

# Minería de datos y Patrones

Version 2024-I

## Características Geométricas

[ Capítulo 2 ]

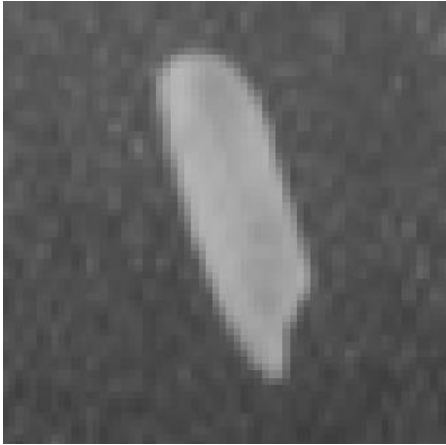
**Dr. José Ramón Iglesias**

DSP-ASIC BUILDER GROUP

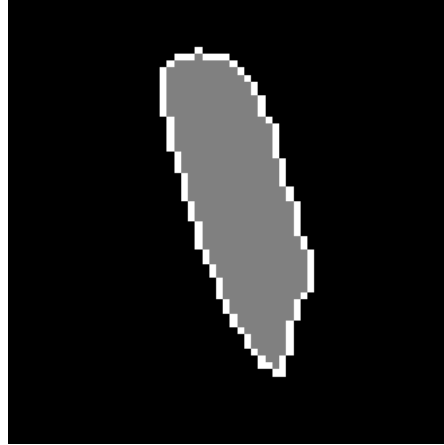
Director Semillero TRIAC

Ingeniería Electronica

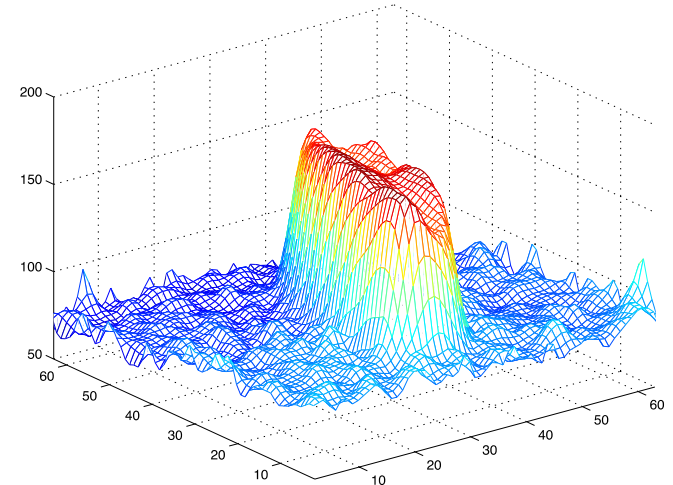
Universidad Popular del Cesar



a) Grayscale image



b) Segmentation



c) 3D representation of a)

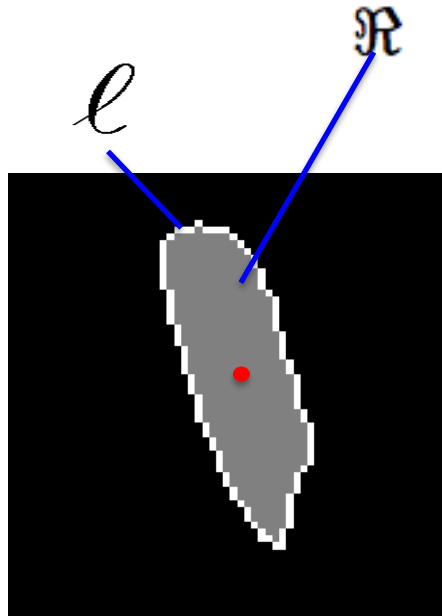
There are two categories of features:

Geometric Features and  
Intensity Features

Geometric Features give information about location, orientation, shape and size.

Intensity Features give information about how are the grayvalues.

# **Geometric Features**



Area and Perimeter

$A$  = # of gray pixels

$L$  = # of white pixels

Height and width of  $\mathcal{R}$

$$h = i_{max} - i_{min} + 1$$

$$w = j_{max} - j_{min} + 1$$

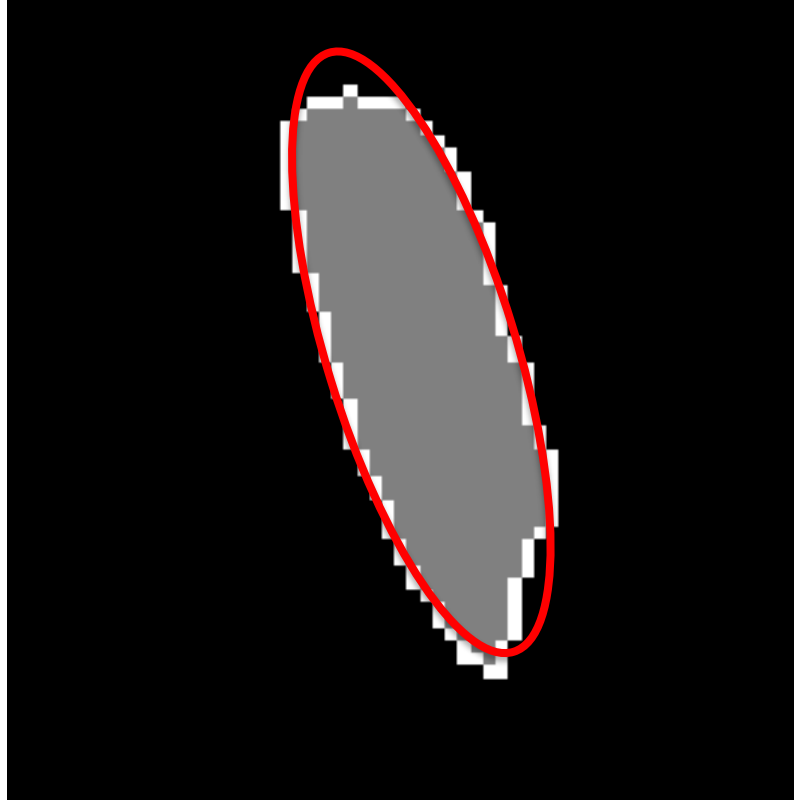
Roundness

$$R = \frac{4 \cdot A \cdot \pi}{L^2}$$

Center of Mass

$$(i_m, j_m)$$

# Ellipses



# Ellipses

Major axis ( $a$ )

Minor axis ( $b$ )

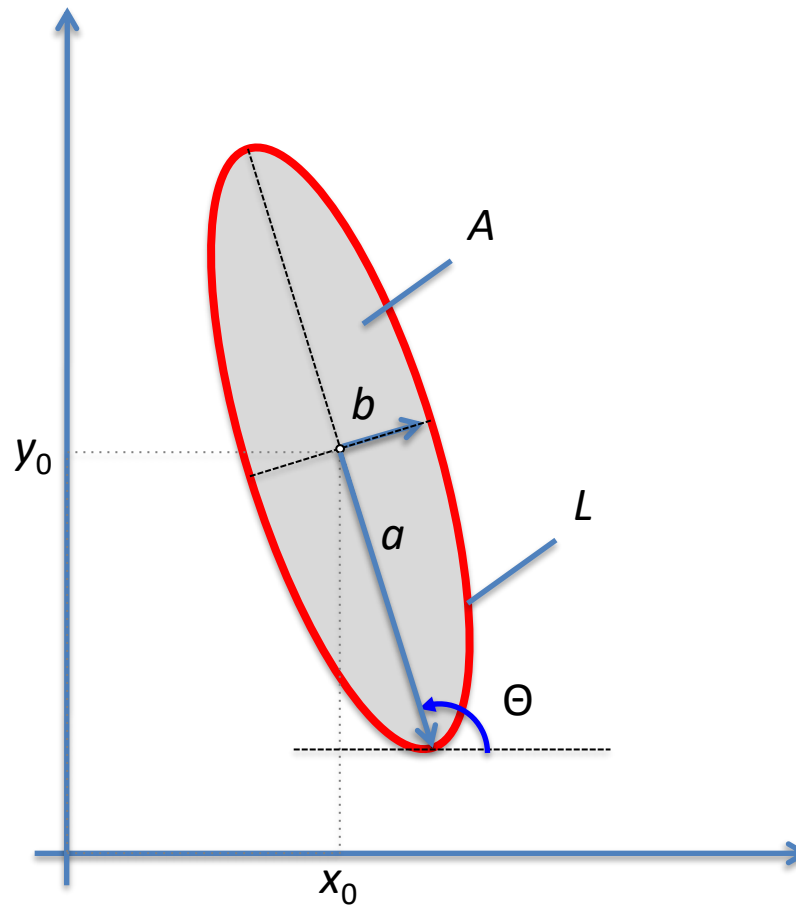
Orientation ( $\theta$ )

Center  $x_0, y_0$

Area  $A$

Perimeter  $L$

Eccentricity  $E = b/a$



# Moments

$$m_{rs} = \sum_{i,j \in \mathfrak{R}} i^r j^s \quad \text{for } r, s \in \mathcal{N}$$

$$\bar{i} = \frac{m_{10}}{m_{00}} \quad \bar{j} = \frac{m_{01}}{m_{00}}$$

$$\mu_{rs} = \sum_{i,j \in \mathfrak{R}} (i - \bar{i})^r (j - \bar{j})^s \quad \text{for } r, s \in \mathcal{N}$$

# Hu - Moments

They are invariant against:

- scale,
- rotation and
- location

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + \\ (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$

$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + \\ 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - \\ (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$

with

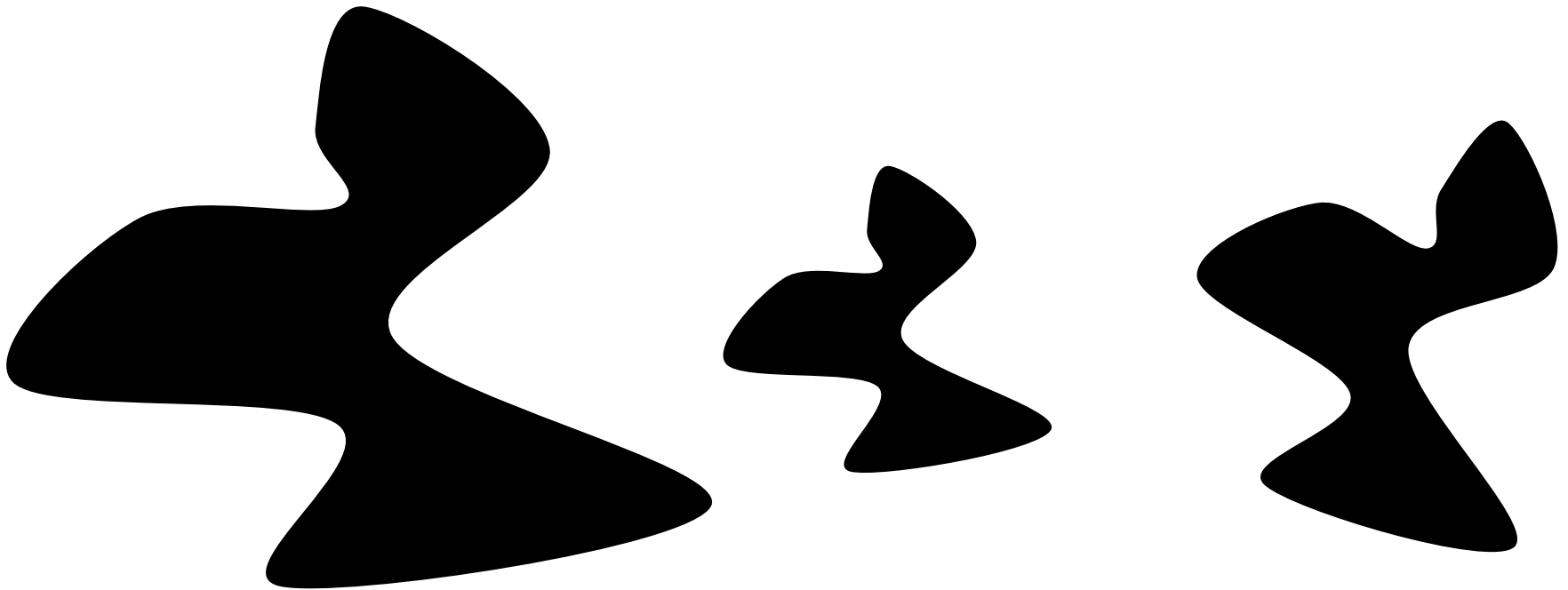
$$\eta_{rs} = \frac{\mu_{rs}}{\mu_{00}^t} \quad t = \frac{r+s}{2} + 1.$$



# Hu - Moments

They are invariant against:

- scale,
- rotation and
- location



They have similar  $\Phi_1, \Phi_2, \dots, \Phi_7$ .

# Hu - Moments

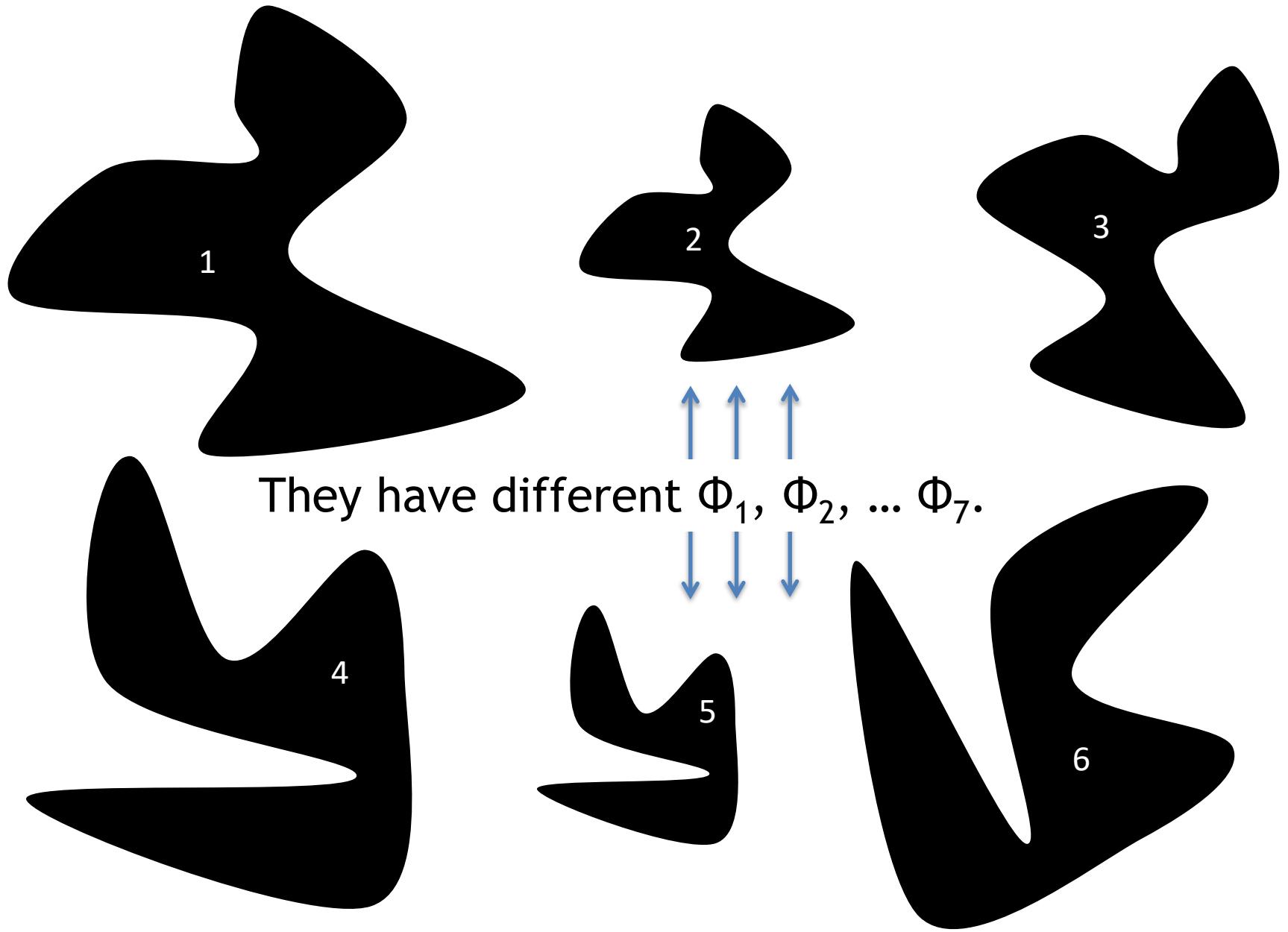
They are invariant against:

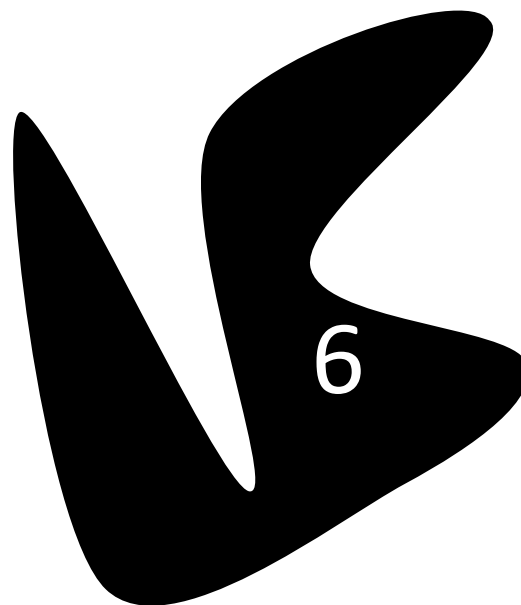
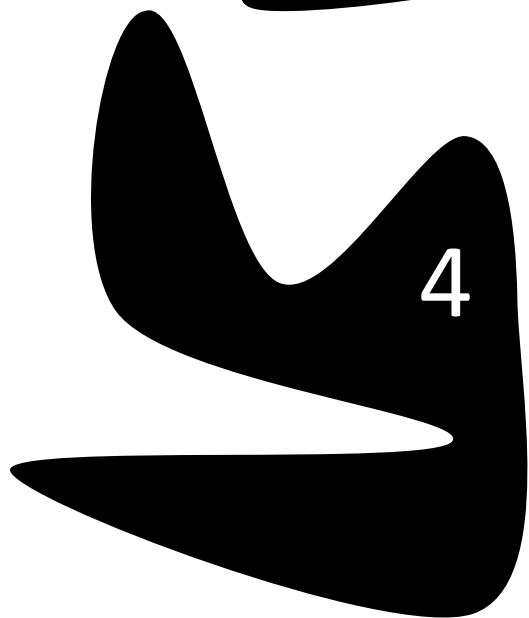
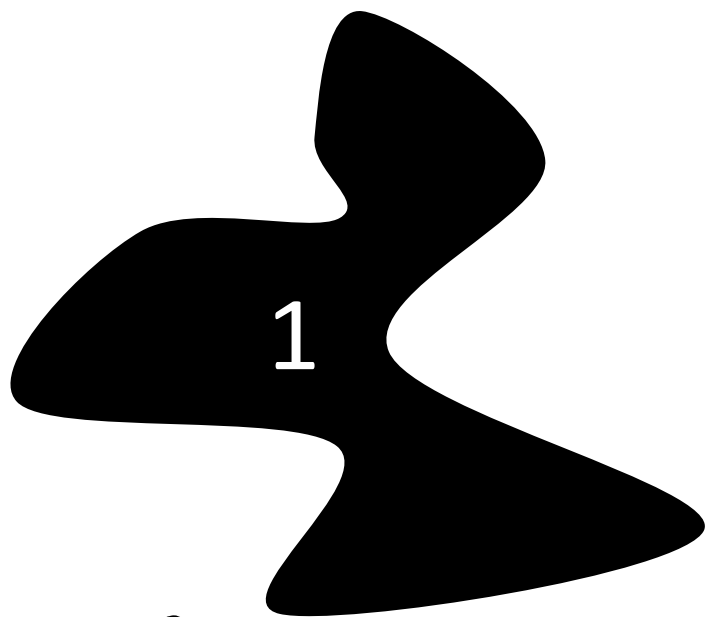
- scale,
- rotation and
- location



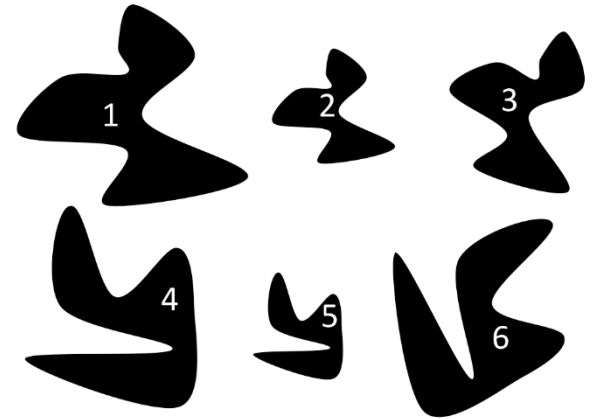
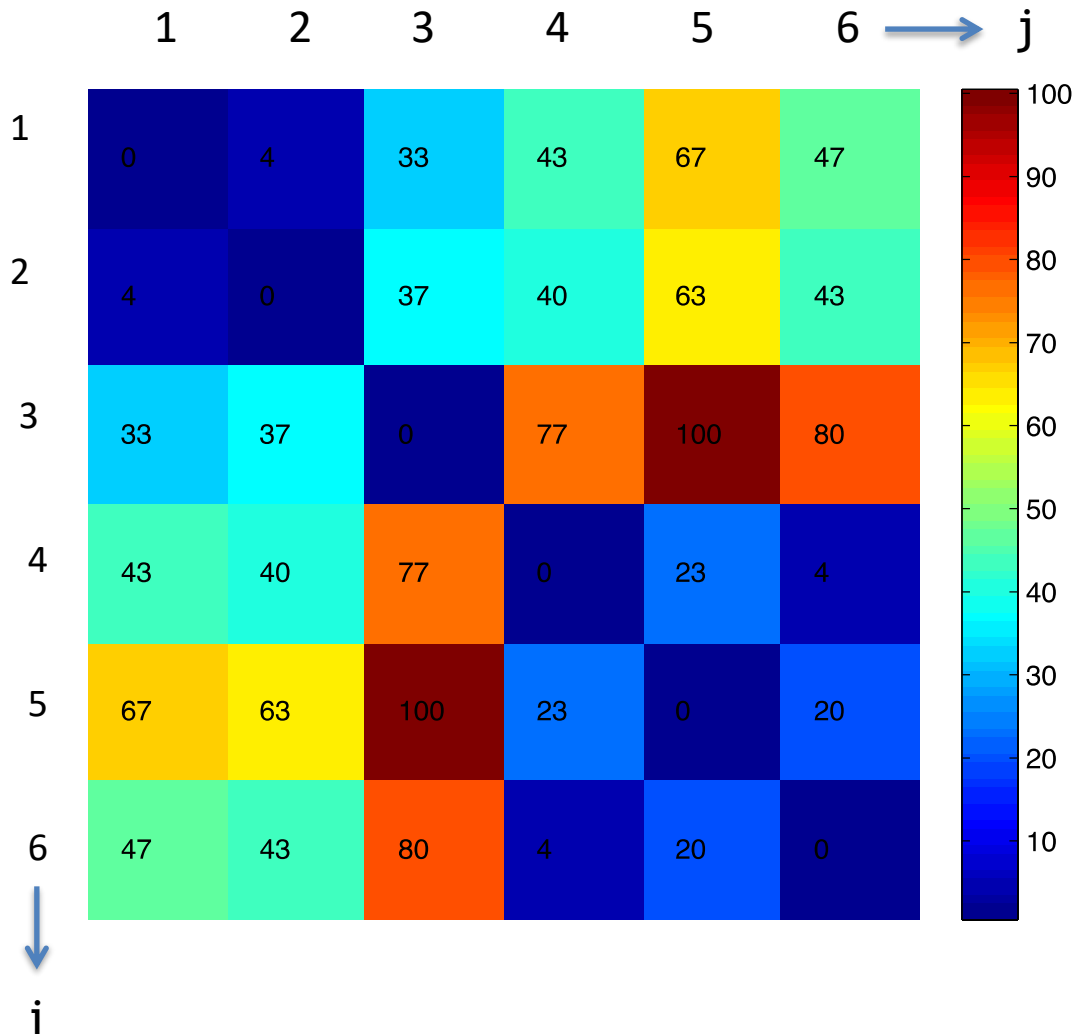
They have similar  $\Phi_1, \Phi_2, \dots, \Phi_7$ .

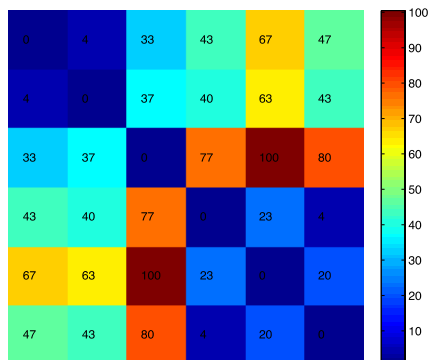
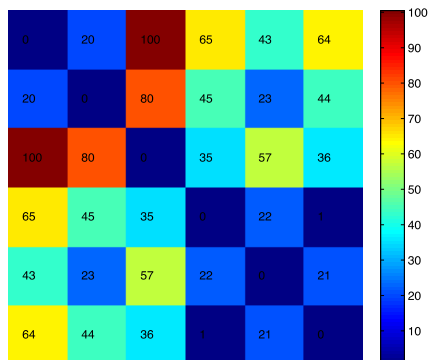
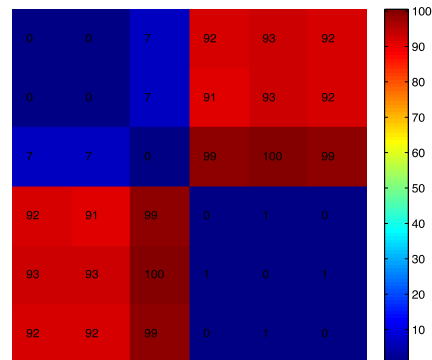
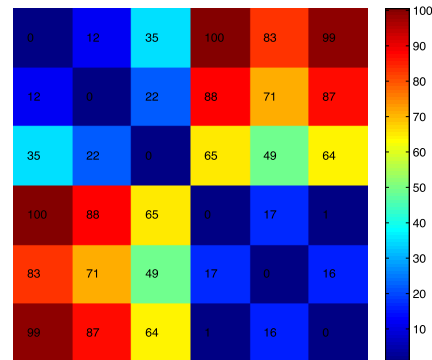
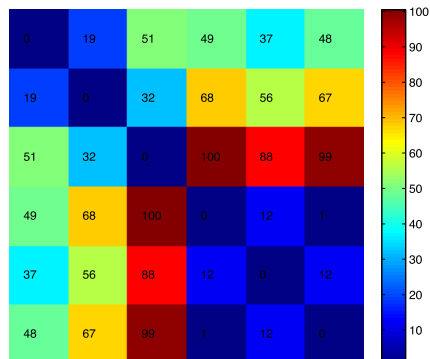
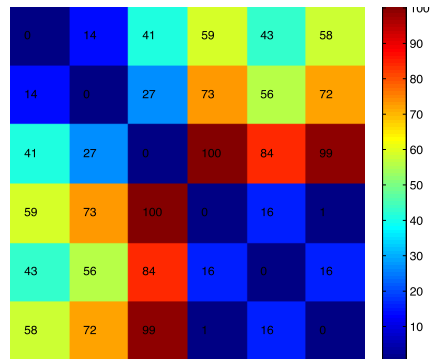
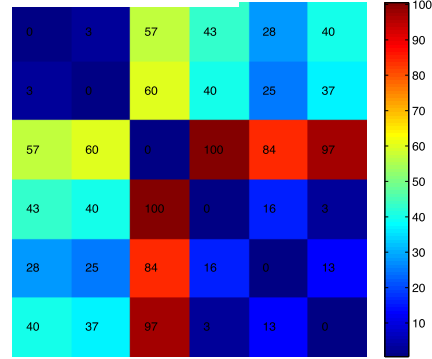
# Hu - Moments



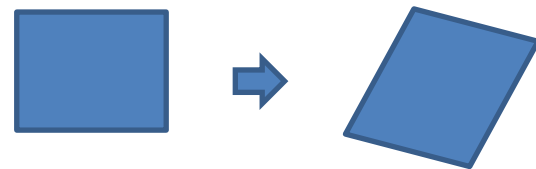


Difference between  $\Phi_1$  of region i and  $\Phi_1$  of region j



$\Phi_1$  $\Phi_2$  $\Phi_3$  $\Phi_4$  $\Phi_5$  $\Phi_6$  $\Phi_7$ 

# Flusser Moments



Invariante a la transformada afín:  
líneas paralelas se transforman  
como líneas paralelas

$$I_1 = \frac{\mu_{20}\mu_{02} - \mu_{11}^2}{\mu_{00}^4}$$

$$I_2 = \frac{\mu_{30}^2\mu_{03}^2 - 6\mu_{30}\mu_{21}\mu_{12}\mu_{03} + 4\mu_{30}\mu_{12}^3 + 4\mu_{21}^3\mu_{03} - 3\mu_{21}^2\mu_{12}^2}{\mu_{00}^{10}}$$

$$I_3 = \frac{\mu_{20}(\mu_{21}\mu_{03} - \mu_{12}^2) - \mu_{11}(\mu_{30}\mu_{03} - \mu_{21}\mu_{12}) + \mu_{02}(\mu_{30}\mu_{12} - \mu_{21}^2)}{\mu_{00}^7}$$

$$I_4 = \frac{(\mu_{20}^3\mu_{03}^2 - 6\mu_{20}^2\mu_{11}\mu_{12}\mu_{03} - 6\mu_{20}^2\mu_{02}\mu_{21}\mu_{03} + 9\mu_{20}^2\mu_{02}\mu_{12}^2 + 12\mu_{20}\mu_{11}^2\mu_{21}\mu_{03} + 6\mu_{20}\mu_{11}\mu_{02}\mu_{30}\mu_{03} - 18\mu_{20}\mu_{11}\mu_{02}\mu_{21}\mu_{12} - 8\mu_{11}^3\mu_{30}\mu_{03} - 6\mu_{20}\mu_{02}^2\mu_{30}\mu_{12} + 9\mu_{20}\mu_{02}^2\mu_{21} + 12\mu_{11}^2\mu_{02}\mu_{30}\mu_{12} - 6\mu_{11}\mu_{02}^2\mu_{30}\mu_{21} + \mu_{02}^3\mu_{30}^2)/\mu_{00}^{11}}$$

Flusser, J., & Suk, T. (1993). [Pattern recognition by affine moment invariants](#). Pattern recognition, 26(1), 167-174.