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Satellite Ephemeris and Coordinates

Exercise 1 – Orbital Parameters

Consider a satellite with the following orbital parameters

- orbit semi-major axis: A = 26559755m;
- orbit eccentricity: e = 0.017545;
- argument of perigee: ω = 1.626021rad.

Assuming 39929 seconds have passed since the satellite was at the perigee, calculate

a) the satellite orbital period, T;
b) the satellite mean angular motion, η;
c) the mean anomaly, M;
d) the eccentric anomaly, E;
e) the radius and true anomaly, (r_o,ϕ_o);
f) the argument of latitude, ϕ.

Exercise 2 – GPS Satellite Ephemeris

Consider a GPS receiver at the following WGS 84 (x,y,z) cartesian coordinates:

$$r_1 = (4918525.18 \text{ m}, -791212.21 \text{ m}, 3969762.19 \text{ m})$$

Consider also that, the ephemerides collected by this receiver are stored in file `ub1.ubx.2056.540000a.eph`, in ASCII format, one line per satellite, with the following tab separated fields:

```
(1) - SV# (1...32)

(2) - IODE sf2 - Issue Of Data Ephemeris (8 bits)
                 [0...255]
                 (consistent with the
                 8 LSbs of the IODC) (Subframe #2)

(3) - IODE sf3 - Issue Of Data Ephemeris (8 bits)
                 [0...255]
                 (consistent with the
                 8 LSbs of the IODC) (Subframe #3)

(4) - WN (Week Number: weeks; 10 bits)

(5..7) - toe - Ephemeris Reference Time
   (5) - 16 MS bits; range: [0...604784];
         hexadecimal notation
   (6) - 16 MS bits; range: [0...604784];
         decimal notation
   (7) - 20 bits; the 4 LS bits are zero; units s

(8) - Fit Interval flag (1 bit; 0 - 4 hours; 1 - 6 hours)

(9..10) - SV Health
    (9) - 6 bits; hexadecimal notation
   (10) - 6 bits; decimal notation
          bit 5: summary of the health of the nav data
                 0 - All navigation data is good
```

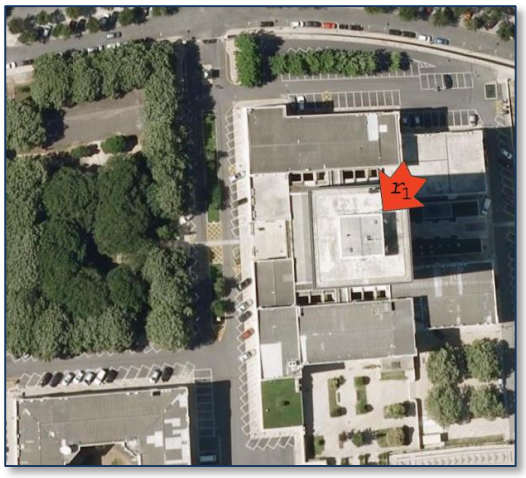

Fig.2: IST North Tower

```
              1 - Some or all navigation data is bad
         bit 0-4: health of satellite signal components
                  0 - All signals OK.
                  1 - All signals week (3 to 6 dB below
                      specified power level due to reduced
                      power output, excess phase noise,
                      satellite attitude, etc).
                  2 - All signals dead.
                  3 - All signals have no data modulation.
                  4 - L1 P signal week.
                  5 - L1 P signal dead.
                  6 - L1 P signal has no data modulation.
                  7 - L2 P signal week.
                  8 - L2 P signal dead.
                  9 - L2 P signal has no data modulation.
                 10 - L1 C/A signal week.
                 11 - L1 C/A signal dead.
                 12 - L1 C/A signal has no data modulation.
                 13 - L2 C/A signal week.
                 14 - L2 C/A signal dead.
                 15 - L2 C/A signal has no data modulation.
                 16 - L1 & L2 P signal week.
                 17 - L1 & L2 P signal dead.
                 18 - L1 & L2 P signal has no data modulation.
                 19 - L1 & L2 C/A signal week.
                 20 - L1 & L2 C/A signal dead.
                 21 - L1 & L2 C/A signal has no data modulation.
                 22 - L1 signal week (3 to 6 dB below
                      specified power level due to reduced
                      power output, excess phase noise,
                      satellite attitude, etc).
                 23 - L1 signal dead.
                 24 - L1 signal has no data modulation.
                 25 - L2 signal week (3 to 6 dB below
                      specified power level due to reduced
                      power output, excess phase noise,
                      satellite attitude, etc).
                 26 - L2 signal dead.
                 27 - L2 signal has no data modulation.
                 28 - Satellite is temporarily out - do not
                      use this satellite during current pass.
                 29 - Satellite will be temporarily out - use
                      with caution.
                 30 - Spare.
                 31 - More than one combination of codes is
                      required to describe anomalies.

(11) - URA index - User Range Accuracy (4 bits)
                  0        0.00 < URA <=    2.40
                  1        2.40 < URA <=    3.40
                  2        3.40 < URA <=    4.85
                  3        4.85 < URA <=    6.85
                  4        6.85 < URA <=    9.65
                  5        9.65 < URA <=   13.65
                  6       13.65 < URA <=   24.00
                  7       24.00 < URA <=   48.00
                  8       48.00 < URA <=   96.00
                  9       96.00 < URA <=  192.00
                 10     192.00 < URA <=  384.00
                 11     384.00 < URA <=  768.00
                 12     768.00 < URA <= 1536.00
                 13    1536.00 < URA <= 3072.00
                 14    3072.00 < URA <= 6144.00
                 15    6144.00 < URA (or no accuracy prediction is available
                                      - unauthorized users are advised to use
                                      the SV at their own risk.)

(12) - "Alert" flag (1 bit)
       1 - URA may be worse than indicated in Subframe 1
       0 - otherwise
```

```
(13) - Anti-Spoof flag (1 bit)
       0 - A-S mode is OFF
       1 - A-S mode is ON

(14) - Code on L2 Channel (2 bits): 00 - reserved
                                    01 - P code ON
                                    10 - C/A code ON

(15) - L2 P Data flag (Data Flag for L2 P-Code; 1 bit)
       1 - the NAV data stream was commanded OFF on the P code of the L2 channel.
       0 - otherwise.

(16..18) - TGD - Estimated Group Delay Differential
    (16) - 8 bits; signed; hexadecimal notation; scale factor: 2**-31; units: s
    (17) - 8 bits; signed; decimal notation; scale factor: 2**-31; units: s
    (18) - No scale factor; units: s

(19) - IODC - Issue Of Data Clock (10 bits) [0...1023]

Satellite Clock Correction Parameters (toc,af2,af1,af0)
(20..22) - toc
    (20) - 16 MS bits; range: [0...604784]; hexadecimal notation
    (21) - 16 MS bits; range: [0...604784]; decimal notation
    (22) - 20 bits; the 4 LS bits are zero; units s

(23..25) - af2
    (23) - 8 bits; signed; hexadecimal notation; scale factor: 2**-55; units: s/s2
    (24) - 8 bits; signed; decimal notation; scale factor: 2**-55; units: s/s2
    (25) - no scale factor; units: s/s2

(26..28) - af1
    (26) - 16 bits; signed; hexadecimal notation; scale factor: 2**-43; units: s/s
    (27) - 16 bits; signed; decimal notation; scale factor: 2**-43; units: s/s
    (28) - no scale factor; units: s/s

(29..31) - af0
    (29) - 22 bits; signed; hexadecimal notation; scale factor: 2**-31; units: s/s2
    (30) - 22 bits; signed; decimal notation; scale factor: 2**-31; units: s/s2
    (31) - no scale factor; units: s/s2

(32..34) - sqrt(A) - Square Root of the Semi-Major Axis
    (32) - 32 bits; hexadecimal notation; scale factor: 2**-19; units: m1/2
    (33) - 32 bits; decimal notation; scale factor: 2**-19; units: m1/2
    (34) - no scale factor; units: m1/2

(35..37) - delta n - Mean Motion Difference From Computed Value
    (35) - 16 bits; signed; hexadecimal notation; scale factor: 2**-43; units: semicircles/s
    (36) - 16 bits; signed; decimal notation; scale factor: 2**-43; units: semicircles/s
    (37) - no scale factor; units: radians/s

(38..40) - Mo - Mean Anomaly at Reference Time
    (38) - 32 bits; signed; hexadecimal notation; scale factor: 2**-31; units: semicircles
    (39) - 32 bits; signed; decimal notation; scale factor: 2**-31; units: semicircles
    (40) - no scale factor; units: radians

(41..43) - e - Eccentricity
    (41) - 32 bits; hexadecimal notation; scale factor: 2**-33; range: [0,0.03]
    (42) - 32 bits; decimal notation; scale factor: 2**-33; range: [0,0.03]
    (43) - no scale factor;

(44..46) - Argument of Perigee
    (44) - 32 bits; signed; hexadecimal notation; scale factor: 2**-31; units: semicircles
    (45) - 32 bits; signed; decimal notation; scale factor: 2**-31; units: semicircles
    (46) - no scale factor; units: radians

(47..49) - io - Inclination Angle at Reference Time
    (47) - 32 bits; signed; hexadecimal notation; scale factor: 2**-31; units: semicircles
    (48) - 32 bits; signed; decimal notation; scale factor: 2**-31; units: semicircles
    (49) - no scale factor; units: radians

(50..52) - IDOT - Rate of Inclination Angle
```

```
    (50) - 14 bits; signed; hexadecimal notation; scale factor: 2**-43; units: semicircles/s
    (51) - 14 bits; signed; decimal notation; scale factor: 2**-43; units: semicircles/s
    (52) - no scale factor; units: radians/s

(53..55) - Omega0 - Longitude of Ascending Node of Orbit Plane at Weekly Epoch
    (53) - 32 bits; signed; hexadecimal notation; scale factor: 2**-31; units: semicircles
    (54) - 32 bits; signed; decimal notation; scale factor: 2**-31; units: semicircles
    (55) - no scale factor; units: radians

(56..58) - OmegaDot - Rate of Right Ascension
    (56) - 24 bits; signed; hexadecimal notation; scale factor: 2**-43; units: semicircles/s
    (57) - 24 bits; signed; decimal notation; scale factor: 2**-43; units: semicircles/s
    (58) - no scale factor; units: radians/s

(59..61) - Cuc - Amplitude of the Cosine Harmonic Correction Term
                 to the Argument of Latitude
    (59) - 16 bits; signed; hexadecimal notation; scale factor: 2**-29; units: radians
    (60) - 16 bits; signed; decimal notation; scale factor: 2**-29; units: radians
    (61) - no scale factor; units: radians

(62..64) - Cus - Amplitude of the Sine Harmonic Correction Term
                 to the Argument of Latitude
    (62) - 16 bits; signed; hexadecimal notation; scale factor: 2**-29; units: radians
    (63) - 16 bits; signed; decimal notation; scale factor: 2**-29; units: radians
    (64) - no scale factor; units: radians

(65..67) - Crc - Amplitude of the Cosine Harmonic Correction Term
                 to the Orbit Radius
    (65) - 16 bits; signed; hexadecimal notation; scale factor: 2**-5; units: m
    (66) - 16 bits; signed; decimal notation; scale factor: 2**-5; units: m
    (67) - no scale factor; units: m

(68..70) - Crs - Amplitude of the Sine Harmonic Correction Term
                 to the Orbit Radius
    (68) - 16 bits; signed; hexadecimal notation; scale factor: 2**-5; units: m
    (69) - 16 bits; signed; decimal notation; scale factor: 2**-5; units: m
    (70) - no scale factor; units: m

(71..73) - Cic - Amplitude of the Cosine Harmonic Correction Term
                 to the Angle of Inclination
    (71) - 16 bits; signed; hexadecimal notation; scale factor: 2**-29; units: radians
    (72) - 16 bits; signed; decimal notation; scale factor: 2**-29; units: radians
    (73) - no scale factor; units: radians

(74..76) - Cis - Amplitude of the Sine Harmonic Correction Term
                 to the Angle of Inclination
    (74) - 16 bits; signed; hexadecimal notation; scale factor: 2**-29; units: radians
    (75) - 16 bits; signed; decimal notation; scale factor: 2**-29; units: radians
    (76) - no scale factor; units: radians

(77..79) - AODO - Age of Data Offset
                  Validity time for the NMCT data (not an ephemeris parameter)
    (77) - 5 bits; range: [0...31]; hexadecimal notation; scale factor: 9000; units: s
    (78) - 5 bits; range: [0...31]; decimal notation; scale factor: 9000; units: s
    (79) - 15bits; no scale factor; units: s
```

Use these ephemerides to calculate each satellite position, in WGS 84 cartesian coordinates, at Time Of Week (*TOW*) 536400 of Week Number (*WN*) 2056.

Exercise 3 – Satellite Positions

Calculate the position of the satellites at the time of transmission of the signal which (eventually) would be received at *TOW* 536400 of *WN* 2056, at the coordinates given by r_1. How many iterations were needed to get the difference between the iterated distances from the satellite to the receiver equal to (or bellow) 1mm?

Exercise 4 – Satellite Direction Cosines

Using the positions calculated in the previous exercise, compute the direction cosines of each satellite, as seen by the receiver at r_1. Take as reference both the WGS 84 reference system and a local (East, North, Up) reference system.

Exercise 5 – Satellite Azimuth and Elevation

Use the results of the previous exercise to compute each satellite azimuth and elevation, as seen by the receiver at r_1.

Exercise 6

Repeat exercises 3, 4 and 5 considering the receiver is now at coordinates r_2 and r_3.

r_2 = (4918532.10 m, -791212.61 m, 3969754.61 m)

r_3 = (38º 44´ 12.46´´ N, 9º 08´ 18.91´´ W, 102 m)

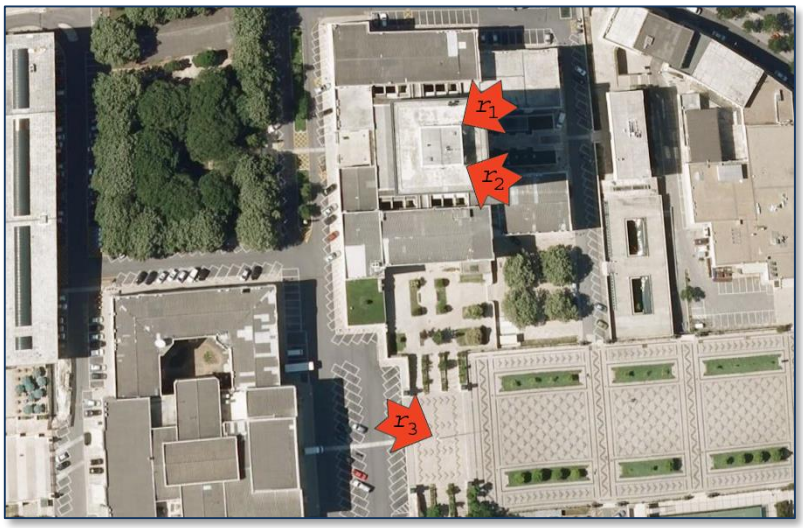

Fig. 3: IST campus - r_1 and r_2 on North Tower roof, r_3 at the flag pole.

Solutions

1.a) T = 43077.158 s = 717.953 min = 11.966 h
1.b) η = 0.0001459 rad/s = 0.008357°/s = 30.086°/h
1.c) M = 5.823999 rad = 333.6905°
1.d) E = 5.816098 rad = 333.2379°
7 iterations for δ = 1e-12 (3 with the Newton-Raphson method) and
4 iterations for δ = 1e-6 (2 with the Newton-Raphson method);
δ is the stopping criterion for the iterative method)

1.e) (r_o,ϕ_o) = (26143679.306 m, -0.475050 rad) =
= (26143679.306 m, -27.2184 °)

1.f) ϕ = 1.150971 rad = 65.9458°

2.

Table I: Satellite positions (*TOW* 536400, *WN* 2056)

SVN	*X*(m)	*Y*(m)	*Z*(m)
2	14300556.710	5726823.974	-21048150.876
5	24017954.368	-254576.219	-11683289.162
9	-337427.234	17906386.146	-19648806.739
10	-5844820.636	-14047605.201	21837695.426
12	23594489.427	-10613395.404	-5810709.924
13	20975774.712	9577789.636	13114921.956
15	19235496.076	-2940584.751	17976624.262
17	13432672.932	21227658.051	9167271.447
19	17813675.552	19604058.388	1008273.556
20	3923216.692	-17848331.095	19121558.707
21	877752.500	-26500879.475	3407085.705
24	14306205.386	-14437110.526	16769402.096
25	13797647.622	-16317824.617	-15822957.823
28	4007666.607	14488235.889	22502789.834
30	266018.223	25193124.683	8118498.929

(The eccentric anomaly was computed, with the Newton-Raphson method, with a stopping criterion of 1e-12)

Intermediate results for SVN2:

Δt = 3600.000 s
A = 26560890.777 m
η = 1.458544e-04 rad/s
M = 0.306625 rad
E = 0.312433 rad (3 iterations for δ = 1e-12, with Newton Raphson method)
ϕ_o = 0.318293 rad
u = -1.418392 rad
r_o = 26083342.541 m
r = 26083080.537 m
i = 0.955199 rad
Ω = -36.007382 rad

3.

Table II: Satellite positions at time of transmission (r_1, *TOW* 536400 *WN* 2056)

SVN	*X*(m)	*Y*(m)	*Z*(m)
2	14300630.183	5726472.320	-21048194.022
5	24018057.807	-254782.683	-11683071.427
9	-337090.265	17906540.787	-19648671.232
10	-5845119.184	-14047493.877	21837688.709
12	23594371.709	-10613530.138	-5810952.034
13	20975754.867	9577583.924	13115102.082
15	19235360.707	-2940779.595	17976732.980
17	13432871.678	21227630.200	9167034.840
19	17813813.714	19603950.293	1008012.193
20	3922943.466	-17848281.181	19121661.970
21	877555.509	-26500859.373	3407354.934
24	14306135.257	-14437336.382	16769266.427
25	13797370.145	-16317871.487	-15823156.273
28	4007961.357	14488158.411	22502786.394
30	266231.678	25193039.635	8118758.268

`#iterations = 3`

(The eccentric anomaly was computed, with the Newton-Raphson method, with a stopping criterion of 1e-12)

Intermediate results for `SVN2`:

Iteration 1 ($s = r_1$, $d_{sr} = \|s - r_1\| = 0$)

t = `536400 s` (assumed time of transmission, $t = TOW - d_{sr}/c$)
s = `( 14300556.710 m, 5726823.974 m, -21048150.876 m)`
$\Delta\Omega$ = `0rad` (correction for Earth rotation during signal travel time)
s = `( 14300556.710 m, 5726823.974 m, -21048150.876 m)`
Δd_{sr} = `27502786.867468 m`

Iteration 2

t = `536399.908260578 s`
s = `( 14300591.874 m, 5726567.988 m, -21048194.022 m)`
$\Delta\Omega$ = `0.000007rad`
s = `( 14300630.183 m, 5726472.320 m, -21048194.022 m)`
Δd_{sr} = `19.025949 m`

Iteration 3

t = `536399.908260641 s`
s = `( 14300591.874 m, 5726567.988 m, -21048194.022 m)`
$\Delta\Omega$ = `0.000007 rad`
s = `( 14300630.183 m, 5726472.320 m, -21048194.022 m)`
Δd_{sr} = `0.000013 m` (< 1 mm)

4.

r_1(WGS-84) =(38.737634° N, 9.138522° W, 195.3 m)
=(38° 44.2581′ N, 9° 8.3113′ W, 195.3 m)
= =(38° 44′ 15.48″ N, 9° 8′ 18.68″ W, 195.3 m)

Table III: Satellite direction cosines (r_1, *TOW* 536400, *WN* 2056)

SVN	WGS 84	ENU
2	[0.341 0.237 -0.910]	[0.288 -0.897 -0.336]
5	[0.773 0.022 -0.634]	[0.144 -0.970 0.196]
9	[-0.172 0.611 -0.772]	[0.576 -0.436 -0.691]
10	[-0.436 -0.536 0.723]	[-0.599 0.780 0.183]
12	[0.803 -0.422 -0.421]	[-0.289 -0.866 0.408]
13	[0.758 0.489 0.432]	[0.603 -0.083 0.793]
15	[0.711 -0.107 0.695]	[0.008 0.093 0.996]
17	[0.352 0.911 0.215]	[0.955 0.041 0.293]
19	[0.530 0.839 -0.122]	[0.913 -0.339 0.228]
20	[-0.044 -0.747 0.663]	[-0.744 0.470 0.474]
21	[-0.155 -0.988 -0.022]	[-1.000 -0.019 -0.011]
24	[0.448 -0.652 0.611]	[-0.572 0.135 0.809]
25	[0.333 -0.582 -0.742]	[-0.522 -0.842 -0.136]
28	[-0.038 0.636 0.771]	[0.622 0.688 0.375]
30	[-0.174 0.972 0.155]	[0.932 0.325 -0.157]

5.

Table IV: Satellite azimuth and elevation (r_1, *TOW* 536400, *WN* 2056)

SVN	*Azimuth*(º)	*Elevation*(º)
2	162.2°	-19.6°
5	171.5°	11.3°
9	127.1°	-43.7°
10	-37.5°	10.6°
12	-161.5°	24.1°
13	97.8°	52.5°
15	4.6°	84.7°
17	87.6°	17.0°
19	110.4°	13.2°
20	-57.7°	28.3°
21	-91.1°	-0.6°
24	-76.7°	54.0°
25	-148.2°	-7.8°
28	42.1°	22.0°
30	70.8°	-9.1°