Subtracting Binary Numbers

0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7	8	-7	-6	-5	-4	-3	-2	-1

Visual Representation of Signed Binary Numbers and Why 1's Complement Works

0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7	-7	-6	-5	-4	-3	-2	-1	-0

One's Compliment

Take the positive numbers and flip(reverse the bits)

Ex. 0101 = +5

1010 = -5

Subtracting 7 – 1 is the same as 7+ (-1)?

	0	1	1	1
-	0	0	0	1
	0	1	1	0

1	1	1			Carry ones
	0	1	1	1	
+	1	1	1	0	
1	0	1	0	1	Overflow bit added
				1	Added the overflow bit
	0	1	1	0	

Two's Compliment

Take the positive number flip (reverse) the bits and add 1

Ex. 1010 = +5

1010+1 = 1011 or -5

Subtracting 7 – 1 is the same as 7+ (-1)?

	0	1	1	1
-	0	0	0	1
	0	1	1	0

	1	1	1		Carry ones
	0	1	1	1	
+	1	1	1	0	Flipped bits
				1	+ one
1	0	1	1	0	Ignore the overflow bit

To do subtraction in the decimal system we normally use the borrow method. Consider the example problem to the right. Here we must borrow a 10 from the tens column in order to complete the subtraction in the ones column. We move 10 to the ones column and subtract 6. Then we copy down the remaining 20 from the tens column to get our answer of 24.

$$\begin{array}{r} 2(10) \\ \cancel{3}\ \cancel{0} \\ -\quad 6 \\ \hline 2\ 4 \end{array}$$

We can also use the borrow method to do binary subtraction. The basic rules for binary subtraction are listed in the table below.

Rule 1	Rule 2	Rule 3	Rule 4
$\begin{array}{r} 0 \\ -\ 0 \\ \hline 0 \end{array}$	$\begin{array}{r} 1 \\ -\ 1 \\ \hline 0 \end{array}$	$\begin{array}{r} 1 \\ -\ 0 \\ \hline 1 \end{array}$	$\begin{array}{r} 0 \\ -\ 1 \\ \hline 1 \end{array}$

Again we see that the first three rules are similar to their decimal counterparts. The fourth rule, however, needs a little more explanation since it defines how we borrow from another column. Let's look at a simple example to see where this rule comes from. Consider the problem of subtracting 1_2 from 10_2.

1. To compute the first column, we need to borrow a 1 from the next column. Recall that two 1s generated a carry in addition. If we reverse this process, we can borrow a 1 from the second column and mark two 1s in the first column.

$$\begin{array}{r} 10 \\ -\ 1 \\ \hline \end{array}$$

2. Once we borrow from the second column, we cross out the 1 and write 0 above it to show this column is now empty. The 1 from the second column is now represented by the two blue 1s in the first column.

$$\begin{array}{r} 1 \\ 01 \\ \cancel{1}0 \\ -\ 1 \\ \hline \end{array}$$

3. To solve our subtraction problem, we take 1 away from our group of two blue 1s. This leaves us with a single 1 which we write below the first column.

$$\begin{array}{r} 1 \\ 0\cancel{1} \\ \cancel{1}0 \\ -\ \cancel{1} \\ \hline 1 \end{array}$$

4. After cleaning up our work, we can see that the first column of our answer is identical to Rule 4. Since we must borrow a 1 from the next column, 0 - 1 = 1.

$$\begin{array}{r} 10 \\ -\ 1 \\ \hline 1 \end{array}$$