Problemas Modelos de Computación

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1. Ejercicio 2

Para el programa de suma desarrollado en clase, evalúe y dé por escrito las computaciones completas correspondientes a las siguientes descripciones instantáneas iniciales. Flexibilice el modelo semántico tanto como sea necesario para poder tratar con las macros involucradas.

```
     Y <- X1
      Z <- X2
(B)  IF Z != 0 GOTO A
      GOTO S
(A)  Z--
      Y++
      GOTO B
```

- (1,<x1 = 2, x2 = 4, z = 0, y = 0>)~(1,<x1 = 2, x2 = 4, z = 0, y = 2>)~
  (2,<x1 = 2, x2 = 4, z = 4, y = 2>)~(3,<x1 = 2, x2 = 4, z = 4, y = 2>)~
  (5,<x1 = 2, x2 = 4, z = 3, y = 2>)~(6,<x1 = 2, x2 = 4, z = 3, y = 3>) ~
  (7,<x1 = 2, x2 = 4, z = 3, y = 3>)~(3,<x1 = 2, x2 = 4, z = 3, y = 3>) ~
  (5,<x1 = 2, x2 = 4, z = 2, y = 3>)~(6,<x1 = 2, x2 = 4, z = 2, y = 4>)~
  (7,<x1 = 2, x2 = 4, z = 2, y = 4>)~(3,<x1 = 2, x2 = 4, z = 2, y = 4>)~
  (5,<x1 = 2, x2 = 4, z = 1, y = 4>)~(6,<x1 = 2, x2 = 4, z = 1, y = 5>)~
  (7,<x1 = 2, x2 = 4, z = 1, y = 5>)~(3,<x1 = 2, x2 = 4, z = 1, y = 5>)~
  (5,<x1 = 2, x2 = 4, z = 0, y = 5>)~(6,<x1 = 2, x2 = 4, z = 0, y = 6>)~
  (7,<x1 = 2, x2 = 4, z = 0, y = 6>)~(3,<x1 = 2, x2 = 4, z = 0, y = 6>)~
  (4,<x1 = 2, x2 = 4, z = 0, y = 6>)~(8,<x1 = 2, x2 = 4, z = 0, y = 6>)

- (1,<x1 = 0, x2 = 3, z = 0, y = 0>)~(1,<x1 = 0, x2 = 3, z = 0, y = 0>)~
  (2,<x1 = 0, x2 = 3, z = 3, y = 0>)~(3,<x1 = 0, x2 = 3, z = 3, y = 0>)~
  (5,<x1 = 0, x2 = 3, z = 2, y = 0>)~(6,<x1 = 0, x2 = 3, z = 2, y = 1>)~
  (7,<x1 = 0, x2 = 3, z = 2, y = 1>)~(3,<x1 = 0, x2 = 3, z = 2, y = 1>)~
  (5,<x1 = 0, x2 = 3, z = 1, y = 1>)~(6,<x1 = 0, x2 = 3, z = 1, y = 2>)~
  (7,<x1 = 0, x2 = 3, z = 1, y = 2>)~(3,<x1 = 0, x2 = 3, z = 1, y = 2>)~
  (5,<x1 = 0, x2 = 3, z = 0, y = 2>)~(6,<x1 = 0, x2 = 3, z = 0, y = 3>)~
  (7,<x1 = 0, x2 = 3, z = 0, y = 3>)~(3,<x1 = 0, x2 = 3, z = 0, y = 3>)~
  (4,<x1 = 0, x2 = 3, z = 0, y = 3>)~(8,<x1 = 0, x2 = 3, z = 0, y = 3>)~

- (1, <x1 = 6, x2 = 0, z = 0, y = 0>)~(1, <x1 = 6, x2 = 0, z = 0, y = 6>)~
  (2, <x1 = 6, x2 = 0, z = 0, y = 6>)~(3, <x1 = 6, x2 = 0, z = 0, y = 6>)~
  (4, <x1 = 6, x2 = 0, z = 0, y = 6>)~(8, <x1 = 6, x2 = 0, z = 0, y = 6>)

- (1, <x1 = 0, x2 = 0, z = 0, y = 0>)~(1, <x1 = 0, x2 = 0, z = 0, y = 0>)~
  (2, <x1 = 0, x2 = 0, z = 0, y = 0>)~(3, <x1 = 0, x2 = 0, z = 0, y = 0>)~
  (4, <x1 = 0, x2 = 0, z = 0, y = 0>)~(8, <x1 = 0, x2 = 0, z = 0, y = 0>)

2. Ejercicio 3

Para el programa de resta"desarrollado en clase, evalúe y dé por escrito las computaciones completas correspondientes a las siguientes descripciones instantáneas iniciales. Flexibilice el modelo semántico tanto como sea necesario para poder tratar con las macros involucradas.

```
        Y <- X1
        Z <- X2
(C)  IF Z !=0 GOTO A
      GOTO S
(A)  IF Y != 0 GOTO B
      GOTO A
(B)  Y—
        Z—
      GOTO C
```

- (1, <x1 = 4, x2 = 2, z = 0, y = 0>)~(1, <x1 = 4, x2 = 2, z = 0, y = 4>)~
  (2, <x1 = 4, x2 = 2, z = 2, y = 4>)~(3, <x1 = 4, x2 = 2, z = 2, y = 4>)~
  (5, <x1 = 4, x2 = 2, z = 2, y = 4>)~(7, <x1 = 4, x2 = 2, z = 2, y = 3>)~
  (8, <x1 = 4, x2 = 2, z = 1, y = 3>)~(9, <x1 = 4, x2 = 2, z = 1, y = 3>)~
  (3, <x1 = 4, x2 = 2, z = 1, y = 3>)~(5, <x1 = 4, x2 = 2, z = 1, y = 3>)~
  (7, <x1 = 4, x2 = 2, z = 1, y = 2>)~(8, <x1 = 4, x2 = 2, z = 0, y = 2>)~
  (9, <x1 = 4, x2 = 2, z = 0, y = 2>)~(3, <x1 = 4, x2 = 2, z = 0, y = 2>)~
  (4, <x1 = 4, x2 = 2, z = 0, y = 2>)~(10, <x1 = 4, x2 = 2, z = 0, y = 2>)

- (1, <x1 = 2, x2 = 2, z = 0, y = 2>)~(2, <x1 = 2, x2 = 2, z = 0, y = 2>)~
  (3, <x1 = 2, x2 = 2, z = 2, y = 2>)~(5, <x1 = 2, x2 = 2, z = 2, y = 2>)~
  (7, <x1 = 2, x2 = 2, z = 2, y = 2>)~(7, <x1 = 2, x2 = 2, z = 2, y = 1>)~
  (8, <x1 = 2, x2 = 2, z = 1, y = 1>)~(9, <x1 = 2, x2 = 2, z = 1, y = 1>)~
  (3, <x1 = 2, x2 = 2, z = 1, y = 1>)~(5, <x1 = 2, x2 = 2, z = 1, y = 1>)~
  (7, <x1 = 2, x2 = 2, z = 1, y = 0->)~(8, <x1 = 2, x2 = 2, z = 0, y = 0>)~
  (9, <x1 = 2, x2 = 2, z = 0, y = 0>)~(3, <x1 = 2, x2 = 2, z = 0, y = 0>)~
  (4, <x1 = 2, x2 = 2, z = 0, y = 0>)~(10, <x1 = 2, x2 = 2, z = 0, y = 0>)

- (1, <x1 = 1, x2 = 3, z = 0, y = 1>)~(2, <x1 = 1, x2 = 3, z = 3, y = 1>)~
  (3, <x1 = 1, x2 = 3, z = 3, y = 1>)~(5, <x1 = 1, x2 = 3, z = 3, y = 1>)~
  (7, <x1 = 1, x2 = 3, z = 3, y = 0>)~(8, <x1 = 1, x2 = 3, z = 2, y = 0>)~
  (9, <x1 = 1, x2 = 3, z = 2, y = 0>)~(3, <x1 = 1, x2 = 3, z = 3, y = 1>)~
  (5, <x1 = 1, x2 = 3, z = 3, y = 1>)~(6, <x1 = 1, x2 = 3, z = 3, y = 1>)~
  (5, <x1 = 1, x2 = 3, z = 3, y = 1>)~(6, <x1 = 1, x2 = 3, z = 3, y = 1>)~
  (5, <x1 = 1, x2 = 3, z = 3, y = 1>)~(6, <x1 = 1, x2 = 3, z = 3, y = 1>)~
  .
  .
  .

- (1, <x1 = 0, x2 = 0, z = 0, y = 0>)~(2, <x1 = 0, x2 = 0, z = 0, y = 0>)~
  (3, <x1 = 0, x2 = 0, z = 0, y = 0>)~(4, <x1 = 0, x2 = 0, z = 0, y = 0>)~
  (10, <x1 = 0, x2 = 0, z = 0, y = 0>)

3. Ejercicio 4

Para el programa de multiplicación desarrollado en clase, evalúe y dé por escrito las computaciones completas correspondientes a las siguientes descripciones instantáneas iniciales. Flexibilice el modelo semántico tanto como sea necesario para poder tratar con las macros involucradas.

```
Z2 <- X2
(B1) IF Z2 !=0 GOTO A1
       GOTO S1
(A1) Z2—
       Z1 <- X1+Y
       Y <- Z1
       GOTO B1
```

- (1,<x1 =2,x2 =4,z1 =0,z2 =4,y =0>)˜(2,<x1 =2,x2 =4,z1 =0,z2 =4,y =0>)˜
  (4,<x1 =2,x2 =4,z1 =0,z2 =3,y =0>)˜(5,<x1 =2,x2 =4,z1 =2,z2 =3,y =0>)˜
  (6,<x1= 2,x2= 4,z1= 2,z2= 3,y= 2>)˜(7,<x1= 2,x2= 4,z1= 2,z2= 3,y= 2>)˜
  (2,<x1= 2,x2= 4,z1= 2,z2= 3,y= 2>)˜(4,<x1= 2,x2= 4,z1= 2,z2= 2,y= 2>)˜
  (5,<x1= 2,x2= 4,z1= 4,z2= 2,y= 2>)˜(6,<x1=2,x2= 4,z1= 4,z2= 2,y = 4>)˜
  (7,<x1= 2,x2= 4,z1= 4,z2= 2,y= 4>)˜(2,<x1= 2,x2= 4,z1= 2,z2= 2,y = 4>)˜
  (4,<x1= 2,x2= 4,z1= 2,z2= 1,y= 4>)˜(5,<x1= 2,x2 = 4,z1= 6,z2= 1,y = 4>)˜
  (6,<x1= 2,x2= 4,z1= 6,z2= 3,y= 6>)˜(7,<x1= 2,x2= 4,z1= 6,z2= 3, y = 6>)˜
  (2,<x1= 2,x2= 4,z1= 6,z2= 3,y= 6>)˜(4,<x1= 2,x2= 4,z1= 6,z2= 0, y = 6>)˜
  (5,<x1= 2,x2= 4,z1= 8,z2= 0,y= 6>)˜(6,<x1= 2,x2= 4,z1= 8,z2= 0,y = 8>)˜
  (7,<x1= 2,x2= 4,z1= 8,z2= 0,y= 8>)˜(2,<x1= 2,x2= 4,z1= 8,z2= 0, y = 8>)˜
  (3,<x1= 2,x2= 4,z1= 8,z2= 0,y= 8>)˜(8,<x1= 2,x2= 4,z1= 8,z2= 0, y = 8>)˜

- (1 <x = 1 x = 1 z1= 0,z2= 1 ,y= 0>)˜(2,<x1= 1,x1= 1,z1= 0,z2= 1,y = 0>)˜
  (4,<x1= 1,x2= 1,z1= 0,z2= 0,y= 0>)˜(5,<x1= 1,x2= 1,z1= 1,z2= 0, y = 1>)˜
  (6,<x1= 1,x2= 1,z1= 1,z2= 0,y= 1>)˜(7,<x1= 1,x2= 1,z1= 1,z2= 0, y = 1>)˜
  (2,<x1= 1,x2= 1,z1= 1,z2= 0,y= 1>)˜(3,<x1= 1,x2 =1,z1= 1,z2= 0, y = 1>)˜
  (8,<x1= 1,x2= 1,z1= 1,z2= 0,y= 1>)

- (1,<x1= 6,x2= 0,z1= 0,z2= 0 ,y= 0>)˜(2,<x1= 6,x2= 0,z1= 0,z2= 0,y = 0>)˜
  (3,<x1= 6,x2= 0,z1= 0,z2= 0,y= 0>)˜(8,<x1= 6,x2= 0,z1= 0,z2= 0 ,y = 0>)

- (1,<x1= 0,x2= 0,z1= 0,z2= 0,y= 0>)˜(2,<x1= 0,x2= 0,z1= 0,z2= 0 ,y = 0>)˜
  (3,<x1= 0,x2= 0,z1= 0,z2= 0,y= 0>)˜(8,<x1= 0,x2= 0,z1= 0,z2= 0 ,y = 0>)

4. Ejercicio 7

Pruebe que para cada función parcialmente computable f(x1, x2,..., xn) existe un número $m \geqslant 0$ tal que f es computada por infinitos programas de longitud mayor que m.

Dado que una funcion parcialmente computable tiene al menos un programa que la resuelve, de una longuitud dada, a ese programa se le pueden añadir tantas instrucciones dummies como deseemos, consiguiendo así un numero de programas infinitos de una longuitud $\geqslant 0$.

5. Ejercicio 13

Para cada número k ¿0, sea fk la función constante fk (x) = k. Muestre que fk es computable para todo k.

Una función es totalmente computable si existe un L-programa que la resuelva en todos los casos. Dado el siguiente L-programa.

Y++
Y++
Y++
Y++
Y++
Y++
.
.
.
Y++
GOTO S

Dado que el incremento es una instrucción del lenguaje,podemos afirmar que resuelve la asignación para cualquier k¿0, luego fk(x) = k es computable.