ECE410 Lab #1 Report

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Output 1

We establish here that given no input (i.e. u = 0) that the initial condition:

$$x = [\theta \ \frac{d\theta}{dt}]^T$$

Has an effect on the output y(t) of the system. Observe Figures 1 and 2 of case 1, where:

$$x_1^{\ 0} = [0 \sqrt{\frac{g}{l}}]^T$$

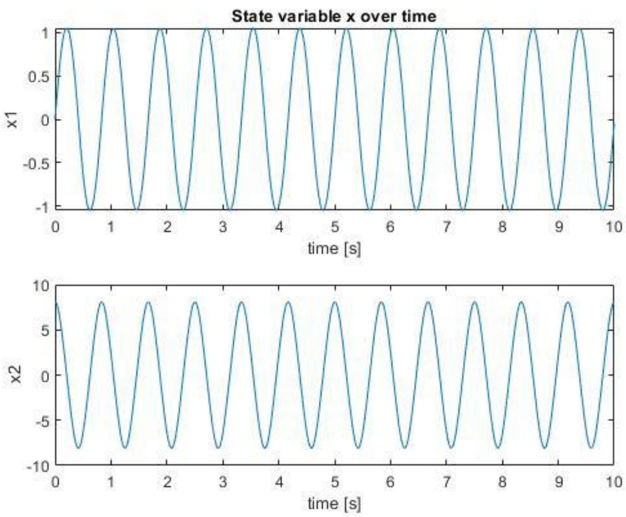

Figure 1: x_1 and x_2 of case 1 initial condition versus time (t)

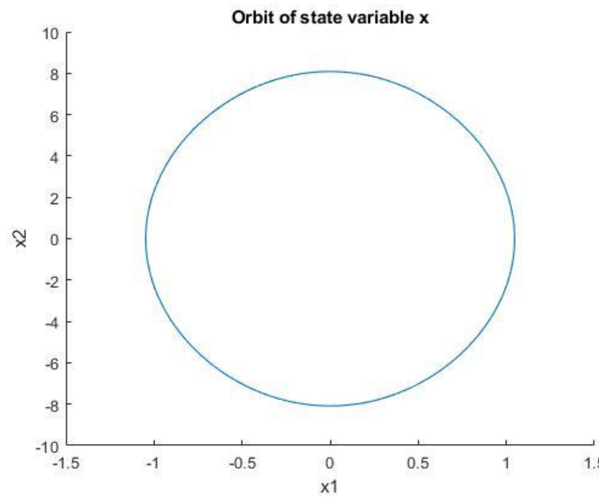

Figure 2: Orbit of x in case 1

Observe Figures 3 and 4 of case 2, where:

$$x_2^{\ 0} = [0 \ 1.99\sqrt{\frac{g}{l}}]^T$$

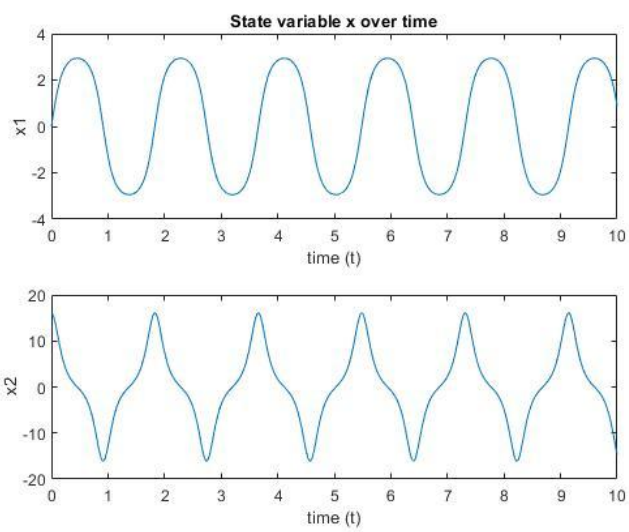

Figure 1: x_1 and x_2 of case 1 initial condition versus time (t)

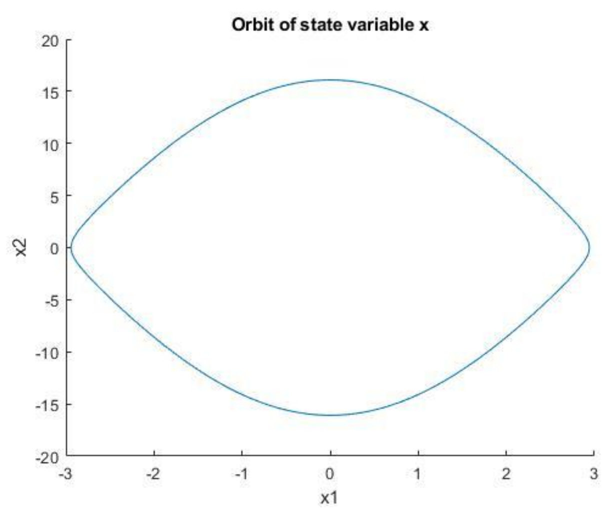

Figure 2: Orbit of x in case 1

The change from x_1^0 to x_2^0 is seemingly trivial: x_2 has been multiplied by a factor of 1.99. This change is a mathematical representation of an increased angular speed $\frac{d\theta}{dt}$ for case 2 at t = 0. This change affects a number of things:

a) **The frequency and shape of x_1:** with a higher angular speed in the pendulum, we see that the peak of the x_2 waveform is higher than the peak of the x_1 waveform (as we might expect). The pendulum is able to travel to a larger angle due to this increased speed. The pendulum of case 2 takes longer to swing due to this, and thus its frequency is lower than that of x_1.

b) **The frequency and shape of x_2:** the initial speed of the pendulum, increased from x_1 to x_2, has an effect on the angular acceleration of the pendulum. In the first case, the angular speed holds a consistent cosine wave, which indicates the acceleration of the pendulum is a negative sine wave. But in case 2, the angular speed is distorted and no longer has a consistent oscillation - indicating its acceleration too is distorted. Additionally, the peak of the waveform is higher due to the pendulum's increased time under the force of gravity (it will reach a higher angular speed).

c) **The orbit:** as we know, the orbit manifests the fact that the solution of the pendulum is periodic. In case 1, we've established that $x_{1max\ case\ 1} < x_{1max\ case\ 2}$, as well as $x_{2max\ case\ 1} < x_{2max\ case\ 2}$ which indicates that the surface area formed by the solution curve is larger in case 2. We also can observe that the solution curve in case 1 is perfectly circular, but in case 2 the solution curve is oblong.

Output 2

As expected, the below output of the lab1.m code produces our linearization for this system at the given $x_0 = [0\ 0]^T$ and $\bar{u} = 0$.

```
>> lab1
A =
[    0, 1]
[ -g/l, 0]
B =
        0
 -1/(l*m)
C =
[ 1, 0]
D =
0
```

Additionally, when input ū = *-m g tan(*θ*)* we see the following:

```
A =
[                 0, 1]
[ -g/(l*cos(theta)), 0]
B =
                 0
 -cos(theta)/(l*m)
```

Which matches our expected A and B matrices.

Output 3

Similarities and differences

Similarity/Difference	**Initial Condition**	**Explanation**
Similarity: Oscillation amplitude	1	Initial condition was closer to just letting the pendulum go thus the innate linear components of the system are more similar to the system itself (oscillation has less effect)
Similarity: Oscillation Frequency	1	Linearization is looking to remove the complexity of a state-space system by equating it to a linear differential equation. This means the system will represent everything in exponentials and thus any harmonics would be captured through Euler's formula (cos(a) + isin(a)) which introduces a more consistent and normalized frequency.
Similarity: Orbital Shape	1	Since the state variable is based upon the velocity (which is likewise based upon the position of the pendulum), the two state variables are closer in similarity due to the lessened effect of the initial condition.
Differences: Oscillation frequency	2	Compliment of (1): See Similarity for Reasoning
Differences: Oscillation amplitude/shape	2	Compliment of (2): See Similarity for Reasoning

Difference: Orbital Shape	2	Compliment of (3): See Similarity for Reasoning

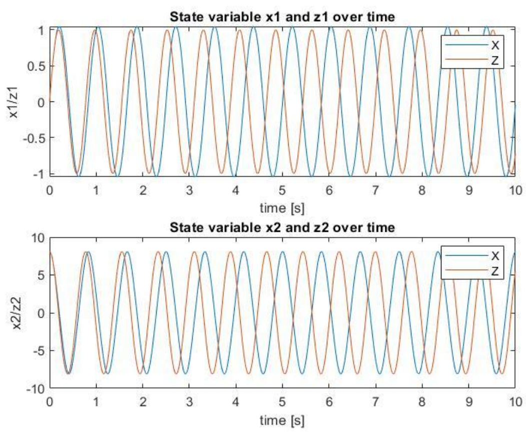

Figure 5. State variable vs time for the first set of initial conditions

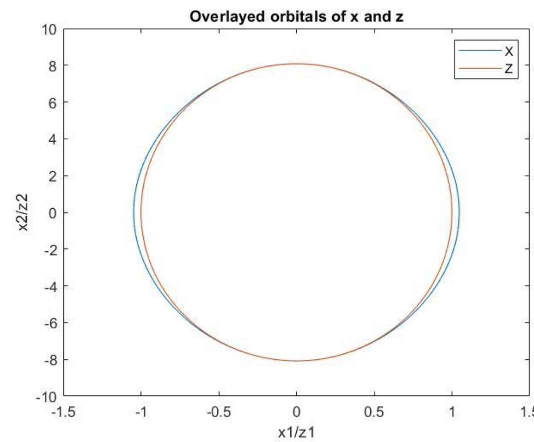

Figure 6. Orbital Diagram for the first set of initial conditions

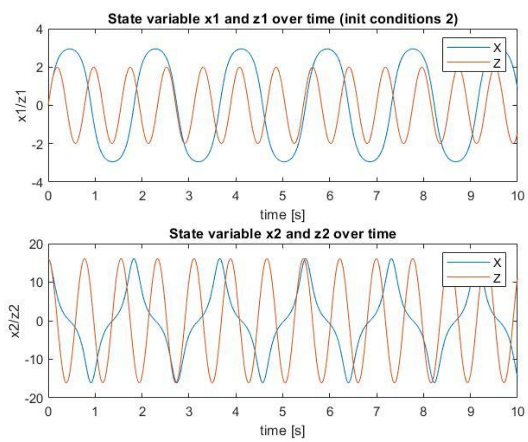

Figure 7. State variable vs time for the first set of initial conditions

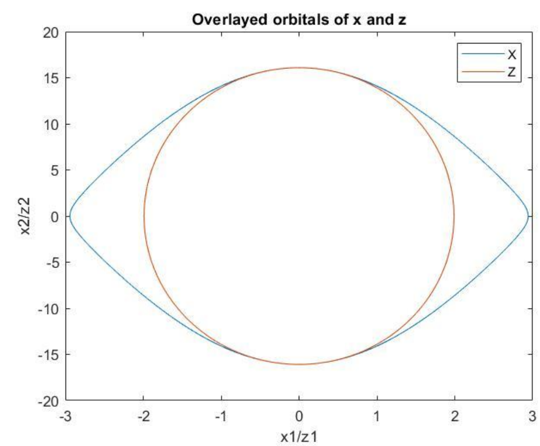

Figure 8. Orbital Diagram for the first set of initial conditions

We can see the full effect of a relatively large initial condition. As the initial conditions for a system are closer to 0, the linearization will be better at approximating the model. In our situation, since one of our state-variable was angular velocity, intuitively the larger that value is the less likely a linearization will be able to approximate that effect.

Output 4

```
Transfer Function of System:
transfer_func =

    -33.333
```

```
 ------------
 (s^2 + 65.4)
```

Further investigation of this system shows us that the pendulum system has equal poles and eigenvectors:

```
Poles of System:
P =
   0.0000 + 8.0870i
   0.0000 - 8.0870i

Eigenvalues of System:
e =
   0.0000 + 8.0870i
   0.0000 - 8.0870i
```

Through the conversion between state-space systems and transfer functions, we learn that the poles of the transfer function is determined by the eigenvalues A. Since the poles of the transfer function are a strict subset of the eigenvectors of A, the fact that they were equal was a good indicator that the conversion was done correctly.

The system linearized at x = $[0\ 0]^T$ has both poles existing along the imaginary axis of the complex plane meaning it for certain is not asymptotically stable, but since the eigenvalues are unique we can assume linearly independent eigenvectors providing consistent algebraic and geometric multiplicity. It is not BIBO stable as the poles are not in the strict left half plane of the complex plane (as they rest on the imaginary axis itself).

Modal decomposition represents the linearized system via a set of exponentials. Since we have imaginary eigenvalues, we can use euler's formula to get a representation of the oscillation of the system (in this case all with the same frequency as well). To scope it down even further, since we have <u>only</u> imaginary eigenvalues, the only information the modal decomposition gives us is on the oscillation of solutions of the linearized system.

Output 5

We've designed a Lead Controller C(s) based on the recommendation in the lab, namely:

$$C(s) \ = \ -\ 30\frac{s+10}{s+1000}$$

To ensure BIBO stability, the pole of C(s) is chosen to be in the strict Right Half Complex Plane. The controller is also designed with the following Bode Plot:

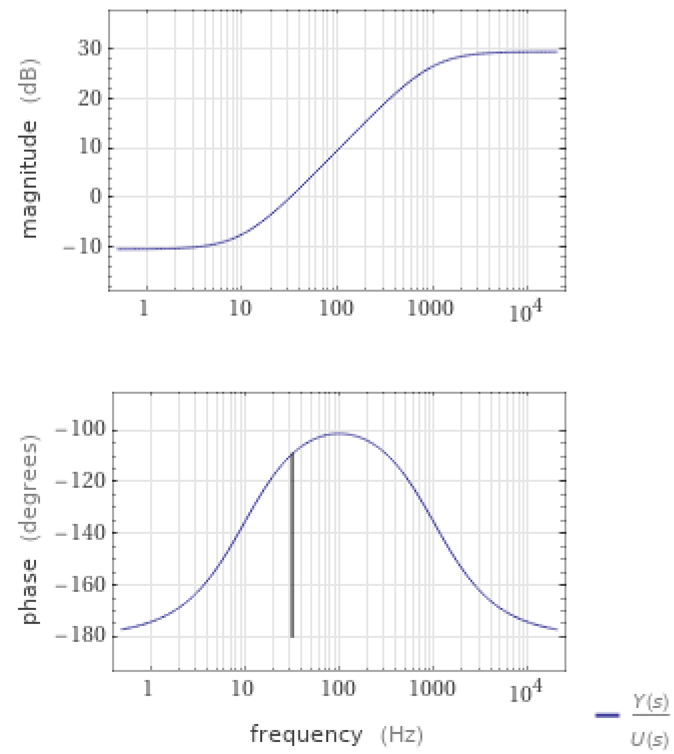

Figure 9: Controller Bode Plots

The zeros of this function are displayed below (from lab1.m code):

```
tf_Zeros =

   1.0e+02 *

  -9.9901 + 0.0000i
  -0.0050 + 0.0867i
  -0.0050 - 0.0867i
```

The first zero is located on the Real axis. The second two are mirrored about the Real axis which is by design. These zeroes are further evidence of the stability of the transfer function.

Choosing any angle between $-\pi$ and $+\pi$ will revert the state x to 0 in any case, but this model will not work if the absolute value of the angle is greater than π.