

4 Applications (exercises)

$$\tau: \mathbb{R}^3 \rightarrow \mathbb{R}^4 \quad B_1 = \{(1, 3, 5), (0, 1, 1), (0, 1, 1)\}$$

$$M(\tau, B_1, B_2)$$

$$M(\tau, B_1, B_2) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & -2 \\ 0 & 2 & 4 \\ 5 & 4 & -3 \end{pmatrix}$$

$$\begin{array}{ccc} & M(\tau, B_1, B_2) & \\ B_1^3 & \xrightarrow{\quad} & B_2^4 \\ & \searrow M(B_1, B_2) & \\ & B_1 & \end{array}$$

$$(1, 0, 0) = a(1, 3, 5) + b(0, 1, 1) + c(0, 1, 1) \quad \begin{matrix} 1 \\ -3 \\ -2 \end{matrix}$$

$$(0, 1, 0) = a(1, 3, 5) + b(0, 1, 1) + c(0, 1, 1) \quad \begin{matrix} 0 \\ 1 \\ -1 \end{matrix}$$

$$(0, 0, 1) = a(1, 3, 5) + b(0, 1, 1) + c(0, 1, 1) \quad \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

$$M(Z, B_C^3, B_C^4) = M(Z, B_1, B_C^4) \cdot M(B_C, B_1)$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & -2 \\ 0 & 2 & 4 \\ 5 & 4 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 6 \\ -3 & 1 & 0 \\ -2 & -1 & 1 \end{pmatrix} = \begin{pmatrix} -11 & -1 & 3 \\ 6 & 2 & -2 \\ -14 & -2 & 4 \\ -1 & 7 & -3 \end{pmatrix}$$

b) Base de  $U$

3-2

$$\begin{cases} x_1 + x_2 - x_3 = 0 \\ x_2 - x_3 = 0 \end{cases}$$

$$\dim(U) = 1$$

$$\begin{cases} x_1 = 0 \\ x_2 = \lambda \\ x_3 = \lambda \end{cases}$$

$$\lambda = 1 = (0, 1, 1)$$

Sistema gerador de  $\tau(U)$  es  $\tau(0, 1, 1)$

$$\tau(0, 1, 1) = (2, 0, 2, 4)$$

$$(a, b, c, d) = \lambda (2, 0, 2, 4)$$

⑥ Diagonalización

②  $\varphi(-1, 4, 1) = b_1(-1, 4, 1)$

$$\varphi(1, -1, -1) = b_2(1, -1, -1)$$

$$\varphi(1, 2, 1) = b_3(1, 2, 1)$$

$$\varphi(1, 1, 1) = (4, 4, 2)$$

$$P \begin{pmatrix} -1 & 1 & 1 \\ 4 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

$A = \text{Matriz base canónica}$

$$A = Q^{-1} D Q$$

