# Math Homework

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### Semester 2 2025

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## 1 Riemann and Trapezoidal sums

#### Left Riemann Sums

$$f(x) = x + 6; [1, 5]; 4 \text{ rectangles}$$

$$\frac{x \mid 1 \mid 2 \mid 3 \mid 4 \mid 5}{y \mid 7 \mid 8 \mid 9 \mid 10 \mid 11}$$

$$\triangle x = \frac{|a - b|}{r}$$

$$\triangle x = \frac{|1 - 5|}{4}$$

$$\triangle x = 1$$

$$A = \sum_{n=a}^{b-1} f(n) \triangle x$$

$$A = \sum_{n=1}^{4} f(n)$$

$$A = 7 + 8 + 9 + 10$$

A = 34

$$f(x) = x + 4; [-2, 2]; 4 \text{ rectangles}$$

$$\frac{x \mid -2 \mid -1 \mid 0 \mid 1 \mid 2}{y \mid 2 \mid 3 \mid 4 \mid 5 \mid 6}$$

$$\triangle x = \frac{|a - b|}{r}$$

$$\triangle x = \frac{|-2 - 2|}{4}$$

$$\triangle x = 1$$

$$A = \sum_{n=a}^{b-1} f(n) \triangle x$$

$$A = \sum_{n=-2}^{1} f(n)$$

$$A = 2 + 3 + 4 + 5$$

$$A = 14$$

## Right Riemann Sums

$$f(x) = -x^{2} - 2x + 9; \ [-3, 2]; 5 \text{ rectangles}$$

$$\frac{x \mid -3 \mid -2 \mid -1 \mid 0 \mid 1 \mid 2}{y \mid 6 \mid 9 \mid 10 \mid 9 \mid 6 \mid 1}$$

$$\triangle x = \frac{|a - b|}{r}$$

$$\triangle x = \frac{|-3 - 2|}{5}$$

$$\triangle x = 1$$

$$A = \sum_{n=a+1}^{b} f(n) \triangle x$$

$$A = \sum_{n=-2}^{2} f(n)$$

$$A = 9 + 10 + 9 + 6 + 1$$

$$A = 35$$

$$f(x) = \frac{2}{x}; [2,7]; 5 \text{ rectangles}$$

$$\frac{x \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7}{y \mid 1 \mid \frac{2}{3} \mid \frac{1}{2} \mid \frac{2}{5} \mid \frac{1}{3} \mid \frac{2}{7}}$$

$$\triangle x = \frac{|a - b|}{r}$$

$$\triangle x = \frac{|2 - 7|}{5}$$

$$\triangle x = 1$$

$$A = \sum_{n=a+1}^{b} f(n) \triangle x$$

$$A = \sum_{n=3}^{7} f(n)$$

$$A = \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7}$$

$$A = 2.19$$

#### Trapezoidal Sums

$$y = -\frac{x^2}{2} + x + 5; [0, 3]; 5 \text{ Trapezoids}$$

$$\frac{x \mid 0 \mid 0.6 \mid 1.2 \mid 1.8 \mid 2.4 \mid 3}{y \mid 5 \mid 5.42 \mid 5.48 \mid 5.18 \mid 4.52 \mid 3.5}$$

$$\triangle x = \frac{|a - b|}{r}$$

$$\triangle x = \frac{|0 - 3|}{5}$$

$$\triangle x = 0.6$$

$$A = \sum_{n=1}^{rectangles} \frac{f(n\triangle x) + f((n - 1)\triangle x)}{2} \triangle x$$

$$A = \sum_{n=1}^{5} \frac{f(0.6n) + f(0.6(n - 1))}{2} 0.6$$

$$A = 3.13 + 3.27 + 3.23 + 2.91 + 2.41$$

$$A = 14.95$$

$$f(x) = \frac{x^2}{2} + x + 1; \ [-2, 1]; 5 \text{ Trapezoids}$$

$$\frac{x \mid -2 \mid -1.4 \mid -0.8 \mid -0.2 \mid 0.4 \mid 1}{y \mid 1 \mid 0.58 \mid 0.52 \mid 0.82 \mid 1,48 \mid 2,5}$$

$$\triangle x = \frac{|a - b|}{r}$$

$$\triangle x = \frac{|-2 - 1|}{5}$$

$$\triangle x = 0.6$$

$$A = \sum_{n=1}^{rectangles} \frac{f(n\triangle x) + f((n-1)\triangle x)}{2} \triangle x$$

$$A = \sum_{n=1}^{5} \frac{f(0.6n) + f(0.6(n-1))}{2} 0.6$$

$$A = 0,474 + 0.33 + 0.402 + 0.69 + 1.194$$

$$A = 3.09$$

## 2 Volume of Solids of Revolutions (Disk Method)

3. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = \sqrt{1 - x^2}$  and y = 0 about the x-axis.

$$V = \pi \int_{b}^{a} \left[ r^{2} \right] dx$$

$$= \pi \int_{-1}^{1} \left[ \left( \sqrt{1 - x^{2}} \right)^{2} \right] dx$$

$$= \pi \int_{-1}^{1} \left[ 1 - x^{2} \right] dx$$

$$= \pi \left[ x - \frac{x^{3}}{3} \right]_{-1}^{1}$$

$$= \pi \left( 1 - \frac{1}{3} - \left( -1 - \frac{-1}{3} \right) \right)$$

$$V = \frac{4\pi}{3}$$

8. Find the volume of the solid formed by revolving the region bounded by the graphs of  $y = \sqrt{\sin x}$  and the x-axis  $(0 \le x \le \pi)$ , about the x-axis.

$$V = \pi \int_0^{\pi} (\sqrt{\sin x})^2 dx$$
$$= \pi \int_0^{\pi} \sin x dx$$
$$= \pi \left[ -\cos x \right]_0^{\pi}$$
$$= \pi \left( -(-1) + 1 \right)$$
$$V = 2\pi$$

11. The region bounded by the parabola  $y = 4x - x^2$  and the x-axis is revolved about the x-axis. Find the volume of the solid.

$$\{a,b\} = x \text{ where } 4x - x^2 = 0$$

$$a = 0; \quad b = 4$$

$$V = \pi \int_0^4 (4x - x^2)^2 dx$$

$$= \pi \int_0^4 (16x^2 - 8x^3 + x^4) dx$$

$$= \pi \left[ \frac{16}{3} x^3 - 2x^4 + \frac{1}{5} x^5 \right]_0^4$$

$$= \pi \left( \left( \frac{1024}{3} - 512 + \frac{1024}{5} \right) - 0 \right)$$

$$V = \frac{512\pi}{15}$$

13a. Find the volume of the solid generated when the area bounded by the curves  $y = x^3 - x + 1$ , x = -1, and x = 1 is revolved around the x-axis.

$$\begin{split} V &= \pi \int_{-1}^{1} (x^3 - x + 1)^2 \, dx \\ &= \pi \int_{-1}^{1} (x^6 - 2x^4 + 2x^3 + x^2 - 2x + 1) \, dx \\ &= \pi \left[ \frac{1}{7} x^7 - \frac{2}{5} x^5 + \frac{1}{2} x^4 + \frac{1}{3} x^3 - x^2 + x \right]_{-1}^{1} \\ &= \pi \left( \left( \frac{1}{7} - \frac{2}{5} + \frac{1}{2} + \frac{1}{3} \right) - \left( -\frac{1}{7} + \frac{2}{5} + \frac{1}{2} - \frac{1}{3} - 2 \right) \right) \\ V &= \frac{226\pi}{105} \text{ source: deepseek I aint doing them fractions :b} \end{split}$$

13b. Find the volume of the solid generated when the area bounded by the curve  $y = x - x^2$  and the x-axis is revolved around the x-axis.

$$V = \pi \int_0^1 (x - x^2)^2 dx$$

$$= \pi \int_0^1 (x^2 - 2x^3 + x^4) dx$$

$$= \pi \left[ \frac{1}{3} x^3 - \frac{1}{2} x^4 + \frac{1}{5} x^5 \right]_0^1$$

$$= \pi \left( \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right)$$

$$V = \frac{\pi}{30}$$

#### Volume of Solids of Revolutions (Washer Method) 3

Find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

$$f(x) = x^2 - 3, \quad g(x) = \sqrt{x} - 3; \quad \text{axis: } y = 2$$

$$f(x) = g(x)$$

$$x^2 - 3 = \sqrt{x} - 3$$

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x(x^3 - 1) = 0$$

$$x = 0 \quad x^3 = 1$$

$$a = 0 \quad b = 1$$

$$V = \pi \int_a^b \left| (f(x) - y)^2 - (g(x) - y)^2 \right| dx$$

$$= \pi \int_0^1 \left| (x^2 - 5)^2 - (\sqrt{x} - 5)^2 \right| dx$$

$$= \pi \left| \frac{x^5}{5} - \frac{10x^3}{3} - \frac{x^2}{2} + \frac{20x^3}{3} \right|_0^1$$

$$= \pi \left| \frac{1}{5} - \frac{10}{3} - \frac{1}{2} + \frac{20}{3} \right|$$

$$V = \frac{91\pi}{30}$$

$$f(x) = x^2 - 3, \quad g(x) = \sqrt{x} - 3;$$

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$$f(x) = x^{2} - 3, \quad g(x) = \sqrt{x} - 3; \quad \text{axis: } y = 1$$

$$f(x) = g(x)$$

$$x^{2} - 3 = \sqrt{x} - 3$$

$$x^{2} = \sqrt{x}$$

$$x^{4} = x$$

$$x(x^{3} - 1) = 0$$

$$x = 0 \quad x^{3} = 1$$

$$a = 0 \quad b = 1$$

$$V = \pi \int_{a}^{b} \left| (f(x) - y)^{2} - (g(x) - y)^{2} \right| dx$$

$$= \pi \int_{0}^{1} \left| (x^{2} - 4)^{2} - (\sqrt{x} - 4)^{2} \right| dx$$

$$= \pi \int_{0}^{1} \left| x^{4} - 8x^{2} + 16 - (x - 8\sqrt{x} + 16) \right| dx$$

$$= \pi \left| \frac{x^{5}}{5} - \frac{8x^{3}}{3} - \frac{x^{2}}{2} + \frac{16x^{\frac{3}{2}}}{3} \right|_{0}^{1}$$

$$= \pi \left| \frac{1}{5} - \frac{8}{3} - \frac{1}{2} + \frac{16}{3} \right|$$

$$V = \frac{71\pi}{30}$$

$$f(x) = x^{2} + 4, \quad g(x) = 2; \ a = 0, \ b = 1;$$

$$\text{axis: } y = -1$$

$$V = \pi \int_{a}^{b} \left| (f(x) - y)^{2} - (g(x) - y)^{2} \right| dx$$

$$= \pi \int_{0}^{1} \left| (x^{2} + 5)^{2} - 3^{2} \right| dx$$

$$= \pi \int_{0}^{1} \left| x^{4} + 10x^{2} + 16 \right| dx$$

$$= \pi \left| \frac{x^{5}}{5} + \frac{10x^{3}}{3} + 16x \right|_{0}^{1}$$

$$= \pi \left| \frac{1}{5} + \frac{10}{3} + 16 \right|$$

$$V = \frac{293\pi}{15}$$

$$f(x) = x^{2} + 3, \quad g(x) = 3; \ a = 0, \ b = 2;$$

$$axis: y = 1$$

$$V = \pi \int_{a}^{b} \left| (f(x) - y)^{2} - (g(x) - y)^{2} \right| dx$$

$$= \pi \int_{0}^{2} \left| (x^{2} + 2)^{2} - 2^{2} \right| dx$$

$$= \pi \int_{0}^{2} \left| x^{4} + 4x^{2} \right| dx$$

$$= \pi \left| \frac{x^{5}}{5} + \frac{4x^{3}}{3} \right|_{0}^{2}$$

$$= \pi \left| \frac{32}{5} + \frac{32}{3} \right|$$

$$V = \frac{256\pi}{15}$$

### 4 Separation of Variables

#### General Solution

1. 2. 3. 4. 
$$\frac{dy}{dx} = \frac{x^3}{y^2} \qquad \frac{dy}{dx} = \frac{1}{\sec^2 y} \qquad \frac{dy}{dx} = 3e^{x-y} \qquad \frac{dy}{dx} = \frac{2x}{e^{2y}}$$

$$y^2 dy = x^3 dx \qquad \sec^2 y dy = dx \qquad e^y dy = 3e^x dx \qquad e^{2y} dy = 2x dx$$

$$\int y^2 dy = \int x^3 dx \qquad \int \sec^2 y dy = \int dx \qquad \int e^y dy = \int 3e^x dx \qquad \int e^{2y} dy = \int 2x dx$$

$$\frac{y^3}{3} = \frac{x^4}{4} + C \qquad \tan y = x + C \qquad e^y = 3e^x + C \qquad \frac{e^{2y}}{2} = x^2 + C$$

$$y^3 = \frac{3x^4}{4} + 3C \qquad y = \tan^{-1}(x + C) \qquad y = \ln(3e^x + C) \qquad e^{2y} = 2\left(x^2 + C\right)$$

$$2y = \ln 2 + \ln\left(x^2 + C\right)$$

$$y = \ln\left(\sqrt{x^2 + C}\right) + \frac{\ln 2}{2}$$

#### Particular Solution

$$\frac{dy}{dx} = \frac{2x}{y^2}, \quad y(2) = \sqrt[3]{13} \qquad \frac{dy}{dx} = 2e^{x-y}, \quad y(-3) = \ln \frac{3e^3 + 2}{e^3}$$

$$y^2 \, dy = 2x \, dx \qquad \sqrt[3]{3(2)^2 + 3C} = \sqrt[3]{13} \qquad e^y \, dy = 2e^x \, dx \qquad \ln(2e^{-3} + C) = \ln \frac{3e^3 + 2}{e^3}$$

$$\int y^2 \, dy = \int 2x \, dx \qquad 3(2)^2 + 3C = \left(\sqrt[3]{13}\right)^3 \qquad \int e^y \, dy = \int 2e^x \, dx \qquad 2e^{-3} + C = \frac{3e^3 + 2}{e^3}$$

$$\frac{y^3}{3} = x^2 + C \qquad 12 + 3C = 13 \qquad e^y = 2e^x + C \qquad C = \frac{3e^3 + 2}{e^3} - 2e^{-3}$$

$$y = \sqrt[3]{3x^2 + 3C} \qquad y = \ln(2e^x + C) \qquad C = 3$$

$$y = \sqrt[3]{3x^2 + 3C} \qquad y = \ln(2e^x + C) \qquad C = 3$$

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$$y = \ln(2e^x + 3) \qquad y = \ln(2e^x + 3)$$

$$y = \ln(2e^x + 4) \qquad 3e^{-1} + 3C = \frac{\sqrt[3]{4} + 3e^2}{e^3}$$

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$$y$$