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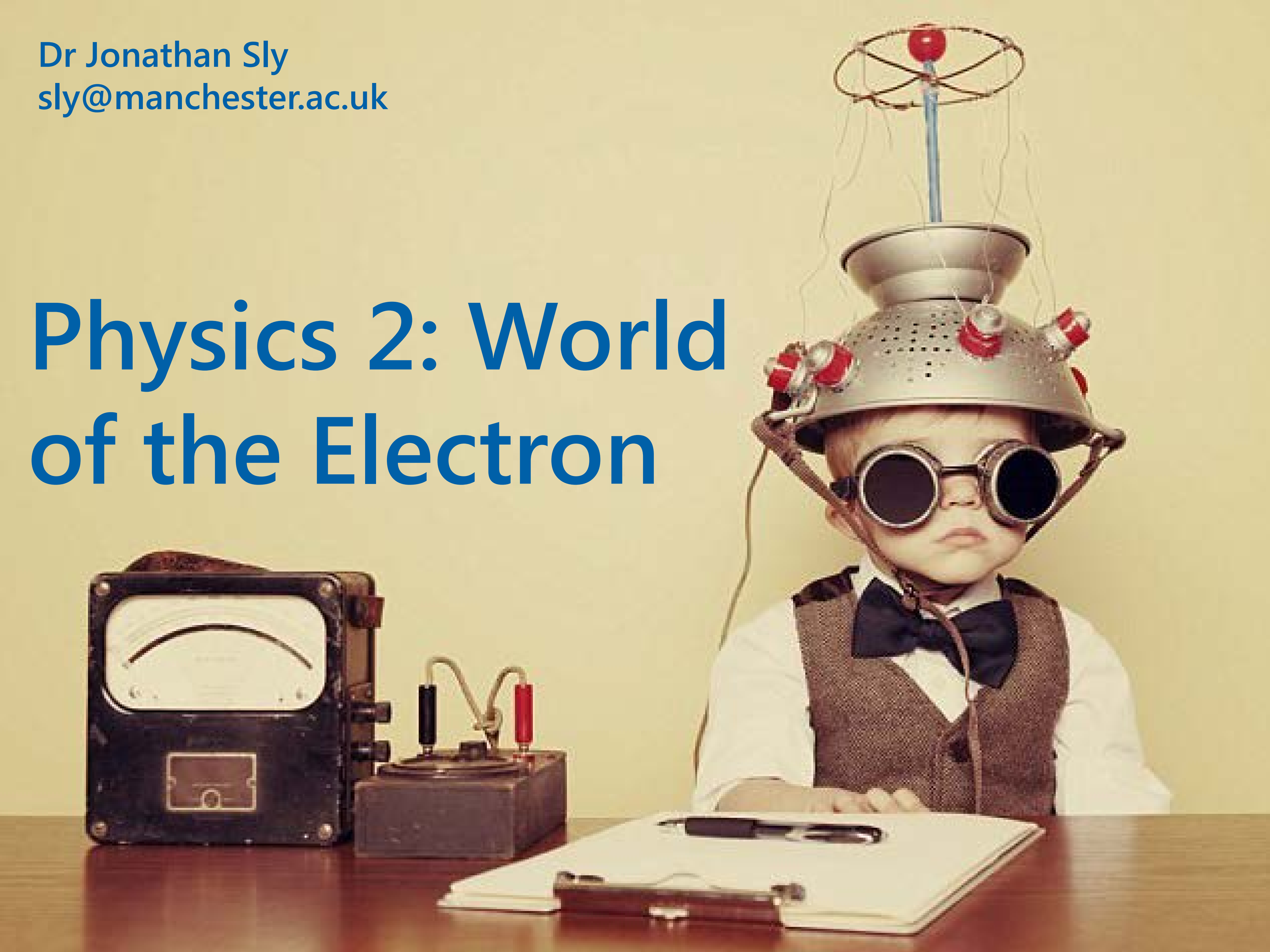
Physics 2: World of the Electron



1.1 electric charge

what is electric charge?

- fundamental property of matter
- we can describe this property by looking at the effects it produces

1.1 electric charge

observations

- there are two types of charge

1.1 electric charge

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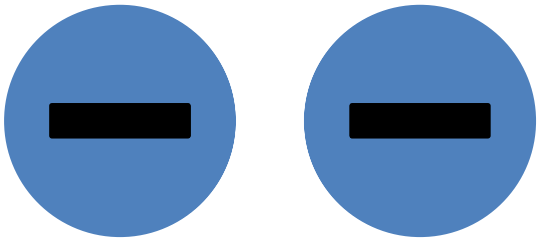
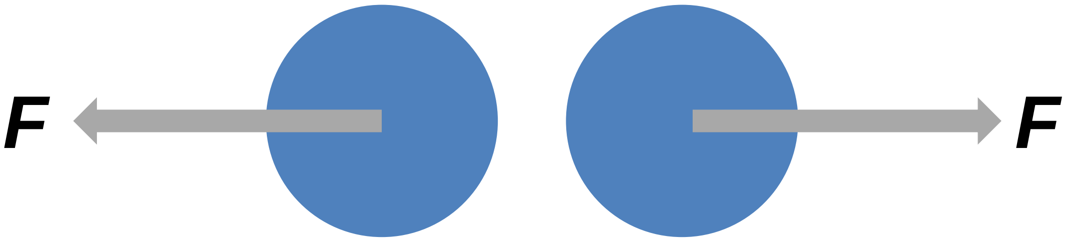


1.1 electric charge

observations

- there are two types of charge



1.1 electric charge

observations

- there are two types of charge – positive and negative
- opposite charges attract one another, like charges repel
- charges give rise to forces
- charge is never created or destroyed

this is the ‘principle of conservation of charge’

1.1 electric charge

conductors and insulators

- an insulating material has tightly bound valence electrons, so charge cannot flow freely through the material
- a conducting material has loosely bound valence electrons that are free to move between atoms, so charge can move freely through the material

1.1 electric charge

units of charge

- the SI unit of charge is the coulomb (C)
- convenient unit for charge transport in circuits (1 C is the amount of charge transported by 1 amp in 1 second)
- the charge on a proton is equal to $+1.6 \times 10^{-19}$ C

  $+1.6 \times 10^{-19}$ C is known as the ‘elementary charge’
- the elementary charge is represented by the symbol ‘e’
- the charge on an electron is equal to $-e$ (-1.6×10^{-19} C)
- in electrostatics, 1 C is a very big charge (more common to be dealing with mC, μC or even nC)

1.2 forces between point charges

the force between two 'point charges' Q_1 and Q_2 that are separated by a distance r can be described by the following expression

$$F \propto \frac{Q_1 Q_2}{r^2}$$

or $$F = k\frac{Q_1 Q_2}{r^2}$$ where $$k = \frac{1}{4\pi\mathcal{E}}$$

and $\mathcal{E}$ is called the permittivity of the material between the charges

1.2 forces between point charges

permittivity

- permittivity, ε, is more properly called the absolute permittivity of a substance
- the permittivity of a vacuum is called the absolute permittivity of free space and is represented by the symbol, ε_0

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2\,N^{-1}\,m^{-2}}$$

- note that the permittivity of air is very close to that of a vacuum

$$\varepsilon_{\mathrm{air}} \approx \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2\,N^{-1}\,m^{-2}}$$

1.2 forces between point charges

relative permittivity, ε_r

- the relative permittivity of a substance is defined as

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

- the relative permittivity of a material is sometimes called its dielectric constant
- examples:

Substance	Relative permittivity
Air	1.00
Polythene	2.3
Glass	4 – 7
Water	80

1.2 forces between point charges

Coulomb's law

The magnitude of the electrostatic force between two charges is given by

$$F = \frac{1}{4\pi\mathcal{E}} \frac{|Q_1||Q_2|}{r^2}$$

where F is force in newtons (N)

r is distance in metres (m)

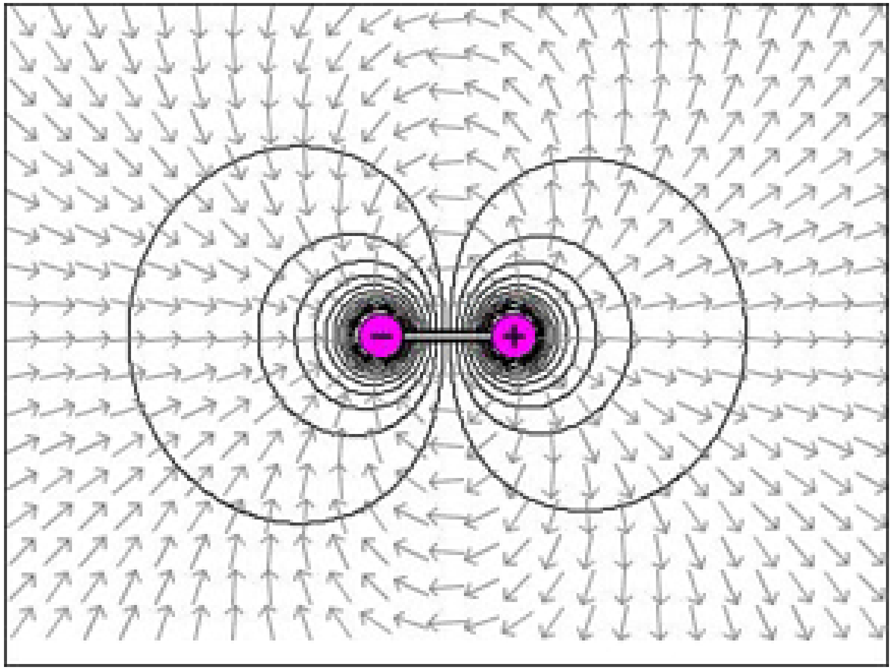
Q is charge in coulombs (C)

$\mathcal{E}$ is permittivity ($C^2\,N^{-1}\,m^{-2}$)

example

point charges A and B have charges of +0.5 C and −2.0 C respectively and are separated in air by a distance of 100 cm. What is the size of the Coulomb force between them? And what is the direction of the force on each charge?

1.3 electric fields



1.3 electric fields

definitions

- an electric field exists at a point if a charged object at that point experiences an electric force
- electric field is a vector quantity, it has a magnitude (field strength) and a direction
- the direction of the field at a given point is the direction of the electric force experienced by a positive test charge placed at that point
- the magnitude of the field at a given point is defined as the force exerted by the field on a unit positive test charge placed at that point

1.3 electric fields

electric field strength

- the electric field strength (E) at a given point is the force exerted by the field on a + 1 C test charge placed at that point
- so electric field strength has units of force per unit charge

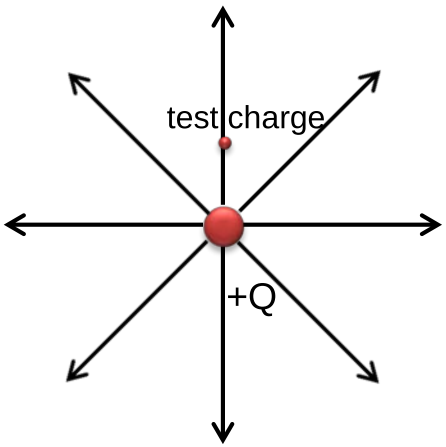
i.e. $$E = \frac{F}{Q}$$ [units are NC^{-1}]

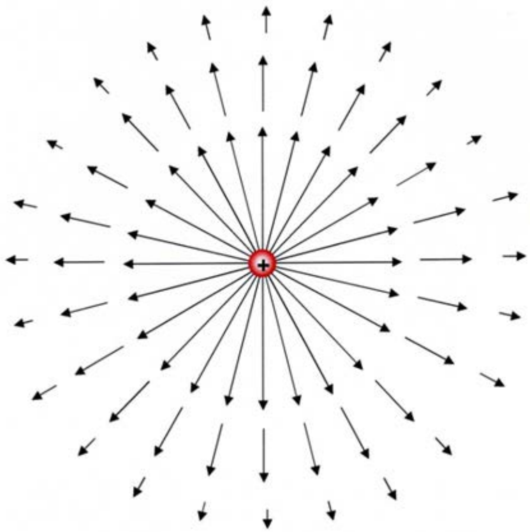
http://www.falstad.com/vector3de/

1.3 electric fields

field diagrams

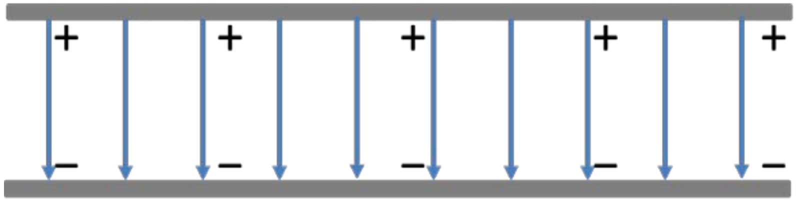
- electric fields can be represented by field lines or vectors
- field direction is indicated by the arrow
- field strength is represented by the density of field lines or the length of the vectors





1.3 electric fields

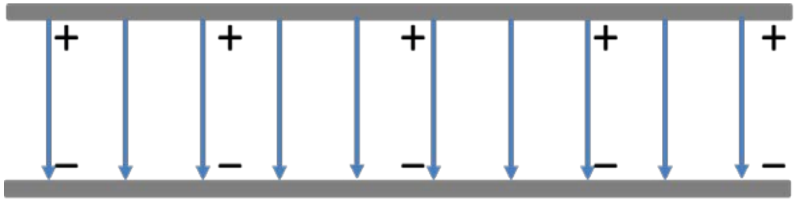
example 1: parallel charged plates



- the field between parallel conducting plates can be represented by the diagram above (neglecting ‘fringing’ effects due to plate edges)
- the field between the plates is **uniform**: the field strength and direction is the same at all points between the plates

1.3 electric fields

example 1: parallel charged plates



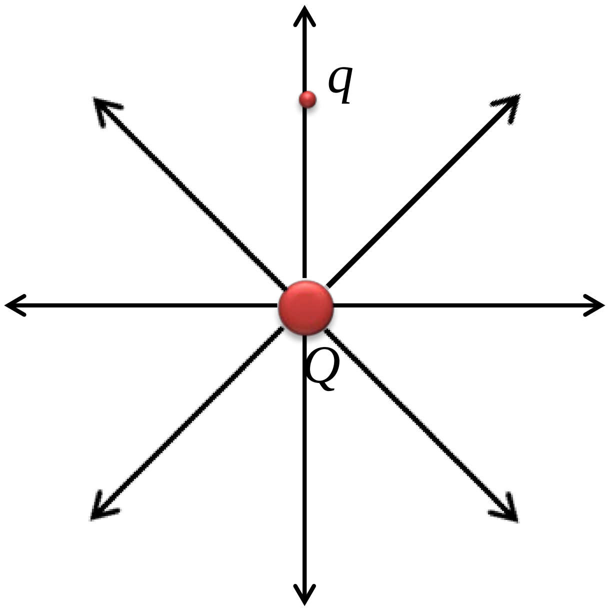
- the field strength anywhere between the plates is given by

$$E = \frac{\sigma}{\varepsilon_0} \quad \left[\mathrm{N\,C^{-1}}\right]$$

- σ is called the sheet charge density (it is the charge per unit area on the plates, so $\sigma = Q/A$)

1.3 electric fields

example 2: point charge



- the field around a point charge is **non-uniform**: the field strength and direction are different at different points

- the magnitude of the field a distance r from a charge Q is given by:

$$E = \frac{|Q|}{4\pi\varepsilon r^2} \qquad [\mathrm{N\,C^{-1}}]$$

1.4 principle of superposition

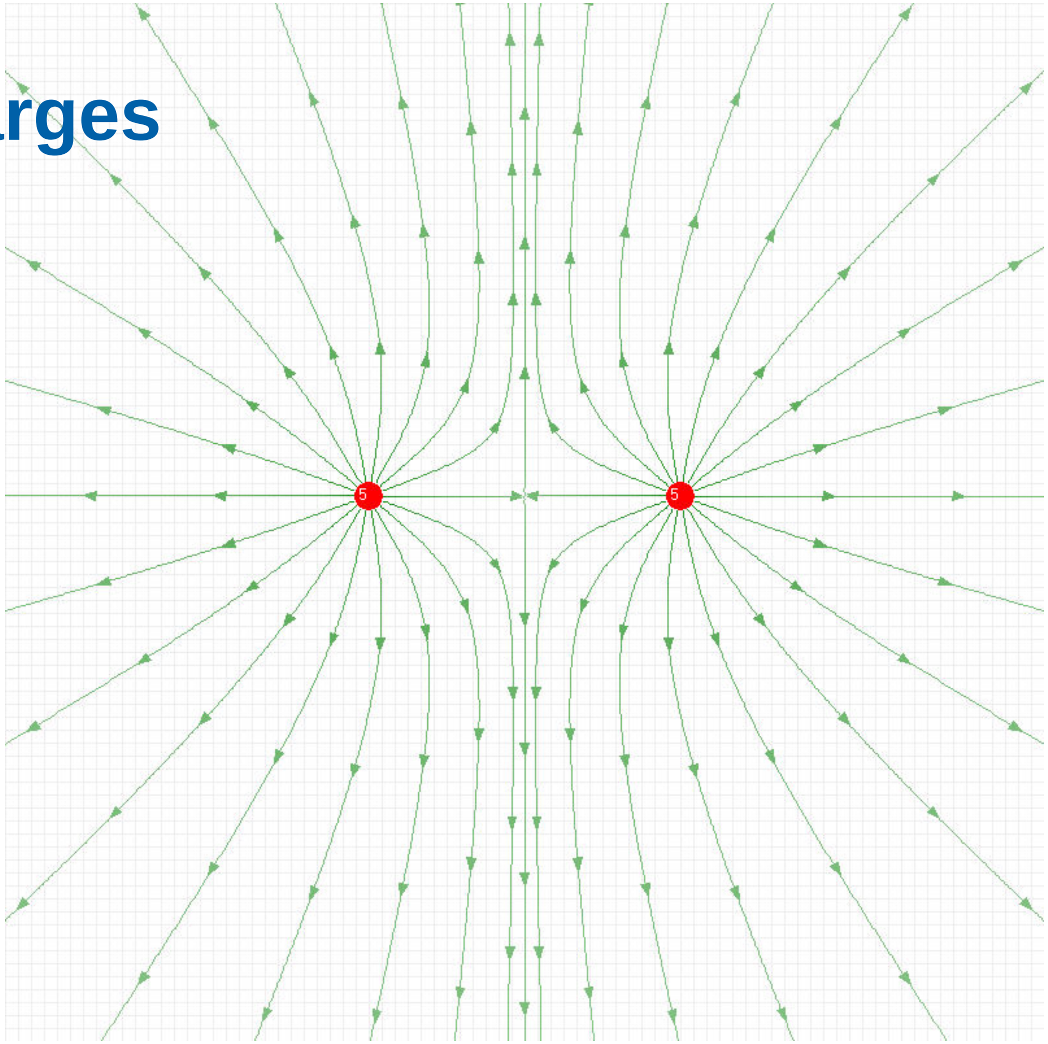
the electric field due to two or more charges

- when there is more than one charge contributing to the field, we can find the resultant field using the principle of superposition:

  - consider the field component due to each individual charge, one at a time (ignoring the others)
  - find the strength and direction – the electric field vector – due to each charge
  - find the resultant field strength and direction by vector addition of the component vectors

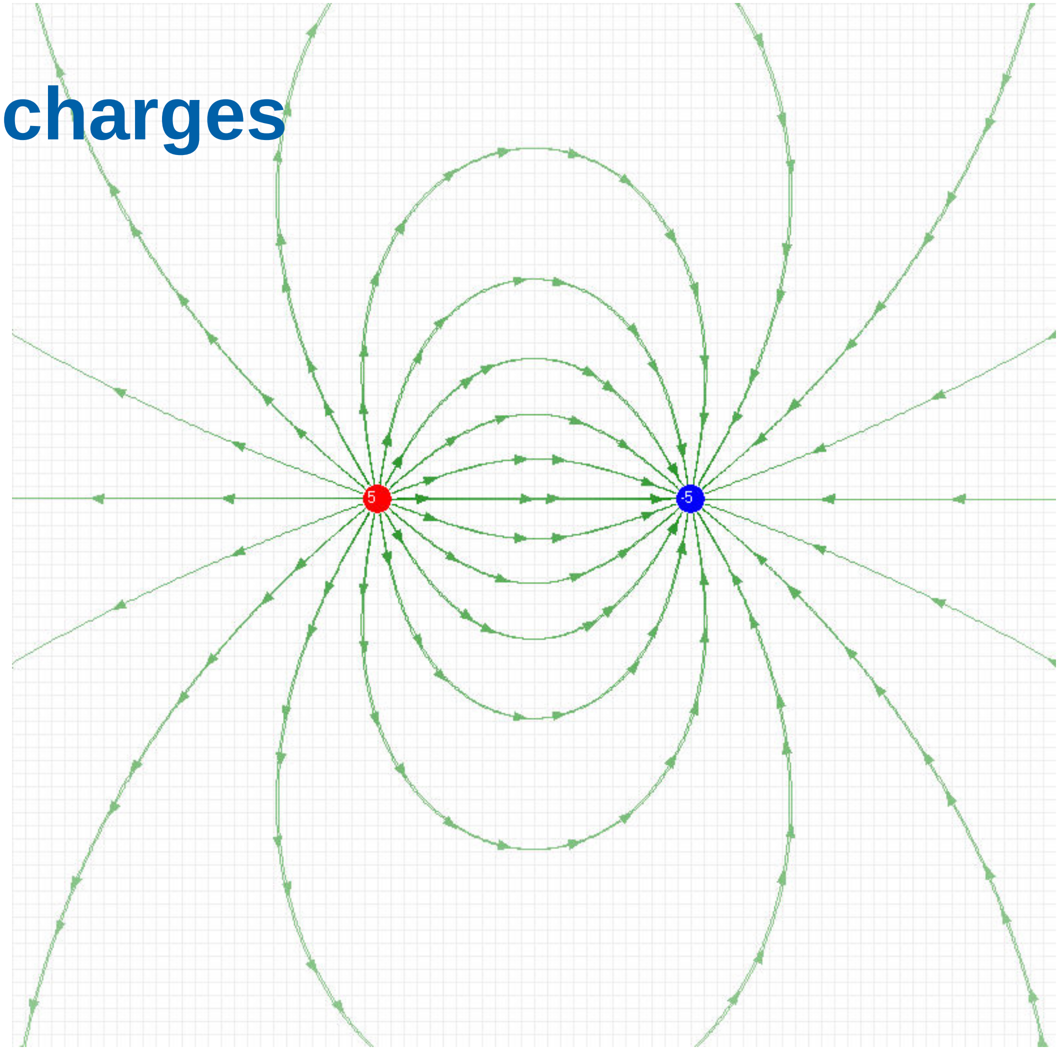
1.4 principle of superposition

equal charges



1.4 principle of superposition

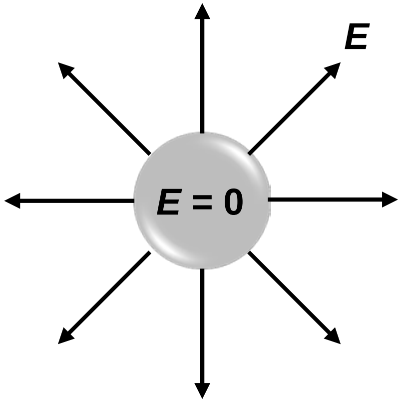
opposite charges



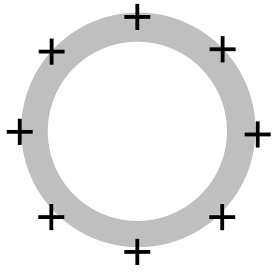
1.5 electric field near charged conductors

the following rules apply to conductors in **electrostatic equilibrium** (in other words when there is no net motion of charge within the conductor)

- there is no electric field within the material of a conductor



- net charge exists entirely on the surface of a solid conductor, and entirely on the outside surface of a hollow conductor



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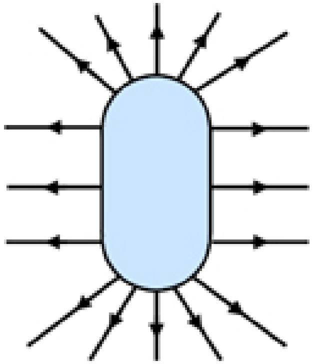
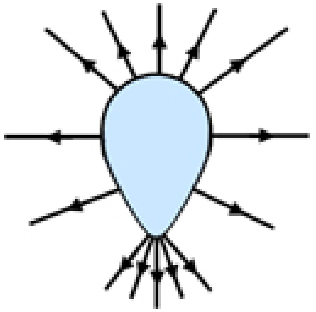


- the electric field just outside a conductor is always perpendicular to the surface



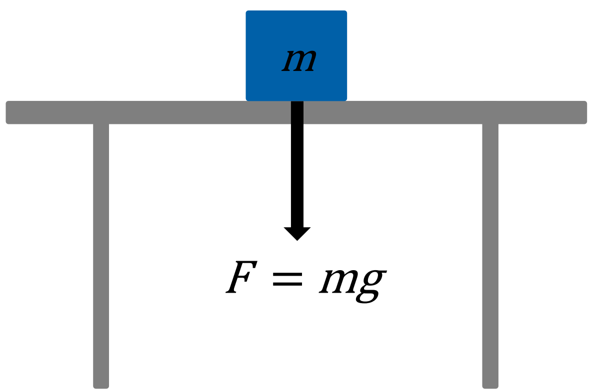
- charge is concentrated at corners, edges and sharp points

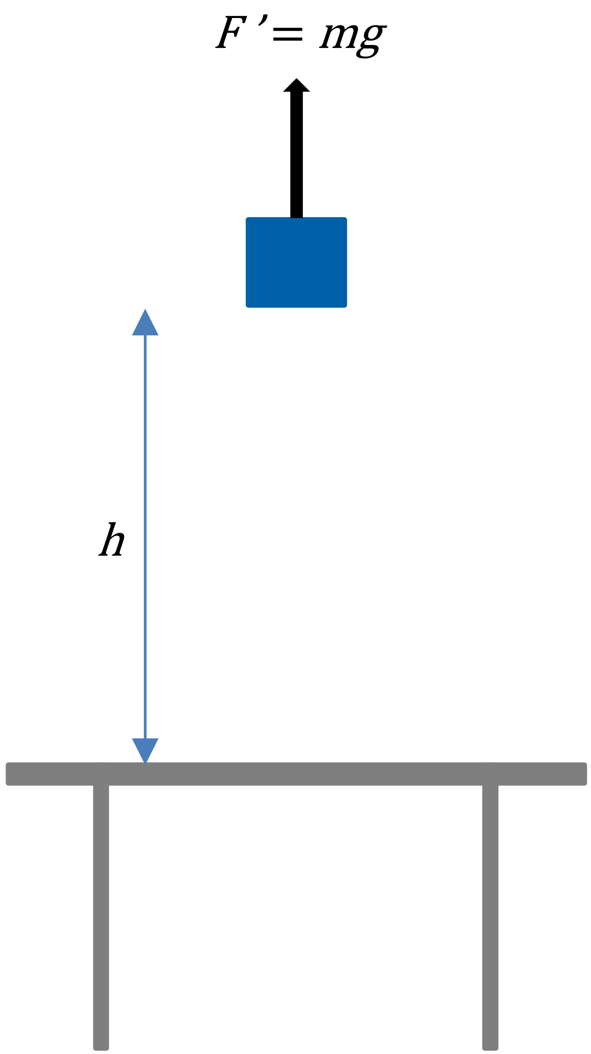
2.1 electrical potential energy & potential

electrical potential energy

- a charged object in an electric field is subject to a force

$$F = QE$$


m
$F = mg$



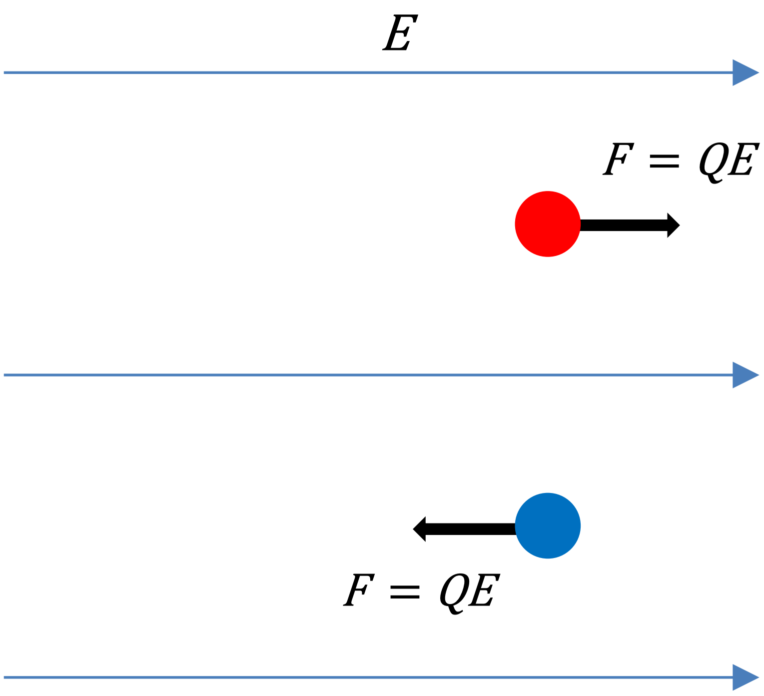
- work done is given by force × distance
- the work done in raising the mass is equal to the change in the potential energy of the mass
- so change in potential energy is given by

$$\Delta E_p = mgh$$

2.1 electrical potential energy & potential

electrical potential energy

- a charged object in an electric field is subject to a force
- as a result of its position in the electric field, the charged object has electrical potential energy
- work (W) is done to move the charged object in the field
- the work done when the charged object moves is equal to the change in the electrical potential energy of the object



2.1 electrical potential energy & potential

electrical potential energy

electrical potential energy (**units: joules**):

the energy an object has due to its position in an electric field; depends on the strength of the field and the size and sign of the charge on the object.

2.1 electrical potential energy & potential

electric potential, V

- electric potential at a given point is the amount of electrical potential energy that a +1 C charge would have at that position in the field
- can be thought of as the electrical potential energy of an object, divided by the amount of charge on the object
- electric potential is not a property of an object, it is the property of a particular place in an electric field (whether or not there is an object there)

2.1 electrical potential energy & potential

electric potential, V

electric potential (**units: J C^{-1} or volts, V**):

the energy per coulomb that a charged object would have at a particular position in an electric field. Depends only on the electric field strength at that point.

2.1 electrical potential energy & potential

- electric potential (volts) is a property of a given position in an electric field

- it is measured with reference to an (imaginary) *positive* test charge, so

  – a place that is 'more positive' is at a **higher** potential
  – a place that is 'more negative' is at a **lower** potential

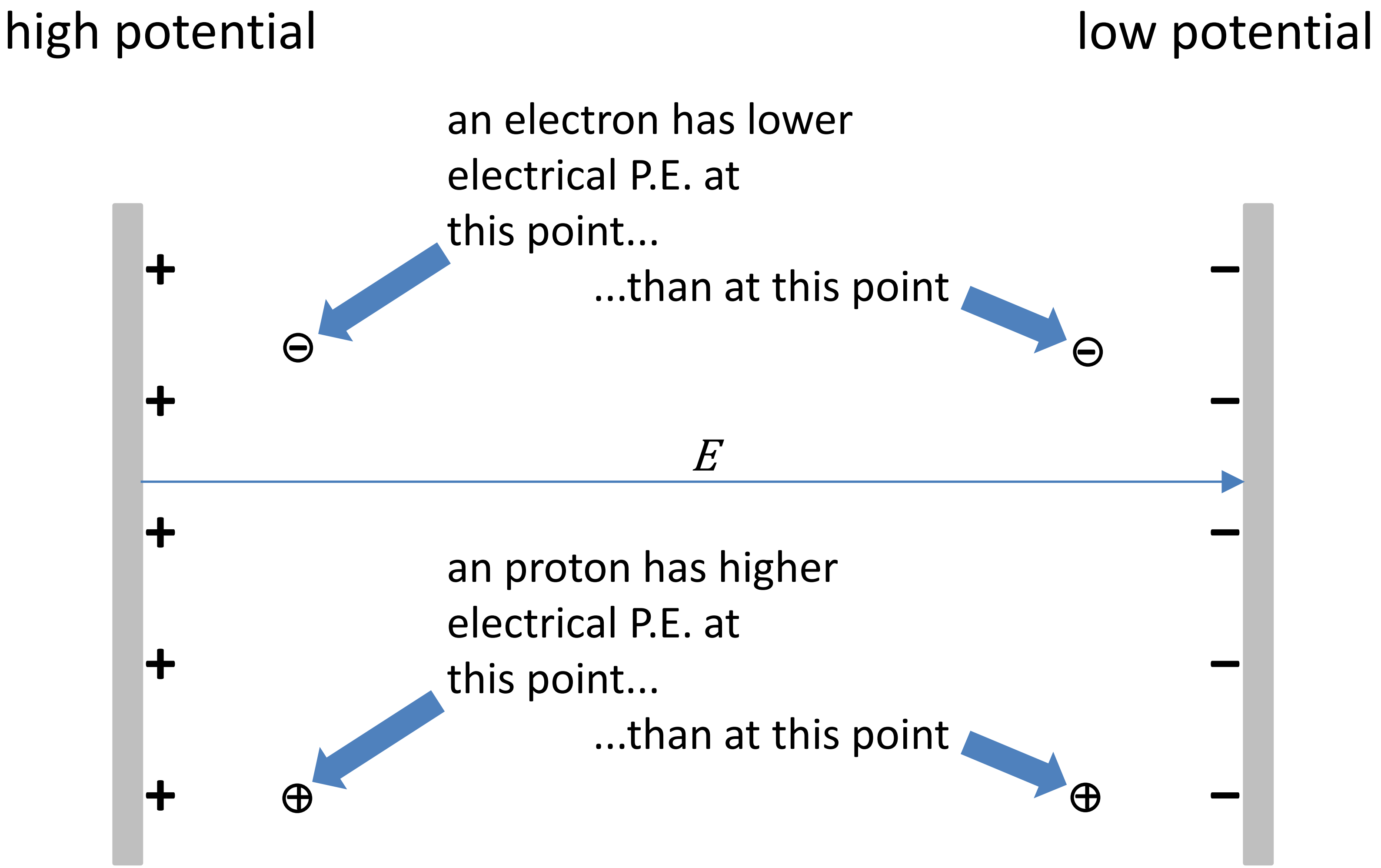
2.1 electrical potential energy & potential

- electrical potential energy (joules) is a property of an object in an electric field

- it depends on the field strength and on the charge of the object

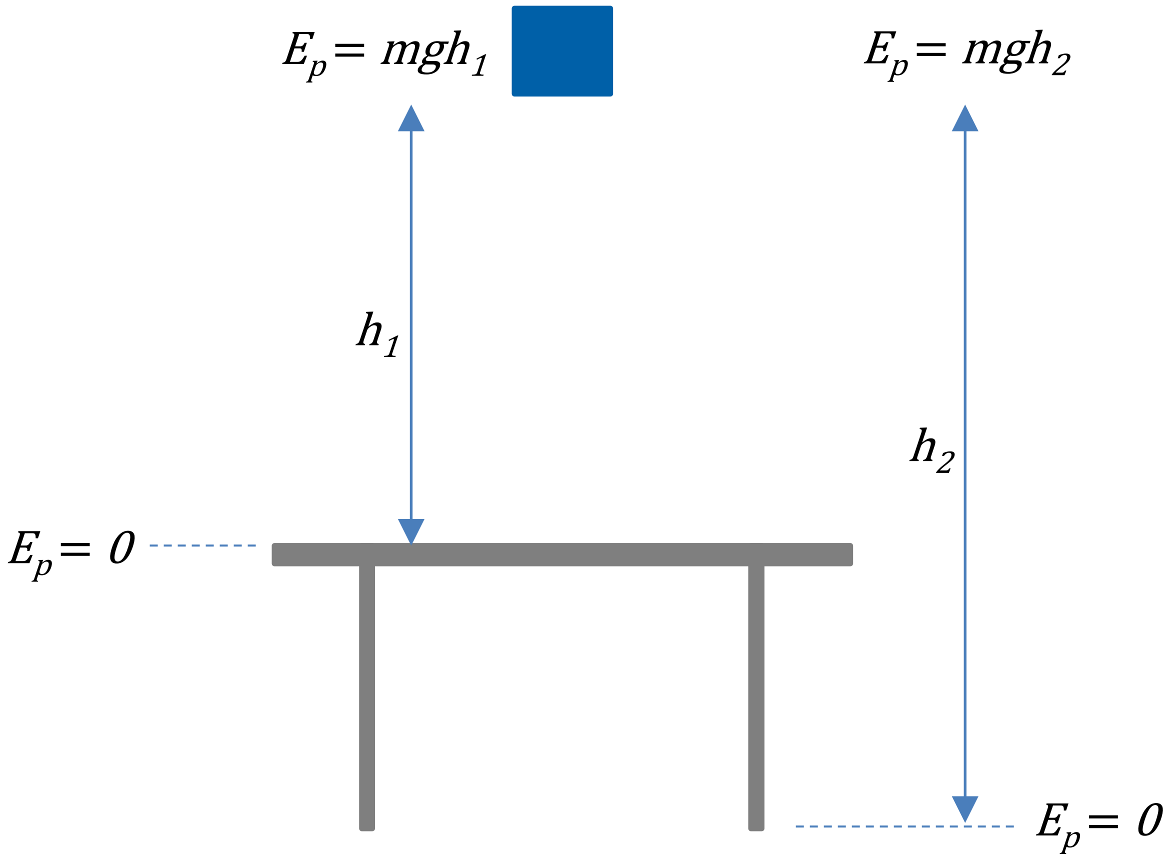
- it depends not only on the magnitude of the charge but also on the type of charge i.e. whether the charge is positive or negative

2.1 electrical potential energy & potential



2.2. potential and potential difference

- to know the absolute potential energy of an object at a given point, we need to define a zero reference point


$E_p = mgh_1$
$E_p = mgh_2$
h_1
h_2
$E_p = 0$
$E_p = 0$

2.2. potential and potential difference

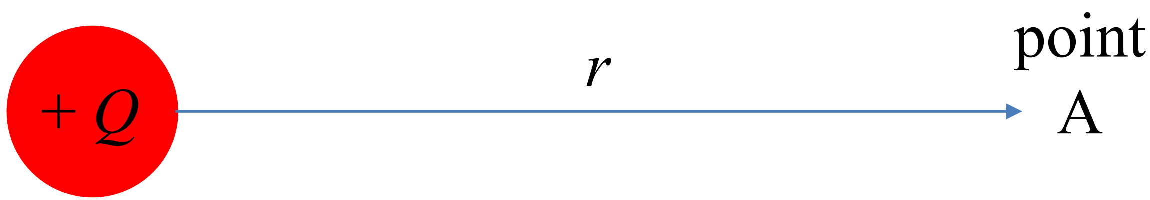
- similarly, to know the absolute electric potential at a given point, we need to define our zero reference
- for a single charge Q, the zero reference (i.e. the point at which the potential due to Q is zero) is a point an infinite distance from Q
- absolute electric potential at a given point is thus formally defined as:

“the amount of energy required to bring a unit positive charge from infinity to that point”

2.2. potential and potential difference

the electric potential a distance r from a point charge Q can be shown (by integration) to be:

$$V = \frac{1}{4\pi\varepsilon}\frac{Q}{r} \qquad [\mathrm{V}]$$



V_A is positive

2.2. potential and potential difference

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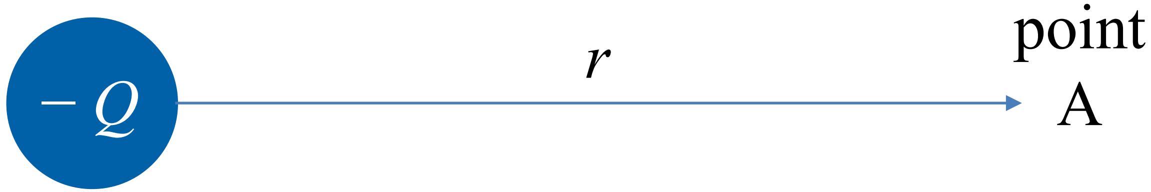


V_{A} is negative

2.2. potential and potential difference

potential difference, V

- often it is more convenient to talk about potential difference rather than absolute potential
- the potential difference between two points is simply the difference in the potential-energy-per-unit-charge at each point

2.2. potential and potential difference

potential difference, V

- imagine an object with charge Q that moves between two points that have a potential difference V between them
- the work done moving the object (W) is equal to the change in the electrical potential energy of the object
- since potential difference is defined as the change in electrical potential energy per unit charge, we can write

$$V = \frac{W}{Q} \qquad \text{[V]}$$

2.2. potential and potential difference

potential difference $$V = \frac{W}{Q}$$ [V]

- if the transfer of 1 C of charge between two points results in an energy change of 1 J, then the potential difference between the points is 1 volt (1 V = 1 J C^{-1})

- rearranged as $$W = QV$$ [J]

  this equation tells us the energy input (or output) when a charge Q is moved across a potential difference of V volts

2.3 potential difference and work done

- previously we have seen that

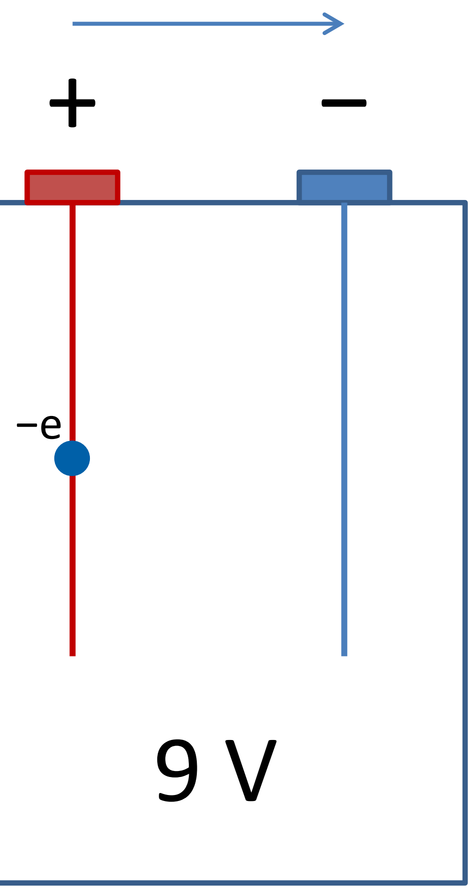
$$W = QV \qquad \text{[J]}$$

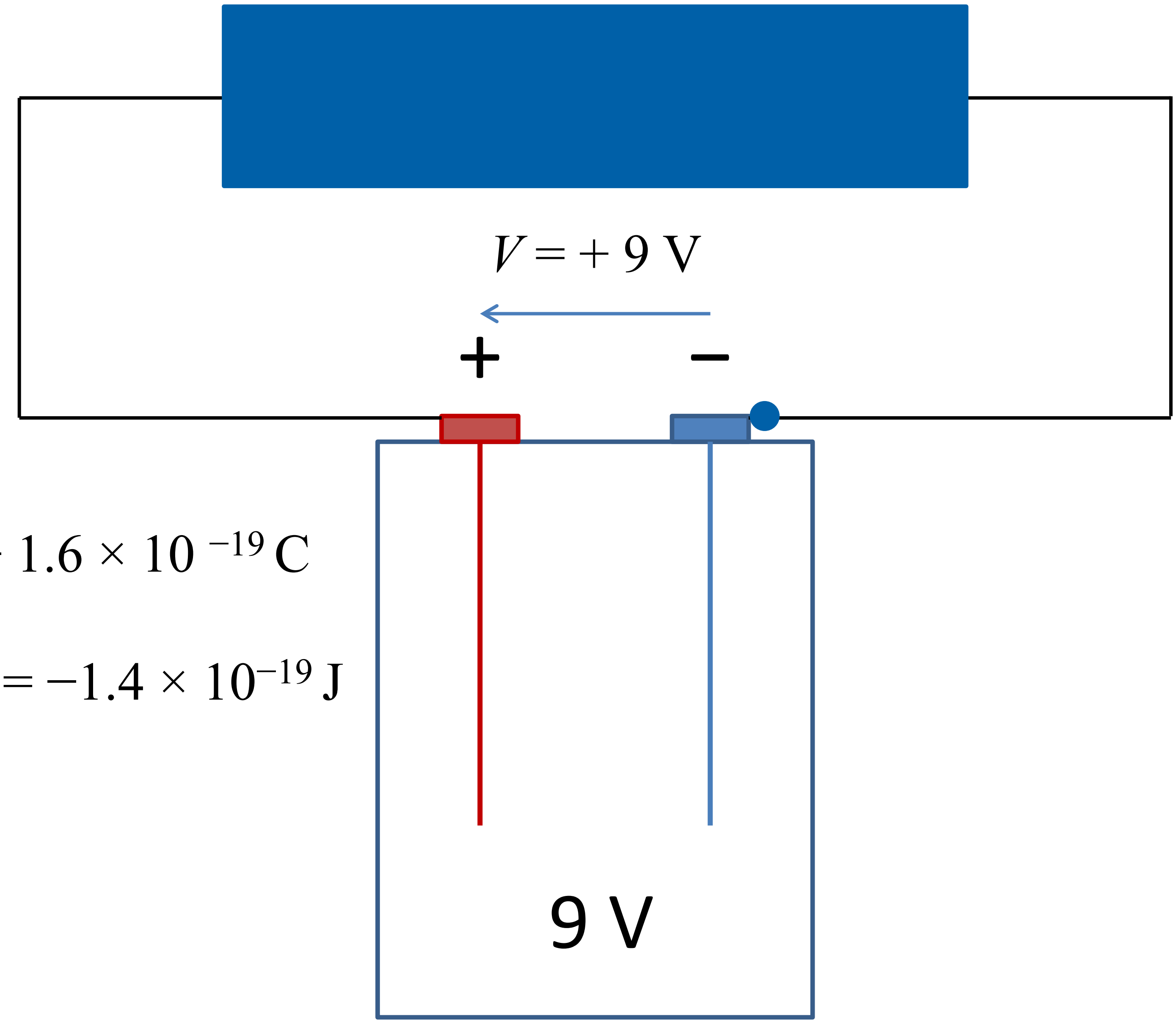
- this is a very useful general equation, which tells us the energy change when a charge Q is moved through a potential difference V

$$W = QV = (-1.6 \times 10^{-19}) \times (-9) = +1.4 \times 10^{-19}\,\mathrm{J}$$

$$Q = -\,1.6 \times 10^{-19}\,\mathrm{C}$$

$$V = -\,9\ \mathrm{V}$$





$$Q = -1.6 \times 10^{-19}\ \mathrm{C}$$

$$W = QV = -1.4 \times 10^{-19}\ \mathrm{J}$$

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there are two significant implications of this equation

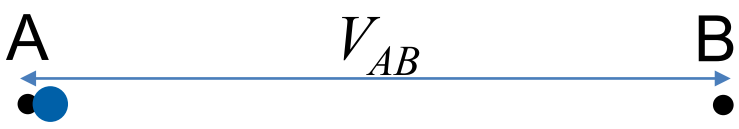
- the work done is independent of the path taken

$W = QV$ $\Rightarrow$ work done is independent of the path taken

$V = V_{AB}$

$Q = -1.6 \times 10^{-19}\,\mathrm{C}$

$W = -\,1.6 \times 10^{-19} \times V_{AB}$ joules

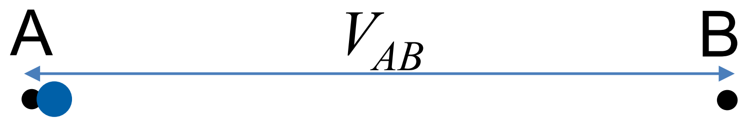


$W = QV$ $\Rightarrow$ work done is independent of the path taken

$V = V_{AB}$

$Q = -1.6 \times 10^{-19}\,\mathrm{C}$

$$W = -\,1.6 \times 10^{-19} \times V_{AB} \text{ joules}$$



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$$W = QV \qquad \text{[J]}$$

- this is a very useful general equation, which tells us the energy change when a charge Q is moved through a potential difference V

there are two significant implications of this equation

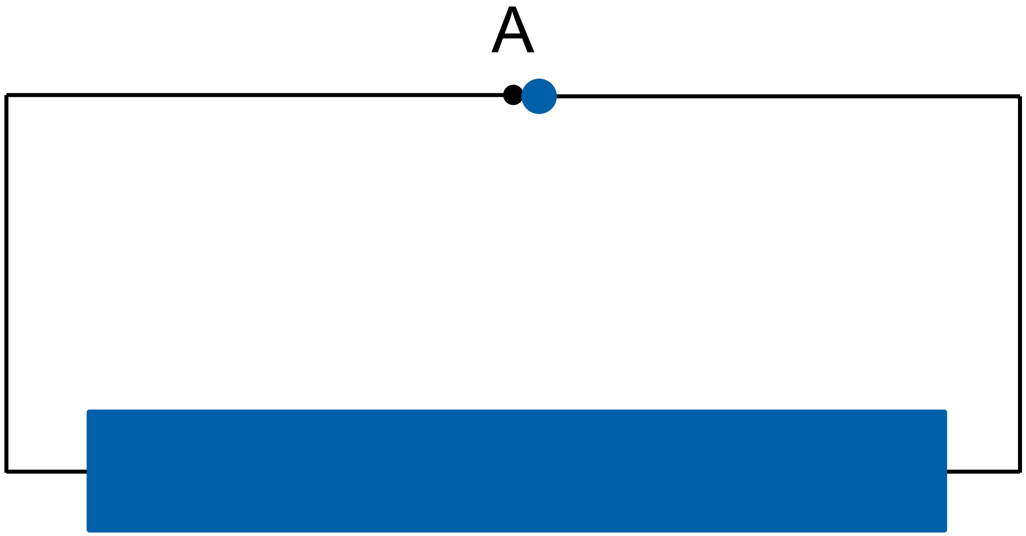
- the work done is independent of the path taken
- the net work done in taking a charged particle around a closed path is zero

$W = QV$ $\Rightarrow$ work done around a closed path is zero

$V = 0$

$W = 0$

$Q = -\mathrm{e}$



2.3 potential difference and work done

- previously we have seen that

$$W = QV \qquad [\mathrm{J}]$$

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there are two significant implications of this equation

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2.3 potential difference and work done

the electron volt (eV)

- the electron volt is an alternative, non-SI unit of energy
- 1 electron volt is the change of energy when 1 unit of elementary charge (e) is moved across a potential difference of 1 volt (or when an electron is moved across a potential difference of −1 volt)

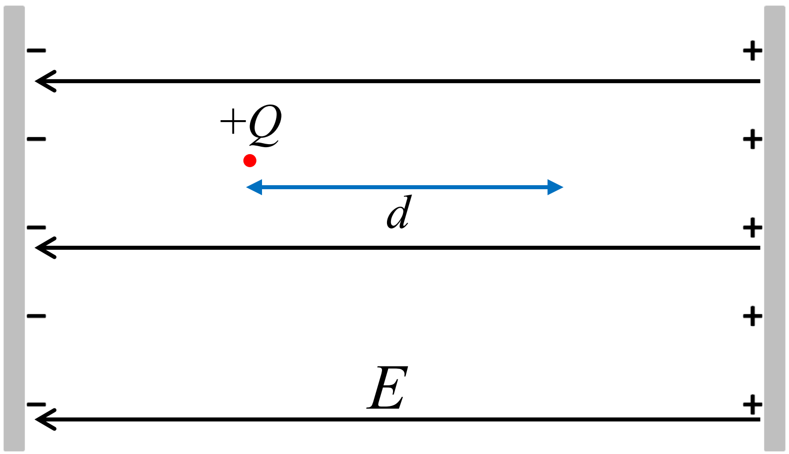
$$W\ [\mathrm{J}] = Q\ [\mathrm{C}] \times V\ [\mathrm{J\ C^{-1}}] \quad \text{remember } 1\ \mathrm{V} = 1\ \mathrm{J\ C^{-1}}$$

$$W\ [\mathrm{eV}] = Q\ [\mathrm{e}] \times V\ [\mathrm{V}] \qquad \mathrm{e} = 1.6 \times 10^{-19}\ \mathrm{C}$$

So $$1\ \mathrm{eV} = +\ 1.6 \times 10^{-19}\ \mathrm{J}$$

2.4 potential difference in a uniform field

- charge Q moves distance d parallel to a uniform electric field E

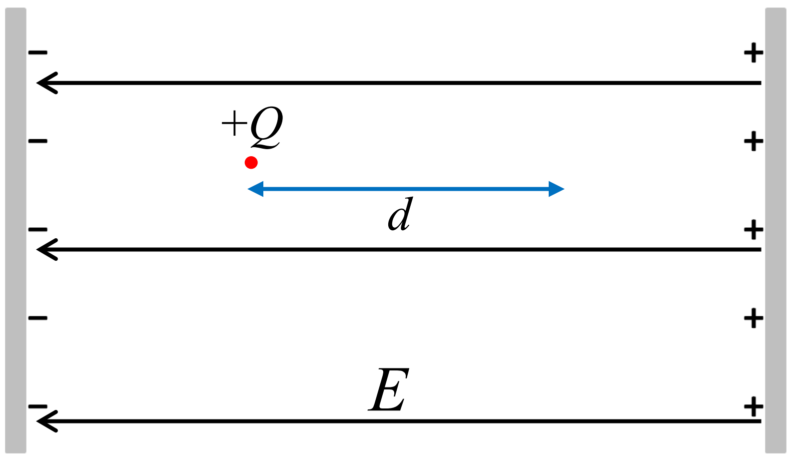


- work done (W) = change in electrical potential energy
- work done = force × distance

$$E = \frac{F}{Q} \Rightarrow F = QE = constant$$

So $$W = F \times d \Rightarrow W = \underbrace{QEd}_{\text{change in electrical potential energy}}$$

2.4 potential difference in a uniform field

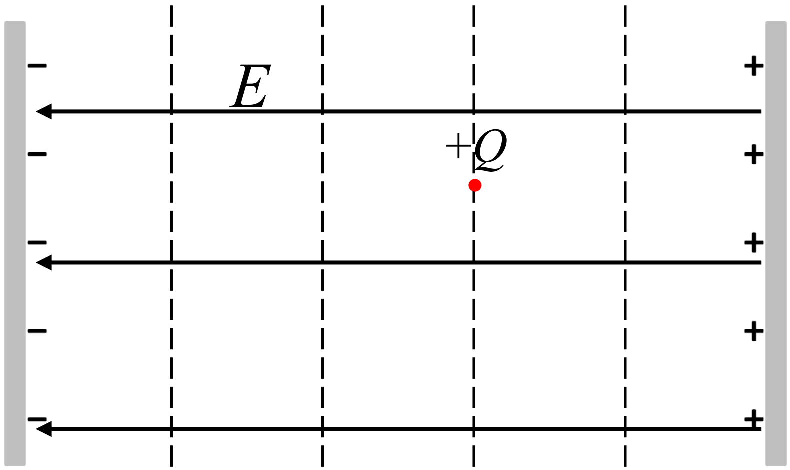


- from previous slide: $W = QEd$
- we also know that: $W = QV$
- equating these gives: $V = Ed$

Rearranging, $E = \dfrac{V}{d}$ [V m^{-1}] where 1 V m^{-1} = 1 N C^{-1}

- this is a useful expression for calculating the strength of a uniform field, given the potential difference between two points separated by a distance d

2.5 equipotential lines and surfaces



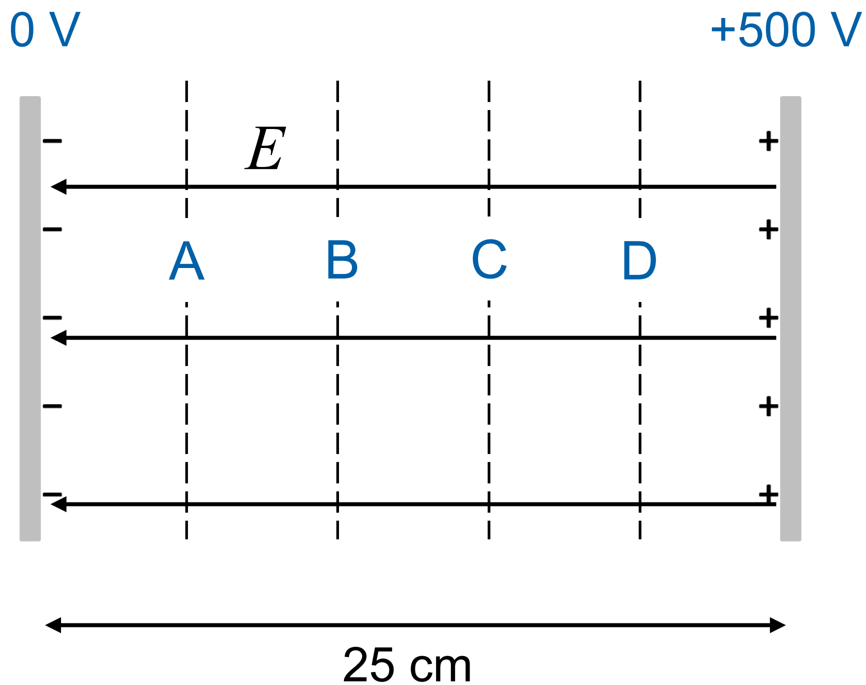
- if Q moves perpendicular to the field lines, d (the distance moved in the direction of the field) is zero

  so $W = Fd = 0$

  and $V = Ed = 0$

- the dotted lines are lines of equipotential

2.5 equipotential lines and surfaces



Let E = 2000 V m^{-1}

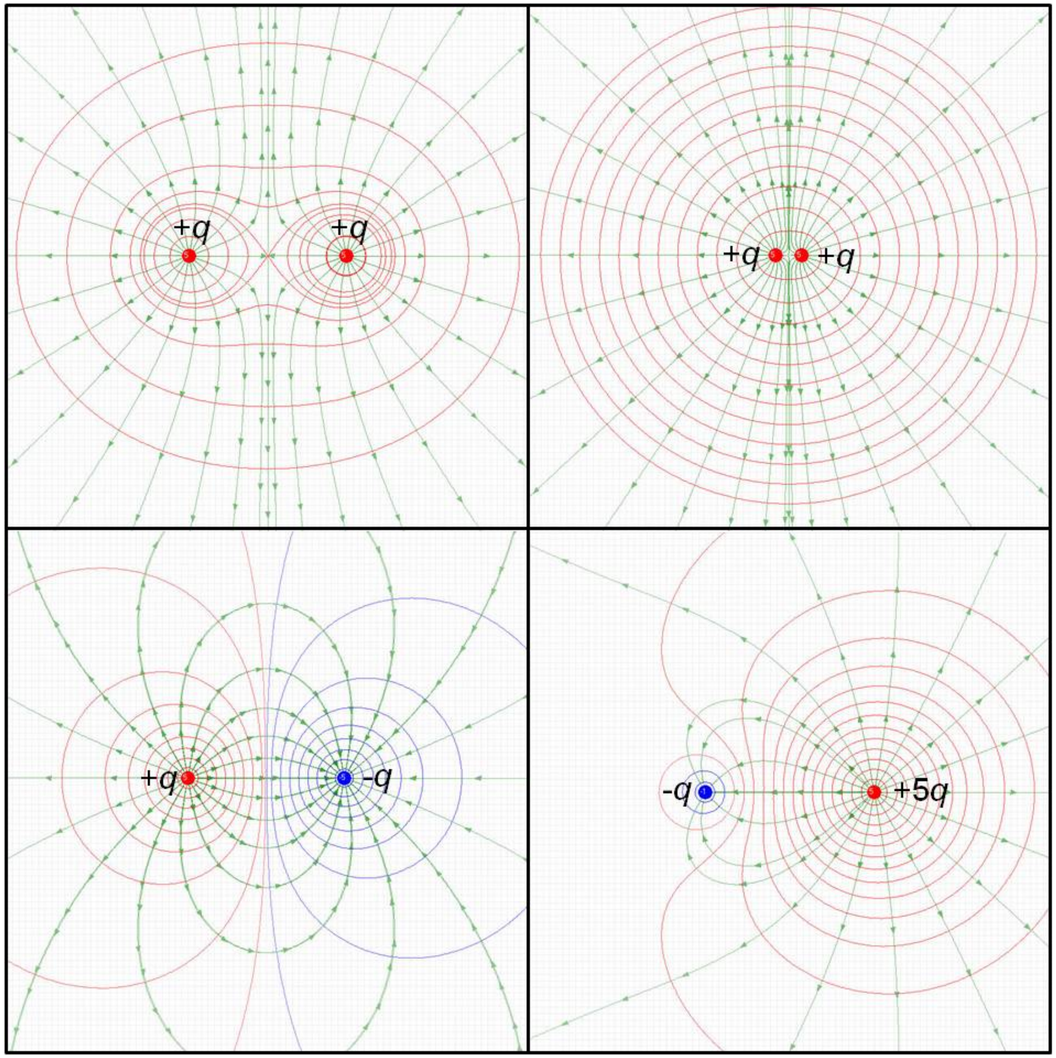
- what is the potential of the right-hand plate?
- what is the potential at points A, B, C and D?
- how much electrical potential energy would a +0.1 C charge have at point C?

$$V = \frac{E_{p,elec}}{Q} \Rightarrow E_{p,elec} = QV = 0.1 \times 300 = 30\ \mathrm{J}$$

2.5 equipotential lines and surfaces

- all points on an equipotential line or surface have the same potential
- a charge can move along an equipotential line or surface without changing its potential energy ($W = 0$)
- equipotential lines and surfaces are always perpendicular to electric field lines
- in electrostatic equilibrium, the surface of a conductor is an equipotential surface

2.5 equipotential lines and surfaces

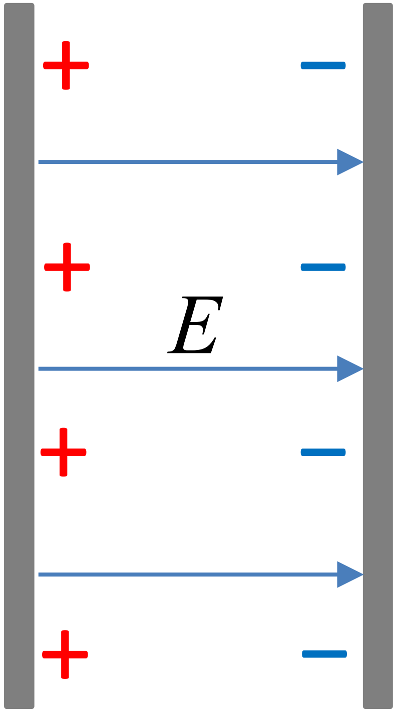


3.1 capacitors

- a capacitor is a device that stores electrical charge
- the most common type (and the best for explaining the principles of capacitance) is the parallel-plate capacitor

3.1 capacitors

a parallel plate capacitor



3.1 capacitors

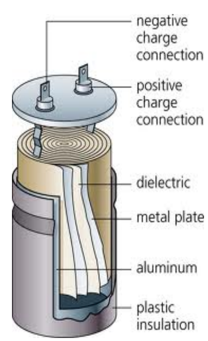
- a capacitor is a device that stores electrical charge

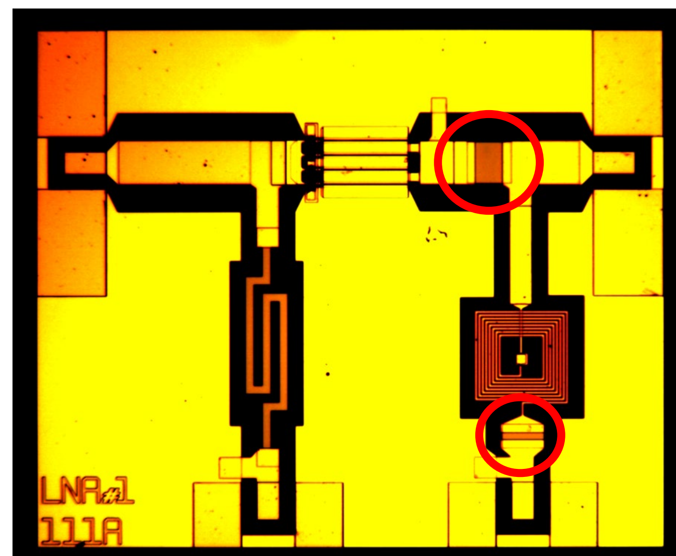
- the most common type (and the best for explaining the principles of capacitance) is the parallel-plate capacitor

- the electric field set up by the charges represents an energy store

- capacitors (along with resistors and inductors) are one of the basic building blocks of circuits

3.1 capacitors





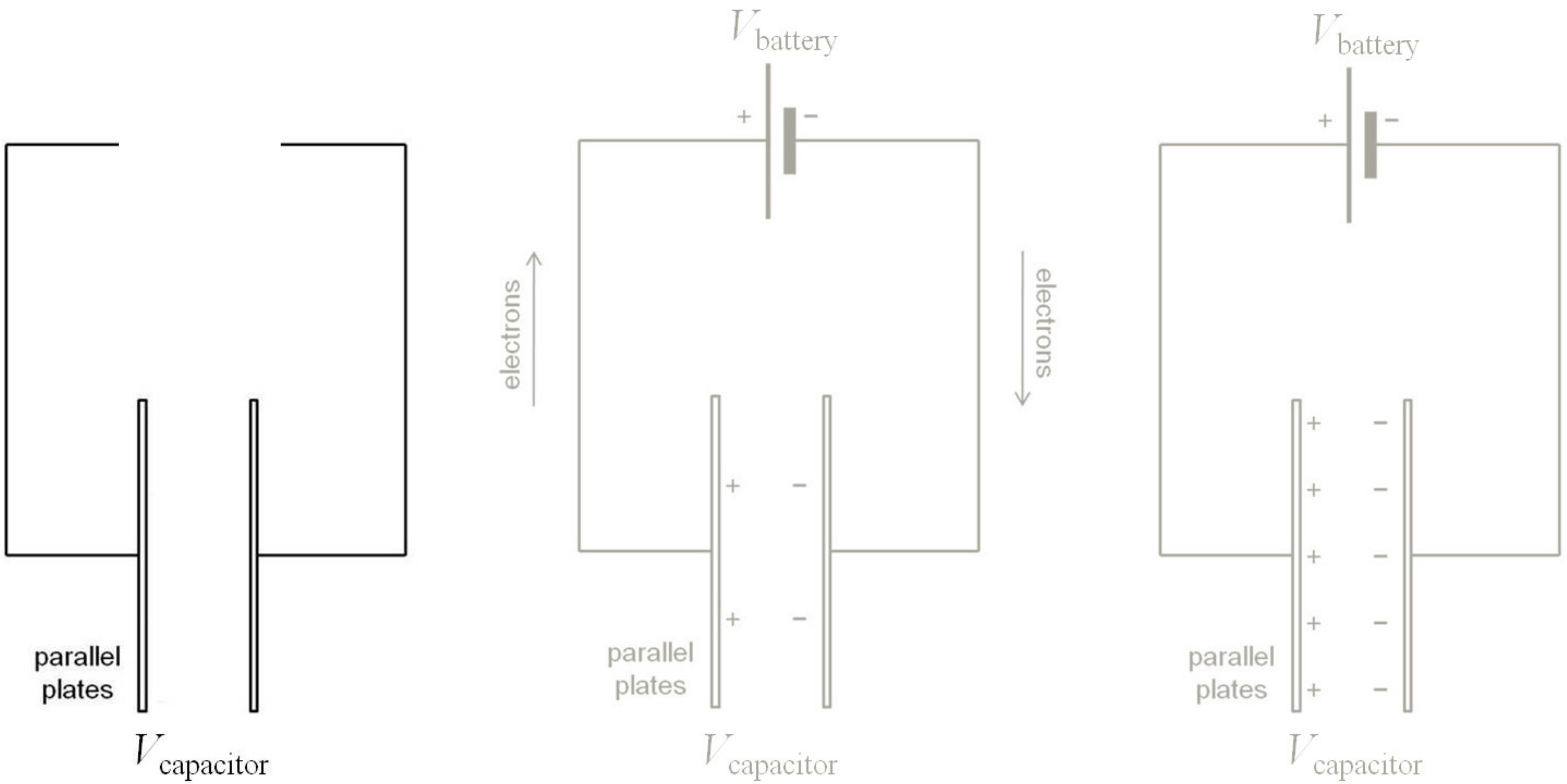
3.2 charging a capacitor

1. initial

2. charging

3. charged

$V_{cap} = \text{zero}$



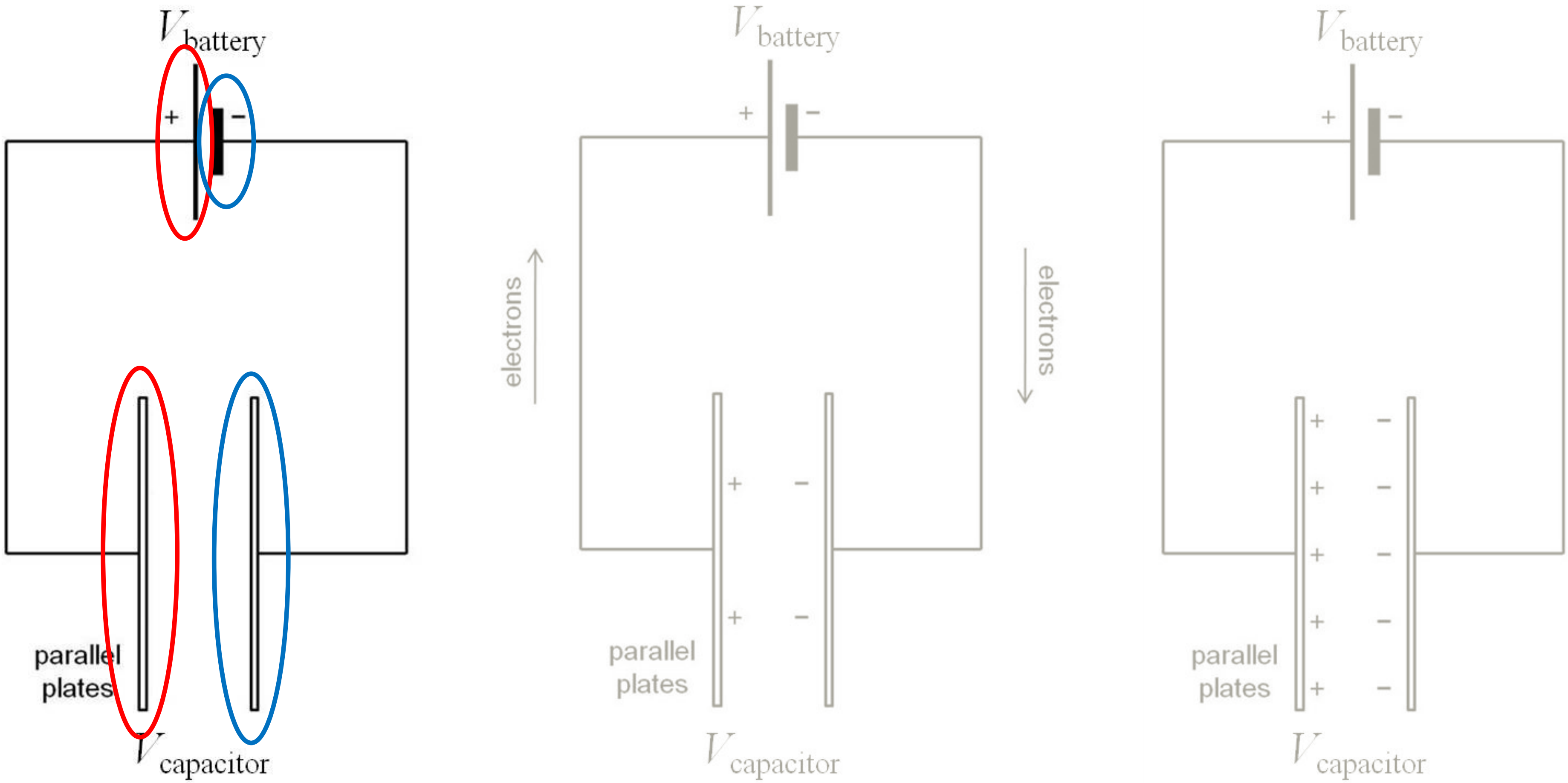
3.2 charging a capacitor

1. initial

2. charging

3. charged

$V_{\text{cap}} = \text{zero}$



3.2 charging a capacitor

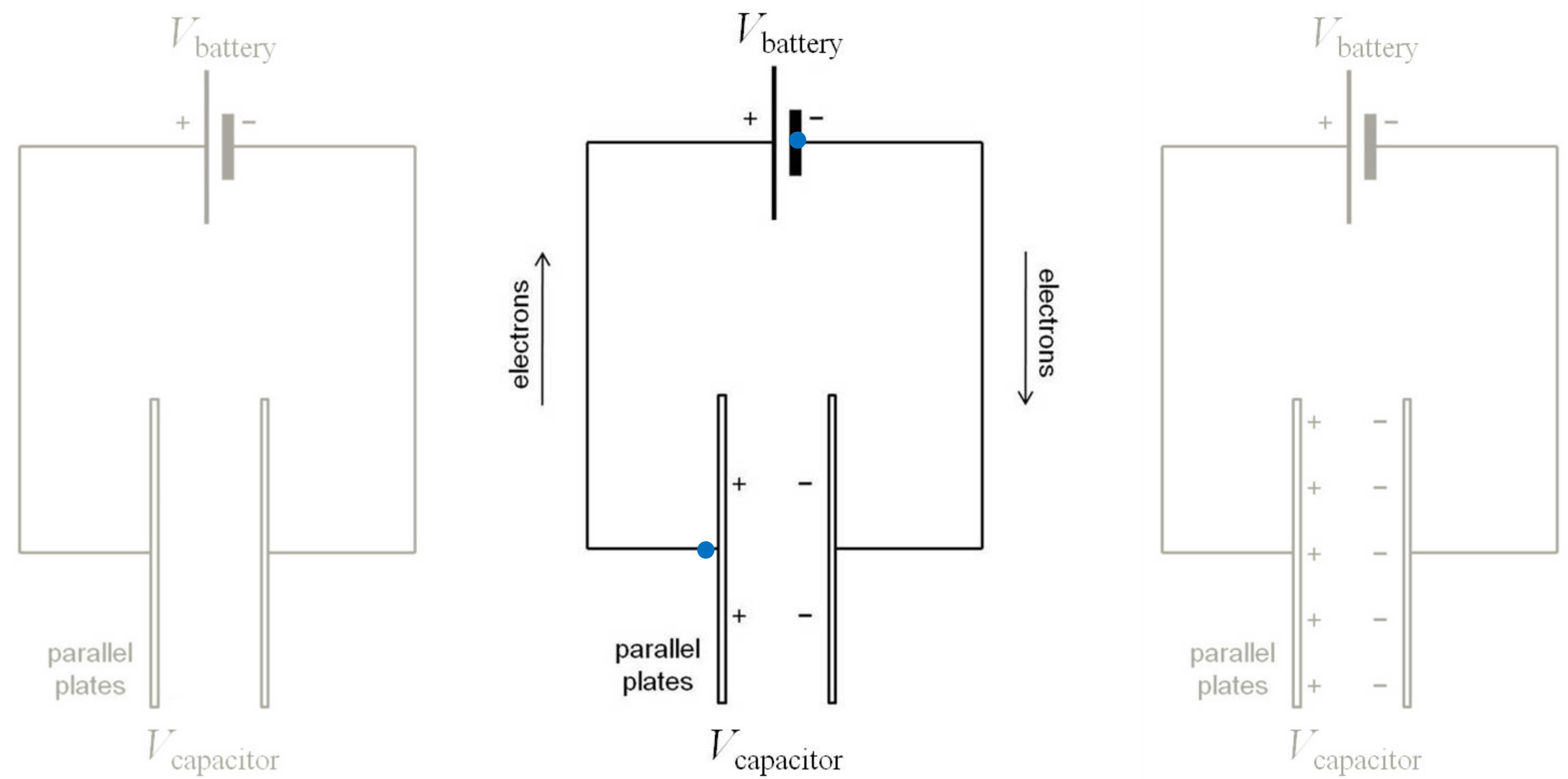
1. initial

$V_{\text{cap}} = \text{zero}$

2. charging

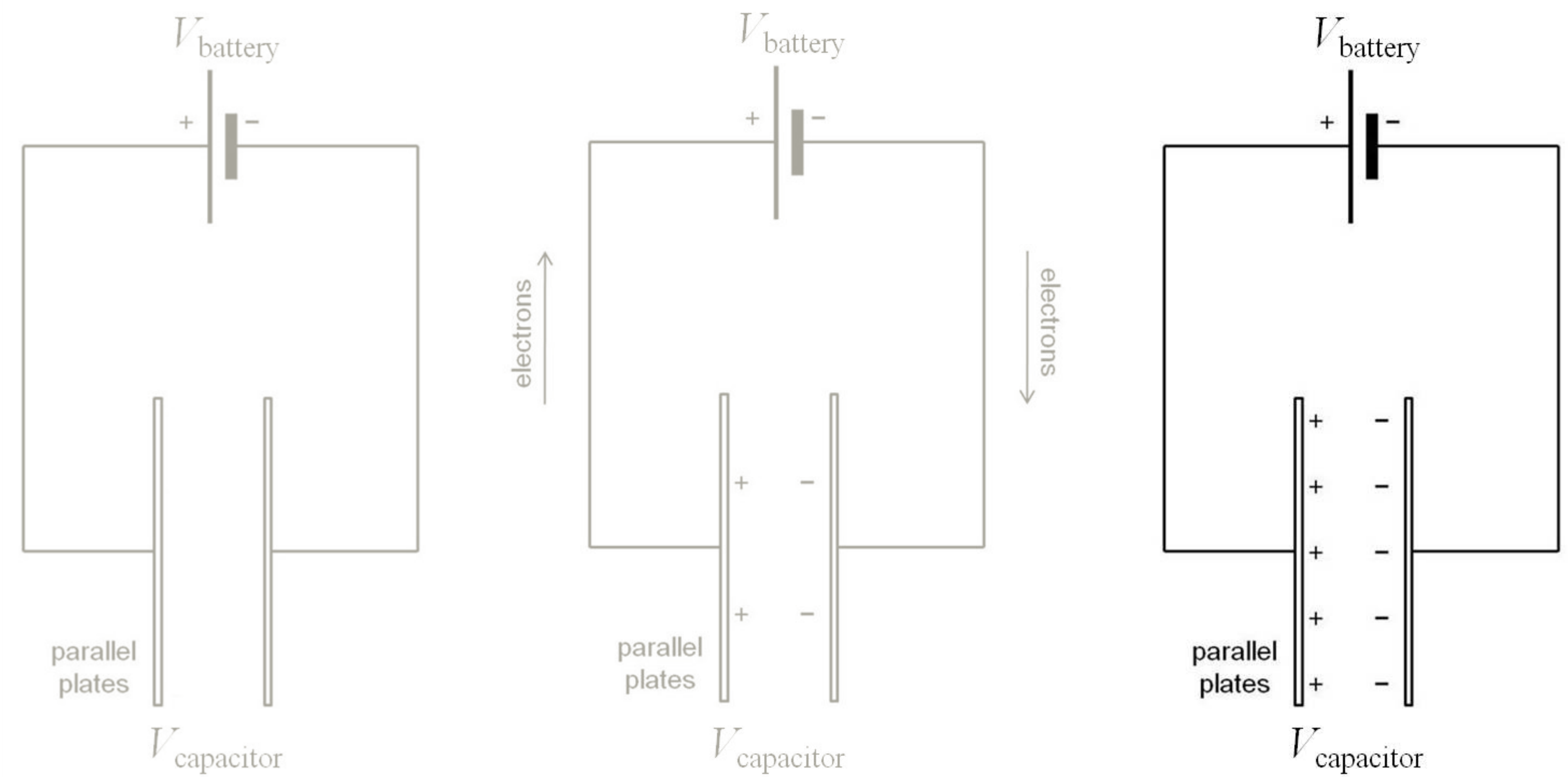
$V_{\text{cap}} < V_{\text{batt}}$

3. charged



3.2 charging a capacitor

1. initial	2. charging	3. charged
$V_{cap} = \text{zero}$	$V_{cap} < V_{batt}$	$V_{cap} = V_{batt}$



3.3 capacitance

- capacitance C measures the ability of a capacitor to store charge

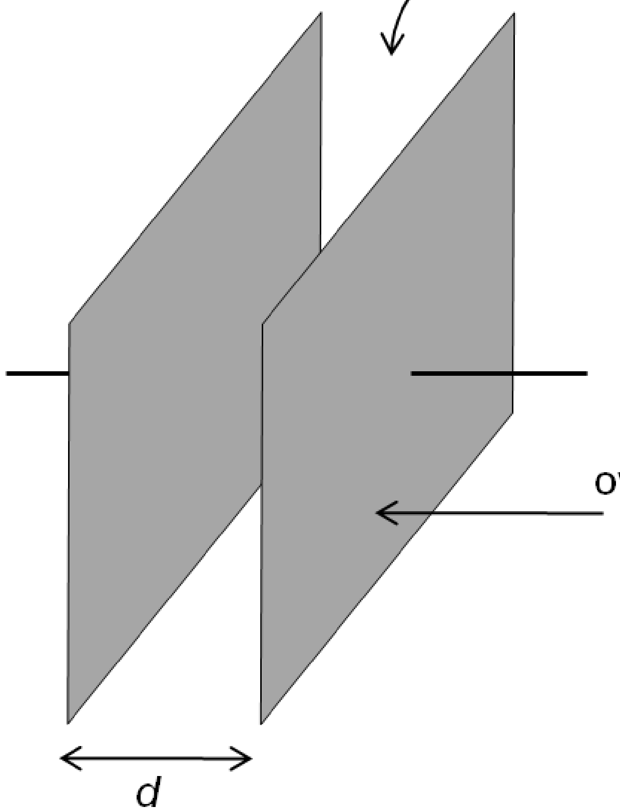
- capacitance is defined as the number of coulombs of charge stored per volt of potential difference ($\mathrm{C\ V^{-1}}$)

$$C = \frac{Q}{V}$$

[Units: $\mathrm{C\ V^{-1}}$ or F]

- rather than $\mathrm{C\ V^{-1}}$ the common unit of capacitance is the farad ($1\ \mathrm{F} = 1\ \mathrm{C\ V^{-1}}$)

3.3 capacitance



- the capacitance of a particular capacitor depends only on its geometry and the permittivity of the material between the plates:

$$C = \frac{\varepsilon A}{d}$$

3.3 capacitance

- the capacitance of a particular capacitor depends only on its geometry and the permittivity of the material between the plates, so

$$C = \frac{Q}{V} = constant$$

or

$$Q = CV$$

- i.e. the charge on the capacitor is directly proportional to the potential difference between the plates, with C as the constant of proportionality

3.4 the dielectric effect

- dielectric materials are insulators
- they respond to, and influence, electric fields
- if a dielectric replaces air between the plates of a capacitor, its capacitance increases by a factor equal to ε_r

3.4 the dielectric effect

With air/vacuum between plates:

$$C_0 = \frac{\varepsilon_0 A}{d}$$

With a dielectric between plates:

$$C = \frac{\varepsilon A}{d} \text{ and } \varepsilon = \varepsilon_r \varepsilon_0 \Rightarrow C = \varepsilon_r \frac{\varepsilon_0 A}{d}$$

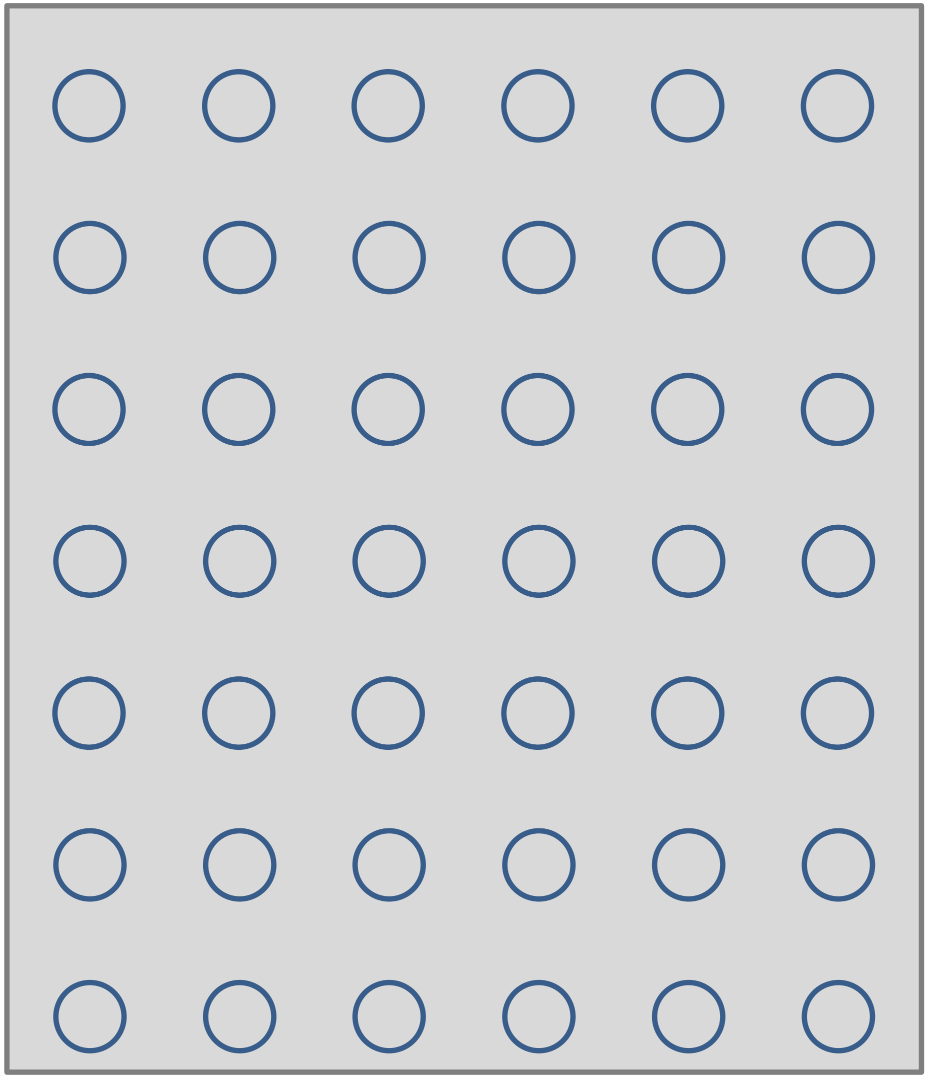
$$\Rightarrow C = \varepsilon_r C_0$$

3.4 the dielectric effect

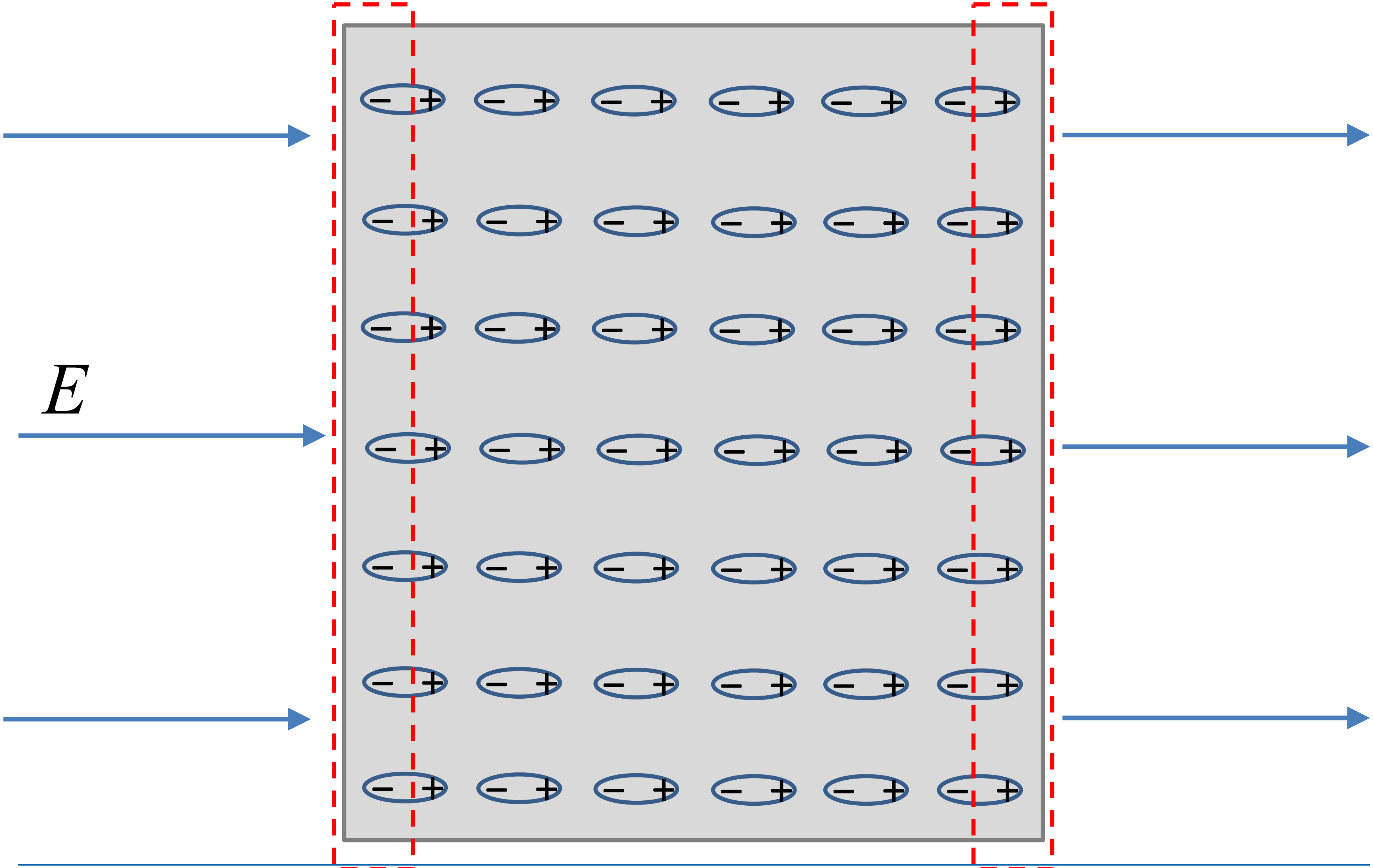
- an electric field distorts the atoms of the dielectric; the positively charged nucleus is pulled in one direction and the orbiting electrons are pulled in the opposite direction, so the atom is slightly polarised

- this produces a localised positive and negative charge on opposite surfaces of the dielectric (without changing the net charge)

3.4 the dielectric effect



3.4 the dielectric effect



3.4 the dielectric effect

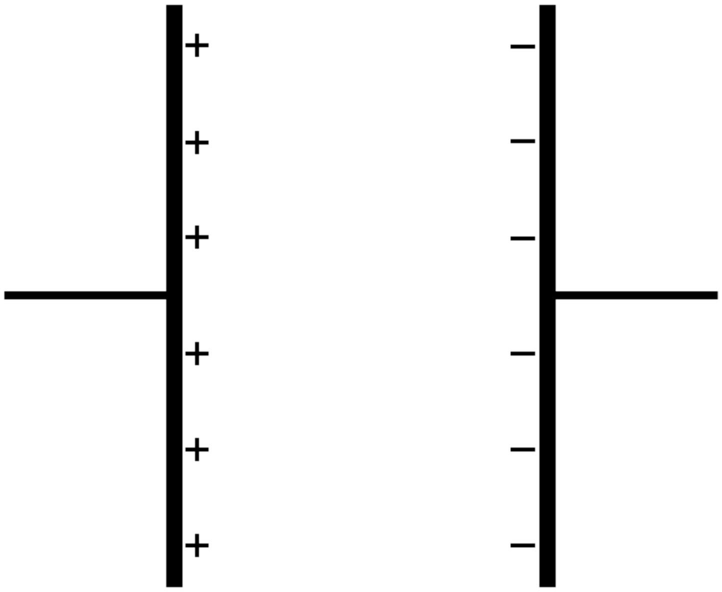
- dielectrics with polar molecules (e.g. water) have a strong dielectric effect (high ε_r)

- it is the localised charge at the surface of the dielectric that increases the capacitance

- the way in which it does this depends on whether the capacitor is isolated, or is connected to a battery

3.4 the dielectric effect

a)



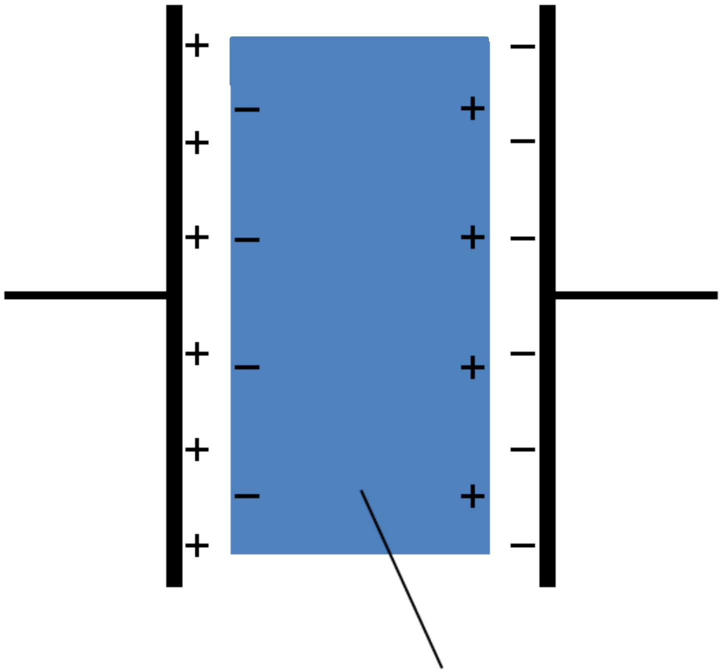
isolated capacitor:

$$C = \frac{Q}{V}$$

- Q cannot change

3.4 the dielectric effect

a)



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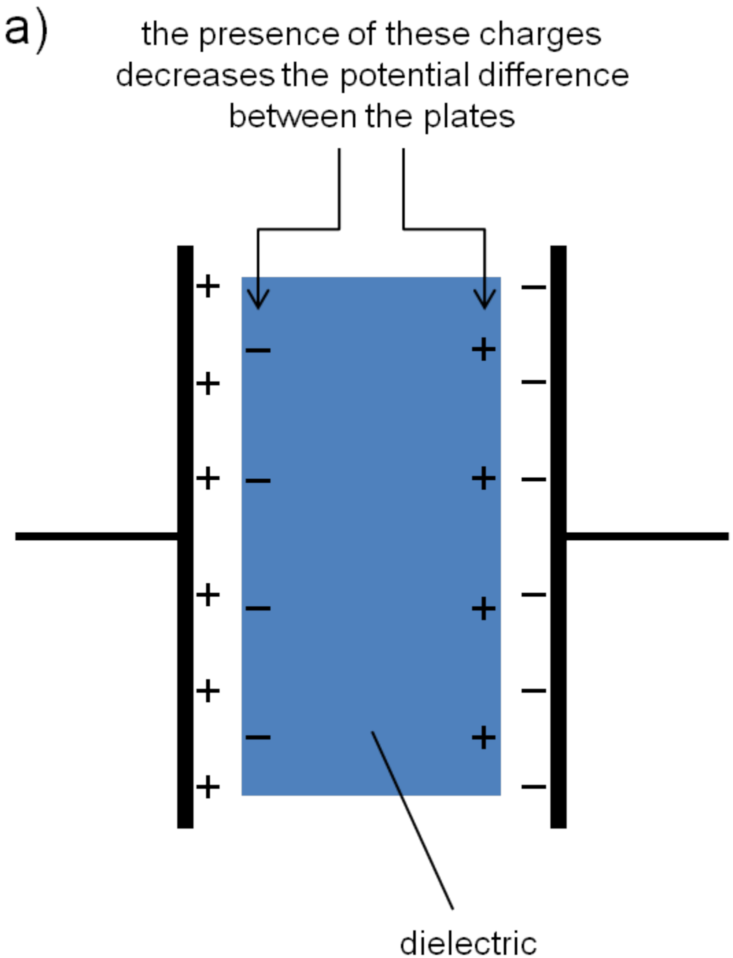
3.4 the dielectric effect

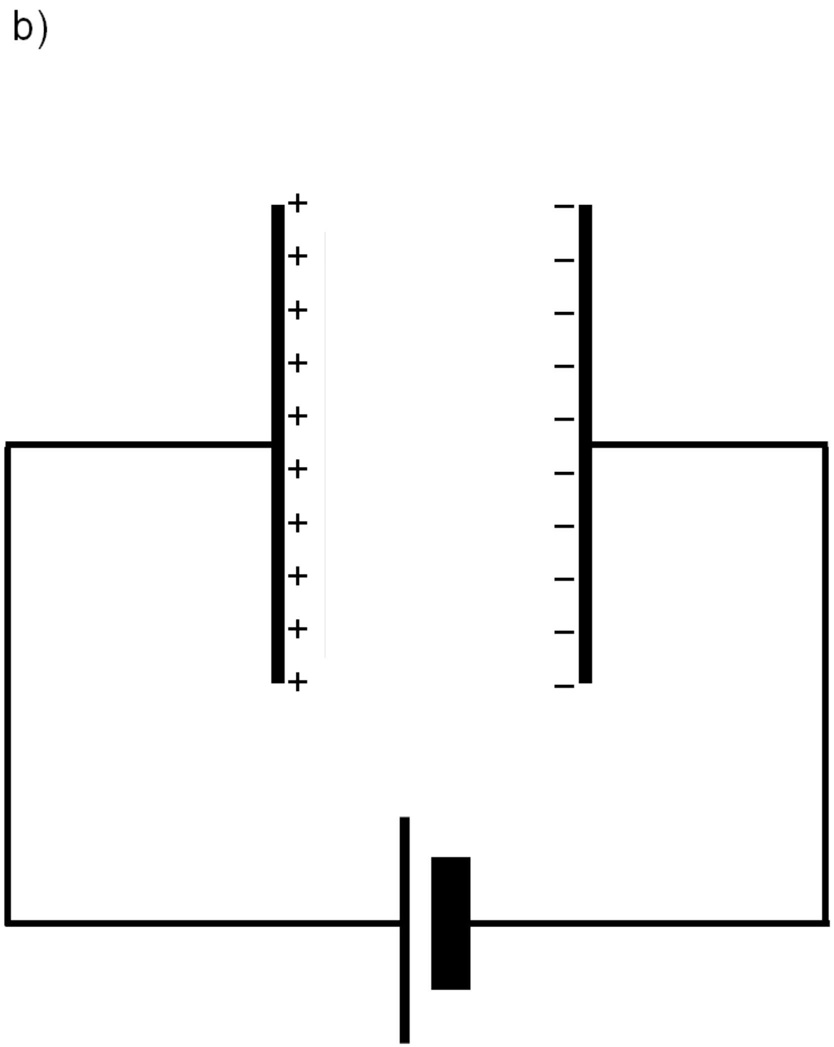


isolated capacitor:

$$C = \frac{Q}{V}$$

- Q cannot change
- charges on the dielectric decrease V across the plates
- therefore C increases

3.4 the dielectric effect

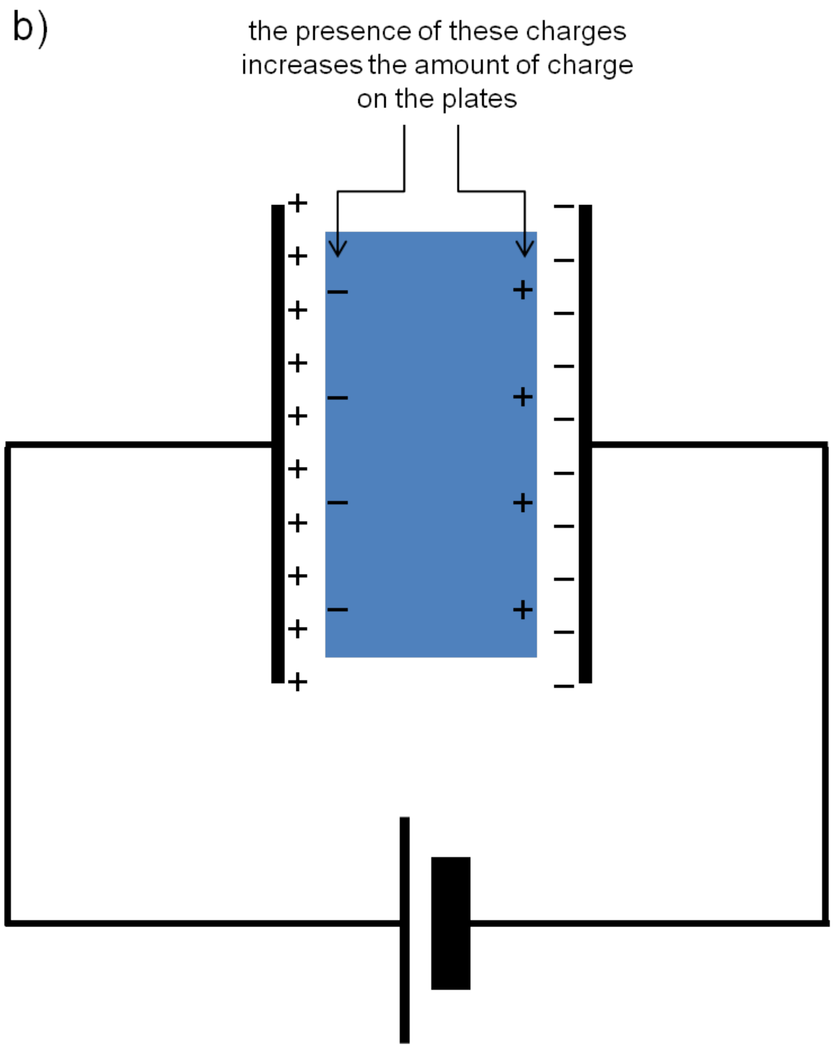


capacitor connected to battery:

$$C = \frac{Q}{V}$$

- battery keeps V_{cap} constant

3.4 the dielectric effect



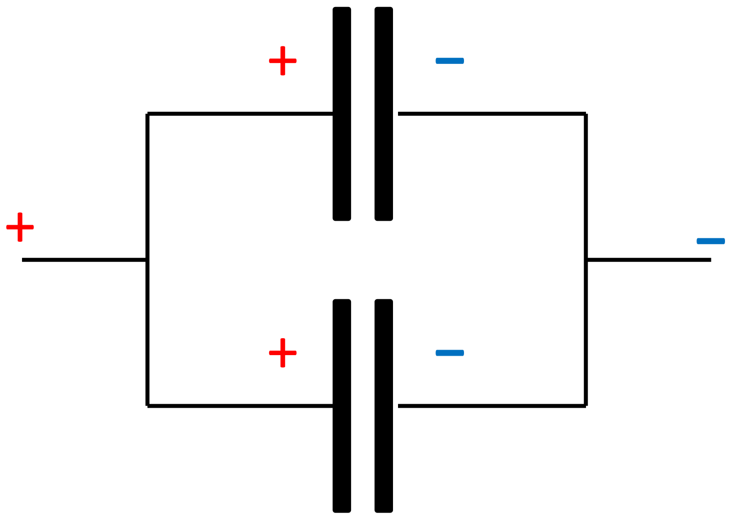
capacitor connected to battery:

$$C = \frac{Q}{V}$$

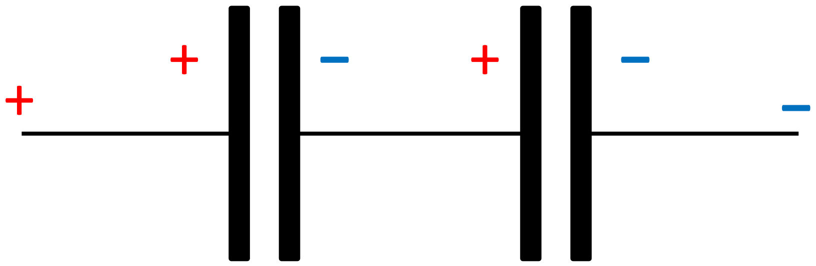
- battery keeps V_{cap} constant
- dielectric charge 'pulls' more charge onto plates, so Q increases
- therefore C increases

3.5 capacitors in parallel

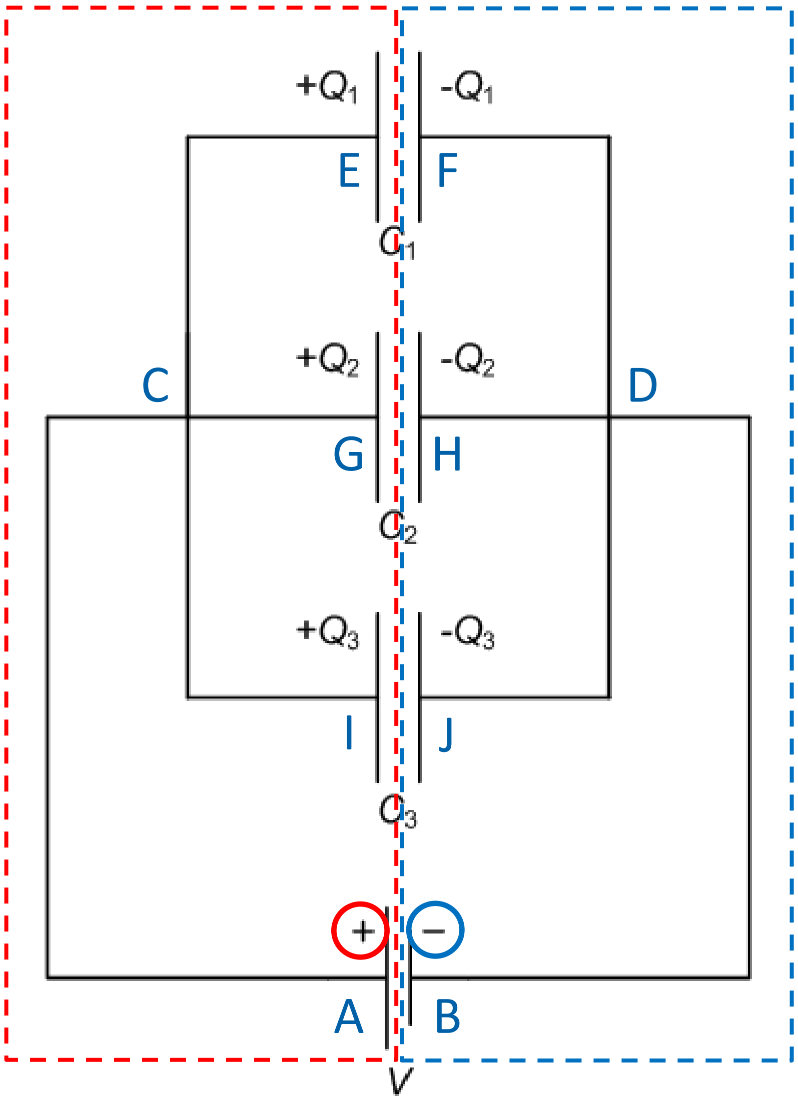
• two capacitors are in **parallel** if the *positive* plate of one capacitor is connected to the *positive* plate of the other (and the negative to the negative)



• two capacitors are in **series** if the *positive* plate of one capacitor is connected to the *negative* plate of the other

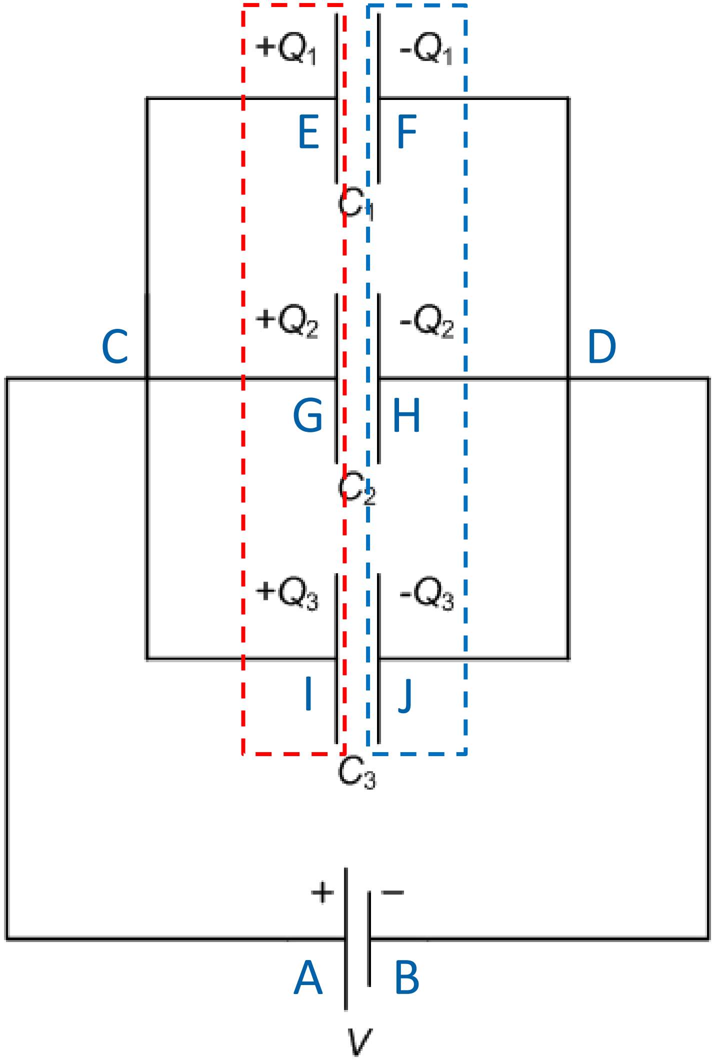


3.5 capacitors in parallel



• the **potential difference is the same** across all capacitors in parallel (and is equal to the total P.D. across the parallel combination)

3.5 capacitors in parallel

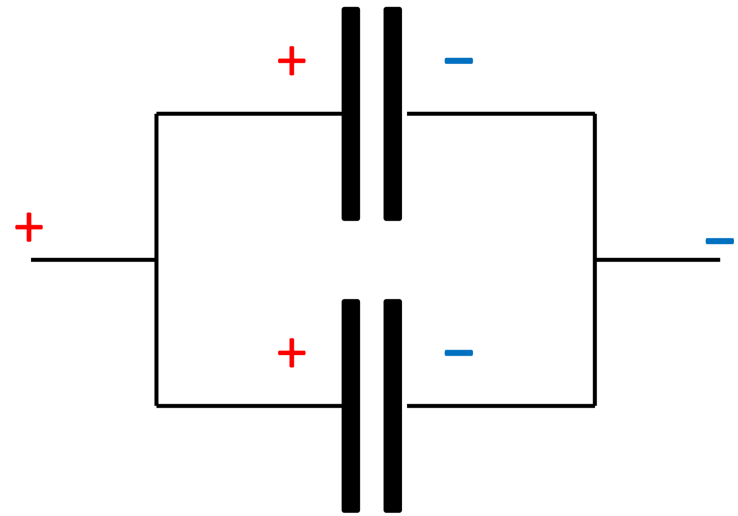


- the **potential difference is the same** across all capacitors in parallel (and is equal to the total P.D. across the parallel combination)
- total **charge is shared** between capacitors in parallel (according to their individual capacitances)
- total capacitance is given by

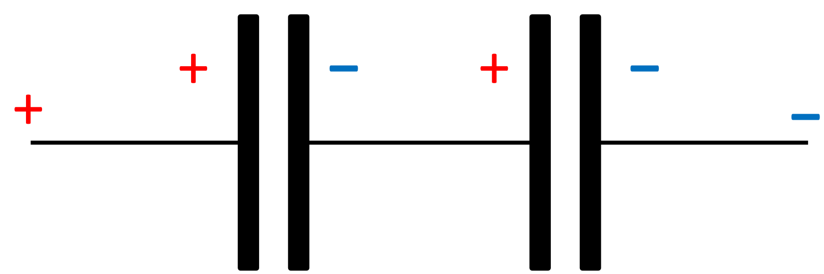
$$C = C_1 + C_2 + C_3 + ... + C_n$$

3.6 capacitors in series

• two capacitors are in **parallel** if the *positive* plate of one capacitor is connected to the *positive* plate of the other (and the negative to the negative)



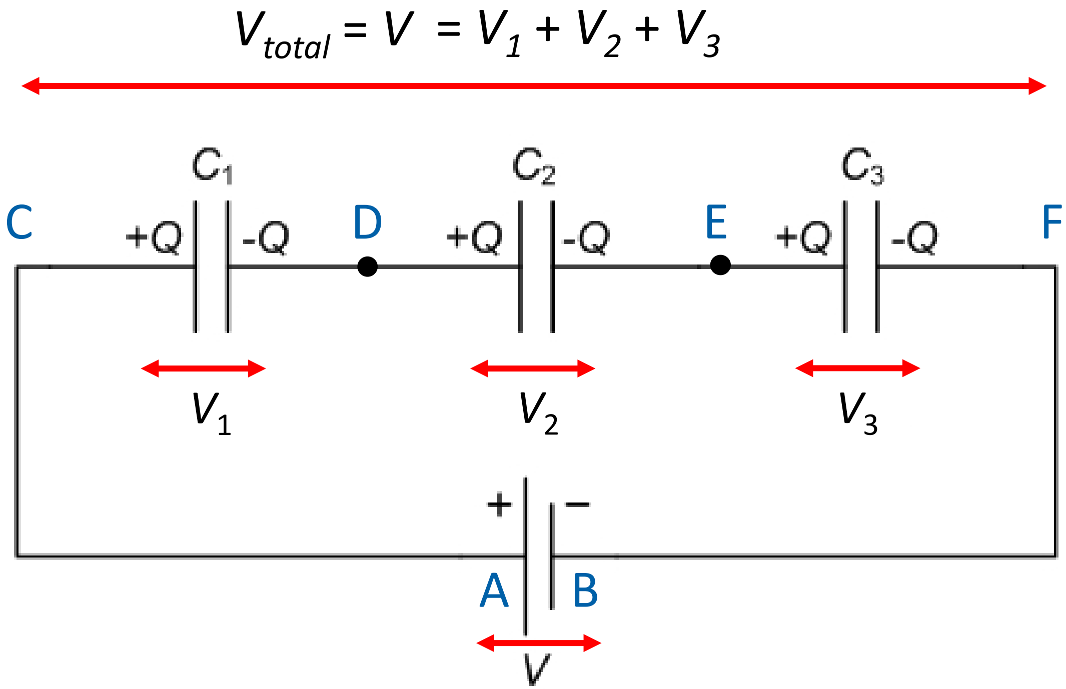
• two capacitors are in **series** if the *positive* plate of one capacitor is connected to the *negative* plate of the other



3.6 capacitors in series

- the **potential difference is split** between capacitors in series (according to the individual capacitances). Total P.D. across the combination = sum of individual P.D.s.

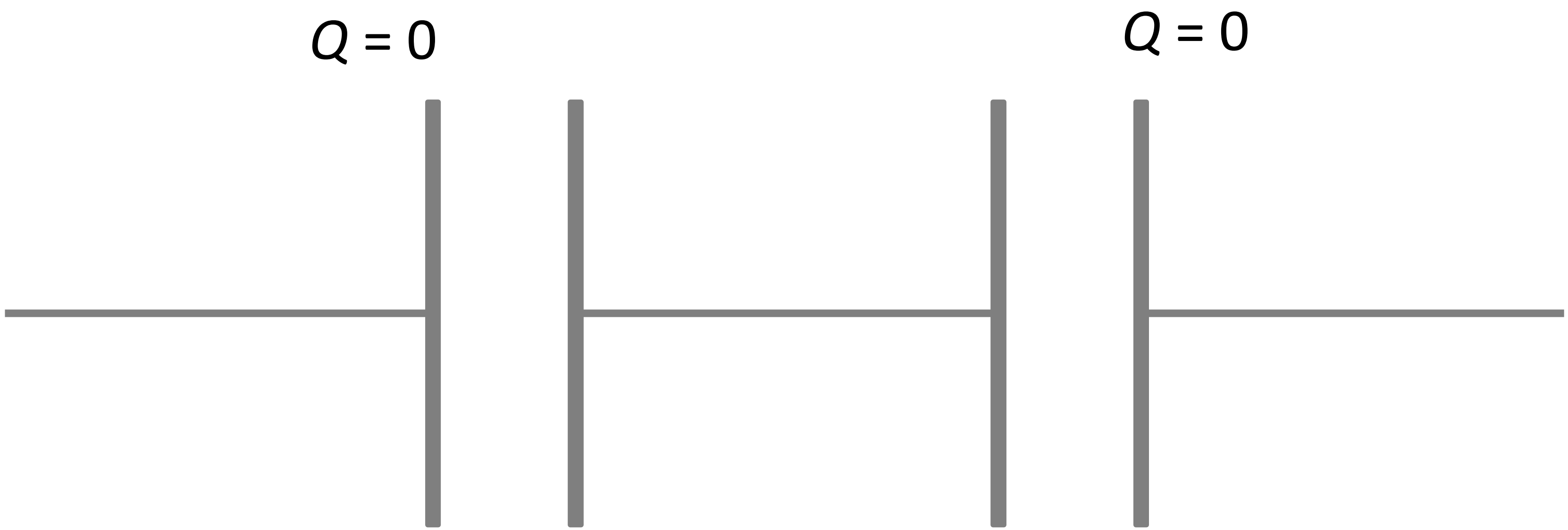
- for capacitors in series, the **charge on each capacitor is the same** (and is equal to the total charge on the series combination)



3.6 capacitors in series

for capacitors in series, the **charge on each capacitor is the same** (and is equal to the total charge on the series combination)

Before charging:



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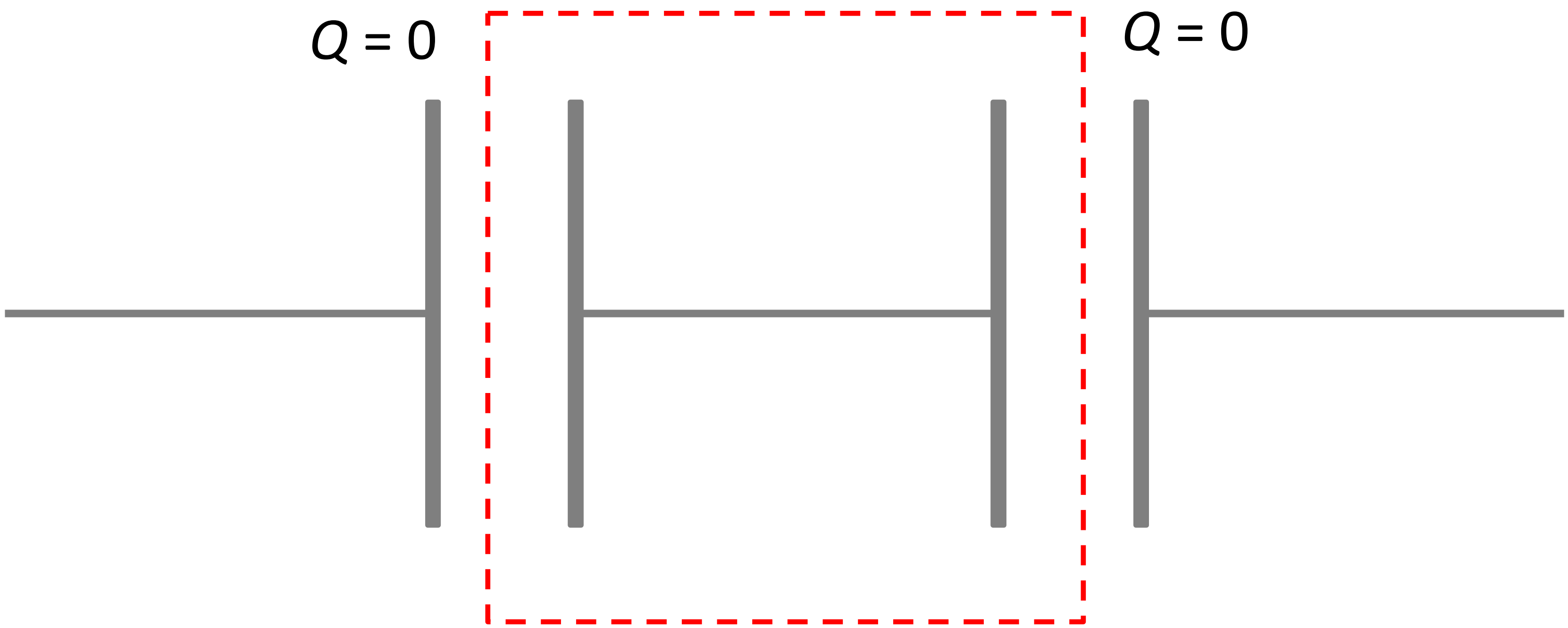

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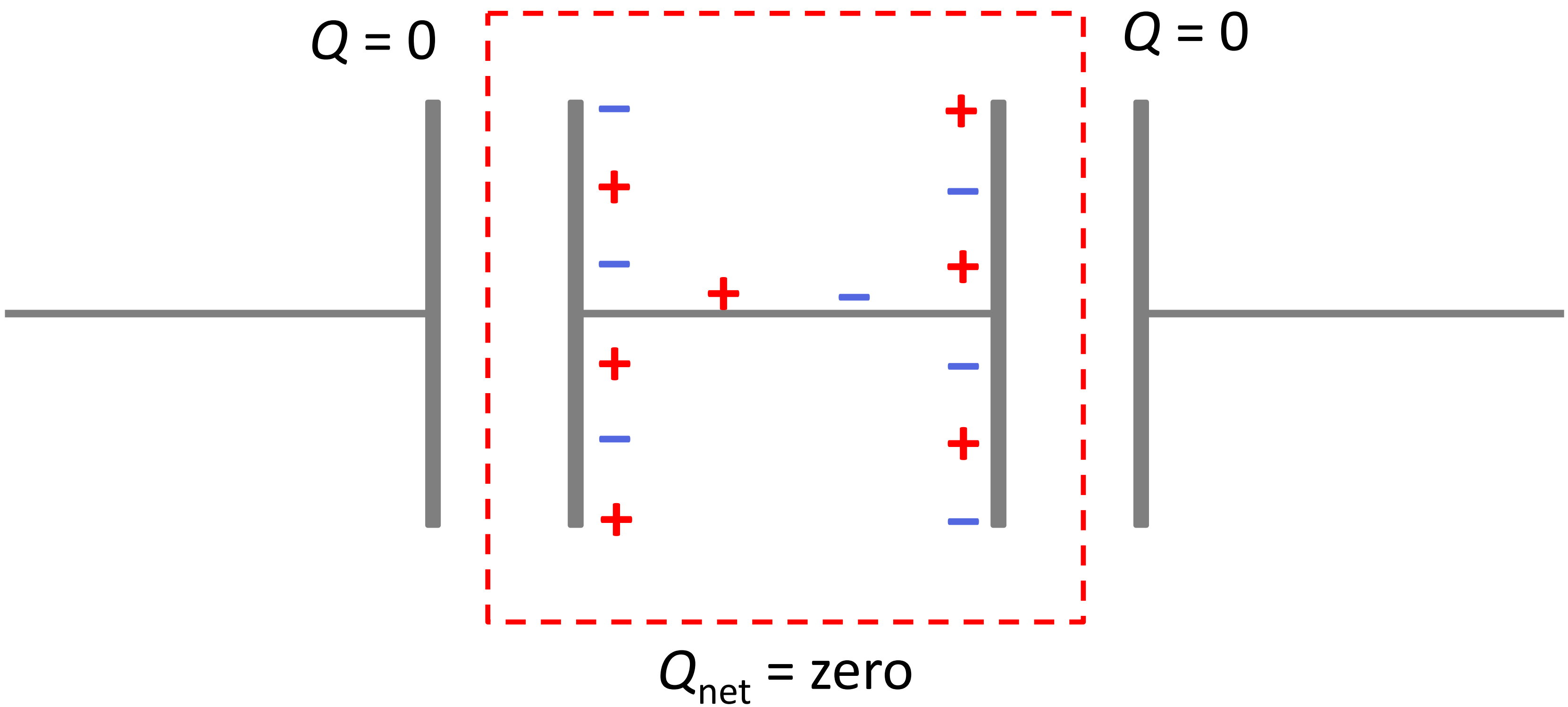

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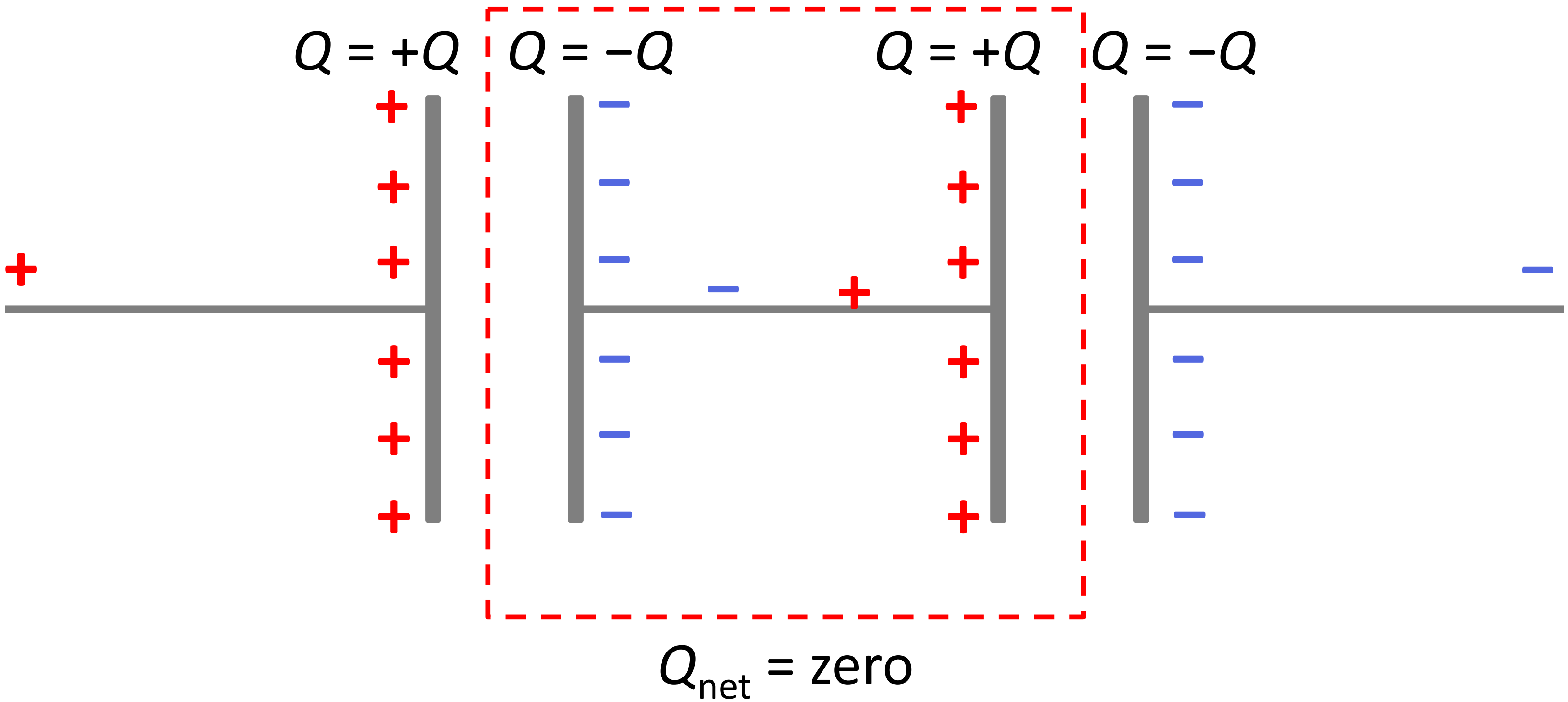

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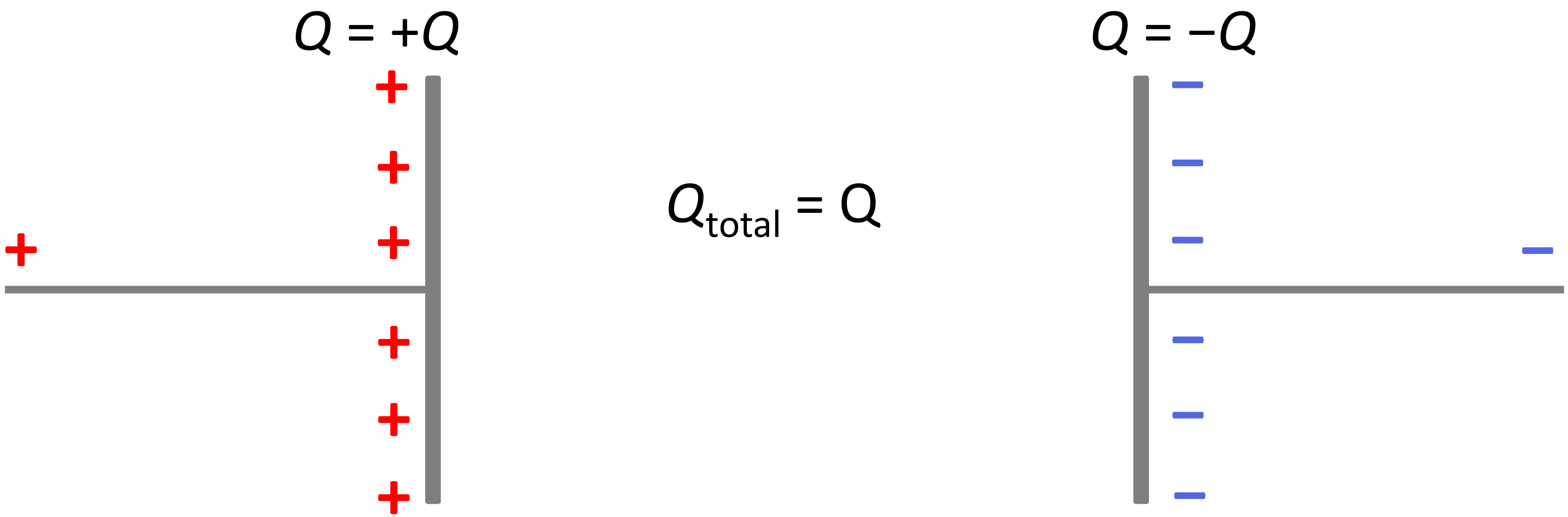

3.6 capacitors in series

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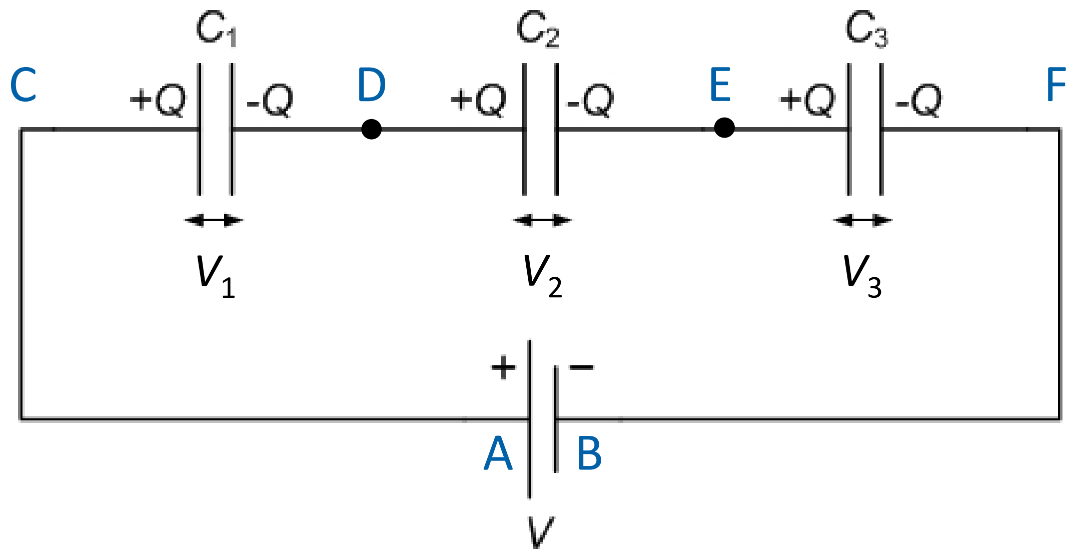


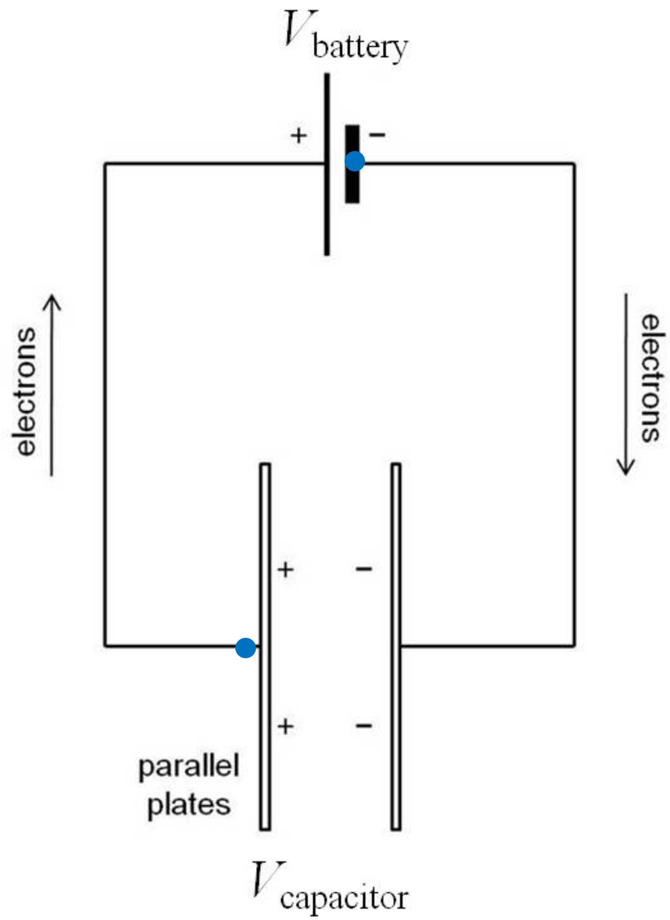
- total capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots + \frac{1}{C_n}$$

3.7 energy stored by a capacitor

- a capacitor stores energy in the form of the electric field between the charged plates
- the process of charging a capacitor is equivalent to moving electrons from the positive plate to the negative plate

3.7 energy stored by a capacitor

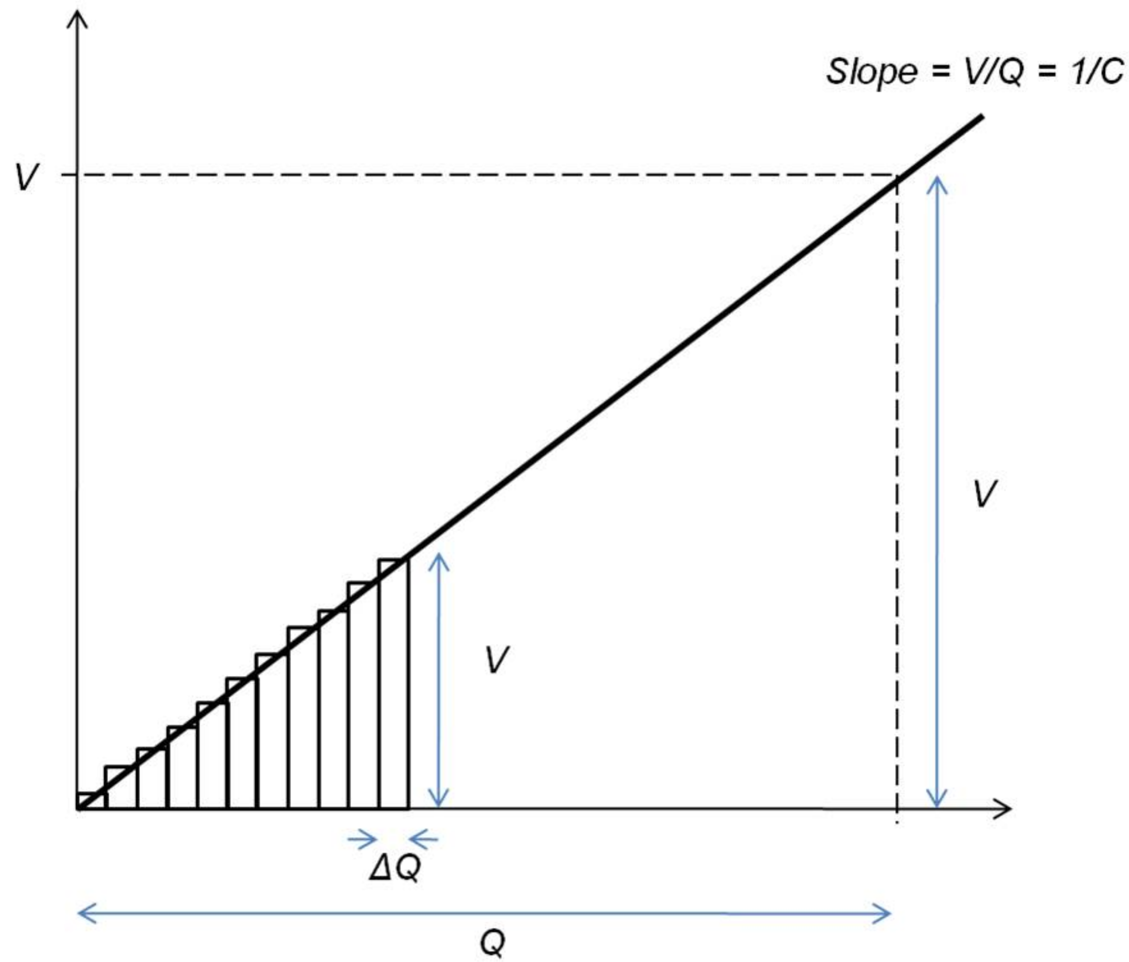


- a battery (or other source of emf) provides the energy
- charging continues until the potential difference between the plates equals the potential difference between the terminals of the emf source

3.7 energy stored by a capacitor

- a capacitor stores energy in the form of the electric field between the charged plates
- the process of charging a capacitor is equivalent to moving electrons from the positive plate to the negative plate
- as the charge builds up, and the potential difference between the plates gets bigger, more and more work is required to move an electron from one plate to the other
- the work input increases the electrical potential energy of the electrons, thus storing energy

3.7 energy stored by a capacitor



$$W = \frac{1}{2}QV$$

or

$$W = \frac{1}{2}CV^2$$

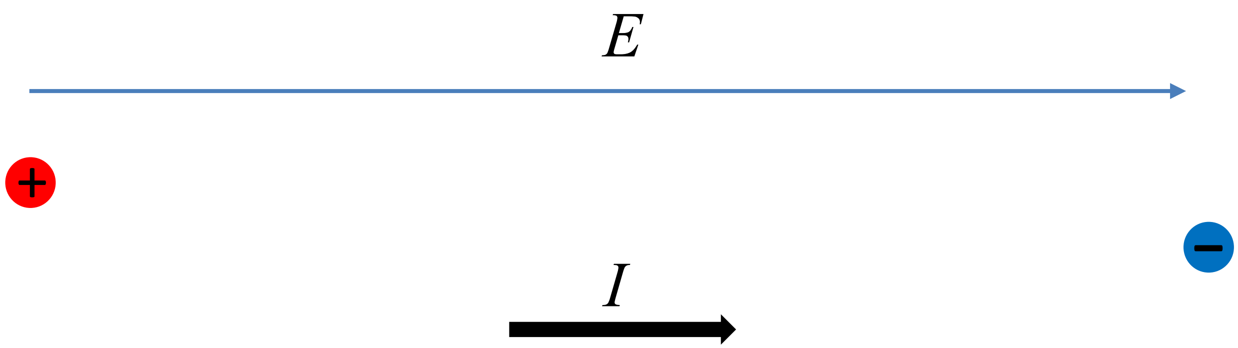
or

$$W = \frac{1}{2}\frac{Q^2}{C}$$

units: joules

4.1 electric current

- all electric currents involve moving charges
- not all moving charge constitutes an electric current
- electric current requires the net transport of charge
- the convention is to describe current direction as the direction in which a positive charge would move



4.1 electric current

- electric current is defined as the **rate of flow** of charge

- for a steady current, I, we can say

$$I = \frac{Q}{t}$$

- the unit of current is the ampere (A) where 1 amp is equal to a flow of 1 coulomb per second

4.1 electric current

“1 amp is the constant current that will produce an attractive force of 2×10^{-7} newtons per metre of length between two straight, parallel conductors of infinite length and negligible circular cross-section placed one metre apart in a vacuum”

(Ampere’s force law)

4.1 electric current

- electric current is defined as the **rate of flow** of charge

- for a steady current, I, we can say

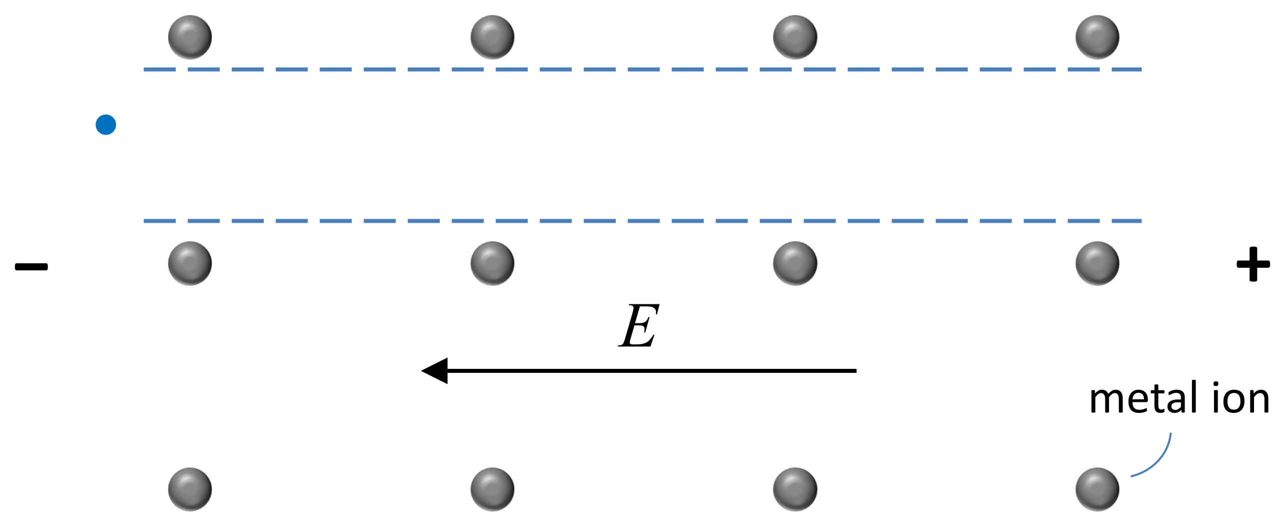
$$I = \frac{Q}{t}$$

- the unit of current is the ampere (A) where 1 amp is equal to a flow of 1 coulomb per second

4.2 conduction in metals

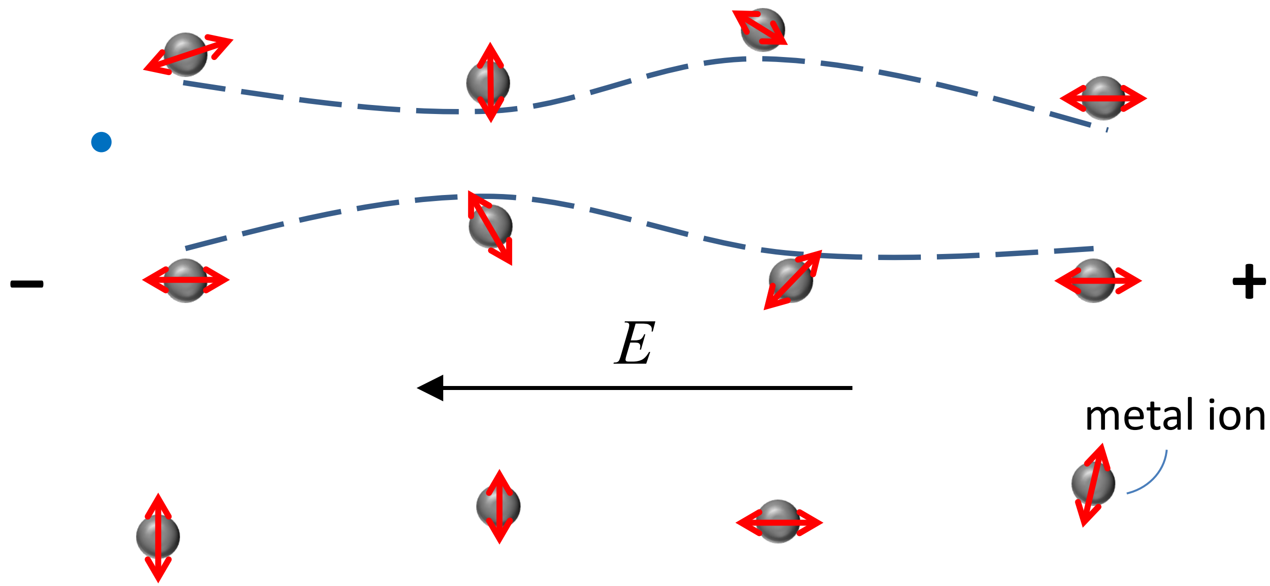
- the atoms in a solid are fixed in place
- in conductors such a metals, the outer (valence) electrons of the atoms become dissociated from the nucleus and are free to move through the conductor
- if a field is applied to the conductor, these free electrons are the primary charge carriers that form a current and rise to conduction

4.2 conduction in metals



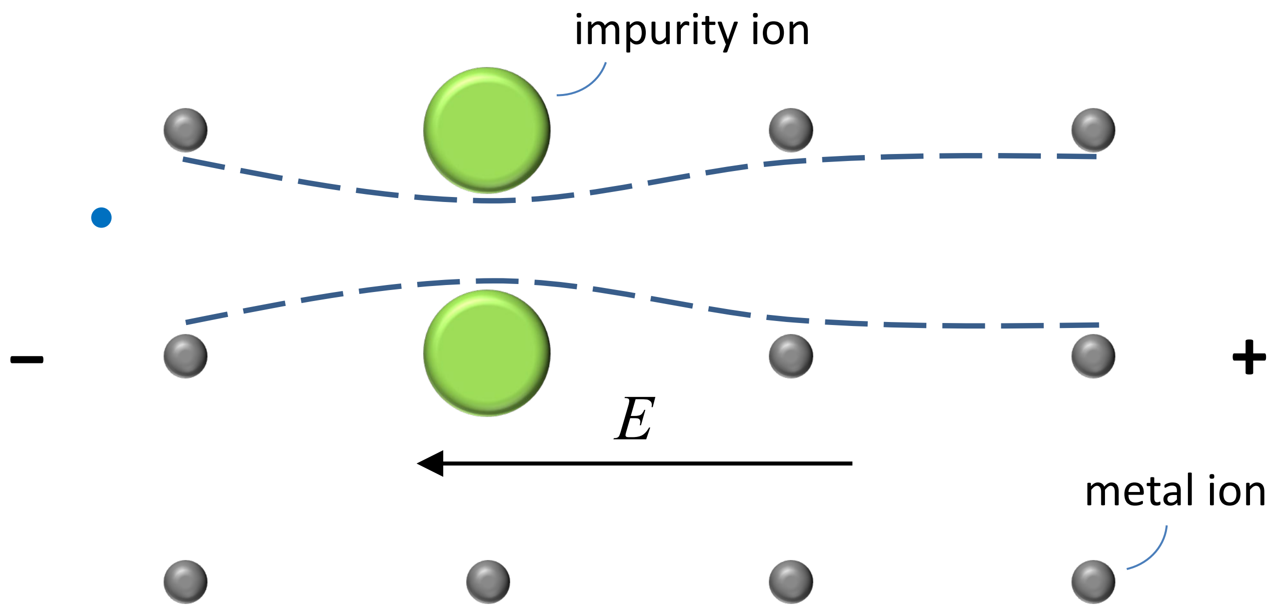
- as electrons move through the conductor they are scattered by the metal ions (and other electrons)
- the electrons don't follow a direct path but the electric field imposes an overall 'drift' of electrons, in the opposite direction to the field

4.2 conduction in metals



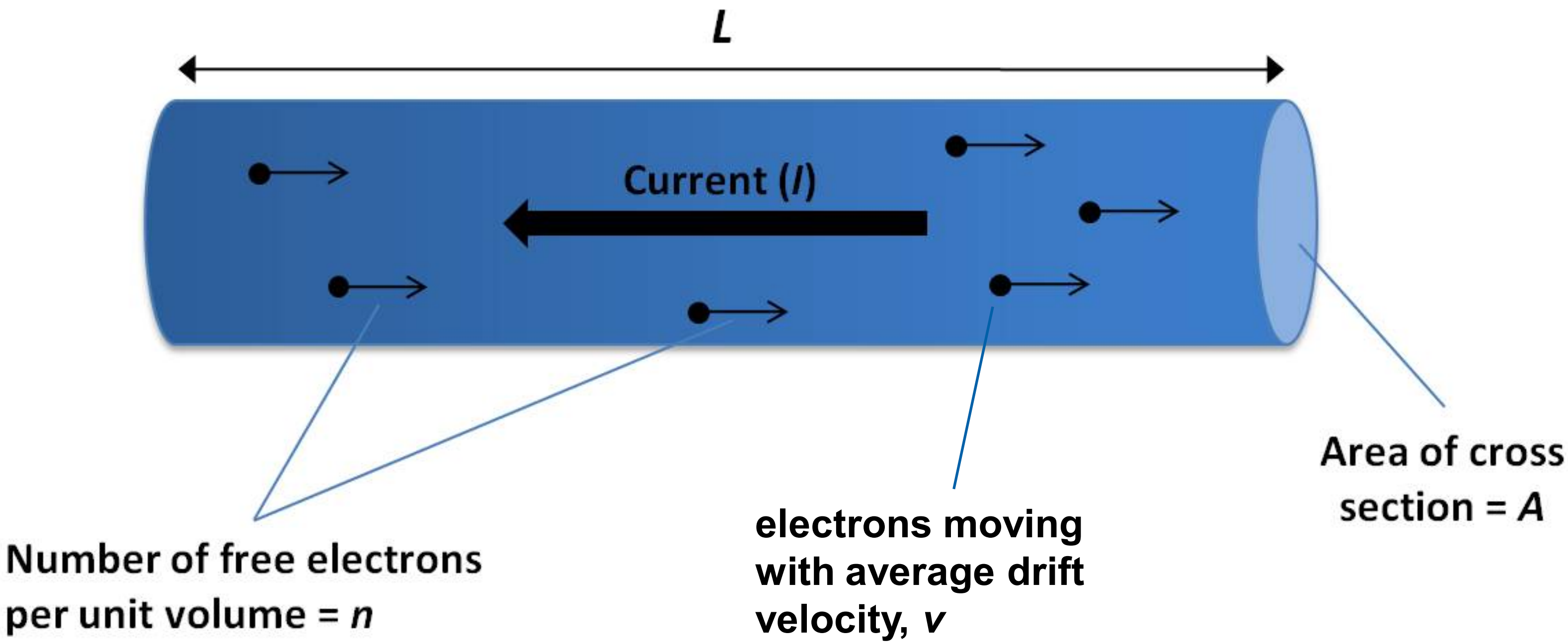
- if the temperature of the conductor increases, the thermal vibration of the ions increases
- the effective cross-sectional area of the ions increases, leading to more collisions as the electrons move through the conductor
- the time taken for the electrons to travel through the conductor increases. In other words, the average 'drift velocity' decreases

4.2 conduction in metals



- the presence of impurities in the metal can disrupt the lattice of metal ions, restricting the flow of electrons
- if the flow of electrons is restricted, the drift velocity increases

4.2 conduction in metals



- for a cylindrical conductor with cross-sectional area *A,* and carrier concentration *n,* the current is given by

$$I = nAve$$

4.3 resistance and resistivity

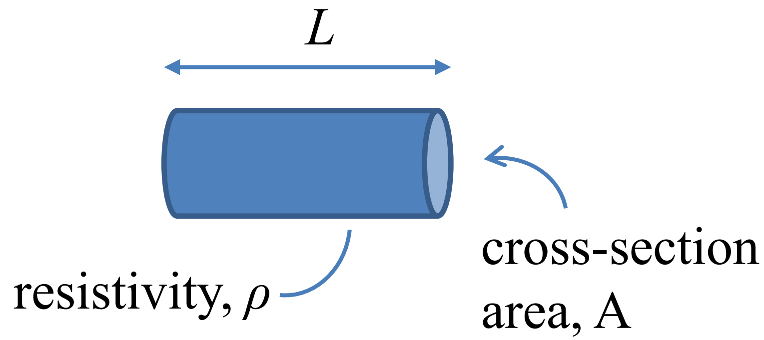
- resistance is defined as the ratio of the potential difference across a conductor to the current passing through it as a result

$$R = \frac{V}{I} \text{ or } V = IR$$

- the unit of resistance is the volt per ampere, which is called the ohm (Ω)

4.3 resistance and resistivity

$$R = \rho \frac{L}{A}$$



- ρ is the resistivity and has units of Ω m (ohm-metre)

- resistance is a property of an object
- resistivity is a property of a material

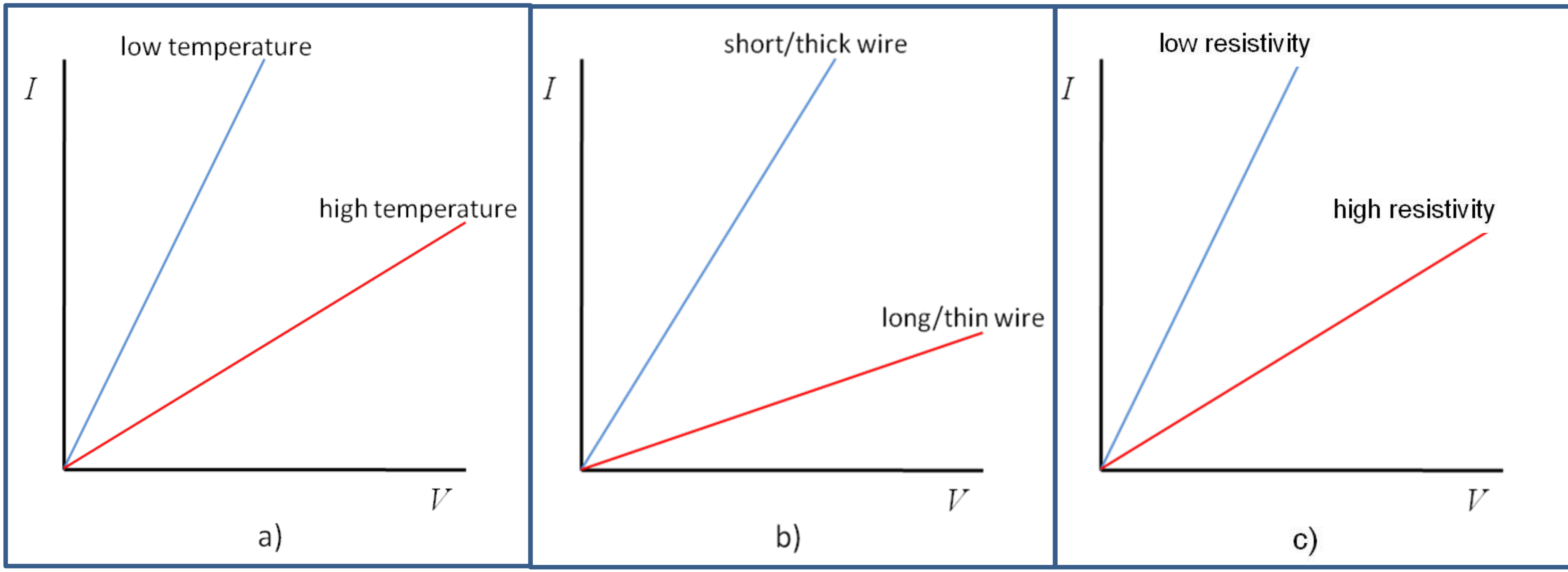
4.3 resistance and resistivity

Material	Resistivity (Ωm)
Typical metals (conductors)	
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}
Aluminium	2.8×10^{-8}
Iron	10×10^{-8}
Tungsten	5.5×10^{-8}
Typical semiconductor	
Silicon	2.5×10^{3}
Typical insulators	
Alumina	10^{9} - 10^{12}
Rubber	approx. 10^{13}
Teflon	10^{22} - 10^{24}

4.3 resistance and resistivity

$$R = \rho \frac{L}{A}$$

I-V graphs



N.B. resistance is given by 1/gradient

4.3 resistance and resistivity

Ohm's law

$$V = IR \quad \text{or} \quad R = \frac{V}{I}$$

This is **NOT** Ohm's law!

4.3 resistance and resistivity

Ohm's law

- for Ohm's law to apply, the potential difference across a device must be directly proportional to the current flowing through the device

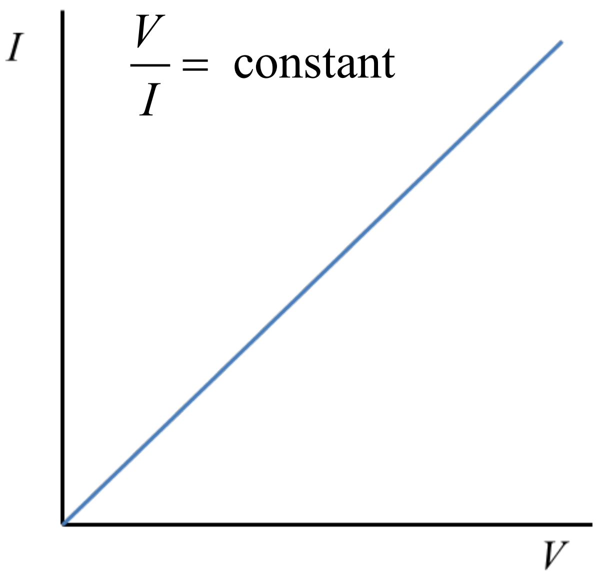
$$V = IR$$

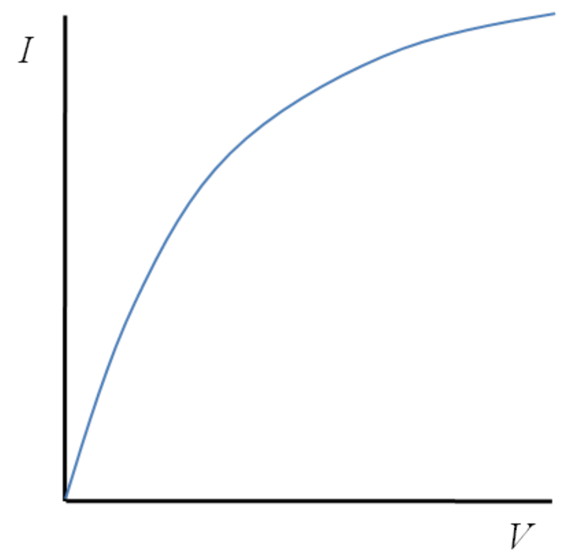
- a conductor obeys Ohm's law when its resistance is independent of the applied potential difference

$$\frac{V}{I} = \text{constant}$$

4.3 resistance and resistivity

Ohm's law

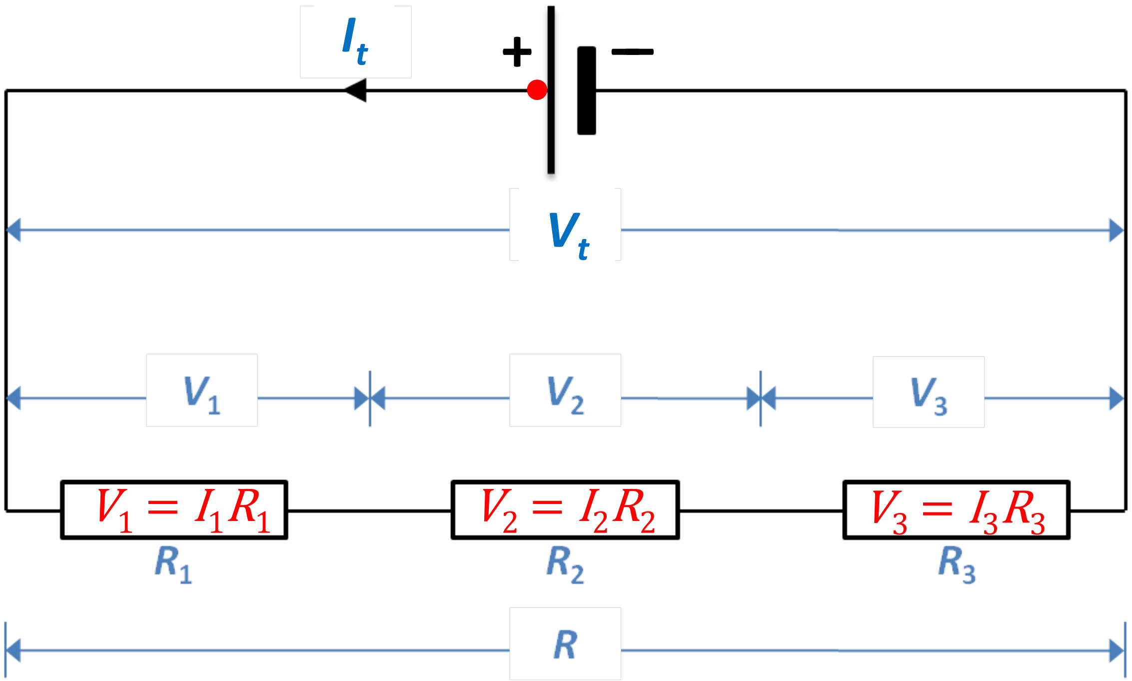




Obeys Ohm's law i.e. "ohmic"

Does not obey Ohm's law i.e. "non-ohmic"

4.4 resistors in series



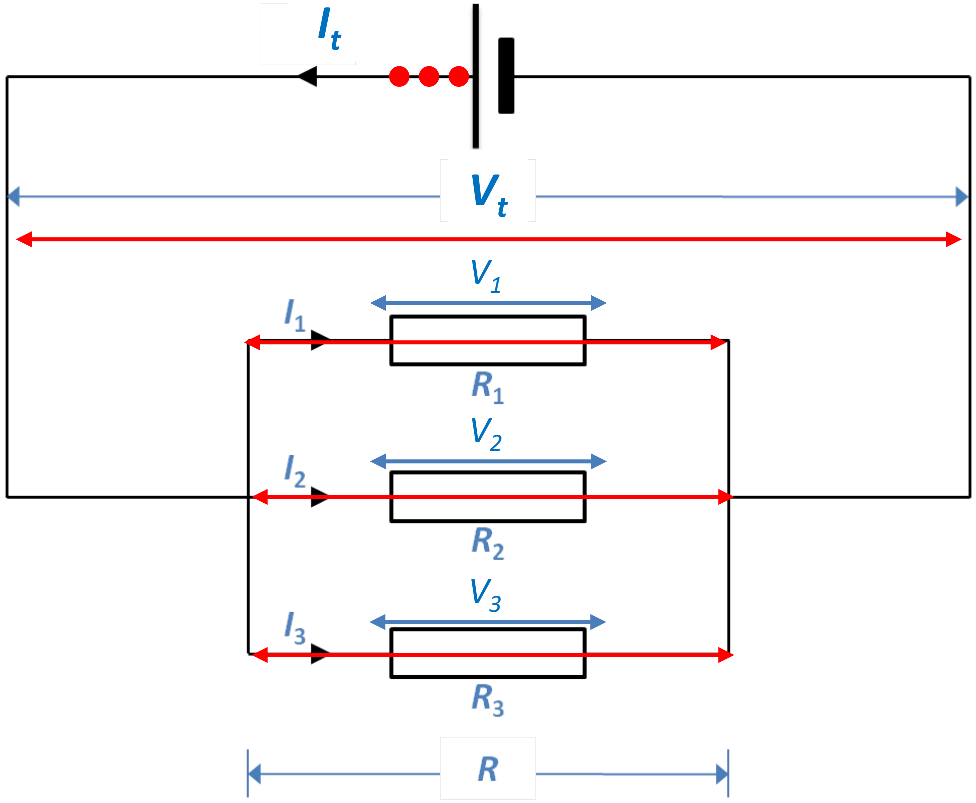
- same current flows through all series resistors
- potential difference is divided across series resistors

$$I_t = I_1 = I_2 = I_3$$

$$V_t = V_1 + V_2 + V_3 = I_1R_1 + I_2R_2 + I_3R_3 = I_t(R_1 + R_2 + R_3)$$

Since $V_t = I_tR_t$ it follows that $R_t = R_1 + R_2 + R_3$

4.5 resistors in parallel



- current is divided between parallel resistors
- same potential difference across parallel resistors

$$I_t = I_1 + I_2 + I_3$$

$$V_t = V_1 = V_2 = V_3$$

$$R_t = \frac{V_t}{I_t}$$

$$\frac{1}{R_t} = \frac{I_t}{V_t} = \frac{I_1 + I_2 + I_3}{V_t} = \frac{I_1}{V_1} + \frac{I_2}{V_2} + \frac{I_3}{V_3} \Rightarrow \frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

4.6 energy and power in DC circuits

- power is defined as the rate of conversion of energy
- the unit of power is the watt (W). 1 W = 1 J s^{-1}
- we know that the work done (in joules) to move a charge Q through a potential difference V is given by

$$W = QV$$

so the power (rate of work) is given by

$$P = \frac{W}{t} = \frac{QV}{t} \quad \text{and since} \quad I = \frac{Q}{t}$$

we can write electrical power as: $P = IV$

4.6 energy and power in DC circuits

$$P = IV$$

- this expression is applicable in all cases
- in the case of resistive loads (any device or situation where the electrical energy is all dissipated as heat) we can use $V = IR$ to eliminate I or V, leading to

$$P = I^2 R = \frac{V^2}{R}$$

- but remember that these equations only tell us the rate at which electrical energy is transferred to heat

5.1 cells, batteries and EMF

- an electric cell is a device that converts chemical energy into electrical potential energy
- this potential energy can be used to do work when the cell is connected in a circuit
- the cell is said to be a source of electromotive force (EMF)
- note that EMF is not a force!
- electromotive force is the action of any source that can cause a current to flow in a circuit
- batteries, generators, solar cells, fuel cells are all examples of sources of EMF

5.1 cells, batteries and EMF

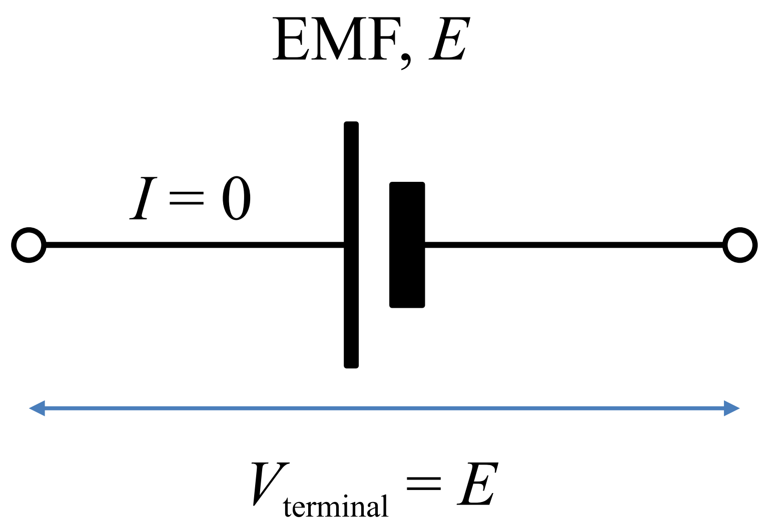
this is the EMF of the battery

- the EMF of a cell is the maximum potential difference produced between its terminals

- when connected to a circuit, however, not all of the cell's EMF may be available to drive current around that circuit

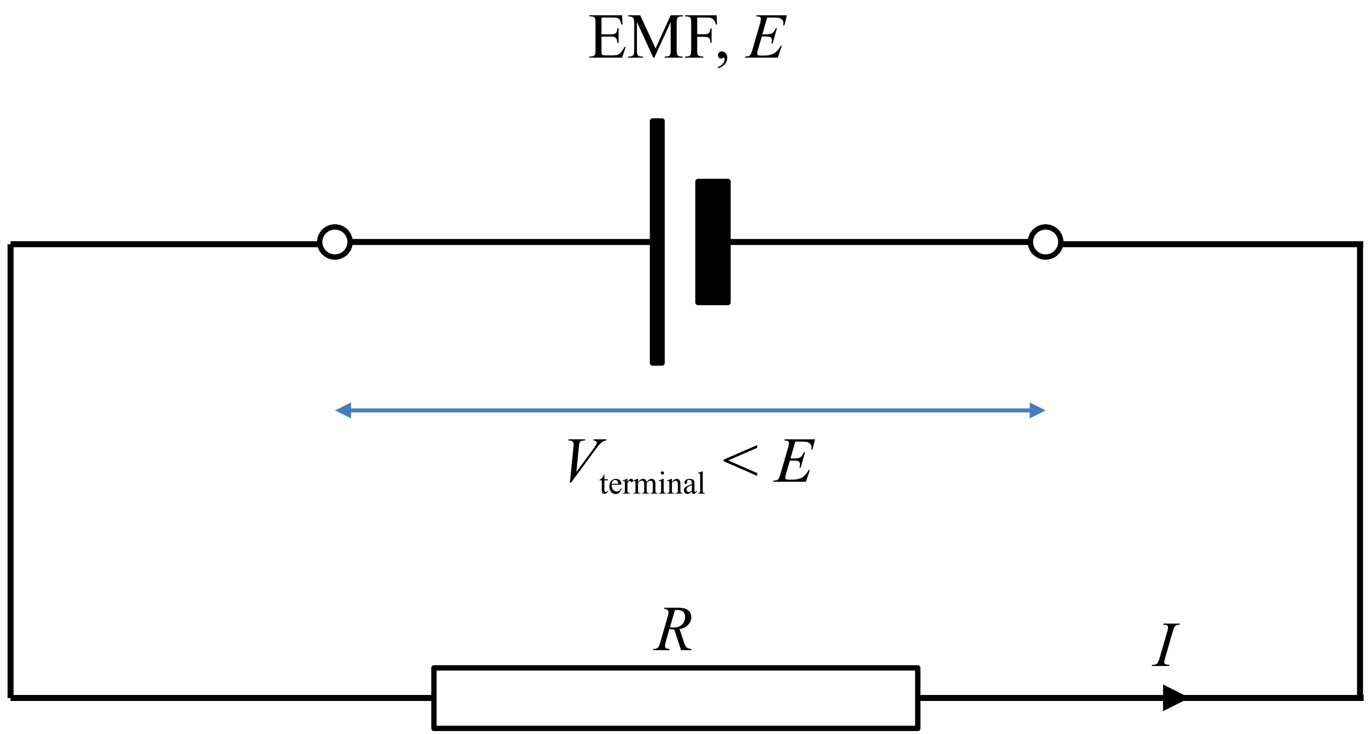
5.1 cells, batteries and EMF

- when a cell is not being used, no current flows $(I = 0)$ and the potential difference between the terminals is equal to the EMF (E) of the cell



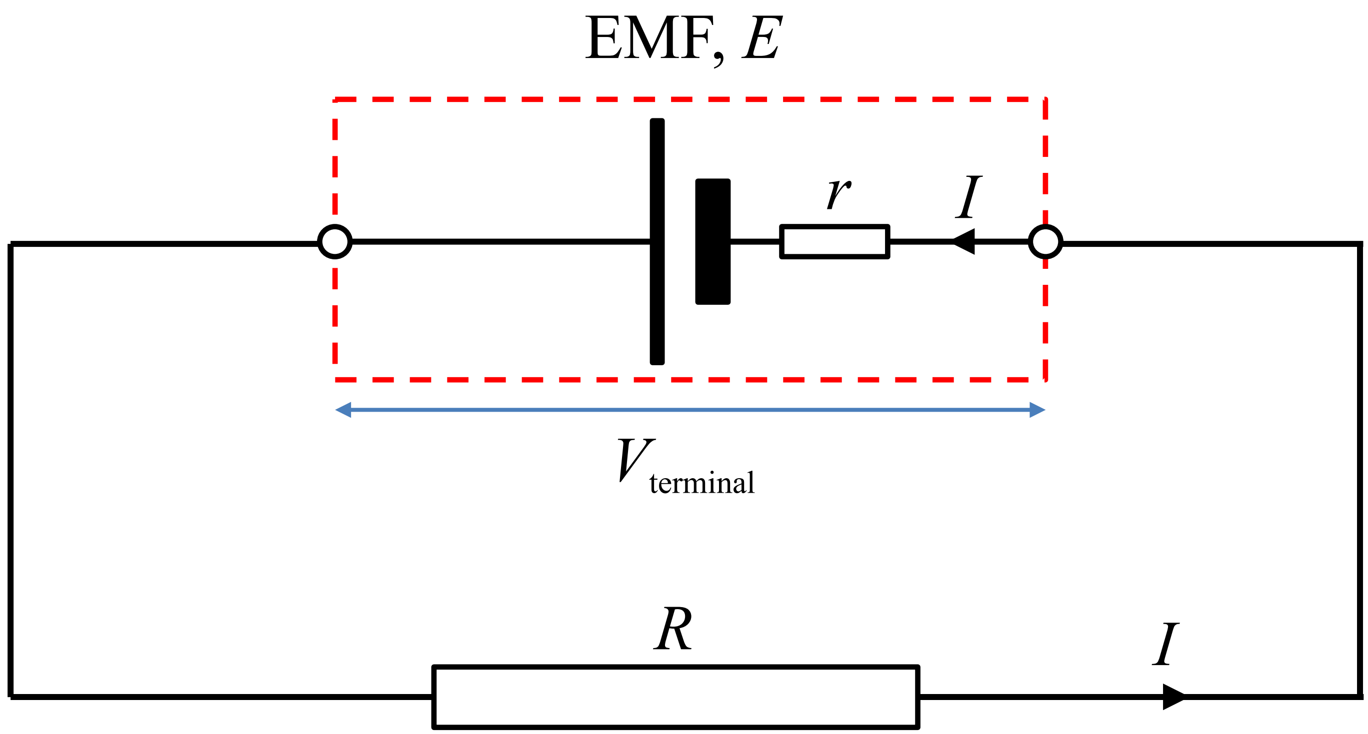
5.1 cells, batteries and EMF

- when a cell is in use, and a current is flowing, the potential difference between the terminals is less than the EMF



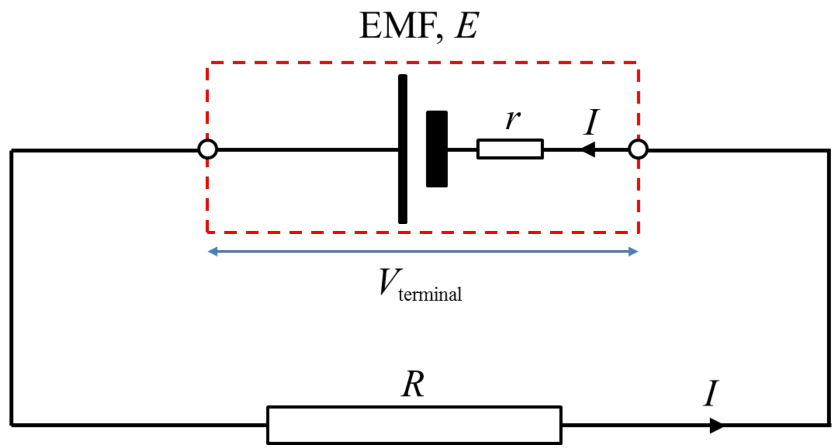
5.1 cells, batteries and EMF

- part of the cell's EMF is used to drive the current through the internal resistance, *r*, of the cell



5.1 cells, batteries and EMF

- for the circuit shown, we can write:



$$E = IR_{total} \text{ where } R_{total} = R + r$$

So: $E = I(R + r)$

We could write this as $E = IR + Ir$ and since $IR = V_{terminal}$

we can also write:

$$E = V + Ir \quad \text{or} \quad E - V = \underbrace{Ir} \qquad (\text{where } V = V_{terminal})$$

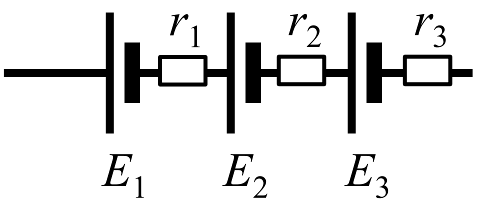
Ir is the 'inaccessible voltage' (lost as heat)

N.B. this shows that the difference between the EMF and the terminal voltage is proportional to the current flowing in the circuit.

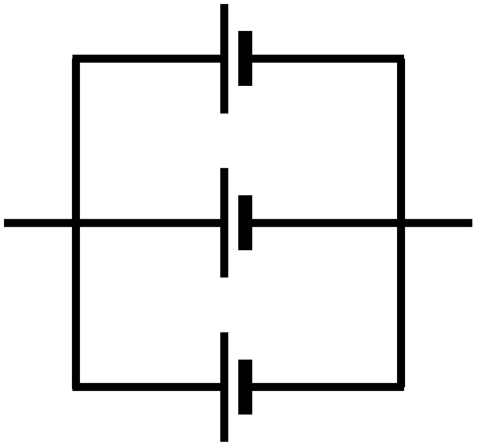
5.1 cells, batteries and EMF

- a battery is an array of cells connected together
- the cells can be connected in series or in parallel

series

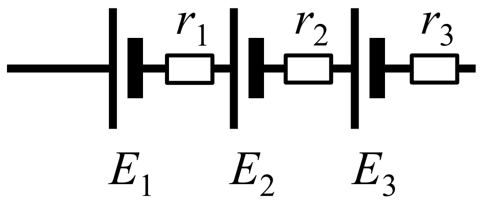


parallel



5.1 cells, batteries and EMF

series



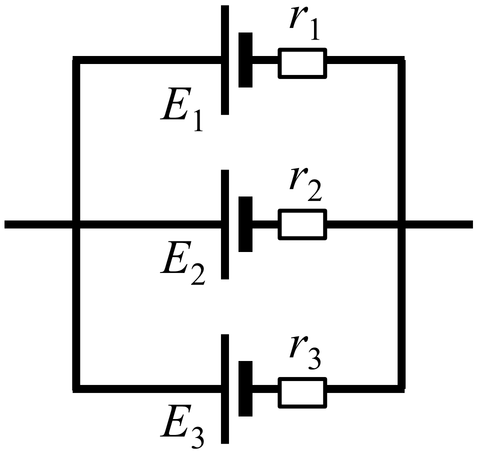
total EMF:

$$E_{\text{total}} = E_1 + E_2 + E_3 \ldots + E_{\text{n}}$$

internal resistance:

$$r_{\text{total}} = r_1 + r_2 + r_3 + \ldots + r_{\text{n}}$$

parallel



total EMF:

$$E_{\text{total}} = E_1 = E_2 = E_3 \ldots = E_{\text{n}}$$

internal resistance:

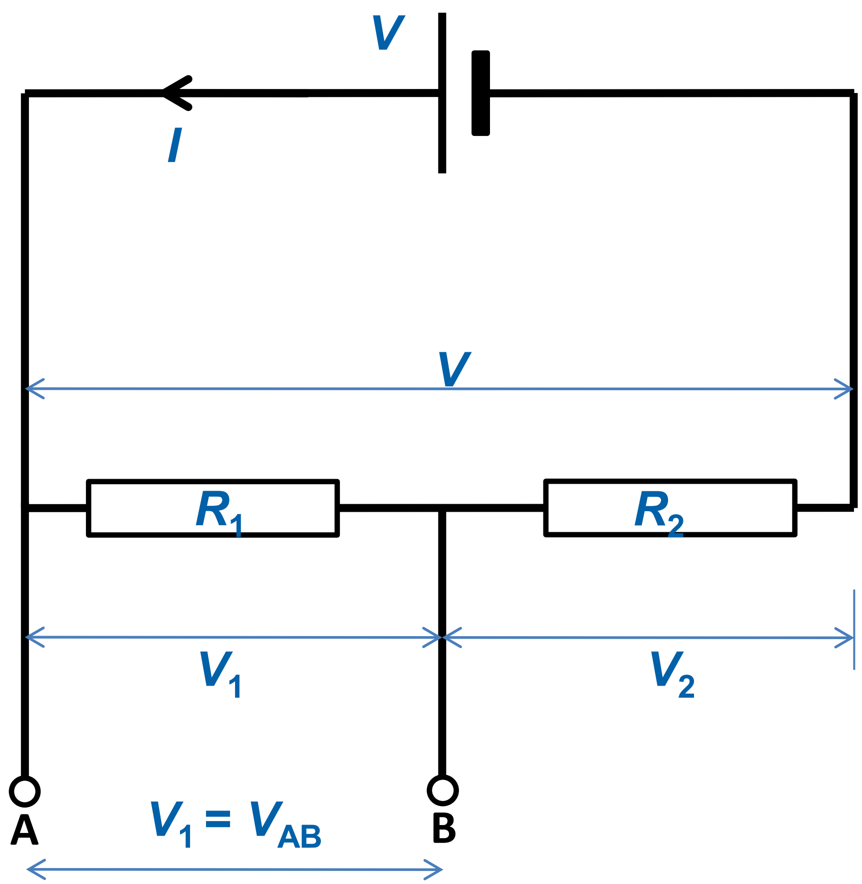
$$\frac{1}{r_{total}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \ldots \frac{1}{r_n}$$

5.2 potential dividers

- simple method for producing a specified potential difference in a circuit
- uses the behaviour of resistors in series
- same current flows through resistors R_1 and R_2 so

$$I = \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{R_1}{R_2}$$



5.2 potential dividers

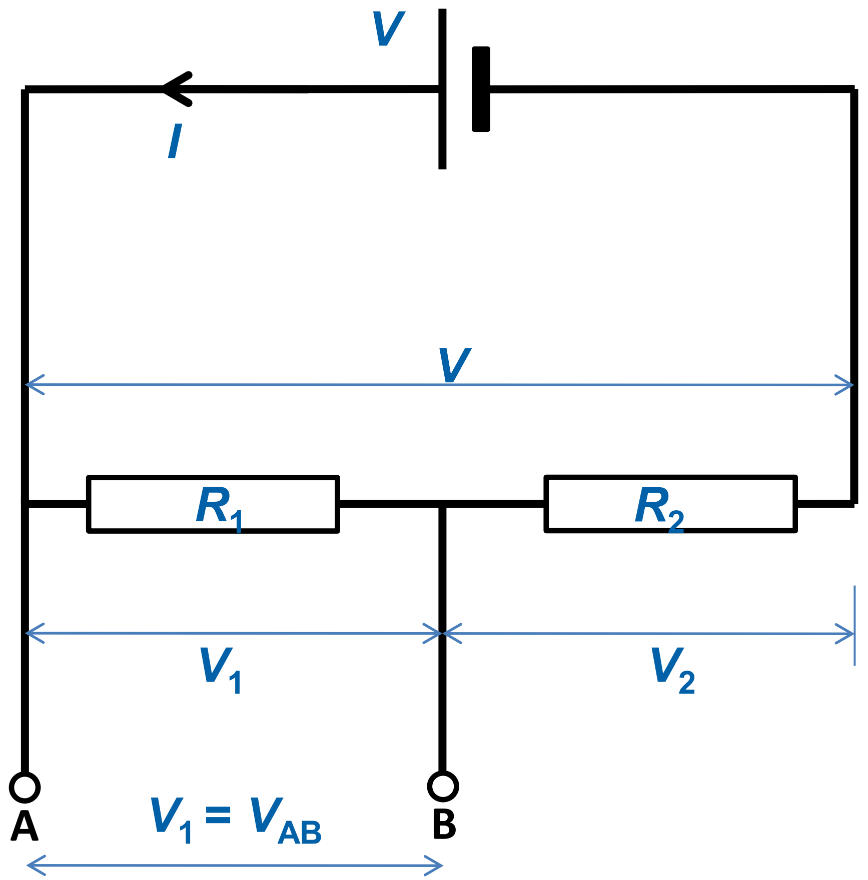
$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

also

$$\frac{V_1}{V_{total}} = \frac{R_1}{R_1 + R_2} = \frac{R_1}{R_{total}}$$

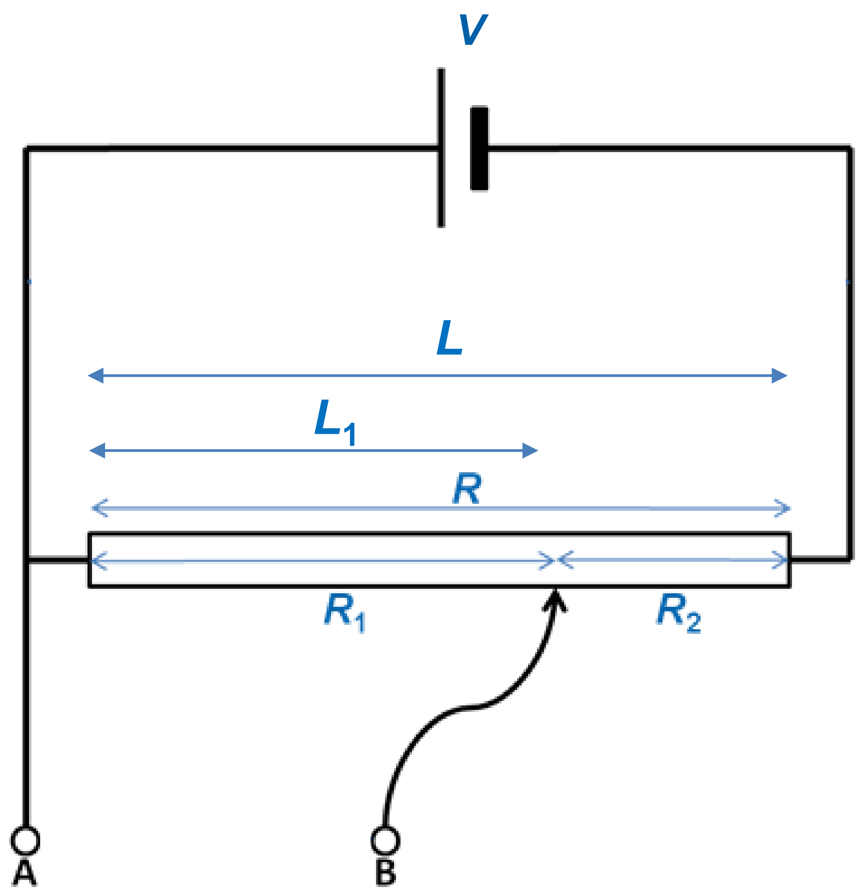
and

$$\frac{V_2}{V_{total}} = \frac{R_2}{R_1 + R_2} = \frac{R_2}{R_{total}}$$



- it is important to note that the accessible voltage V_{AB} will drop when a component is connected between A and B and draws a current

5.2 potential dividers



- we can replace the 2 series resistors with a single resistor that can be split (tapped off) with a sliding contact
- this gives a continuously adjustable voltage source between A and B

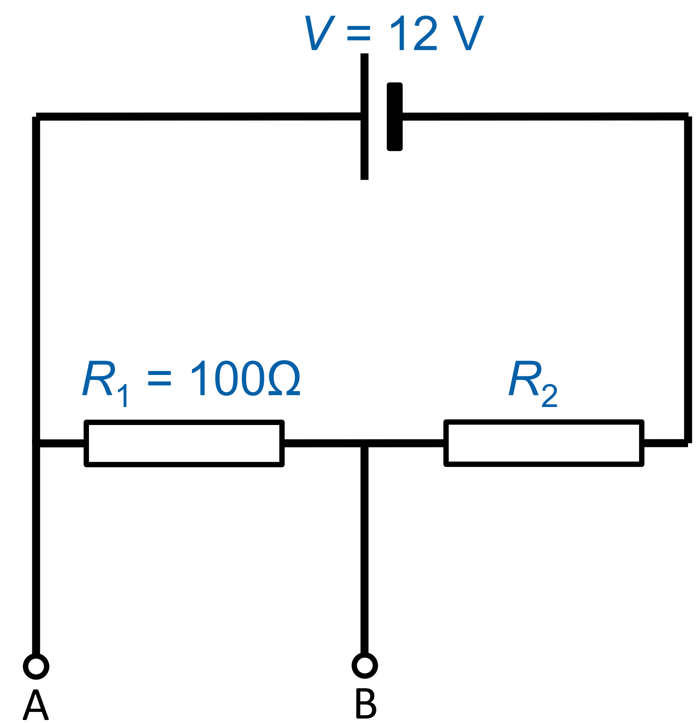
$$\frac{V_{AB}}{V} = \frac{R_1}{R} = \frac{L_1}{L}$$

5.2 potential dividers

a) what value resistor R_2 will give a 5 V output across terminals AB ?

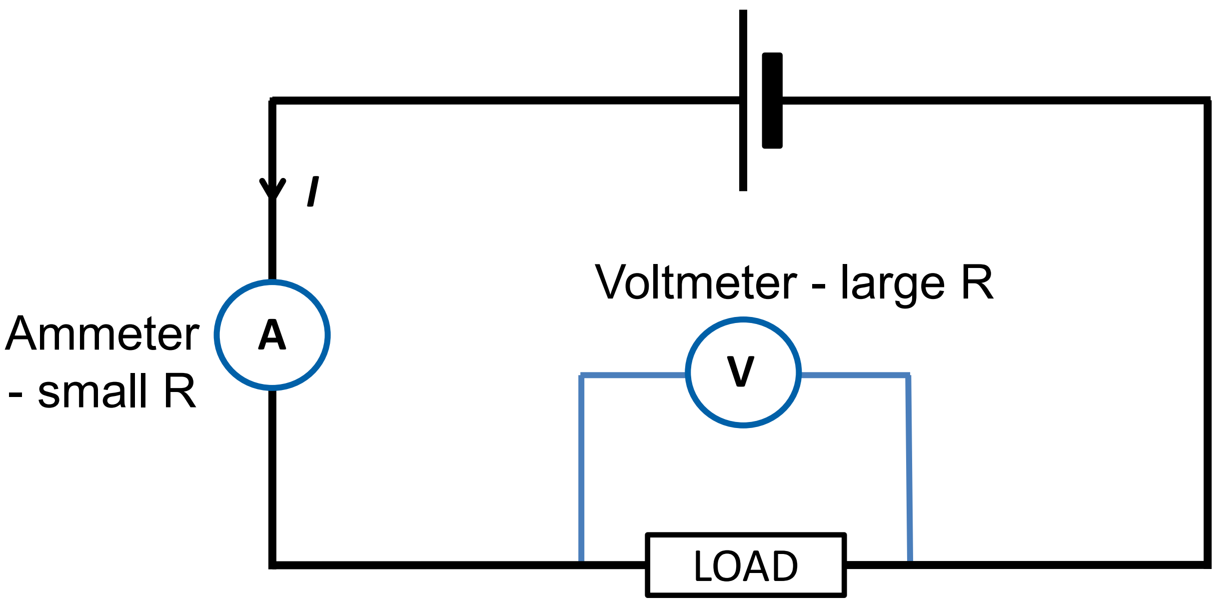
b) if a load with 25 Ω resistance is connected between A and B, what is the potential difference V_{AB}?

c) what new value for R_2 will restore a potential difference of 5 V between A and B?



5.3 measuring voltage and current

- a general principle of measurement is that the act of measuring should have the smallest possible influence on the quantity being measured



5.3 measuring voltage and current

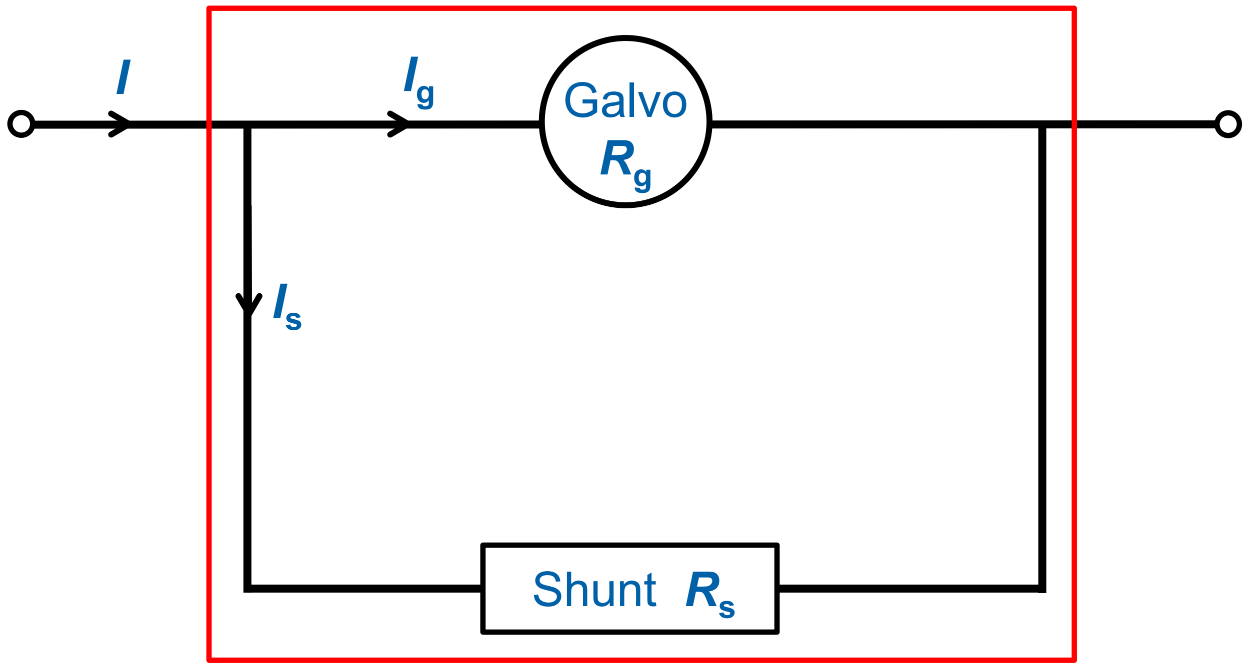
galvanometers

- a galvanometer is an instrument that measures small electric currents
- galvanometers can be used as a component to make ammeters or voltmeters
- galvanometers are sensitive, and so require the use of shunts or multipliers to prevent them being damaged by overloading

5.3 measuring voltage and current

galvanometer as an ammeter

- a shunt resistor is used to bypass current around the galvanometer, preventing it from exceeding its maximum current (I_g)

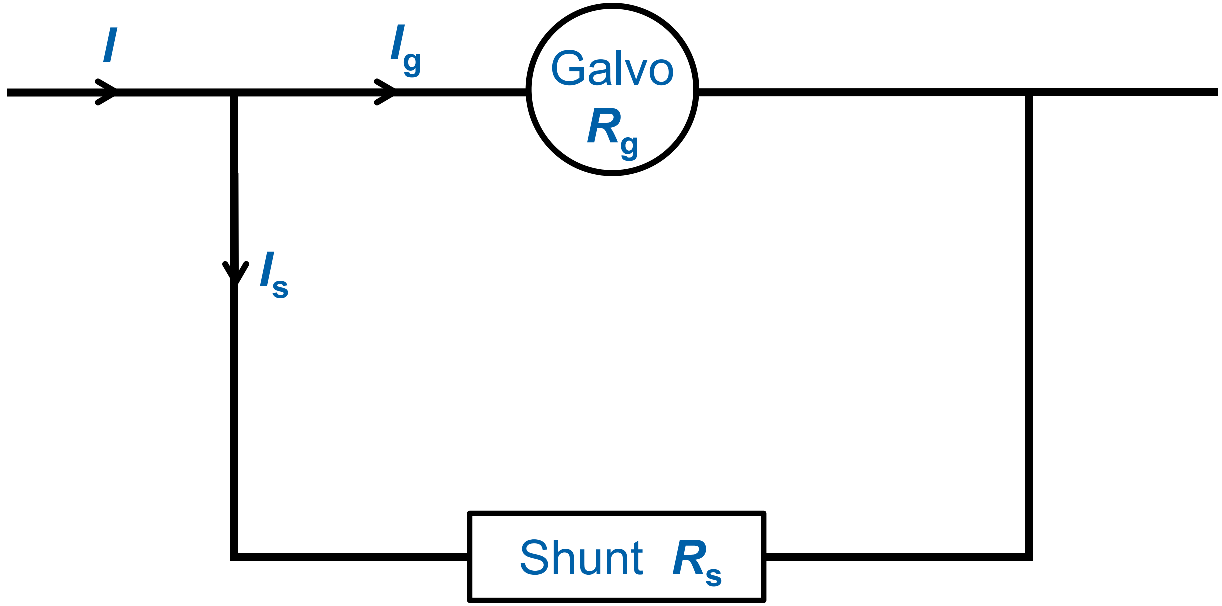


5.3 measuring voltage and current

galvanometer as an ammeter

$$R_s = \frac{I_g R_g}{I - I_g}$$

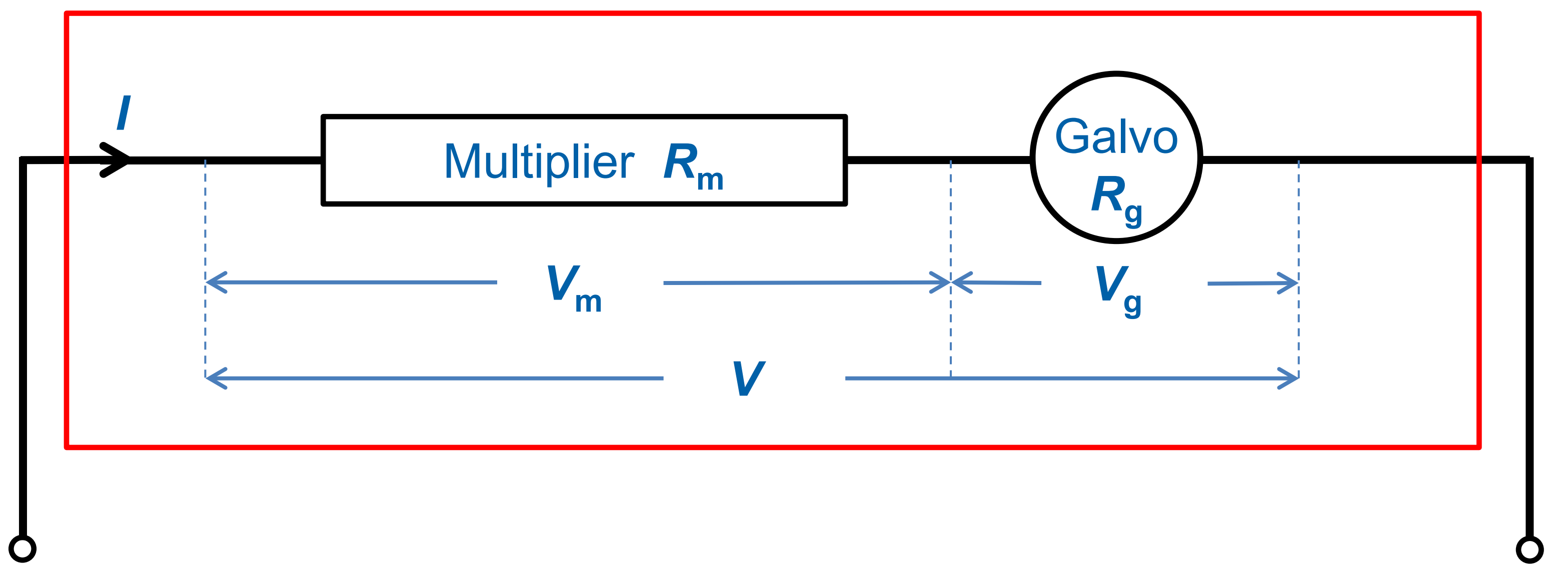
- I is maximum current to be measured
- I_g is the current limit of the galvanometer
- R_g is resistance of the galvanometer
- R_s is the resistance of the shunt required



5.3 measuring voltage and current

galvanometer as a voltmeter

- a multiplier resistor is used to reduce the potential drop across the galvanometer, preventing it from exceeding its maximum potential difference (V_g)

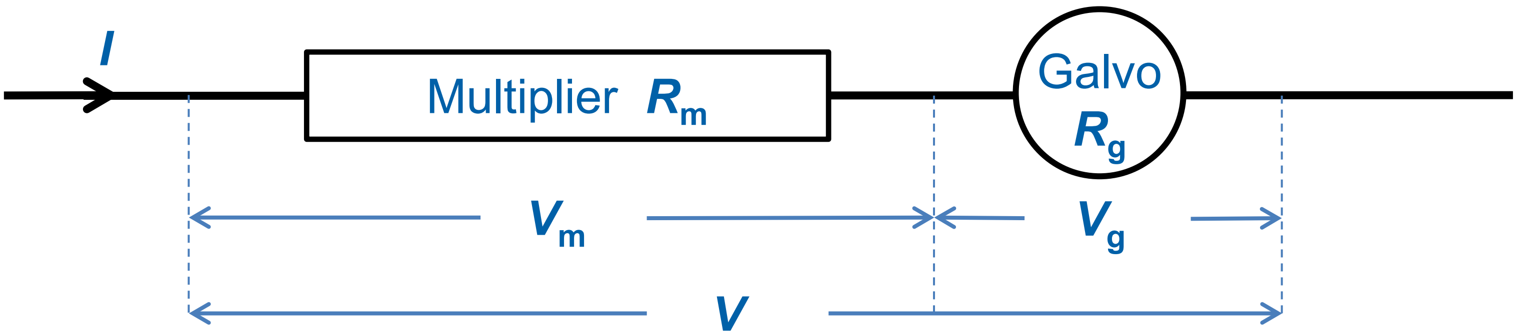


5.3 measuring voltage and current

galvanometer as a voltmeter

$$R_m = \frac{\left(V - V_g\right) R_g}{V_g}$$

- V is maximum voltage to be measured
- V_g is the voltage limit of the galvanometer
- R_g is resistance of the galvanometer
- R_m is the resistance of the multiplier required



5.4 null methods of measurement

- null measurement methods can be used to make very precise measurements of resistance, capacitance, inductance etc

- a null method of measurement is one in which there is zero current flowing in a part of the circuit

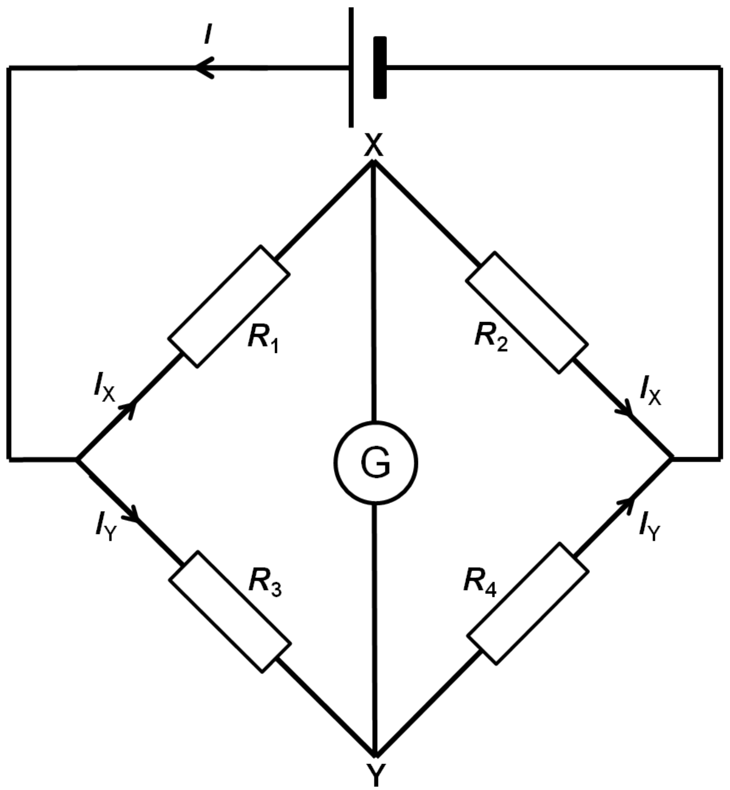
- bridge or balance circuits are commonly used for making null measurements

5.4 null methods of measurement

wheatstone bridge

- used for the accurate measurement of resistance
- when the galvanometer shows zero current flowing in the bridge, it is said to be balanced
- in this state, the potential difference between X and Y is zero and it can be shown that

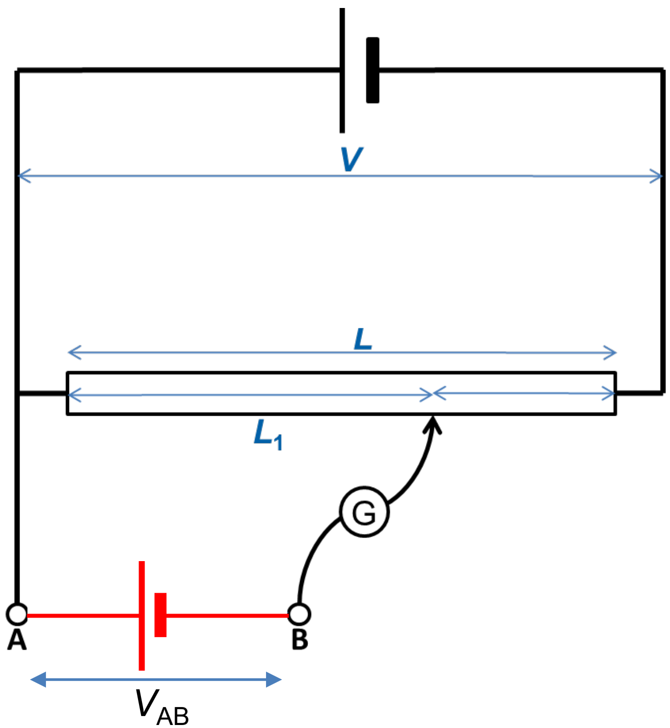
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$



- so, one unknown resistance can be found in terms of three, known resistances, without the need to measure current or voltage

5.4 null methods of measurement

potentiometer



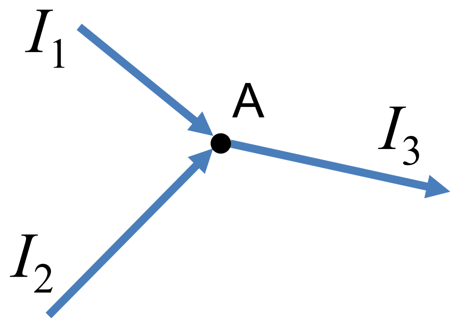
$$V_{AB} = V\left(\frac{L_1}{L}\right)$$

- the potentiometer is a form of potential divider
- a galvanometer indicates when it is balanced (zero current flowing through it)
- the balanced potentiometer draws no current from the device/circuit being measured; it acts like a voltmeter with infinite resistance
- it can be used to measure the EMF of a test cell connected between A and B
- with the potentiometer balanced, no current flows through the test cell and so the measured V_{AB} is equal to the EMF of the test cell

5.5 Kirchhoff's circuit laws

- Kirchhoff's circuit laws form the basis of a variety of different methods of circuit analysis
- these methods allow us to calculate the current flowing in different branches of a circuit, and the voltages at different points in the circuit
- the first law, the junction rule, is a consequence of the law of conservation of charge
- the second law, the loop rule, follows from the equation $W = QV$ (which tells us that the net work done in moving a charge around a closed loop is zero)

5.5 Kirchhoff's circuit laws



the junction rule

- the sum of the currents entering a junction must equal the sum of the currents leaving a junction

$$\sum I_{IN} = \sum I_{OUT} \qquad I_1 + I_2 = I_3$$

- alternatively, if we define currents **entering** a junction as **positive** and currents **leaving** a junction as **negative**, this means that

$$\sum I = 0 \qquad +I_1 + I_2 - I_3 = 0$$

5.5 Kirchhoff's circuit laws

the loop rule

- around a closed loop, the sum of the potential rises must equal the sum of the potential drops
- in terms of conventional current, EMFs represent a potential rise (given by E) and resistors represent a potential drop (given by $V = IR$)
- so, the loop rule can be written as

$$\sum E = \sum IR$$

5.5 Kirchhoff's circuit laws

the loop rule

- around a closed loop, the sum of the potential rises must equal the sum of the potential drops

- in terms of conventional current, EMFs represent a potential rise (given by E) and resistors represent a potential drop (given by $V = IR$)

- so, the loop rule can be written as

$$\sum E = \sum IR$$

5.5 Kirchhoff's circuit laws

the loop rule

- in terms of conventional current, EMFs represent a potential rise (given by E) and resistors represent a potential drop (given by $V = IR$)
- however, depending on which way we are moving around the loop (with respect to the current), the potential rises due to EMFs, and the potential drops due to resistors, can have either positive or negative values
- when applying the loop rule we must follow a consistent sign convention…

5.5 Kirchhoff's circuit laws

the loop rule

Sign convention:

- an EMF is positive if we enter through the negative terminal and exit through the positive terminal of the cell. It is negative if we enter through the positive terminal and exit through the negative terminal.

- the drop in potential due to resistance is positive if we travel through the resistor in the same direction as the current. It is negative if we travel through the resistor in the opposite direction to the current.

5.5 Kirchhoff's circuit laws

If we are trying to find '*n*' unknown currents, we need a total of '*n*' unique simultaneous equations to solve.

Steps:

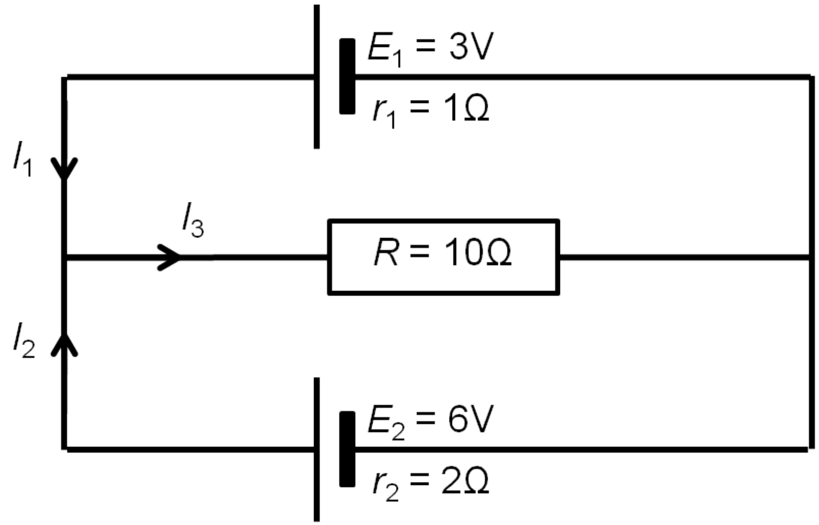
1. label the current direction in each branch of the circuit. If the direction is not known, assume a direction and mark this on the diagram.
2. apply the junction rule to get an equation linking the currents entering and leaving each junction.
3. for each loop, travel clockwise around the loop, applying the loop rule:
   - first, sum all the EMFs (positive and negative, according to the sign convention) in one complete loop. This is the left side of the equation.
   - second, go round the same loop, summing all the potential changes across resistances (positive and negative, according to the sign convention). e.g. $I_x R_1 + I_y R_2 - I_z R_3$. This is the right side of the equation.
4. when you have enough equations, solve to find the unknown currents (if a current comes out as negative, it means the actual direction of that current is opposite to the assumed direction).

5.5 Kirchhoff's circuit laws

Note: if you are not told the internal resistance of a cell, you can assume the cell is ideal (i.e. $r = 0$). If you are told the internal resistance, r, of a cell, this potential change ($I_x r$) must be included in the right-hand side of the appropriate loop equation.

example

for the circuit shown, find the current in each branch of the circuit.

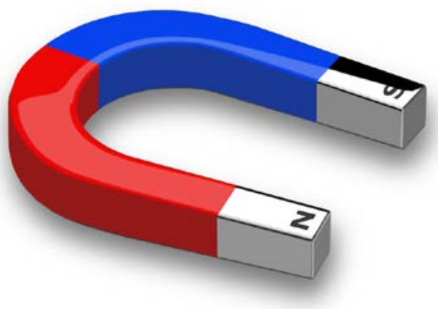


6.1 magnetism and magnetic fields

- we have seen that electric charges give rise to electric fields
- in this part of the course, we will see that moving electric charges give rise to magnetic fields
- whether we are considering electromagnets or permanent magnets , the origin of the magnetic field is moving electric charge

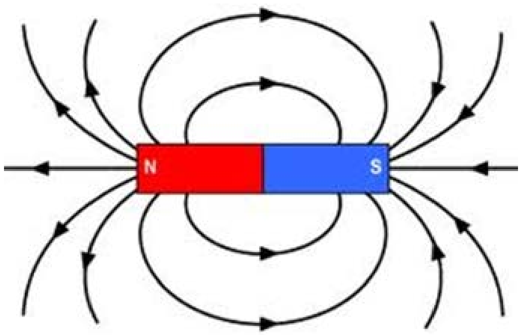
6.1 magnetism and magnetic fields

- there are two types of magnetic pole: north and south
- like poles repel one another, unlike poles attract
- unlike positive and negative electric charges, magnetic poles (as far as we know) do not exist as monopoles



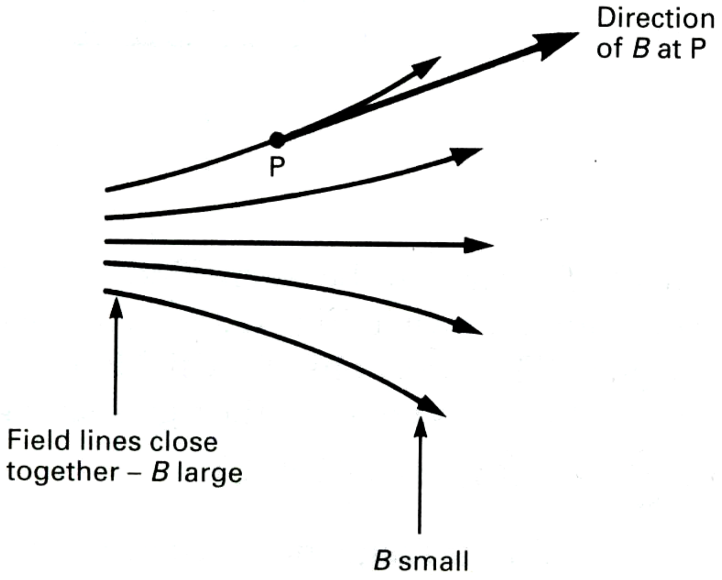
6.1 magnetism and magnetic fields

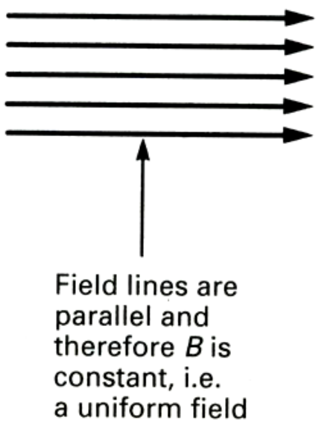
- magnetic field strength is a vector quantity, it has direction and magnitude
- the direction of a magnetic field at a point is taken as the direction of the force acting on a 'north pole' placed there
- the magnitude of the magnetic field strength can be expressed by the magnetic flux density, B
- the unit of magnetic flux density is the tesla (T)



6.1 magnetism and magnetic fields

- magnetic fields can be represented by field lines
- the magnitude of the field strength is greater where concentration of field lines is higher (i.e. more field lines per unit area)





6.1 magnetism and magnetic fields

- the permeability, μ, of a material is a constant that indicates how the material responds to an applied magnetic field
- absolute permeability: $\mu = \mu_r \mu_0$

where

- μ_0 is the permeability of a vacuum ($4\pi \times 10^{-7}$ T m A^{-1})
- and μ_r is the relative permeability:

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B}{B_0}$$

➡ field strength in the presence of the material

➡ field strength in vacuum (or air)

- for air, $\mu_r \approx 1$, in other words, $\mu_{air} \approx 4\pi \times 10^{-7}$ T m A^{-1}

6.2 magnetic materials

- recall our definition of relative permeability

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B}{B_0}$$

➡ field strength in the presence of the material

➡ field strength in vacuum (or air)

- so the magnetic flux density in the presence of a material is equal to the flux density in a vacuum (or air) multiplied by the relative permeability of the material

- therefore $\mu_r > 1$ means the material increases an applied magnetic field

- and $\mu_r < 1$ means the material decreases an applied magnetic field

6.2 magnetic materials

- by considering relative permeability, we can define 3 distinct categories of magnetic material
- diamagnetic materials: cause a very slightly reduction in the flux density of a magnetic field into which they are placed i.e. μ_r is very slightly less than 1 (typically about 0.99999)
- paramagnetic materials: cause a slight increase in the flux density of a magnetic field i.e. μ_r is slightly greater than 1 (typically about 1.001)
- ferromagnetic materials: cause a very large increase in the flux density of a magnetic field i.e. μ_r is much greater than 1 (typically > 10^3)

6.2 magnetic materials

- whether we are considering electromagnets or permanent magnets, the origin of the magnetic field is moving electric charge

- the moving charge in electromagnets is obvious, it is in the form of an electric current
- the moving charge in diamagnetic, paramagnetic and ferromagnetic materials is less obvious, but it is moving charge that is responsible for their defining characteristics

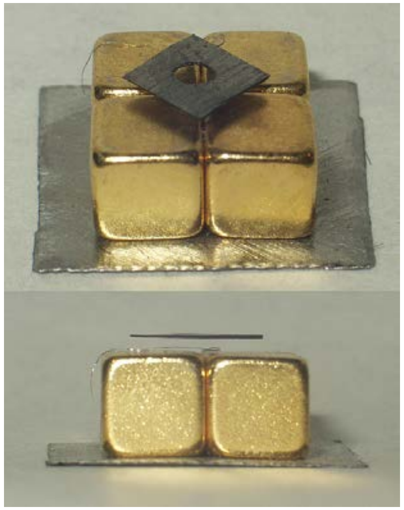
6.2 magnetic materials

diamagnetic materials

- the orbital motion of atomic electrons produce small magnetic fields (which oppose the external field)
- exhibited by all materials, but sometimes masked by larger effects
- more evident in materials with electron pairing in atomic structure

examples

- water
- copper
- pyrolytic carbon
- bismuth



A small (6mm) piece of pyrolytic graphite levitating over a permanent neodymium magnet array

6.2 magnetic materials

paramagnetic materials

- the magnetic field due to the spin of atomic electrons gives the atom a permanent magnetic moment, the electron acts like a little bar magnet (a magnetic dipole)
- the magnetic dipole tends to align with an external field, slightly increasing the overall flux density
- occurs in materials containing unpaired atomic electrons

examples

- aluminium
- oxygen
- tin

liquid oxygen (a paramagnetic material) suspended by the magnetic field between the poles of a permanent magnet

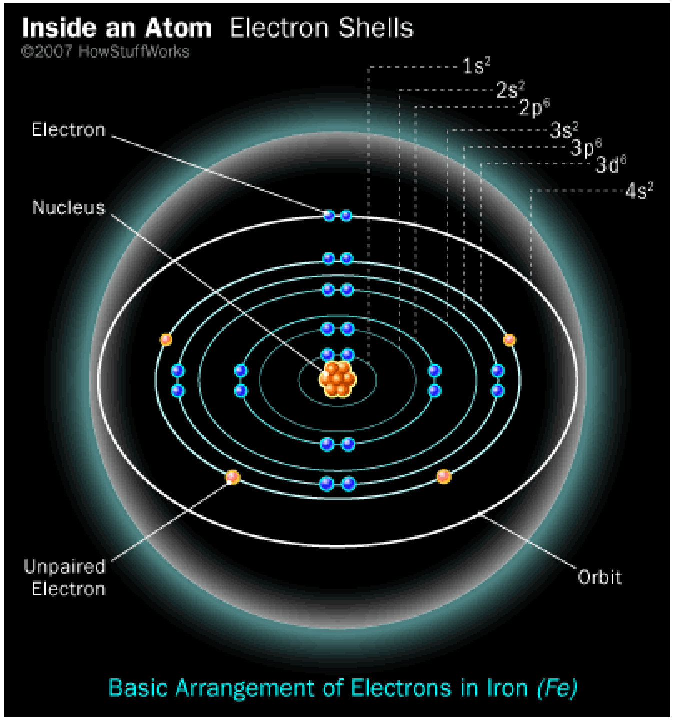
6.2 magnetic materials

ferromagnetic materials

- due to spin of unpaired electrons in inner atomic orbitals
- magnetic dipoles of neighbouring atoms align spontaneously
- causes magnetization of the material, even with no external field (permanent magnets)
- alignment of dipoles is due to quantum mechanical exchange interaction

examples

- iron
- nickel
- cobalt



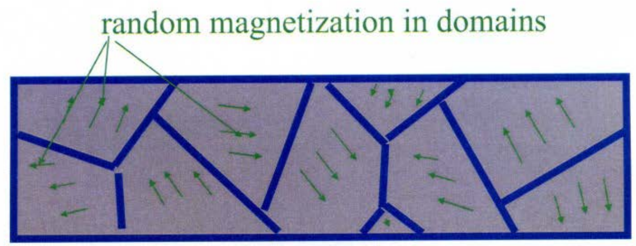
electron arrangement in an iron atom, showing unpaired electrons in 3d orbital

6.3 ferromagnetism

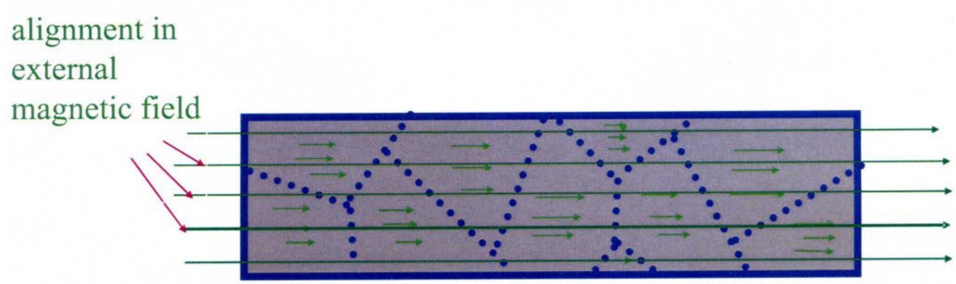
- ferromagnetism is a macroscopic manifestation of quantum physics
- the quantum mechanical exchange interaction causes the magnetic dipoles of neighbouring atoms to align spontaneously
- this leads to the formation of sub-millimetre sized regions with aligned magnetic fields
- these regions of spontaneous magnetisation are called magnetic domains

6.3 ferromagnetism

- in the absence of an external magnetic field, the domains are magnetised in random directions with respect to one another



- in an external field, the domains align with the applied field (and so with one another)

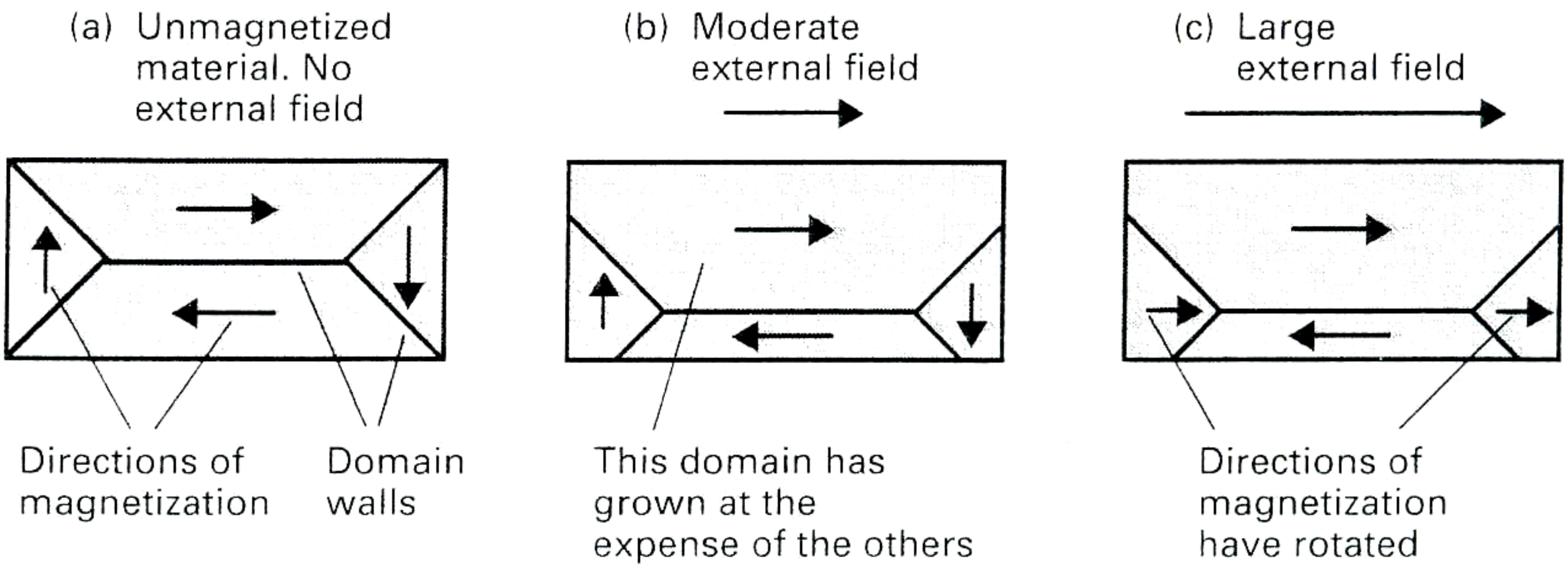


6.3 ferromagnetism

- ferromagnetic domains align in an external magnetic field
- when the applied field is removed, most of the domains remain in alignment with one another, the material is magnetised
- the magnetisation of these so-called permanent magnets can be lost, for example by heating or by physical shock
- when a permanent magnet is heated above its Curie temperature (~ 760 °C for iron), the increased thermal energy destroys the alignment of the domains

6.3 ferromagnetism

magnetisation of iron



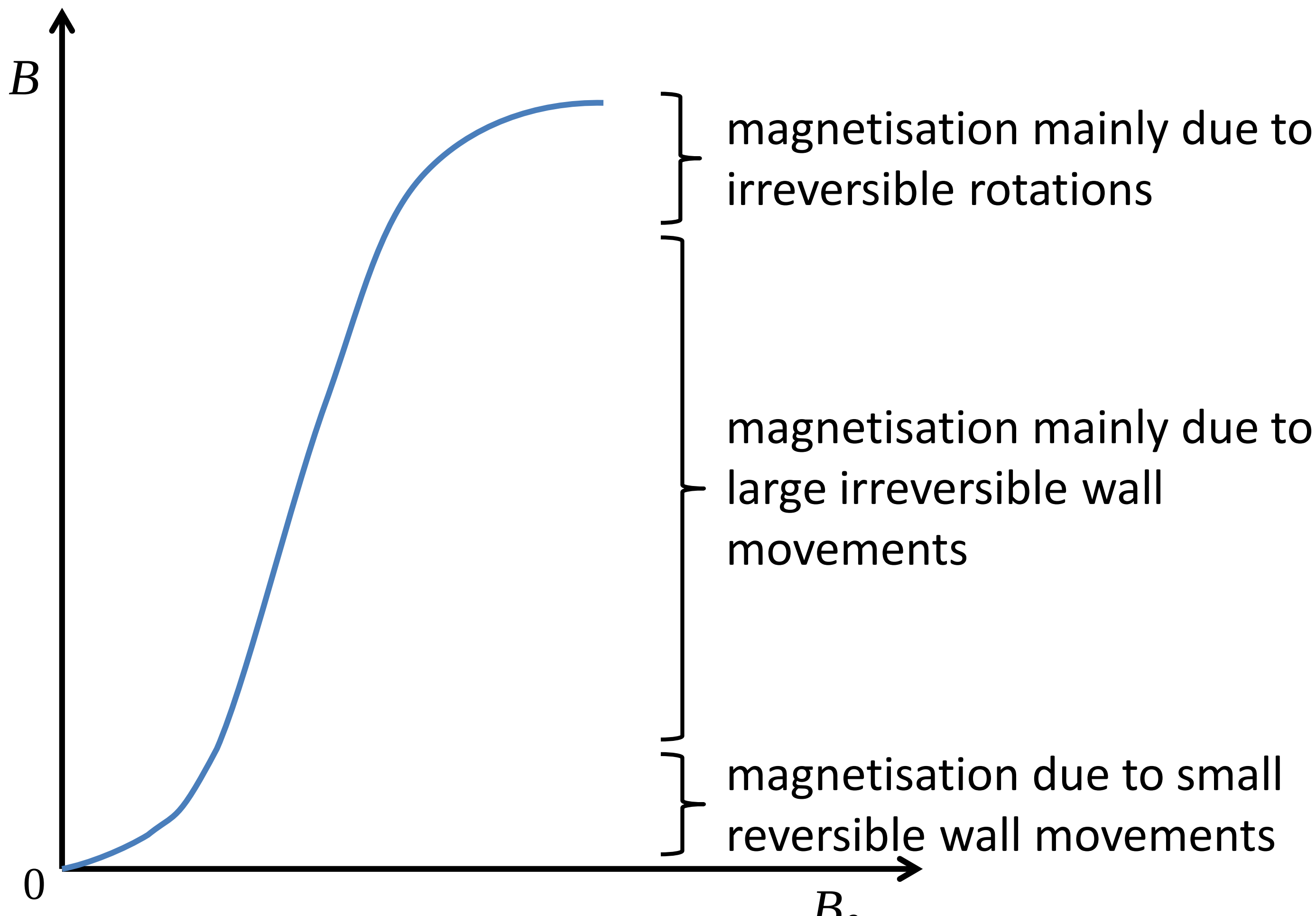
(a) no applied field, domains cancel out

(b) domains that are in alignment with external field begin to grow, domain walls move further as external field increases

(c) in large external field, domains may rotate into alignment

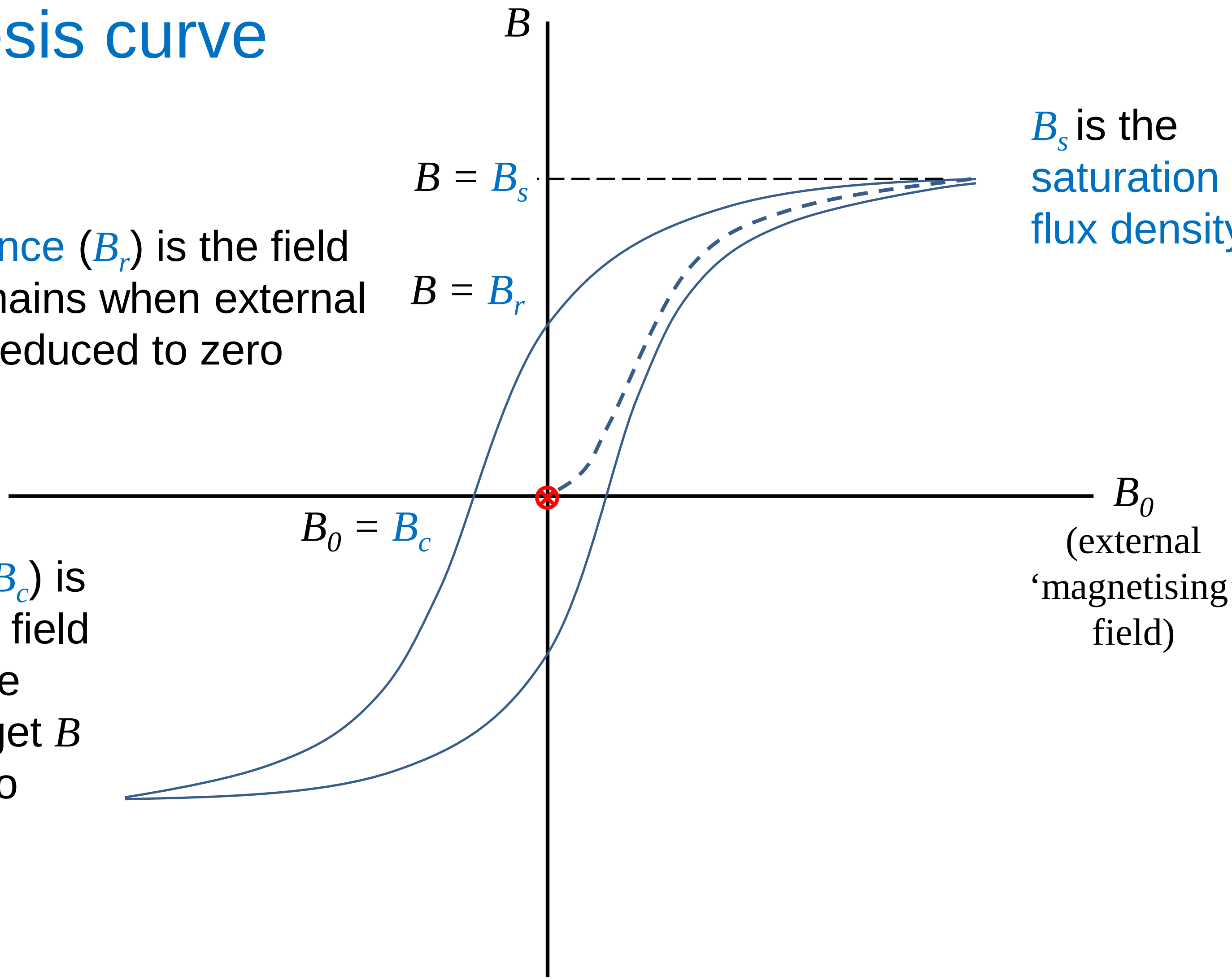
6.3 ferromagnetism

magnetisation curve of iron



6.3 ferromagnetism

hysteresis curve



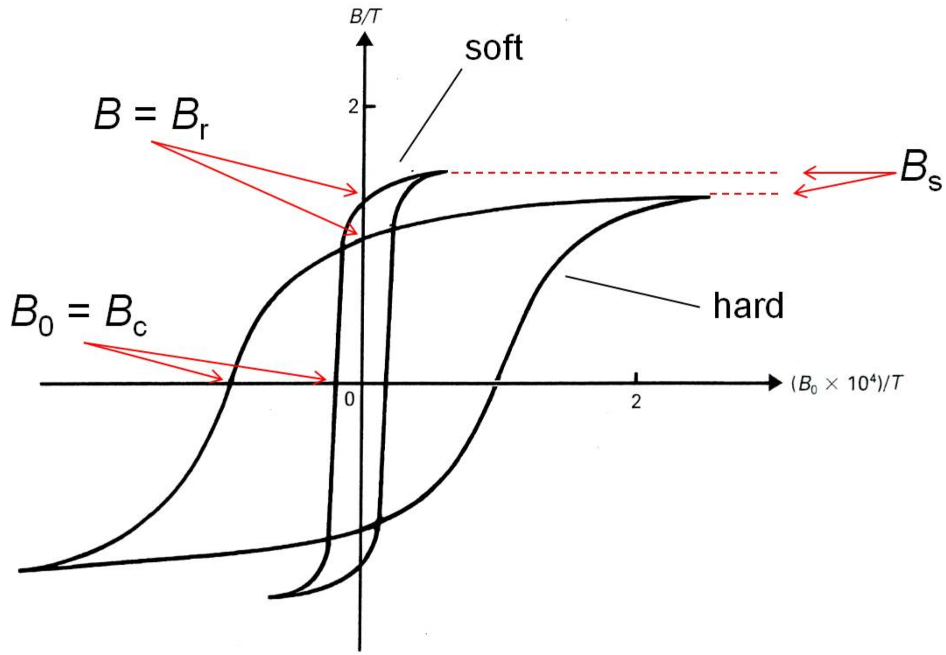
B_s is the saturation flux density

remanence (B_r) is the field that remains when external field is reduced to zero

coercivity (B_c) is the reverse field that must be applied to get B back to zero

6.3 ferromagnetism

hysteresis curve



- when the external field is removed, a magnetically hard material (e.g. steel) retains its magnetism more then a magnetically soft material (e.g. iron)
- hard materials are best suited as permanent magnets
- the area enclosed by the hysteresis loop is greater for a magnetically hard material, compared to a soft one
- the area is also proportional to the energy expended in cycling a material around the loop (magnetising and demagnetising)
- the core of electromagnets, transformers etc are best made from soft materials (less energy loss as the field is cycled)

7.1 superconductors: part 1

overview

Superconductors are materials in which electrical current can flow without experiencing any resistance.

Many common metals (conductors at normal temperatures) are superconductors when cooled to very low temperatures. There are also materials that are insulators at room temperature but are superconductors at very low temperatures.

It's not just the electrical properties of superconductors that are unique, they also exhibit remarkable magnetic properties.

In a superconductor that's below its critical temperature (T_c)

- the electrical resistance drops to zero
- magnetic fields are expelled from its interior (the 'Meissner' effect)

7.1 superconductors: part 1

a brief history

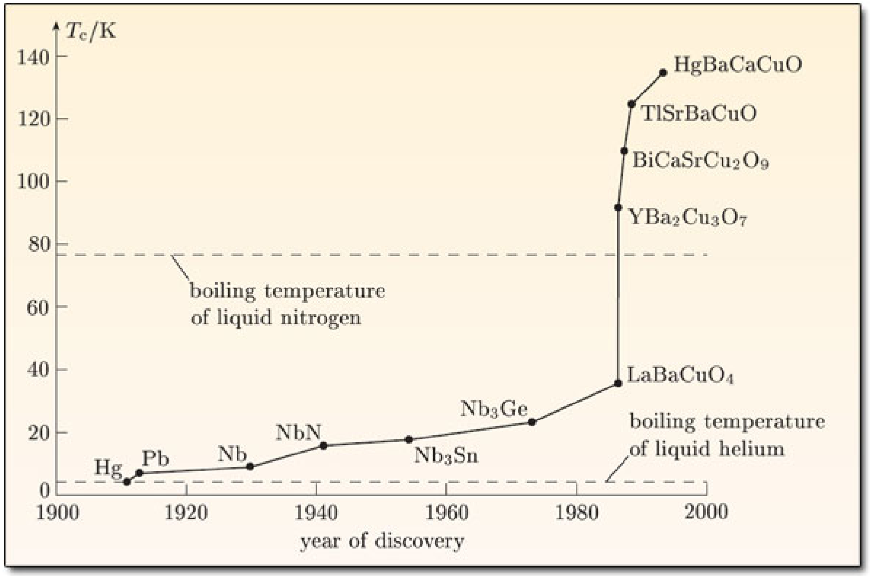
- 1911: first example of superconductivity discovered, by Heike Onnes, when pure Mercury was cooled below 4.2 K
- further examples followed rapidly, of pure metals exhibiting superconductivity below a critical temperature

Substance	Critical Temperature, T_c (K)
Mercury	4.2
Lead	7.2
Tin	3.7
Indium	3.4

7.1 superconductors: part 1

a brief history

- as research continued, more superconducting materials were discovered, including metallic alloys
- a major focus was to find the materials with the highest critical temperature
- in the 1980s, a range of ceramic materials known as cuprates (copper oxides) were found to have critical temperatures above the temperature of liquid nitrogen
- these ceramic materials are insulators at room temperature



highest superconducting transition temperatures as a function of year

7.1 superconductors: part 1
applications

established applications

- superconducting magnets – MRI scanners, particle accelerators
- wires and films for power transmission
- superconducting quantum interference devices (SQUIDS) for super sensitive magnetic detectors

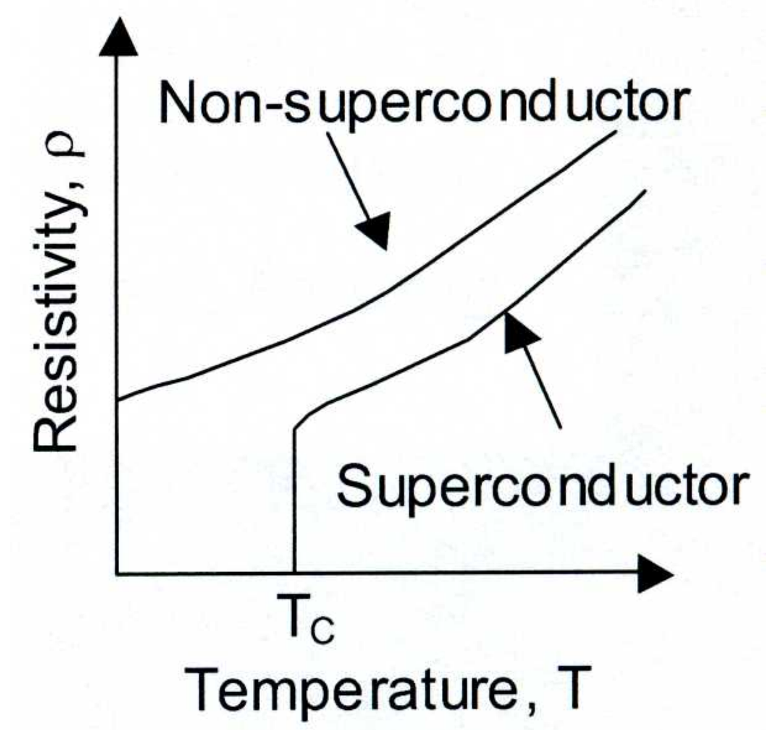
potential applications

- electric motors, generators, propulsion units e.g. Maglev
- need to overcome problems of cryogenic cooling and material robustness – ceramic (high temp) superconductors tend to be brittle

7.2 superconductors: part 2

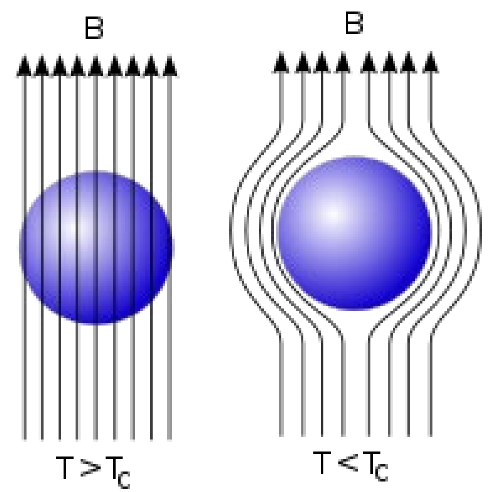
electrical characteristics

- resistance drops abruptly to zero
- current flows without generating heat
- no energy loss, current flows without degradation with no power source
- these so-called persistent currents can flow for many years
- for certain semiconductors, this behaviour can be described by 'BCS theory' (in which conducting electrons existing as 'Cooper pairs')



7.2 superconductors: part 2

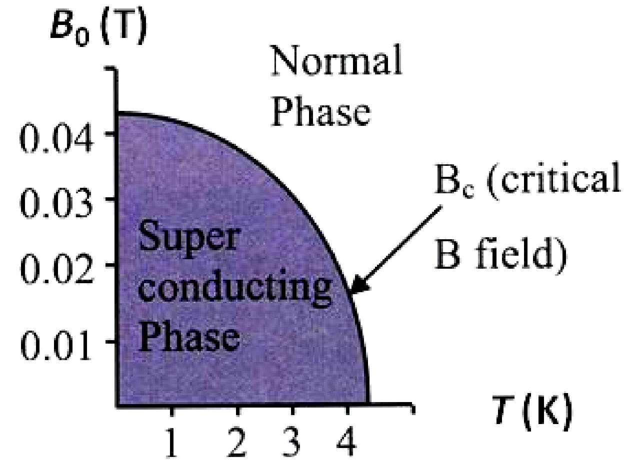
magnetic characteristics



- as the material goes through the transition to become superconducting, all magnetic flux passing through the material is abruptly expelled
- this is the Meissner effect - it's a quantum mechanical effect (the superconductor acts like a giant atom, with electrons orbiting around the edges producing 'shielding' currents)
- once in the superconducting state, a superconductor repels external magnetic fields, exhibiting superdiamagnetism (as demonstrated by magnetic levitation)
- superdiamagnetism is a result of the external magnetic field inducing currents on the surface of the superconductor, which produce their own magnetic field that is equal and opposite to the external field

7.2 superconductors: part 2

type I superconductors



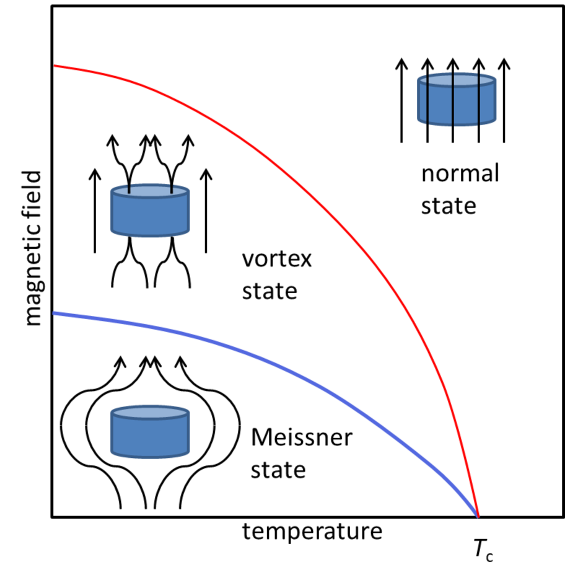
superconducting phase diagram for mercury – type I

- type I superconductors are pure metals
- they have a single superconducting phase
- temperatures greater than the critical temperature T_c cause the loss of superconductivity
- magnetic fields greater than the critical magnetic field B_c cause the loss of superconductivity
- type I superconductors are well described by BCS theory
- they have limited practical uses due to low T_c and B_c

7.2 superconductors: part 2

type II superconductors

- higher T_c than type 1 (often referred to as 'high temperature superconductors')
- higher B_c – can carry higher current densities while remaining in superconducting state (more practical uses)
- 2 superconducting phases: below lower critical field (blue line) they're in the Meissner state – behave like type I (zero resistance, completely expel magnetic fields from interior)

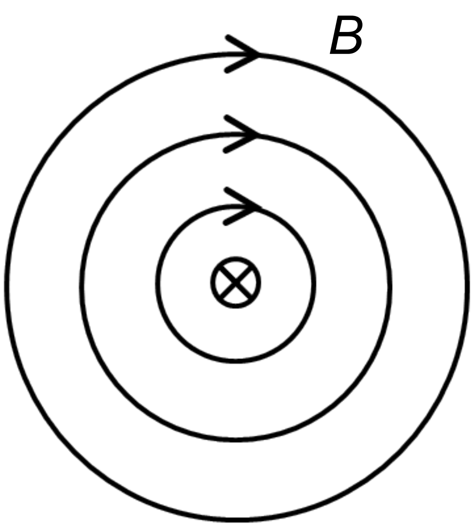


phase diagram for type II superconductor

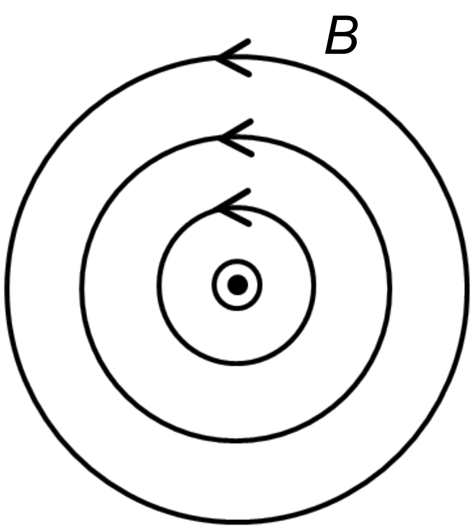
- between lower critical field (blue line) and upper critical field (red line) the superconductor is in a mixed state (or vortex state)
- in mixed state, filaments of non-superconducting material run through the superconductor; flux lines pass through these vortices but are trapped in the vortices by shielding currents (known as flux pinning)

7.3 magnetic fields around currents

- moving charges such as currents create magnetic fields
- the magnetic field is always perpendicular to the direction of the current



a) current **into** page

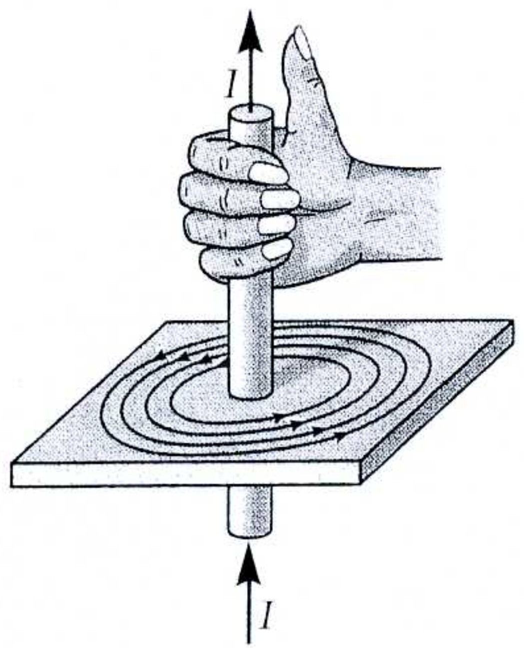


b) current **out of** page

7.3 magnetic fields around currents

right hand ‘grip’ rule

- there is a right-hand rule for determining the direction of the magnetic field lines around a current



grasp the conductor in your right hand with your extended thumb pointing in the direction of the (conventional) current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that current element

7.3 magnetic fields around currents

- the magnetic field produced by a current can be quantified by the law of Biot and Savart
- the law is experimentally deduced
- it is an inverse-square law

$$dB = \frac{\mu_0}{4\pi}\frac{i\ ds\sin\theta}{r^2}$$ *(scalar form)*

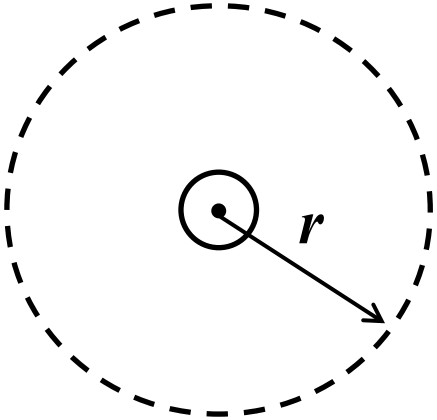
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\ d\vec{s}\times\hat{\mathrm{r}}}{r^2}$$ *(vector form)*

- can be used to determine (by integration) the net magnetic field produced at a point by various current arrangements

7.3 magnetic fields around currents

long straight wire

in air

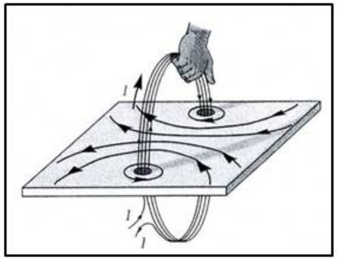


$$B = \frac{\mu_0 I}{2\pi r}$$

- B is the magnetic flux density (in tesla)
- r is the distance from the centre of the conductor
- μ_0 (permeability of free space) = $4\pi \times 10^{-7}$ T m A^{-1}

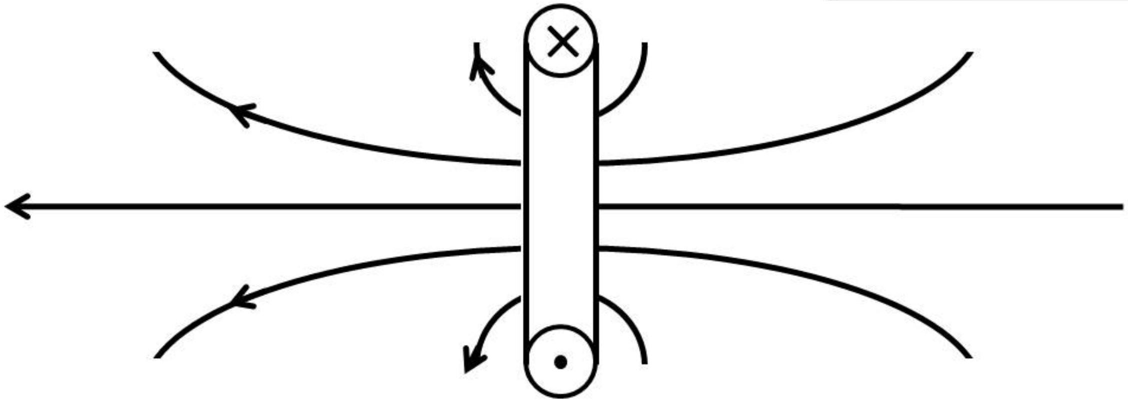
7.3 magnetic fields around currents

centre of a flat coil



in air

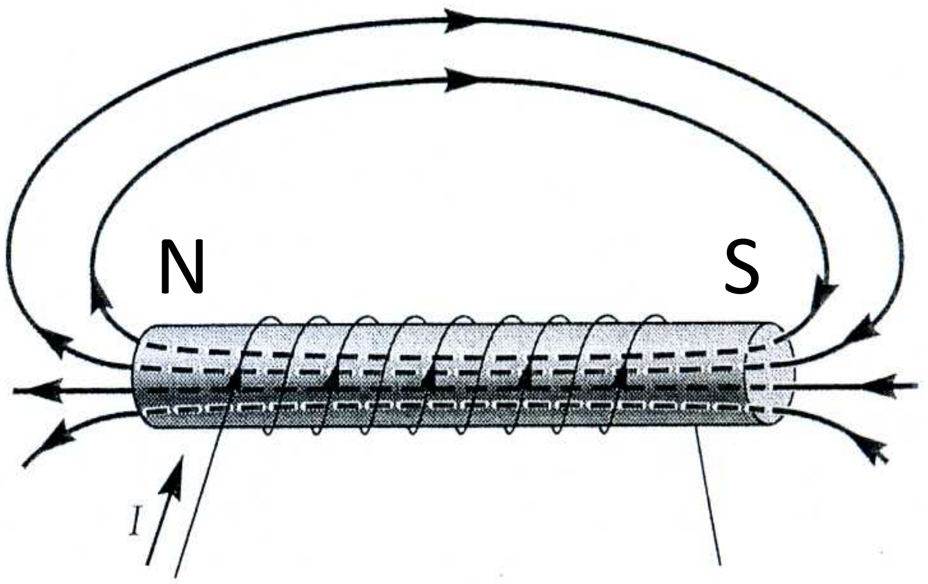
$$B = \frac{N \mu_0 I}{2a}$$



N = number of loops and a = radius of the loop

7.3 magnetic fields around currents

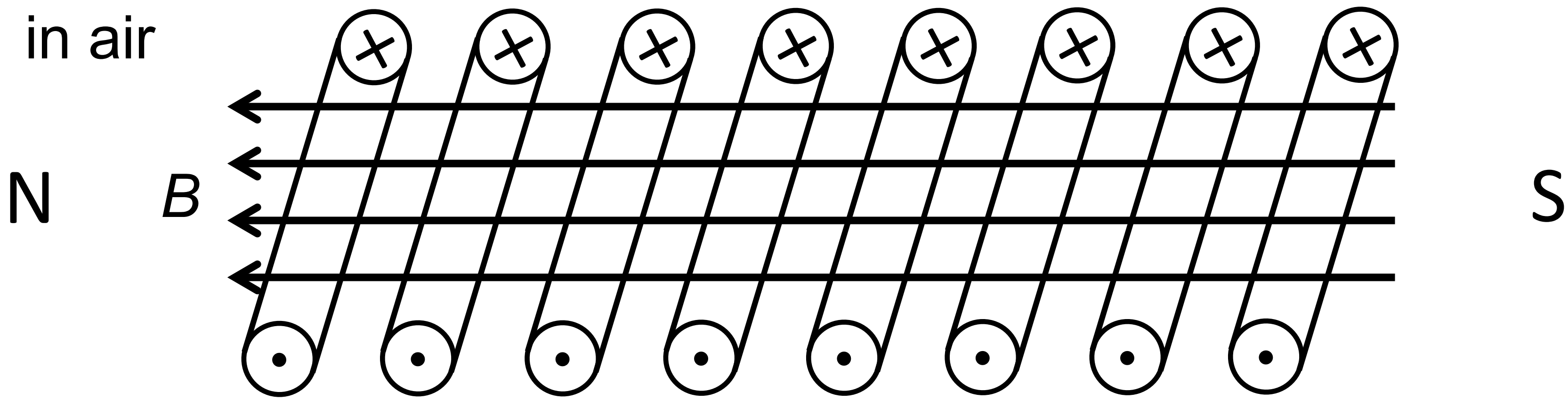
outside a solenoid



- outside the solenoid the field resembles that of a bar magnet (the solenoid has a north and a south pole)

7.3 magnetic fields around currents
magnetic field inside solenoid

in air



$$B = n\mu_0 I$$

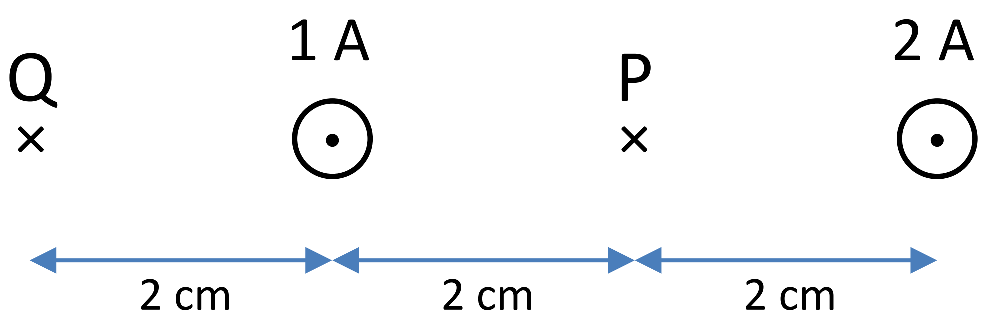
where n = number of loops per metre

- note that the magnetic field is uniform inside the solenoid

7.3 magnetic fields around currents

example

two long, straight, parallel conductors are separated in air by a distance of 4 cm and carry currents of 1 and 2 amps, in the same direction, as shown below. What are the magnetic flux densities at points P and Q?



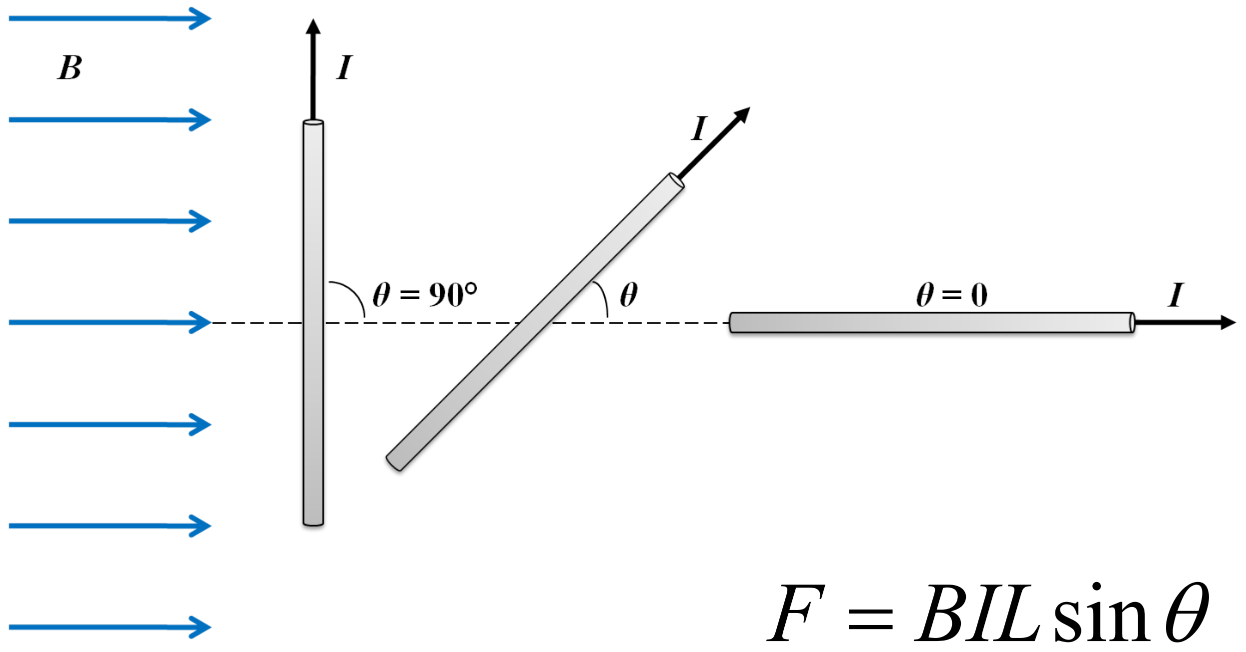
8.1 magnetic force on a conductor

- when a current flows through a wire that is at some angle, other than parallel, to a magnetic field, the wire experiences a force (the Laplace force)
- the size of the force is proportional to:
  - the magnitude (B) of the magnetic field
  - the size of the current (I)
  - the length (L) of the conductor in the field

$$F \propto BIL$$

8.1 magnetic force on a conductor

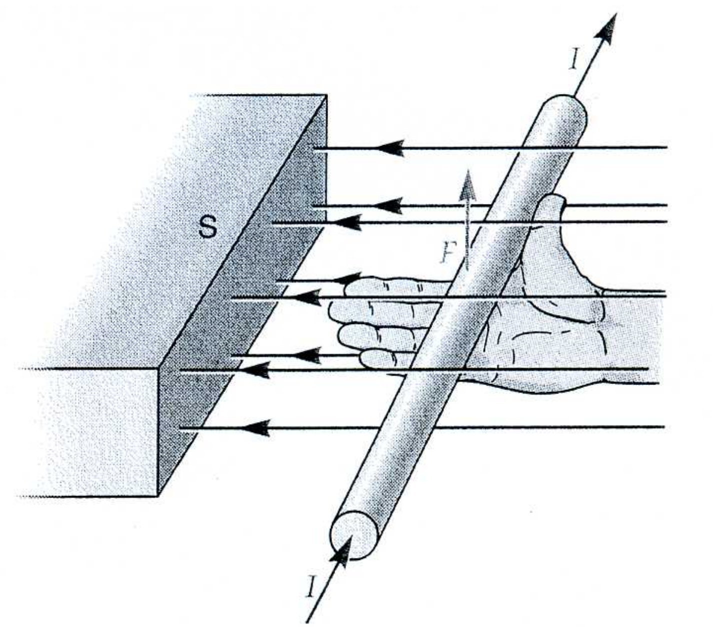
- the magnitude of the force also depends on the angle between the current and the magnetic field



$$F = BIL\sin\theta$$

8.1 magnetic force on a conductor

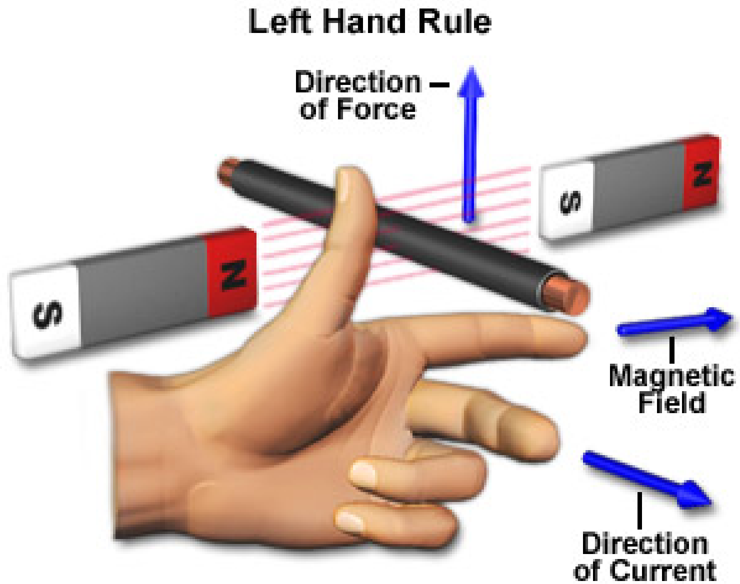
- there is a right-hand rule for determining the direction of the force on a current-carrying wire in a magnetic field



- remember, the direction of the current is (by convention) the direction a positive charge would flow (i.e. opposite to the flow of electrons)

8.1 magnetic force on a conductor

- alternatively, there is a left-hand rule for determining the direction of the force on a conductor in a magnetic field (known as Fleming's left hand rule)



8.2 forces between parallel conductors

the ampere is defined as the steady current in each of two straight, parallel conductors of infinite length and negligible cross-sectional area, 1 metre apart in vacuum, that produces a force between them of 2 x 10^{-7} N per metre

- now we can understand the origin of the force that each conductor experiences, which is the basis of this definition of the amp

8.2 forces between parallel conductors

- assume currents are flowing in the same direction

$I_1 \otimes$ $\otimes I_2$

8.2 forces between parallel conductors

- current I_2 generates a magnetic field (B_2). The field line passing through I_1 is shown below

8.2 forces between parallel conductors

- the conductor carrying I_1 experiences a force due to B_2, whose direction is given by Fleming's left-hand rule

8.2 forces between parallel conductors

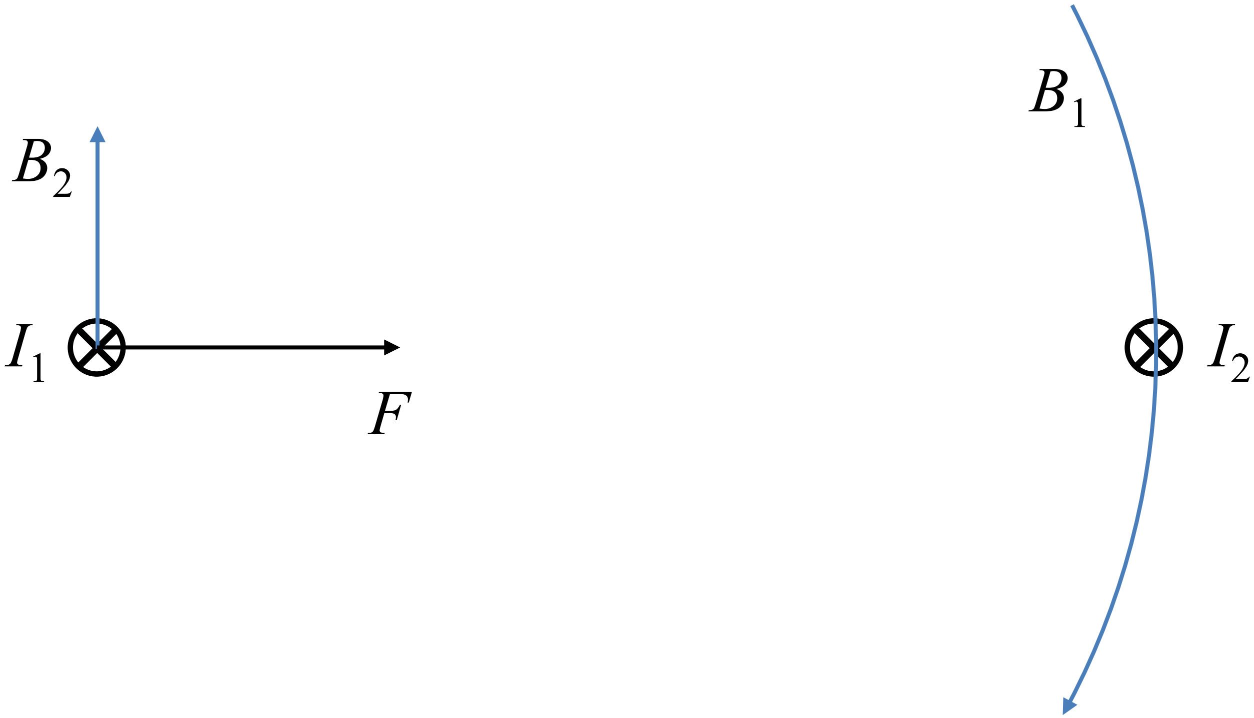
- at the position of I_1 the direction of B_2 is a tangent to the field line at that point

8.2 forces between parallel conductors

- applying the left-hand rule shows that the force acts towards I_2

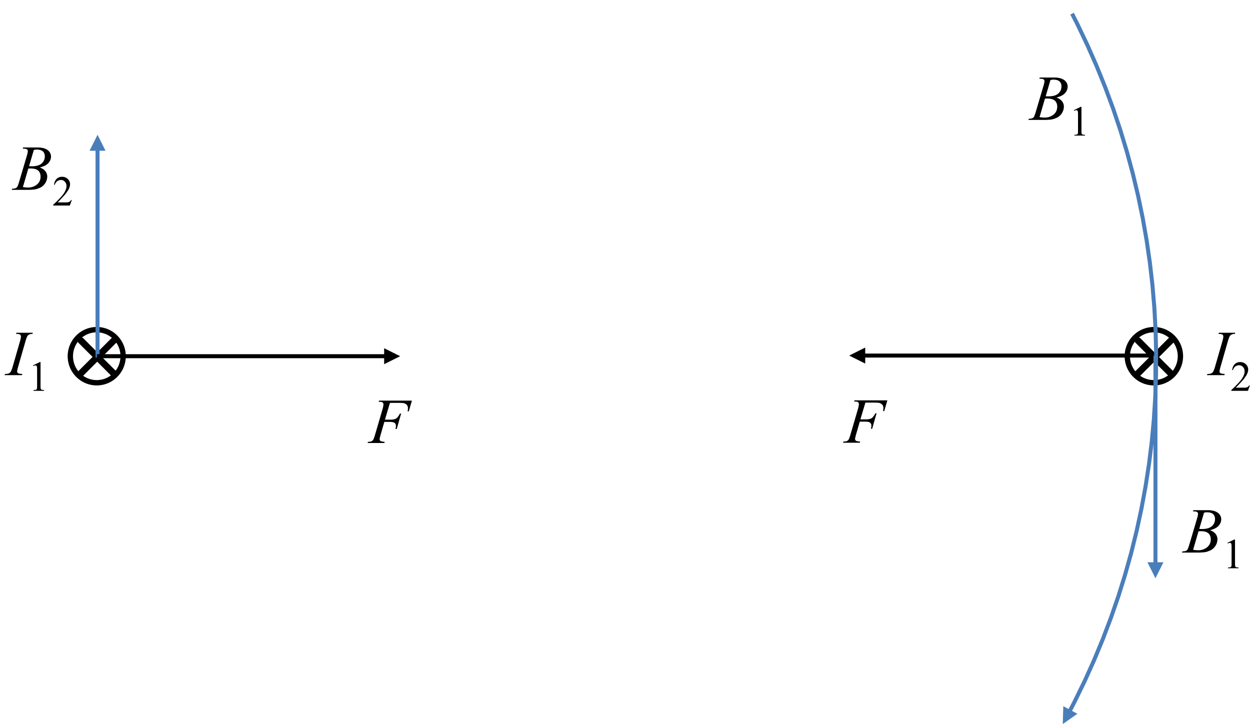
8.2 forces between parallel conductors

- similarly, I_1 will produce a magnetic field (B_1) passing through I_2



8.2 forces between parallel conductors

- at I_2 the direction of B_1 will be ‘down’, causing a force on the conductor carrying I_2, as shown



8.2 forces between parallel conductors

- the conductors experience an attractive force

8.2 forces between parallel conductors

- if the currents flow in opposite directions to one another, similar arguments show that each conductor would experience a force directed away from the other

  - currents in opposite direction → repulsive force
  - currents in same direction → attractive force

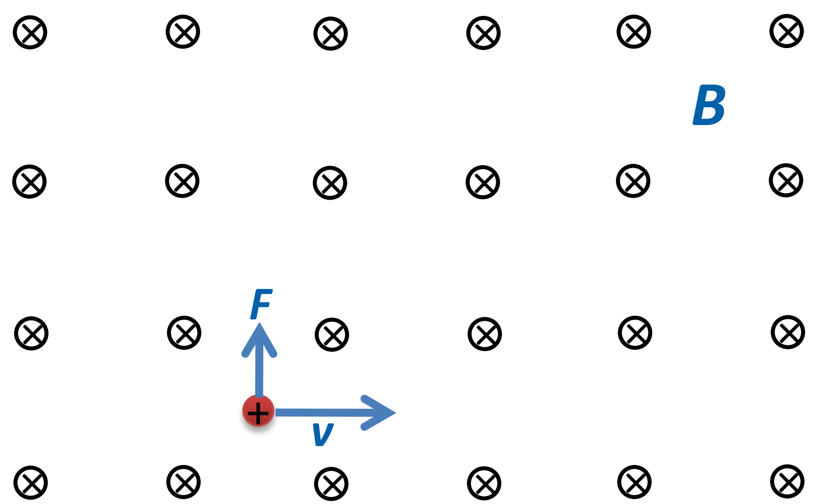
8.3 magnetic force on a moving charge

- a charged particle moving in a magnetic field experiences a force due to the field (Lorentz force)
- the force is perpendicular to the field and to the direction of motion of the charge (right/left hand rules apply)
- because the magnetic force is always perpendicular to the velocity of the particle, the magnetic field does no work – the speed of the particle doesn't change, only its direction

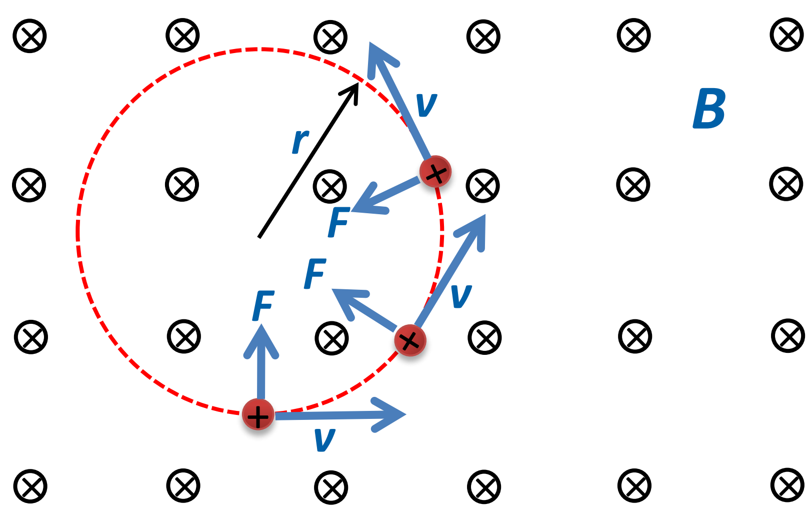
$$F = qvB\sin\theta$$

8.3 magnetic force on a moving charge

- the magnetic force doesn't change the speed of the particle, but it does change the direction of motion



8.3 magnetic force on a moving charge



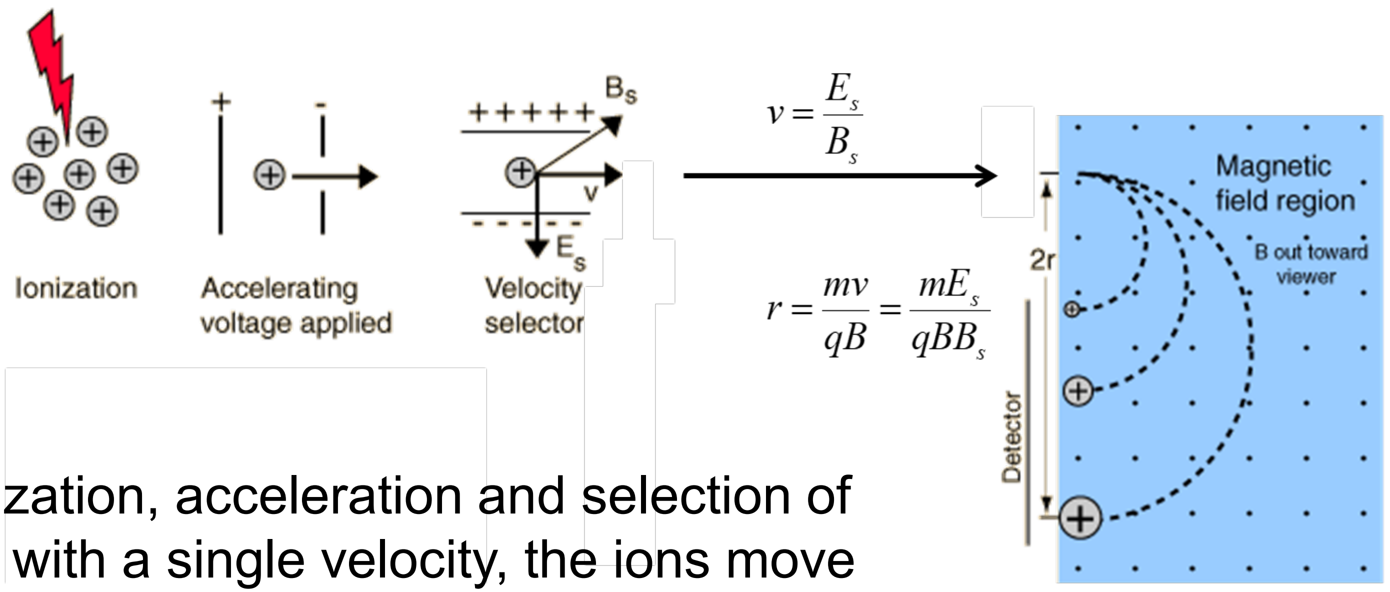
- we can equate the centripetal and magnetic forces:

$$\frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB}$$

8.3 magnetic force on a moving charge

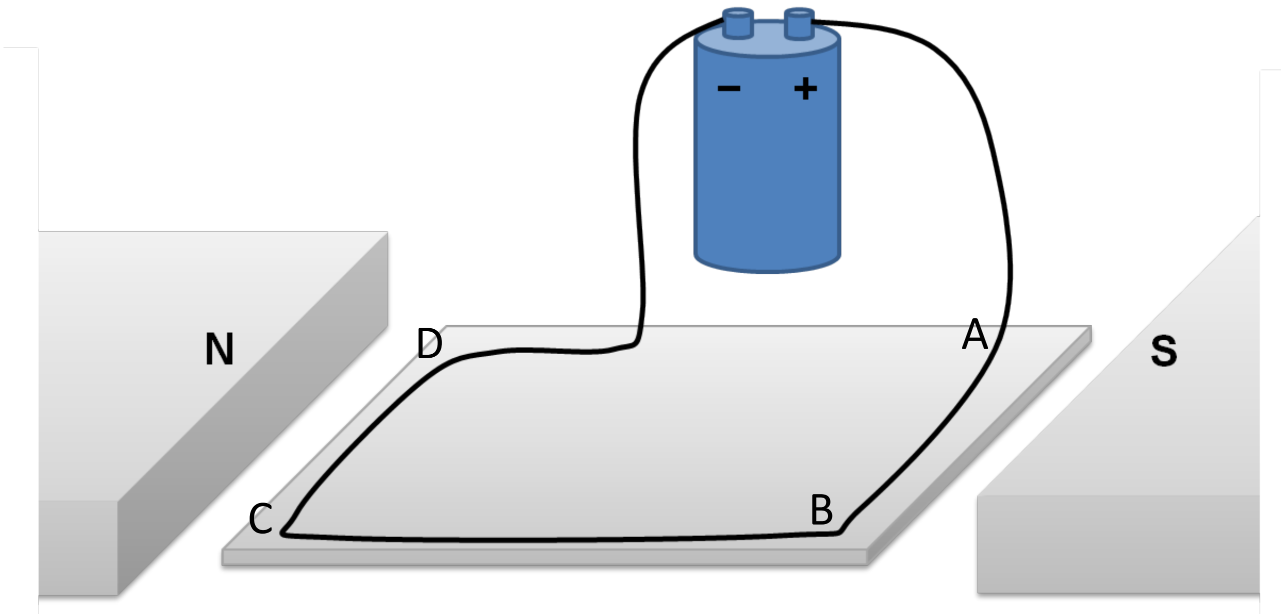
$$r = \frac{mv}{qB}$$

- particles can be separated by their mass
- this is the principle of the mass spectrometer



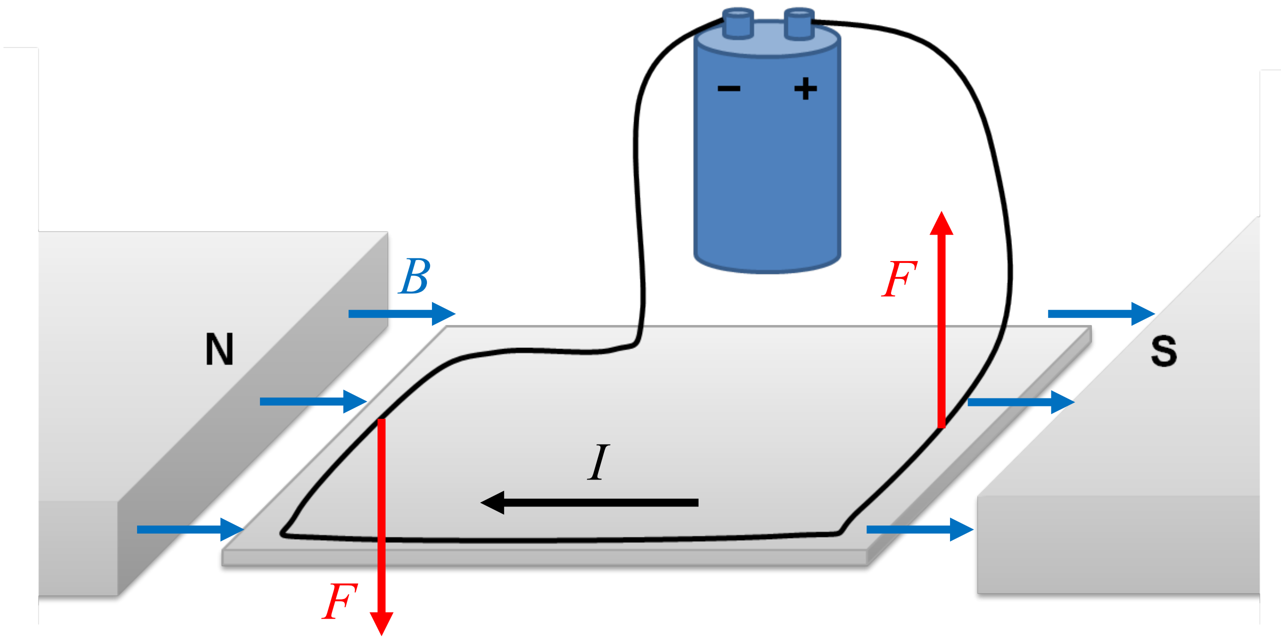
- after ionization, acceleration and selection of particles with a single velocity, the ions move into the magnetic field where the radius of the path, and thus the position on the detector, is a function of the mass

8.4 simple DC motor
magnetic force on a current loop



- what is the direction of the magnetic force on sides AB and CD of the loop?

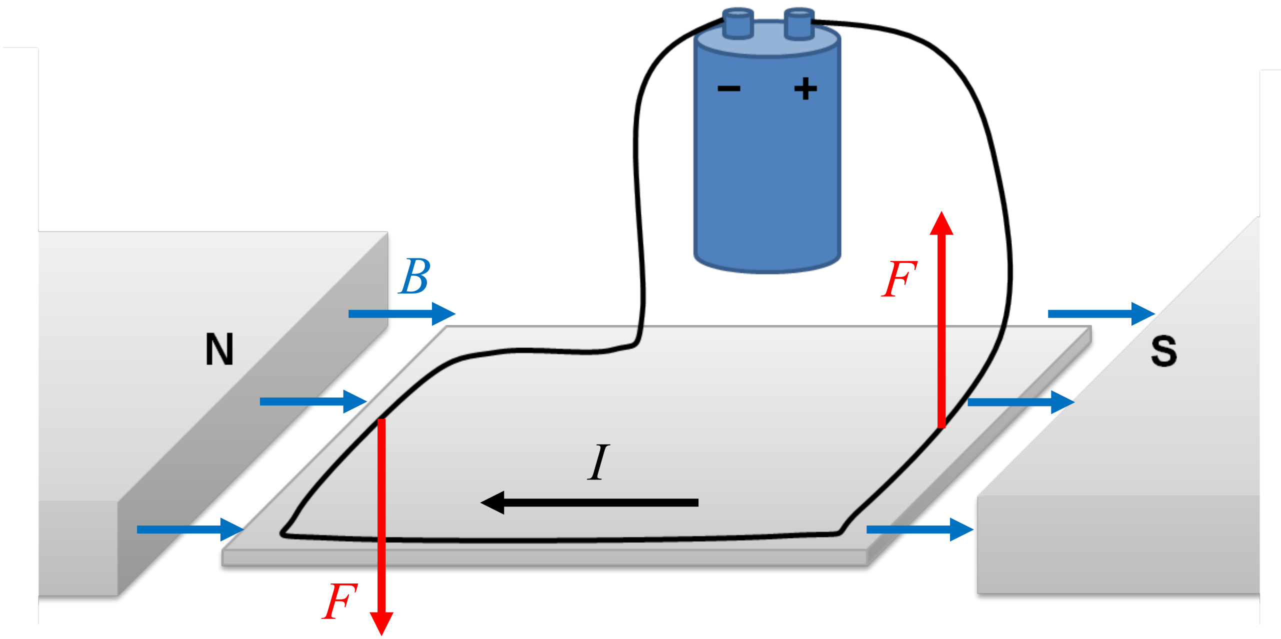
8.4 simple DC motor
magnetic force on a current loop



- the forces on the left and right sides of the loop create a turning force, or torque

8.4 simple DC motor

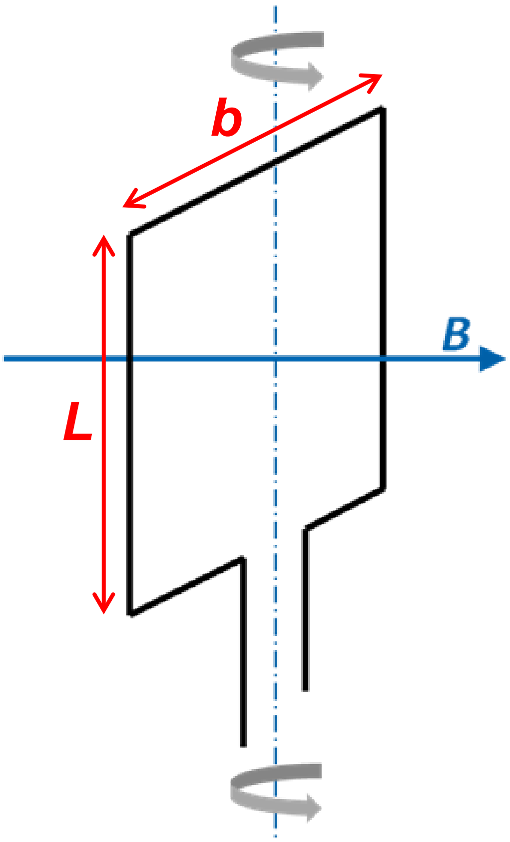
magnetic force on a current loop

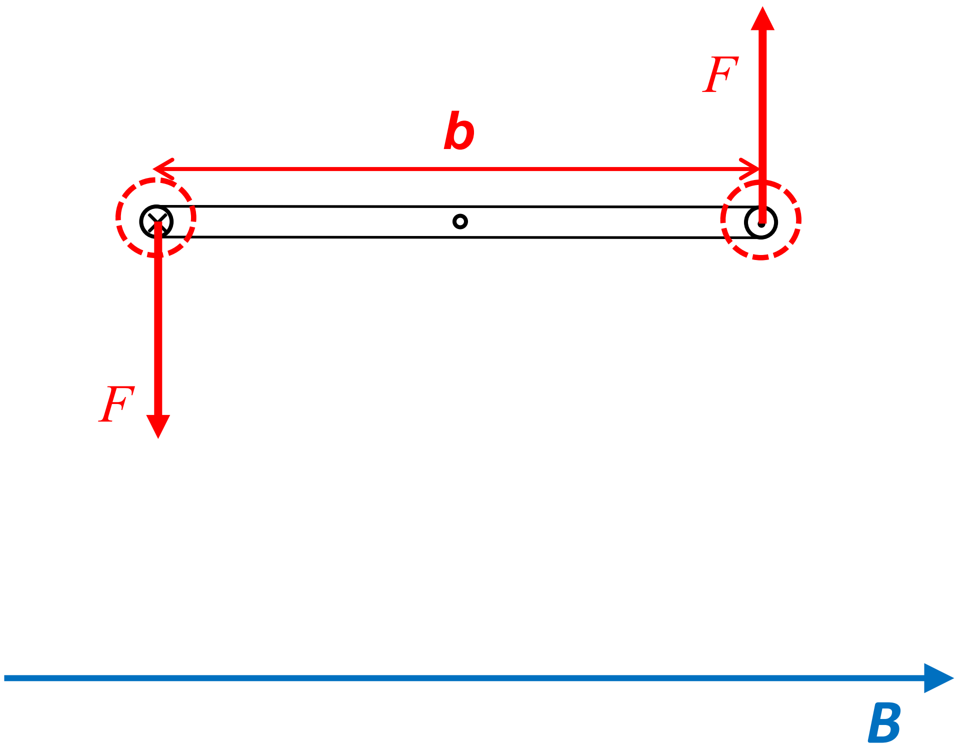


- if we arrange for the loop to be free to rotate around an axis, we have the basis of a simple electric motor

8.4 simple DC motor

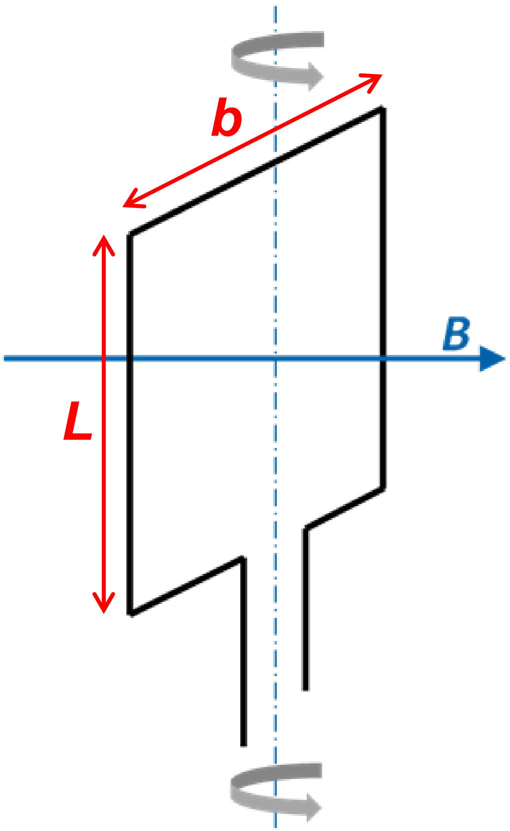
torque on a coil in a magnetic field



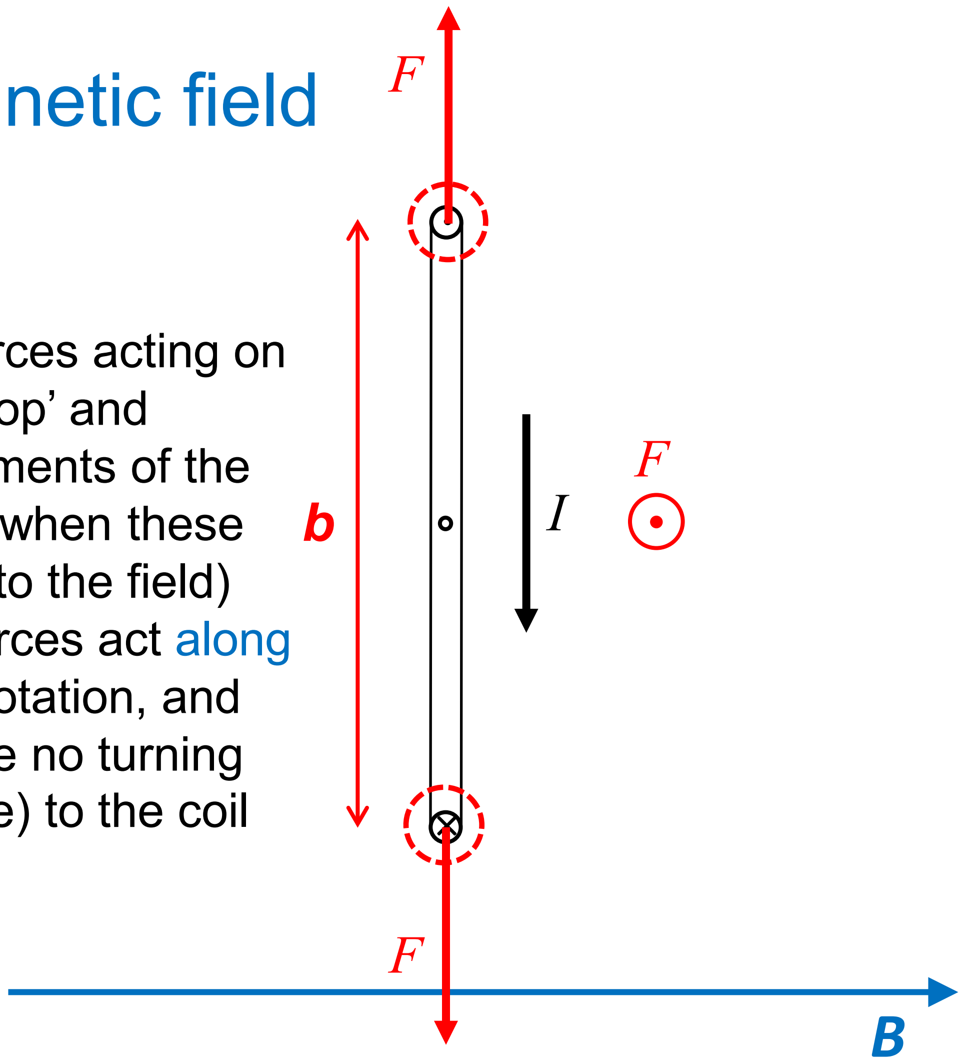


8.4 simple DC motor

torque on a coil in a magnetic field

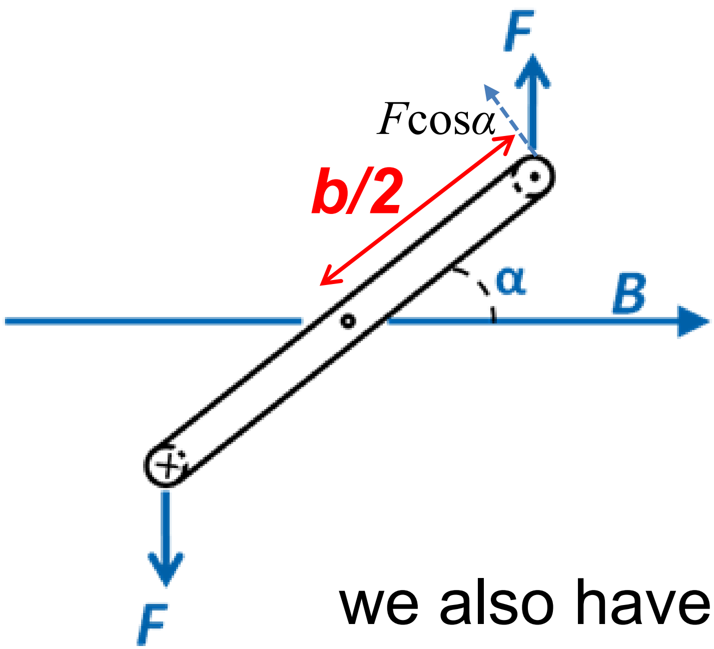


there are forces acting on the on the 'top' and 'bottom' elements of the coil (except when these are parallel to the field) but these forces act along the axis of rotation, and so contribute no turning force (torque) to the coil



8.4 simple DC motor

torque on a coil in a magnetic field



torque due to ‘upward’ force F is

$$b/2 \times F\cos\alpha$$

where $F = BINL$

so, the torque is

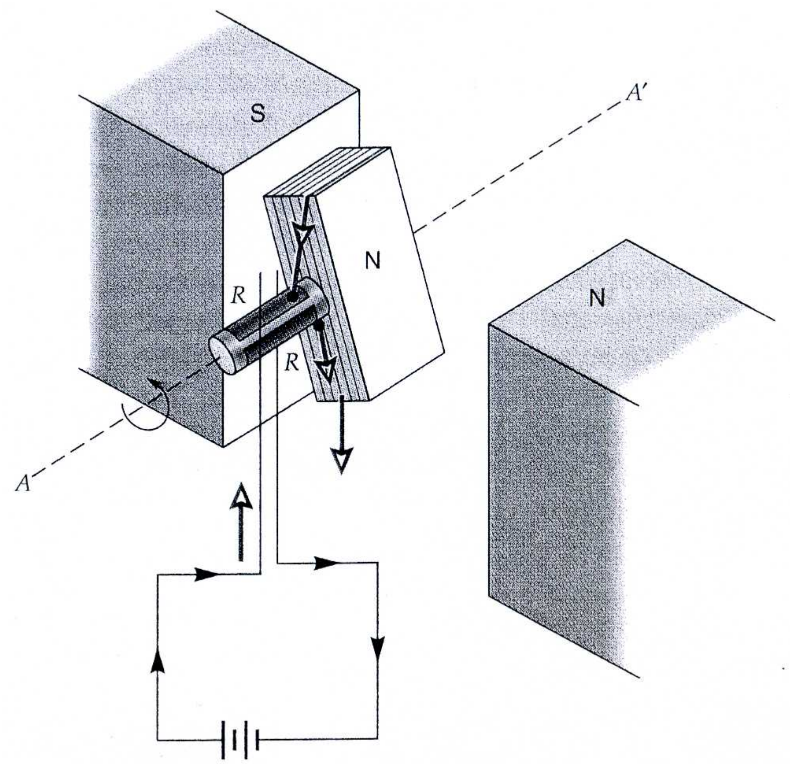
$$(BINLb\cos\alpha)/2$$

or $$(BINA\cos\alpha)/2$$

we also have an equal ‘downward’ force F acting to rotate the coil in an anticlockwise direction, so the total torque T on the coil is given by

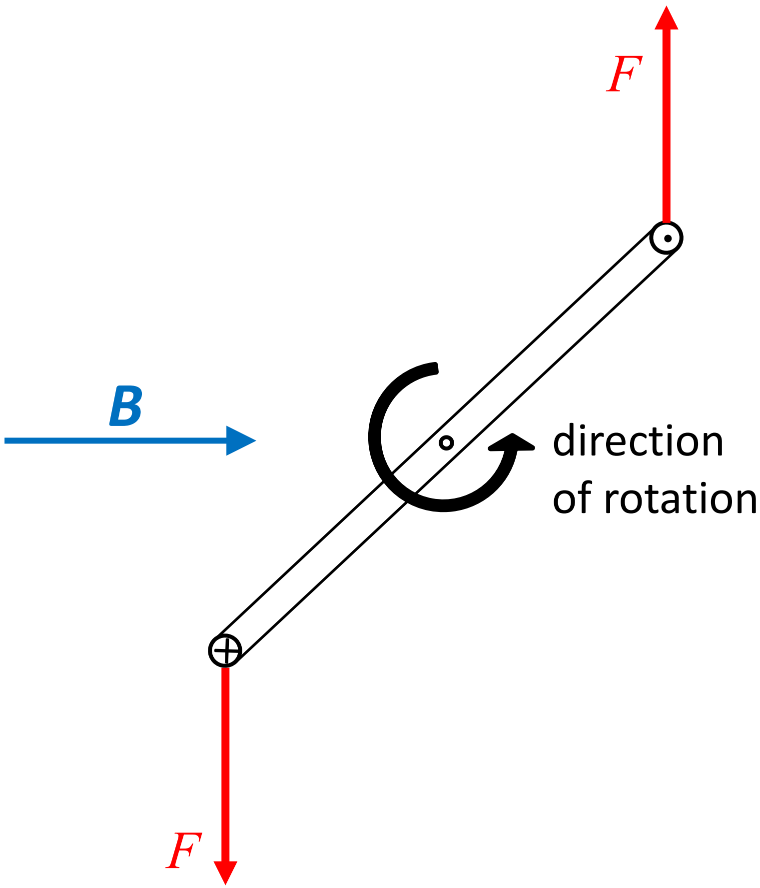
$$T = BINA\cos\alpha \quad [\mathrm{N\ m}]$$

8.4 simple DC motor



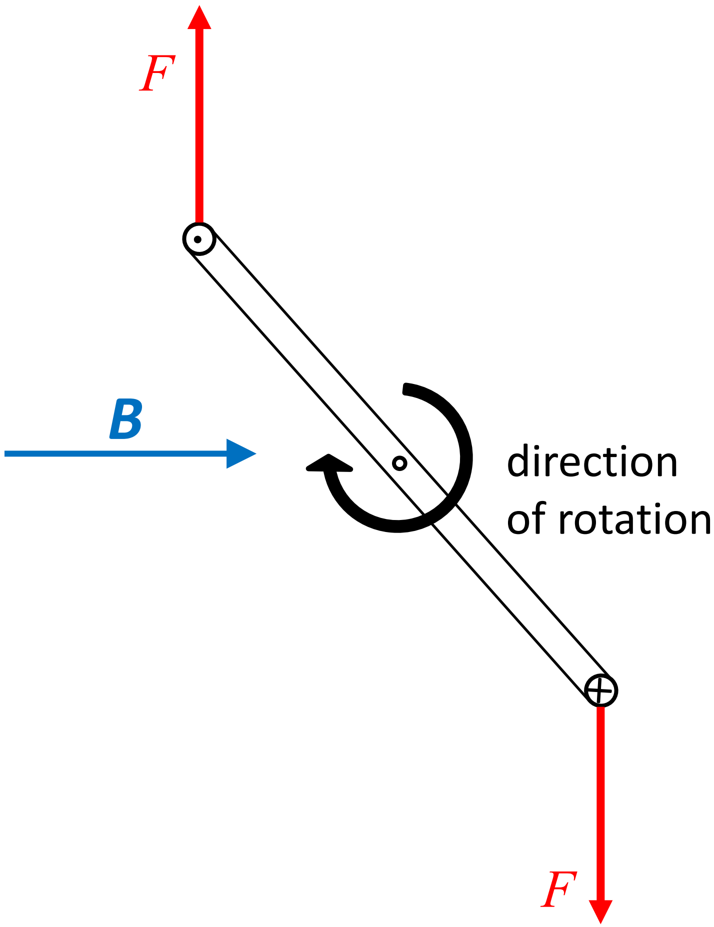
There is one problem to solve before we have a working DC motor

8.4 simple DC motor



There is one problem to solve before we have a working DC motor

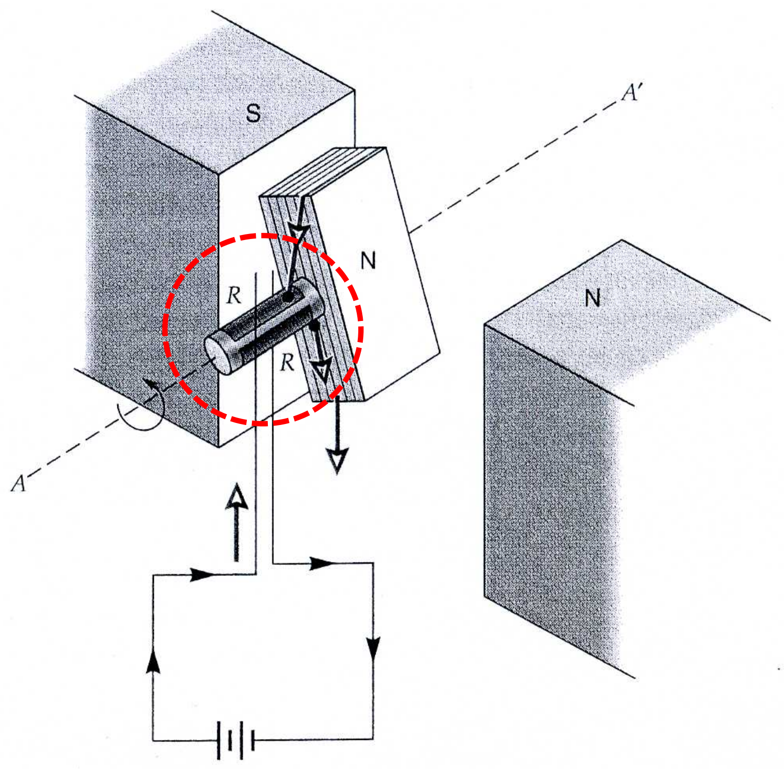
8.4 simple DC motor



There is one problem to solve before we have a working DC motor

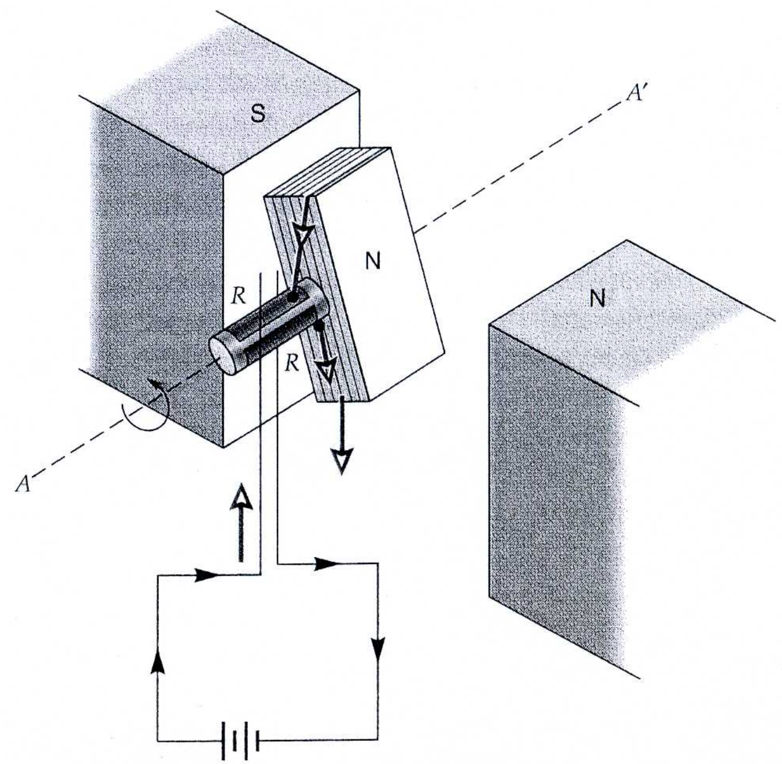
- as the coil rotates past the vertical, the direction of the forces on the top and bottom edges of the coil do not change
- so, the direction of the torque (and therefore the direction of rotation) will change

8.4 simple DC motor



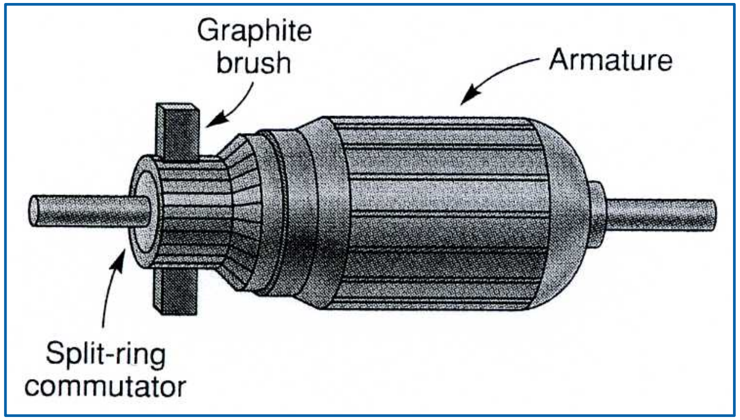
- commutation is required to prevent the torque from reversing every half cycle
- a split ring commutator and brushes are used to connect the motor coil to the DC power supply
- the commutator reverses the current direction in the coil every half cycle, causing the force to change direction as the coil passes vertical

8.4 simple DC motor



- the torque on a single coil varies sinusoidally from zero (when it's perpendicular to the field) to maximum (when it's parallel to the field)
- a single coil motor will not run smoothly and may stall
- a flywheel can be used to prevent stalling and smooth the running of the motor

8.4 simple DC motor



- for each additional coil, the commutator is split into two additional segments, such that the current is supplied to the coil which is in its position of maximum torque

- the torque on a single coil varies sinusoidally from zero (when it's perpendicular to the field) to maximum (when it's parallel to the field)

- a single coil motor will not run smoothly and may stall

- a flywheel can be used to prevent stalling and smooth the running of the motor

- multiple coils wound onto the armature produce a more uniform torque

9.1 magnetic flux

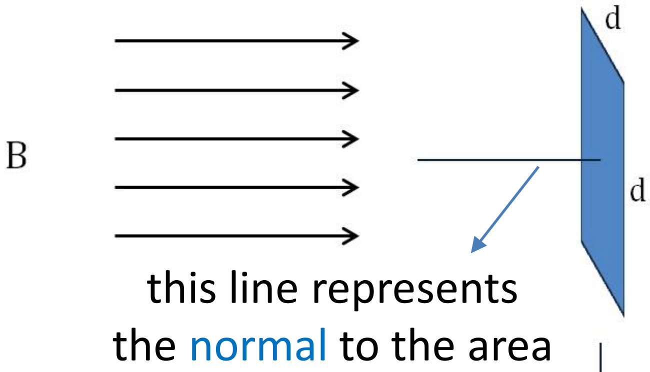
- the magnetic flux through an area, denoted Φ_B, is a measure of the number of magnetic field lines passing through that area
- magnetic flux is found by multiplying the magnetic flux density B by the effective area A.

$$\Phi_B = \underbrace{B}_{\text{flux lines per m}^2\text{ (T)}} \underbrace{A\cos\theta}_{\text{effective area (m}^{-2}\text{)}}$$

- the unit of magnetic flux is the weber (Wb), where 1 T = 1 Wb m^{-2}

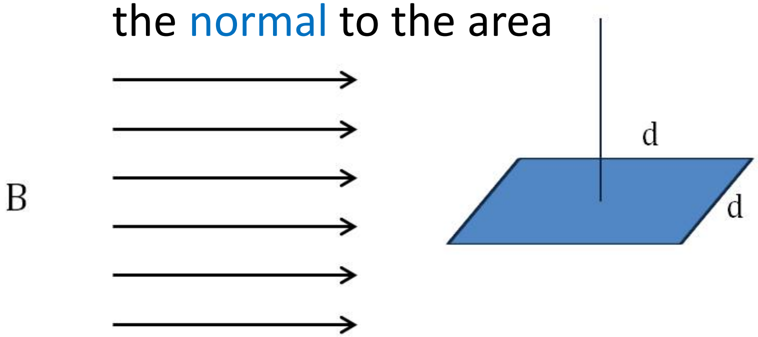
9.1 magnetic flux

θ is the angle between the normal to the area and the magnetic field



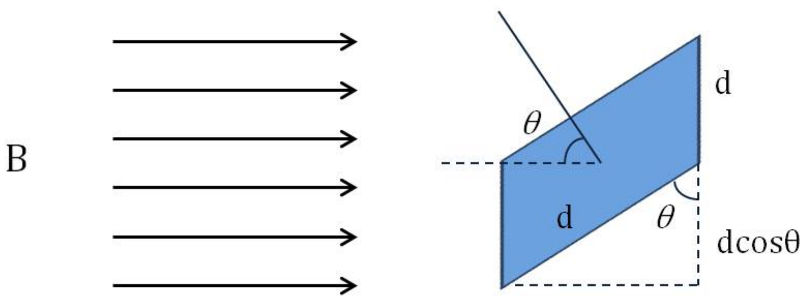
area $A = d^2$
$\theta = 0°$
$\cos\theta = 1$
effective area $= d^2$

$$\Phi_B = BA\cos\theta = BA$$



area $A = d^2$
$\theta = 90°$
$\cos\theta = 0$
effective area $= 0$

$$\Phi_B = BA\cos\theta = 0$$



area $A = d^2$
angle $= \theta$
effective area $= d \times d\cos\theta$
effective area $= A\cos\theta$

$$\Phi_B = BA\cos\theta$$

9.1 magnetic flux

$$\Phi_B = BA\cos\theta$$

where θ is the angle between the normal to the area and the direction of the magnetic field

and if there are N loops in a coil of area A, each loop has the same flux passing through it and we can say the total flux through the coil is:

$$\Phi_B = NBA\cos\theta$$

where θ is the angle between the normal to the plane of the coil and the direction of the magnetic field

9.1 magnetic flux

example

a circular coil contains 50 loops of wire. The coil is in a magnetic field with a flux density of 65 mT and the plane of the coil makes an angle of 20° with the direction of the field. The magnetic flux through the coil is 0.079 Wb. What is the radius of the coil?

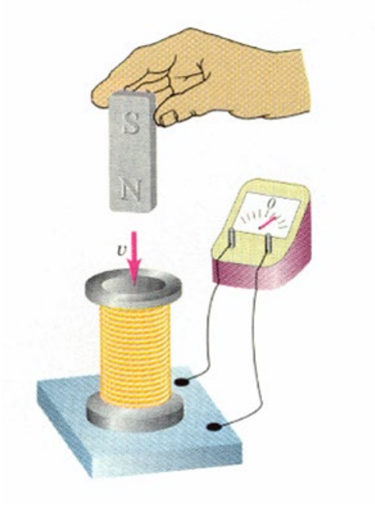
[answer 15 cm]

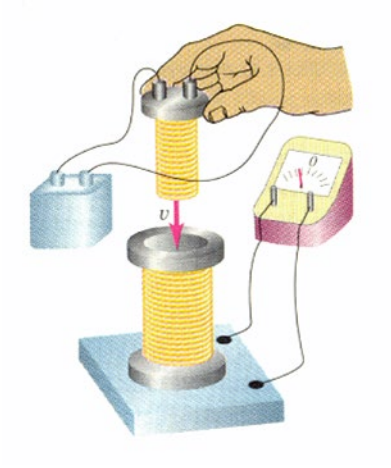
9.2 electromagnetic induction

experimental observations

we can induce a current to flow in a coil by:

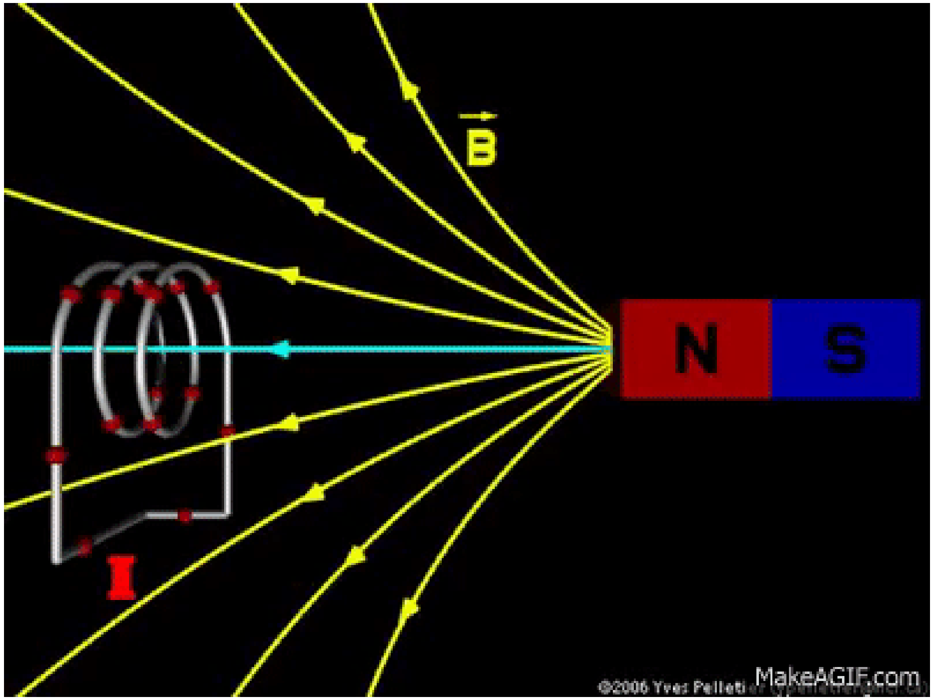
- moving a magnet near the coil
- moving a current-carrying coil near the coil
- changing the current in another coil, near the coil





9.2 electromagnetic induction

- we can induce a current to flow in a coil by moving a magnet near the coil

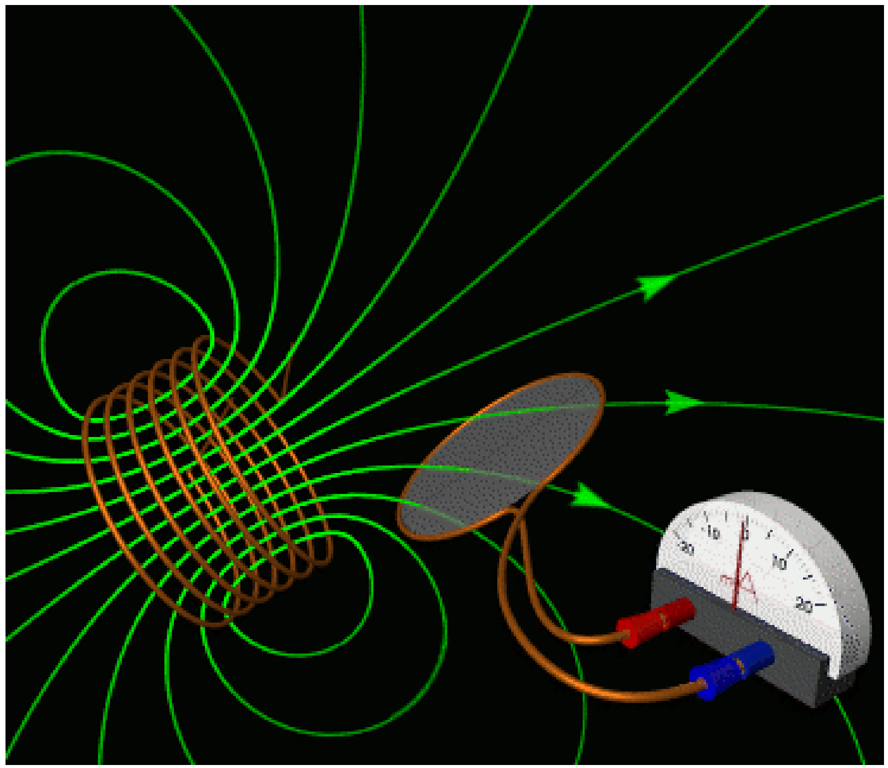


while the magnet is moving, the amount of magnetic flux passing through the loops of the coil is changing

9.2 electromagnetic induction

- we can induce a current to flow in a conducting loop by changing the current in coil, or solenoid, near the loop

an alternating current is passed through the solenoid



which induces a current to flow in the loop

9.2 electromagnetic induction

- the current observed in all these cases is driven by an induced EMF (ε)
- the induced EMF is effectively a voltage produced by the changing magnetic flux passing through the conducting loop
- the size of the resulting current will depend on the resistance of the loop, according to

$$\mathcal{E} = IR$$

9.2 electromagnetic induction

Faraday's law

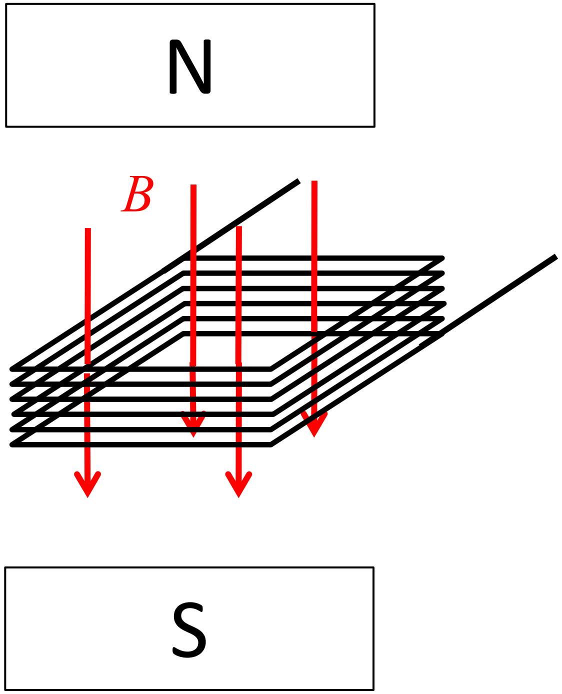
- the induced EMF (ε) in a conducting loop is equal to the negative of the rate at which the magnetic flux through the loop changes with time:

$$\varepsilon = -\frac{d\Phi_B}{dt} \qquad \text{[volts]}$$

9.2 electromagnetic induction

Faraday’s law

A rectangular coil consists of 100 turns, measuring 0.10 m x 0.10 m. The resistance of the coil is 2 Ω. The coil is placed between the poles of an electromagnet, perpendicular to the magnetic field (as shown). When the electromagnet is turned off the magnetic field strength decreases at a steady rate of 20 T per second.



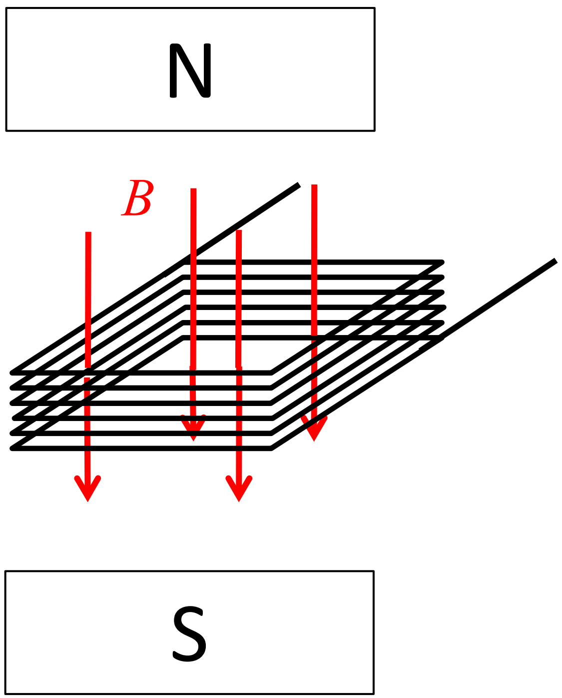
What is the **magnitude** of the current induced in the coil?

9.3 Lenz's law

- the direction of any magnetic induction effect is such as to oppose the change that's causing it
- the changes that cause the induction effect could be
  - » motion of a conducting loop in a field
  - » changing magnetic flux though a stationary circuit due to a time varying magnetic field
  - » or a combination of the above
- Lenz's law is analogous to Newton's third law and is a consequence of the law of conservation of energy

9.3 Lenz's law

A rectangular coil consists of 100 turns, measuring 0.10 m x 0.10 m. The resistance of the coil is 2 Ω. The coil is placed between the poles of an electromagnet, perpendicular to the magnetic field (as shown). When the electromagnet is turned off the magnetic field strength decreases at a steady rate of 20 T per second.

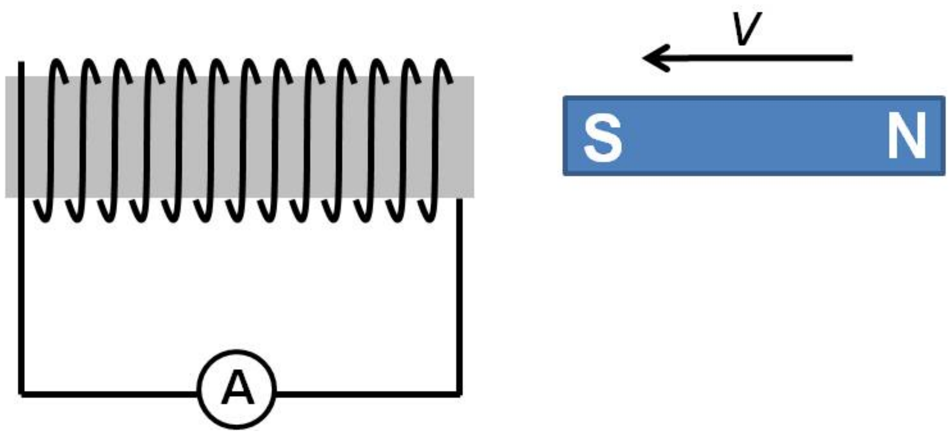


What is the **direction** of the current induced in the coil?

9.3 Lenz's law

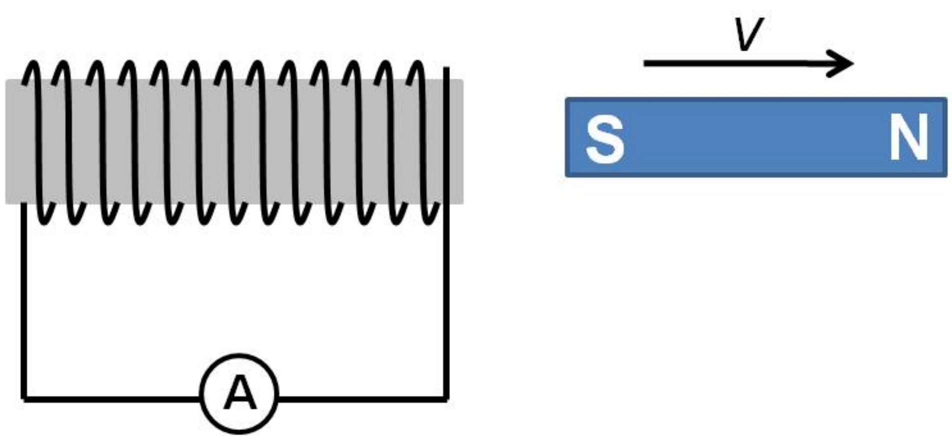
example

what is the direction of the induced current through the ammeter when the magnet is moved towards the solenoid?



and away from a different* solenoid?

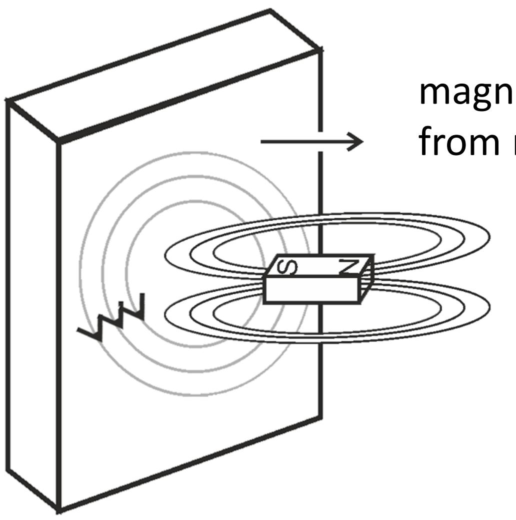
* the difference is significant



9.4 eddy currents

- so far, we have only considered currents induced in circuits, loops or coils
- but induced currents, called eddy currents, flow in all metal objects that are in a changing magnetic field

- eddy currents are induced as a consequence of Faraday's law
- the direction of the eddy current (and the magnetic field it produces) is determined by Lenz's law – it will try to oppose the change producing it



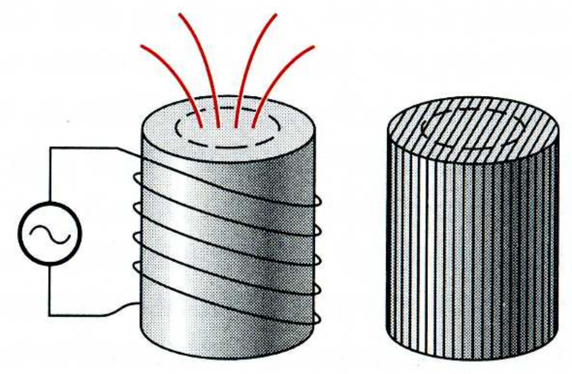
magnet moving away from metal block

9.4 eddy currents

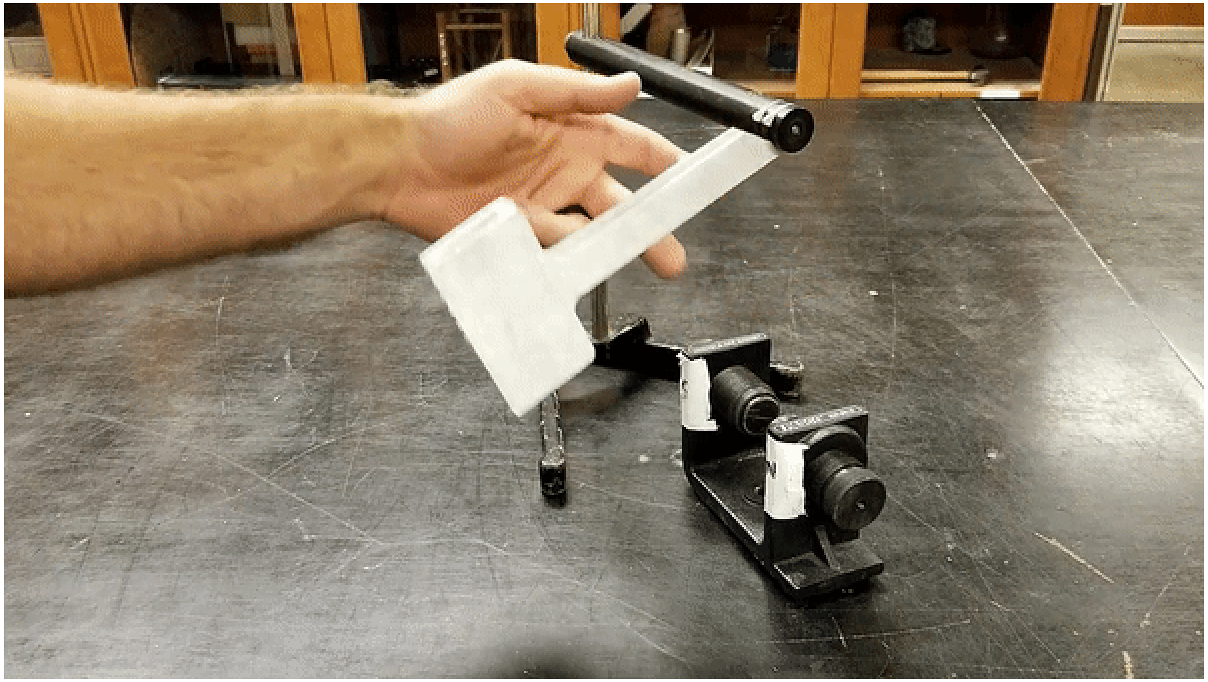
- remember, all AC devices will produce a continuously changing magnetic field

problems:

- unwanted heating effects
- energy loss in transformers, electric motors etc (mitigated by use of laminated cores or ferrites)



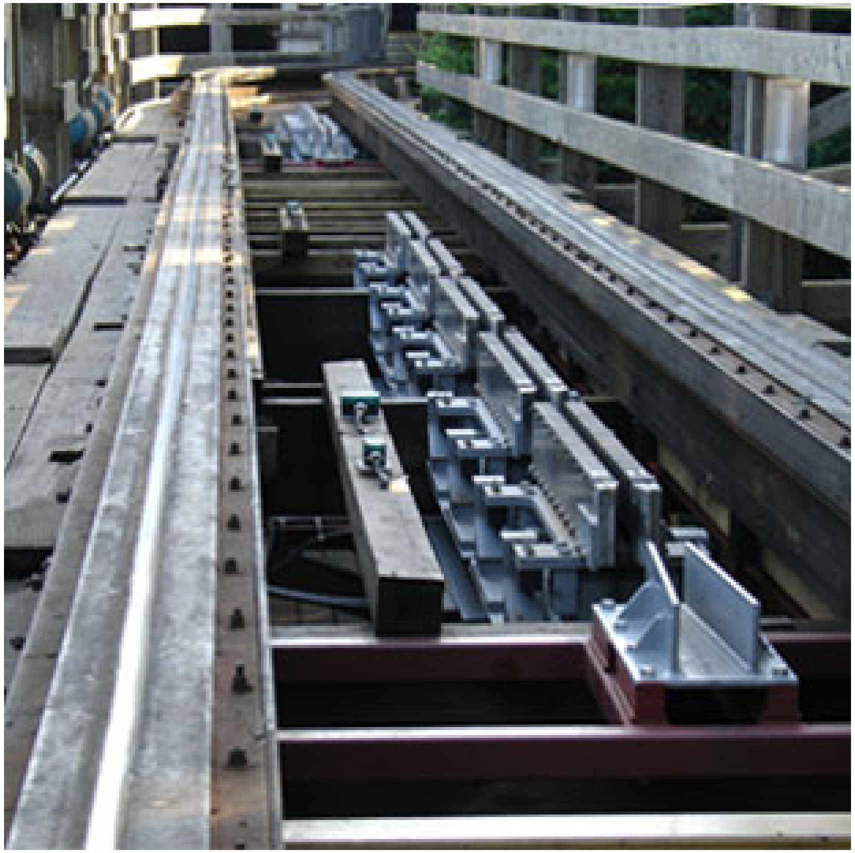
9.4 eddy currents



9.4 eddy currents

applications:

- induction heating
- metal detection/sorting
- non-contact braking systems



9.5 self-inductance and inductors

- a current flowing through a coil will generate a magnetic field, which passes through the coil
- if the current changes, so does the magnetic flux through the coil, so an EMF is induced in the coil (Faraday's law)
- this is self-induction (because the EMF is induced in the same coil in which the current is changing)
- the induced EMF opposes the change in current producing it (Lenz's law) and so is called back EMF
- if the current increases, the back EMF opposes the increase, if the current decreases the back EMF opposes the decrease.

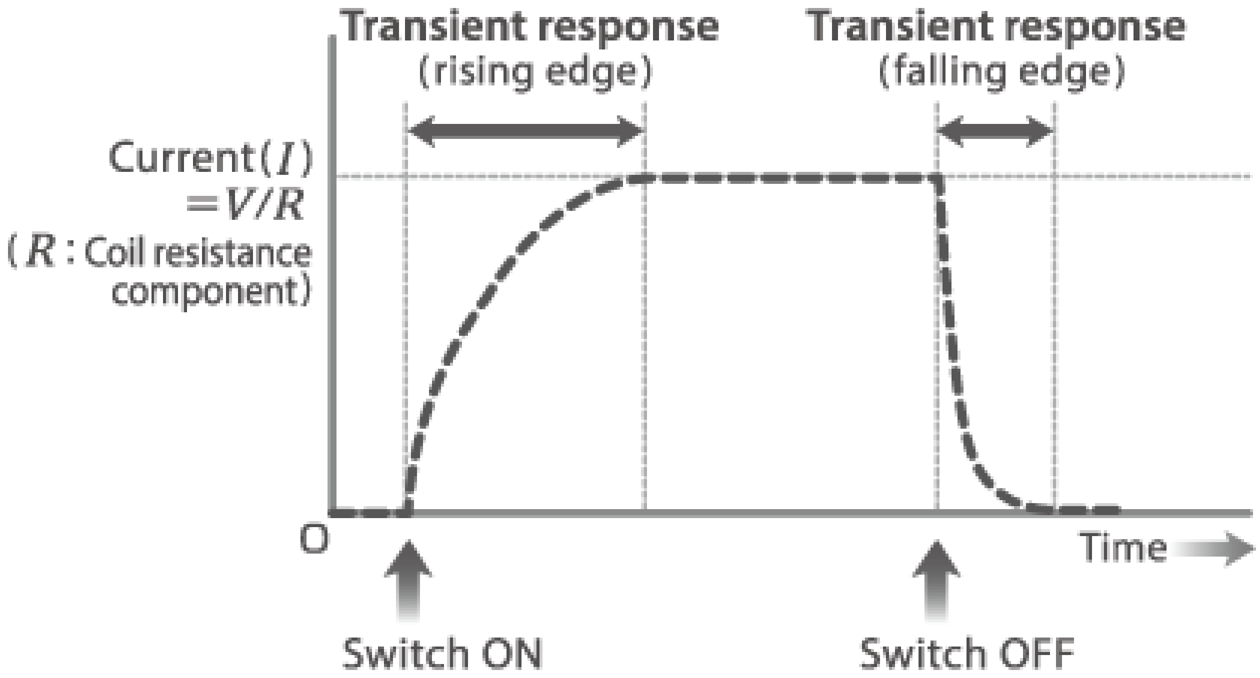
9.5 self-inductance and inductors

inductors

- one of the three basic building blocks of all AC circuits
- store energy in the form of a magnetic field
- the ability of a coil to give rise to back EMF is known as the self-inductance, L, of the coil
- inductance of a coil depends on its geometry and the material in its core
- the unit of inductance is the henry (H)

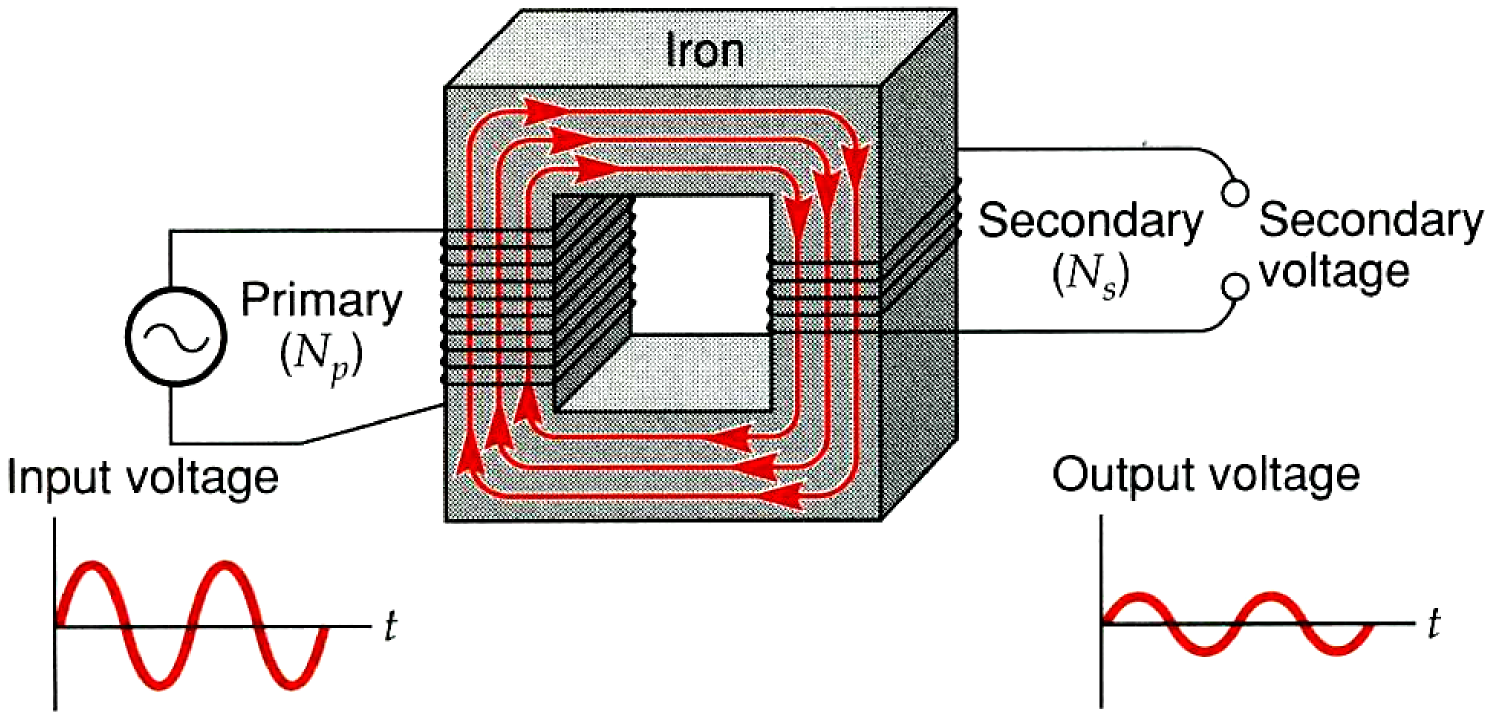
9.5 self-inductance and inductors

energising and de-energising an inductor



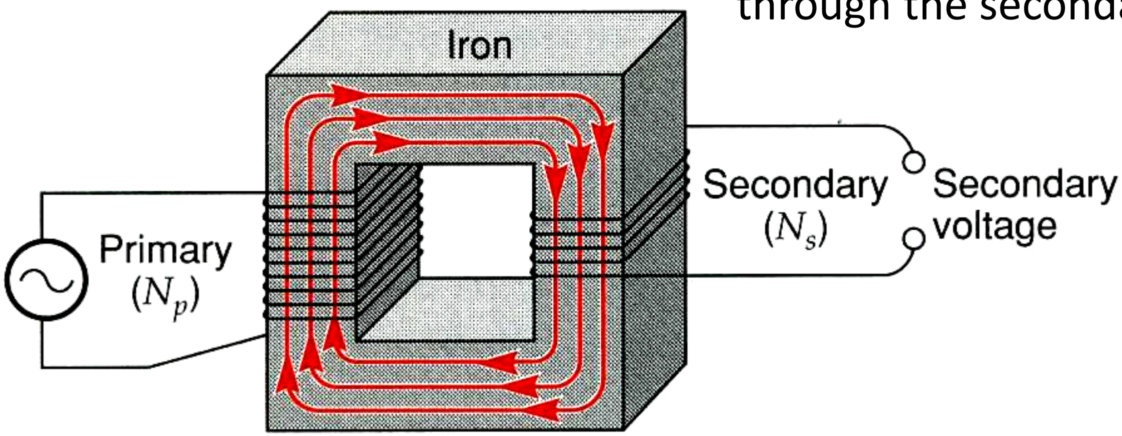
9.6 transformers

- transformers use magnetic induction to change one AC voltage into another



9.6 transformers

the time-varying magnetic field produced by the primary coil passes through the secondary coil



a changing voltage applied to the primary coil generates a back emf (ε_p) given by Faraday's law:

$$\varepsilon_p = -\frac{d\Phi}{dt} = -N_p A \frac{dB}{dt}$$

the induced emf in the secondary coil (ε_s) is also given by Faraday's law:

$$\varepsilon_s = -\frac{d\Phi}{dt} = -N_s A \frac{dB}{dt}$$

9.6 transformers

$$\varepsilon_p = -\frac{d\Phi}{dt} = -N_p A \frac{dB}{dt} \quad \text{and} \quad \varepsilon_s = -\frac{d\Phi}{dt} = -N_s A \frac{dB}{dt}$$

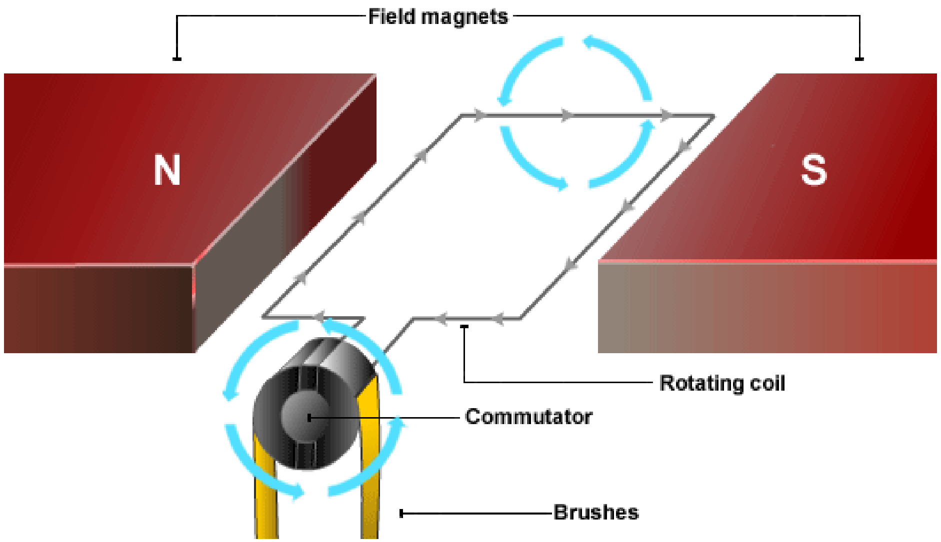
- dividing one of these equations by the other gives us the simple transformer equation:

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

- **step-up** transformers ($N_s > N_p$) increase the supply voltage
- **step-down** transformers ($N_s < N_p$) decrease the supply voltage
- note that the frequency of the signal is not changed

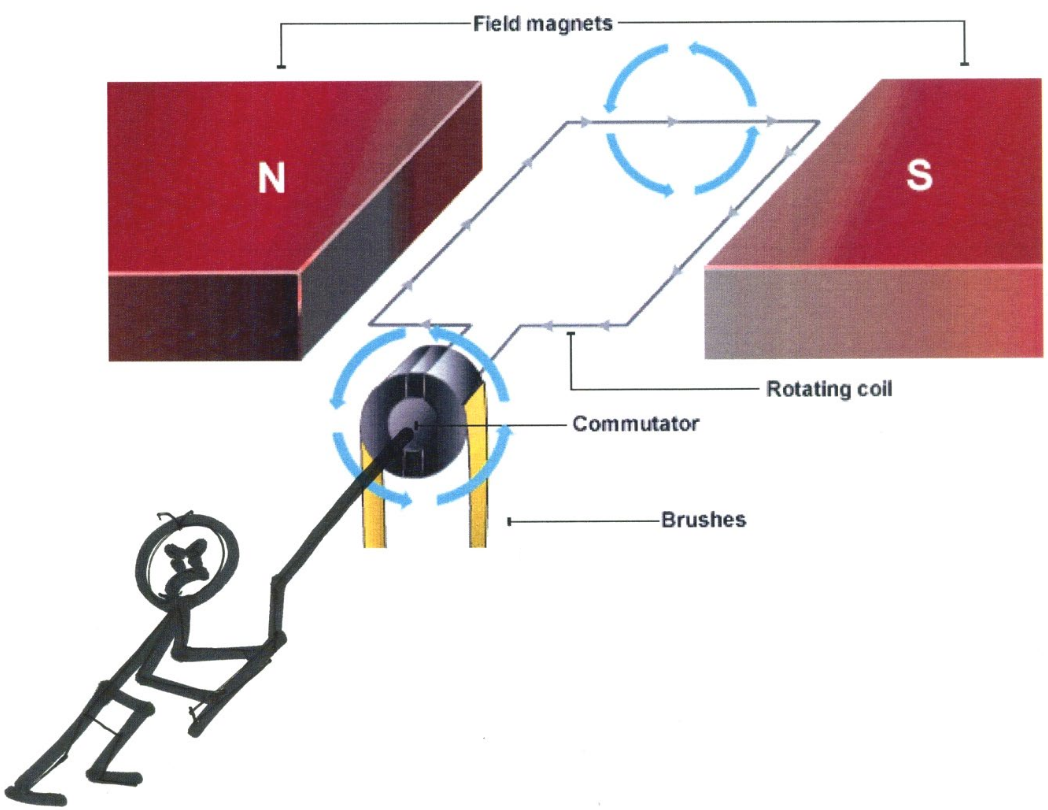
10.1 electric motors and back emf

- in an electric motor, electrical energy is converted to mechanical energy
- current flowing around a coil in a magnetic field is subject to a magnetic force
- the magnetic force gives rise to torque on the coil
- the torque causes the coil to rotate, and can be used to do mechanical work



10.1 electric motors and back emf

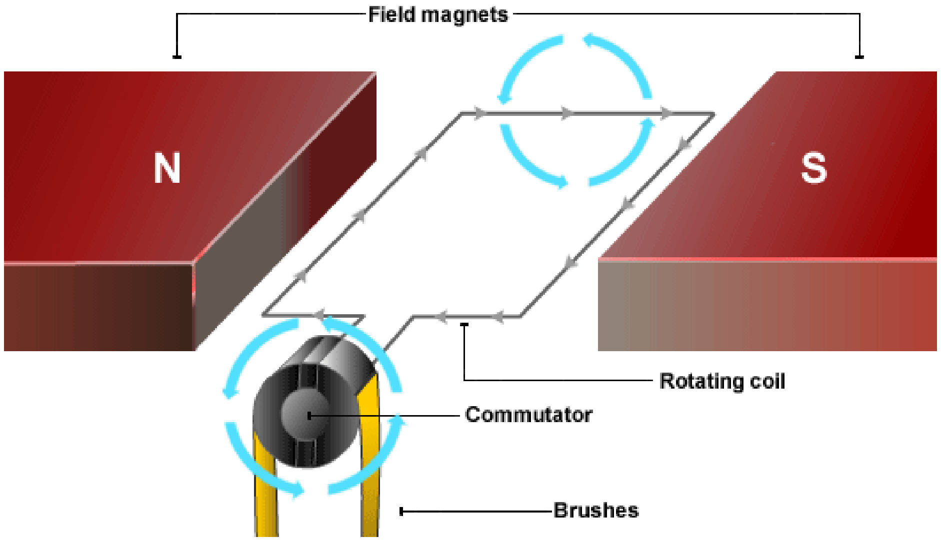
- in an electric generator, mechanical energy is converted to electrical energy
- a coil is rotated in a magnetic field
- the flux through the coil changes, so an emf is induced in the coil (Faraday's law)
- the induced emf causes a current to flow in the coil



10.1 electric motors and back emf

- in both cases (motor and generator), the arrangement of the magnetic field and the rotating coil is the same

- so, whenever an electric motor is running, we would expect an emf (different to the 'supply' voltage) to be induced in the coil



- this induced emf opposes the supply voltage, reducing the effective potential difference across the coil and so reducing the current that flows through the coil (and the motor)

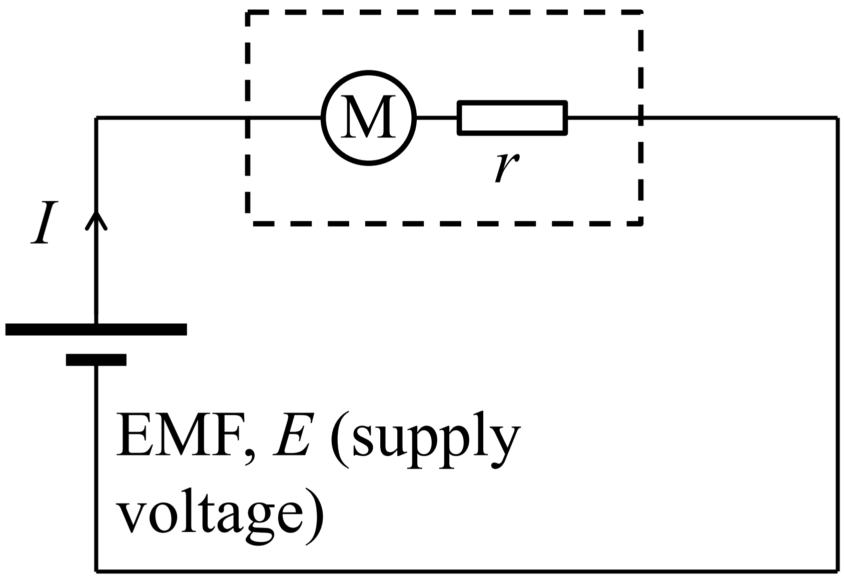
10.1 electric motors and back emf

- the induced emf generated by an electric motor in use is called the back emf
- in accordance with Lenz's law, the back emf acts in opposition to the emf driving the motor (the supply voltage)
- the size of the back emf is proportional to the rate of change of flux through the coil (according to Faraday's law)
- in other words, the back emf increases as the motor runs faster

10.1 electric motors and back emf

with the motor not running:

- no induced back emf
- $E = Ir$ or $I = E/r$



r represents the resistance of the coil of the motor

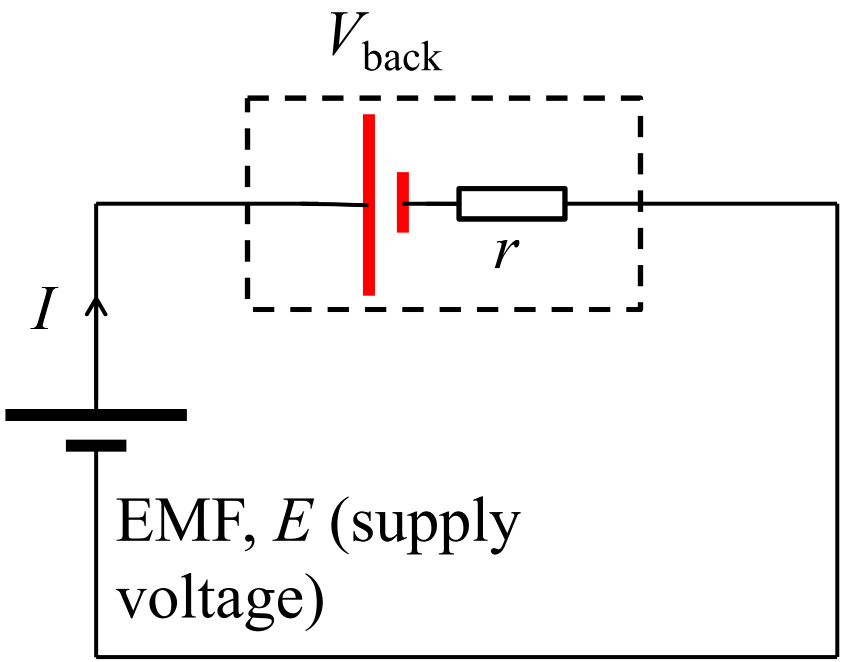
with the motor running:

- we get an induced emf (= V_{back})

10.1 electric motors and back emf

with the motor not running:

- no induced back emf
- $E = Ir$ or $I = E/r$



with the motor running:

- we get an induced emf (= V_{back})
- now, according to Kirchhoff

$$E - V_{\text{back}} = Ir$$

or $$E = V_{\text{back}} + Ir$$

or $$I = (E - V_{\text{back}})/r$$

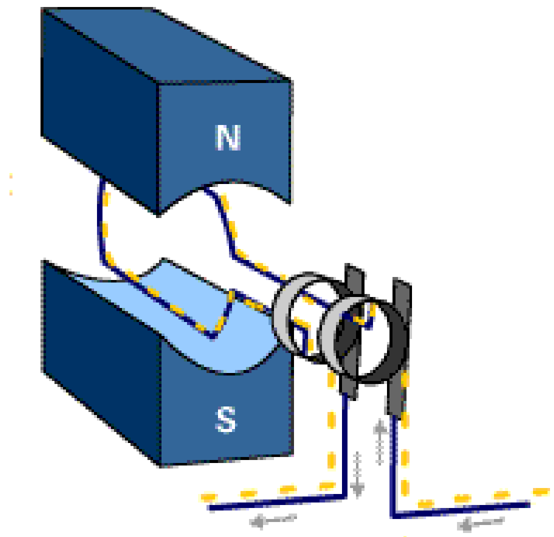
r represents the resistance of the coil of the motor

10.1 electric motors and back emf

$$I = (E - V_{\mathrm{back}})/r$$

- at start-up V_{back} is zero, so the current drawn by the motor ($I = E/r$) is at its maximum, and power consumed ($P = I \times E$) is at its highest
- as the motor runs faster, V_{back} increases, so the current, I, flowing through the motor decreases, and less power is consumed
- if a load is applied to the motor, the motor slows; the back emf decreases, the current increases and so more power is consumed
- if an electric motor jams suddenly, the back emf drops abruptly to zero and the full supply voltage is applied across coil, rapidly increasing the current flowing through the coil (beware meltdown)!

10.2 generators



- an external energy source rotates a coil in a magnetic field
- the flux through the coil changes as it rotates
- this induces an EMF in the coil (according to Faraday's law), which causes a current to flow

10.2 generators

- for a coil with N loops:

$$\Phi_B = NBA\cos\theta$$

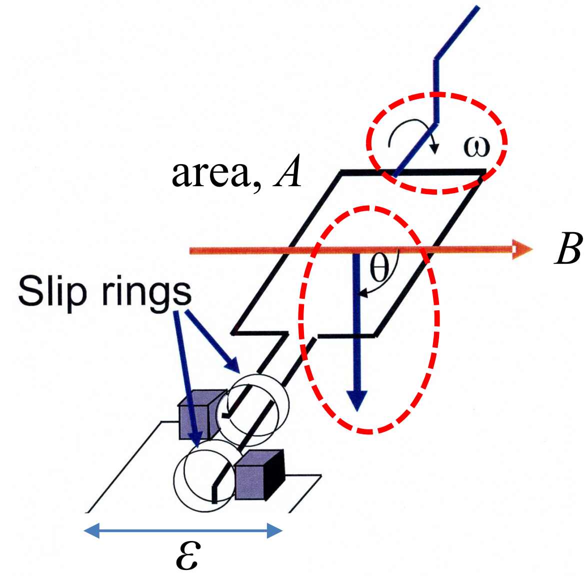
- we can define θ in terms of a parameter known as the angular frequency, ω, and time, t, using the following relationship:

$$\theta = \omega t$$

- angular frequency is the rate of rotation measured in radians per second (or degrees per second), so the product ωt is just the angle in radians (or degrees)

- so we can write the expression for the flux through the coil as:

$$\Phi_B = NBA\cos\omega t$$



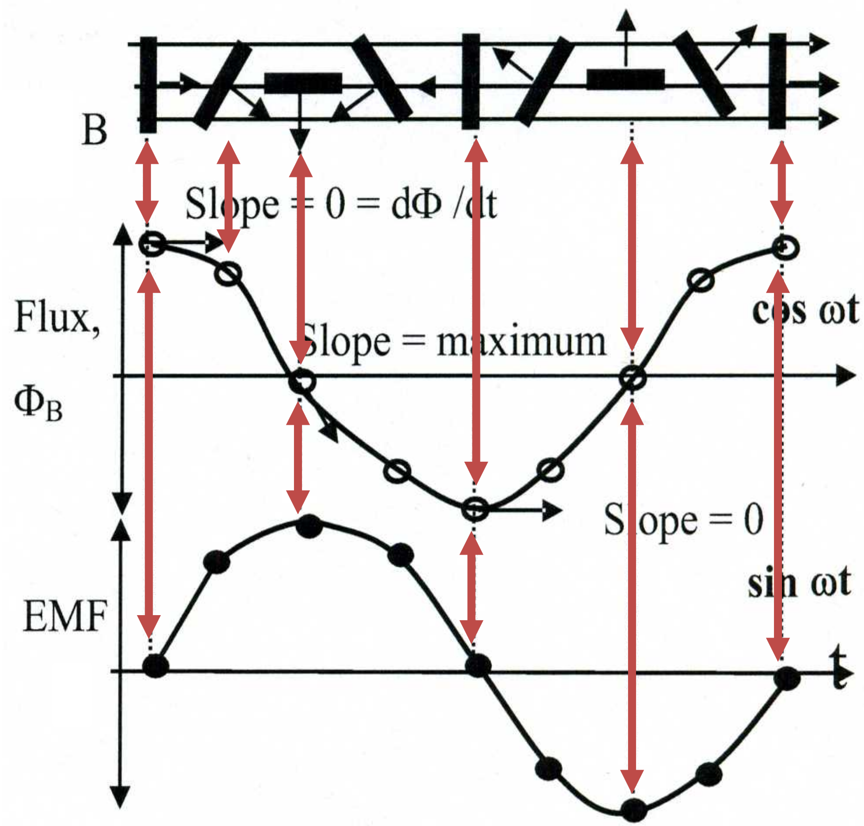
10.2 generators

$$\Phi_B = NBA\cos\omega t$$

- Faraday's law gives us the induced EMF:

$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\varepsilon = NBA\omega\sin\omega t$$



- this simple generator is an AC generator. It produces a sinusoidally alternating EMF, which would produce an alternating current

10.2 generators

- Faraday's law gives us the output of the generator

$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\varepsilon = NBA\omega \sin \omega t$$

- generators are also subject to Lenz's law
  - at very low rotation rates (very low output) the generator turns relatively easily, the main energy losses are due to friction
  - as the rotation rate (and output) increases, it gets harder to turn the generator, in accordance with Lenz's law
  - as the work required to turn the generator becomes greater (and more current flows in the coils), energy losses due to heating become more significant

10.2 generators

peak output

- the output of an AC generator is given by:

$$\varepsilon = NBA\omega \sin \omega t$$

- so, the peak output (when $sin\omega t = \pm 1$) is given by:

$$\varepsilon_{\text{peak}} = \pm \omega NBA$$

and since $\omega = 2\pi f$

$$\varepsilon_{\text{peak}} = \pm 2\pi f NBA$$

- which means the peak output of a generator depends on frequency and, for a given generator, only on frequency

10.2 generators

RMS output

- the peak output from a generator is given by:

$$\varepsilon_{peak} = \pm\,\omega NBA = \pm\,2\pi fNBA$$

- the RMS value of the output is given by:

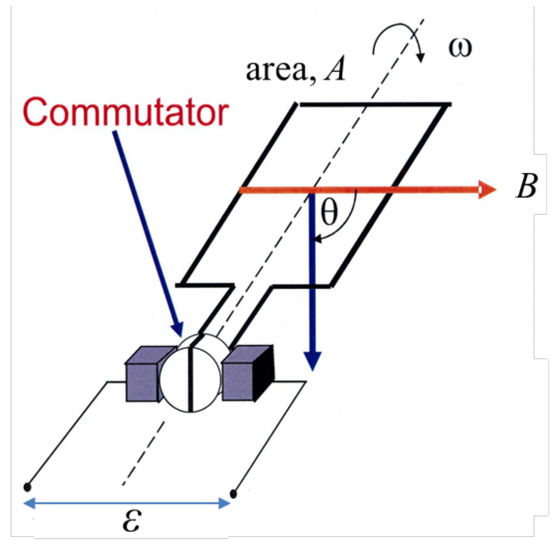
$$\varepsilon_{RMS} = \frac{\varepsilon_{peak}}{\sqrt{2}}$$

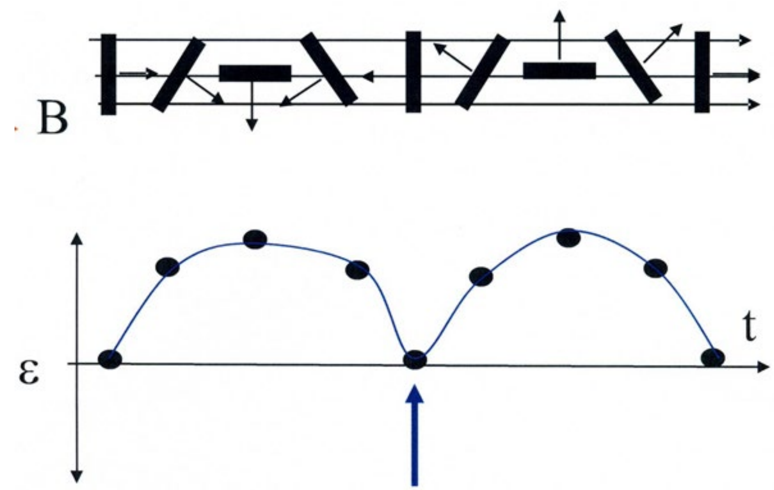
10.2 generators

DC generator

- the commutator reverses the connection to the coil at angular position where the induced EMF reverses its sign

- additional coils could be added (with appropriate commutation) to provide a more constant output



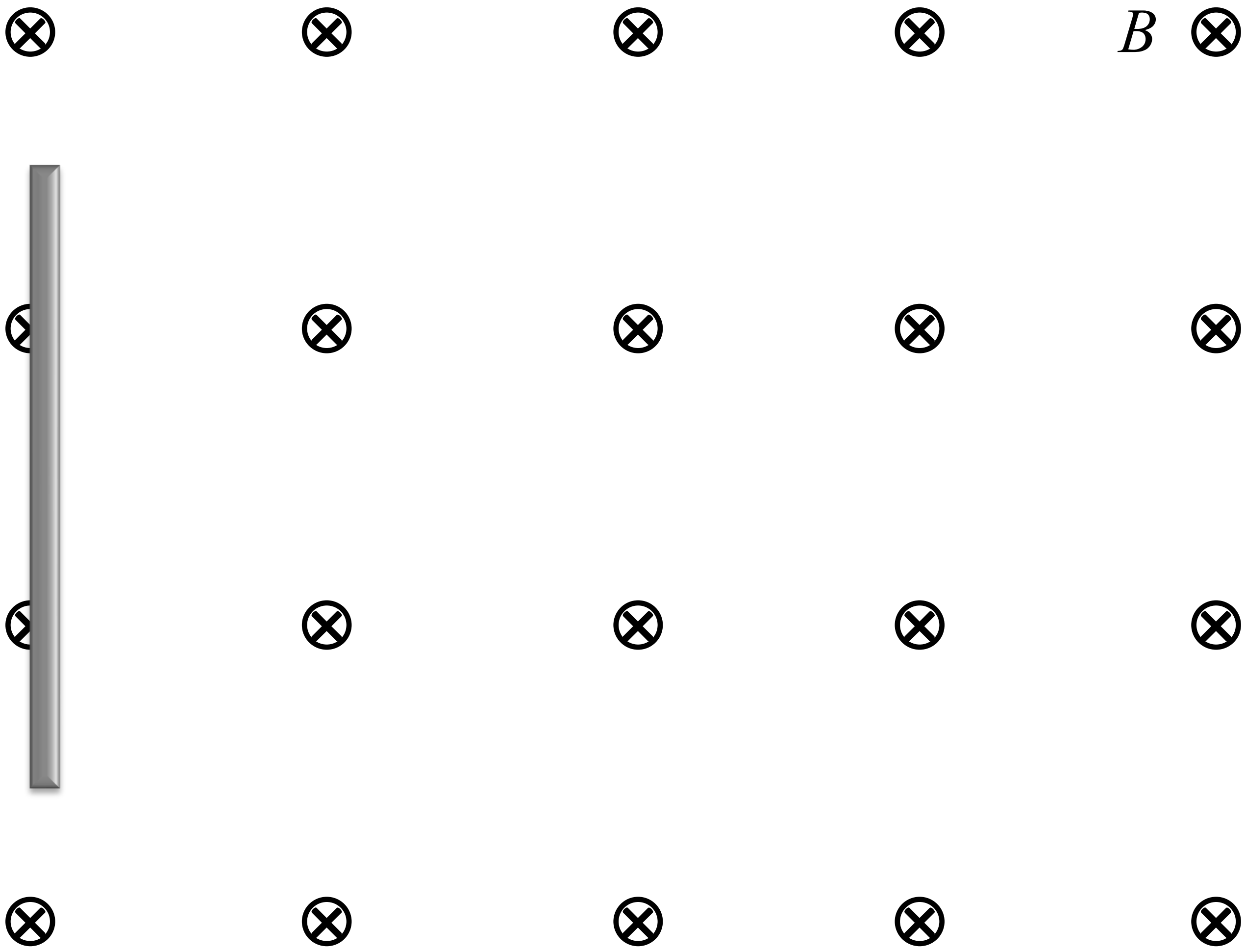


connection to coil is
reversed by commutator

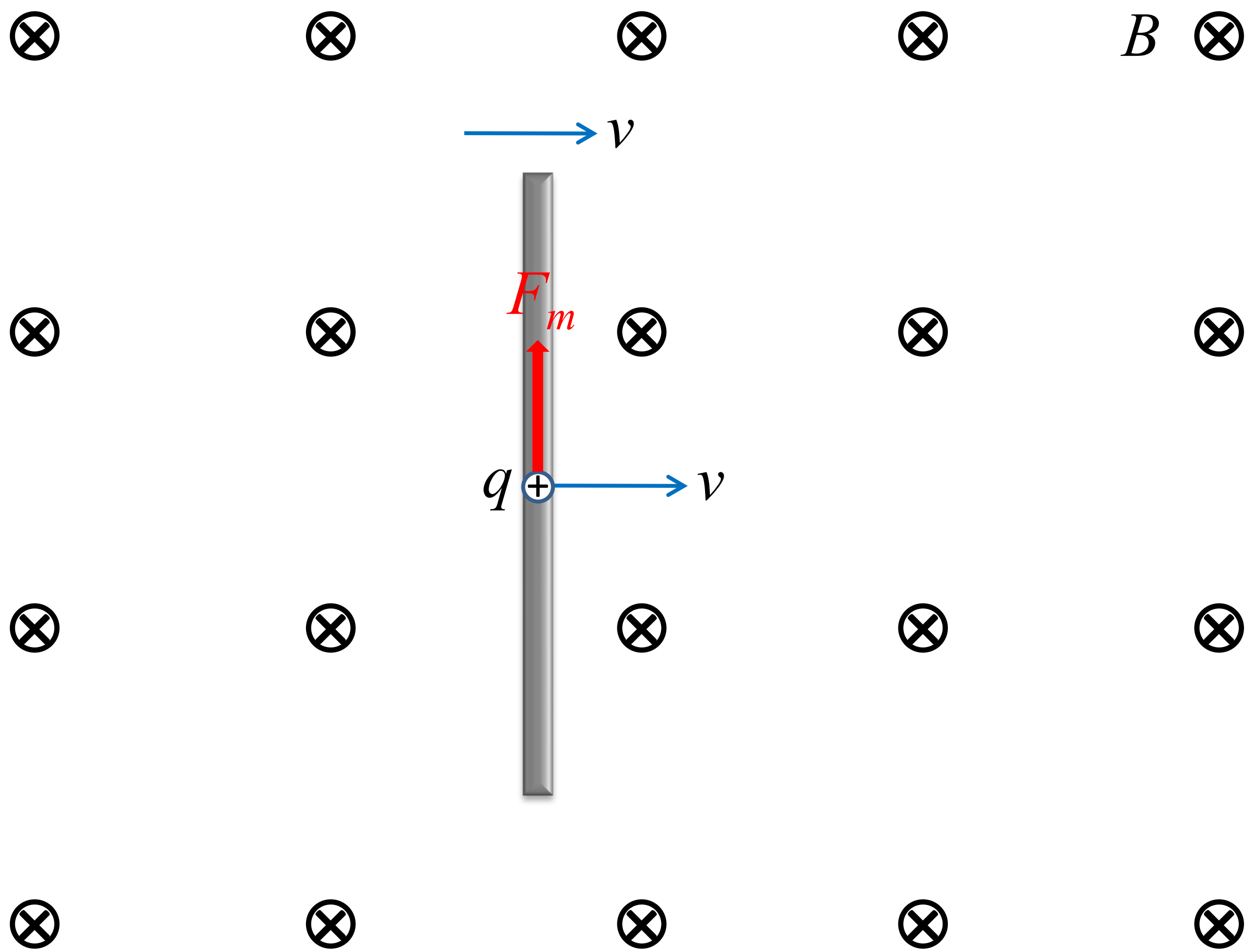
10.3 motional EMF

- a conductor moving in a magnetic field will experience an induced emf, this is known as motional EMF
- here we explore this phenomenon by considering a metal rod, perpendicular to a uniform magnetic field, moving with constant speed at right angles to the field
- we will analyse the magnetic (Lorentz) and electric forces acting on hypothetical positive charges within the conductor (even though we know it is actually free electrons that are the charge carriers in metals)
- note that in this arrangement, the sinθ term in the magnetic force equations will always equal 1 (because the angle between the magnetic field and the direction of motion/current is 90°

10.3 motional EMF

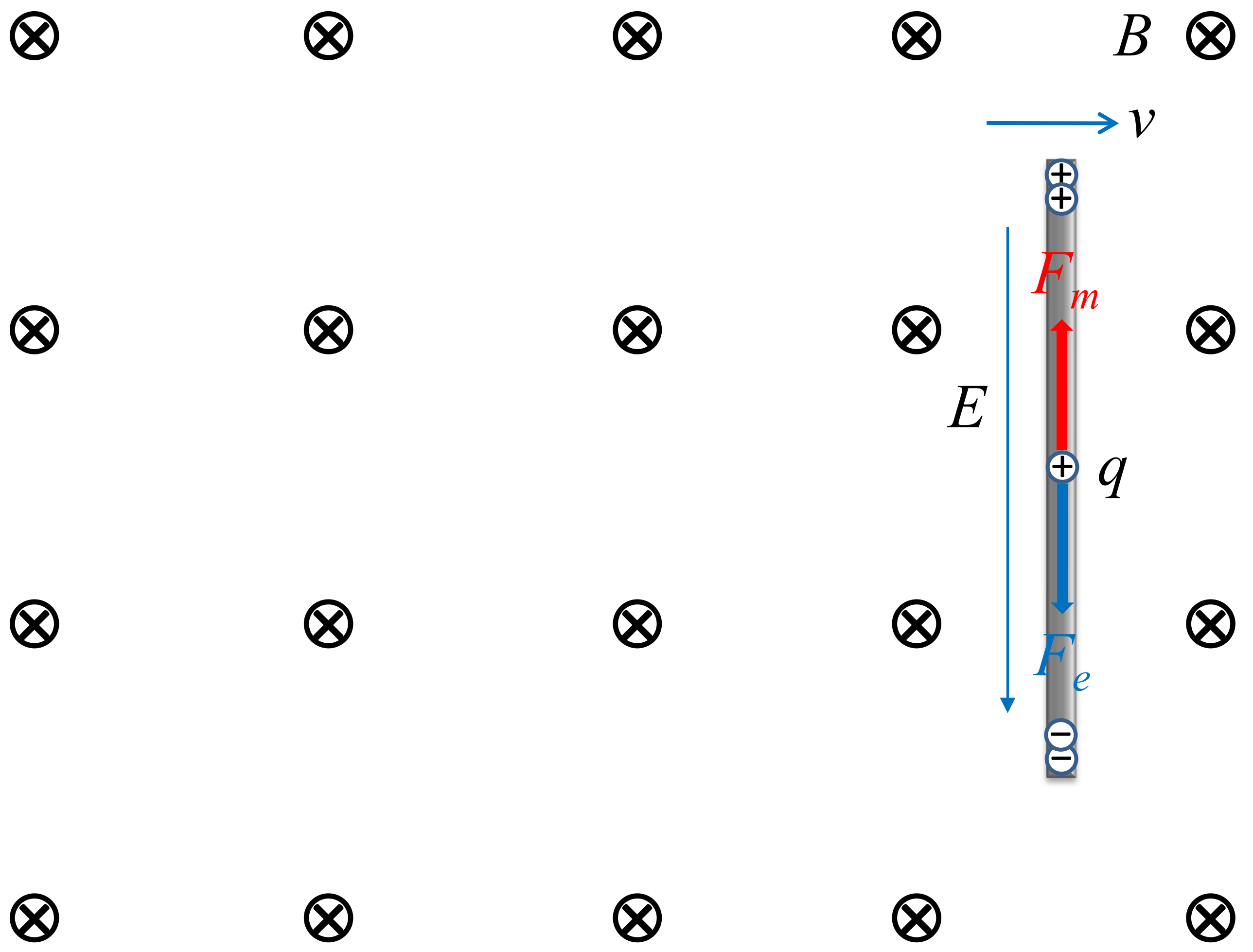


10.3 motional EMF



$$F_m = qvB$$

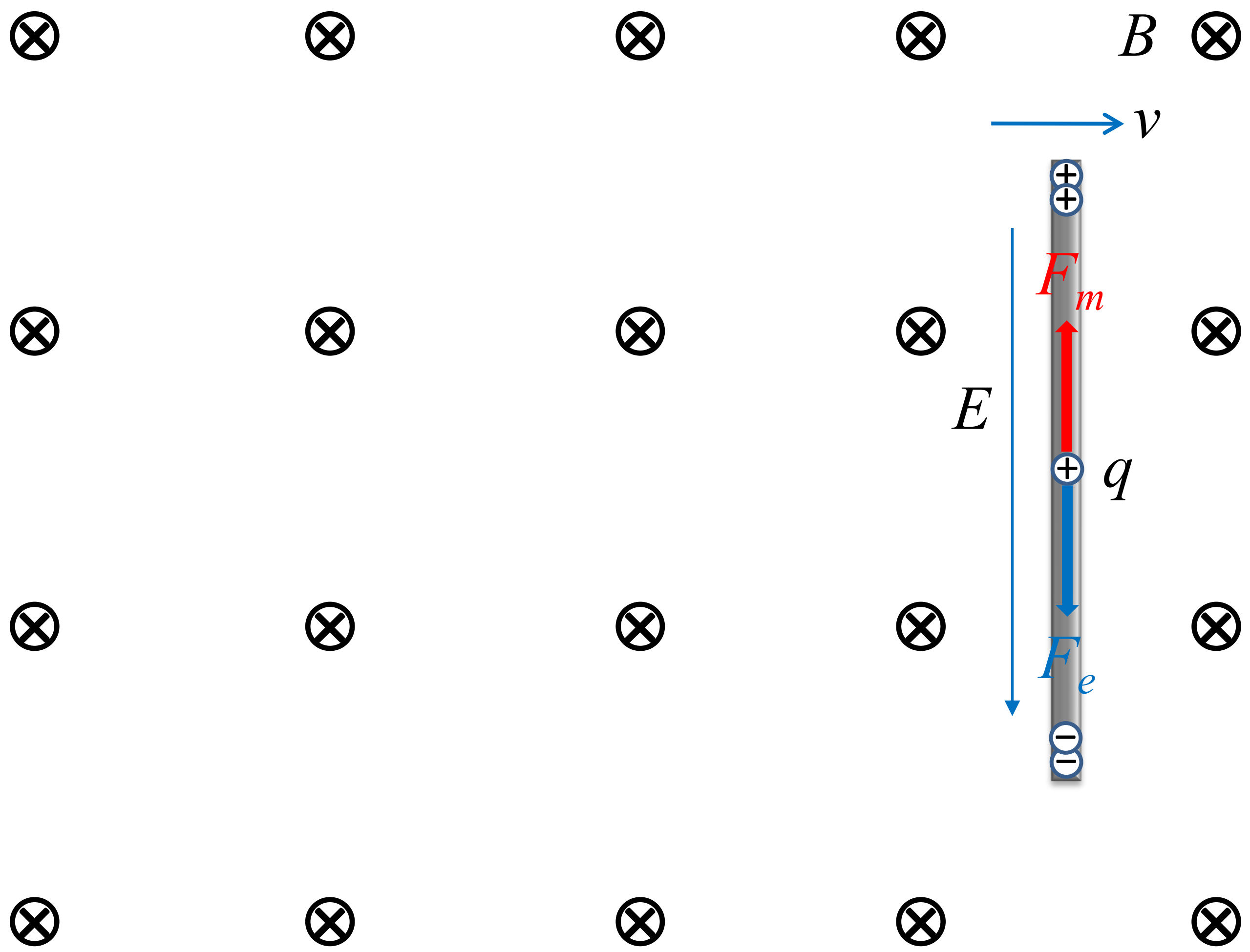
10.3 motional EMF



$F_m = qvB$

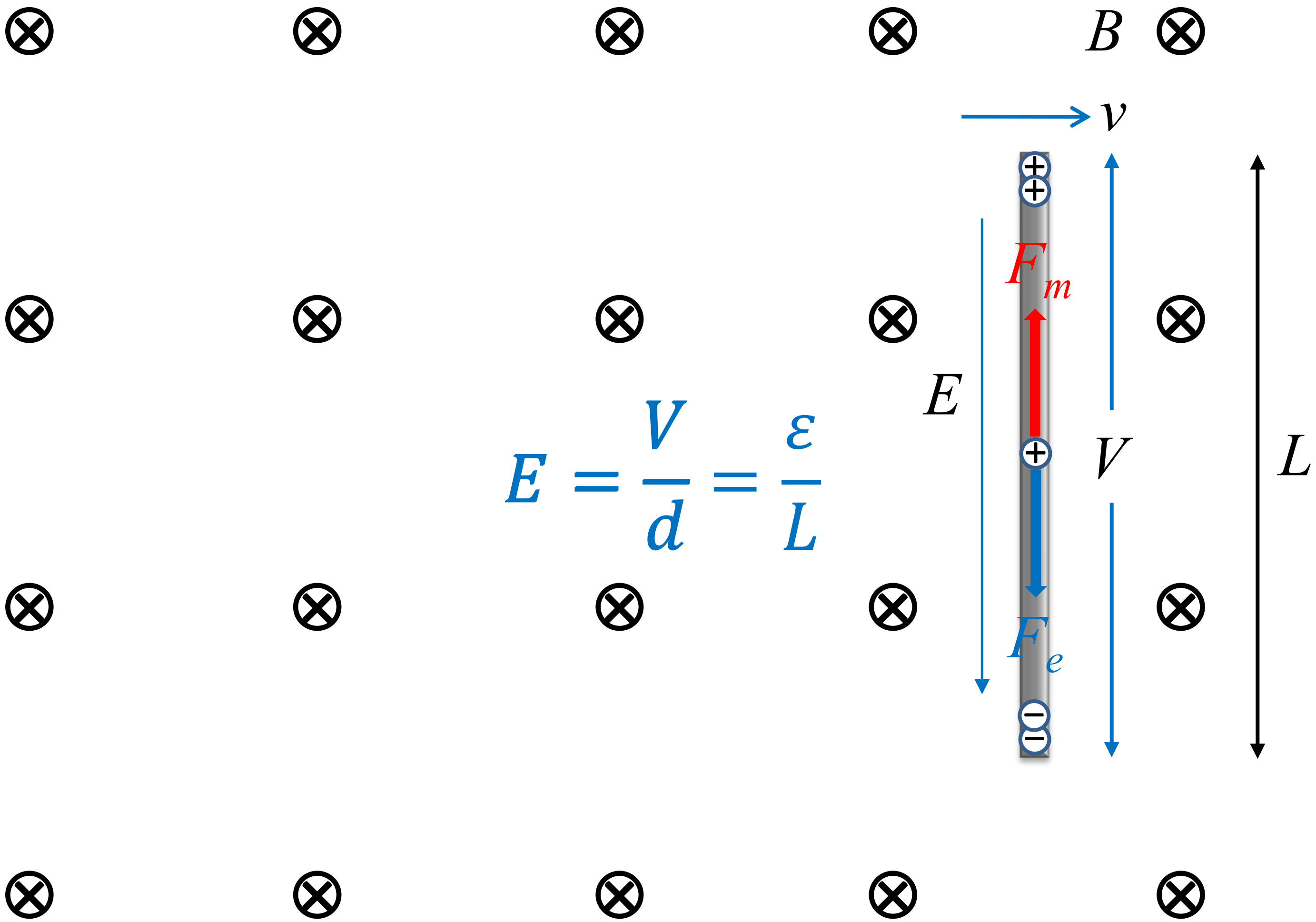
$F_e = qE$

10.3 motional EMF



$$|F_m| = |F_e| \quad \Rightarrow qvB = qE \quad \Rightarrow E = vB$$

10.3 motional EMF



$$E = \frac{V}{d} = \frac{\varepsilon}{L}$$

$$E = vB \qquad \Rightarrow \frac{\varepsilon}{L} = vB \qquad \Rightarrow \varepsilon = vBL$$

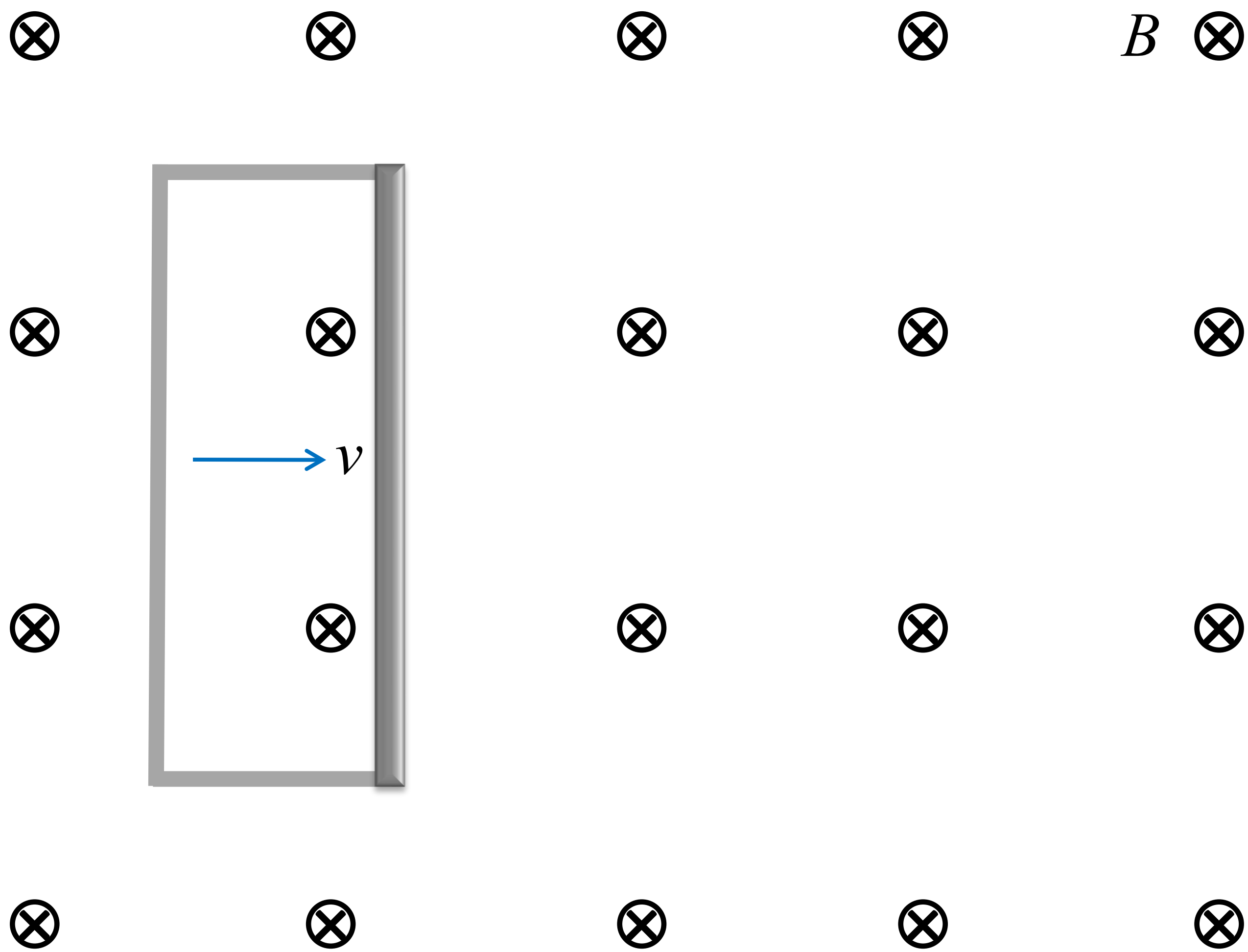
10.3 motional EMF

- the moving rod is a source of EMF
- the induced, motional EMF is given by

$$\varepsilon = vBL$$

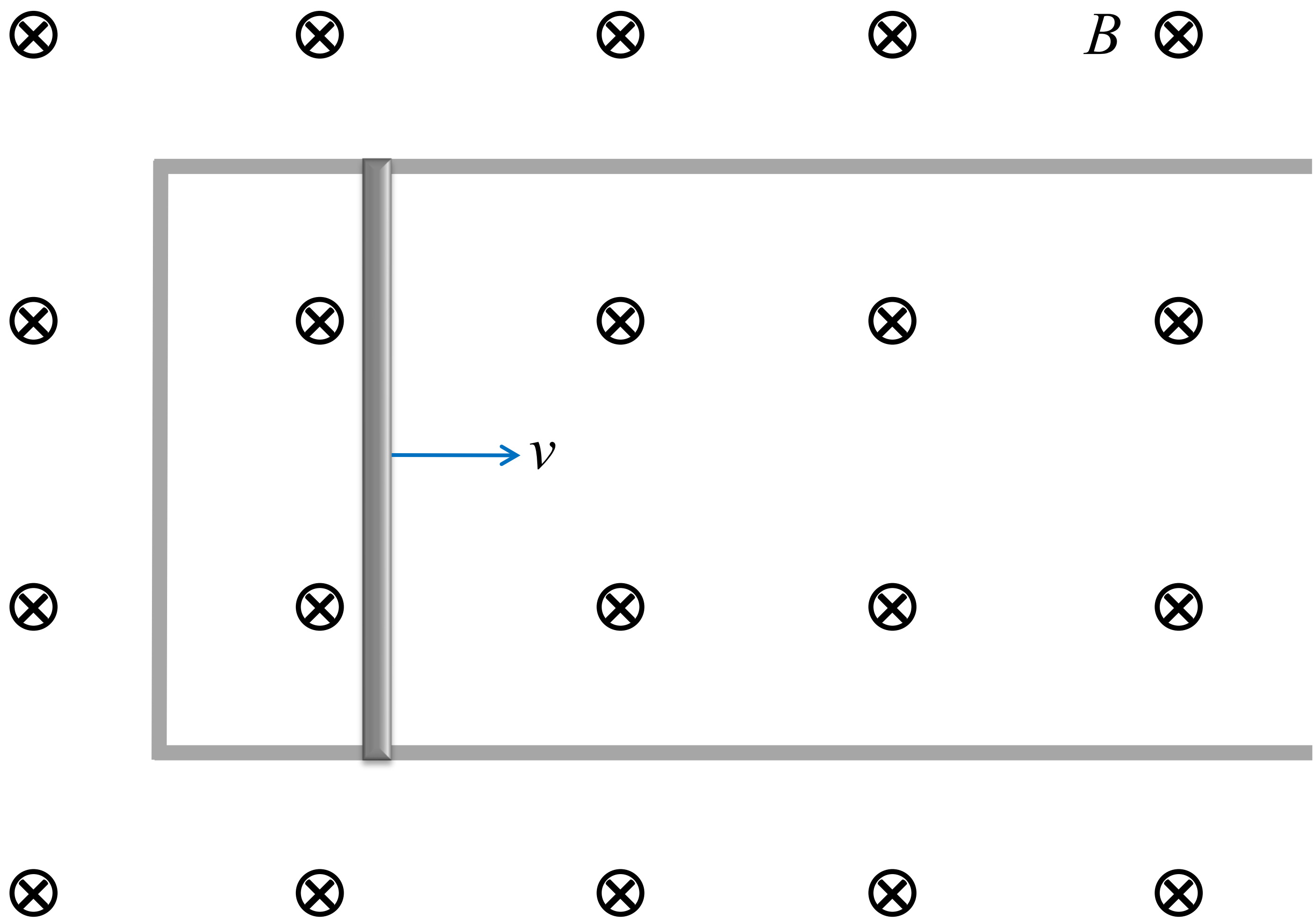
- in order to use this EMF to drive a current around a circuit, we would have to connect the circuit to the ends of the rod
- look at the next slide and decide whether this scenario would cause a current to flow in the circuit…

10.3 motional EMF



as we would expect from Faraday's law, there is no induced current in this circuit because there is no change of flux through the circuit (magnetic field is uniform)

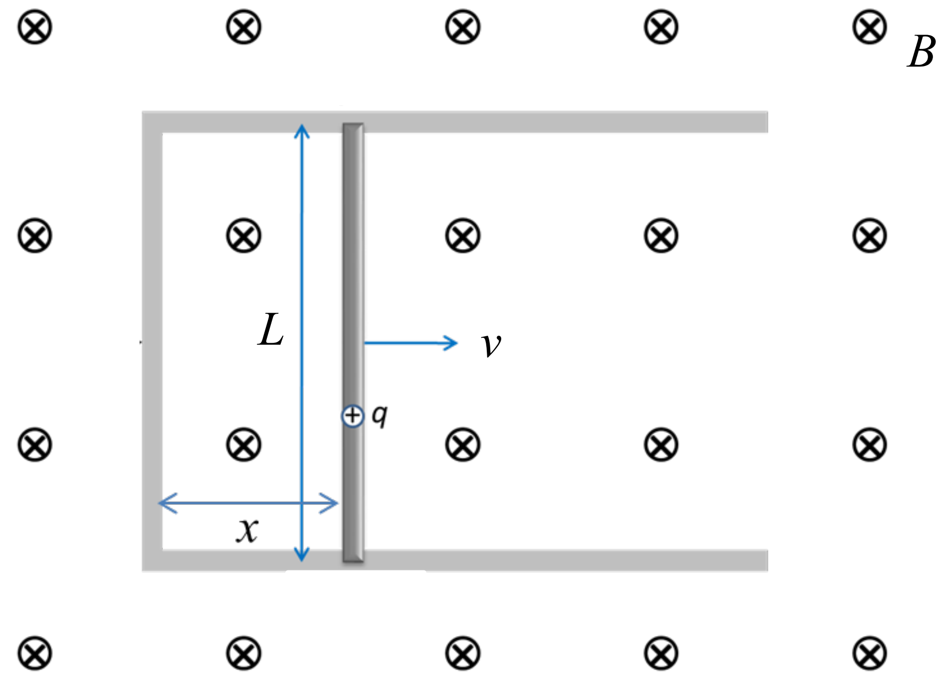
10.3 motional EMF



in this case, an EMF is induced and a current will flow in the circuit as the rod moves – the flux through the circuit changes as the effective area increases

10.3 motional EMF

- for the case where the rod moves but the circuit is static, we can use Faraday's law to find the expression for the induced EMF
- the flux through the circuit increases as the area Lx increases



$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\theta)$$

$N = 1$, $\theta = 0°$, B is constant

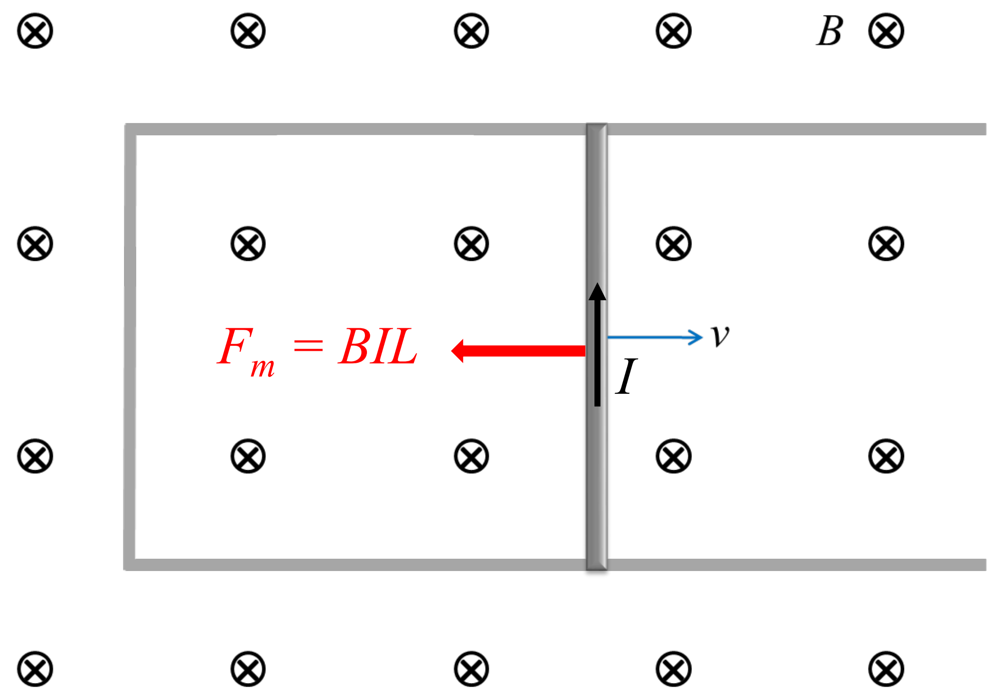
$$\Rightarrow \varepsilon = -B\frac{dA}{dt} = -BL\frac{dx}{dt}$$

note that $\dfrac{dx}{dt} = v$

$$\Rightarrow \varepsilon = -vBL$$

10.3 motional EMF

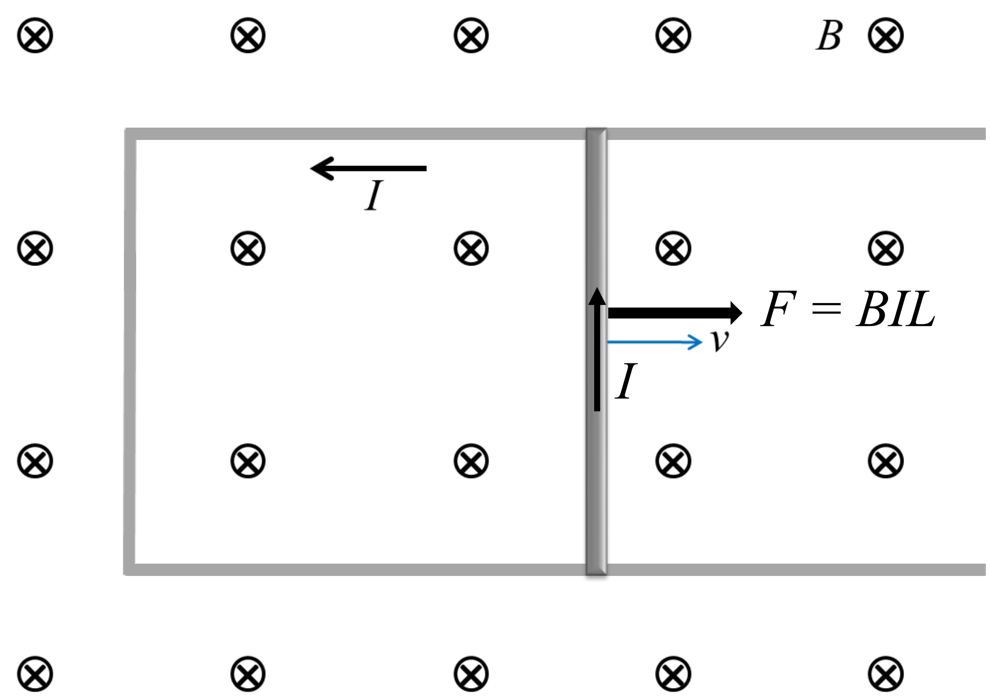
- Lenz's law tells us that the current will flow in an anticlockwise direction (to oppose the increasing flux through the circuit directed into the screen)
- we now have a current-carrying conductor moving in a magnetic field, which will be subject to a Laplace force, as shown



- this force opposes the motion of the rod (as we would expect, from Lenz's law), so to move the rod with constant velocity v we need to apply a force of the same size in the direction of v

10.3 motional EMF

- Lenz's law tells us that the current will flow in an anticlockwise direction (to oppose the increasing flux through the circuit directed into the screen)
- we now have a current-carrying conductor moving in a magnetic field, which will be subject to a Laplace force, as shown



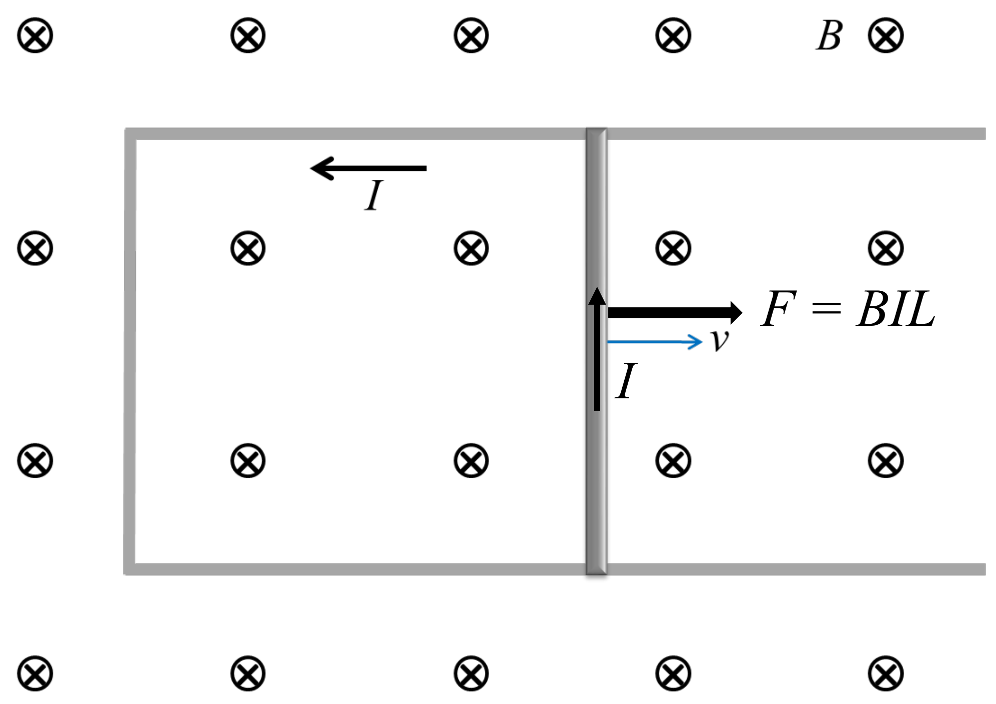
- this force opposes the motion of the rod (as we would expect, from Lenz's law), so to move the rod with constant velocity v we need to apply a force of the same size in the direction of v

10.3 motional EMF

- we can work out the mechanical power (P_{m}) needed to keep the rod moving at constant velocity
- power is rate of work and the work done is force multiplied by distance

$$P_m = \frac{force \times distance}{time} = F_m \times v$$

$$P_m = BIL \times v = I \times vBL = I\varepsilon$$



- compare this to our usual expression for electrical power, in terms of current and potential difference

$$P_e = IV = I\varepsilon$$

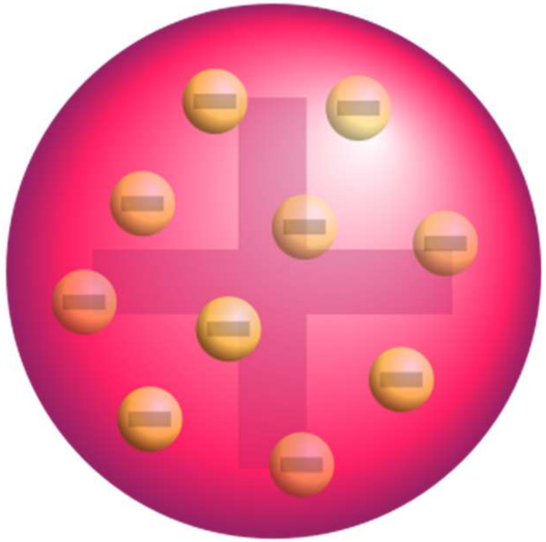
- so, as expected for an (ideal) generator, power in = power out

11.1 classical models of the atom

the plum pudding model

JJ Thomson (1856-1940)

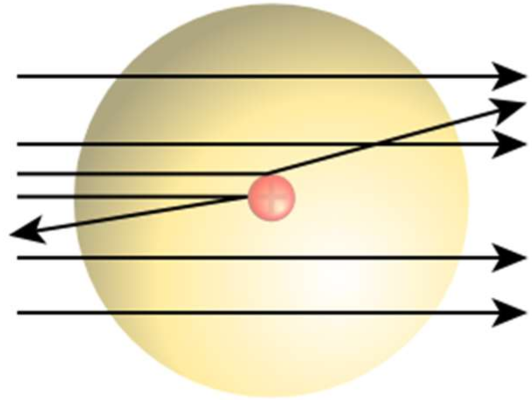
- ‘discovered’ the electron (1897)
- atoms were not indivisible
- proposed the plum pudding model
- corpuscles distributed in a sea of positive charge

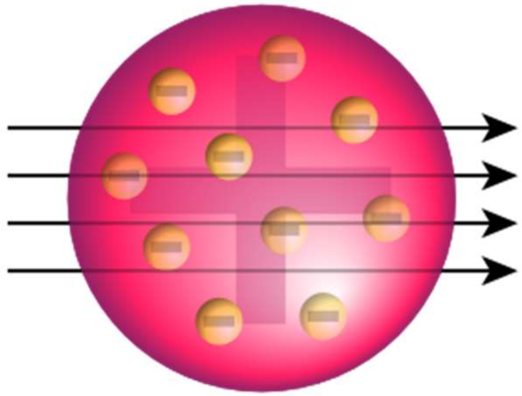


11.1 classical models of the atom

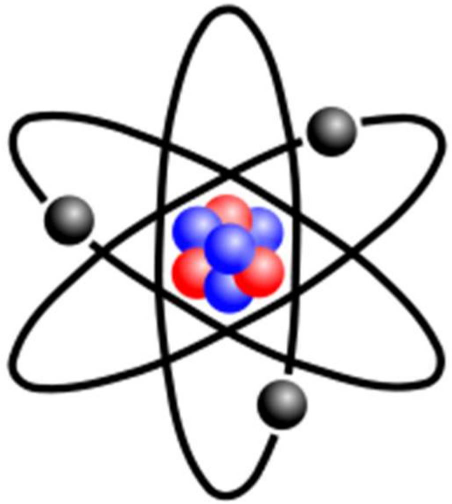
Rutherford (1871-1937)

- Geiger-Marsden alpha particle scattering experiments (1909)
- looked at how He nuclei (i.e. positively charged particles, also referred to as α-particles) were deflected by gold foil
- observed that some α-particles had very high deflection angles – they 'bounced back' towards the source
- results were not compatible with the distribution of charge and mass predicted by the plum pudding model





11.1 classical models of the atom



Rutherford's (planetary) model

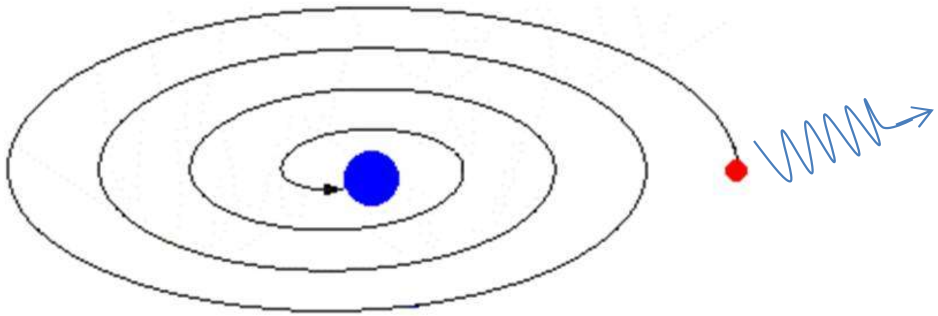
to explain the data from the α-particle scattering experiments, Rutherford proposed the following model:

- the atom contains a very small, central nucleus, which contains almost all of the mass of the atom
- the nucleus is positively charged
- the nucleus is surrounded by a 'cloud' of orbiting electrons (he didn't try to explain the detailed electron-structure)

11.1 classical models of the atom

problems with classical models

- accelerating charges (such as orbiting electrons) should emit electromagnetic radiation (EMR)
- this would cause them to lose energy
- as they lose energy, they would spiral in towards the nucleus, colliding with it after about 10^{-10} seconds

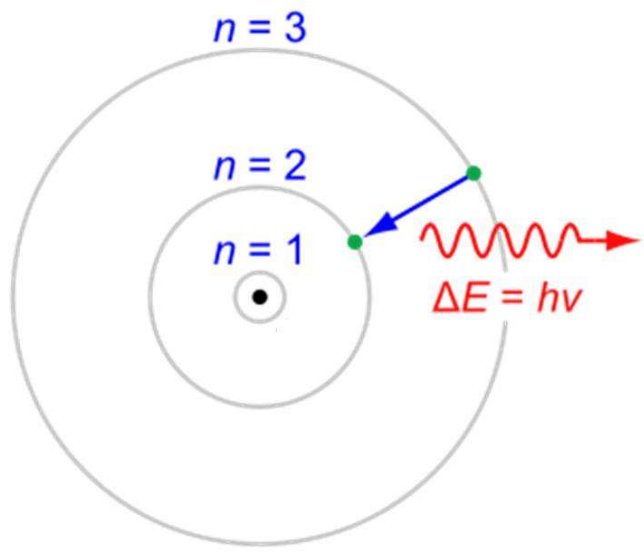


electromagnetic (synchrotron) radiation

11.2 Bohr's hydrogen atom

Niels Bohr (1885-1962)

- Bohr developed a model of the hydrogen atom (1913) by adapting Rutherford's planetary model
- he focused on the electron structure rather than the nucleus
- it was the first quantum physics based model of an atom
- electrons can only exist in certain allowed orbits, each of which has a specific quantised energy



11.2 Bohr's hydrogen atom

Bohr's postulates

- a postulate is something that is assumed to be true in order for the proposed model to work
- Bohr based his model on 4 postulates:

1. the allowed orbits and energies of atomic electrons are quantized and are known as stationary states. Electrons move between stationary states by quantum jumps

2. classical mechanics applies to the orbital motion of the electrons in a stationary state, but these laws do not apply during quantum jumps

11.2 Bohr's hydrogen atom

Bohr's postulates

3. when an electron makes a jump from one stationary state to another, the difference in energy ΔE is emitted or absorbed as a single packet or quantum of light called a photon, which has a frequency, f, given by:

$$f = \frac{\Delta E}{h}$$

h = Plank's constant = 6.626 x 10^{-34} J s

4. the allowed orbits are characterised by quantized values of the orbital angular momentum, L. This angular momentum is always an integer multiple of ($h/2\pi$)

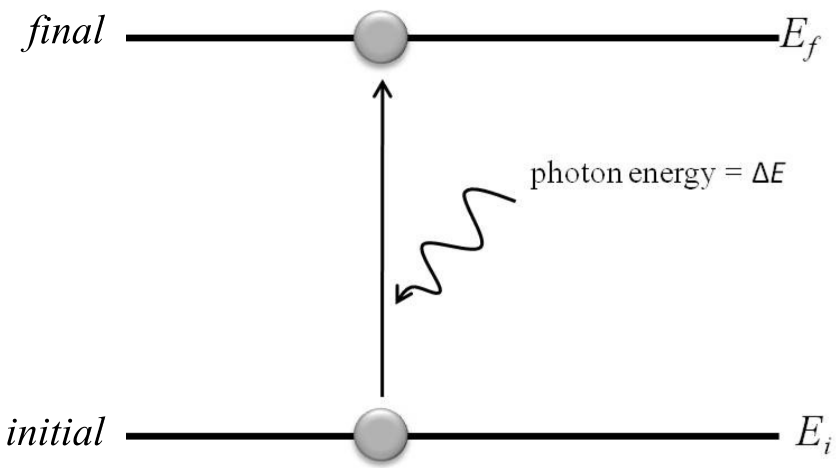
11.2 Bohr's hydrogen atom

postulate 3: absorption and emission

absorption

- electron moves to higher energy level by absorbing a photon
- photon energy is equal to the energy difference, ΔE, between levels

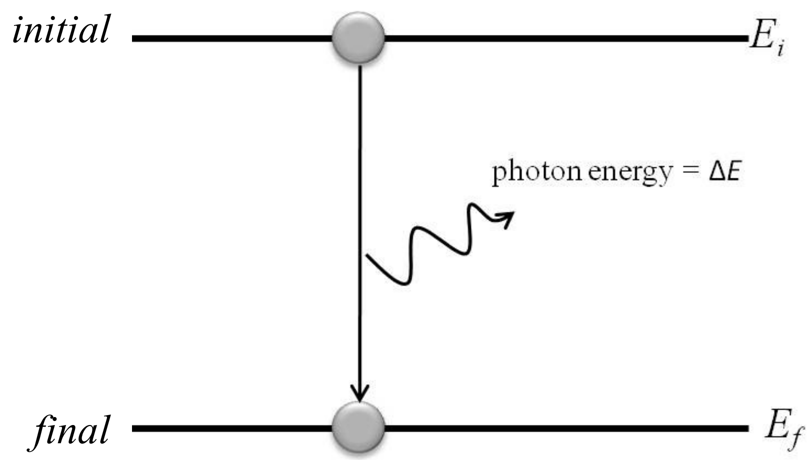
$$\Delta E = E_\mathrm{f} - E_\mathrm{i} = hf = hc/\lambda$$



emission

- electron drops to lower energy level by emitting a photon
- photon energy is equal to the energy difference, ΔE, between levels

$$\Delta E = E_\mathrm{i} - E_\mathrm{f} = hf = hc/\lambda$$



11.2 Bohr's hydrogen atom

constructing the model

- start with the simplest atom i.e. hydrogen – one proton and one electron
- use postulate 4 (quantisation of angular momentum) to work out the allowed electron orbits
- calculate the electron energy associated with each orbital
- consider the energy differences between the allowed states

11.2 Bohr's hydrogen atom

energy of the electron in orbital

electrical potential energy ($W = qV$) of the electron is given by :

$$E_P = -q\frac{Q}{4\pi\varepsilon_0 r} = -\frac{e^2}{4\pi\varepsilon_0 r}$$

kinetic energy is given by: $E_k = \frac{1}{2}mv^2$

so, total energy is given by

$$E = E_p + E_k = -\frac{e^2}{4\pi\varepsilon_0 r} + \frac{1}{2}mv^2$$

11.2 Bohr's hydrogen atom

energy of the electron in orbital

total energy is given by: $$E = E_p + E_k = -\frac{e^2}{4\pi\varepsilon_0 r} + \frac{1}{2}mv^2$$

but quantisation of angular momentum gives: $$v = \frac{n\hbar}{mr}$$

and equating centripetal force to Coulomb force gives:

$$r = n^2\left(\frac{4\pi\varepsilon_0}{e^2 m}\hbar^2\right)$$ (= $n^2 a_0$ where a_0 is the bohr radius)

so, the energy E_n of the electron in the n^{th} orbit is given by:

$$E_n = -\frac{1}{2}\frac{e^4 m}{(4\pi\varepsilon_0)^2}\frac{1}{\hbar^2}\frac{1}{n^2}$$

11.2 Bohr’s hydrogen atom

hydrogen energy levels

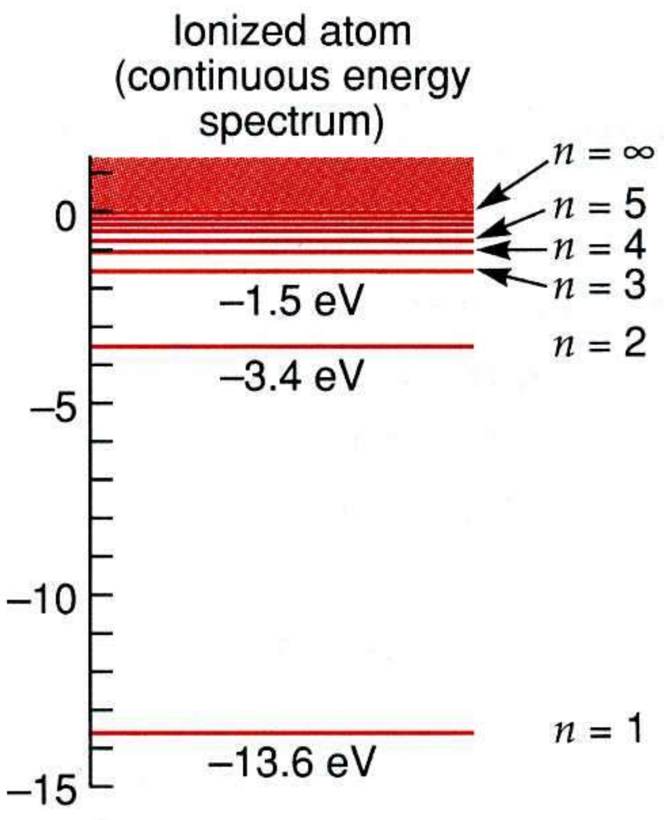
- the energy of an electron in the n^{th} orbit is:

$$E_n = -\frac{1}{2}\frac{e^4 m}{(4\pi\varepsilon_0)^2}\frac{1}{\hbar^2}\frac{1}{n^2}$$

- substituting constants and converting joules to eV gives:

$$E_n = -\frac{13.6}{n^2}\ [\text{eV}]$$

- $n = 1$ gives the lowest energy level, or ground state, of the hydrogen atom ($E_1 = -13.6$ eV)



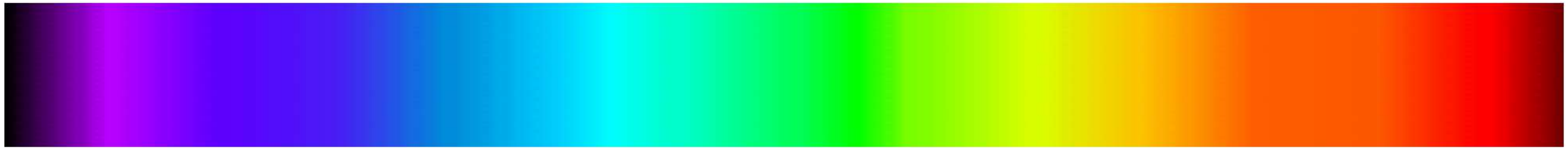
11.2 Bohr's hydrogen atom

- Bohr's quantum model avoids the problems with the classical planetary model of Rutherford (the unstable electron structure)
- applied to the simplest atom, hydrogen, it explains very well the observed hydrogen spectra (emission and absorption lines)
- applied to more complex atoms, however, the model breaks down
    - it incorrectly predicts that the ground state angular momentum would be $L = \hbar$ (it actually equals zero)
    - it can't explain the *Stark* or *Zeeman* effects (splitting of spectral lines in electric and magnetic fields)
- Bohr's model is supplanted by models based on Schrodinger's wave equations (which treat electrons as matter waves and electron orbitals as standing, or stationary, waves).

11.3 hydrogen spectra

emission spectra

- light can be separated into its component wavelengths, or spectrum, using a prism or diffraction grating
- the spectrum of white light is a continuum, containing all the visible colours, from violet to red



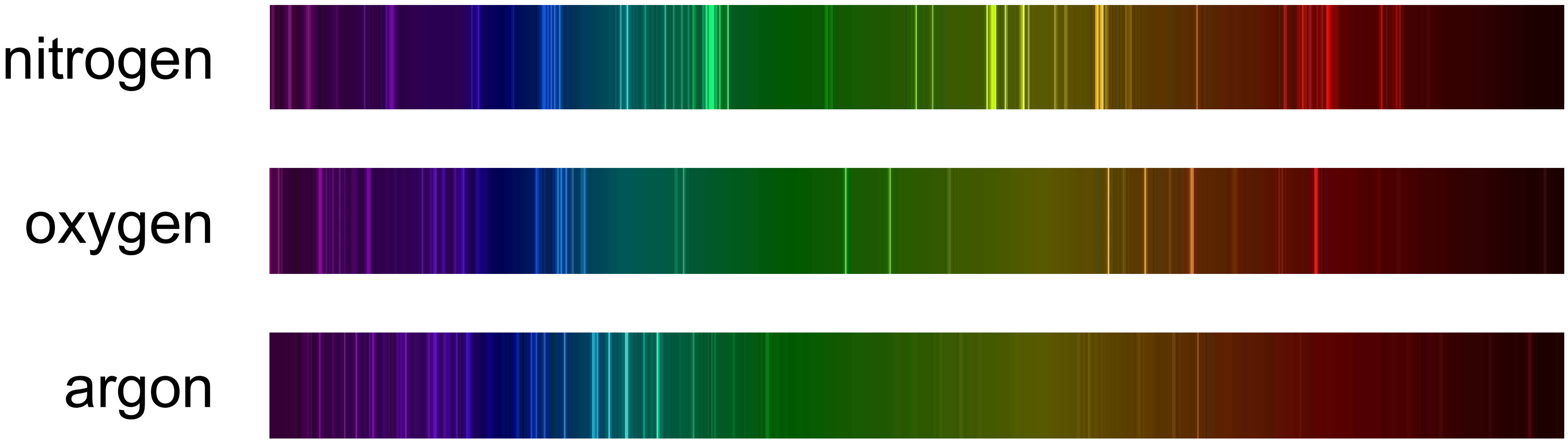
white light (continuous) spectrum

11.3 hydrogen spectra

emission spectra

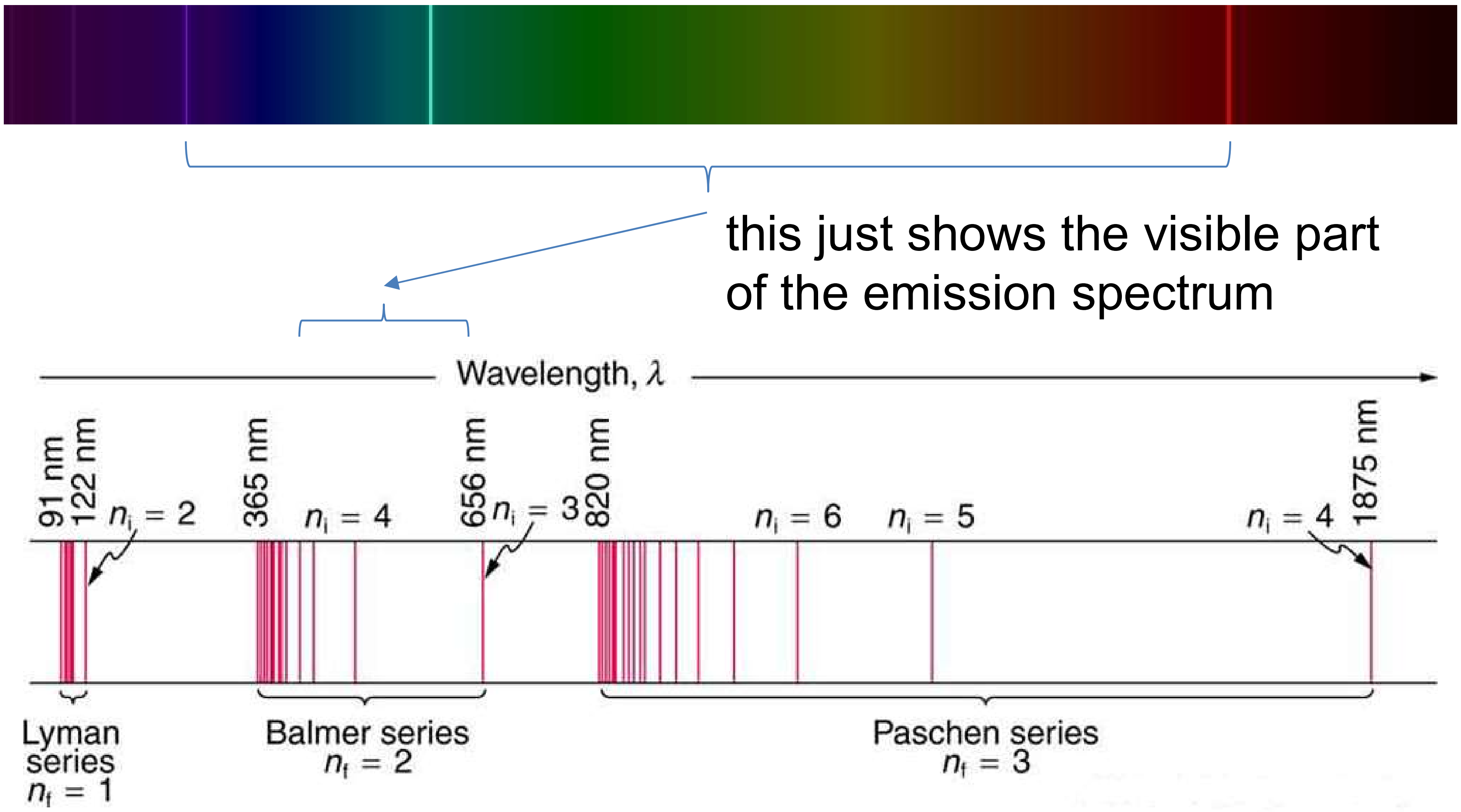
- when gases containing a single type of atom are excited, they emit a spectrum that is characteristic of the element

- the emission spectrum is not continuous, it consists of a series of narrow lines with well-defined wavelengths



11.3 hydrogen spectra

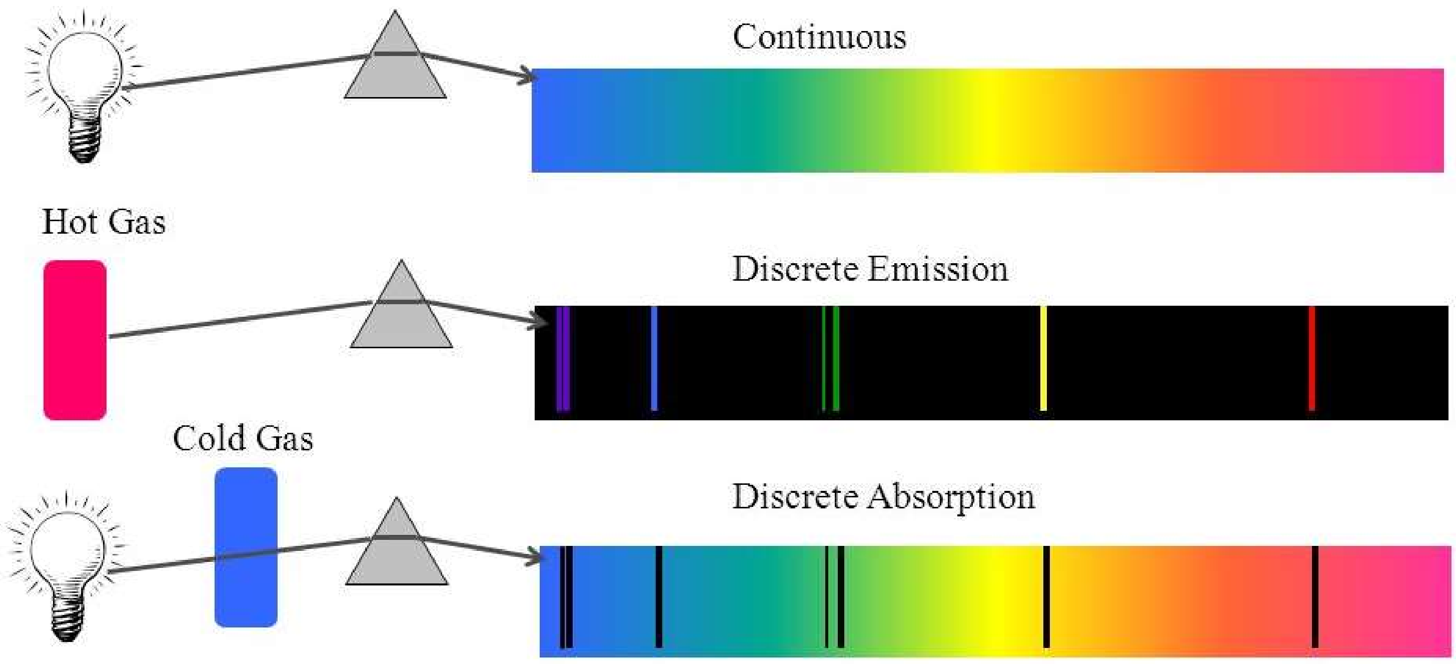
hydrogen emission spectrum



11.3 hydrogen spectra

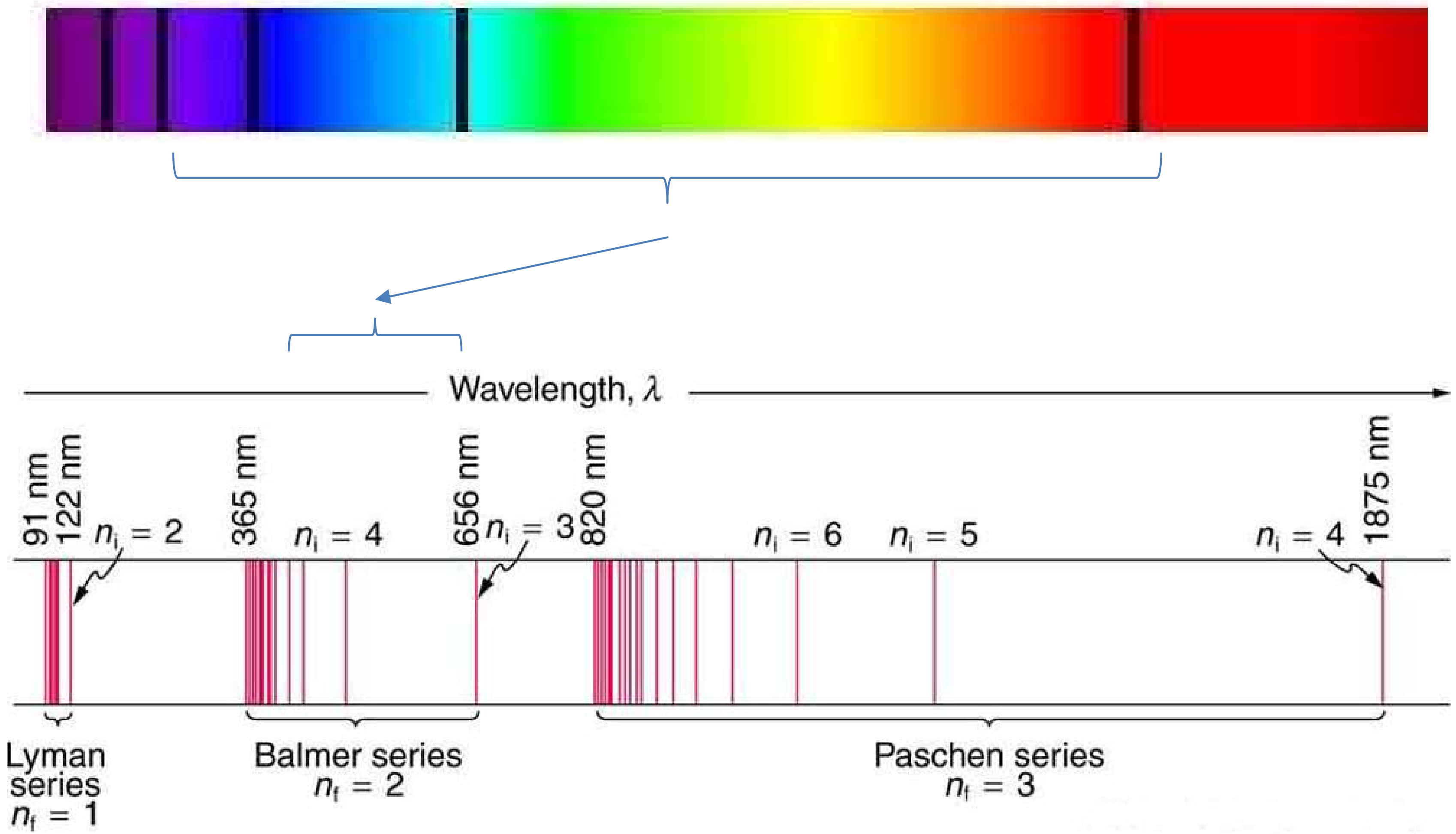
absorption spectra

- white light shining through a gas produces a continuous spectrum with dark 'missing' lines, corresponding to the wavelengths seen in the emission spectrum of the gas

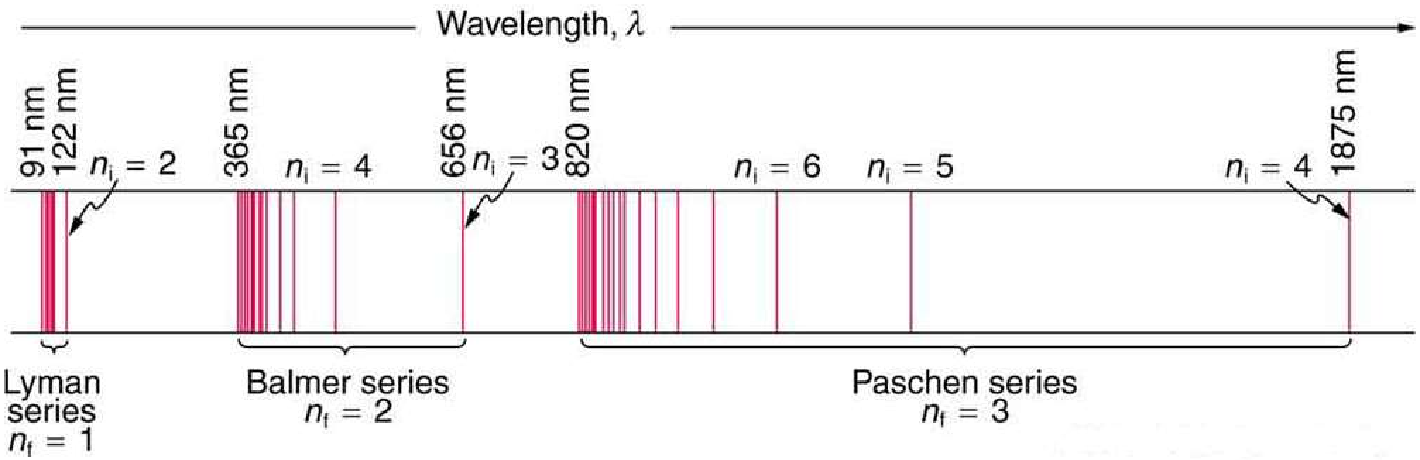


11.3 hydrogen spectra

hydrogen absorption spectrum



11.3 hydrogen spectra



Balmer series: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$ with n = 3, 4, 5, 6...

R is the *Rydberg* constant = 1.0974 × 10^7 m^{-1}

Lyman series: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)$ with n = 2, 3, 4, 5...

Paschen series: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2} - \dfrac{1}{n^2}\right)$ with n = 4, 5, 6, 7...

11.3 hydrogen spectra

- we can explain the hydrogen spectra by considering the difference in energy between two levels, n_i and n_f , as given by Bohr's model

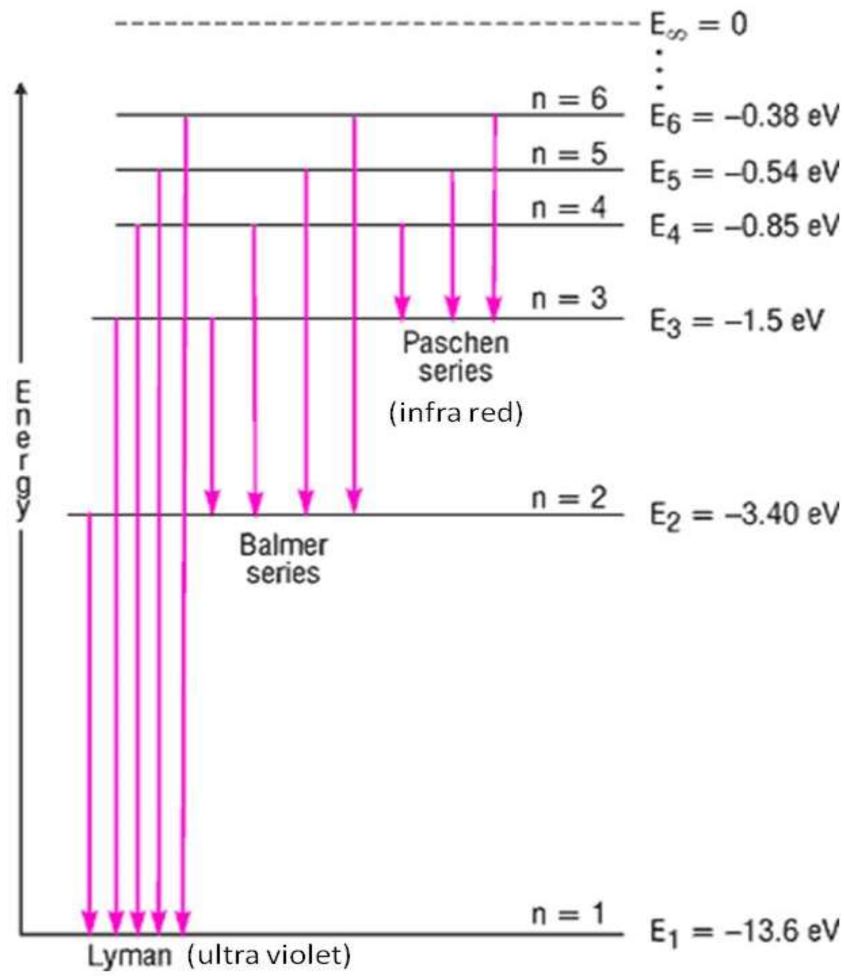
$$\Delta E = E_i - E_f = \frac{hc}{\lambda} = \frac{1}{2}\frac{e^4 m}{(4\pi\varepsilon_0)^2}\frac{1}{\hbar^2}\left(-\frac{1}{n_i^{\,2}} + \frac{1}{n_f^{\,2}}\right)$$

- with $n_f = 2$, we can rearrange this expression so it looks just like the equation for the Balmer series:

$$\frac{1}{\lambda} = \frac{1}{2}\frac{e^4 m}{(4\pi\varepsilon_0)^2}\frac{1}{\hbar^2}\frac{1}{hc}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

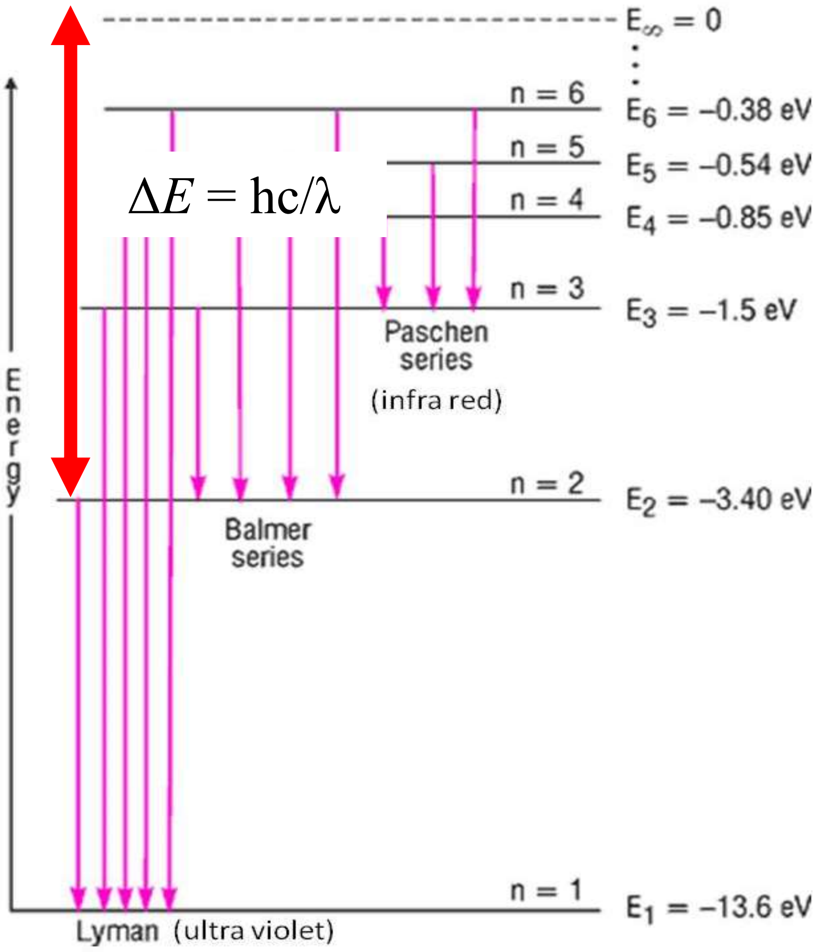
11.3 hydrogen spectra

- spectral lines arise from electrons jumping between energy levels
- emission lines represent the photon emitted when an electron moves from a higher to a lower energy level
- absorption lines represent the photon absorbed when an electron moves from a lower to a higher energy level
- e.g. the Balmer emission series results from electron transitions, from higher levels, to the n = 2 level

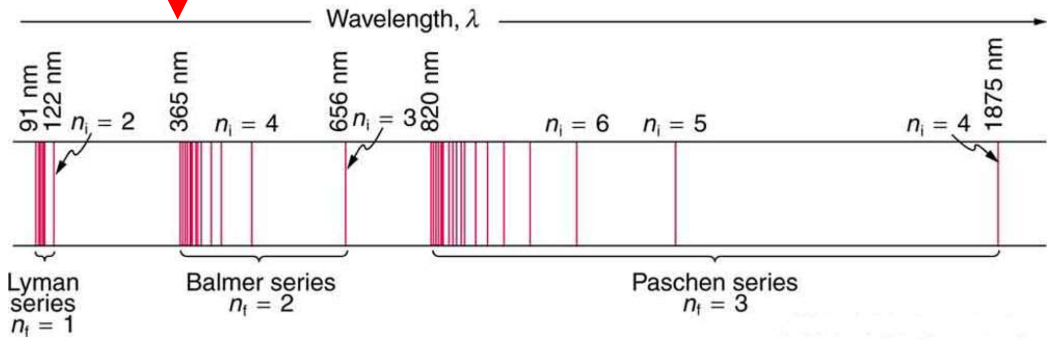


11.3 hydrogen spectra

- the series limit is the shortest wavelength (highest energy) transition for a given series
- e.g the Balmer series limit is given by:

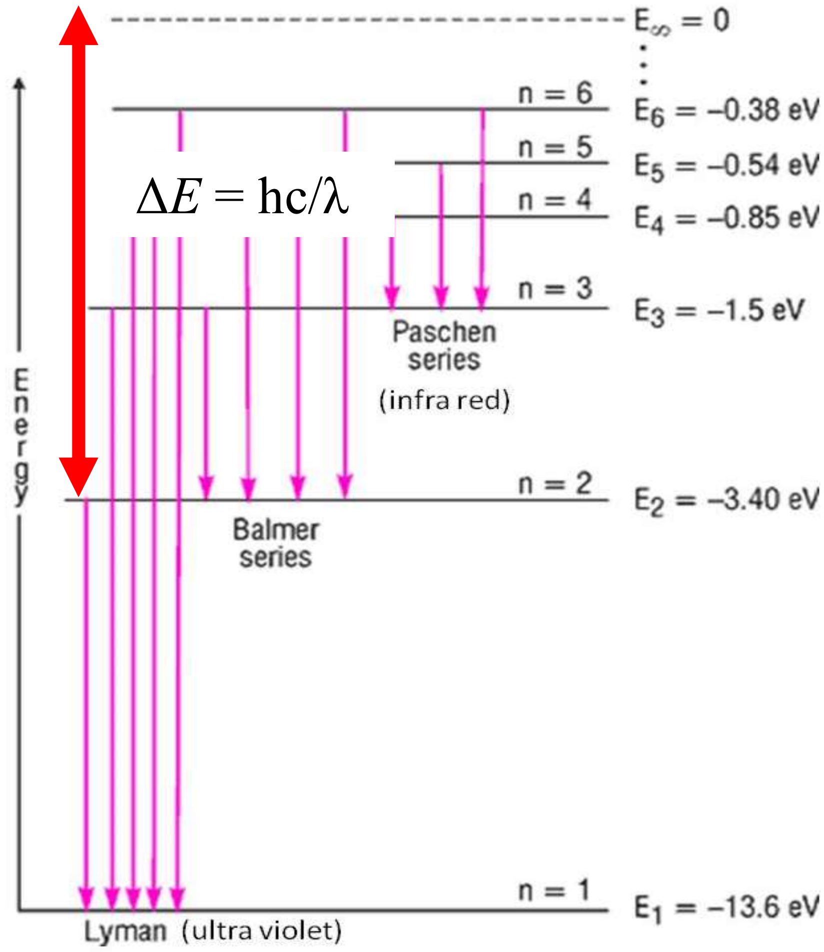


Balmer series limit



11.3 hydrogen spectra

- the series limit is the shortest wavelength (highest energy) transition for a given series

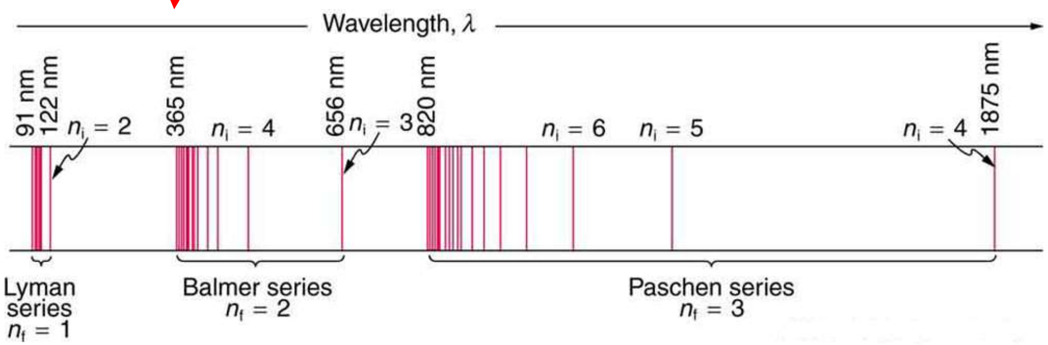


- e.g the Balmer series limit is given by:

$$\Delta E_{\lim} = \frac{hc}{\lambda_{\lim}} = -13.6\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right) = 3.40 \text{ eV}$$

$$\Rightarrow \frac{hc}{\lambda_{\lim}} = 3.40 \times 1.60 \times 10^{-19} \text{ J}$$

$$\Rightarrow \lambda_{\lim} = 366 \text{ nm}$$



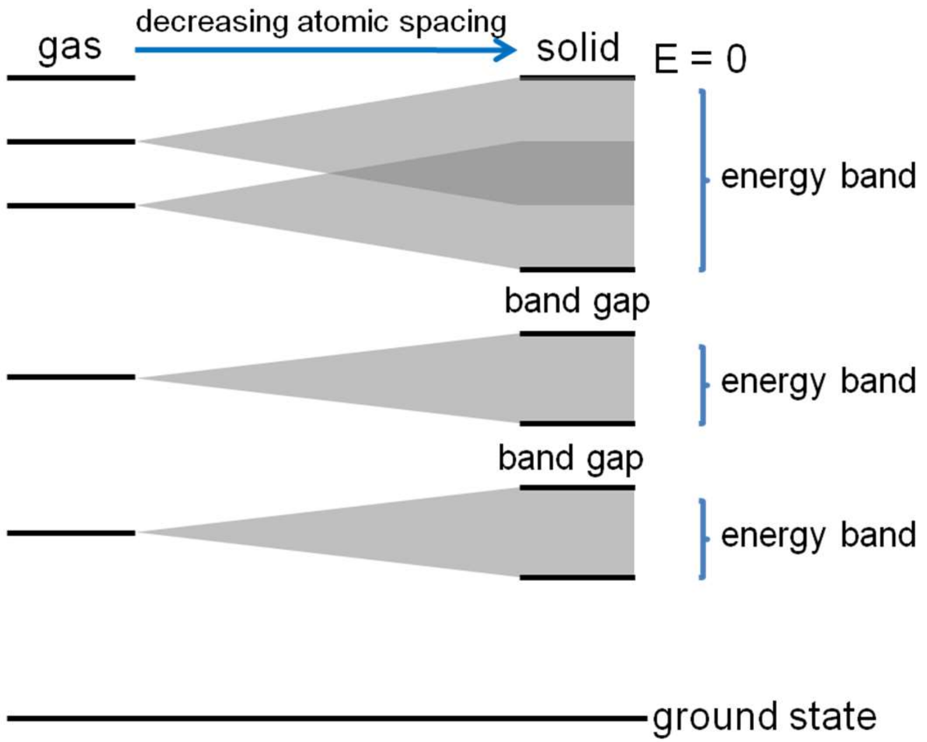
11.4 band theory of solids

- the band theory of solids is effective for explaining the electrical (and other) properties of solid materials
- band theory explains the defining characteristics conductors, insulators and semiconductors
- band theory provides the basis for understanding and utilising semiconductor devices, such as p-n junctions, transistors, LEDs, solid-state lasers, photodetectors, photovoltaic cells etc

11.4 band theory of solids

energy levels in solids

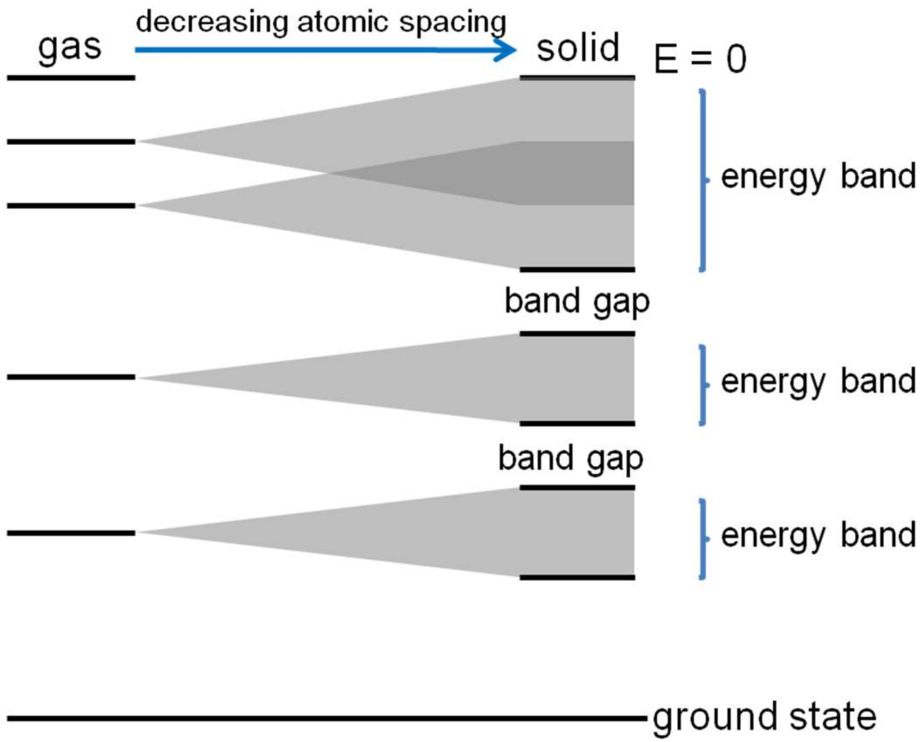
- in individual atoms, or gases, the energy levels associated with each atom are narrow and well-defined (the atoms are too far apart for the electron orbitals to interact)
- in solids, the atoms are closely packed and electron orbitals of neighbouring atoms overlap, causing energy levels to interact
- the laws of quantum physics do not allow electrons in the same space to have the same energy



11.4 band theory of solids

energy levels in solids

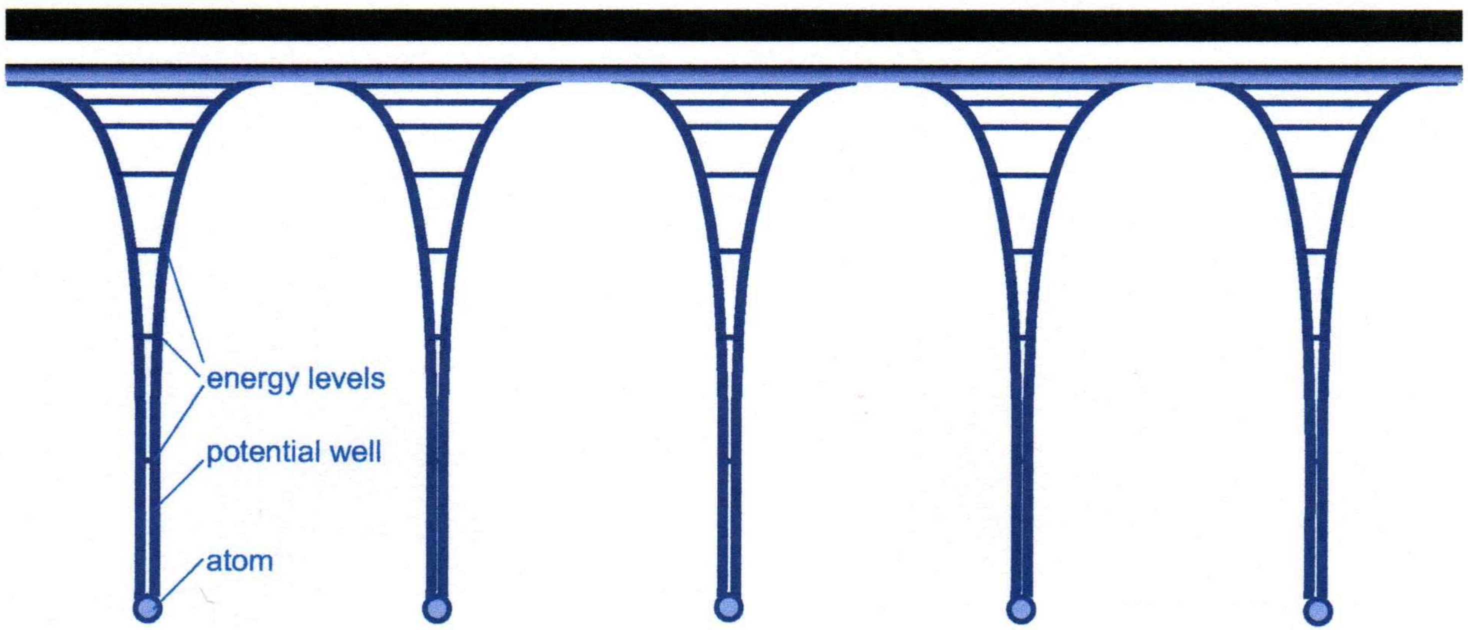
- if two identical atoms are brought close together, each individual level will shift slightly, either up or down in energy
- a single level effectively becomes two closely spaced levels
- if 1000 atoms are packed together, each energy level ‘splits’ into 1000 closely spaced energy levels
- the result is a broadening of energy levels into energy bands, within which electrons can have an (almost) continuous range of energies



11.4 band theory of solids

- higher-energy bands (associated with the outer orbitals) ‘spread out’, forming a continuous band of allowed electron energies throughout the solid

- energy bands may overlap, or be separated by ‘forbidden’ band gaps

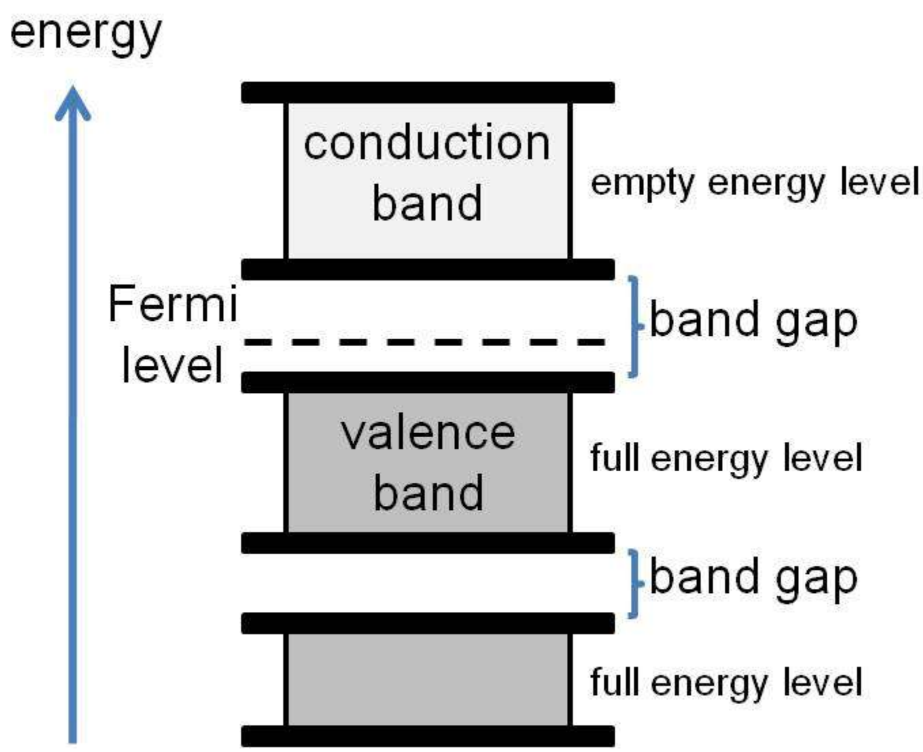


energy bands

11.4 band theory of solids

band structure – definitions

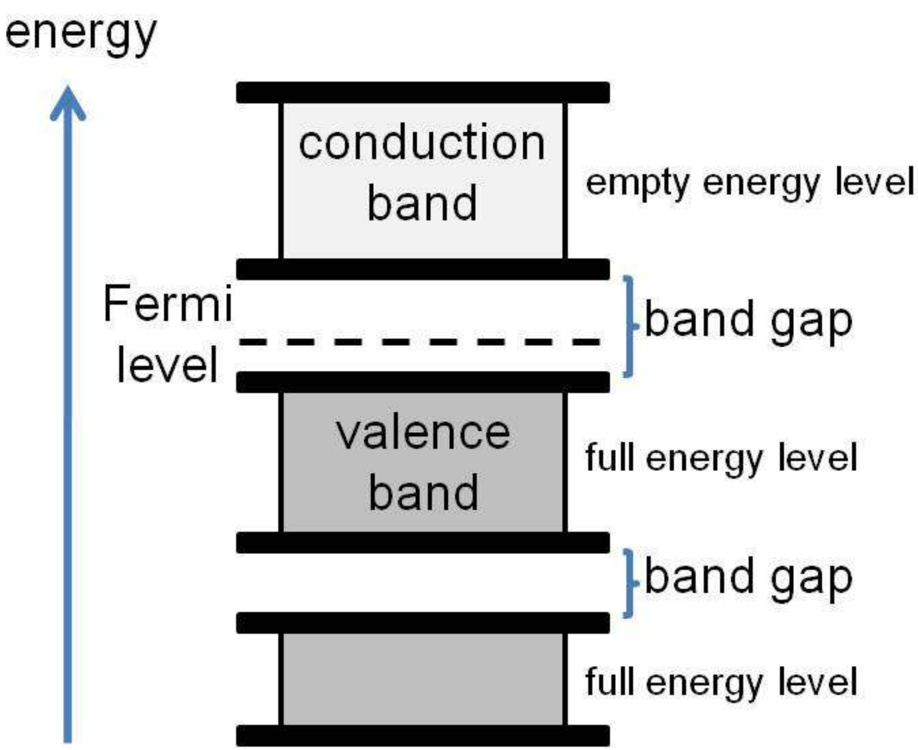
- **valence band** – highest energy band that is full of electrons
- **conduction band** – lowest energy band that is empty
- **band gap** – range of energies that are not ‘allowed’, electrons can’t be found in a band gap
- **fermi level** – *theoretical* level of highest energy electrons at absolute zero (when temperature = 0 K)



11.4 band theory of solids

band structure – definitions

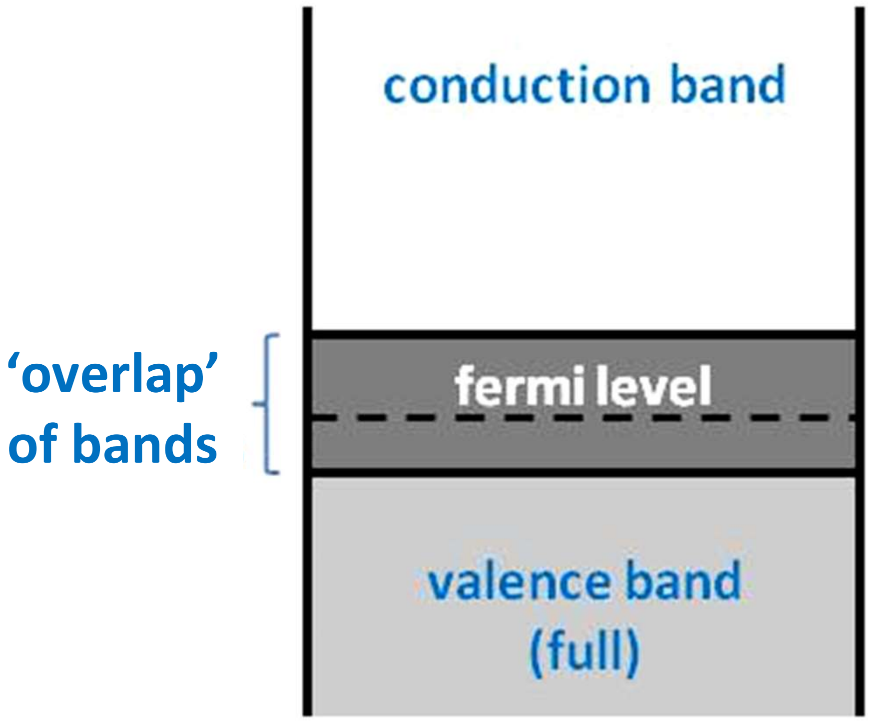
- **fermi level** – *theoretical* level of highest energy electrons at absolute zero (when temperature = 0 K)
- at 0 K, energy levels below the fermi level are full, above it are empty
- above 0 K, thermal energy means some energy levels below the fermi level may be empty and some levels above it may be occupied
- it is a theoretical energy level; if it lies in a band gap (where there are no allowed energies) electrons cannot have that energy



11.4 band theory of solids

conductors

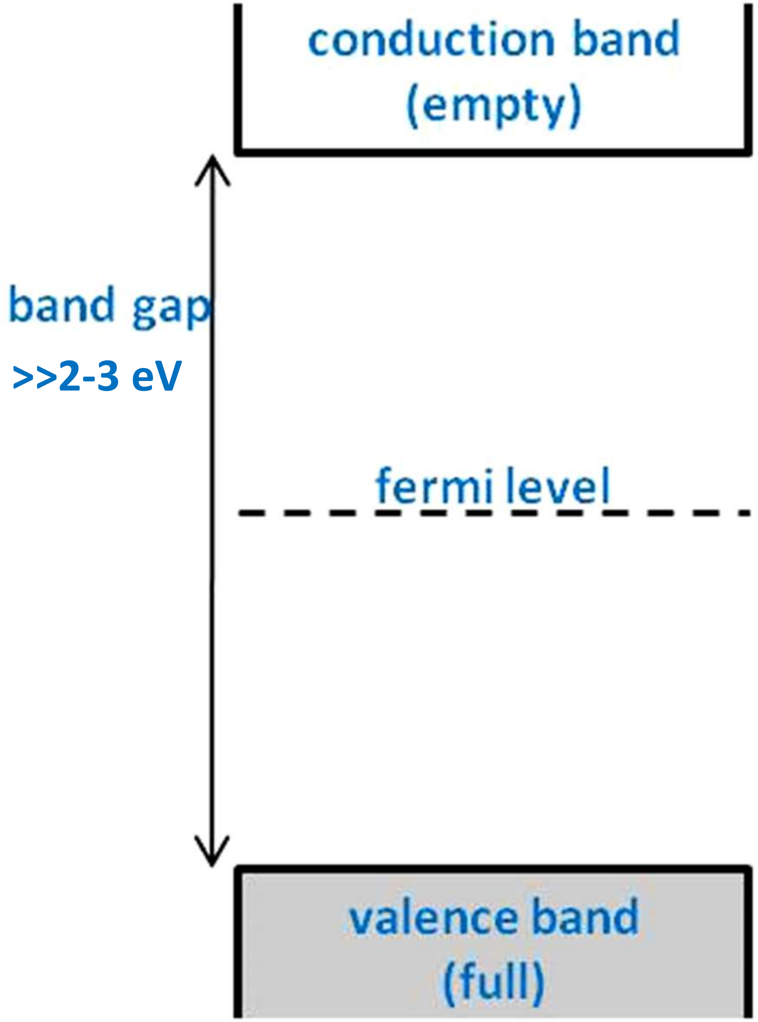
- the fermi level lies within an energy band
- there is no band gap between valence and conduction bands
- conduction band has multiple available energy levels for electrons
- electrons can easily move to an 'empty' energy level and so are free to move
- conduction is possible



11.4 band theory of solids

insulators

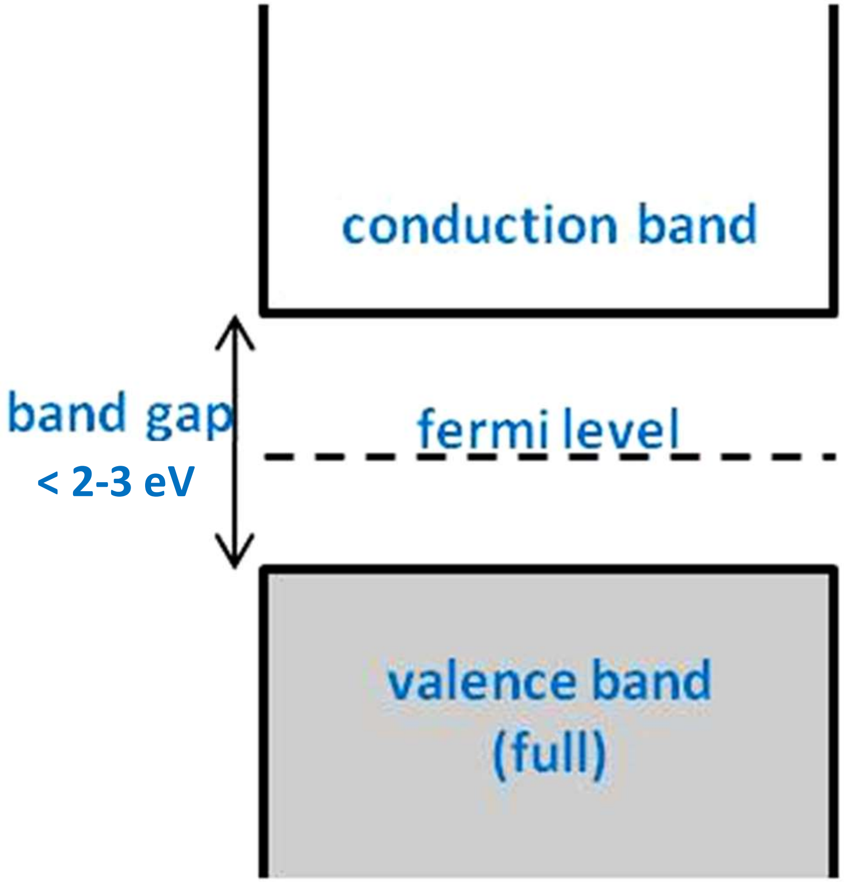
- the fermi level lies within the principal band gap
- very large band gap between valence and conduction bands (typically >> 2-3 eV)
- valence band is completely full, net transport of charge not possible
- conduction band is empty
- band gap too big for electrons to jump across – no conduction



11.4 band theory of solids

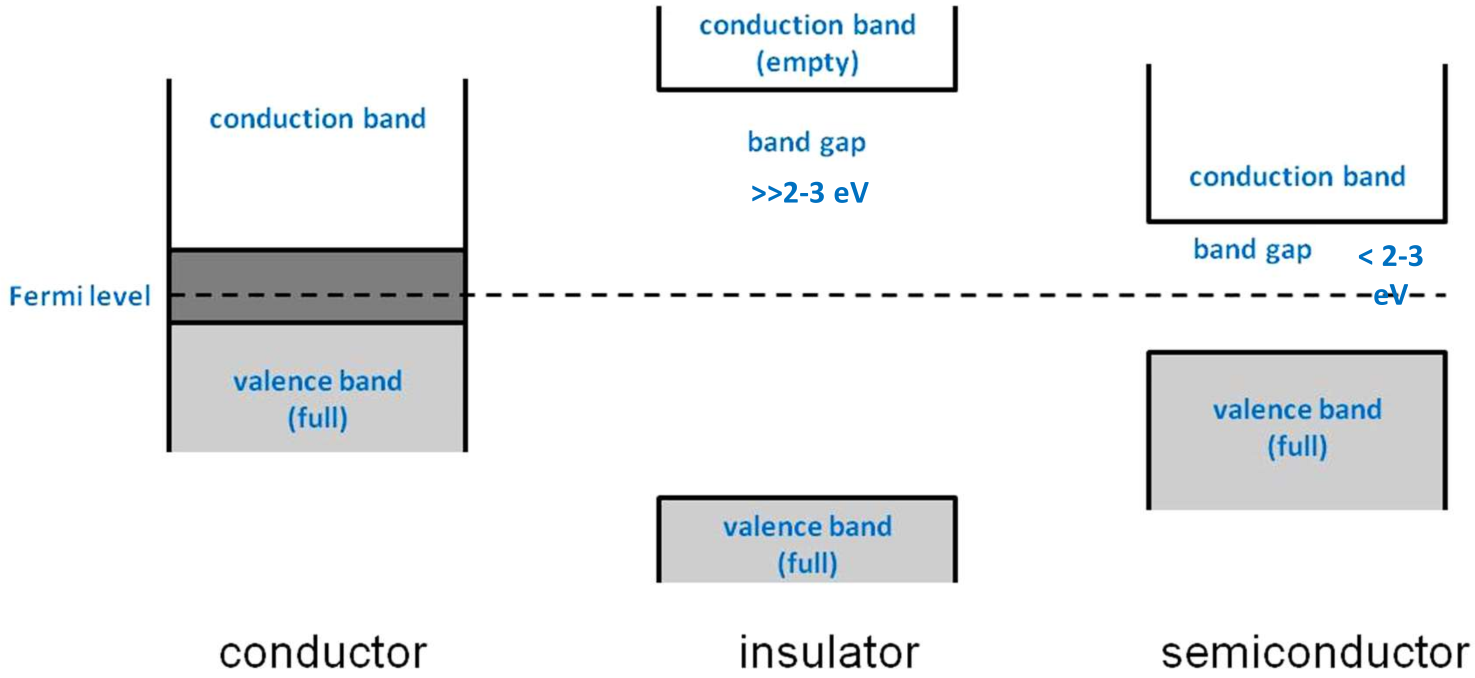
semiconductors

- the fermi level lies within the principal band gap
- relatively small band gap between the valence and conduction bands (typically < 2-3 eV)
- only a small amount of energy is required for electrons to jump across the band gap
- thermal, electrical or photon (light) energy can promote electrons into the conduction band – conductivity depends on conditions and can be ‘engineered’



11.4 band theory of solids

comparison and summary



- conductors – no band gap between valence and conduction bands
- insulators – large band gap between valence and conduction bands
- semiconductors – conductivity strongly influenced by temperature, electric field, illumination etc

11.4 band theory of solids

comparison and summary (typical figures)

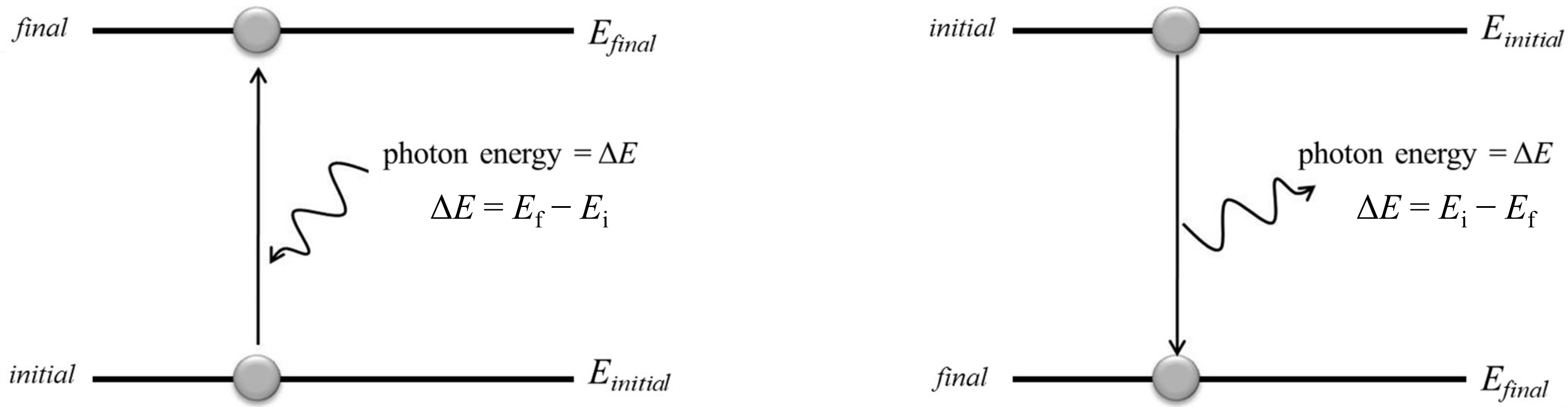
	Insulators	Semiconductors	Conductors
Resistivity, ρ (Ω m)	Up to 10^{15}	~3×10^{3}	~2×10^{-8}
Temperature dependence of ρ	No effect	ρ **decreases** as temperature increases	ρ **increases** as temperature increases
Carrier density, n (m^{-3})	~0	~10^{16}	~10^{28}

12.1 photoluminescence

- there are a several processes by which a semiconductor can emit a photon of light as an electron drops from the conduction band into the valence band (a process known as luminescence)
- if the energy that caused the electron to be in the conduction band was obtained by the absorption of a photon, the process is known as photoluminescence (if the energy came from an electric field, it is known as electroluminescence and if it came from heat, it is called thermoluminescence)
- photoluminescence is a demonstration of how light interacts with semiconductors and is one example of how semiconductors can produce light
- photoluminescence is also a very accurate way of measuring the band gap of semiconductor materials

12.1 photoluminescence

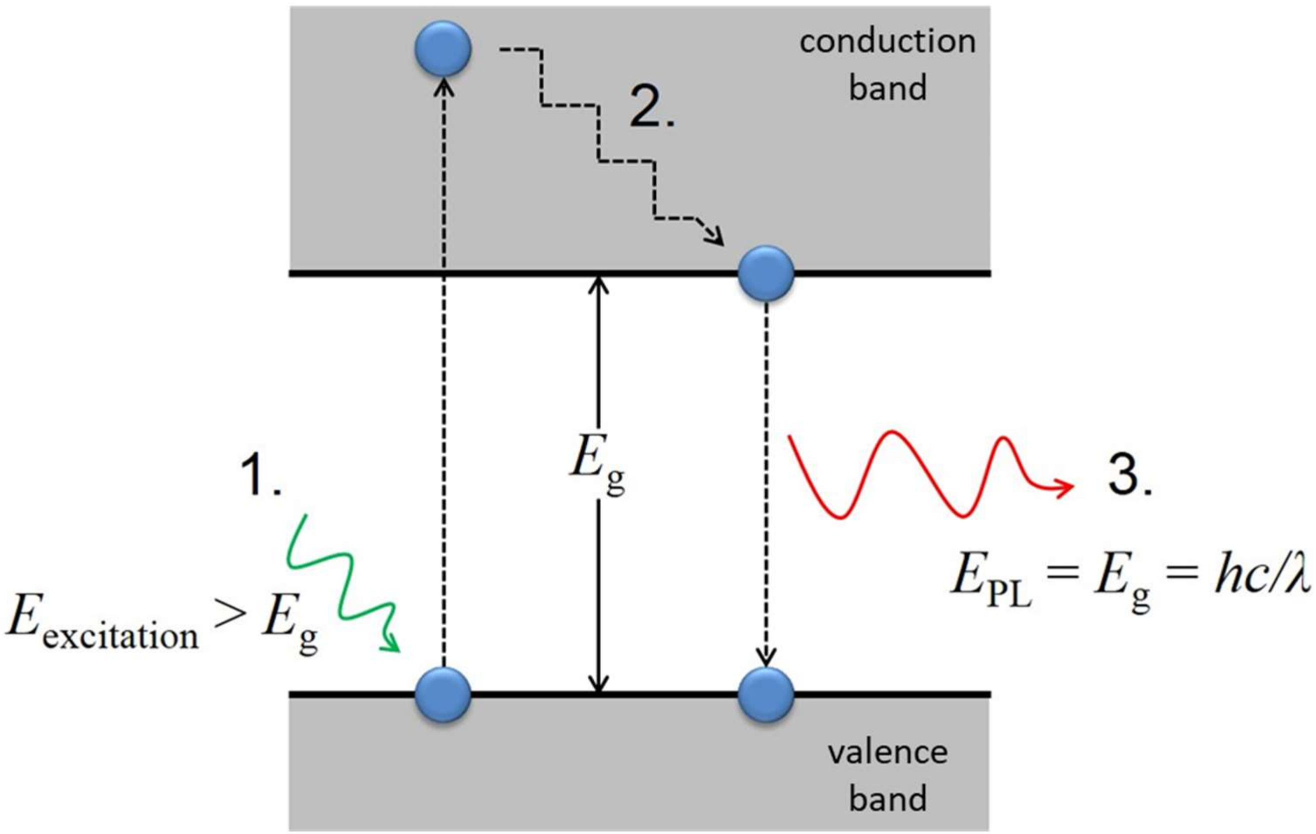
- photoluminescence involves the movement of electrons between (and within) energy bands in solids
- before looking at photoluminescence, recall Bohr's 3rd postulate, which explains how an electron can move between atomic energy levels by absorbing or emitting a photon whose energy is equal to the energy difference between the levels:



where $$\Delta E = hf = \frac{hc}{\lambda}$$

12.1 photoluminescence

1. excitation by a light source with photon energy $E_\gamma > E_g$
2. thermalisation – excess energy lost as heat (in form of phonons)
3. emission – photoluminescence energy, $E_{\mathrm{PL}} = E_g$ (band gap)



12.1 photoluminescence

example

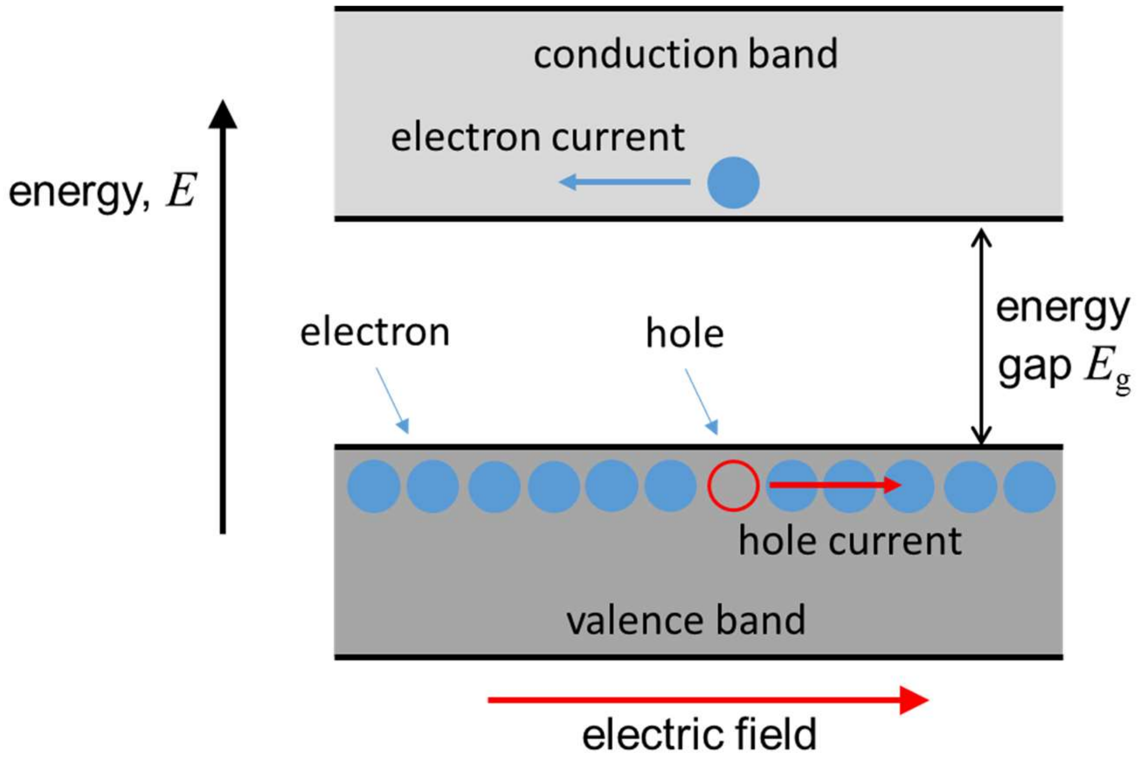
A sample of the semiconductor, InGaAlP, is illuminated with a green laser which has wavelength λ = 514 nm. The sample emits photoluminescence with a wavelength of 611 nm.

Calculate the band gap of the semiconductor, giving your answer in electron volts?

12.2 doping of semiconductors

- the conductivity of semiconductors depends on the number of charge carriers, given by the carrier concentration (or carrier density), and on the availability of unfilled energy bands within which they can move
- the carrier concentration of semiconductors can be manipulated by a process known as doping
- there are three categories of semiconductor
  - intrinsic semiconductors - 'pure' or undoped
  - n-type semiconductors - doped
  - p-type semiconductors - doped
- before looking at each of these in turn, we must consider exactly what we mean by a charge carrier

12.2 doping of semiconductors

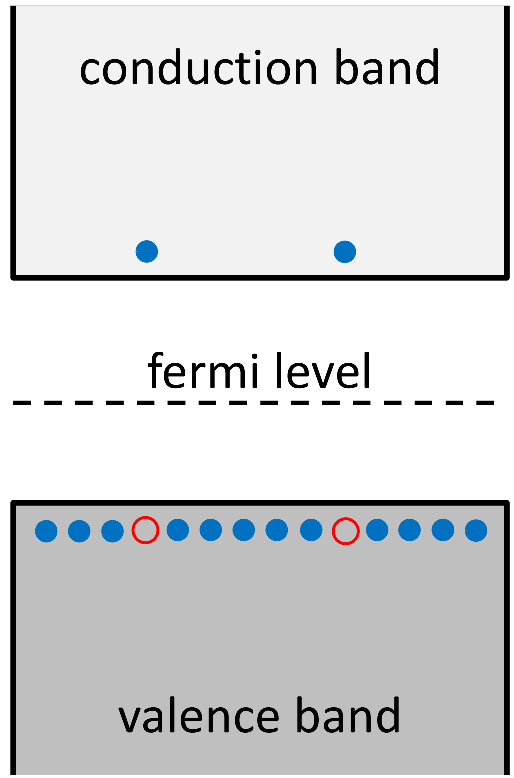


- charge can be carried by electrons and holes
- a hole is created when an electron jumps into the conduction band
- holes act like electrons with a positive charge

12.2 doping of semiconductors

intrinsic semiconductors

- pure (undoped) semiconductors that have no charge carriers added are known as intrinsic semiconductors
- intrinsic semiconductors have equal numbers of mobile (conducting) electrons and holes
- the concentration of charge carriers is low
- the conductivity of an intrinsic semiconductor increases as the temperature is increased (a 40° increase in temperature will typically increase conductivity by a factor of 10)
- intrinsic semiconductors are used to make thermistors (thermal resistors), thermal fuses etc



12.2 doping of semiconductors

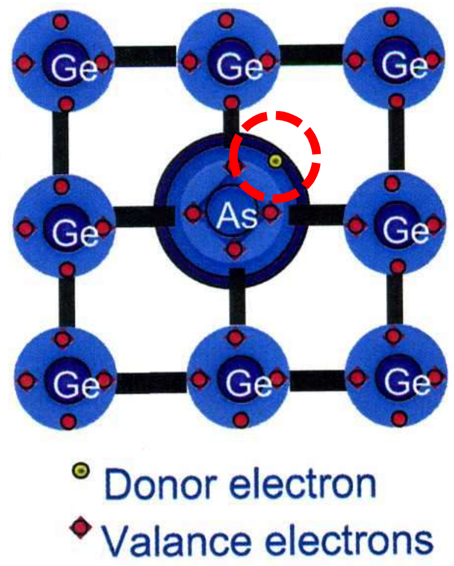
doped semiconductors

- the doping of semiconductors involves adding impurities to an intrinsic semiconductor to modify its electrical characteristics
- the impurities change the carrier concentration of the material, which changes its conductivity
- only a very small concentration of impurity atoms (e.g. 1 in 5 million atoms) are needed to significantly change the carrier concentration
- there are two types of doping:
  - n-type doping – adding negative charge carriers i.e. adding electrons to an intrinsic semiconductor
  - p-type doping – adding positive charge carriers i.e. adding holes to an intrinsic semiconductor

12.2 doping of semiconductors

n-type semiconductors

- an n-type dopant fits into the semiconductor crystal structure and donates an electron
- the majority charge carriers are electrons (holes are the minority carriers)



example: arsenic-doped germanium

- Ge is a group IV element (4 valence electrons)
- As is a group V element (5 valence electrons)
- As fits into the Ge lattice with 4 of its valence electrons bonding to its nearest neighbours
- the fifth electron is free to conduct charge

III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb
Tl	Pb	Bi

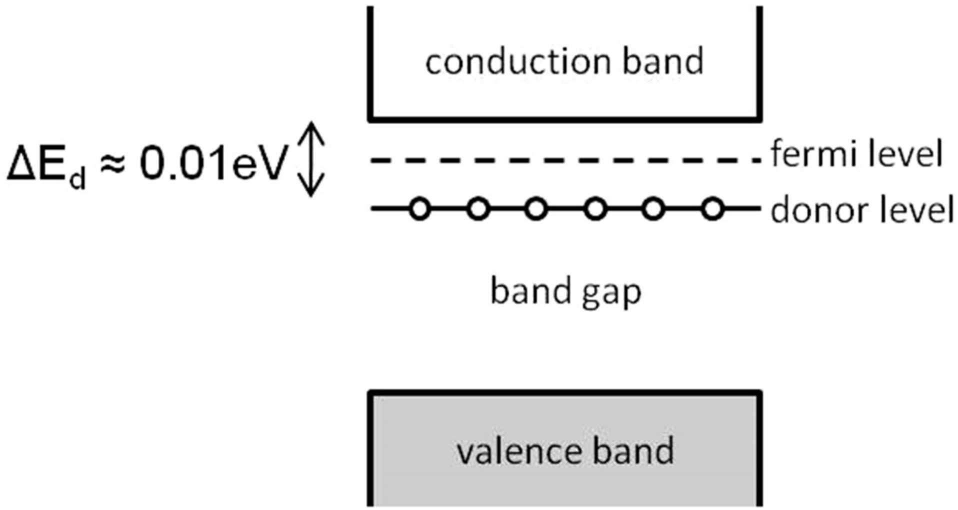
12.2 doping of semiconductors

n-type semiconductors

- the energy levels of the dopant atoms are similar to those of the host atoms, so they form narrow (atom-like) energy levels, close to the energy bands of the semiconductor

- a donor level, containing electrons, lies just below the conduction band

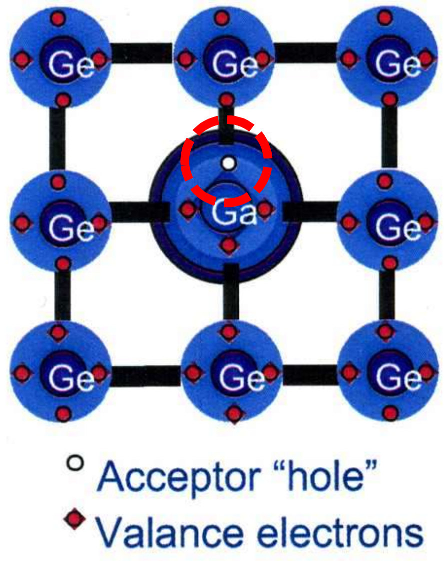
- thermal energy promotes electrons from the donor level into conduction band, where they act as free charge carriers



12.2 doping of semiconductors

p-type semiconductors

- a p-type dopant fits into the semiconductor by accepting an electron, which creates a hole
- the majority charge carriers are holes (electrons are the minority carriers)



example: gallium-doped germanium

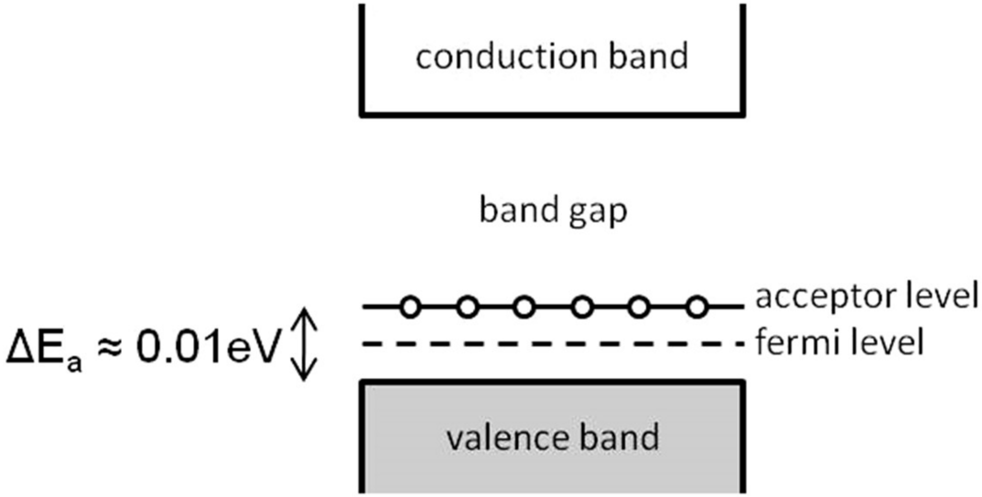
- Ge is a group IV element (4 valence electrons)
- Ga is a group III element (3 valence electrons)
- Ga fits into the Ge lattice with its valence electrons bonding to 3 of its 4 nearest neighbours
- a vacancy, or hole, is introduced at the 4th bond site

III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb
Tl	Pb	Bi

12.2 doping of semiconductors

p-type semiconductors

- an acceptor level, containing vacancies (holes), lies just above the valance band, which is full of electrons
- thermal energy promotes electrons into the acceptor level, creating holes in valence band
- the holes act as free charge carriers in the valence band



12.3 the p-n junction

the significance of p-n junctions

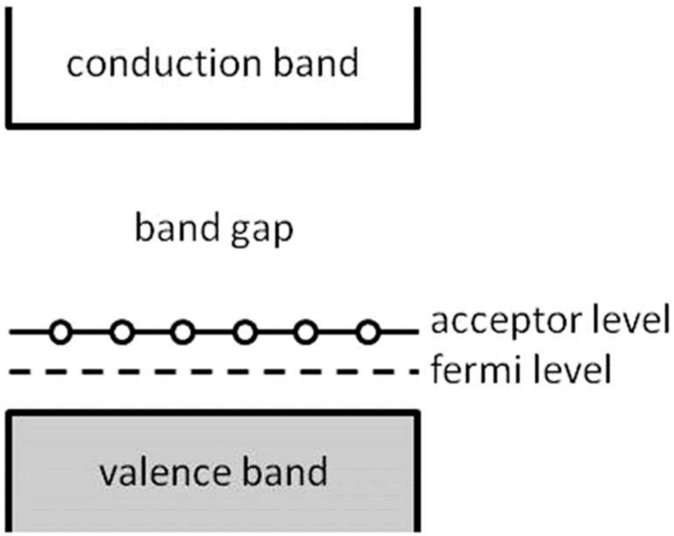
- p-n junctions form the basis of most solid-state diodes
  - diodes are electronic devices that control current flow (current can only flow in one direction through a diode)
  - diodes are one of the most important components in modern electronic circuits and devices, used to make LEDs, photodiodes, solid state lasers, solar cells, rectifiers etc.

- the technology of n and p doped semiconductors, and p-n junctions in particular, is fundamental to the design and operation of the majority of transistors; for example field effect transistors, such as MOSFETS, which are the key building blocks of digital logic circuits

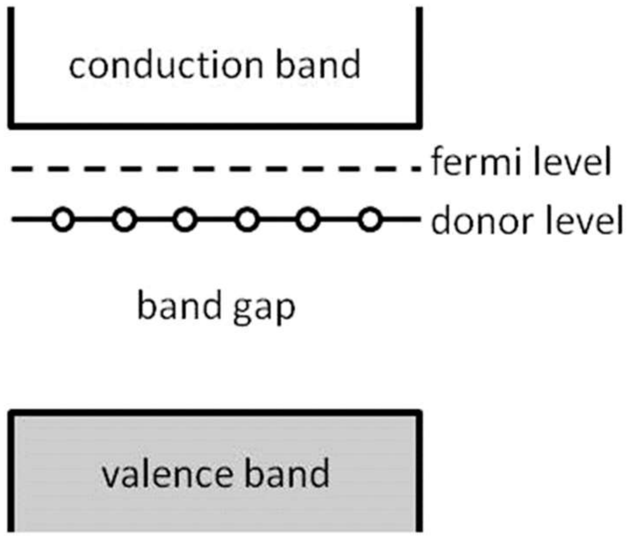
12.3 the p-n junction

p and n-type band structure - reminder

p-type



n-type

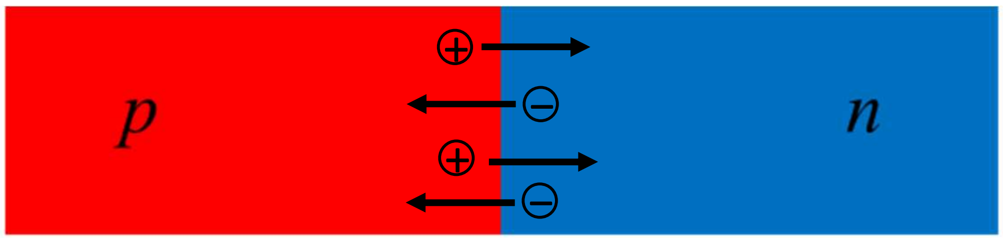


- majority carriers are holes in the valence band, created when electrons move into the acceptor level (due to thermal energy)

- majority carriers are electrons, promoted from the donor level into the conduction band by thermal energy

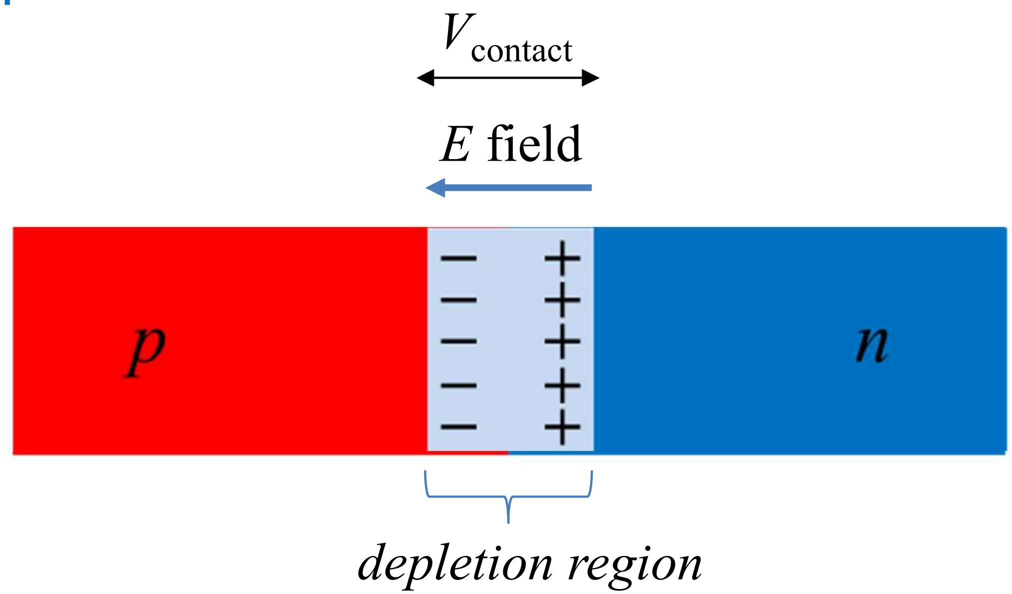
12.3 the p-n junction

- a p-n junction is the junction between a p-type region of semiconductor and an n-type region of the same semiconductor
- to understand the nature of a p-n junction, imagine what happens when p and n-type material are brought into contact (forming the junction)
- p-type material has a high concentration of holes, the n-type material has a high concentration of electrons
- diffusion occurs at the junction; electrons diffuse out of the n region and holes diffuse out of the p region



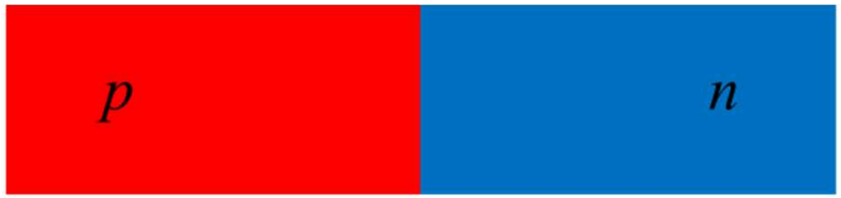
12.3 the p-n junction

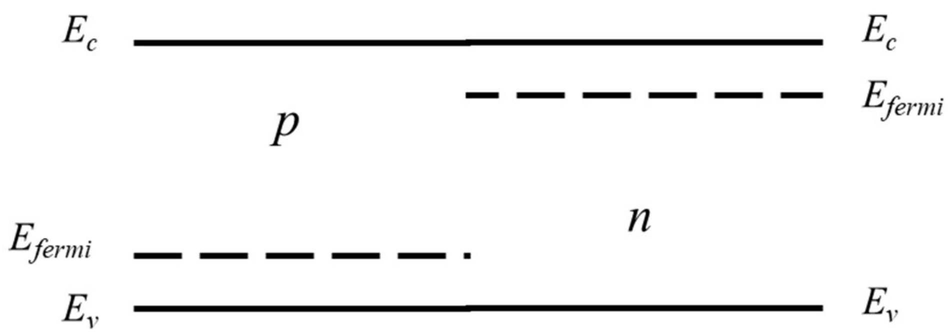
- as the diffusion continues, electrons leave behind a localised region of positive charge and holes leave behind a region of negative charge
- an electric field is set up, with a so-called contact potential difference across the junction, which eventually prevents further net diffusion (the drift current due to the field equals the current due to diffusion)
- a region containing no mobile charge carriers is created, known as the depletion region



12.3 the p-n junction

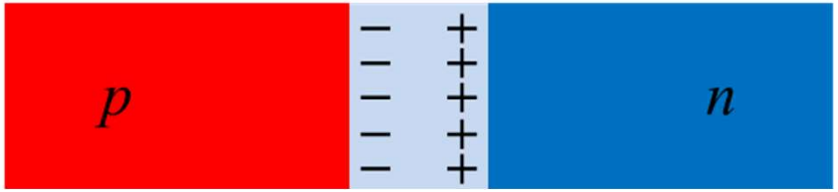
- now consider the band structure as the junction is formed
- the p and n materials have the same band structure, so at the moment of contact the bands are in alignment (but the fermi level is different)

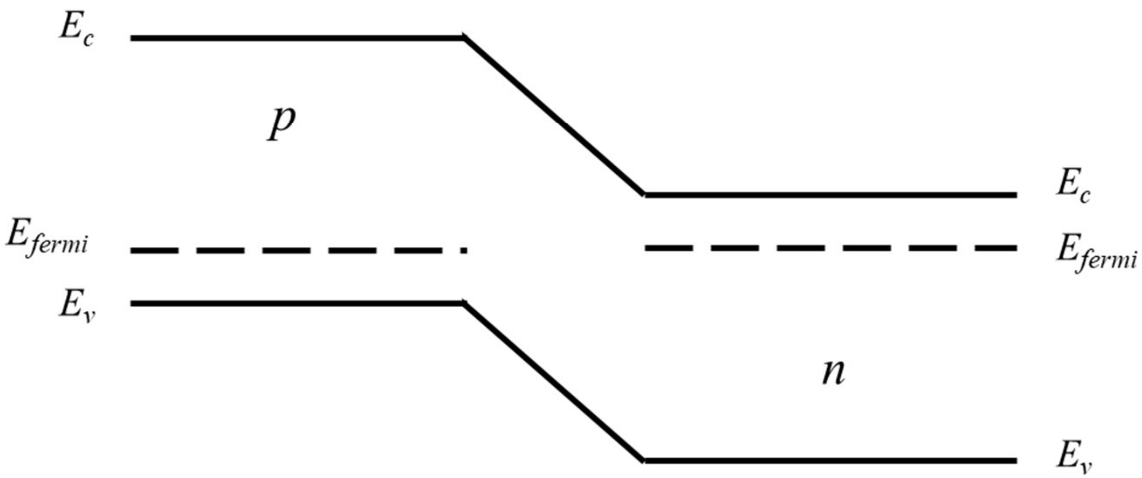




12.3 the p-n junction

- now consider the band structure as the junction is formed
- the p and n materials have the same band structure, so at the moment of contact the bands are in alignment (but the fermi level is different)
- however, as diffusion occurs, the increasing contact potential difference across the junction moves the energy levels up in the p region and down in the n region

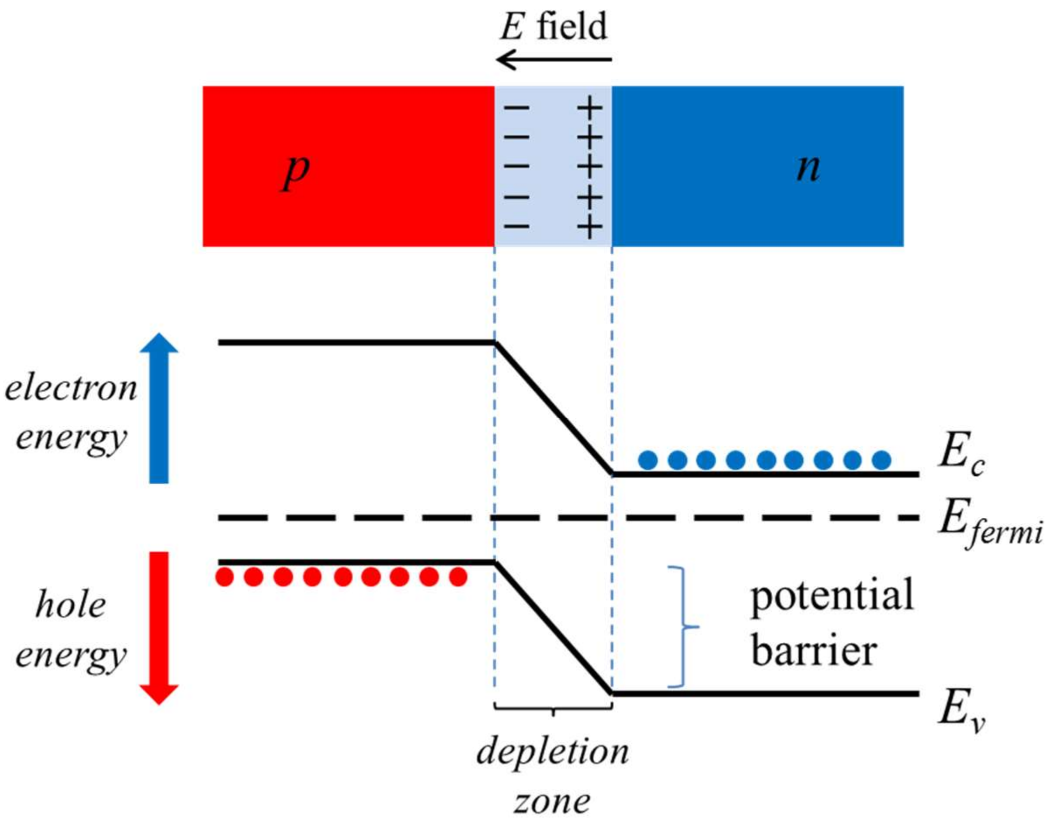




12.3 the p-n junction

unbiased

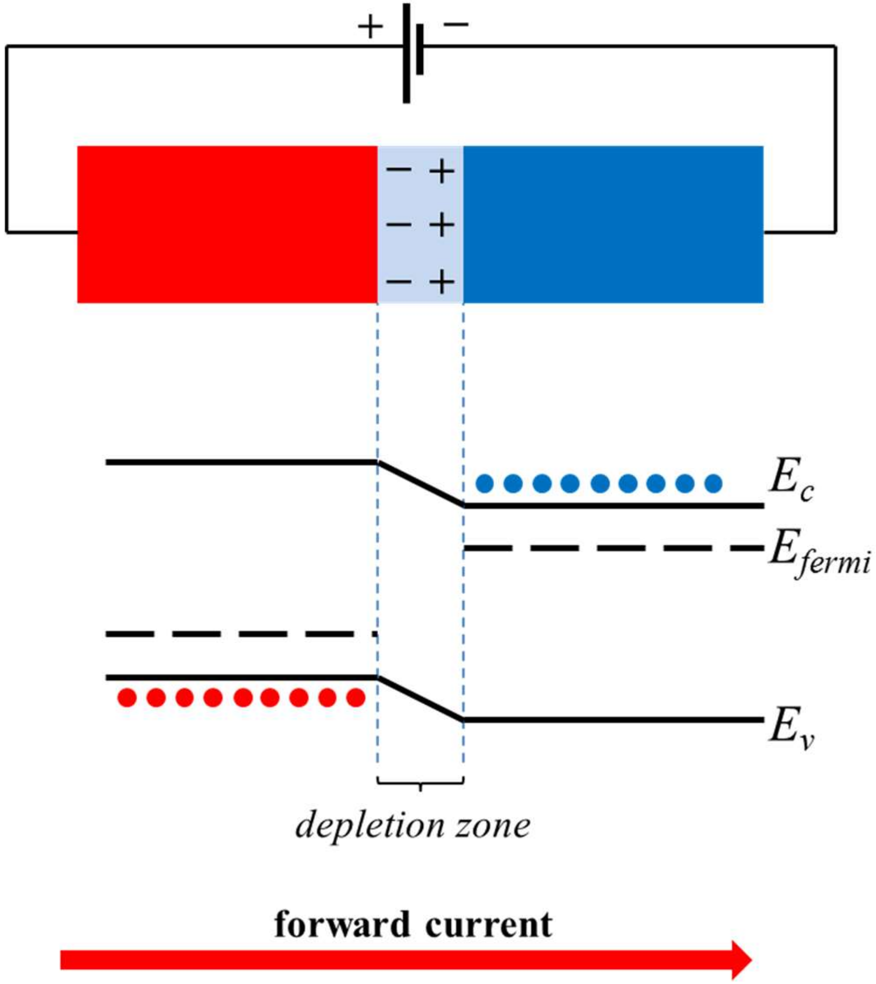
- a p-n junction not connected to a power source is unbiased
- the step in the band edges caused by the contact potential difference is a potential barrier that must be overcome by electrons or holes wanting to flow across the junction
- **note**: for electrons, moving ‘up’ on the band diagram represents an energy input, but for holes (with the opposite charge), moving ‘down’ requires an energy input



12.3 the p-n junction

forward biased

- if a power supply is attached, with the positive connected to the p side and the negative to the n side, the junction is forward biased
- forward bias adds holes the p side and electrons to the n side
- the contact potential difference is reduced which lowers the potential barrier, so carriers can overcome it
- forward bias narrows the depletion zone (reduces its resistance)
- **forward current can flow**



12.3 the p-n junction

reverse biased

- if a power supply is attached, with the positive connected to the n side and the negative to the p side, the junction is reverse biased
- reverse bias adds holes the n side and electrons to the p side
- the contact potential difference is increased which increases the potential barrier, so carriers cannot get over it
- reverse bias widens the depletion zone (increases its resistance)
- **NO forward current flows**