

Inteligencia Artificial - Deep Learning

# Lecture 4: Neural Networks and Backpropagation

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Universidad Popular del Cesar

Neural Network - Backpropagation

Lecture 4 - 1

# Administrative: Discussion Section

Discussion section tomorrow:

Backpropagation

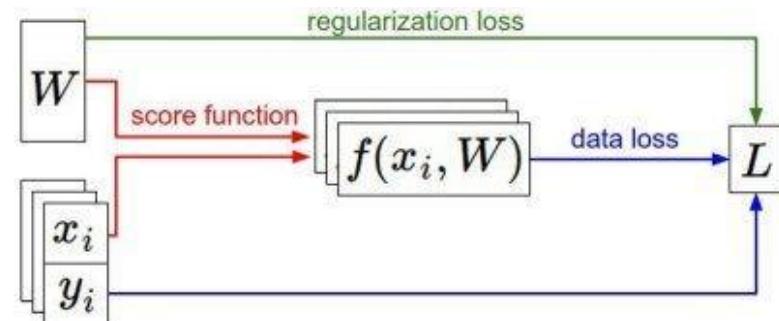
# Recap

- We have some dataset of  $(x, y)$
- We have a **score function**:  $s = f(x; W) = Wx$  e.g.
- We have a **loss function**:

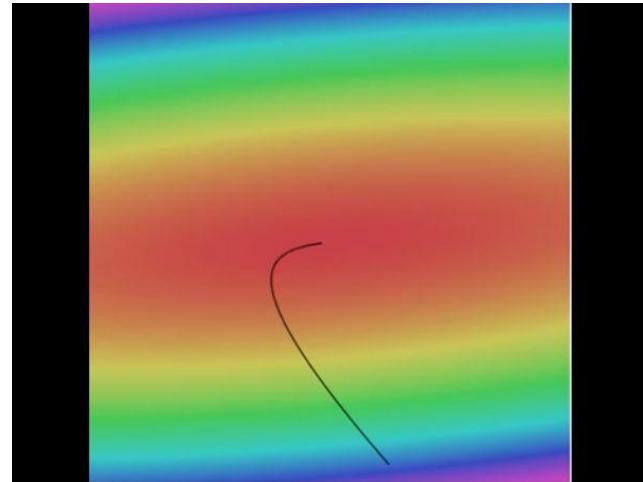
$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{sj}}\right) \text{ Softmax}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \text{ SVM}$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + R(W) \text{ Full loss}$$



# Finding the best W: Optimize with Gradient Descent



```
# Vanilla Gradient Descent

while True:
    weights_grad = evaluate_gradient(loss_fun, data, weights)
    weights += - step_size * weights_grad # perform parameter update
```

Landscape image is CC0 1.0 public domain  
Walking man image is CC0 1.0 public domain

## Neural Network - Backpropagation

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# Gradient descent

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

**Numerical gradient:** slow :, approximate :, easy to write :)

**Analytic gradient:** fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

# Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N} \sum_{i=1}^N L_i(x_i, y_i, W) + \lambda R(W)$$

Full sum expensive  
when N is large!

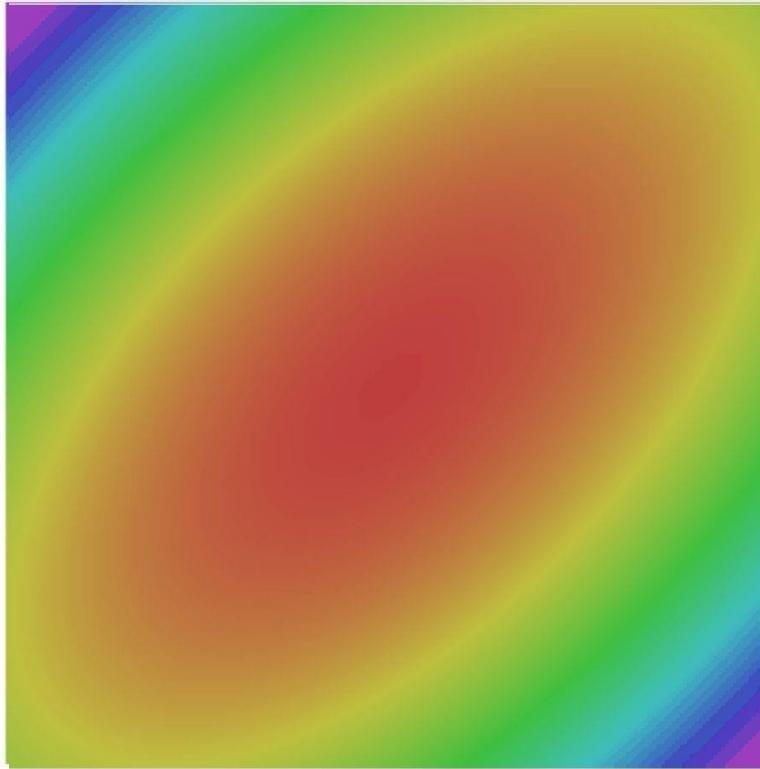
$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^N \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Approximate sum  
using a **minibatch** of  
examples  
32 / 64 / 128 common

```
# Vanilla Minibatch Gradient Descent
```

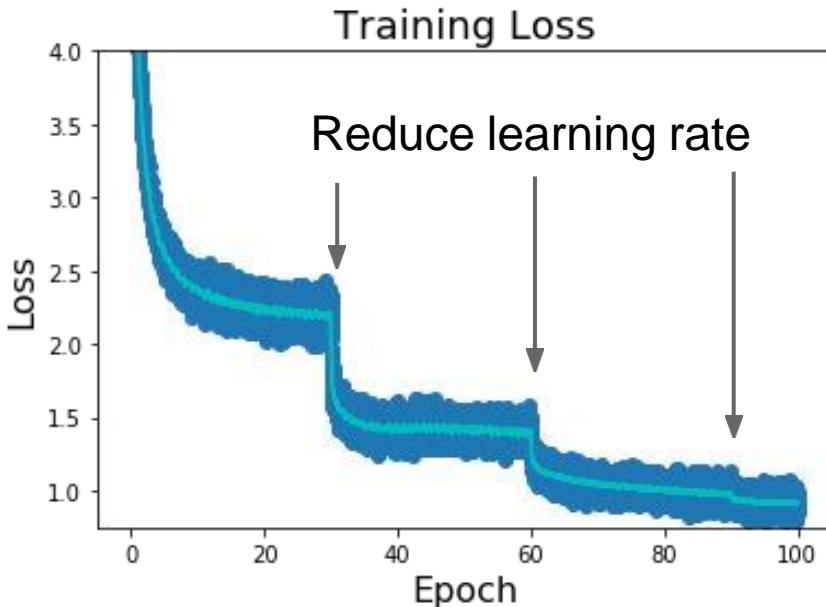
```
while True:  
    data_batch = sample_training_data(data, 256) # sample 256 examples  
    weights_grad = evaluate_gradient(loss_fun, data_batch, weights)  
    weights += - step_size * weights_grad # perform parameter update
```

# Last time: fancy optimizers



- SGD
- SGD+Momentum
- RMSProp
- Adam

# Last time: learning rate scheduling



**Step:** Reduce learning rate at a few fixed points. E.g. for ResNets, multiply LR by 0.1 after epochs 30, 60, and 90.

**Cosine:**  $\alpha_t = \frac{1}{2}\alpha_0 (1 + \cos(t\pi/T))$

**Linear:**  $\alpha_t = \alpha_0(1 - t/T)$

**Inverse sqrt:**  $\alpha_t = \alpha_0/\sqrt{t}$

$\alpha_0$  : Initial learning rate

$\alpha_t$  : Learning rate at epoch  $t$

$T$  : Total number of epochs

# Today:

# Deep Learning

Neural Network - Backpropagation

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# Dall-E 2



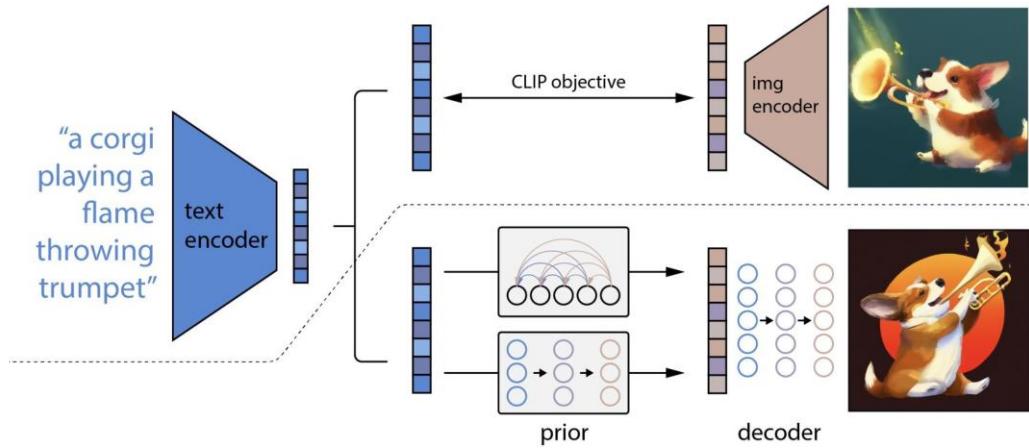
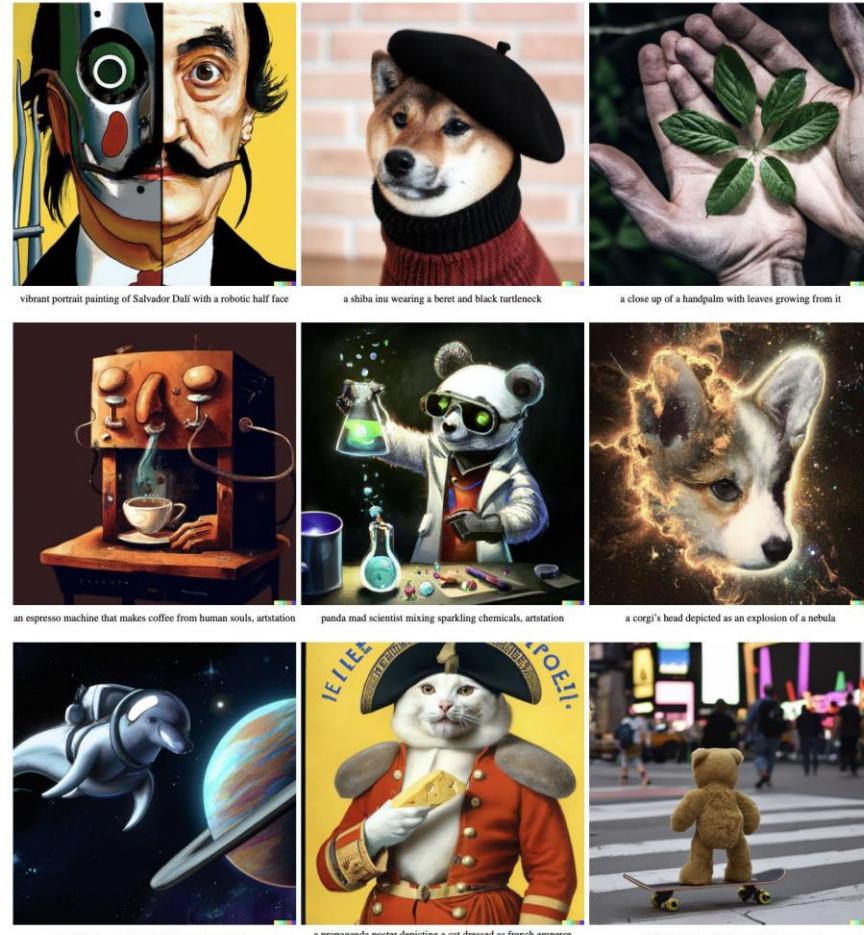
"Teddy bears working on new AI research on the moon in the 1980s."

"Rabbits attending a college seminar on human anatomy."

"A wise cat meditating in the Himalayas searching for enlightenment."

Image source: Sam Altman, <https://openai.com/dall-e-2/>, <https://twitter.com/sama/status/1511724264629678084>

## Neural Network - Backpropagation



Ramesh et al., Hierarchical Text-Conditional Image Generation with CLIP Latents, 2022.

## Neural Network - Backpropagation

# GPT-4

User What is unusual about this image?



Source: [Barnorama](#)

GPT-4 The unusual thing about this image is that a man is ironing clothes on an ironing board attached to the roof of a moving taxi.

User Can you explain this meme?

Sometimes I just look at pictures of the earth from space and I marvel at how beautiful it all is.



GPT-4 This meme is a joke that combines two unrelated things: pictures of the earth from space and chicken nuggets. The text of the meme suggests that the image below is a beautiful picture of the earth from space. However, the image is actually of chicken nuggets arranged to vaguely resemble a map of the world. The humor in this meme comes from the unexpected juxtaposition of the text and the image. The text sets up an expectation of a majestic image of the earth, but the image is actually something mundane and silly.

Image source: <https://openai.com/research/gpt-4>

## Neural Network - Backpropagation

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# Segment Anything Model (SAM)

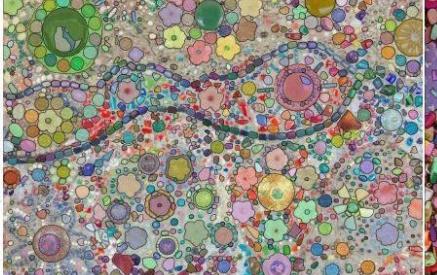
300-400 masks



400-500 masks



>500 masks



# Neural Networks

Neural Network - Backpropagation

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# Neural networks: the original linear classifier

(Before) Linear score function:  $f = Wx$

$$x \in \mathbb{R}^D, W \in \mathbb{R}^{C \times D}$$

# Neural networks: 2 layers

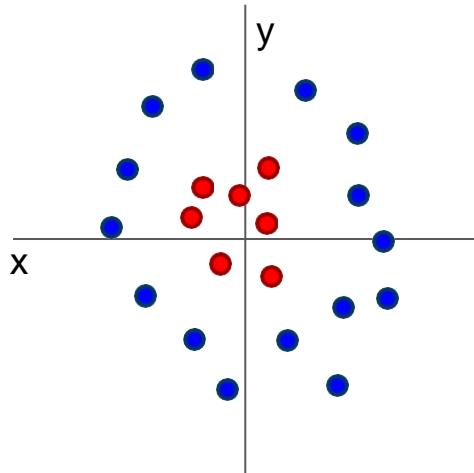
**(Before)** Linear score function:  $f = Wx$

**(Now)** 2-layer Neural Network  $f = W_2 \max(0, W_1 x)$

$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

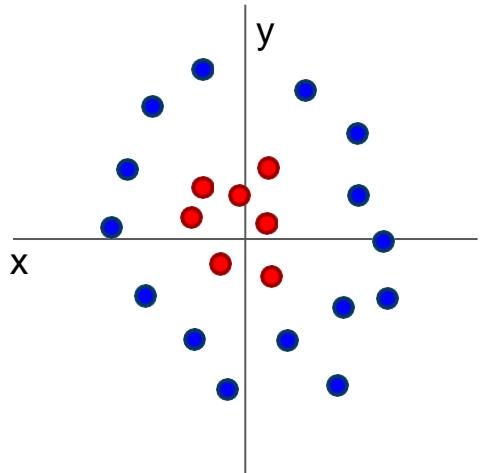
(In practice we will usually add a learnable bias at each layer as well)

# Why do we want non-linearity?



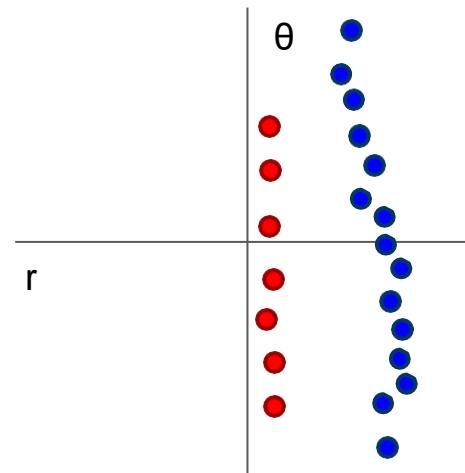
Cannot separate red  
and blue points with  
linear classifier

# Why do we want non-linearity?



Cannot separate red  
and blue points with  
linear classifier

$$f(x, y) = (r(x, y), \theta(x, y))$$



After applying feature  
transform, points can  
be separated by linear  
classifier

# Neural networks: also called fully connected network

**(Before)** Linear score function:  $f = Wx$

**(Now)** 2-layer Neural Network  $f = W_2 \max(0, W_1 x)$

$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

“Neural Network” is a very broad term; these are more accurately called “fully-connected networks” or sometimes “multi-layer perceptrons” (MLP)

(In practice we will usually add a learnable bias at each layer as well)

# Neural networks: 3 layers

(Before) Linear score function:

$$f = Wx$$

(Now) 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$

or 3-layer Neural Network

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

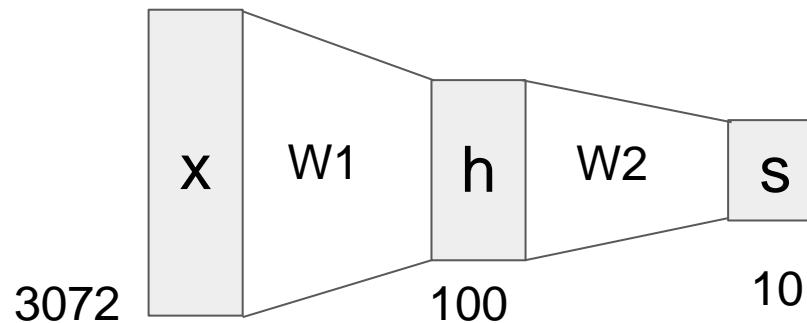
$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H_1 \times D}, W_2 \in \mathbb{R}^{H_2 \times H_1}, W_3 \in \mathbb{R}^{C \times H_2}$$

(In practice we will usually add a learnable bias at each layer as well)

# Neural networks: hierarchical computation

(Before) Linear score function:  $f = Wx$

(Now) 2-layer Neural Network  $f = W_2 \max(0, W_1 x)$

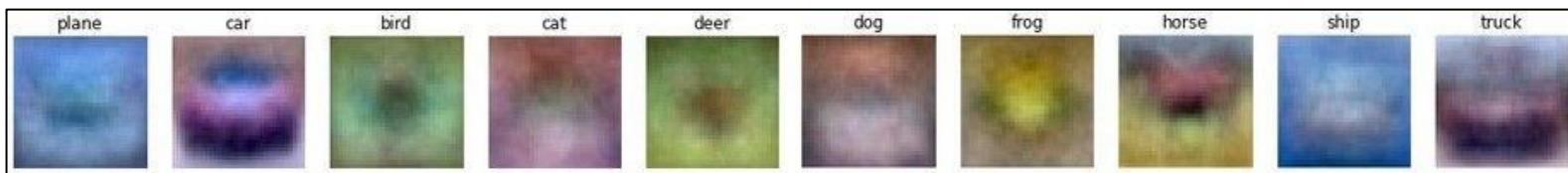
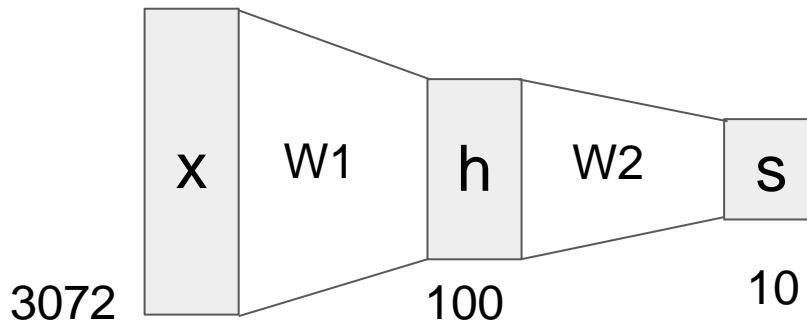


$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

# Neural networks: learning 100s of templates

(Before) Linear score function:  $f = Wx$

(Now) 2-layer Neural Network  $f = W_2 \max(0, W_1 x)$



Learn 100 templates instead of 10

Share templates between classes

## Neural Network - Backpropagation

# Neural networks: why is max operator important?

(Before) Linear score function:  $f = Wx$

(Now) 2-layer Neural Network  $f = W_2 \max(0, W_1 x)$

The function  $\max(0, z)$  is called the **activation function**.

**Q:** What if we try to build a neural network without one?

$$f = W_2 W_1 x$$

# Neural networks: why is max operator important?

(Before) Linear score function:  $f = Wx$

(Now) 2-layer Neural Network  $f = W_2 \max(0, W_1 x)$

The function  $\max(0, z)$  is called the **activation function**.

Q: What if we try to build a neural network without one?

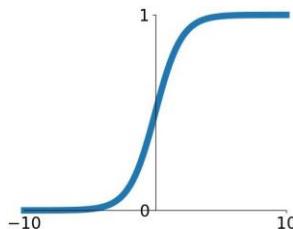
$$f = W_2 W_1 x \quad W_3 = W_2 W_1 \in \mathbb{R}^{C \times H}, f = W_3 x$$

A: We end up with a linear classifier again!

# Activation functions

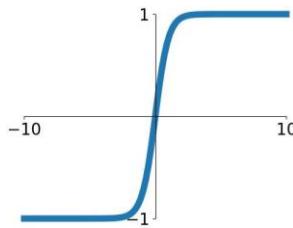
## Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



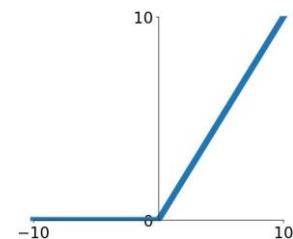
## tanh

$$\tanh(x)$$



## ReLU

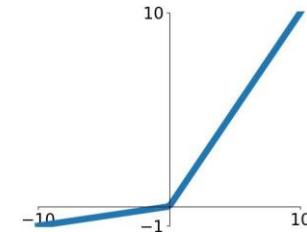
$$\max(0, x)$$



ReLU is a good default choice for most problems

## Leaky ReLU

$$\max(0.1x, x)$$

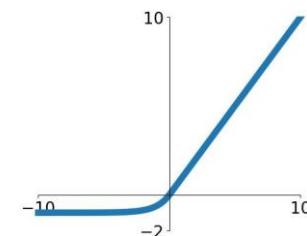


## Maxout

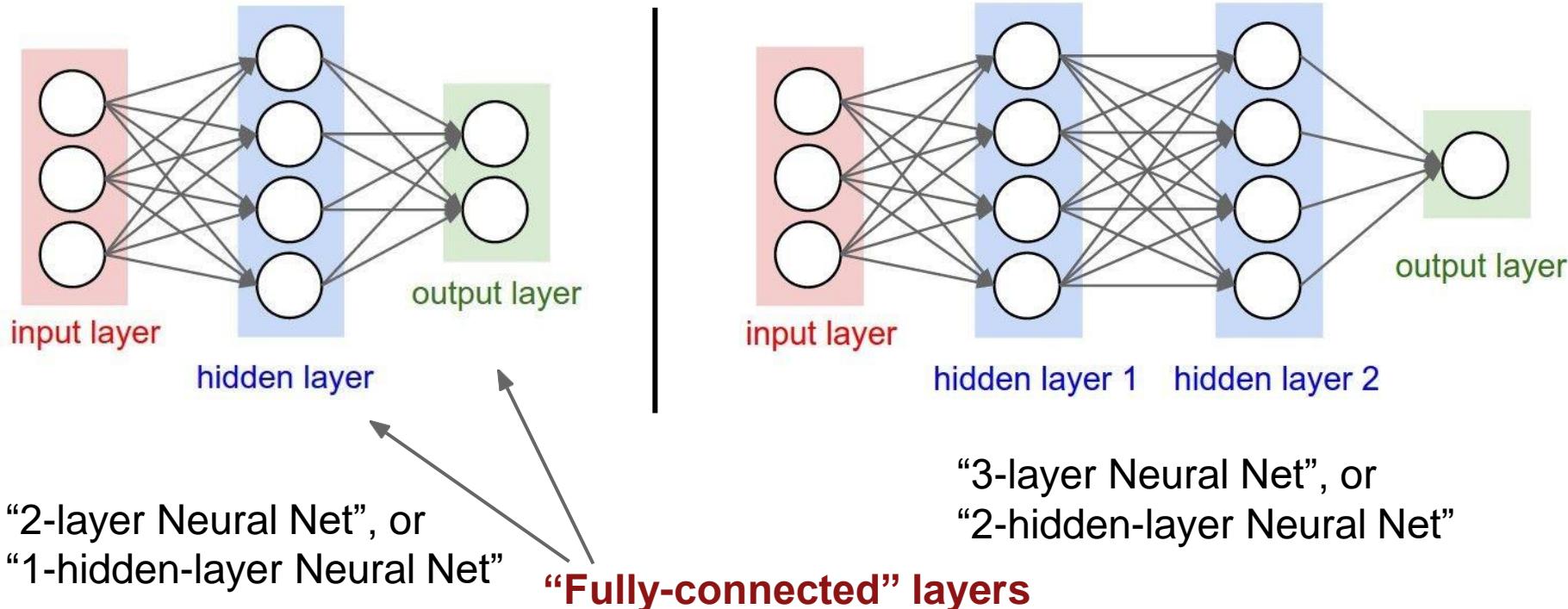
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

## ELU

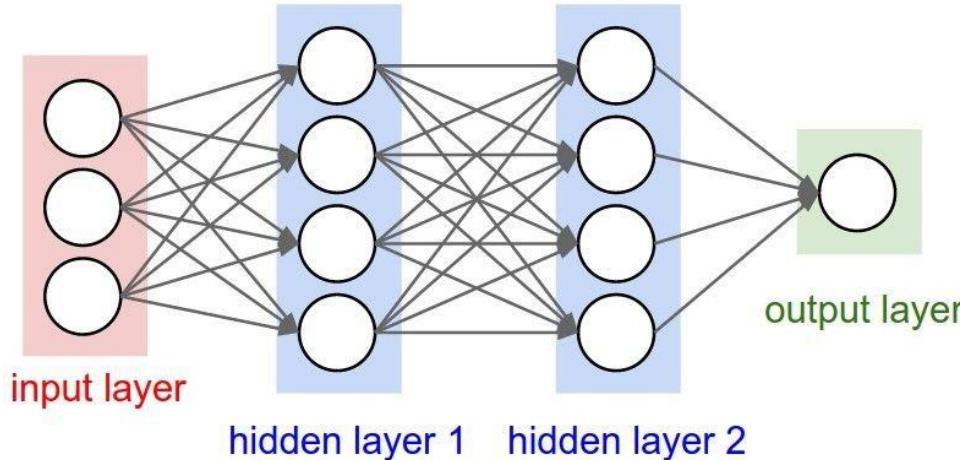
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



# Neural networks: Architectures



# Example feed-forward computation of a neural network



```
# forward-pass of a 3-layer neural network:  
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)  
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)  
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)  
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)  
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

# Full implementation of training a 2-layer Neural Network needs ~20 lines:

```
1 import numpy as np
2 from numpy.random import randn
3
4 N, D_in, H, D_out = 64, 1000, 100, 10
5 x, y = randn(N, D_in), randn(N, D_out)
6 w1, w2 = randn(D_in, H), randn(H, D_out)
7
8 for t in range(2000):
9     h = 1 / (1 + np.exp(-x.dot(w1)))
10    y_pred = h.dot(w2)
11    loss = np.square(y_pred - y).sum()
12    print(t, loss)
13
14    grad_y_pred = 2.0 * (y_pred - y)
15    grad_w2 = h.T.dot(grad_y_pred)
16    grad_h = grad_y_pred.dot(w2.T)
17    grad_w1 = x.T.dot(grad_h * h * (1 - h))
18
19    w1 -= 1e-4 * grad_w1
20    w2 -= 1e-4 * grad_w2
```

## Neural Network - Backpropagation

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Define the network

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Define the network

Forward pass

## Neural Network - Backpropagation

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Define the network

Forward pass

Calculate the analytical gradients

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```

Define the network

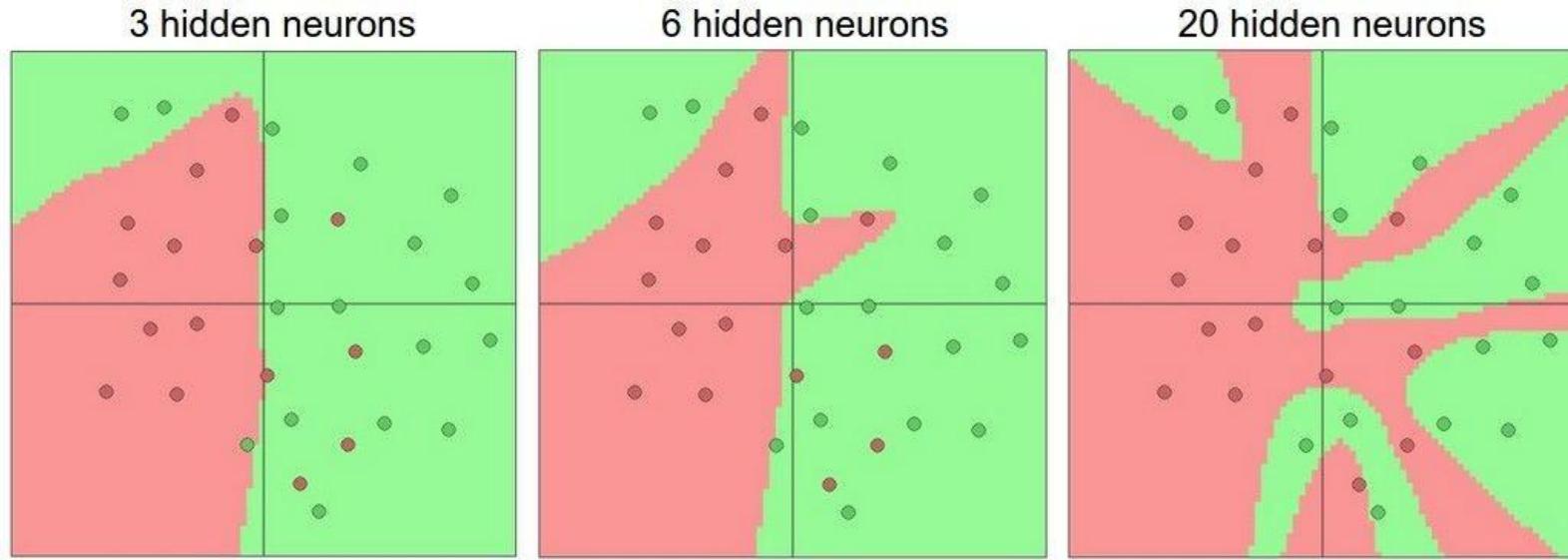
Forward pass

Calculate the analytical gradients

Gradient descent

## Neural Network - Backpropagation

# Setting the number of layers and their sizes



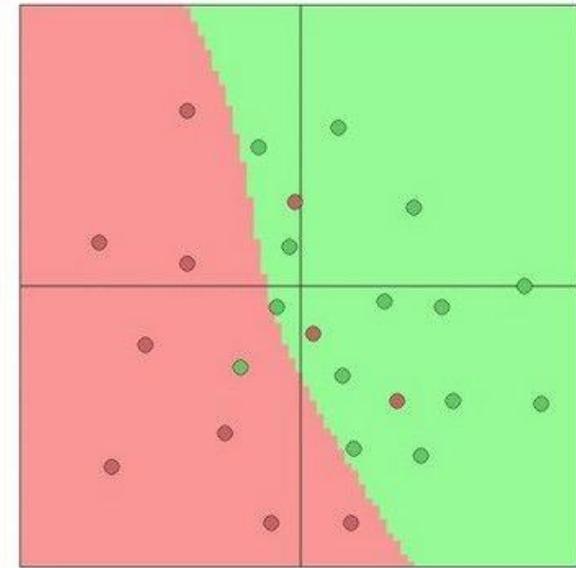
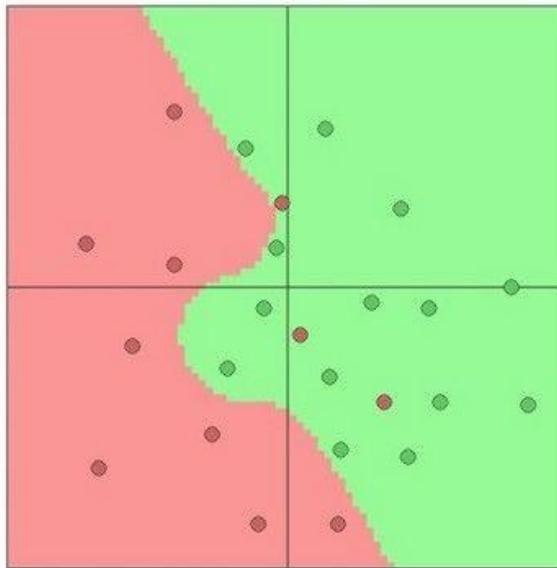
more neurons = more capacity

Do not use size of neural network as a regularizer. Use stronger regularization instead:

$\lambda = 0.001$

$\lambda = 0.01$

$\lambda = 0.1$



(Web demo with ConvNetJS:  
<http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html>)

$$L(W) = \frac{1}{N} \sum_{i=1}^N L_i(f(x_i, W), y_i) + \lambda R(W)$$

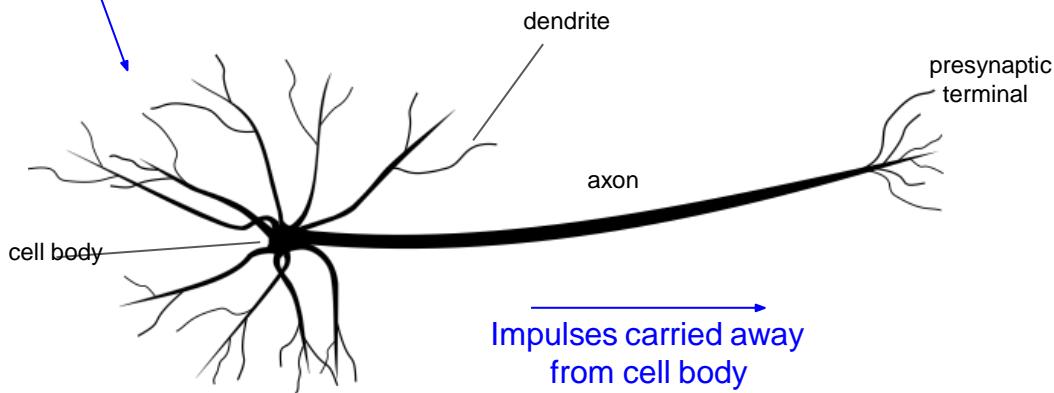


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# Neural Network - Backpropagation

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Impulses carried toward cell body

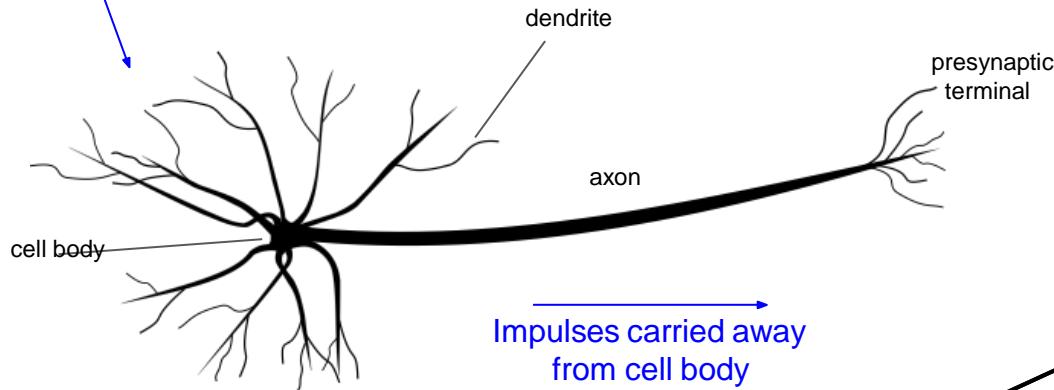


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## Neural Network - Backpropagation

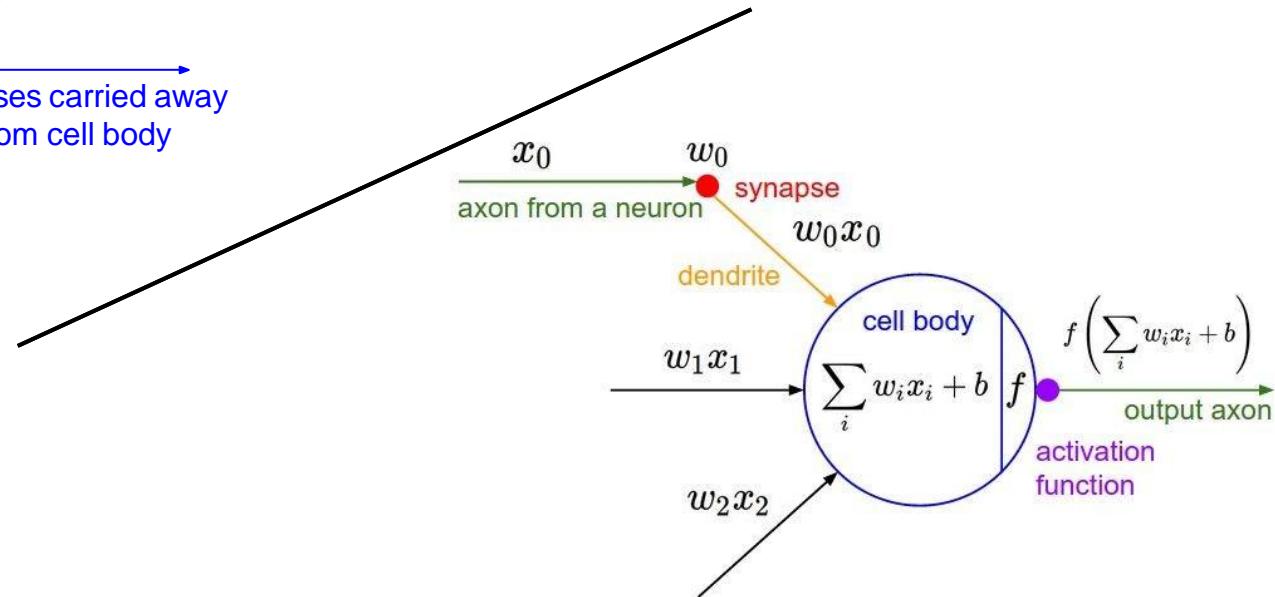
Lecture 4 - 36

Impulses carried toward cell body



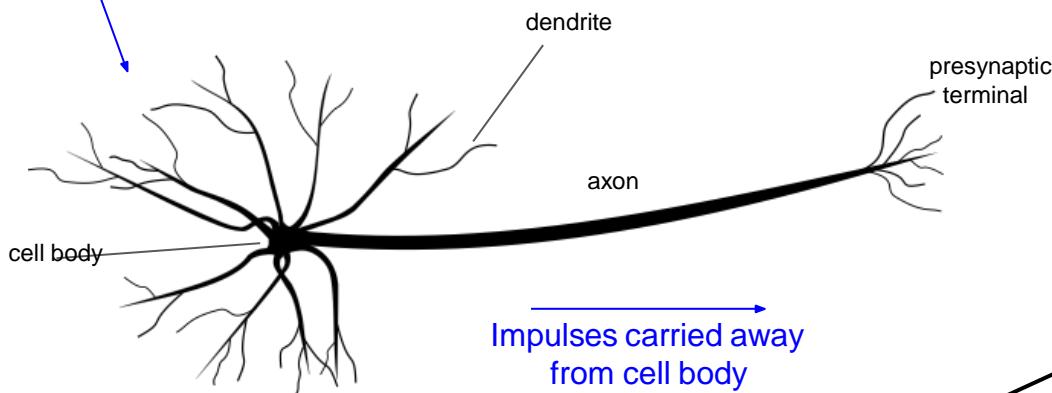
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Impulses carried away  
from cell body

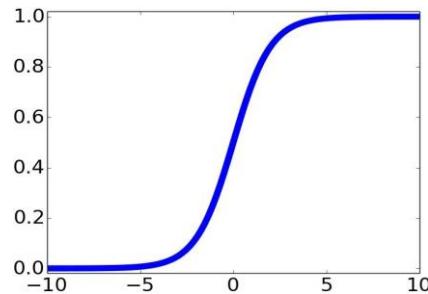


## Neural Network - Backpropagation

Impulses carried toward cell body



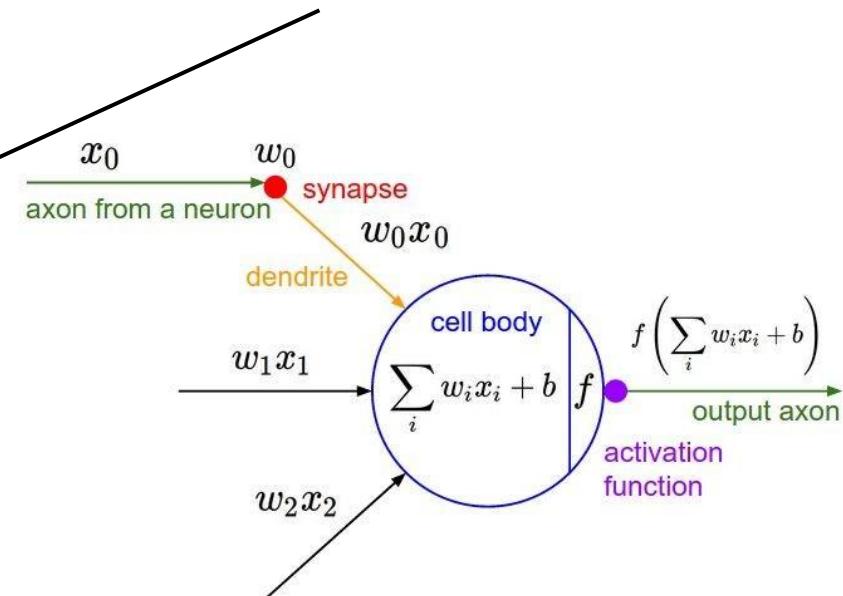
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Impulses carried away  
from cell body

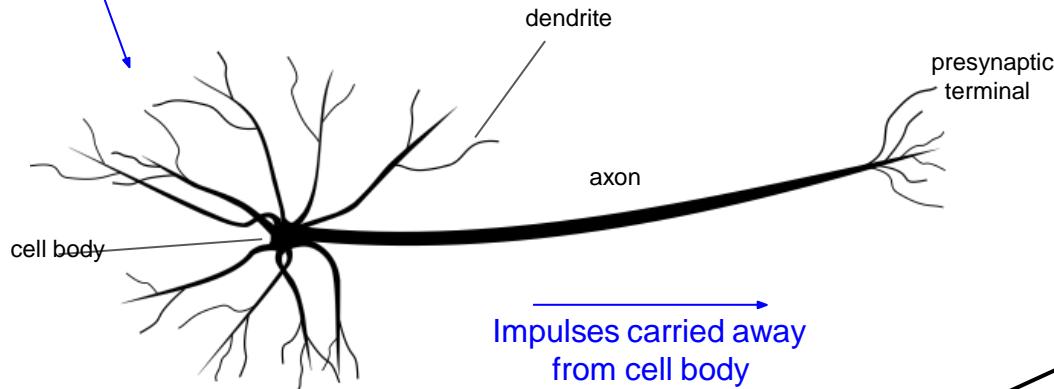
sigmoid activation function

$$\frac{1}{1 + e^{-x}}$$



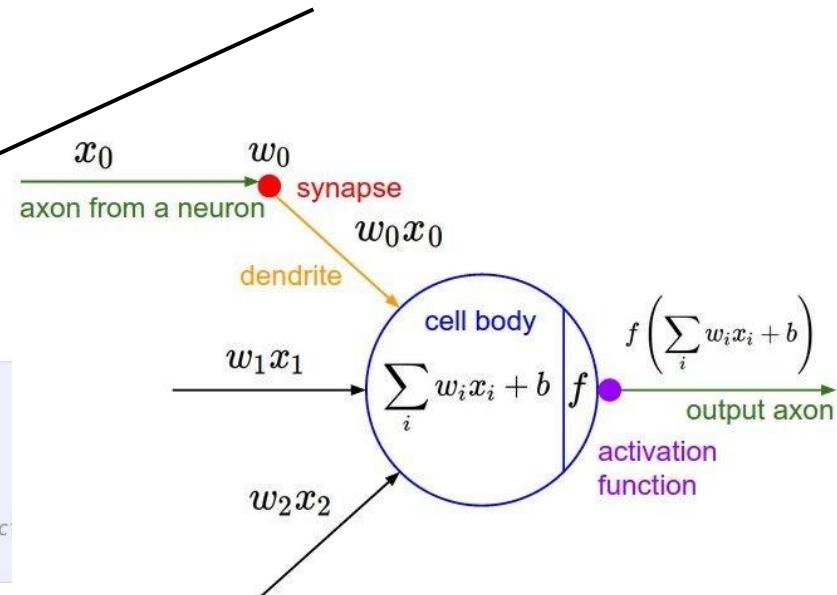
## Neural Network - Backpropagation

Impulses carried toward cell body



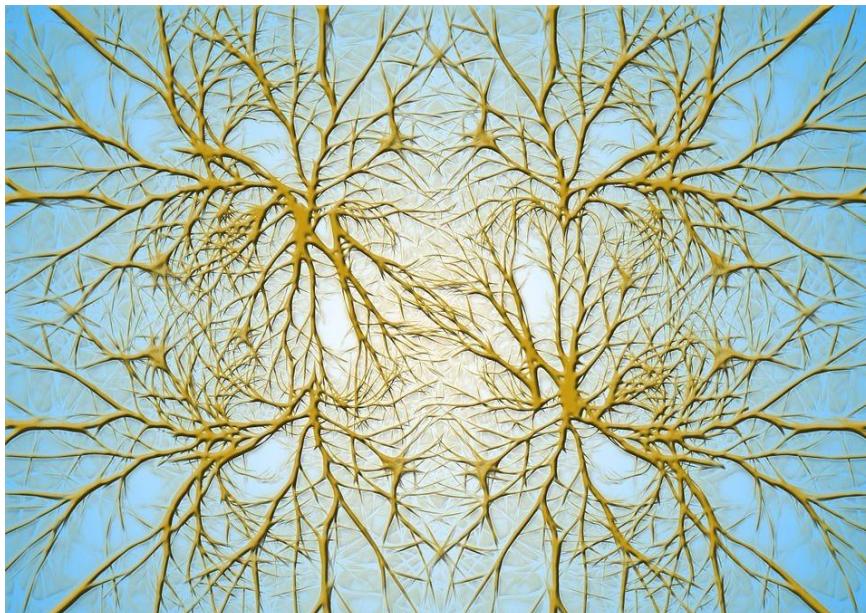
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```
class Neuron:  
    # ...  
    def neuron_tick(inputs):  
        """ assume inputs and weights are 1-D numpy arrays and bias is a number """  
        cell_body_sum = np.sum(inputs * self.weights) + self.bias  
        firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation func  
        return firing_rate
```



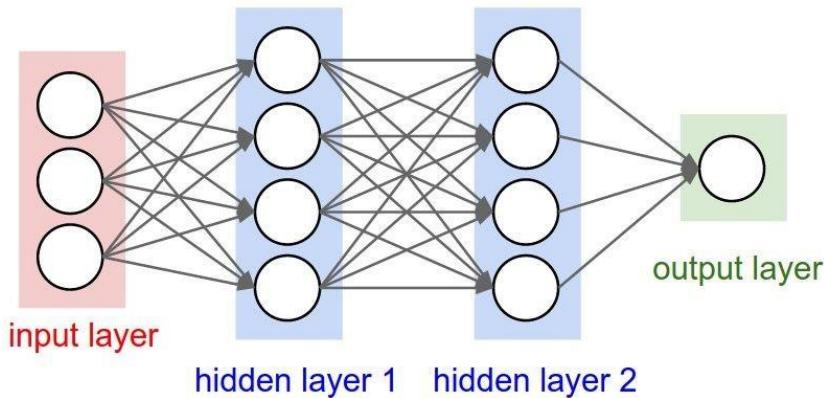
## Neural Network - Backpropagation

## Biological Neurons: Complex connectivity patterns

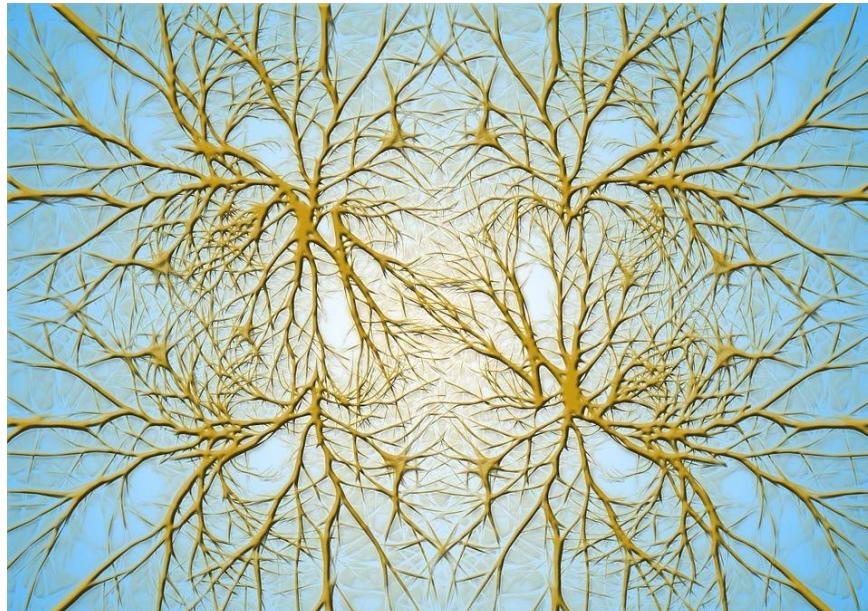


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Neurons in a neural network:  
Organized into regular layers for  
computational efficiency

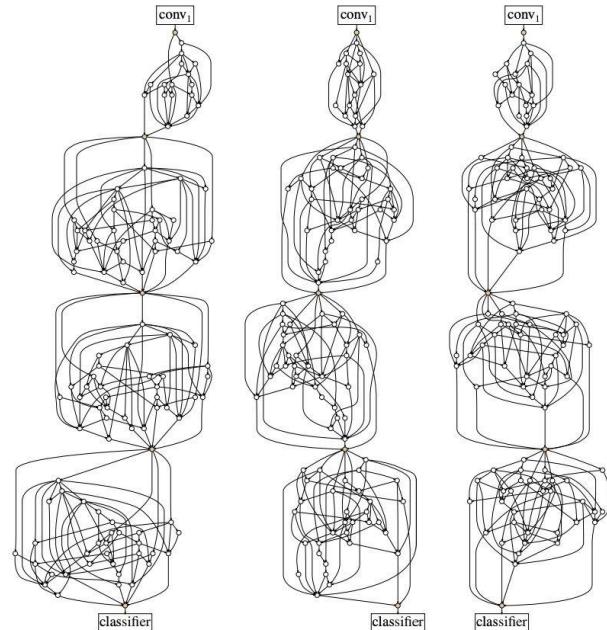


## Biological Neurons: Complex connectivity patterns



[This image is CC0 Public Domain](#)

But neural networks with random connections can work too!



Xie et al, "Exploring Randomly Wired Neural Networks for Image Recognition", arXiv 2019

## Neural Network - Backpropagation

# Be very careful with your brain analogies!

## Biological Neurons:

- Many different types
- Dendrites can perform complex non-linear computations
- Synapses are not a single weight but a complex non-linear dynamical system

[Dendritic Computation. London and Häusser]

# Plugging in neural networks with loss functions

$s = f(x; W_1, W_2) = W_2 \max(0, W_1 x)$  Nonlinear score function

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$  SVM Loss on predictions

$R(W) = \sum_k W_k^2$  Regularization

$L = \frac{1}{N} \sum_{i=1}^N L_i + \lambda R(W_1) + \lambda R(W_2)$  Total loss: data loss + regularization

# Problem: How to compute gradients?

$$s = f(x; W_1, W_2) = W_2 \max(0, W_1 x) \quad \text{Nonlinear score function}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \quad \text{SVM Loss on predictions}$$

$$R(W) = \sum_k W_k^2 \quad \text{Regularization}$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \lambda R(W_1) + \lambda R(W_2) \quad \text{Total loss: data loss + regularization}$$

If we can compute  $\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_2}$  then we can learn  $W_1$  and  $W_2$

# (Bad) Idea: Derive $\nabla_W L$ on paper

$$s = f(x; W) = Wx$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$= \sum_{j \neq y_i} \max(0, W_{j,:} \cdot x + W_{y_i,:} \cdot x + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \lambda \sum_k W_k^2$$

$$= \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, W_{j,:} \cdot x + W_{y_i,:} \cdot x + 1) + \lambda \sum_k W_k^2$$

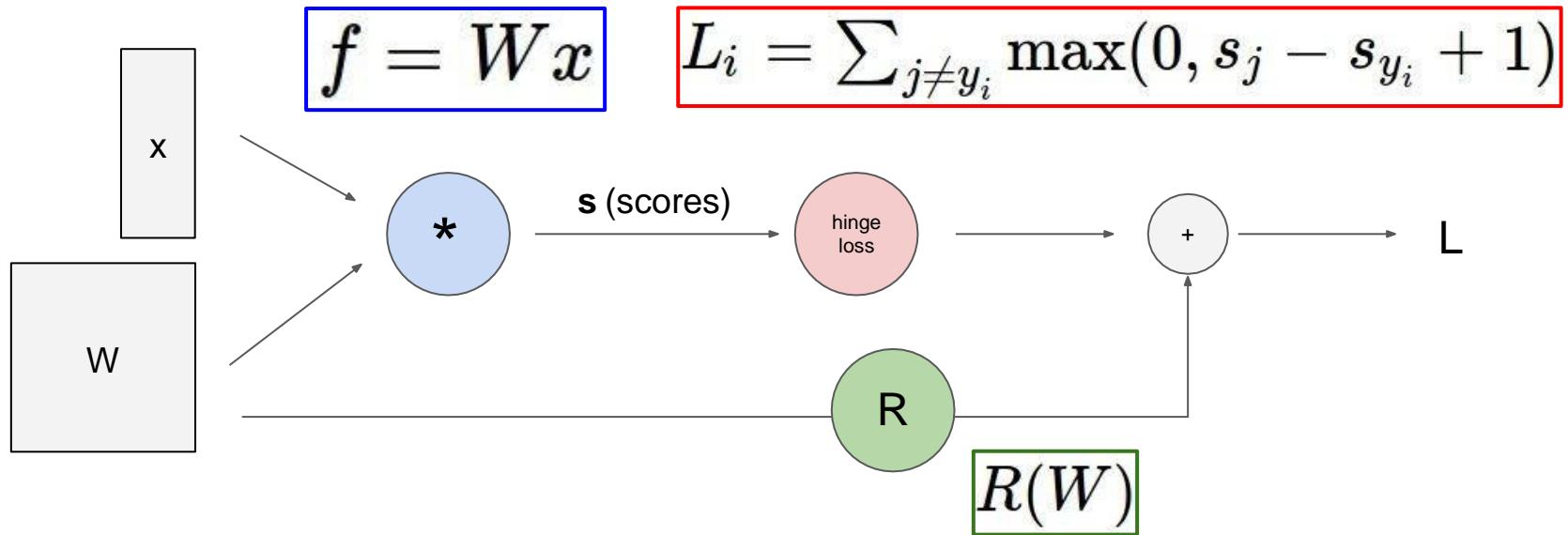
$$\nabla_W L = \nabla_W \left( \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, W_{j,:} \cdot x + W_{y_i,:} \cdot x + 1) + \lambda \sum_k W_k^2 \right)$$

**Problem:** Very tedious: Lots of matrix calculus, need lots of paper

**Problem:** What if we want to change loss? E.g. use softmax instead of SVM? Need to re-derive from scratch =(

**Problem:** Not feasible for very complex models!

# Better Idea: Computational graphs + Backpropagation



# Convolutional network (AlexNet)

input image

weights

loss

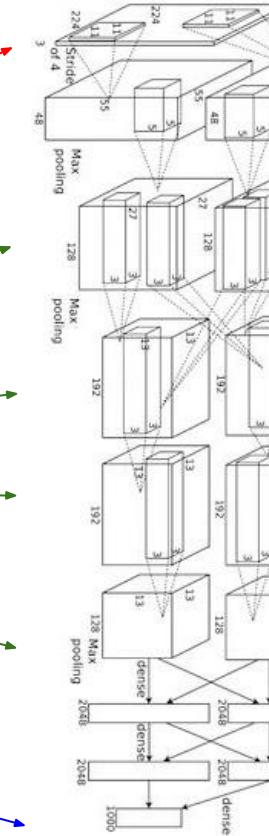


Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

# Really complex neural networks!!

input image

loss

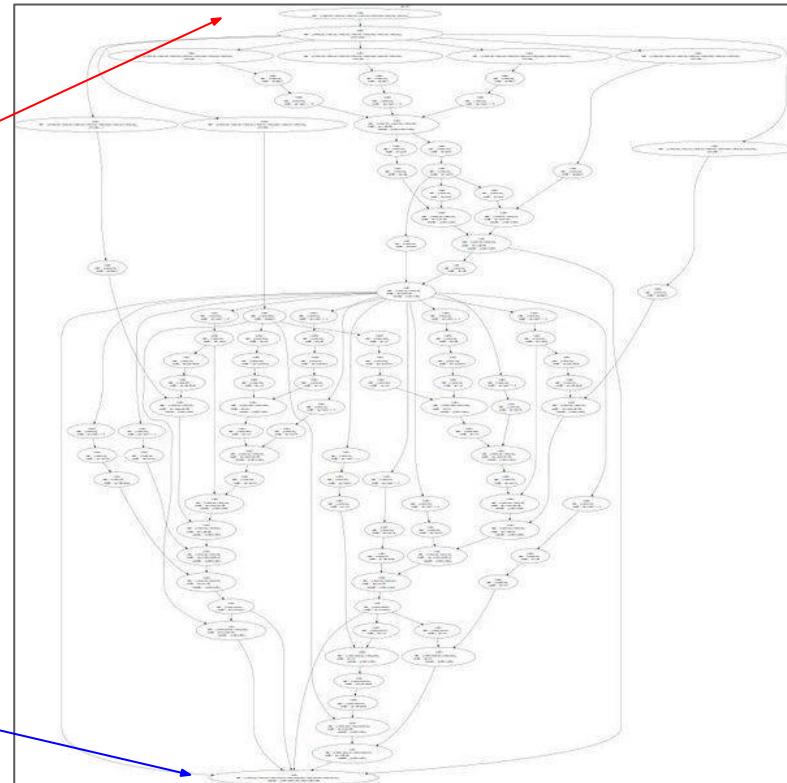


Figure reproduced with permission from a [Twitter post](#) by Andrej Karpathy.

# Neural Turing Machine

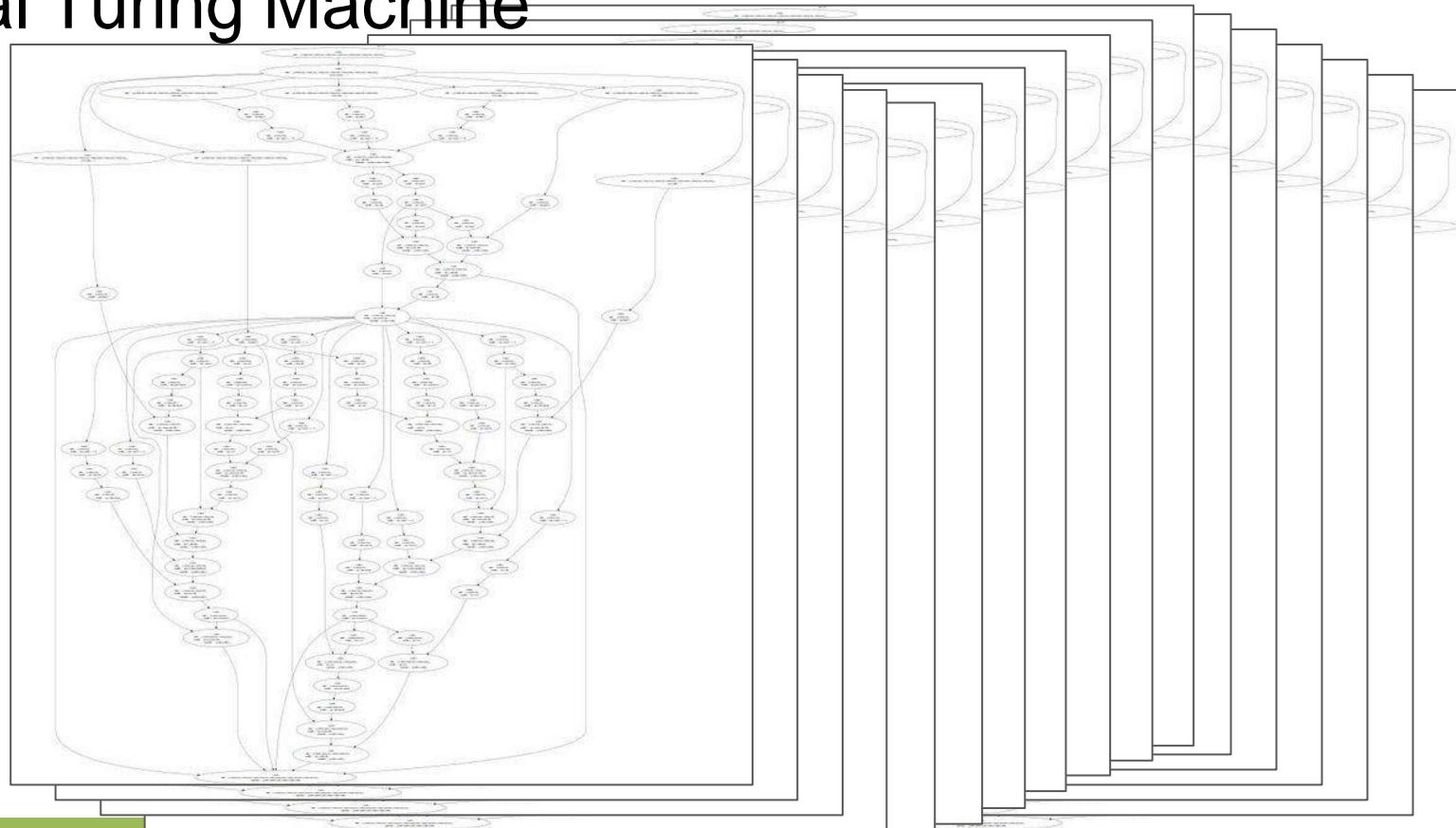


Figure reproduced with permission from a [Twitter post](#) by Andrej Karpathy.

Neural network - Backpropagation

Lecture 4 -



Universidad  
Popular del Cesar

# Solution: Backpropagation

Neural Network - Backpropagation

Lecture 4 - 50

## Backpropagation: a simple example

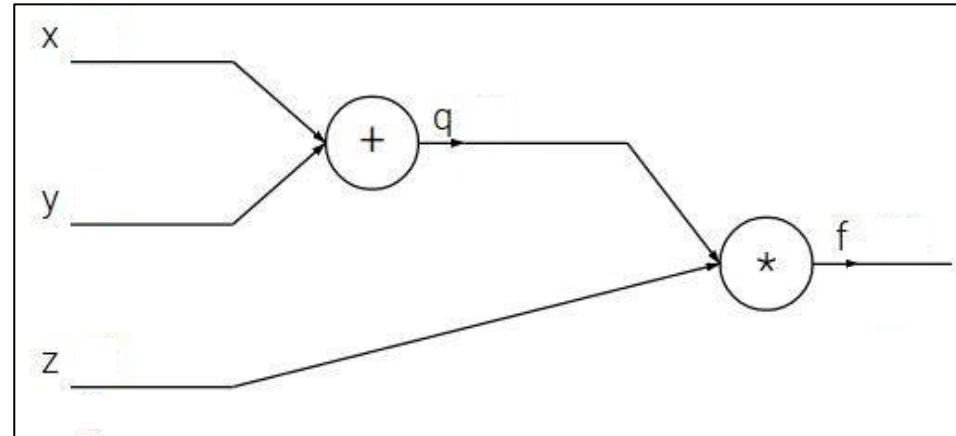
$$f(x, y, z) = (x + y)z$$

Neural Network - Backpropagation

Lecture 4 - 51

## Backpropagation: a simple example

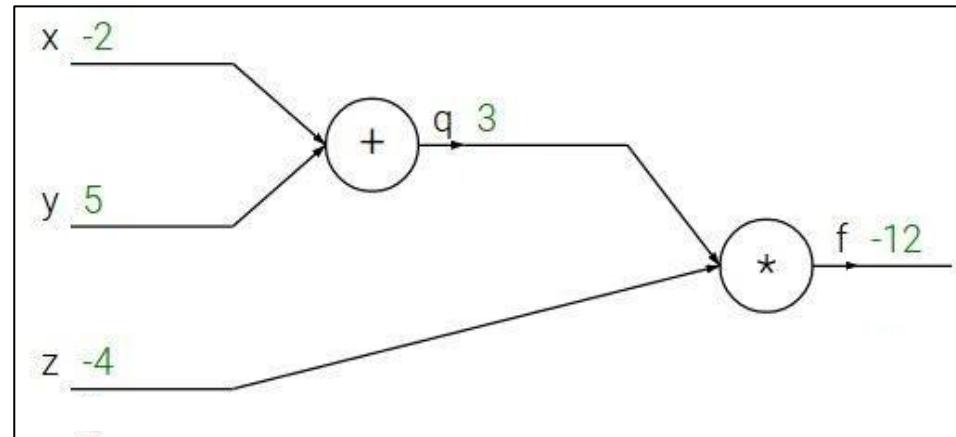
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## Backpropagation: a simple example

$$f(x, y, z) = (x + y)z$$

e.g.  $x = -2$ ,  $y = 5$ ,  $z = -4$

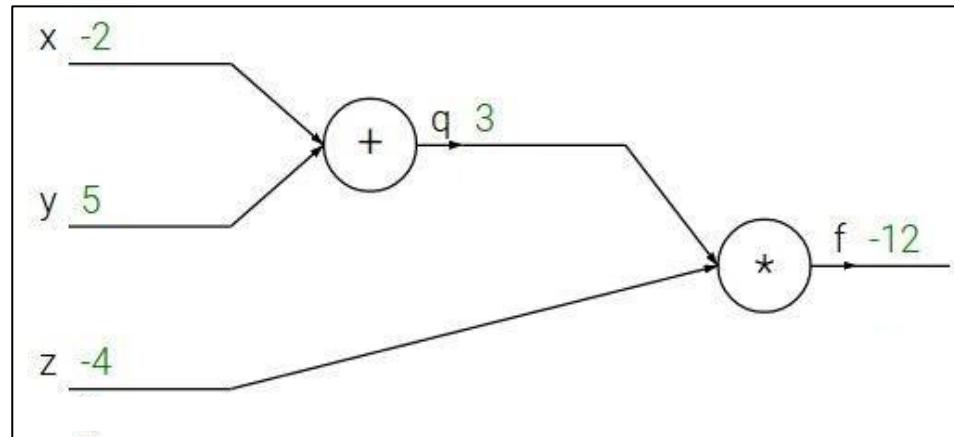


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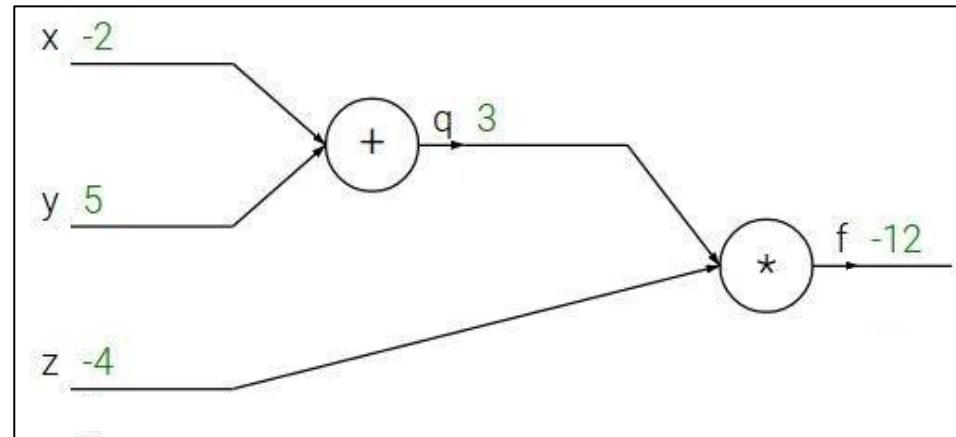
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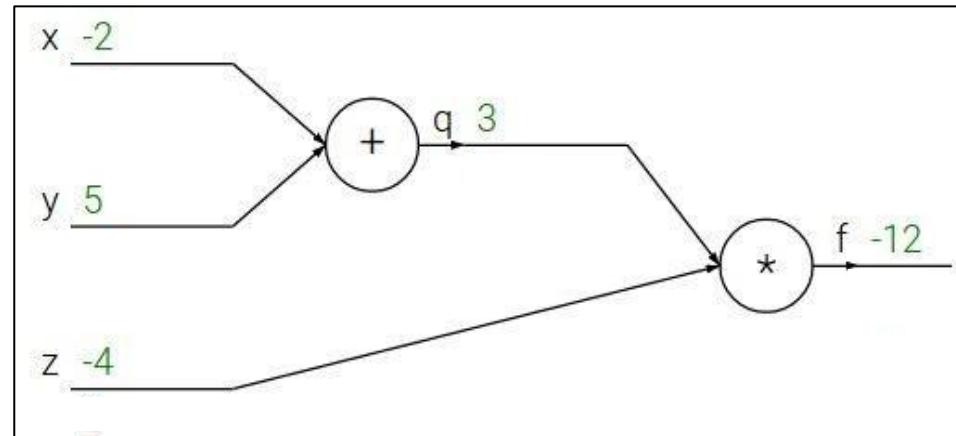
$$f(x, y, z) = (x + y)z$$

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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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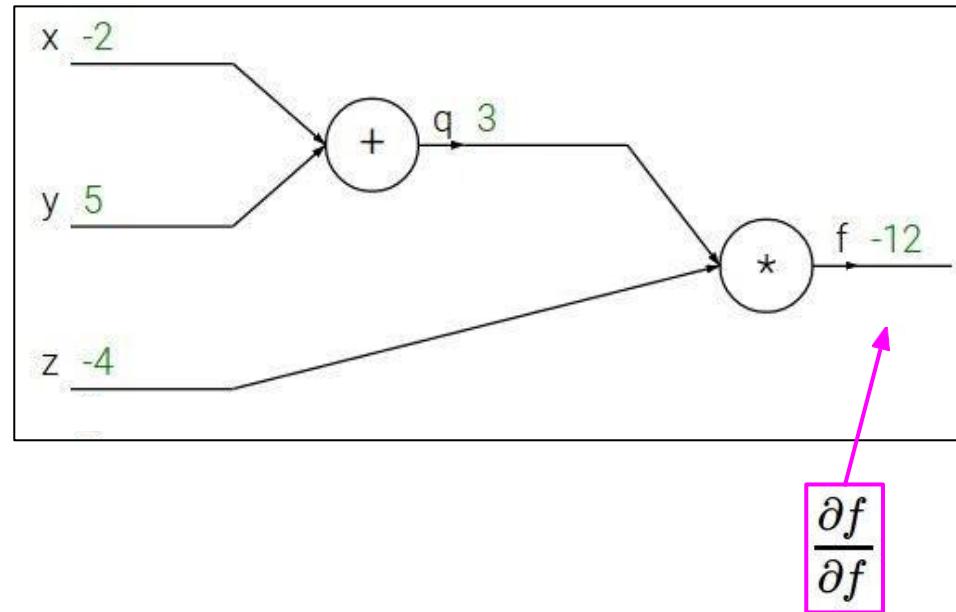
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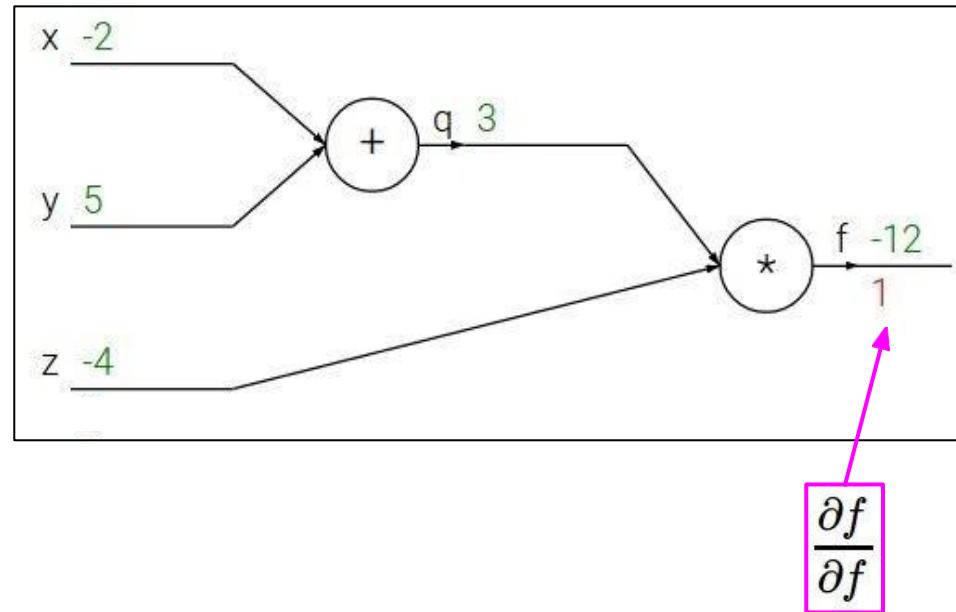
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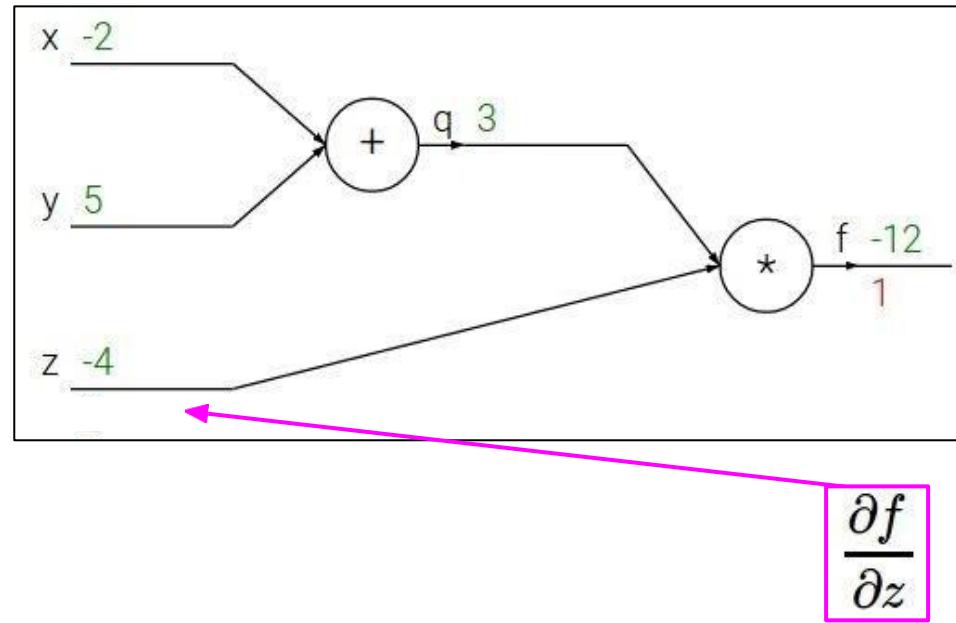
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$$\frac{\partial f}{\partial z}$$

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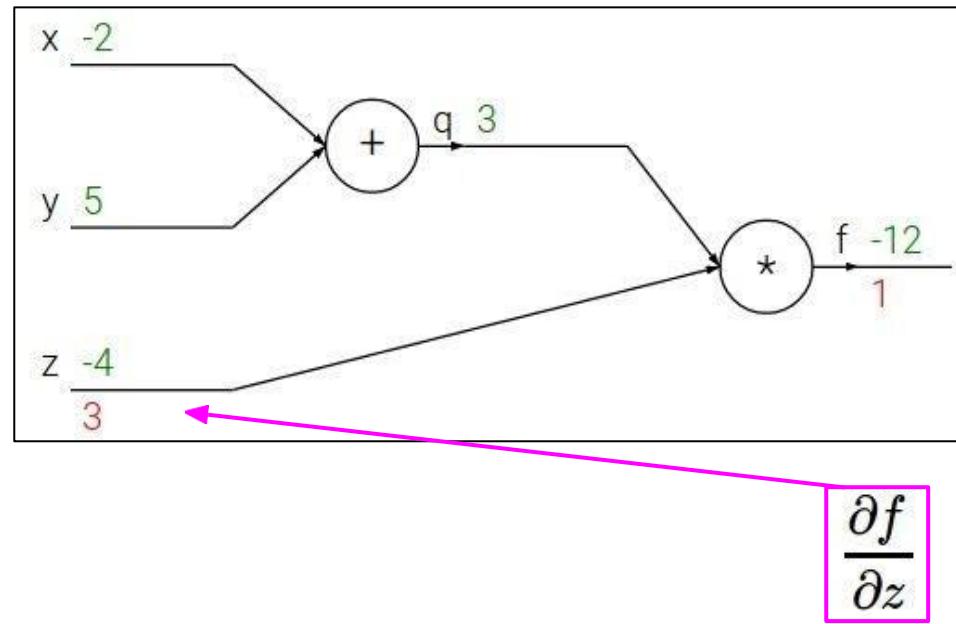
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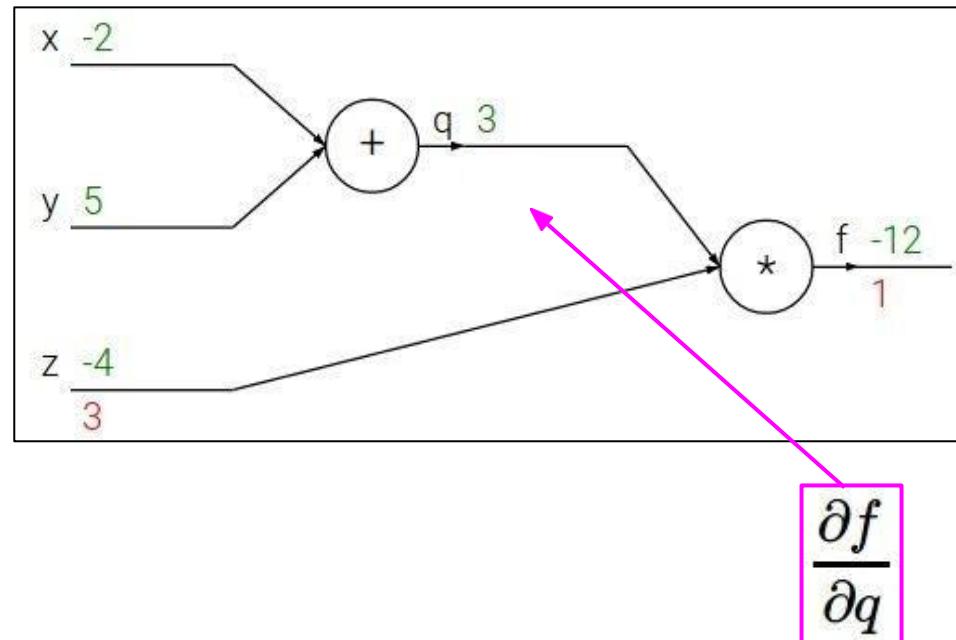
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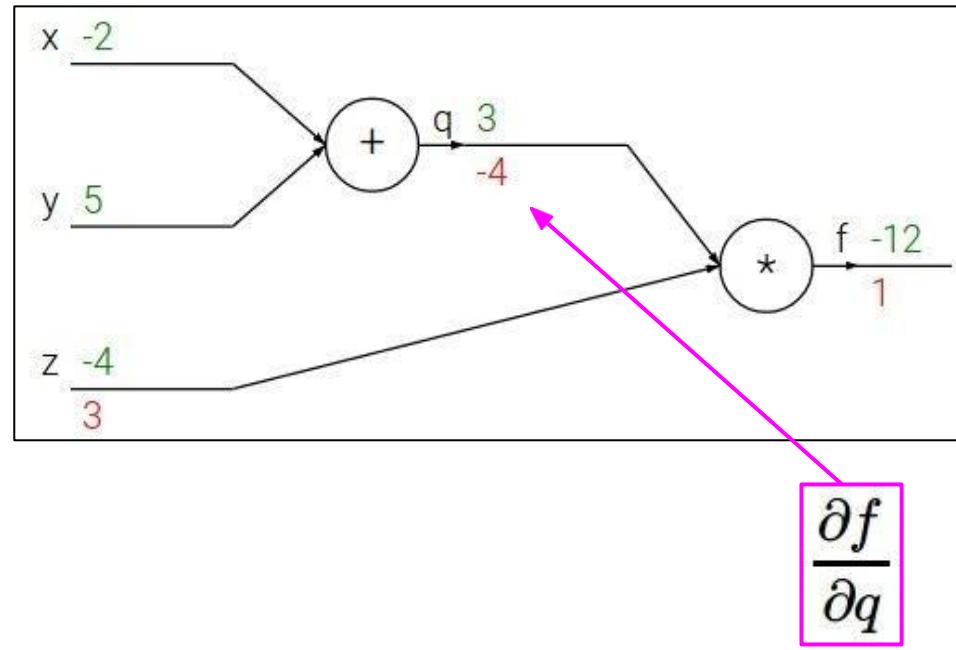
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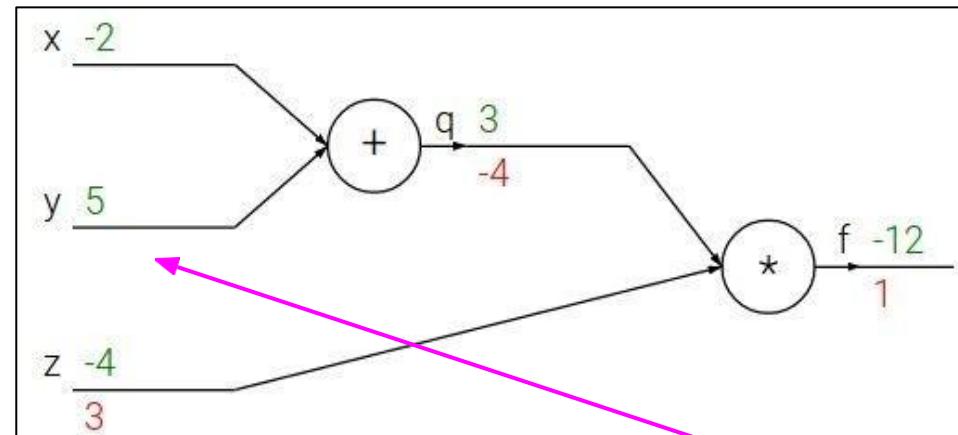
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Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

Upstream  
gradient

Local  
gradient

$$\frac{\partial f}{\partial y}$$

## Backpropagation: a simple example

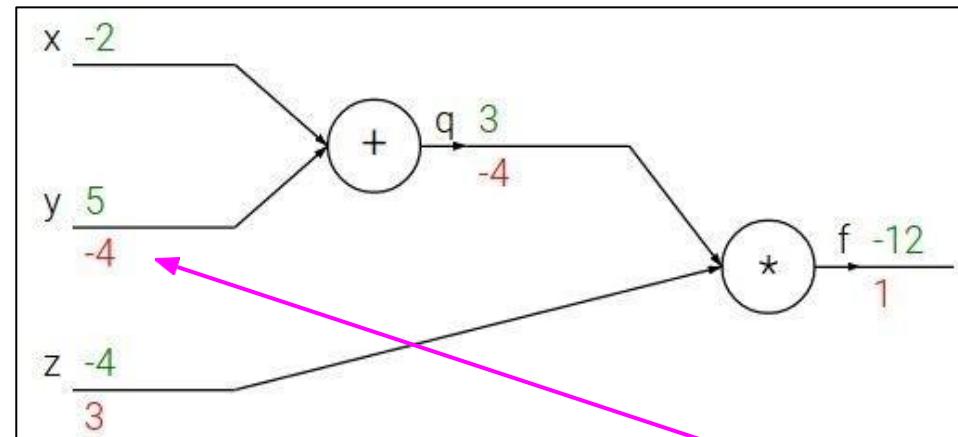
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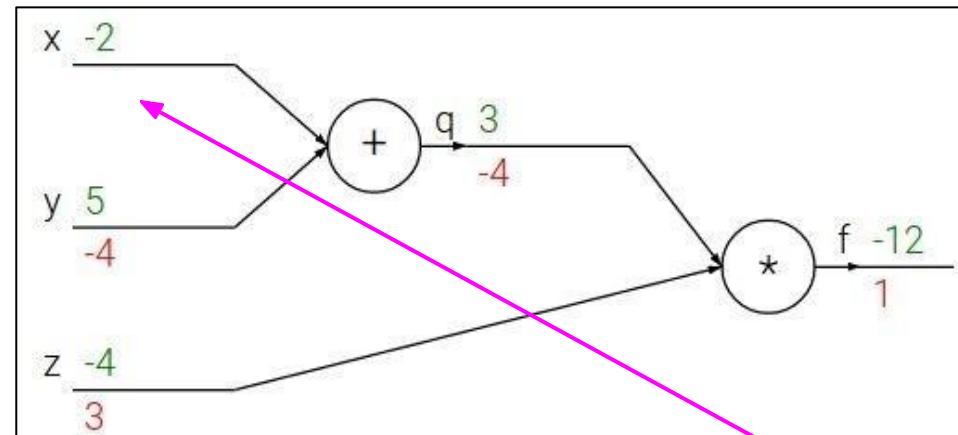
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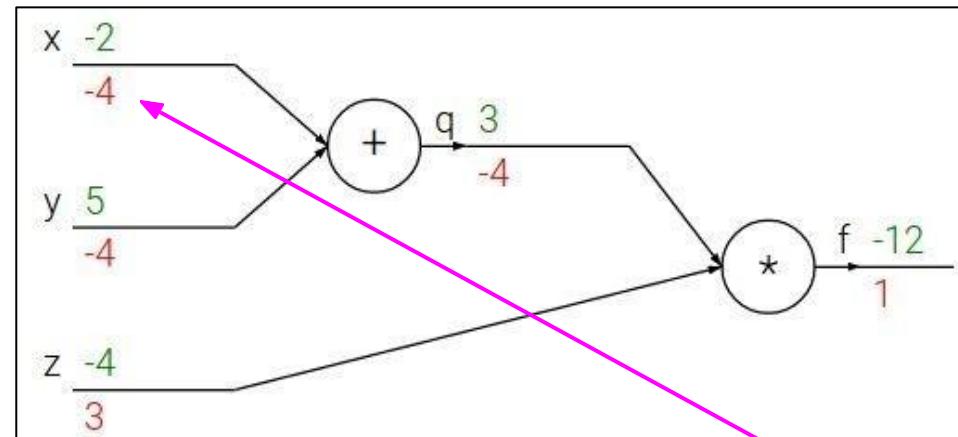
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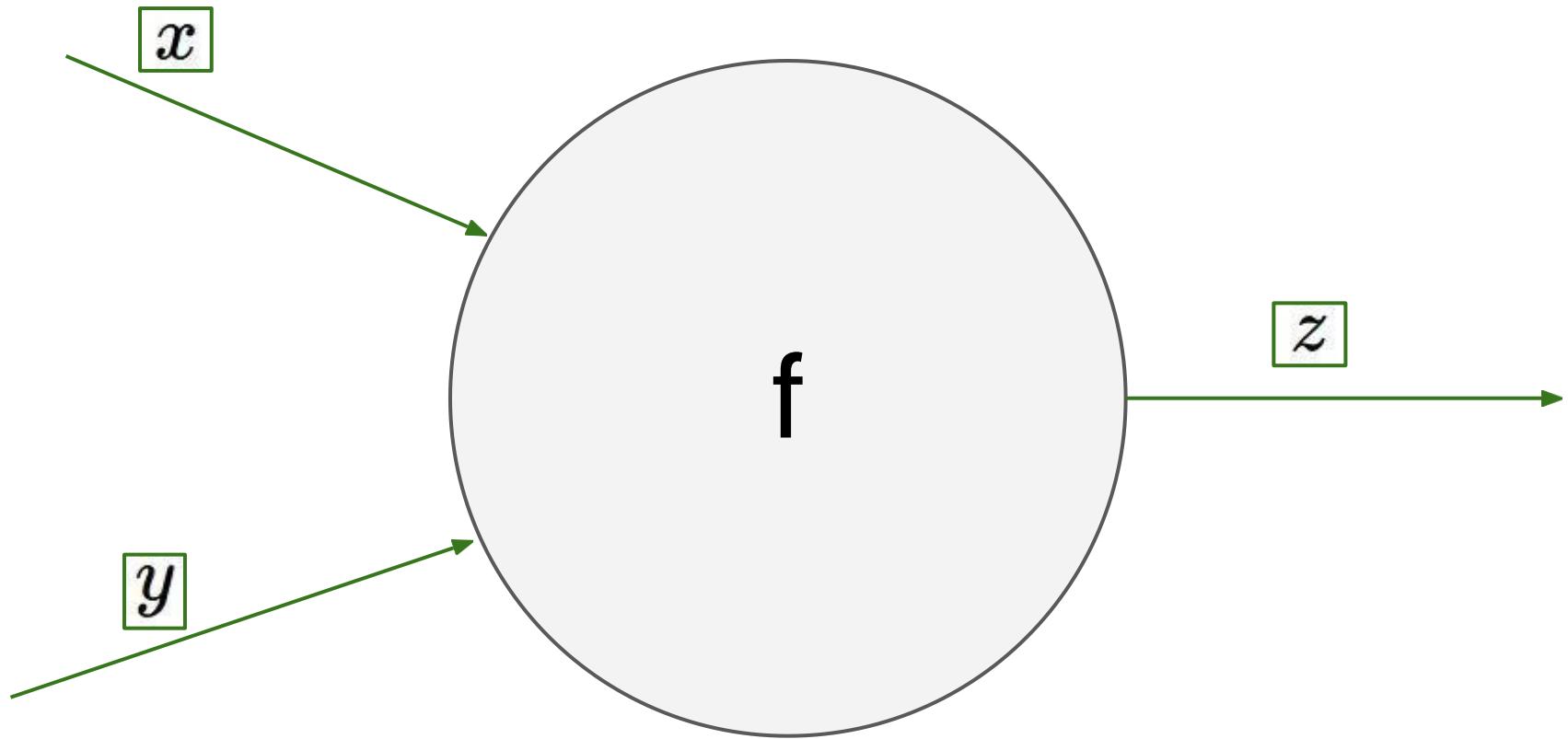


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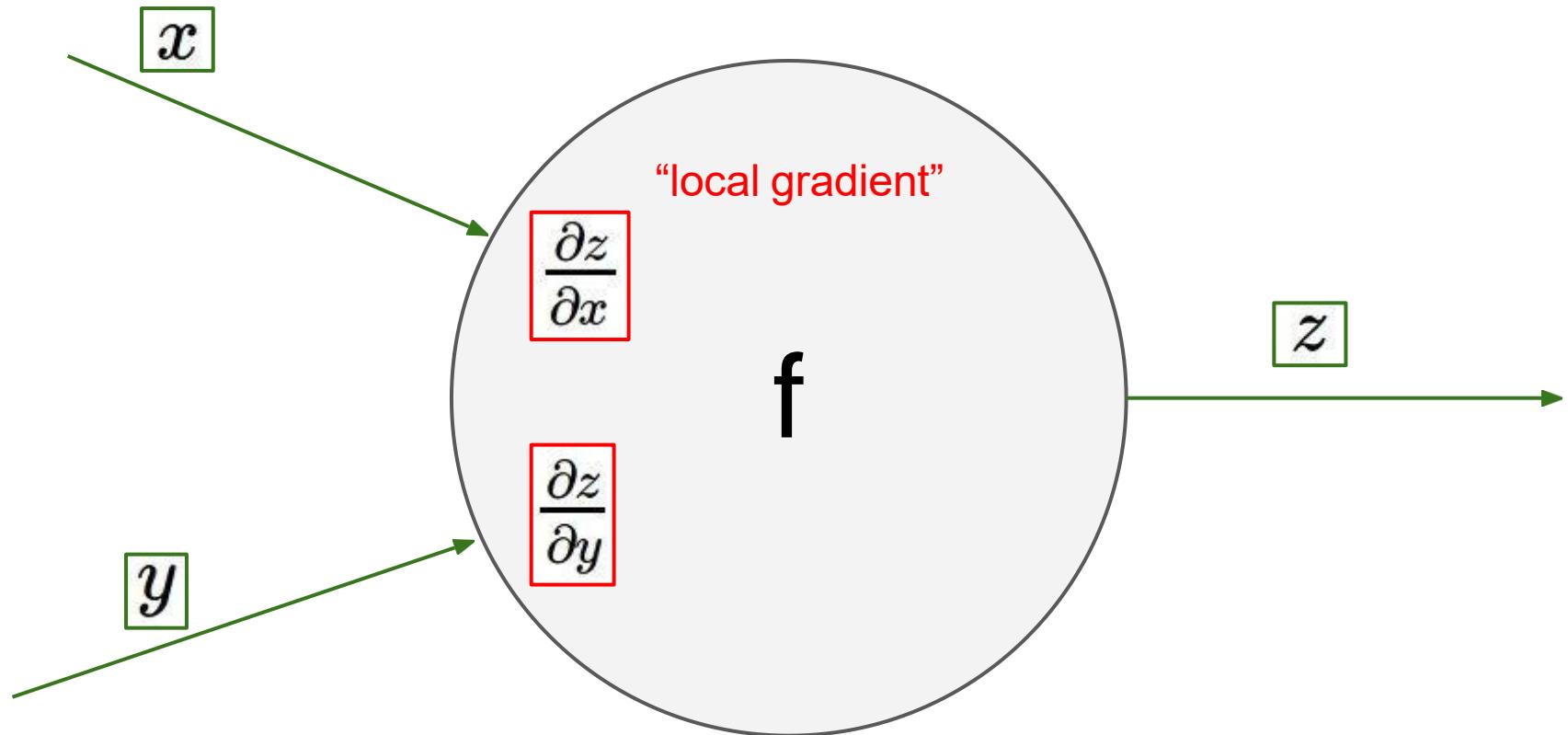
Upstream  
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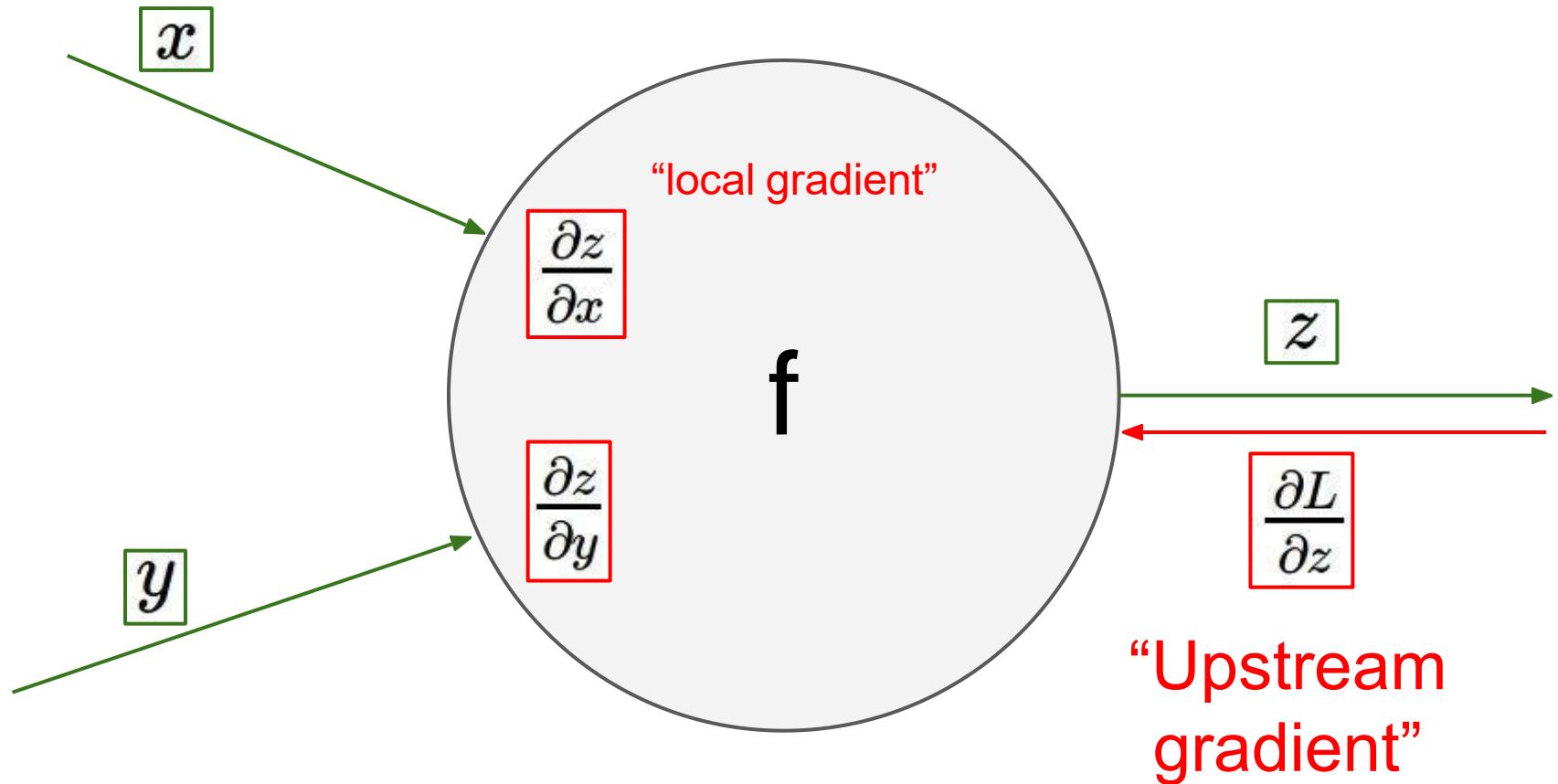
Local  
gradient

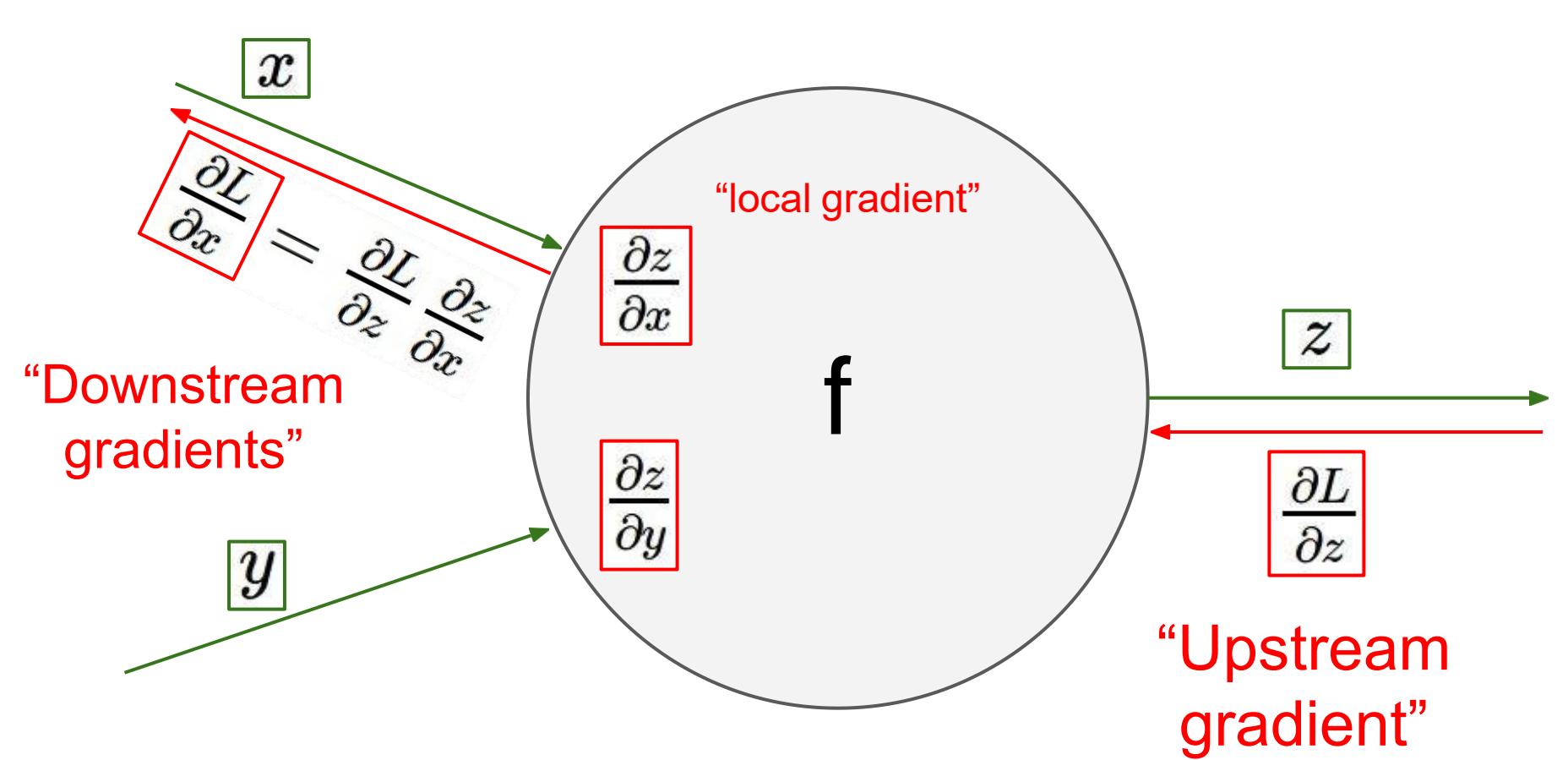


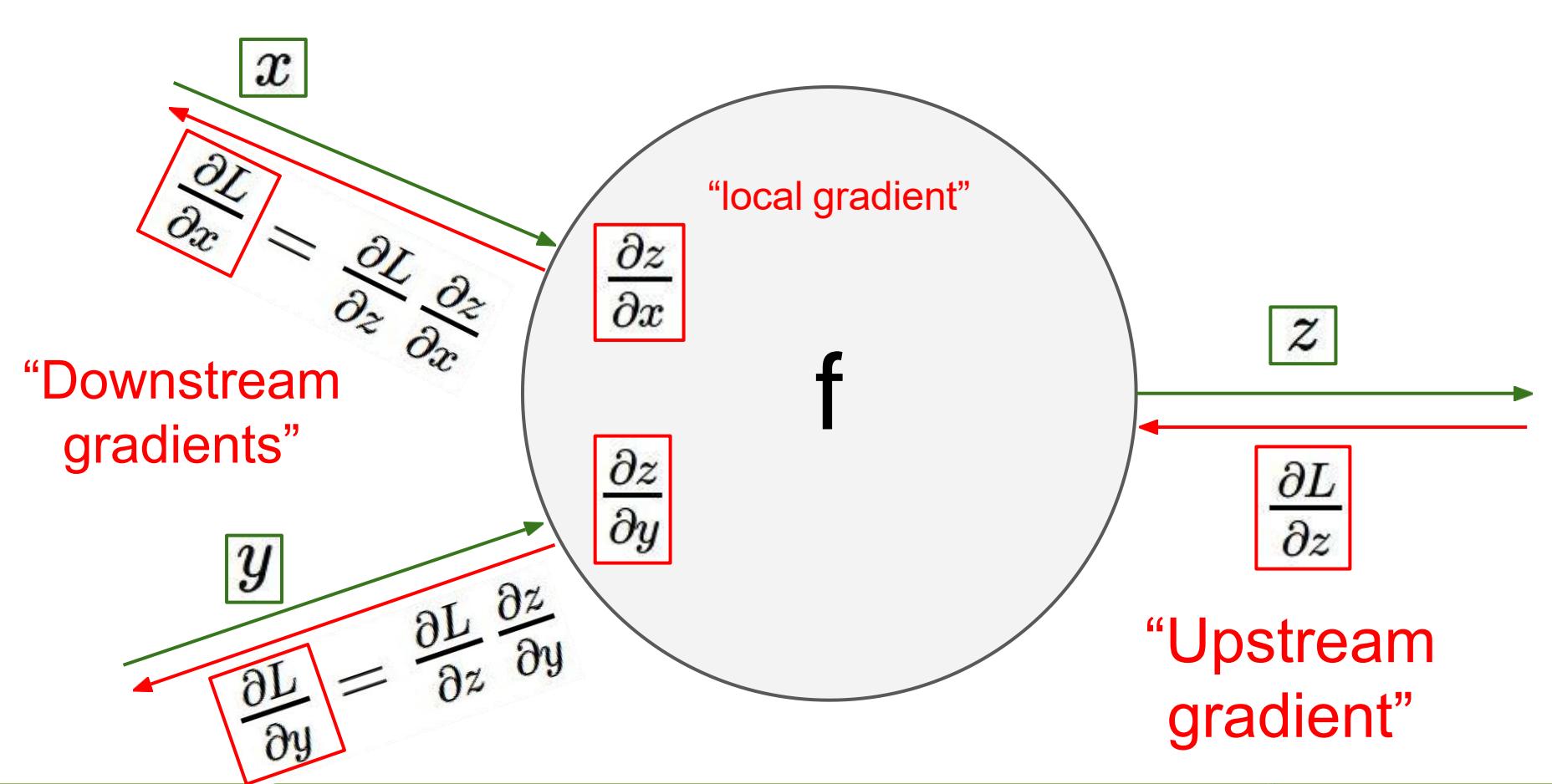
## Neural Network - Backpropagation

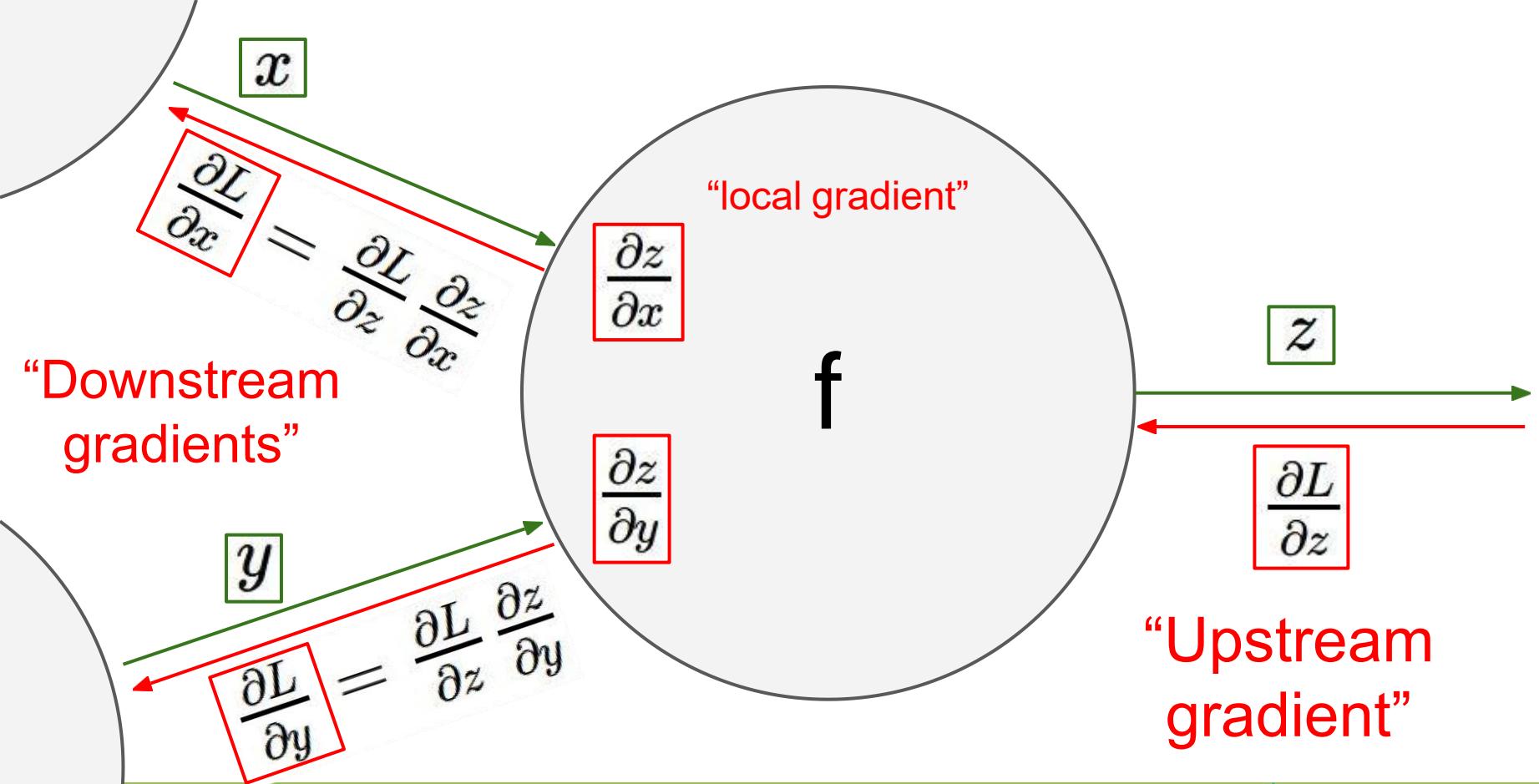
Lecture 4 - 67





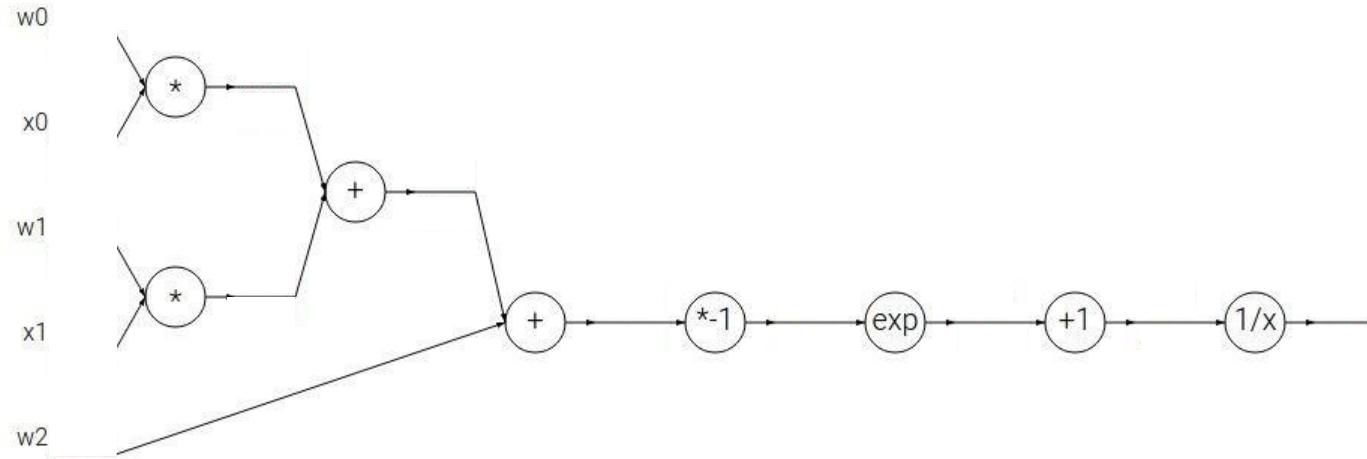






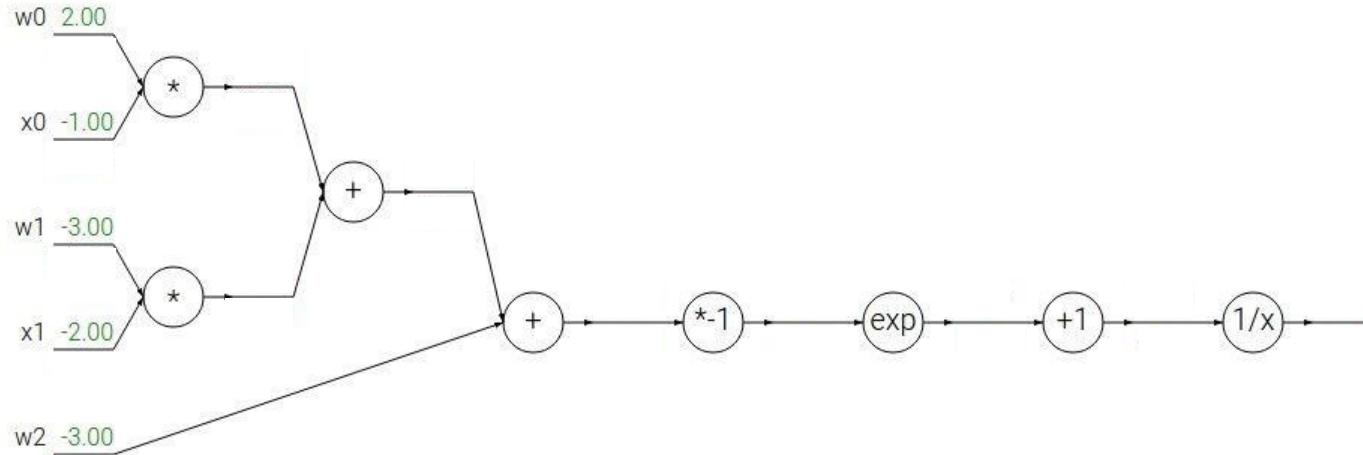
Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



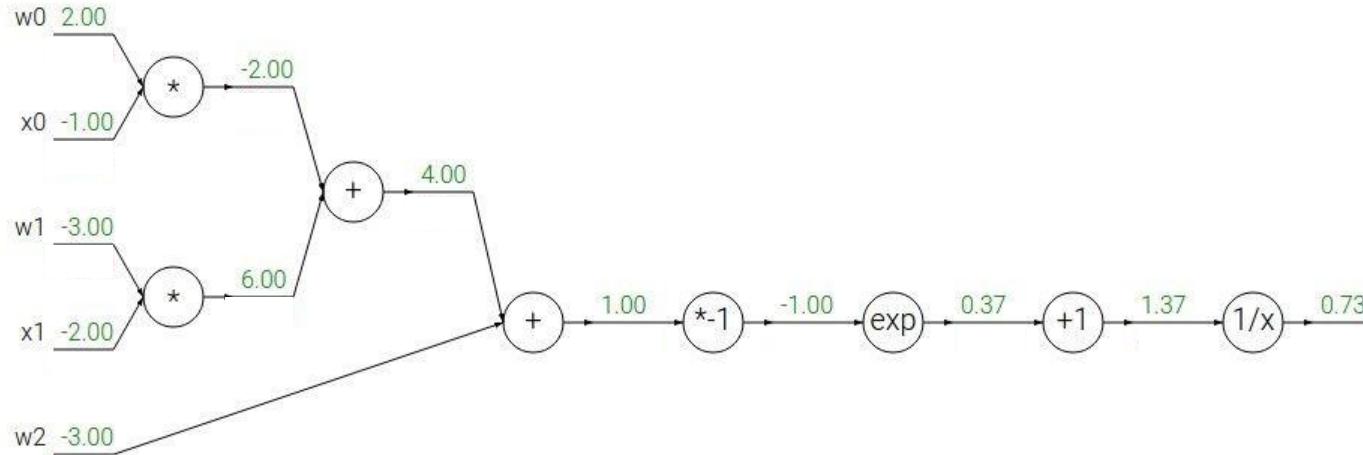
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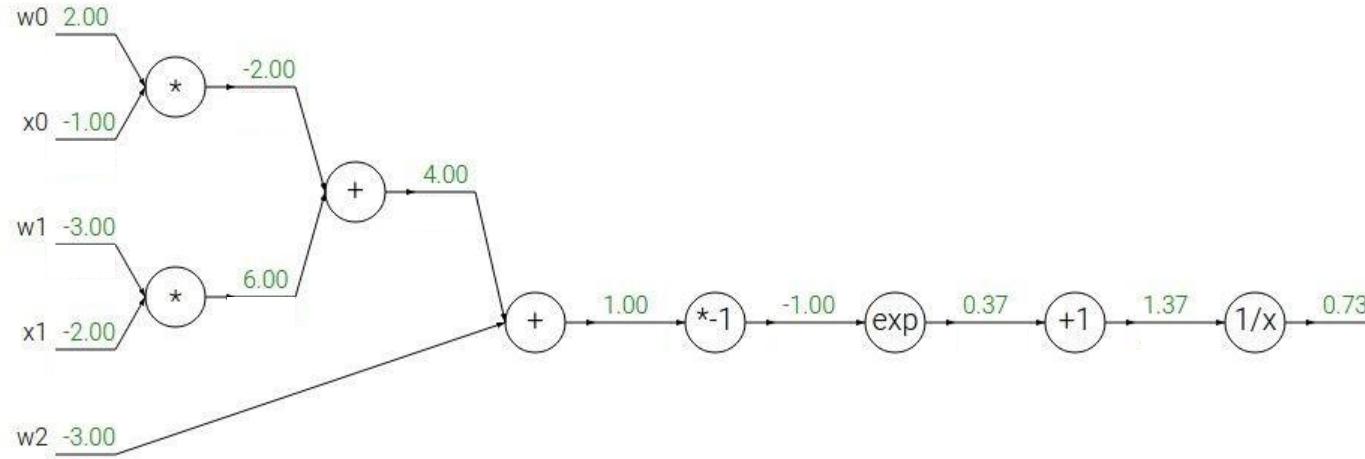


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Another example:  $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



$$f(x) = e^x$$

$\rightarrow$

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

$\rightarrow$

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

$$f_c(x) = c + x$$

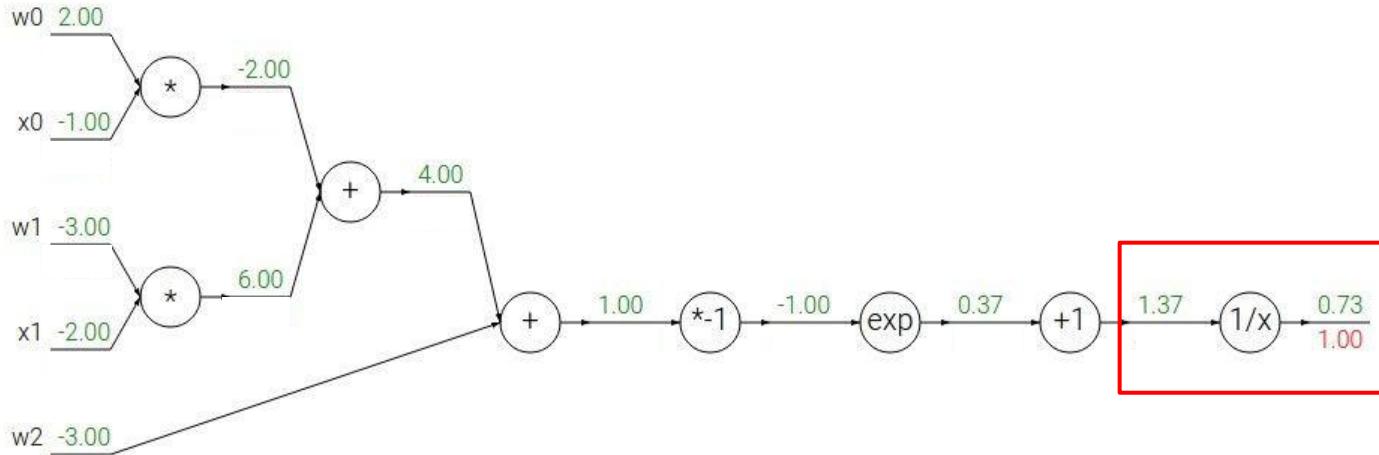
$\rightarrow$

$$\frac{df}{dx} = -1/x^2$$

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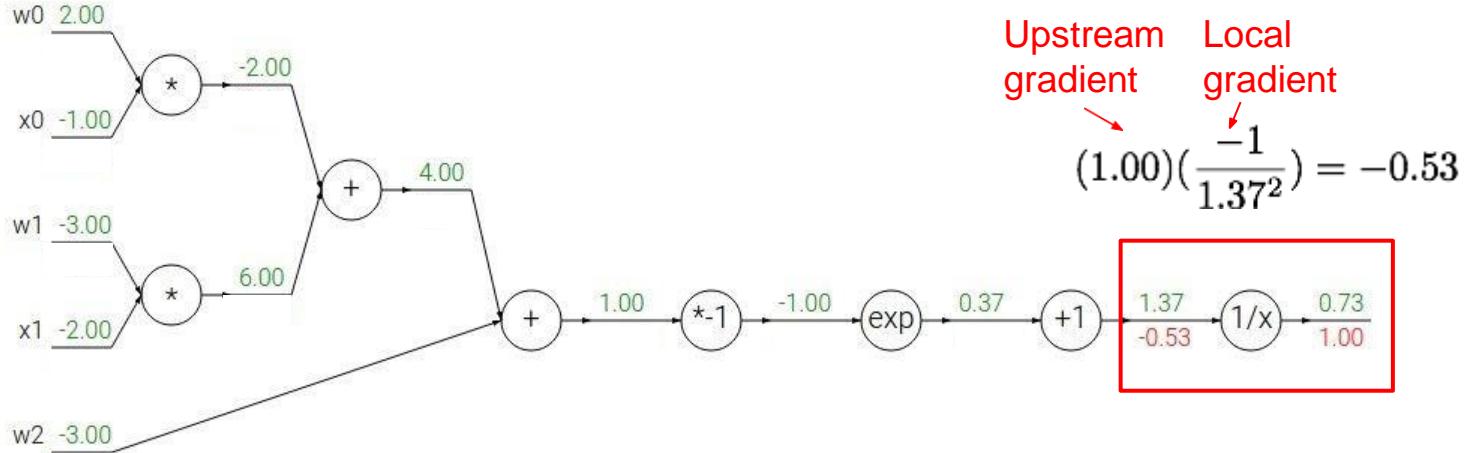
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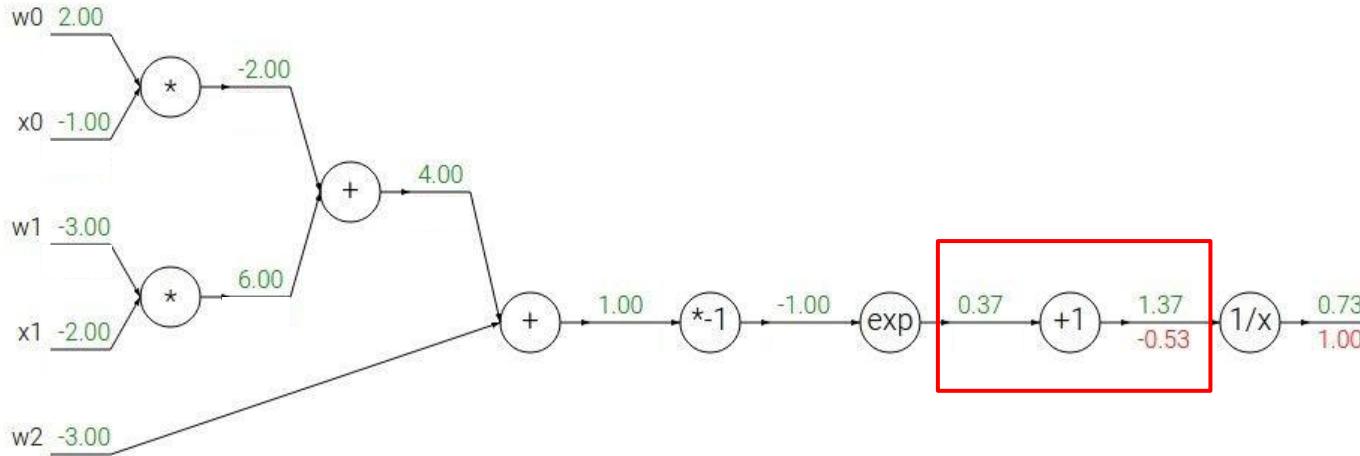
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## Neural Network - Backpropagation

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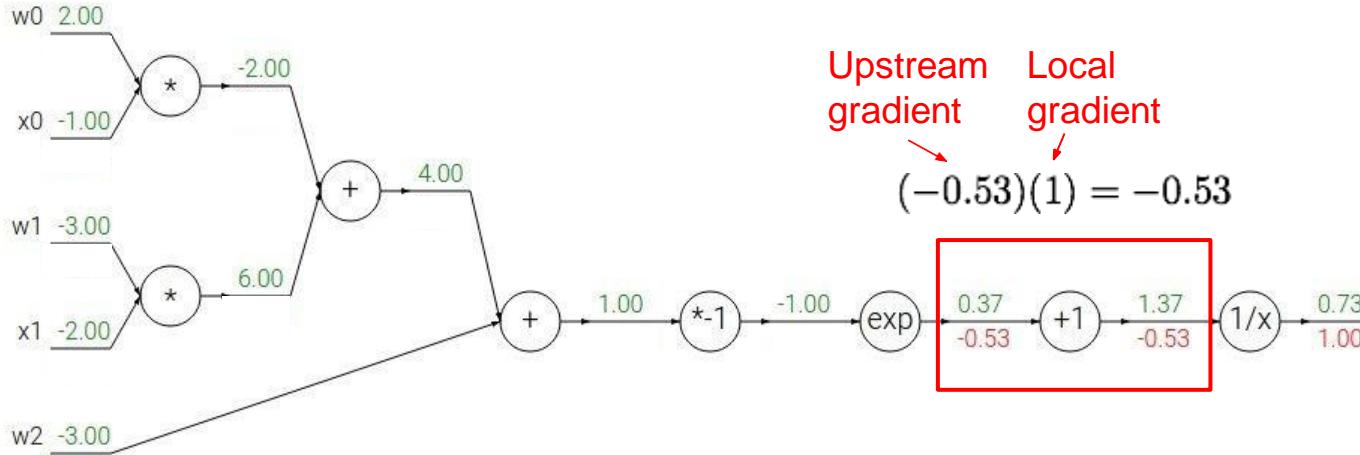
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→

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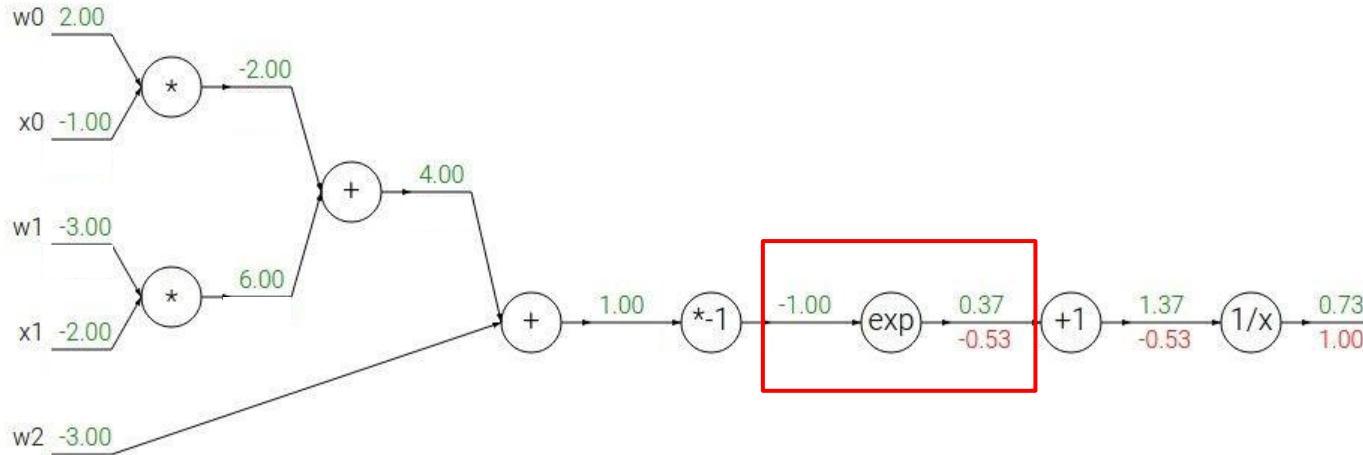
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## Neural Network - Backpropagation

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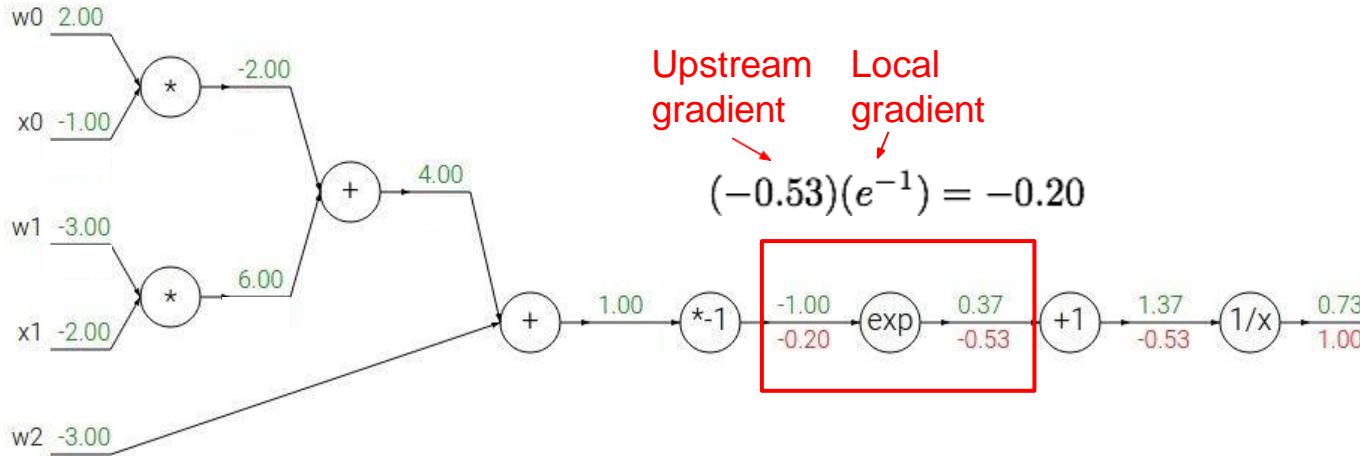
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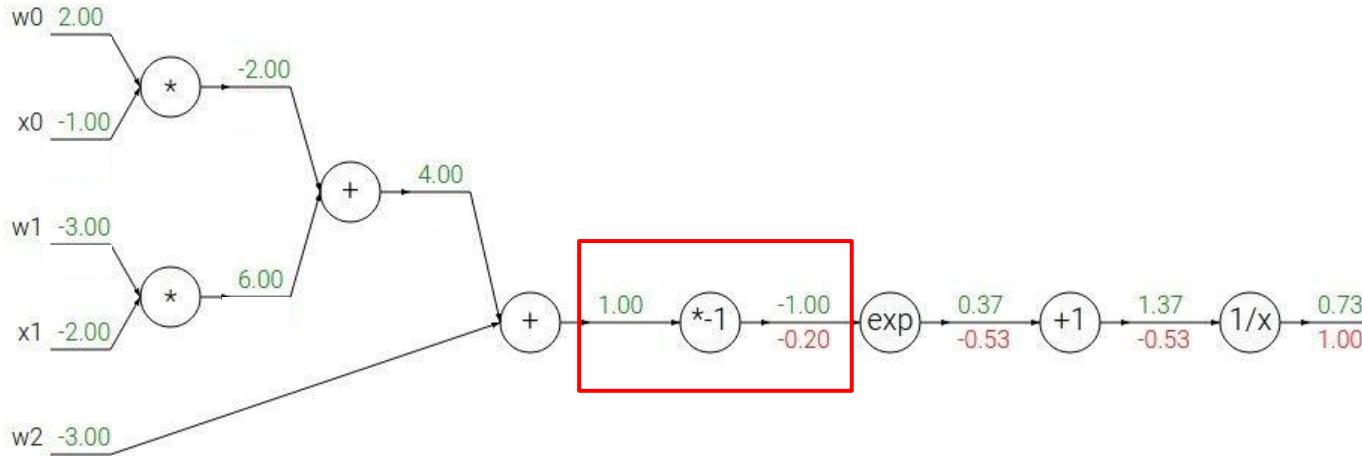
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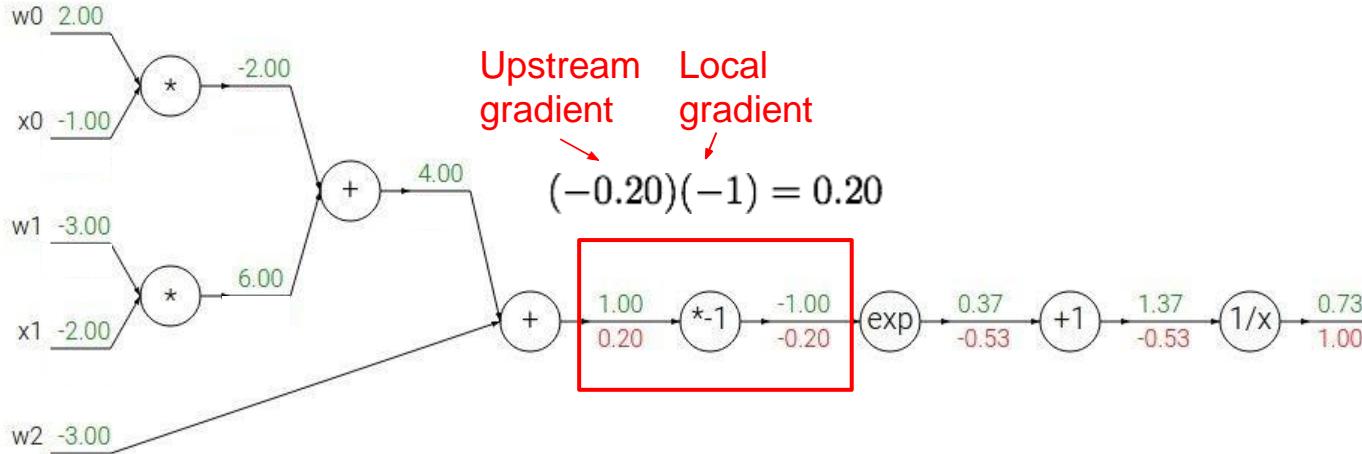
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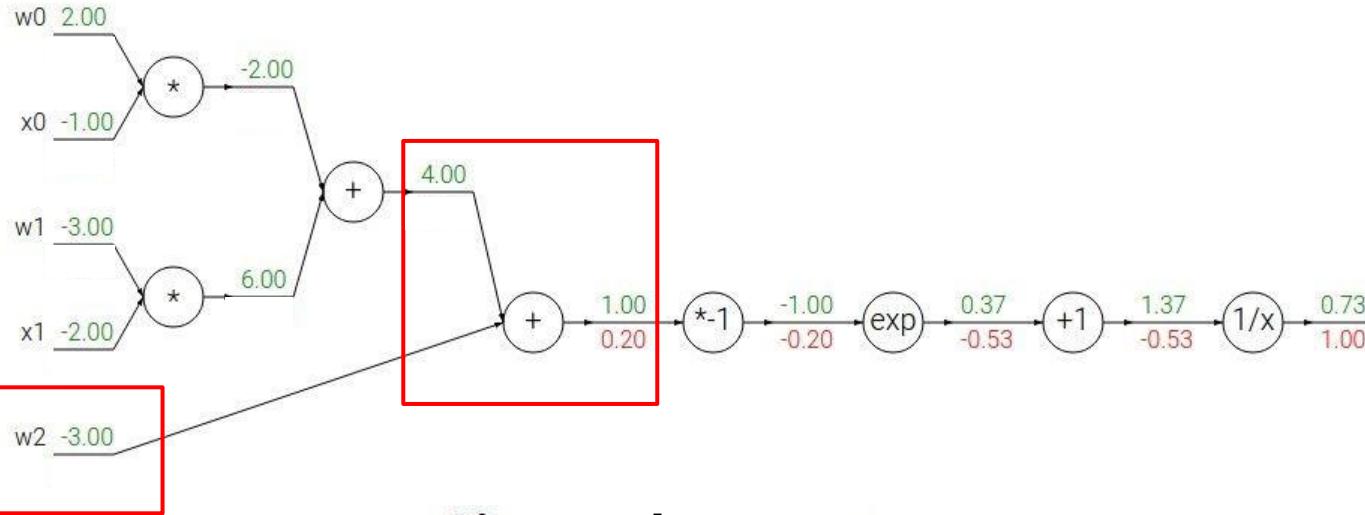
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$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

$$f_c(x) = c + x$$

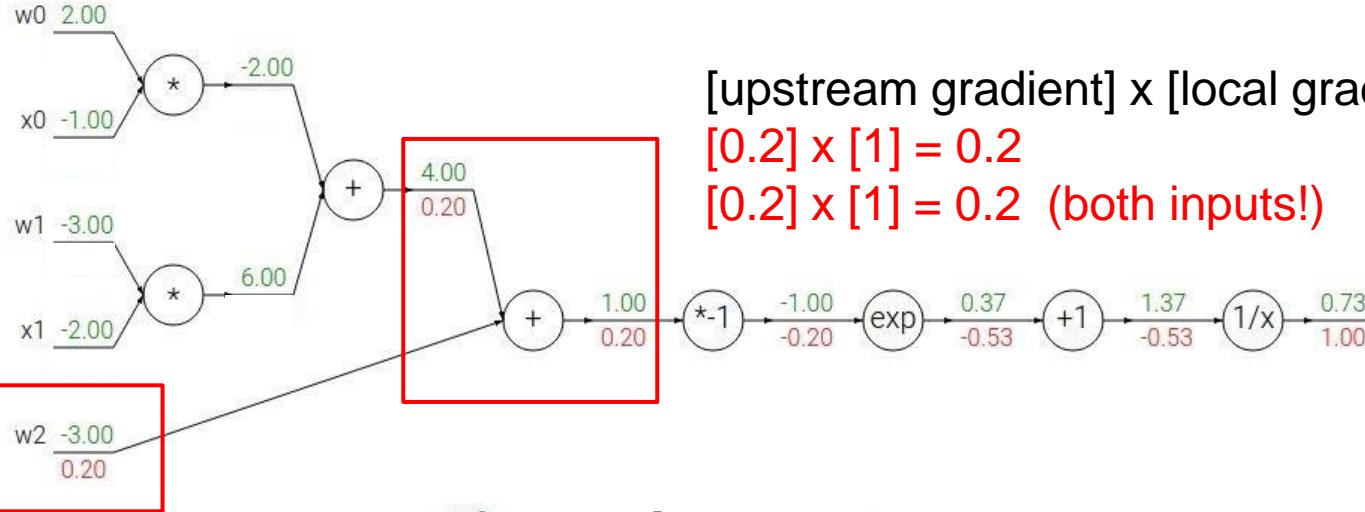
→

$$\frac{df}{dx} = -1/x^2$$

$$\frac{df}{dx} = 1$$

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



[upstream gradient] x [local gradient]

$$[0.2] \times [1] = 0.2$$

[0.2] x [1] = 0.2 (both inputs!)

$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

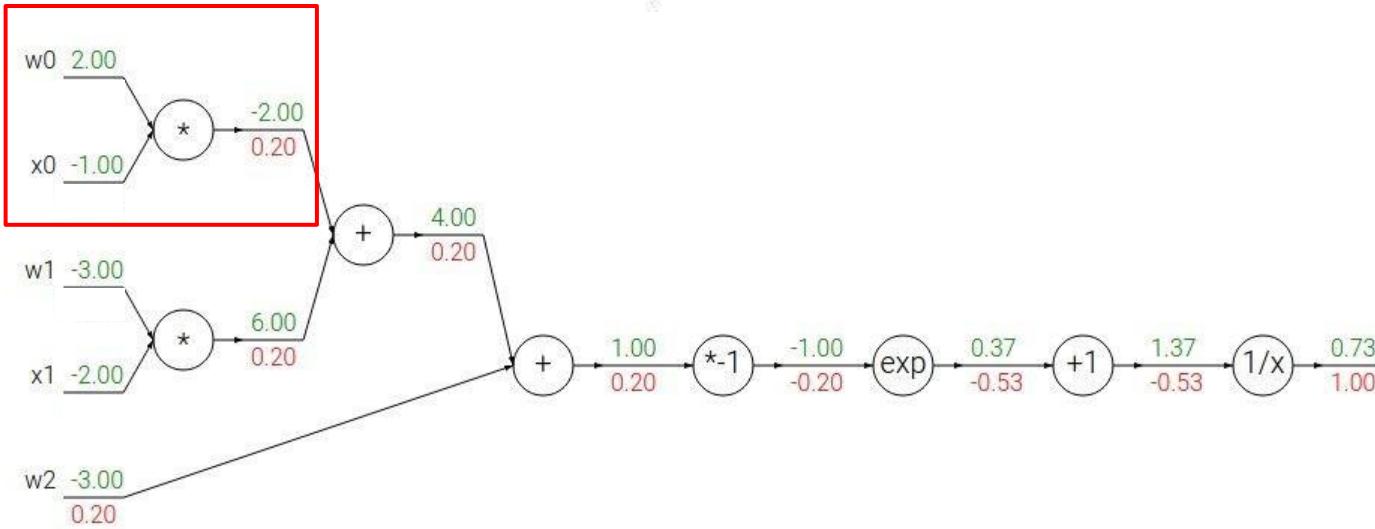
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

Neural Network - Backpropagation

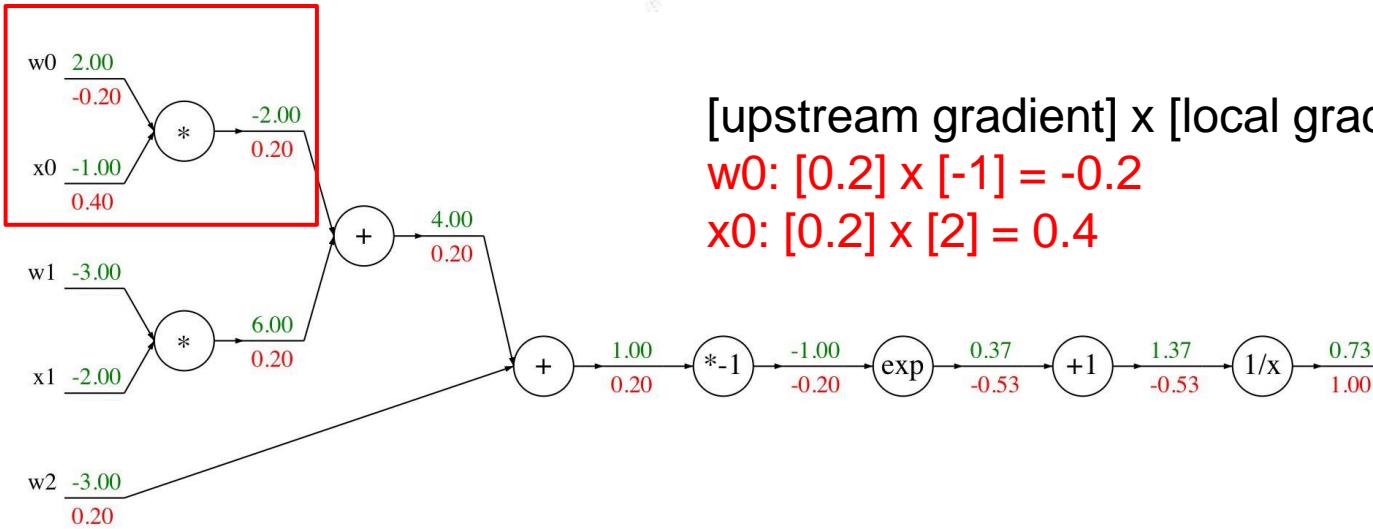
Lecture 4 - 87



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Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$f(x) = e^x$$

$\rightarrow$

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

$\rightarrow$

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

$\rightarrow$

$$\frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x$$

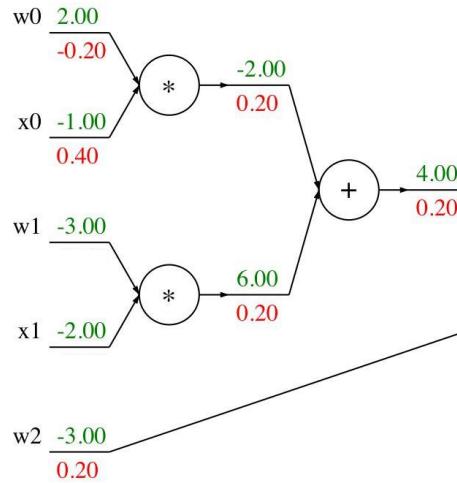
$\rightarrow$

$$\frac{df}{dx} = 1$$

## Neural Network - Backpropagation

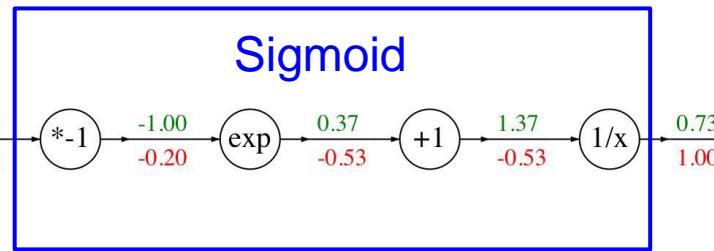
# Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



Sigmoid  
function

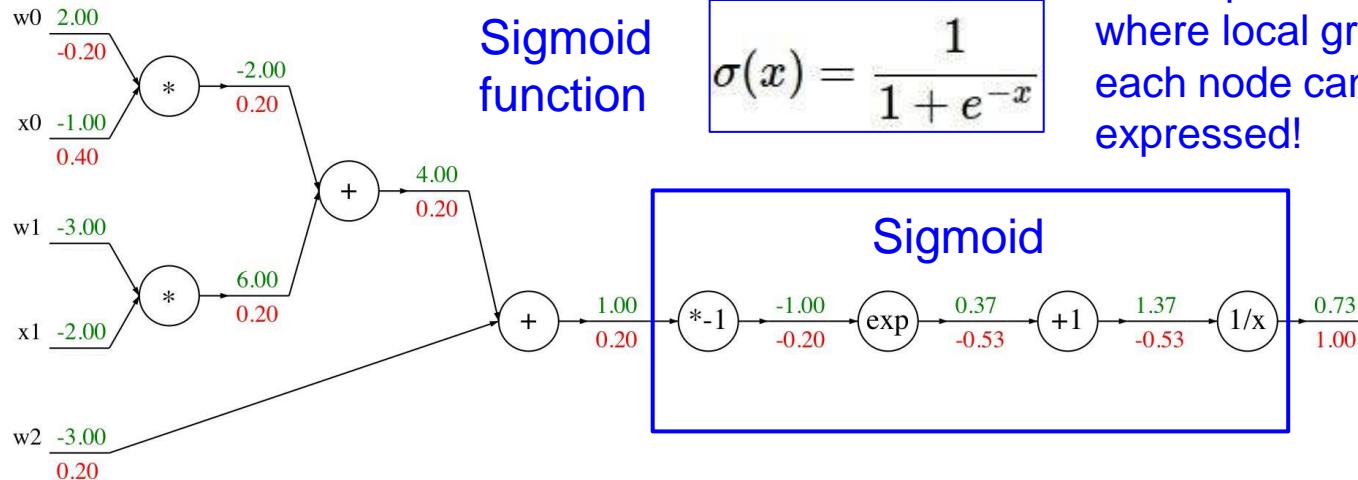
$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

# Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



Sigmoid local  
gradient:

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left( \frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left( \frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$

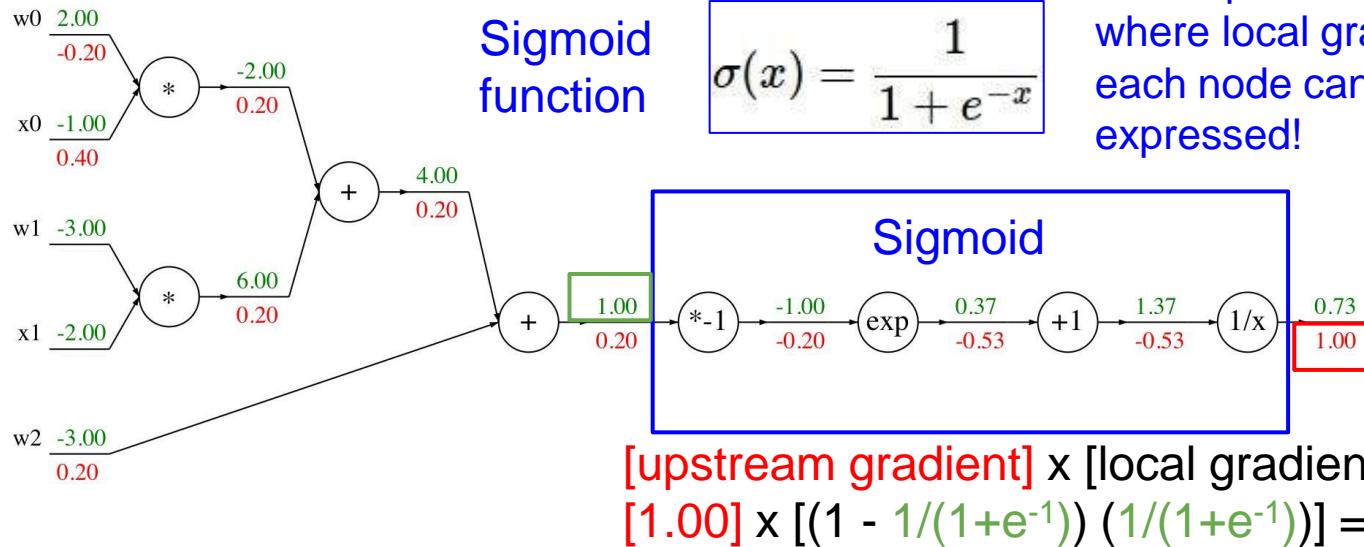
Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

## Neural Network - Backpropagation

Lecture 4 - 90

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



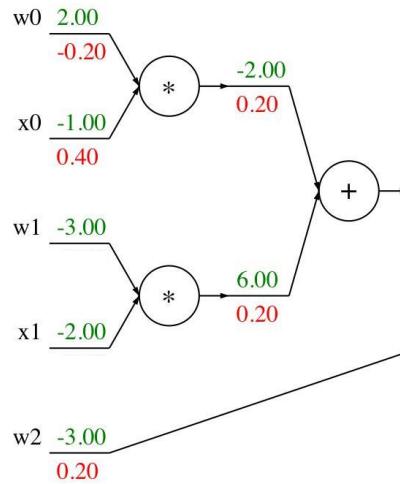
Sigmoid local  
gradient:

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left( \frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left( \frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$

Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

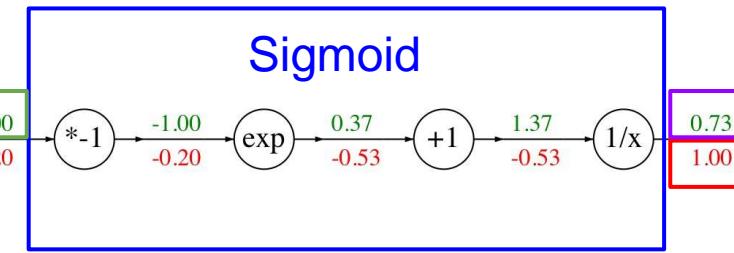
# Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



Sigmoid  
function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



$$\begin{aligned} &[\text{upstream gradient}] \times [\text{local gradient}] \\ &[1.00] \times [(1 - 0.73) (0.73)] = 0.2 \end{aligned}$$

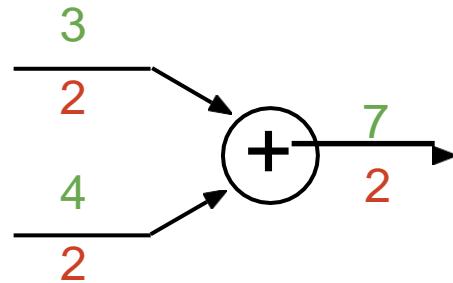
Sigmoid local  
gradient:

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left( \frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left( \frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x)) \sigma(x)$$

Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

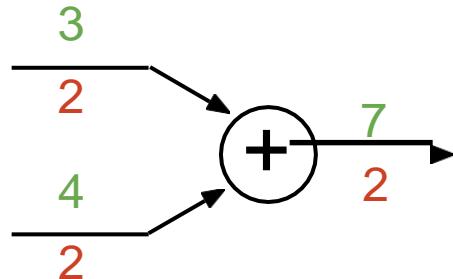
# Patterns in gradient flow

**add** gate: gradient distributor

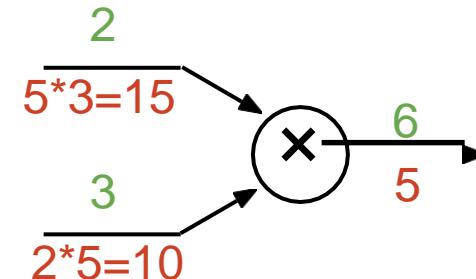


# Patterns in gradient flow

**add** gate: gradient distributor

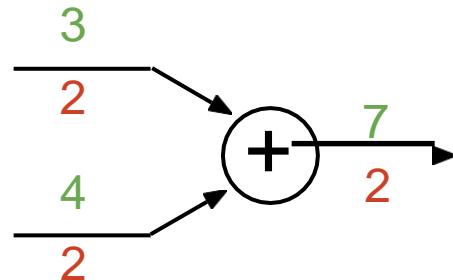


**mul** gate: “swap multiplier”

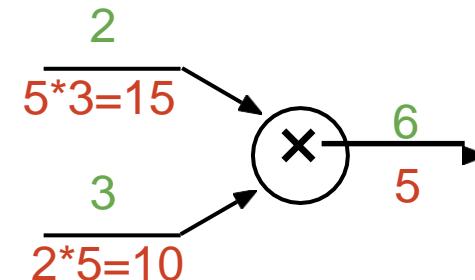


# Patterns in gradient flow

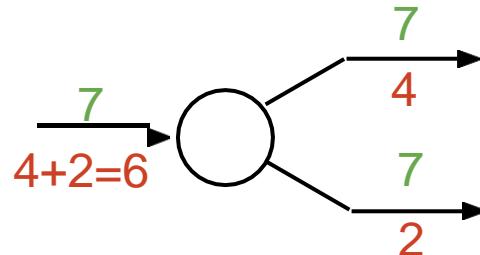
**add** gate: gradient distributor



**mul** gate: “swap multiplier”

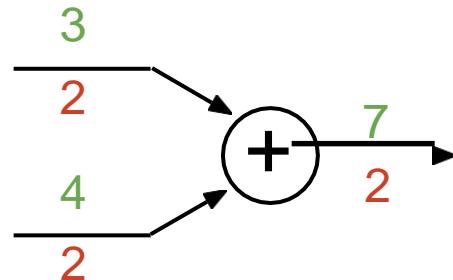


**copy** gate: gradient adder

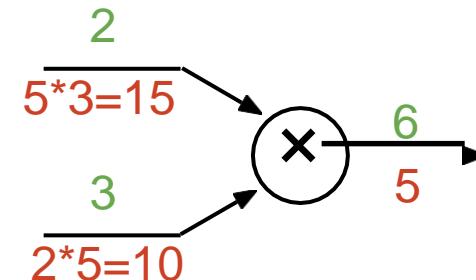


# Patterns in gradient flow

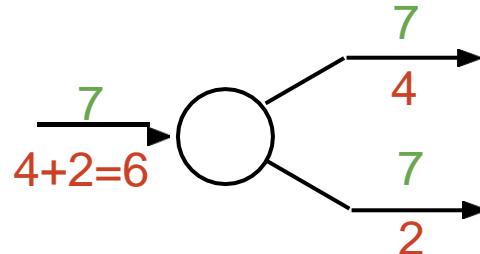
**add** gate: gradient distributor



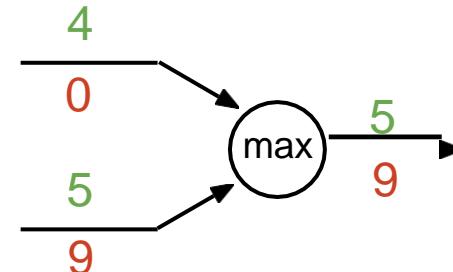
**mul** gate: “swap multiplier”



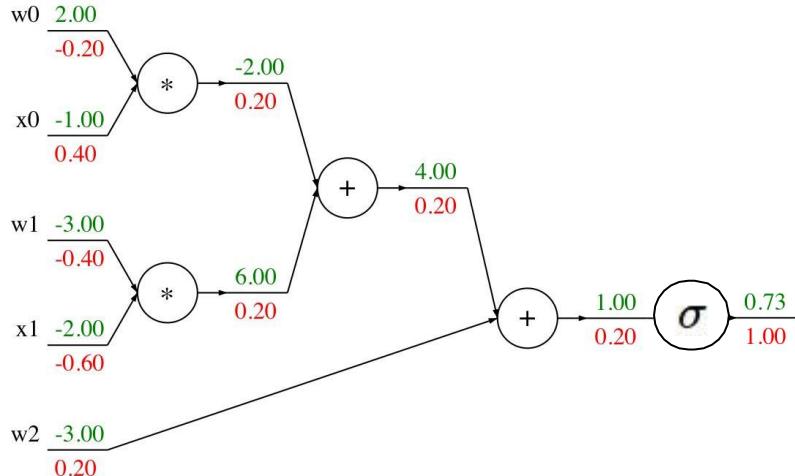
**copy** gate: gradient adder



**max** gate: gradient router



# Backprop Implementation: “Flat” code



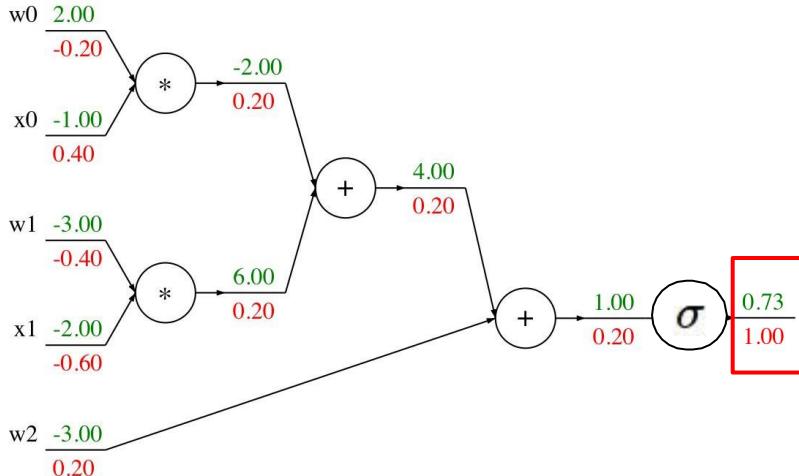
Forward pass:  
Compute output

```
def f(w0, x0, w1, x1, w2):  
    s0 = w0 * x0  
    s1 = w1 * x1  
    s2 = s0 + s1  
    s3 = s2 + w2  
    L = sigmoid(s3)
```

Backward pass:  
Compute grads

```
grad_L = 1.0  
grad_s3 = grad_L * (1 - L) * L  
grad_w2 = grad_s3  
grad_s2 = grad_s3  
grad_s0 = grad_s2  
grad_s1 = grad_s2  
grad_w1 = grad_s1 * x1  
grad_x1 = grad_s1 * w1  
grad_w0 = grad_s0 * x0  
grad_x0 = grad_s0 * w0
```

# Backprop Implementation: “Flat” code



Forward pass:  
Compute output

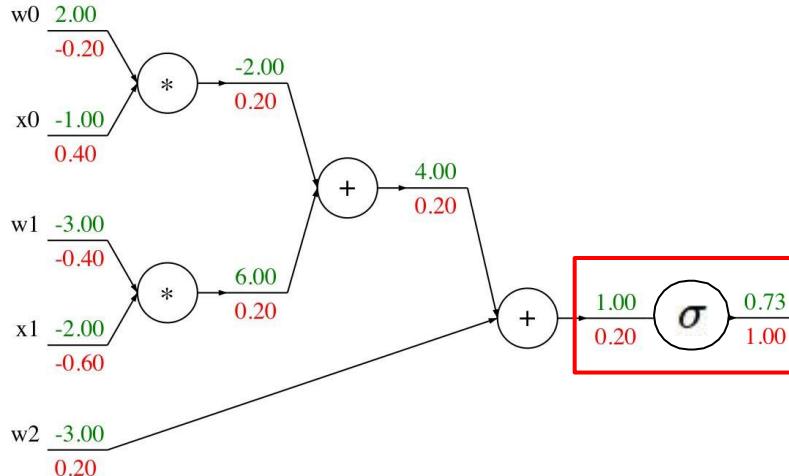
```
def f(w0, x0, w1, x1, w2):  
    s0 = w0 * x0  
    s1 = w1 * x1  
    s2 = s0 + s1  
    s3 = s2 + w2  
    L = sigmoid(s3)
```

Base case

```
grad_L = 1.0
```

```
grad_s3 = grad_L * (1 - L) * L  
grad_w2 = grad_s3  
grad_s2 = grad_s3  
grad_s0 = grad_s2  
grad_s1 = grad_s2  
grad_w1 = grad_s1 * x1  
grad_x1 = grad_s1 * w1  
grad_w0 = grad_s0 * x0  
grad_x0 = grad_s0 * w0
```

# Backprop Implementation: “Flat” code



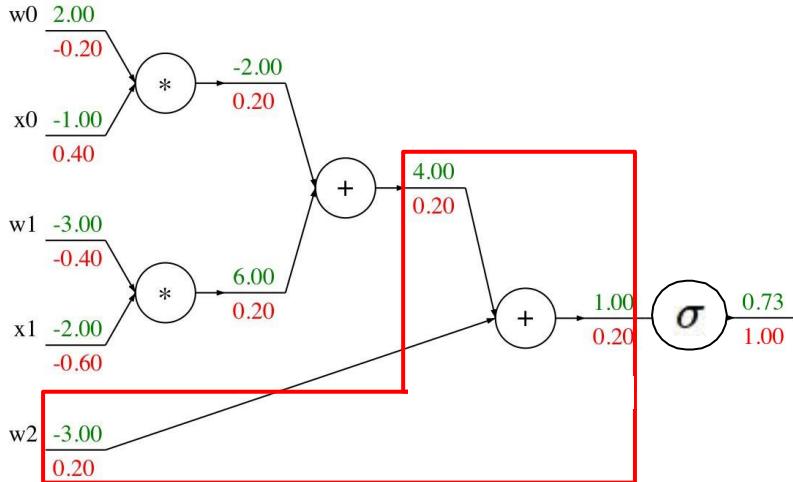
Forward pass:  
Compute output

Sigmoid

```
def f(w0, x0, w1, x1, w2):  
    s0 = w0 * x0  
    s1 = w1 * x1  
    s2 = s0 + s1  
    s3 = s2 + w2  
    L = sigmoid(s3)
```

```
grad_L = 1.0  
grad_s3 = grad_L * (1 - L) * L  
grad_w2 = grad_s3  
grad_s2 = grad_s3  
grad_s0 = grad_s2  
grad_s1 = grad_s2  
grad_w1 = grad_s1 * x1  
grad_x1 = grad_s1 * w1  
grad_w0 = grad_s0 * x0  
grad_x0 = grad_s0 * w0
```

# Backprop Implementation: “Flat” code



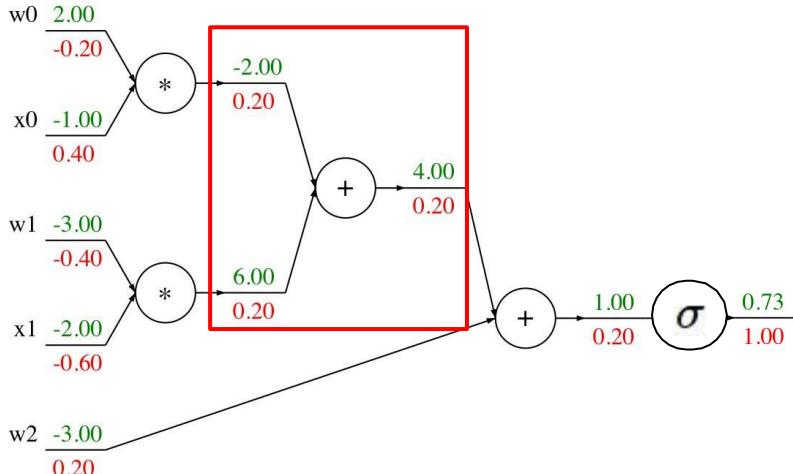
Forward pass:  
Compute output

Add gate

```
def f(w0, x0, w1, x1, w2):  
    s0 = w0 * x0  
    s1 = w1 * x1  
    s2 = s0 + s1  
    s3 = s2 + w2  
    L = sigmoid(s3)
```

```
grad_L = 1.0  
grad_s3 = grad_L * (1 - L) * L  
grad_w2 = grad_s3  
grad_s2 = grad_s3  
grad_s0 = grad_s2  
grad_s1 = grad_s2  
grad_w1 = grad_s1 * x1  
grad_x1 = grad_s1 * w1  
grad_w0 = grad_s0 * x0  
grad_x0 = grad_s0 * w0
```

# Backprop Implementation: “Flat” code



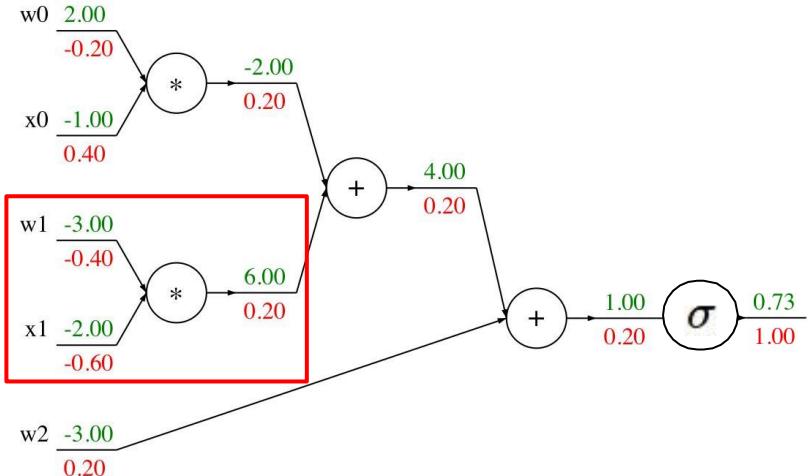
Forward pass:  
Compute output

Add gate

```
def f(w0, x0, w1, x1, w2):  
    s0 = w0 * x0  
    s1 = w1 * x1  
    s2 = s0 + s1  
    s3 = s2 + w2  
    L = sigmoid(s3)
```

```
grad_L = 1.0  
grad_s3 = grad_L * (1 - L) * L  
grad_w2 = grad_s3  
grad_s2 = grad_s3  
grad_s0 = grad_s2  
grad_s1 = grad_s2  
grad_w1 = grad_s1 * x1  
grad_x1 = grad_s1 * w1  
grad_w0 = grad_s0 * x0  
grad_x0 = grad_s0 * w0
```

# Backprop Implementation: “Flat” code



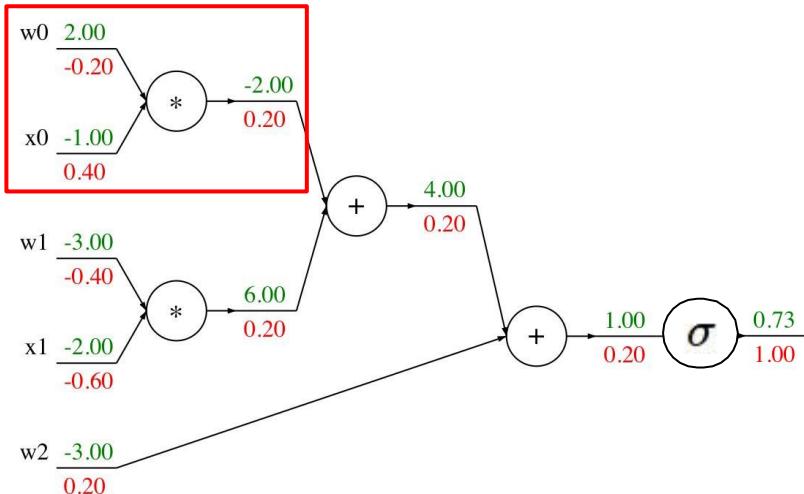
Forward pass:  
Compute output

```
def f(w0, x0, w1, x1, w2):  
    s0 = w0 * x0  
    s1 = w1 * x1  
    s2 = s0 + s1  
    s3 = s2 + w2  
    L = sigmoid(s3)
```

```
grad_L = 1.0  
grad_s3 = grad_L * (1 - L) * L  
grad_w2 = grad_s3  
grad_s2 = grad_s3  
grad_s0 = grad_s2  
grad_s1 = grad_s2  
grad_w1 = grad_s1 * x1  
grad_x1 = grad_s1 * w1  
grad_w0 = grad_s0 * x0  
grad_x0 = grad_s0 * w0
```

Multiply gate

# Backprop Implementation: “Flat” code



Forward pass:  
Compute output

```
def f(w0, x0, w1, x1, w2):  
    s0 = w0 * x0  
    s1 = w1 * x1  
    s2 = s0 + s1  
    s3 = s2 + w2  
    L = sigmoid(s3)
```

```
grad_L = 1.0  
grad_s3 = grad_L * (1 - L) * L  
grad_w2 = grad_s3  
grad_s2 = grad_s3  
grad_s0 = grad_s2  
grad_s1 = grad_s2  
grad_w1 = grad_s1 * x1  
grad_x1 = grad_s1 * w1  
grad_w0 = grad_s0 * x0  
grad_x0 = grad_s0 * w0
```

Multiply gate

Neural Network - Backpropagation

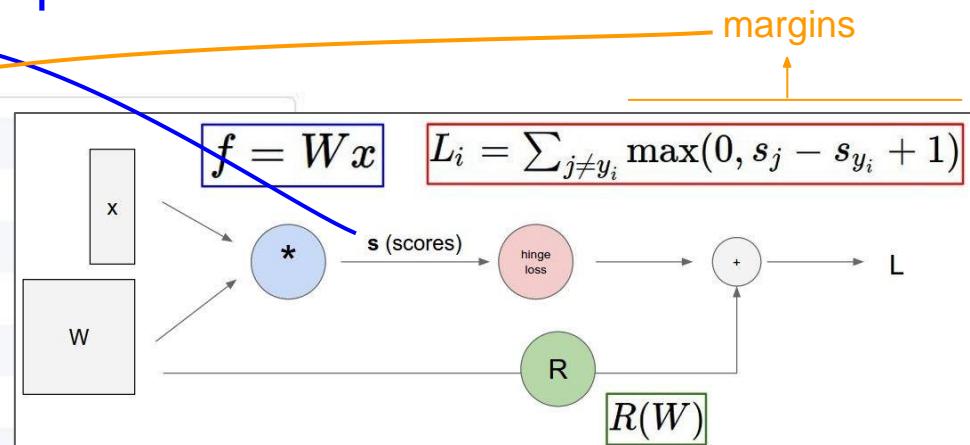
Lecture 4 - 103

# “Flat” Backprop: Do this for assignment 1!

Stage your forward/backward computation!

E.g. for the SVM:

```
# receive W (weights), X (data)
# forward pass (we have 6 lines)
scores = #...
margins = #...
data_loss = #...
reg_loss = #...
loss = data_loss + reg_loss
# backward pass (we have 5 lines)
dmargins = # ... (optionally, we go direct to dscores)
dscores = #...
dW = #...
```



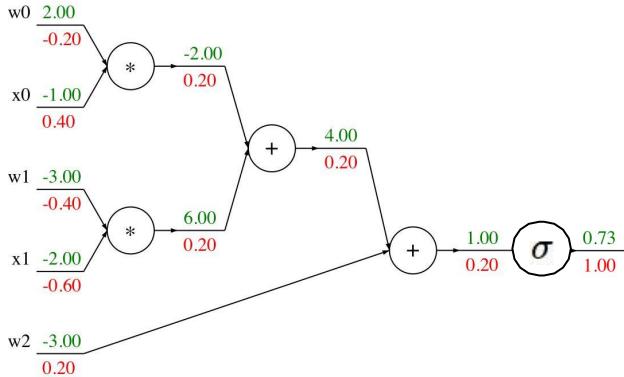
# “Flat” Backprop: Do this for assignment 1!

E.g. for two-layer neural net:

```
# receive W1,W2,b1,b2 (weights/biases), X (data)
# forward pass:
h1 = #... function of X,W1,b1
scores = #... function of h1,W2,b2
loss = #... (several lines of code to evaluate Softmax loss)
# backward pass:
dscores = #...
dh1,dW2,db2 = #...
dW1,db1 = #...
```

# Backprop Implementation: Modularized API

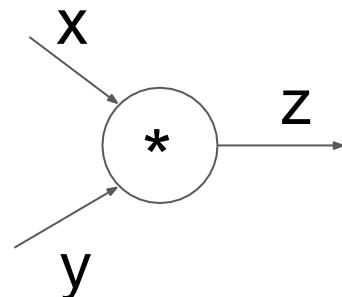
Graph (or Net) object (*rough pseudo code*)



```
class ComputationalGraph(object):  
    ...  
    def forward(inputs):  
        # 1. [pass inputs to input gates...]  
        # 2. forward the computational graph:  
        for gate in self.graph.nodes_topologically_sorted():  
            gate.forward()  
        return loss # the final gate in the graph outputs the loss  
    def backward():  
        for gate in reversed(self.graph.nodes_topologically_sorted()):  
            gate.backward() # little piece of backprop (chain rule applied)  
        return inputs_gradients
```

# Modularized implementation: forward / backward API

Gate / Node / Function object: Actual PyTorch code



( $x, y, z$  are scalars)

```
class Multiply(torch.autograd.Function):  
    @staticmethod  
    def forward(ctx, x, y):  
        ctx.save_for_backward(x, y) ← Need to cache some values for use in backward  
        z = x * y  
        return z  
  
    @staticmethod  
    def backward(ctx, grad_z): ← Upstream gradient  
        x, y = ctx.saved_tensors  
        grad_x = y * grad_z # dz/dx * dL/dz  
        grad_y = x * grad_z # dz/dy * dL/dz  
        return grad_x, grad_y
```

Need to cache some values for use in backward

Upstream gradient

Multiply upstream and local gradients

# Example: PyTorch operators

pytorch / pytorch

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ezyang and facebook-github-bot Canonicalize all includes in PyTorch. (#14849) Latest commit 517c7c9 on Dec 8, 2018

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AbsCriterion.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
BCECriterion.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
ClassNLLCriterion.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
Col2Im.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
ELU.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
FeatureLPPooling.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
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SpatialDilatedMaxPooling.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
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SpatialFullDilatedConvolution.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
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TemporalRowConvolution.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
TemporalUpSamplingLinear.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
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VolumetricDilatedMaxPooling.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
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VolumetricReplicationPadding.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
VolumetricUpSamplingNearest.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
linear_upsampling.h	Implement nn.functional.interpolate based on upsample. (#8591)	9 months ago
pooling_shape.h	Use integer math to compute output size of pooling operations (#14405)	4 months ago
unfold.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago

Neural Network - Backpropagation

Lecture 4 - 108

# PyTorch sigmoid layer

```
1 #ifndef TH_GENERIC_FILE
2 #define TH_GENERIC_FILE "THNN/generic/Sigmoid.c"
3 #else
4
5 void THNN_(Sigmoid_updateOutput)(
6     THNNState *state,
7     THTensor *input,
8     THTensor *output)
9 {
10    THTensor_(sigmoid)(output, input);
11 }
12
13 void THNN_(Sigmoid_updateGradInput)(
14     THNNState *state,
15     THTensor *gradOutput,
16     THTensor *gradInput,
17     THTensor *output)
18 {
19     THNN_CHECK_NELEMENT(output, gradOutput);
20     THTensor_(resizeAs)(gradInput, output);
21     TH_TENSOR_APPLY3(scalar_t, gradInput, scalar_t, gradOutput, scalar_t, output,
22         scalar_t z = *output_data;
23         *gradInput_data = *gradOutput_data * (1. - z) * z;
24     );
25 }
26
27 #endif
```

Forward

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

[Source](#)

## Neural Network - Backpropagation

Lecture 4 - 109

# PyTorch sigmoid layer

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1 #ifndef TH_GENERIC_FILE
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23         *gradInput_data = *gradOutput_data * (1. - z) * z;
24     );
25 }
26
27 #endif
```

Forward

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

```
static void sigmoid_kernel(TensorIterator& iter) {
    AT_DISPATCH_FLOATING_TYPES(iter.dtype(), "sigmoid_cpu", [&]() {
        unary_kernel_vec(
            iter,
            [=](scalar_t a) -> scalar_t { return (1 / (1 + std::exp((-a)))); },
            [=](Vec256<scalar_t> a) {
                a = Vec256<scalar_t>((scalar_t)(0)) - a;
                a = a.exp();
                a = Vec256<scalar_t>((scalar_t)(1)) + a;
                a = a.reciprocal();
                return a;
            });
    });
}
```

Forward actually defined [elsewhere...](#)

`return (1 / (1 + std::exp((-a))));`

[Source](#)

## Neural Network - Backpropagation

Lecture 4 - 110

# PyTorch sigmoid layer

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1 #ifndef TH_GENERIC_FILE
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15     THTensor *gradOutput,
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23         *gradInput_data = *gradOutput_data * (1. - z) * z;
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27 #endif
```

Forward

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

```
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            [=](Vec256<scalar_t> a) {
                a = Vec256<scalar_t>((scalar_t)(0)) - a;
                a = a.exp();
                a = Vec256<scalar_t>((scalar_t)(1)) + a;
                a = a.reciprocal();
                return a;
            });
    });
}
```

Forward actually defined [elsewhere...](#)

Backward

$$(1 - \sigma(x)) \sigma'(x)$$

[Source](#)

## Neural Network - Backpropagation

Lecture 4 - 111



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So far: backprop with scalars

What about vector-valued functions?

# Recap: Vector derivatives

## Scalar to Scalar

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If  $x$  changes by a small amount, how much will  $y$  change?

# Recap: Vector derivatives

Scalar to Scalar

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If  $x$  changes by a small amount, how much will  $y$  change?

Vector to Scalar

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

Derivative is **Gradient**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N \quad \left( \frac{\partial y}{\partial x} \right)_n = \frac{\partial y}{\partial x_n}$$

For each element of  $x$ , if it changes by a small amount then how much will  $y$  change?

# Recap: Vector derivatives

## Scalar to Scalar

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If  $x$  changes by a small amount, how much will  $y$  change?

## Vector to Scalar

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

Derivative is **Gradient**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N \quad \left( \frac{\partial y}{\partial x} \right)_n = \frac{\partial y}{\partial x_n}$$

For each element of  $x$ , if it changes by a small amount then how much will  $y$  change?

## Vector to Vector

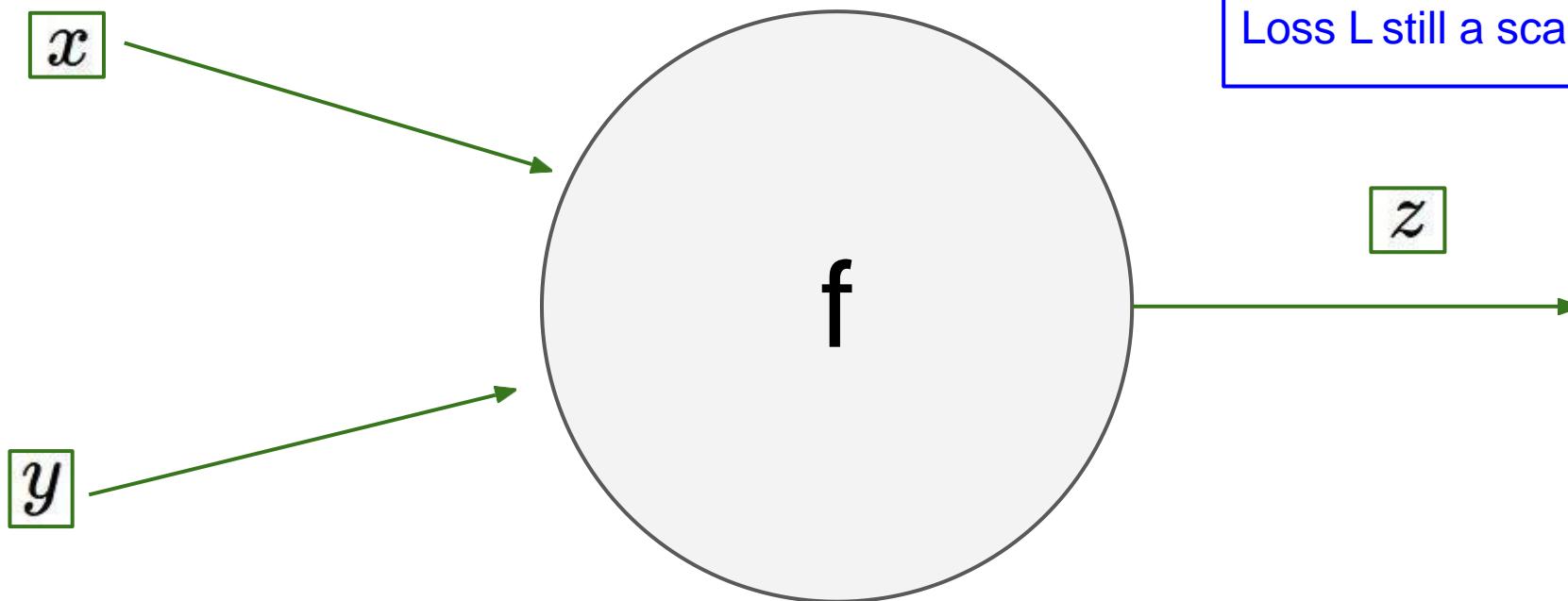
$$x \in \mathbb{R}^N, y \in \mathbb{R}^M$$

Derivative is **Jacobian**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^{N \times M} \quad \left( \frac{\partial y}{\partial x} \right)_{n,m} = \frac{\partial y_m}{\partial x_n}$$

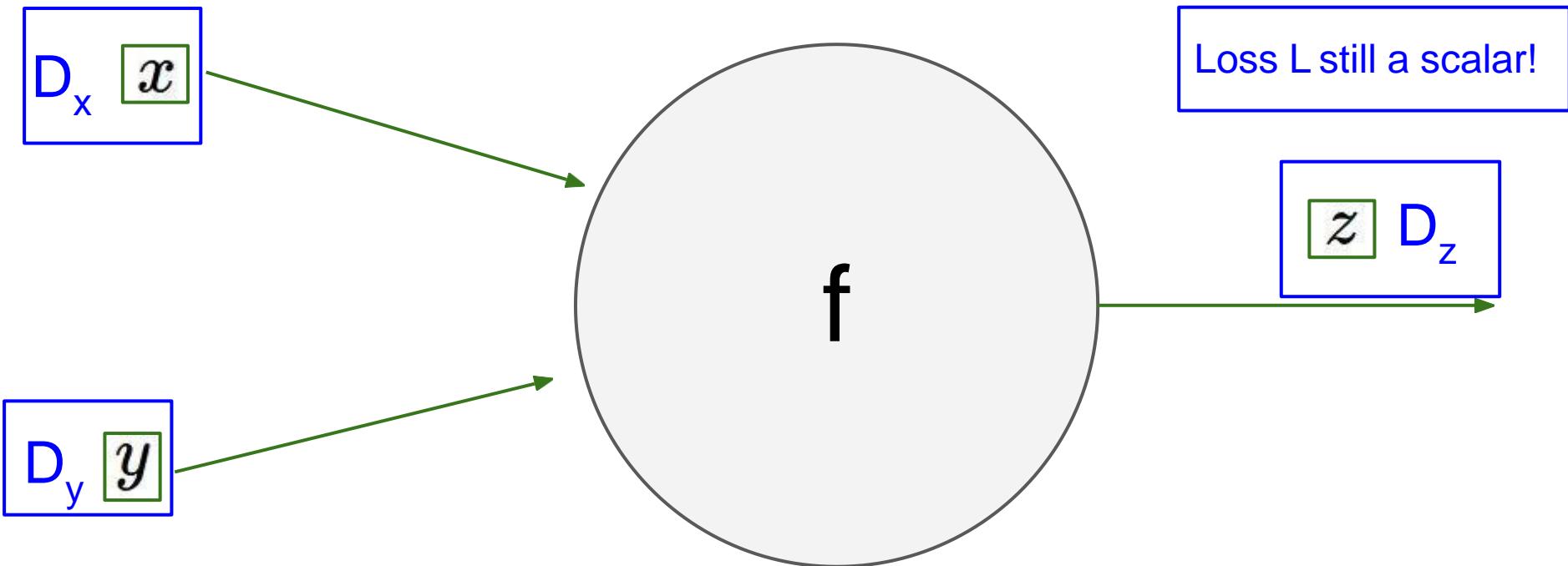
For each element of  $x$ , if it changes by a small amount then how much will each element of  $y$  change?

# Backprop with Vectors

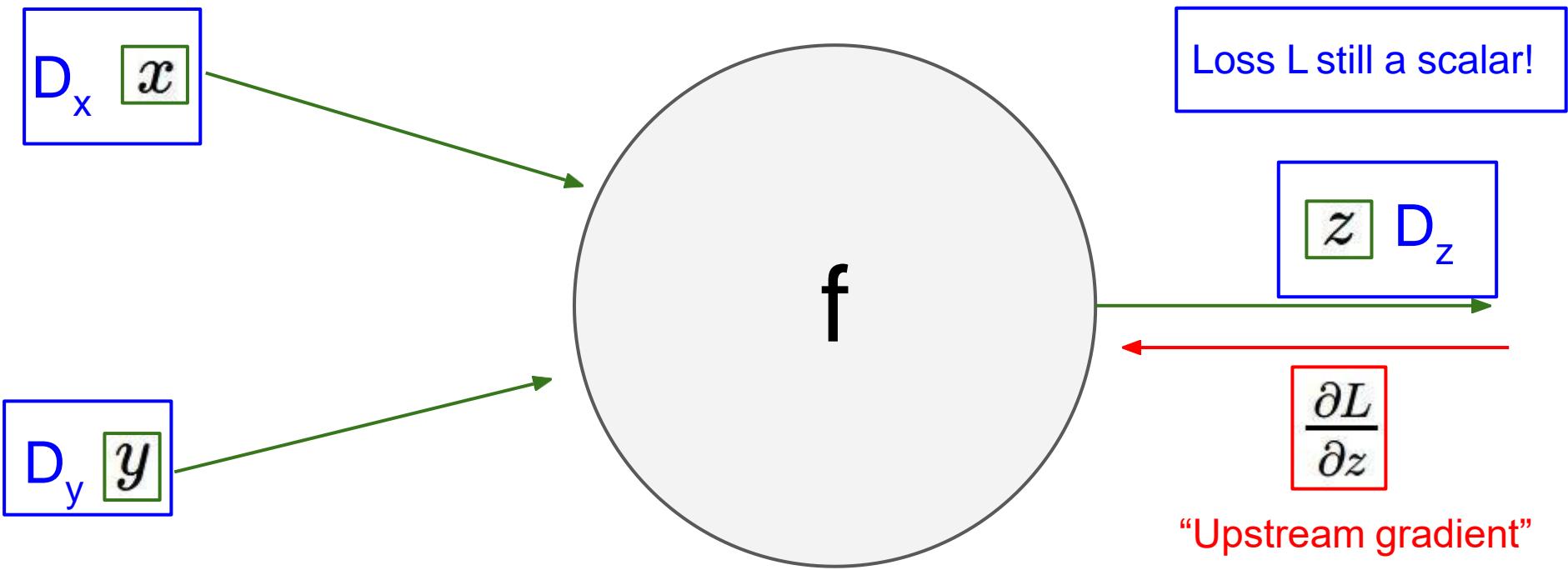


Loss  $L$  still a scalar!

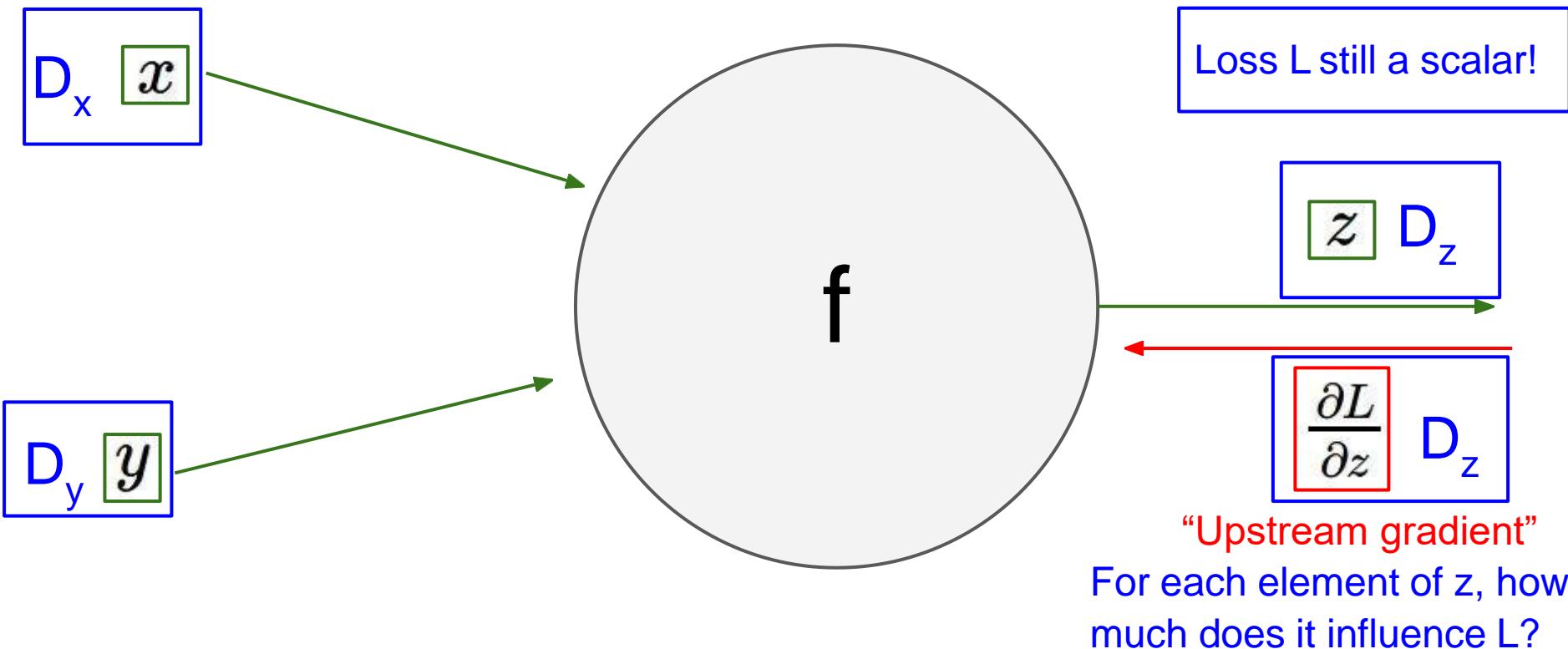
# Backprop with Vectors



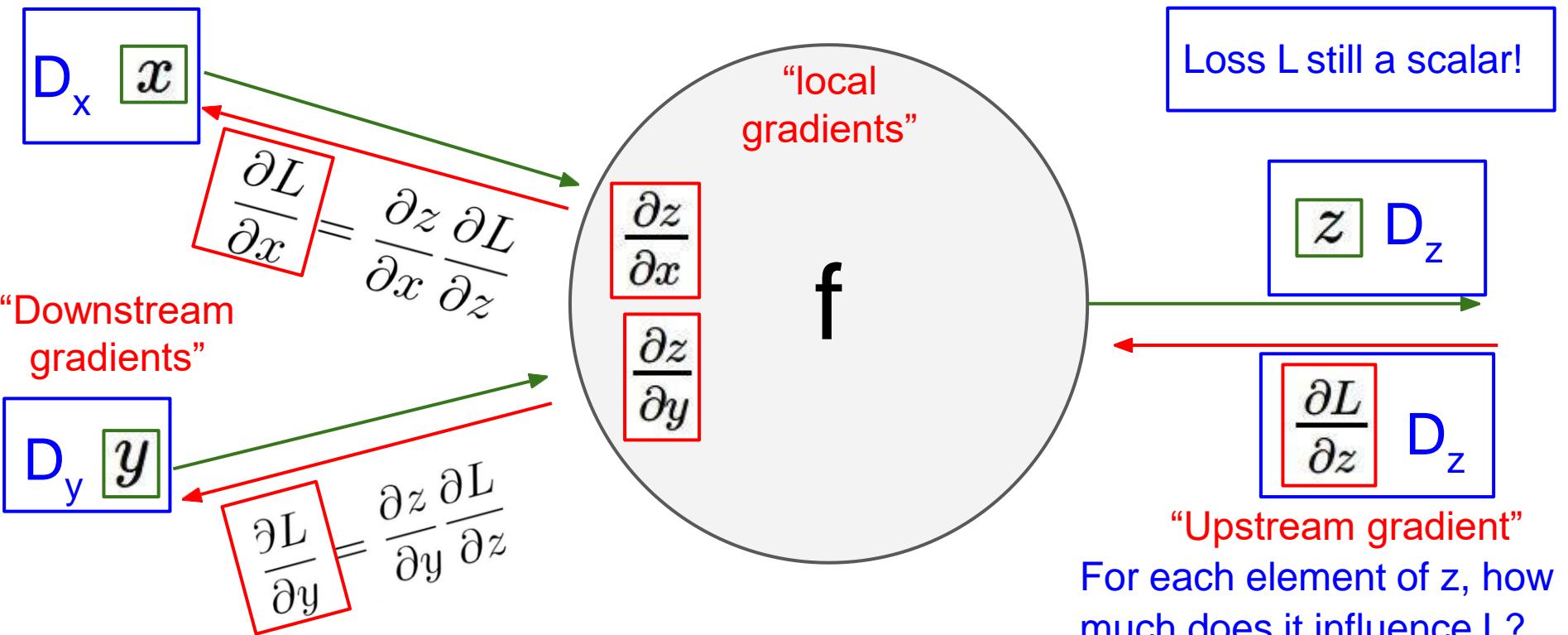
# Backprop with Vectors



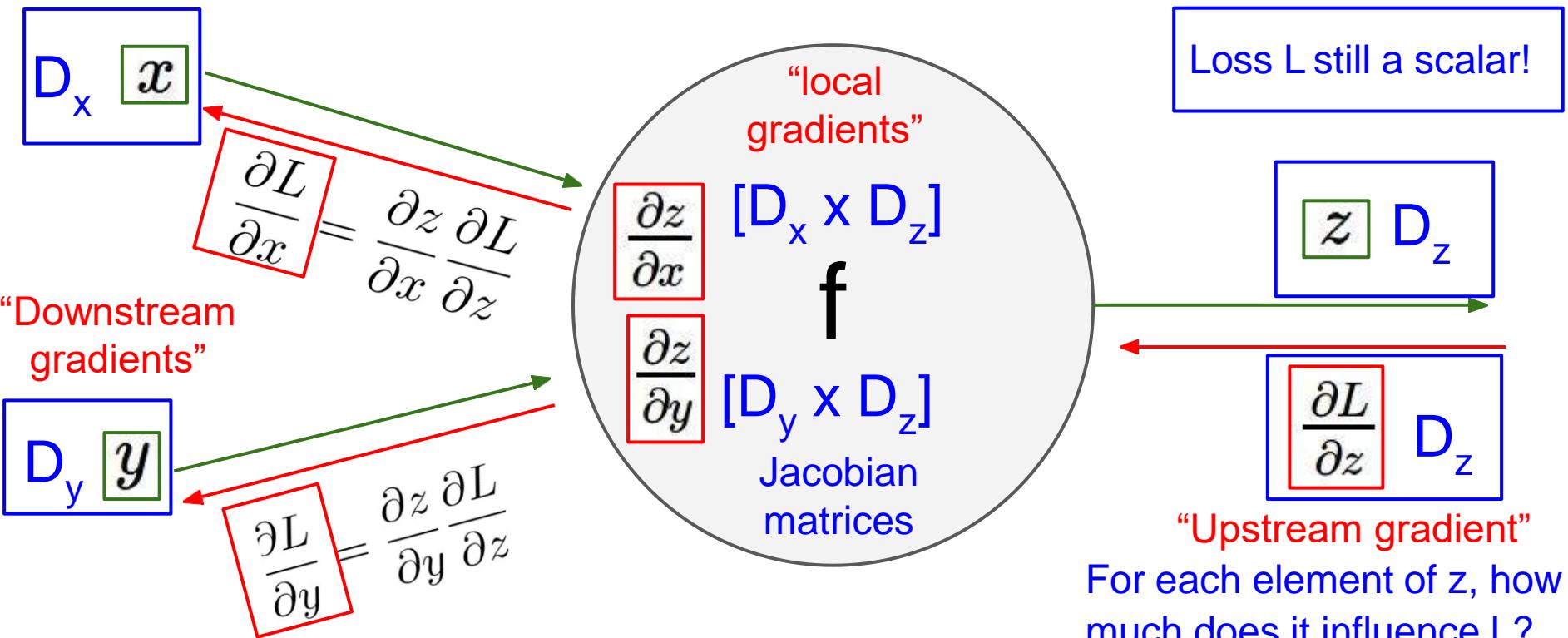
# Backprop with Vectors



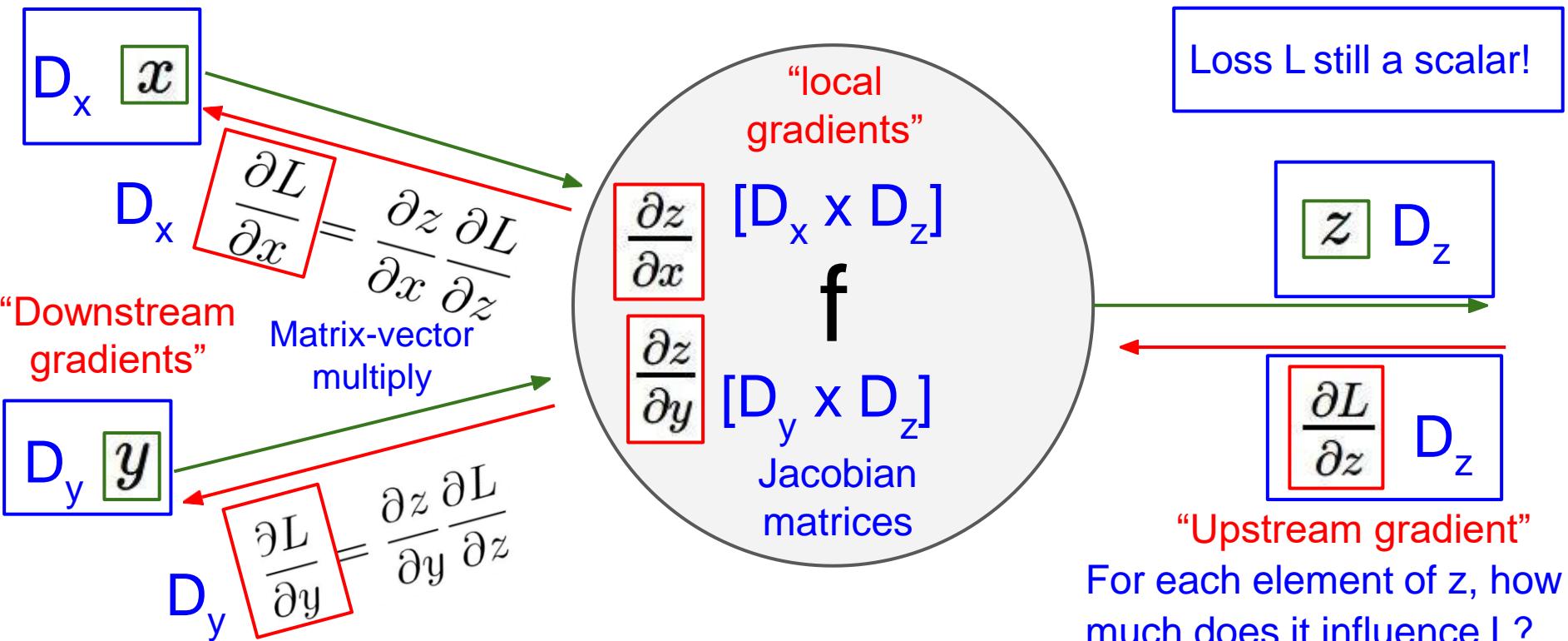
# Backprop with Vectors



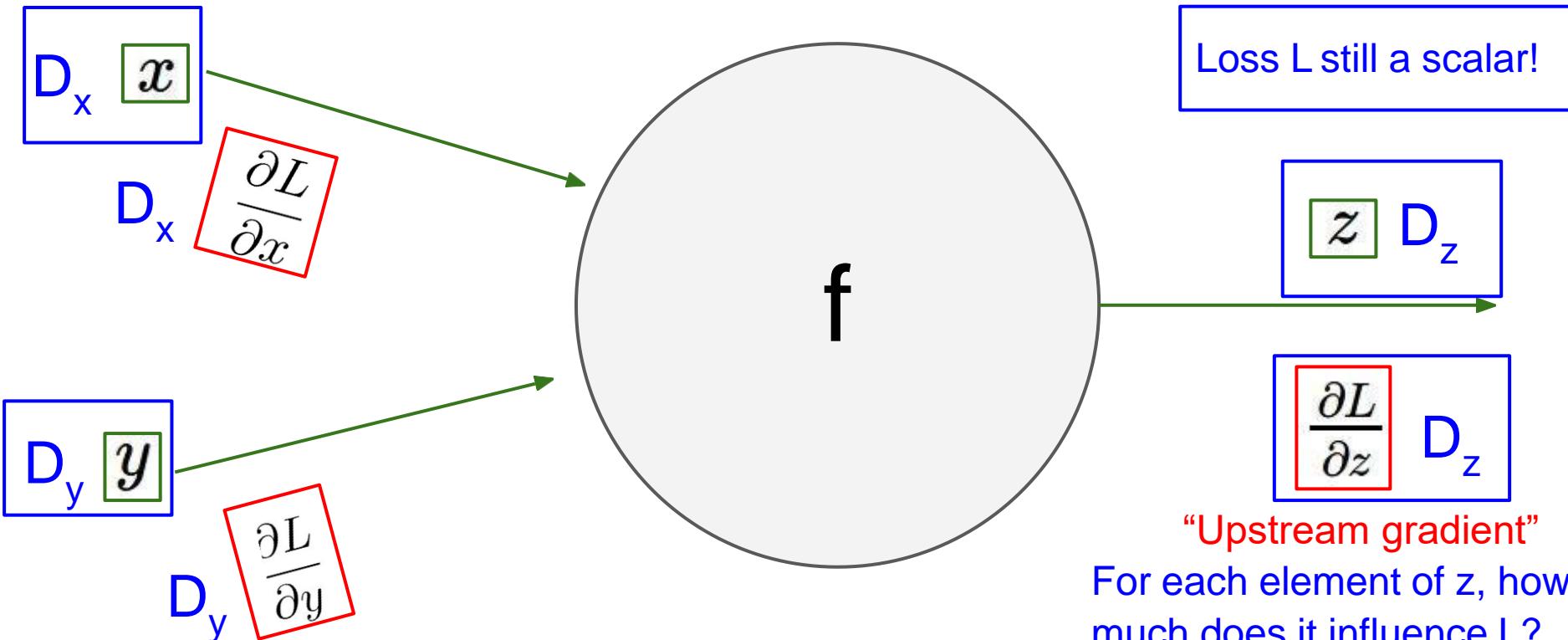
# Backprop with Vectors



# Backprop with Vectors



Gradients of variables wrt loss have same dims as the original variable



## Neural Network - Backpropagation

# Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \xrightarrow{\hspace{1cm}} \begin{array}{l} \text{[ 1 ]} \\ \text{[ -2 ]} \\ \text{[ 3 ]} \\ \text{[ -1 ]} \end{array}$$

$$f(x) = \max(0, x)$$

*(elementwise)*

4D output z:

$$\xrightarrow{\hspace{1cm}} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

Neural Network - Backpropagation

Lecture 4 - 124

# Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \xrightarrow{\hspace{1cm}}$$

$$f(x) = \max(0, x)$$

(elementwise)

4D output z:

$$\begin{array}{l} \xrightarrow{\hspace{1cm}} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} \\ \xrightarrow{\hspace{1cm}} \\ \xrightarrow{\hspace{1cm}} \\ \xrightarrow{\hspace{1cm}} \end{array}$$

4D  $dL/dz$ :

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix} \leftarrow$$

Upstream  
gradient

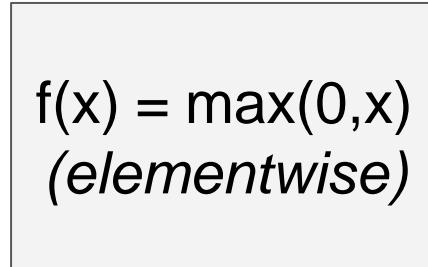
Neural Network - Backpropagation

Lecture 4 - 125

# Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}$$



4D output z:

$$\begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

Jacobian  $dz/dx$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4D  $dL/dz$ :

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

Upstream  
gradient

Neural Network - Backpropagation

Lecture 4 - 126

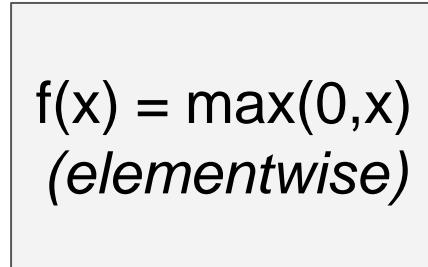


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# Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \longrightarrow$$



4D output z:

$$\longrightarrow \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

[ $\frac{dz}{dx}$ ] [ $\frac{dL}{dz}$ ]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} [4]$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} [-1]$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} [5]$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} [9]$$

4D  $\frac{dL}{dz}$ :

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix} \longleftarrow$$

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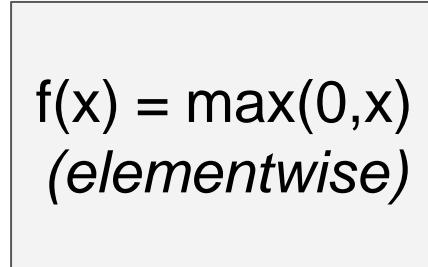
$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix} \longleftarrow$$

Upstream  
gradient

# Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}$$



4D output z:

$$\begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

4D  $dL/dx$ :

$$\begin{bmatrix} 4 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$

$[dz/dx]$   $[dL/dz]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

4D  $dL/dz$ :

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

Upstream  
gradient

Neural Network - Backpropagation

Lecture 4 - 130



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# Backprop with Vectors

4D input  $x$ :

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}$$

Jacobian is **sparse**:  
off-diagonal entries  
always zero! Never  
**explicitly** form  
Jacobian -- instead  
use **implicit**  
multiplication

4D output  $z$ :

$$\begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

$$f(x) = \max(0, x)$$

(elementwise)

4D  $dL/dx$ :

$$\begin{bmatrix} 4 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$

$[dz/dx]$   $[dL/dz]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

4D  $dL/dz$ :

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

Upstream  
gradient

Neural Network - Backpropagation

Lecture 4 - 129



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# Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \xrightarrow{\quad} \begin{array}{c} f(x) = \max(0, x) \\ (\text{elementwise}) \end{array}$$

Jacobian is **sparse**:  
off-diagonal entries  
always zero! Never  
**explicitly** form  
Jacobian -- instead  
use **implicit**  
multiplication

4D output z:

$$\begin{array}{c} \xrightarrow{\quad} [1] \\ \xrightarrow{\quad} [0] \\ \xrightarrow{\quad} [3] \\ \xrightarrow{\quad} [0] \end{array}$$

4D  $dL/dx$ :

$$\begin{bmatrix} 4 \\ 0 \\ 5 \\ 0 \end{bmatrix} \leftarrow$$

$$\left( \frac{\partial L}{\partial x} \right)_i = \begin{cases} \left( \frac{\partial L}{\partial z} \right)_i & \text{if } x_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

$[dz/dx]$   $[dL/dz]$

4D  $dL/dz$ :

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix} \leftarrow$$

Upstream  
gradient

Neural Network - Backpropagation

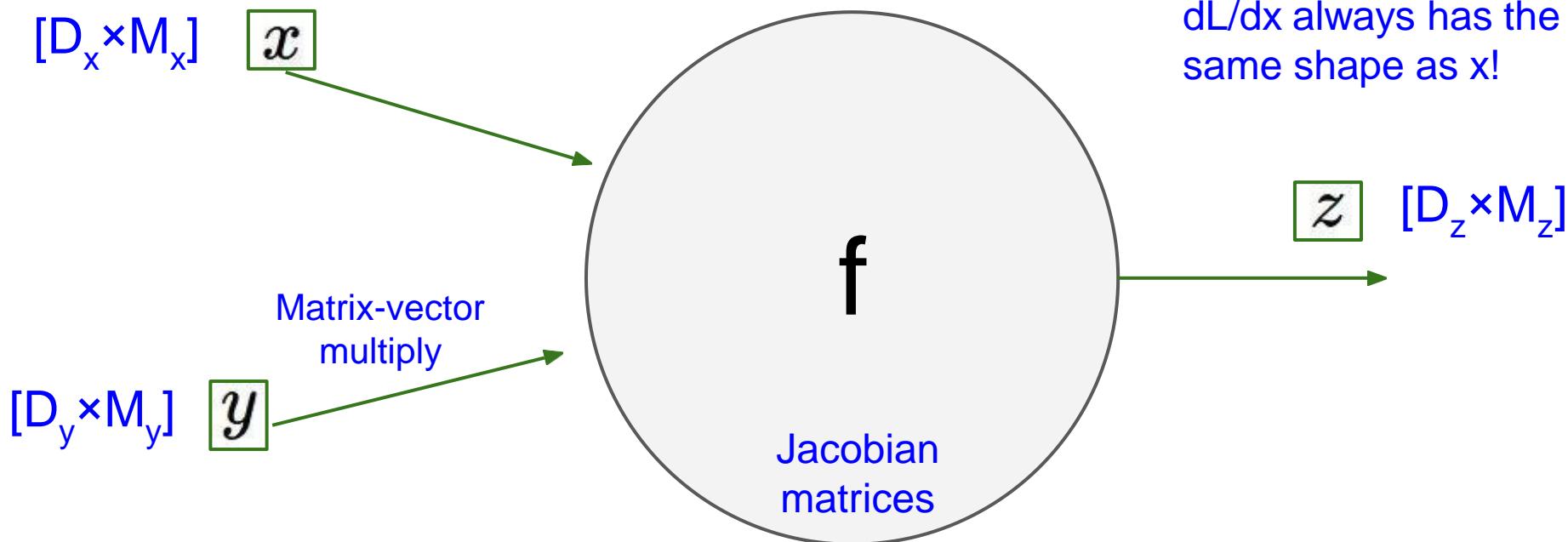
Lecture 4 - 130



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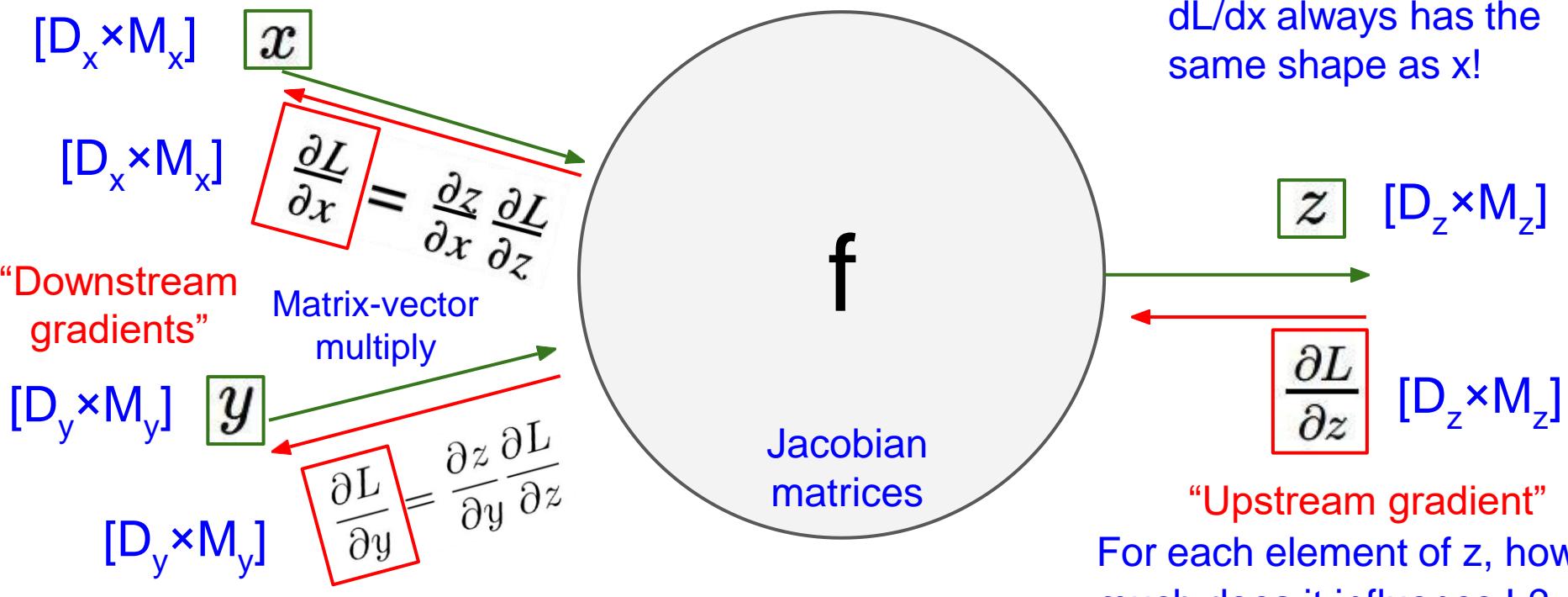
# Backprop with Matrices (or Tensors)

Loss L still a scalar!



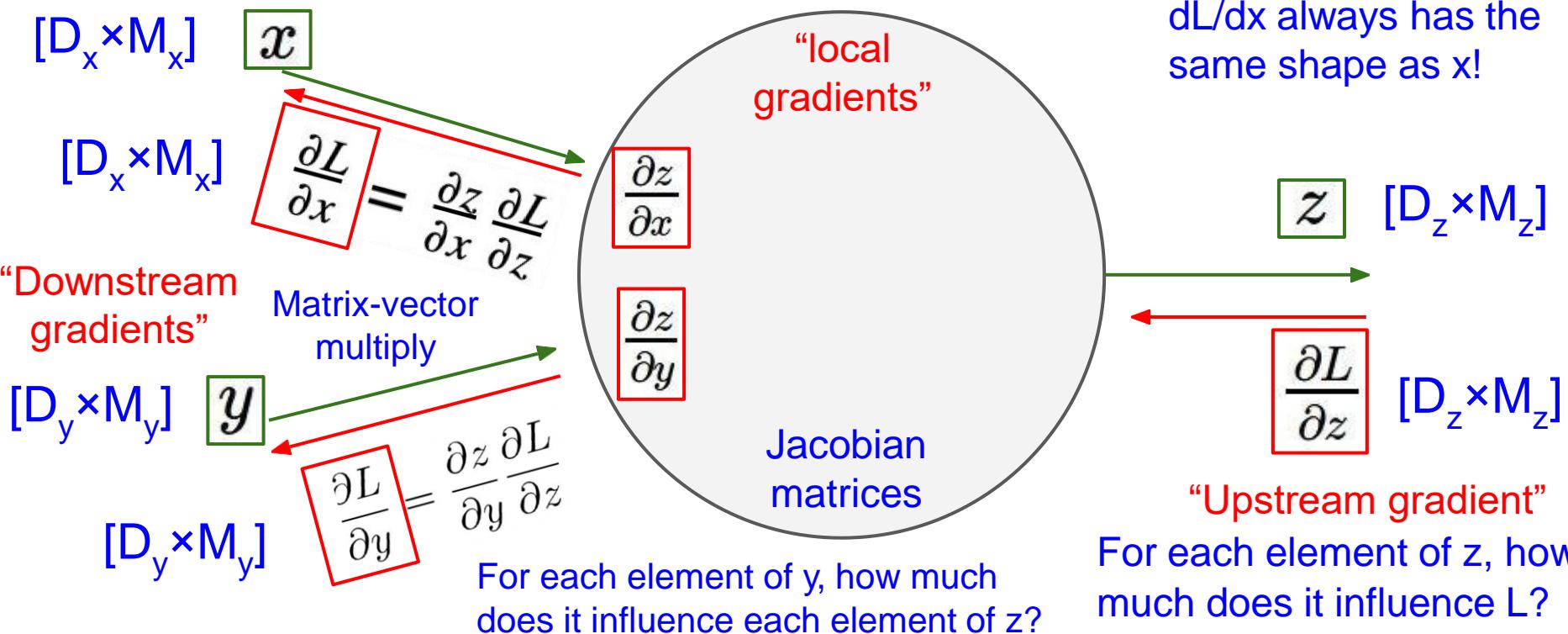
# Backprop with Matrices (or Tensors)

Loss L still a scalar!



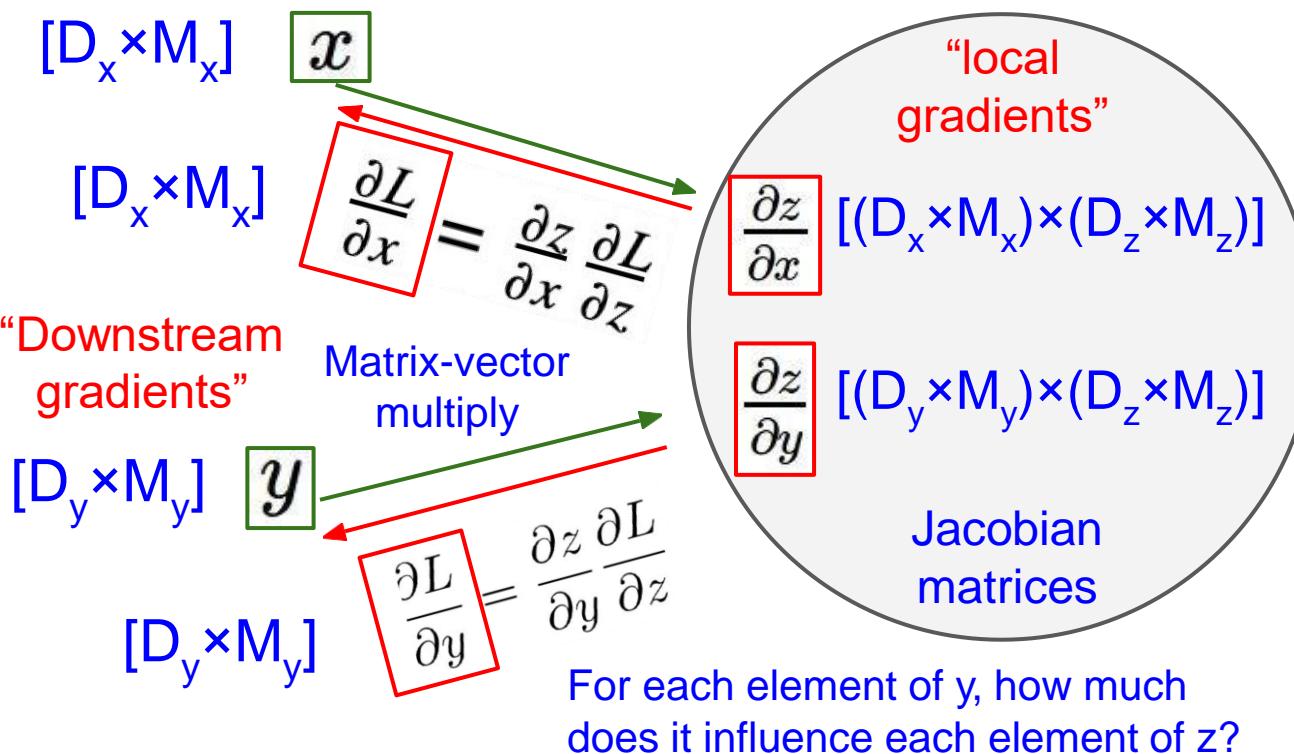
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Loss L still a scalar!

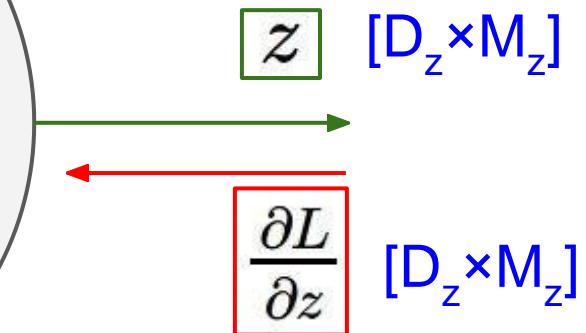


# Backprop with Matrices (or Tensors)

Loss L still a scalar!



$dL/dx$  always has the same shape as  $x$ !



"Upstream gradient"

For each element of  $z$ , how much does it influence  $L$ ?

# Backprop with Matrices

x: [N×D]

[ 2 1 -3 ]

[ -3 4 2 ]

w: [D×M]

[ 3 2 1 -1 ]

[ 2 1 3 2 ]

[ 3 2 1 -2 ]

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]

[ 13 9 -2 -6 ]

[ 5 2 17 1 ]

dL/dy: [N×M]

[ 2 3 -3 9 ]

[ -8 1 4 6 ]

Also see derivation in the course notes:

<http://cs231n.stanford.edu/handouts/linear-backprop.pdf>

# Backprop with Matrices

x: [N×D]

$$\begin{bmatrix} 2 & 1 & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

Jacobians:

$$\begin{aligned} dy/dx &: [(N \times D) \times (N \times M)] \\ dy/dw &: [(D \times M) \times (N \times M)] \end{aligned}$$

y: [N×M]

$$\begin{bmatrix} 13 & 9 & -2 & -6 \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

For a neural net we may have

$$N=64, D=M=4096$$

Each Jacobian takes ~256 GB of  
memory! Must work with them implicitly!

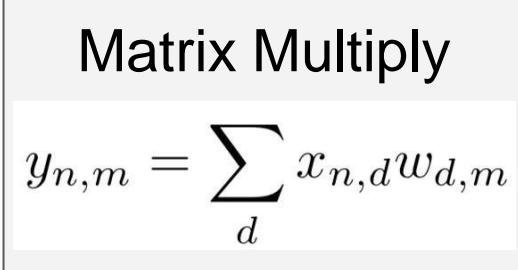
# Backprop with Matrices

x: [N×D]

$$\begin{bmatrix} 2 & \boxed{1} & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$



Q: What parts of y  
are affected by one  
element of x?

y: [N×M]

$$\begin{bmatrix} 13 & 9 & -2 & -6 \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

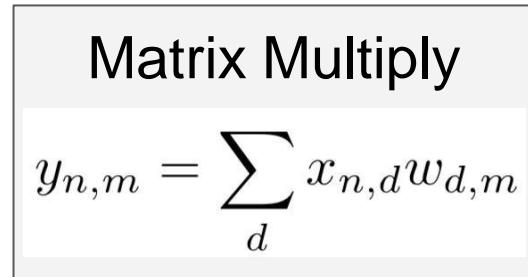
# Backprop with Matrices

x: [N×D]

$$\begin{bmatrix} 2 & \boxed{1} & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$



**Q:** What parts of y  
are affected by one  
element of x?

**A:**  $x_{n,d}$  affects the  
whole row  $y_{n,\cdot}$ .

$$\frac{\partial L}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} \frac{\partial y_{n,m}}{\partial x_{n,d}}$$

y: [N×M]

$$\begin{bmatrix} 13 & 9 & -2 & -6 \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

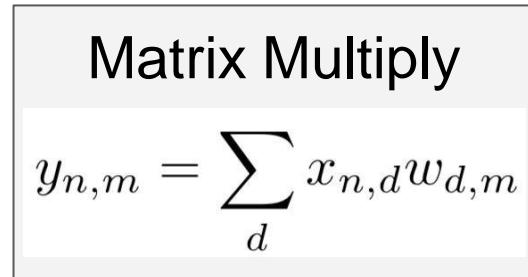
# Backprop with Matrices

$x: [N \times D]$

$$\begin{bmatrix} 2 & 1 & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

$w: [D \times M]$

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$



**Q:** What parts of  $y$  are affected by one element of  $x$ ?

**A:**  $x_{n,d}$  affects the whole row  $y_{n,\cdot}$ .

$$\frac{\partial L}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} \frac{\partial y_{n,m}}{\partial x_{n,d}}$$

$y: [N \times M]$

$$\begin{bmatrix} 13 & 9 & \boxed{-2} & -6 \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

$dL/dy: [N \times M]$

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

**Q:** How much does  $x_{n,d}$  affect  $y_{n,m}$ ?

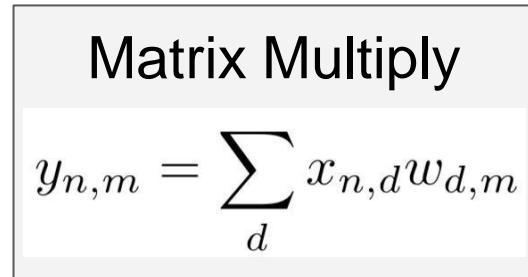
# Backprop with Matrices

x: [N×D]

[ 2 1 -3 ]  
[-3 4 2]

w: [D×M]

[ 3 2 1 -1 ]  
[ 2 1 3 2 ]  
[ 3 2 1 -2 ]



y: [N×M]  
[ 13 9 -2 -6 ]  
[ 5 2 17 1 ]

dL/dy: [N×M]  
[ 2 3 -3 9 ]  
[ -8 1 4 6 ]

Q: What parts of y  
are affected by one  
element of x?  
A:  $x_{n,d}$  affects the  
whole row  $y_{n,:}$ .

Q: How much  
does  $x_{n,d}$   
affect  $y_{n,m}$ ?  
A:  $w_{d,m}$

$$\frac{\partial L}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} \frac{\partial y_{n,m}}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} w_{d,m}$$

# Backprop with Matrices

x: [N×D]

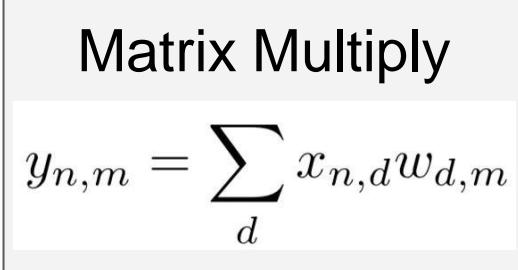
$$\begin{bmatrix} 2 & \boxed{1} & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & \boxed{3} & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$

[N×D] [N×M] [M×D]

$$\frac{\partial L}{\partial x} = \left( \frac{\partial L}{\partial y} \right) w^T$$



**Q:** What parts of y are affected by one element of x?  
**A:**  $x_{n,d}$  affects the whole row  $y_{n,:}$ .

**Q:** How much does  $x_{n,d}$  affect  $y_{n,m}$ ?  
**A:**  $w_{d,m}$

y: [N×M]

$$\begin{bmatrix} 13 & 9 & \boxed{-2} & -6 \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

# Backprop with Matrices

$x: [N \times D]$

[ 2 1 -3 ]  
[-3 4 2]

$w: [D \times M]$

[ 3 2 1 -1 ]  
[ 2 1 3 2 ]  
[ 3 2 1 -2 ]

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

$y: [N \times M]$

[ 13 9 -2 -6 ]  
[ 5 2 17 1 ]

$dL/dy: [N \times M]$

[ 2 3 -3 9 ]  
[-8 1 4 6 ]

By similar logic:

$[N \times D] [N \times M] [M \times D]$

$[D \times M] [D \times N] [N \times M]$

$$\frac{\partial L}{\partial x} = \left( \frac{\partial L}{\partial y} \right) w^T$$

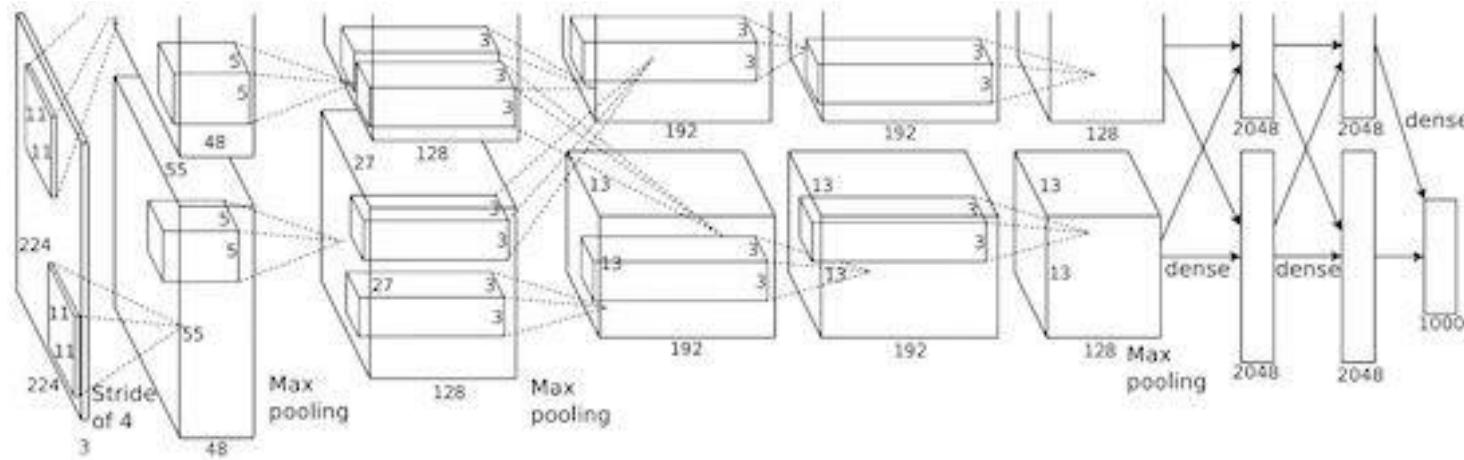
$$\frac{\partial L}{\partial w} = x^T \left( \frac{\partial L}{\partial y} \right)$$

These formulas are easy to remember: they are the only way to make shapes match up!

# Summary for today:

- **(Fully-connected) Neural Networks** are stacks of linear functions and nonlinear activation functions; they have much more representational power than linear classifiers
- **backpropagation** = recursive application of the chain rule along a computational graph to compute the gradients of all inputs/parameters/intermediates
- implementations maintain a graph structure, where the nodes implement the **forward()** / **backward()** API
- **forward**: compute result of an operation and save any intermediates needed for gradient computation in memory
- **backward**: apply the chain rule to compute the gradient of the loss function with respect to the inputs

# Next Time: Convolutional Neural Networks!



Neural Network - Backpropagation

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