Minería de datos y Patrones

Version 2024-I

Bayes

[Capítulo 4]

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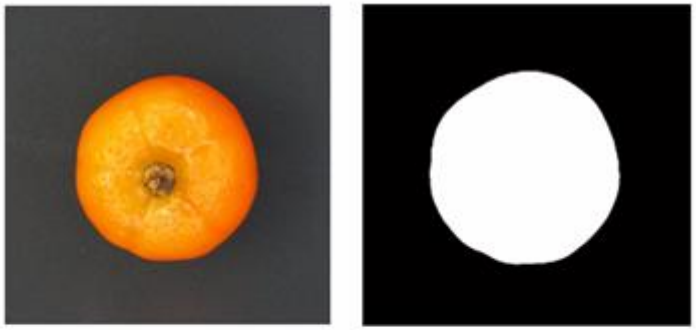
Mandarinas vs. Naranjas

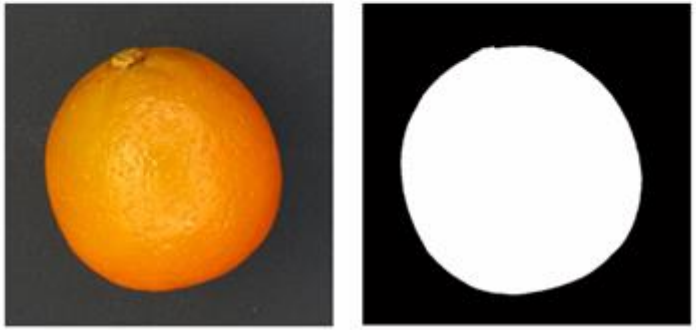
¿cómo separar las mandarinas de las naranjas?

Mandarinas vs. Naranjas

Clasificación por tamaño: (*las mandarinas son más pequeñas*)



Área = 15.457 pixeles



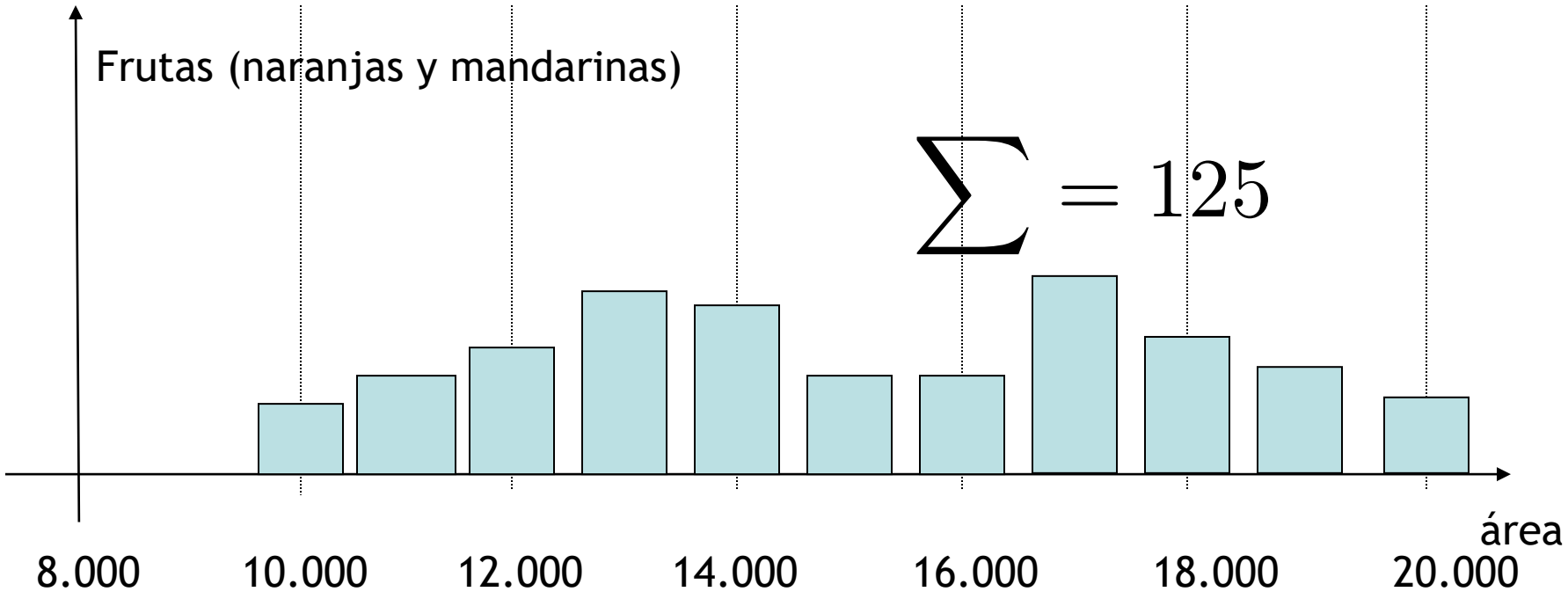
Área = 18.583 pixeles

Extracción de Característica: Área en Pixeles

Naranja-01	19.327	Mandarina-01	13.221
Naranja-02	18.265	Mandarina-02	14.987
Naranja-03	17.456	Mandarina-03	15.321
Naranja-04	19.341	Mandarina-04	15.987
Naranja-05	16.342	Mandarina-05	16.345
Naranja-06	16.987	Mandarina-06	15.965
Naranja-07	17.001	Mandarina-07	16.341
:	19.056	:	
Naranja-75	15.900	Mandarina-50	13.439

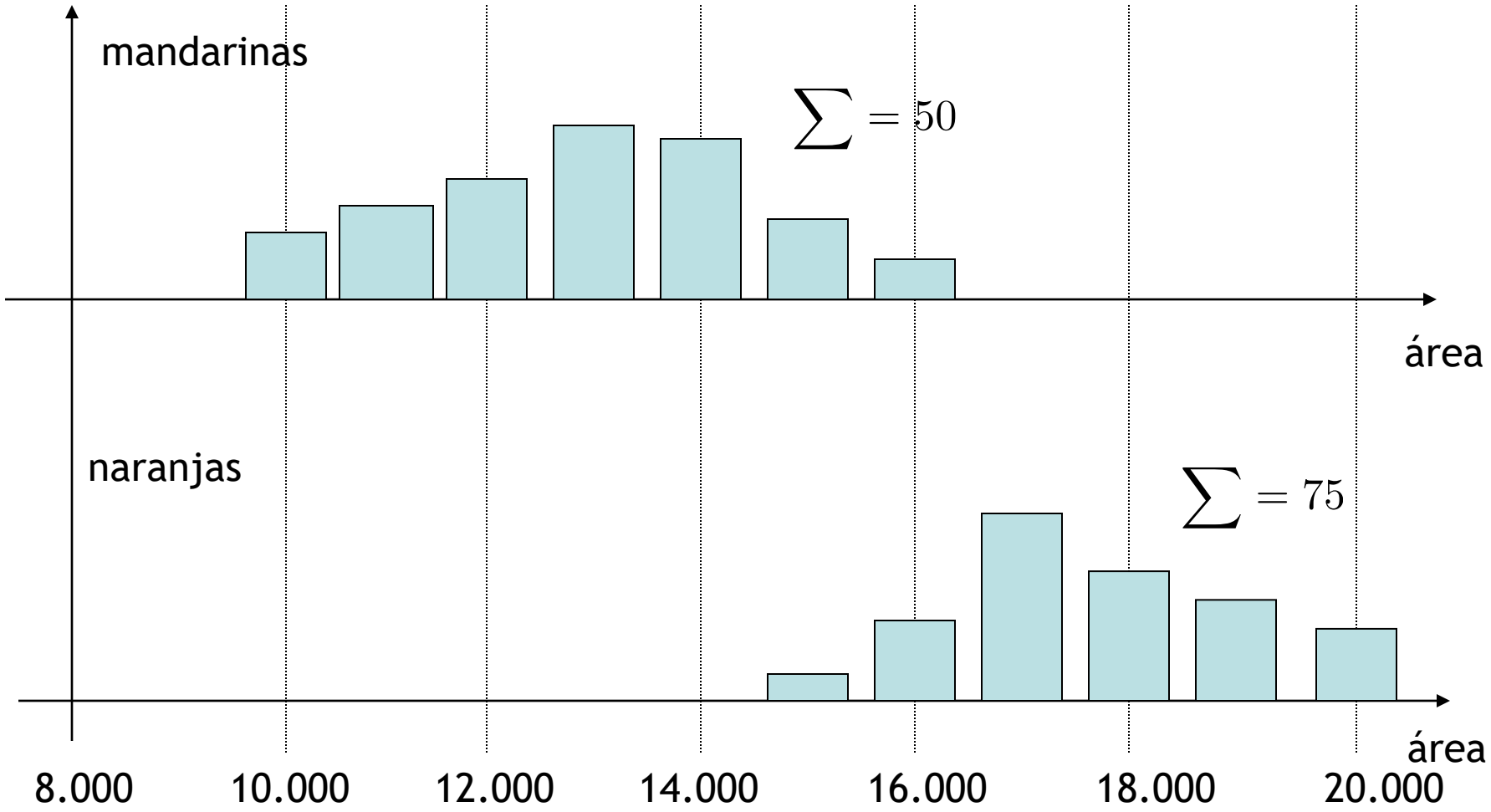
Mandarinas vs. Naranjas

Histogramas:



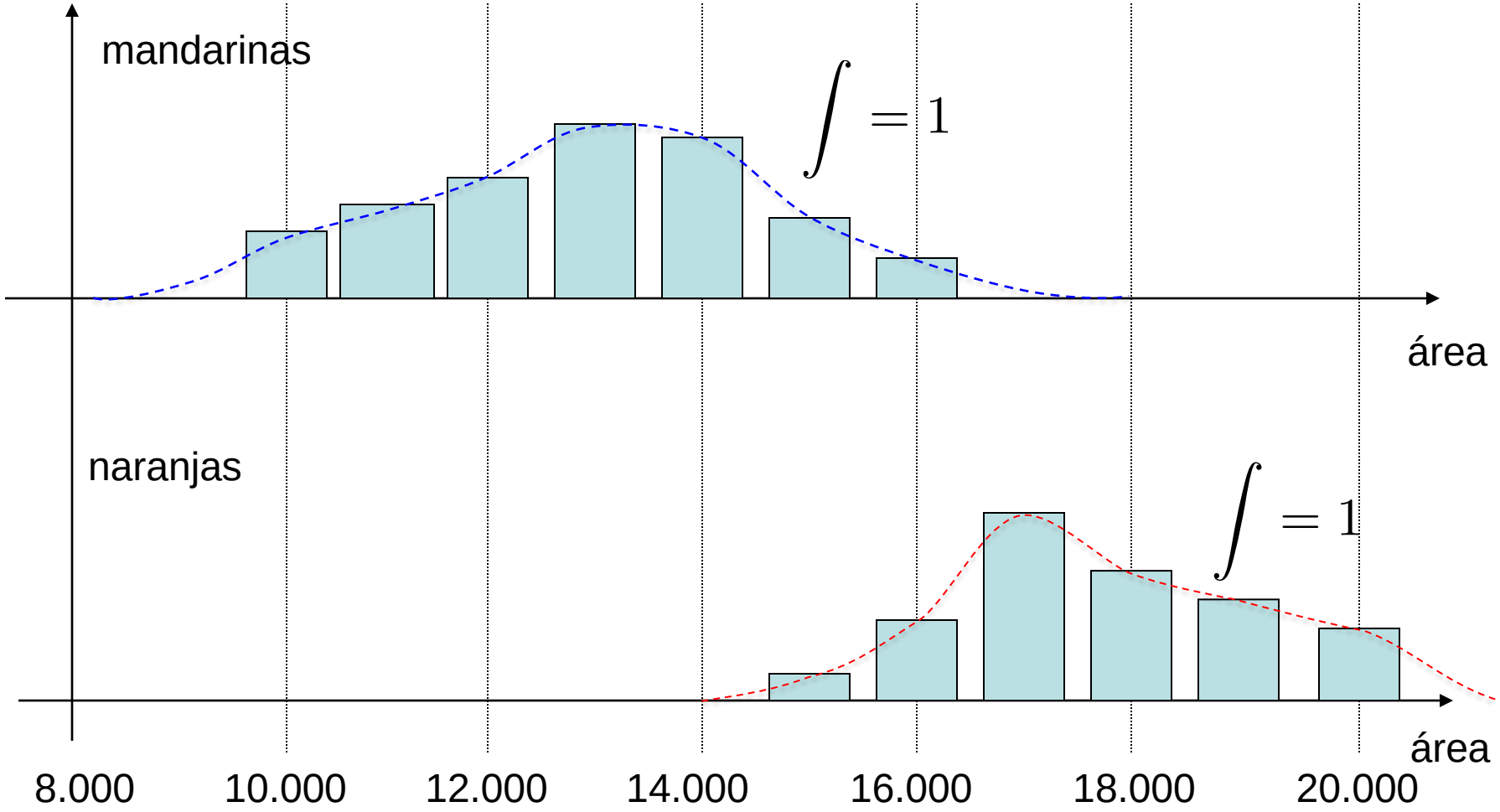
Mandarinas vs. Naranjas

Histogramas:



Mandarinas vs. Naranjas

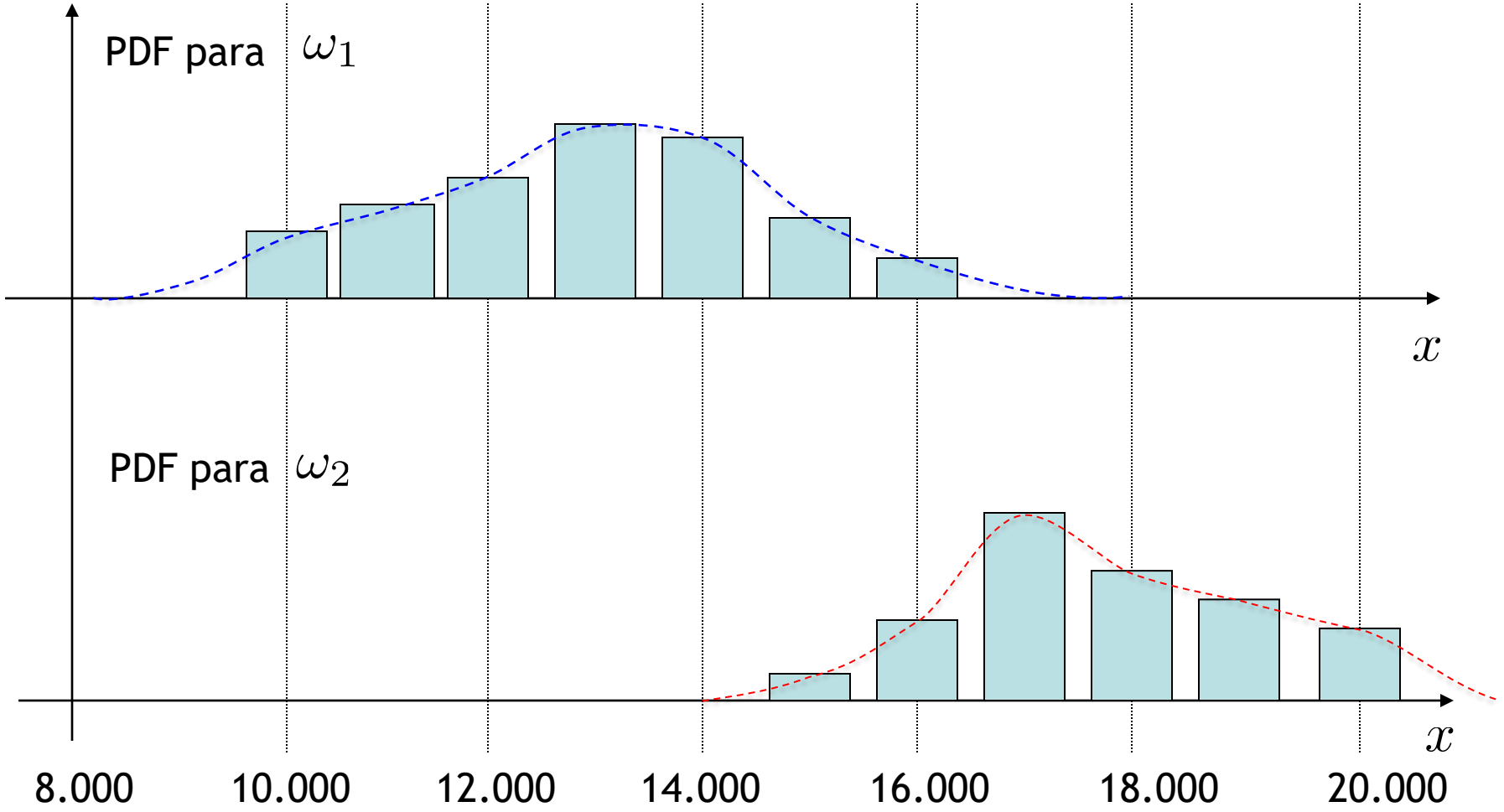
PDF: Probability Density Function



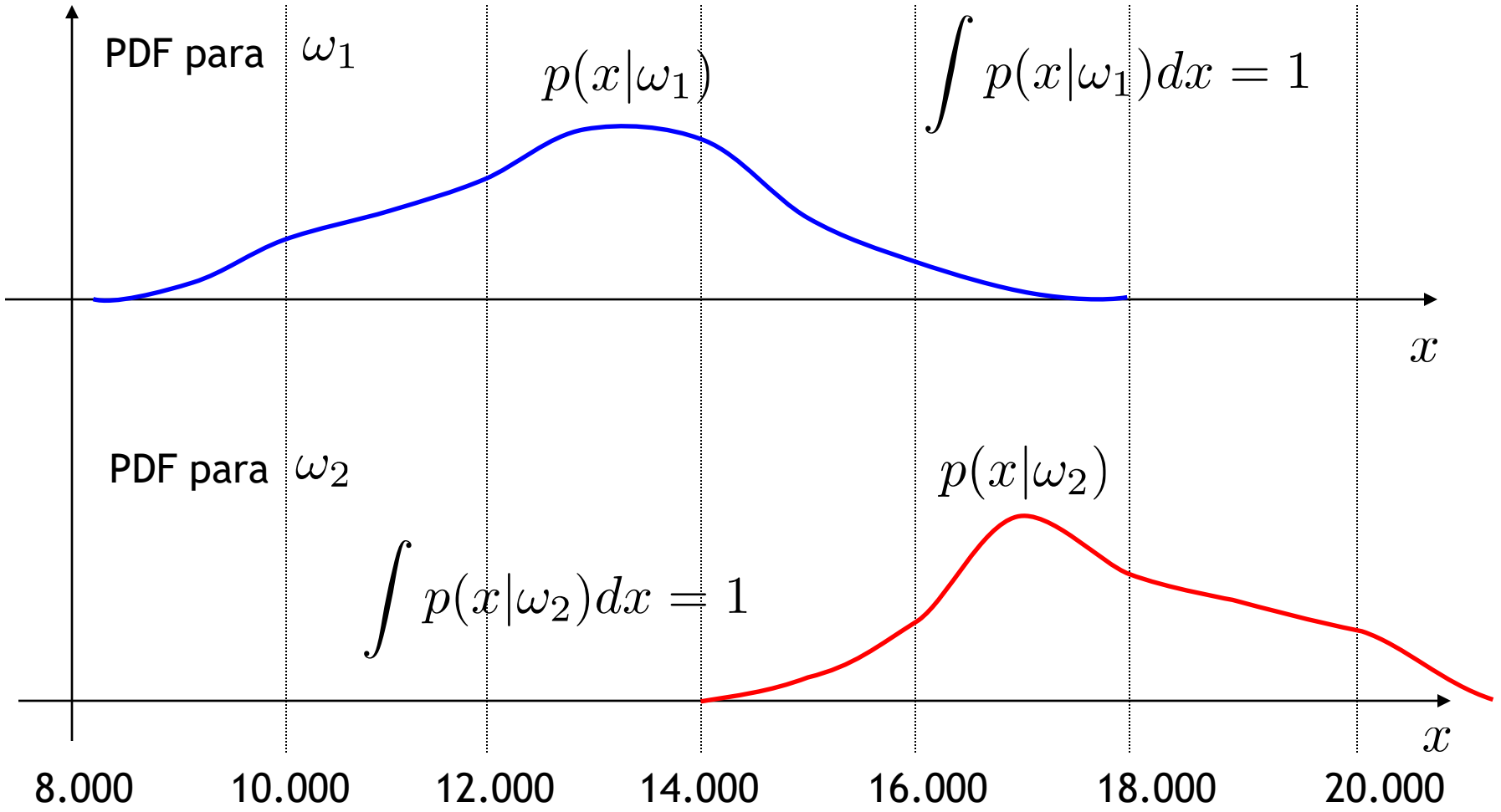
Definiciones

ω_1	Clase Mandarina
ω_2	Clase Naranja
x	Característica (área)

PDF: Probability Density Function



PDF: Probability Density Function



Definiciones

ω_1	Clase Mandarina
ω_2	Clase Naranja
x	Característica (área)
$p(x\|\omega_1)$	Distribución de x para Mandarinas
$p(x\|\omega_2)$	Distribución de x para Naranjas

Clasificación según Bayes

Dado el valor ***x*** (área), calculamos

- p_1, la probabilidad de que sea ω_1 (mandarina)
- p_2, la probabilidad de que sea ω_2 (naranja)

Clasificamos ***x*** como ω_1 si $p_1 > p_2$,

en caso contrario clasificamos ***x*** como ω_2

Clasificación según Bayes

Dado el valor x (área), calculamos

- p_1, la probabilidad de que sea ω_1 (mandarina)
- p_2, la probabilidad de que sea ω_2 (naranja)

Clasificamos x como ω_1 si $p_1 > p_2$,

en caso contrario clasificamos x como ω_2

ALGORITMO:

$p(\omega_1|x) > p(\omega_2|x) \rightarrow \omega_1$ (mandarina)

else $\rightarrow \omega_2$ (naranja)

$p_1 = p(\omega_1|x)$

$p_2 = p(\omega_2|x)$

Clasificación según Bayes

ALGORITMO:

$p(\omega_1|x) > p(\omega_2|x) \rightarrow \omega_1$ (mandarina)

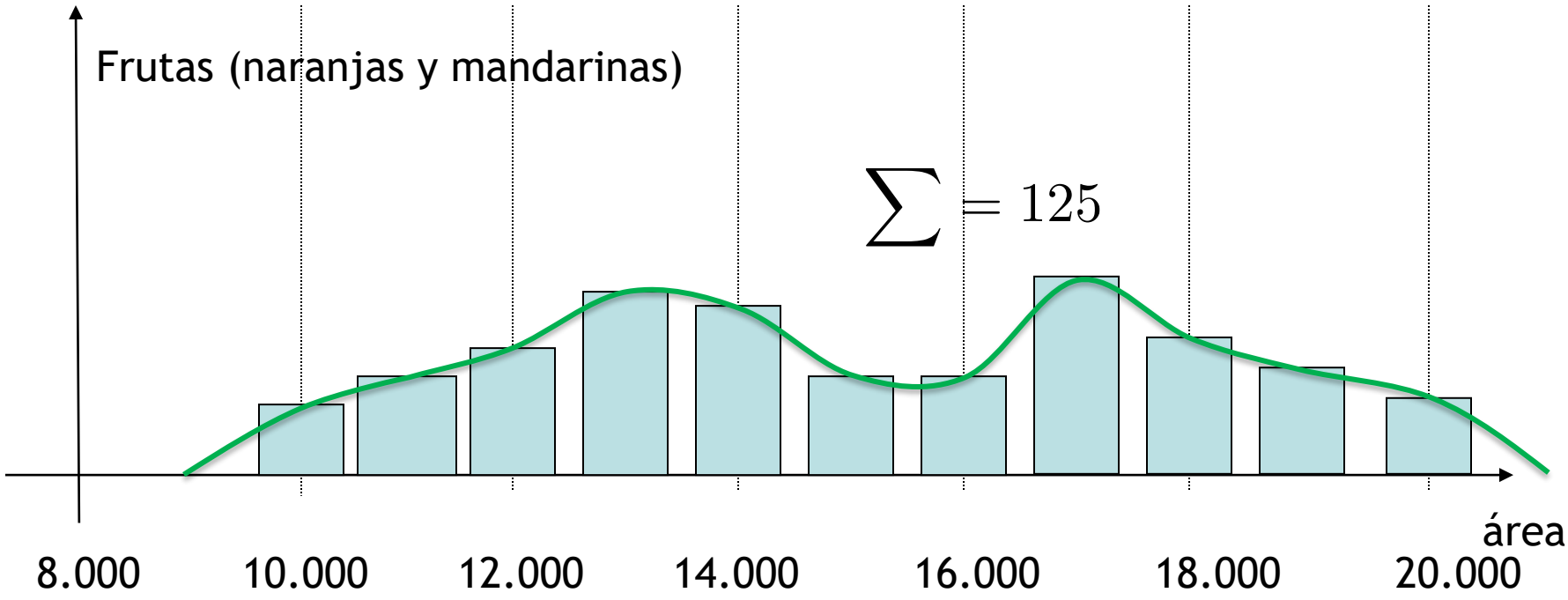
else $\rightarrow \omega_2$ (naranja)

$$p(\omega_1|x) = \frac{p(\omega_1)p(x|\omega_1)}{p(x)}$$

$$p(\omega_2|x) = \frac{p(\omega_2)p(x|\omega_2)}{p(x)}$$

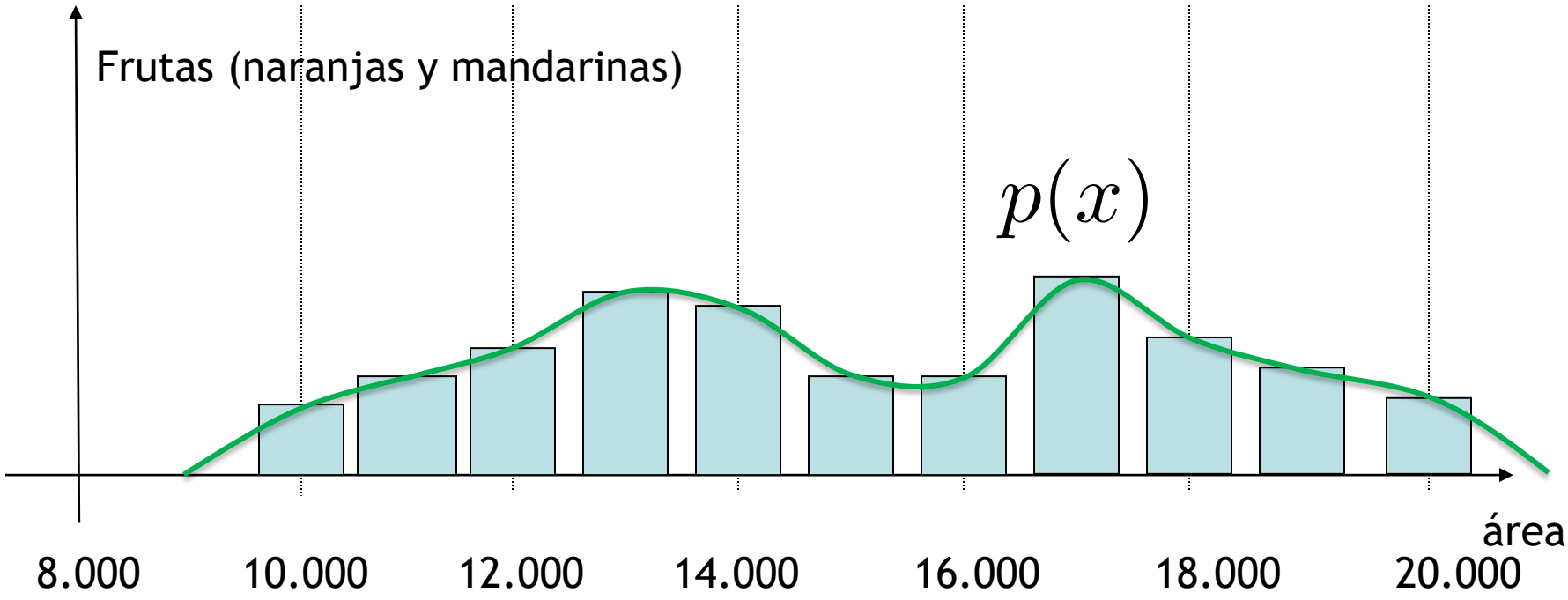
Mandarinas vs. Naranjas

Histogramas:



Mandarinas vs. Naranjas

PDF:



Clasificación según Bayes

ALGORITMO:

$p(\omega_1|x) > p(\omega_2|x) \rightarrow \omega_1$ (mandarina)

else $\rightarrow \omega_2$ (naranja)

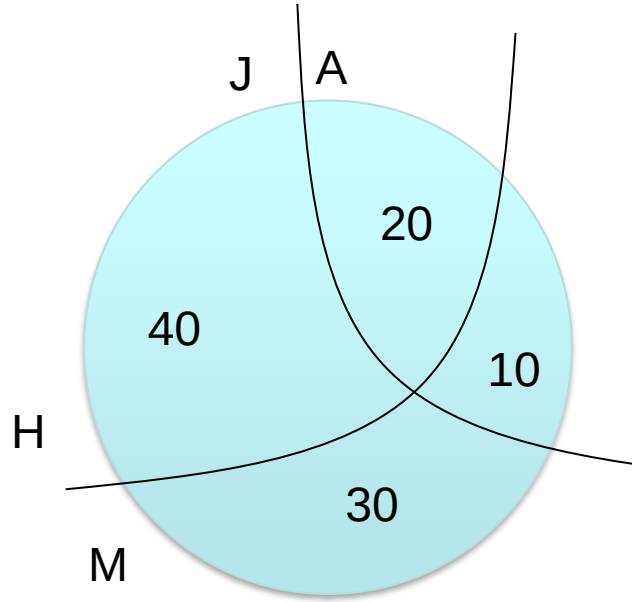
$$p(\omega_1|x) = \frac{p(\omega_1)p(x|\omega_1)}{p(x)}$$

$$p(\omega_2|x) = \frac{p(\omega_2)p(x|\omega_2)}{p(x)}$$

EJEMPLO: Una clase de cocina que imparte la municipalidad

Se inscriben 100 personas

- 60 hombres (40 jóvenes y 20 adultos)
- 40 mujeres (30 jóvenes y 10 adultos)



$$P(H)P(J|H) = P(J)P(J|H)$$

$$\frac{60}{100}\frac{40}{60} = \frac{70}{100}\frac{40}{70}$$

Clasificación según Bayes

ALGORITMO:

$p(\omega_1|x) > p(\omega_2|x) \rightarrow \omega_1$ (mandarina)

else $\rightarrow \omega_2$ (naranja)

$$\frac{p(\omega_1)p(x|\omega_1)}{\cancel{p(x)}} > \frac{p(\omega_2)p(x|\omega_2)}{\cancel{p(x)}}$$

Clasificación según Bayes

ALGORITMO:

$p(\omega_1|x) > p(\omega_2|x) \rightarrow \omega_1$ (mandarina)

else $\rightarrow \omega_2$ (naranja)

$$p(\omega_1)p(x|\omega_1) > p(\omega_2)p(x|\omega_2)$$

Clasificación según Bayes

ALGORITMO:

$$p(\omega_1|x) > p(\omega_2|x) \rightarrow \omega_1 \quad \text{(mandarina)}$$

$$\text{else} \rightarrow \omega_2 \quad \text{(naranja)}$$

ALGORITMO:

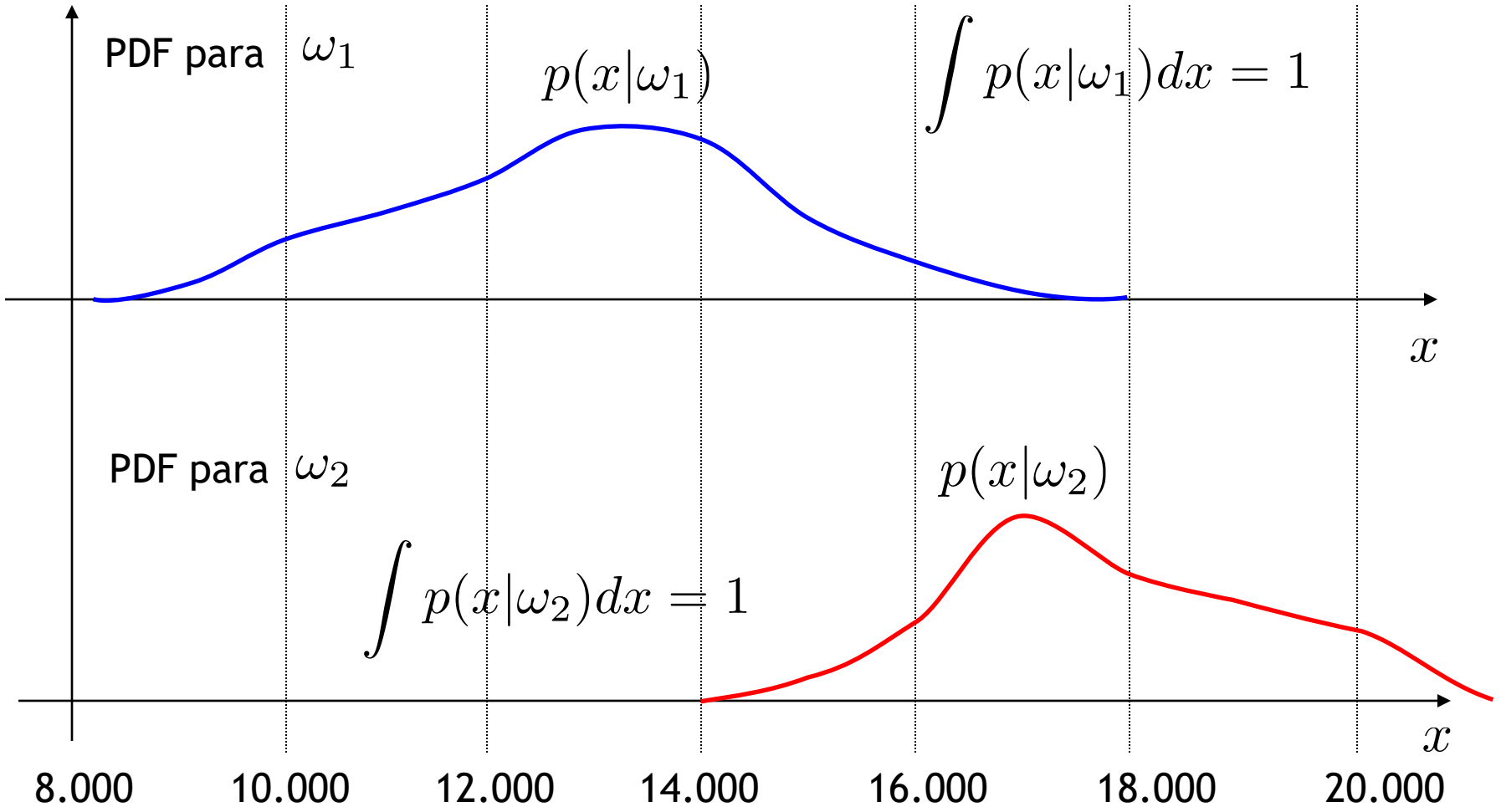
$$p(\omega_1)p(x|\omega_1) > p(\omega_2)p(x|\omega_2) \rightarrow \omega_1 \quad \text{(mandarina)}$$

$$\text{else} \rightarrow \omega_2 \quad \text{(naranja)}$$

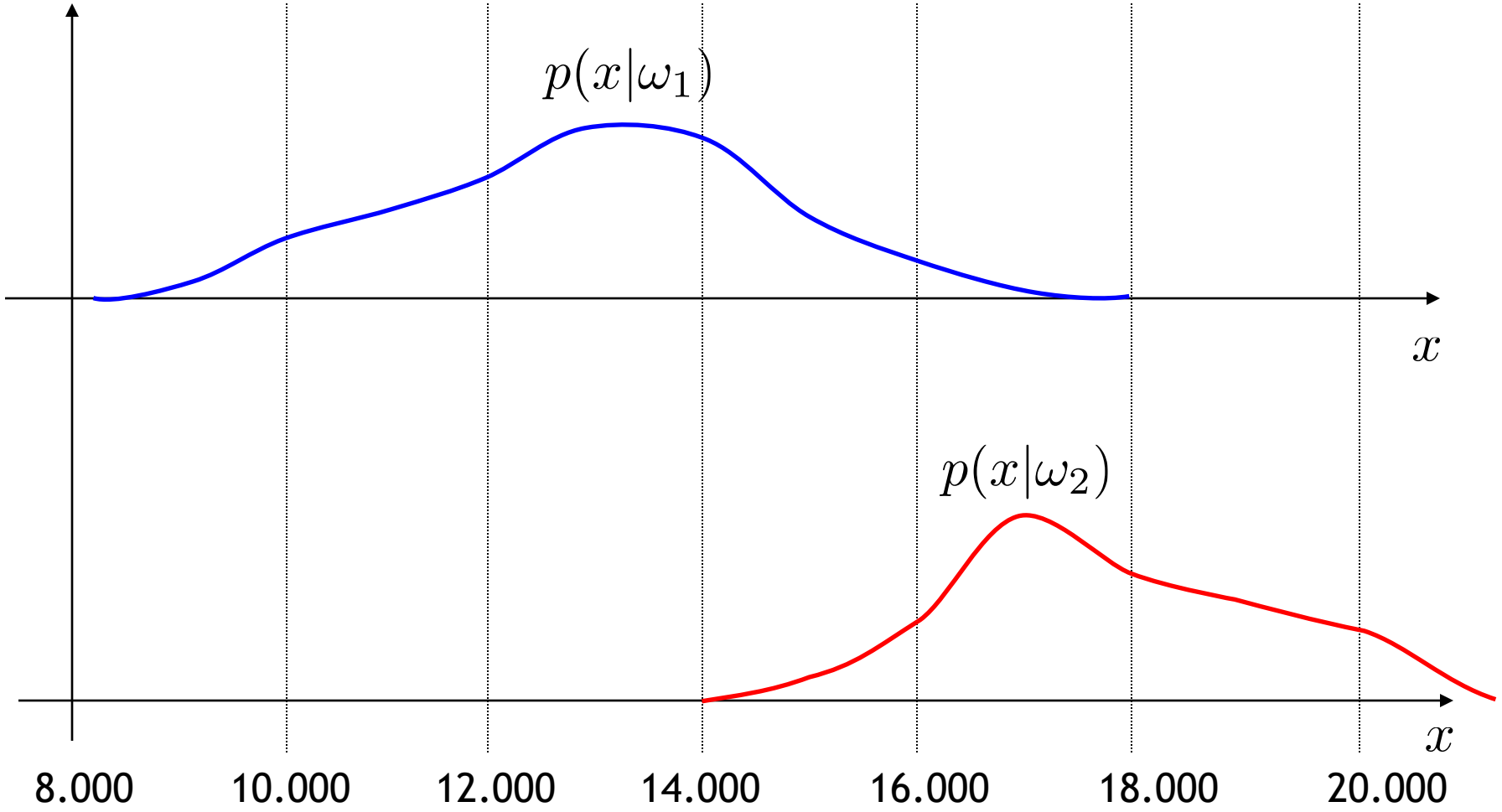
Definiciones

ω_1	Clase Mandarina
ω_2	Clase Naranja
x	Característica (área)
$p(x\|\omega_1)$	Distribución de x para Mandarinas
$p(x\|\omega_2)$	Distribución de x para Naranjas
$p(\omega_1)$	Probabilidad a priori de clase ω_1
$p(\omega_2)$	Probabilidad a priori de clase ω_2

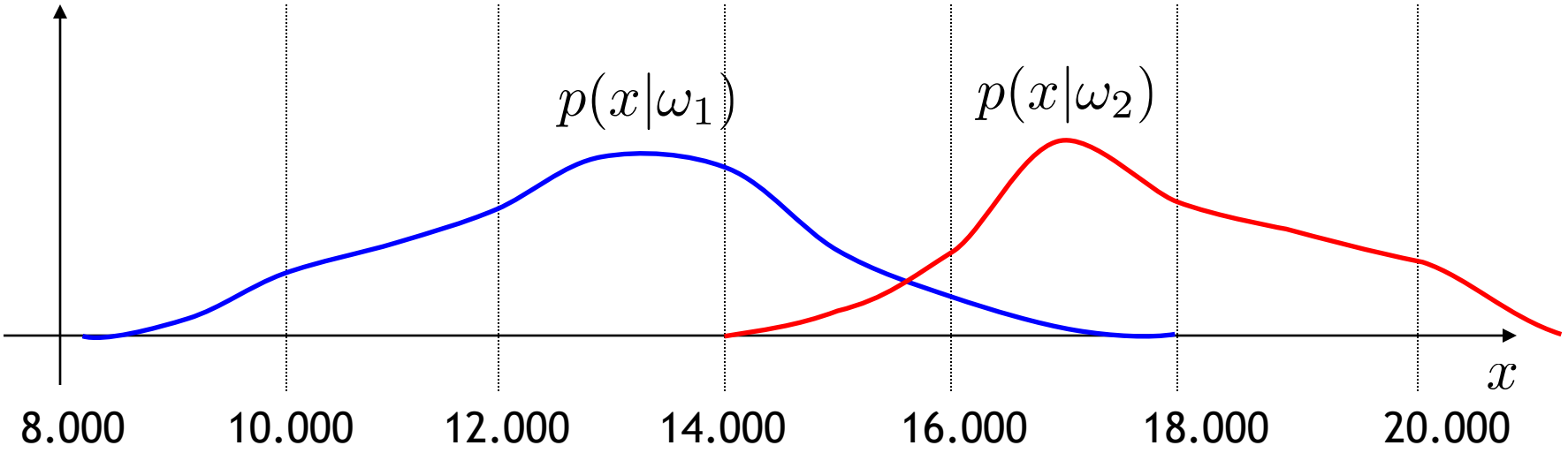
PDF: Probability Density Function



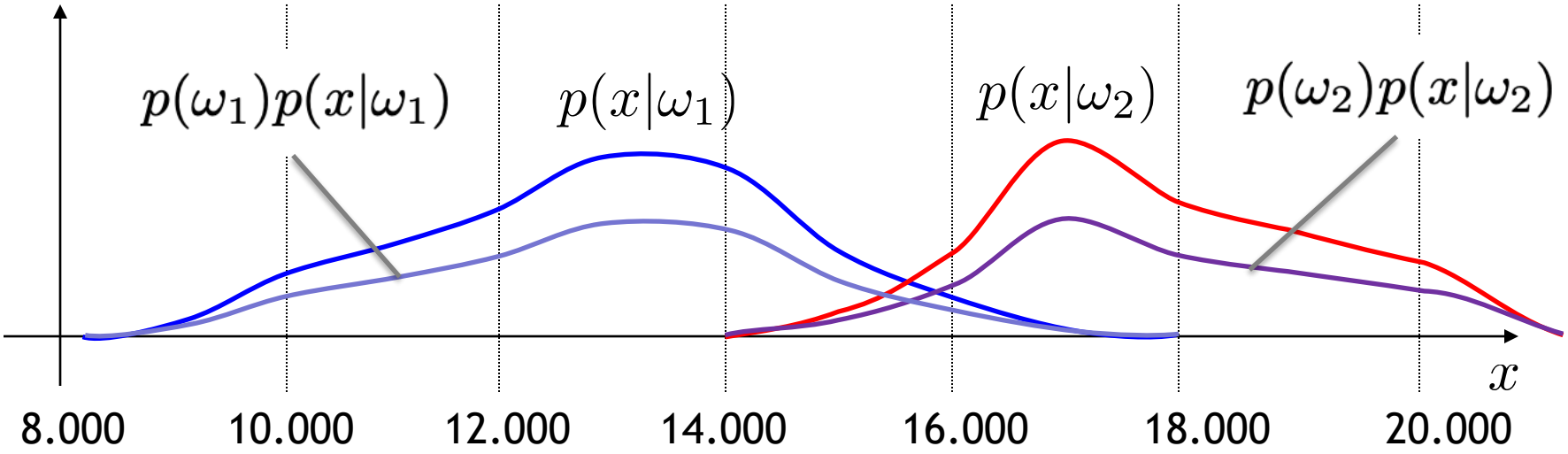
PDF: Probability Density Function



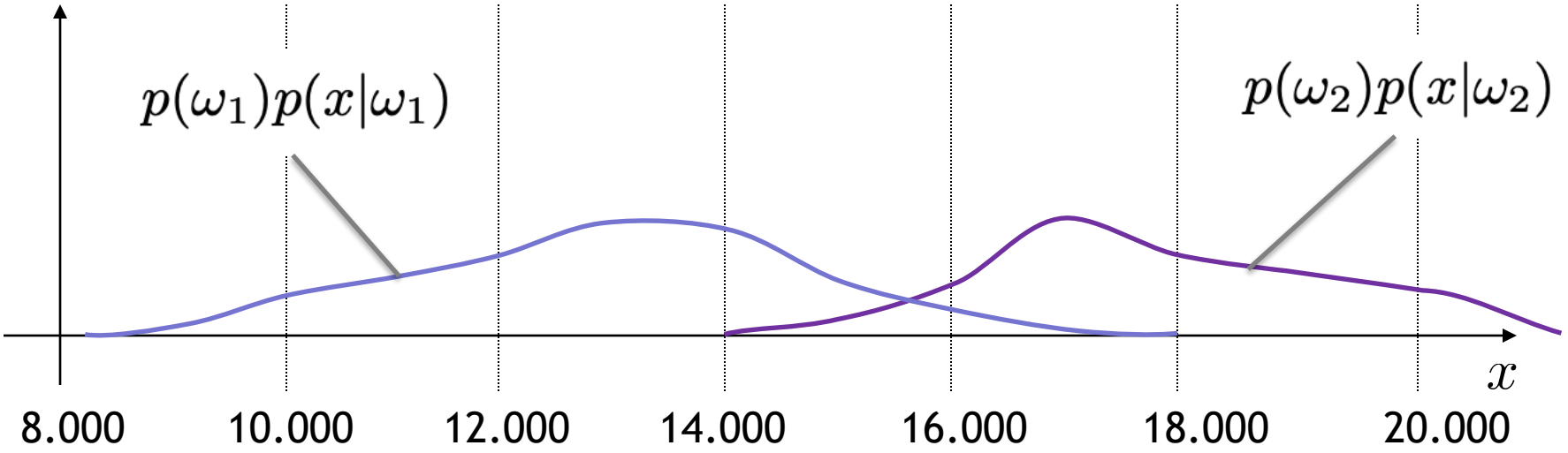
PDF: Probability Density Function



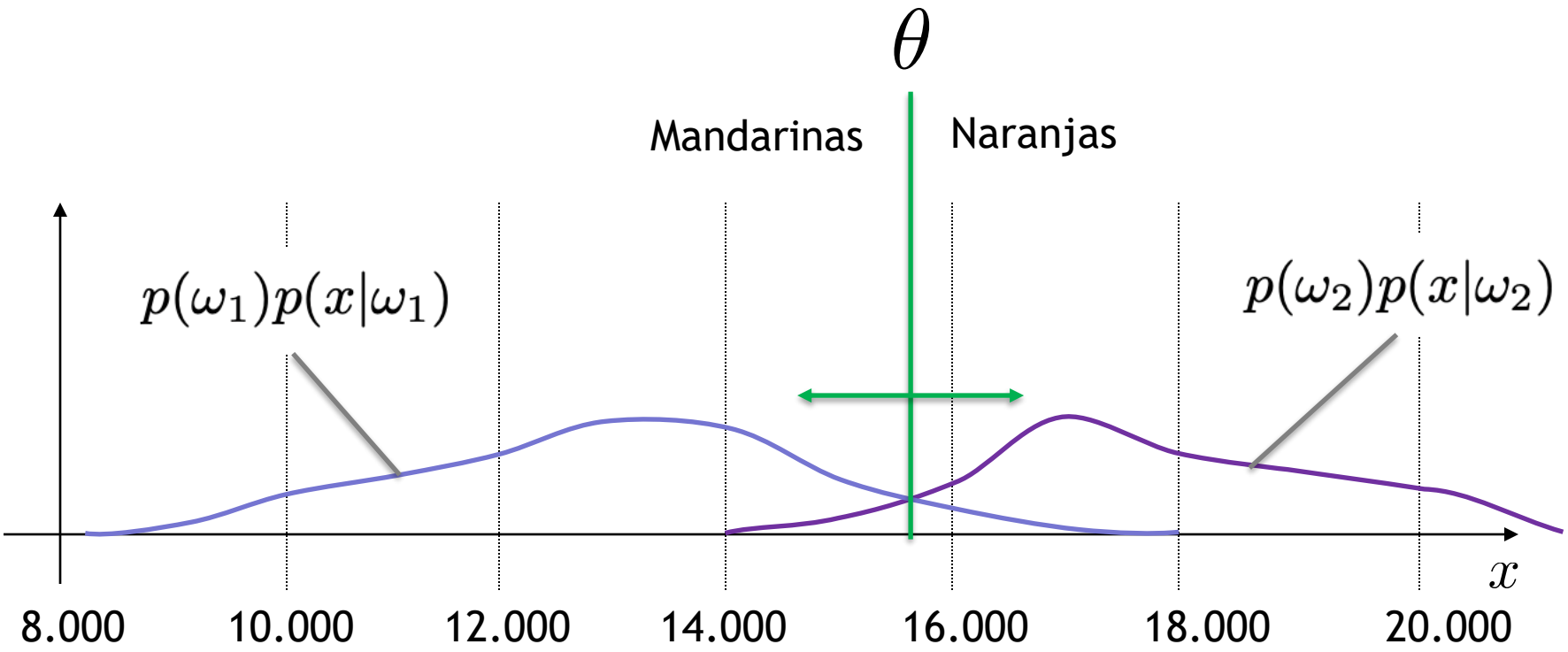
PDF: Probability Density Function



PDF: Probability Density Function



PDF: Probability Density Function



Clasificación según Bayes

ALGORITMO:

$$p(\omega_1|x) > p(\omega_2|x) \rightarrow \omega_1 \quad \text{(mandarina)}$$

$$\text{else} \rightarrow \omega_2 \quad \text{(naranja)}$$

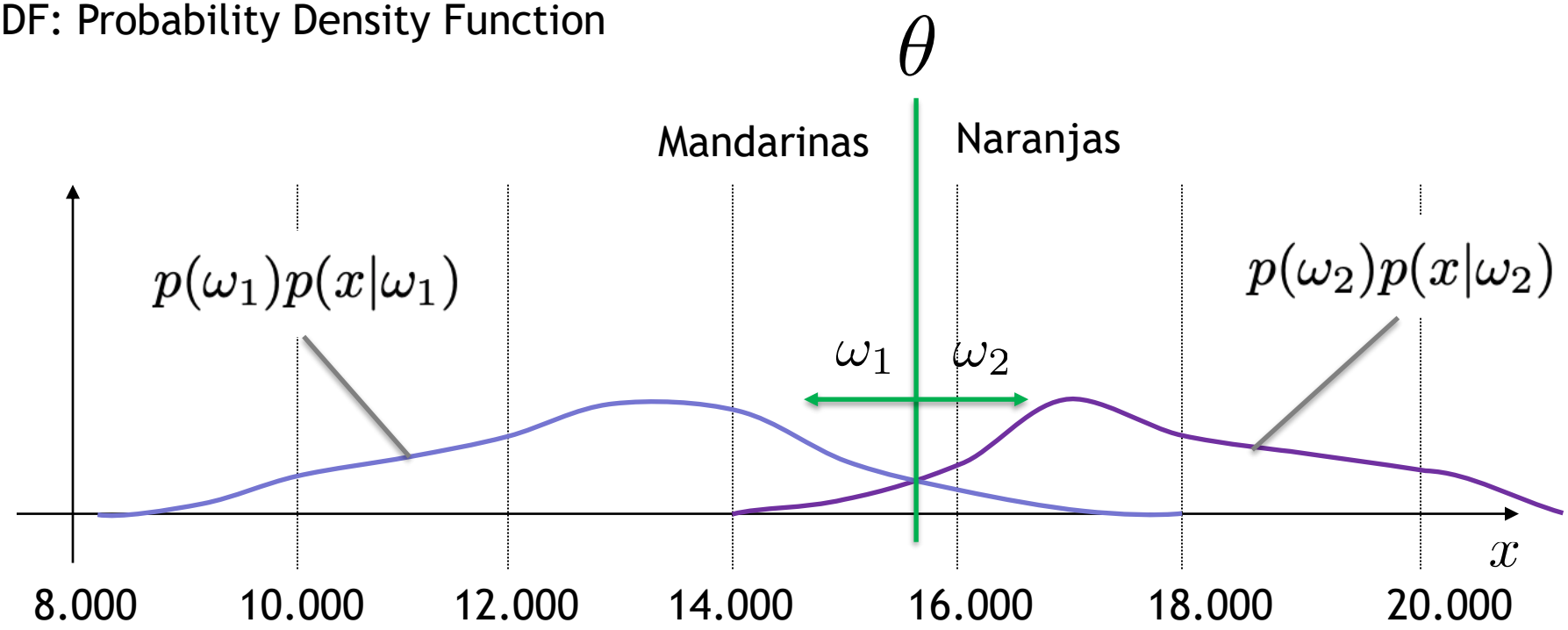
ALGORITMO:

$$p(\omega_1)p(x|\omega_1) \gtrless p(\omega_2)p(x|\omega_2) \rightarrow \omega_1 \quad \text{(mandarina)}$$

$$\text{else} \rightarrow \omega_2 \quad \text{(naranja)}$$

Mandarinas vs. Naranjas

PDF: Probability Density Function



ALGORITMO:

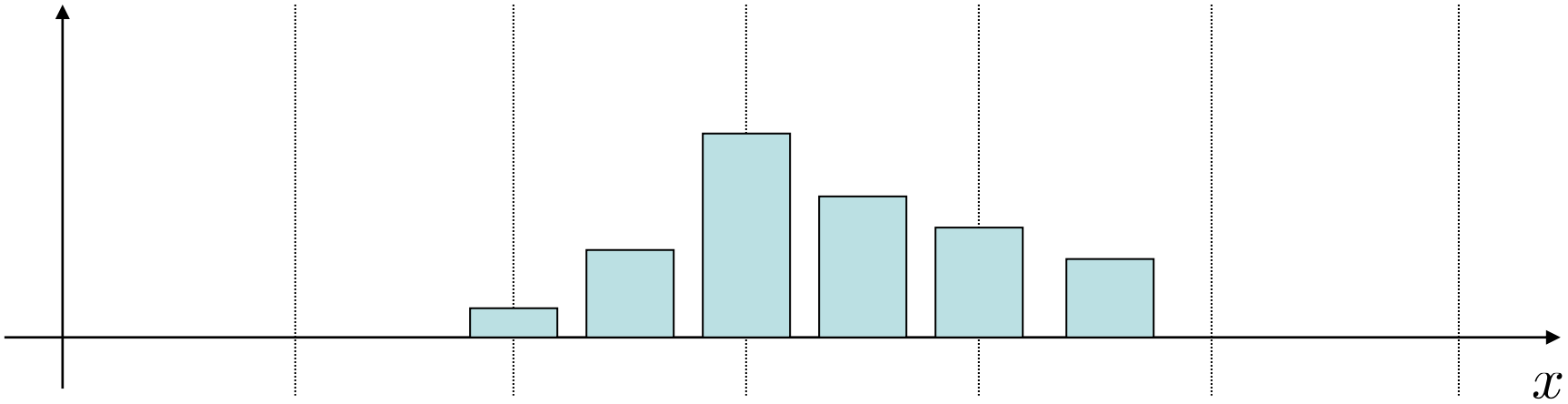
$x < \theta \rightarrow \omega_1$ (mandarina)

else $\rightarrow \omega_2$ (naranja)

Cómo calcular la PDF en 1D? KDE

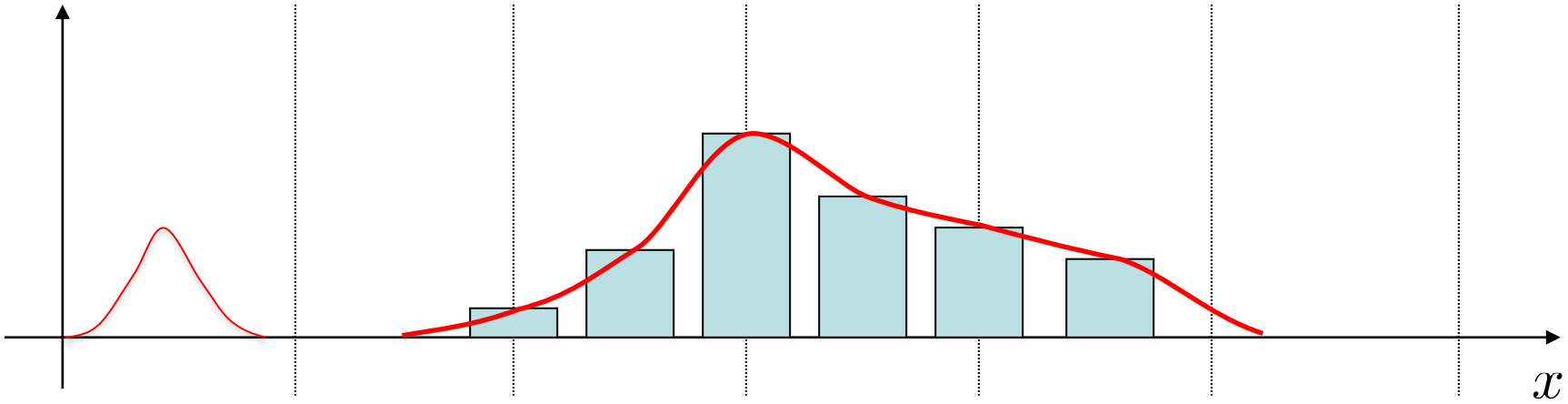
Cómo calcular la PDF?

PASO 1: Histograma



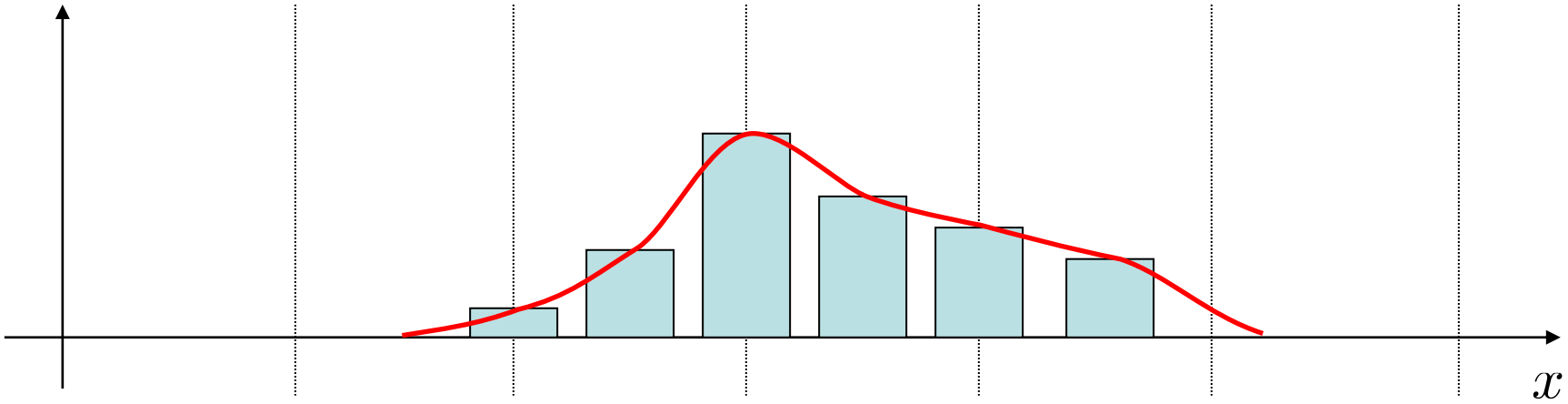
Cómo calcular la PDF?

PASO 2: Convolución con una Gaussiana (Kernel)



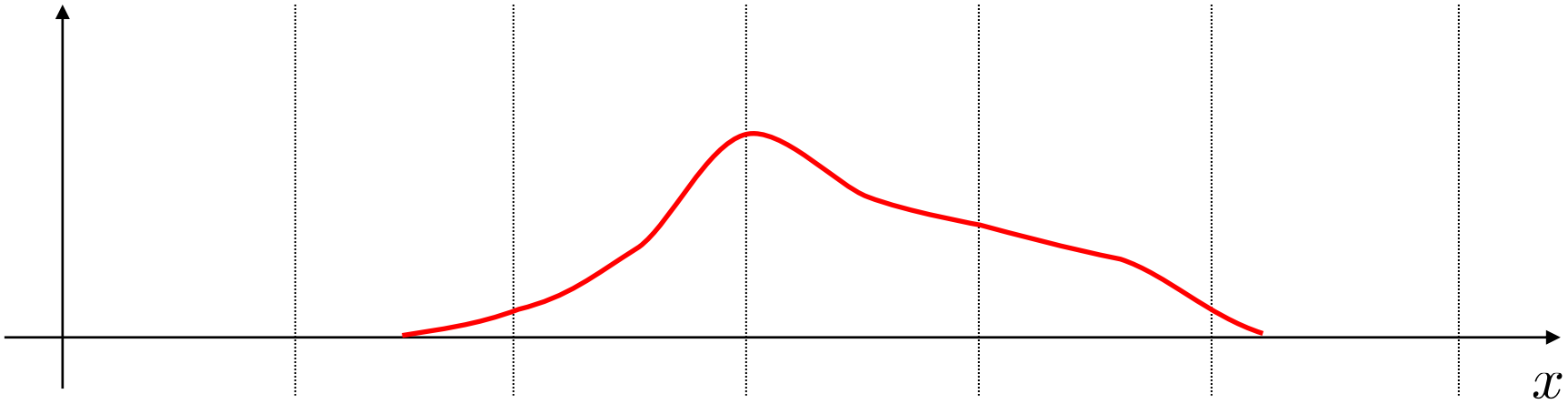
Cómo calcular la PDF?

PASO 2: Convolución con una Gaussiana (Kernel)



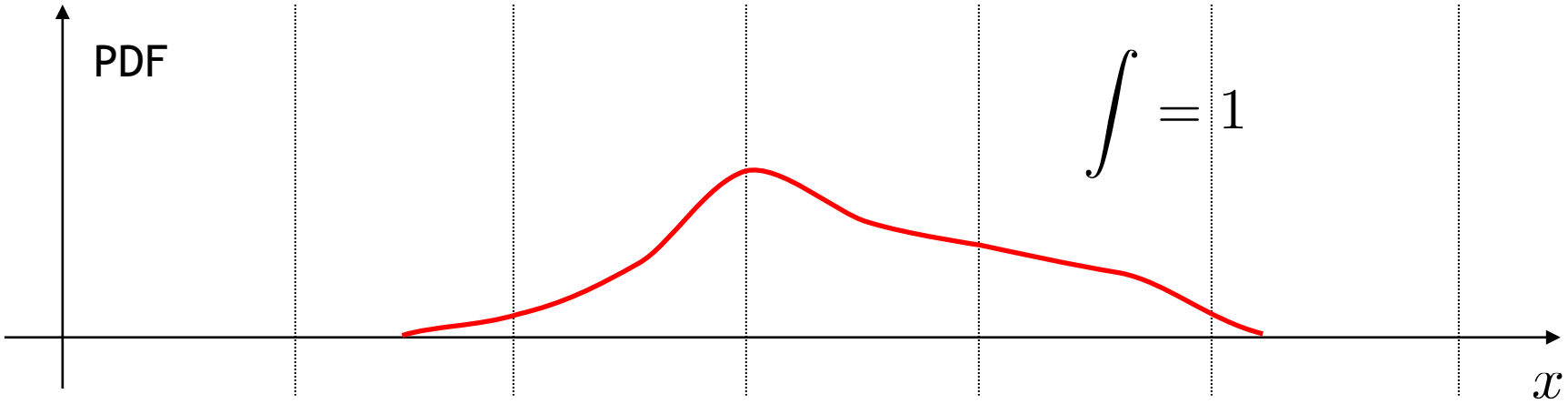
Cómo calcular la PDF?

PASO 2: Convolución con una Gaussiana (Kernel)

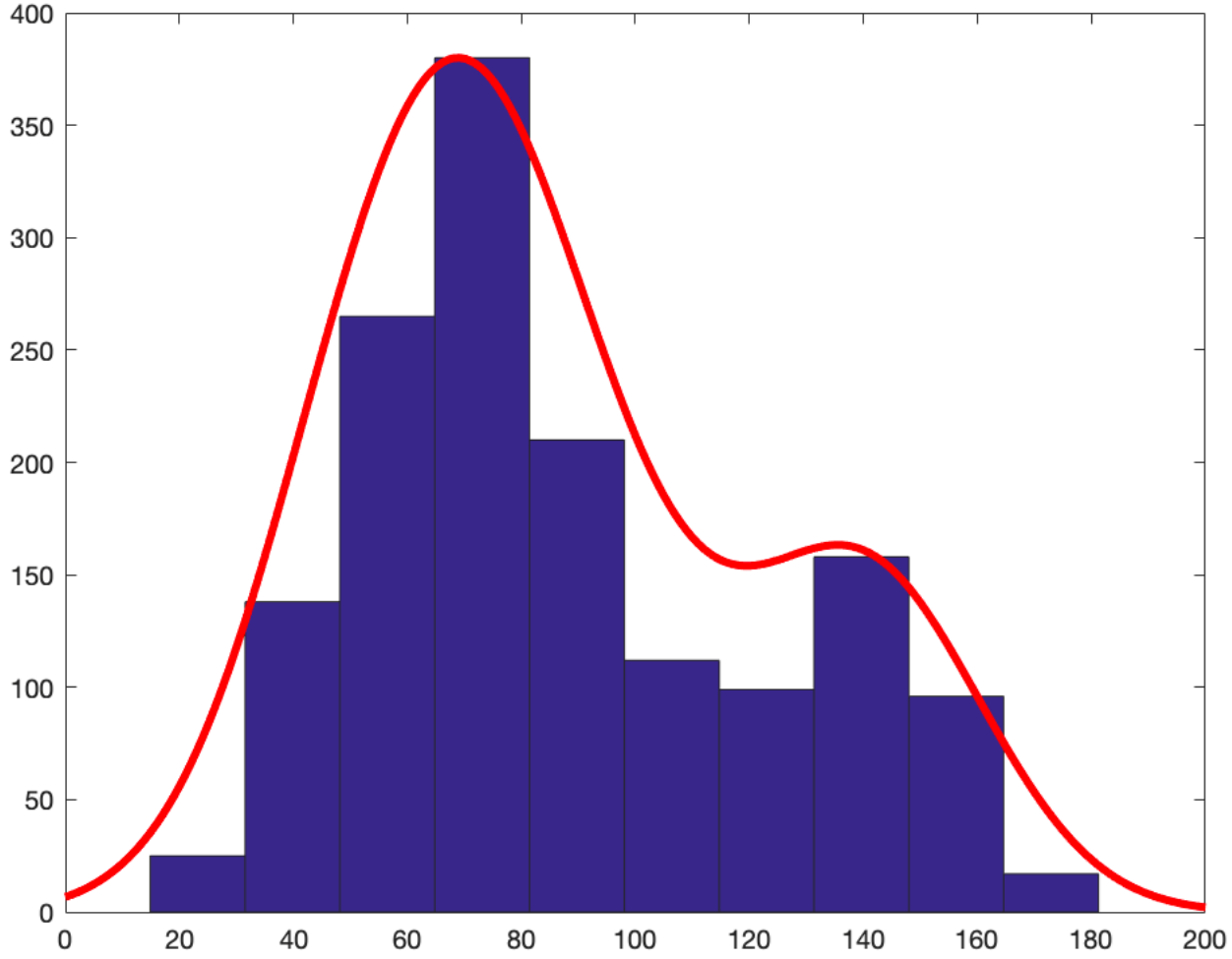


Cómo calcular la PDF?

PASO 3: División por el área

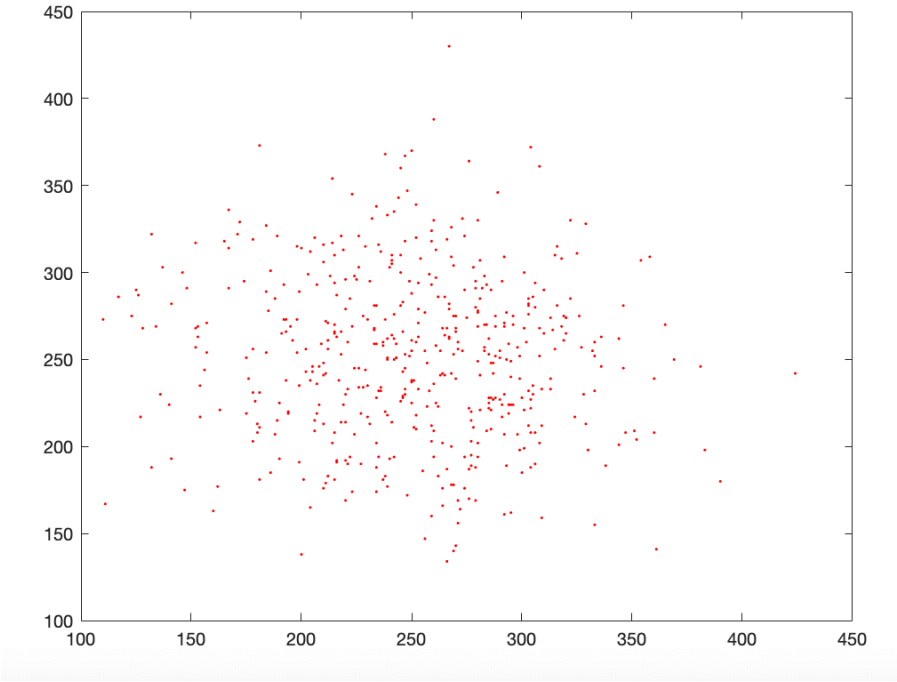


PDF en 1D

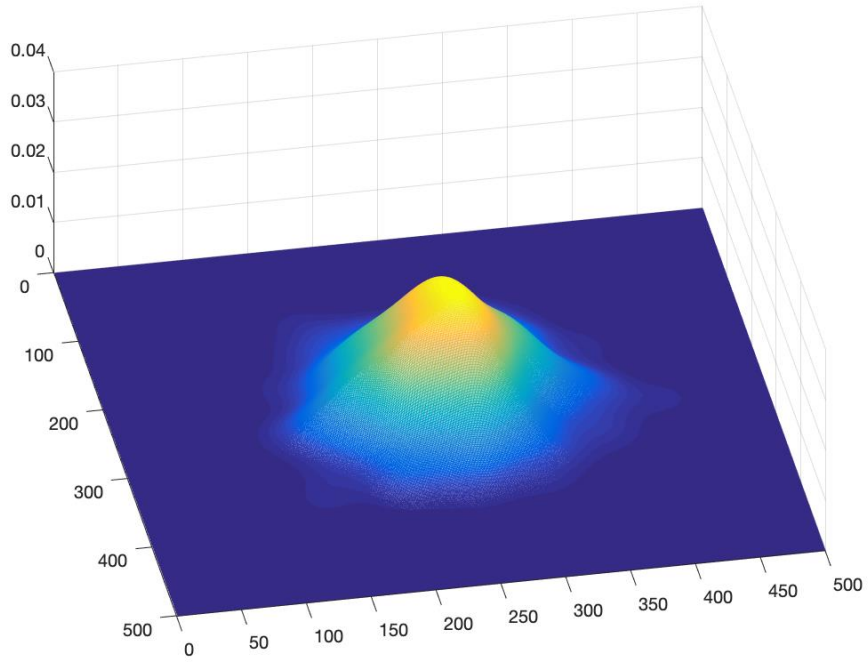

400
350
300
250
200
150
100
50
0
0
20
40
60
80
100
120
140
160
180
200

Cómo calcular la PDF en 2D? KDE

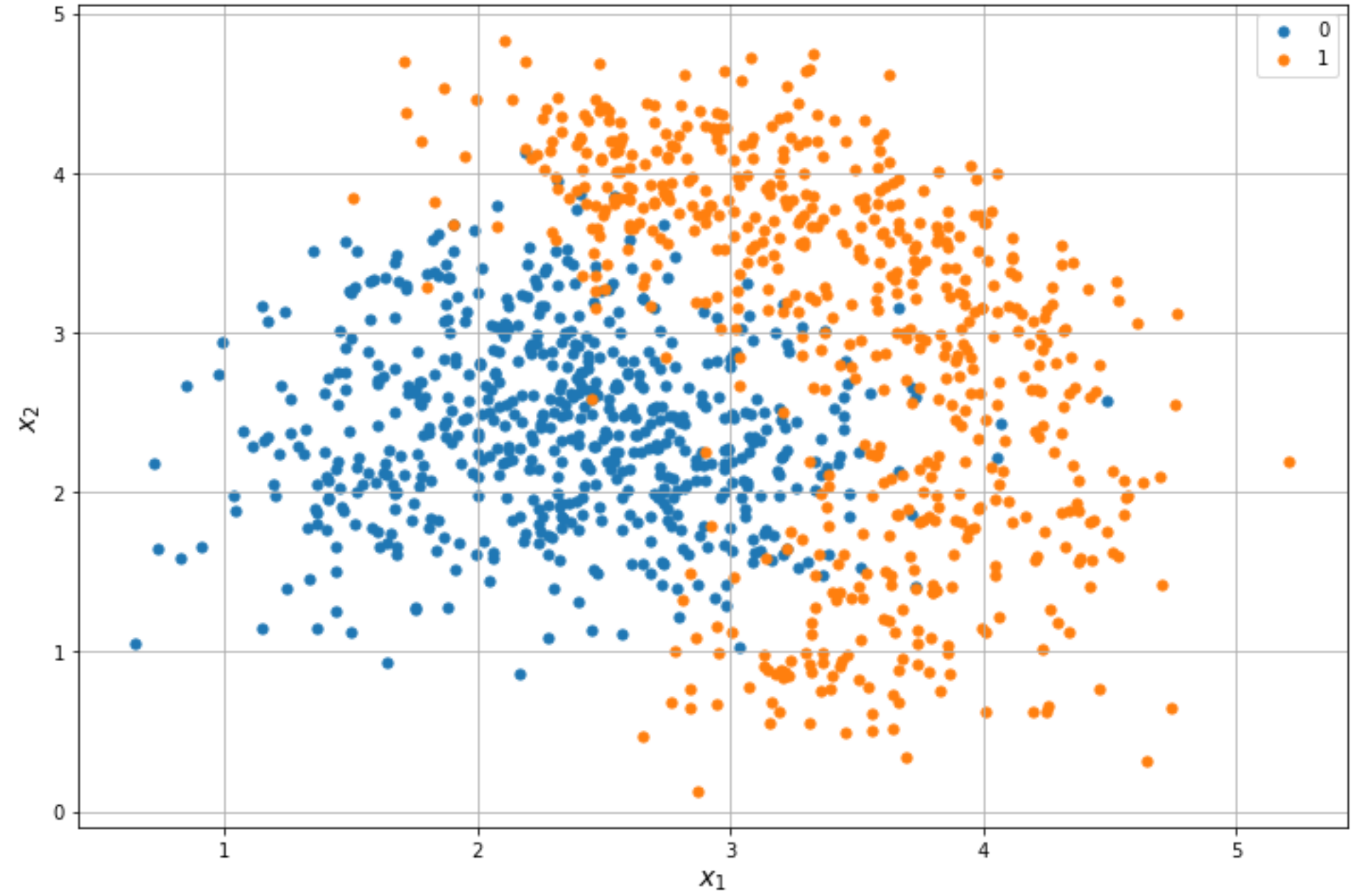
PDF en 2D

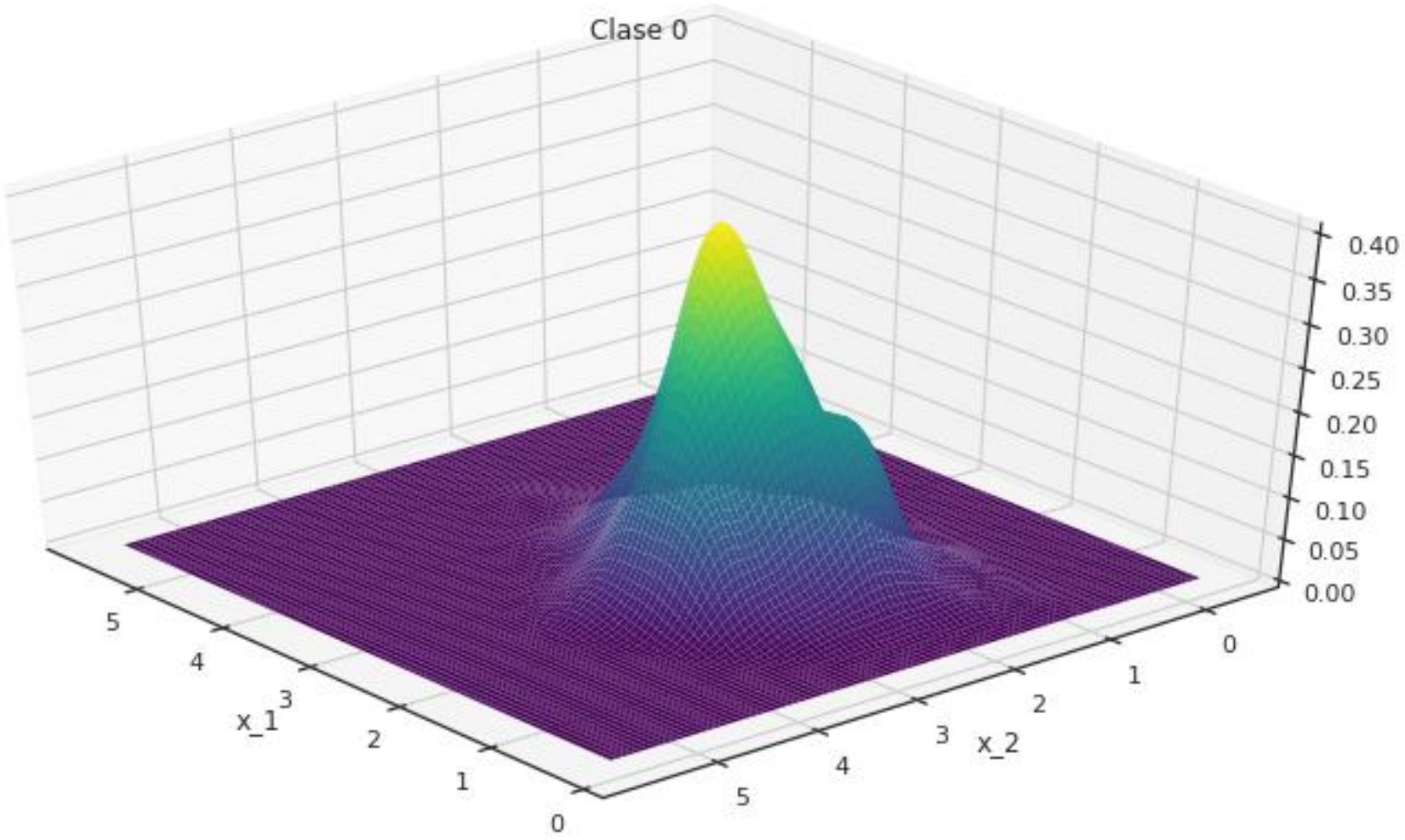


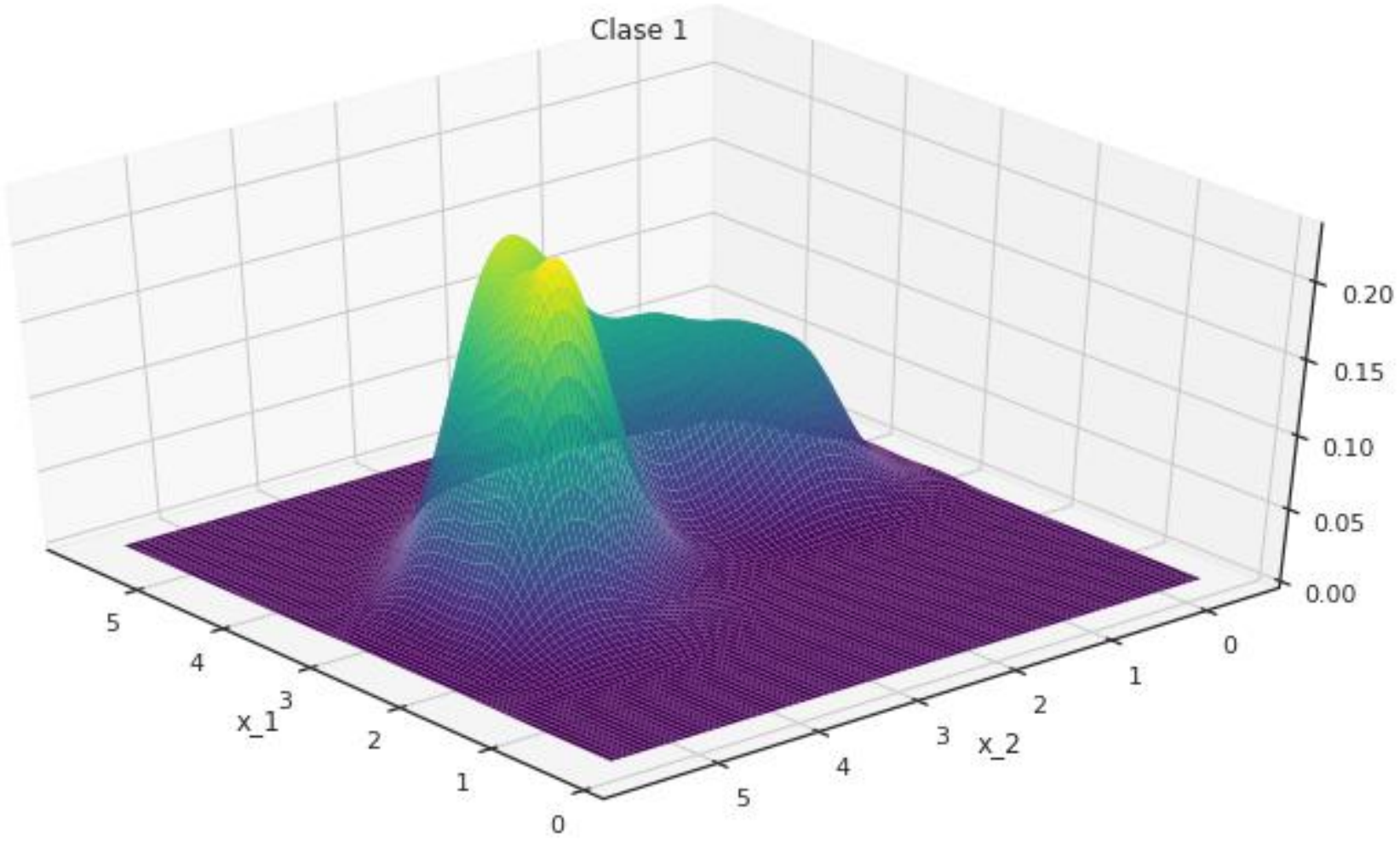
Muestras

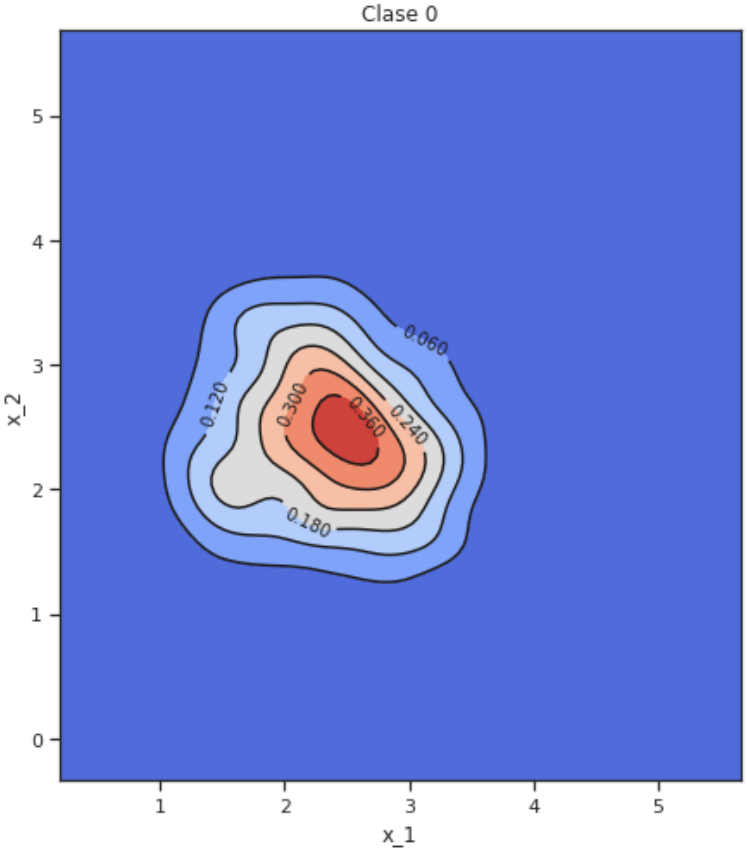


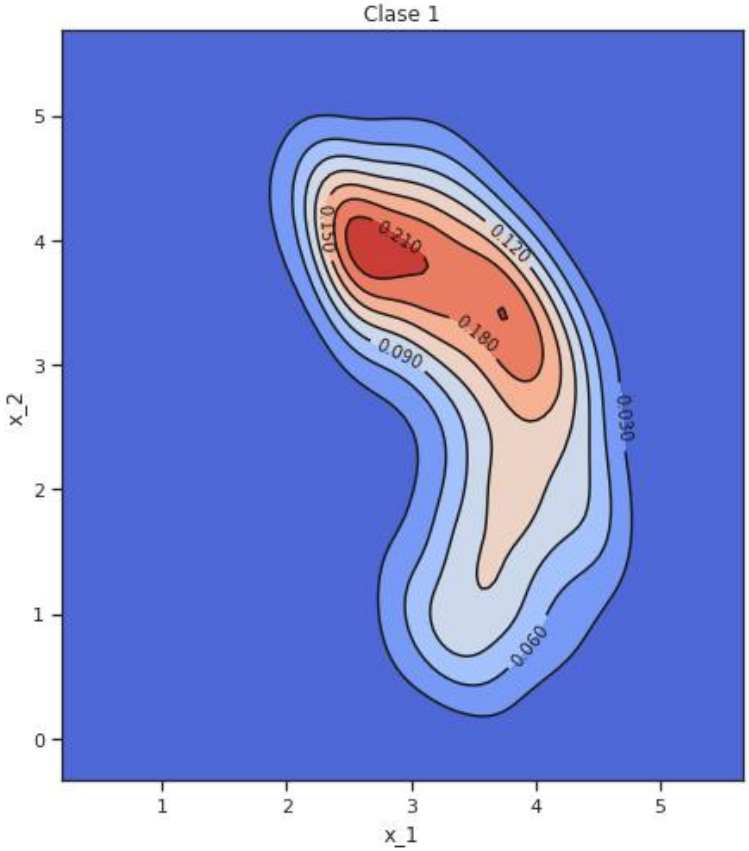
PDF

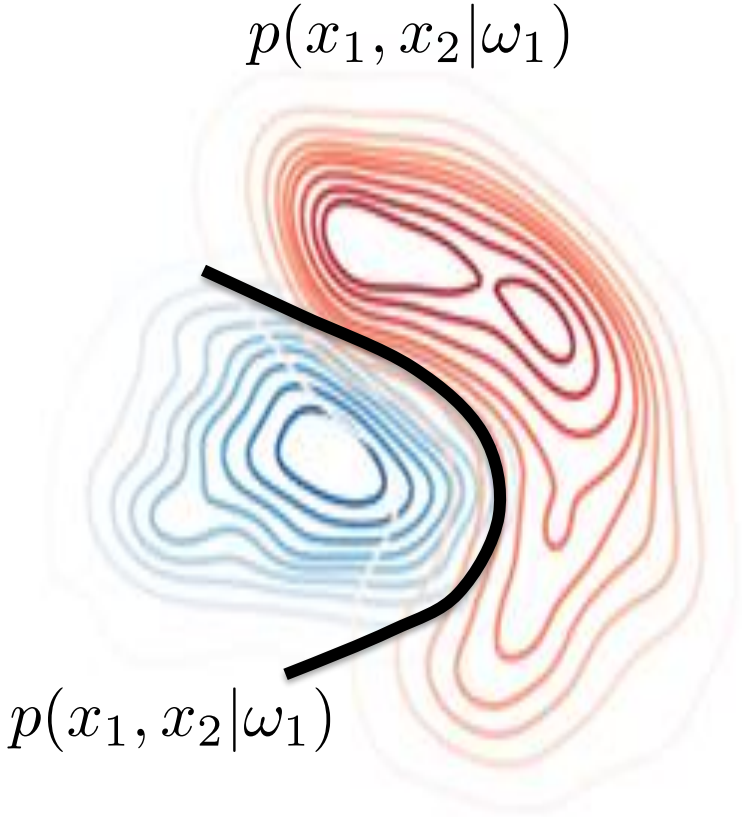

0
1
x_1
x_2
0
1
2
3
4
5
1
2
3
4
5

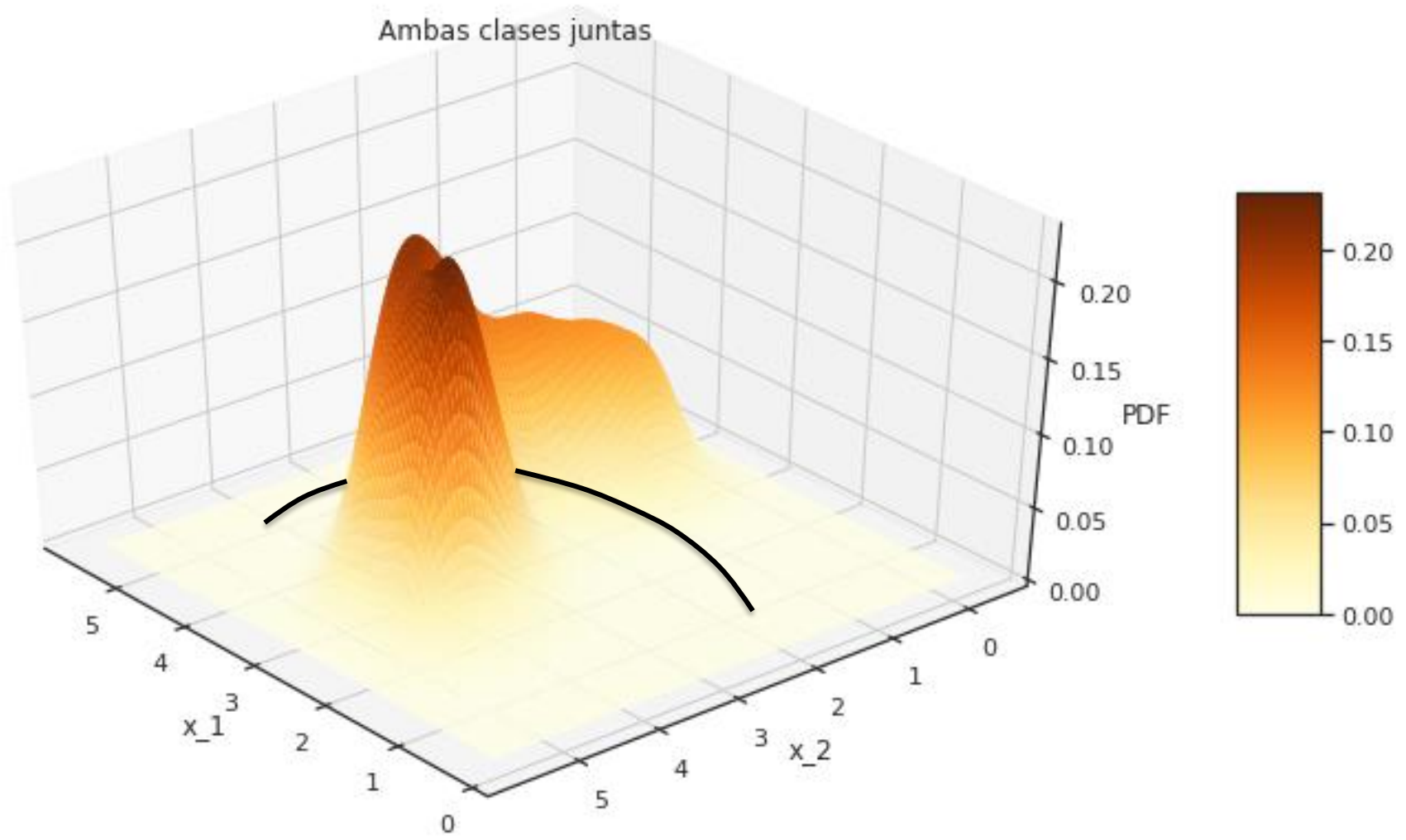

Clase 0
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
x_1
x_2
0
1
2
3
4
5


Clase 1
x_1
x_2
0
1
2
3
4
5
0.00
0.05
0.10
0.15
0.20


Clase 0
x_2
x_1
0.060
0.120
0.300
0.360
0.240
0.180


Clase 1
x_2
x_1
0.150
0.210
0.120
0.180
0.090
0.030
0.060


$p(x_1, x_2|\omega_1)$
$p(x_1, x_2|\omega_1)$

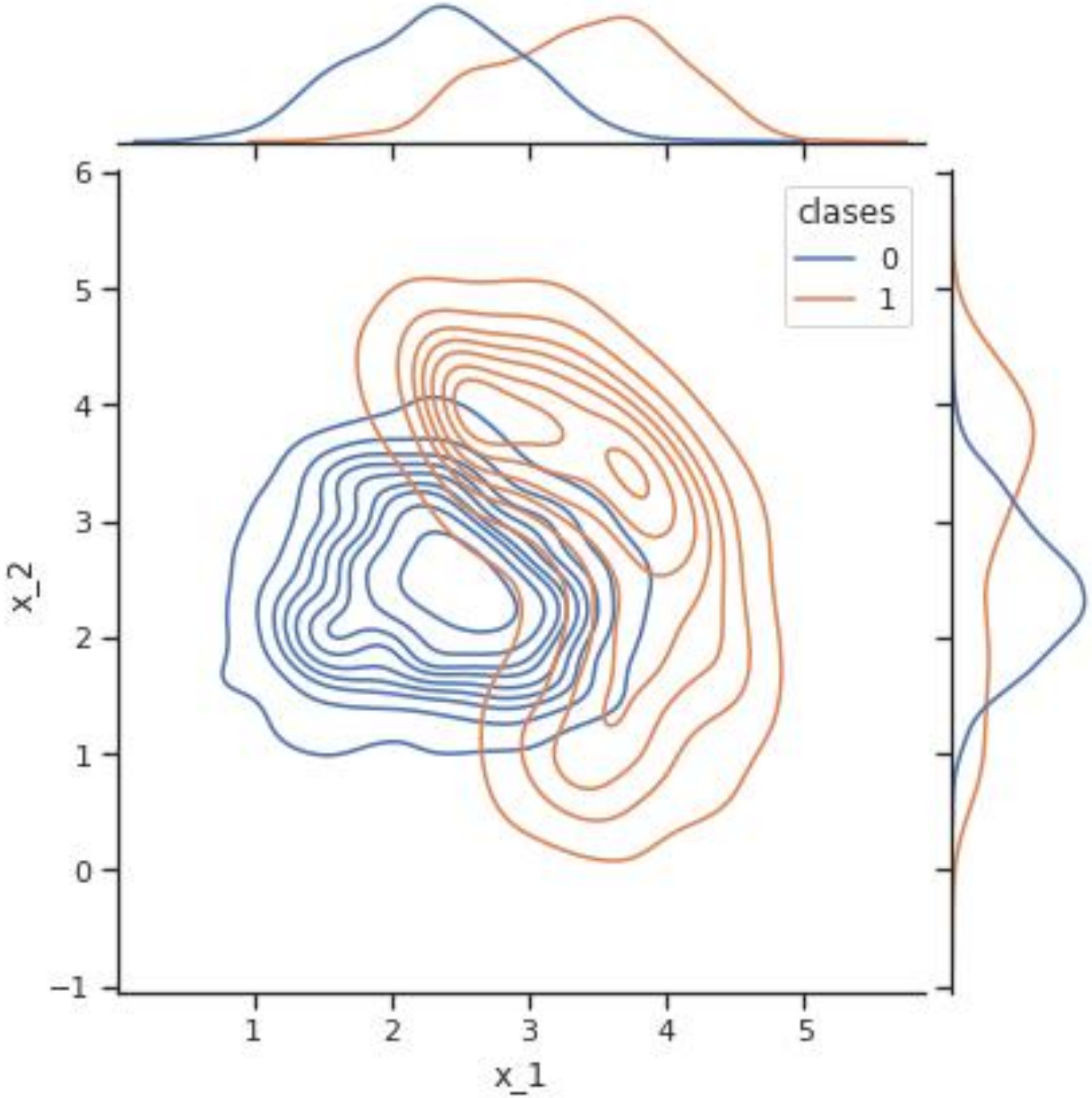

Ambas clases juntas
x_1
x_2
PDF
0
1
2
3
4
5
0.00
0.05
0.10
0.15
0.20

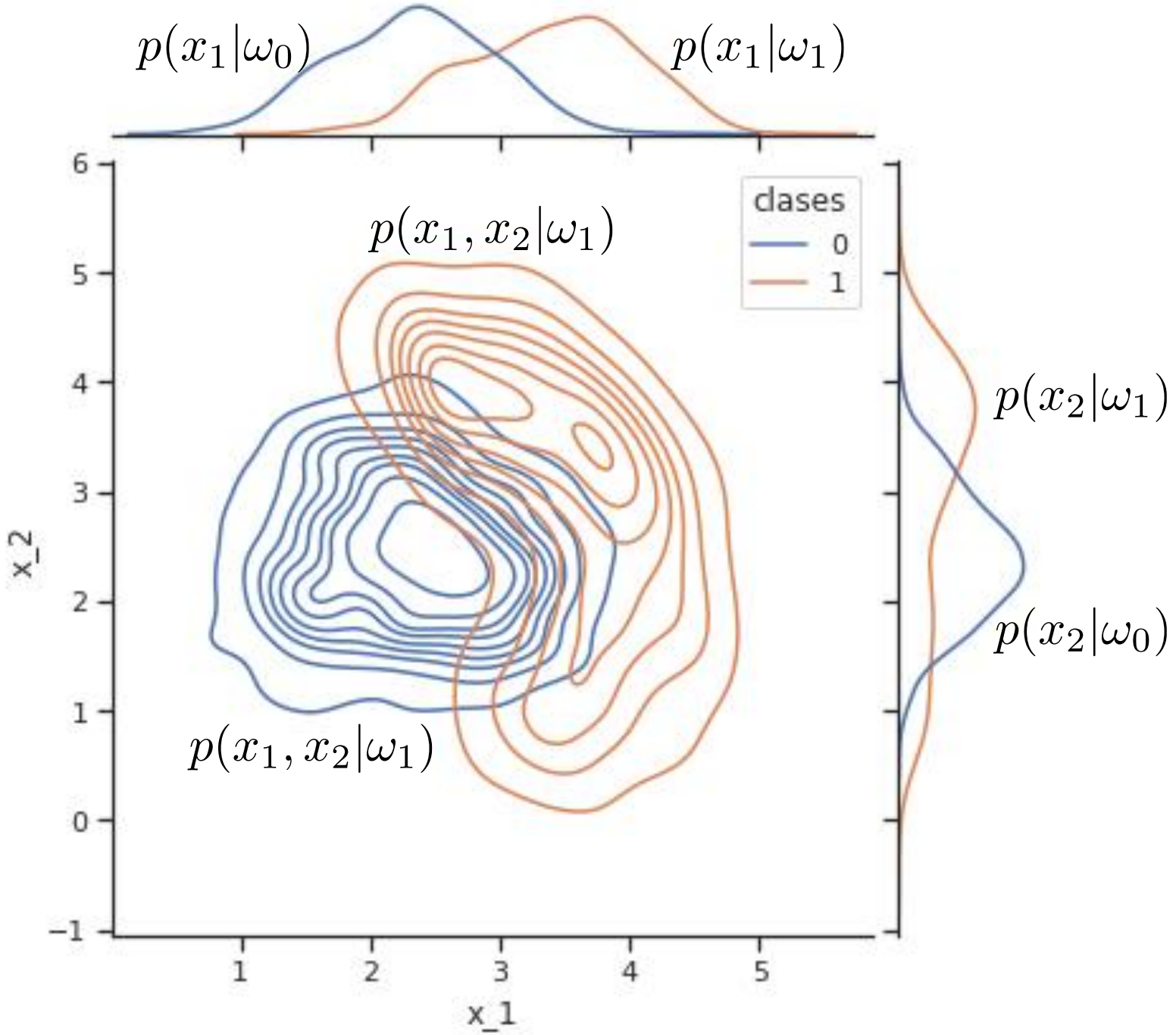
Naïve Bayes

$$p(x_1, \cdots, x_n|\omega_0) = p(x_1|\omega_0)p(x_2|\omega_0)\cdots p(x_n|\omega_0)$$

$$p(x_1, \cdots, x_n|\omega_1) = p(x_1|\omega_1)p(x_2|\omega_1)\cdots p(x_n|\omega_1)$$

Eventos independientes


clases
0
1
x_1
x_2
1
2
3
4
5
−1
0
1
2
3
4
5
6


$p(x_1|\omega_0)$
$p(x_1|\omega_1)$
clases
0
1
$p(x_1, x_2|\omega_1)$
$p(x_1, x_2|\omega_1)$
$p(x_2|\omega_1)$
$p(x_2|\omega_0)$
x_2
x_1
6
5
4
3
2
1
0
−1
1
2
3
4
5

Naïve Bayes

$$p(x_1, \cdots, x_n|\omega_0) = p(x_1|\omega_0)p(x_2|\omega_0)\cdots p(x_n|\omega_0)$$

$$p(x_1, \cdots, x_n|\omega_1) = p(x_1|\omega_1)p(x_2|\omega_1)\cdots p(x_n|\omega_1)$$

Eventos independientes