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Score Initially Received: 4/5

If all graphs of H are complete graphs, then at any instance H is a subgraph of another graph, say G, H would also be an induced subgraph of G. In other words, complete graphs are the only graphs for which the conditional "if H is a subgraph of another graph(say G), then H must actually be an induced subgraph" works. Complete graphs are graphs that have all of their vertices adjacent to every other vertex except to itself that are available inside its vertex set. Since H contains every possible edge (due to being a complete graph) in between all its vertices and G contains all of H, then G would'nt have any additional edges that H doesn't already have in between the vertices of the exact copy of H inside G. In other words, there is an injection for all vertices from H inside G to the point that G is incapable of having any other additional edge inside itself(graph G). Any additional edge in G would pertain to another vertice that isn't considered to have all its endpoints in H, allowing an induced subgraph to become possible. So anytime H is a subgraph of G, H also exists as an induced subgraph of G, if all graphs of H are complete graphs. Now, if all graphs of H are not complete graphs, and G is the complete graph of H, in other words G has all the edges possible H could have and H doesn't have all possible edges G could have, then no matter what subgraph H is of G, there will always be at least one additional edge **minimum** in G not in H, making the conditional "if H is a subgraph of another graph (say G), then H must actually be an induced subgraph" not work.