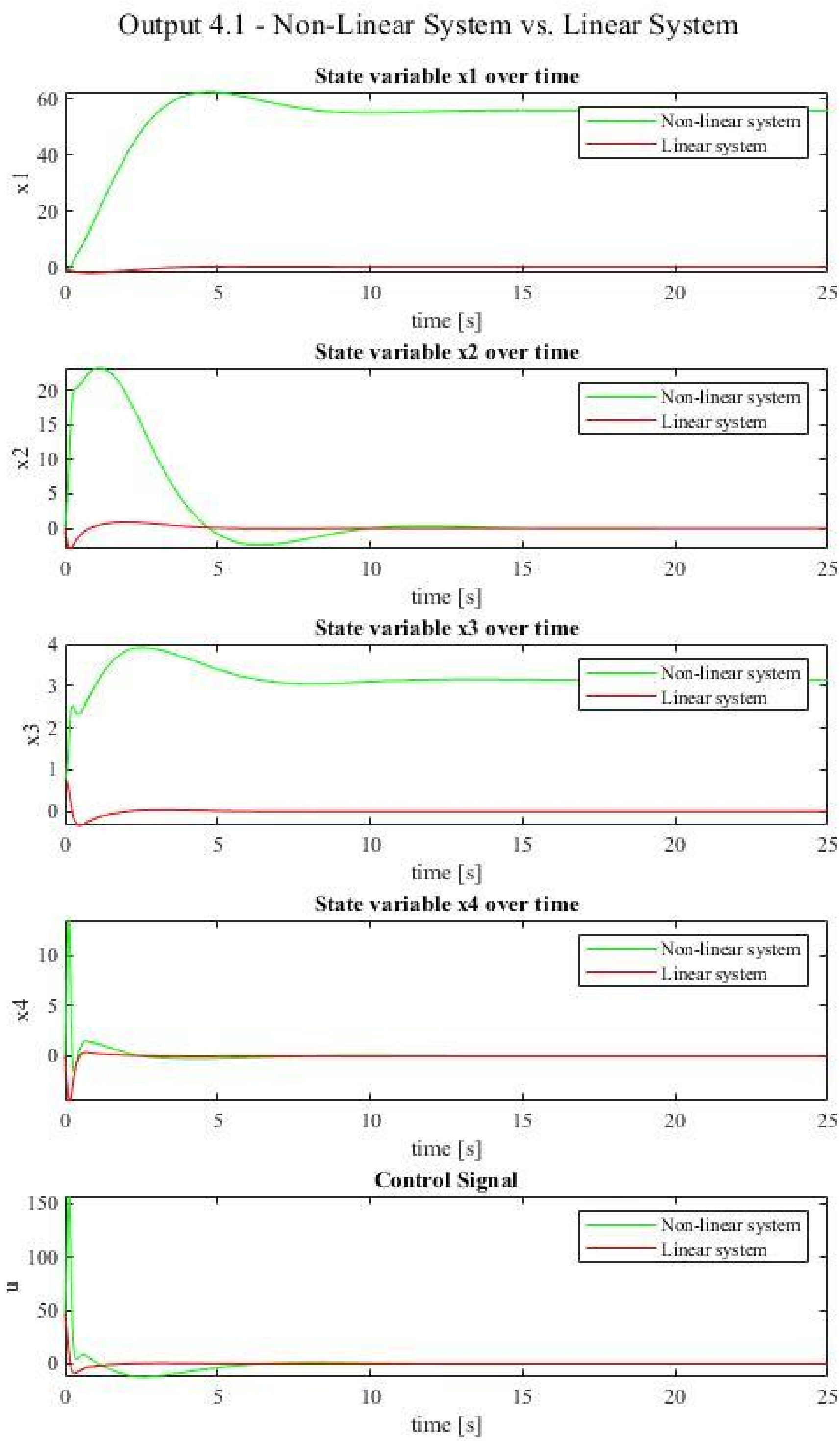


Comment on the results. How does changing R compare to changing q1 or q2? What difference is there between changing q1 and q2?

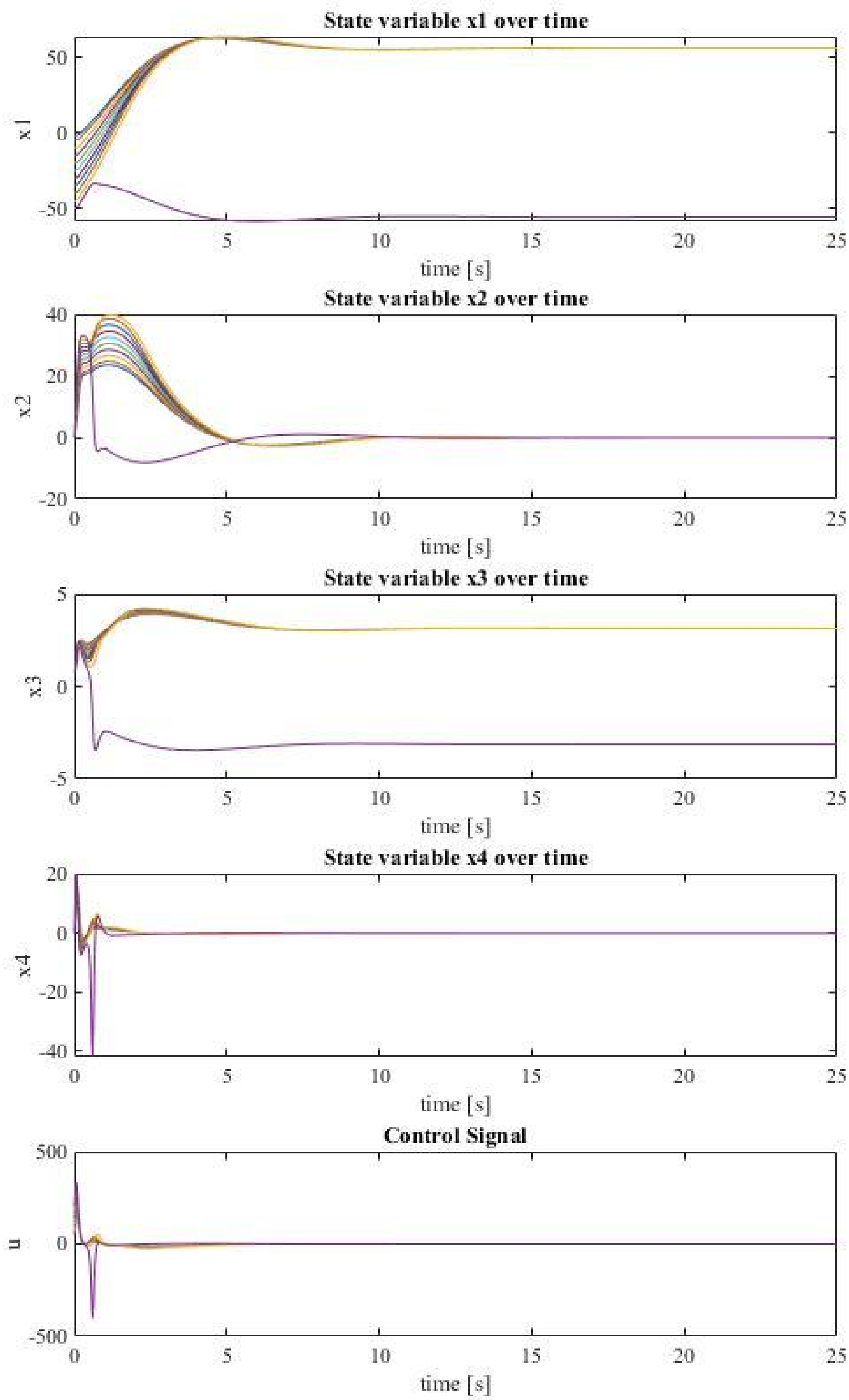
Cost function variable modulated	Affect on x1	Affect on x2	Affect on x3	Affect on x4	Affect on control output, $u = Kx$
q1	When q1 is reduced, this state converges slower. The cart moves further in this scenario.	When q1 is reduced, this state converges slower. The cart has a less drastic acceleration in this scenario.	When q1 is reduced, this state converges slightly faster. The pendulum overshoots equilibrium less in this scenario.	When q1 is reduced, this state converges slightly faster. The pendulum swings slower in this scenario.	When q1 is reduced, the control input stabilizes at 0 faster and does not overshoot.
q2	When q2 is increased, this state takes much longer to converge. The cart does not move any further in this scenario, however.	When q2 is increased, this state converges much faster. The cart slows down in a gradual fashion in this scenario.	When q2 is increased, this state converges faster and with less overshoot. The pendulum overshoots equilibrium in this scenario.	When q2 is increased, this state reaches a much larger value and overshoots a little, but converges faster. The pendulum swings much faster and accelerates much more aggressively, but balances faster in this scenario.	When q2 is increased, the control input is more aggressive (as we see a decent amount of overshoot), but stabilizes at 0 quicker.
R	When R is increased, this state takes much longer to converge and has some overshoot. The cart goes a much further distance in this scenario.	When R is increased, this state converges to 0 much slower, and there is a large amount of overshoot. The cart moves faster and accelerates / decelerates slower in this scenario.	When R is increased, this state converges faster and has less overshoot. The pendulum does not overshoot equilibrium, and balances much quicker in this scenario.	When R is increased, this state does not reach as high a value and does not overshoot, but converges slower. The pendulum swings much faster and accelerates more aggressively, but balances faster in this scenario.	When R is increased, the control system converges faster to equilibrium and has less overshoot.

It would appear that, from the experiments, adjusting q1 changes how aggressively the cart moves and accelerates, adjusting q2 changes how much the pendulum oscillates and how quickly it balances, and adjusting R affects how much u(t) influences the system.

Output 4


Output 4.1 - Non-Linear System vs. Linear System
State variable x1 over time
State variable x2 over time
State variable x3 over time
State variable x4 over time
Control Signal
Non-linear system
Linear system
x1
x2
x3
x4
u
time [s]

Output 4.2 - Non-Linear System with decreasing x1 initial condition



Comment on how the behaviour of the nonlinear system differs from the linear system

The nonlinear system is prone to more uncertainty at earlier times, resulting in a more erratic behaviour from the states and control input in that window. Observe the spikes in x2, x3, x4, and u at the beginning of the plot. Because of this erratic behaviour, the nonlinear system takes longer to stabilize than the linear system.

Additionally, we see two differences in the equilibrium of states x1 and x3. x1 appears to stabilize at y = 58 (approximately): this makes sense, as while the equilibrium point is (0,0,0,0) the cart can reasonably be at any y value when it stabilizes. The state x3 stabilizes at π radians: this too is to be expected. The initial condition of x3 is $\pi/4$ radians, which means at instantaneous time t = 0, gravity will take hold and it is very difficult for the cart to push the pendulum back towards $\theta = 0$. Instead, the cart cannot catch the pendulum and it falls, which explains the sudden spike in x4 as the angular velocity rises and falls with gravity taking hold. The cart attempts to stop this fall, as can be seen in control input u and it's velocity, x2, but eventually it gives up and stabilizes the system at its stable equilibrium (i.e. hanging straight down).

What significant differences appear for initial conditions farther from equilibrium?

As we increase the initial y = x1 condition, we see that x2 must reach higher values (i.e. move faster) to return the cart to zero to achieve the desired pole placement. This in turn causes x4 to overshoot equilibrium by more and more with increasing x1 initial condition, making the pendulum swing with higher angular velocity. That affects x3; it overshoots equilibrium more and more, meaning the pendulum passes equilibrium further and further with higher x1 values. The control input u, of course, is a function of all these state variables, and we can see that the force applied to the cart is increased with every larger x1 position. It's important to note, to achieve the desired eigenvalues that are encoded in K, the convergence of the system does note change despite the initial condition.