Ficha 8 - Intervalos de confiança

$n=20 \quad \sigma^2=225 \quad e \ \bar{x}=64.3$

a) $1-\alpha=0.95$

$\Leftrightarrow \alpha=0.05$

$$P\left(-z_{1-\frac{\alpha}{2}}<\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}<z_{1-\frac{\alpha}{2}}\right)=0.95$$

$$\Leftrightarrow P\left(\bar{x}-z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}<\mu<\bar{x}+z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)=0.95$$

$\Rightarrow$ IC:

$$\bar{x} \pm 1.96\frac{15}{\sqrt{20}}$$

$\Leftrightarrow \bar{x} \pm 6.574$

$\Leftrightarrow 64.3 \pm 6.574$

b) $1-\alpha=0.9$

$\Leftrightarrow \alpha=0{,}10$

$$IC: \bar{x} \pm z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

$$\Leftrightarrow \bar{x} \pm z_{0.95}\frac{15}{\sqrt{20}}=64.3 \pm 1{,}65\frac{15}{\sqrt{20}}$$

$$=64{,}3 \pm 5.53$$

2. $n = 12$

σ desconhecido

$$\bar{X} = \frac{2.3 + 1.9 + 2.1 + 2.8 + 2.3 + 3.6 + 1.4 + 1.8 + 2.1 + 3.2 + 2.0 + 1.9}{12}$$

$\Leftrightarrow \bar{X} \simeq 2{,}28$

$$S^2 = \frac{1}{12-1}\left((2.3-2.28)^2 + \dots + (1.9-2.28)^2\right)$$

$\Leftrightarrow S^2 \simeq 0.39 \Rightarrow S \simeq 0{,}62$

a) $1-\alpha = 0{,}99$

$\Leftrightarrow \alpha = 0.01$

$IC: \bar{X} \pm t_{0.005,11} \frac{0{,}62}{\sqrt{12}}$

$= 2.28 \pm 3.106 \frac{0{,}62}{\sqrt{12}}$

$= 2.28 \pm 0{,}56$

b) (i) $1-\alpha = 0{,}9 \Leftrightarrow \alpha = 0.1$

$IC_1: \bar{X} \pm t_{0.05,11} \frac{0{,}62}{\sqrt{12}}$

$= 2.28 \pm 0.32$

(ii) $1-\alpha = 0.95 \Leftrightarrow \alpha = 0.05$

$IC_2: \bar{X} \pm t_{0.025,11} \frac{0{,}62}{\sqrt{12}}$

$= 2.28 \pm 0.39$

3.-

$n = 100$

$\bar{x} = 177500$ e $S = 9000$

$1 - \alpha = 0.95$

$\Rightarrow \alpha = 0.05$

$IC: 177500 \pm Z_{0.975} \frac{900\cancel{0}}{1\cancel{0}}$

$= 177500 \pm 1764$

4.-

$\bar{x} = 45$ e $S = 5.8$

$1 - \alpha = 0.95 \Leftrightarrow \alpha = 0.05$

a) $n = 30$

$IC: 45 \pm Z_{0.975} \frac{5{,}8}{\sqrt{30}}$

$= 45 \pm 2.08$

b) $n = 60$

$IC: 45 \pm Z_{0.975} \frac{5.8}{\sqrt{60}}$

$= 45 \pm 1.47$

c) $n = 90$

$IC: 45 \pm Z_{0.975} \frac{5.8}{\sqrt{90}}$

$= 45 \pm 1{,}20$

à medida que n aumenta, o intervalo de confiança diminui, ou seja há mais precisão sobre o valor da média. μ

5.- Sabe-se σ_1 e σ_2 e que ambas X_1 e X_2 são normais

$$\sigma_1^2 = \sigma_2^2 = \sigma^2$$

Logo, $1-\alpha = 0.9$

$\Leftrightarrow \alpha = 0.1$.

$$P\left(\bar{X}_1 - \bar{X}_2 - Z_{0.95}\sqrt{\frac{2\sigma^2}{m}} < \mu_1 - \mu_2 < \bar{X}_1 - \bar{X}_2 + Z_{0.95}\sqrt{\frac{2\sigma^2}{m}}\right) = 0.9$$

$$\Rightarrow Z_{0.95}\sqrt{\frac{2\sigma^2}{m}} = \frac{\sigma}{5}$$

$$\Leftrightarrow 1,65\cancel{\sigma}\sqrt{\frac{2}{m}} = \frac{\cancel{\sigma}}{5}$$

$$\Leftrightarrow \sqrt{\frac{2}{m}} = \frac{1}{8.25} \Leftrightarrow \frac{2}{m} = \frac{1}{68,06}$$

$$\Leftrightarrow m \approx 136$$

6.- tratam-se de variáveis aleatórias dependentes

Logo	Antes	Depois	Dif
A	2750	2550	100
B	2360	2380	20
C	2950	2930	-20
D	2830	28 60	30
E	2250	2320	70

$\overline{Dif} = 40$

$S_{Dif} \approx 46,37$

IC:

$$\left]40 - t_{4;0.05}\frac{46.37}{\sqrt{5}} ; 40 + t_{4;0.05}\ldots\right[$$

$$= \left]-4.21 ; 84,21\right[$$

4

a) $1-\alpha = 0.99$

$\alpha = 0.01$

$$\bar{A} = \frac{8500 + \dots + 8030}{5} = 8260$$

$$\bar{B} = \frac{7710 + \dots + 7860}{5} = 7930$$

$$\Rightarrow \bar{A} - \bar{B} = 330$$

$$S_A^2 = \frac{1}{5-1}\left((8500-8260)^2 + \dots + (8030-8260)^2\right)$$

$$\Leftrightarrow S_A^2 = 63450$$

$$S_B^2 = \frac{1}{4}\left((7710-7930)^2 + \dots + (7860-7930)^2\right)$$

$$\Leftrightarrow S_B^2 = 42650$$

$$S_p^2 = \frac{(n_1-1)S_A^2 + (n_2-1)S_A^2}{n_1+n_2-2}$$

$$\Rightarrow S_p^2 = \frac{4 \times 63450 + 4 \times 42650}{10-2}$$

$$\Leftrightarrow S_p^2 = \frac{63450 + 42650}{2} = 53050$$

G.L.: 8

$$\Rightarrow S_p = \sqrt{53050} \approx 230,33$$

IC: $330 \pm t_{8,\,0,005}\, S_p \sqrt{\frac{2}{5}}$

$= 330 \pm 488.73$

b) $1-\alpha=0.9$

$\Leftrightarrow \alpha=0.1$

$t_{8,0.05}$

$\Rightarrow$ IC:

330 ± 270.95

Observa-se que à medida que ao diminuir a certeza, o intervalo de confiança diminui

8.-

a) $1-\alpha=0.95$ $\quad Z_{1-\frac{\alpha}{2}}=Z_{0.9750}=1.96$

$\Leftrightarrow \alpha=0.05$

IC:

$$(91.1-92.3) \pm 1.96\sqrt{\frac{5.4^2}{50}+\frac{7.6^2}{50}}$$

$$=-1,2 \pm 2.58$$

b) 2.58

9.- $n_A = 250$

Corresponde a um exercício de proporção binomial.

a) $\pi = \frac{x}{n} = \frac{70}{250} = 0.28$

b) $1-\alpha = 0{,}954$ limite de erro cometido $\rightarrow f$

$\Leftrightarrow \alpha = 0{,}046$

$$f = Z_{1-\frac{\alpha}{2}} \sqrt{\frac{0.28(1-0.28)}{250}}$$

$$f \approx 0.0568$$

10. $p = \frac{41}{500} = 0.082$ $\quad n = 500$

$1-\alpha = 0.95 \Leftrightarrow \alpha = 0{,}05$

IC:

$$p \pm Z_{0.975} \sqrt{\frac{p(1-p)}{500}}$$

$\approx 0.082 \pm 0.025$

11. $p = \frac{30}{150} = 0.2$ $\quad \Rightarrow n = 150$

IC: 0.2 ± 0.064

12.- a) $p = \frac{x}{n} = \frac{35}{60} \approx 0.58$

b) IC

$$0.58 \pm 1.96 \sqrt{\frac{0.58(1-0.58)}{60}}$$

$= 0.58 \pm \underbrace{0.125}_{}$ $\rightarrow$ erro da estimativa.

13:- $p = \frac{140}{400} = 0.35$ $n = 400$

a) $1-\alpha = 0.9$

$\Leftrightarrow \alpha = 0.1$

IC

$$p \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$$

$$= 0.35 \pm 1.65 \sqrt{\frac{0.35(1-0.35)}{400}}$$

$\approx 0.35 \pm 0.039$ ✓

b) $1-\alpha = 0.95$ (i)

$\Leftrightarrow \alpha = 0.05$

IC

$$0.35 \pm 1.96 \sqrt{\frac{0.35(1-0.35)}{400}}$$

$\approx 0.35 \pm 0.047$ ✓

(ii)

$1-\alpha = 0.98$

$\Leftrightarrow \alpha = 0.02$

IC $0.35 \pm Z_{0.99} \sqrt{\frac{0.35(1-0.35)}{400}}$

$\approx 0.35 \pm 2.33 \sqrt{\frac{0.35(1-0.35)}{400}}$

$\approx 0.35 \pm 0.056$ ✓

14. $x_A = 132$ $n_A = 400$ $p_A = 0.33$

$x_B = 90$ $n_B = 150$ $p_B = 0.6$

$1-\alpha = 0.99$
$\Leftrightarrow \alpha = 0.01$

IC:

$$(p_A - p_B) \pm z_{0.995}\sqrt{\frac{0.33(1-0.33)}{400} + \frac{0.6(1-0.6)}{150}}$$

$$= -0{,}27 \pm 2.58\sqrt{\frac{0.33(1-0.33)}{400} + \frac{0.6(1-0.6)}{150}}$$

$$\simeq -0.27 \pm 0.120$$

15. $n_1 = n_2 = 1000$

$x_1 = 825$ $x_2 = 760$

$p_1 = 0{,}825$ $p_2 = 0{,}76$

IC Assumido $\alpha = .05$

$$0.065 \pm 1{,}96\sqrt{\frac{0.825(1-0.825)}{1000} + \frac{0.76(1-0.76)}{1000}}$$

$$\simeq 0.065 \pm 0.0354$$

16. $n = 10$

$\overline{x} = 7$ seg. $s = 4$ seg.

$$\sigma \in \left]\sqrt{\frac{9 \times 4^2}{\chi^2_{0{,}05,9}}} ; \sqrt{\frac{9 \times 4^2}{\chi^2_{0.95,9}}}\right[$$

$$\Rightarrow \sigma \in \left]2.92 ; 6.58\right[\longleftarrow \text{IC}$$

17. $n = 10$ $\bar{x} = 32.98\ cl$

$s = 0.04\ cl$

$1 - \alpha = 0.9$

$\Leftrightarrow \alpha = 0.1$

IC:
$$\left] \frac{0.0144}{\chi^2_{0.05,9}} ; \frac{0.0144}{\chi^2_{0.95,9}} \right[$$

$$= \left] 8.57 \times 10^{-4} ; 4.33 \times 10^{-3} \right[$$

18.- $n_1 = 21$ $s_1^2 = 1432$ $1 - \alpha = 0.95$

$n_2 = 25$ $s_2^2 = 3761$ $\Leftrightarrow \alpha = 0.05$

$V_1 = 20$ $V_2 = 24$ $\frac{1}{F_{0.975,20,24}} = F_{0.025,24,20}$

IC:
$$\left] \frac{1432}{3761} \frac{1}{F_{0.025,20,24}} ; \frac{1432}{3761} \frac{1}{F_{0.975,20,24}} \right[$$

$$\approx \left] 0.163 ; 0.914 \right[$$

— FIM —