Math Homework

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Contents

1 Riemann and Trapezoidal sums

Left Riemann Sums

$f(x) = x + 6;\ [1, 5];$ 4 rectangles

x	1	2	3	4	5
y	7	8	9	10	11

$$\triangle x = \frac{|a-b|}{r}$$
$$\triangle x = \frac{|1-5|}{4}$$
$$\triangle x = 1$$
$$A = \sum_{n=a}^{b-1} f(n)\triangle x$$
$$A = \sum_{n=1}^{4} f(n)$$
$$A = 7 + 8 + 9 + 10$$
$$A = 34$$

$f(x) = x + 4;\ [-2, 2];$ 4 rectangles

x	-2	-1	0	1	2
y	2	3	4	5	6

$$\triangle x = \frac{|a-b|}{r}$$
$$\triangle x = \frac{|-2-2|}{4}$$
$$\triangle x = 1$$
$$A = \sum_{n=a}^{b-1} f(n)\triangle x$$
$$A = \sum_{n=-2}^{1} f(n)$$
$$A = 2 + 3 + 4 + 5$$
$$A = 14$$

Right Riemann Sums

$f(x) = -x^2 - 2x + 9;\ [-3, 2];$ 5 rectangles

x	-3	-2	-1	0	1	2
y	6	9	10	9	6	1

$$\triangle x = \frac{|a-b|}{r}$$
$$\triangle x = \frac{|-3-2|}{5}$$
$$\triangle x = 1$$
$$A = \sum_{n=a+1}^{b} f(n)\triangle x$$
$$A = \sum_{n=-2}^{2} f(n)$$
$$A = 9 + 10 + 9 + 6 + 1$$
$$A = 35$$

$f(x) = \frac{2}{x};\ [2, 7];$ 5 rectangles

x	2	3	4	5	6	7
y	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$

$$\triangle x = \frac{|a-b|}{r}$$
$$\triangle x = \frac{|2-7|}{5}$$
$$\triangle x = 1$$
$$A = \sum_{n=a+1}^{b} f(n)\triangle x$$
$$A = \sum_{n=3}^{7} f(n)$$
$$A = \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7}$$
$$A = 2.19$$

Trapezoidal Sums

$y = -\frac{x^2}{2} + x + 5;\ [0,3];$ 5 Trapezoids

x	0	0.6	1.2	1.8	2.4	3
y	5	5.42	5.48	5.18	4.52	3.5

$$\triangle x = \frac{|a-b|}{r}$$
$$\triangle x = \frac{|0-3|}{5}$$
$$\triangle x = 0.6$$
$$A = \sum_{n=1}^{rectangles} \frac{f(n\triangle x) + f((n-1)\triangle x)}{2}\triangle x$$
$$A = \sum_{n=1}^{5} \frac{f(0.6n) + f(0.6(n-1))}{2}0.6$$
$$A = 3.13 + 3.27 + 3.23 + 2.91 + 2.41$$
$$A = 14.95$$

$f(x) = \frac{x^2}{2} + x + 1;\ [-2,1];$ 5 Trapezoids

x	-2	-1.4	-0.8	-0.2	0.4	1
y	1	0.58	0.52	0,82	1,48	2,5

$$\triangle x = \frac{|a-b|}{r}$$
$$\triangle x = \frac{|-2-1|}{5}$$
$$\triangle x = 0.6$$
$$A = \sum_{n=1}^{rectangles} \frac{f(n\triangle x) + f((n-1)\triangle x)}{2}\triangle x$$
$$A = \sum_{n=1}^{5} \frac{f(0.6n) + f(0.6(n-1))}{2}0.6$$
$$A = 0,474 + 0.33 + 0.402 + 0.69 + 1.194$$
$$A = 3.09$$

2 Volume of Solids of Revolutions (Disk Method)

3. Find the volume of the solid of revolution generated by revolving the region bounded by $y = \sqrt{1 - x^2}$ and $y = 0$ about the x-axis.

$$\begin{aligned}
V &= \pi \int_b^a \left[r^2\right] dx \\
&= \pi \int_{-1}^{1} \left[\left(\sqrt{1-x^2}\right)^2\right] dx \\
&= \pi \int_{-1}^{1} \left[1 - x^2\right] dx \\
&= \pi \left[x - \frac{x^3}{3}\right]_{-1}^{1} \\
&= \pi \left(1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right)\right) \\
V &= \frac{4\pi}{3}
\end{aligned}$$

8. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{\sin x}$ and the x-axis ($0 \leq x \leq \pi$), about the x-axis.

$$\begin{aligned}
V &= \pi \int_0^{\pi} (\sqrt{\sin x})^2\, dx \\
&= \pi \int_0^{\pi} \sin x\, dx \\
&= \pi \left[-\cos x\right]_0^{\pi} \\
&= \pi \left(-(-1) + 1\right) \\
V &= 2\pi
\end{aligned}$$

11. The region bounded by the parabola $y = 4x - x^2$ and the x-axis is revolved about the x-axis. Find the volume of the solid.

$$\begin{aligned}
\{a, b\} &= x \quad \text{where } 4x - x^2 = 0 \\
a = 0&; \quad b = 4 \\
V &= \pi \int_0^4 (4x - x^2)^2\, dx \\
&= \pi \int_0^4 (16x^2 - 8x^3 + x^4)\, dx \\
&= \pi \left[\frac{16}{3}x^3 - 2x^4 + \frac{1}{5}x^5\right]_0^4 \\
&= \pi \left(\left(\frac{1024}{3} - 512 + \frac{1024}{5}\right) - 0\right) \\
V &= \frac{512\pi}{15}
\end{aligned}$$

13a. Find the volume of the solid generated when the area bounded by the curves $y = x^3 - x + 1$, $x = -1$, and $x = 1$ is revolved around the x-axis.

$$\begin{aligned}
V &= \pi \int_{-1}^{1} (x^3 - x + 1)^2 \, dx \\
&= \pi \int_{-1}^{1} (x^6 - 2x^4 + 2x^3 + x^2 - 2x + 1) \, dx \\
&= \pi \left[\frac{1}{7}x^7 - \frac{2}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3 - x^2 + x \right]_{-1}^{1} \\
&= \pi \left(\left(\frac{1}{7} - \frac{2}{5} + \frac{1}{2} + \frac{1}{3} \right) - \left(-\frac{1}{7} + \frac{2}{5} + \frac{1}{2} - \frac{1}{3} - 2 \right) \right) \\
V &= \frac{226\pi}{105} \text{ source: deepseek I aint doing them fractions :b}
\end{aligned}$$

13b. Find the volume of the solid generated when the area bounded by the curve $y = x - x^2$ and the x-axis is revolved around the x-axis.

$$\begin{aligned}
V &= \pi \int_{0}^{1} (x - x^2)^2 \, dx \\
&= \pi \int_{0}^{1} (x^2 - 2x^3 + x^4) \, dx \\
&= \pi \left[\frac{1}{3}x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5 \right]_{0}^{1} \\
&= \pi \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) \\
V &= \frac{\pi}{30}
\end{aligned}$$

3 Volume of Solids of Revolutions (Washer Method)

Find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

$f(x) = x^2 - 3, \quad g(x) = \sqrt{x} - 3; \quad$ axis: $y = 2$

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 3 &= \sqrt{x} - 3 \\ x^2 &= \sqrt{x} \\ x^4 &= x \\ x\left(x^3 - 1\right) &= 0 \\ x = 0 &\qquad x^3 = 1 \\ a = 0 &\qquad b = 1 \end{aligned}$$

$$\begin{aligned} V &= \pi \int_a^b \left|(f(x) - y)^2 - (g(x) - y)^2\right| dx \\ &= \pi \int_0^1 \left|(x^2 - 5)^2 - (\sqrt{x} - 5)^2\right| dx \\ &= \pi \int_0^1 \left|x^4 - 10x^2 + 25 - (x - 10\sqrt{x} + 25)\right| dx \\ &= \pi \left|\frac{x^5}{5} - \frac{10x^3}{3} - \frac{x^2}{2} + \frac{20x^{\frac{3}{2}}}{3}\right|_0^1 \\ &= \pi \left|\frac{1}{5} - \frac{10}{3} - \frac{1}{2} + \frac{20}{3}\right| \\ V &= \frac{91\pi}{30} \end{aligned}$$

$f(x) = x^2 - 3, \quad g(x) = \sqrt{x} - 3; \quad$ axis: $y = 1$

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 3 &= \sqrt{x} - 3 \\ x^2 &= \sqrt{x} \\ x^4 &= x \\ x\left(x^3 - 1\right) &= 0 \\ x = 0 &\qquad x^3 = 1 \\ a = 0 &\qquad b = 1 \end{aligned}$$

$$\begin{aligned} V &= \pi \int_a^b \left|(f(x) - y)^2 - (g(x) - y)^2\right| dx \\ &= \pi \int_0^1 \left|(x^2 - 4)^2 - (\sqrt{x} - 4)^2\right| dx \\ &= \pi \int_0^1 \left|x^4 - 8x^2 + 16 - (x - 8\sqrt{x} + 16)\right| dx \\ &= \pi \left|\frac{x^5}{5} - \frac{8x^3}{3} - \frac{x^2}{2} + \frac{16x^{\frac{3}{2}}}{3}\right|_0^1 \\ &= \pi \left|\frac{1}{5} - \frac{8}{3} - \frac{1}{2} + \frac{16}{3}\right| \\ V &= \frac{71\pi}{30} \end{aligned}$$

$f(x) = x^2 + 4, \quad g(x) = 2; \; a = 0, \; b = 1;$ axis: $y = -1$

$$\begin{aligned} V &= \pi \int_a^b \left|(f(x) - y)^2 - (g(x) - y)^2\right| dx \\ &= \pi \int_0^1 \left|(x^2 + 5)^2 - 3^2\right| dx \\ &= \pi \int_0^1 \left|x^4 + 10x^2 + 16\right| dx \\ &= \pi \left|\frac{x^5}{5} + \frac{10x^3}{3} + 16x\right|_0^1 \\ &= \pi \left|\frac{1}{5} + \frac{10}{3} + 16\right| \\ V &= \frac{293\pi}{15} \end{aligned}$$

$f(x) = x^2 + 3, \quad g(x) = 3; \; a = 0, \; b = 2;$ axis: $y = 1$

$$\begin{aligned} V &= \pi \int_a^b \left|(f(x) - y)^2 - (g(x) - y)^2\right| dx \\ &= \pi \int_0^2 \left|(x^2 + 2)^2 - 2^2\right| dx \\ &= \pi \int_0^2 \left|x^4 + 4x^2\right| dx \\ &= \pi \left|\frac{x^5}{5} + \frac{4x^3}{3}\right|_0^2 \\ &= \pi \left|\frac{32}{5} + \frac{32}{3}\right| \\ V &= \frac{256\pi}{15} \end{aligned}$$

4 Separation of Variables

General Solution

1.

$$\frac{dy}{dx} = \frac{x^3}{y^2}$$
$$y^2\,dy = x^3\,dx$$
$$\int y^2\,dy = \int x^3\,dx$$
$$\frac{y^3}{3} = \frac{x^4}{4} + C$$
$$y^3 = \frac{3x^4}{4} + 3C$$
$$y = \sqrt[3]{\frac{3x^4}{4} + 3C}$$

2.

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$
$$\sec^2 y\,dy = dx$$
$$\int \sec^2 y\,dy = \int dx$$
$$\tan y = x + C$$
$$y = \tan^{-1}(x + C)$$

3.

$$\frac{dy}{dx} = 3e^{x-y}$$
$$e^y\,dy = 3e^x\,dx$$
$$\int e^y\,dy = \int 3e^x\,dx$$
$$e^y = 3e^x + C$$
$$y = \ln\left(3e^x + C\right)$$

4.

$$\frac{dy}{dx} = \frac{2x}{e^{2y}}$$
$$e^{2y}\,dy = 2x\,dx$$
$$\int e^{2y}\,dy = \int 2x\,dx$$
$$\frac{e^{2y}}{2} = x^2 + C$$
$$e^{2y} = 2\left(x^2 + C\right)$$
$$2y = \ln 2 + \ln\left(x^2 + C\right)$$
$$y = \ln\left(\sqrt{x^2 + C}\right) + \frac{\ln 2}{2}$$

Particular Solution

$$\frac{dy}{dx} = \frac{2x}{y^2}\,, \quad y(2) = \sqrt[3]{13}$$

$$y^2\,dy = 2x\,dx$$
$$\int y^2\,dy = \int 2x\,dx$$
$$\frac{y^3}{3} = x^2 + C$$
$$y^3 = 3x^2 + 3C$$
$$y = \sqrt[3]{3x^2 + 3C}$$

$$\sqrt[3]{3(2)^2 + 3C} = \sqrt[3]{13}$$
$$3(2)^2 + 3C = \left(\sqrt[3]{13}\right)^3$$
$$12 + 3C = 13$$
$$C = \frac{1}{3}$$

$$y = \sqrt[3]{3x^2 + 1}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}\,, \quad y(3) = 0$$

$$\sec^2 y\,dy = dx$$
$$\int \sec^2 y\,dy = \int dx$$
$$\tan y = x + C$$
$$y = \tan^{-1}(x + C)$$

$$\tan^{-1}(3 + C) = 0$$
$$3 + C = \tan(0)$$
$$3 + C = 0$$
$$C = -3$$

$$y = \tan^{-1}(x - 3)$$

$$\frac{dy}{dx} = 2e^{x-y}\,, \quad y(-3) = \ln\frac{3e^3 + 2}{e^3}$$

$$e^y\,dy = 2e^x\,dx$$
$$\int e^y\,dy = \int 2e^x\,dx$$
$$e^y = 2e^x + C$$
$$y = \ln(2e^x + C)$$

$$\ln(2e^{-3} + C) = \ln\frac{3e^3 + 2}{e^3}$$
$$2e^{-3} + C = \frac{3e^3 + 2}{e^3}$$
$$C = \frac{3e^3 + 2}{e^3} - 2e^{-3}$$
$$C = 3$$

$$y = \ln(2e^x + 3)$$

$$\frac{dy}{dx} = \frac{e^x}{y^2}\,, \quad y(-1) = \frac{\sqrt[3]{e^3 + 3e^2}}{e}$$

$$y^2\,dy = e^x\,dx$$
$$\int y^2\,dy = \int e^x\,dx$$
$$\frac{y^3}{3} = e^x + C$$
$$y^3 = 3e^x + 3C$$
$$y = \sqrt[3]{3e^x + 3C}$$

$$\sqrt[3]{3e{-}1 + 3C} = \frac{\sqrt[3]{e^3 + 3e^2}}{e}$$
$$3e{-}1 + 3C = \frac{e^3 + 3e^2}{e^3}$$
$$3C = \frac{e^3 + 3e^2 - 3e^2}{e^3}$$
$$C = \frac{1}{3}$$

$$y = \sqrt[3]{3(e^x + \frac{1}{3})}$$

$$\frac{dy}{dx} = -\frac{1}{\sin y}\,, \quad y(3) = \frac{\pi}{2}$$

$$\begin{aligned}
\sin y\, dy &= -1\, dx \\
\int \sin y\, dy &= \int -1\, dx \\
-\cos y &= -x + C \\
y &= \cos^{-1}(x - C)
\end{aligned}
\qquad\qquad
\begin{aligned}
\cos^{-1}(3 - C) &= \frac{\pi}{2} \\
3 - C &= \cos\frac{\pi}{2} \\
3 - C &= 0 \\
C &= 3
\end{aligned}$$

$$y = \cos^{-1}(x - 3)$$

$$\frac{dy}{dx} = \frac{3x^2}{e^{2y}}\,, \quad y(1) = 0$$

$$\begin{aligned}
e^{2y}\, dy &= 3x^2\, dx \\
\int e^{2y}\, dy &= \int 3x^2\, dx \\
\frac{e^{2y}}{2} &= x^3 + C \\
e^{2y} &= 2x^3 + 2C \\
2y &= \ln\left(2\left(x^2 + C\right)\right) \\
y &= \frac{\ln\left(2\left(x^2 + C\right)\right)}{2}
\end{aligned}
\qquad\qquad
\begin{aligned}
\frac{\ln\left(2\left(1^2 + C\right)\right)}{2} &= 0 \\
2\left(1^2 + C\right) &= 1 \\
1 + C &= \frac{1}{2} \\
C &= -\frac{1}{2}
\end{aligned}$$

$$y = \ln\left(\sqrt{x^2 - \frac{1}{2}}\right) + \ln\sqrt{2}$$

$$\frac{dy}{dx} = \frac{2x}{e^{2y}}\,, \quad y(2) = \frac{\ln 5}{2}$$

$$\begin{aligned}
y^2\, dy &= e^x\, dx \\
\int y^2\, dy &= \int e^x\, dx \\
\frac{y^3}{3} &= e^x + C \\
y^3 &= 3e^x + 3C \\
y &= \sqrt[3]{3e^x + 3C}
\end{aligned}
\qquad\qquad
\begin{aligned}
\sqrt[3]{3e{-1} + 3C} &= \frac{\sqrt[3]{e^3 + 3e^2}}{e} \\
3e{-1} + 3C &= \frac{e^3 + 3e^2}{e^3} \\
3C &= \frac{e^3 + 3e^2 - 3e^2}{e^3} \\
C &= 1
\end{aligned}$$

$$y = \sqrt[3]{3(e^x + 1)}$$

$$\frac{dy}{dx} = \frac{1 + x^2}{y^2}\,, \quad y(-1) = -\sqrt[3]{4}$$

$$\begin{aligned}
y^2\, dy &= e^x\, dx \\
\int y^2\, dy &= \int e^x\, dx \\
\frac{y^3}{3} &= e^x + C \\
y^3 &= 3e^x + 3C \\
y &= \sqrt[3]{3e^x + 3C}
\end{aligned}
\qquad\qquad
\begin{aligned}
\sqrt[3]{3e{-1} + 3C} &= \frac{\sqrt[3]{e^3 + 3e^2}}{e} \\
3e{-1} + 3C &= \frac{e^3 + 3e^2}{e^3} \\
3C &= \frac{e^3 + 3e^2 - 3e^2}{e^3} \\
C &= 1
\end{aligned}$$

$$y = \sqrt[3]{3(e^x + 1)}$$