Universidad Popular del Cesar

Reconocimiento de Patrones

Version 2024-1

Texturas de Haralick

[Capítulo 2]

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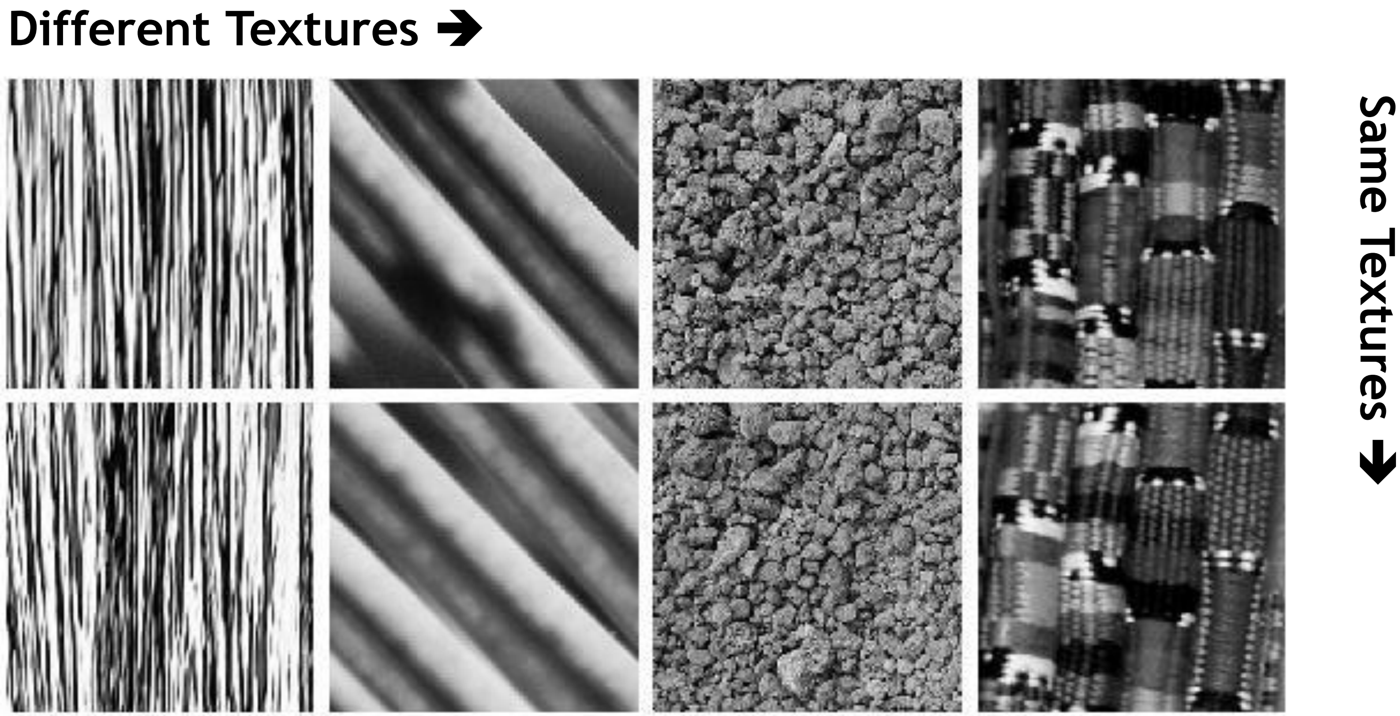
Different Textures
Same Textures

What are texture images?

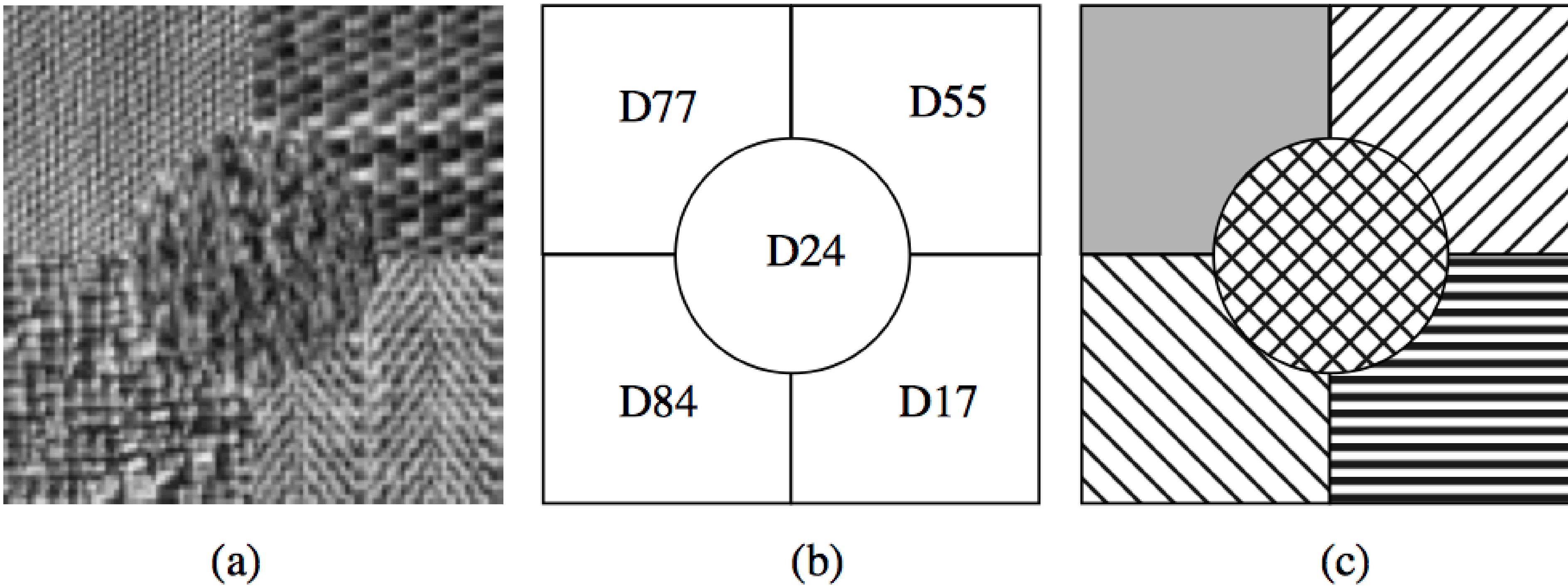

FIGURE 1. (a) An image consisting of five different textured regions: cotton canvas (D77), straw matting (D55), raffia (D84), herringbone weave (D17), and pressed calf leather. [8]. (b) The goal of texture classification is to label each textured region with the proper category label: the identities of the five texture regions present in (a). (c) The goal of texture segmentation is to separate the regions in the image which have different textures and identify the boundaries between them. The texture categories themselves need not be recognized. In this example, the five texture categories in (a) are identified as separate textures by the use of generic category labels (represented by the different fill patterns).

The Handbook of Pattern Recognition and Computer Vision (2nd Edition), by C. H. Chen, L. F. Pau, P. S. P. Wang (eds.), pp. 207-248, World Scientific Publishing Co., 1998.

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Haralick, R.M., K. Shanmugan, and I. Dinstein (1973): Textural Features for Image Classification, IEEE Transactions on Systems, Man, and Cybernetics, SMC-3:610-621.

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$
I = \begin{array}{cccc} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{array}
$$

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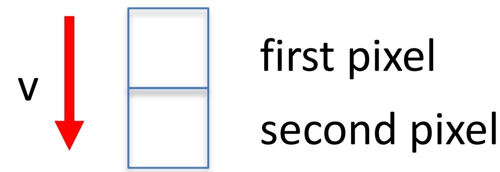
$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

10 indicates 1 ↓ and 0 →:



second pixel is located 1 pixel down,
and 0 pixels to the right of the first pixel.

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10 indicates 1 ↓ and 0 →:

v

first pixel

second pixel

second pixel is located 1 pixel down, and 0 pixels to the right of the first pixel.

For direction 10, there are 12 possible pairs pixels that can be analized.

Co-occurrence matrices

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$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

Co-occurrence matrices

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Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

value of the second pixel

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \qquad P_{10} = \frac{1}{12} \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

normalization

Co-occurrence matrices

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Example:

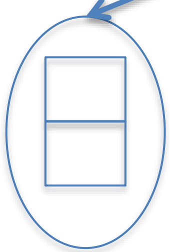
Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many  there are in image I ?

Co-occurrence matrices

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Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

value of the second pixel

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \qquad P_{10} = \frac{1}{12} \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\begin{array}{|c|}\hline 0 \\ \hline 0 \\ \hline \end{array}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{matrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & \boxed{0} \\ \boxed{0} & 2 & 1 & \boxed{0} \\ \boxed{0} & 2 & 0 & 2 \end{matrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{ccc} 0 & 1 & 2 \\ \left[\begin{matrix} 2 & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{matrix} \right. & & \left. \right] \end{array}$$

value of the first pixel

¿how many $\begin{matrix} \boxed{0} \\ \boxed{0} \end{matrix}$ there are in image I ?

Co-occurrence matrices

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Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{matrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{matrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 2 & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\begin{matrix} 0 \\ 1 \end{matrix}$ there are in image I ?

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value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 2 & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\begin{array}{|c|}\hline 0 \\ \hline 1 \\ \hline \end{array}$ there are in image I ?

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$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \hline \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 2 & 1 \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{bmatrix} 1 & 0 & \boxed{1} & 2 \\ \boxed{1} & 1 & \boxed{0} & 0 \\ \boxed{0} & 2 & \boxed{1} & 0 \\ 0 & 2 & \boxed{0} & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 2 & 2 & 1 \\ 1 & 3 & ? & ? \\ 2 & ? & ? & ? \end{array}$$

value of the first pixel

¿how many $\begin{array}{|c|}\hline 1 \\ \hline 0 \\ \hline\end{array}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{bmatrix} \boxed{1} & 0 & 1 & 2 \\ \boxed{1} & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & ? \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \end{array}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & \boxed{1} & 0 & 0 \\ 0 & \boxed{2} & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 1 \\ ? & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline \end{array}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{bmatrix} 1 & 0 & 1 & \boxed{2} \\ 1 & 1 & 0 & \boxed{0} \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & ? & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\begin{array}{|c|}\hline 2 \\ \hline 0 \\ \hline \end{array}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 0 & ? \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\boxed{\begin{array}{c} 2 \\ \hline 1 \end{array}}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is computed as follows:

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & \boxed{2} & 1 & 0 \\ 0 & \boxed{2} & 0 & 2 \end{bmatrix}$$

value of the second pixel

$$P_{10} = \frac{1}{12} \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

value of the first pixel

¿how many $\boxed{\begin{matrix} 2 \\ 2 \end{matrix}}$ there are in image I ?

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{10} is:

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

$$P_{10} = \frac{1}{12} \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{11} is:

$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

$$P_{11} = \frac{1}{?} \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

v

Co-occurrence matrices

They measure how is the distribution of co-occurring of pairs of pixels in an image.

Example:

Given an imagen I, the co-ccurrence matrix P_{11} is:

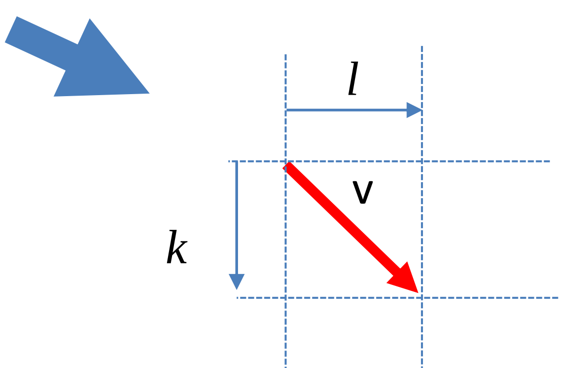
$$I = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \qquad P_{11} = \frac{1}{9} \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

v

Definition of the Co-ocurrence Matrix

$$P_{kl}(i,j) = \frac{1}{nm} \sum_{p=1}^{n} \sum_{q=1}^{m} \begin{cases} 1, & \text{if } I(p,q) = i \text{ and } I(p+k, q+l) = j \\ 0, & \text{otherwise} \end{cases}$$

Vector **v**

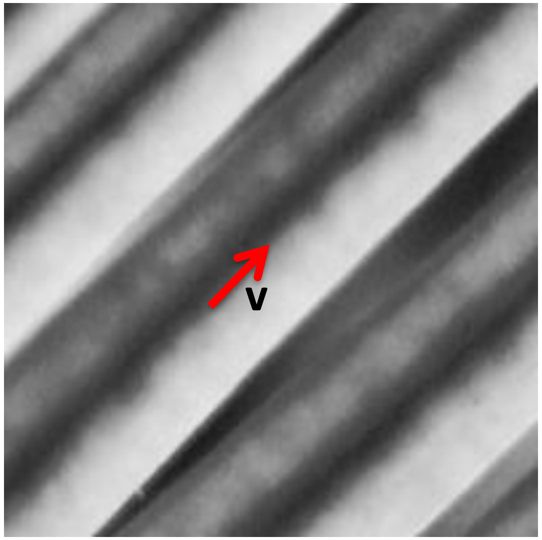


What happens if
the diagonal of the co-occurrence matrix is high ?

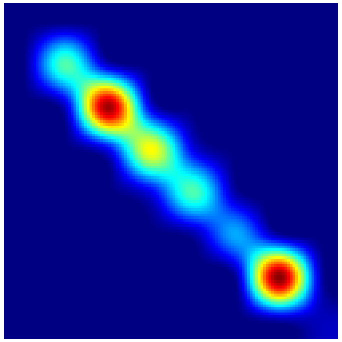
That means, that in the selected direction the pixel values do not change significantly.

In this direction the pixel values do not change significantly.

I



P_v

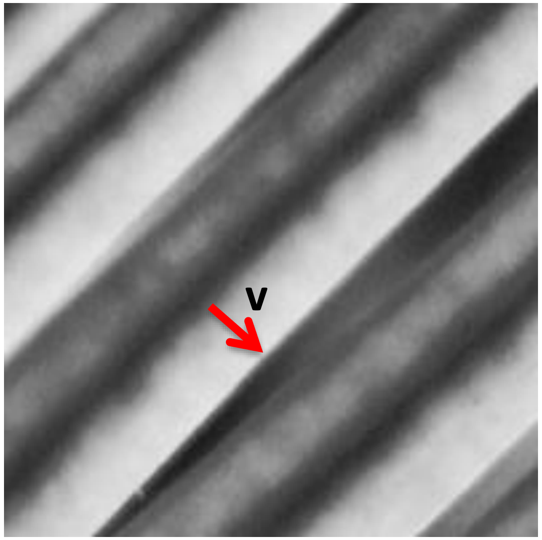


What happens if
the diagonal of the co-occurrence matrix is low?

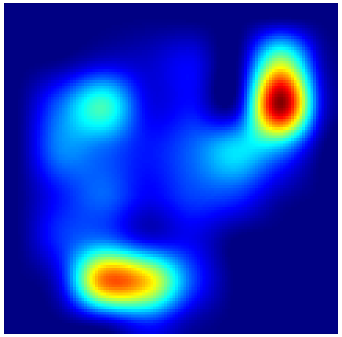
That means, that in the selected direction the pixel values do change significantly.

In this direction the pixel values do change significantly from low to high and from high to low.

I

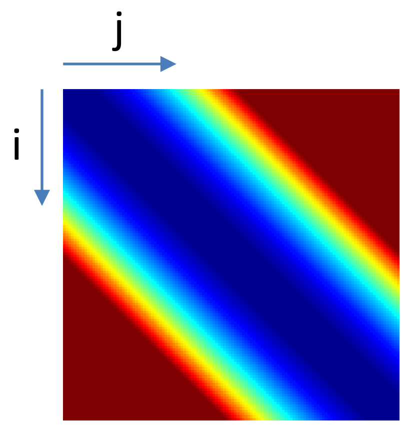


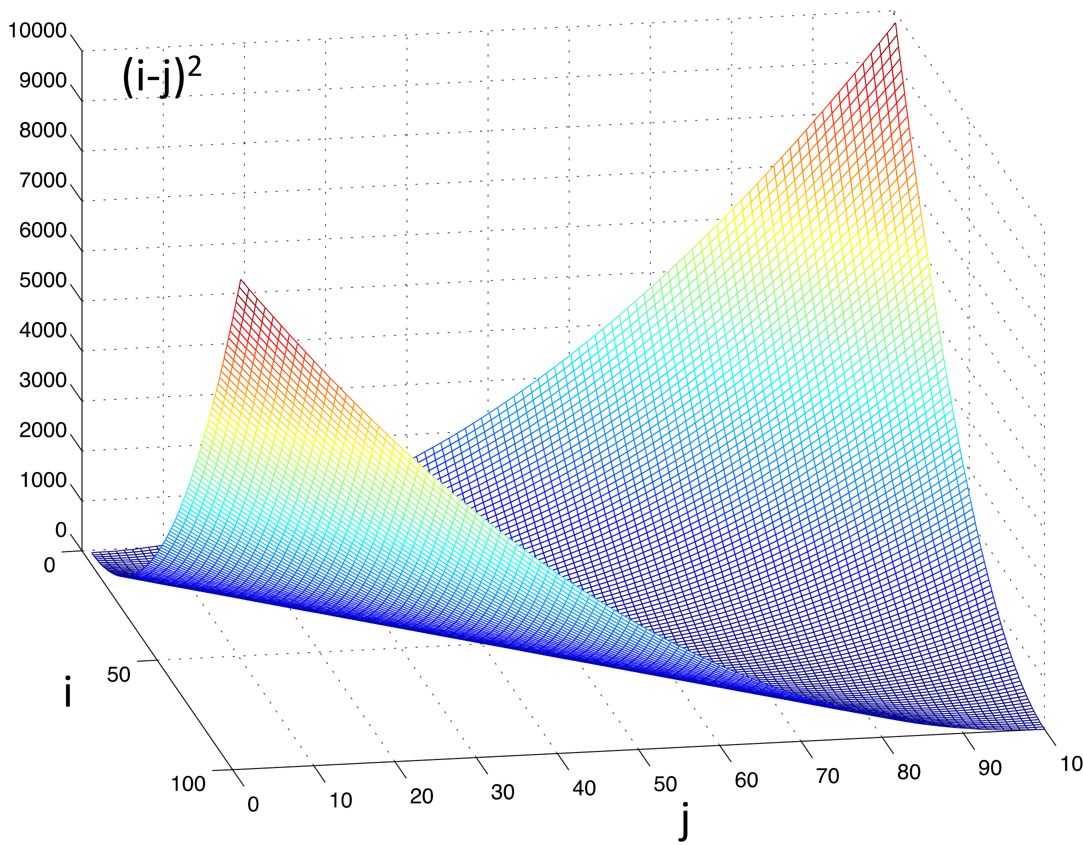
P_v



Contrast

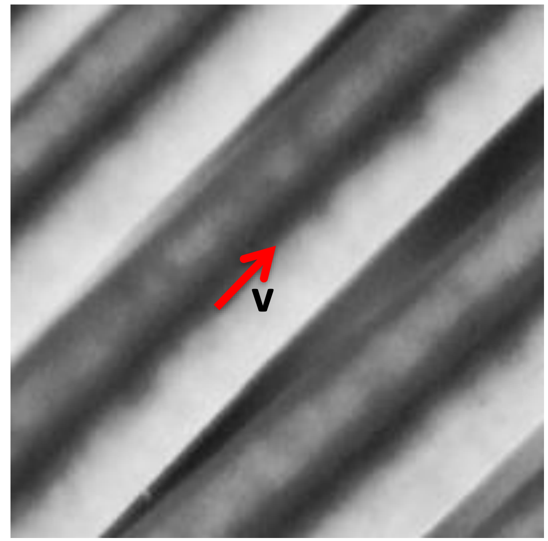
$$I_{kl} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} (i-j)^2 P_{kl}[i,j]$$

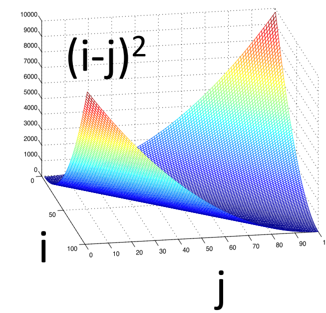




In this direction the pixel values do not change significantly.

I

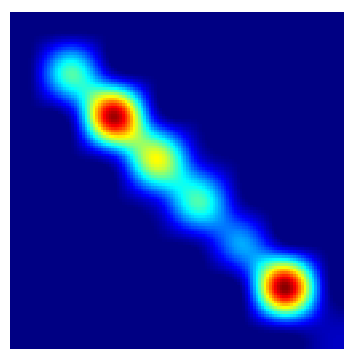




Contrast

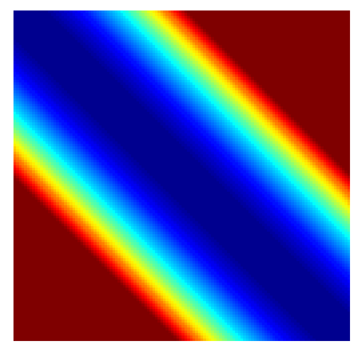
$$I_{kl} = \sum_{i=1}^{N_x}\sum_{j=1}^{N_x}(i-j)^2 P_{kl}[i,j] \quad = \sum$$

P_v



x

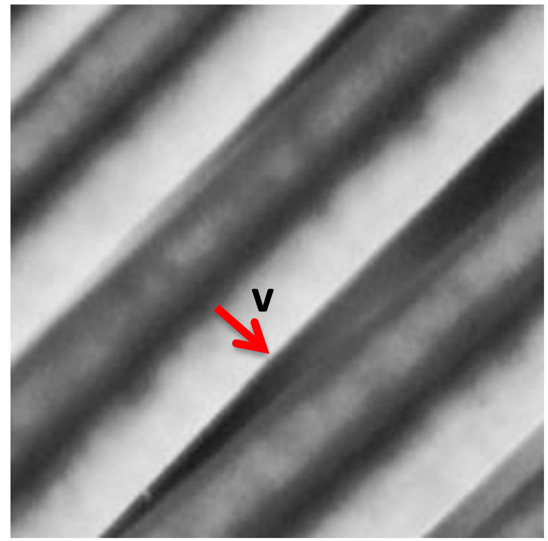
$(i - j)^2$

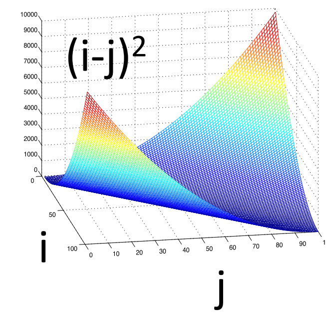


= low

In this direction the pixel values do change significantly from low to high and from high to low.

I



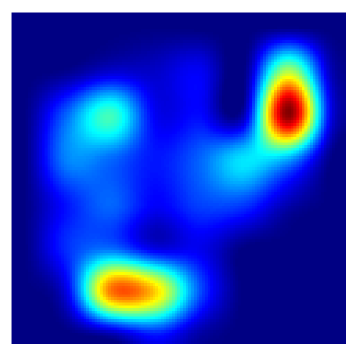


Contrast

$$I_{kl} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} (i-j)^2 P_{kl}[i,j]$$

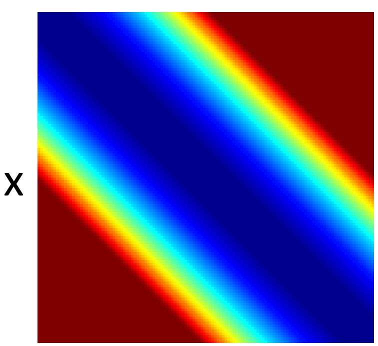
$= \sum$

$\mathbf{P_v}$



x

$(i - j)^2$



= high

Features based on Co-occurrence Matrix

Contrast

$$I_{kl} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} (i - j)^2 P_{kl}[i, j]$$

Moment of inverse difference

$$Z_{kl} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} \frac{P_{kl}[i, j]}{1 + (i - j)^2}$$

Energy

$$E_{kl} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} [P_{kl}[i, j]]^2$$

Entropy

$$H_{kl} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} P_{kl}[i, j] \log(P_{kl}[i, j])$$

Features based on Co-occurrence Matrix

Angular second moment: $f_1 = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} [p(i,j)]^2$

Contrast: $f_2 = \sum_{n=0}^{N_x-1} n^2 \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} p(i,j) \text{for } |i-j| = n$

Correlation: $f_3 = \frac{1}{\sigma_x \sigma_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} [ij \cdot p(i,j) - \mu_x \mu_y]^2$

Sum of squares: $f_4 = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} (i-j)^2 p(i,j)$

Inverse difference moment: $f_5 = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} \frac{p(i,j)}{1+(i-j)^2}$

Sum average: $f_6 = \sum_{i=2}^{2N_x} i \cdot p_{x+y}(i)$

Sum variance: $f_7 = \sum_{i=2}^{2N_x} (i - f_8) \cdot p_{x+y}(i)$

Sum entropy: $f_8 = -\sum_{i=2}^{2N_x} p_{x+y}(i) \cdot \log(p_{x+y}(i))$

Entropy: $f_9 = -\sum_{i=1}^{N_x} \sum_{j=1}^{N_x} p(i,j) \log(p(i,j))$

Difference variance: $f_{10} = \text{var}(\mathbf{p}_{x+y})$

Difference entropy: $f_{11} = -\sum_{i=0}^{N_x-1} p_{x-y}(i) \cdot \log(p_{x-y}(i))$

Information measures of correlation 1: $f_{12} = \frac{f_9 - HXY1}{\max(HX, HY)}$

Information measures of correlation 2: $f_{13} = \sqrt{1 - \exp(-2(HXY2 - HXY))}$

Maximal correlation coefficient: $f_{14} = \sqrt{\lambda_2}$

where μ_x, μ_y, σ_x and σ_y are the means and standard deviations of p_x and p_y respectively with

$$\begin{aligned} p_x &= \sum_{j=1}^{N_x} p(i,j) \\ p_y &= \sum_{i=1}^{N_x} p(i,j) \\ p_{x+y}(k) &= \sum_{i=1}^{N_x} \sum_{j=1 i+j=k}^{N_x} p(i,j) \;\text{ for } k = 2,3,...2N_x \\ p_{x-y}(k) &= \sum_{i=1}^{N_x} \sum_{j=1 |i-j|=k}^{N_x} p(i,j) \;\text{ for } k = 0,1,...N_x - 1 \end{aligned},$$

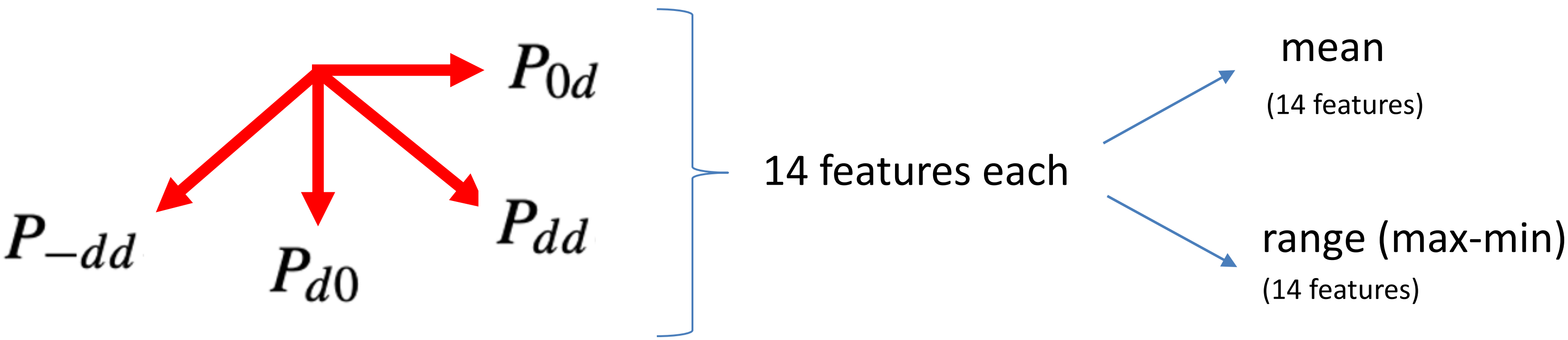
and

$$\begin{aligned} HX &= -\sum_{i=1}^{N_x} p_x(i) \log\left(p_x(i)\right) \\ HY &= -\sum_{j=1}^{N_x} p_y(j) \log\left(p_y(j)\right) \\ HXY1 &= -\sum_{i=1}^{N_x} \sum_{j=1}^{N_x} p(i,j) \log\left(p_x(i) p_y(j)\right) \\ HXY2 &= -\sum_{i=1}^{N_x} \sum_{j=1}^{N_x} p_x(i) p_y(j) \log\left(p_x(i) p_y(j)\right) \end{aligned}.$$

In f_14, λ_2 is the second largest eigenvalue of Q defined by

$$Q(i,j) = \sum_{k=1}^{N_x} \frac{p(i,k)p(j,k)}{p_x(i)p_y(k)}$$

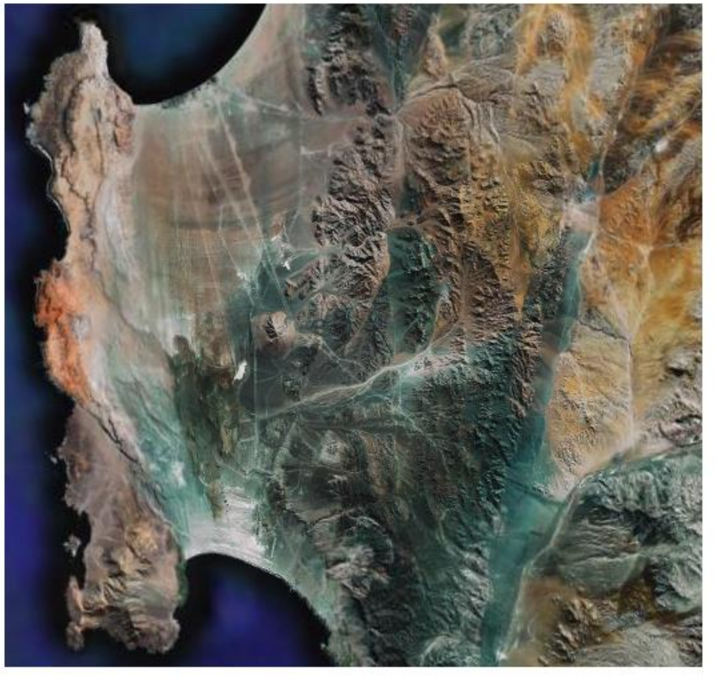
For a given ‘d’ in pixels,
we extract 4 co-occurrence matrices:



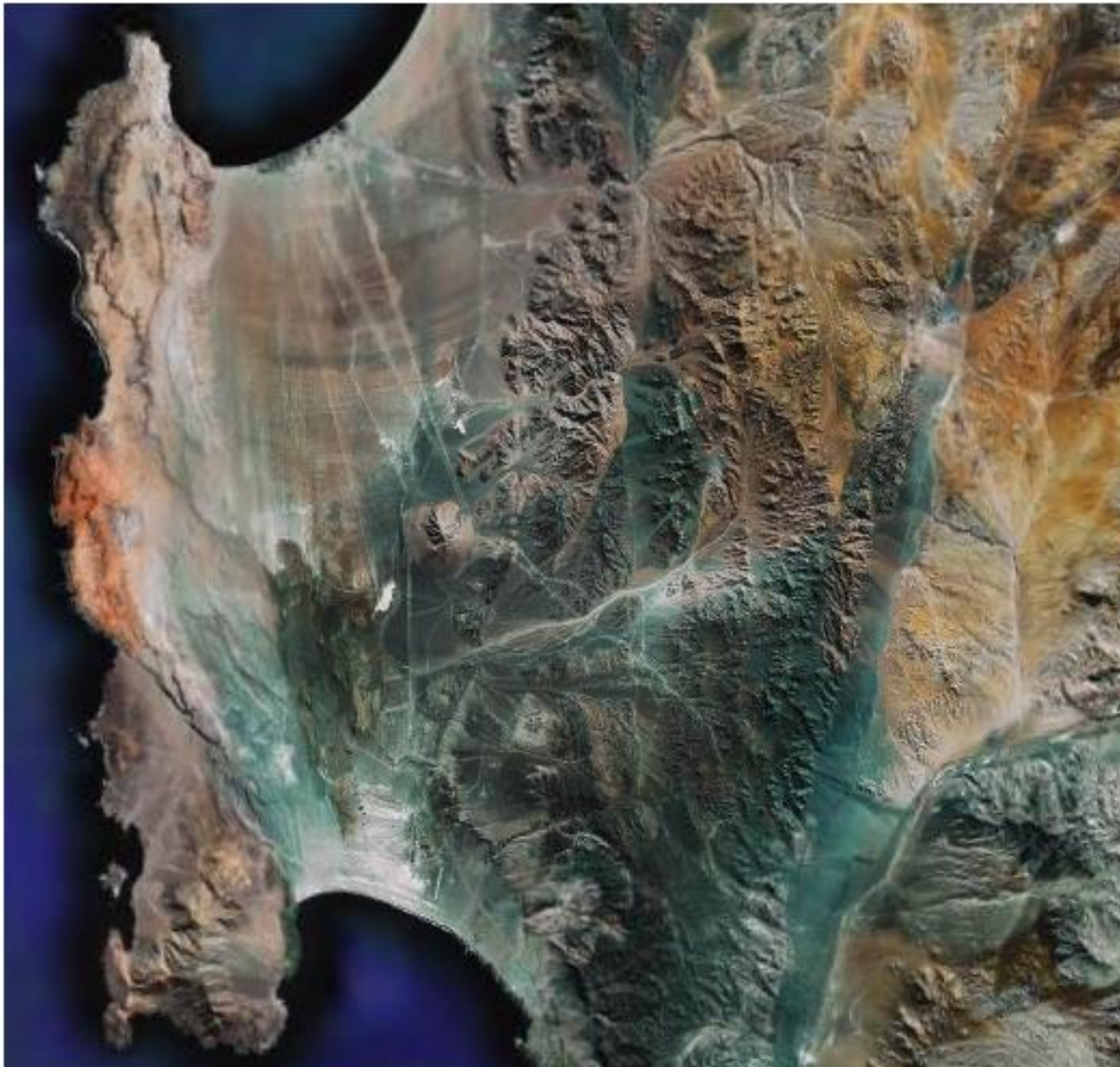
The texture features are extracted for four directions (0°–180°, 45°–225°, 90°–270° and 135°–315°) in different distances $d = \max(k, l)$. That is, for a given distance d we have four possible co-occurrence matrices: P_{0d}, P_{dd}, P_{d0} and P_{-dd}. For example, for $d = 1$, we have (k, l) = (0, 1); (1, 1); (1, 0); and (−1, 1). After Haralick, 14 texture features using each co-occurrence matrix are computed and the mean and range for each feature are calculated, i.e., we obtain $14 \times 2 = 28$ texture features for each distance d. The features will be denoted as $\bar{f}_i$ for the mean and f_i^{Δ} for the range, for $i = 1 \ldots 14$.

Example

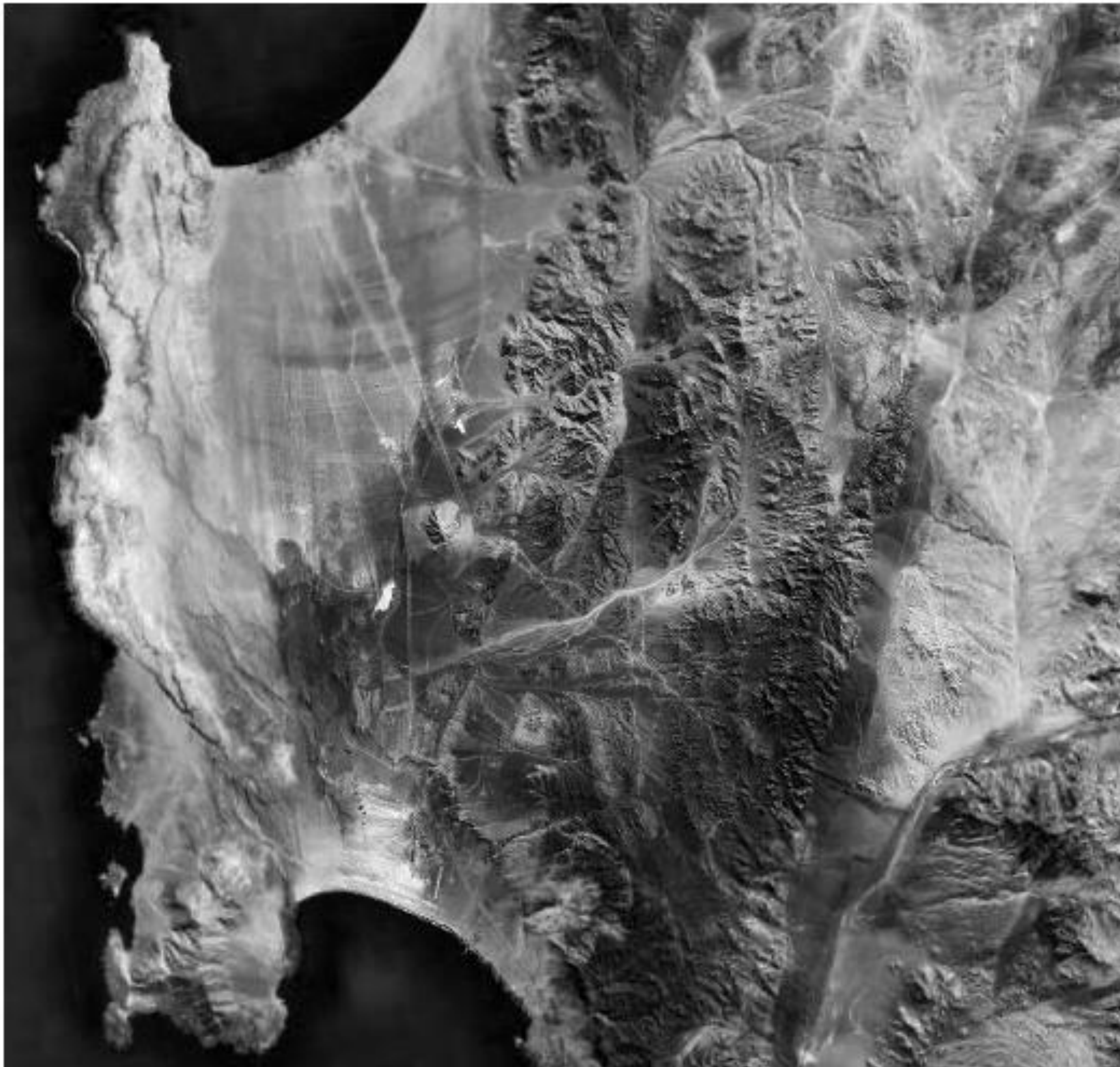
Detection of mountains and desert areas



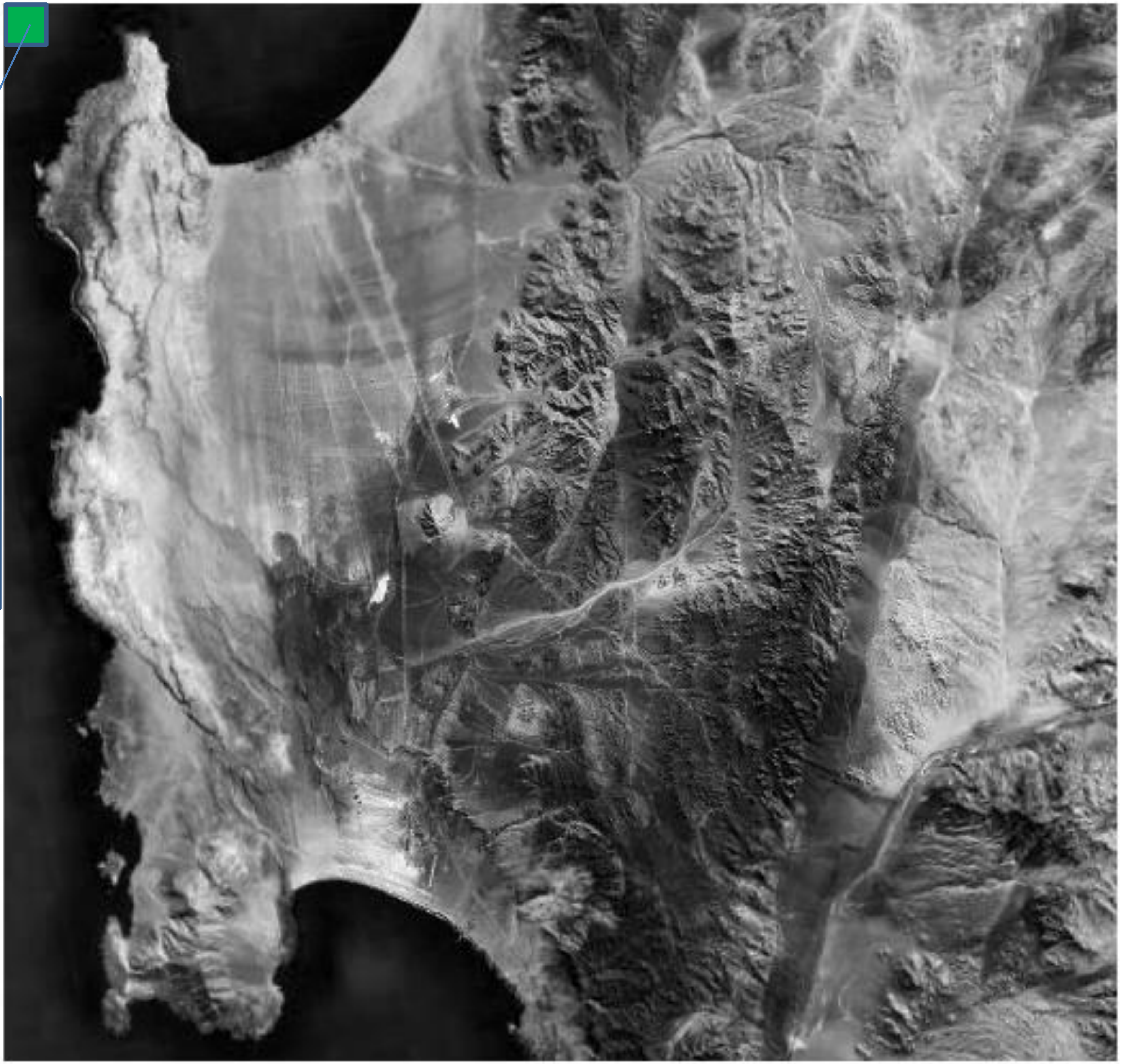
Original



Red
channel

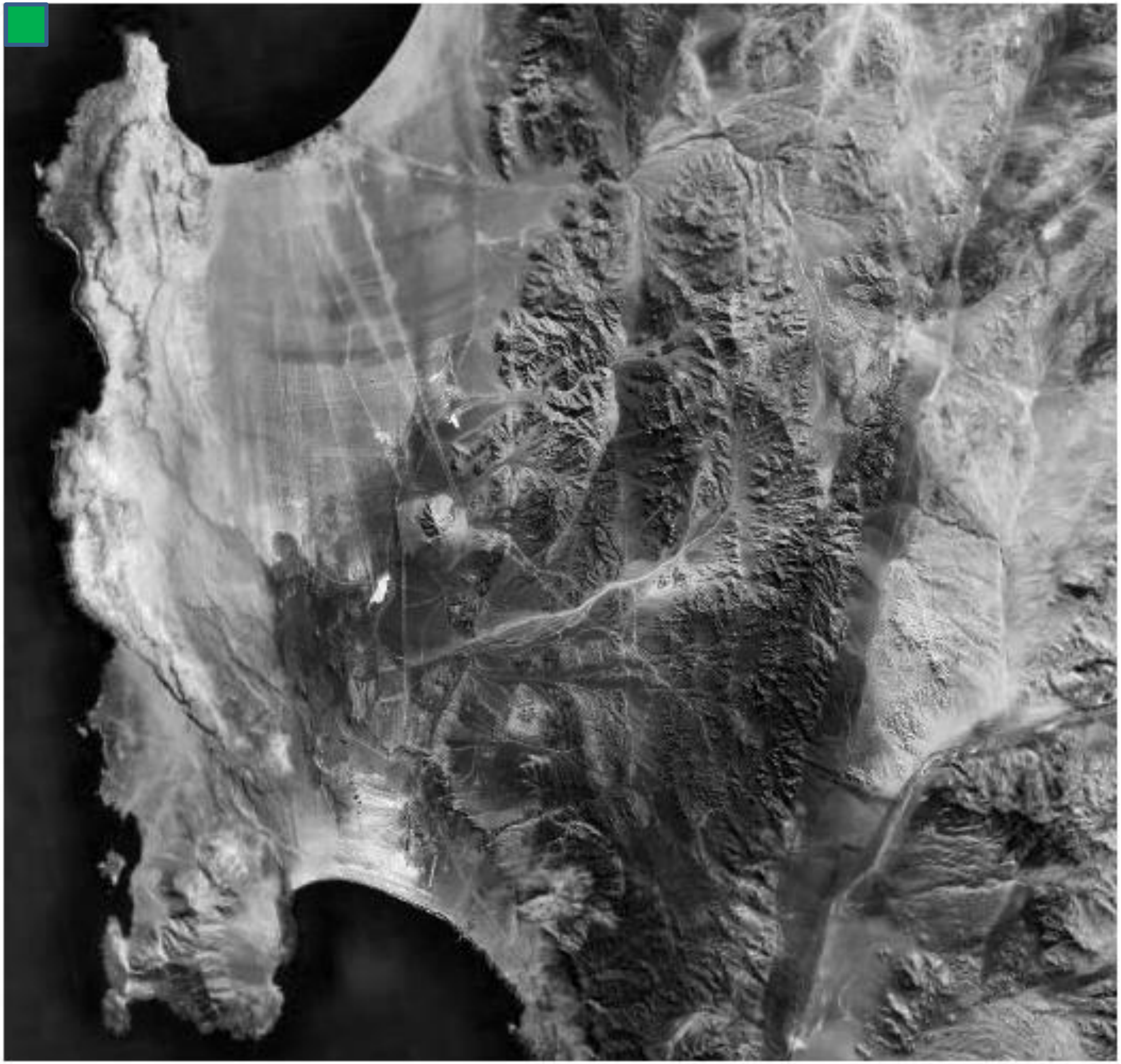


Red channel

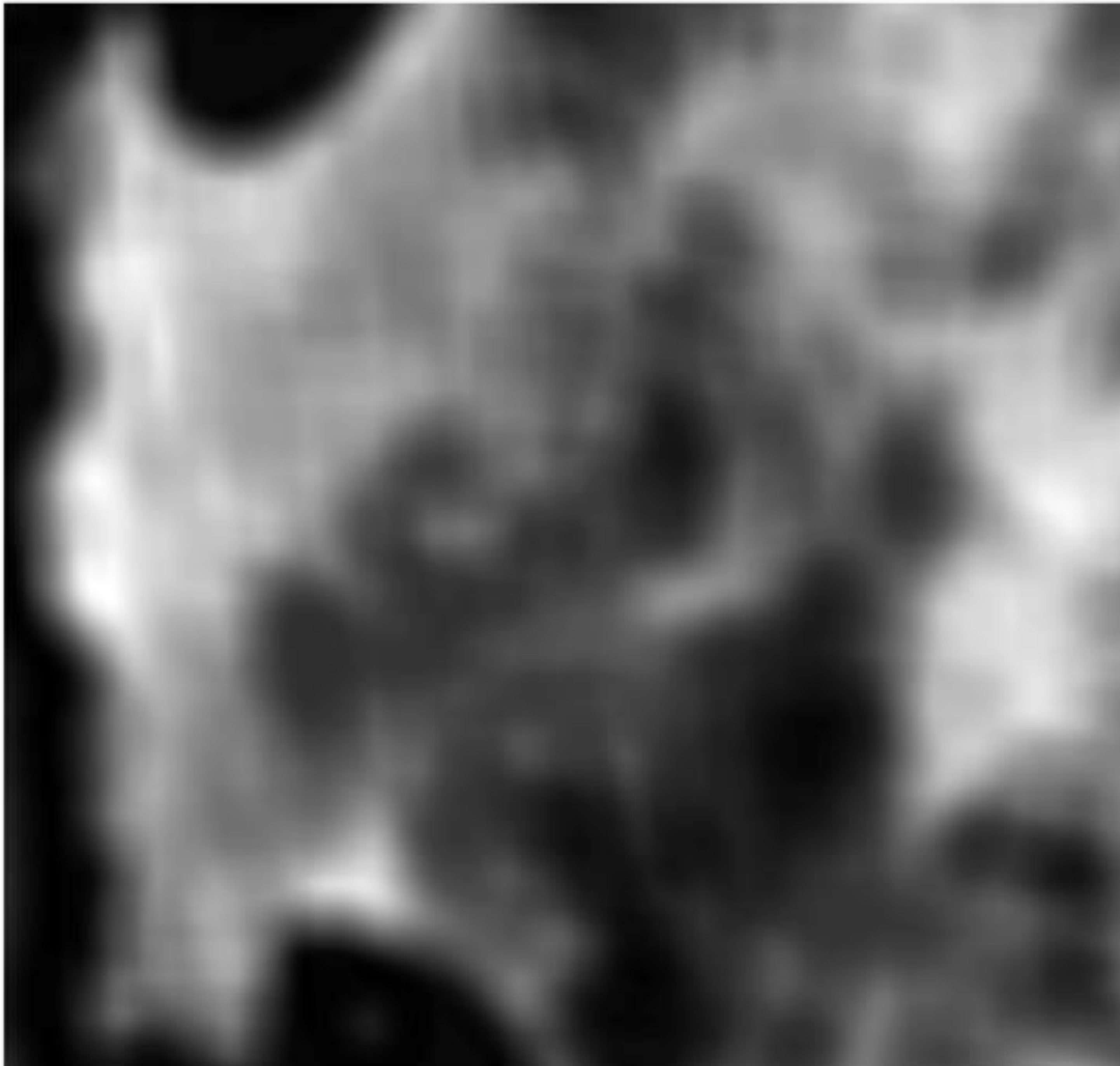


Extraction of Haralick Features

Red channel

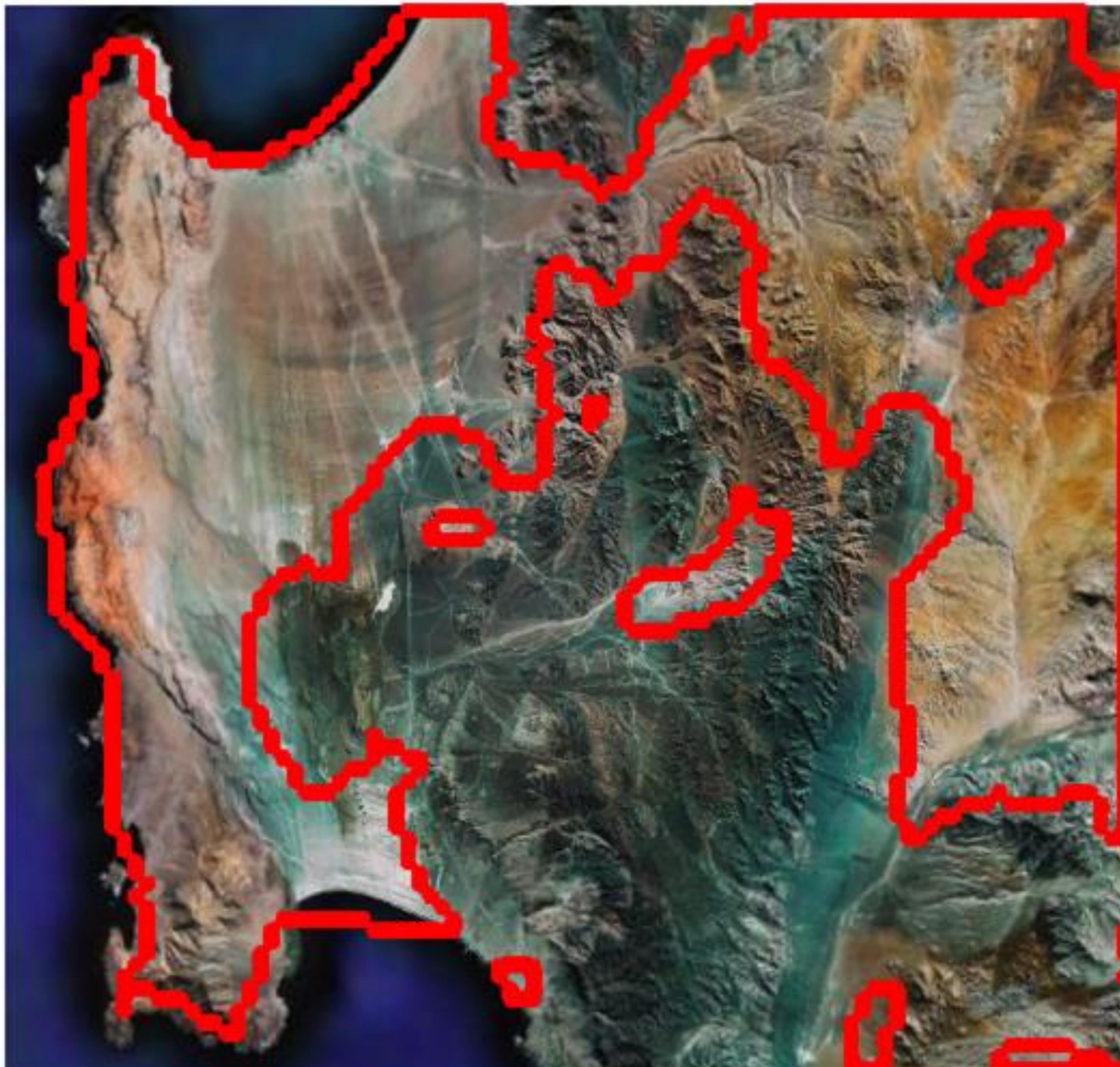


Sum
Variance



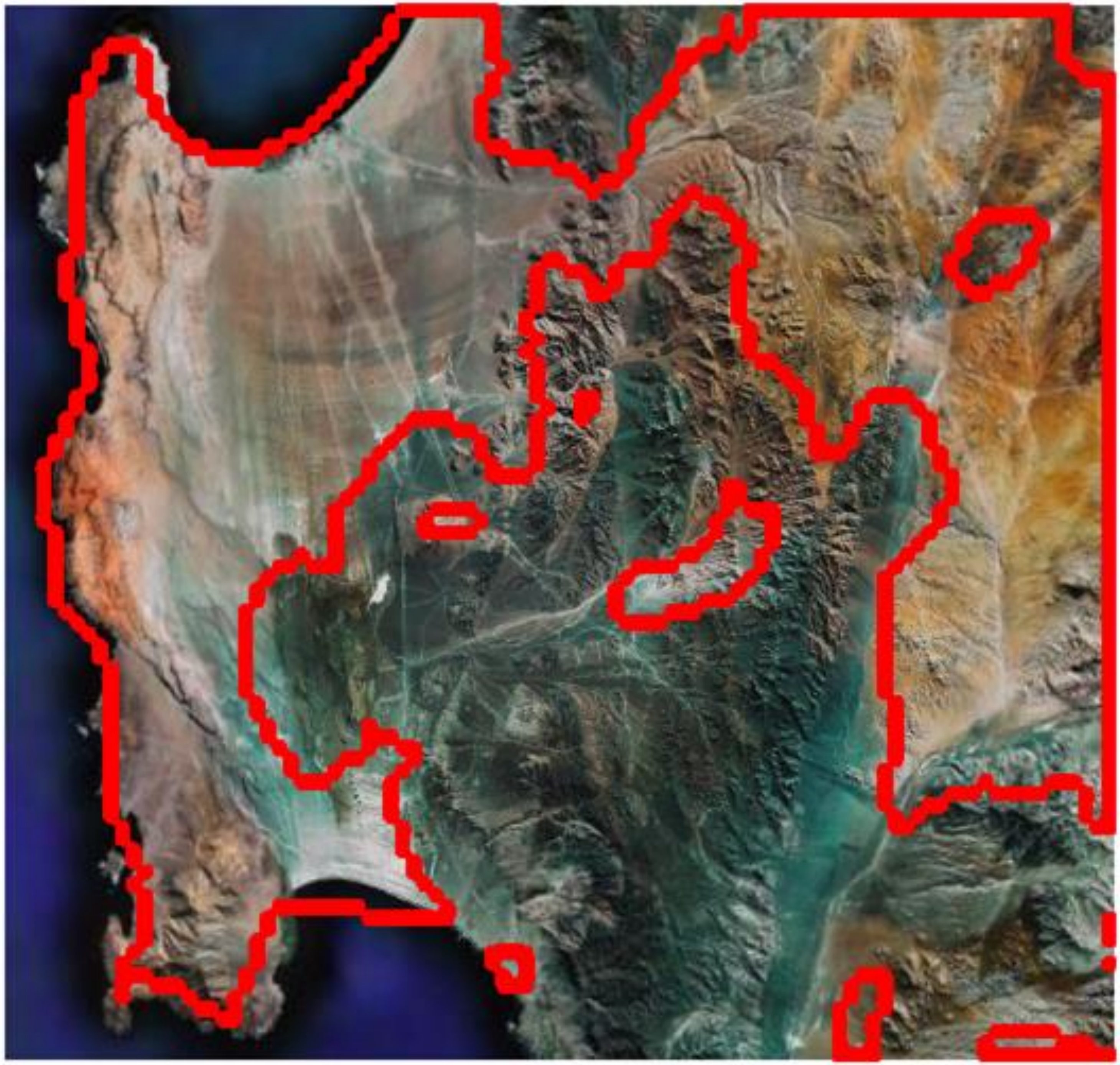
Sum
Variance
> Threshold

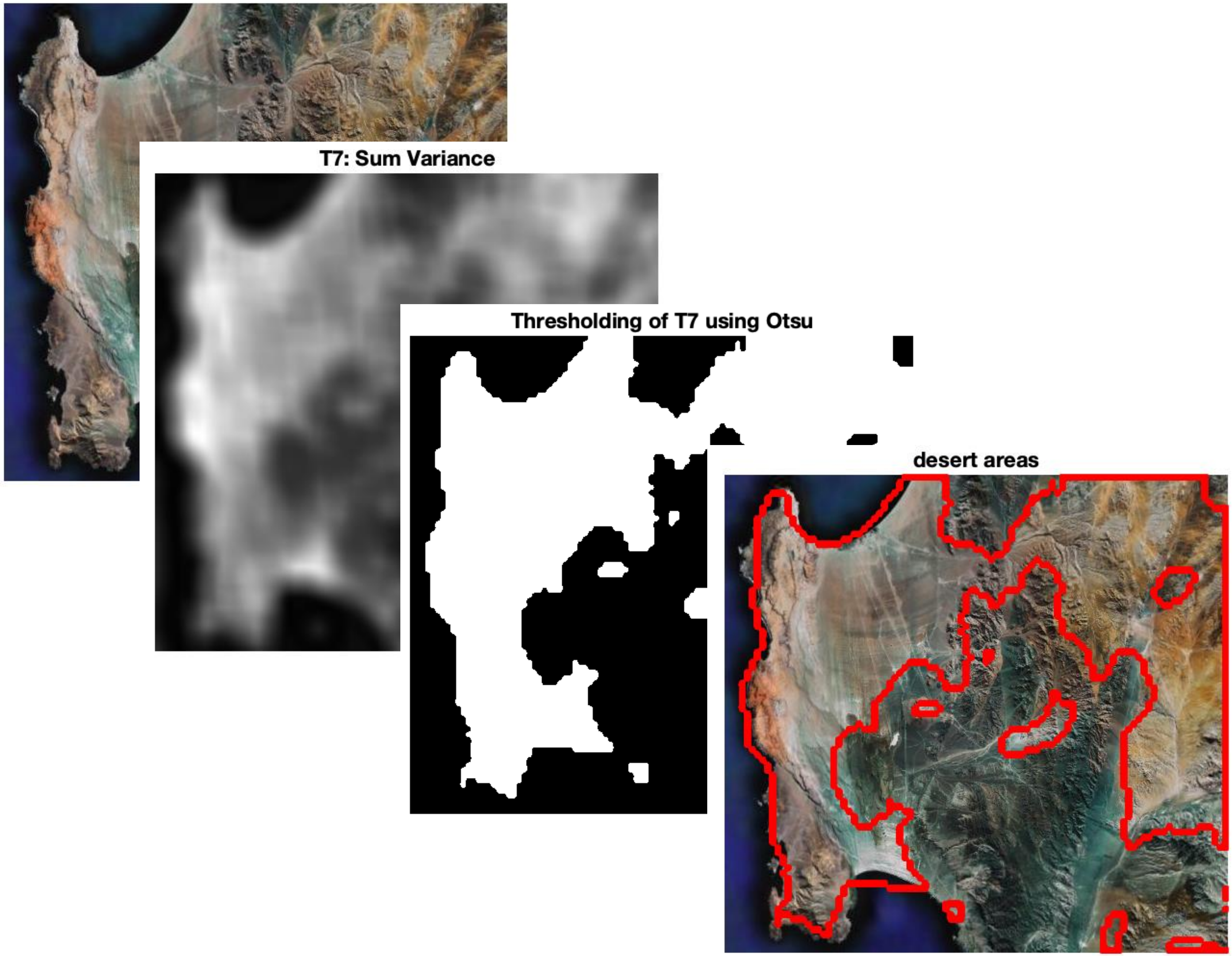
Sum
Variance
> Threshold



Sum
Variance
> Threshold

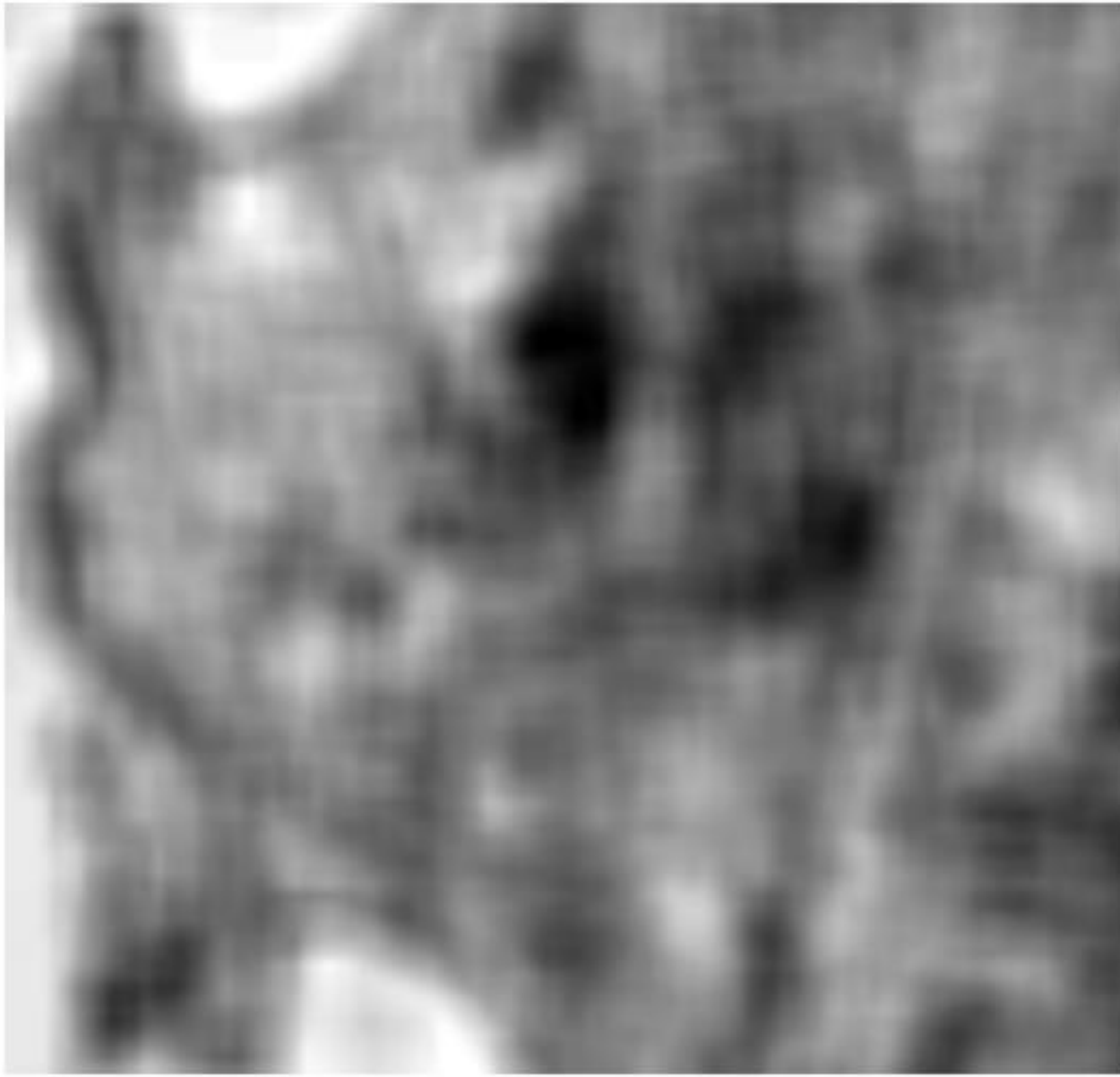
= desert
areas





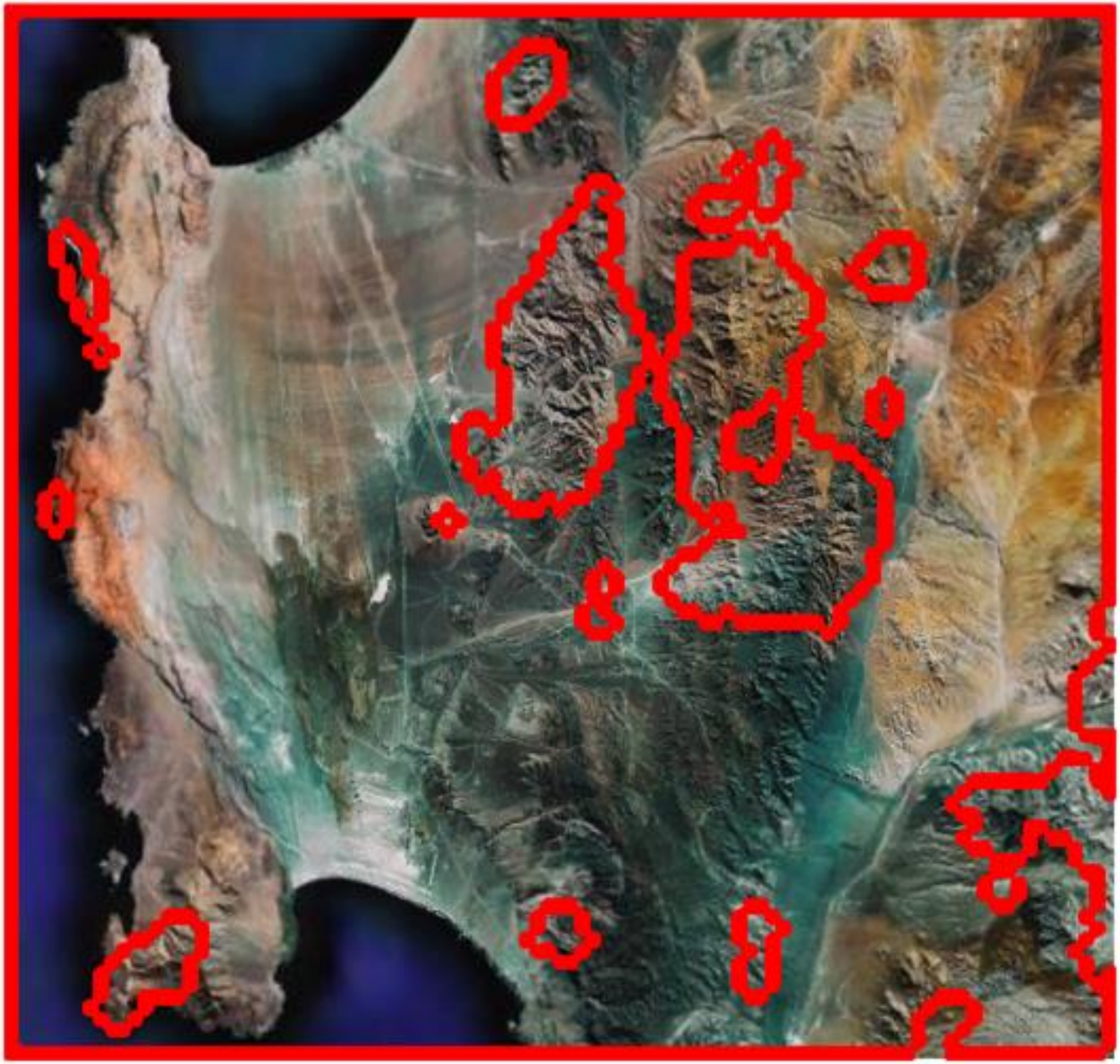
T7: Sum Variance
Thresholding of T7 using Otsu
desert areas

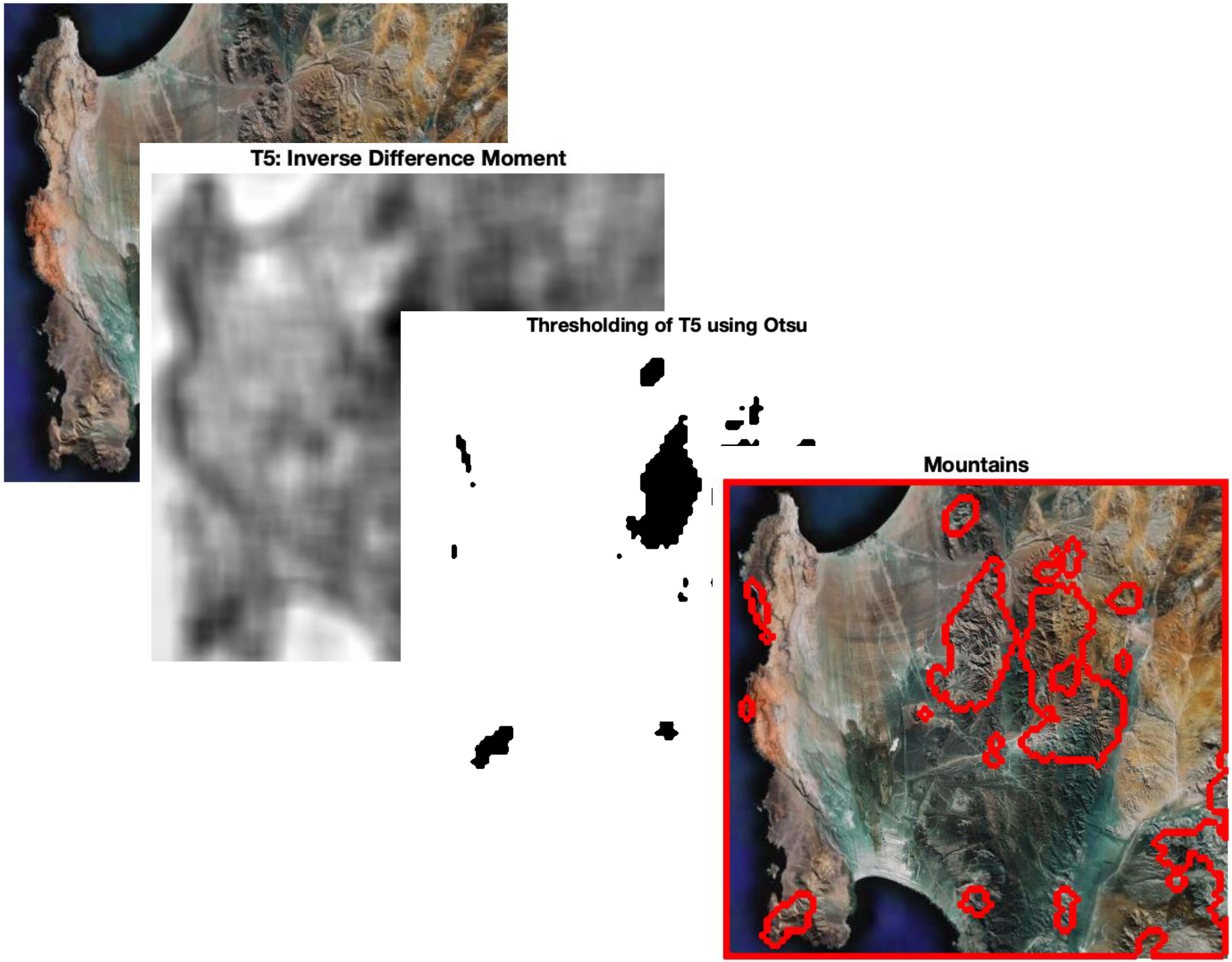
Inverse
Moment
Difference

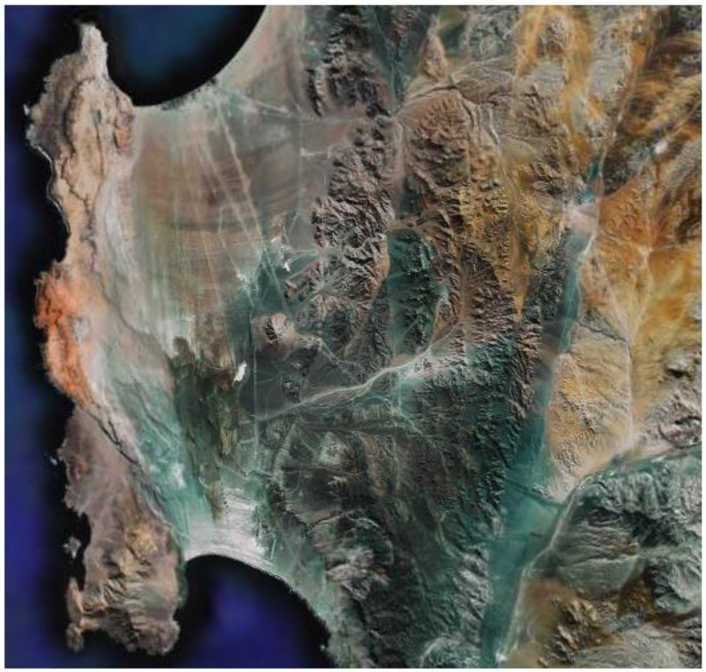


Inverse
Moment
Difference
< Threshold

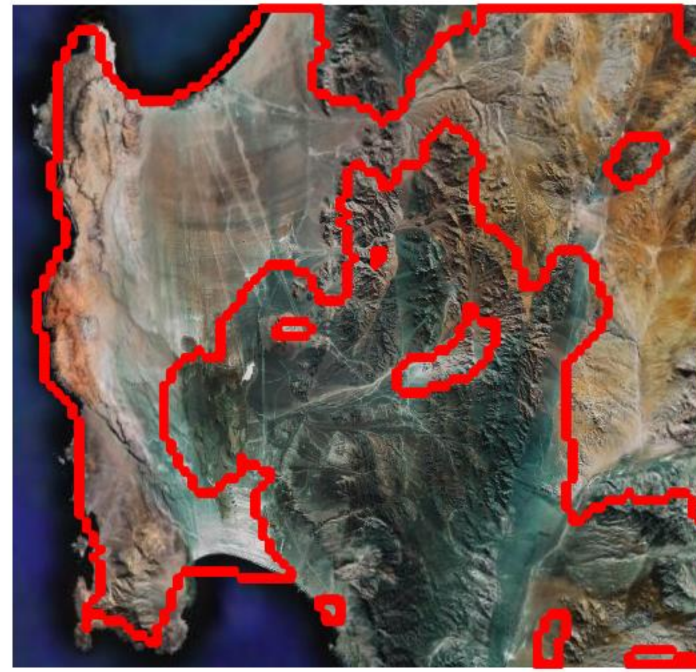
= Mountains




T5: Inverse Difference Moment
Thresholding of T5 using Otsu
Mountains



desert areas



Mountains