Universidad Popular del Cesar

Tratamiento de Señales

Version 2022-I

Wavelets

[Capítulo 4]

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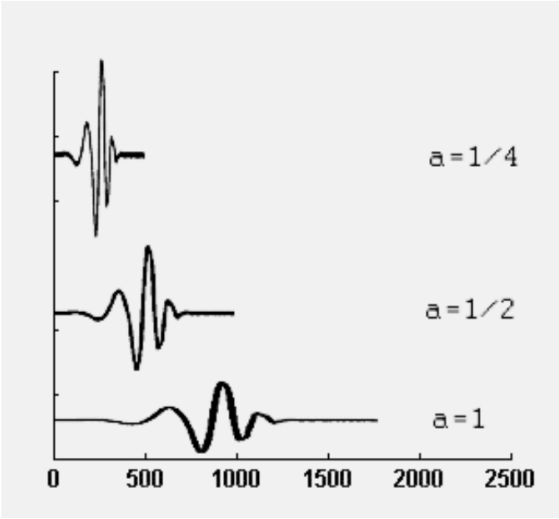
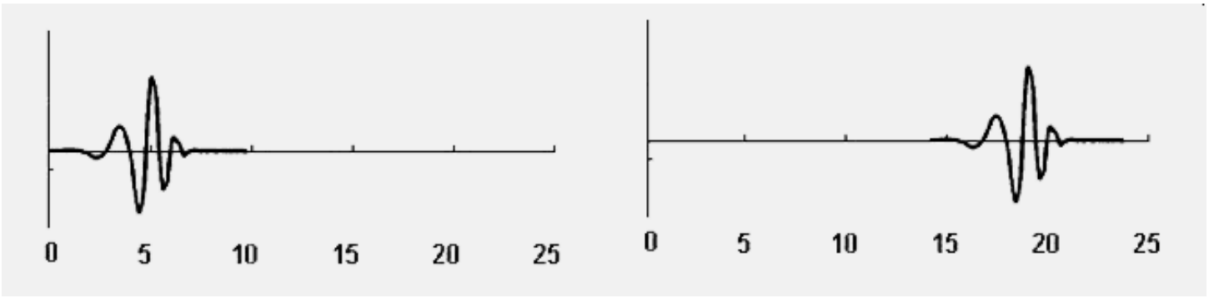
Wavelets are functions in the set of real numbers to the set of real numbers, each of which is derived from the mother using translation and scaling:

$$\Psi_{s,x}(t) = \Psi(2s * t + x)$$

where: s,x – real numbers, Ψ – mother wavelet, Ψs,x – wavelet of scale s and translation x.



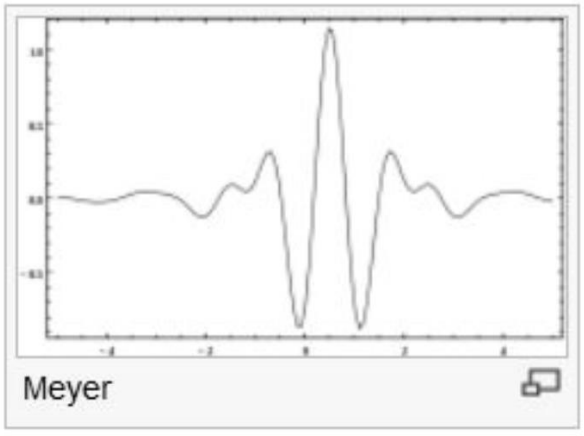


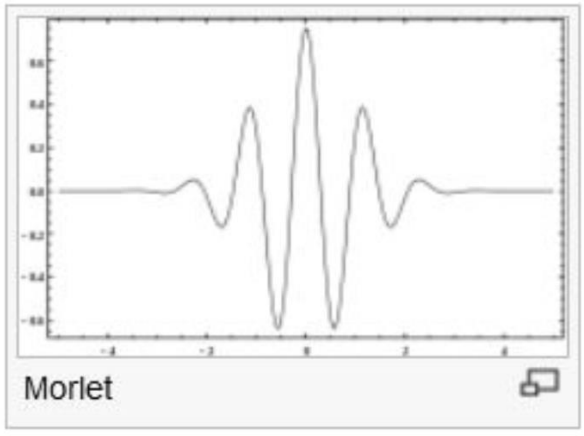
[wikipedia]

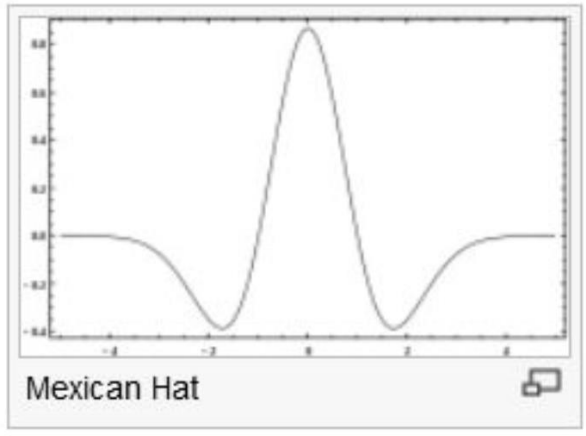
$$\psi^{s,x}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-x}{s}\right), \quad s \neq 0 .$$

$\int \psi(t)dt = 0$ At least several oscillations of mother wavelet (basis function)

$f = \sum_{m,n \in C} c_{m,n}(f)\psi_{m,n}$ Decomposition of *f* using base function

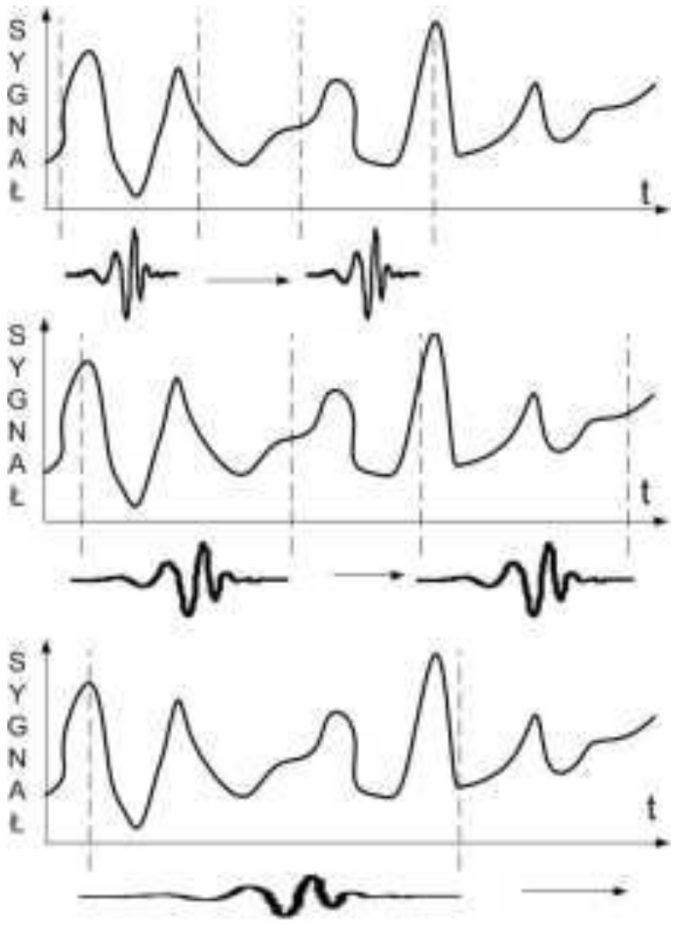

Meyer


Morlet

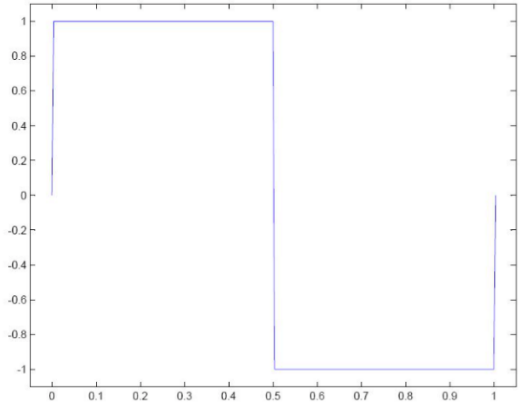

Mexican Hat

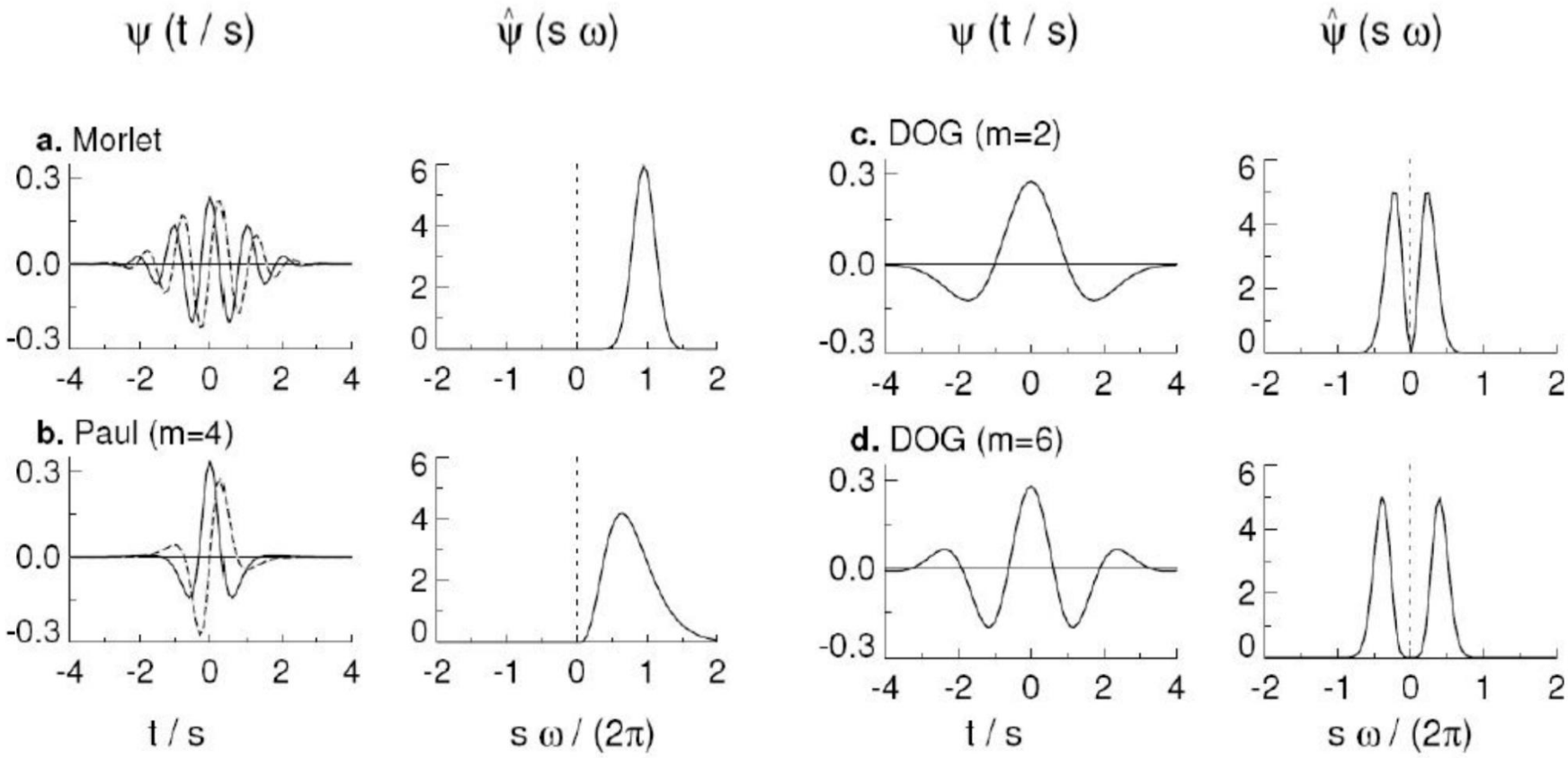
[wikipedia]

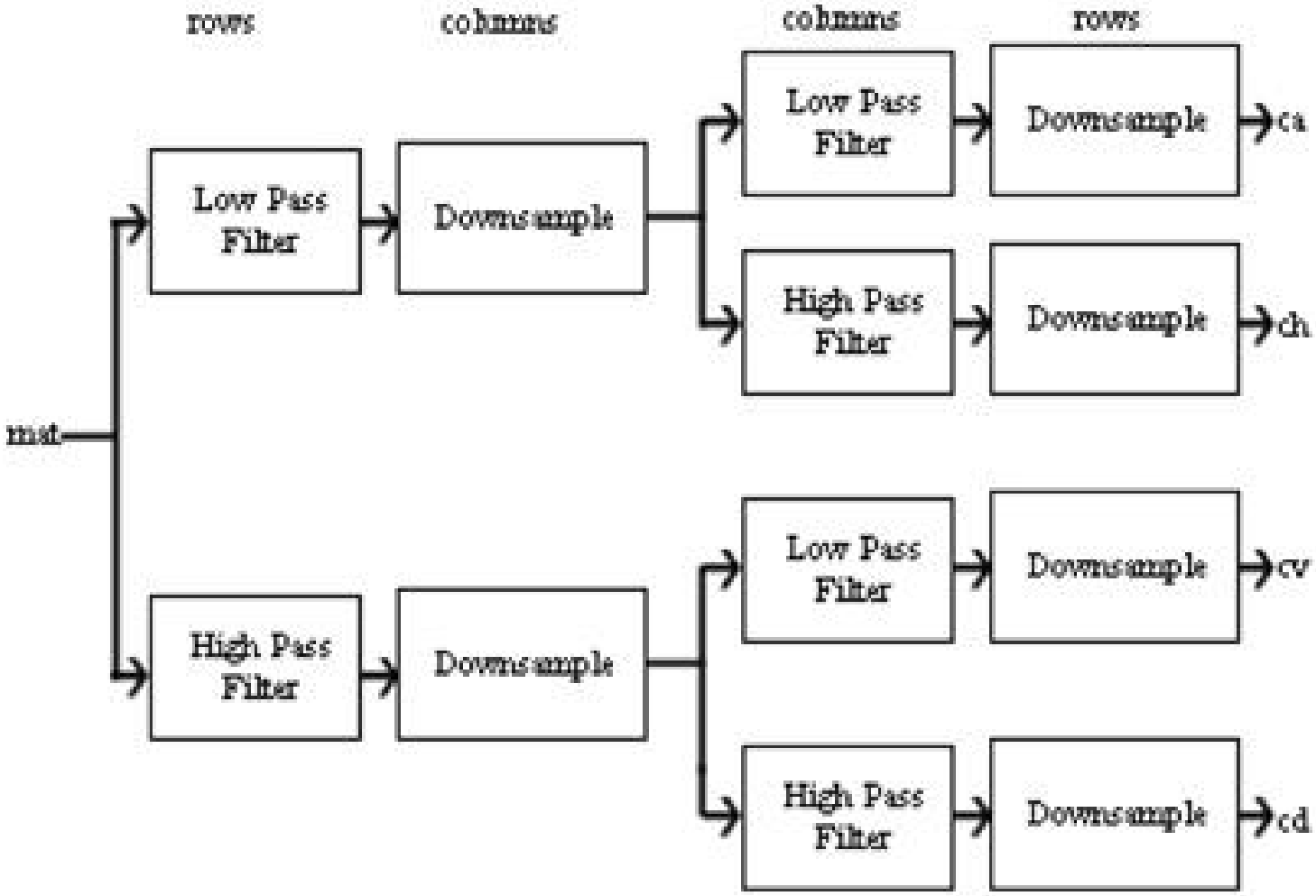
The sum of the weighted functions Ψs,x can represent with any accuracy any continuous function like the cosine functions of different periods allow to represent any periodic function with arbitrary precision



- Mean value = 0 $$\int_{-\infty}^{+\infty} \psi(t)\, dt = 0$$
- normalization $$\|\psi\| = 1$$
- Set around $t=0$
- Finite range of transmision




ψ (t / s)
ψ̂ (s ω)
ψ (t / s)
ψ̂ (s ω)
a. Morlet
c. DOG (m=2)
b. Paul (m=4)
d. DOG (m=6)
0.3
0.0
-0.3
-4 -2 0 2 4
6
4
2
0
-2 -1 0 1 2
t / s
s ω / (2π)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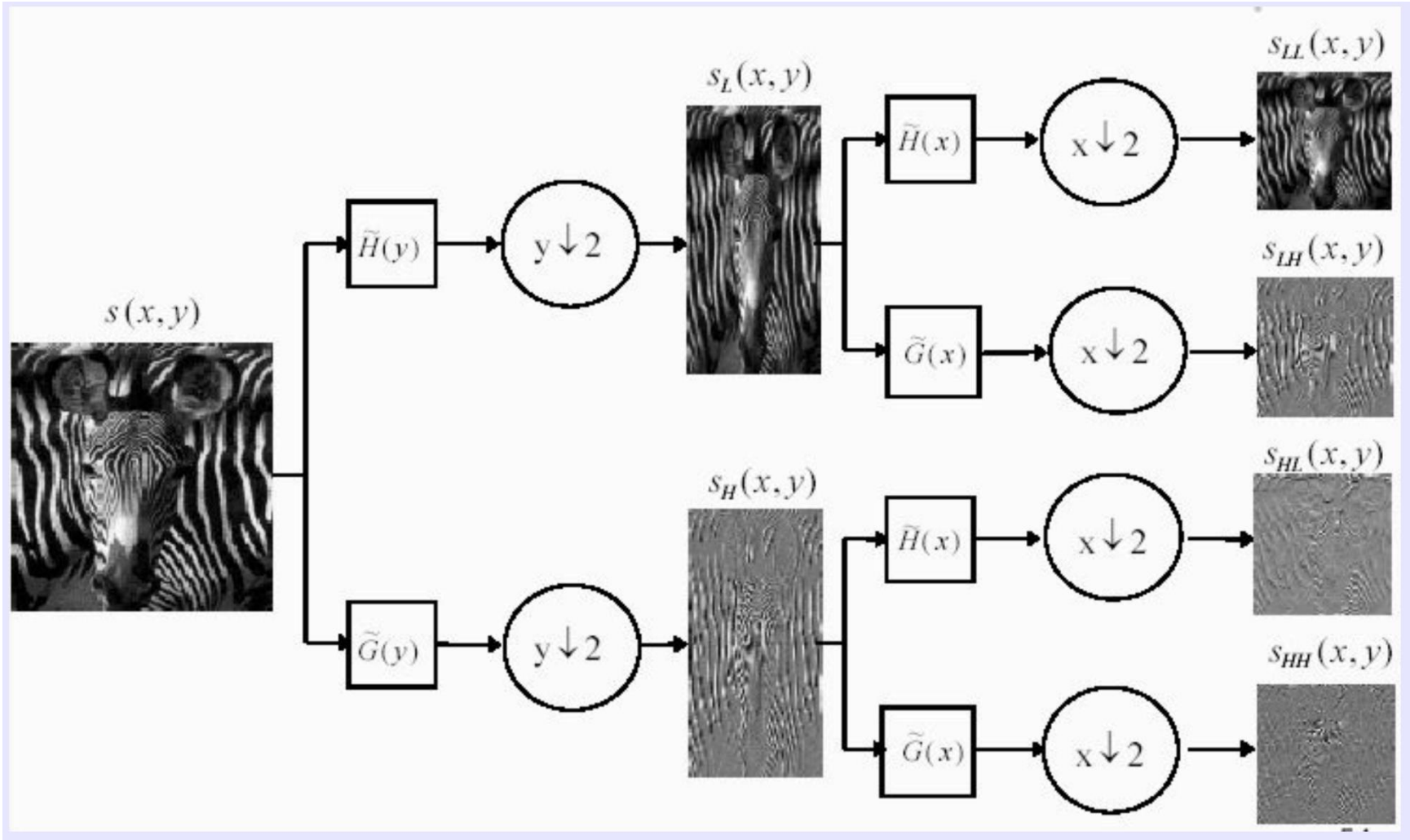
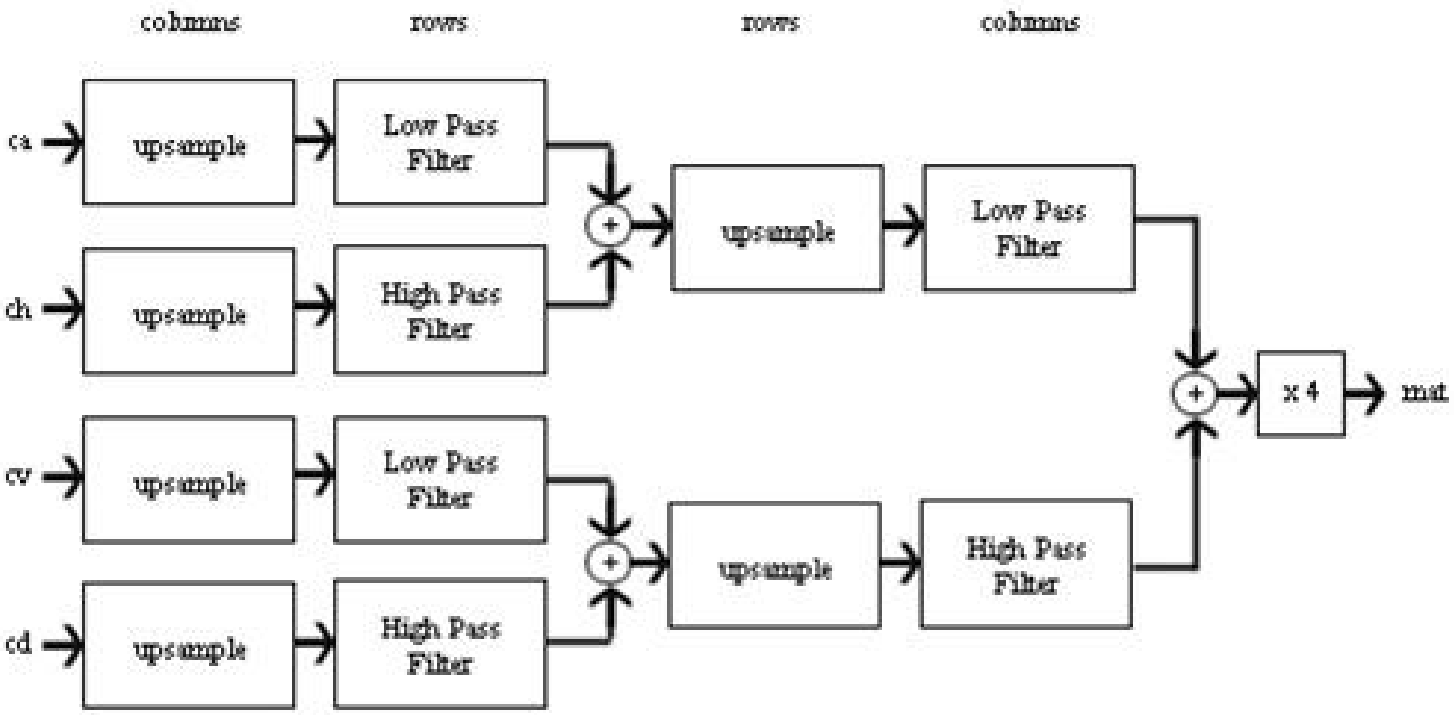

$s(x,y)$
$\widetilde{H}(y)$
$\widetilde{G}(y)$
y↓2
y↓2
$s_L(x,y)$
$s_H(x,y)$
$\widetilde{H}(x)$
$\widetilde{G}(x)$
$\widetilde{H}(x)$
$\widetilde{G}(x)$
x↓2
x↓2
x↓2
x↓2
$s_{LL}(x,y)$
$s_{LH}(x,y)$
$s_{HL}(x,y)$
$s_{HH}(x,y)$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