4 $U = \{(1,2,-1,0), (0,1,1,0), (1,0,-2,1)\}$

Base ortonormal

$$\begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -2 & 1 \end{pmatrix} \xrightarrow{F_3 - F_1} \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 1 & 1 \end{pmatrix} \xrightarrow{F_3 + 2F_2}$$

$$\begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \qquad B = \{(1,2,-1,0), (0,1,1,0), (0,0,1,1)\}$$

$e_1 = (0,0,1,1)$

$$e_2 = (0,1,1,0) - \frac{\langle (0,1,1,0), (0,0,1,1) \rangle}{\langle (0,0,1,1), (0,0,1,1) \rangle} (0,0,1,1) =$$

$(0,1,1,0) - (0,0,1/2,1/2) = (0,1,1/2,-1/2)$

$$e_3 = (1,2,-1,0) - \frac{\langle (1,2,-1,0),(0,0,1,1)\rangle}{\langle (0,0,1,1),(0,0,1,1)\rangle}(0,0,1,1) -$$

$$\frac{\langle (1,2,-1,0),(0,1,1/2,-1/2)\rangle}{\langle (0,1,1/2,-1/2),(0,1,1/2,-1/2)\rangle}(0,1,1/2,-1/2) =$$

$$= (1,2,-1,0) - (0,0,-1/2,-1/2) - \ - - -$$

$$f_1 = \frac{e_1}{\|e_1\|} = (0,0,\tfrac{1}{\sqrt{2}},\tfrac{1}{\sqrt{2}}$$

5) a) $\varphi(\vec{x},\vec{y}) = 4x_1y_1 + 2x_3y_1 + 6x_2y_2 + 2x_1y_3 + 4x_3y_3$

$B = \{e_1, e_2, e_3\}$ base canonica

$\varphi(e_1,e_1) = \varphi((1,0,0),(1,0,0)) = 4$

$\varphi(e_1,e_2) = \varphi((1,0,0),(0,1,0)) = 0$

$\varphi(e_1,e_3) = \varphi((1,0,0),(0,0,1)) = 2$

$\varphi(e_2,e_1) = \varphi((0,1,0),(1,0,0)) = 0$
$\varphi(e_2,e_2) = \varphi((0,1,0),(0,1,0)) = 6$
$\varphi(e_2,e_3) = \varphi((0,1,0),(0,0,1)) = 0$

$\varphi(e_3,e_1) = \varphi((0,0,1),(1,0,0)) = 2$
$\varphi(e_3,e_2) = \varphi((0,0,1),(0,1,0)) = 0$
$\varphi(e_3,e_3) = \varphi((0,0,1),(0,0,1)) = 4$

$$M = \begin{pmatrix} 4 & 0 & 2 \\ 0 & 6 & 0 \\ 2 & 0 & 4 \end{pmatrix}$$

(1) φ Es simetrico? ✓ $\varphi(e_1, e_2) = \varphi(e_2, e_1)$

(2) Definida positiva $\varphi(x, x) > 0$

Que la diagonal sea positiva no indica que lo sea

$$\varphi((x_1, x_2, x_3), (x_1, x_2, x_3)) = 4x_1^2 + 6x^2 + \underbrace{4x_1x_3 + 4x_3^2}$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$2x_3 = a$$
$$b = x_1$$

$$4x_1^2 + 6x_2^2 + (2x_3 + x_1)^2 - x_1^2$$

(3) $\varphi(x, y+z) = \varphi(x, y) + \varphi(x, z)$

$$\varphi((x_1, x_2 + x_3), (y_1, y_2, y_3) + (z_1 + z_2 + z_3))$$

b) Gram - Schmidt a $B_c = \{(1,0,0)$
$(0,1,0)$
$(0,0,1\}$

$e_1 = (1,0,0)$

$$e_2 = (0,1,0) - \frac{\varphi(e_2,e_1)}{\varphi(e_1,e_1)} e_1$$

⋮