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GNSS Positioning

Exercise 1 – True Ranges

Consider a GPS receiver at the following WGS 84 (x,y,z) cartesian coordinates:

$$r_1 = (4918525.18 \text{ m}, -791212.21 \text{ m}, 3969762.19 \text{ m})$$

and the satellites' ephemerides, collected by this receiver, stored in file `ub1.ubx.2056.540000b.eph`.
Assuming a receiver clock offset of zero, and for the satellites above an elevation angle of 10º (elevation mask), compute the ranges, measured by r_1, at Week Number (*WN*) 2056, for every second between Time Of Week (TOW) 536400 s and (536400 s + 3600 s). Plot the true ranges, for the time interval in consideration.

Exercise 2 – Noiseless Pseudoranges

For the same receiver position of the previous exercise, and for the same ephemerides, consider now a clock offset of 500 µs, at TOW 536400 s, and a clock drift of 0.4 µs/s. In these conditions, and for an elevation mask of 10º, compute the pseudoranges, measured at r_1, on Week Number (*WN*) 2056, for every second between Time Of Week (TOW) 536400 s and (536400 s +3600 s). Plot the true ranges and the computed pseudoranges, for the time interval in consideration.

Exercise 3 – Linearized Single Epoch LS Solution

Consider the simulated pseudoranges, stored in the ASCII file `npr.txt`, as have been measured by a receiver, whose true position is r_1, and for which all satellites, with ephemeris stored in `ub1.ubx.2056.540000b.eph`, are visible. Measurements refer to *WN*=2056, and *TOW* between 536400 s and (536400 s + 3600 s). Each line contains the pseudoranges of a given satellite. Pseudoranges were generated based on the true range computed on exercise 1, plus the clock offset computed on exercise 2, and a normal disturbance of zero mean and 5 m standard deviation.
Based on the simulated measurements for *TOW* = 536400 s, estimate the receiver position using the least squares (LS) solution to the linearized pseudorange measurement equation. Use r_3 as the initial guess for the receiver position. Use the position estimate as the initial guess on the next iteration. How many iterations were needed to get the difference between the position estimates equal to (or below) 1mm? Repeat the exercise using p_1 and the origin of the reference system as initial guesses.

Exercise 4 - Dilution Of Precision (DOP)

Considering the satellite ephemerides stored in the `ub1.ubx.2056.540000b.eph` file and using subsets of the available satellite constellation, compute the minimum and maximum *PDOP* and *HDOP* that can be obtained, with a receiver at r_1, at *TOW* = 536400 s, *WN* = 2056.

Exercise 5

Compute the average error using all the measurements in `npr.txt`.

Solutions

1. Satellites above the 10º elevation mask, during the interval [536400 s, 536400 s + 3600 s]: SVN10, SVN12, SVN13, SVN15, SVN17, SVN19, SVN20, SVN24.

Table I: The three first ranges (in m) for each satellite
(*WN*=2056, *TOW*=536400 s, 536401 s, 536402 s)

SVN10	SVN12	SVN13	SVN15	SVN17	SVN19	SVN20	SVN24
24715336.164	23257849.079	21189237.840	20144171.617	24173025.470	24310966.310	22836699.669	20932540.137
24714759.783	23257126.847	21189555.716	20144166.307	24172755.859	24310456.754	22836403.776	20932277.937
24714183.465	23256404.655	21189873.661	20144161.089	24172486.328	24309947.241	22836107.995	20932015.798

2.

Table II: The three first pseudoranges (in m) for each satellite
(*WN*=2056, *TOW*=536400 s, 536401 s, 536402 s)

SVN10	SVN12	SVN13	SVN15	SVN17	SVN19	SVN20	SVN24
24865232.393	23407745.308	21339134.069	20294067.846	24322921.699	24460862.539	22986595.898	21082436.366
24864775.929	23407142.993	21339571.862	20294182.453	24322772.005	24460472.900	22986419.922	21082294.083
24864319.528	23406540.718	21340009.724	20294297.152	24322622.391	24460083.304	22986244.058	21082151.861

3. With $r_0 = r_3$:

```
Partial results - first iteration ----------

sat =     -5845119.18516113          -14047493.8764067          21837688.7089306
           23594371.7090322          -10613530.1377868          -5810952.03388924
           20975754.8671346           9577583.92302812           13115102.0828325
           19235360.7061089          -2940779.59594573            17976732.980128
           13432871.6787895           21227630.1995618           9167034.83991089
           17813813.7139722           19603950.2927382           1008012.19289924
           3922943.46465993          -17848281.1803692           19121661.9708362
           14306135.2566974          -14437336.3833629           16769266.4262718

H =        0.435502535066798          0.536356536011209        -0.722951732986683        1
         -0.802993312254272          0.422323380486984          0.420529074819994        1
         -0.757799416573639         -0.489341114312847         -0.431607829034314        1
         -0.710715672492193          0.106708450900497         -0.695339154212091        1
         -0.352225203613741         -0.910883598071656         -0.215008085204469        1
         -0.530431670573514         -0.838928812846971          0.121822369980901        1
         0.0435948677722797          0.746911914854038         -0.663492335263183        1
         -0.448468609346169          0.651906818485253         -0.611468238294965        1

z =       -1002389.19302295
          -2464384.42100814
          -4903578.35272855
          -6190532.50381175
          -1715359.45234217
          -1311675.27877403
          -2860556.09674423
          -4999086.33261234
```

$\hat{x}$= [4918525.64937606 -791206.172177836 3969768.48545315 149898.378469716]^T

```
Partial results - second iteration ----------
sat =    -5845119.18410339        -14047493.8768011         21837688.7089544
          23594371.7087836        -10613530.1380715        -5810952.03440041
          20975754.8672054         9577583.92376275         13115102.0821893
          19235360.7068188        -2940779.59492384         17976732.9795578
          13432871.6784323         21227630.1996119          9167034.8403362
          17813813.7139761         19603950.2927351         1008012.19289174
          3922943.46566009        -17848281.1805519         19121661.9704583
          14306135.2569821        -14437336.3824459         16769266.4268226

H =      0.435504710300743        0.536358810215315        -0.722948735395725        1
        -0.802990944051085        0.422322885349311         0.420534094078209        1
         -0.75780138515305       -0.489342425799111        -0.431602885733124        1
        -0.710718624712668        0.10670946899486          -0.69533598045289        1
        -0.352225190507561       -0.910884853556758        -0.215002787732616        1
        -0.530431012577863       -0.838928408669343         0.121828018218911        1
        0.0435957114095053        0.746915053039467        -0.663488747070927        1
        -0.448469684118982        0.651909906850723        -0.611464157392813        1

z =      -1002368.39370274
         -2464352.45638754
         -4903567.37427329
         -6190535.23101958
         -1715337.36516416
         -1311649.94039157
          -2860540.1860617
         -4999077.86235998
```

$\hat{x}$= [4918525.64956072 -791206.172110281 3969768.48524072 149898.37819337]^T

$\hat{r}_1$ = (4918525.650 m, -791206.172 m, 3969768.485 m)

$\| \hat{r}_1 - r_1 \|$ = 8.735 m

#iterations = 2

With $r_0 = p_1$:

#iterations = 3

With r_0 = (0,0,0):

#iterations = 5

4.

```
Ephemerides loaded  #sat= 8
sat01(SVN10) = ( -5845119.184 m, -14047493.877 m,  21837688.709 m)
sat02(SVN12) = ( 23594371.709 m, -10613530.138 m,  -5810952.034 m)
sat03(SVN13) = ( 20975754.867 m,   9577583.924 m,  13115102.082 m)
sat04(SVN15) = ( 19235360.707 m,  -2940779.595 m,  17976732.980 m)
sat05(SVN17) = ( 13432871.678 m,  21227630.200 m,   9167034.840 m)
sat06(SVN19) = ( 17813813.714 m,  19603950.293 m,   1008012.193 m)
sat07(SVN20) = (  3922943.466 m, -17848281.181 m,  19121661.970 m)
sat08(SVN24) = ( 14306135.257 m, -14437336.382 m,  16769266.427 m)

 4 satellites min(PDOP) =   2.03  11010100      SVN = [10 12 15 19 ]
              max(PDOP) = 388.22  00110101      SVN = [13 15 19 24 ]

 4 satellites min(HDOP) =   1.23  11011000      SVN = [10 12 15 17 ]
              max(HDOP) = 366.99  00110101      SVN = [13 15 19 24 ]
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 5 satellites min(PDOP) =   1.88  11110100      SVN = [10 12 13 15 19 ]
              max(PDOP) =   8.55  10101011      SVN = [10 13 17 20 24 ]

 5 satellites min(HDOP) =   1.12  11101010      SVN = [10 12 13 17 20 ]
              max(HDOP) =   7.27  00111011      SVN = [13 15 17 20 24 ]

 6 satellites min(PDOP) =   1.75  11111100      SVN = [10 12 13 15 17 19 ]
              max(PDOP) =   5.44  10111011      SVN = [10 13 15 17 20 24 ]

 6 satellites min(HDOP) =   1.05  11101110      SVN = [10 12 13 17 19 20 ]
              max(HDOP) =   4.55  10111011      SVN = [10 13 15 17 20 24 ]

 7 satellites min(PDOP) =   1.66  11111101      SVN = [10 12 13 15 17 19 24 ]
              max(PDOP) =   3.03  10111111      SVN = [10 13 15 17 19 20 24 ]

 7 satellites min(HDOP) =   1.04  11101111      SVN = [10 12 13 17 19 20 24 ]
              max(HDOP) =   2.57  10111111      SVN = [10 13 15 17 19 20 24 ]

 8 satellites min(PDOP) =   1.63  11111111      SVN = [10 12 13 15 17 19 20 24 ]
              max(PDOP) =   1.63  11111111      SVN = [10 12 13 15 17 19 20 24 ]

 8 satellites min(HDOP) =   1.03  11111111      SVN = [10 12 13 15 17 19 20 24 ]
              max(HDOP) =   1.03  11111111      SVN = [10 12 13 15 17 19 20 24 ]
```

5. Mean error = 8.964 m ($r_0 = r_{3,}$ 3601 measurements)