

Math Homework

José Porcar

Semester 2 2025

Contents

1	Riemann and Trapezoidal sums	2
2	Volume of Solids of Revolutions (Disk Method)	4
3	Volume of Solids of Revolutions (Washer Method)	6
4	Separation of Variables	7

1 Riemann and Trapezoidal sums

Left Riemann Sums

$f(x) = x + 6$; $[1, 5]$; 4 rectangles

x	1	2	3	4	5
y	7	8	9	10	11

$$\Delta x = \frac{|a - b|}{r}$$

$$\Delta x = \frac{|1 - 5|}{4}$$

$$\Delta x = 1$$

$$A = \sum_{n=a}^{b-1} f(n) \Delta x$$

$$A = \sum_{n=1}^4 f(n)$$

$$A = 7 + 8 + 9 + 10$$

$$A = 34$$

$f(x) = x + 4$; $[-2, 2]$; 4 rectangles

x	-2	-1	0	1	2
y	2	3	4	5	6

$$\Delta x = \frac{|a - b|}{r}$$

$$\Delta x = \frac{|-2 - 2|}{4}$$

$$\Delta x = 1$$

$$A = \sum_{n=a}^{b-1} f(n) \Delta x$$

$$A = \sum_{n=-2}^1 f(n)$$

$$A = 2 + 3 + 4 + 5$$

$$A = 14$$

Right Riemann Sums

$f(x) = -x^2 - 2x + 9$; $[-3, 2]$; 5 rectangles

x	-3	-2	-1	0	1	2
y	6	9	10	9	6	1

$$\Delta x = \frac{|a - b|}{r}$$

$$\Delta x = \frac{|-3 - 2|}{5}$$

$$\Delta x = 1$$

$$A = \sum_{n=a+1}^b f(n) \Delta x$$

$$A = \sum_{n=-2}^2 f(n)$$

$$A = 9 + 10 + 9 + 6 + 1$$

$$A = 35$$

$f(x) = \frac{2}{x}$; $[2, 7]$; 5 rectangles

x	2	3	4	5	6	7
y	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$

$$\Delta x = \frac{|a - b|}{r}$$

$$\Delta x = \frac{|2 - 7|}{5}$$

$$\Delta x = 1$$

$$A = \sum_{n=a+1}^b f(n) \Delta x$$

$$A = \sum_{n=3}^7 f(n)$$

$$A = \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7}$$

$$A = 2.19$$

Trapezoidal Sums

$$y = -\frac{x^2}{2} + x + 5; [0, 3]; 5 \text{ Trapezoids}$$

x	0	0.6	1.2	1.8	2.4	3
y	5	5.42	5.48	5.18	4.52	3.5

$$\Delta x = \frac{|a - b|}{r}$$

$$\Delta x = \frac{|0 - 3|}{5}$$

$$\Delta x = 0.6$$

$$A = \sum_{n=1}^{rectangles} \frac{f(n\Delta x) + f((n-1)\Delta x)}{2} \Delta x$$

$$A = \sum_{n=1}^5 \frac{f(0.6n) + f(0.6(n-1))}{2} 0.6$$

$$A = 3.13 + 3.27 + 3.23 + 2.91 + 2.41$$

$$A = 14.95$$

$$f(x) = \frac{x^2}{2} + x + 1; [-2, 1]; 5 \text{ Trapezoids}$$

x	-2	-1.4	-0.8	-0.2	0.4	1
y	1	0.58	0.52	0.82	1.48	2.5

$$\Delta x = \frac{|a - b|}{r}$$

$$\Delta x = \frac{|-2 - 1|}{5}$$

$$\Delta x = 0.6$$

$$A = \sum_{n=1}^{rectangles} \frac{f(n\Delta x) + f((n-1)\Delta x)}{2} \Delta x$$

$$A = \sum_{n=1}^5 \frac{f(0.6n) + f(0.6(n-1))}{2} 0.6$$

$$A = 0.474 + 0.33 + 0.402 + 0.69 + 1.194$$

$$A = 3.09$$

2 Volume of Solids of Revolutions (Disk Method)

3. Find the volume of the solid of revolution generated by revolving the region bounded by $y = \sqrt{1 - x^2}$ and $y = 0$ about the x -axis.

$$\begin{aligned} V &= \pi \int_b^a [r^2] dx \\ &= \pi \int_{-1}^1 \left[\left(\sqrt{1 - x^2} \right)^2 \right] dx \\ &= \pi \int_{-1}^1 [1 - x^2] dx \\ &= \pi \left[x - \frac{x^3}{3} \right]_{-1}^1 \\ &= \pi \left(1 - \frac{1}{3} - \left(-1 - \frac{-1}{3} \right) \right) \\ V &= \frac{4\pi}{3} \end{aligned}$$

8. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{\sin x}$ and the x -axis ($0 \leq x \leq \pi$), about the x -axis.

$$\begin{aligned} V &= \pi \int_0^\pi (\sqrt{\sin x})^2 dx \\ &= \pi \int_0^\pi \sin x dx \\ &= \pi [-\cos x]_0^\pi \\ &= \pi (-(-1) + 1) \\ V &= 2\pi \end{aligned}$$

11. The region bounded by the parabola $y = 4x - x^2$ and the x -axis is revolved about the x -axis. Find the volume of the solid.

$$\begin{aligned} \{a, b\} &= x \quad \text{where } 4x - x^2 = 0 \\ a &= 0; \quad b = 4 \\ V &= \pi \int_0^4 (4x - x^2)^2 dx \\ &= \pi \int_0^4 (16x^2 - 8x^3 + x^4) dx \\ &= \pi \left[\frac{16}{3}x^3 - 2x^4 + \frac{1}{5}x^5 \right]_0^4 \\ &= \pi \left(\left(\frac{1024}{3} - 512 + \frac{1024}{5} \right) - 0 \right) \\ V &= \frac{512\pi}{15} \end{aligned}$$

- 13a. Find the volume of the solid generated when the area bounded by the curves $y = x^3 - x + 1$, $x = -1$, and $x = 1$ is revolved around the x -axis.

$$\begin{aligned}
 V &= \pi \int_{-1}^1 (x^3 - x + 1)^2 dx \\
 &= \pi \int_{-1}^1 (x^6 - 2x^4 + 2x^3 + x^2 - 2x + 1) dx \\
 &= \pi \left[\frac{1}{7}x^7 - \frac{2}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3 - x^2 + x \right]_{-1}^1 \\
 &= \pi \left(\left(\frac{1}{7} - \frac{2}{5} + \frac{1}{2} + \frac{1}{3} \right) - \left(-\frac{1}{7} + \frac{2}{5} + \frac{1}{2} - \frac{1}{3} - 2 \right) \right) \\
 V &= \frac{226\pi}{105} \text{ source: deepseek I aint doing them fractions :b}
 \end{aligned}$$

- 13b. Find the volume of the solid generated when the area bounded by the curve $y = x - x^2$ and the x -axis is revolved around the x -axis.

$$\begin{aligned}
 V &= \pi \int_0^1 (x - x^2)^2 dx \\
 &= \pi \int_0^1 (x^2 - 2x^3 + x^4) dx \\
 &= \pi \left[\frac{1}{3}x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5 \right]_0^1 \\
 &= \pi \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) \\
 V &= \frac{\pi}{30}
 \end{aligned}$$

3 Volume of Solids of Revolutions (Washer Method)

Find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

$$f(x) = x^2 - 3, \quad g(x) = \sqrt{x} - 3; \quad \text{axis: } y = 2$$

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 3 &= \sqrt{x} - 3 \end{aligned}$$

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x(x^3 - 1) = 0$$

$$x = 0 \quad x^3 = 1$$

$$a = 0 \quad b = 1$$

$$\begin{aligned} V &= \pi \int_a^b \left| (f(x) - y)^2 - (g(x) - y)^2 \right| dx \\ &= \pi \int_0^1 \left| (x^2 - 5)^2 - (\sqrt{x} - 5)^2 \right| dx \\ &= \pi \int_0^1 \left| x^4 - 10x^2 + 25 - (x - 10\sqrt{x} + 25) \right| dx \\ &= \pi \left| \frac{x^5}{5} - \frac{10x^3}{3} - \frac{x^2}{2} + \frac{20x^{\frac{3}{2}}}{3} \right|_0^1 \\ &= \pi \left| \frac{1}{5} - \frac{10}{3} - \frac{1}{2} + \frac{20}{3} \right| \\ V &= \frac{91\pi}{30} \end{aligned}$$

$$f(x) = x^2 - 3, \quad g(x) = \sqrt{x} - 3; \quad \text{axis: } y = 1$$

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 3 &= \sqrt{x} - 3 \end{aligned}$$

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x(x^3 - 1) = 0$$

$$x = 0 \quad x^3 = 1$$

$$a = 0 \quad b = 1$$

$$\begin{aligned} V &= \pi \int_a^b \left| (f(x) - y)^2 - (g(x) - y)^2 \right| dx \\ &= \pi \int_0^1 \left| (x^2 - 4)^2 - (\sqrt{x} - 4)^2 \right| dx \\ &= \pi \int_0^1 \left| x^4 - 8x^2 + 16 - (x - 8\sqrt{x} + 16) \right| dx \\ &= \pi \left| \frac{x^5}{5} - \frac{8x^3}{3} - \frac{x^2}{2} + \frac{16x^{\frac{3}{2}}}{3} \right|_0^1 \\ &= \pi \left| \frac{1}{5} - \frac{8}{3} - \frac{1}{2} + \frac{16}{3} \right| \\ V &= \frac{71\pi}{30} \end{aligned}$$

$$f(x) = x^2 + 4, \quad g(x) = 2; \quad a = 0, \quad b = 1; \\ \text{axis: } y = -1$$

$$\begin{aligned} V &= \pi \int_a^b \left| (f(x) - y)^2 - (g(x) - y)^2 \right| dx \\ &= \pi \int_0^1 \left| (x^2 + 5)^2 - 3^2 \right| dx \\ &= \pi \int_0^1 \left| x^4 + 10x^2 + 16 \right| dx \\ &= \pi \left| \frac{x^5}{5} + \frac{10x^3}{3} + 16x \right|_0^1 \\ &= \pi \left| \frac{1}{5} + \frac{10}{3} + 16 \right| \\ V &= \frac{293\pi}{15} \end{aligned}$$

$$f(x) = x^2 + 3, \quad g(x) = 3; \quad a = 0, \quad b = 2; \\ \text{axis: } y = 1$$

$$\begin{aligned} V &= \pi \int_a^b \left| (f(x) - y)^2 - (g(x) - y)^2 \right| dx \\ &= \pi \int_0^2 \left| (x^2 + 2)^2 - 2^2 \right| dx \\ &= \pi \int_0^2 \left| x^4 + 4x^2 \right| dx \\ &= \pi \left| \frac{x^5}{5} + \frac{4x^3}{3} \right|_0^2 \\ &= \pi \left| \frac{32}{5} + \frac{32}{3} \right| \\ V &= \frac{256\pi}{15} \end{aligned}$$

4 Separation of Variables

General Solution

1.	2.	3.	4.
$\frac{dy}{dx} = \frac{x^3}{y^2}$	$\frac{dy}{dx} = \frac{1}{\sec^2 y}$	$\frac{dy}{dx} = 3e^{x-y}$	$\frac{dy}{dx} = \frac{2x}{e^{2y}}$
$y^2 dy = x^3 dx$	$\sec^2 y dy = dx$	$e^y dy = 3e^x dx$	$e^{2y} dy = 2x dx$
$\int y^2 dy = \int x^3 dx$	$\int \sec^2 y dy = \int dx$	$\int e^y dy = \int 3e^x dx$	$\int e^{2y} dy = \int 2x dx$
$\frac{y^3}{3} = \frac{x^4}{4} + C$	$\tan y = x + C$	$e^y = 3e^x + C$	$\frac{e^{2y}}{2} = x^2 + C$
$y^3 = \frac{3x^4}{4} + 3C$	$y = \tan^{-1}(x + C)$	$y = \ln(3e^x + C)$	$e^{2y} = 2(x^2 + C)$
$y = \sqrt[3]{\frac{3x^4}{4} + 3C}$			$2y = \ln 2 + \ln(x^2 + C)$
			$y = \ln(\sqrt{x^2 + C}) + \frac{\ln 2}{2}$

Particular Solution

5.	6.
$\frac{dy}{dx} = \frac{2x}{y^2}$	$\frac{dy}{dx} = 2e^{x-y}$
$y^2 dy = 2x dx$	$e^y dy = 2e^x dx$
$\int y^2 dy = \int 2x dx$	$\int e^y dy = \int 2e^x dx$
$\frac{y^3}{3} = x^2 + C$	$e^y = 2e^x + C$
$y^3 = 3x^2 + 3C$	$y = \ln(2e^x + C)$
$y = \sqrt[3]{3x^2 + 3C}$	
$y(2) = \sqrt[3]{13}$	$y(-3) = \ln \frac{3e^3 + 2}{e^3}$
$\sqrt[3]{3(2)^2 + 3C} = \sqrt[3]{13}$	$\ln(2e^{-3} + C) = \ln \frac{3e^3 + 2}{e^3}$
$3(2)^2 + 3C = (\sqrt[3]{13})^3$	$2e^{-3} + C = \frac{3e^3 + 2}{e^3}$
$12 + 3C = 13$	$C = \frac{3e^3 + 2}{e^3} - 2e^{-3}$
$C = \frac{1}{3}$	$C = 3$
$y = \sqrt[3]{3x^2 + 1}$	$y = \ln(2e^x + 3)$

$$\begin{aligned}
7. \quad & \frac{dy}{dx} = \frac{1}{\sec^2 y} \\
& \sec^2 y \, dy = dx \\
& \int \sec^2 y \, dy = \int dx \\
& \tan y = x + C \\
& y = \tan^{-1}(x + C)
\end{aligned}$$

$$\begin{aligned}
& y(3) = 0 \\
& \tan^{-1}(3 + C) = 0 \\
& 3 + C = \tan(0) \\
& 3 + C = 0 \\
& C = -3 \\
& y = \tan^{-1}(x - 3)
\end{aligned}$$

$$\begin{aligned}
8. \quad & \frac{dy}{dx} = \frac{e^x}{y^2} \\
& y^2 \, dy = e^x \, dx \\
& \int y^2 \, dy = \int e^x \, dx \\
& \frac{y^3}{3} = e^x + C \\
& y^3 = 3e^x + 3C \\
& y = \sqrt[3]{3e^x + 3C}
\end{aligned}$$

$$\begin{aligned}
& y(-1) = \frac{\sqrt[3]{e^3 + 3e^2}}{e} \\
& \sqrt[3]{3e - 1 + 3C} = \frac{\sqrt[3]{e^3 + 3e^2}}{e} \\
& 3e - 1 + 3C = \frac{e^3 + 3e^2}{e^3} \\
& 3C = \frac{e^3 + 3e^2 - 3e^2}{e^3} \\
& C = \frac{1}{3}
\end{aligned}$$

$$y = \sqrt[3]{3 \left(e^x + \frac{1}{3} \right)}$$

$$\begin{aligned}
9. \quad & \frac{dy}{dx} = -\frac{1}{\sin y} \\
& \sin y \, dy = -1 \, dx \\
& \int \sin y \, dy = \int -1 \, dx \\
& -\cos y = -x + C \\
& y = \cos^{-1}(x - C)
\end{aligned}$$

$$\begin{aligned}
& y(3) = \frac{\pi}{2} \\
& \cos^{-1}(3 - C) = \frac{\pi}{2} \\
& 3 - C = \cos \frac{\pi}{2} \\
& 3 - C = 0 \\
& C = 3 \\
& y = \cos^{-1}(x - 3)
\end{aligned}$$

$$\begin{aligned}
10. \quad & \frac{dy}{dx} = \frac{2x}{e^{2y}} \\
& e^{2y} \, dy = 2x \, dx \\
& \int e^{2y} \, dy = \int 2x \, dx \\
& \frac{e^{2y}}{2} = x^2 + C \\
& e^{2y} = 2(x^2 + C) \\
& 2y = \ln(x^2 + C) + \ln 2 \\
& y = \frac{\ln(2(x^2 + C))}{2}
\end{aligned}$$

$$\begin{aligned}
& y(2) = \frac{\ln 5}{2} \\
& \frac{\ln(2(2^2 + C))}{2} = \frac{\ln 5}{2} \\
& 2(4 + C) = 5 \\
& C = -1.5
\end{aligned}$$

$$y = \ln \left(\sqrt{x^2 - 1.5} \right) + \frac{\ln 2}{2}$$

$$\begin{aligned}
11. \quad & \frac{dy}{dx} = \frac{3x^2}{e^{2y}} \\
& e^{2y} dy = 3x^2 dx \\
& \int e^{2y} dy = \int 3x^2 dx \\
& \frac{e^{2y}}{2} = x^3 + C \\
& e^{2y} = 2x^3 + 2C \\
& 2y = \ln(2(x^3 + C)) \\
& y = \frac{\ln(2(x^3 + C))}{2}
\end{aligned}$$

$$\begin{aligned}
& y(1) = 0 \\
& \frac{\ln(2(1^3 + C))}{2} = 0 \\
& \ln(2(1^3 + C)) = 0 \\
& 2(1^3 + C) = 1 \\
& 1 + C = \frac{1}{2} \\
& C = -\frac{1}{2}
\end{aligned}$$

$$y = \ln\left(\sqrt{x^3 - \frac{1}{2}}\right) + \frac{\ln 2}{2}$$

$$\begin{aligned}
12. \quad & \frac{dy}{dx} = \frac{1+x^2}{y^2} \\
& y^2 dy = (1+x^2) dx \\
& \int y^2 dy = \int (1+x^2) dx \\
& \frac{y^3}{3} = x + \frac{x^3}{3} + C \\
& y^3 = 3x + x^3 + 3C \\
& y = \sqrt[3]{x^3 + 3x + 3C}
\end{aligned}$$

$$\begin{aligned}
& y(-1) = -\sqrt[3]{4} \\
& \sqrt[3]{(-1)^3 + 3(-1) + 3C} = -\sqrt[3]{4} \\
& (-1)^3 + 3(-1) + 3C = -4 \\
& -1 - 3 + 3C = -4 \\
& 3C = 0 \\
& C = 0
\end{aligned}$$

$$y = \sqrt[3]{x^3 + 3x + 3C}$$