Assignment 2

October 17, 2018

1 Assignment 2 - Josephine Tan

2 Question 1

```
In [121]: #Import packages
    import pandas as pd
```

I chose to open the two datasets with the following labels. I labelled the variables of weight and total income from the SurveyIncome dataset differently from the BestIncome dataset to differentiate between these two variables which are from two datasets.

Out[122]:		labor income	capital income	height	weight
	count	10000.000000	10000.000000	10000.000000	10000.000000
	mean	57052.925133	9985.798563	65.014021	150.006011
	std	8036.544363	2010.123691	1.999692	9.973001
	min	22917.607900	1495.191896	58.176154	114.510700
	25%	51624.339880	8611.756679	63.652971	143.341979
	50%	56968.709935	9969.840117	65.003557	149.947641
	75%	62408.232277	11339.905773	66.356915	156.724586
	max	90059.898537	19882.320069	72.802277	185.408280

Out[123]:		total_income1	weight1	age	iemale
	count	1000.000000	1000.000000	1000.000000	1000.00000
	mean	64871.210860	149.542181	44.839320	0.50000
	std	9542.444214	22.028883	5.939185	0.50025
	min	31816.281649	99.662468	25.741333	0.00000
	25%	58349.862384	130.179235	41.025231	0.00000
	50%	65281.271149	149.758434	44.955981	0.50000
	75%	71749.038000	170.147337	48.817644	1.00000
	max	92556.135462	196.503274	66.534646	1.00000

(a) I will use a linear regression twice to derive the coefficients of the following equations from the SurveyIncome dataset: Equations I will use: age = B0 + total income1B1 + weight1B2 and female = BO + total income1B1 + weight1B2

In [124]: #For the linear regression for age:

```
outcome = 'age'
         features = ['total_income1', 'weight1']
         x, y = SurveyIncome[features], SurveyIncome[outcome]
In [125]: #Testing if the X and Y variables are correct:
         x.head()
Out[125]:
           total_income1
                             weight1
           63642.513655 134.998269
         0
         1 49177.380692 134.392957
           67833.339128 126.482992
            62962.266217 128.038121
             58716.952597 126.211980
In [126]: y.head()
Out[126]: 0
              46.610021
              48.791349
         1
         2
             48.429894
         3
             41.543926
             41.201245
         Name: age, dtype: float64
In [127]: #Running the linear regression for age
         import statsmodels.api as sm
         x = sm.add_constant(x, prepend=False)
         x.head()
         m = sm.OLS(y, x)
         res = m.fit()
         print(res.summary())
                          OLS Regression Results
_____
Dep. Variable:
                                     R-squared:
                                                                    0.001
                               age
Model:
                               OLS
                                     Adj. R-squared:
                                                                   -0.001
Method:
                      Least Squares F-statistic:
                                                                   0.6326
Date:
                   Tue, 16 Oct 2018 Prob (F-statistic):
                                                                    0.531
Time:
                           20:10:45
                                    Log-Likelihood:
                                                                  -3199.4
No. Observations:
                               1000
                                    ATC:
                                                                    6405.
Df Residuals:
                               997
                                     BIC:
                                                                    6419.
Df Model:
                                 2
```

Covariance Type:		nonrobust				
	coef	std err	t	P> t	[0.025	0.975]
total_income1	2.52e-05	2.26e-05	1.114	0.266	-1.92e-05	6.96e-05
weight1	-0.0067	0.010	-0.686	0.493	-0.026	0.013
const	44.2097	1.490	29.666	0.000	41.285	47.134
	.======			=======		======
Omnibus:		2.460	Durbin-Wa	tson:		1.921
Prob(Omnibus):		0.292	Jarque-Be	ra (JB):		2.322
Skew:		-0.109	Prob(JB):			0.313
Kurtosis:		3.092	Cond. No.		5	.20e+05
==========	=======			=======		======

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 5.2e+05. This might indicate that there are strong multicollinearity or other numerical problems.

```
In [128]: #Saving prediction of model for age
         ols_df = pd.concat([y, x], axis=1)
         ols_df.head()
Out[128]:
                  age total_income1
                                        weight1 const
         0 46.610021 63642.513655 134.998269
                                                   1.0
         1 48.791349 49177.380692 134.392957
                                                   1.0
         2 48.429894 67833.339128 126.482992
                                                   1.0
         3 41.543926 62962.266217 128.038121
                                                   1.0
         4 41.201245
                       58716.952597 126.211980
                                                   1.0
In [129]: #Adding model prediction of age to the SurveyIncome dataset
         ols_df['age_pred'] = res.predict(x)
         ols_df.head()
Out[129]:
                  age total_income1
                                        weight1 const
                                                         age_pred
         0 46.610021
                        63642.513655 134.998269
                                                   1.0 44.906121
         1 48.791349 49177.380692 134.392957
                                                   1.0 44.545636
         2 48.429894 67833.339128 126.482992
                                                   1.0 45.068980
         3 41.543926 62962.266217 128.038121
                                                   1.0 44.935764
         4 41.201245 58716.952597 126.211980
                                                   1.0 44.841048
In [130]: #Running linear regression on gender (female) as gender
         outcome = 'female'
         features = ['total_income1', 'weight1']
         X, y = SurveyIncome[features], SurveyIncome[outcome]
         import statsmodels.api as sm
```

```
X = sm.add_constant(X, prepend=False)
X.head()

m = sm.OLS(y, X)

res = m.fit()
print(res.summary())
```

OLS Regression Results

Dep. Variable: Model:		female OLS	R-squared: Adj. R-squared:			0.834 0.834	
Method:		Least Squares		F-statistic:		2513.	
Date:	Tue,	16 Oct 2018	Prob (F-s	tatistic):	:	0.00	
Time:		20:10:50	Log-Likel	ihood:		173.49	
No. Observation	ns:	1000	AIC:			-341.0	
Df Residuals:		997	BIC:			-326.3	
Df Model:		2					
Covariance Typ	e:	nonrobust					
========	=======		=======			========	
	coef	std err	t	P> t	[0.025	0.975]	
total_income1	-5.25e-06	7.76e-07	-6.765	0.000	-6.77e-06	-3.73e-06	
weight1	-0.0195	0.000	-58.098	0.000	-0.020	-0.019	
const	3.7611	0.051	73.600	0.000	3.661	3.861	
Omnibus:	=======	0.170	======= Durbin-Wa	======= tson:		1.634	
Prob(Omnibus):		0.918	Jarque-Be	ra (JB):		0.114	
Skew:		-0.022	-			0.945	
Kurtosis: 3.029			Cond. No.			5.20e+05	
=========	========		========	=======		======	

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 5.2e+05. This might indicate that there are strong multicollinearity or other numerical problems.

Out[131]:		female	total_income1	weight1	const
	0	1.0	63642.513655	134.998269	1.0
	1	1.0	49177.380692	134.392957	1.0
	2	1.0	67833.339128	126.482992	1.0
	3	1.0	62962.266217	128.038121	1.0
	4	1.0	58716.952597	126.211980	1.0

```
In [132]: #Adding model prediction for gender to the SurveyIncome dataset
         ols_df['gender_pred'] = res.predict(x)
         ols_df.head()
Out [132]:
            female total_income1
                                     weight1 const gender_pred
               1.0 63642.513655 134.998269
                                                1.0
                                                        0.790496
               1.0 49177.380692 134.392957
                                                1.0
         1
                                                        0.878254
                                                1.0 0.934802
1.0 0.930001
         2
               1.0 67833.339128 126.482992
               1.0 62962.266217 128.038121
         3
               1.0 58716.952597 126.211980
                                                1.0
                                                        0.987952
(b) Imputing the variables into the BestIncome dataset
In [133]: #For us to estimate age and gender (female) on the BestIncome dataset, and use the f
          \#age = B0 + total incomeB1 + weightB2
         #female = BO + total incomeB1 + weightB2
          #We need to add labor income and capital income to equate to total income in the Bes
         BestIncome['total_income'] = BestIncome['labor income'] + BestIncome['capital income
         BestIncome.head()
Out[133]:
            labor income capital income
                                            height
                                                        weight total_income
                             9279.509829 64.568138 152.920634 61935.115336
         0 52655.605507
         1 70586.979225
                             9451.016902 65.727648 159.534414 80037.996127
         2 53738.008339
                           8078.132315 66.268796 152.502405 61816.140654
         3 55128.180903 12692.670403 62.910559 149.218189 67820.851305
         4 44482.794867 9812.975746 68.678295 152.726358 54295.770612
In [134]: #Adding the equation of predicted age (age_pred) from the SurveyIncome dataset to th
         def age_pred(row):
             total_income = row[0]
             weight = row[1]
             age_pred = 44.2097+(total_income*-2.52e-05)+(weight*-0.0067)
             return age_pred
         age_pred([63642.513655, 134.998269])
Out[134]: 41.701420253594
In [135]: #Adding the imputed age variable (imputed_age) into the BestIncome dataset
         BestIncome['imputed_age'] = BestIncome[['total_income', 'weight']].apply(age_pred, age)
         BestIncome.head()
Out[135]:
            labor income capital income
                                            height
                                                        weight total_income \
         0 52655.605507
                             9279.509829 64.568138 152.920634 61935.115336
         1 70586.979225
                             9451.016902 65.727648 159.534414 80037.996127
         2 53738.008339
                           8078.132315 66.268796 152.502405 61816.140654
```

```
3 55128.180903
                             12692.670403 62.910559 149.218189 67820.851305
          4 44482.794867
                              9812.975746 68.678295 152.726358 54295.770612
             imputed_age
               41.624367
          0
          1
               41.123862
          2
               41.630167
               41.500853
               41.818180
In [136]: #Adding the equation of female (gender_pred) from the SurveyIncome dataset to the Be
          def gender_pred(row):
              total_income = row[0]
              weight = row[1]
              gender_pred = 3.7611+(total_income*-5.25e-06)+(weight*-0.0195)
              if gender_pred < 0.5:</pre>
                  return 0
              elif gender_pred >= 0.5:
                  return 1
          gender_pred([63642.513655, 134.998269])
Out[136]: 1
In [137]: BestIncome['imputed_gender_pred'] = BestIncome[['total_income', 'weight']].apply(generation)
          BestIncome.head()
Out[137]:
             labor income capital income
                                                          weight total_income \
                                              height
          0 52655.605507
                              9279.509829 64.568138 152.920634 61935.115336
                              9451.016902 65.727648 159.534414 80037.996127
          1 70586.979225
          2 53738.008339
                              8078.132315 66.268796 152.502405 61816.140654
          3 55128.180903
                             12692.670403 62.910559 149.218189 67820.851305
          4 44482.794867
                              9812.975746 68.678295 152.726358 54295.770612
             imputed_age imputed_gender_pred
          0
               41.624367
                                            0
                                            0
          1
               41.123862
               41.630167
                                            0
          3
               41.500853
                                            0
               41.818180
(c) Descriptive Statistics of age and gender
In [138]: BestIncome['imputed_age'].describe()
```

10000.000000 41.515284

Out[138]: count

mean

```
std 0.219817
min 40.683056
25% 41.365293
50% 41.515710
75% 41.664056
max 42.373440
Name: imputed_age, dtype: float64
```

The mean is 41.52, standard deviation is 0.22, maximum is 42.37, minimum is 40.68, number of observations is 10,000.

```
In [139]: BestIncome['imputed_gender_pred'].describe()
Out[139]: count
                   10000.000000
                       0.470500
          mean
          std
                       0.499154
                       0.00000
          min
          25%
                       0.00000
          50%
                       0.00000
          75%
                       1.000000
                       1.000000
          max
          Name: imputed_gender_pred, dtype: float64
```

The mean is 0.47, standard deviation is 0.50, maximum is 1.00, minimum is 0.00, number of observations is 10,000.

(d) Correlation Matrix

2001.0 735.002861 51737.324165

3 Question 2

(a) Obtaining the coefficients of the regression of Salary on GRE quantitative scores

```
In [142]: #Linear regression on Salary on GRE quantitative scores
    outcome = 'salary'
    features = ['gre_qnt']

X, y = IncomeIntel[features], IncomeIntel[outcome]

import statsmodels.api as sm

X = sm.add_constant(X, prepend=False)
X.head()

m = sm.OLS(y, X)

res = m.fit()
    print(res.summary())
```

OLS Regression Results

uared: 0.263
R-squared: 0.262
atistic: 356.3
(F-statistic): 3.43e-68
Likelihood: -10673.
2.135e+04
2.136e+04
P> t [0.025 0.975]
0.000 -28.442 -23.085
0.000 8.78e+04 9.13e+04
in-Watson: 1.424
ue-Bera (JB): 9.100
(JB): 0.0106
. No. 1.71e+03
· · · · · · · · · · · · · · · · · · ·

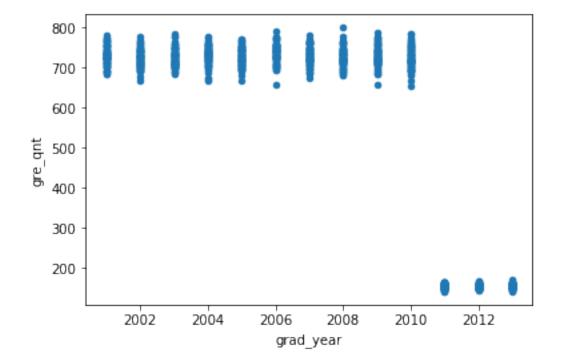
Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.71e+03. This might indicate that there are strong multicollinearity or other numerical problems.

The coefficient for GRE quantitative score is -25.76 and the constant is 8.95e+04, and the standard error is 1.37.

(b) Scatter plot of graduation year and GRE quantitative score

```
In [143]: #Running a simple scatter plot of the graduation year and GRE quantitative scores
    import matplotlib.pyplot as plt
    grad_year = IncomeIntel['grad_year']
    gre_qnt = IncomeIntel['gre_qnt']
    IncomeIntel.plot(x='grad_year', y='gre_qnt', kind='scatter')
    plt.show()
```



As seen in the scatter plot, the three variables in 2011, 2012 and 2013 have a very low GRE quantitative score of 200, while the rest of the GRE scores are around 800. This drop in GRE scores are because the GRE tests have changed in testing format since 2011. As such, we cannot compare GRE scores for 2011 and after 2011 based on absolute numbers alone. One solution is to change all the GRE scores to Z-scores (as these scores have not changed over the years), for us to compare between all the years.

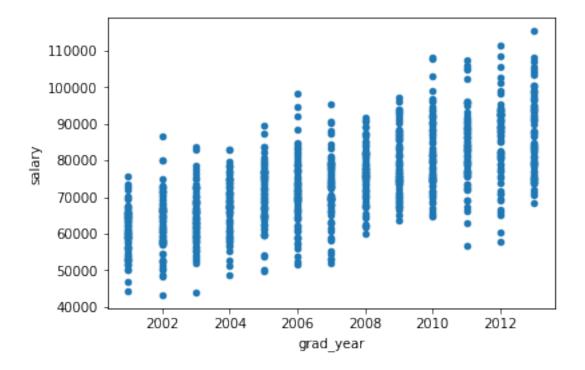
```
In [144]: #Implementing solution proposed above
          df = IncomeIntel.copy()
          gre2 = df.groupby('grad_year').transform(lambda x : (x - x.mean()) / x.std())
          gre2.head()
          IncomeIntel['zscore'] = gre2['gre_qnt']
          IncomeIntel.head
Out[144]: <bound method NDFrame.head of
                                             grad_year
                                                                           salary
                                                           gre_qnt
                                                                                      zscore
                  2001.0 739.737072
                                       67400.475185 0.406740
          1
                  2001.0 721.811673
                                       67600.584142 -0.356635
```

```
736.277908
2
        2001.0
                              58704.880589
                                             0.259427
3
        2001.0
                770.498485
                              64707.290345
                                             1.716750
4
                              51737.324165
        2001.0
                735.002861
                                             0.205128
5
        2001.0
                763.876037
                              64010.822579
                                             1.434726
6
        2001.0
                738.758659
                              60080.107481
                                            0.365073
7
        2001.0
                706.407471
                              56263.309815 -1.012641
8
        2001.0
                705.886037
                              62109.859243 -1.034847
9
        2001.0
                700.971986
                              50189.704747 -1.244117
10
        2001.0
                709.754522
                              58721.753127 -0.870103
11
        2001.0
                734.854582
                              65380.594586 0.198813
12
        2001.0
                753.384151
                              52857.212365 0.987916
13
        2001.0
                690.312090
                              63572.217765 -1.698081
14
        2001.0
                774.154371
                              65892.177035 1.872441
15
        2001.0
                726.377225
                              67454.545201 -0.162205
16
        2001.0
                702.735945
                              59346.670232 -1.168997
17
        2001.0
                723.806542
                              70031.012603 -0.271681
18
        2001.0
                758.051159
                              53441.672888 1.186666
19
        2001.0
                711.063082
                              61008.652046 -0.814376
20
        2001.0
                702.975969
                              50065.932451 -1.158775
21
        2001.0
                733.877837
                              75612.225369 0.157217
22
        2001.0
                735.918767
                              59580.620375
                                            0.244133
23
        2001.0
                749.069115
                              57825.611782
                                            0.804156
24
        2001.0
                732.581793
                              52809.225854 0.102024
25
        2001.0
                728.050446
                              57492.084316 -0.090949
26
        2001.0
                690.265988
                              64686.224351 -1.700045
27
        2001.0
                732.448836
                              53067.021394 0.096361
28
                724.755887
                              58902.707320 -0.231252
        2001.0
29
        2001.0
                721.739038
                              62094.061567 -0.359728
. .
            . . .
                        . . .
                                        . . .
970
        2013.0
                158.578197
                              79263.470892 0.576381
971
                147.667305
        2013.0
                             104782.627567 -1.440563
972
        2013.0
                160.086274
                              94013.946074 0.855158
973
        2013.0
                156.289493
                              74032.543183 0.153300
        2013.0
                150.340044
                              84220.290724 -0.946491
974
975
        2013.0
                163.054596
                              74940.546965
                                            1.403870
976
        2013.0
                 157.624151
                              83293.343135 0.400020
977
        2013.0
                 150.927266
                              78340.908128 -0.837940
978
        2013.0
                157.393763
                              91066.889575 0.357431
        2013.0
                              87169.012509 -0.186810
979
                154.449630
980
        2013.0
                153.756644
                              90033.601423 -0.314912
        2013.0
                              98650.768576 -0.862137
981
                150.796371
982
        2013.0
                150.691700
                              70455.885421 -0.881486
        2013.0
                              91133.301177 -0.336494
983
                153.639896
984
        2013.0
                 150.374470
                              91796.617819 -0.940128
985
        2013.0
                162.350725
                              73780.832249
                                             1.273755
986
        2013.0
                155.803279
                              96927.925237
                                             0.063421
987
        2013.0
                159.111662
                              71875.246552
                                             0.674995
988
        2013.0
                158.338350
                             103357.966587
                                             0.532044
```

```
989
        2013.0
                162.308518
                             73780.472319
                                            1.265953
990
        2013.0
               156.651125
                             79055.571295
                                           0.220150
991
        2013.0
               153.836045
                             91529.313046 -0.300235
992
        2013.0
               149.542467
                             75940.200168 -1.093928
        2013.0 155.349020
993
                             97688.397380 -0.020552
994
        2013.0
               161.767399
                             75260.194609
                                            1.165924
995
        2013.0
               160.441025
                            100430.166532
                                           0.920736
996
        2013.0 160.431891
                             82198.200872
                                           0.919047
997
        2013.0 154.254526
                             84340.214218 -0.222876
998
               162.036321
        2013.0
                             87600.881985
                                           1.215636
999
        2013.0 156.946735
                             82854.576903 0.274795
```

[1000 rows x 4 columns]>

(c) Scatter plot of income and graduation year



As the data spans over time, and time increases constantly, we need to stationarize the data and control for time for us to complete the regression. Because these data are not panel data, I cannot use differencing or log differencing methods to detrend them. Instead, I plan to use growth rate to stationarize the data.

```
In [146]: #Implementing solution proposed above
                   #To calculate the average growth rate in salary,
                  avg_inc_by_year = IncomeIntel['salary'].groupby(IncomeIntel['grad_year']).mean().val
In [147]: #To calculate the average growth rate in salaries across all 13 years,
                  avg_growth_rate = ((avg_inc_by_year[1:] - avg_inc_by_year[:-1]) / avg_inc_by_year[:-
In [148]: #Divide each salary by the growth rate
                  IncomeIntel['adj_salary'] = IncomeIntel['salary'] / (1 + avg_growth_rate)**(IncomeIntel['salary'] / (1 + avg_growth_rate)
(d) Reestimating the coefficients with updated variables
In [149]: #Linear regression on revised Salary on revised GRE quantitative scores (Z scores)
                  outcome = 'adj_salary'
                  features = ['zscore']
                  X, y = IncomeIntel[features], IncomeIntel[outcome]
                  import statsmodels.api as sm
                  X = sm.add_constant(X, prepend=False)
                  X.head()
                  m = sm.OLS(y, X)
                  res = m.fit()
                  print(res.summary())
                                                  OLS Regression Results
______
Dep. Variable:
                                                 adj_salary R-squared:
                                                                                                                                       0.000
Model:
                                                               OLS Adj. R-squared:
                                                                                                                                   -0.001
                                         Least Squares F-statistic:
Method:
                                                                                                                                  0.4395
                                                                                                                     0.508
-10291.
                          Tue, 16 Oct 2018 Prob (F-statistic): 20:12:11 Log-Likelihood:
Date:
Time:
No. Observations:
                                                             1000 AIC:
                                                                                                                             2.059e+04
Df Residuals:
                                                               998
                                                                        BIC:
                                                                                                                                2.060e+04
Df Model:
                                                                   1
Covariance Type:
                                                 nonrobust
______
                              coef std err t P>|t| [0.025
_____
                                                                  -0.663 0.508 -596.440
zscore
                     -150.6097 227.193
                  6.142e+04 225.711 272.117 0.000 6.1e+04 6.19e+04
______
Omnibus:
                                                           0.776 Durbin-Watson:
                                                                                                                                        2.025
                                                           0.678 Jarque-Bera (JB):
Prob(Omnibus):
                                                                                                                                      0.687
Skew:
                                                           0.059 Prob(JB):
                                                                                                                                       0.709
```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The estimated coefficients for Z score (for GRE quantitative score) is -150.61 and the constant is now 6.142e+04. The new estimated coefficients for zscore on revised salary is much higher than before and is no longer significant (P>0.05) as compared to the regression ran between the unrevised GRE scores and unrevised salary data. This means that after accounting for the test format changes in the GRE, and time drift in the salary data, there is no longer a significant relationship between these two variables. Thus the hypothesis that that higher intelligence, as operationalized as quantitative GRE scores, is associated with higher salary, is not supported.