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Seminar Report

Linear and Nonlinear Filters

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Abstract

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As its name suggests, the function which is used to pass the image through must be linear and shift invariant. A common formula for linear filtering is the *Correlation Filtering*.

$$g(i,j) = \sum_{l \in \mathcal{M}} \sum_{k \in \mathcal{N}} f(i+k, j+l) \cdot h(k,l)$$

or commonly notated as $g = f \otimes h$.

The desired output pixel $g(i,j)$, where i and j specify the coordinates of it, is based on a $M \times N$ sized neighborhood, meaning not only does one pixel define an output pixel, but also a specified number of its neighbors. The influence of each pixel in the neighborhood is defined by the filter coefficient $h(k,l)$, also called its *kernel* or *mask*.

$$\underbrace{\begin{bmatrix} 128 & 34 & 123 \\ 68 & 54 & 73 \\ 100 & 95 & 17 \end{bmatrix}}_{\text{input neighborhood}} \otimes \underbrace{\begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}}_{\text{kernel}} = 75$$

As the above example with a 3×3 kernel, a total of 9 pixels is needed to calculate a single output pixel.

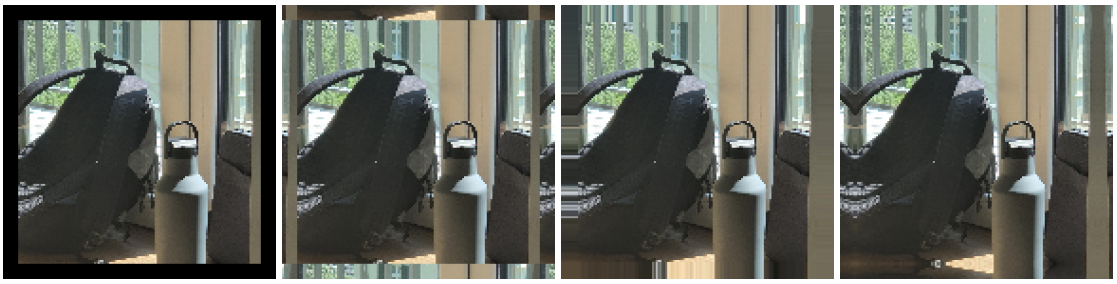
Another common variant on the formula is having the signs of the offsets reversed.

$$g = f * h$$

$$g(i,j) = \sum_{l \in \mathcal{M}} \sum_{k \in \mathcal{N}} f(i-k, j-l) \cdot h(k,l)$$

With this formula, $*$ is called the *convolutional* operator, and the kernel h is called the *impulse response function*. An interesting note is that, when the kernel h is convolved with an impulse signal δ (an image with 0 everywhere except the origin), it reproduces the kernel itself $h * \delta = h$, whereas with correlational filtering, it produces the reflected signal (inverted signal in both dimensions).

An apparent problem from neighborhood filtering is that on the edges, the neighbors simply does not exist in one or two directions (e.g. a 1000×1000 image passed through a 3×3 kernel would produce a 998×998 image). There are a couple method to alleviate the calculation of the nonexistent neighbors.



(a) zero (b) wrap (c) clamp (d) mirror

3 example page from template

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4 Section Title

I am a section. LaTeX will give me a number *automatically* and put me into the table of contents. Using `\label` and `\ref` you can write that this is Section 4. Another section is Section 5.
You can use the commands `\eg`, `\ie`, `\etal` to get *e.g.*, *i.e.*, *et al.* And "this is a quote."

4.1 Subsection Title With Capitalized Words

We can make bulleted lists as follows.

- I am an item,
- I am another item.

4.1.1 Subsubsection with only the first word capitalized

I am a subsubsection, an even smaller subsection. Let's see a table.

Method	Accuracy (%)
Boring old method	86.6
Shiny new method	86.7

Table 1: This is the caption for the table.

Tab. 1 is an example table. The table also got a number automatically and will be placed where LaTeX thinks it looks good. You can specify a preference with h(ere), t(op), b(ottom), p(age).

5 Another Section

Figure 1: Insert caption here. Image from [DDS+09].

6 Equations

LaTeX is also really good at printing equations. You can do it inline, such as $E = mc^2$, or centered, like

$$\mathcal{L}_{\mathcal{T}}(\vec{\lambda}) = \sum_{(\mathbf{x},\mathbf{s}) \in \mathcal{T}} \log P(\mathbf{s} \mid \mathbf{x}) - \sum_{i=1}^{m} \frac{\lambda_i^2}{2\sigma^2}. \tag{1}$$

Equations are numbered as well, *e.g.*, above we have Eq. 1.[KB14]

References

[DDS+09] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. ImageNet: A large-scale hierarchical image database. In *IEEE Conf on Computer Vision and Pattern Recognition*, 2009.

[KB14] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv:1412.6980*, 2014.