Reconocimiento de Patrones

Version 2022-2

Confusion Matrix

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[CONFUSION MATRIX]

The confusion matrix, $\mathbf{T}$, is a $K \times K$ matrix, where K is the number of classes of our data. The element $T(i, j)$ of the confusion matrix is defined as the number of samples that belong to class ω_i and were classified as ω_j. A perfect classification means that $T(i,i)$ is N_i and $T(i, j) = 0$ for $i \neq j$, where N_i is the number of samples of class ω_i.

[CONFUSION MATRIX]

predicted

	ω_1	ω_2	ω_3	...	ω_K
ω_1					
ω_2					
ω_3					
:					
ω_K					

actual

[CONFUSION MATRIX]

predicted

actual

	ω_1	ω_2	ω_3	...	ω_K
ω_1	1230	90	78		9
ω_2	66	890	120		5
ω_3	59	95	1527		90
:					
ω_K	40	67	129		912

Example: there are 78 samples of class 1 that have been classified as class 3.

[CONFUSION MATRIX: TWO CLASSES]

		predicted	
		ω_1	ω_0
actual	ω_1		
	ω_0		

[CONFUSION MATRIX: TWO CLASSES]

TARGET DETECTION

ω_1 Target

ω_0 No-Target

predicted

actual

	ω_1	ω_0
ω_1	TP	FN
ω_0	FP	TN

- True Positive (TP): number of targets correctly classified.
- True Positive (TN): number of non–targets correctly classified.
- False Positive (FP): number of non–targets classified as targets. The false positives are known as 'false alarms' and 'Type I error'.
- False Negative (FN): number of targets classified as no-targets. The false negatives are known as 'Type II error'.

[CONFUSION MATRIX: TWO CLASSES]

predicted

actual

	ω_1	ω_0
ω_1	TP	FN
ω_0	FP	TN

P = TP+FN (positive instances)

N = FP+TN (negative instances)

D = TP+FP (detections)

- True Positive (TP): number of targets correctly classified.
- True Negative (TN): number of non–targets correctly classified. .
- False Positive (FP): number of non–targets classified as targets. The false positives are known as 'false alarms' and 'Type I error'.
- False Negative (FN): number of targets classified as no-targets. The false negatives are known as 'Type II error'.

[DEFINITIONS]

True positive rate, known as Sensitivity or Recall:

$$TPR = S_n = Re = \frac{TP}{P} = \frac{TP}{TP+FN}$$

Precision or Positive Predictive Value:

$$Pr = \frac{TP}{D} = \frac{TP}{TP+FP}$$

True negative rate, known as Specificity:

$$TNR = Sp = \frac{TN}{N} = \frac{TN}{TN+FP}$$

False positive rate, known as 1-Specificity:

$$FPR = 1 - Sp = \frac{FP}{N} = \frac{FP}{TN+FP}$$

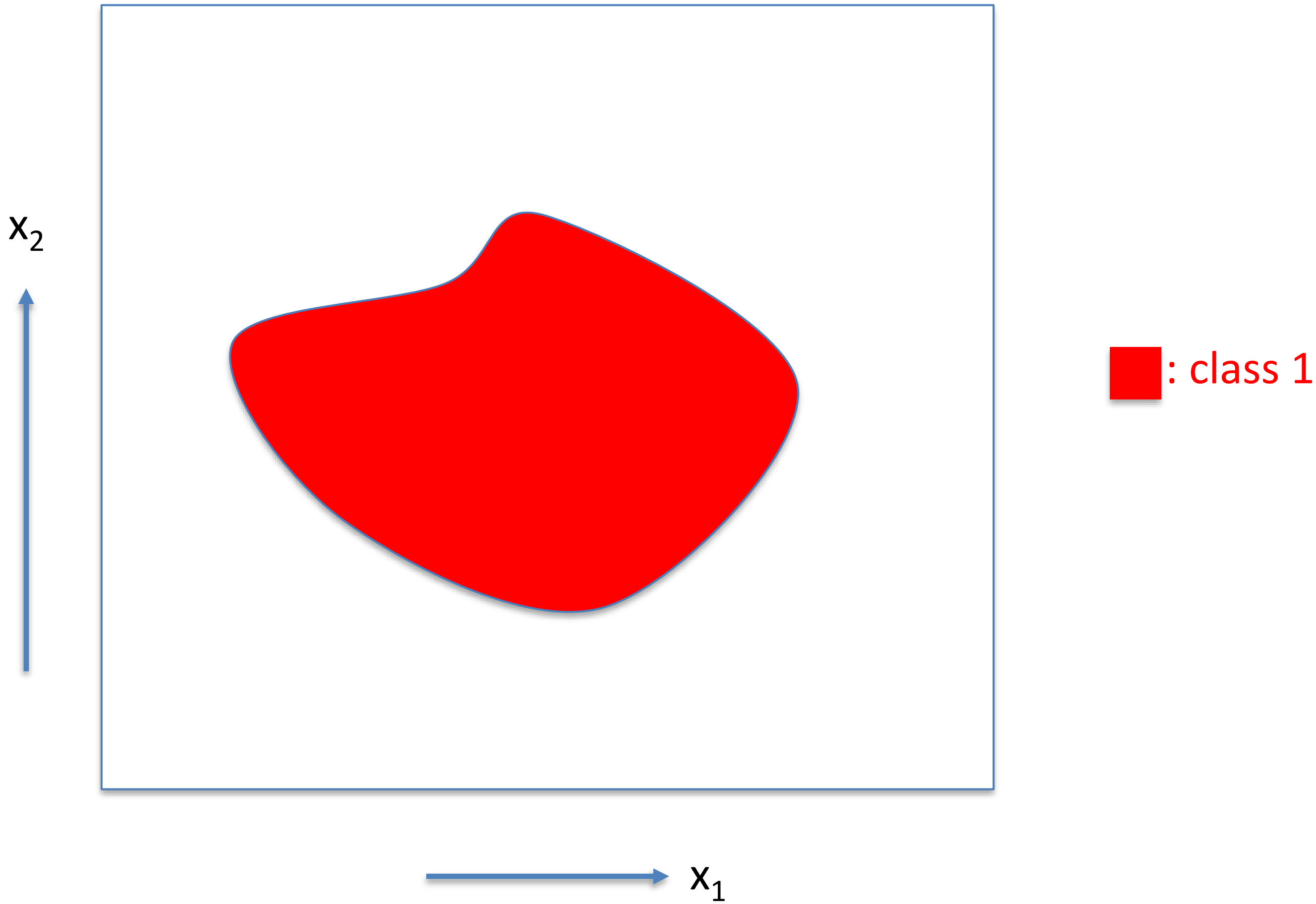
False negative rate, known as Miss Rate:

$$FNR = MR = \frac{FN}{P} = \frac{FN}{TP+FN}$$

Accuracy:

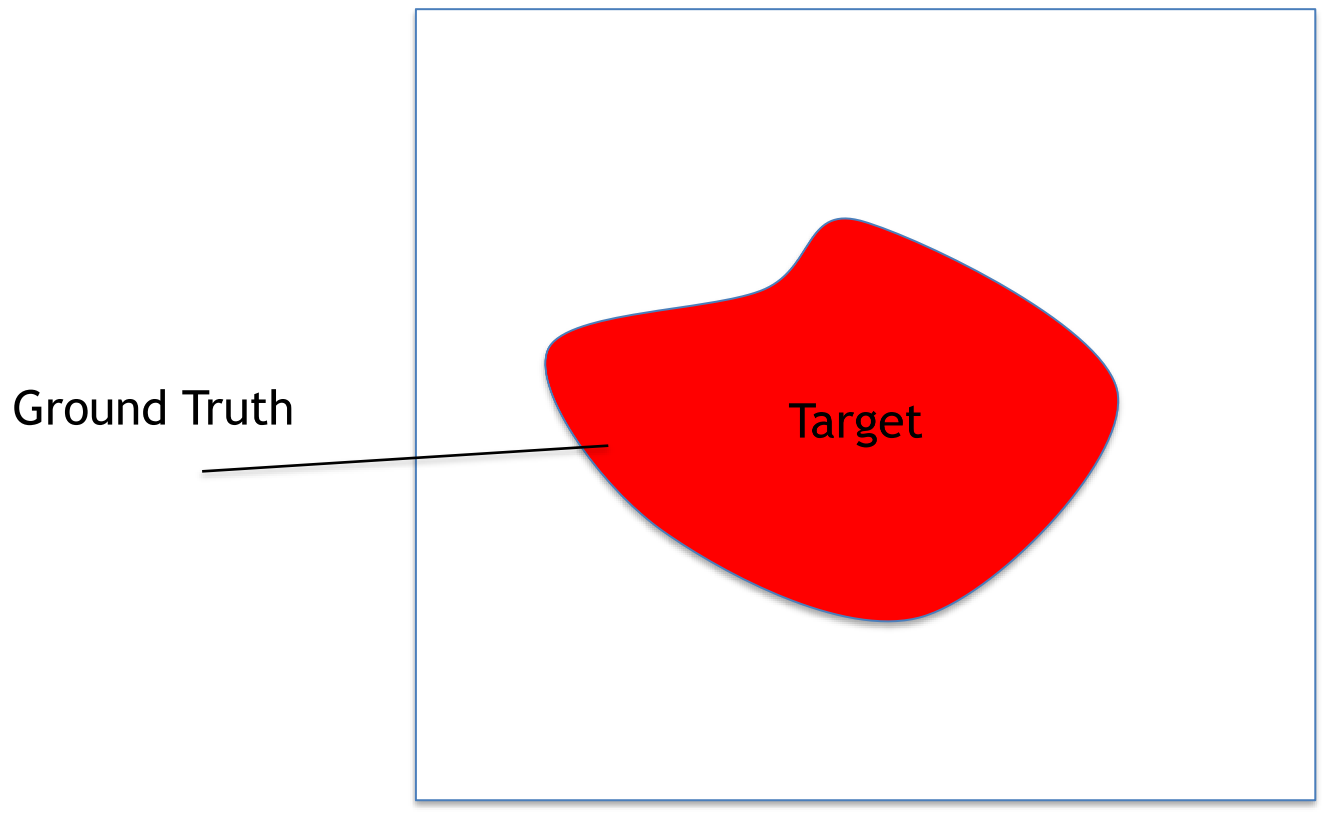
$$ACC = \frac{TP+TN}{P+N}$$

[EXAMPLE]

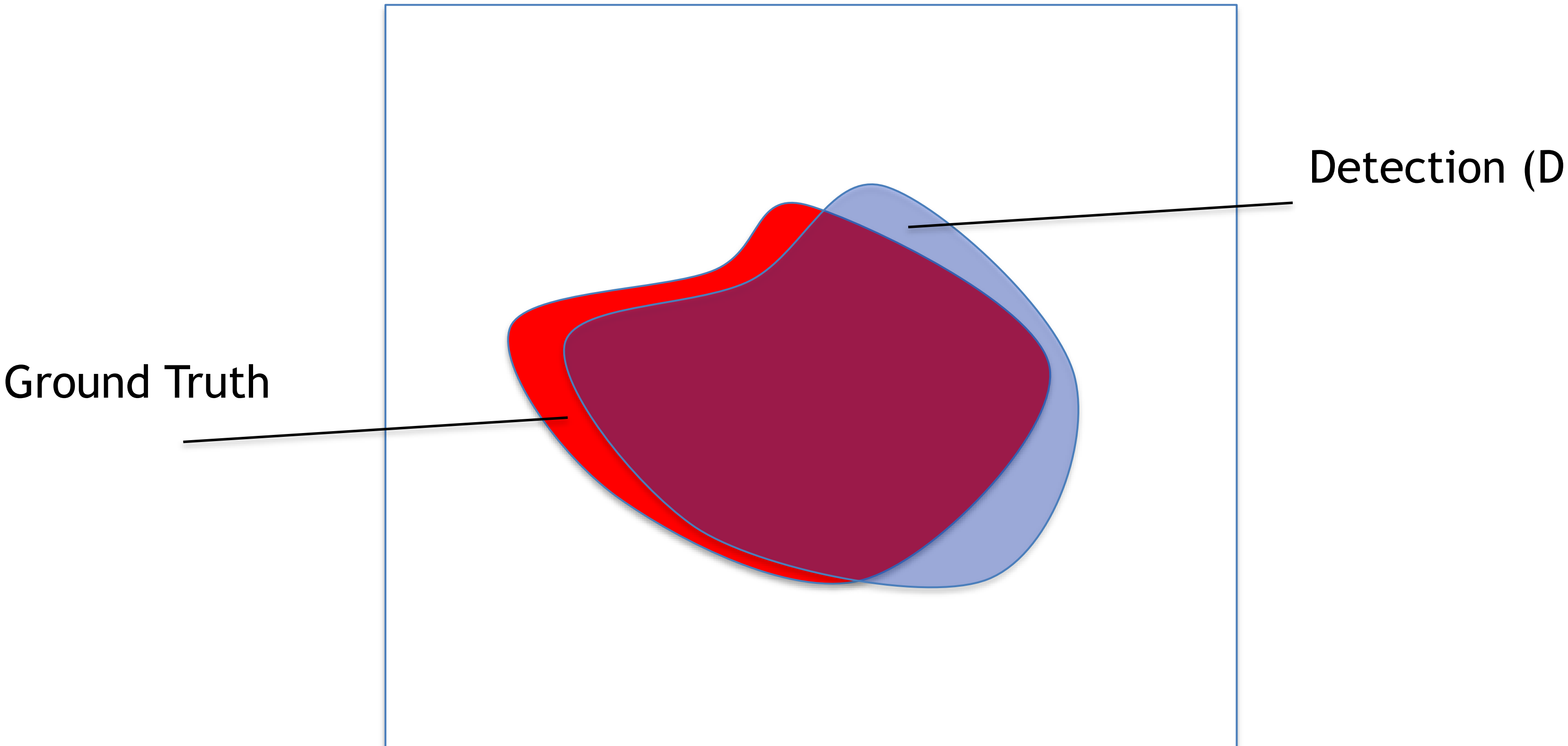


[EXAMPLE]

Red pixels: Positive instances (P)
White pixels: Negative instances (N)

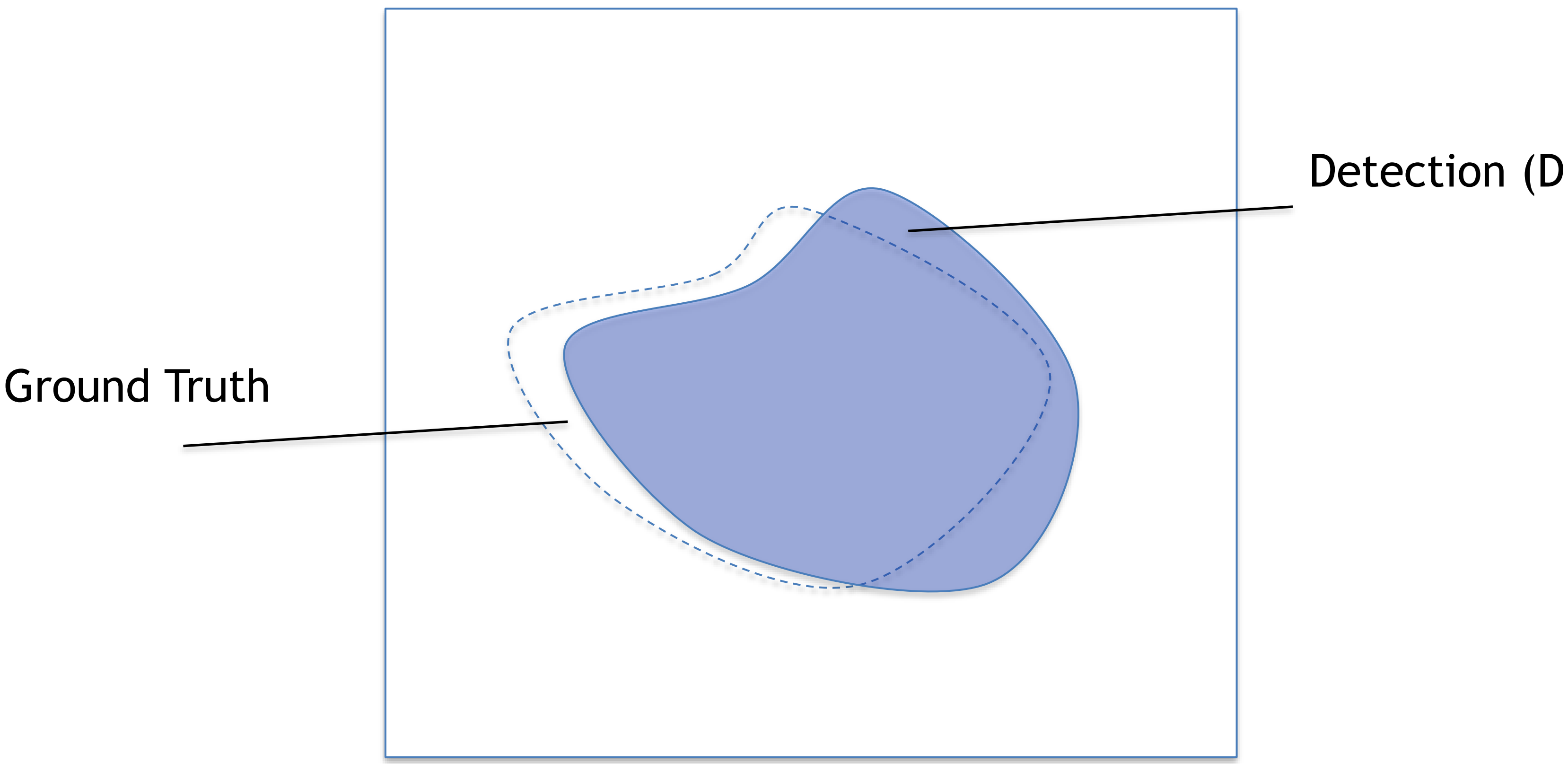


[EXAMPLE]



Detection using a classifier

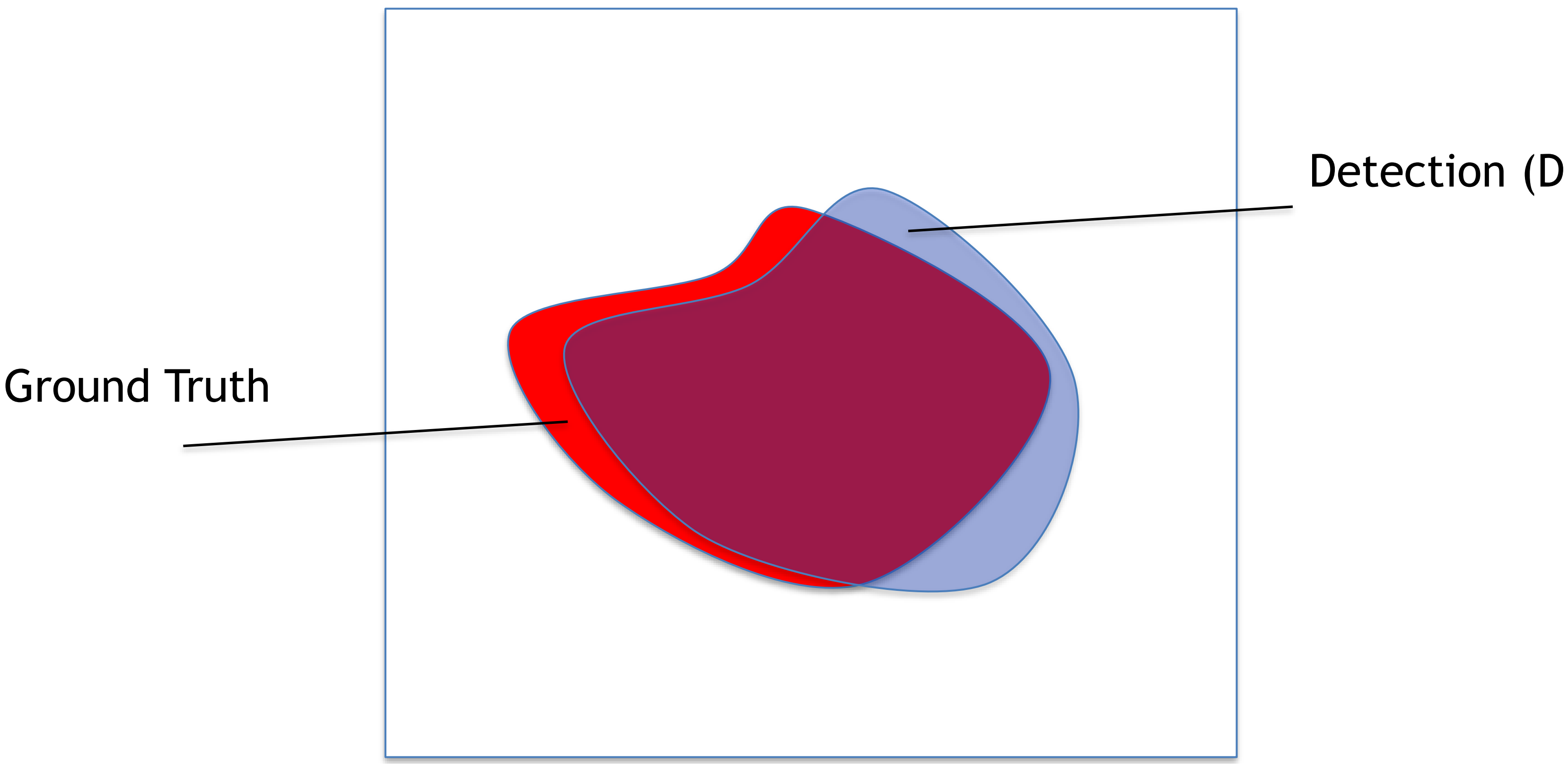
[EXAMPLE]



Detection using a classifier

[EXAMPLE]

Magenta pixels: True positives (TP)
Blue pixels: False positives (FP)
Red pixels: False negatives (FN)
White pixels: True negatives (TN)



[EXAMPLE]

Magenta pixels: True positives (TP)
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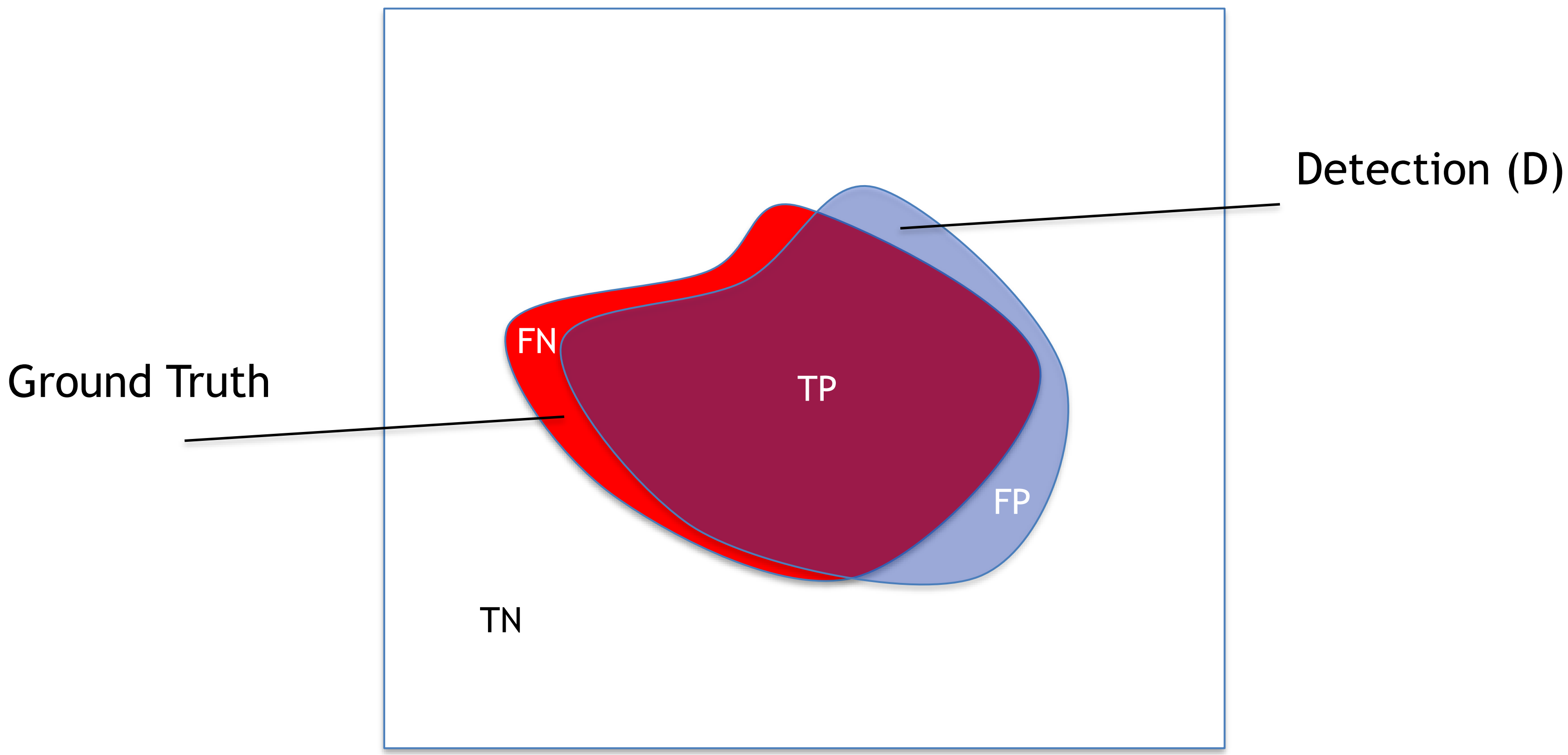
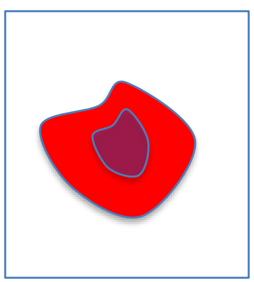
Red pixels: False negatives (FN)
White pixels: True negatives (TN)



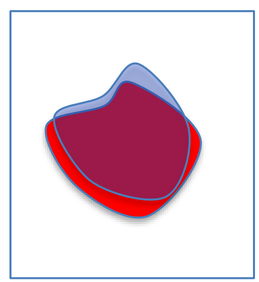
IDEAL
TPR = 100%
FPR = 0%

Magenta pixels: True positives (TP)
Blue pixels: False positives (FP)
Red pixels: False negatives (FN)
White pixels: True negatives (TN)

Extreme
No false positive



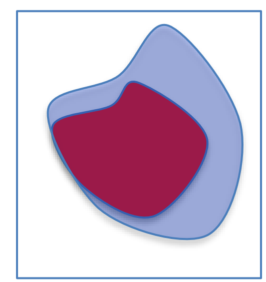
No false alarm



Reality:
Trade-off between
FPR and TPR

Extreme

All positive samples
are detected

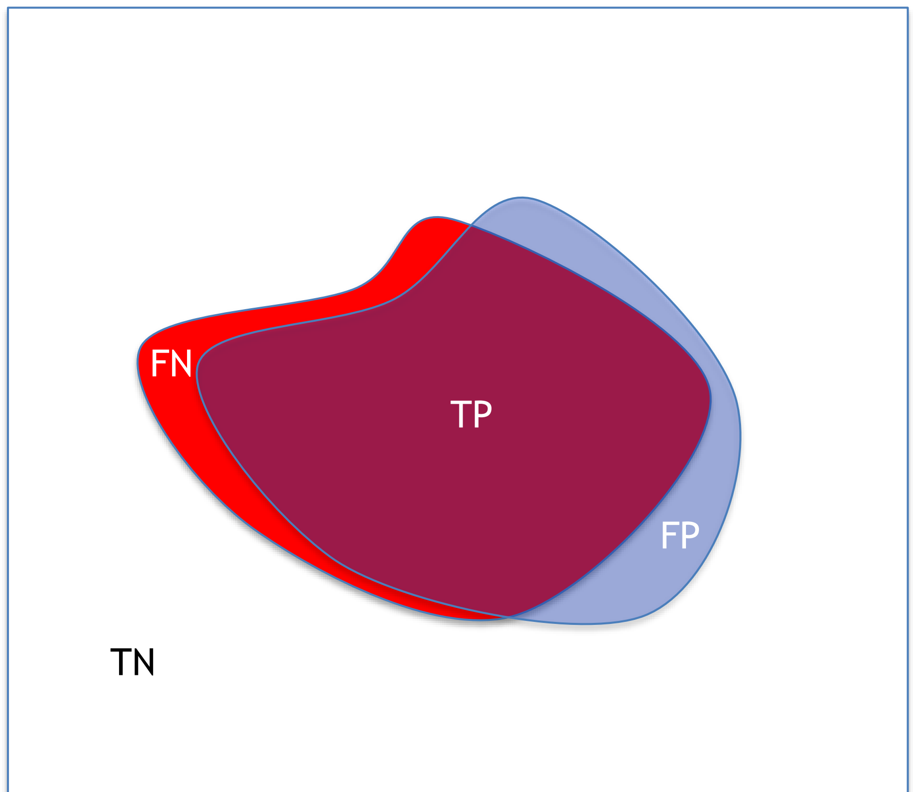


All targets detected

Curve
Precision – Recall

[EXAMPLE]

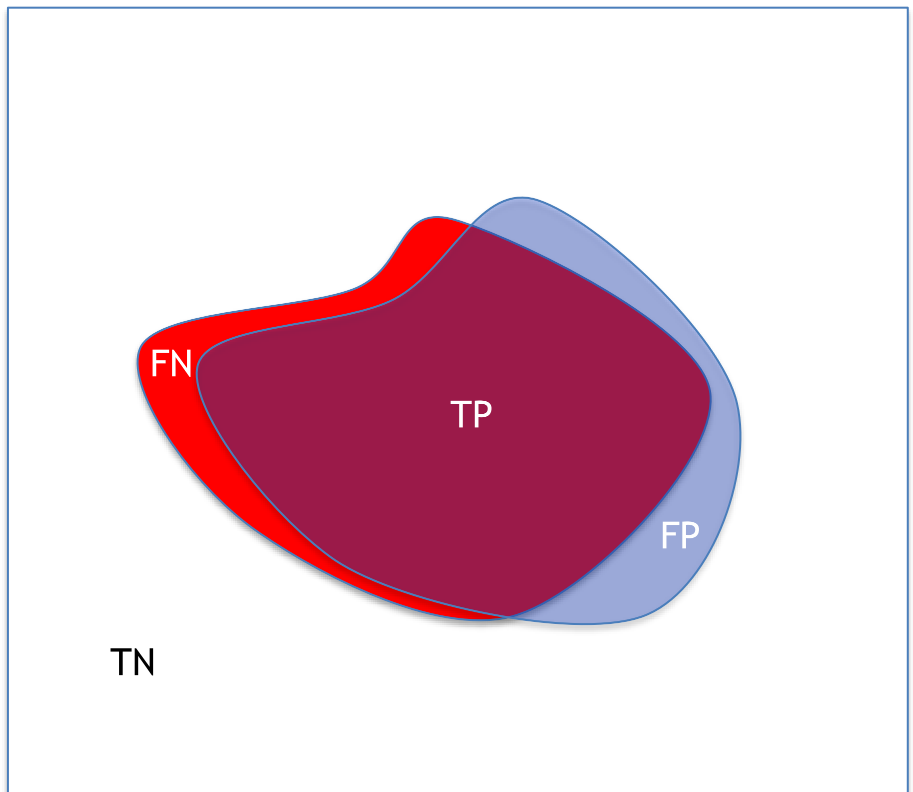
Magenta pixels: True positives (TP)
Blue pixels: False positives (FP)
Red pixels: False negatives (FN)
White pixels: True negatives (TN)

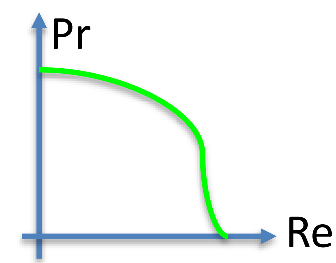


[EXAMPLE]

Magenta pixels: True positives (TP)
Blue pixels: False positives (FP)
Red pixels: False negatives (FN)
White pixels: True negatives (TN)

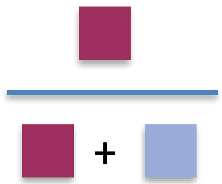
Pr = TP/(TP+FP)
Re = TP/(TP+FN)



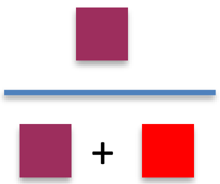


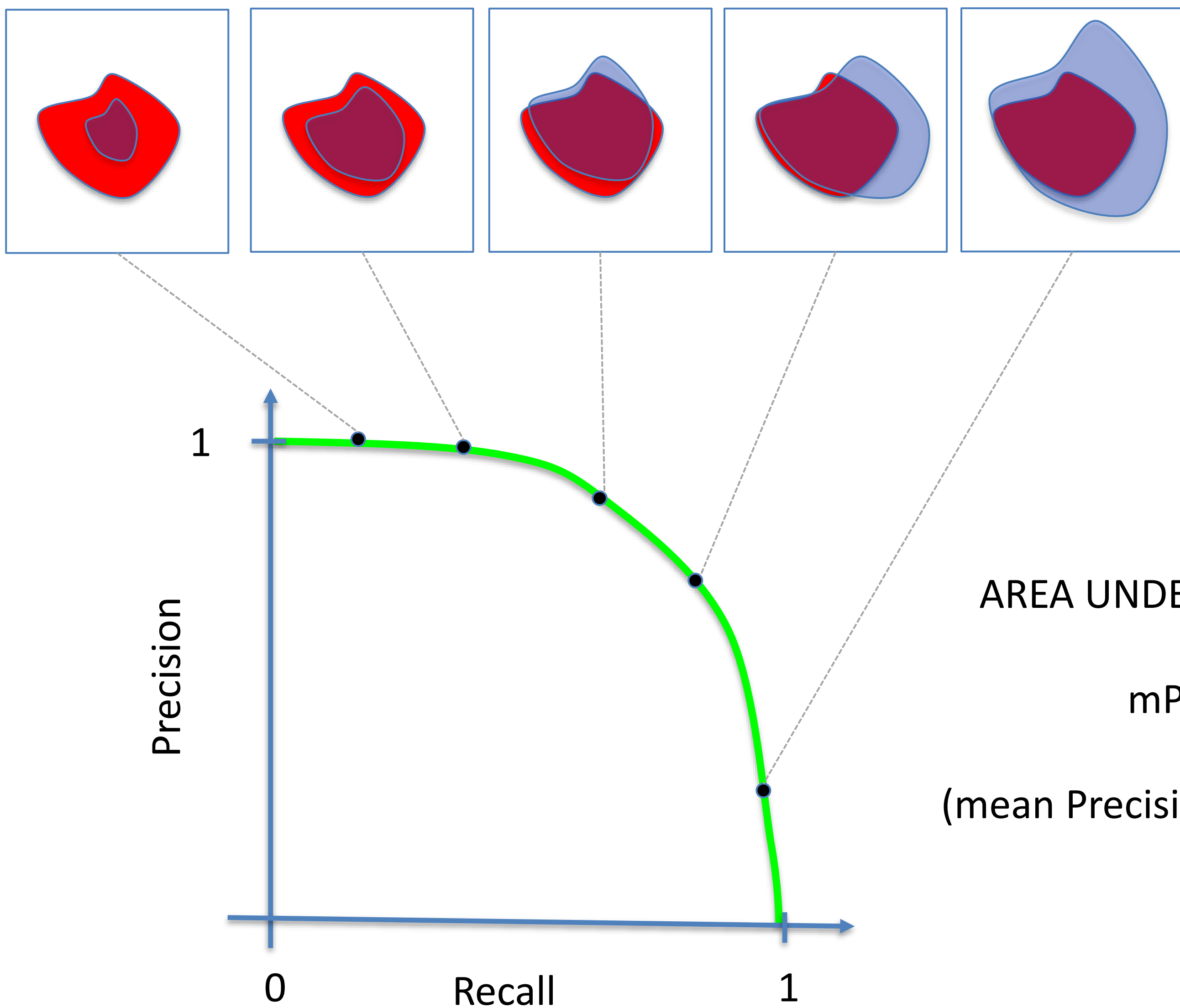
Precision-Recall curve

$$\text{Precision} = \frac{TP}{TP+FP} =$$



$$\text{Recall} = \frac{TP}{TP+FN} =$$





AREA UNDER CURVE:

mPA

(mean Precision Average)

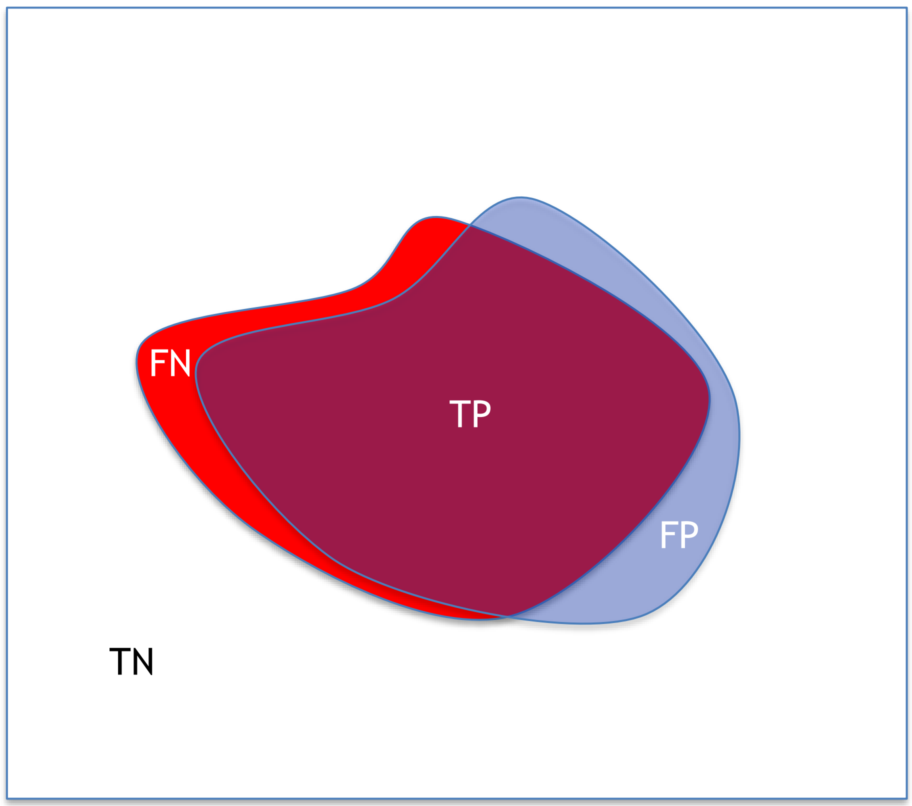
Curve
ROC
(Receiver Operation Characteristic)

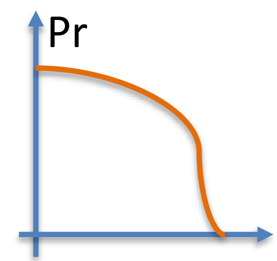
[EXAMPLE]

Magenta pixels: True positives (TP)
Blue pixels: False positives (FP)
Red pixels: False negatives (FN)
White pixels: True negatives (TN)

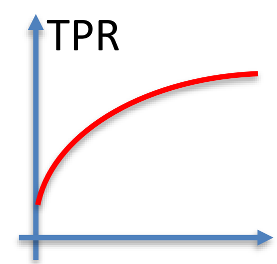
Pr = TP/(TP+FP)
Re = TP/(TP+FN)

TPR = TP/(TP+FN)
FPR = FP/(TN+FP)





Precision-Recall curve



ROC curve

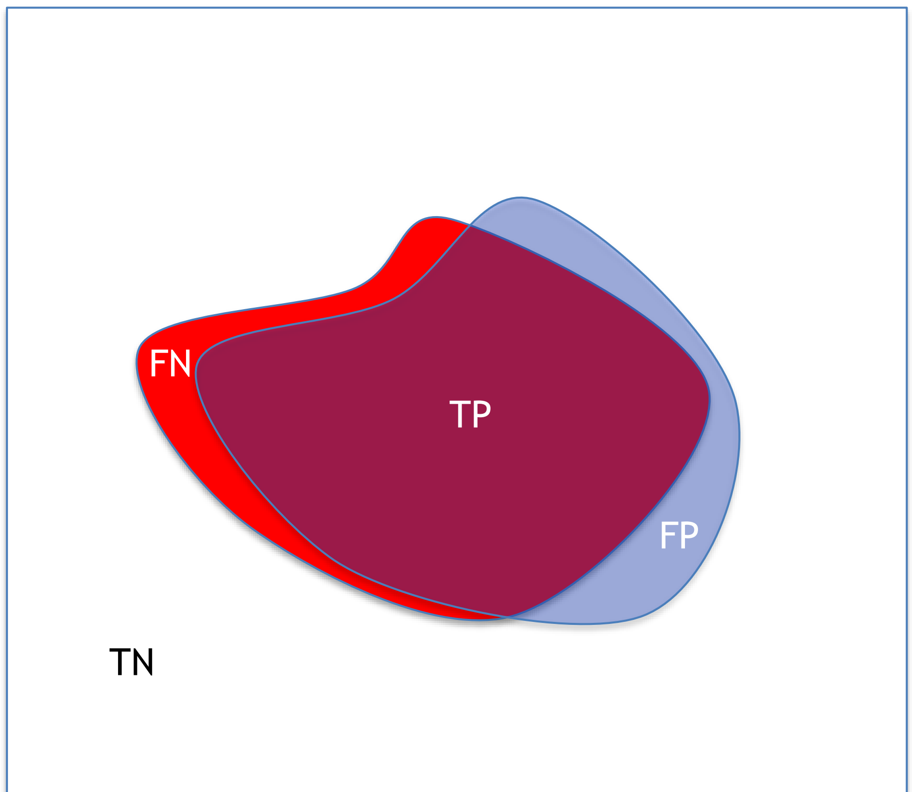
[EXAMPLE]

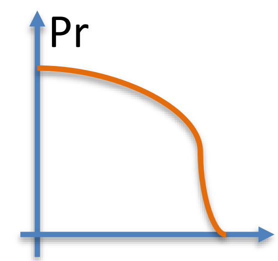
Magenta pixels: True positives (TP)
Blue pixels: False positives (FP)
Red pixels: False negatives (FN)
White pixels: True negatives (TN)

Pr = TP/(TP+FP)
Re = TP/(TP+FN)

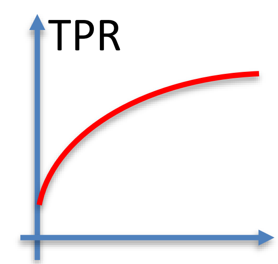
TPR = TP/(TP+FN)
FPR = FP/(TN+FP)

Sensitivity = TPR
Specificity = 1-FPR





Precision-Recall curve

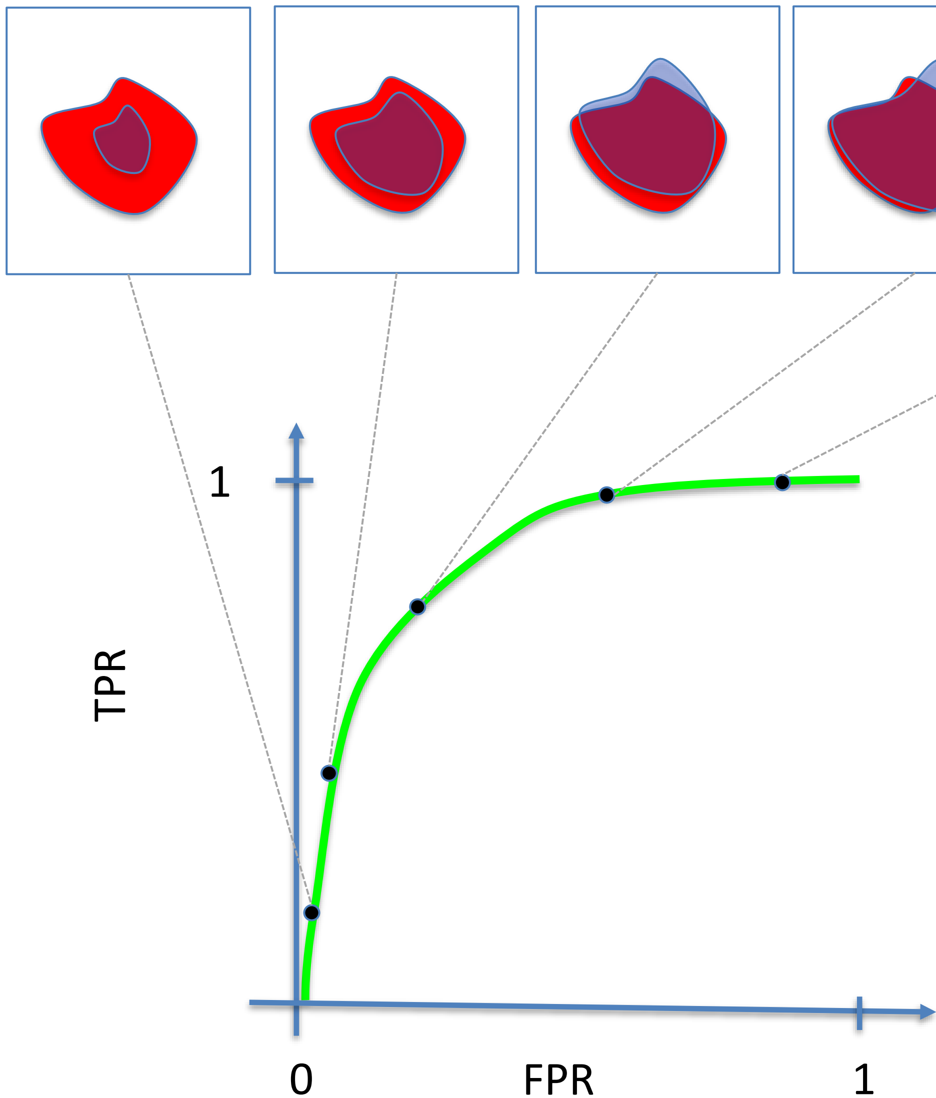


ROC curve

[ROC curve – Example]

$$TPR = \frac{TP}{TP+FN} =$$

$$FPR = \frac{FP}{TN+FP} =$$

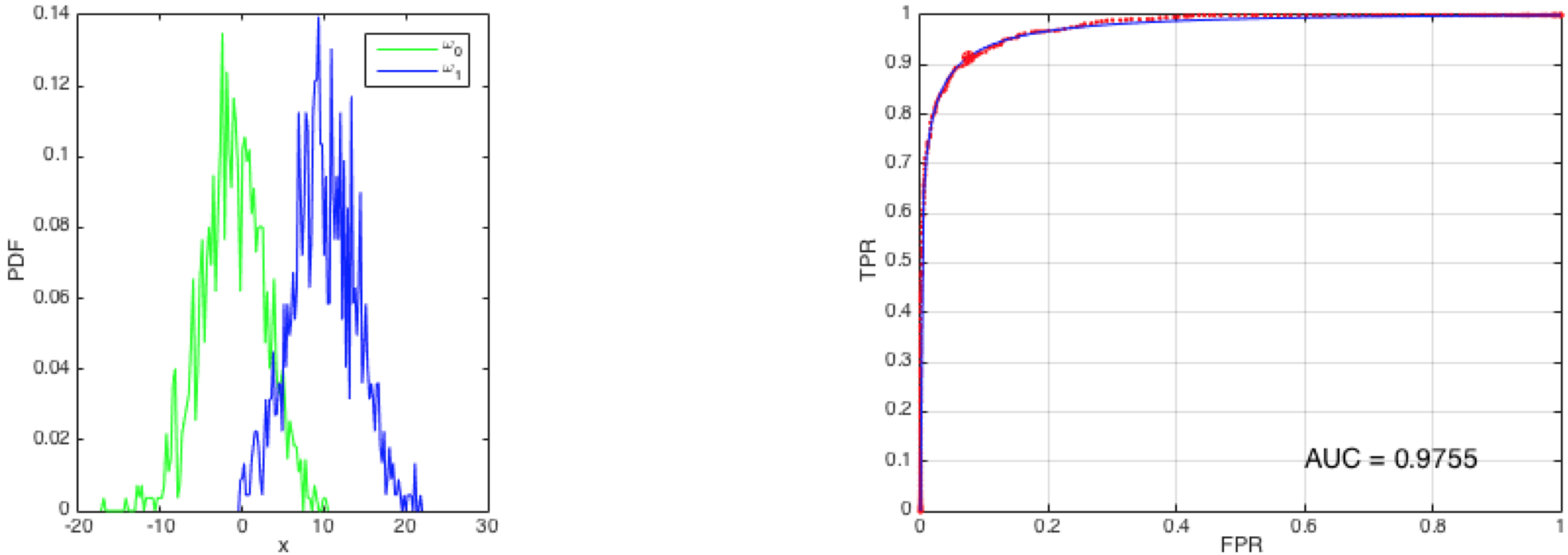


AREA UNDER CURVE:

Az

(AUC)

[ROC curve – Example]



ROC curves (right) for different class distributions (left). The area under the curve (AUC) give a good measure of the performance of the detection. The obtained points (x_i, y_i) are used to fit the ROC curve to $y = (1 - a^{\gamma x^b})/(1 - a^{\gamma})$. In each ROC curve, the 'best operation point' is shown as *. This point is defined as the closest point to ideal operation point (0,1).

[ROC curve – Example]