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Reference Coordinate Systems - Exercises

Keywords

angular units, degrees, minutes, seconds, radians, cartesian coordinates, geodetic coordinates, latitude, longitude, altitude, WGS 84 reference system, datum transformations, local datum, Molodensky parameters, azimuth, elevation, orthodromes, loxodromes

Introduction

This section of exercises deals with reference coordinate systems and coordinate transformations usually found when working with data from satellite navigation systems. Use GNU Octave or Matlab® to solve each of the following exercises, and create your own toolbox with the code that you think will be re-used later.

Exercise 1 – Degrees, Minutes and Seconds

Consider the following WGS 84 geographic coordinates (latitude, longitude, altitude):

$$p_1 = (38^o\ 46´\ 49.61´´\ \text{N},\ 9^o\ 29´\ 56.19´´\ \text{W},\ 103\ \text{m})$$

The above format of the latitude and longitude will be referred to as *DDº MM´ SS.ss´´*. Rewrite those coordinates in the following formats: a) *DDº MM.mmm´*; b) *DD.dddº*.

Exercise 2 – WGS 84 Geodetic to Cartesian Coordinates

Convert p_1 to WGS 84 cartesian coordinates (x, y, z).

Exercise 3 – WGS 84 Cartesian to Geodetic Coordinates

Consider the following WGS 84 cartesian coordinates (x, y, z):

$$p_2 = (4910384.3\ \text{m},\ -821478.6\ \text{m},\ 3973549.6\ \text{m})$$

Convert p_2 to WGS 84 geographic coordinates (latitude, longitude, altitude). Write the result with the latitude and longitude in the following formats: a) *DD.dddº*; b) *DDº MM.mmm´*; c) *DDº MM´ SS.ss´´*.

Exercise 4 – Distances

Compute the distance between p_1 and p_2.

Exercise 5 – Datum Transformations

Consider the following Molodensky parameters for the conversion from the European 1950 datum to the WGS

Fig. 1: Cape Roca

84 datum: Δa = - 251 m, Δf = - $0.14192702 \times 10^{-4}$, Δx = -84 m, Δy = -107 m, Δz = -120 m. Using a Molodensky transformation, compute the p_1 and p_2 geodetic coordinates in the European 1950 Datum. For those conversions, compare the values of $\Delta\phi$, $\Delta\lambda$ and Δh. From those geodetic coordinates, compute the cartesian coordinates of p_1 (European 1950 datum) and compare them with the cartesian coordinates obtained from a translation of ($-\Delta x$, $-\Delta y$, $-\Delta z$) applied to the WGS 84 cartesian coordinates of p_1.

Exercise 6 – Azimuth and Elevation

Compute the azimuth and elevation angles from p_1 to p_2.

Exercise 7 – Orthodromes

Consider the following coordinates for New York and London:

New York	(40° 45´ N, 73° 58´ W)
London	(51° 32´ N, 0° 10´ W)

For the orthodrome from New York to London, compute the distance, the departure heading and the approaching heading. Assume a spherical model for the Earth, with a radius of 6378 km.

Exercise 8 – Loxodromes

For the loxodrome from New York to London, compute the distance and bearing. Compare the results with the previous exercise.

Solutions

1.a) p_1 = (38°46.8268´N, 9°29.9365´W, 103.0 m)
1.b) p_1 = (38.780447°N, 9.498942°W, 103.0 m)

2) R_N = 6386528.7 m
p_1 = (4910445.1 m, -821633.4 m, 3973410.6 m)

3) Intermediate results, with the Bowring (1985) method:

b = 6356752.3 m
e^2 = 0.00669437999
e'^2 = 0.00673949674
p = 4978624.4 m
β = 0.675232785 radians
R_N = 6386529.2 m

Intermediate results, with the Heikkinen's method:

b = 6356752.3 m
e^2 = 0.00669437999
e'^2 = 0.00673949674
r = 4978624.4 m
F = 3.44525694e+028
G = 40468276185189
c = 0.000577453410
s = 1.01139183
P = 0.779096570
Q = 1.00003491
r_0 = 4978528.2 m
U = 6343898.8 m
V = 6335562.6 m
z_0 = 3973472.3

3.a) p_2 = (38.781906°N, 9.497299°W, 123.4 m)
3.b) p_2 = (38° 46.9143´N, 9° 29.8380´W, 123.4 m)
3.c) p_2 = (38° 46' 54.86´´N, 9° 29´ 50.28´´W, 123.4 m)

4) $d = \| p_1 - p_2 \| = 216.8$ m

5.a) Intermediate results of the Molodensky transformation:

R_N = 6386528.661 m
R_M = 6360478.694 m

p_1(WGS 84) → p_1(ED 50)
($\Delta\phi$, $\Delta\lambda$, Δh) = (4.60´´, 4.95´´, -89.3 m)
p_1(ED 50) = (38.781725°N, 9.497568°W, 13.7 m)
= (38°46.9035´N, 9°29.8541´W, 13.7 m)
= (38°46´54.21´´N, 9°29´51.24´´W, 13.7 m)

5.b) Intermediate results of the Molodensky transformation:

R_N = 6386529.194 m
R_M = 6360480.286 m

p_2(WGS 84) → p_2(ED 50)
($\Delta\phi$, $\Delta\lambda$, Δh) = (4.60´´, 4.95´´, −89.2 m)
p_2(ED 50) = (38.783184°N, 9.495925°W, 34.2 m)
= (38°46.9910´N, 9°29.7555´W, 34.2 m)
= (38°46´59.46´´N, 9°29´45.33´´W, 34.2 m)

5.c) $p_1^{ED\,50}(molodensky)$ = (4910529.1 m, -821526.4 m, 3973530.6 m)

$p_1^{ED\,50}$ = (4910529.1 m, -821526.4 m, 3973530.6 m)

$\| p_1^{ED\,50}(molodensky) - p_1^{ED\,50} \| = 0.004$ m

6) p_2 - p_1 = (-60.783 m, 154.849 m, 139.016 m)
enu = (142.695 m, 161.925 m, 20.416 m)
az. = 41.4° el. = 5.4°

7) $d_{\text{orthodrome}}$ = 5567699m $\psi_{\text{departure}}$ = 51.2° $\psi_{\text{approaching}}$ = 108.3°

8) $d_{\text{loxodrome}}$ = 5791189m ψ = 78.0°

$| d_{\text{loxodrome}} - d_{\text{orthodrome}} |$ = 223490 m