Análisis de Señales

Sistemas LTI

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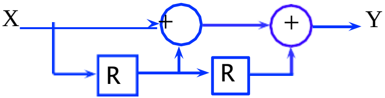
Director Semillero TRIAC
Ingenieria Electronica
Universidad Popular del Cesar

Agenda

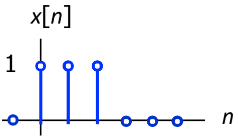
1. **Sistemas LIT:La suma de convolución**
2. **Sistema LIT continuos: La integral de convolución**

Ejercicio

Busque $y[3]$



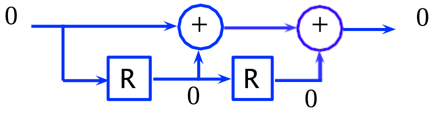
Cuando la entrada es

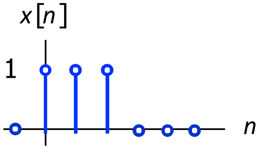


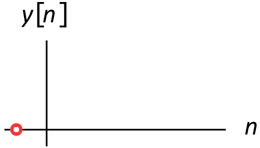
Respuesta

a)1 **b**)2 **c**)3 **d**)4 **e**)5

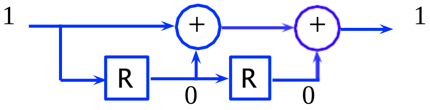
Ejercicio

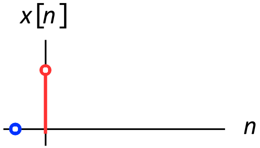


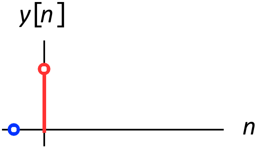




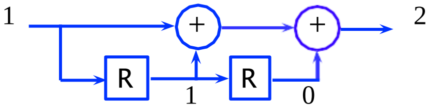
Ejercicio

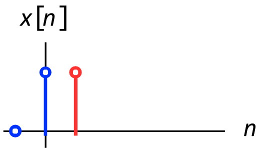


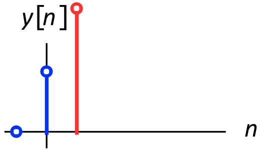




Ejercicio







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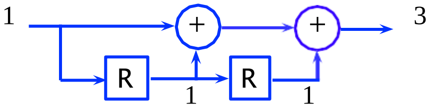

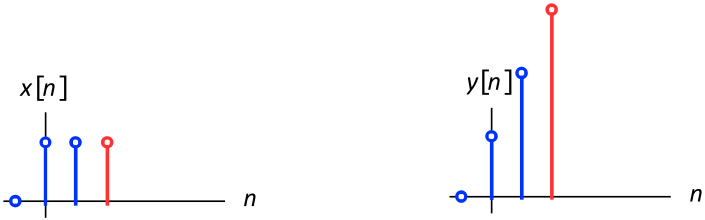

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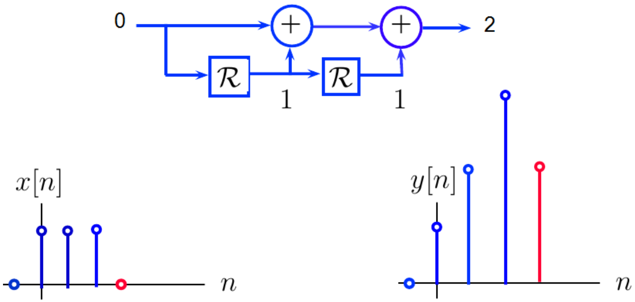

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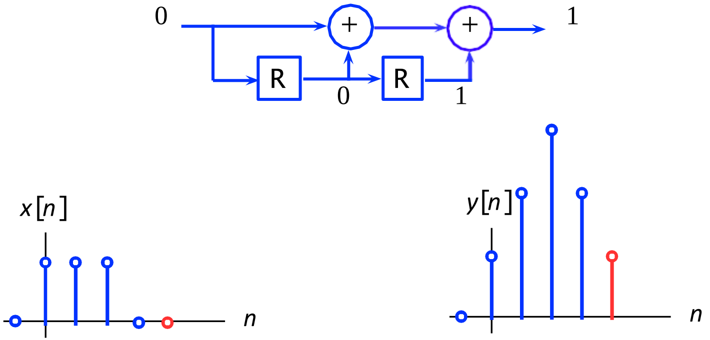

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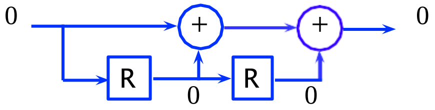
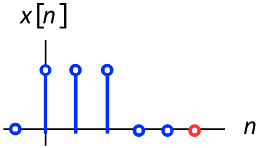
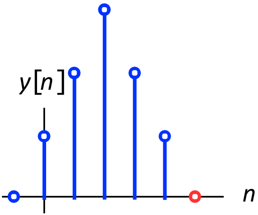
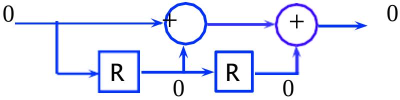
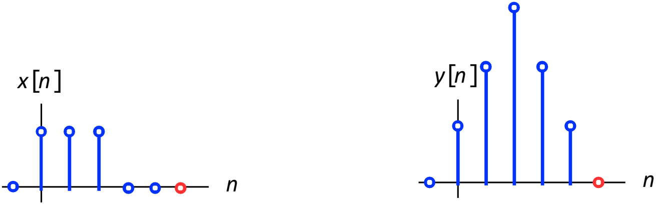






$y[3] = 2$

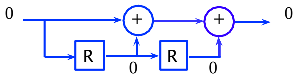


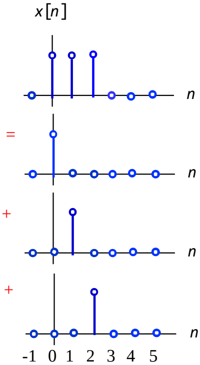


Respuesta

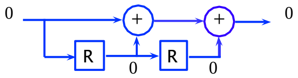
a)1 b)2 c)3 d.)3 e.)5

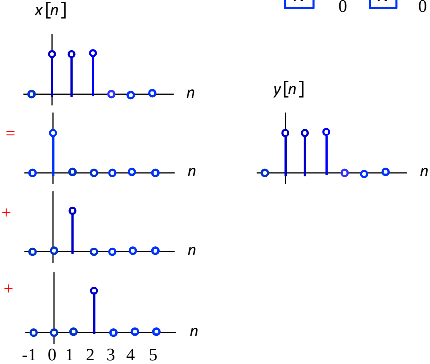
Ejercicio





Ejercicio





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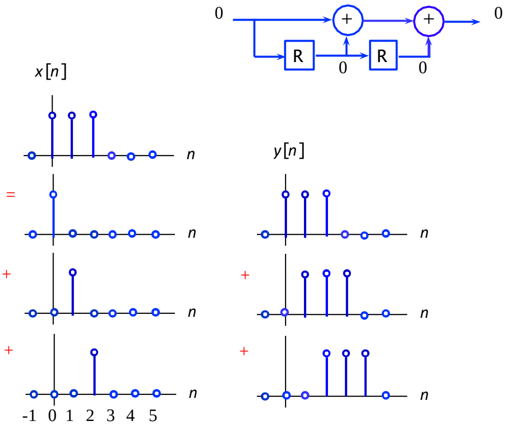

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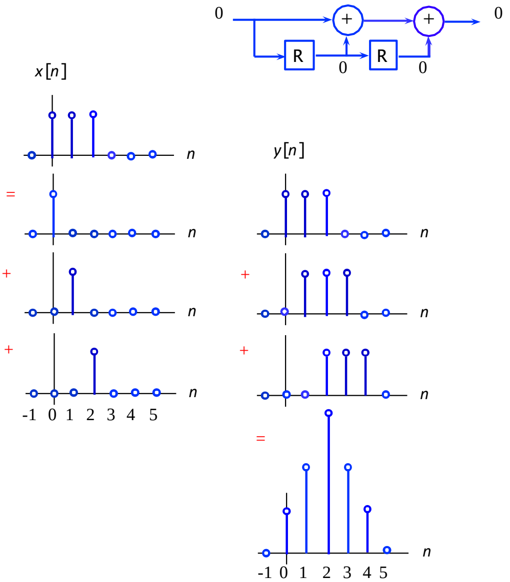

Representación de señales discretas en términos de impulsos

El impulso unitario discreto se puede usar para construir cualquier señal discreta

$$x[-1]\delta[n+1] = \begin{cases} x[-1], & n = -1 \\ 0, & n \neq -1 \end{cases}$$

$$x[0]\delta[n] = \begin{cases} x[0], & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$x[1]\delta[n-1] = \begin{cases} x[1], & n = 1 \\ 0, & n \neq 1 \end{cases}$$

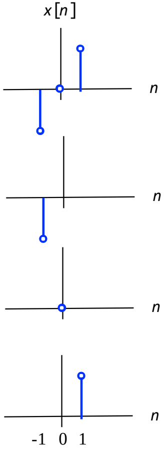

Figura: Representación de señal en términos de impulsos

Respuesta al impulso unitario y representación de sistemas LIT como suma de convolución

Si un sistema es lineal e invariante en el tiempo (LIT), su salida es la sumatoria ponderada de cada respuesta.

$$\delta[n] \longrightarrow \boxed{\text{Sistema}} \longrightarrow h[n]$$

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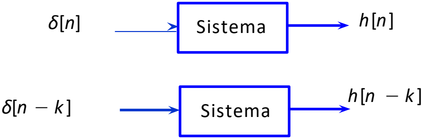

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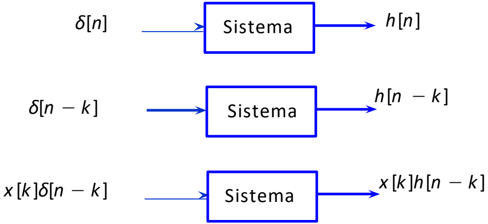

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Respuesta al impulso unitario y representación de sistemas LIT como suma de convolución

Si un sistema es lineal e invariante en el tiempo (LIT), su salida es la sumatoria ponderada de cada respuesta.

$$\delta[n] \longrightarrow \boxed{\text{Sistema}} \longrightarrow h[n]$$

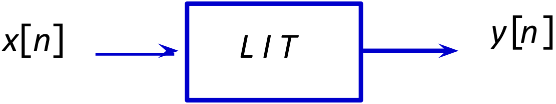
$$\delta[n-k] \longrightarrow \boxed{\text{Sistema}} \longrightarrow h[n-k]$$

$$x[k]\delta[n-k] \longrightarrow \boxed{\text{Sistema}} \longrightarrow x[k]h[n-k]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \longrightarrow \boxed{\text{Sistema}} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * y[n]$$

Respuesta al impulso unitario y representación de sistemas LIT como suma de convolución

La respuesta de un sistema LIT a una entrada arbitraria

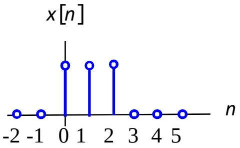


$$y[n] = x[n] * h[n] = \sum_{k=}^{\infty} x[k]h[n-k]$$

Donde h[n] es la respuesta al impulso del sistema.
Esta operación es llamada **convolución**

Ejemplo

$$y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



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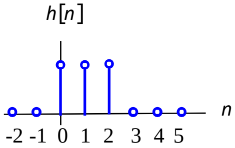
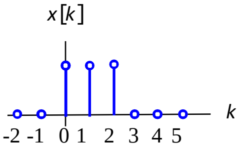


Ejemplo

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k]$$



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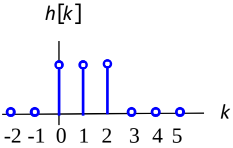

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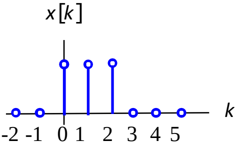

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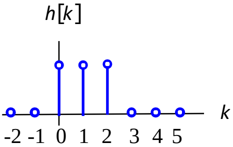
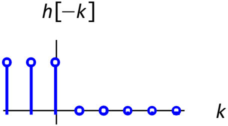
Desplazada y escalada



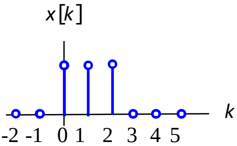


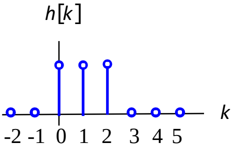
Ejemplo

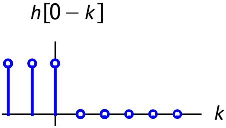
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k]$$

$*$

Multiplicada







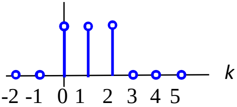
Ejemplo

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k]$$

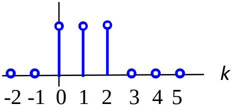
$*$

Multiplicada

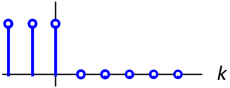
$x[k]$



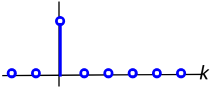
$h[k]$



$h[0-k]$



$x[k]h[0-k]$



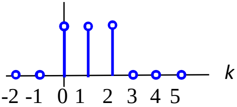
Ejemplo

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k]$$

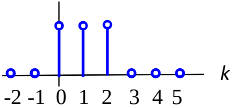
$*$

Sumatoria

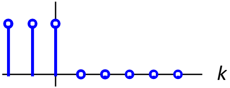
$x[k]$



$h[k]$

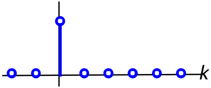


$h[0-k]$



$x[k]h[0-k]$

$\sum_{k=-\infty}^{\infty}$

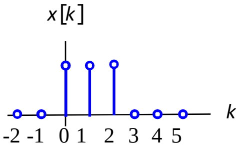


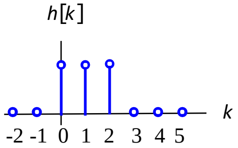
Ejemplo

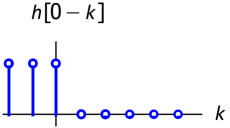
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k]$$

$*$

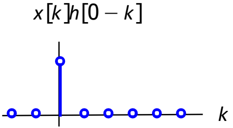
Sumatoria







$$\sum_{k=-\infty}^{\infty}$$



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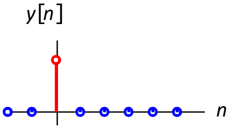
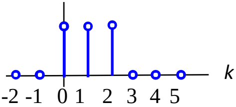


Ejemplo

$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k]$$

$x[k]$



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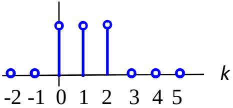
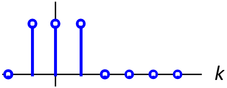
$h[k]$

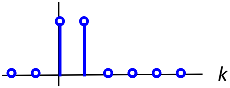


$h[1-k]$



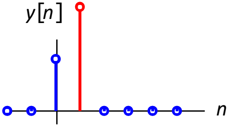
$x[k]h[1-k]$

$\sum_{k=-\infty}^{\infty}$



$= 2$

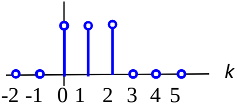
$y[n]$



Ejemplo

$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k]$$

$x[k]$



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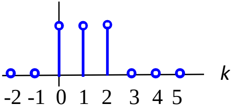
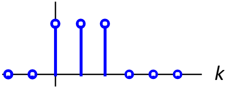
$h[k]$

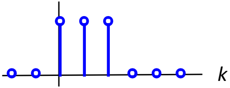


$h[2-k]$



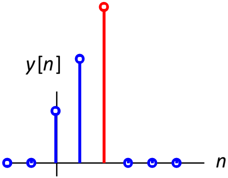
$x[k]h[2-k]$

$\sum_{k=-\infty}^{\infty}$



$= 3$

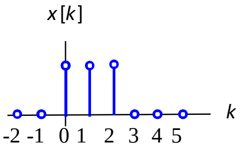
$y[n]$

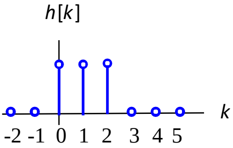


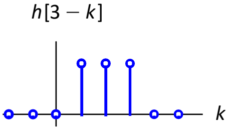
Ejemplo

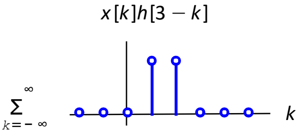
$$y[3] = \sum_{k=-\infty}^{\infty} x[k]h[3-k]$$

$*$

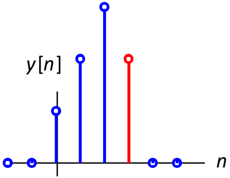






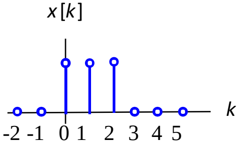


= 2

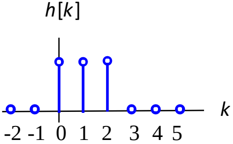


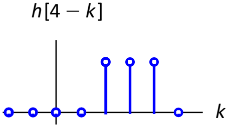
Ejemplo

$$y[4] = \sum_{k=-\infty}^{\infty} x[k]h[4-k]$$

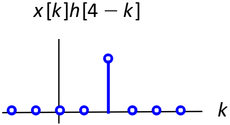


$*$

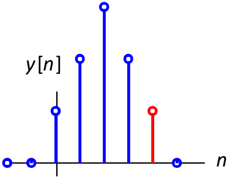




$\sum_{k=-\infty}^{\infty}$

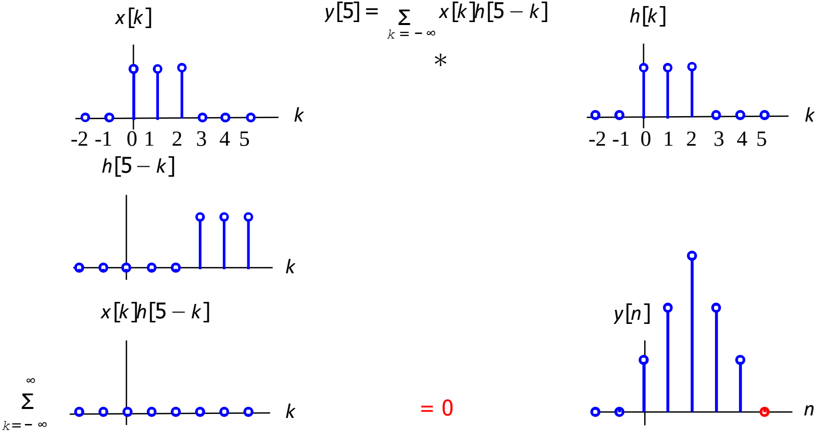


$= 1$



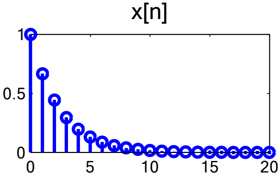
Ejemplo

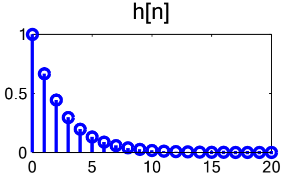
$$y[5] = \sum_{k=-\infty}^{\infty} x[k]h[5-k]$$



$$\sum_{k=-\infty}^{\infty} x[k]h[5-k] = 0$$

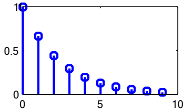
Ejercicio



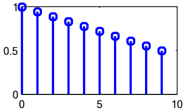


Si $x[n] = \frac{2}{3}^{n} u[n]$ y $h[n] = \frac{2}{3}^{n} u[n]$, determine $y[n]$

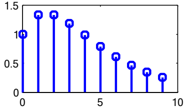
1.



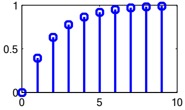
2.



3.

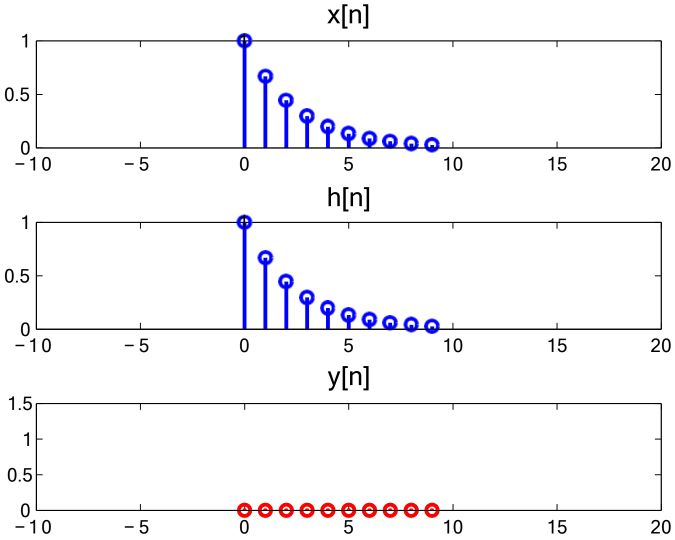


4.



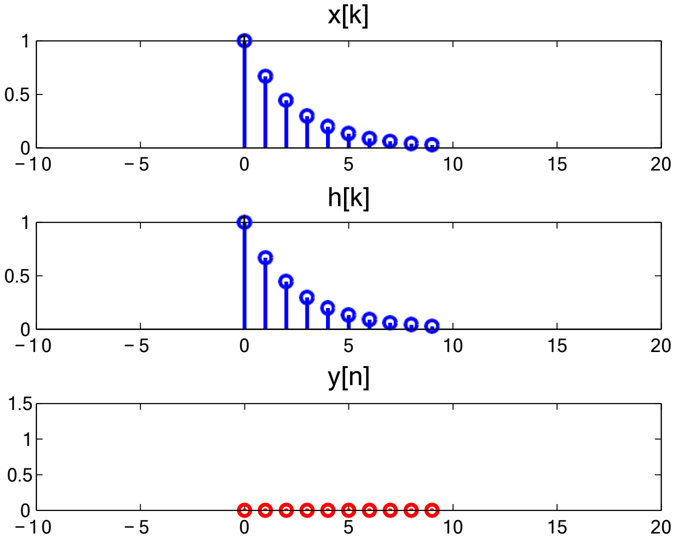
5. Ninguno de los anteriores.

Ejercicio



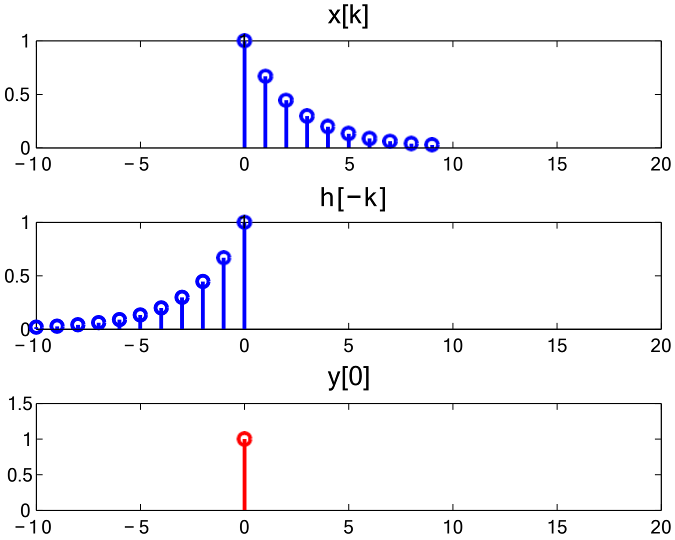
x[n]
1
0.5
0
−10
−5
0
5
10
15
20
h[n]
1
0.5
0
−10
−5
0
5
10
15
20
y[n]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



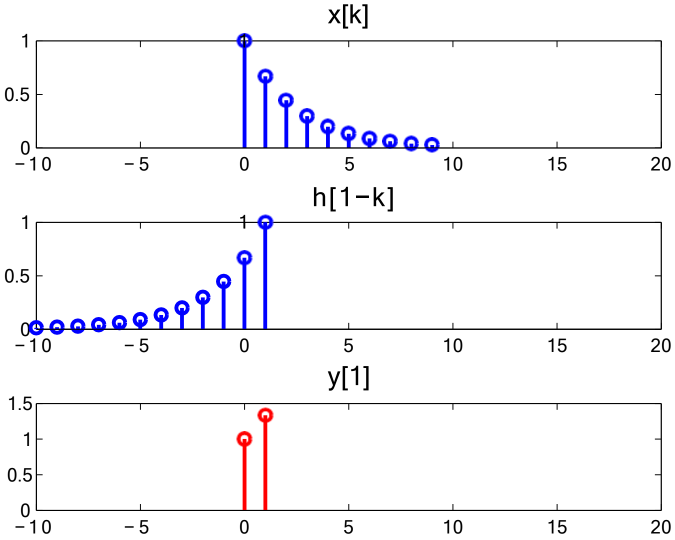
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[k]
1
0.5
0
−10
−5
0
5
10
15
20
y[n]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



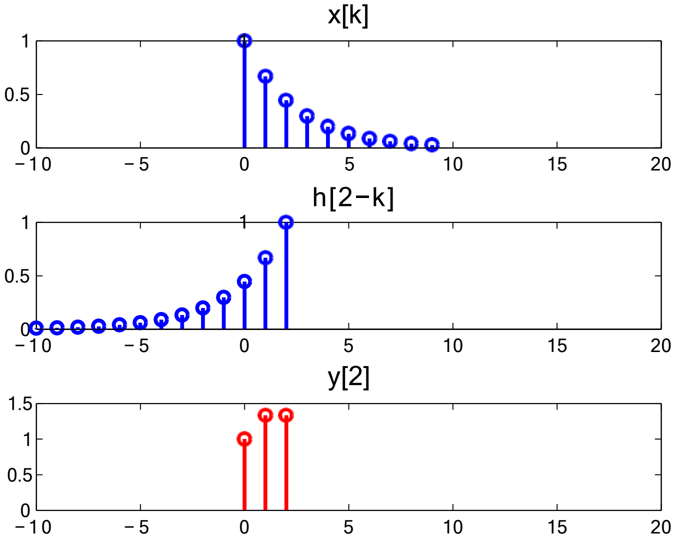
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[0]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



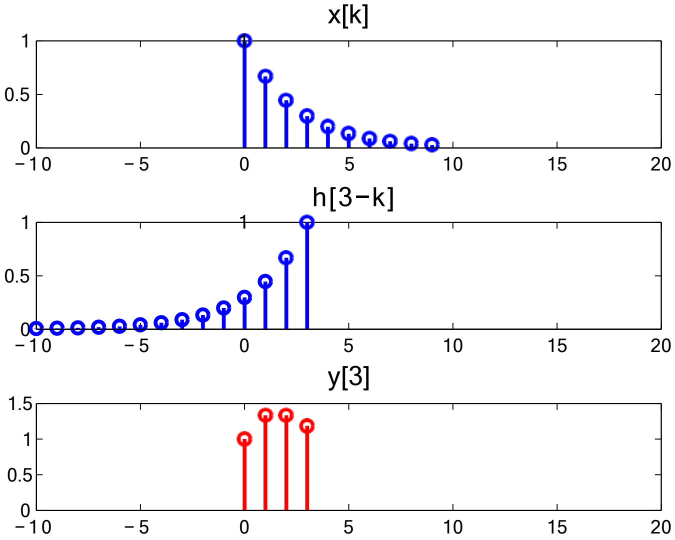
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[1−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[1]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



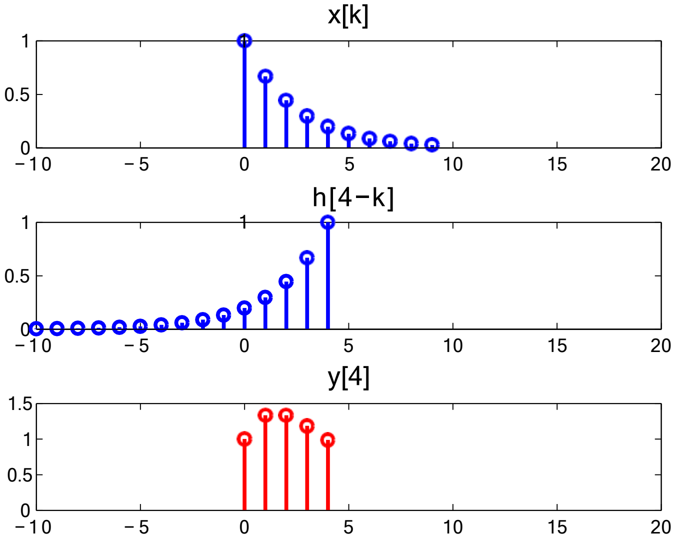
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[2−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[2]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

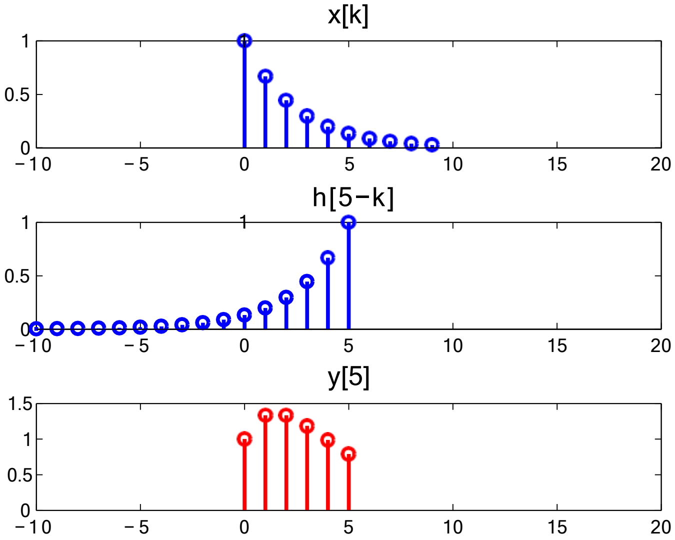


x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[3−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[3]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

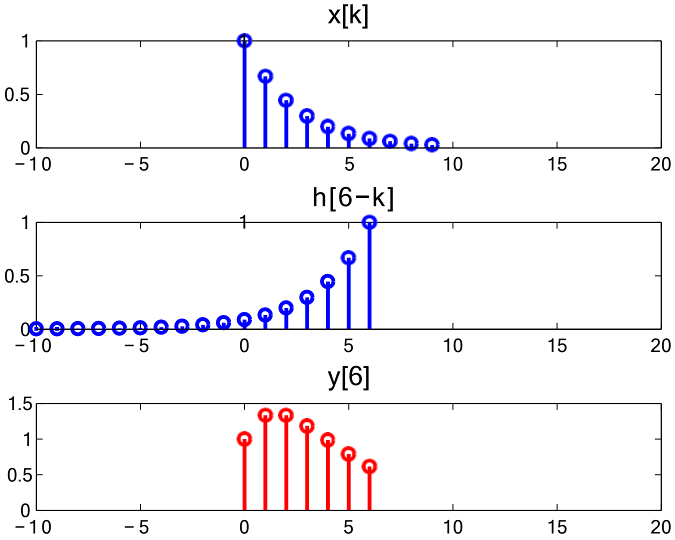

x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[4−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[4]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



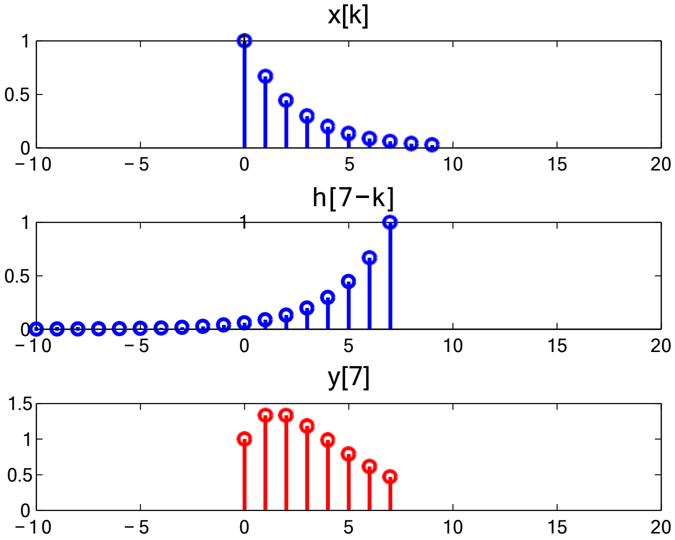
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[5−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[5]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

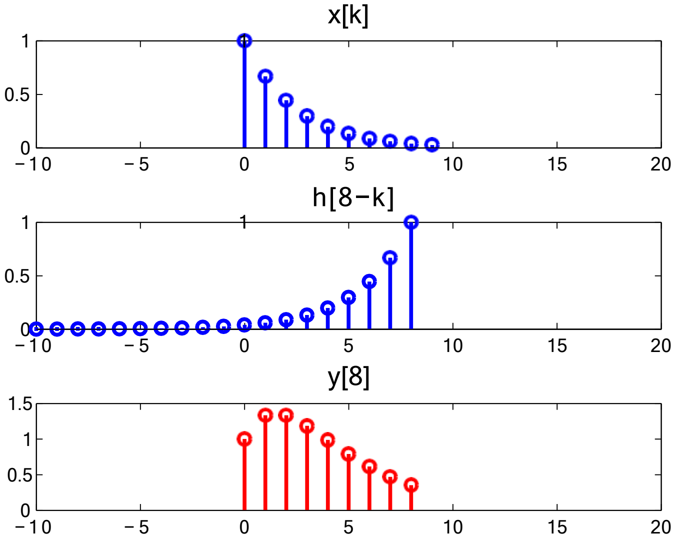


x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[6−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[6]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

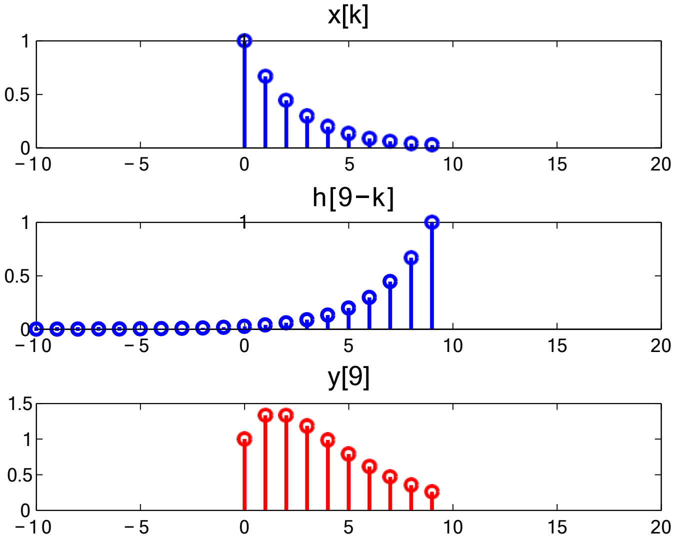


Ejercicio



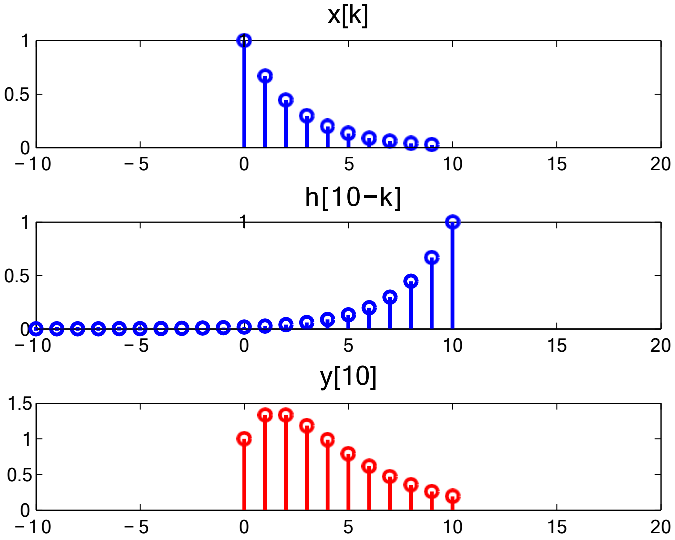
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[8−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[8]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



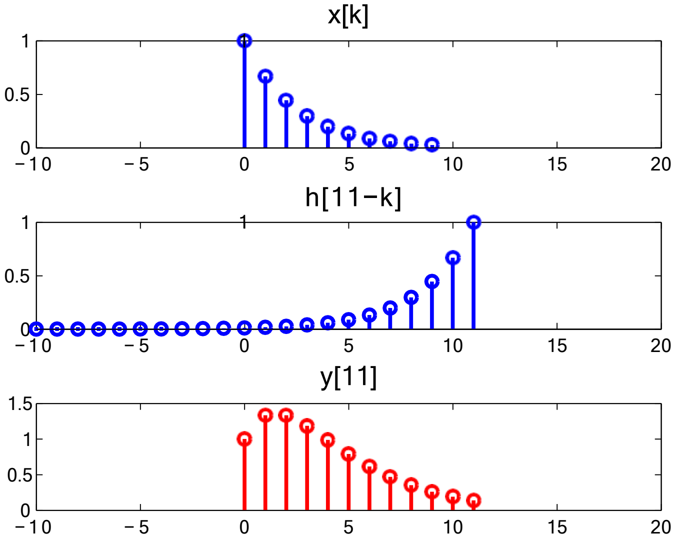
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[9−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[9]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



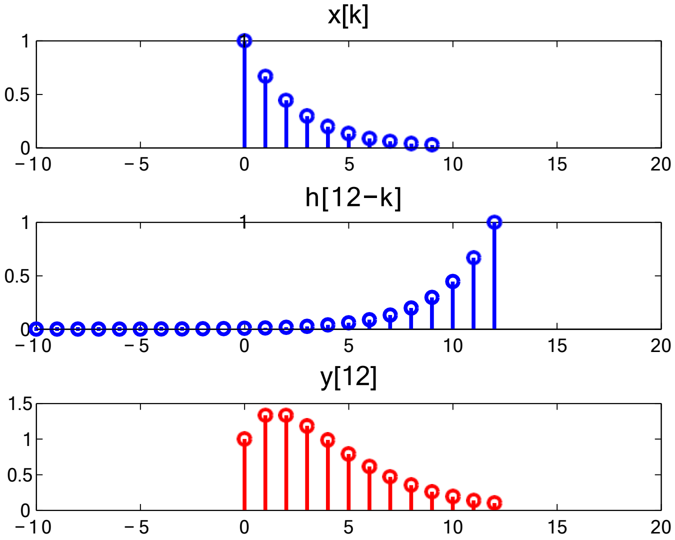
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[10−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[10]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

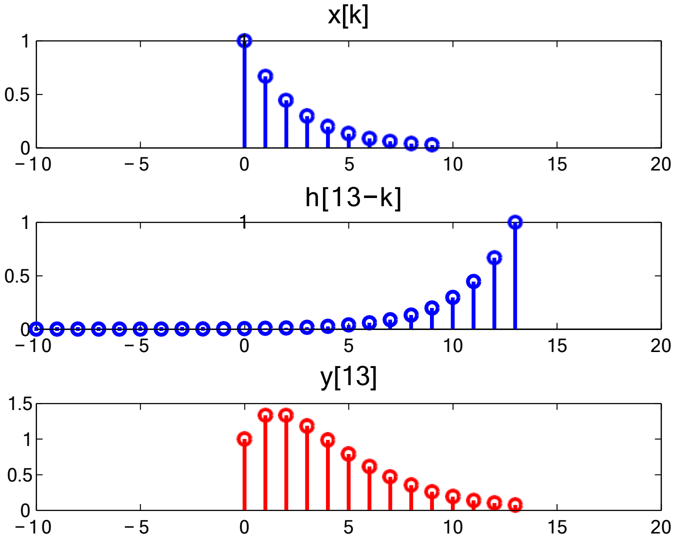


x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[11−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[11]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

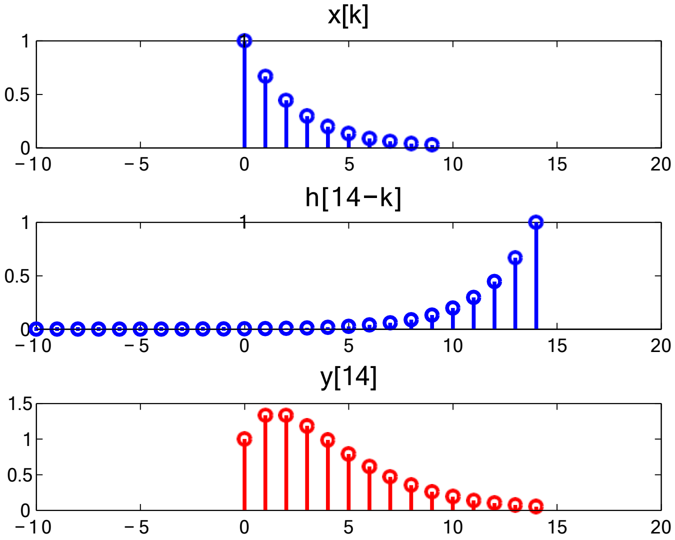

x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[12−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[12]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



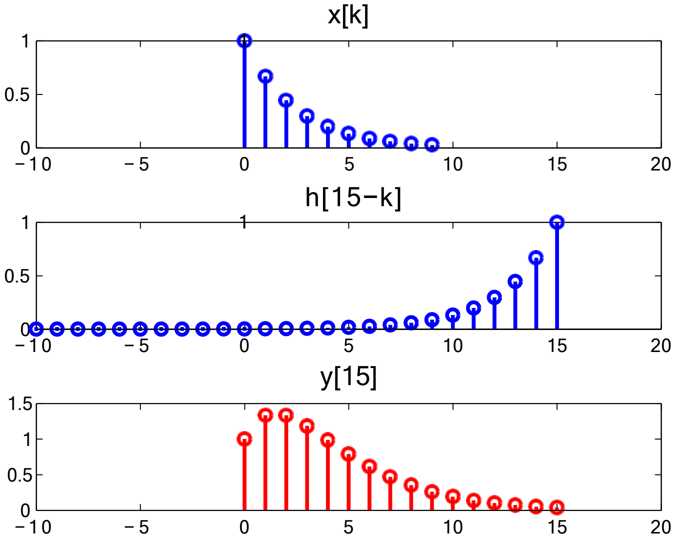
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[13−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[13]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



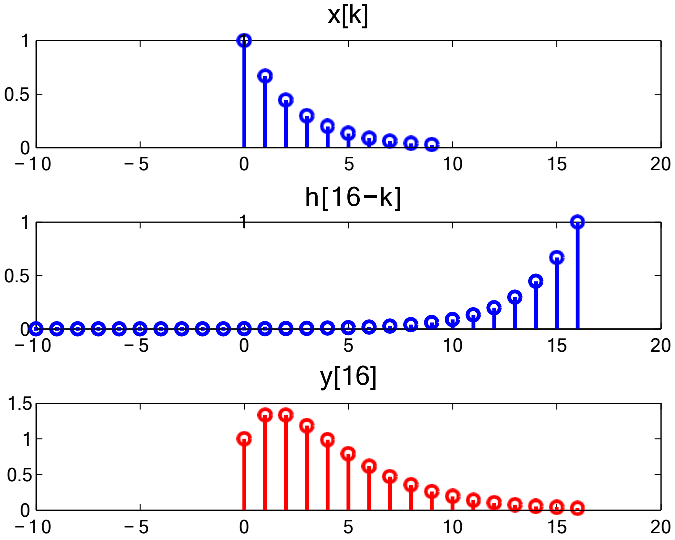
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[14−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[14]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



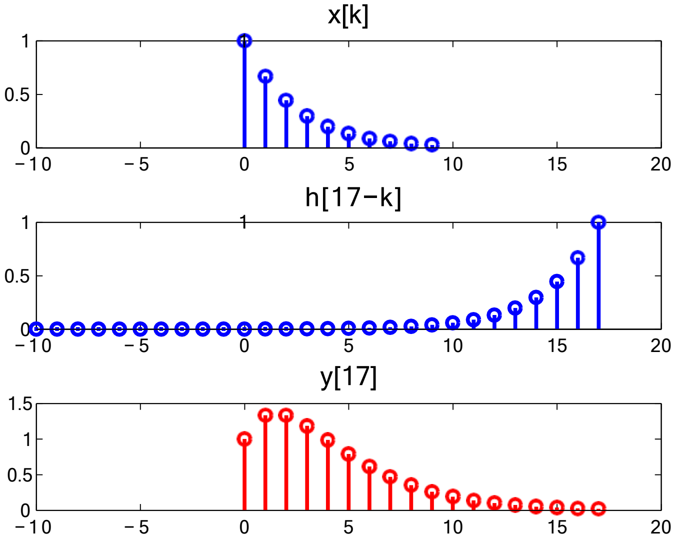
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[15−k]
1
1
0.5
0
−10
−5
0
5
10
15
20
y[15]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

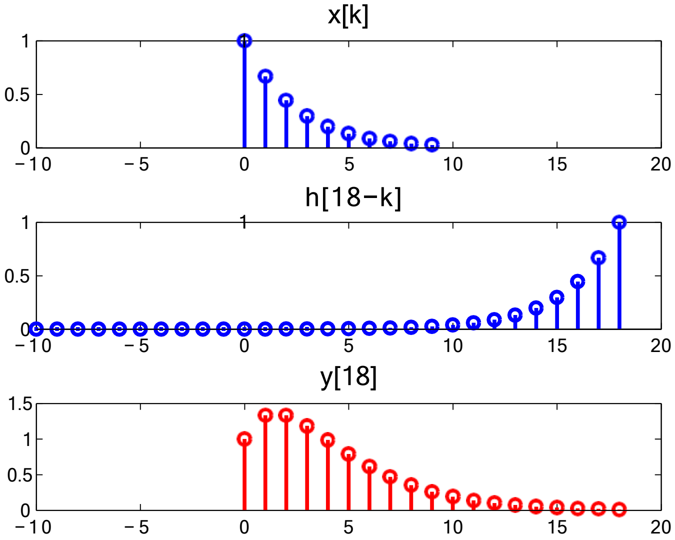


x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[16−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[16]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio

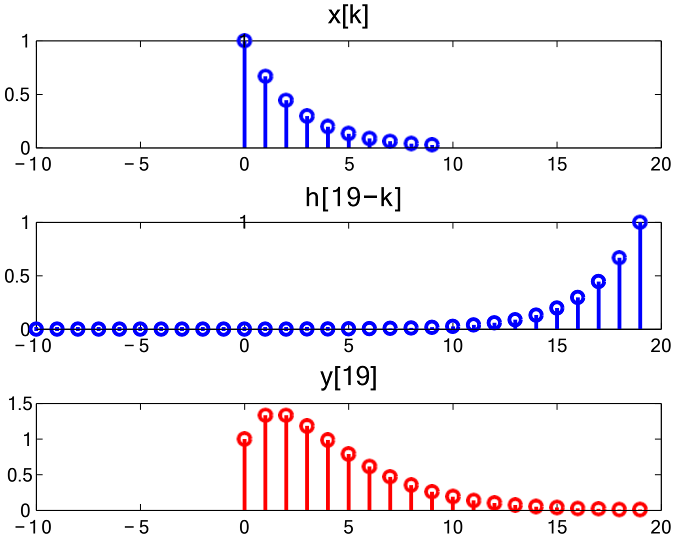


Ejercicio



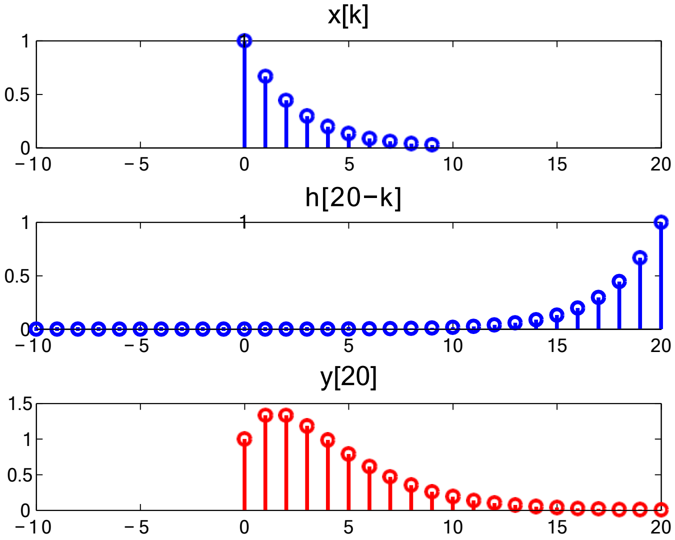
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[18−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[18]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



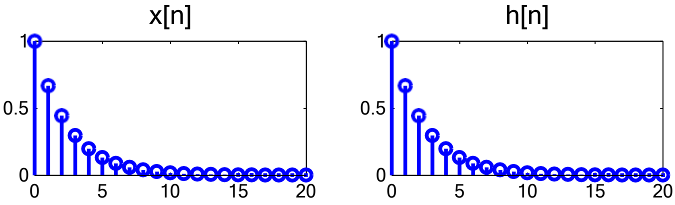
x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[19−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[19]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



x[k]
1
0.5
0
−10
−5
0
5
10
15
20
h[20−k]
1
0.5
0
−10
−5
0
5
10
15
20
y[20]
1.5
1
0.5
0
−10
−5
0
5
10
15
20

Ejercicio



Expresado matemáticamente como:

$$\left(\frac{2}{3}\right)^n u[n] * \left(\frac{2}{3}\right)^n u[n] = \sum_{k=-\infty}^{\infty} \left(\frac{2}{3}\right)^k u[k] \times \left(\frac{2}{3}\right)^{n-k} u[n-k]$$

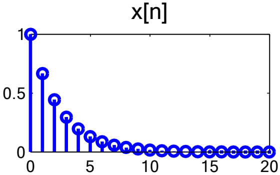
$$= \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k \times \left(\frac{2}{3}\right)^{n-k}$$

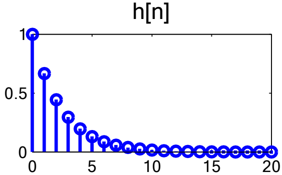
$$= \sum_{k=0}^{n} \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^n \sum_{k=0}^{n} 1$$

$$= (n+1) \left(\frac{2}{3}\right)^n u[n]$$

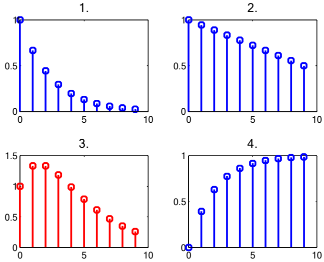
$$= 1, \frac{4}{3}, \frac{4}{3}, \frac{32}{27}, \frac{80}{81}, \ldots$$

Ejercicio





Si $x[n] = \left(\frac{2}{3}\right)^n u[n]$ y $h[n] = \left(\frac{2}{3}\right)^n u[n]$, determine $y[n]$. respuesta 3



5. Ninguno de los anteriores.

Resumen convolución discreta

La representación de un sistema LIT dado por una señal

$$x[n] \longrightarrow \boxed{\;h[n]\;} \longrightarrow y[n]$$

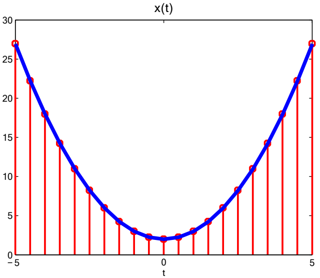
La respuesta al impulso unitario $h[n]$ es una descripción completa de un sistema LIT.

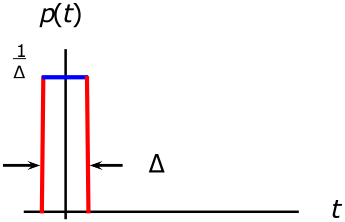
Dado $h[n]$ se puede determinar la respuesta $y[n]$ a una entrada arbitraria $x[n]$:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Representación de señales continuas en términos de impulsos

De manera similar que en tiempo discreto se hace una aproximación por medio de impulsos.





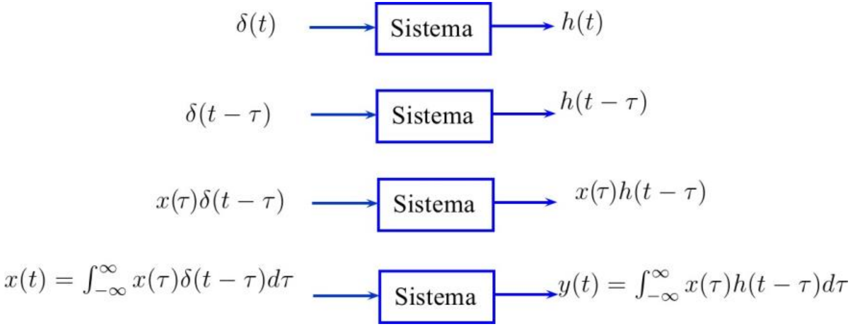
$$x(t) = \lim_{\Delta \to 0} \sum_k x(k\Delta)p(t - k\Delta)\Delta$$

Como $\Delta \to 0$, $k\Delta \to \tau$, $\Delta \to d\tau$, y $p(t) \to \delta(t)$

$$x(t) \to \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau$$

Estructura de superposición

Si un sistema es lineal e invariante en el tiempo (LIT), su salida es la integral desplazada y escalada de la respuesta de impulsos unitarios unitarios



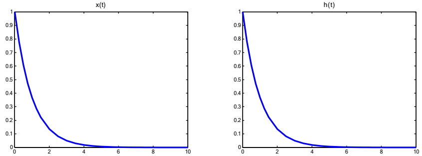
Convolución en tiempo continuo

La convolución de una señal en tiempo continuo es análoga a la convolución de una señal en tiempo discreto.

$$\textbf{DT:}\, y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$\textbf{CT:}\, y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

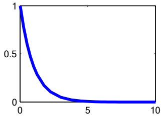
Ejercicio



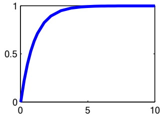
¿Cuál gráfico muestra el resultado de la convolución de las señales?

$$x(t) = h(t) = e^{-t}u(t)$$

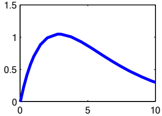
1.



2.



3.



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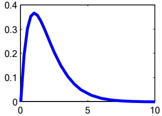
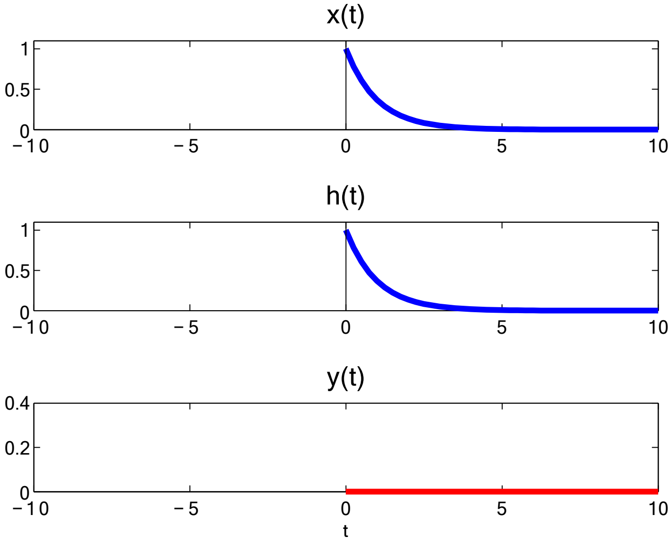


5. Ninguno de los anteriores.

Ejercicio



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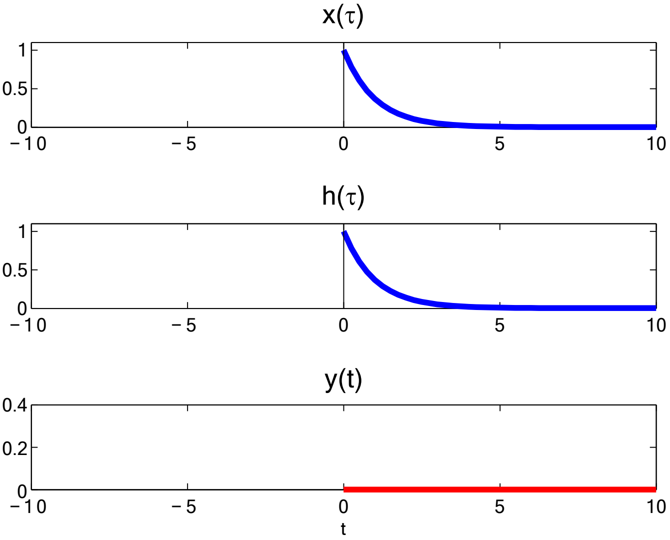

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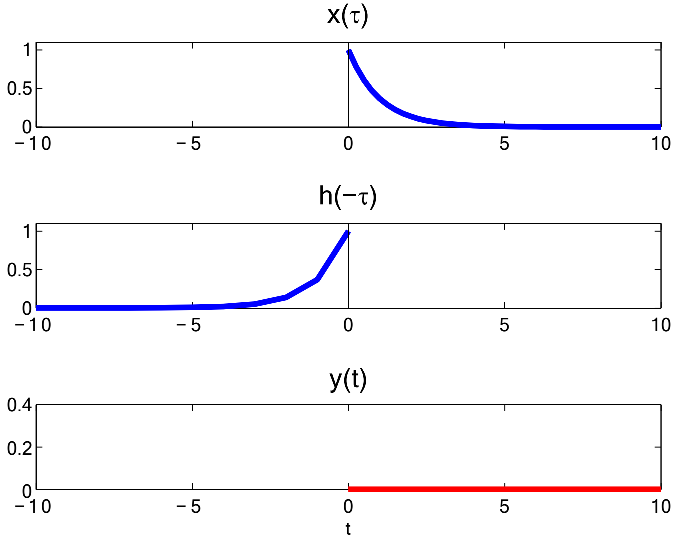

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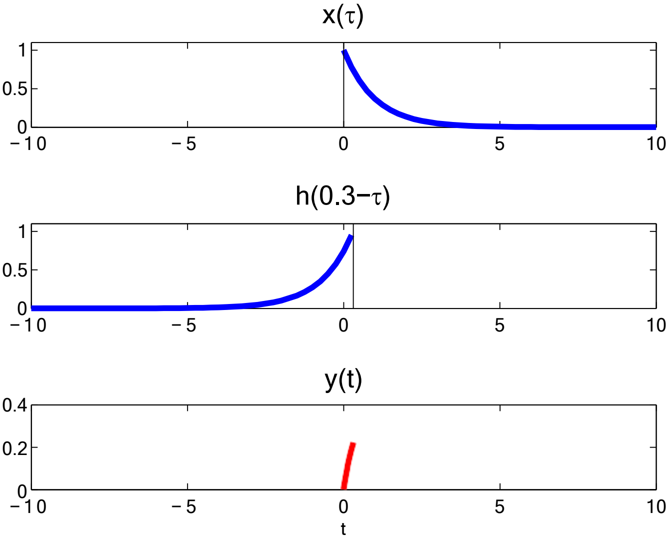

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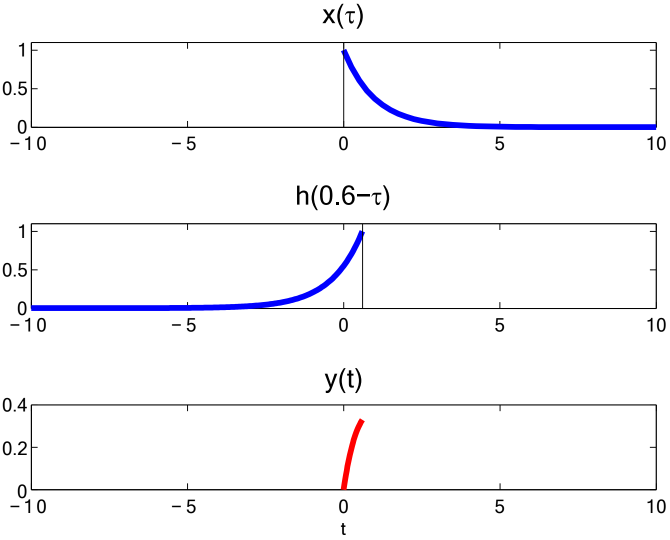

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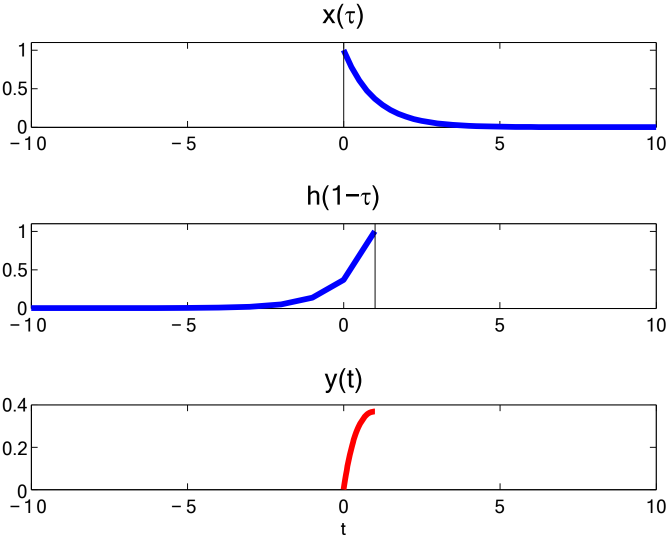

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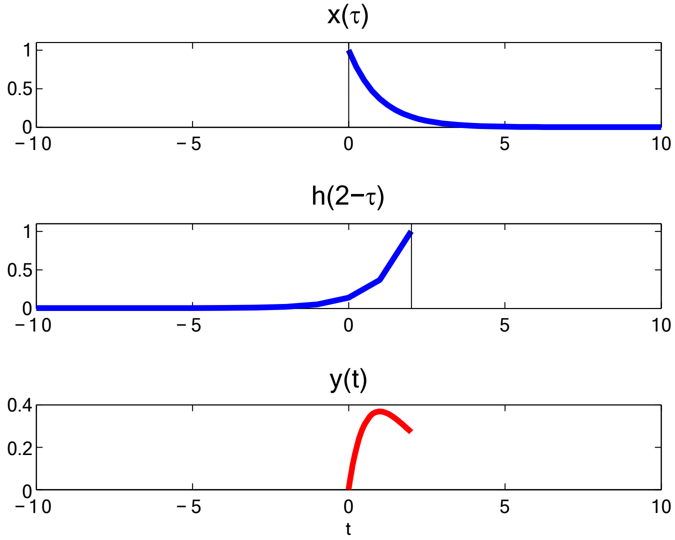

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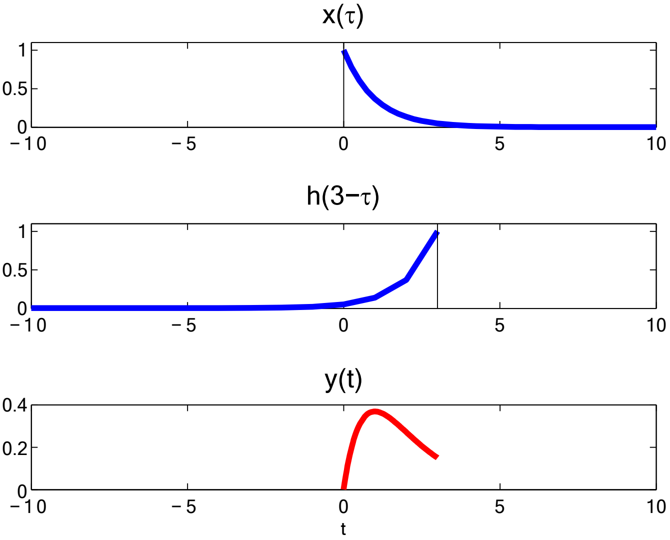

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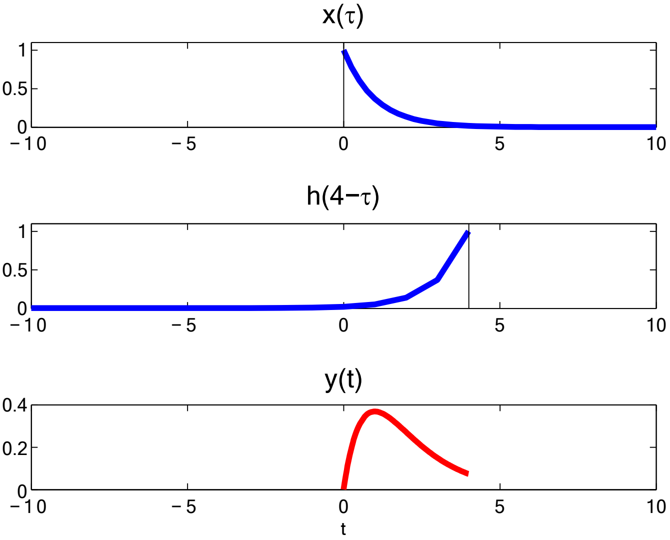

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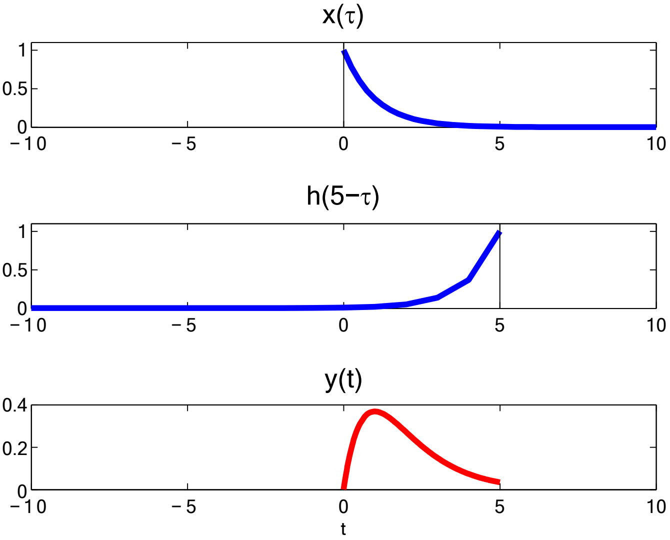

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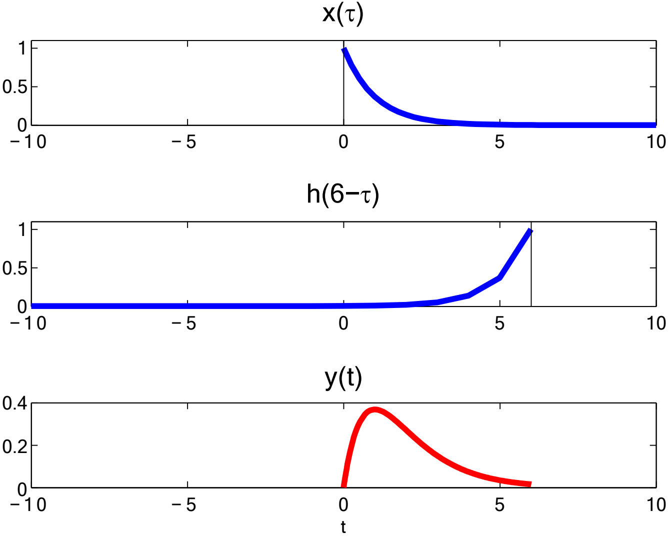

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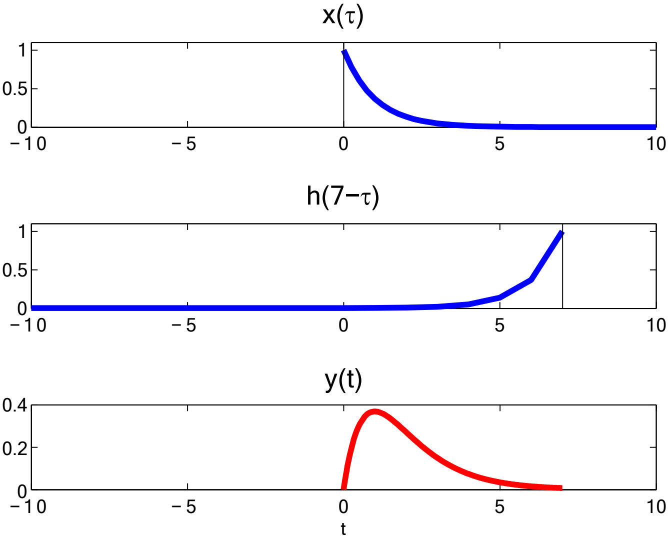

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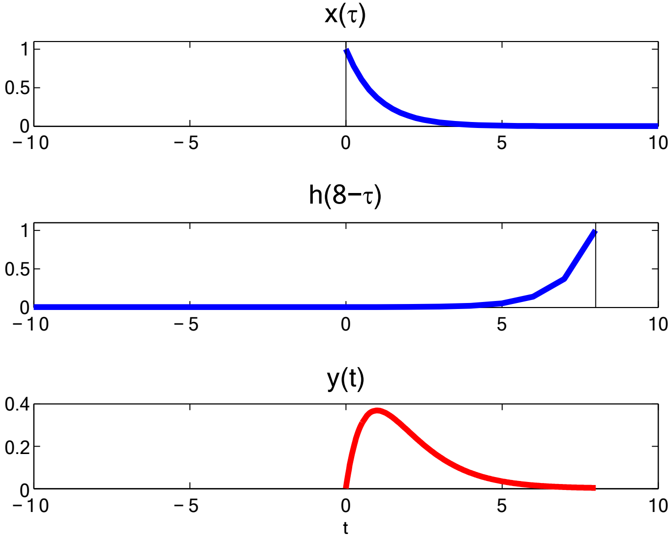

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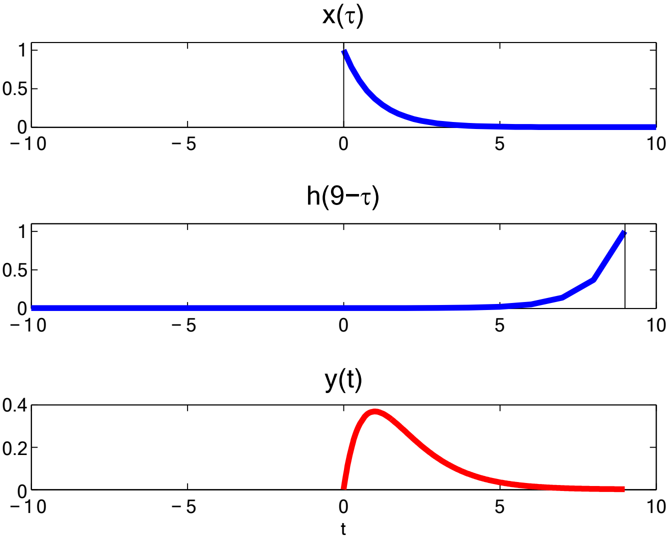
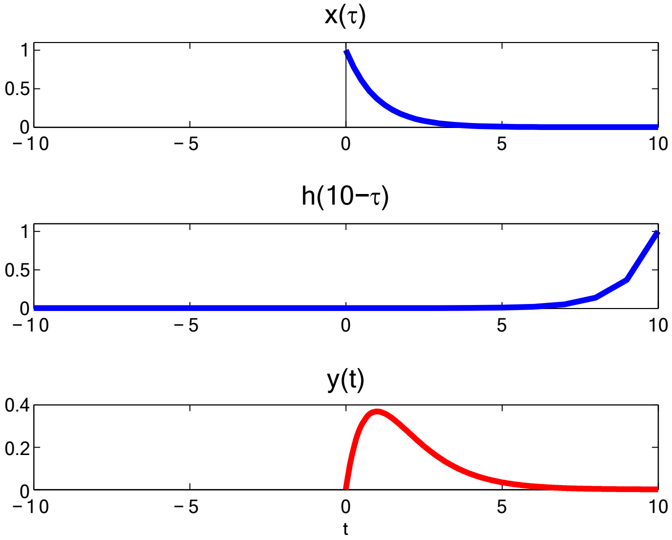


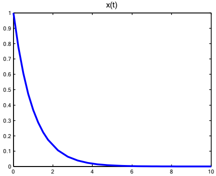
Ejercicio

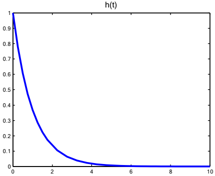


Ejercicio

¿Cuál gráfico muestra el resultado de la convolución de las señales?

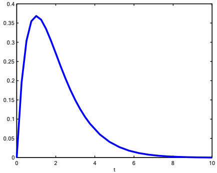
$x(t) = h(t) = e^{-t}u(t)$



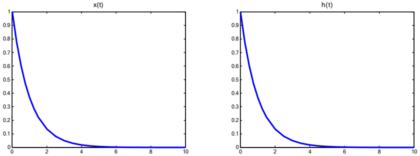


$$(e^{-t}u(t)) * (e^{-t}u(t)) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau$$

$$= \int_0^t e^{-\tau}e^{-(t-\tau)}d\tau = e^{-t}\int_0^t d\tau = te^{-t}u(t)$$

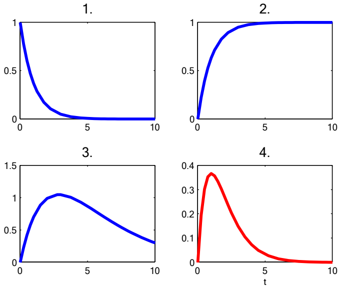


Ejercicio



¿Cuál gráfico muestra el resultado de la convolución de las señales?

$$x(t) = h(t) = e^{-t}u(t) \; 4$$



5. Ninguno de los anteriores.

Convolución

La convolución es una importante herramienta computacional Ejemplo:

Caracterización de un sistema LIT

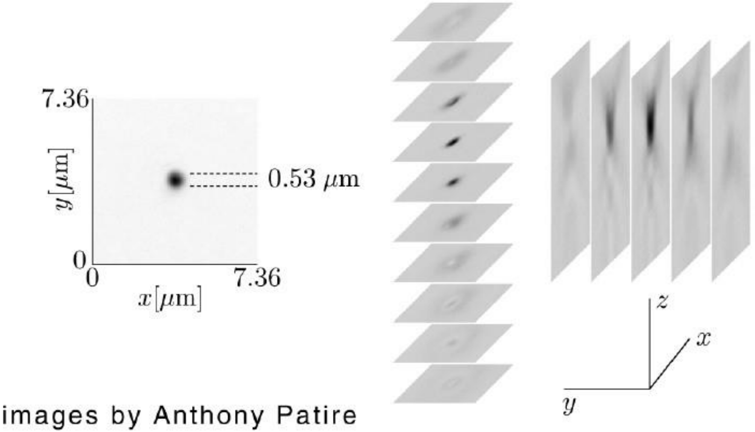
- Determinar la respuesta al impulso unitario $h[n]$.
- Calcular la salida para una entrada arbitraria usando la convolución:

$$y[n] = x[n] * h[n] = \sum x[k]h[n-k]$$

Además es una importante herramienta conceptual: Es un nuevo camino para pensar acerca del comportamiento de los sistemas.

Respuesta al impulso de una lente

La señal borrosa a lo largo del eje óptico se visualiza mejor mediante un nuevo muestreo de la respuesta de impulso de tres dimensiones.



Fuente: MIT OpenCourseWare fall 2011.

Microscopio

Las imgenes de incluso el mejor microscopio son borrosas

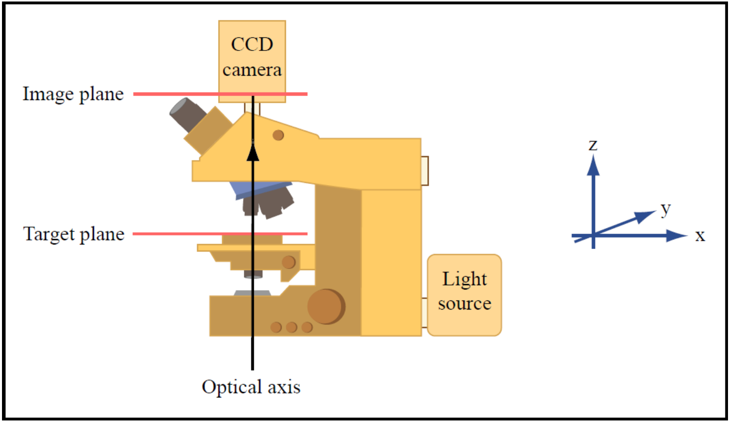


Image by MIT OpenCourseWare.

Microscopio

Un lente perfecto transforma una onda de luz esférica de un objetivo en una onda esférica que converge a la imagen

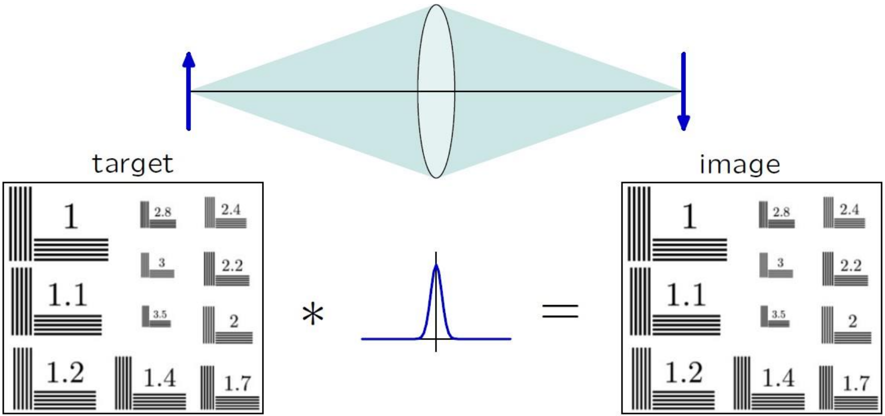


Image by MIT OpenCourseWare.

La imagen borrosa es inversamente relacionada con el diámetro del lente.

Microscopio

La imagen borrosa puede ser representada por la convolución de la imagen con el punto de difusión de función (respuesta al impulso 3D).

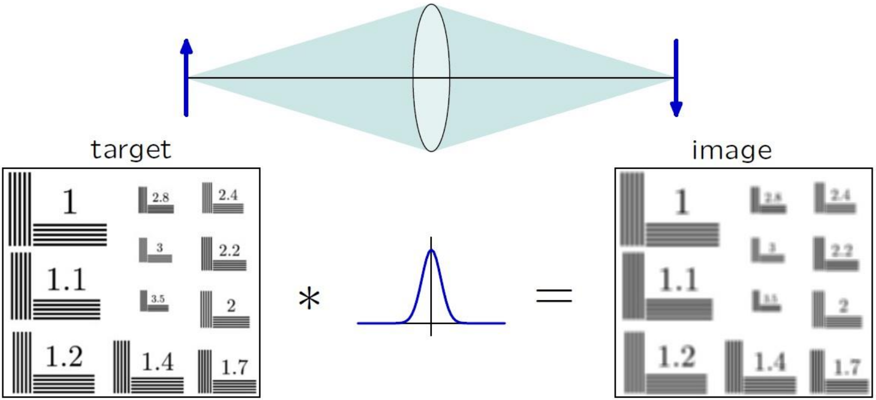


Image by MIT OpenCourseWare.

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Microscopio

La imagen borrosa puede ser representada por la convolución de la imagen con el punto de difusión de función (respuesta al impulso 3D).

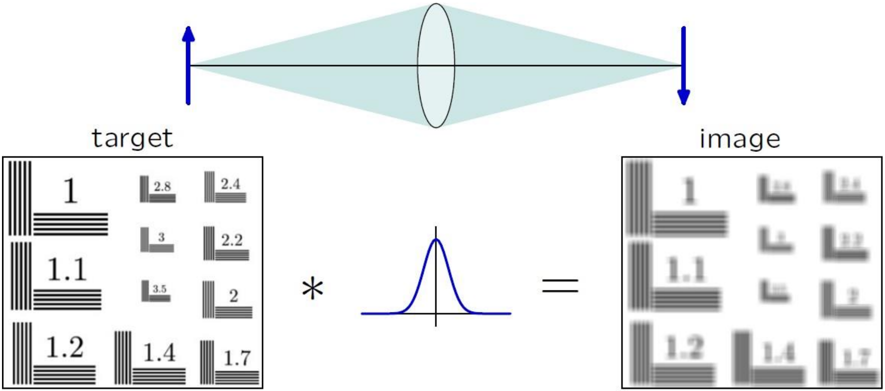

Image by MIT OpenCourseWare.

La imagen borrosa es inversamente relacionada con el diámetro del lente.

Resumen de la clase

- Sistemas LIT discretos: La suma de convolución
- Sistemas LIT continuos: La integral de convolución

Siguiente clase

- Sistemas LIT causales descritos por ecuaciones diferenciales.
- Lecturas recomendadas
  - Secciones 2.4.1
  - Secciones 2.4.2
- del libro *Señales y Sistemas, Alan V. Oppenheim, Segunda Edici´on*.