Chapter 6

Validation results for the thermaal solver

This chapter provides validation results for the thermal solver implemented in this work. The appropriate examples are sourced from DIN EN 1991-1-2/NA:2010-12: Anhang CC - Prüfung und Validierung von Rechenprogramm für Brandschutznachweise mittels allgemeiner Rechenverfahren and the linear thermo-elastic test in the standard NAFEMS benchmarks . They include thermal effects such as variable conductivity, heat convection and radiation at the boundary. The numerical solutions are obtained using the thermal solver in LINKS, employing TRI3, TRI6, QUAD4, QUAD8 elements, in two-dimensions, and TETRA4 and TETRA10 elements in three-dimensions. No convergence study was performed, however the mesh size was chosen small enough so that assuming convergence of the FEM solution is reasonable. Moreover the good agreement with reference solutions supports this assumption.

6.1 Validation example 1 - DIN EN 1991-1-2/NA:2010-12: Anhang CC - Prüfung und Validierung von Rechenprogramm für Brandschutznachweise mittels allgemeiner Rechenverfahren - Beispiel 1)

6.1.1 Description

The geometry examined is a square plate with side length equal to 1 m, as shown in Figure 6.1. The boundary conditions considered are as follows: the left, upper and right edges are assumed to be adiabatic. At the lower edge there is heat transfer by convection with a heat convection coefficient h equation to $1\,\mathrm{Wm^{-2}K^{-1}}$ and an environment temperature equal to $0\,^{\circ}\mathrm{C}$ (see Equation **??**). The initial temperature for the entire plate is $1000\,^{\circ}\mathrm{C}$. Reference values for the temperature at the middle of the upper edge are supplied to determine performance. The relevant properties of the material making up the plate are its conductivity k, equal to $1\,\mathrm{Wm^{-1}K^{-1}}$, its specific heat c_p, equal to $1\,\mathrm{Jkg^{-1}K^{-1}}$, and its density ρ, set equal to $1000\,\mathrm{kgm^{-3}}$. Table 6.1 summarizes all the information regarding initial and boundary conditions, geometry and material properties.

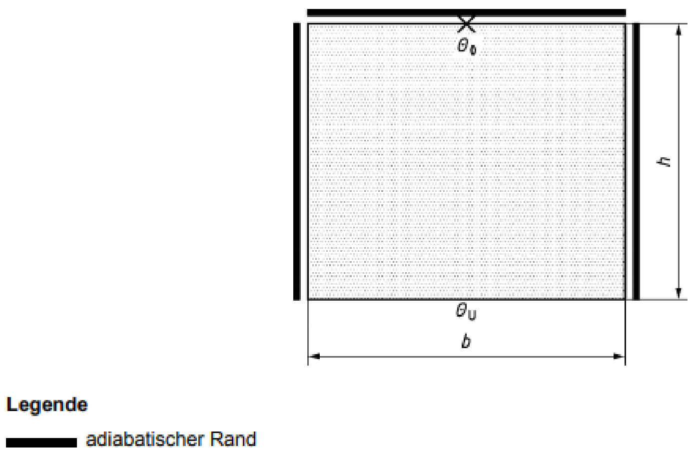


Figure 6.1: Geometry and boundary conditions considered in the validation example 1.

Table 6.1: Material properties, and initial and boundary conditions for validation example 1.

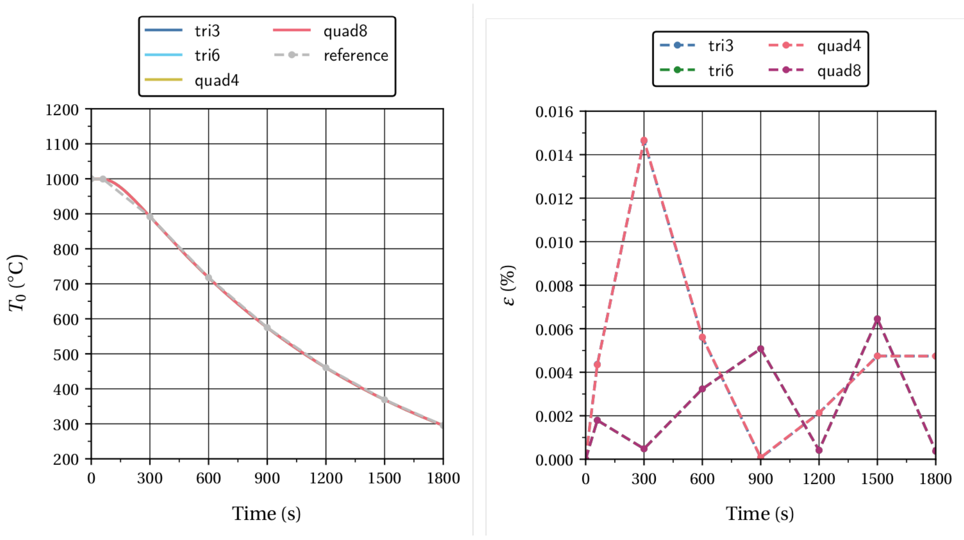
Material Properties		Effective value
Conductivity k	($\mathrm{Wm^{-1}K^{-1}}$)	1
Specific heat c_p	($\mathrm{J\,kg^{-1}\,K^{-1}}$)	1
Density ρ	($\mathrm{kg/m^3}$)	1000
Boundary Conditions		
Dimensions h, b	(m)	1
Heat convection coefficient h	($\mathrm{W/m^2/K}$)	1
Initial Conditions		
Ambient temperature T_{env}	(°C)	0
Temperature in cross-section	(°C)	1000
Reference value		
Temperature T_0 in point X	(°C)	

6.1.2 Results

The numerical solutions obtained using FEM are presented in Table 6.2, as well as, the reference values and the corresponding relative difference. Figure 6.2 presents the same results in graphical form. It can seen that that agreement between the numerical results and the reference solutions is very good, with relative error always smaller than 0.02%. recomends a relative difference ±1% and an absolute difference ±5 °C. Figure 6.3 shows different time instants of the numerical solution using TRI3 elements. The evolution of the temperature field depicted seems reasonable given the description of the problem.

Table 6.2: Reference and computed values for T_0 concerning the validation example 1.

Time (s)	Reference value T_0 (°C)	Element Type	Computed value T'_0 (°C)	Relative difference ε (%)
0	1000.0	TRI3	1000.0000	0.00
		TRI6	1000.0000	0.00
		QUAD4	1000.0000	0.00
		QUAD8	1000.0000	0.00
60	999.3	TRI3	999.3436	4.36×10^{-3}
		TRI6	999.2821	1.79×10^{-3}
		QUAD4	999.3434	4.34×10^{-3}
		QUAD8	999.2821	1.79×10^{-3}
300	891.8	TRI3	891.9305	1.46×10^{-2}
		TRI6	891.7957	4.82×10^{-4}
		QUAD4	891.9308	1.47×10^{-2}
		QUAD8	891.7957	4.82×10^{-4}
600	717.7	TRI3	717.7402	5.60×10^{-3}
		TRI6	717.6768	3.23×10^{-3}
		QUAD4	717.7403	5.62×10^{-3}
		QUAD8	717.6768	3.23×10^{-3}
900	574.9	TRI3	574.9004	6.96×10^{-5}
		TRI6	574.8708	5.08×10^{-3}
		QUAD4	574.9005	8.70×10^{-5}
		QUAD8	574.8708	5.08×10^{-3}
1200	460.4	TRI3	460.4098	2.13×10^{-3}
		TRI6	460.4019	4.13×10^{-4}
		QUAD4	460.4098	2.13×10^{-3}
		QUAD8	460.4019	4.13×10^{-4}
1500	368.7	TRI3	368.7175	4.75×10^{-3}
		TRI6	368.7238	6.46×10^{-3}
		QUAD4	368.7175	4.75×10^{-3}
		QUAD8	368.7238	6.46×10^{-3}
1800	295.3	TRI3	295.2860	4.74×10^{-3}
		TRI6	295.3011	3.73×10^{-4}
		QUAD4	295.2860	4.74×10^{-3}
		QUAD8	295.3011	3.73×10^{-4}



(a) (b)

Figure 6.2: Numerical results for the validation example 1. (a) Temperature values at X as a function of time. (b) Relative error in percentage as function of time.

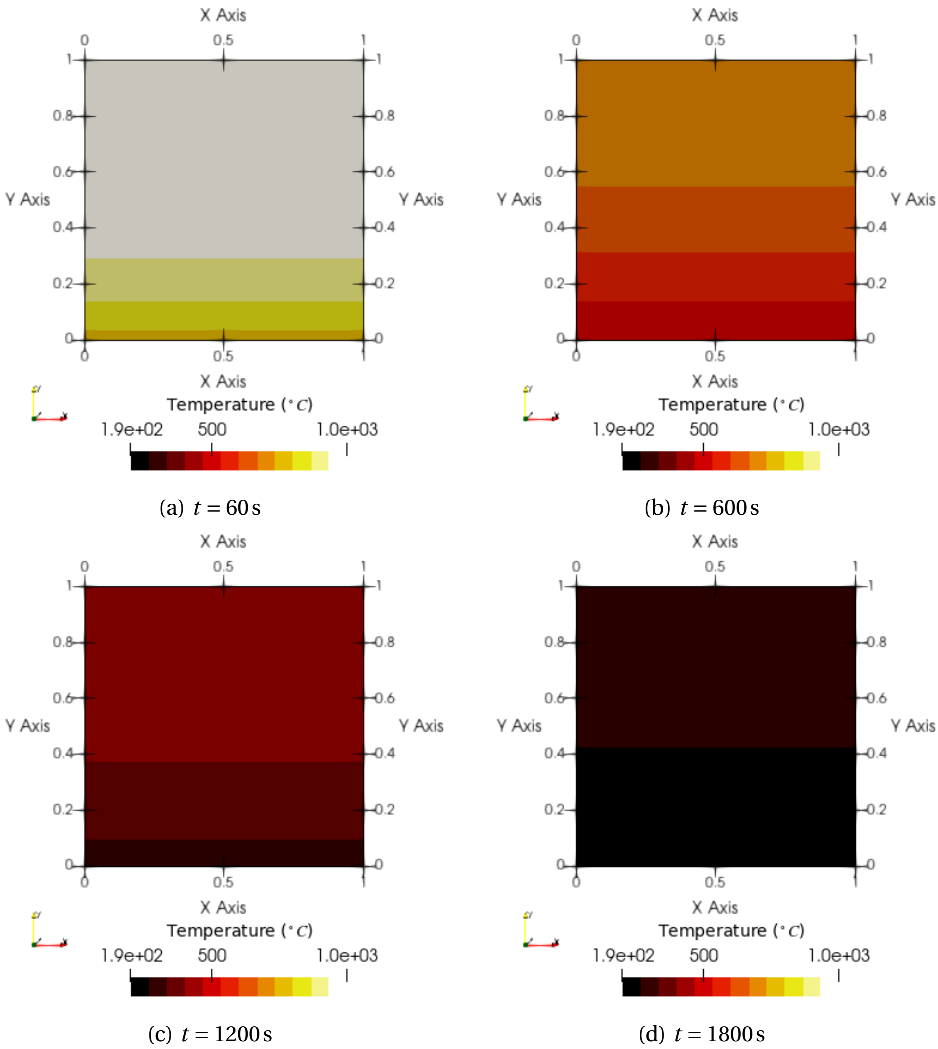


Figure 6.3: Numerical results for the validation example 1 using a TRI3 mesh.

6.2 Validation example 2 - DIN EN 1991-1-2/NA:2010-12: Anhang CC - Prüfung und Validierung von Rechenprogramm für Brandschutznachweise mittels allgemeiner Rechenverfahren - Beispiel 2)

6.2.1 Description

The geometry examined is a square plate with side length equal to 0.2 m, as shown in Figure 6.4. There is heat transfer by convection along all the edges with a heat convection coefficient h equation to $10\,\mathrm{Wm^{-2}K^{-1}}$ and an environment temperature equal to 1000 °C (see Equation **??**). Thre is also heat transfer through radition, with the emissivity $\varepsilon_{\mathrm{res}}$ equal to 0.8. The initial temperature for the entire plate is 0 °C. Reference values for the temperature in the middle of the plate are supplied to determine performance. The relevant properties of the material making up the plate are its conductivity k, which follows a linear behavior (see Table **??**), its specific heat c_p, equal to $1000\,\mathrm{Jkg^{-1}K^{-1}}$, and its density ρ, set equal to $2400\,\mathrm{kgm^{-3}}$. Table 6.3 summarizes all the information regarding initial and boundary conditions, geometry and material properties.

6.2.2 Results

The numerical solutions obtained using FEM are presented in Table 6.4, as well as, the reference values and the corresponding relative difference. Figure 6.5 presents the same results in graphical form. recomends for $t \leq 60\,\mathrm{min}$ an absolute difference smaller than ±5 °C, and for $t > 60\,\mathrm{min}$, a relative difference smaller than ±2%. It can be seen that that agreement between the numerical results and the reference solutions is acceptable. For $t \leq 60\,\mathrm{min}$ the linear elements do not satisfy the recomendation set forth by . Otherwise the requirements are completly fullfiled. Figure 6.6 shows different time instants of the numerical solution using TRI3 elements. The evolution of the temperature field depicted seems reasonable given the description of the problem.

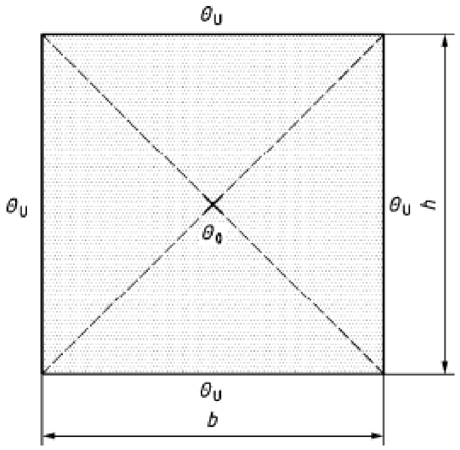


Figure 6.4: Geometry and boundary conditions considered in the validation example 2.

Table 6.3: Material properties, and initial and boundary conditions for validation example 2.

Material Properties		Effective value	
		T	$\lambda(T)$
Conductivity k (Linear behavior)	$(\mathrm{W\,m^{-1}\,K^{-1}})$	0	1.5
		200	0.7
		1000	0.5
Specific heat c_p	$(\mathrm{J\,kg^{-1}\,K^{-1}})$	1000	
Density ρ	$(\mathrm{kg/m^3})$	2400	
Boundary Conditions			
Dimensions h, b	(m)	0.2	
Heat convection coefficient h	$(\mathrm{W/m^2/K})$	10	
Emissivity $\varepsilon_{\mathrm{res}}$		0.8	
Initial Conditions			
Ambient temperature T_{env}	(°C)	1000	
Temperature in cross-section	(°C)	0	
Reference value			
Temperature T_0 in point X	(°C)		

Table 6.4: Reference and computed values for T_0 concerning the validation example 2.

Time (min)	Reference value T_0 (°C)	Element Type	Computed value T_0' (°C)	Relative difference ε (%)
0	0.0	TRI3	0.0000	0.00
		TRI6	0.0000	0.00
		QUAD4	0.0000	0.00
		QUAD8	0.0000	0.00
30	36.9	TRI3	29.7312	1.94×10^{1}
		TRI6	33.5906	8.97
		QUAD4	30.4248	1.75×10^{1}
		QUAD8	33.8503	8.26
60	137.4	TRI3	130.0251	5.37
		TRI6	133.7875	2.63
		QUAD4	131.0145	4.65
		QUAD8	133.8905	2.55
90	244.6	TRI3	240.0627	1.85
		TRI6	242.8709	7.07×10^{-1}
		QUAD4	240.4040	1.72
		QUAD8	242.9500	6.75×10^{-1}
120	361.1	TRI3	362.2362	3.15×10^{-1}
		TRI6	363.4852	6.61×10^{-1}
		QUAD4	361.9427	2.33×10^{-1}
		QUAD8	363.5435	6.77×10^{-1}
150	466.2	TRI3	470.0065	8.16×10^{-1}
		TRI6	470.2503	8.69×10^{-1}
		QUAD4	469.3439	6.74×10^{-1}
		QUAD8	470.2947	8.78×10^{-1}
180	554.8	TRI3	560.5277	1.03
		TRI6	560.1557	9.65×10^{-1}
		QUAD4	559.6558	8.75×10^{-1}
		QUAD8	560.1907	9.72×10^{-1}

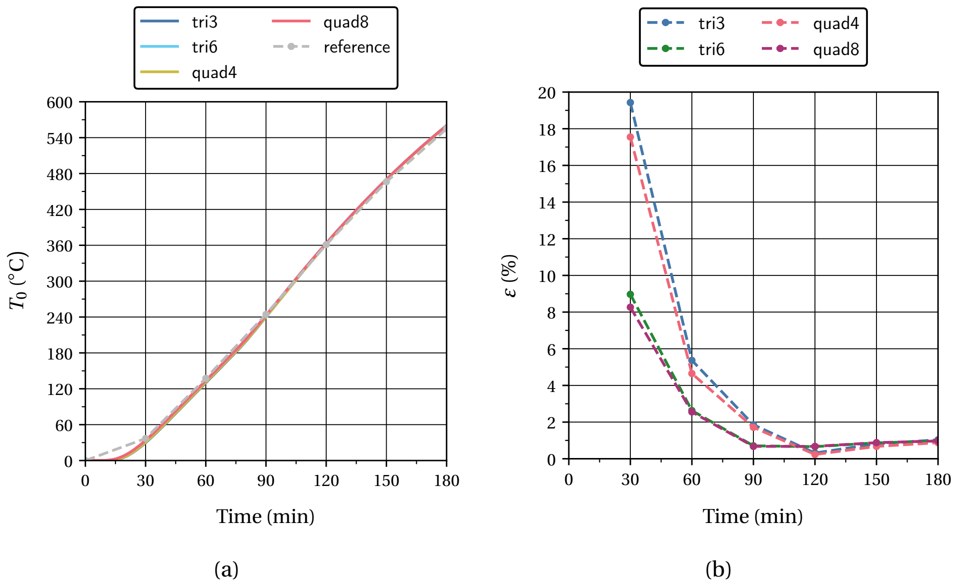
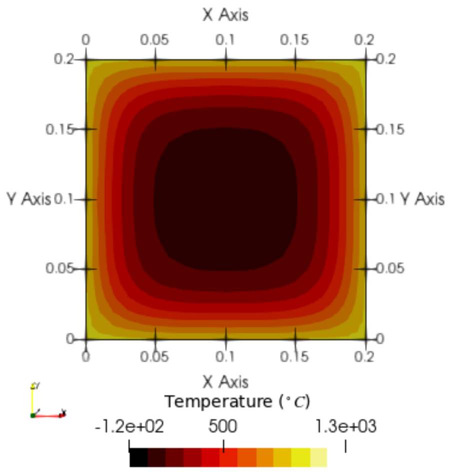
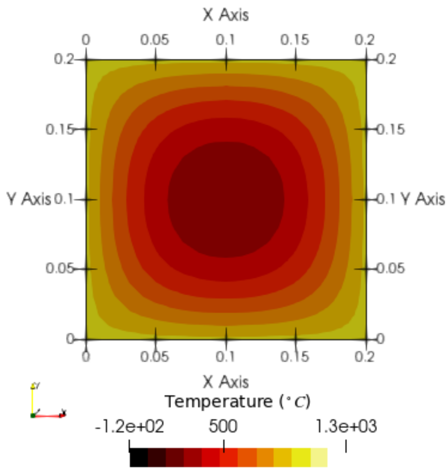


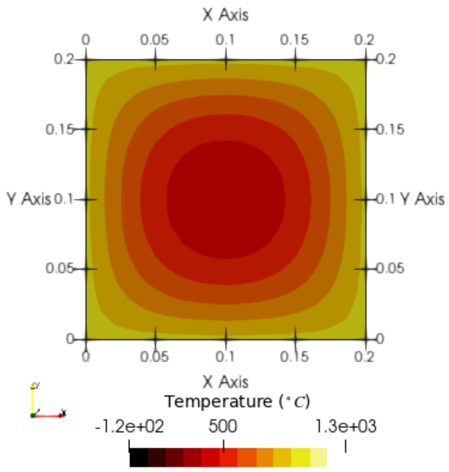
Figure 6.5: Numerical results for the validation example 2. (a) Temperature values at X as a function of time. (b) Relative error in percentage as function of time.



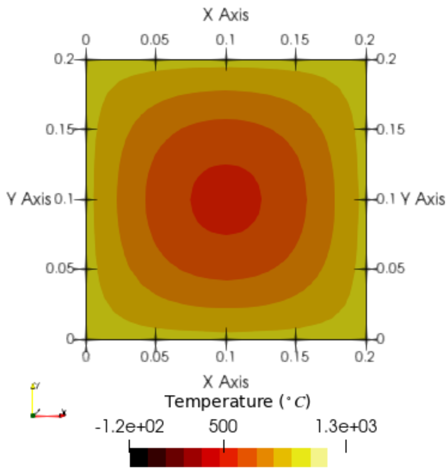
(a) $t = 30\,\mathrm{min}$



(b) $t = 90\,\mathrm{min}$



(c) $t = 120\,\mathrm{min}$



(d) $t = 180\,\mathrm{min}$

Figure 6.6: Numerical results for the validation example 2 using a TRI3 mesh.

6.3 Validation example 3 - The Standard NAFEMS Benchmarks: linear thermo-elastic tests - Two dimensional heat transfer with convection

6.3.1 Description

The geometry examined is a rectangular plate with width equal to 0.6 m and length equal to 1 m, as shown in Figure 6.7. A corresponding three-dimensional geometry is also considered with a thickness equal to 1 m. The boundary conditions considered are as follows: the left edge is assumed to be adiabatic. At the lower edge the temperature is prescribed to be 100 °C and along the upper and right edges there is heat transfer by convection and radiation. The heat convection coefficient h is equal to 70 $\mathrm{Wm^{-2}K^{-1}}$, and the ambient temperature is equal to 0 °C (see Equation **??**). The initial temperature for the entire plate is 0 °C. The relevant properties of the material making up the plate are its conductivity k, equal to 52 $\mathrm{Wm^{-1}K^{-1}}$, its specific heat c_p, equal to 1 $\mathrm{Jkg^{-1}K^{-1}}$, and its density ρ, set equal to 1 $\mathrm{kgm^{-3}}$. Table 6.5 summarizes all the information regarding initial and boundary conditions, geometry and material properties. The expected temperature at E (see Figure 6.7) is 18.3 °C.

6.3.2 Results

The numerical solutions obtained using FEM are presented in Table 6.6 for two dimensions and in Table **??** for three-dimensions, as well as, the reference values and the corresponding relative difference. It can be seen that that agreement between the numerical results and the reference solutions is acceptable. It is below 1% for all elements employed, except for the TETRA4 element. There is no significant difference between the two integrators tested. Figure 6.8 shows the temperature distribution obtained using TRI3 and TETRA10 elements, which is reasonable given the description of the problem.

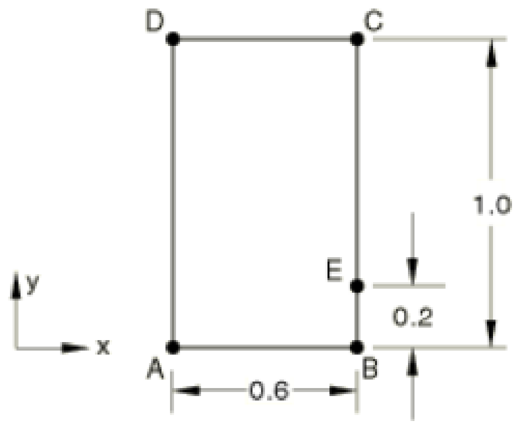


Figure 6.7: Geometry and boundary conditions considered in the validation example 3.

Table 6.5: Material properties, and initial and boundary conditions for validation example 3.

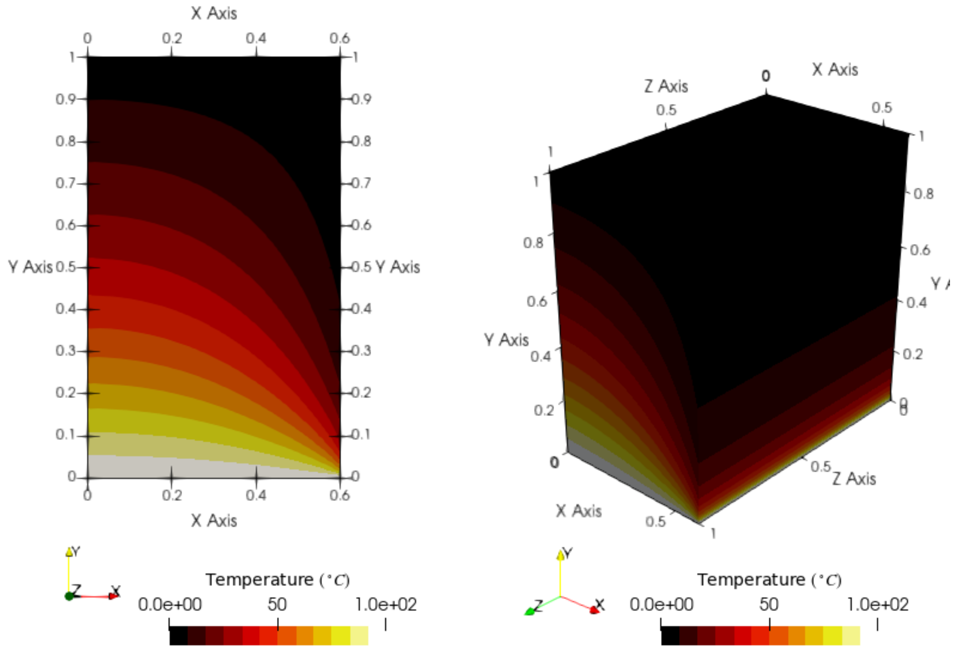
Material Properties		Effective value
Conductivity k	$(\mathrm{Wm^{-1}K^{-1}})$	52
Specific heat c_p	$(\mathrm{J\,kg^{-1}\,K^{-1}})$	1
Density ρ	$(\mathrm{kg/m^3})$	1
Boundary Conditions		
Dimension h	(m)	1
Dimension b	(m)	0.6
Thickness t	(m)	1
Heat convection coefficient h	$(\mathrm{W/m^2/K})$	70
Initial Conditions		
Ambient temperature T_{env}	(°C)	100
Temperature in cross-section	(°C)	0
Reference value		
Temperature T_0 at point E	(°C)	

Table 6.6: Reference and computed values for T_0 concerning the validation example 3 in two-dimensions.

Element	Temperature T_0 at E °C	Relative difference ε (%)
Alpha integrator ($\rho = 1$)		
TRI3	18.1890	6.07×10^{-1}
TRI6	18.2553	2.44×10^{-1}
QUAD4	18.2286	3.90×10^{-1}
QUAD8	18.2532	2.56×10^{-1}
Quasi static integrator		
TRI3	18.1895	6.04×10^{-1}
TRI6	18.2548	2.47×10^{-1}
QUAD4	18.2281	3.93×10^{-1}
QUAD8	18.2536	2.54×10^{-1}

Table 6.7: Reference and computed values for T_0 concerning the validation example 3 in three-dimensions.

Element	Temperature T_0 at E °C	Relative difference ε (%)
Alpha integrator ($\rho = 1$)		
TETRA4	17.9501	1.91
TETRA10	18.2556	2.43×10^{-1}
Quasi static integrator		
TETRA4	17.9497	1.91
TETRA10	18.2548	2.47×10^{-1}


X Axis
Y Axis
Temperature (°C)
0.0e+00
50
1.0e+02
Z Axis
X Axis
Y Axis
Temperature (°C)
0.0e+00
50
1.0e+02

(a) (b)

Figure 6.8